

Title: Dynamical Emergence of Universal Horizons during the formation of Black Holes

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Abstract: In many theories with fundamental preferred frame, such as Einstein-Aether or Gravitational Aether theories, K-essence, Cuscuton theory, Shape Dynamics, or (non-projectable) Horava-Lifshitz gravity, the low energy theory contains a fluid with superluminal or incompressible excitations. In this talk, I study the formation of black holes in the presence of such a fluid. In particular, I focus on the incompressible limit of the fluid (or Constant Mean Curvature foliation) in the space-time of a spherically collapsing shell within an asymptotically cosmological space-time. In this case, I show that an observer inside $3/4$ of the Schwarzschild radius cannot send a signal outside, after a stage in collapse, even using signals that propagate infinitely fast in the preferred frame. This confirms the dynamical emergence of universal horizons that have been previously found in static solutions [arXiv:1110.2195, arXiv:1104.2889, arXiv:1212.1334].

Dynamical Emergence of Universal Horizons during the formation of Black Holes

Mehdi Saravani

Perimeter Institute

arXiv:1310.4143 with N. Afshordi and R. B. Mann

1-minute summary

- Light (luminal) signals cannot escape event horizon of black holes (BH). What about super-luminal signals? They probably can.
- If there were superluminal signals, would BHs be “black”? It seems the answer is still yes!

Otherwise the classical theory would be unpredictable (naked singularity)

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- Even for signals with infinite sound speed there is a horizon (universal horizon). In spherically symmetric solutions, universal horizon is at $r = \frac{3}{4} r_s$
- We show emergence of **universal horizon** in a **dynamical setting**.

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Outline

- Invitation: theories with superluminal excitations
- Signal propagation on a curved background
- Emergence of universal horizon in collapse of a spherical shell
- Concluding remarks

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Superluminal excitations

Example 1)

Scalar field theory with non-canonical term

$$S = \int d^4x \sqrt{-g} \mathcal{L}(X, \phi) \quad X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

$$c_s^2 = \frac{1}{1 + 2X \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}}$$

- Limit of infinite sound speed \rightarrow Cuscuton

$$S = \int d^4x \sqrt{-g} (\mu^2 \sqrt{\partial_\nu \phi \partial^\nu \phi} - V(\phi))$$

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$$v^2 = \frac{1}{1 + 2X \frac{\partial \mathcal{L}}{\partial X}}$$

• Limit of infinite sound speed \rightarrow Causation

$$S = \int d^4x \sqrt{-g} (p^2 \sqrt{G_{\mu\nu}} \nabla^\mu \phi \nabla^\nu \phi - V(\phi))$$

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Superluminal excitations

Example 2)

Einstein-aether (EA) theories

- u is constrained to be a unit time-like vector.

$$S = -\frac{1}{16\pi G} \int \sqrt{-g} (R + K_{mn}^{ab} \nabla_a u^m \nabla_b u^n) d^4x$$

$$K_{mn}^{ab} = c_1 g^{ab} g_{mn} + c_2 \delta_m^a \delta_n^b + c_3 \delta_n^a \delta_m^b + c_4 u^a u^b g_{mn}$$

- Experimental constraints imply that aether disturbances must propagate (super)luminally.

J. W. Elliott, G. D. Moore, and H. Stoica, Constraining the new Aether: Gravitational Cerenkov radiation, [arxiv:hep-ph/0505211].

Superluminal excitations

Example 3)

Horava-Lifshitz (HL) gravity

- IR limit of HL is equivalent to EA with hypersurface orthogonal vector field.

T. Jacobson, Extended Horava gravity and Einstein-aether theory, [arXiv:1001.4823].

- IR limit of Non-projectable HL is equivalent to Cuscuton gravity.

N. Afshordi, Cuscuton and low energy limit of Horava-Lifshitz gravity, [arXiv:0907.5201].

- EA \longleftrightarrow (IR) Non-projectable HL \longleftrightarrow Cuscuton

$$c_2 = \lambda - 1 \qquad \mu^2 = -V''(\phi) = \frac{\lambda - 1}{16\pi G_N}$$

- In this limit, perturbations propagate infinitely fast.

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Signal Propagation on a Curved Background

- Consider scalar field with the following action

metric signature (+ - - -)

$$S = \int d^4x \sqrt{-g} \mathcal{L}(X, \phi) \quad X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

- g is the metric of background spacetime.
- It satisfies

E. Babichev et al. [arXiv:0708.0561]

$$\tilde{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi + 2X \mathcal{L}_{,X\phi} - \mathcal{L}_{,\phi} = 0 \quad \tilde{G}^{\mu\nu} = \mathcal{L}_{,X} g^{\mu\nu} + \mathcal{L}_{,XX} \nabla^\mu \phi \nabla^\nu \phi$$

- Linear perturbations π satisfy

$$\frac{1}{\sqrt{-G}} \partial_\mu \left(\sqrt{-G} G^{\mu\nu} \partial_\nu \pi \right) + M_{eff}^2 \pi = 0$$

$$G^{\mu\nu} = \frac{c_s^2}{\mathcal{L}_{,X}^2} \tilde{G}^{\mu\nu}, \quad (G^{-1})_{\mu\nu} G^{\nu\rho} = \delta_\mu^\rho, \quad \sqrt{-G} = \sqrt{-\det(G^{-1})_{\mu\nu}}$$

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$$\nabla^\mu \nabla_\mu \phi = 2X \mathcal{L}_X - \mathcal{L}_\phi = 0 \quad \nabla^\mu = \mathcal{L}_X \phi^\mu - \mathcal{L}_{X, X} \nabla^\mu \phi$$

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$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \nabla^\mu \pi) = \mathcal{L}_X \pi = 0$$

$$\nabla^\mu = \partial^\mu - \partial_\nu \xi^\mu \cdot (\nabla^\nu)^{\text{old}} = \partial^\mu - \sqrt{-g}^{-1} \partial_\nu (\sqrt{-g} \xi^\mu)$$

$$M_{\mu\nu}^{\alpha\beta} = \partial_\nu (2X \mathcal{L}_{X, X} \xi^\alpha - \frac{1}{\sqrt{-g}} \nabla_\mu \nabla_\nu \pi)$$

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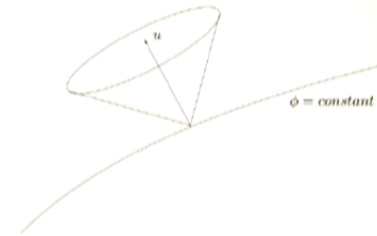
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- Linear perturbations satisfy Klein-Gordon equation with effective metric G . As a result, propagation cone is given by the effective metric G .

$$(G^{-1})_{\mu\nu} = \frac{\mathcal{L},X}{c_s} \left(g_{\mu\nu} - c_s^2 \frac{\mathcal{L},XX}{\mathcal{L},X} \nabla_\mu \phi_0 \nabla_\nu \phi_0 \right)$$

- At any point, perturbations propagate inside the cone defined by vectors “ v ” that are null with respect to G

$$(G^{-1})_{\mu\nu} v^\mu v^\nu = 0$$



- It reduces to

$$g_{\mu\nu} v^\mu v^\nu = (1 - c_s^2) (g_{\mu\nu} u^\mu v^\nu)^2 \quad u_\mu = \frac{\nabla_\mu \phi}{\sqrt{\nabla_\alpha \phi \nabla^\alpha \phi}}$$

- $C_s=1 \rightarrow v$ is null. Propagation cone is independent of background field.
- $C_s<1 \rightarrow v$ is time-like. Propagation cone at any point depends on the constant field surfaces.
- $C_s>1 \rightarrow v$ is space-like. Propagation cone at any point depends on the constant field surfaces.
- Large $C_s \rightarrow$ Propagation cone is almost tangent to constant field surfaces.

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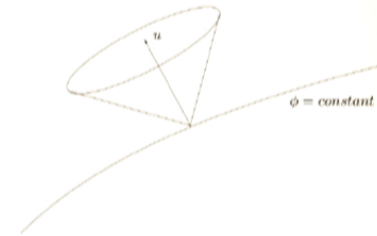
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Choosing the Lagrangian to be

$$\mathcal{L} = aX^n - V(\phi)$$

where n is very close to $1/2$

$$n = \frac{1}{2}(1 + \epsilon^2)$$

up to first order in ϵ , we get

Background field equation $\rightarrow \frac{a\sqrt{2}}{2} \nabla_\mu u^\mu + V'(\phi) = 0$ $u_\mu = \frac{\nabla_\mu \phi}{\sqrt{\nabla_\alpha \phi \nabla^\alpha \phi}}$

Propagation cone $\rightarrow u_\mu v^\mu = \epsilon$

More on Cuscuton

- Perfect fluid with the following energy-momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$$

$$\rho = V(\phi)$$

$$p = a\sqrt{X} - V(\phi)$$

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{\nabla_\alpha \phi \nabla^\alpha \phi}}$$

- EOM from last page $K = u^\alpha{}_{;\alpha} = -\frac{\sqrt{2}}{a} V'(\phi)$
 Mean extrinsic curvature

- Constant field surfaces \longleftrightarrow Comoving surfaces \longleftrightarrow CMC (constant mean curvature) surfaces
- Ignoring backreaction (verify later), we only need to find CMC surfaces of the background spacetime.

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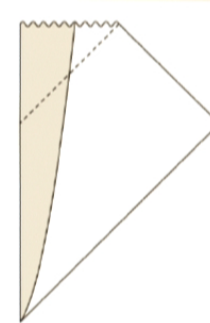
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- Background spacetime: spherical thin shell collapse

$$\text{I} : ds^2 = A^2(t)dt^2 - dr^2 - r^2d\Omega^2, \quad r < R(t)$$

$$\text{II} : ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2d\Omega^2, \quad r > R(t)$$

$$f(r) = 1 - \frac{2M}{r}$$



- Shell at $r=R(t)$
- $R=R(t)$ satisfies the geodesic equation

$$\dot{R} = -f(R)\sqrt{1 - \frac{f(R)}{e^2}} \quad \cdot = \frac{d}{dt}$$

- $A(t)$ is given by the continuity of the induced metric on shell surface.

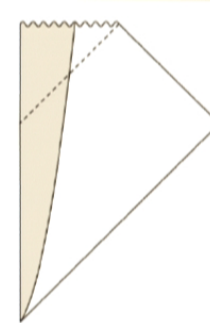
$$A = f(R)\sqrt{1 + \frac{2M}{e^2 R}}$$

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CMC surfaces: $K = \nabla_\mu u^\mu$

- Region I

$$u_I^r = \frac{K}{3}r$$

$$u_I^t = \frac{\sqrt{(\frac{Kr}{3})^2 + 1}}{A}$$

- Region II

$$u_{II}^r = \frac{K}{3}r - \frac{B}{r^2}$$

$$u_{II}^t = \frac{1}{f(r)} \sqrt{(\frac{K}{3}r - \frac{B}{r^2})^2 + f(r)}$$

$$t_{CMC} = T(r) \quad T'(r) = \frac{\frac{K}{3}r - \frac{B}{r^2}}{f(r)\sqrt{f(r) + (\frac{K}{3}r - \frac{B}{r^2})^2}}$$

Constant of integration, fixed by matching conditions

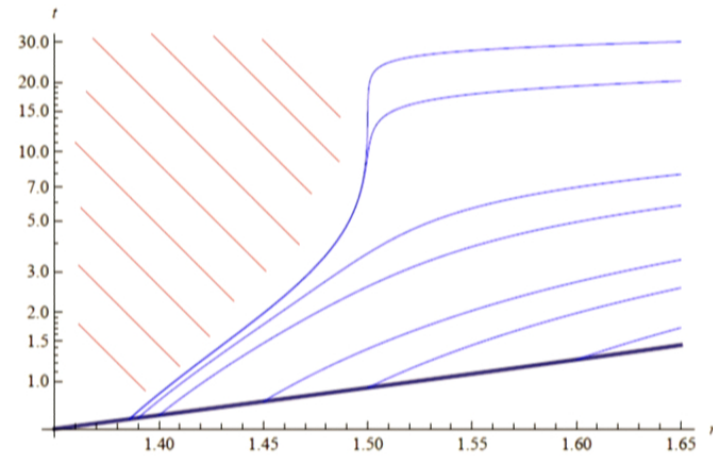
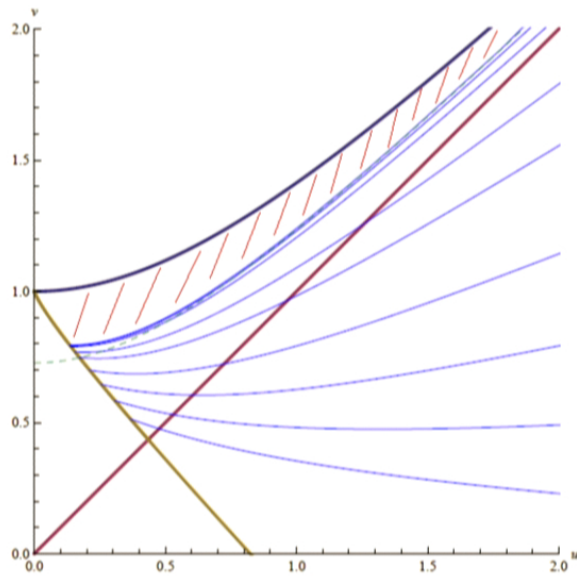
- B is constant on each surface, fixed by matching conditions.

- Non-singularity of K on shell fixes the value of $B=B(R,K)$
- Each CMC surface is labeled by the value of K or corresponding radius of shell R .

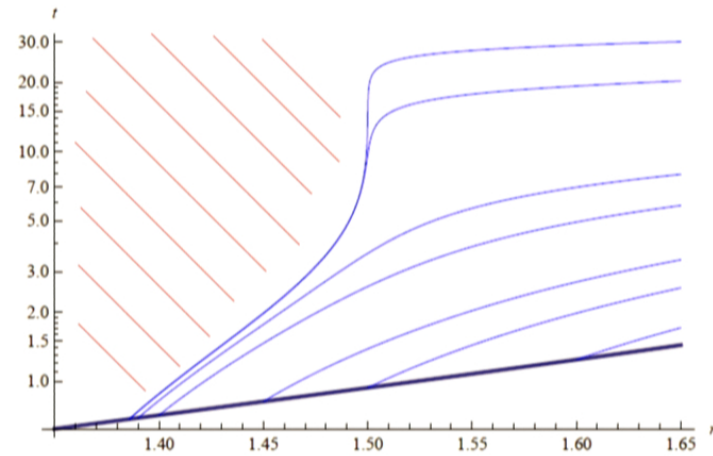
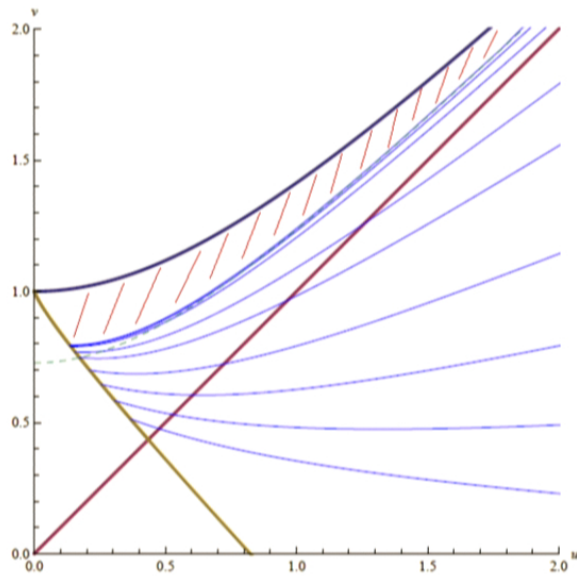


$$K(x) = \lambda_{max}(R) = \int_x dx \frac{\sqrt{x - \frac{R}{2}}}{f(x) \sqrt{f(x) + (\sqrt{x - \frac{R}{2}})^2}}$$

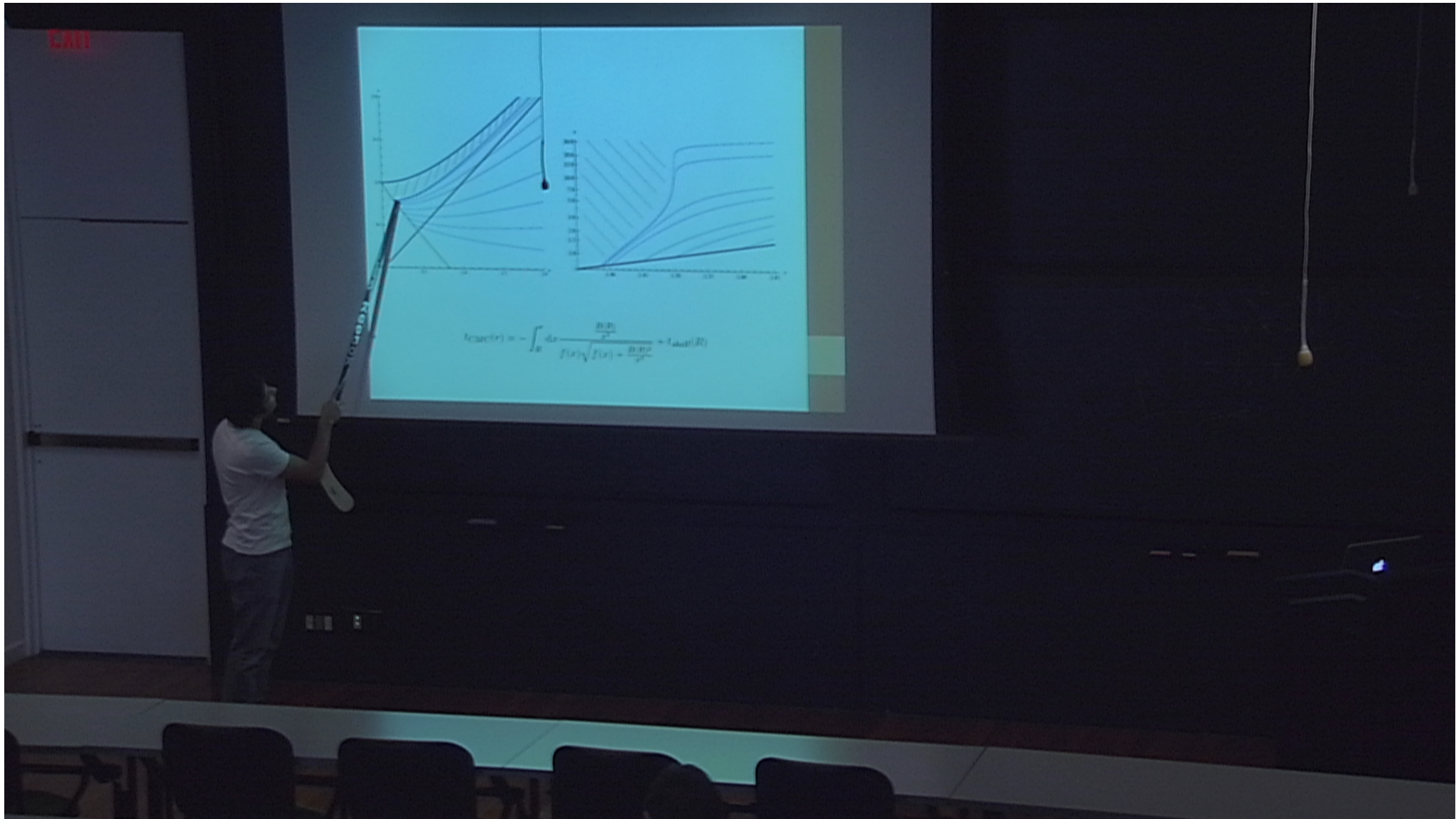
- K is fixed by matching these solutions to CMC surfaces of constant K .

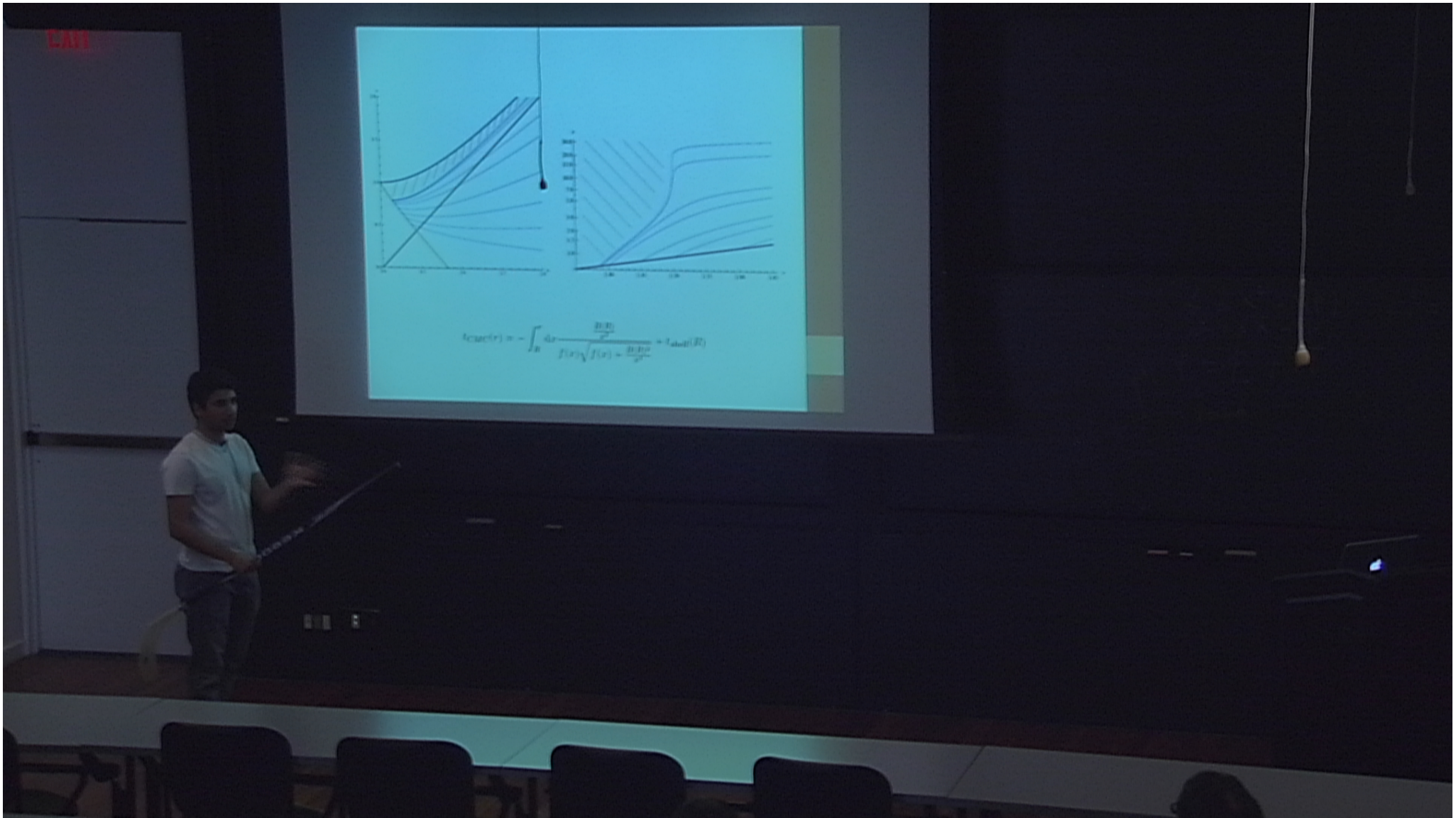


$$t_{CMC}(r) = - \int_R^r dx \frac{\frac{B(R)}{x^2}}{f(x) \sqrt{f(x) + \frac{B(R)^2}{x^4}}} + t_{shell}(R)$$



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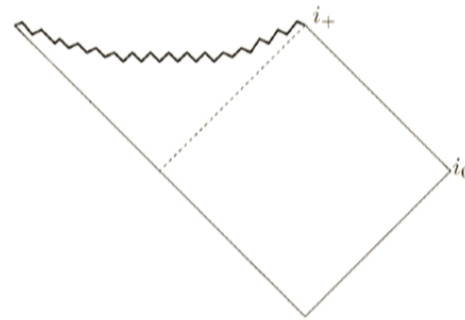


Concluding remarks

- This result is independent of the initial conditions (initial position of shell, value of e , ...)
- Cuscuton's energy density and pressure remains small everywhere (suppressed by Hubble constant) \rightarrow universal horizon is regular.
- Aspherical perturbations may change universal horizon to a singular surface.

D. Blas and S. Sibiryakov, Horava gravity versus thermodynamics: The Black hole case, [arXiv:1110.2195].

- We argue that this universal horizon should be considered as the future boundary of the classical space-time.



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Li Bao and S. Shreyas, Horava gravity versus thermodynamics: The Black hole case, [arXiv:1310.2195]

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