Title: From the symmetric group to giant gravitons in super Yang-Mills theory

Date: Nov 05, 2013 02:00 PM

URL: http://pirsa.org/13110084

Abstract: In this talk we will discuss how giant gravitons and their open string interactions emerge from super Yang-Mills Theory. This is accomplished by diagonalizing the one loop dilatation operator on a class of operators with bare dimension of order N. From the result of this diagonalization, the Gauss Law governing the allowed open string excitations of giant gravitons is clearly visible. In addition, we show that this sector of the theory is integrable.

From the symmetric group to giant gravitons in super Yang-Mills theory

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November 5, 2013

Goal of the talk

Problem: Compute the spectrum of anomalous dimensions, to one loop, in a large N but non-planar limit of $\mathcal{N} = 4$ SYM.

$$\langle \mathcal{O}_A(x)\mathcal{O}_B(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta_A}}$$

$\mathcal{O}_A(x)$ has free field dimension of $O(N)$. Non-planar ribbon graphs contribute to the large N limit.

The action of the dilatation operator reduces to a set of decoupled oscillators.

This limit in the gauge theory is dual to systems of excited giant gravitons.

Matrix Model Two Point Function

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j = \langle Y^i{}_j (Y^\dagger)^k{}_l \rangle$$

$$\langle Y^i{}_j (Z^\dagger)^k{}_l \rangle = \langle Z^i{}_j (Y^\dagger)^k{}_l \rangle = 0$$

Operators built using n Z fields and m Y fields. $n, m \sim O(N)$; $m \ll n$. Drop $O(\frac{m}{n})$.

$m = 0$, $n \neq 0$ - $\frac{1}{2}$-BPS sector: Gauge-invariant BPS operators are traces and products of traces built using a single matrix.

$n = 1 : \mathrm{Tr}(Z)$
$n = 2 : \mathrm{Tr}(Z^2); \ (\mathrm{Tr} Z)^2$
$n = 3 : \mathrm{Tr}(Z^3); \ \mathrm{Tr}(Z^2)\mathrm{Tr}(Z); \ (\mathrm{Tr} Z)^3$

m, n are $O(N)$

drop $O\left(\frac{m}{n}\right)$.

Planar Limit

Distinct multi-trace structures are orthogonal in the large N limit.

$$\langle O_{\text{trace structure 1}} O_{\text{trace structure 2}} \rangle \propto \delta_{\text{trace structure 1;trace structure 2}}$$

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j$$

$$\langle \frac{\mathrm{Tr}(Z^J)}{\sqrt{JN^J}} \frac{\mathrm{Tr}(Z^{\dagger\, J})}{\sqrt{JN^J}} \rangle = 1$$

$$\langle \frac{\mathrm{Tr}(Z^{J_1})}{\sqrt{J_1 N^{J_1}}} \frac{\mathrm{Tr}(Z^{J_2})}{\sqrt{J_2 N^{J_2}}} \frac{\mathrm{Tr}(Z^{\dagger\, J_3})}{\sqrt{J_3 N^{J_3}}} \rangle = \frac{\sqrt{J_1 J_2 J_3}}{N} \delta_{J_1+J_2;J_3}$$

Orthogonality breaks down at $J_i \sim N^{\frac{2}{3}}$

(Balasubramanian, Berkooz, Naqvi, Strassler, hep-th/0107119)

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Multitraces

At $n = 2$: $\mathrm{Tr}Z\mathrm{Tr}Z = Z^{i_1}_{i_1} Z^{i_2}_{i_2}$ $\qquad$ $\mathrm{Tr}(Z^2) = Z_{i_2}{}^{i_1} Z^{i_2}_{i_1}$

Lower labels are permuted with respect to upper labels.

$$\mathrm{Tr}Z\mathrm{Tr}Z = Z^{i_1}_{i_\sigma(1)} Z^{i_2}_{i_\sigma(2)} \equiv \mathrm{Tr}(\sigma Z^{\otimes 2}) \qquad \sigma = (1)(2)$$

Language for discussing the complete set of multitrace operators

$$\mathrm{Tr}(\sigma Z^{\otimes n}) = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

Any multitrace operator composed from k fields corresponds to a $\sigma \in S_k$. Permutations in the same conjugacy class determine the same operator.

Schur Polynomials

$$\chi_R(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) \mathrm{Tr}(\sigma Z^{\otimes n})$$

R specifies an irrep of S_n. $\chi_R(\sigma)$ is the character of σ in irrep R.

$$\chi_{\yng(3)} = \frac{1}{6}\left[\mathrm{Tr}(Z)^3 + 3\mathrm{Tr}(Z)\mathrm{Tr}(Z^2) + 2\mathrm{Tr}(Z^3)\right]$$

$$\chi_{\yng(2,1)} = \frac{1}{6}\left[2\mathrm{Tr}(Z)^3 - 2\mathrm{Tr}(Z^3)\right]$$

$$\chi_{\yng(1,1,1)} = \frac{1}{6}\left[\mathrm{Tr}(Z)^3 - 3\mathrm{Tr}(Z)\mathrm{Tr}(Z^2) + 2\mathrm{Tr}(Z^3)\right]$$

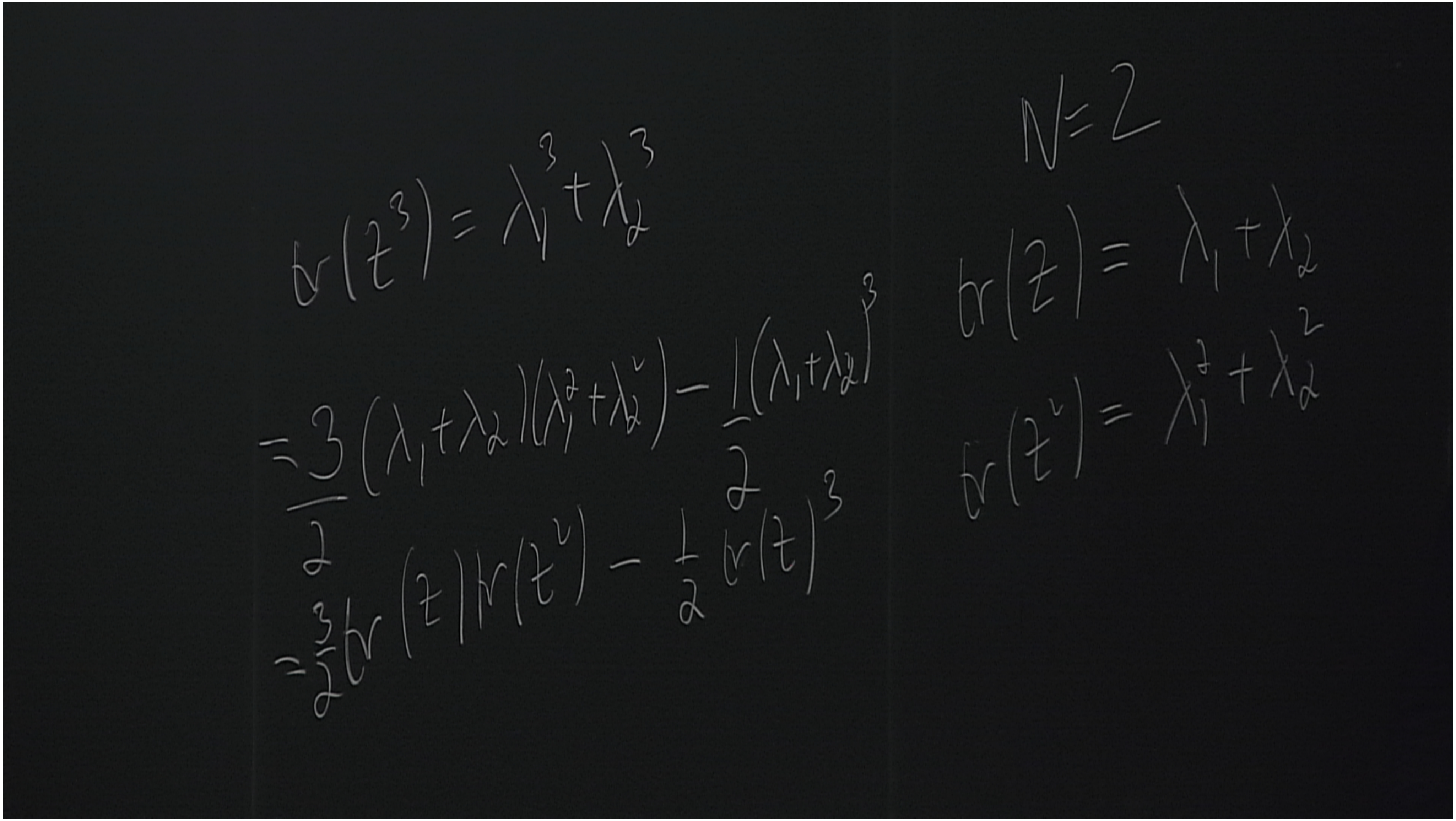
Schur Polynomials

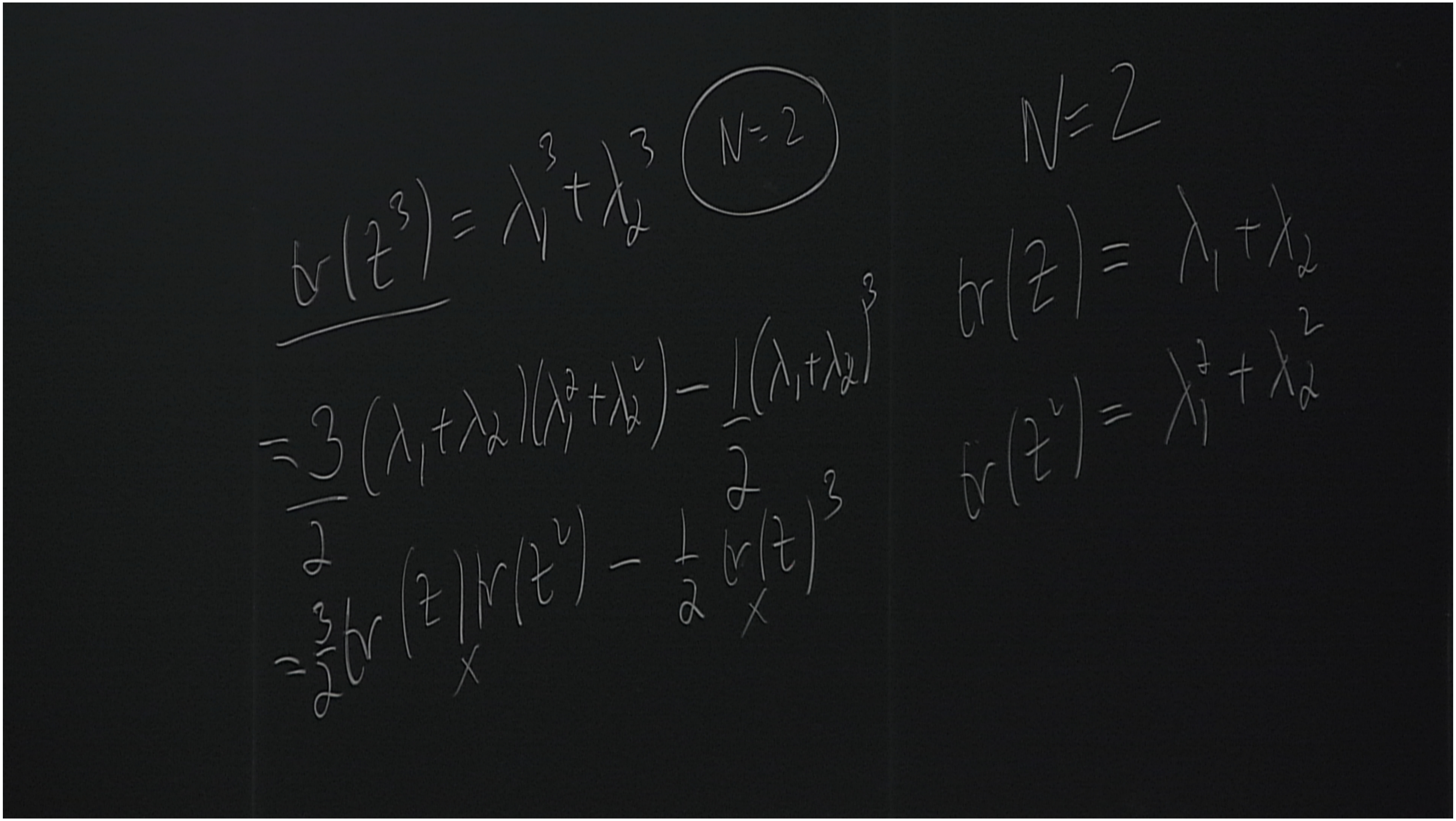
Number of Schur polynomials agrees with finite N counting.

$$\langle \chi_R(Z)\chi_S(Z)^\dagger \rangle = f_R \delta_{RS}$$

$$\mathrm{Tr}(\sigma Z^{\otimes n}) = \sum_R \chi_R(\sigma)\chi_R(Z)$$

(Corley, Jevicki, Ramgoolam, hep-th/0111222)


N=2
tr(Z) = λ1 + λ2
tr(Z^2) = λ1^2 + λ2^2
tr(Z^3) = λ1^3 + λ2^3
= 3/2 (λ1+λ2)(λ1^2+λ2^2) − 1/2 (λ1+λ2)^3
= 3/2 tr(Z) tr(Z^2) − 1/2 tr(Z)^3


$tr(z^3) = \lambda_1^3 + \lambda_2^3$
$N=2$
$= \frac{3}{2}(\lambda_1+\lambda_2)(\lambda_1^2+\lambda_2^2) - \frac{1}{2}(\lambda_1+\lambda_2)^3$
$= \frac{3}{2} tr(z) tr(z^2) - \frac{1}{2} tr(z)^3$
$N=2$
$tr(z) = \lambda_1 + \lambda_2$
$tr(z^2) = \lambda_1^2 + \lambda_2^2$

Key Ideas

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j$$

$$\langle\, A^{i_1\, i_2 \cdots i_n}_{j_1\, j_2 \cdots j_n} Z^{j_1}_{i_1} Z^{j_2}_{i_2} \cdots Z^{j_n}_{i_n} \;\; B^{k_1\, k_2 \cdots k_n}_{l_1\, l_2 \cdots l_n} (Z^\dagger)^{l_1}_{k_1} (Z^\dagger)^{l_2}_{k_2} \cdots (Z^\dagger)^{l_n}_{k_n} \,\rangle$$
$$= \sum_{\sigma \in S_n} \mathrm{Tr}(A \sigma B \sigma^{-1})$$

Projection operators obey

$$[P_A, \sigma] = 0$$

$$P_A P_B = \delta_{AB} P_A$$

The Schur polynomials

$$\chi_R(Z) \propto (P_R)^{i_1\, i_2 \cdots i_n}_{j_1\, j_2 \cdots j_n} Z^{j_1}_{i_1} Z^{j_2}_{i_2} \cdots Z^{j_n}_{i_n}$$

If $n = 3$ and $A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} = \delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}$ then

$$A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} Z^{j_1}_{i_1} Z^{j_2}_{i_2} Z^{j_3}_{i_3} = \mathrm{Tr}(Z)^3$$

If $n = 3$ and $A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} = \delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}$ then

$$A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} Z^{j_1}_{i_1} Z^{j_2}_{i_2} Z^{j_3}_{i_3} = \mathrm{Tr}(Z)\mathrm{Tr}(Z)^2$$

If $n = 3$ and $A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} = \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}$ then

$$A^{i_1\,i_2\,i_3}_{j_1\,j_2\,j_3} Z^{j_1}_{i_1} Z^{j_2}_{i_2} Z^{j_3}_{i_3} = \mathrm{Tr}(Z^3)$$

Including Y: $m \neq 0$

How much of the $\frac{1}{2}$-BPS story can be generalized?

$$\mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m}) = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_n}_{i_{\sigma(n)}} Y^{i_{n+1}}_{i_{\sigma(n+1)}} Y^{i_{n+2}}_{i_{\sigma(n+2)}} \cdots Y^{i_{m+n}}_{i_{\sigma(m+n)}}$$

Any multitrace operator built using n Zs and m Ys corresponds to a $\sigma \in S_{n+m}$. Permutations related by $\gamma\sigma_1\gamma^{-1} = \sigma_2$ with $\sigma_1, \sigma_2 \in S_{n+m}$ and $\gamma \in S_n \times S_m$ determine the same operator.

Restricted Schur Polynomials

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\sigma)) \mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m})$$

R is an irrep of S_{n+m}. We can subduce the $S_n \times S_m$ irrep (r,s) from R. α, β keep track of which copy we subduce.

$$\chi_{\square\square,(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) + \mathrm{Tr}(ZY)$$

$$\chi_{\begin{smallmatrix}\square\\\square\end{smallmatrix},(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) - \mathrm{Tr}(ZY)$$

(Berenstein, Balasubramanian, Feng, Huang, hep-th/0411205; Bhattacharyya, Collins, dMK, arXiv:0801.2061)

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\sigma)) \mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m})$$

Consider the **3** of $SO(3)$ $\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Restrict $SO(3)$ to

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \in SO(2)$$

we find that the **3** of $SO(3)$ decomposes $\mathbf{3} \to \mathbf{2} \oplus \mathbf{1}$ of $SO(2)$

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$$

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\sigma))\mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m})$$

The **3** of $SO(3)$ decomposes **3** $\rightarrow$ **2** $\oplus$ **1** of $SO(2)$

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$$

$$\mathrm{Tr}_{\mathbf{2}} \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{bmatrix} = m_{xx} + m_{yy}$$

$$\mathrm{Tr}_{\mathbf{1}} \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{bmatrix} = m_{zz}$$

Restricted Schur Polynomials

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\sigma)) \mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m})$$

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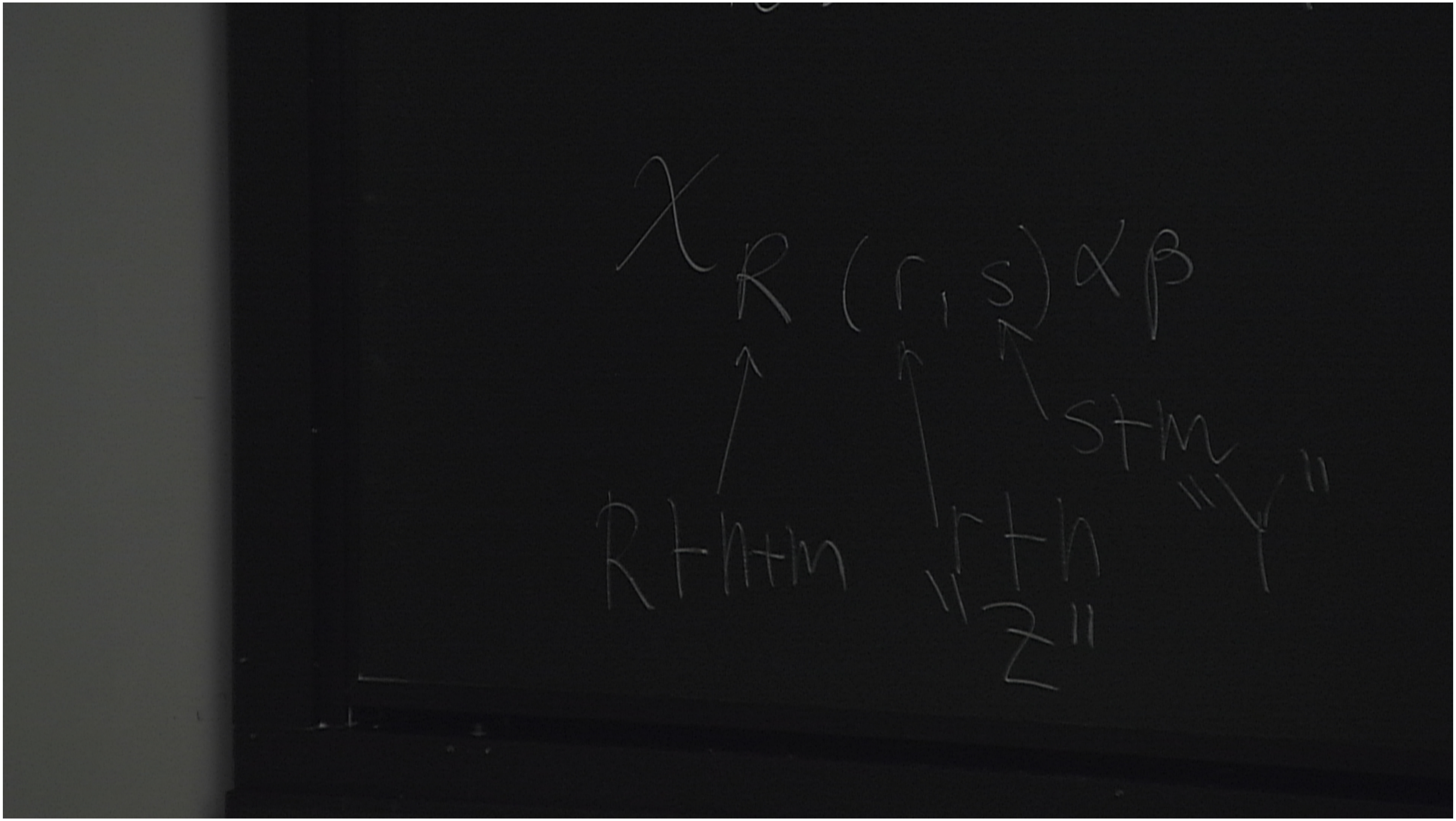
$$\chi_{\square\square,(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) + \mathrm{Tr}(ZY)$$

$$\chi_{\substack{\square\\\square},(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) - \mathrm{Tr}(ZY)$$

(Berenstein, Balasubramanian, Feng, Huang, hep-th/0411205; Bhattacharyya, Collins, dMK, arXiv:0801.2061)


χ R(r, s)αβ
Rthm
rth
stm
"Y"
"Z"

Restricted Schur Polynomials

Number of restricted Schur polynomials agrees with finite N counting.

$$\langle \chi_{R,(r,s)\mu\nu}(Z,Y)\chi_{S,(t,u)\alpha\beta}(Z,Y)^{\dagger}\rangle = N(R,r,s)\delta_{RS}\delta_{rt}\delta_{su}\delta_{\mu\alpha}\delta_{\nu\beta}$$

$$\mathrm{Tr}(\sigma Z^{\otimes n} Y^{\otimes m}) = \sum_R \mathrm{Tr}_{(r,s)\beta\alpha}(\Gamma^R(\sigma))\chi_{R,(r,s)\beta\alpha}(Z,Y)$$

(Collins, arXiv:0810.4217; Bhattacharyya, Collins, dMK, arXiv:0801.2061; Bhattacharyya, dMK, Stephanou, arXiv:0805.3025)

There are other ways to organize multi-matrix operators. (Ramgoolam, Kimura, arXiv:0709.2158; Brown, Heslop, Ramgoolam arXiv:0711.0176, arXiv:0806.1911.)

Dilatation Operator

$$D = -g_{YM}^2 \mathrm{Tr}\left(\left[Z, Y\right]\left[\frac{d}{dZ}, \frac{d}{dY}\right]\right)$$

(Beisert, Kristjansen, Staudacher, hep-th/0303060)

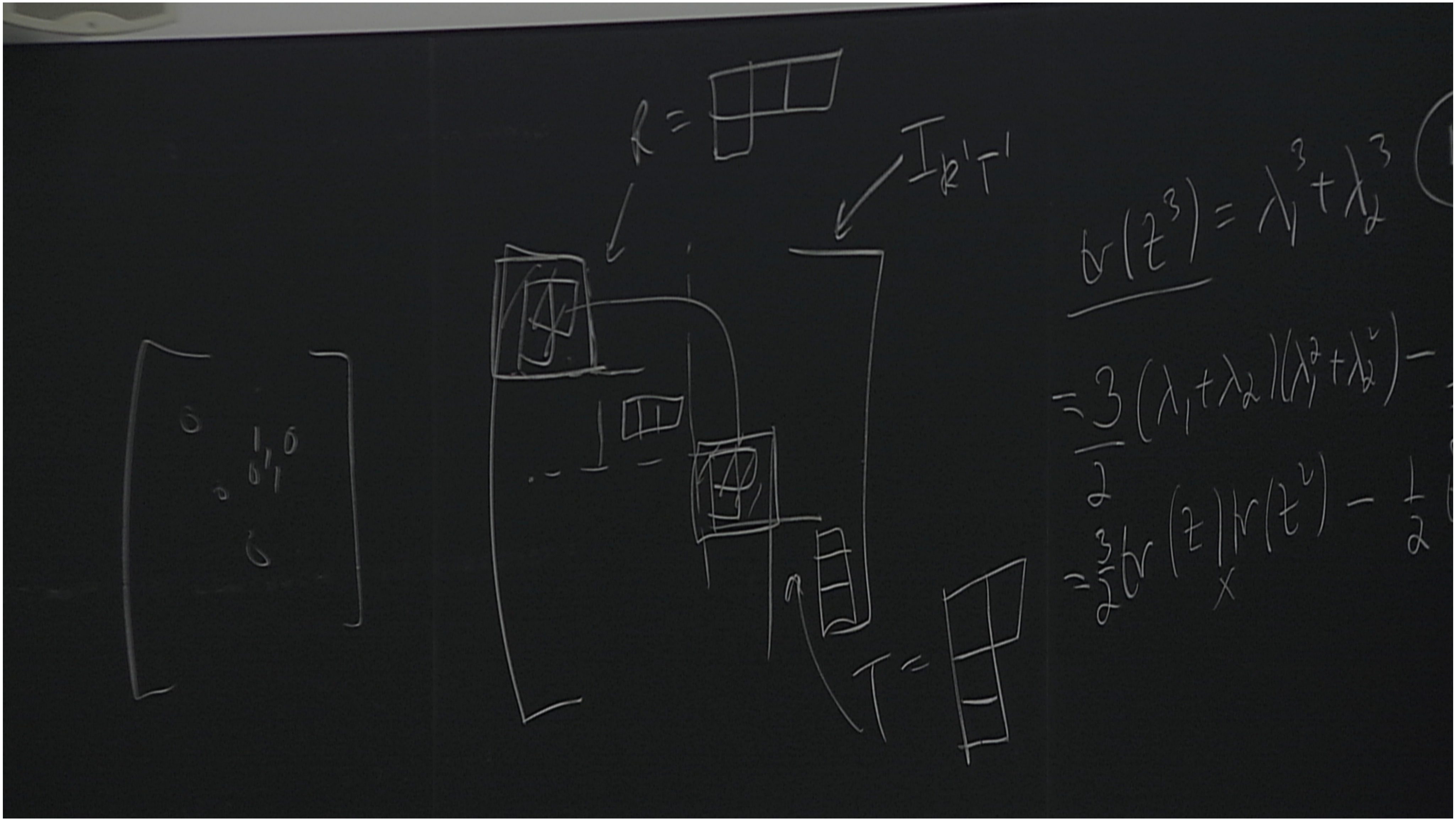
$$D\chi_{R,(r,s)\alpha\beta} = \sum_{T,(t,u)kl} M_{R,(r,s)\alpha\beta;T,(t,u)\gamma\delta}\chi_{T,(t,u)\delta\gamma}$$

$$M_{R,(r,s)\alpha\beta;T,(t,u)\delta\gamma} = -g_{YM}^2 \sum_{R'} N_{R,R',r,s,T,t,u}$$

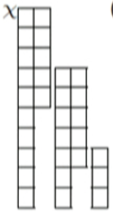
$$\times \mathrm{Tr}_{R\oplus T}\left(\left[\Gamma^R(1,m+1), P_{R,(r,s)\alpha\beta}\right] I_{R'\,T'}\left[\Gamma^T(1,m+1), P_{T,(t,u)\delta\gamma}\right] I_{T'\,R'}\right).$$

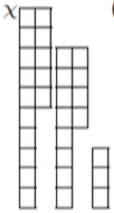
$$\mathrm{Tr}_{(r,s)\alpha\beta}(*) = \mathrm{Tr}_R(P_{R,(r,s)\alpha\beta} *)$$

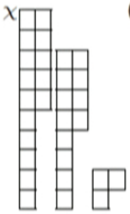
(De Comarmond, dMK, Jefferies, arXiv:1012.3884)

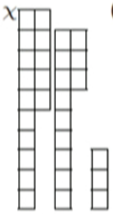

R =
$I_{R'T'}$
$tr(t^3) = \lambda_1^3 + \lambda_2^3$
$= \frac{3}{2}(\lambda_1 + \lambda_2)(\lambda_1^2 + \lambda_2^2) -$
$= \frac{3}{2} tr(t) tr(t^2) - \frac{1}{2}$
T =

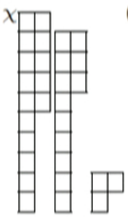
$m = 3$ sector: $D\chi_{*,(b_0,b_1)}$

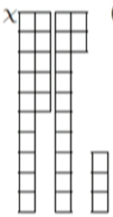
$\chi_A(b_0, b_1) = \chi$  (Z, Y)

$\chi_B(b_0, b_1) = \chi$  (Z, Y)

$\chi_C(b_0, b_1) = \chi$  (Z, Y)

$\chi_D(b_0, b_1) = \chi$  (Z, Y)

$\chi_E(b_0, b_1) = \chi$  (Z, Y)

$\chi_F(b_0, b_1) = \chi$  (Z, Y)

One term

$$
\begin{aligned}
\hat{D} O_B(b_0, b_1) = {} & \sqrt{(N-b_0-b_1-1)(N-b_0)}\left[-\frac{4}{3}\sqrt{\frac{(b_1+2)(b_1-1)}{b_1\,(b_1+1)}}\,\frac{(b_1-2)(b_1+3)}{b_1\,(b_1+1)}\,O_B(b_0+1, b_1-2)\right. \\
& +\frac{2}{3}\,\frac{b_1+3}{b1}\sqrt{\frac{(b_1+2)(b_1-1)}{(b_1+1)\,b_1}}\,\sqrt{2}O_C(b_0+1, b_1-2) - \frac{32}{3}\,\frac{{b_1}^2+2\,b_1-3}{b_1(b_1+1)(b_1+2)^2}\sqrt{\frac{b_1+2}{b_1}}\,O_D(b_0, b_1) \\
& \left. -\frac{2\sqrt{2}}{3}\sqrt{\frac{b_1+2}{b_1}}\,\frac{(b_1+3)(3b_1-2)}{b_1(b_1+2)(b_1+1)}\,O_E(b_0, b_1) + 8\sqrt{\frac{(b_1+3)\,b_1}{(b_1+2)(b_1+1)}}\,\frac{1}{(b_1+1)(b_1+2)}\,O_F(b_0-1, b_1+2)\right] \\
& +\sqrt{(N-b_0-b_1-2)(N-b_0+1)}\left[\frac{2}{3}\sqrt{\frac{(b_1+4)(b_1+1)}{(b_1+2)(b_1+3)}}\,\frac{\sqrt{2}b_1}{(b_1+3)}\,O_C(b_0-1, b_1+2)\right. \\
& -\frac{4}{3}\sqrt{\frac{(b_1+4)(b_1+1)}{(b_1+3)(b_1+2)}}\,\frac{(b_1+5)\,b_1}{(b_1+3)(b_1+2)}\,O_B(b_0-1, b_1+2) \\
& \left. +4\sqrt{\frac{b_1+4}{b_1+2}}\,\frac{b_1}{(b_1+3)(b_1+2)}\,O_A(b_0, b_1)\right] + (N-b_0-b_1-1)\left[-4\sqrt{\frac{b_1-1}{b_1+1}}\,\frac{(b_1+3)}{(b_1+1)\,b_1}\,O_A(b_0+1, b_1-2)\right. \\
& +\frac{4}{3}\,\frac{(b_1+3)(b_1^3+5b_1^2+8b_1-12)}{(b_1+1)b_1(b_1+2)^2}\,O_B(b_0, b_1) - \frac{2\sqrt{2}}{3}\,\frac{(b_1^2+2b_1-4)(b_1+3)}{(b_1+1)(b_1+2)^2}\,O_C(b_0, b_1) \\
& \left. -\frac{8}{3}\sqrt{\frac{b_1+3}{b_1+1}}\,\frac{(b_1+4)\,b_1}{(b_1+2)^2\,(b_1+1)}\,O_D(b_0-1, b_1+2) + \frac{4}{3}\sqrt{2}\sqrt{\frac{b_1+3}{b_1+1}}\,\frac{b_1}{(b_1+2)^2}\,O_E(b_0-1, b_1+2)\right] \\
& +(N-b_0+1)\left[\frac{4}{3}\,\frac{(b_1+4)\,{b_1}^2}{(b_1+3)(b_1+2)^2}\,O_B(b_0, b_1) + \frac{8}{3}\,\frac{\sqrt{(b_1+1)(b_1+3)}b_1\,(b_1+4)}{(b_1+3)^2\,(b_1+2)^2}\,O_D(b_0-1, b_1+2)\right. \\
& \left. -\frac{2}{3}\,\frac{\sqrt{2}\,(b_1+4)\,b_1}{(b_1+2)^2}\,O_C(b_0, b_1) + \frac{2}{3}\,\frac{\sqrt{2}\sqrt{(b_1+1)(b_1+3)}b_1\,(b_1+4)}{(b_1+3)^2\,(b_1+2)^2}\,O_E(b_0-1, b_1+2)\right]
\end{aligned}
$$

Results

$m = 2$; 4 operators mix; $\omega = 0\ (\times 3)$ $\quad \omega = 8g_{YM}^2\ (\times 1)$

$m = 3$; 6 operators mix; $\omega = 0\ (\times 4)$ $\quad \omega = 8g_{YM}^2\ (\times 2)$

$m = 4$; 9 operators mix; $\omega = 0\ (\times 5)$ $\quad \omega = 8g_{YM}^2\ (\times 3)$
$\omega = 16g_{YM}^2\ (\times 1)$

$m = 5$; 12 operators mix; $\omega = 0\ (\times 6)$ $\quad \omega = 8g_{YM}^2\ (\times 4)$
$\omega = 16g_{YM}^2\ (\times 2)$

$m = 6$; 16 operators mix; $\omega = 0\ (\times 7)$ $\quad \omega = 8g_{YM}^2\ (\times 5)$
$\omega = 16g_{YM}^2\ (\times 3)$ $\quad \omega = 24g_{YM}^2\ (\times 1)$

The Displaced Corners Approximation

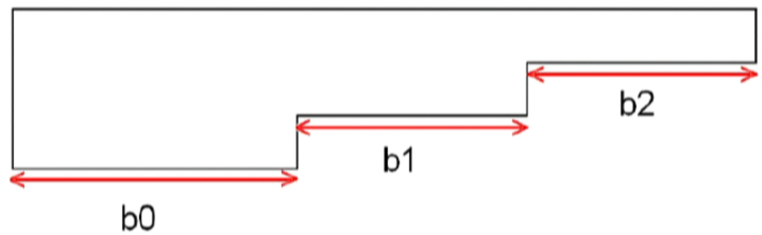


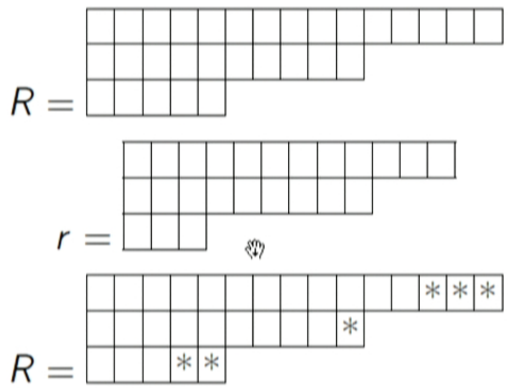
Figure: Example of a three row Young diagram.

In the displaced corners approximation we assume that b_0, b_1, b_2 are all of order N.

This limit simplifies the action of the symmetric group which is responsible for a new $U(p)$ symmetry. (dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

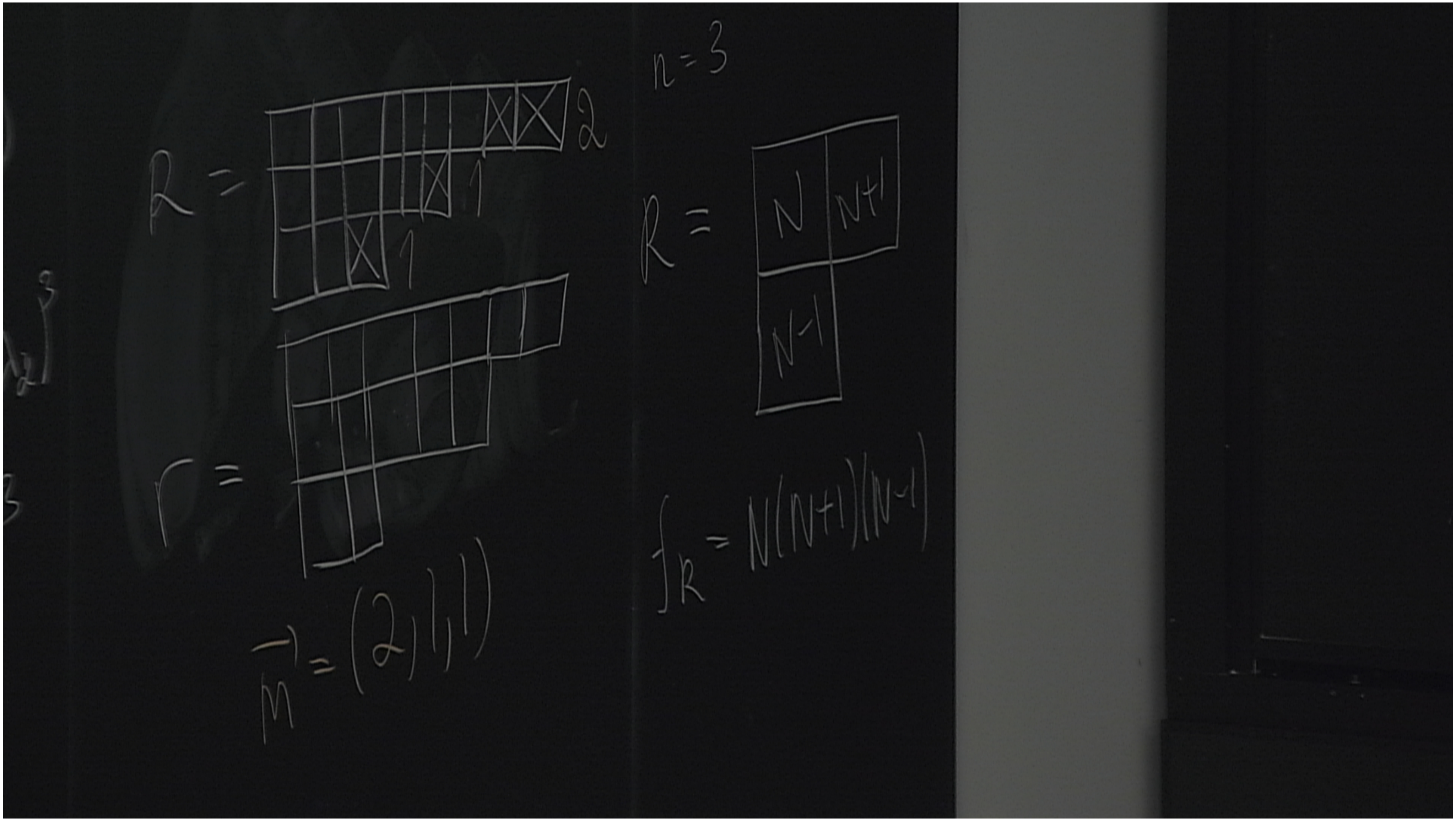
A New Symmetry

$\chi_{R,(r,s)\mu\nu}$



$$\vec{m} = (3, 1, 2)$$

New symmetry leads to a further conservation law - the dilatation operator does not mix operators with different $\vec{m}$.


n = 3
R =
2
1
1
r =
m = (2, 1, 1)
R =
N
N+1
N-1
f_R = N(N+1)(N-1)

D in the Displaced Corners Approximation: Factorization

$$D\chi_{R,(r,s)\mu_1\mu_2} = -g_{YM}^2 \sum_{u\nu_1\nu_2} \sum_{i<j} M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2} \Delta_{ij} \chi_{R,(r,u)\nu_1\nu_2}$$

Δ_{ij} acts only on the Young diagrams R, r and $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ acts only on the labels $s\mu_1\mu_2$.

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Action of Δ_{12}

$$\Delta_{12}\chi(b_0, b_1, b_2) = (2N + 2b_0 + 2b_1 + b_2)\chi(b_0, b_1, b_2)$$
$$-\sqrt{(N + b_0 + b_1)(N + b_0 + b_1 + b_2)}(\chi(b_0, b_1 - 1, b_2 + 2) + \chi(b_0, b_1 + 1, b_2 - 2))$$

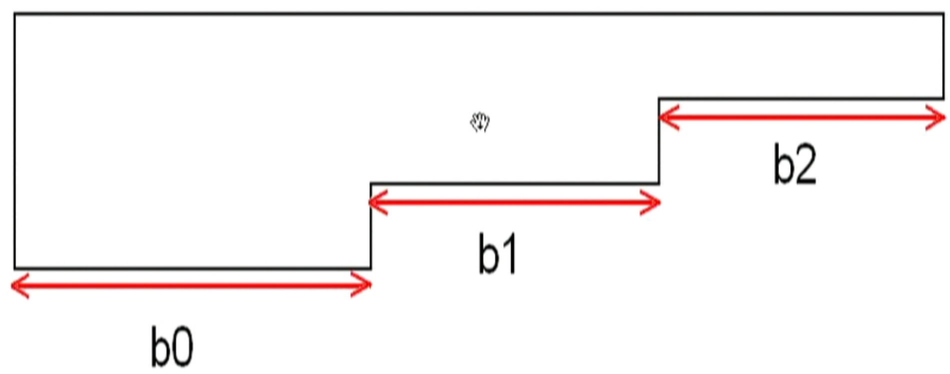


Figure: Example of labeling for a three row Young diagram.

Δ_{ij} eigenproblem

Δ_{ij} is an element of the $u(p)$ Lie algebra.

Basis for fundamental rep of $u(p)$ Lie algebra: $p \times p$ matrices $(E_{kl})_{ab} = \delta_{ak}\delta_{bl}$.

Extract an $su(2)$ subalgebra.

$$Q_{ij} = \frac{E_{ii} - E_{jj}}{2}, \qquad Q_{ij}^{+} = E_{ij}, \qquad Q_{ij}^{-} = E_{ji}\,,$$

$$[Q_{ij}, Q_{ij}^{+}] = Q_{ij}^{+}, \qquad [Q_{ij}, Q_{ij}^{-}] = -Q_{ij}^{-}, \qquad [Q_{ij}^{+}, Q_{ij}^{-}] = 2Q_{ij}\,.$$

Δ_{ij} eigenproblem

A completely standard raising/lowering operator discussion gives

$$Q_{ij}^{+}|\lambda,\Lambda\rangle = c_{+}|\lambda+1,\Lambda\rangle\,, \qquad c_{+} = \sqrt{(\Lambda+\lambda+1)(\Lambda-\lambda)}\,,$$

and

$$Q_{ij}^{-}|\lambda,\Lambda\rangle = c_{-}|\lambda-1,\Lambda\rangle\,, \qquad c_{-} = \sqrt{(\Lambda+\lambda)(\Lambda-\lambda+1)}\,.$$

$$\Delta_{12}\chi(b_0,b_1,b_2) = (2N+2b_0+2b_1+b_2)\chi(b_0,b_1,b_2)$$
$$-\sqrt{(N+b_0+b_1)(N+b_0+b_1+b_2)}(\chi(b_0,b_1-1,b_2+2)+\chi(b_0,b_1+1,b_2-2))$$

Δ_{ij} eigenproblem

$$\Delta_{12}\chi(b_0,b_1,b_2) = (2N+2b_0+2b_1+b_2)\chi(b_0,b_1,b_2)$$
$$-\sqrt{(N+b_0+b_1)(N+b_0+b_1+b_2)}(\chi(b_0,b_1-1,b_2+2)+\chi(b_0,b_1+1,b_2-2))$$

$$Q^{\pm}_{ij}|\lambda,\Lambda\rangle = c_{\pm}|\lambda+1,\Lambda\rangle\,, \qquad c_{+} = \sqrt{(\Lambda+\lambda+1)(\Lambda-\lambda)}\,,$$
$$c_{-} = \sqrt{(\Lambda+\lambda)(\Lambda-\lambda+1)}\,.$$

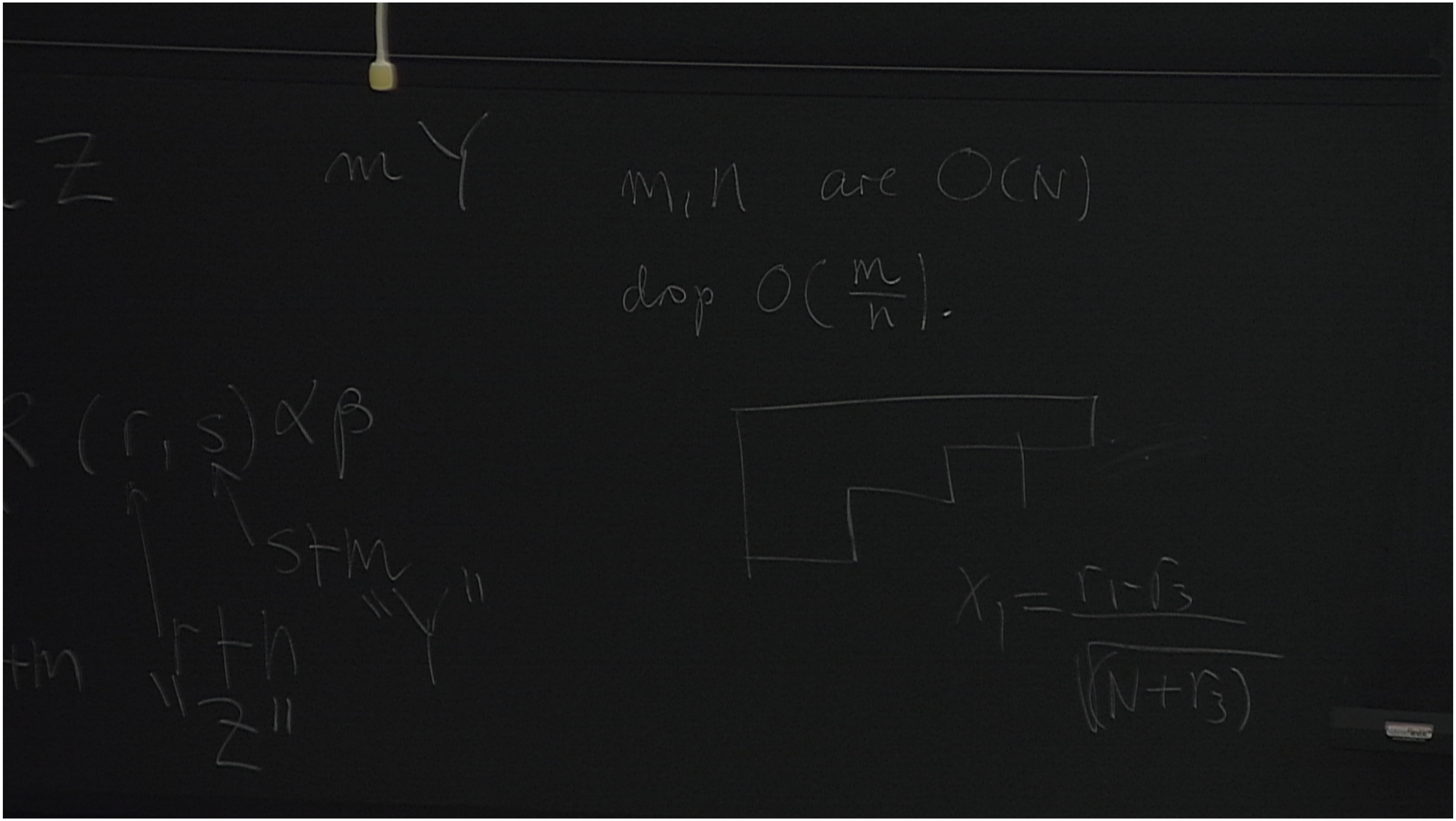
$$\Delta_{ij} = -\frac{1}{2}(E_{ii}+E_{jj}) + Q^{-}_{ij} + Q^{+}_{ij}\,.$$
$$c_{-} = \sqrt{(N+b_0+b_1)(N+b_0+b_1+b_2+1)},$$
$$c_{+} = \sqrt{(N+b_0+b_1+1)(N+b_0+b_1+b_2)}$$
$$\Lambda = 2N+r_i+r_j, \qquad \lambda = \frac{1}{2}b_i\,.$$

Δ_{ij} eigenproblem at large N

$$\Delta_{ij} \to \left(\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_j} \right)^2 - \frac{(x_i - x_j)^2}{4}$$

$$x_i = \frac{r_i - r_p}{\sqrt{N + r_p}}$$

(dMK, Kemp, Smith, arXiv:1111.1058)


m Y
m, n are O(N)
drop O(m/n).
R (r, s) α β
s+m
r+n
"Y"
"Z"
x1 = r1-r3 / √(N+r3)

D in the Displaced Corners Approximation: Factorization

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Δ_{ij} acts only on the Young diagrams R, r and $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ acts only on the labels $s\mu_1\mu_2$.

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Y Eigenproblem: $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$

Example: (from Young diagrams with 4 rows and 8 boxes removed; $\vec{m} = (3, 2, 2, 1)$)

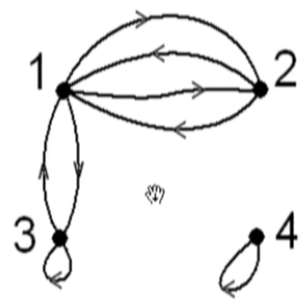


Figure: Example of a pictorial labeling.

$$DO(b_0, b_1, b_2, b_3) = -g^2_{YM}(4\Delta_{12} + 2\Delta_{13})O(b_0, b_1, b_2, b_3)$$

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

D in the Displaced Corners Approximation: Factorization

$$D\chi_{R,(r,s)\mu_1\mu_2} = -g_{YM}^2 \sum_{u\nu_1\nu_2} \sum_{i<j} M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2} \Delta_{ij} \chi_{R,(r,u)\nu_1\nu_2}$$

Δ_{ij} acts only on the Young diagrams R, r and $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ acts only on the labels $s\mu_1\mu_2$.

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Enumerate graphs with double coset

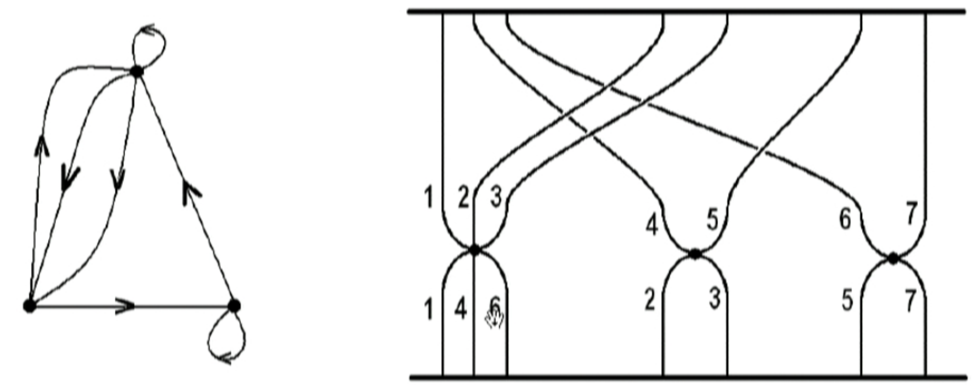
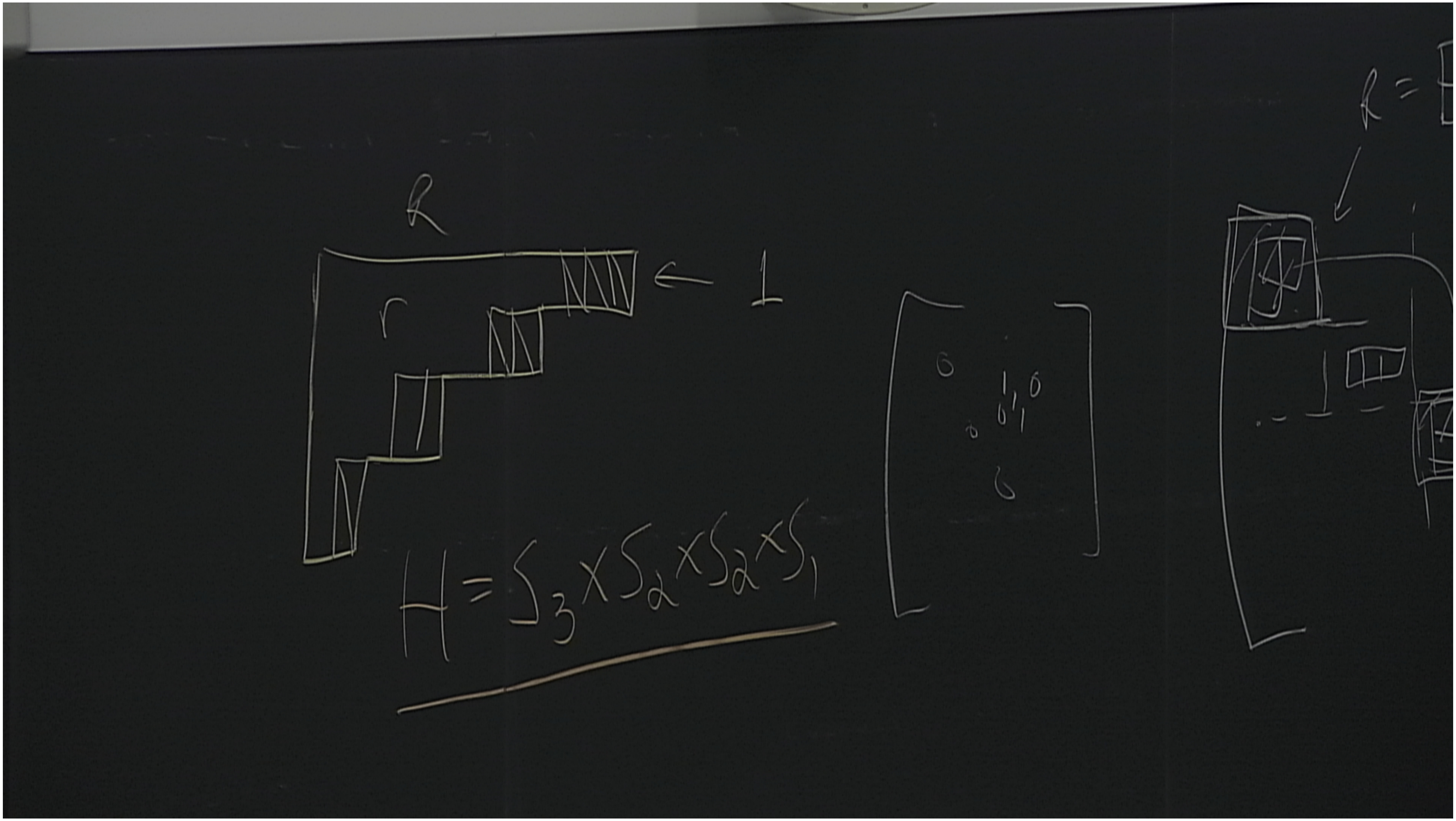


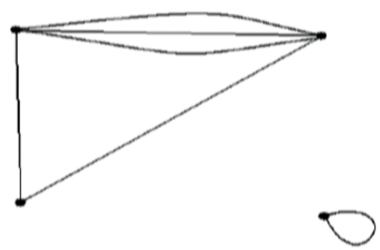
Figure: The graph determines an element of $H \setminus S_{m_1+m_2+m_3}/H$ where $H = S_{m_1} \times S_{m_2} \times S_{m_3}$. $\sigma = (1)(24)(356)(7)$

Cardinality of the double coset
$= \frac{1}{|H|^2} \sum_{\alpha_1,\alpha_2 \in H} \sum_{\sigma \in S_n} \delta(\alpha_2 \sigma^{-1} \alpha_1 \sigma) = \sum_{s \vdash m} (M^s_{1_H})^2.$

(dMK, Ramgoolam, arXiv:1204.2153) based on earlier work dMK, Ramgoolam arXiv:1110.4858


R
r
1
H = S3 x S2 x S2 x S1
R =

Open Spring Theory



$$\Delta = \Delta_0 + g_{YM}^2 \sum_i n_i \omega_i = \Delta_0 + \lambda \sum_i \frac{n_i}{N} \omega_i$$

Continuous spectrum at large N!

(dMK, Kemp, Smith, arXiv:1111.1058)

Gauss Law

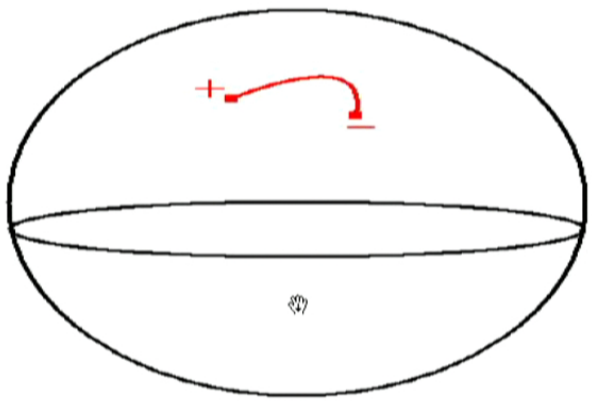


Figure: The Gauss Law forbids a net charge on the giant's worldvolume

$$Q = \int_{WV} d^3x\rho = \int_{WV} d^3x\vec{\nabla}\cdot\vec{E} = \int_{\partial WV} \vec{E}\cdot d\vec{A} = 0$$

(Berenstein, Balasubramanian, Feng, Huang, hep-th/0411205)

Gauss Law

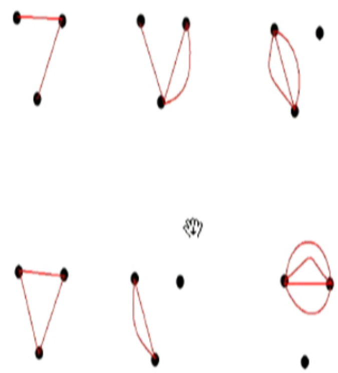


Figure: Forbidden and allowed configurations

Gauss Law

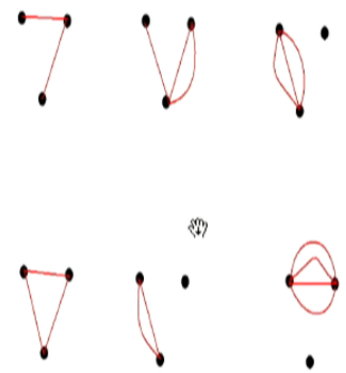


Figure: Forbidden and allowed configurations

Summary

Families of operators with a definite scaling dimension are labelled by a permutation - visible connection to giant gravitons with open strings attached.

The action of the dilatation operator on each family reduces to a set of decoupled harmonic oscillators.

$$O_{\vec{n}}(\sigma) = \sum_{s,\mu,\nu}\sum_{ij}\sqrt{d_s}\sum_{r}\Gamma^{s}_{ij}(\sigma)B_{j\mu}B_{i\nu}\psi_{HO,\vec{n}}(r)\chi_{R,(r,s)\mu\nu}(Z,Y)$$

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!}\sum_{\psi\in S_{n+m}}\mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^{R}(\psi))\mathrm{Tr}(\psi Z^{\otimes n}\otimes Y^{\otimes m})$$

$$R\vdash m+n \quad r\vdash n \quad s\vdash m \quad \sigma\in H\setminus S_m/H$$

The action of the dilatation operator is tightly constrained by symmetry.