Title: A note on emergent classical behavior and approximations to decoherence functionals

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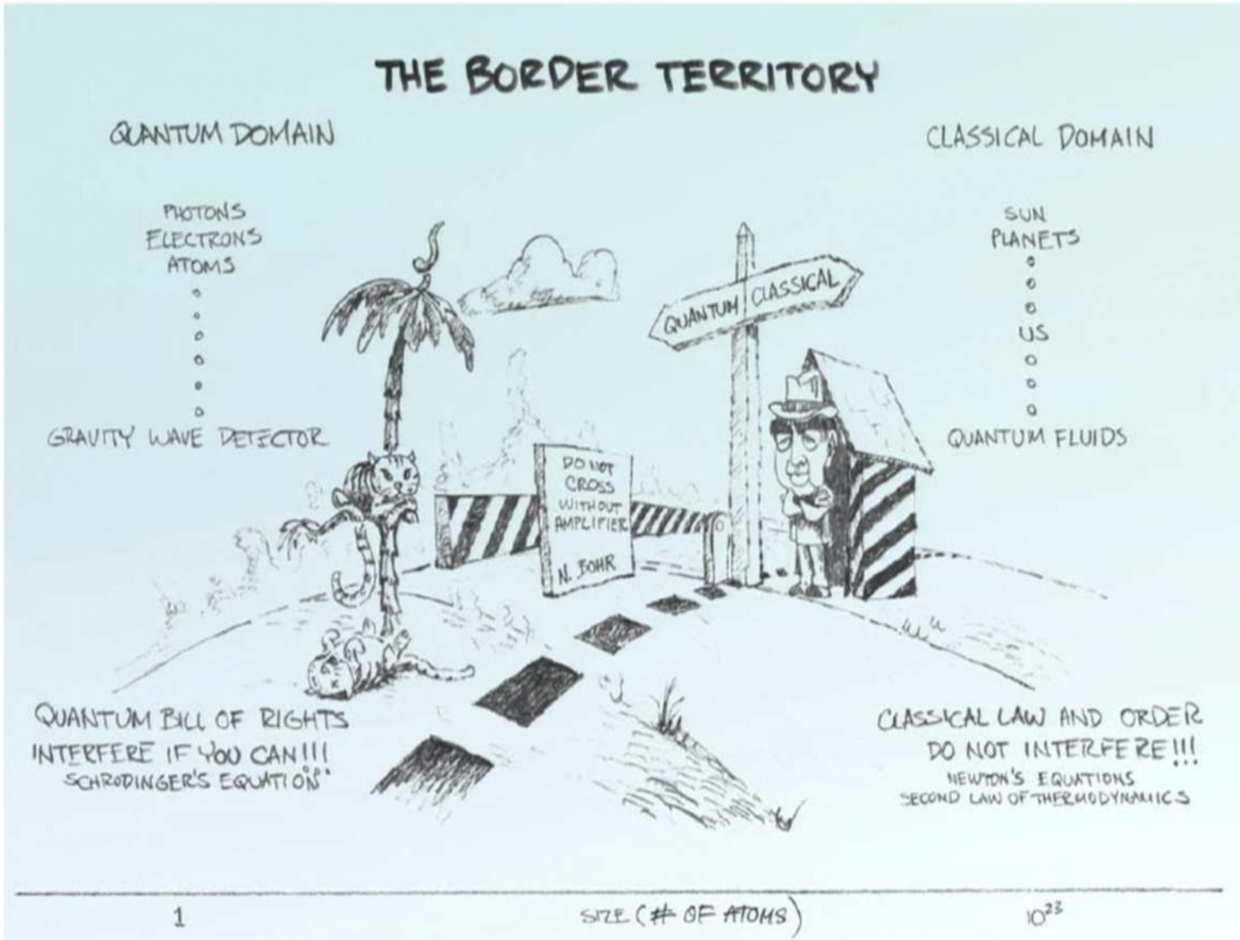
Abstract: Although it can only be argued to have become consequential in the study of quantum cosmology, the question ``Why do we observe a classical world? " has been one of the biggest preoccupations of quantum foundations. In the consistent histories formalism, the question is shifted to an analysis of the telltale sign of quantum mechanics: superposition of states. In the consistent histories formalism, histories of the system which ``decohere", i.e. fall out of superposition or have negligible interference can be subjected to a notion of classical probability. In this paper we use an extension of Kirchoff's diffraction formula for wave functions on configuration spaces to give a different analysis and an approximation of decoherence. The Kirchoff diffraction formula lies conveniently at the midway between path integrals, wave equations, and classical behavior. By using it, we formulate an approximate dampening of the amplitude of superposition of histories. The dampening acts on each middle element of the fine-grained history {c_\alpha}, and is a function of the angle formed between {c_{n-1},c_n} and {c_n,c_{n+1}}, as classical trajectories in configuration space. As an example we apply the formalism to a modified gravity theory in the ADM gravitational conformal superspace.

The Bohrder territory



The classical/quantum divide

$$\hat{H}|\psi\rangle = i\hbar\frac{d}{dt}|\psi\rangle \qquad (1)$$

Given almost any initial condition, the Universe described by $|\psi\rangle$ evolves into a state containing many alternatives that are never seen to coexist in our world.

Thus, at the root of our unease with quantum theory is the clash between the principle of superposition – the basic tenet of the theory reflected in the linearity of equation (1) – and everyday classical reality in which this principle appears to be violated.

This is one aspect of the *measurement problem*.

For Bohr, classical apparatus measuring the quantum system was a *necessity*, and quantum theory was not universal.

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Decoherence

Decoherence [Zeh, Zurek, Joos, etc]

Decoherence is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition. One consequence of this 'dephasing' is classical or probabilistically additive behavior.

E.g.: Let $|n\rangle$ be the states of the measured system, and $|\Phi_0\rangle$ the initial 'apparatus' state.

$$H_{\text{int}} : |n\rangle|\Phi_0\rangle \xrightarrow{t} |n\rangle|\Phi_n(t)\rangle$$

$$\rho = \sum_{n,m} c_m^* c_n |m\rangle\langle n| \xrightarrow{t} \sum_{n,m} c_m^* c_n \langle\Phi_m|\Phi_n\rangle|m\rangle\langle n|$$

If $\langle\Phi_m|\Phi_n\rangle \approx \delta_{mn}$ then $\rho \approx |c_n|^2|n\rangle\langle n|$.

Approximately diagonal in the pointer basis $\Rightarrow$ classical probabilities.

Consistent histories

Consistent histories [Griffiths, Gell-Mann and Hartle, Omnes]

Condition that allows probabilities to be assigned to various alternative histories of a system such that the probabilities for each history obey the rules of classical probability while being consistent with the Schrödinger equation. The *absence of quantum mechanical interference between histories* is the sufficient condition.

Let $\{P_{\alpha_k}(t_k)\}$ be a set of orthogonal projection operators. For example, α_k could be the position interval an electron might arrive at a screen at time t_k.

The projectors are taken to be exhaustive and exclusive:

$$\sum_{\alpha_k} P_{\alpha_k}(t_k) = 1 \ , \quad P_{\alpha_k}(t_k) P_{\alpha'_k}(t_k) = \delta_{\alpha_k \alpha'_k} P_{\alpha_k}(t_k)$$

Consistent histories II

A particular history corresponds to a particular sequence:

$(\alpha_1, \alpha_2, \cdots, \alpha_n) \equiv \alpha$,

and a corresponding chain of projection operators:

$C_\alpha = P_{\alpha_n}(t_n) \cdots P_{\alpha_1}(t_1)$

Ex: two slits (upper and lower), one observer of the slits (measure or not measure), and one screen $(x_1, \cdots x_n)$: $C_i = P^3_{x_i}(t_3) P^2_{\text{upper}}(t_2) P^1_{\text{meas}}(t_1)$

When the branches corresponding to a set of histories are sufficiently orthogonal the set of histories is said to decohere, i.e. for all $\alpha \neq \alpha'$:

$$\langle\psi| C_\alpha C'_\alpha |\psi\rangle \approx 0 \quad \text{(for pure initial states)}$$

Then, assign probabilities to histories: $p(\alpha) = ||C_\alpha \psi\rangle||^2$.

Path integral form: $D(\alpha, \alpha') = \int_{\alpha,\alpha'} \mathcal{D}\gamma(t)\mathcal{D}\gamma'(t) e^{i(S[\gamma(t)] - S[\gamma'(t)]/\hbar)}$

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Path integrals in configuration space

- **Configuration space**: The space of all possible **classical** configurations of an entire **closed system.**

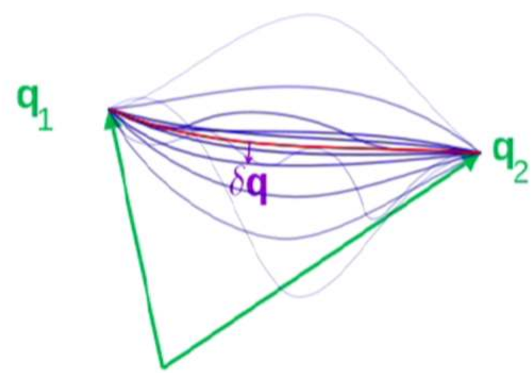
  E.g. $\mathbb{R}^{3n}$ for the dynamics of 3-particles in $\mathbb{R}^3$.

  Partial configurations (e.g. just the position of particle 1) defines submanifolds in configuration space.

- **Path integrals in configuration space**:

Propagator:

$$\langle q_1 | q_2 \rangle = \int_{q_1}^{q_2} \mathcal{D}\gamma \, e^{iS[\gamma]}$$



γ are curves between q_1, q_2

Classical behavior: stationary phase (steepest descent).

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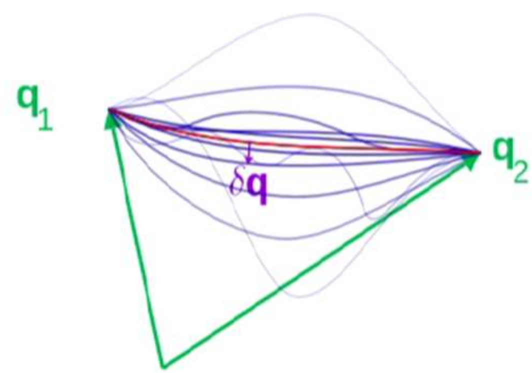


Path integrals in configuration space II

Two further notes:

- Approximations: segmented classical paths (DeWitt-Morette 05) capture the full theory arbitrarily well.
- We will usually consider classical paths as just geodesics for some metric in configuration space:

  the 'Jacobi metric'.

E.g. for particles with zero total energy, involves an incorporation of the potential into kinetic term (which is now position dependent).

This is not always possible (see Lanczos 1970').

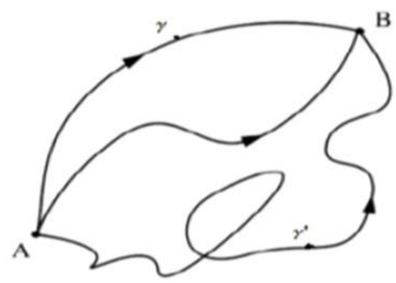
Superposition and decoherence in configuration space

How do we represent superposition and decoherence in this picture?

Only partially. All paths contribute, or superpose.

As we saw, path integral form of decoherence functional:

$D(\alpha, \alpha') = \int_{\alpha,\alpha'} \mathcal{D}\gamma(t)\mathcal{D}\gamma'(t)e^{i(S[\gamma(t)]-S[\gamma'(t)]/\hbar)}$



Approximate decoherence

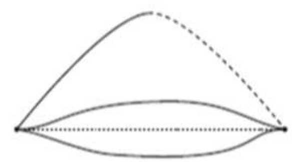
Given a set $\{\gamma_k\}_{q_1,q_2}$ of paths in configuration space between q_1 and q_2 representing $\{\alpha_k\}$ we will say γ_i and γ_j *'approximately' decohere* if the difference in lengths is not 'approximately' extremal.

Decoherence in configuration space II

Fading

Given a course graining $\{\gamma_k\}_{q_1,q_2}$ of paths in configuration space between q_1 and q_2, we will say γ_i *fades* to order $\epsilon << 1$ if, due to dephasing, the contribution to the propagator $\langle q_1|q_2\rangle$ is of order ϵ (e.g. compared to the largest contribution γ_{max}).

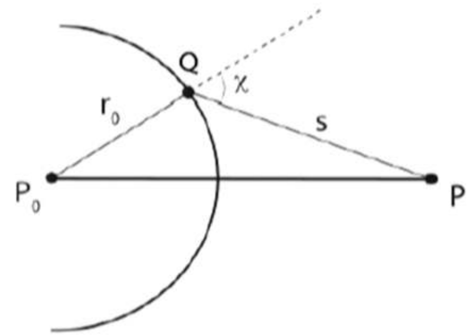
We will say that coarse grained paths between q_1, q_2 are *in superposition* if they don't fade. E.g. all the paths connecting q_1 to its cut locus.



We can approximate curves piecewise by geodesics. Can successive approximations of contributions to $\langle q_1|q_2\rangle$ be parametrized by the number of breaks in geodesics?

The Kirchoff diffraction formula

Consider a point source in $\mathbb{R}^3$ emitting wave at frequency ν and wavelength λ.



$U(r_o) = \frac{U_o e^{ikr_o}}{r_o}$. Considering a new point of emission at every point of the secondary wavefront, we get:

$$U(P) = -\frac{i}{\lambda} U(r_o) \int_{\text{sphere}} \frac{e^{iks}}{s} \frac{(1+\cos(\chi))}{2} d\Omega$$

- Describes waves from sum over contributions of classical paths (with phase).
- Gives an angle-dependent decrease for each broken geodesic path.

Consistent histories and Kirchoff in configuration space

Can we implement something similar in configuration space?

This would be well adapted to estimating the extra fading (and thus decoherence) between histories, from just geodesic properties in configuration space.

Given a sequence of points in configuration space (a coarse grained history), $\{C_\alpha\}_{\alpha\in I}$, connected by geodesics and denoting the angle in configuration space formed at C_α by $\Theta(\{C_{\alpha-1}, C_{\alpha+1}\})$ we could tentatively write:

$$\text{Fading}(\{C_\alpha\}) = \prod_{\alpha=2,\ldots n-1} \frac{1+\cos(\Theta(\{C_{\alpha-1}, C_{\alpha+1}\}))}{2}$$

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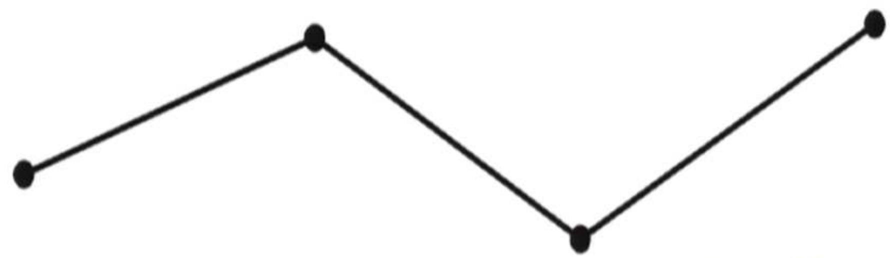
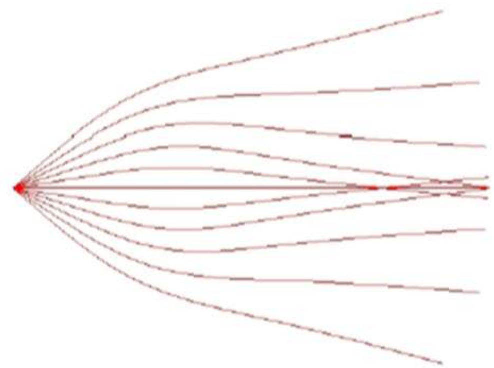
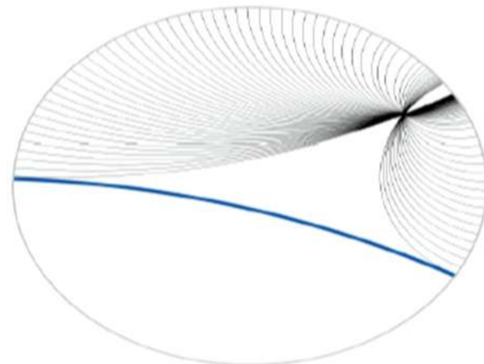


Decoherence time

Decoherence time of different orders.





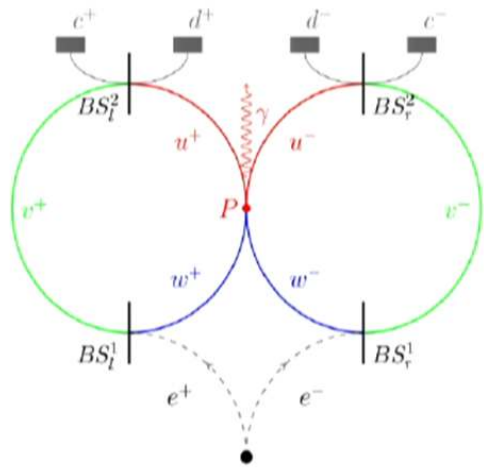
Depending on the initial angle, no decoherence.

Fast decoherence. Large number of dim. After length τ, decoherence to 2^{nd} order.

- The geometric character of the Jacobi metric should for complex systems is expected to be highly hyperbolic. 'Defocusing theorem' in configuration space?

Qualitative example I

- **Hardy's paradox**



We parametrize the 4 *geodesics* leaving $q_1 = (BS_l^1, BS_r^1)$ by: (v^+, v^-), $(w^+, v^-), (v^+, w^-)$ and (w^+, w^-). Of these, only (w^+, w^-) diverges from reaching the final point $q_2 = (BS_l^2, BS_r^2)$. Only fading trajectory would get (w^+, w^-) back to q_2. The others interfere.

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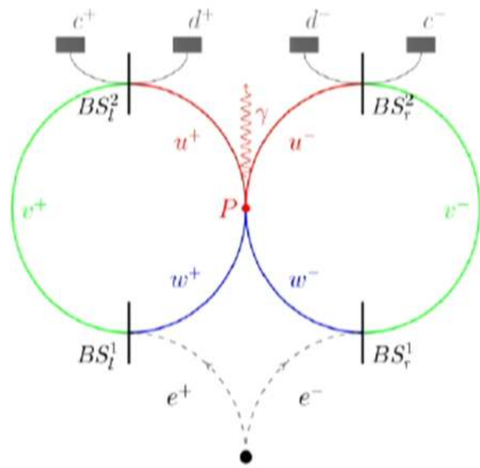

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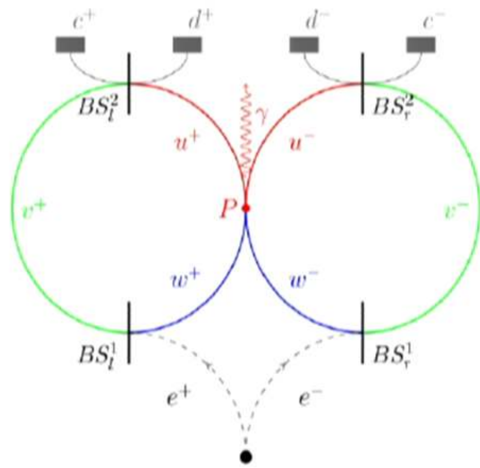

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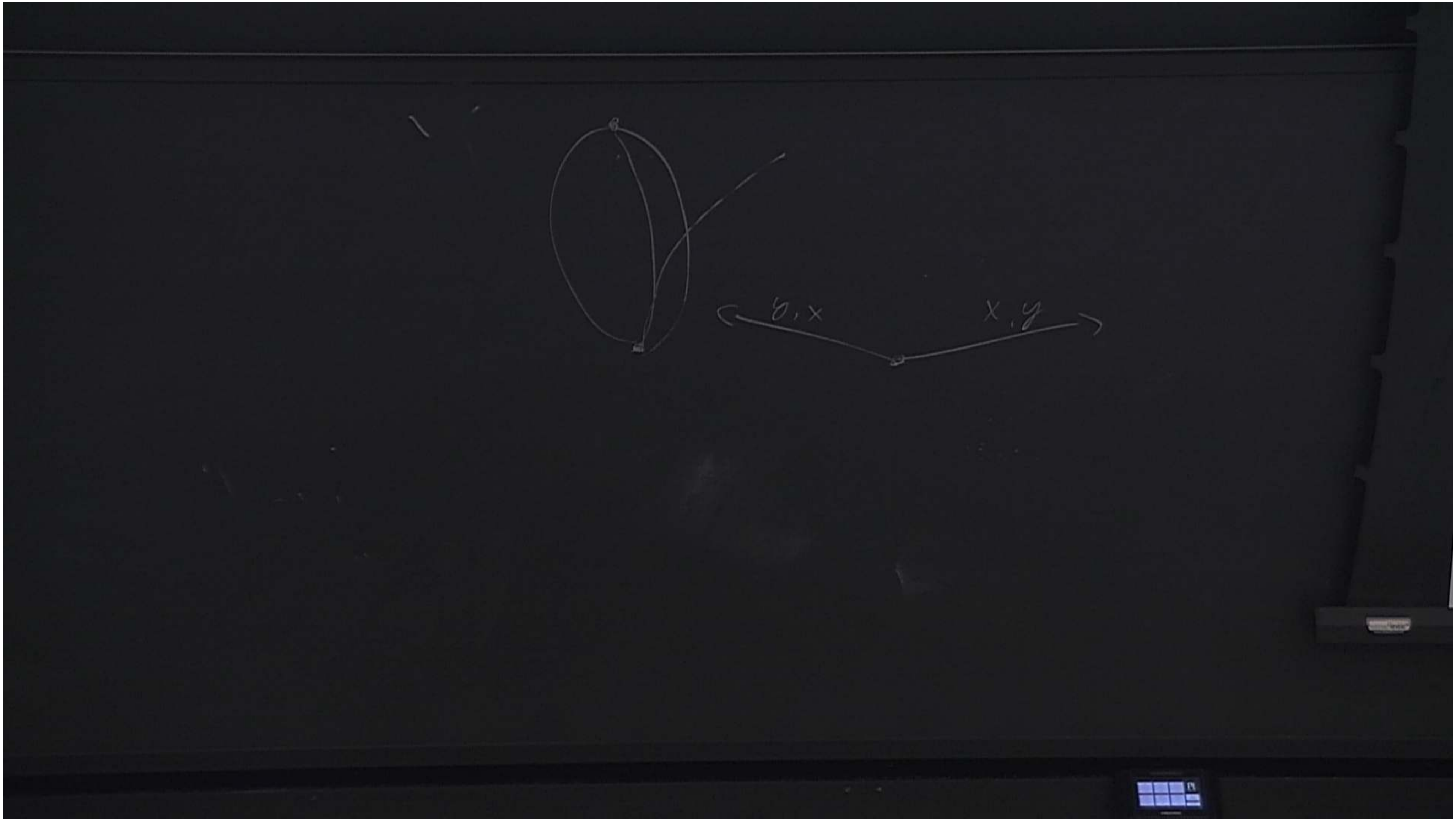
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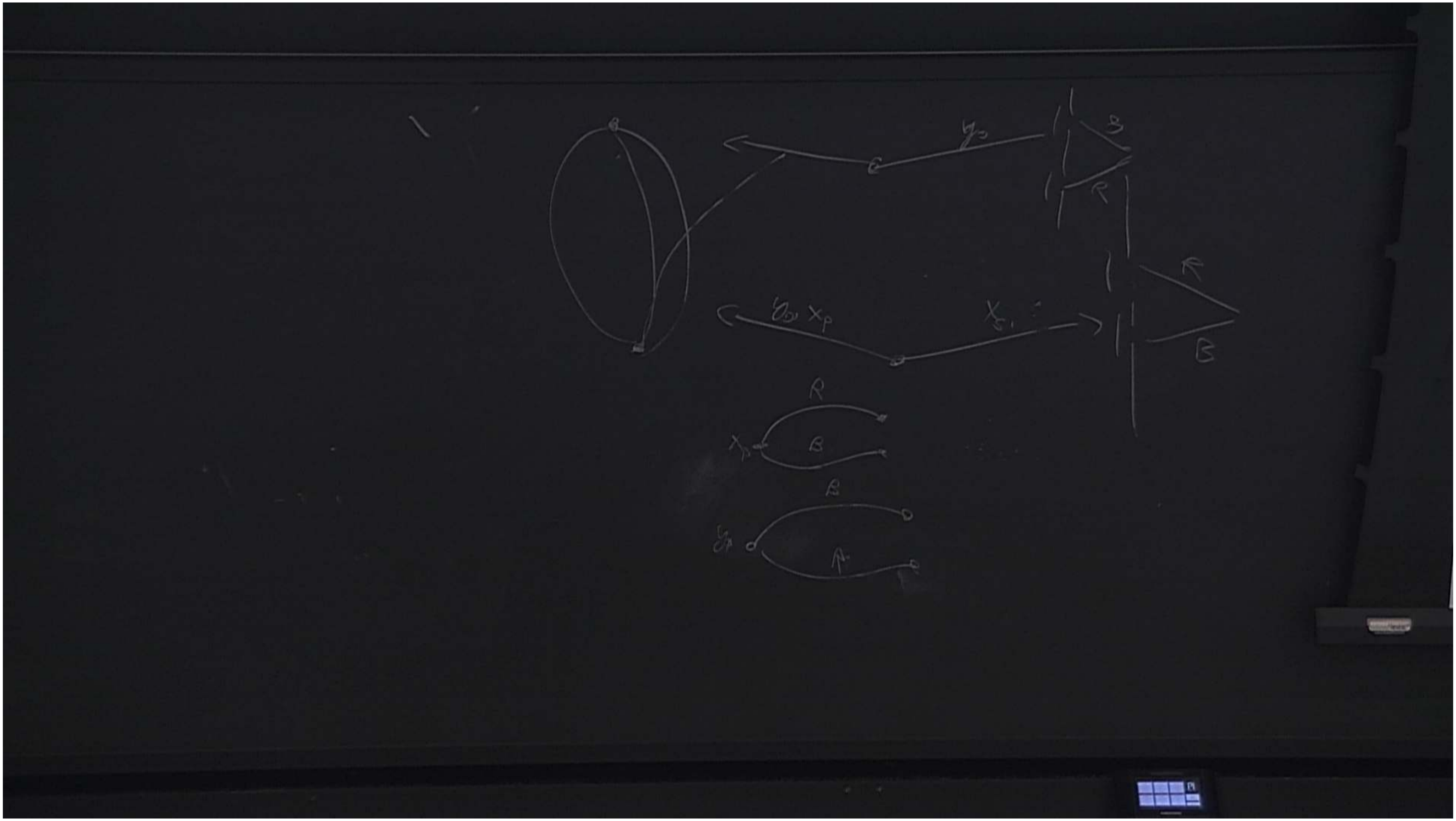
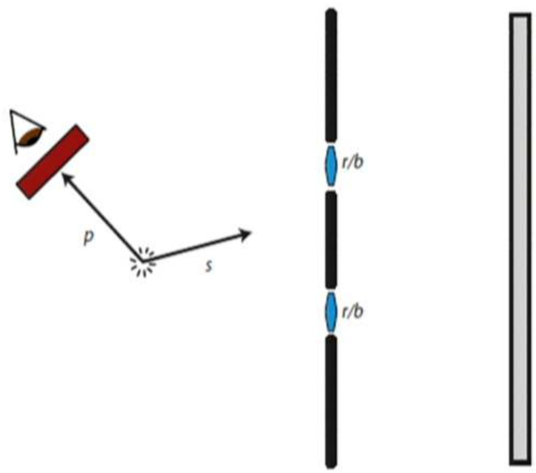
x, y



Qualitative example II

- **Walborn et al 02'**



- Orthogonal polarized photon pair production.
- Originally we have interference.
- Quarter wave plates with $\pm 45^o$ on slits (circ. pol.).

Outline

1. Introduction

2. Decoherence in configuration space

3. Discussions

Qualitative example II

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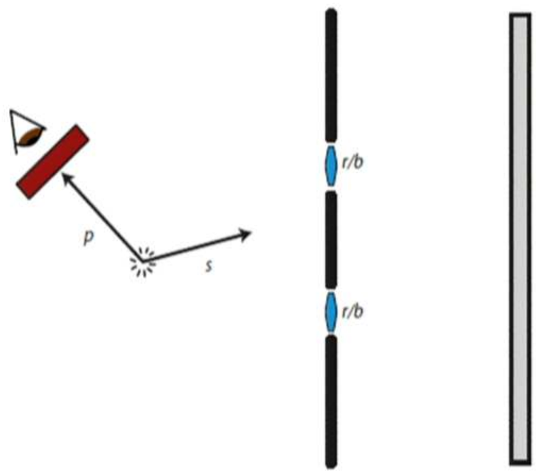

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- No interference (with/out p detection).
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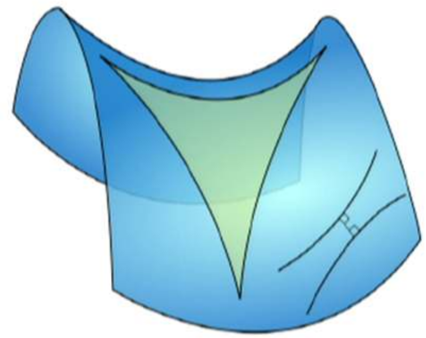
- Interference recovered.

Application to gravity

'Gravitationally induced decoherence'. Sure. In this respect nothing special about the gravitational field. Geodesics in its configuration space diverge rapidly due to non-linearity.



- Problem: non-relativistic description. Paths γ_1 and γ_2 that intersect in one foliation might not intersect in another.

  Requires a preferred notion of *simultaneity, to work in this simple form*. Requires geodesics in reduced configuration space.

- Einstein-Aether, Hořava, Shape Dynamics are such theories.

Experiments for preferred foliation theories?

- The path integral for gravity involves a gauge integration over foliations (the lapse), which complicates the reduced configuration space interpretation.
  - Theories whose solutions are uniquely determined by a point and direction in reduced configuration space are said to obey the *Poincaré principle* [Barbour].
- Even for a preferred–foliation theory that is *classically indistinguishable from gr*, can we use this formalism to infer qualitative quantum differences – such as different interference patterns for *non-local* measurements– due to an ontological preferred simultaneity surface? (without resolving all the issues with field quantization)
  - (There could be qualitative differences between considering quantum subsystems in a fixed background.)

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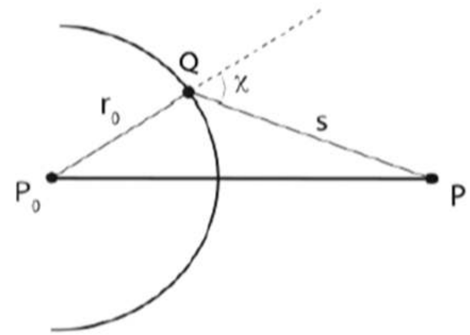
Last comments

- We have discussed a quantum mechanical description of closed Universes. The description is spatially non-local, since each point is a configuration of the entire universe.
  - In that context we have 'formally' relied on a path integral formulation in (reduced) configuration space.
  - We assume that the formalism of consistent histories in this context is rich enough to detect decoherence and superposition, to different orders of approximation.
- Each *configuration* is completely classical. The quantum character makes itself felt by interference between different histories.
- All the points (configurations) exist 'simultaneously'. All we can talk about are conditional probabilities based on records.
- Can we derive qualitative predictions from a quantum theoretic description of preferred foliation theories, *without* going through the full field quantization scheme?

The Kirchoff diffraction formula

Consider a point source in $\mathbb{R}^3$ emitting wave at frequency ν and wavelength λ.



$U(r_o) = \frac{U_o e^{ikr_o}}{r_o}$. Considering a new point of emission at every point of the secondary wavefront, we get:

$$U(P) = -\frac{i}{\lambda} U(r_o) \int_{\text{sphere}} \frac{e^{iks}}{s} \frac{(1+\cos(\chi))}{2} d\Omega$$

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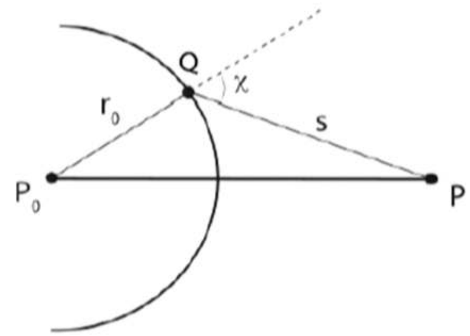

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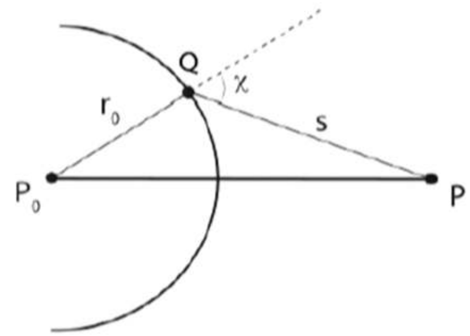

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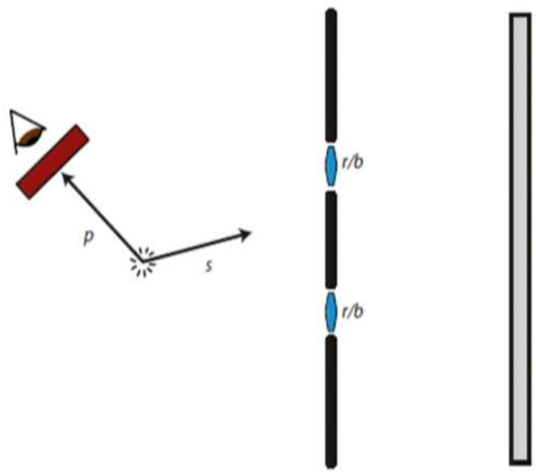

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