Title: BiGravity: from Cosmological Solutions to Dual Galileons

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Abstract: I will present Cosmological FRW Solutions in BiGravity Theories and discuss their stability. After deriving the stability bound, one realizes that in Bigravity (in contradistinction to the FRW massive gravity case) the tension between requirements stemming from stability and those set by observations is resolved. The stability bound can also be derived in the decoupling limit of Bigravity. In this context an intriguing duality between Galilean interactions has emerged.

BiGravity

From Cosmological Solutions to Dual Galileons

Matteo Fasiello

Case Western Reserve University

based on work with Claudia de Rham and Andrew J. Tolley

(ArXiv: 1308.2702, ArXiv: 1308.1647, JCAP 1211 (2012) 035)

Nov 26, 2013, Perimeter Institute

Tuesday, November 26, 13

Stability bound

$$H = \alpha\, p^2 + \beta\, q^2 + \gamma(\nabla q)^2 + ...$$

coefficient of kinetic term > 0

tachyon inst.

gradient inst.

Quickest route to the Higuchi bound in dS:

"In the the linear (massive) theory there exist a unitary spin 2 representation of the dS group iff:"

	$m^2 = 0$	G.R.
	$m^2 = 2H^2$	Partially massless theory
Higuchi bound in massive g. →	$m^2 > 2H^2$	Massive

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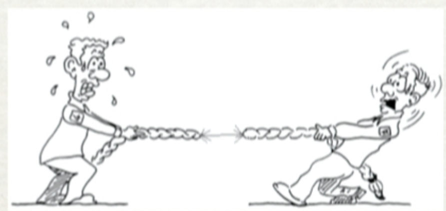
Bound from Observations

Before Dark Energy epoch sets in, G.R. good description:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1) + \ldots$$

$$m^2 \lesssim H^2$$

combining Stability and Observations then:



want our theory to be stable

$$m^2 > 2H^2$$

GR over many cosmo epochs

$$m^2 \lesssim H^2$$

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dRGT Theory of Massive Gravity

de Rham, Gabadadze, Tolley

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

$$\mathrm{Det}[1 + \lambda X] = \sum_{n=0}^{D} \lambda^n \mathcal{U}_n(X) \qquad X^\mu_\nu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

* No Boulware-Deser Ghost, at all orders

* Screening mechanism in the non-linear regime that restores continuity with G.R.

* High enough cutoff so that the theory hierarchy of scales: linear, non-linear, quantum

* Two free parameters $\beta_n = \beta_n(\alpha_3, \alpha_4)$

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! No FRW solutions in dRGT if "f" Minkowski

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley

N.B. Different routes from here: introduce inhomogeneities, use "f"

✓ Allow different "f": solutions exist for "f" dS, FRW

Fasiello, Tolley (2012)

<u>Our setup is 2nd Route</u>

Ghost-free theory {

Add matter content:

$$\mathcal{L}_M \sim \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$$

"f" is now FRW

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Inhomogeneities/Anisotropies

Vainshtein mechanism should guarantee
inhomogeneities unobservable before late times

Inhomogenities only appear on scale set by inverse graviton mass

Inhomogeneities/Anisotropies can be hidden inside Stueckelberg fields which do not directly couple to matter, only indirectly through Mp suppressed terms

Even if metric is perfectly homogeneous+isotropic, inhomogeneities show up in cosmological perturbations, but can easily be small

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley

- Volkov 2011, 2012, 2013,
- Koyama 2011,
- Gumrukcuoglu et al 2011,
- Gratia, Hu, Wyman 2012,
- Kobayashi et al 2012,
- DeFelice 2011/2013,
- Gumrukcuoglu 2012,
- Tasinato et al :2012.2013,
- Maeda + Volkov 2013

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What now? Go bigravity !

Hassan, Rosen

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left[M_P^2\, R(g) - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\left(g^{-1}f\right)\right] + \frac{1}{2}\sqrt{-f} M_f^2\, R(f) + \mathcal{L}_M$$

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu},\ M_P \leftrightarrow M_f,\ \beta_n \leftrightarrow \beta_{4-n}$$

This fact must be reflected on the bound itself

Will be crucial when we get to Galileon Duality later

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Stability Bound in Bigravity

Soon in a more symmetric form

Stability bound

$$\tilde{m}^2\left[1+\left(\frac{H_f/M_f}{H/M_P}\right)^2\right]\geq 2H^2$$

→ Minisuperspace action ✓

→ Decoupling limit ✓

→ PM check ✓

→ Symmetric ✓

Recover Massive gravity bound in the limit $M_f \to \infty,\ M_P, H_f$ finite.

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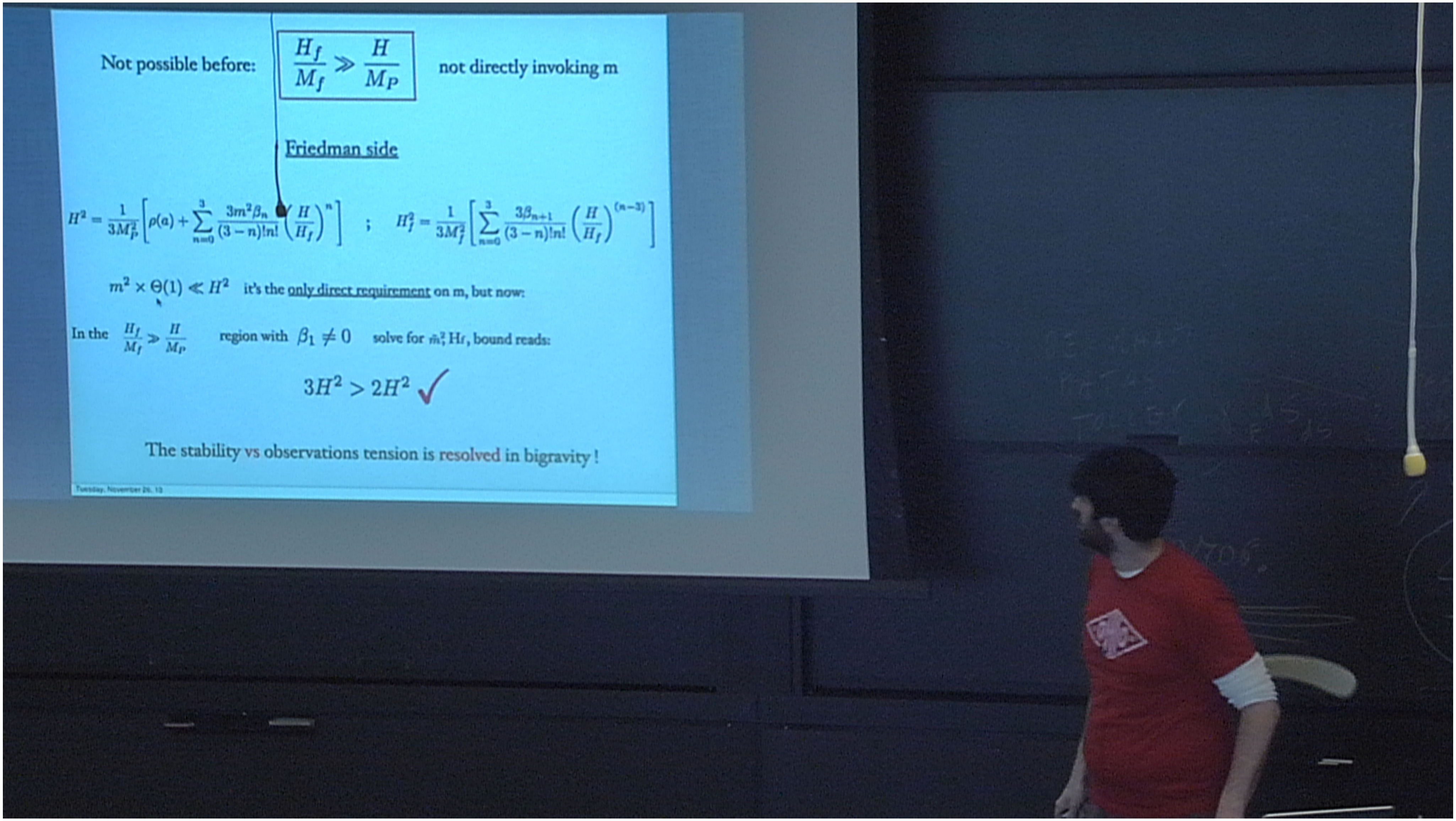
Not possible before: $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ not directly invoking m
Friedman side
$H^2 = \frac{1}{3M_P^2}\left[\rho(a) + \sum_{n=0}^{3} \frac{3m^2\beta_n}{(3-n)!n!}\left(\frac{H}{H_f}\right)^n\right]$; $H_f^2 = \frac{1}{3M_f^2}\left[\sum_{n=0}^{3} \frac{3\beta_{n+1}}{(3-n)!n!}\left(\frac{H}{H_f}\right)^{(n-3)}\right]$
$m^2 \times \Theta(1) \ll H^2$ it's the only direct requirement on m, but now:
In the $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ region with $\beta_1 \neq 0$ solve for m^2; Hf, bound reads:
$3H^2 > 2H^2$ ✓
The stability vs observations tension is resolved in bigravity !

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Stable Self-accelerating Solution

Akrami, Koivisto, Sandstad
(2012,2013)

Set: $\beta_2 = 0 = \beta_3;\ \beta_1 = 2M_P^2$

$$H^2 = \frac{1}{6M_P^2}\left(\rho(a) + \sqrt{\rho(a)^2 + \frac{12\, m^4 M_P^6}{M_f^2}}\right)$$

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73

Observationally viable ! Small part of the whole table

Stability bound? It reduces to

$$\left(\frac{1}{M_P^2} + \frac{12M_f^2}{m^4\beta_1^2}H^4\right) > 0 \quad ✓$$

Stable as well.

ArXiv:1302.5268 Akrami, Koivisto, Sandstad

...The data we use include the position of the first peak on the cosmic microwave background angular power spectrum, the ratio of the sound horizon at the drag epoch to the dilation scale at six different redshifts, the luminosity distances to 580 Type Ia Supernovae,23 and the present value of the Hubble parameter...

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Summary and Conclusions

No FRW solutions in dRGT if "f" Minkowski. Where does one go from there?

Possibilities are manyfold...

Stability and Observational constraints combined rule out FRW on FRW in massive gravity

N.B. not a general no-go, just for exact FRW on FRW

Bigravity: stability bound relaxed if $H_f/M_f >> H/M_P$. Combined constraints not too stringent on $\tilde{m}^2$

Not shown here but vector sector ok if $\tilde{m}^2 > 0$

<u>Stability vs Observations tension resolved in bigravity</u>

Understanding of the dynamics

Self-Accelerating solution

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Understanding of the dynamics

Self-Accelerating solution

Further work on Cosmo Solutions

Perturbations in bigravity solutions also performed in:

Comelli, Crisostomi, Pilo

- Naively different dS result: upon closer inspection, we agree ✓

- Gradient instability... Our focus has been on unitarity (Higuchi)

Etc Etc...

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FRW on FRW dRGT Stability

exact

re-derived in minisuperspace

but not viable

FRW on FRW BiGravity Stability

...

minisuperspace ✓

symmetry check ✓

decoupling should capture ✓

$$\frac{m^2}{2}\left[\beta_1 H_f^2 + 2\beta_2 H H_f + \beta_3 H^2\right]\left(\frac{H^2}{M_P^2} + \frac{H_f^2}{M_f^2}\right) \geq 2H_f^3 H^3$$

It does

Viable + self-accelerating solutions

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It does

<u>Viable + self-accelerating solutions</u>

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Decoupling limit of BiGravity

$$S_{\text{helicity }2/0} = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} + \frac{\Lambda_3^3}{2} h^{\mu\nu}(x) X^{\mu\nu} + \frac{M_P \Lambda_3^3}{2M_f} v_{\mu A} [x^a + \Lambda_3^{-3} \partial^a \pi] (\eta^A_\nu + \Pi^A_\nu) Y^{\mu\nu} \right]$$

$$X^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(3-n)!n!} \epsilon^{\mu\cdots} \epsilon^{\nu\cdots} (\eta + \Pi)^n \eta^{3-n} \,,$$

$$Y^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(4-n)!(n-1)!} \epsilon^{\mu\cdots} \epsilon^{\nu\cdots} (\eta + \Pi)^{(n-1)} \eta^{4-n}$$

$$\Pi_{ab} = \frac{\partial_a \partial_b \pi}{\Lambda_3^3}$$

Two massless spin-2 and a funny looking Galileon contribution

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Decoupling limit of BiGravity

$$M_P \to \infty \; ; \quad M_F \to \infty \; ; \quad m \to 0$$

$$\Lambda_3 = (m^2 M_P)^{1/3} \to \text{constant} \; ; \quad M_P/M_F \to \text{constant} \; ; \quad \hat{\beta}_n = \beta_n / M_P^2 \to \text{constant}$$

$$\lim_{M_P \to \infty, \Lambda_3 \to \text{const}} S_{\text{bigravity}} = \underline{S_{\text{helicity } 2/0}} + S_{\text{helicity } 1/0} + \ldots$$

$$E^a_\mu = \delta^a_\mu + \frac{1}{2M_P} h^a_\mu \,, \qquad F^a_\mu = \delta^a_\mu + \frac{1}{2M_f} v^a_\mu$$

$$\Lambda^a{}_b = e^{\hat{\omega}^a{}_b} = \delta^a{}_b + \hat{\omega}^a{}_b + \frac{1}{2} \hat{\omega}^a{}_c \hat{\omega}^c{}_b + \cdots$$

$$\hat{\omega}^a{}_b = \frac{\omega^a{}_b}{m M_P}$$

$$\partial_\mu \Phi^a = \partial_\mu \left(x^a + \frac{B^a}{m M_P} + \frac{\partial^a \pi}{\Lambda_3^3} \right)$$

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Decoupling limit of BiGravity

Can we put the ~Y piece in a simpler form ? Yes: coordinate transformation

$$S_{\text{helicity}-2/0} = \dots + \int d^4x \left[\frac{M_P \Lambda_3^3}{2M_f} v_{\mu\nu}(x^a) \tilde{Y}^{\mu\nu} \right]$$

$$(\partial Z)^a_\nu = \partial_\mu Z^a(x)$$

$$Z^a(x^b + \Lambda_3^{-3} \partial^b \pi(x)) = x^a$$

$$\tilde{Y}^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(4-n)!(n-1)!} \epsilon^{\mu \dots} \epsilon^{\nu \dots} \eta^{(n-1)} (\tilde{\partial} Z)^{4-n},$$

Z is the inverse map, Z is $\Phi^{-1}(x)$

$$\phi(x)^A = x^A + \partial^A \pi(x)$$

$$\tilde{\partial} Z = (\tilde{\partial} Z)^T$$

$$Z(\tilde{x}) = \tilde{x}^a + \tilde{\partial}^a \rho(\tilde{x})$$

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Decoupling limit of BiGravity

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Decoupling limit of BiGravity

MF, Tolley (2013)

Where did we put the Stueckelbergs?

$$\mathrm{Diff}(M) \;\times\; \mathrm{Diff}(M) \quad\rightarrow\quad \mathrm{Diff}(M)_{diag}$$

The broken Diffs Stueckebergs will give us the Galileons in the decoupling limit

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu\phi^A\partial_\nu\phi^B$$

$$\tilde{x}^A = \phi(x)^A = x^A + \frac{1}{m\,M_p}\,\partial^A\pi(x)$$

here

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Bigravity Duality

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu},\ M_P \leftrightarrow M_f,\ \beta_n \leftrightarrow \beta_{4-n}$$

Dynamical metric I

$g_{\mu\nu}(x)$

Dynamical metric II

$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu\phi^A\partial_\nu\phi^B$

$$\tilde{x}^A = \phi(x)^A = x^A + \partial^A\pi(x)$$

OR

Dynamical metric I

$\tilde{G}(\tilde{x}) = g_{\mu\nu}(Z)\partial_A Z^\mu \partial_B Z^\nu$

Dynamical metric II

$f_{AB}(\tilde{x})$

$$x^\mu = Z(\tilde{x})^\mu = \tilde{x}^\mu + \partial^\mu\rho(\tilde{x})$$

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de Rham, MF, Tolley

Galileon Duality

For every Galileon π(x) there exists a dual Galileon ρ(x)

Inverting order by order the implicit relation for Z

$$Z^a(x^b + \Lambda_3^{-3}\partial^b\pi(x)) = x^a$$

The ρ(π) map

$$\rho(x) = -\pi(x) + \frac{1}{2\Lambda_3^3}(\partial_b\pi(x))^2 - \frac{1}{2\Lambda_3^6}\partial^a\pi(x)\partial^b\pi(x)\partial_a\partial_b\pi(x) + \dots$$

It can be inverted (Bigravity Duality) and looks analogous

local

$$\pi(x) = -\rho(x) + \frac{1}{2\Lambda_3^3}(\partial_b\rho(x))^2 + -\frac{1}{2\Lambda_3^6}\partial^a\rho(x)\partial^b\rho(x)\partial_a\partial_b\rho(x) + \dots$$

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Galileon Duality at Work

$$\mathcal{L}_n[\pi] = \pi\, \mathcal{L}^{\rm der}_{n-1}[\Pi]\ ; \qquad \mathcal{L}_n[\rho] = \rho\, \mathcal{L}^{\rm der}_{n-1}[\Sigma],$$

$$\mathcal{L}^{\rm der}_n[X] = \Lambda^{2\sigma} \epsilon^{\mu_1\cdots\mu_d} \epsilon^{\nu_1\cdots\nu_d} \prod_{j=1}^{n} X_{\mu_j\nu_j} \prod_{k=n+1}^{d} \eta_{\mu_k\nu_k}$$

Duality toolbox

$$\delta\pi(x) = -\delta\rho(\tilde{x})\ ;$$

$$\Pi_{\mu\nu}(x) = \partial_\mu\partial_\nu\pi(x)\,; \quad \Sigma_{\mu\nu}(\tilde{x}) = \partial_\mu\partial_\nu\rho(\tilde{x})$$

$$(\eta + \Pi(x)) = (\eta + \Sigma(\tilde{x}))^{-1}$$

Generic Galileon
$$S = \int d^dx \sum_{n=2}^{d+1} c_n \mathcal{L}_n[\pi(x)]$$

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Galileon Duality at Work

$$S = \int d^d x \sum_{n=2}^{d+1} c_n \mathcal{L}_n[\pi(x)]$$

$$\delta S = \int d^d x \left(\sum_{n=1}^{d} (n+1) c_{n+1} \mathcal{L}_n^{\rm der}[\Pi(x)] \right) \delta\pi(x)$$

$$\delta S = -\int d^d\tilde{x}\, |\eta + \Sigma(\tilde{x})| \sum_{n=2}^{d+1} n\, c_n \mathcal{L}_{n-1}^{\rm der}\left[\frac{-\Sigma(\tilde{x})}{\eta + \Sigma(\tilde{x})} \right] \delta\rho(\tilde{x}),$$

$$S_{\rm dual} = \int d^d\tilde{x} \sum_{n=2}^{d+1} p_n \mathcal{L}_n[\rho(\tilde{x})] \equiv \int d^d x \sum_{n=2}^{d+1} p_n \mathcal{L}_n[\rho(x)],$$

Dictionary:

$$p_n = \frac{1}{n} \sum_{k=2}^{d+1} (-1)^k c_k \frac{k(d-k+1)!}{(n-k)!(d-n+1)!}$$

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Free Theory dual to a Quintic

$p_2 = -1/12\,,\ p_3 = -1/6\,,\ p_4 = -1/8\,,\ p_5 = -1/30$ ⟷ $\frac{1}{12} = c_2 \neq 0 = c_{n>2}$

$$S = \int d^dx \sum_{n=2}^{d+1} p_n \mathcal{L}_n[\rho(x)]$$

Plane wave solution for <u>unsourced</u> eom

$F[(x_1 - t)/\sqrt{2}]$

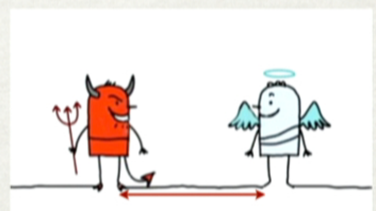
Linear fluctuations propagate at

$$c_s = 1 \quad \text{and} \quad c_s = \frac{1 - F''}{1 + F''}\,,$$

$c_s > 1$ as soon as $F'' < 0$

$$S = \int d^4x \left(-\frac{1}{2}(\partial\pi)^2\right)$$

Free theory ! !



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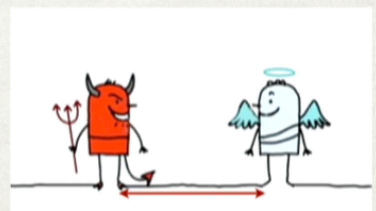
Linear fluctuations propagate at

$$c_s = 1 \quad \text{and} \quad c_s = \frac{1 - F''}{1 + F''}\,,$$

$c_s > 1$ as soon as $F^{''} < 0$



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Free Theory dual to a Quintic

$p_2 = -1/12\,,\; p_3 = -1/6\,,\; p_4 = -1/8\,,\; p_5 = -1/30$ ⟷ $\frac{1}{12} = c_2 \neq 0 = c_{n>2}$

$$S = \int d^dx \sum_{n=2}^{d+1} p_n \mathcal{L}_n[\rho(x)]$$

$$S = \int d^4x (-\frac{1}{2}(\partial\pi)^2)$$

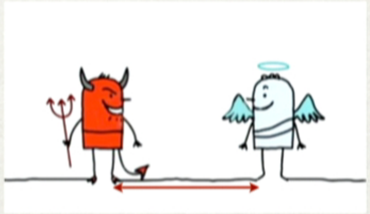
Plane wave solution for <u>unsourced</u> eom

Free theory ! !

$F[(x_1 - t)/\sqrt{2}]$

Linear fluctuations propagate at

$$c_s = 1 \quad \text{and} \quad c_s = \frac{1 - F''}{1 + F''}\,,$$



$c_s > 1$ as soon as $F^{''} < 0$

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