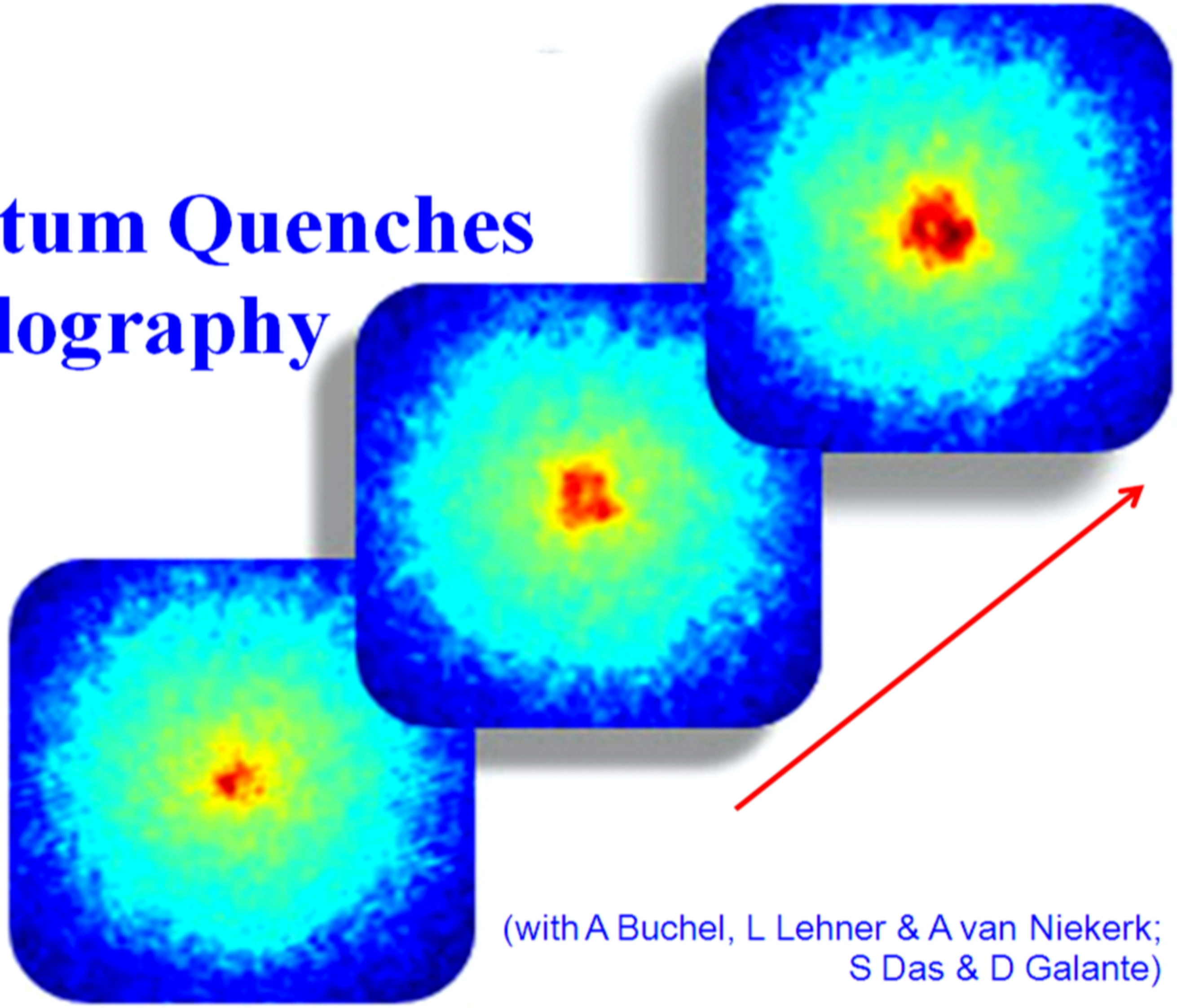
Title: Quantum quenches & holography

Date: Nov 07, 2013 04:15 PM

URL: http://pirsa.org/13110074

Abstract: We employ holographic techniques to study quantum quenches at finite temperature, where the quenches involve varying the coupling of the boundary theory to a relevant operator with an arbitrary conformal dimension. The evolution of the system is studied by evaluating the expectation value of the quenched operator and the stress tensor throughout the process. The time dependence of the new coupling is characterized by a fixed timescale and the response of the observables depends on the ratio of the this timescale to the initial temperature. The observables exhibit universal scaling behaviours when the transitions are either fast or slow, i.e., when this ratio is very small or very large. For fast quenches, we uncover a universal scaling behaviour in the response of the system, which depends only on the conformal dimension of the quenched operator in the vicinity of the ultraviolet fixed point of the theory.

Quantum Quenches & Holography



(with A Buchel, L Lehner & A van Niekerk; S Das & D Galante)

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

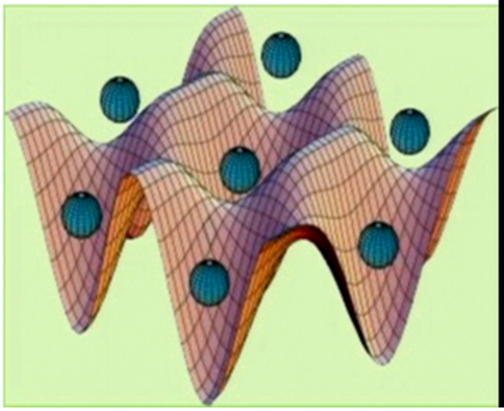
Quantum Quenches:

- consider quantum system with Hamiltonian:

$$H = H_0 + \lambda(t)\,\delta H$$

- prepare system in eigenstate $|\psi_0\rangle$ of Hamiltonian H_0
- abruptly turn on λ ; system evolves *unitarily* according to H
- Question: How do observables, eg, expectation values and correlation functions, evolve in time?
- for most systems, coupling to environment is unavoidable ⟶ decoherence, dissipation
- effects minimized for, eg, cold atoms in optical lattice

⟶ is there "universal" behaviour?



Quantum Quenches:

⟶ is there "universal" behaviour?

what are organizing principles for out-of-equilibrium systems?

- theoretical progress made for variety systems: d=2 CFT, (nearly) free fields, integrable models, ...
- still seeking broadly applicable and efficient techniques

AdS/CFT Correspondence:

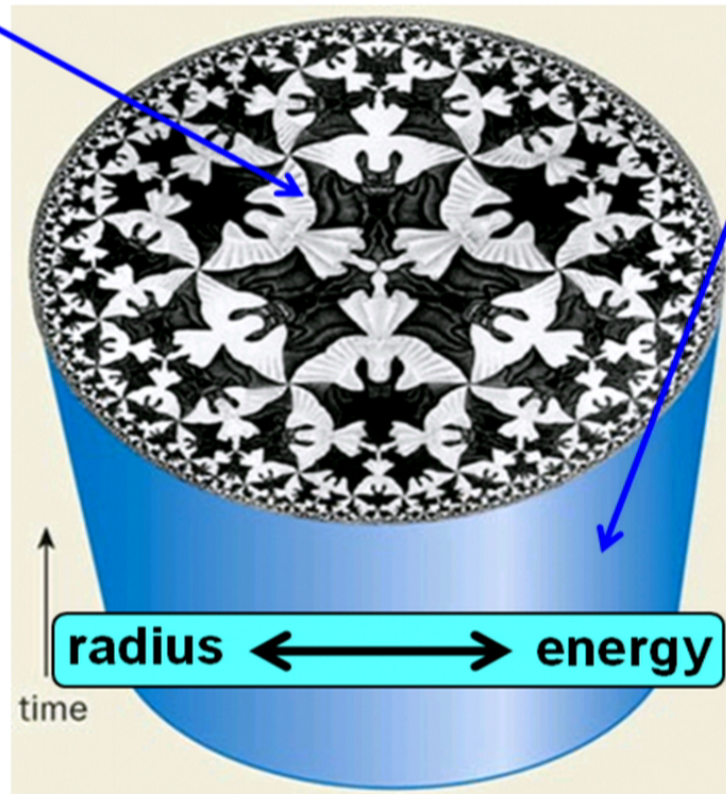
Bulk: gravity with negative Λ
in **d+1** dimensions

Boundary: quantum field theory
in **d** dimensions

↔
"holography"

anti-de Sitter
space

conformal
field theory



Quantum Quenches & Holography:

→ is there "universal" behaviour?

what are organizing principles for out-of-equilibrium systems?

- theoretical progress made for variety systems: d=2 CFT, (nearly) free fields, integrable models, ...
- still seeking broadly applicable and efficient techniques
- what can AdS/CFT correspondence offer?
  - → strongly coupled field theories
  - → real-time analysis
  - → finite temperature (if desired)
  - → general spacetime dimension
- perhaps re-organization of problem will lead to new insights

Quantum Quenches & Holography:

- AdS/CFT lends itself to the study quantum quenches for a new class of strongly coupled field theories
- there has been a great deal of interest in the past few years

  Chesler, Yaffe; Das, Nishioka, Takayanagi, Basu; Bhattacharyya, Minwalla; Abajo-Arrastia, Aparicio, Lopez; Albash, Johnson; Ebrahim, Headrick; Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Mueller, Schafer, Shigemori, Staessens, Galli; Allias, Tonni; Keranen, Keski-Vakkuri, Thorlacius; Galante, Schvellinger; Carceres, Kundu; Wu; Garfinkle, Pando Zayas, Reichmann; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; . . . . . .

- much of work aimed at “thermalization” (eg, quark-gluon plasma)

Quantum Quenches & Holography:

- AdS/CFT lends itself to the study quantum quenches for a new class of strongly coupled field theories

Where are control parameters in AdS/CFT framework?

AdS/CFT dictionary:

gravity fields ⟷ boundary operators

Φ $\qquad\qquad$ δH

eg, consider some scalar field in AdS:

equation of motion: $(\nabla^2 - m^2)\Phi + \cdots = 0$

asymptotic solutions: $\Phi \sim \dfrac{C_1}{r^{d-\Delta}} + \dfrac{C_2}{r^{\Delta}} + \cdots$

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 L^2}$$

Quantum Quenches & Holography:

- AdS/CFT lends itself to the study quantum quenches for a new class of strongly coupled field theories

Where are control parameters in AdS/CFT framework?

AdS/CFT dictionary:

gravity fields ⟷ boundary operators

Φ $\qquad$ δH

eg, consider some scalar field in AdS:

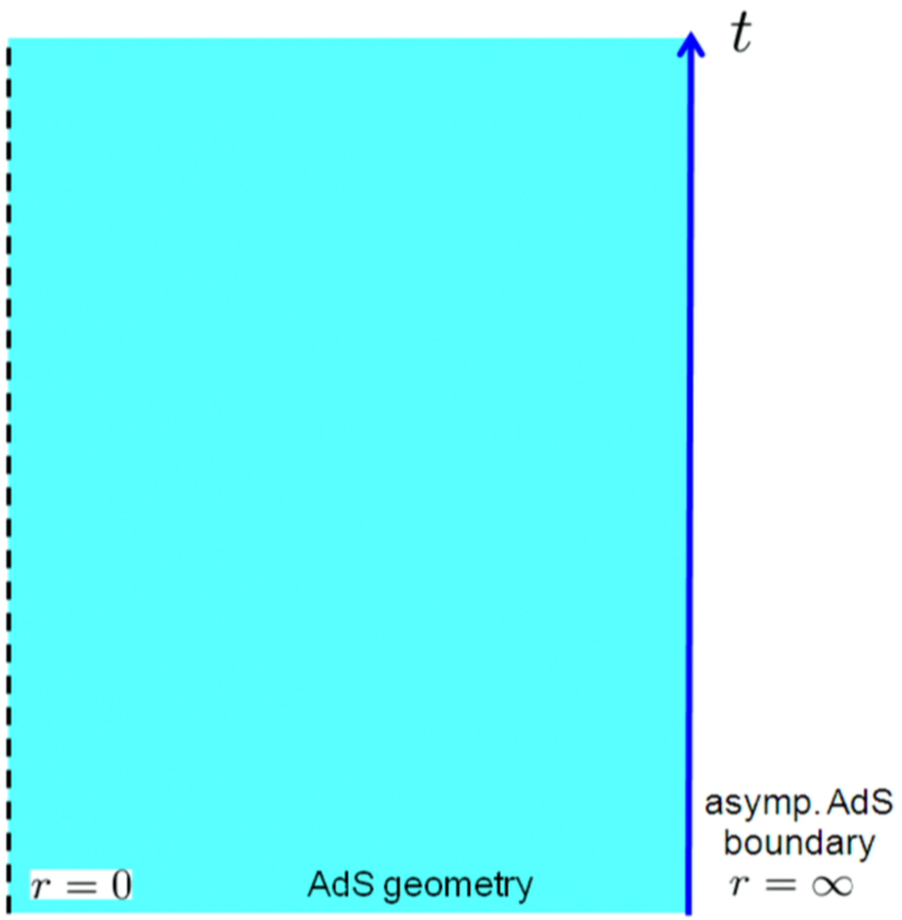
equation of motion: $(\nabla^2 - m^2)\Phi + \cdots = 0$

asymptotic solutions: $\Phi \sim \dfrac{\lambda}{r^{d-\Delta}} + \dfrac{\langle \delta H \rangle}{r^{\Delta}} + \cdots$

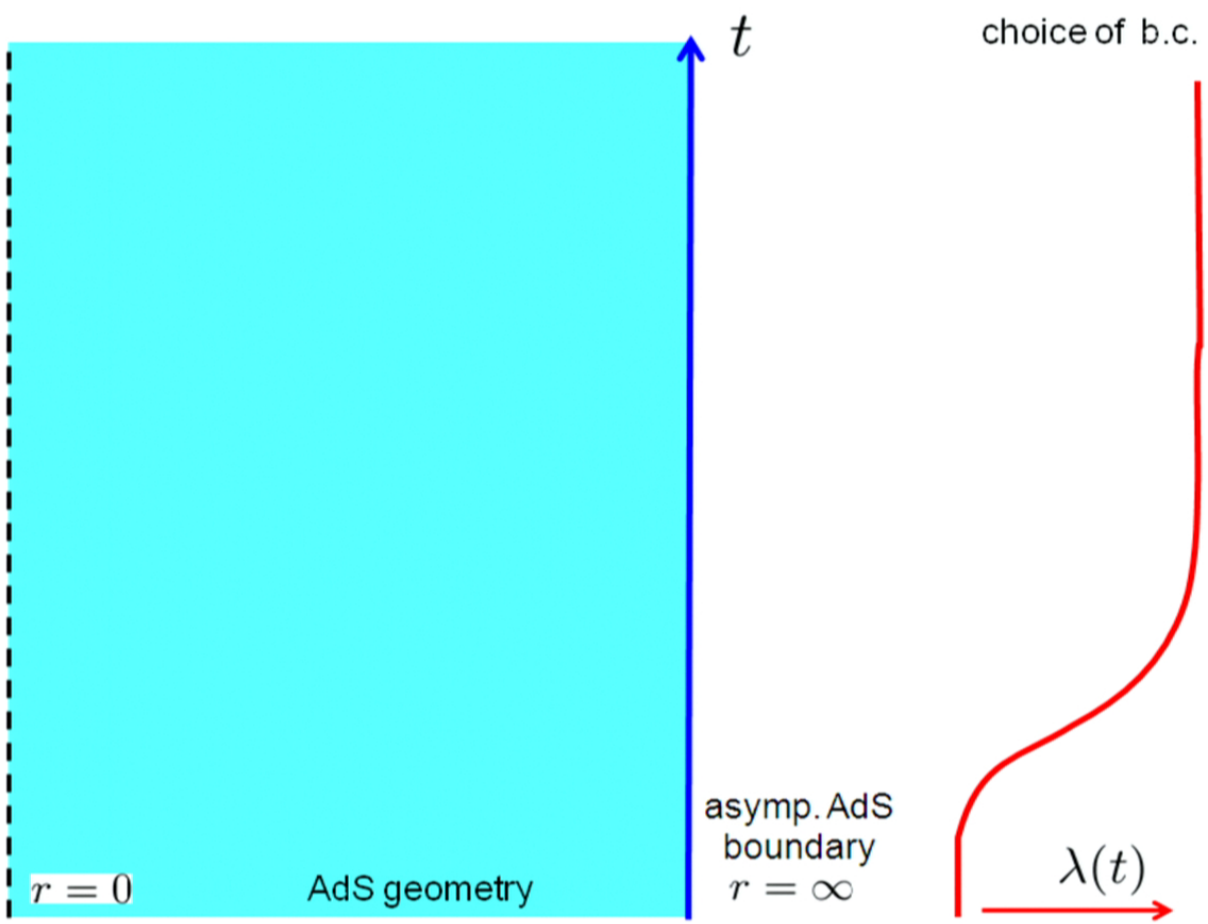
⟶ integration constants become coupling and expectation value

with conformal dimension: $\Delta = \dfrac{d}{2} + \sqrt{\dfrac{d^2}{4} + m^2 L^2}$

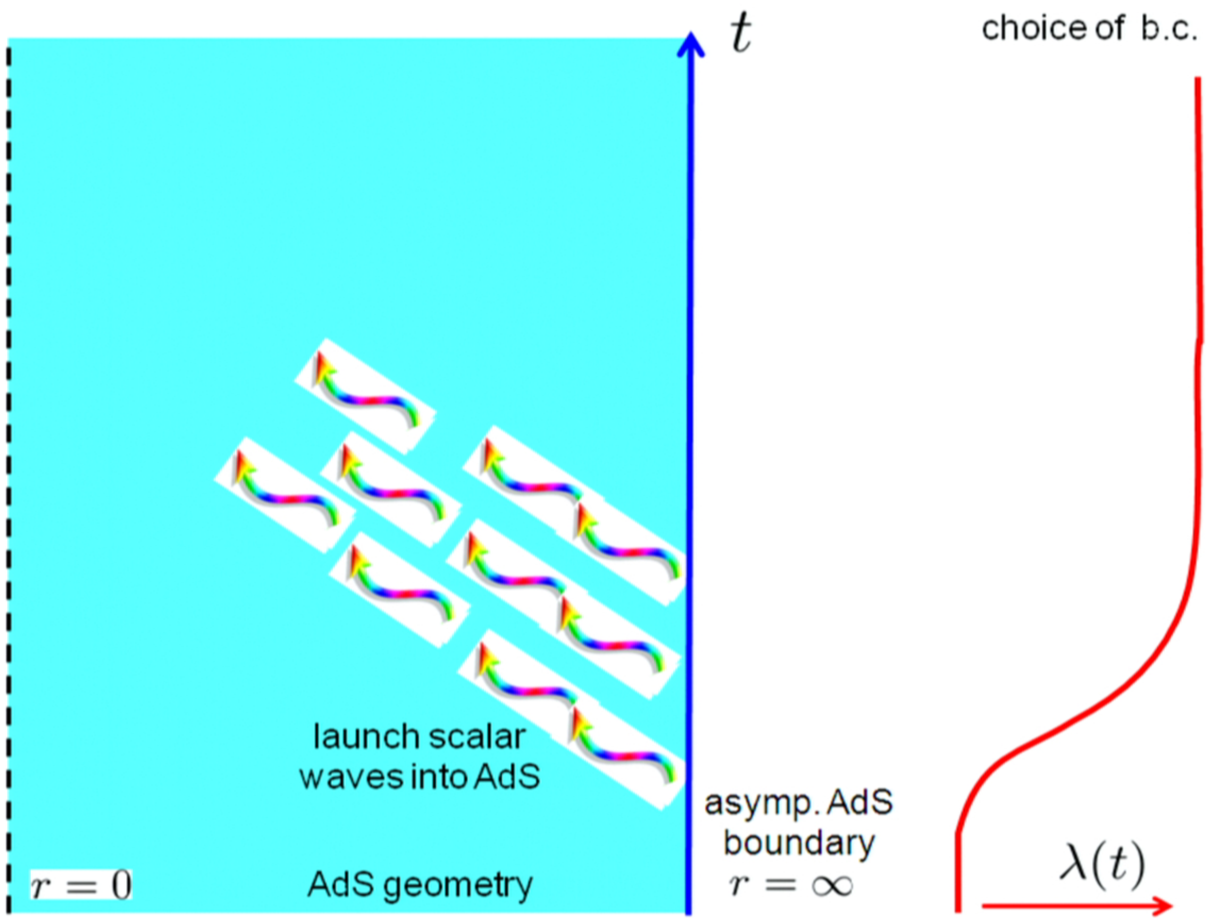
Holographic Quantum Quench (cartoon):



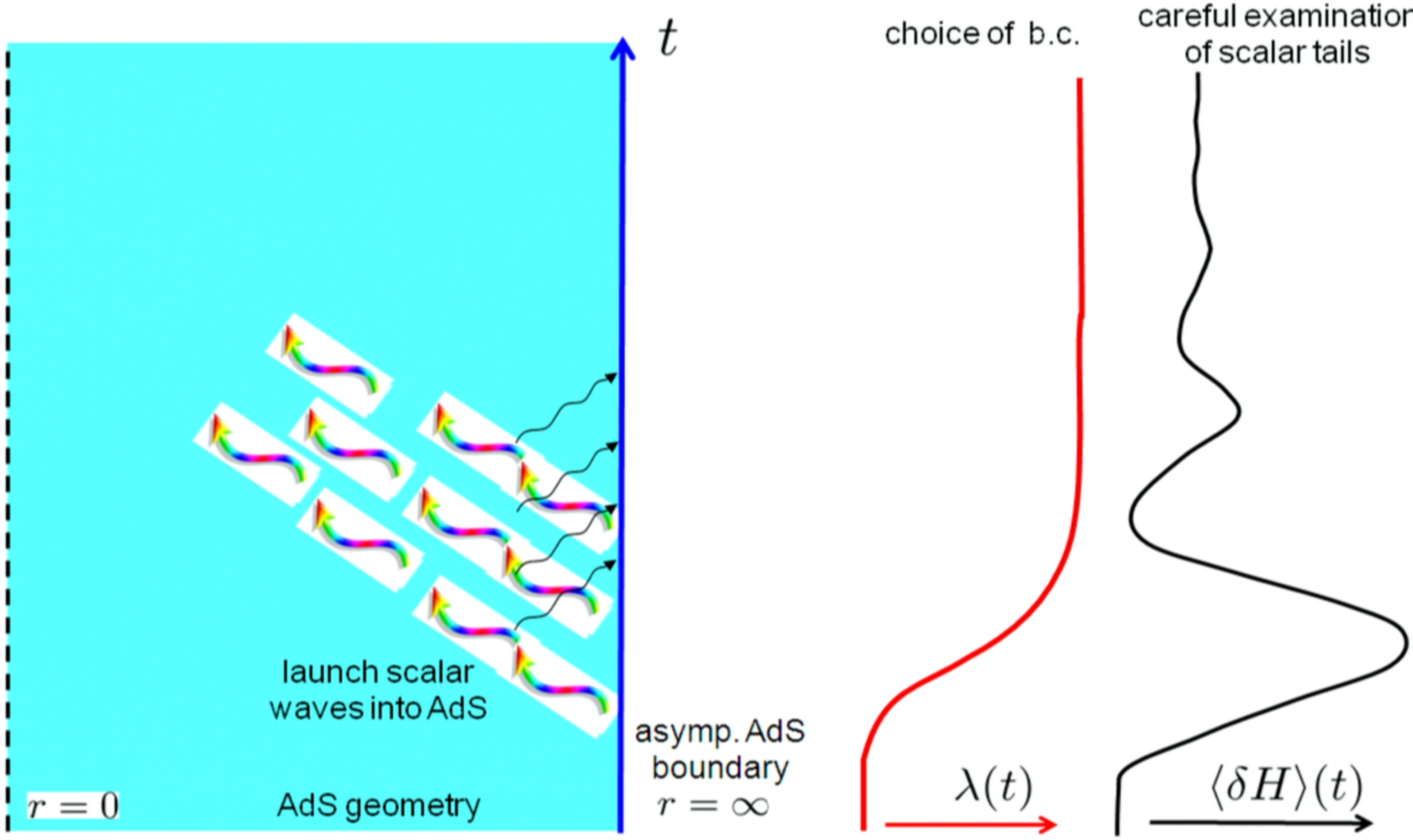
Holographic Quantum Quench (cartoon):



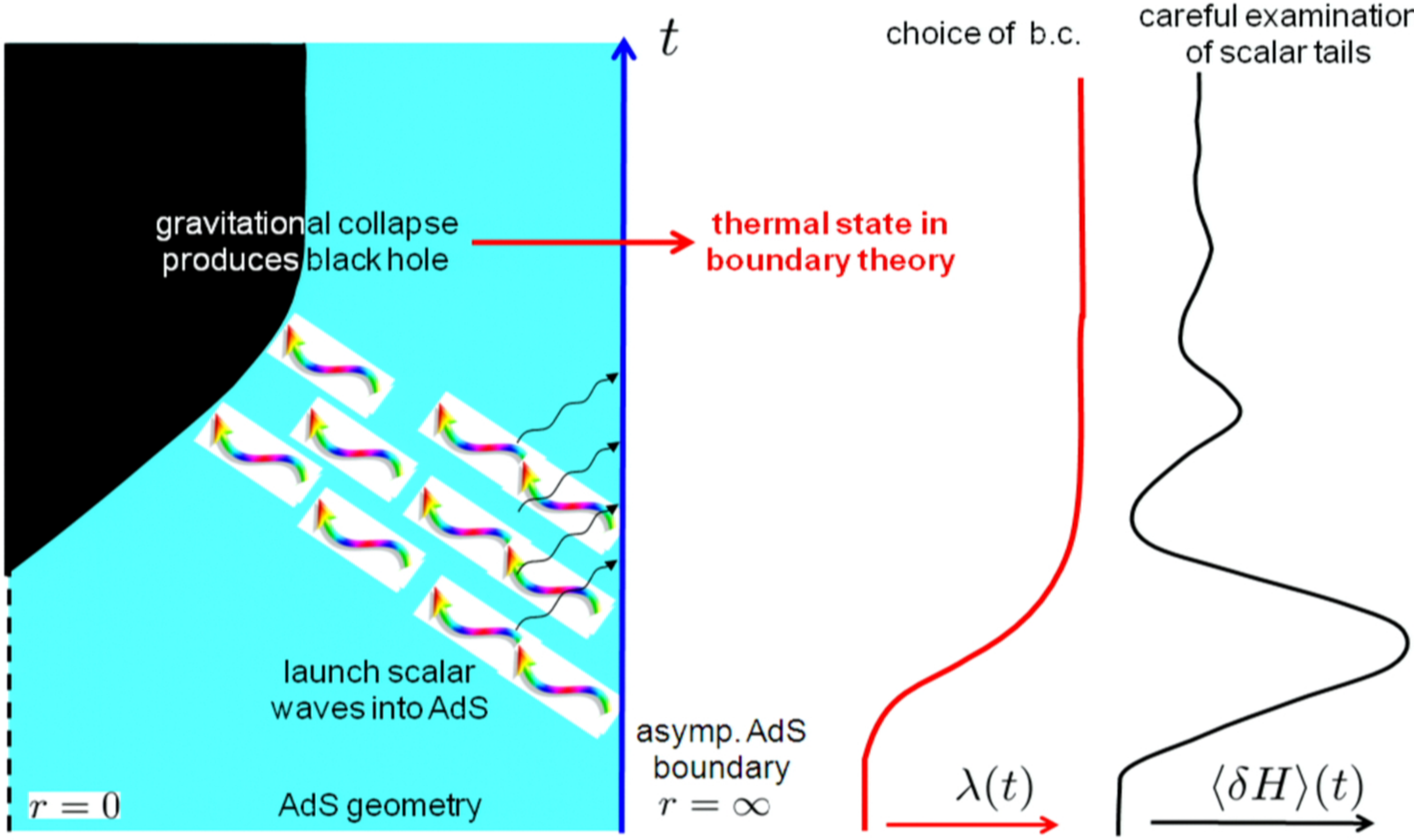
Holographic Quantum Quench (cartoon):



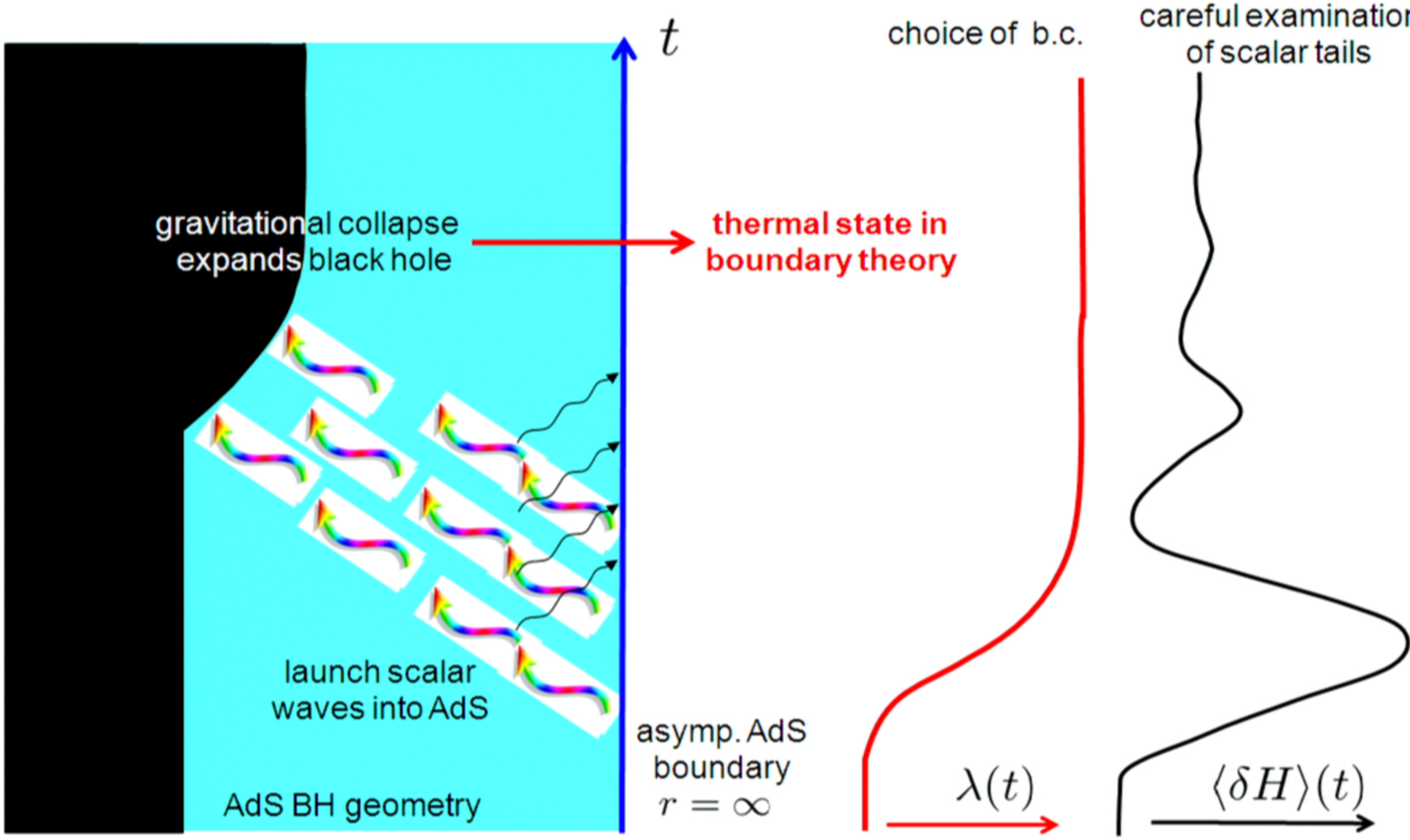
Holographic Quantum Quench (cartoon):



Holographic Quantum Quench (cartoon):

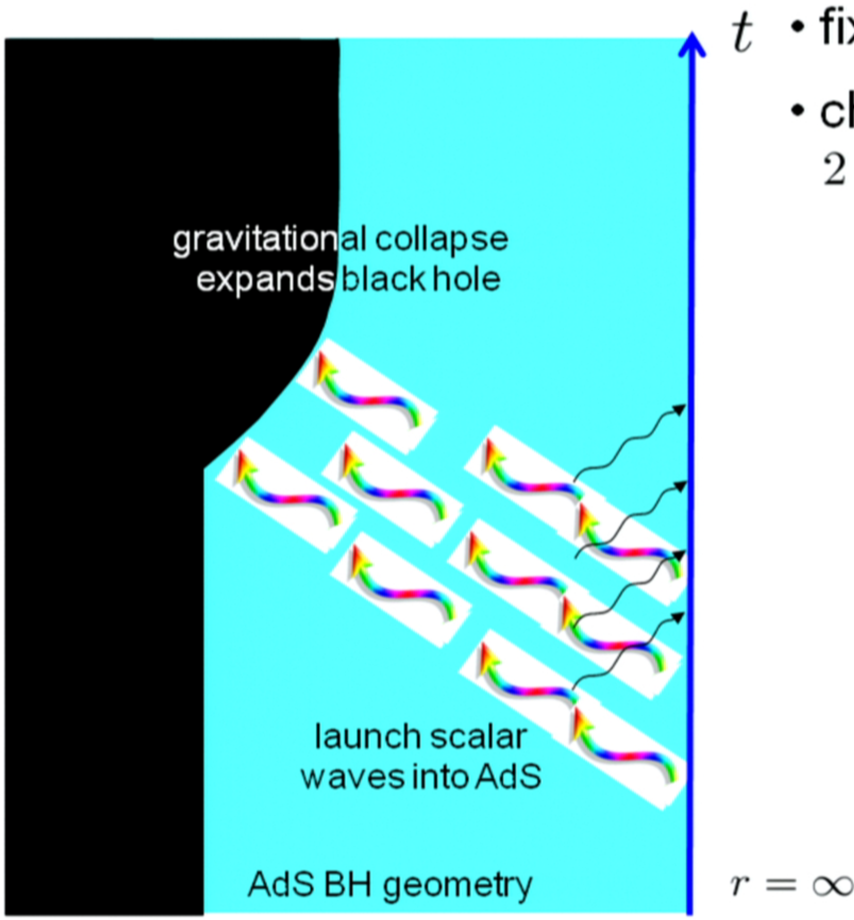


Holographic Quantum Quench (cartoon):



- **"thermal quench"**: quantum quench at finite temperature

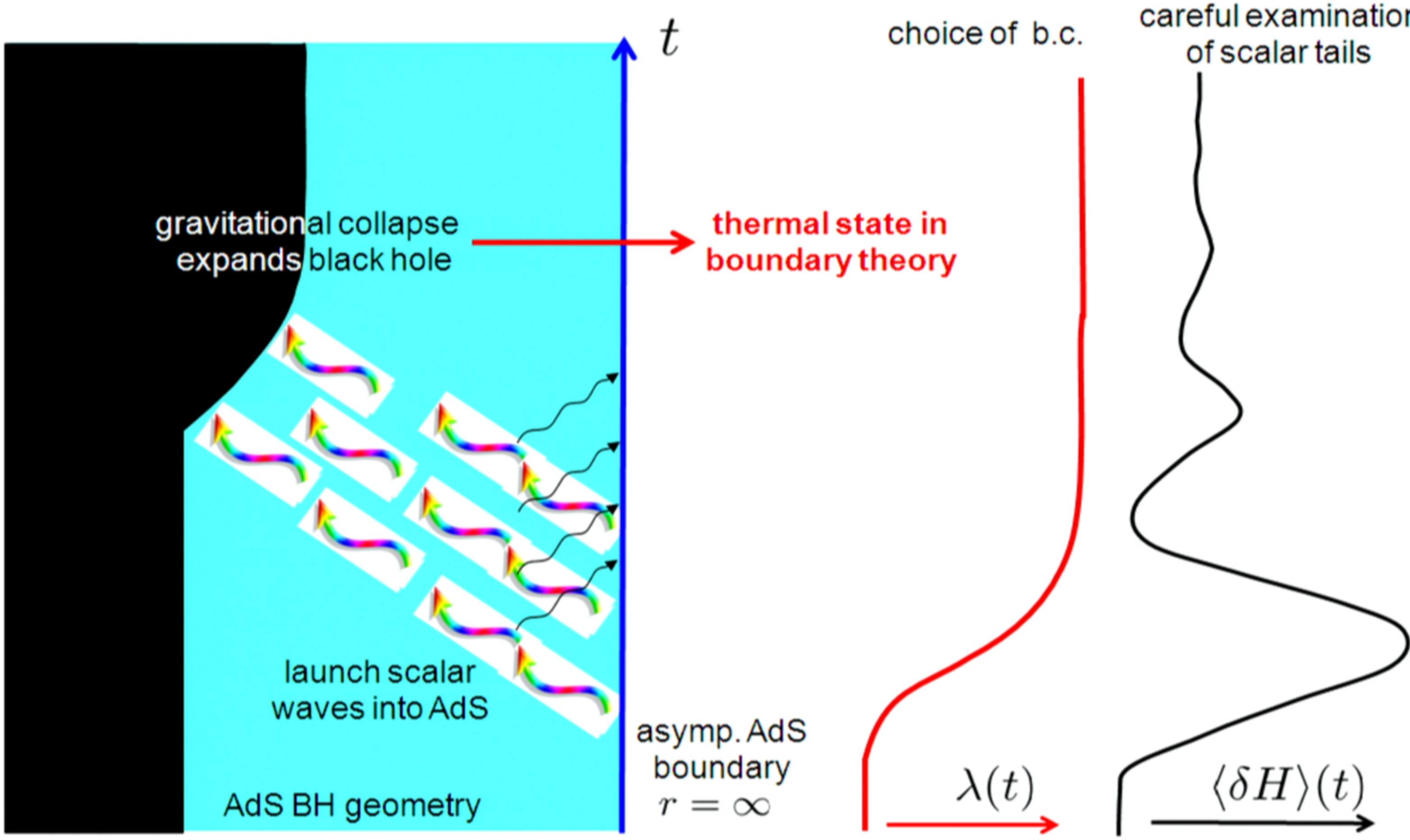
Holographic Thermal Quench:



- fix boundary dimension: $d = 4$
- choose conformal dimension, $2 \leq \Delta \leq 4$ and profile

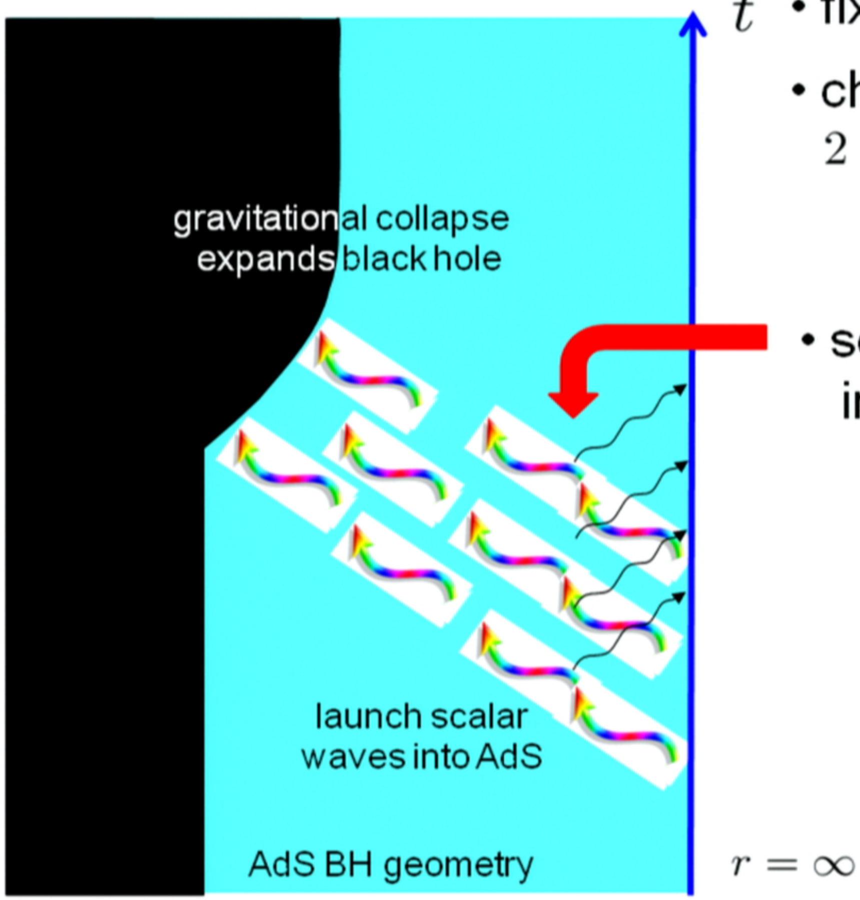
$$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$

Holographic Quantum Quench (cartoon):



- **"thermal quench"**: quantum quench at finite temperature

Holographic Thermal Quench:



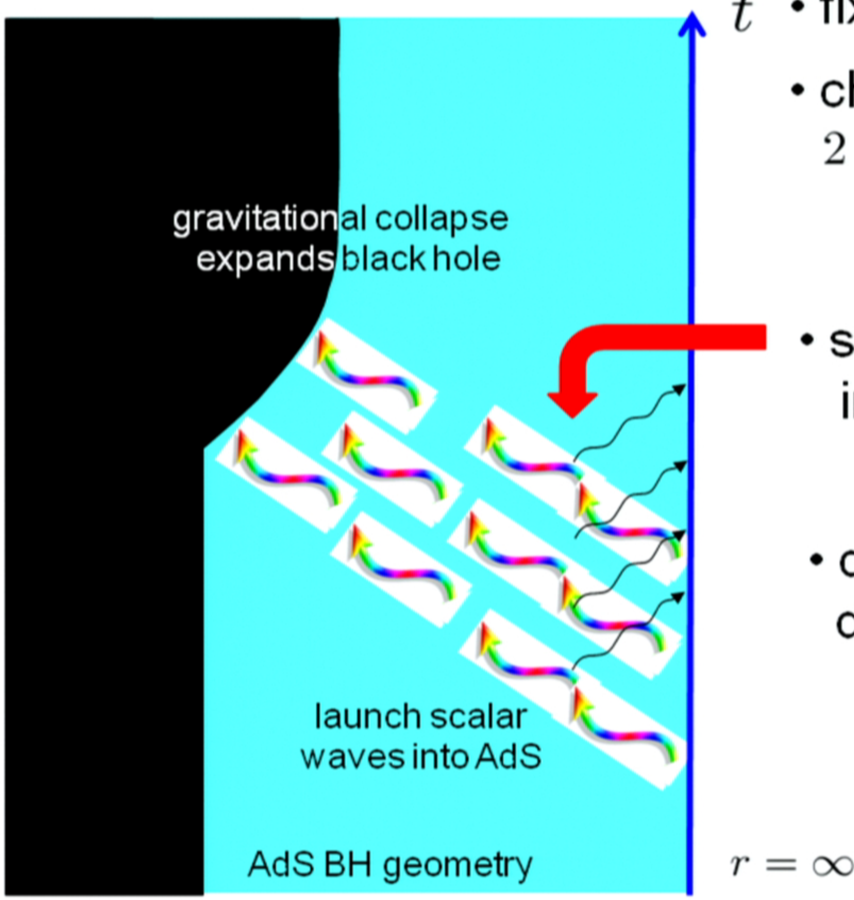
- fix boundary dimension: $d = 4$
- choose conformal dimension, $2 \leq \Delta \leq 4$ and profile

$$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$

- solve linearized scalar eom in fixed BH geometry

  $\longrightarrow$ determines $\langle \mathcal{O}_\Delta \rangle(t)$

Holographic Thermal Quench:



- fix boundary dimension: $d = 4$
- choose conformal dimension, $2 \le \Delta \le 4$ and profile

$$\lambda(t) = \frac{\Delta\lambda}{2}\left[1 + \tanh(t/\Delta t)\right]$$

- solve linearized scalar eom in fixed BH geometry
  → determines $\langle \mathcal{O}_\Delta \rangle(t)$
- determine "BH mass" $\mathcal{E}(t)$ with diffeomorphism Ward identity*:

$$\partial^i \langle T_{ij} \rangle \ = \ \langle \mathcal{O}_\Delta \rangle \ \partial_j \lambda$$

→ integrate for $\mathcal{E}(t)$, ie,

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \ \partial_t \lambda$$

* boundary constraint from Einstein eq's

Holographic Thermal Quench:

- lessons learned:

  1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way

- holography gives well-defined approach to renormalize bdry QFT
- bdry theory has new divergences: (Λ = UV cut-off scale)

$$I_{ct} \simeq \int d^4x\sqrt{-g}\Big(\Lambda^4 + \Lambda^{2\Delta-4}\,\lambda^2(t) + \cdots$$
$$+\Lambda^{2\Delta-6} g^{ij}\,\partial_i\lambda\,\partial_j\lambda + \Lambda^{2\Delta-6}\mathcal{R}(g)\,\lambda^2 + \cdots \Big)$$

- familiar in the context of QFT in curved backgrounds
- new log divergences lead to new scheme dependent ambiguities

(Bianchi, Freedman & Skenderis; Aharony, Buchel & Yarom; Petkou & Skenderis; Emparan, Johnson & Myers; . . .)

Holographic Thermal Quench:

- lessons learned:

  2. Response to "fast" quenches exhibits universal scaling

- for example: $(d=4)$

$$\left.\begin{aligned} \max\langle \mathcal{O}_\Delta \rangle &\sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\ \Delta\mathcal{E} &\sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{aligned}\right\} \quad \Delta t \to 0$$

Holographic Thermal Quench:

- lessons learned:

2. Response to “fast” quenches exhibits universal scaling

- for example: $(d=4)$

$$\max\langle\mathcal{O}_\Delta\rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}}$$

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}}$$

$\Delta t \to 0$

yields physical divergence!!

- seems to indicate instantaneous quench is problematic
  - ► physical problem?
  - ► simply an issue with perturbative expansion?

Holographic Thermal Quench:

- lessons learned:

  2. Response to "fast" quenches exhibits universal scaling

- for example: $(d=4)$

$$\max\langle\mathcal{O}_\Delta\rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}}$$

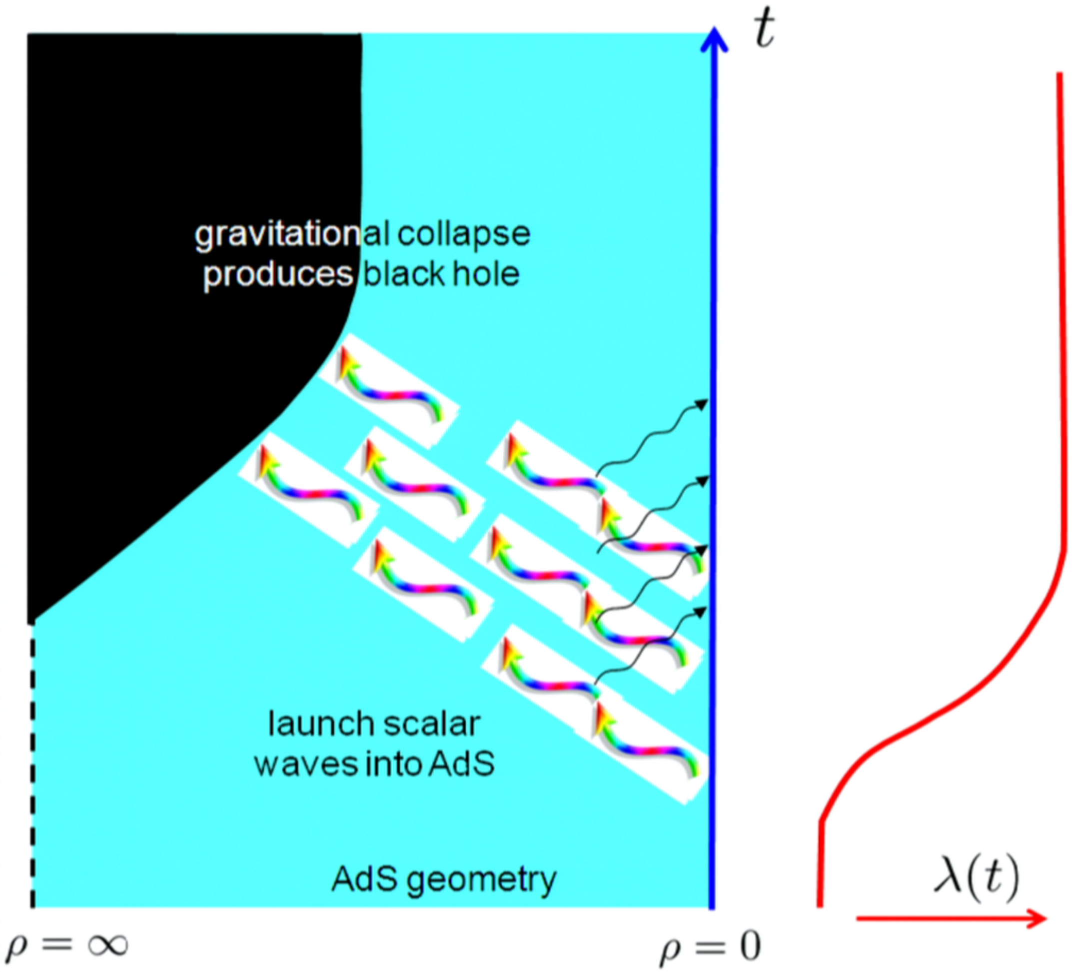
$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}}$$

$\Delta t \to 0$ **yields physical divergence!!**

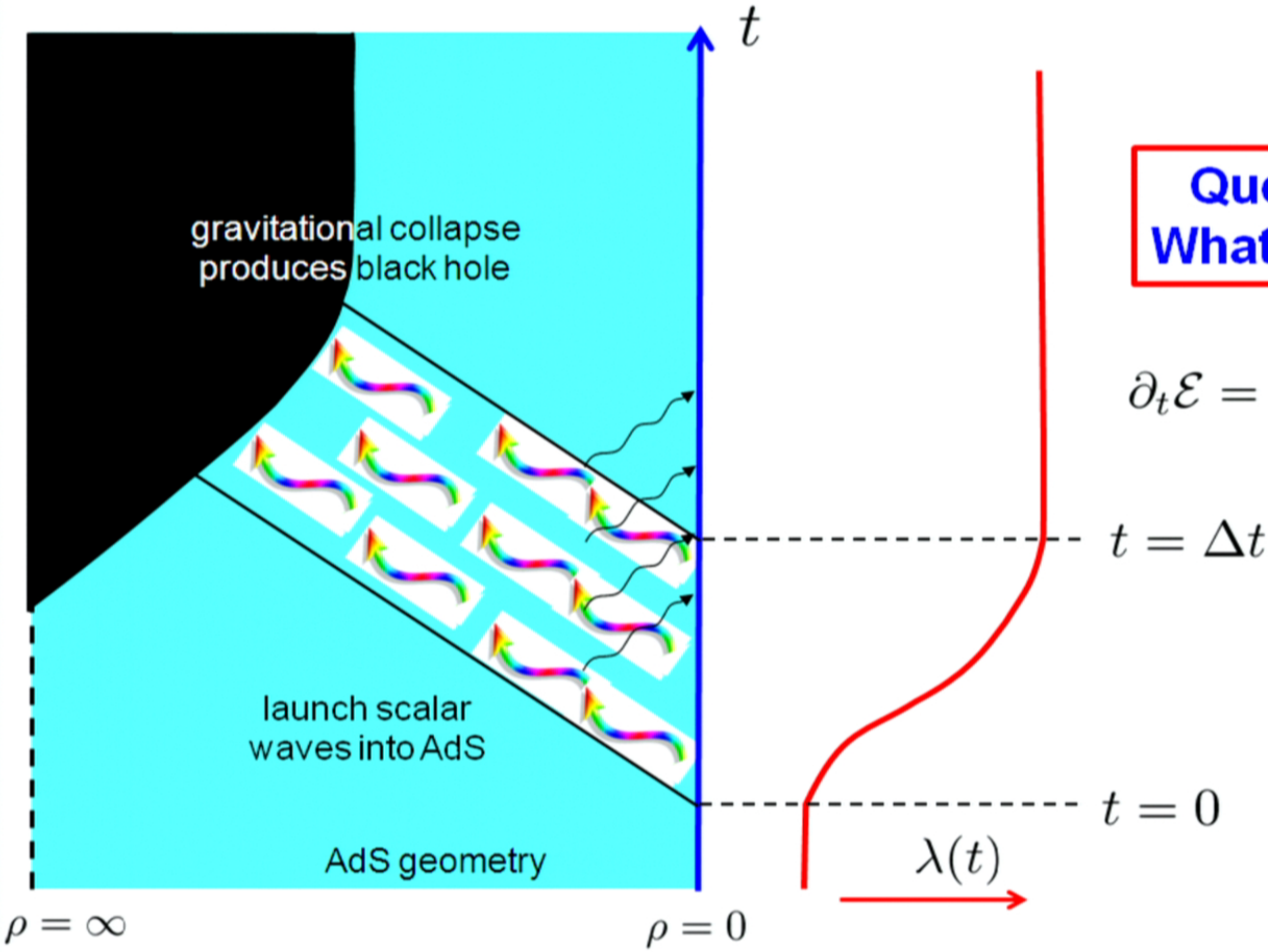
- seems to indicate instantaneous quench is problematic
  - ► physical problem?
  - ► simply an issue with perturbative expansion?
- compare to seminal work of, eg, Calabrese & Cardy

  ⟶ "instantaneous quench" is basic starting point

Holographic Quantum Quench (cartoon):



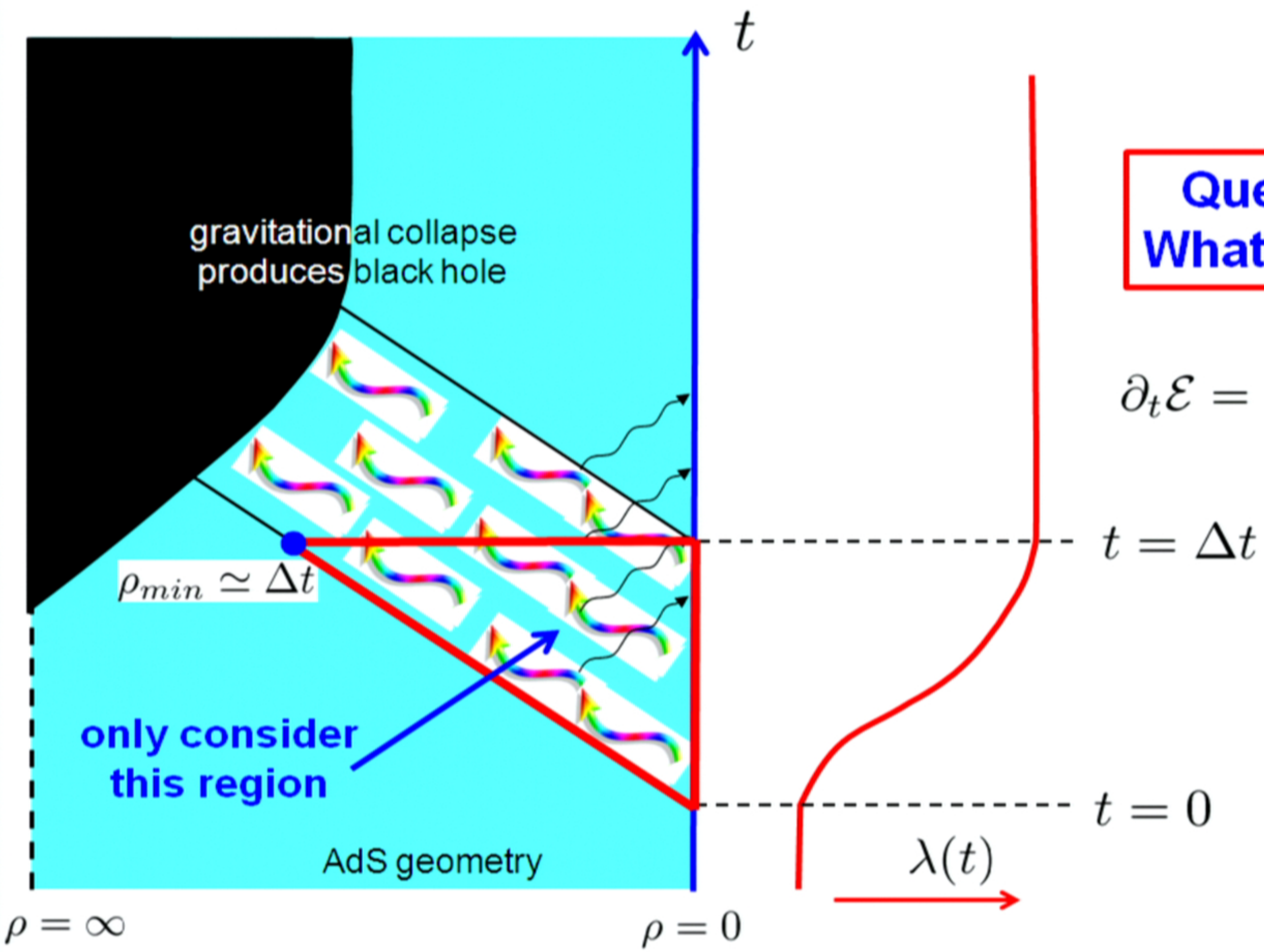
Holographic Quantum Quench (cartoon):



Question: What is $\Delta\mathcal{E}$?

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \; \partial_t \lambda$$

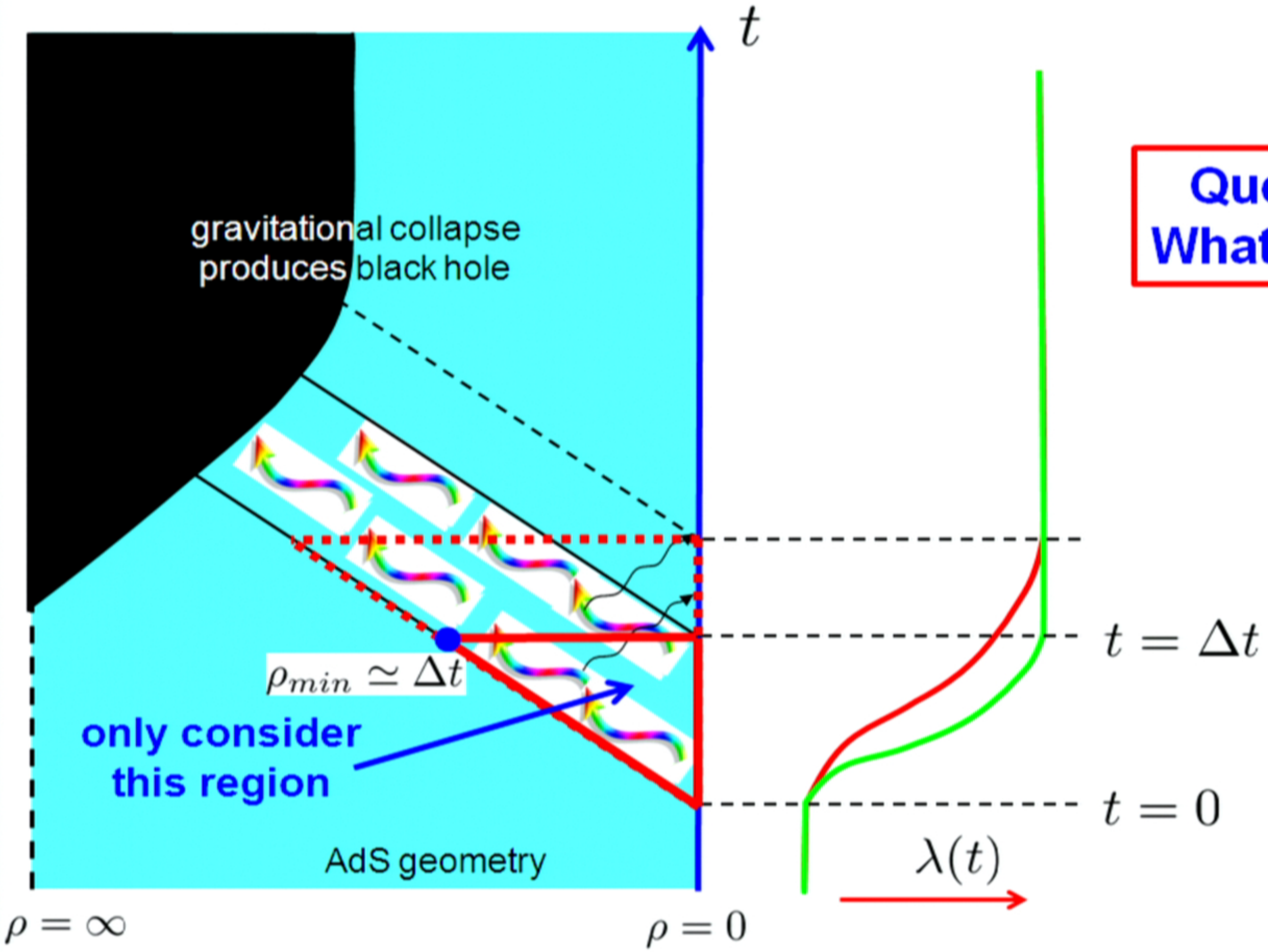
Holographic Quantum Quench (cartoon):



Question:
What is $\Delta\mathcal{E}$?

$$\partial_t \mathcal{E} = -\langle \mathcal{O}_\Delta \rangle \; \partial_t \lambda$$

Holographic Quantum Quench (cartoon):



Question:
What is $\Delta\mathcal{E}$?

Generalizing "Fast" Quenches:

→ **Question: What is $\Delta\mathcal{E}$?**

- focus: full details of evolution, eg, approach to final state, are not determined but allows us to understand scaling behaviour
- as we scale $\Delta t \to 0$, only "tiny" region of solution in asymptotic AdS relevant for this question

→ certainly full numerical simulations are not needed

Generalizing "Fast" Quenches:

→ **Question: What is $\Delta\mathcal{E}$?**

- focus: full details of evolution, eg, approach to final state, are not determined but allows us to understand scaling behaviour
- as we scale $\Delta t \to 0$, only "tiny" region of solution in **asymptotic** AdS relevant for this question

  → certainly full numerical simulations are not needed

  → solvable with purely analytic approach!!

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion) → nonlinearities unimportant!

Generalizing "Fast" Quenches:

→ **Question: What is $\Delta\mathcal{E}$?**

- focus: full details of evolution, eg, approach to final state, are not determined but allows us to understand scaling behaviour
- as we scale $\Delta t \to 0$, only "tiny" region of solution in **asymptotic** AdS relevant for this question

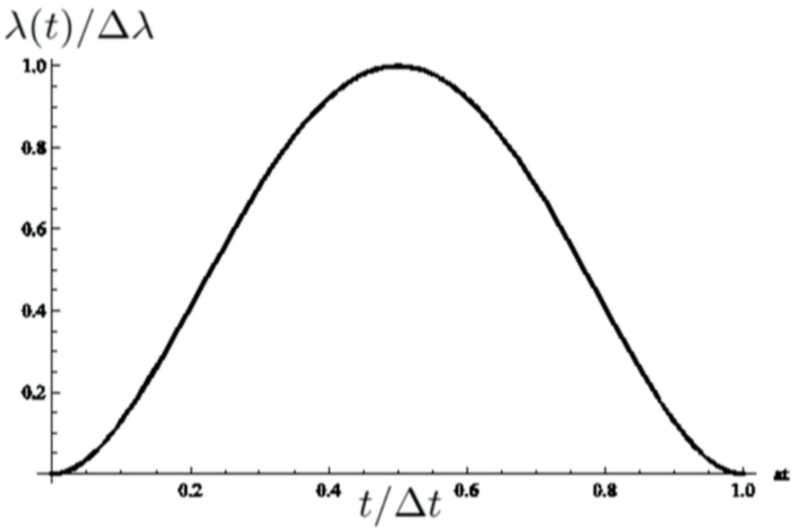
  → certainly full numerical simulations are not needed

  → solvable with purely analytic approach!!

- key: asymptotic fields in AdS decay in precise manner (ie, Fefferman-Graham expansion) → nonlinearities unimportant!
- **relevant solution = linearized scalar solution in (pure) AdS!**
- result $\Delta\mathcal{E}$ applies for full nonlinear solution!!

Generalizing "Fast" Quenches:

- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

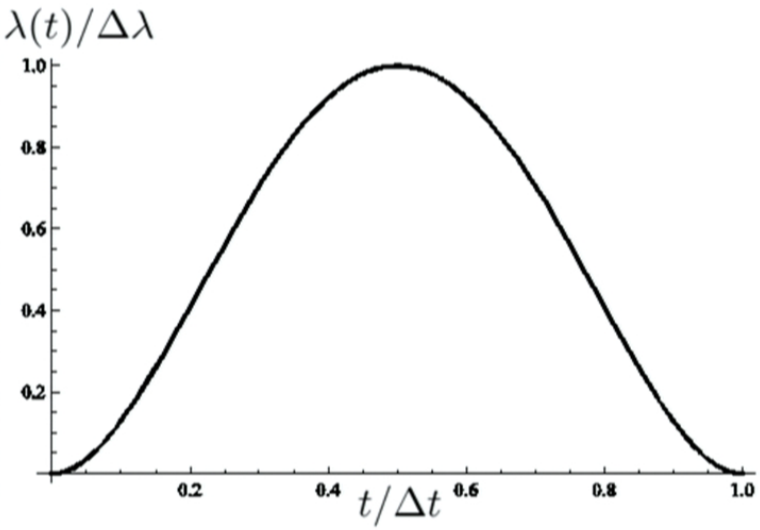


Generalizing "Fast" Quenches:

- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

$$\longrightarrow \langle \mathcal{O}_\Delta \rangle(t) = \frac{16\,\Delta\lambda}{(\Delta t)^{2\Delta - d}} \left(\frac{t}{\Delta t}\right)^{d-2\Delta} \left(b_2 \left(\frac{t}{\Delta t}\right)^2 - 2b_3 \left(\frac{t}{\Delta t}\right)^3 + b_4 \left(\frac{t}{\Delta t}\right)^4 \right)$$

where $b_\kappa = -\dfrac{2^{d-2\Delta}\,\Gamma(\kappa+1)\,\Gamma(\frac{d+2}{2} - \Delta)}{\Gamma(d+1+\kappa-2\Delta)\,\Gamma(\Delta - \frac{d-2}{2})}$

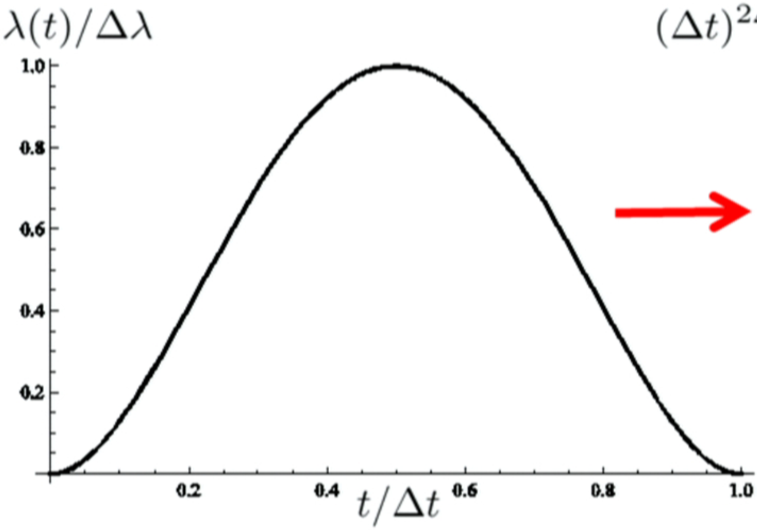


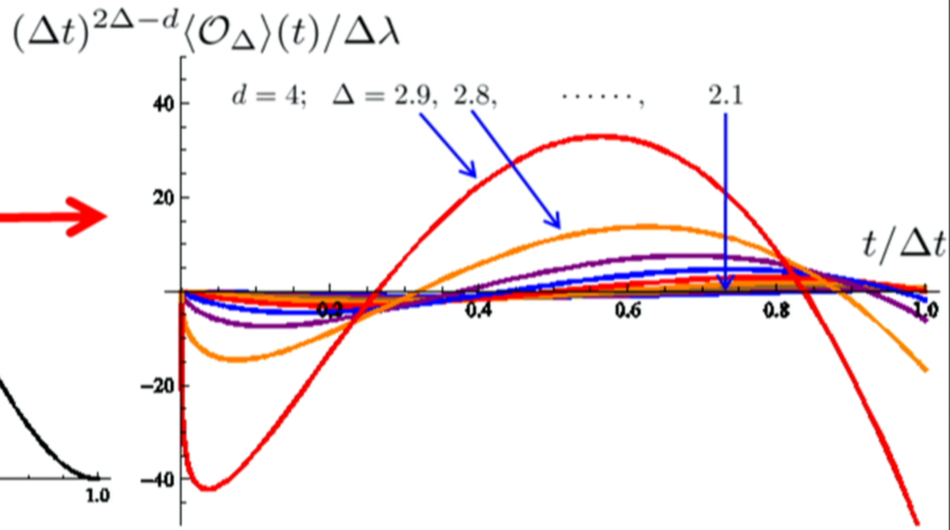
Generalizing "Fast" Quenches:

- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

$$\rightarrow \langle \mathcal{O}_\Delta \rangle(t) = \frac{16\,\Delta\lambda}{(\Delta t)^{2\Delta-d}} \left(\frac{t}{\Delta t}\right)^{d-2\Delta} \left(b_2 \left(\frac{t}{\Delta t}\right)^2 - 2b_3 \left(\frac{t}{\Delta t}\right)^3 + b_4 \left(\frac{t}{\Delta t}\right)^4 \right)$$

where $b_\kappa = -\dfrac{2^{d-2\Delta}\,\Gamma(\kappa+1)\,\Gamma(\frac{d+2}{2}-\Delta)}{\Gamma(d+1+\kappa-2\Delta)\,\Gamma(\Delta-\frac{d-2}{2})}$



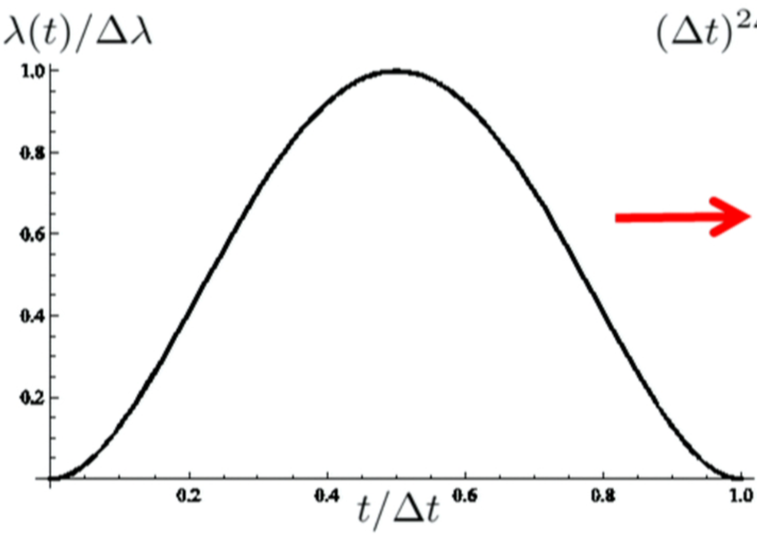


Generalizing "Fast" Quenches:

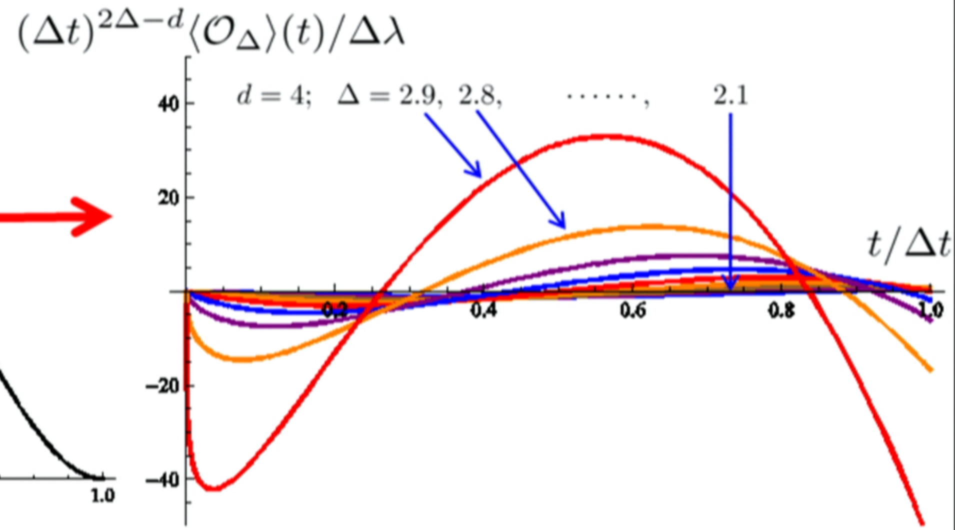
- analytic solutions, eg: $\lambda(t) = 16\Delta\lambda\big((t/\Delta t)^2 - 2(t/\Delta t)^3 + (t/\Delta t)^4\big)$

$$\rightarrow \langle \mathcal{O}_\Delta\rangle(t) = \frac{16\,\Delta\lambda}{(\Delta t)^{2\Delta-d}}\left(\frac{t}{\Delta t}\right)^{d-2\Delta}\left(b_2\left(\frac{t}{\Delta t}\right)^2 - 2b_3\left(\frac{t}{\Delta t}\right)^3 + b_4\left(\frac{t}{\Delta t}\right)^4\right)$$

where $b_\kappa = -\dfrac{2^{d-2\Delta}\,\Gamma(\kappa+1)\,\Gamma(\frac{d+2}{2}-\Delta)}{\Gamma(d+1+\kappa-2\Delta)\,\Gamma(\Delta-\frac{d-2}{2})}$



$\rightarrow$



Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- **relevant solution = linearized scalar solution in AdS space!**
- general scaling $\langle \mathcal{O}_\Delta \rangle \sim \Delta\lambda/(\Delta t)^{2\Delta-d}$ with holographic dictionary, ie, "energy conservation":

$$\Delta\mathcal{E} \simeq \int_0^{\Delta t} dt\, \langle \mathcal{O}_\Delta \rangle\, \partial_t \lambda$$

$$\Rightarrow \quad \langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}} \; ; \qquad \Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}$$

- matches previous perturbative numerical calc's (for d=4)

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- **relevant solution = linearized scalar solution in AdS space!**
- general scaling $\langle \mathcal{O}_\Delta \rangle \sim \Delta\lambda/(\Delta t)^{2\Delta-d}$ with holographic dictionary, ie, "energy conservation":

$$\Delta\mathcal{E} \simeq \int_0^{\Delta t} dt\, \langle \mathcal{O}_\Delta \rangle\, \partial_t \lambda$$

$$\Rightarrow \quad \langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}} \; ; \qquad \Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}$$

- matches previous perturbative numerical calc's (for d=4)
- result here applies for full nonlinear solution!!
  - ► physical problem?
  - ► simply an issue with perturbative expansion?

Generalizing "Fast" Quenches:

- as we scale $\Delta t \to 0$, only "tiny" region in asymptotic AdS relevant
- **relevant solution = linearized scalar solution in AdS space!**
- general scaling $\langle \mathcal{O}_\Delta \rangle \sim \Delta\lambda/(\Delta t)^{2\Delta - d}$ with holographic dictionary, ie, "energy conservation":

$$\Delta\mathcal{E} \simeq \int_0^{\Delta t} dt\, \langle \mathcal{O}_\Delta \rangle\, \partial_t \lambda$$

$$\Rightarrow \quad \langle \mathcal{O}_\Delta \rangle \sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta - d}} \; ; \qquad \Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta - d}}$$

- matches previous perturbative numerical calc's (for d=4)
- result here applies for full nonlinear solution!!
  - ► physical ~~problem?~~ effect
  - ~~► simply an issue with perturbative expansion?~~

Generalizing "Fast" Quenches:

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta - d}}$$

- $\Delta t \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

  $\longrightarrow$ "instantaneous" quench seems problematic!?!

Generalizing "Fast" Quenches:

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}$$

- $\Delta t \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

  ⟶ "instantaneous" quench seems problematic!?!

- can consider various scaling limits:

  ⟶ $\Delta t = \alpha\Delta t_0\,;\ \Delta\lambda = \alpha^{\Delta-d/2}\Delta\lambda_0$

  as $\alpha \to 0$, $\Delta\mathcal{E}$ finite but $\langle\mathcal{O}_\Delta\rangle$ divergent

  ⟶ $\Delta t = \alpha\Delta t_0\,;\ \Delta\lambda = \alpha^{2\Delta-d}\Delta\lambda_0$

  as $\alpha \to 0$, $\langle\mathcal{O}_\Delta\rangle$ finite but $\Delta\mathcal{E}$ vanishes

  but would not be "standard" protocol

Generalizing "Fast" Quenches:

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}$$

- $\Delta t \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

  ⟶ "instantaneous" quench seems problematic!?!

- can consider various scaling limits:

  ⟶ $\Delta t = \alpha\Delta t_0\,;\ \Delta\lambda = \alpha^{\Delta-d/2}\Delta\lambda_0$

  as $\alpha \to 0$, $\Delta\mathcal{E}$ finite but $\langle\mathcal{O}_\Delta\rangle$ divergent

  ⟶ $\Delta t = \alpha\Delta t_0\,;\ \Delta\lambda = \alpha^{2\Delta-d}\Delta\lambda_0$

  as $\alpha \to 0$, $\langle\mathcal{O}_\Delta\rangle$ finite but $\Delta\mathcal{E}$ vanishes

  but would not be "standard" protocol

- operators in range $\frac{d}{2} - 1 \leq \Delta < \frac{d}{2}$ seem to be okay
- note UV fixed point, ie, CFT, is source of divergence
- strongly coupled holographic QFT versus free fields???

Generalizing "Fast" Quenches:

$$\Delta\mathcal{E} \sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-d}}$$

- $\Delta t \to 0$ yields physical divergence for $\frac{d}{2} < \Delta < d$

  ⟶ "instantaneous" quench seems problematic!?!

- can consider various scaling limits:

  ⟶ $\Delta t = \alpha \Delta t_0\,;\ \Delta\lambda = \alpha^{\Delta-d/2}\Delta\lambda_0$

  as $\alpha \to 0$, $\Delta\mathcal{E}$ finite but $\langle\mathcal{O}_\Delta\rangle$ divergent

  ⟶ $\Delta t = \alpha \Delta t_0\,;\ \Delta\lambda = \alpha^{2\Delta-d}\Delta\lambda_0$

  as $\alpha \to 0$, $\langle\mathcal{O}_\Delta\rangle$ finite but $\Delta\mathcal{E}$ vanishes

  but would not be "standard" protocol

- operators in range $\frac{d}{2} - 1 \leq \Delta < \frac{d}{2}$ seem to be okay
- note UV fixed point, ie, CFT, is source of divergence
- strongly coupled holographic QFT versus free fields???

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda = m^2 \; ; \qquad \mathcal{O}_\Delta = \phi^2 \; ; \qquad \Delta = d - 2$$

- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $\left[\nabla^2 - \frac{m^2}{2}\left(1 + \tanh(t/\Delta t)\right)\right]\phi = 0$

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda = m^2 \;; \qquad \mathcal{O}_\Delta = \phi^2 \;; \qquad \Delta = d-2$$

- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $$\left[\nabla^2 - \frac{m^2}{2}\left(1+\tanh(t/\Delta t)\right)\right]\phi = 0$$

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda = m^2 \ ; \qquad \mathcal{O}_\Delta = \phi^2 \ ; \qquad \Delta = d-2$$

- quench with finite Δt and examine limit $\Delta t \to 0$

eq. of motion: $$\left[\nabla^2 - \frac{m^2}{2}\left(1+\tanh(t/\Delta t)\right)\right]\phi = 0$$

- example in: Birrell & Davies, "*Quantum Fields in Curved Space*"

eg, "in" modes:

$$f_k(t) = \frac{1}{\sqrt{4\pi k}} \exp\left[-i(\omega_+ + \omega_- \Delta t \log(2\cosh(t/\Delta t))\right] \times {}_2F_1\left(1+i\omega_-\Delta t, i\omega_-\Delta t, 1-ik\Delta t; \frac{1}{2}(1+\tanh(t/\Delta t)\right)$$

with $\omega_\pm = \frac{1}{2}\left(\pm k + \sqrt{k^2+m^2}\right)$

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda(t) = m^2(t) = \frac{m^2}{2}\left[1 + \tanh(t/\Delta t)\right] \; ;$$

$$\mathcal{O}_\Delta = \phi^2 \; ; \qquad\qquad \Delta = d - 2$$

- given individual modes, consider two point correlator

$$G_k(t_1, t_2) = {}_{in}\langle 0|\, \phi_k(t_1)\, \phi_{-k}(t_2)\, |0\rangle_{in}$$

Generalizing "Fast" Quenches:

- compare directly to C&C, ie, quench mass of a free scalar:

$$\lambda(t) = m^2(t) = \frac{m^2}{2}\left[1 + \tanh(t/\Delta t)\right] \; ;$$

$$\mathcal{O}_\Delta = \phi^2 \; ; \qquad\qquad \Delta = d - 2$$

- given individual modes, consider two point correlator

$$G_k(t_1, t_2) = {}_{in}\langle 0|\, \phi_k(t_1)\, \phi_{-k}(t_2)\, |0\rangle_{in}$$

- yields simple result in the limit $\Delta t \to 0$:

$$G_k(t_1, t_2) \quad \longrightarrow \quad \frac{1}{4\pi\omega}\, e^{-i\omega(t_1 - t_2)} + \frac{(\omega - \omega_0)^2}{8\pi\omega_0\omega}\, \cos\omega(t_1 - t_2)$$
$$+ \frac{\omega^2 - \omega_0^2}{8\pi\omega_0\omega}\, \cos\omega(t_1 + t_2)$$

$$(\, \omega = \sqrt{k^2 + m^2}\,, \;\; \omega_0 = k\,)$$

Generalizing "Fast" Quenches:

- consider response: $\langle\phi^2\rangle \simeq \int_0^{k_{max}} dk\, k^{d-3}\, |{}_2F_1|^2$

Generalizing "Fast" Quenches:

- consider response: $\langle \phi^2 \rangle \simeq \int_0^{k_{max}} dk\, k^{d-3}\, |{}_2F_1|^2$
- following holographic example, UV divergences are removed by adding appropriate counterterms in effective action

Generalizing "Fast" Quenches:

- consider response: $\langle \phi^2 \rangle \simeq \int_0^{k_{max}} dk\, k^{d-3}\, |{}_2F_1|^2$
- following holographic example, UV divergences are removed by adding appropriate counterterms in effective action
- UV divergences: eg, consider a constant mass

$$\langle \phi^2 \rangle \simeq \int_0^{k_{max}} dk\, \frac{k^{d-2}}{\sqrt{k^2+m^2}} = \int_0^{k_{max}} dk\, \left[k^{d-3} - \frac{1}{2} m^2 k^{d-5} + \cdots \right]$$
$$= \frac{1}{d-2} k_{max}^{d-2} - \frac{m^2}{2(d-4)} k_{max}^{d-4} + \cdots$$

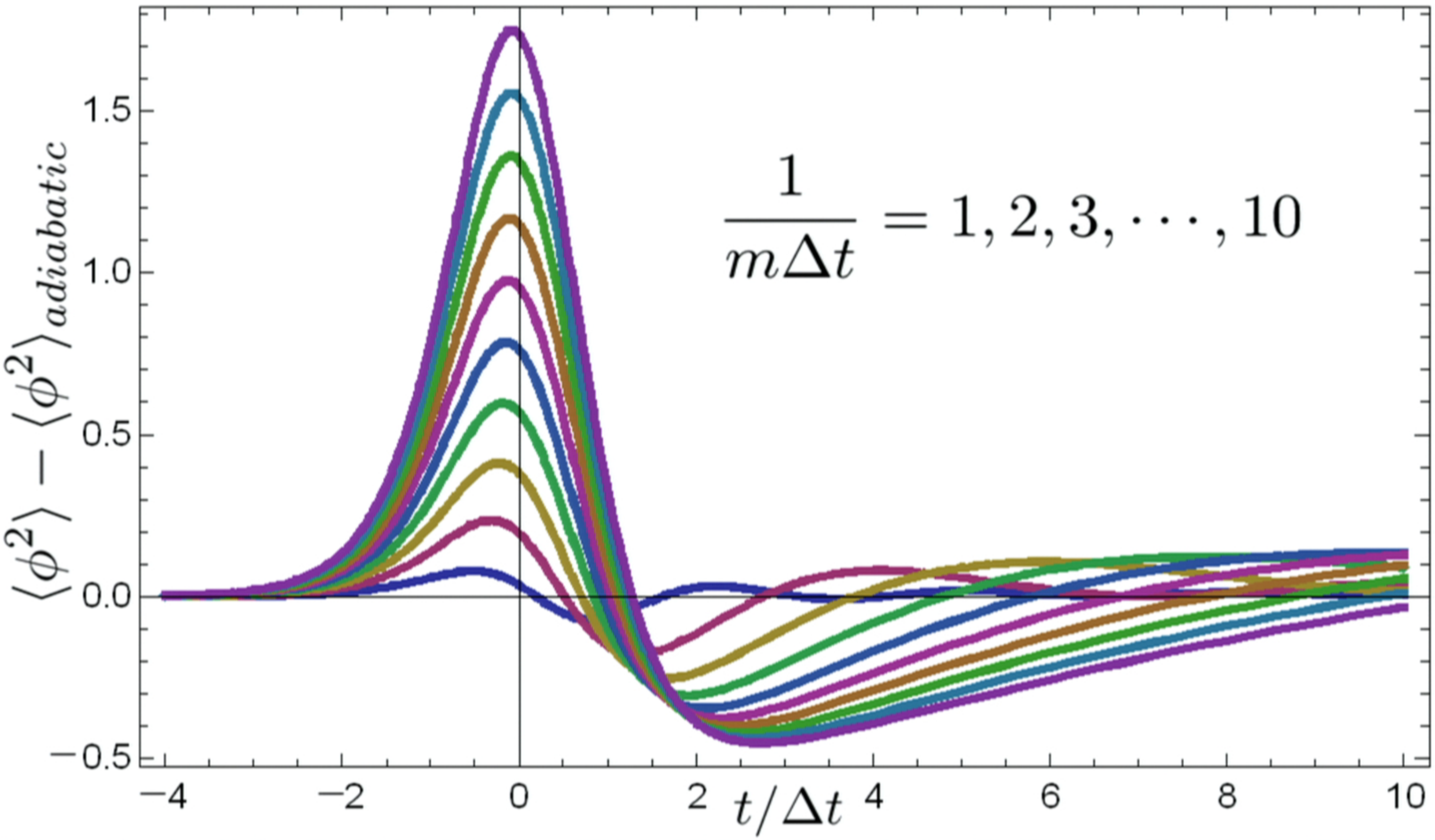
- regulated response (d=5):

$$\langle \phi^2 \rangle \simeq \int_0^{\infty} dk\, \left[k^2\, |{}_2F_1|^2 - k^2 + \frac{1}{2} m^2(t) \right]$$

where $m^2(t) = \frac{m^2}{2}\left[1 + \tanh(t/\Delta t)\right]$

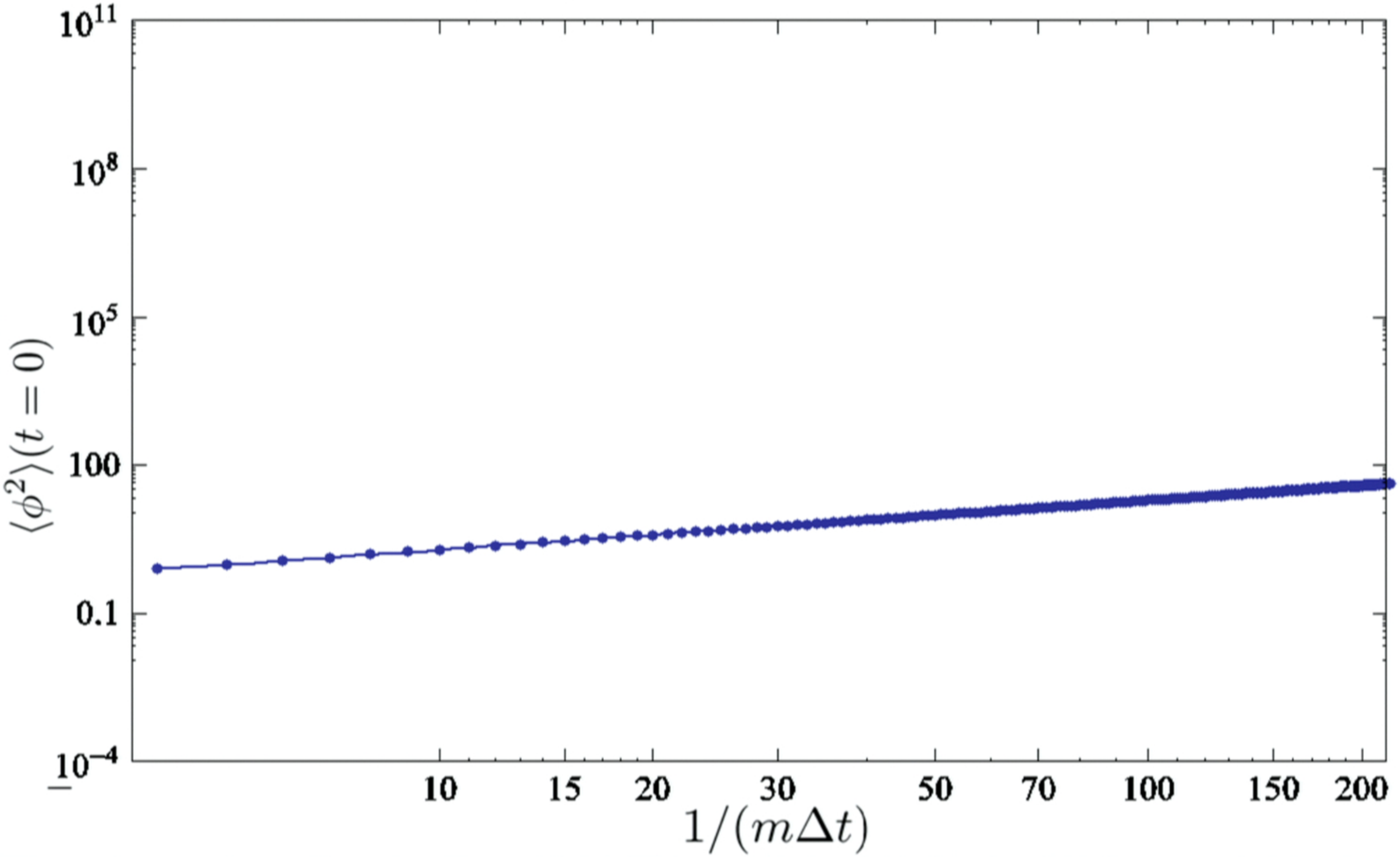
Generalizing "Fast" Quenches:

- regulated response (d=5): $\langle\phi^2\rangle \simeq \int_0^\infty dk \left[k^2\,|{}_2F_1|^2 - k^2 - \frac{1}{2}m^2(t)\right]$



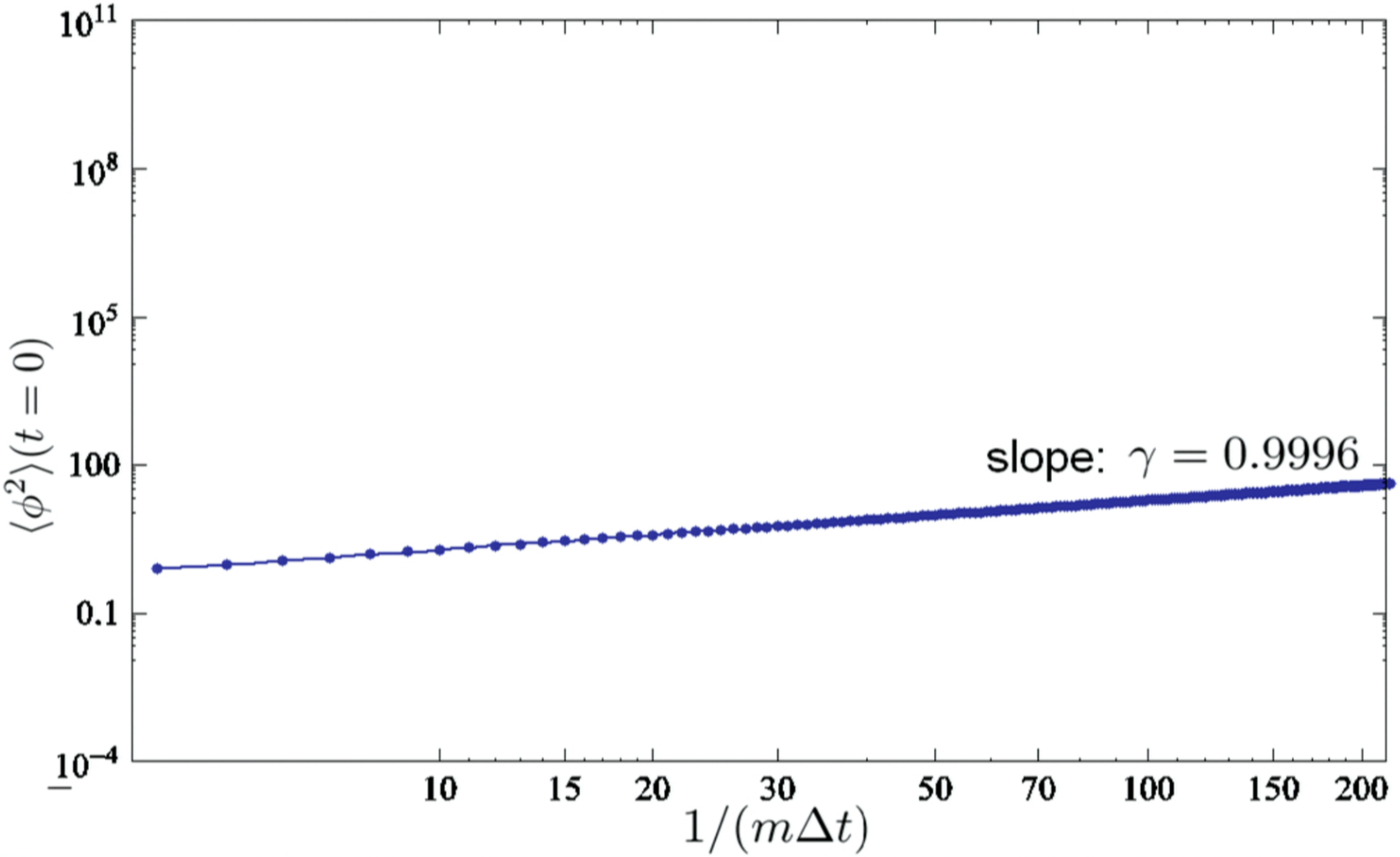
Generalizing "Fast" Quenches:

- regulated response (d=5):



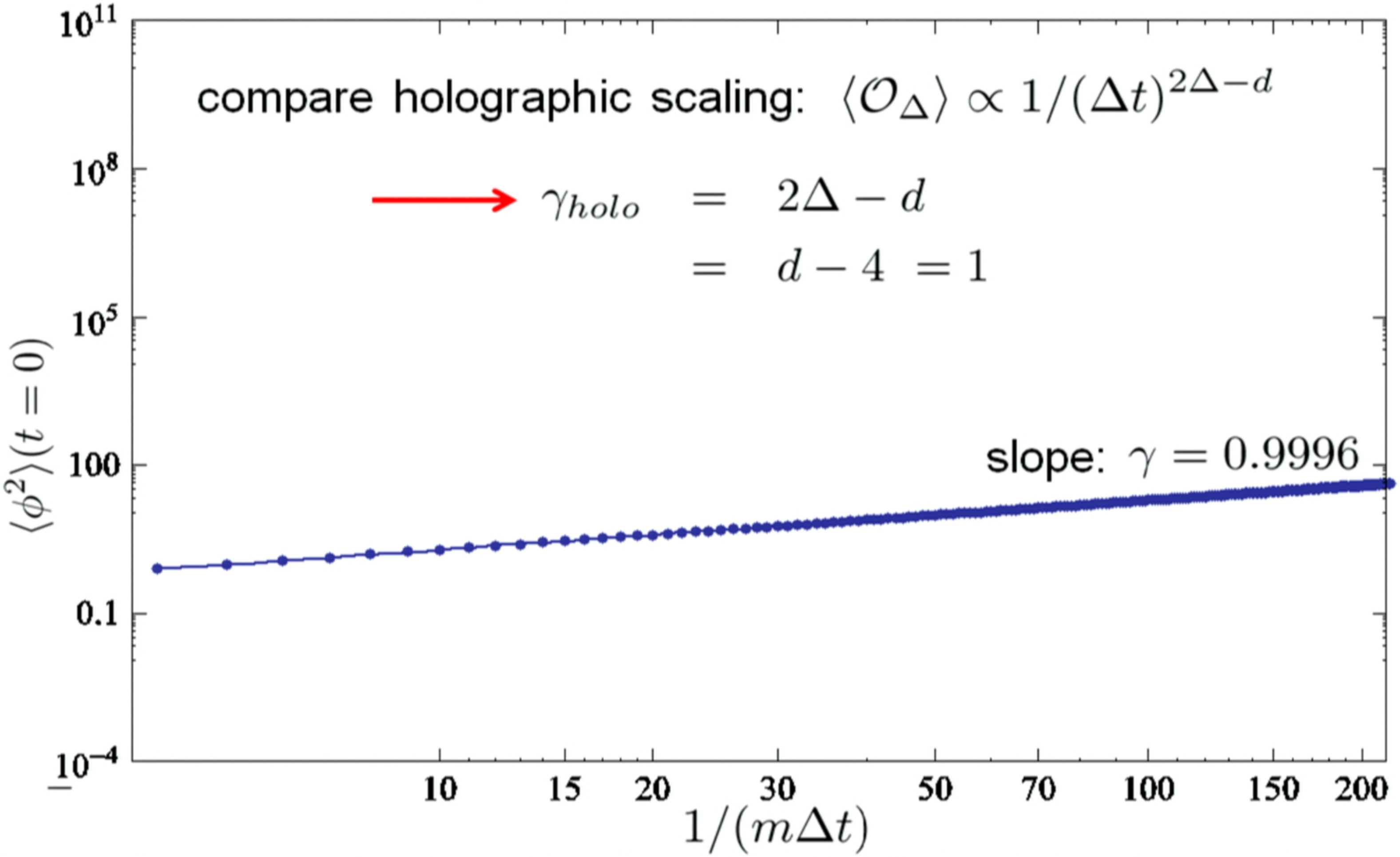
Generalizing "Fast" Quenches:

- regulated response (d=5):



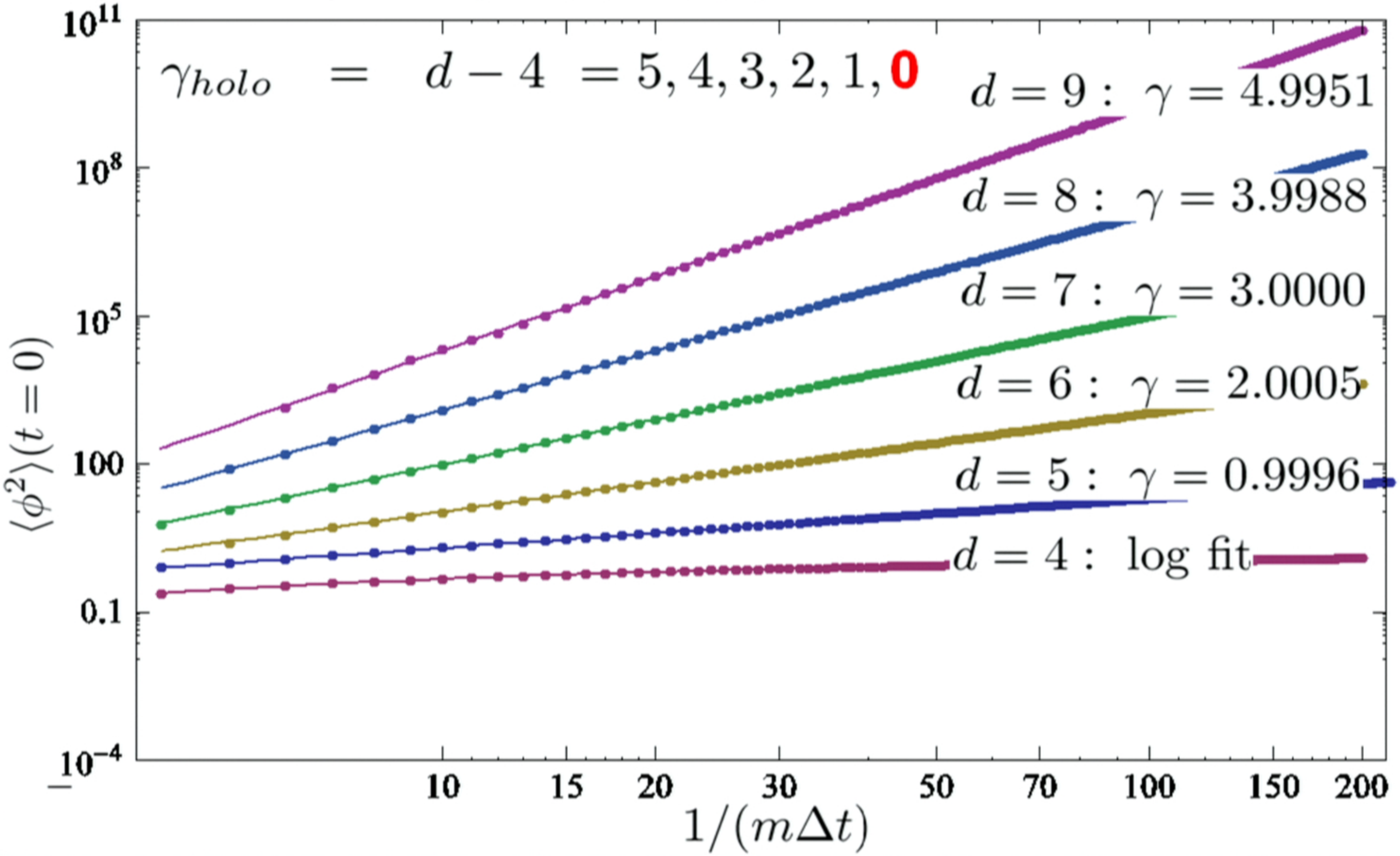
Generalizing "Fast" Quenches:

- regulated response (d=5):



Generalizing "Fast" Quenches:

- regulated response (d=9,8,7,6,5,4):



Generalizing “Fast” Quenches:

→ why is holographic scaling reproduced by free field?!?!?

Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^dx\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\; f(t/\Delta t)$
- apply conformal perturbation theory

$$\begin{aligned}
\langle \mathcal{O}_\Delta(0) \rangle &= \langle \mathcal{O}_\Delta(0) \exp[i \int d^dx \lambda(t) \mathcal{O}_\Delta(x)] \rangle_{\text{CFT}} \\
&= \cancel{\langle \mathcal{O}_\Delta(0) \rangle_{\text{CFT}}} + i\Delta\lambda \langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t) \mathcal{O}_\Delta(x) \rangle_{\text{CFT}} \\
&\quad - \frac{\Delta\lambda^2}{2} \langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t) \mathcal{O}_\Delta(x) \int d^dx' f(t'/\Delta t) \mathcal{O}_\Delta(x') \rangle_{\text{CFT}} + \cdots \\
&= b_1 \frac{\Delta\lambda}{(\Delta t)^{2\Delta - d}} + b_2 \frac{\Delta\lambda^2}{(\Delta t)^{3\Delta - 2d}} + \cdots
\end{aligned}$$

Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^dx\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\ f(t/\Delta t)$

- apply conformal perturbation theory

$$\begin{aligned}
\langle \mathcal{O}_\Delta(0)\rangle &= \langle \mathcal{O}_\Delta(0)\, \exp[i\int d^dx \lambda(t)\mathcal{O}_\Delta(x)]\rangle_{\rm CFT} \\
&= \cancel{\langle \mathcal{O}_\Delta(0)\rangle_{\rm CFT}} + i\Delta\lambda \langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x)\rangle_{\rm CFT} \\
&\quad - \frac{\Delta\lambda^2}{2}\langle \mathcal{O}_\Delta(0) \int d^dx f(t/\Delta t)\mathcal{O}_\Delta(x) \int d^dx' f(t'/\Delta t)\mathcal{O}_\Delta(x')\rangle_{\rm CFT} + \cdots \\
&= b_1\, \frac{\Delta\lambda}{(\Delta t)^{2\Delta-d}} + b_2\, \frac{\Delta\lambda^2}{(\Delta t)^{3\Delta-2d}} + \cdots
\end{aligned}$$

- organized with dimensionless effective coupling: $g = \Delta\lambda\, (\Delta t)^{d-\Delta}$

Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^d x \, \lambda(t) \, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda \; f(t/\Delta t)$
- apply conformal perturbation theory

$$
\begin{aligned}
\langle \mathcal{O}_\Delta(0) \rangle &= \langle \mathcal{O}_\Delta(0) \exp[i \int d^d x \lambda(t) \mathcal{O}_\Delta(x)] \rangle_{\rm CFT} \\
&= \cancel{\langle \mathcal{O}_\Delta(0) \rangle_{\rm CFT}} + i\Delta\lambda \langle \mathcal{O}_\Delta(0) \int d^d x f(t/\Delta t) \mathcal{O}_\Delta(x) \rangle_{\rm CFT} \\
&\quad - \frac{\Delta\lambda^2}{2} \langle \mathcal{O}_\Delta(0) \int d^d x f(t/\Delta t) \mathcal{O}_\Delta(x) \int d^d x' f(t'/\Delta t) \mathcal{O}_\Delta(x') \rangle_{\rm CFT} + \cdots \\
&= \frac{1}{(\Delta t)^\Delta} \left(b_1 \, g + \; b_2 \, g^2 + \; \cdots \right)
\end{aligned}
$$

- organized with dimensionless effective coupling: $g = \Delta\lambda \, (\Delta t)^{d-\Delta}$

Generalizing "Fast" Quenches:

→ why is holographic scaling reproduced by free field?!?!?

- consider $S = S_{CFT} + \int d^d x\, \lambda(t)\, \mathcal{O}_\Delta(x)$ with $\lambda(t) = \Delta\lambda\ f(t/\Delta t)$

- apply conformal perturbation theory

$$\begin{aligned}
\langle \mathcal{O}_\Delta(0) \rangle &= \langle \mathcal{O}_\Delta(0) \exp[i \int d^d x \lambda(t) \mathcal{O}_\Delta(x)] \rangle_{\rm CFT} \\
&= \cancel{\langle \mathcal{O}_\Delta(0) \rangle_{\rm CFT}} + i\Delta\lambda \langle \mathcal{O}_\Delta(0) \int d^d x f(t/\Delta t) \mathcal{O}_\Delta(x) \rangle_{\rm CFT} \\
&\quad - \frac{\Delta\lambda^2}{2} \langle \mathcal{O}_\Delta(0) \int d^d x f(t/\Delta t) \mathcal{O}_\Delta(x) \int d^d x' f(t'/\Delta t) \mathcal{O}_\Delta(x') \rangle_{\rm CFT} + \cdots \\
&= \frac{1}{(\Delta t)^\Delta} \left(b_1\, g + b_2\, g^2 + \cdots \right)
\end{aligned}$$

- organized with dimensionless effective coupling: $g = \Delta\lambda\, (\Delta t)^{d-\Delta}$

- in limit $\Delta\lambda$ fixed and $\Delta t \to 0$: $g \to 0$ **!!**

→ leading term dominates: $\langle \mathcal{O}_\Delta(0) \rangle \simeq b_1 \dfrac{\Delta\lambda}{(\Delta t)^{2\Delta - d}}$

Generalizing "Fast" Quenches:

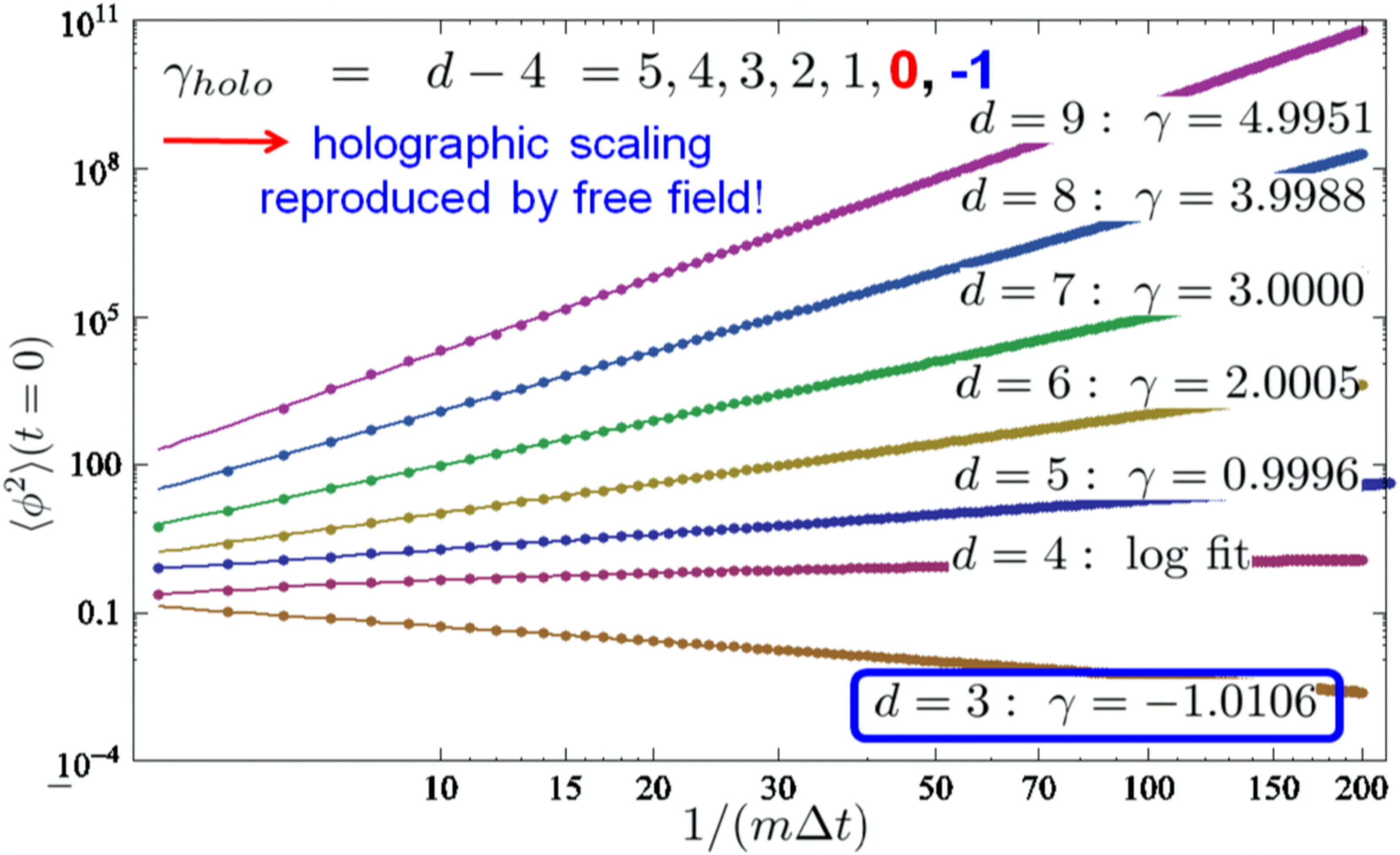
- **holographic scaling should appear quite generally!!**

- for example:

$$\left.\begin{aligned} \langle \mathcal{O}_\Delta \rangle &\sim \frac{\Delta\lambda}{(\Delta t)^{2\Delta-4}} \\ \Delta\mathcal{E} &\sim \frac{\Delta\lambda^2}{(\Delta t)^{2\Delta-4}} \end{aligned}\right\} \quad \Delta t \to 0$$

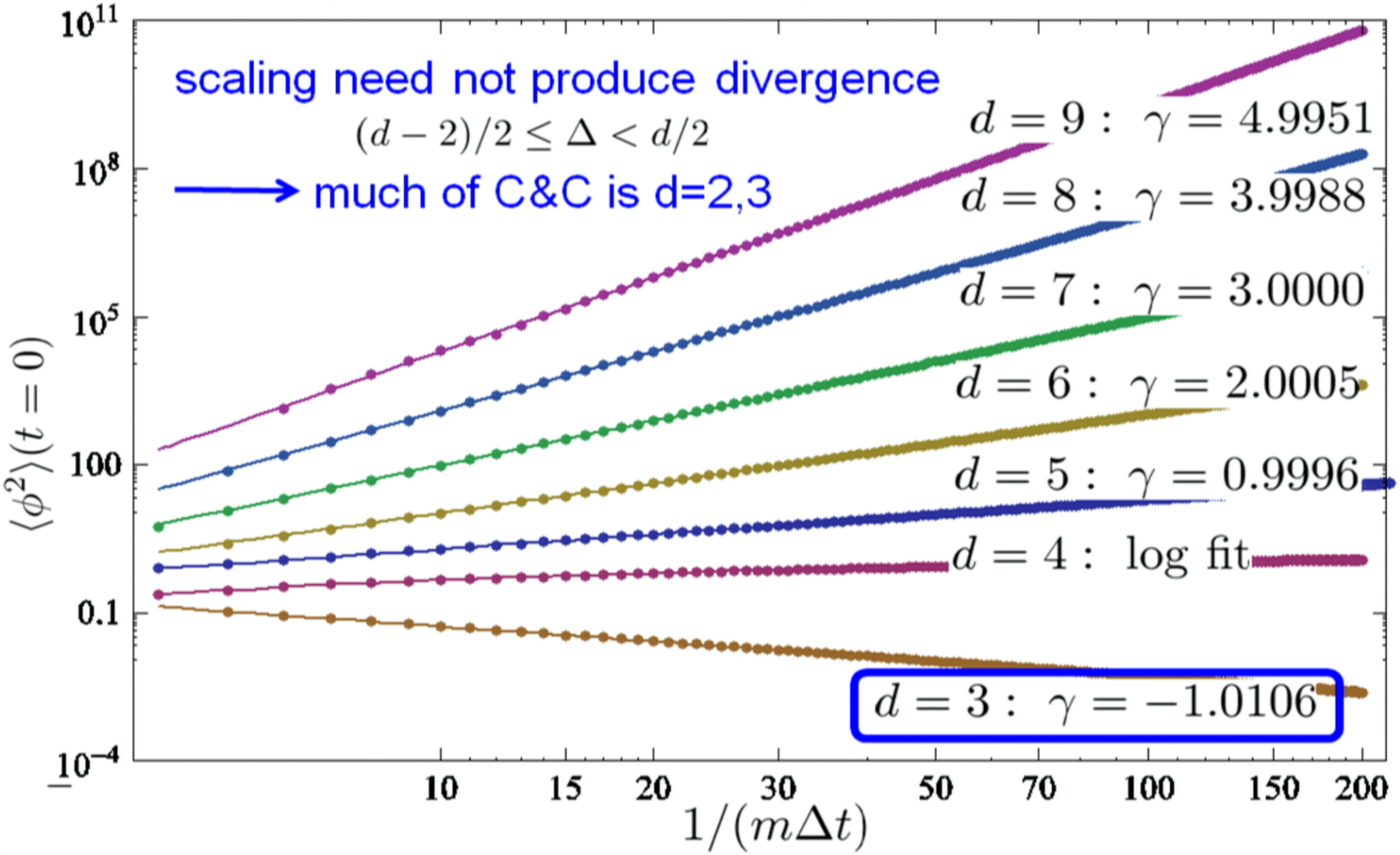
Generalizing "Fast" Quenches:

• regulated response (d=9,8,7,6,5,4,**3**):



Generalizing "Fast" Quenches:

- regulated response (d=9,8,7,6,5,4,**3**):



Conclusions:

- quantum quenches: interesting arena for holographic study
- lessons learned:
  1. Renormalization of (strongly coupled) boundary QFT with time-dependent couplings works in a straightforward way
  2. Response to fast quenches exhibits universal scaling

► much of fast holographic quenches analytically accessible

► both lessons 1 & 2 apply beyond holographic arena!!

Lots to explore!