Title: Twist Defects in Topological Systems with Anyonic Symmetries

Date: Nov 08, 2013 11:15 AM

URL: http://pirsa.org/13110078

Abstract: Twist defects are point-like objects that support robust non-local storage of quantum information and non-abelian unitary operations. Unlike quantum deconfined anyonic excitations, they rely on symmetry rather than a non-abelian topological order. Zero energy Majorana bound states can arise at lattice defects, such as disclinations and dislocations, in a topological crystalline superconductor. More general parafermion bound state can appear as twist defects in a topological phase with an anyonic symmetry, such as a bilayer fractional quantum Hall state and the Kitaev toric code. They are however fundamentally different from quantum anyonic excitations in a true topological phase. This is demonstrated by their unconventional exchange and braiding behavior, which is characterized by a modified spin statistics theorem and modular invariance.

Outline

- Motivation: Ising quasiparticles
  - What are they? Why desirable? How to realize?
- Twist Defects in Topological Phases
  [**JT**, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)]

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  - Review on abelian topological phases

    K-matrix,   modular *S*,*T* transformation

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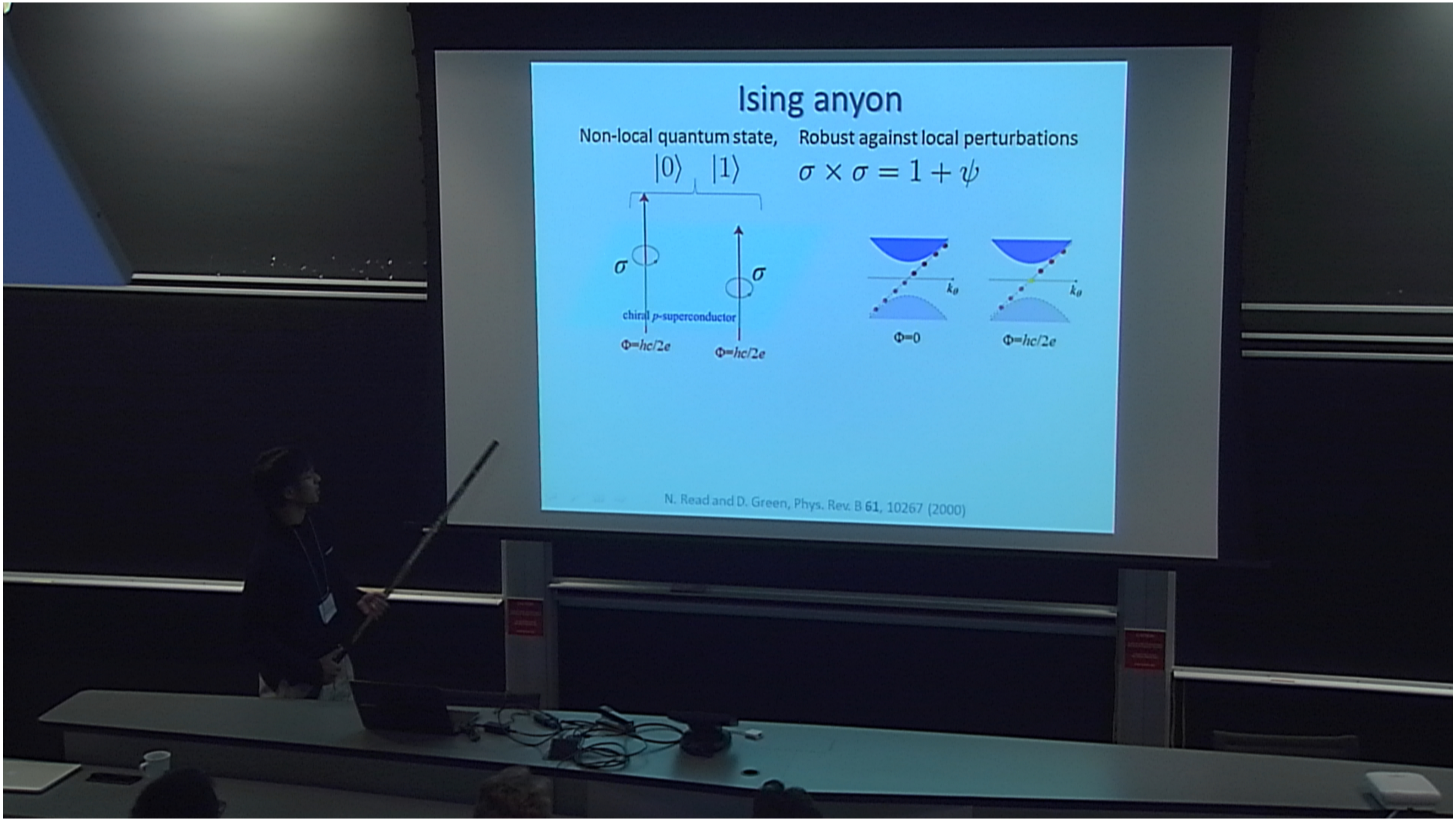
  - Review on abelian topological phases
    K-matrix, modular *S*,*T* transformation
  - Anyonic symmetry
    anyon relabeling, e.g. toric code, bilayer FQH, *A*-*D*-*E* Lie algebra
  - Topological twist defect (theoretical examples)
  - Multi-channel fusion (i.e. non-abelian)
  - Exchange and double twist
    modified spin-statistics theorem



Ising anyon
Non-local quantum state,
Robust against local perturbations
$|0\rangle$ $|1\rangle$
$\sigma \times \sigma = 1 + \psi$
σ
σ
chiral p-superconductor
Φ=hc/2e
Φ=hc/2e
Φ=0
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N. Read and D. Green, Phys. Rev. B 61, 10267 (2000)

Ising anyon

Non-local quantum state, $|0\rangle$ $|1\rangle$

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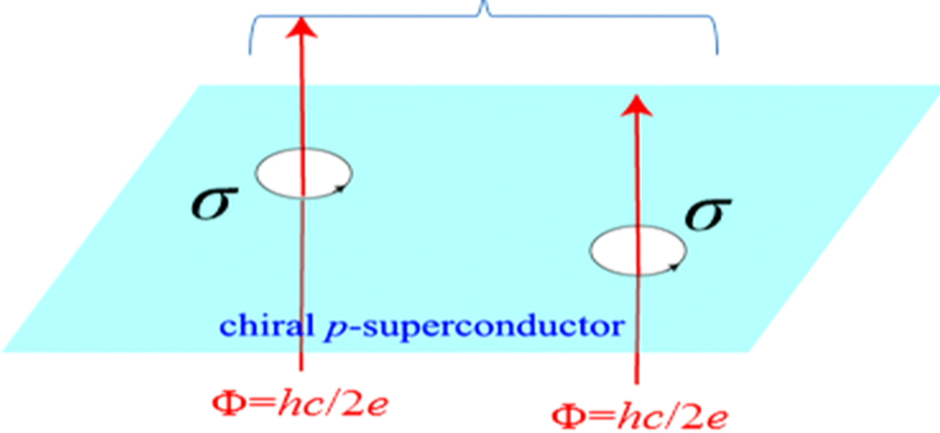

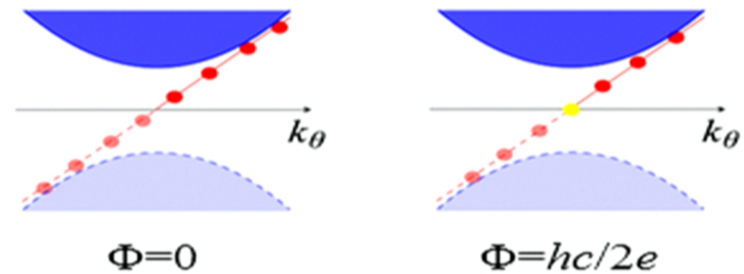

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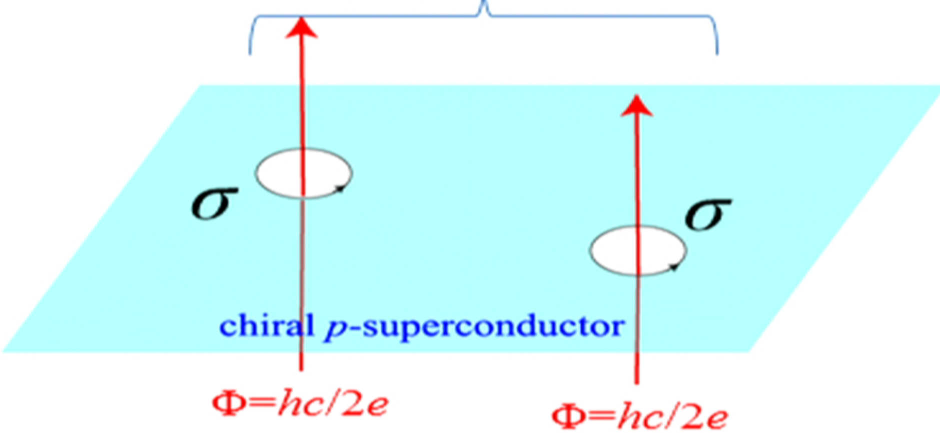
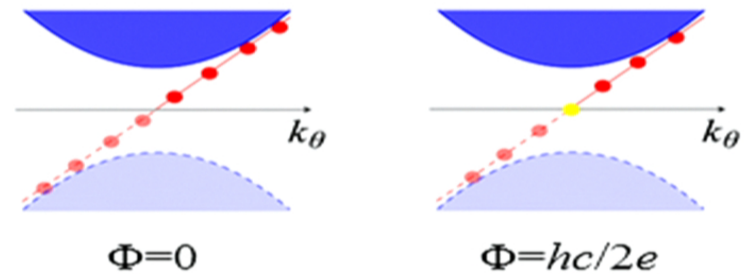
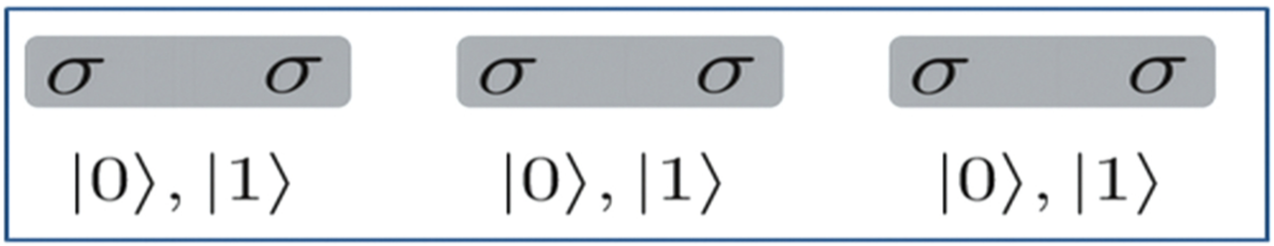
Ising anyon

Non-local quantum state, Robust against local perturbations

$$|0\rangle \quad |1\rangle \qquad \sigma \times \sigma = 1 + \psi$$

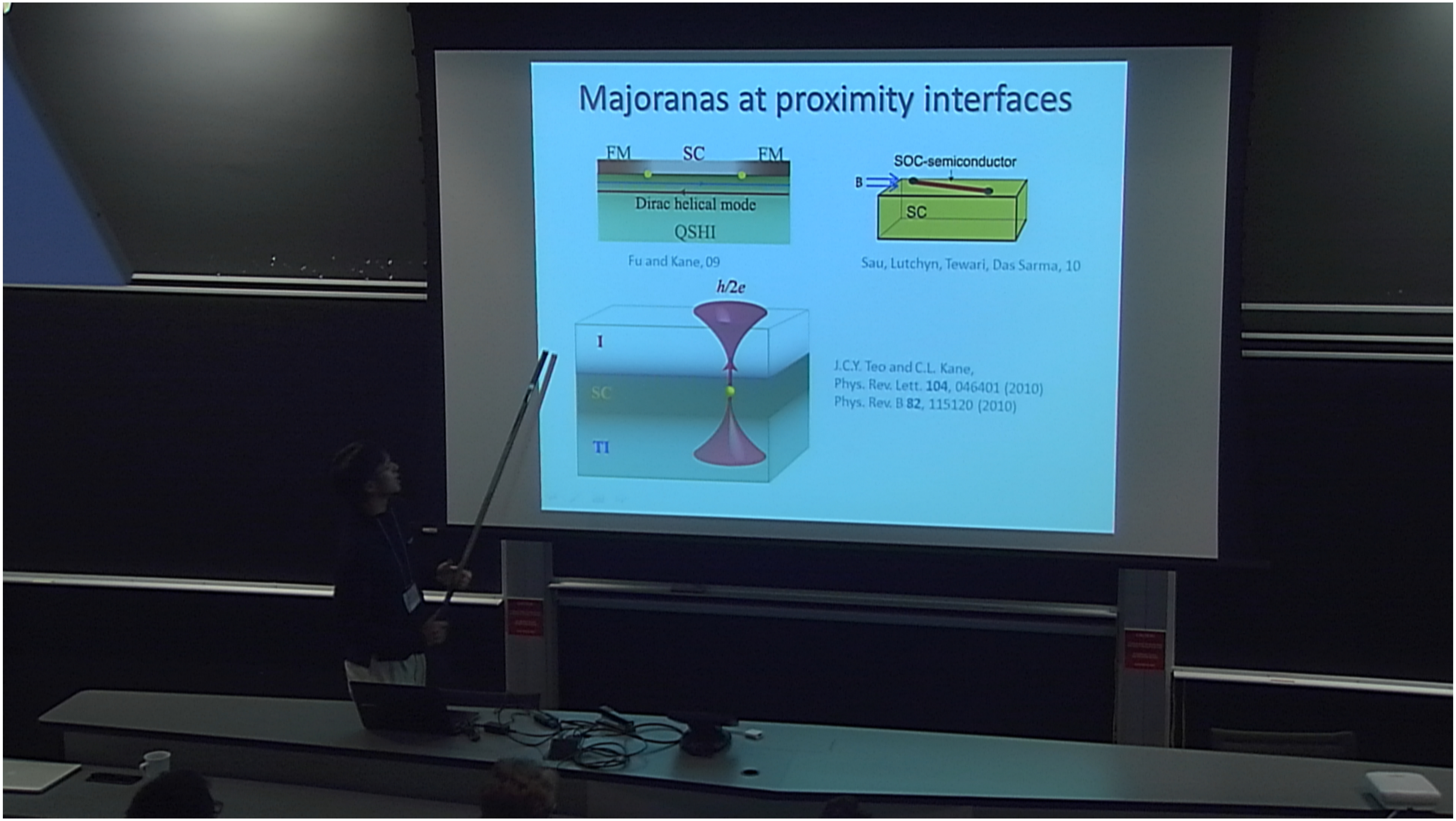






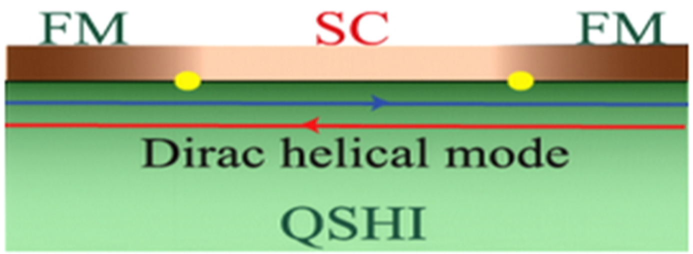
$$|000\rangle, |110\rangle, |101\rangle, \ldots$$

N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000)

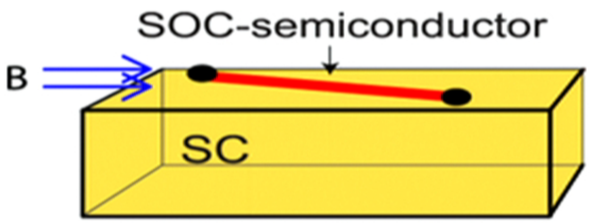


Majoranas at proximity interfaces
FM
SC
FM
Dirac helical mode
QSHI
Fu and Kane, 09
SOC-semiconductor
B
SC
Sau, Lutchyn, Tewari, Das Sarma, 10
h/2e
I
SC
TI
J.C.Y. Teo and C.L. Kane,
Phys. Rev. Lett. 104, 046401 (2010)
Phys. Rev. B 82, 115120 (2010)

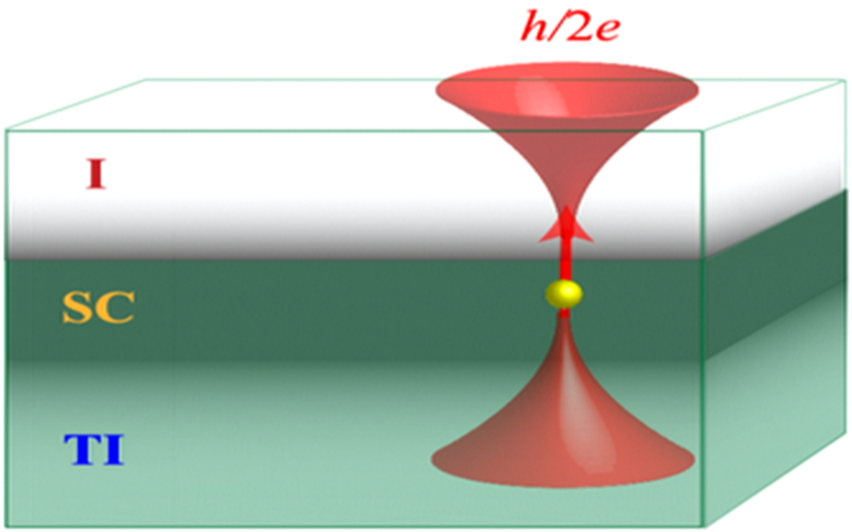
Majoranas at proximity interfaces



Fu and Kane, 09



Sau, Lutchyn, Tewari, Das Sarma, 10



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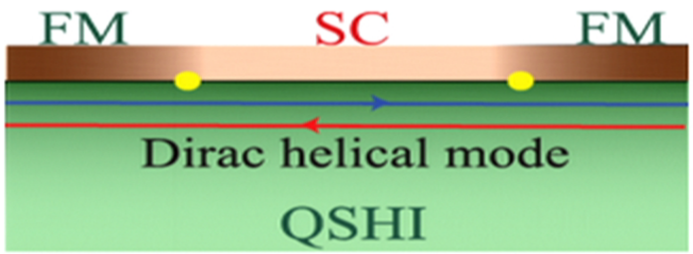

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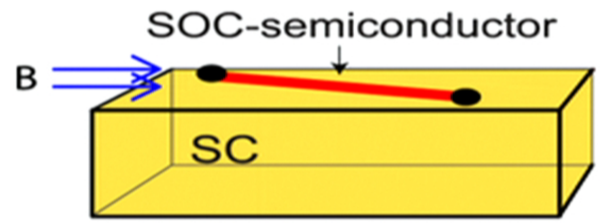

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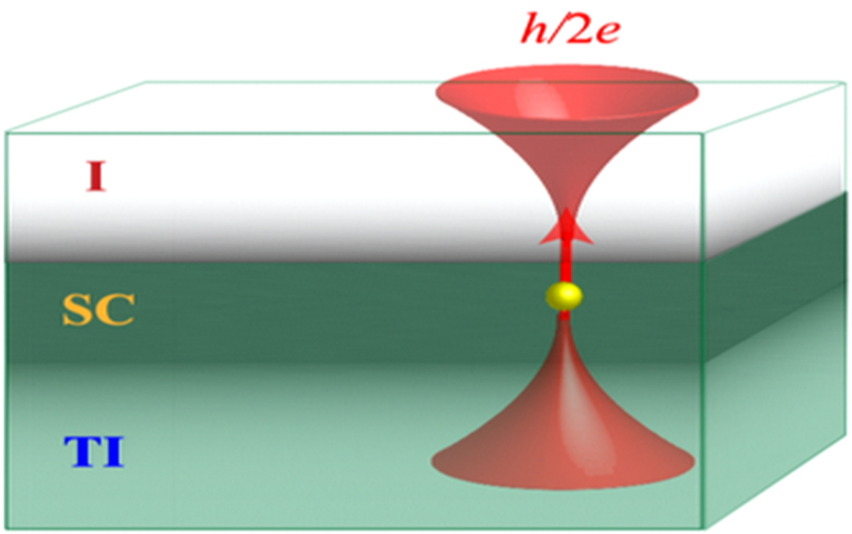

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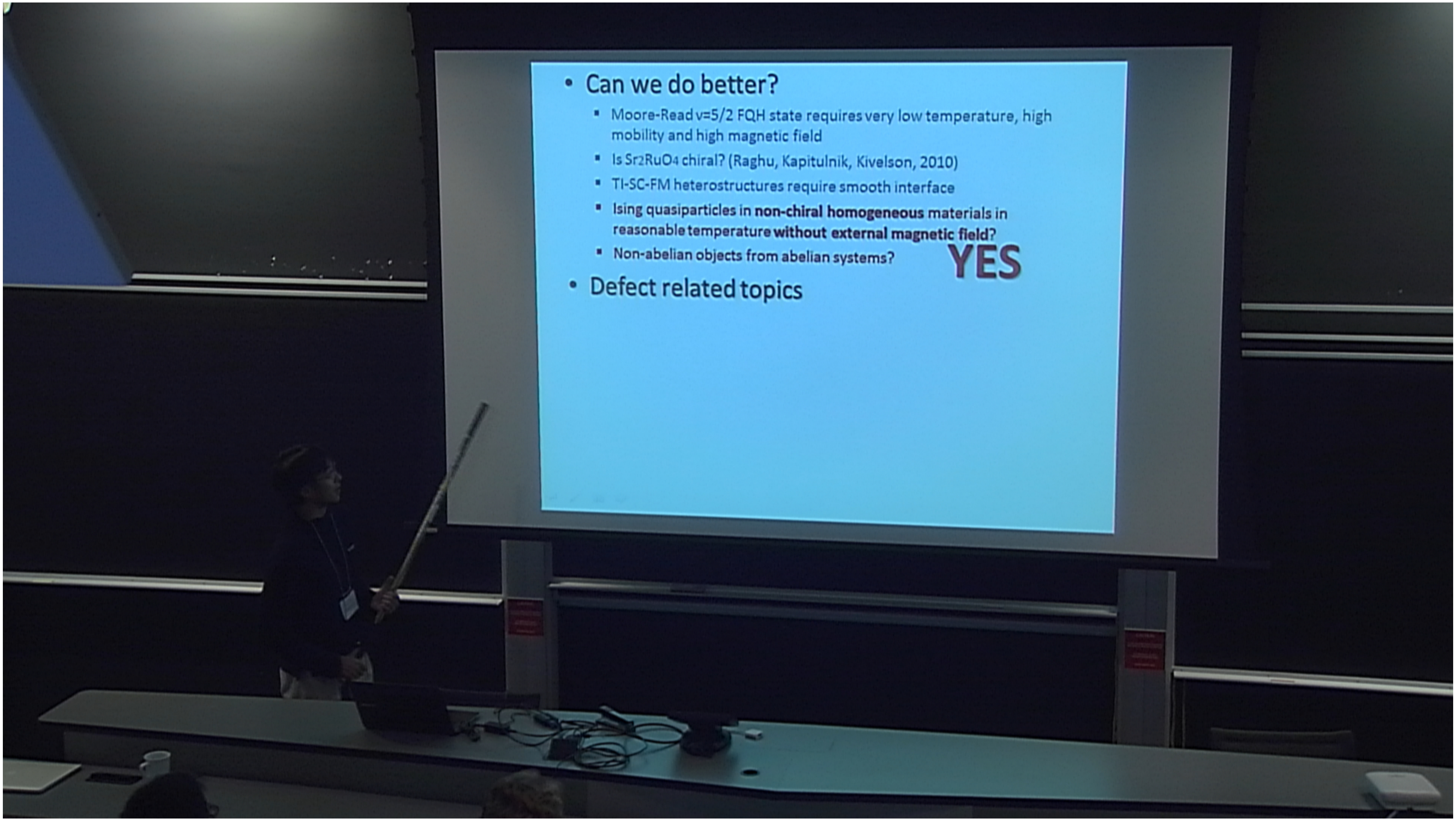
Phys. Rev. Lett. **104**, 046401 (2010)
Phys. Rev. B **82**, 115120 (2010)

- Can we do better?
  - Moore-Read $\nu=5/2$ FQH state requires very low temperature, high mobility and high magnetic field
  - Is Sr_2RuO_4 chiral? (Raghu, Kapitulnik, Kivelson, 2010)
  - TI-SC-FM heterostructures require smooth interface
  - Ising quasiparticles in **non-chiral homogeneous** materials in reasonable temperature **without external magnetic field**?
  - Non-abelian objects from abelian systems?

YES

- Defect related topics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YES
Defect related topics

TWIST DEFECTS IN TOPOLOGICAL PHASES

JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

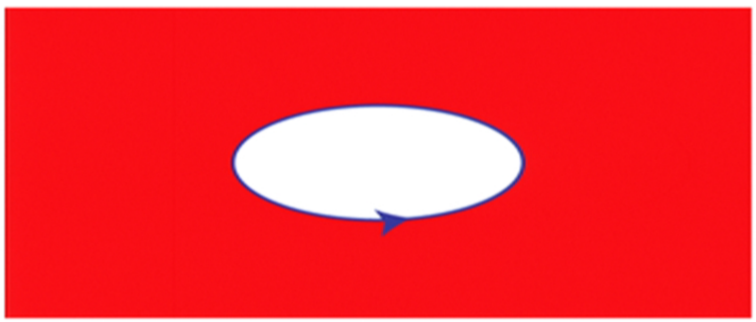
(Reivew) Abelian Topological States

- (2+1)d Chern-Simons theory (Wen and Zee, 92)

$$\mathcal{L} = \frac{1}{4\pi} K_{IJ} a_I \wedge da_J \qquad K = (K_{IJ})_{N \times N}$$

- Gapless edge

$$\mathcal{L}_{edge} = \frac{1}{4\pi} K_{IJ} \partial_x \phi_I \partial_t \phi_J$$



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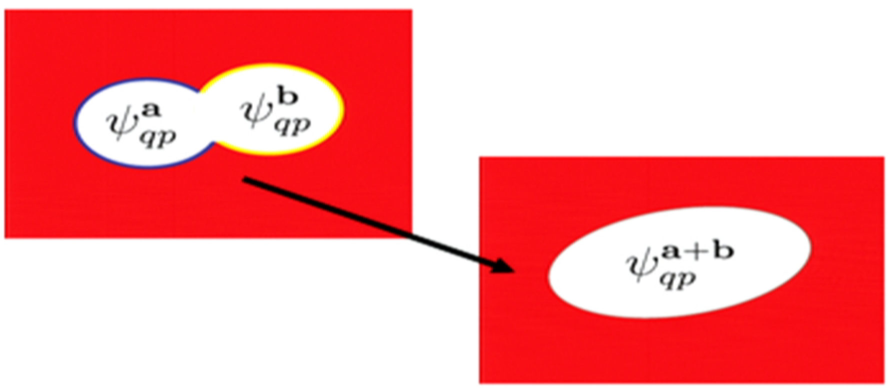


Quasiparticle operator $\psi_{qp}^{\mathbf{a}} = e^{i\mathbf{a}\cdot\vec{\phi}}$

$$\vec{\phi} = (\phi_1, \ldots, \phi_N)$$

Anyon lattice $\mathbf{a} \in \Gamma^* = \mathbb{Z}^N$

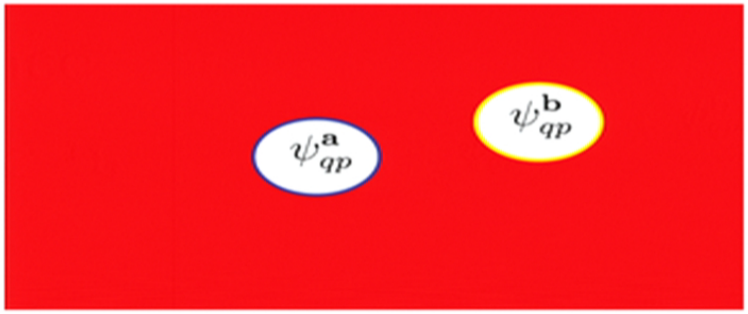
- Fusion $\psi_{qp}^{\mathbf{a}} \times \psi_{qp}^{\mathbf{b}} = \psi_{qp}^{\mathbf{a+b}}$

(Reivew) Abelian Topological States

- Bulk boundary correspondence

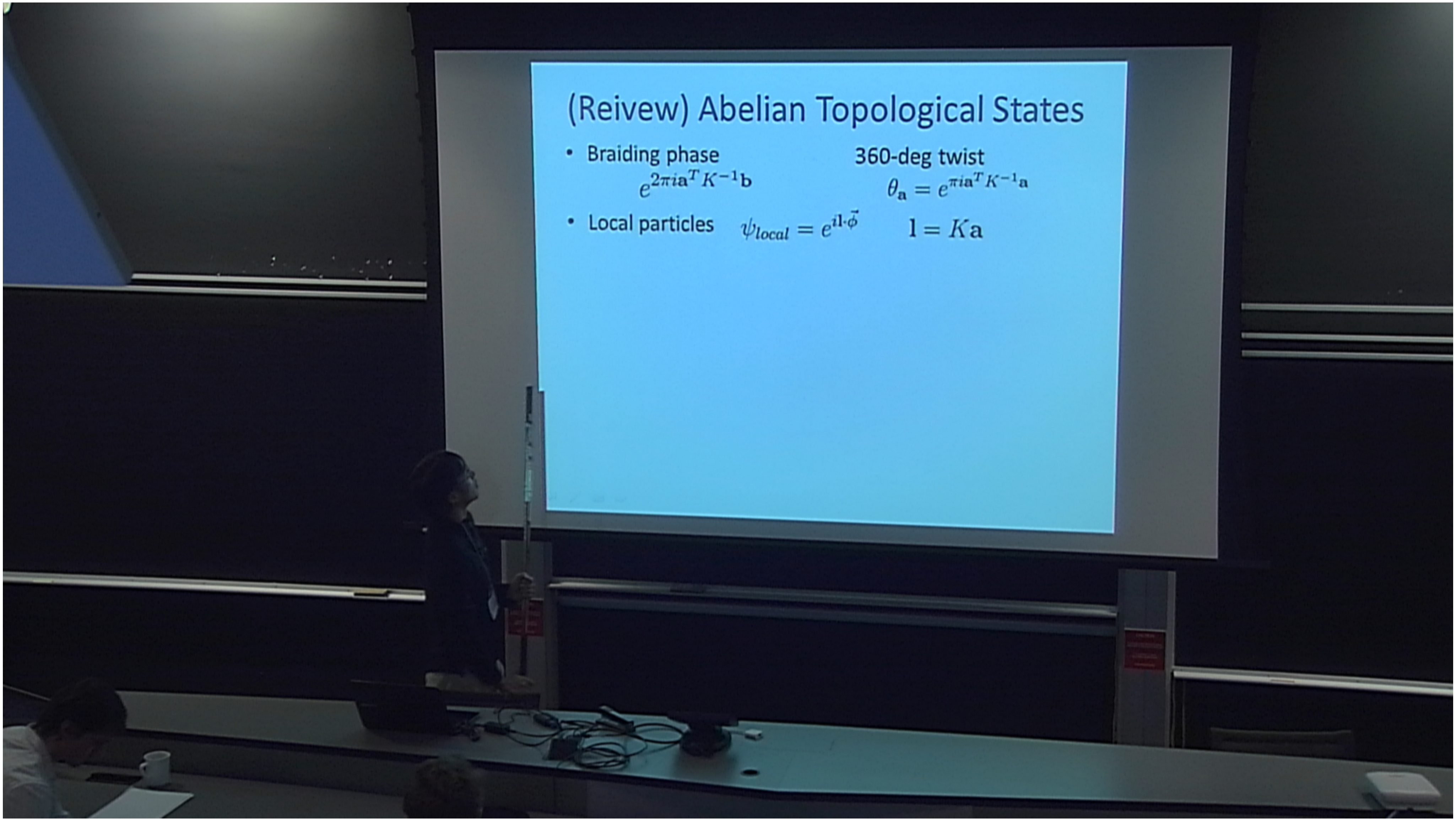
$$\langle \psi_{qp}^{\mathbf{a}}(z)\psi_{qp}^{\mathbf{b}}(w)\rangle = (z-w)^{\mathbf{a}^T K^{-1}\mathbf{b}}$$

$$z \sim x + iy$$



(Reivew) Abelian Topological States

- Braiding phase $e^{2\pi i \mathbf{a}^T K^{-1} \mathbf{b}}$ 360-deg twist $\theta_{\mathbf{a}} = e^{\pi i \mathbf{a}^T K^{-1} \mathbf{a}}$
- Local particles $\psi_{local} = e^{i \mathbf{l} \cdot \vec{\phi}}$ $\mathbf{l} = K \mathbf{a}$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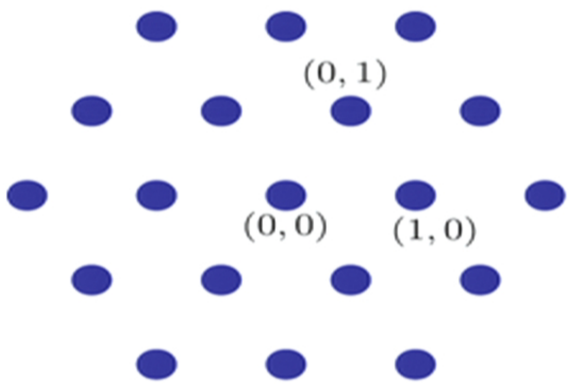
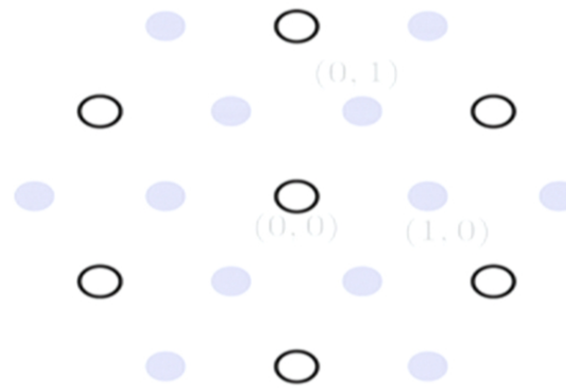
$$K = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^T K^{-1} \mathbf{b}$$



$$\Gamma^* = \mathbb{Z}^2$$



$$\Gamma = K\mathbb{Z}^2$$

Bose condensed

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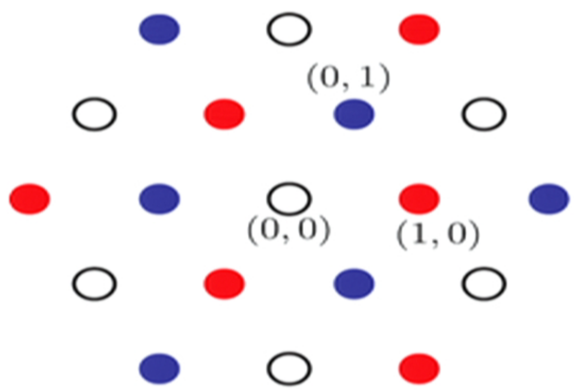

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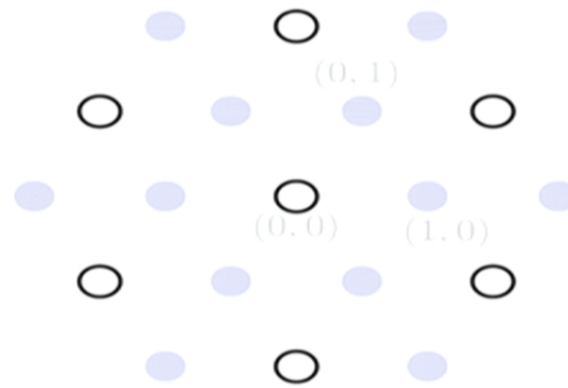
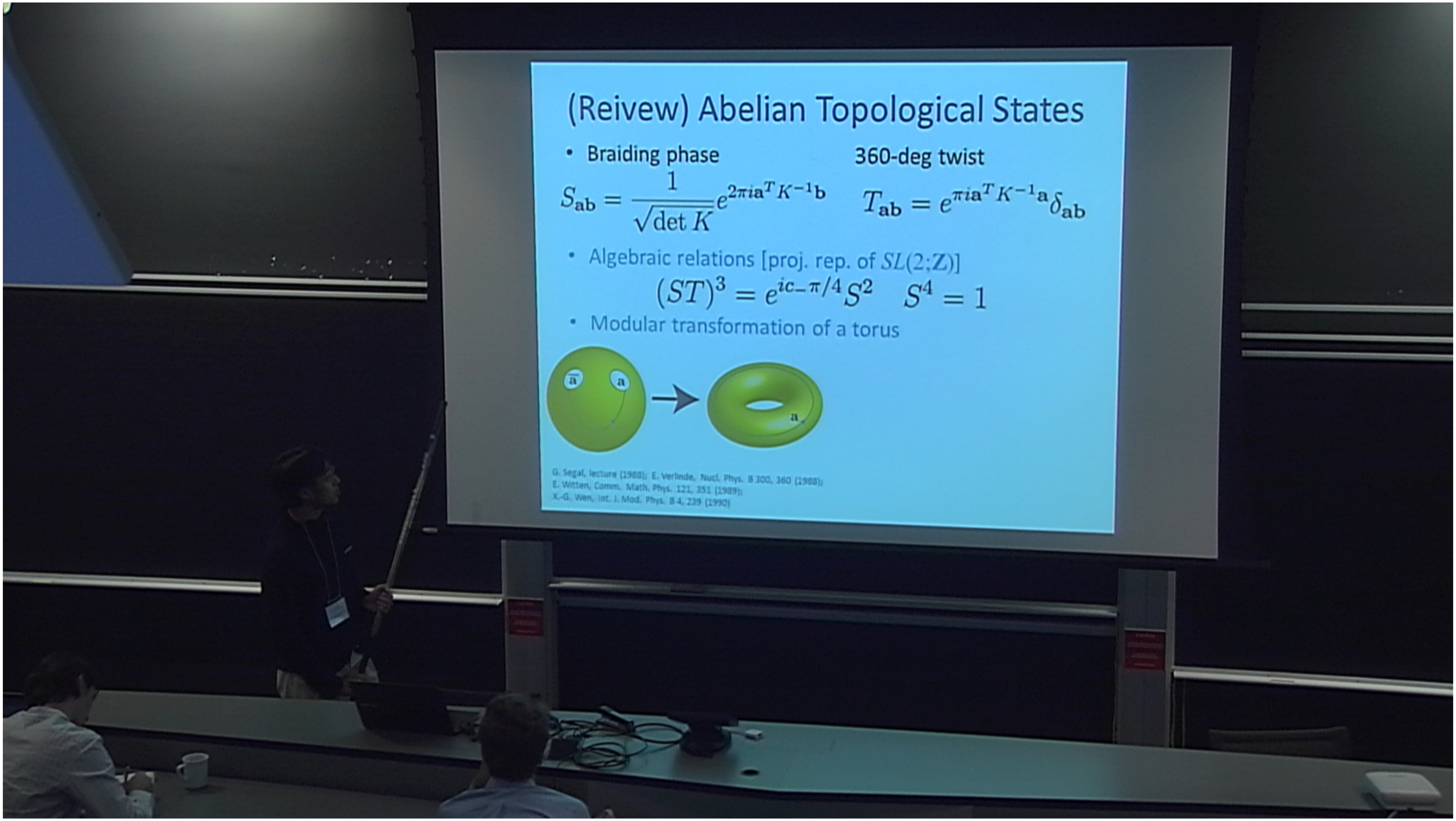


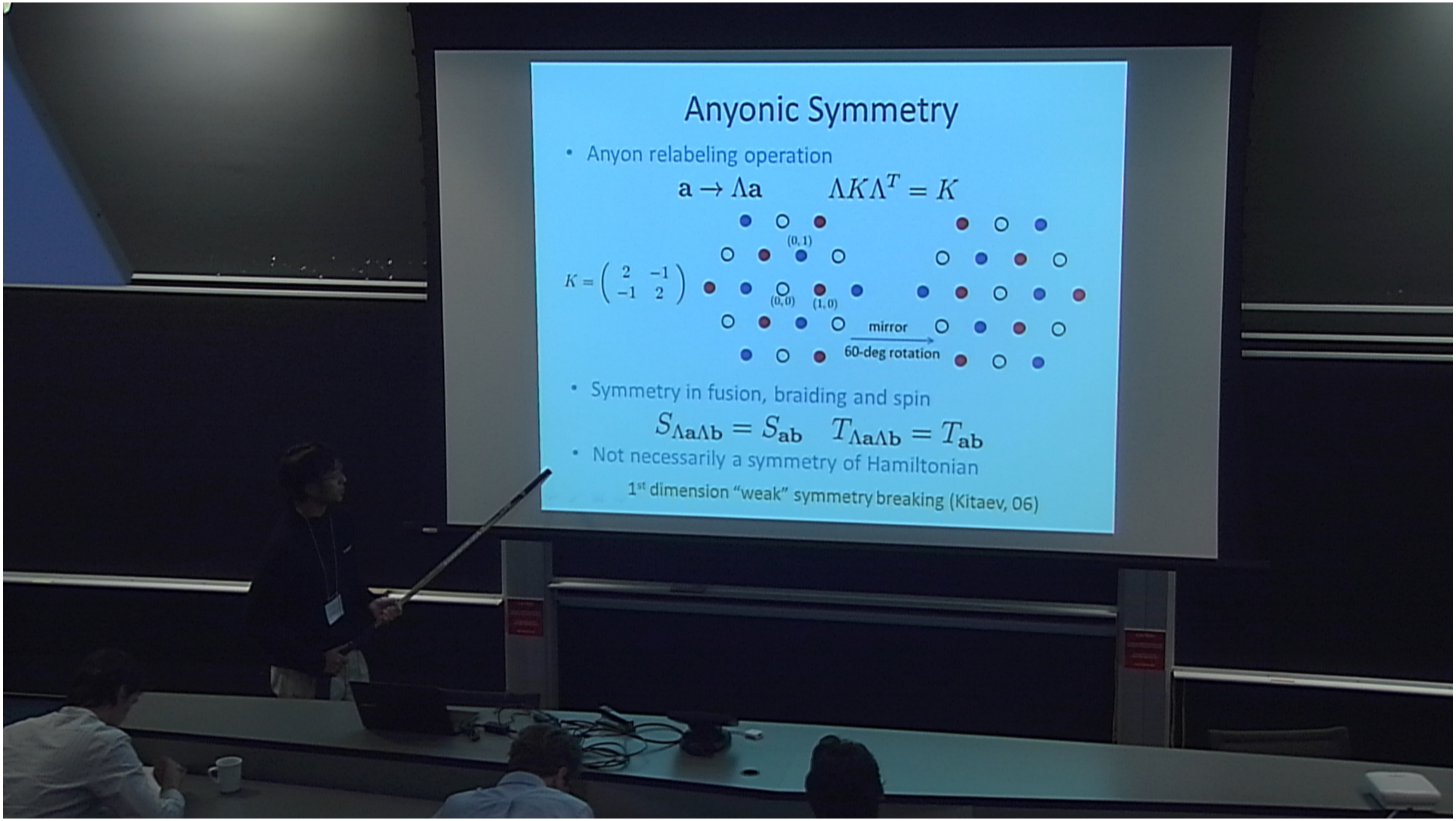
$$\Gamma = K\mathbb{Z}^2$$

Bose condensed

- Anyon content

$$\boxed{\mathcal{A} = \Gamma^*/\Gamma}$$


(Reivew) Abelian Topological States
• Braiding phase
360-deg twist
$S_{\mathbf{ab}} = \frac{1}{\sqrt{\det K}} e^{2\pi i \mathbf{a}^T K^{-1} \mathbf{b}}$
$T_{\mathbf{ab}} = e^{\pi i \mathbf{a}^T K^{-1} \mathbf{a}} \delta_{\mathbf{ab}}$
• Algebraic relations [proj. rep. of $SL(2;Z)$]
$(ST)^3 = e^{ic_{-}\pi/4} S^2 \quad S^4 = 1$
• Modular transformation of a torus
G. Segal, lecture (1988); E. Verlinde, Nucl. Phys. B 300, 360 (1988);
E. Witten, Comm. Math. Phys. 121, 351 (1989);
X.-G. Wen, Int. J. Mod. Phys. B 4, 239 (1990)

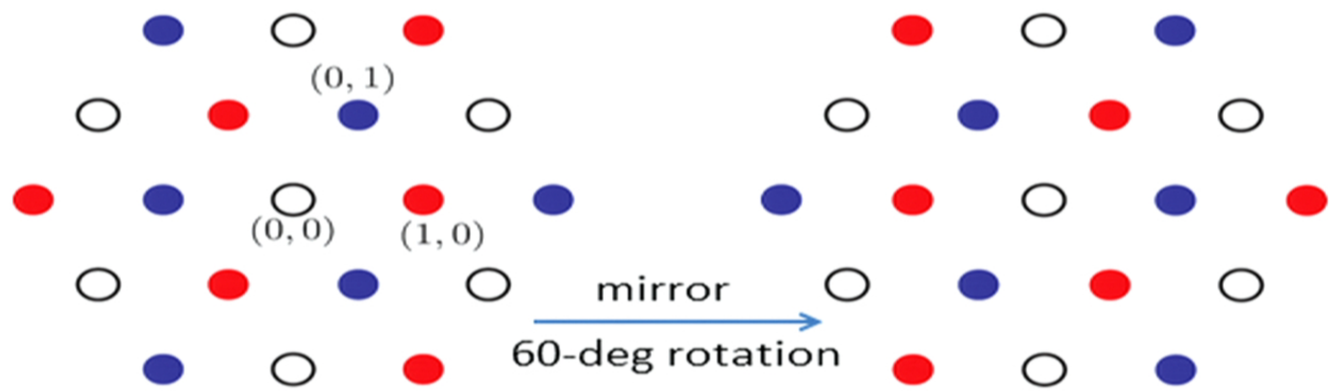

Anyonic Symmetry
Anyon relabeling operation
$\mathbf{a} \to \Lambda \mathbf{a}$ $\Lambda K \Lambda^T = K$
$K = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$
(0, 1)
(0, 0)
(1, 0)
mirror
60-deg rotation
Symmetry in fusion, braiding and spin
$S_{\Lambda\mathbf{a}\Lambda\mathbf{b}} = S_{\mathbf{ab}}$ $T_{\Lambda\mathbf{a}\Lambda\mathbf{b}} = T_{\mathbf{ab}}$
Not necessarily a symmetry of Hamiltonian
1st dimension "weak" symmetry breaking (Kitaev, 06)

Anyonic Symmetry

- Anyon relabeling operation

$$\mathbf{a} \rightarrow \Lambda \mathbf{a} \qquad \Lambda K \Lambda^T = K$$

$$K = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$



- Symmetry in fusion, braiding and spin

$$S_{\Lambda \mathbf{a} \Lambda \mathbf{b}} = S_{\mathbf{ab}} \quad T_{\Lambda \mathbf{a} \Lambda \mathbf{b}} = T_{\mathbf{ab}}$$

- Not necessarily a symmetry of Hamiltonian

1st dimension "weak" symmetry breaking (Kitaev, 06)

Anyonic Symmetry (Examples)

- Anyonic symmetry $\Lambda K \Lambda^T = K$
- Kitaev toric code ($\mathbf{Z}_2$ gauge theory) electric-magnetic duality

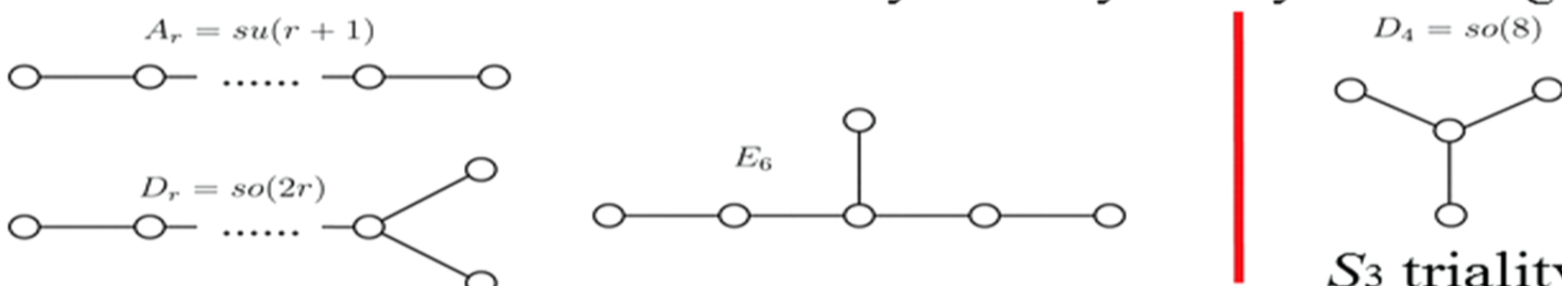
$$K = 2\sigma_x \quad \Lambda = \sigma_x \quad e \leftrightarrow m$$

- Bilayer fractional quantum Hall (mmn)-state

$$K = \begin{pmatrix} m & n \\ n & m \end{pmatrix} \quad \Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- *A-D-E* level 1 Lie algebra state outer automorphisms

$K =$ Cartan matrix $\Lambda =$ symmetry of Dynkin diagrams



Mirror symmetry

S_3 triality symmetry

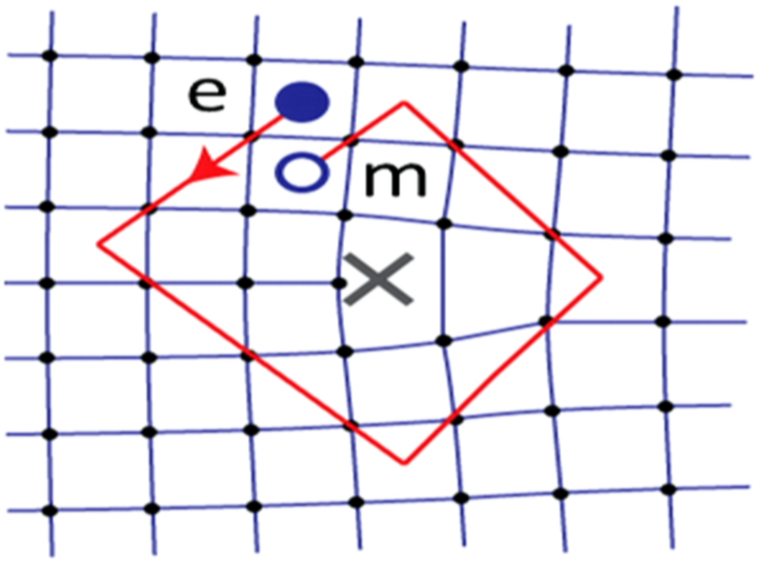
Non-abelian Defects in abelian systems

- Semiclassical topological point defect

JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Non-abelian Defects in abelian systems

- "Dislocations" in Kitaev toric code



H. Bombin, PRL 105, 030403 (2010)
A. Kitaev and L. Kong,
Comm. Math. Phys. 313, 351 (2012)
You and Wen, PRB 86, 161107(R) (2012)

Non-abelian Defects in abelian systems

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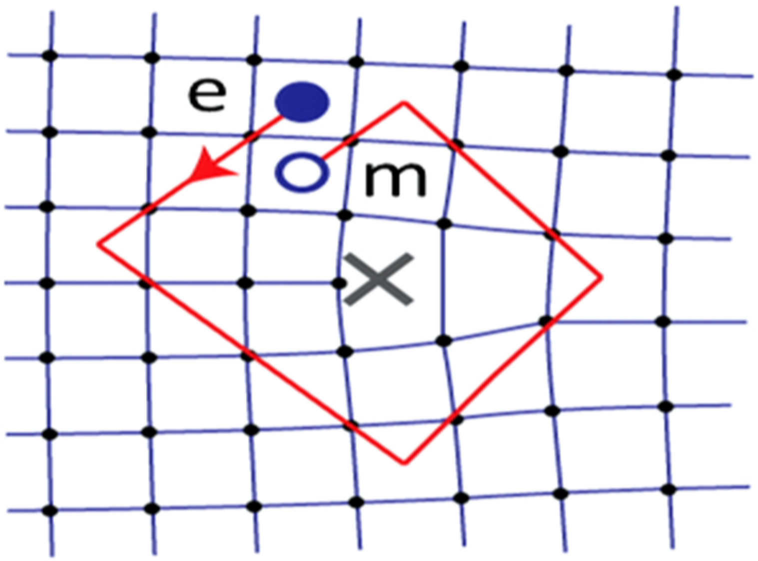

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A. Kitaev and L. Kong, Comm. Math. Phys. 313, 351 (2012)
You and Wen, PRB 86, 161107(R) (2012)

- Majorana zero mode at QSHI-AFM-SC

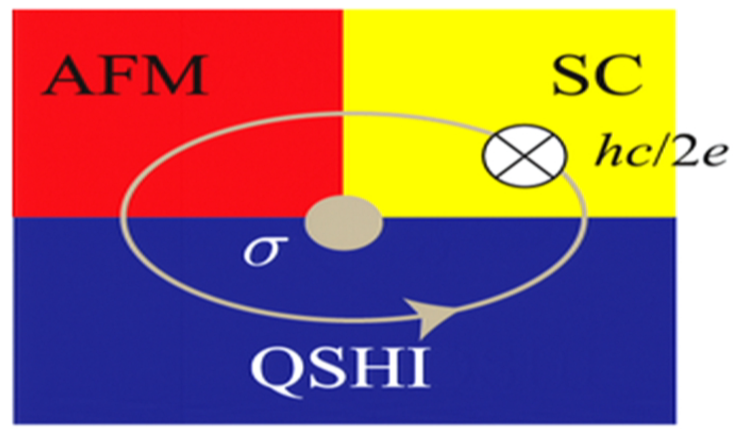
Quasi-2D s-superconductor

m = flux vortex

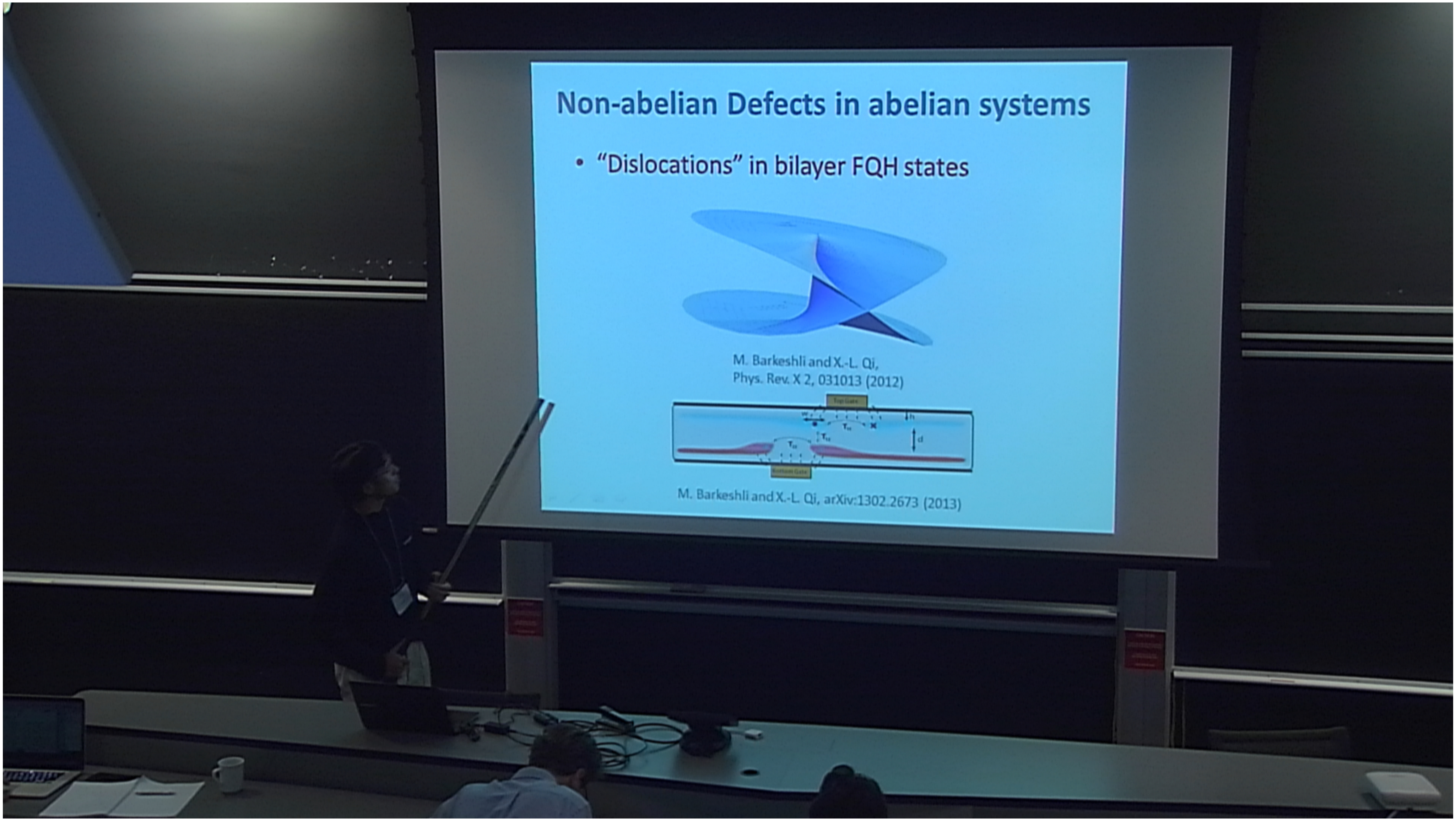
ψ = fermion

$e = m \times \psi$

Fermion parity pump



L. Fu and C.L. Kane, 09
JT and C.L. Kane, 10


Non-abelian Defects in abelian systems
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M. Barkeshli and X.-L. Qi,
Phys. Rev. X 2, 031013 (2012)
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Non-abelian Defects in abelian systems

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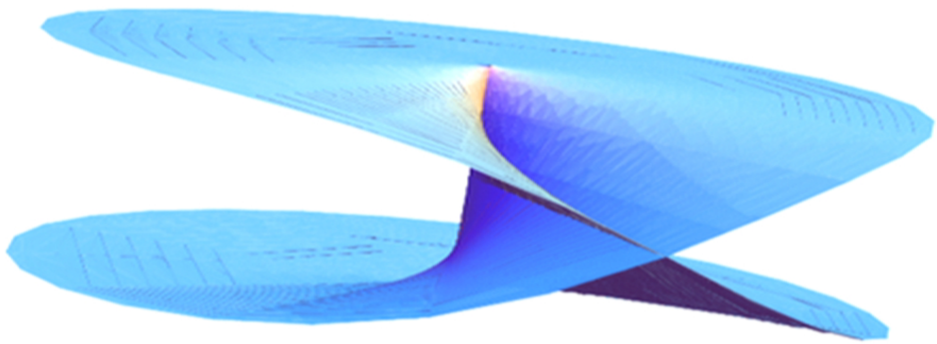

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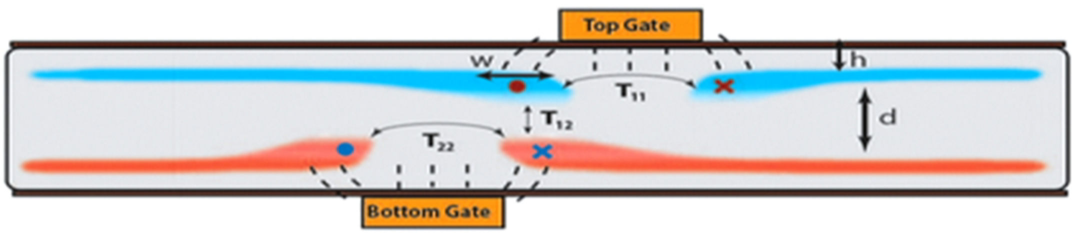
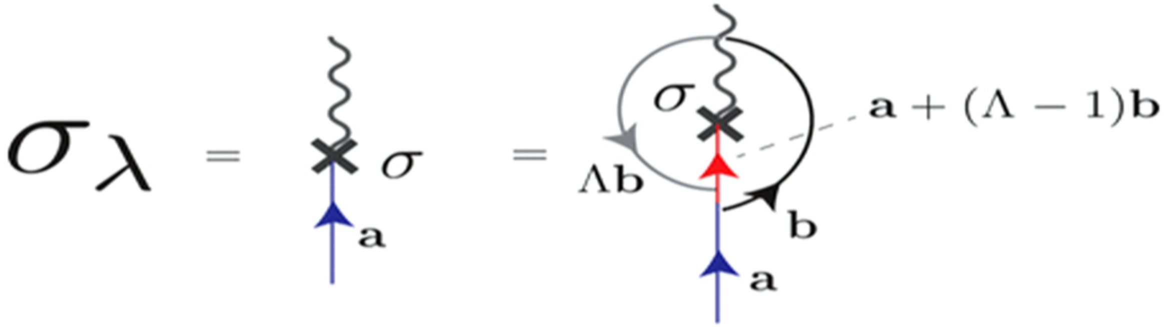


M. Barkeshli and X.-L. Qi, arXiv:1302.2673 (2013)

Defect – anyon composite

- Defect species

$$\sigma_\lambda = \sigma \times \mathbf{a} = \sigma \times (\mathbf{a} + (\Lambda - 1)\mathbf{b})$$



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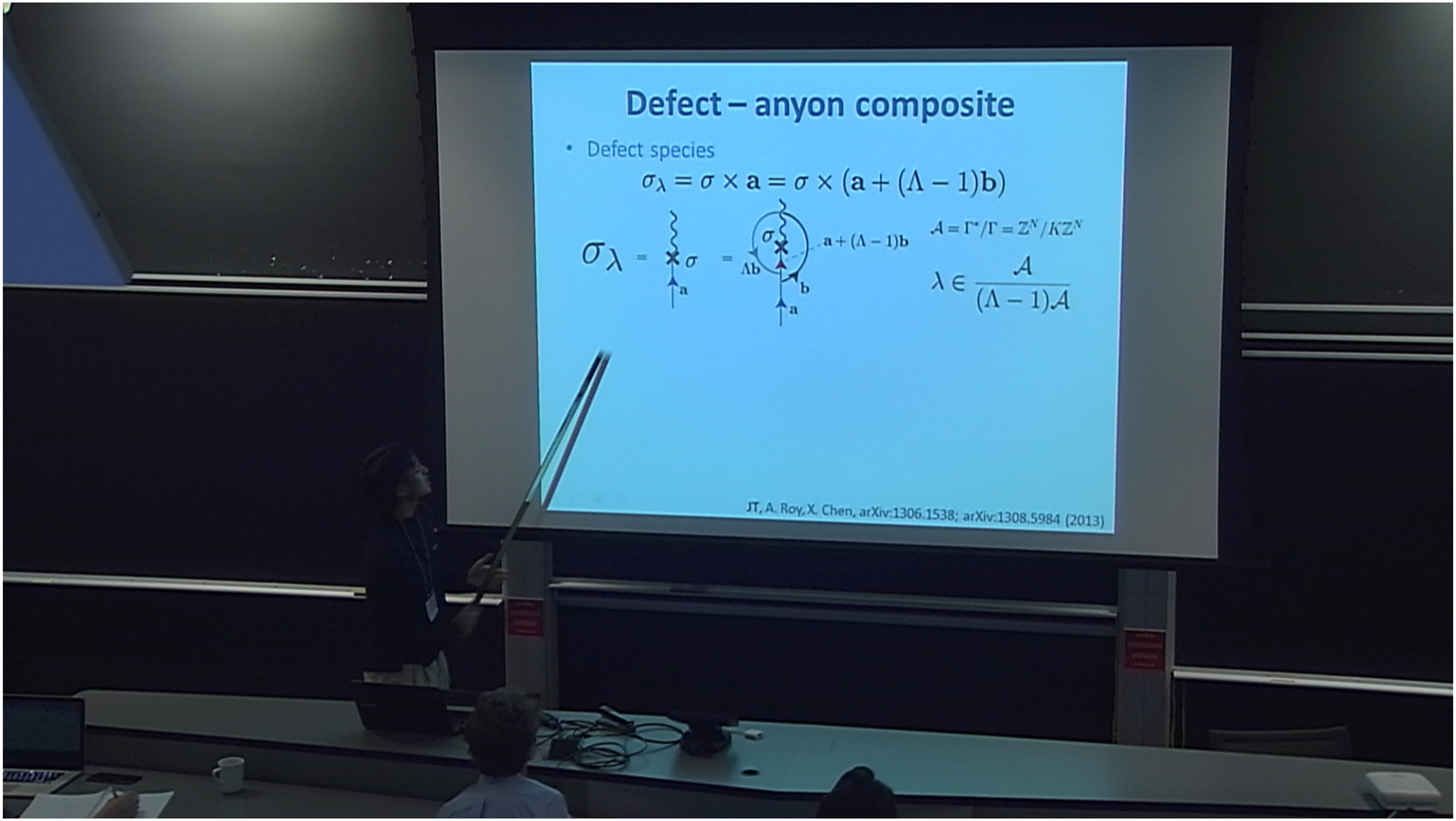
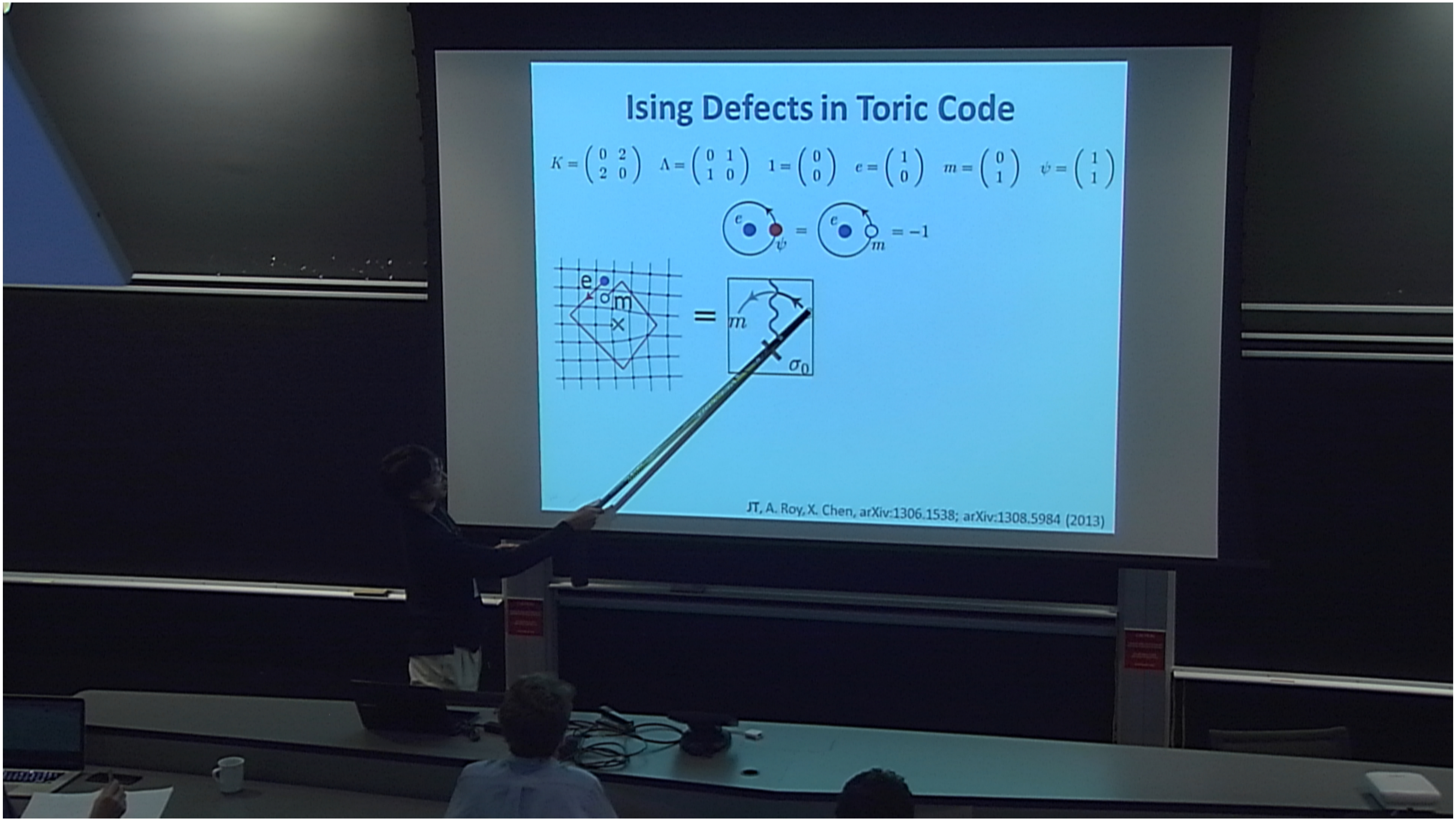

Defect – anyon composite
• Defect species
$\sigma_\lambda = \sigma \times \mathbf{a} = \sigma \times (\mathbf{a} + (\Lambda - 1)\mathbf{b})$
σ_λ
σ
a
$\Lambda\mathbf{b}$
$\mathbf{a} + (\Lambda - 1)\mathbf{b}$
b
$\mathcal{A} = \Gamma^*/\Gamma = \mathbb{Z}^N/K\mathbb{Z}^N$
$\lambda \in \frac{\mathcal{A}}{(\Lambda - 1)\mathcal{A}}$
JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)



Ising Defects in Toric Code
$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ $\Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $e = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\psi = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
e ψ = e m = −1
e
m
m
σ_0
JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Ising Defects in Toric Code

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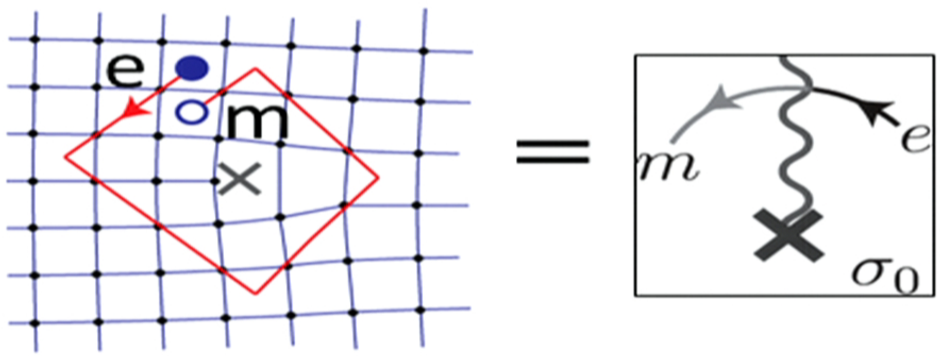
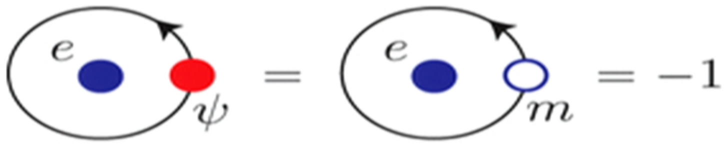
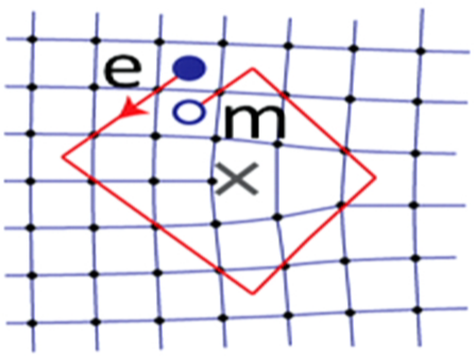


JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

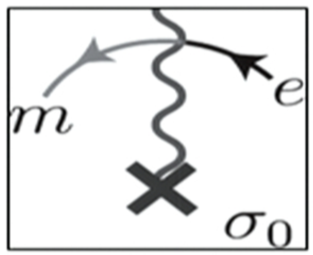
Ising Defects in Toric Code

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad \Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad 1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad e = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \psi = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



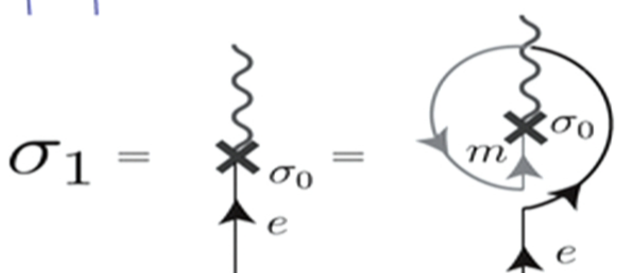


=

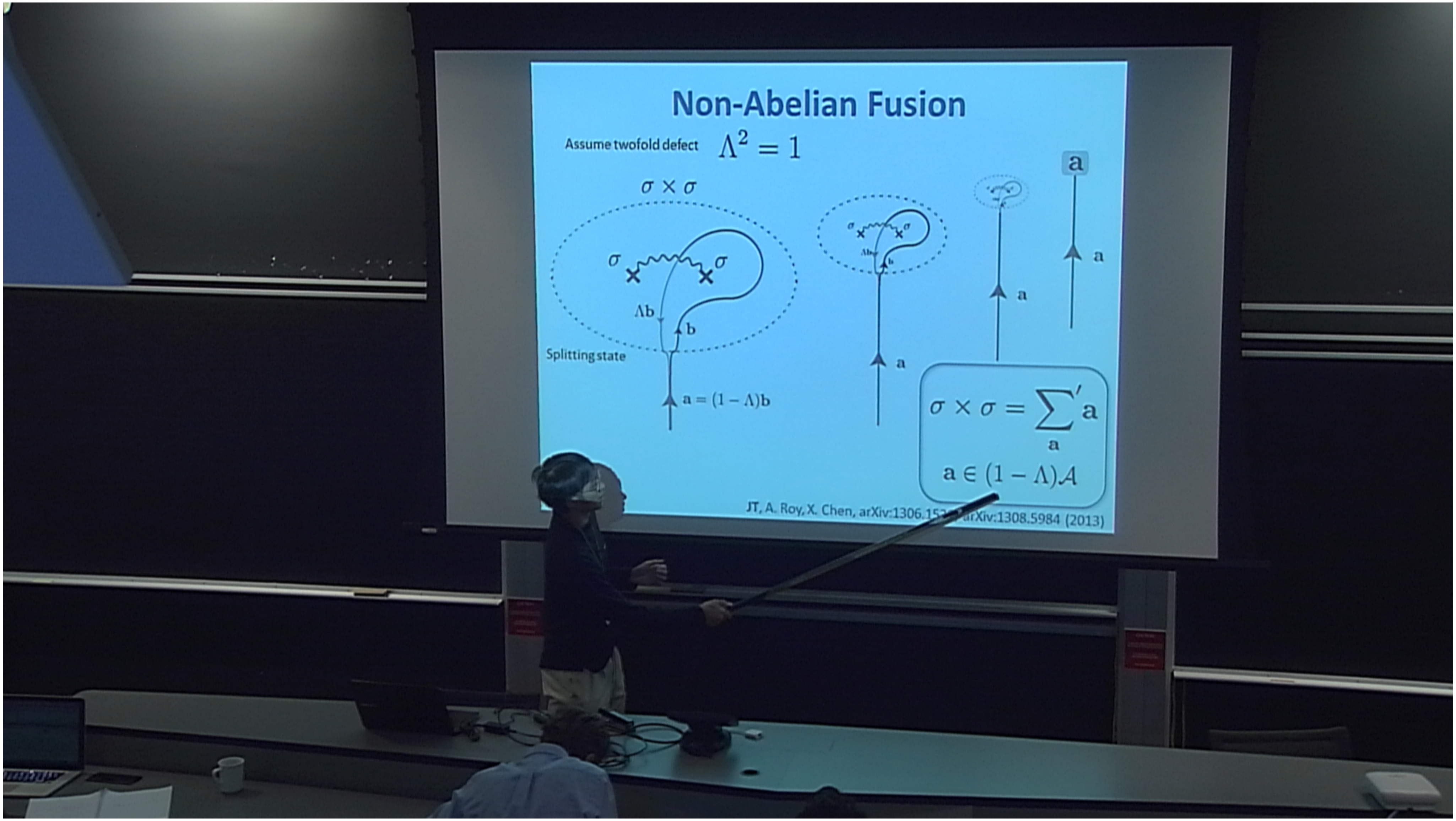


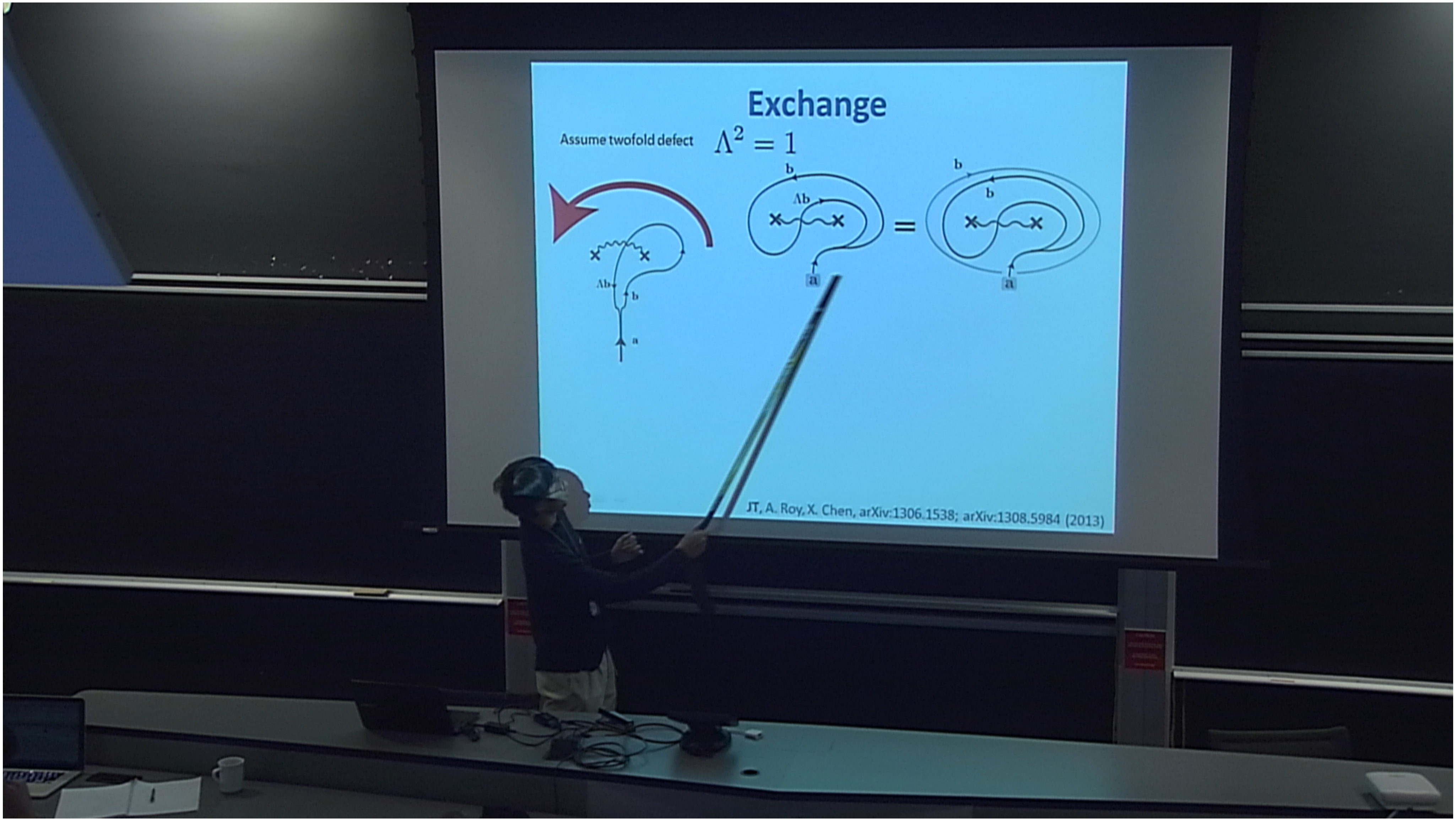
$$\sigma_1 = \sigma_0 \times e = \sigma_0 \times m$$

$$\sigma_0 = \sigma_0 \times \psi$$



JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

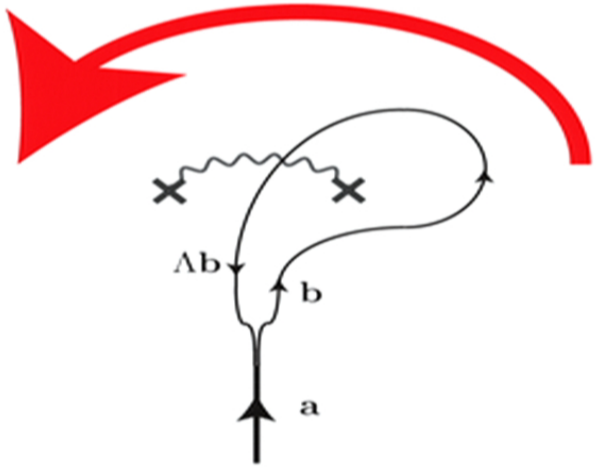

Non-Abelian Fusion
Assume twofold defect $\Lambda^2 = 1$
$\sigma \times \sigma$
σ
σ
Λb
b
Splitting state
a = (1 − Λ)b
a
a
a
a
$\sigma \times \sigma = \sum_{a}' a$
$a \in (1 - \Lambda)\mathcal{A}$
JT, A. Roy, X. Chen, arXiv:1306.15 arXiv:1308.5984 (2013)

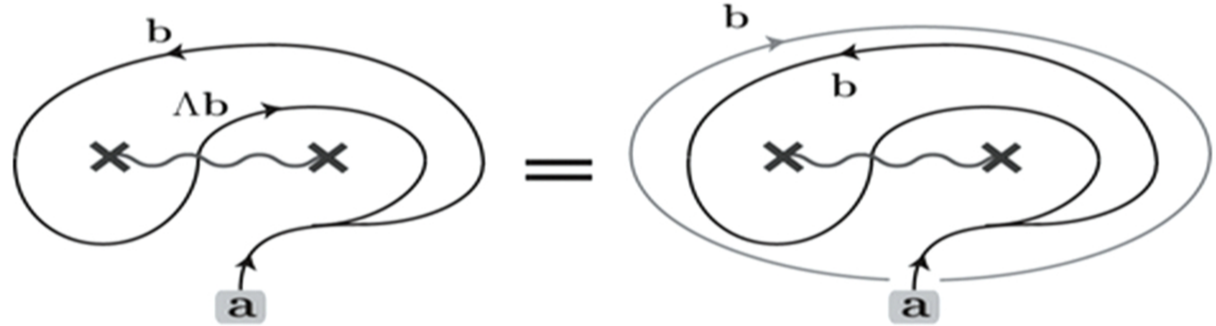


Exchange
Assume twofold defect $\Lambda^2 = 1$
Λb
b
a
=
JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Exchange

Assume twofold defect $\Lambda^2 = 1$

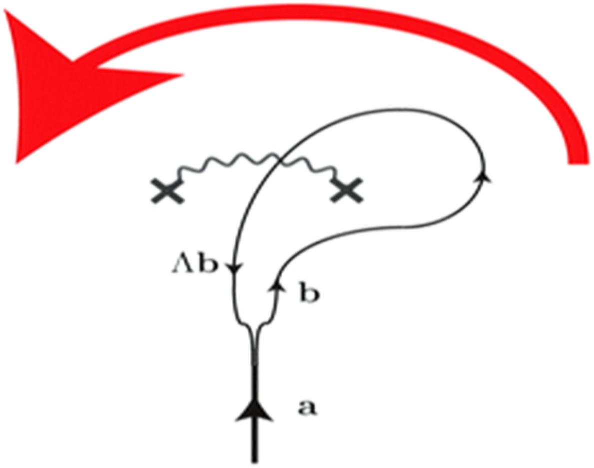


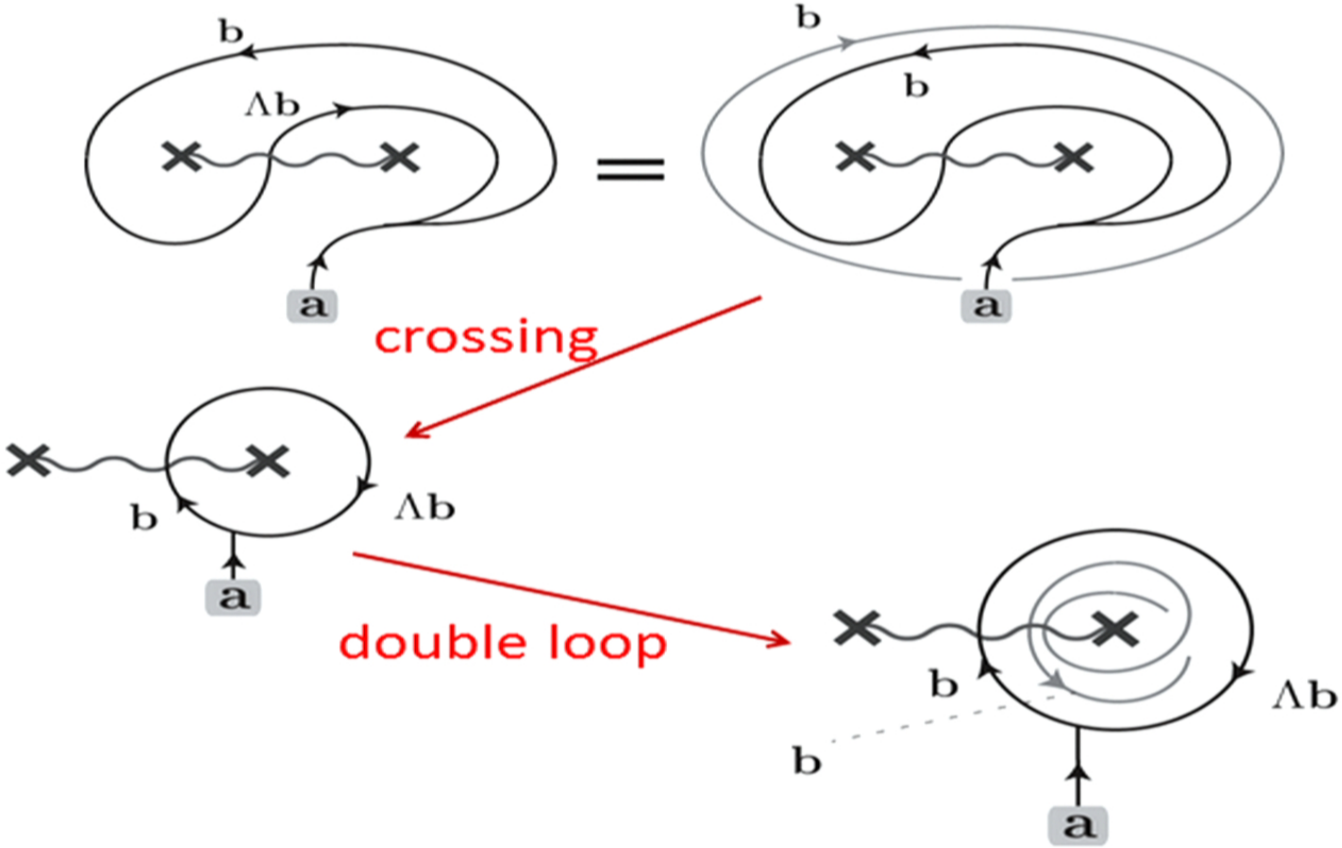


JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Exchange

Assume twofold defect $\Lambda^2 = 1$

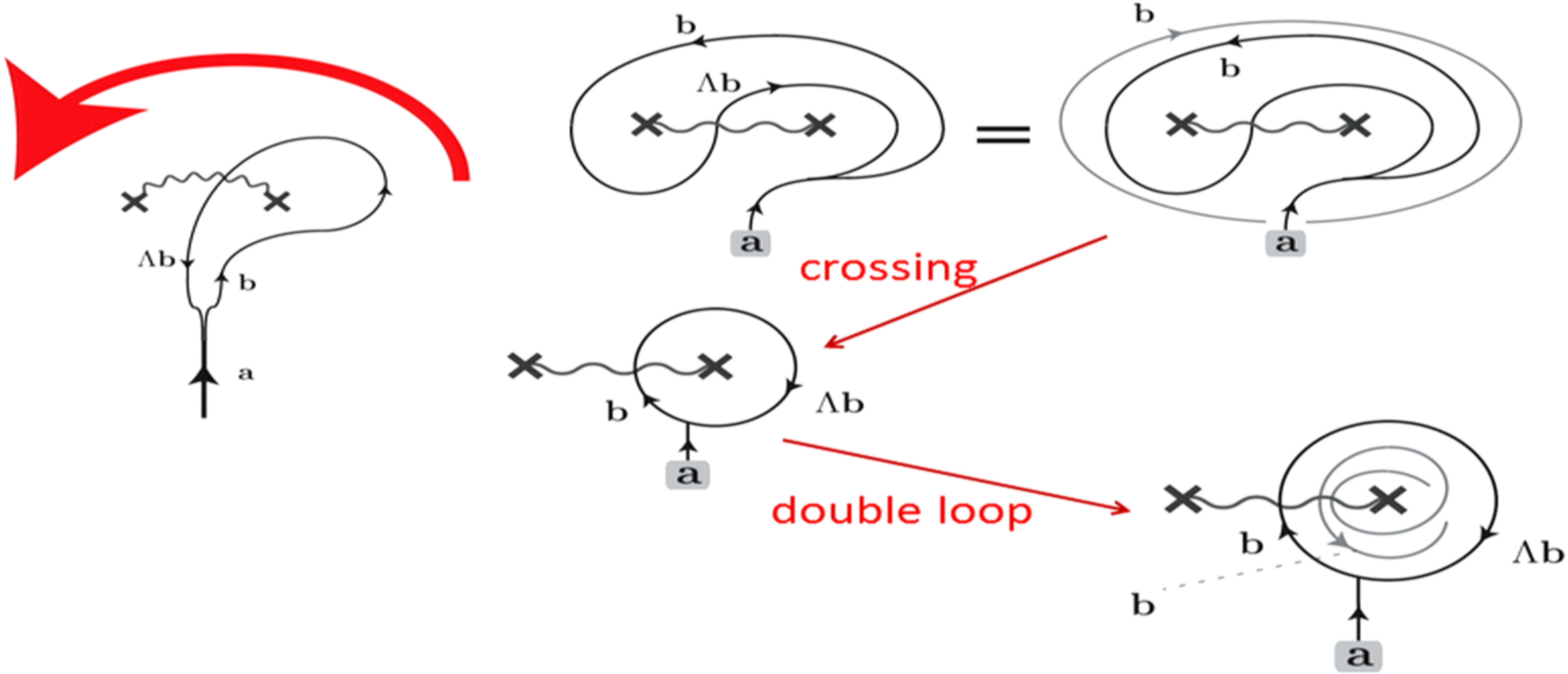




JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Exchange

Assume twofold defect $\Lambda^2 = 1$

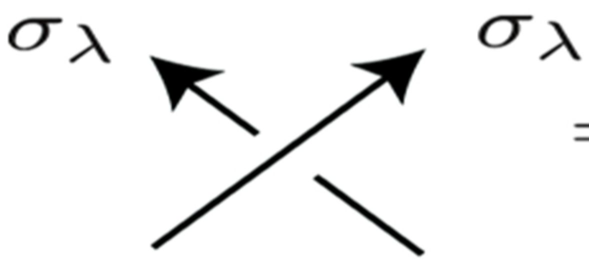


JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

Exchange

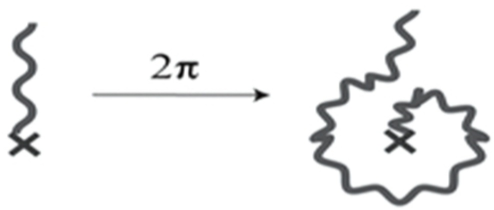
Assume twofold defect $\Lambda^2 = 1$

$$R_{\mathbf{a}}^{\sigma\sigma} = (\text{crossing}) \times (\text{double loop})$$



$$= \frac{1}{d_\sigma} {\sum_{\mathbf{a}}}' R_{\mathbf{a}}^{\sigma_\lambda \sigma_\lambda} = \theta_{\sigma_\lambda}$$

360 rotation – not a closed cycle

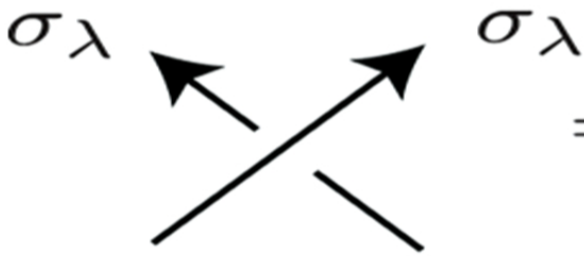


JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

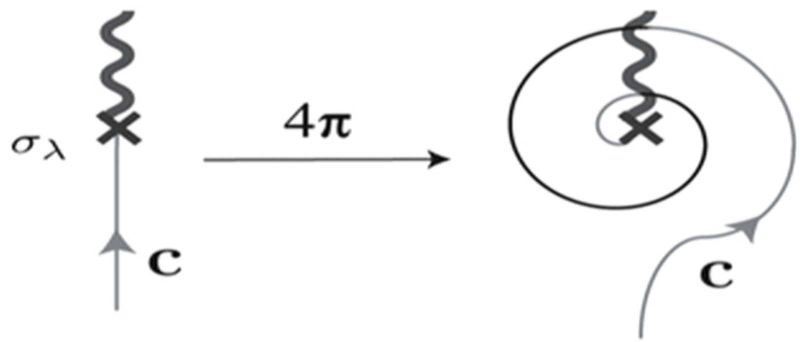
Modified Spin-Statistics Theorem

Assume twofold defect $\Lambda^2 = 1$

$$R^{\sigma\sigma}_{\mathbf{a}} = (\text{crossing}) \times (\text{double loop})$$

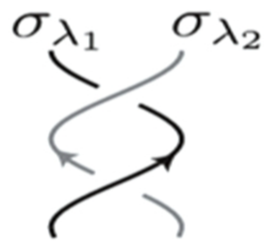


$$= \frac{1}{d_\sigma} \sum_{\mathbf{a}}{}' R^{\sigma\lambda\sigma\lambda}_{\mathbf{a}} = \theta_{\sigma\lambda}$$



JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

$SL(2;\mathbf{Z})$ Modular Transformation?

$S_{\sigma_{\lambda_1}\sigma_{\lambda_2}} =$ 

$T_{\sigma_{\lambda_1}\sigma_{\lambda_2}} = \delta_{\sigma_{\lambda_1}\sigma_{\lambda_2}}$ 

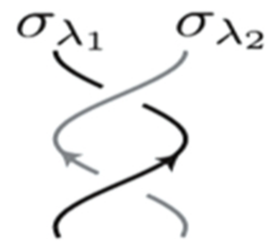
- Algebraic relations [proj. rep. of $SL(2;\mathbf{Z})$]

$$(ST)^3 = e^{ic_-\pi/4} S^2 \quad S^4 = 1$$

JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

$SL(2;\mathbf{Z})$ Modular Transformation?

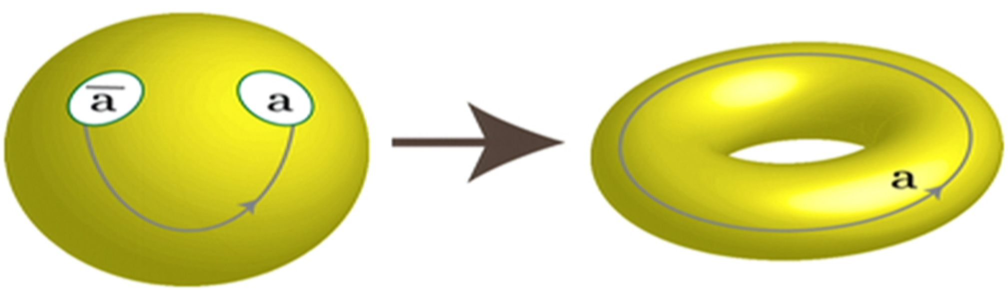
$S_{\sigma_{\lambda_1}\sigma_{\lambda_2}} =$

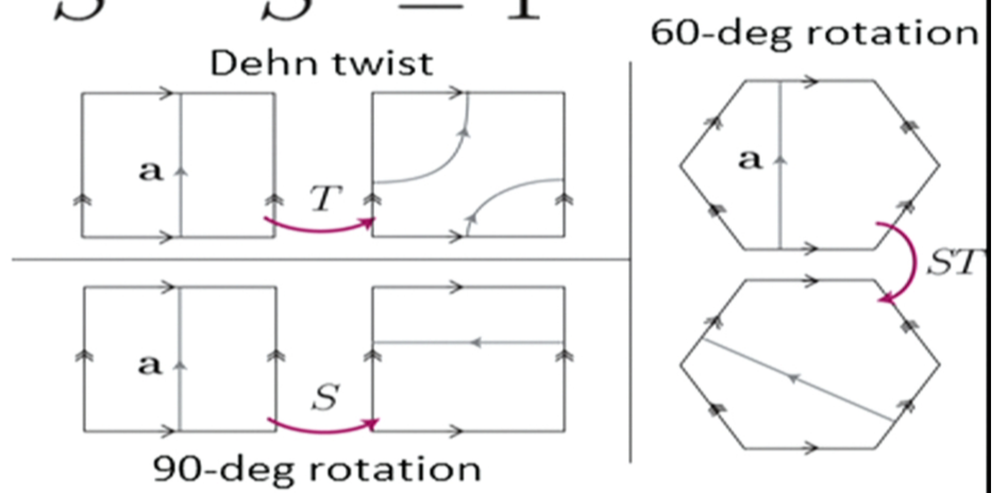


$T_{\sigma_{\lambda_1}\sigma_{\lambda_2}} = \delta_{\sigma_{\lambda_1}\sigma_{\lambda_2}}$

- Algebraic relations [proj. rep. of $SL(2;\mathbf{Z})$]

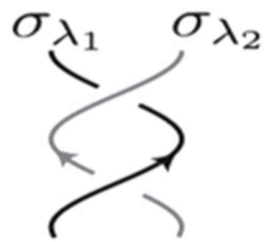
$$(ST)^3 = e^{ic_-\pi/4}S^2 \quad S^4 = 1$$





JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)

$\Gamma_0(2)$ Congruent Transformation

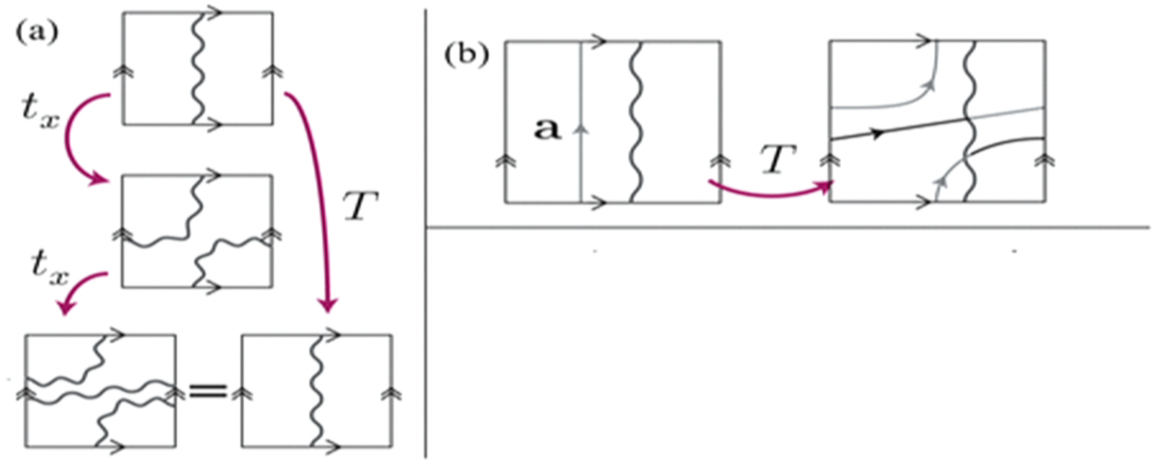
$S_{\sigma_{\lambda_1}\sigma_{\lambda_2}} =$ 

$T_{\sigma_{\lambda_1}\sigma_{\lambda_2}} = \delta_{\sigma_{\lambda_1}\sigma_{\lambda_2}}$ 

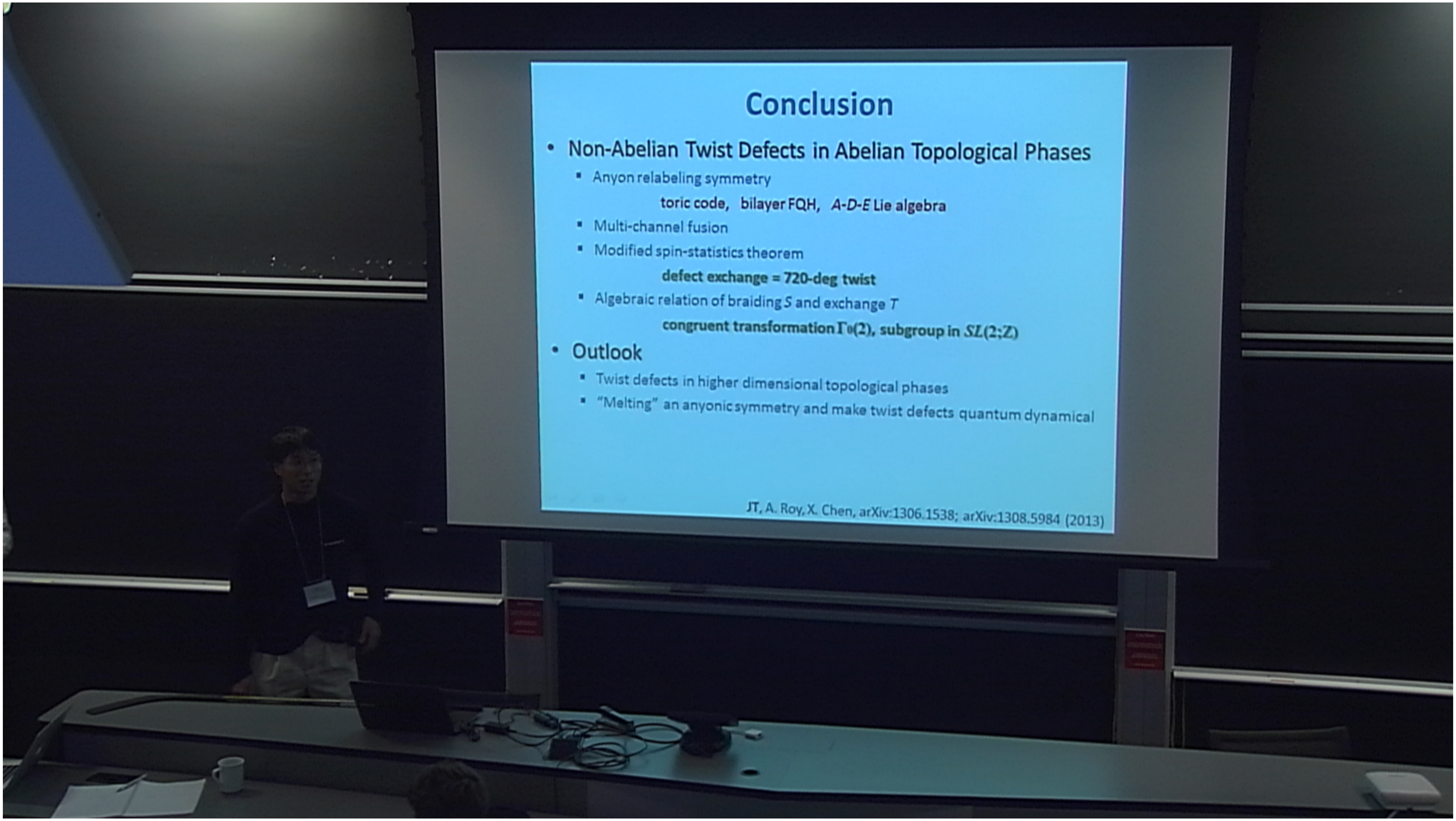
- Algebraic relations

$$(ST^{-1})^2 = C \qquad C^2 = 1 \qquad [S, C] = [T, C] = 0$$

***T* = double *x*-Dehn twist**



JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)


Conclusion
Non-Abelian Twist Defects in Abelian Topological Phases
Anyon relabeling symmetry
toric code, bilayer FQH, A-D-E Lie algebra
Multi-channel fusion
Modified spin-statistics theorem
defect exchange = 720-deg twist
Algebraic relation of braiding S and exchange T
congruent transformation Γ0(2), subgroup in SL(2;Z)
Outlook
Twist defects in higher dimensional topological phases
"Melting" an anyonic symmetry and make twist defects quantum dynamical
JT, A. Roy, X. Chen, arXiv:1306.1538; arXiv:1308.5984 (2013)