

Title: Many-body localization: entanglement, emergent conservation laws, and the structure of eigenstates

Date: Nov 07, 2013 03:00 PM

URL: <http://pirsa.org/13110073>

Abstract:

Many-body localization: local conservation laws, dynamics, and entanglement

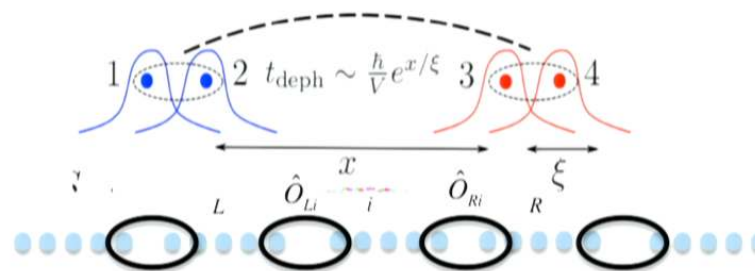
Collaborators

Max Serbyn

Zlatko Papić

Dima Abanin

Perimeter Institute



Based on: Phys. Rev. Lett. 110, 260601 (2013); Phys. Rev. Lett. 111, 127201 (2013)

Perimeter-Urbana Workshop
Waterloo, November 7, 2013

Many-body localization: local conservation laws, dynamics, and entanglement

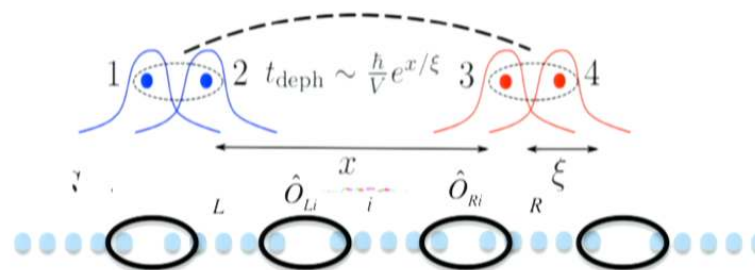
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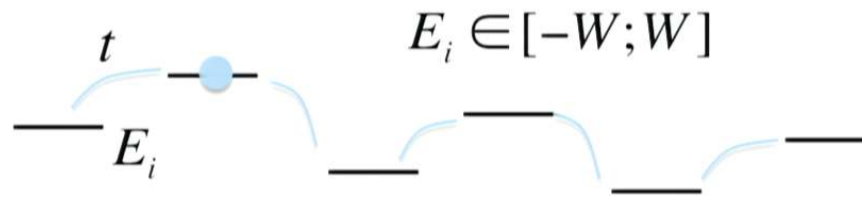


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Single-particle Anderson localization

One quantum particle in 1D
disordered crystal



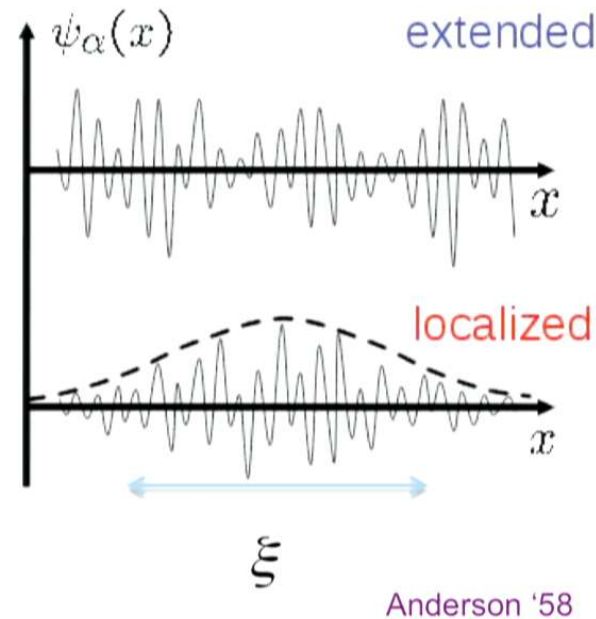
Wave functions become **localized**

$$\psi(r) \sim \exp(-r / \xi)$$

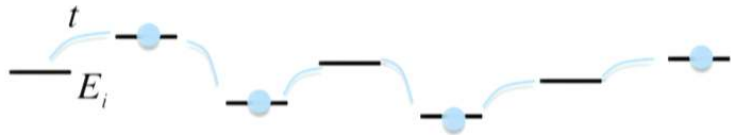
Contrast to extended Bloch states

Absence of diffusion \rightarrow Anderson
insulator

Origin: quantum interference
enhancement of backscattering



Many interacting particles in a disorder potential



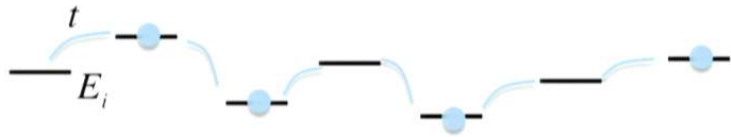
The diagram shows a 1D lattice of sites represented by blue dots. A hopping parameter t is indicated by a blue arrow between two adjacent sites. The energy levels are shown as horizontal lines, with the energy of the i -th site labeled E_i . The disorder potential V is represented by a horizontal axis with a tick mark at W .

$$H = \sum_i E_i c_i^\dagger c_i + t \sum_i c_i^\dagger c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

Zero interactions

- Anderson localization
- No diffusion
- Locally, spectrum looks discrete

Many interacting particles in a disorder potential



The diagram shows a 1D lattice of sites represented by blue dots. Horizontal lines represent energy levels E_i at each site. Curved arrows labeled t indicate hopping between adjacent sites. The energy levels are shown to be discrete and localized.

$$H = \sum_i E_i c_i^\dagger c_i + t \sum_i c_i^\dagger c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$



Zero interactions

Strong interactions

What happens at $V \sim W$?

Does localization survive at small V ?

Localization-delocalization phase transition?

-And

-No dimension

-Locally, spectrum looks discrete

-Fast transport of energy and particle number

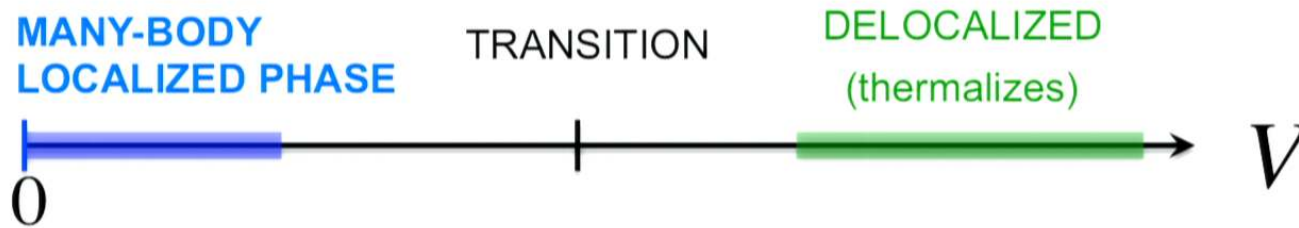
-Locally continuous spectrum

Many-body localization

Anderson, Fleishman'80;
Basko, Aleiner, Altshuler'05
Gornyi, Mirlin, Polyakov'05

Hypothesis: localization survives at $V \ll W$

Basic argument: matrix element for decay of a localized quasiparticle into many particle-hole pairs decays faster than the number of available final configurations. Far resonances absent.

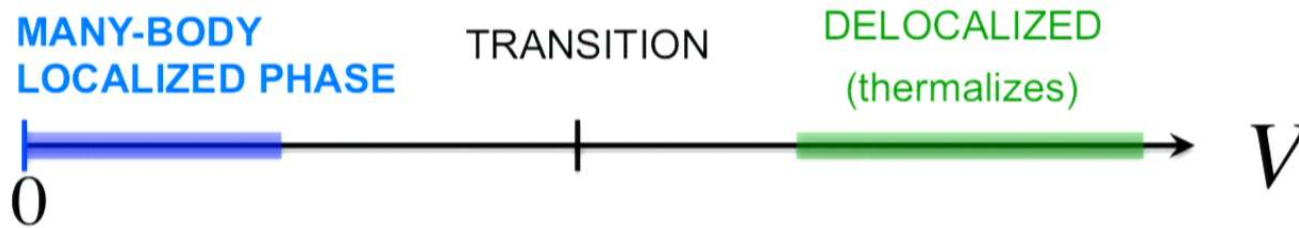


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Supported by numerics
(exact diagonalization)

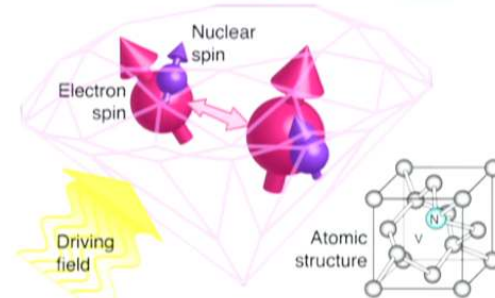
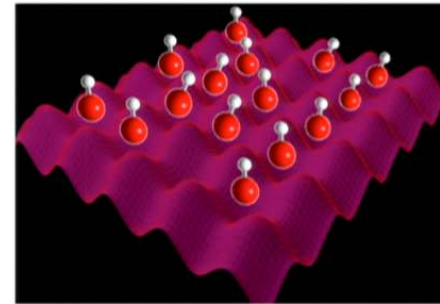
Oganesyan, Huse'07; Znidaric, Prosen'08; Pal, Huse'10;
Monthus, Garel '10, ...

Experimental developments

-In semiconductors, difficult to disentangle effects of e-e interactions from phonons (variable-range hopping) → MBL out of reach

-**Recently:** Isolated, quantum-coherent systems with controllable disorder available

-Cold atoms, polar molecules, spin systems (NV-centers in diamond)



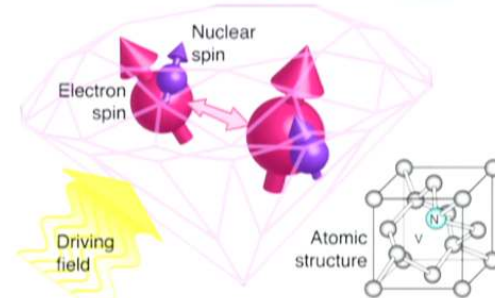
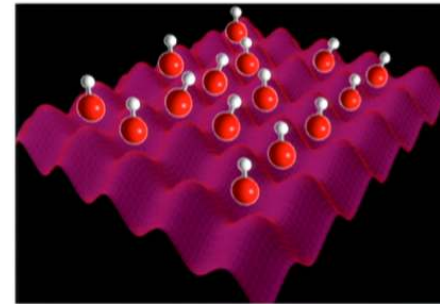
Studying many-body localization experimentally now possible

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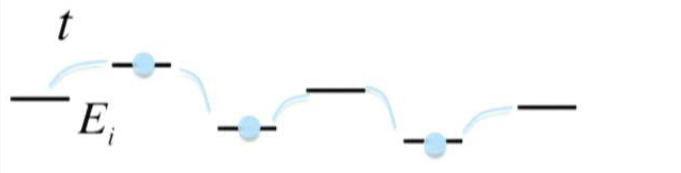


Studying many-body localization experimentally now possible

Model: 1D disordered chain

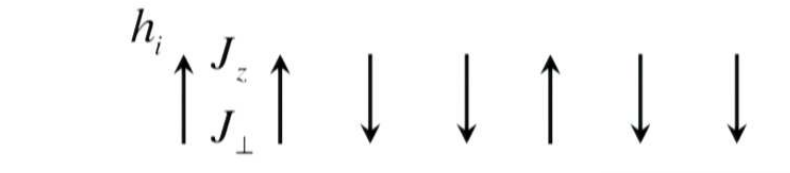
Jordan-Wigner

Spinless interacting 1D fermions \approx Random-field XXZ spin-1/2 chain



The diagram shows a 1D chain of sites represented by blue dots. Energy levels E_i are indicated by horizontal lines. Blue arrows represent hopping t between adjacent sites.

$$H = \sum_i E_i c_i^\dagger c_i + t \sum_i c_i^\dagger c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$



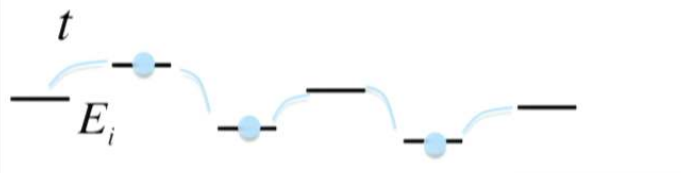
The diagram shows a 1D chain of sites with magnetic field h_i and exchange interactions J_z and J_\perp between adjacent sites. The field h_i is represented by upward arrows, and the exchange interactions are represented by vertical arrows pointing up or down.

$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c.) + J_z \sum_i S_i^z S_{i+1}^z$$

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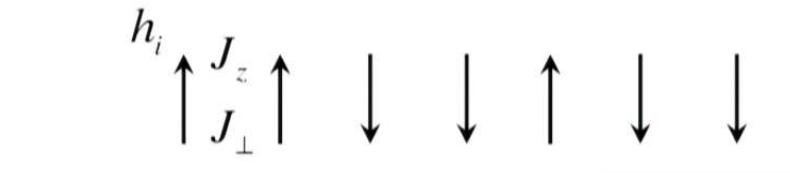
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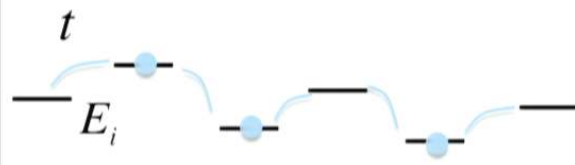
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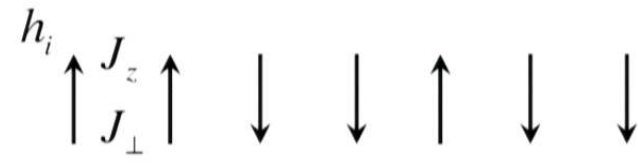
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Will use fermionic and spin language interchangeably

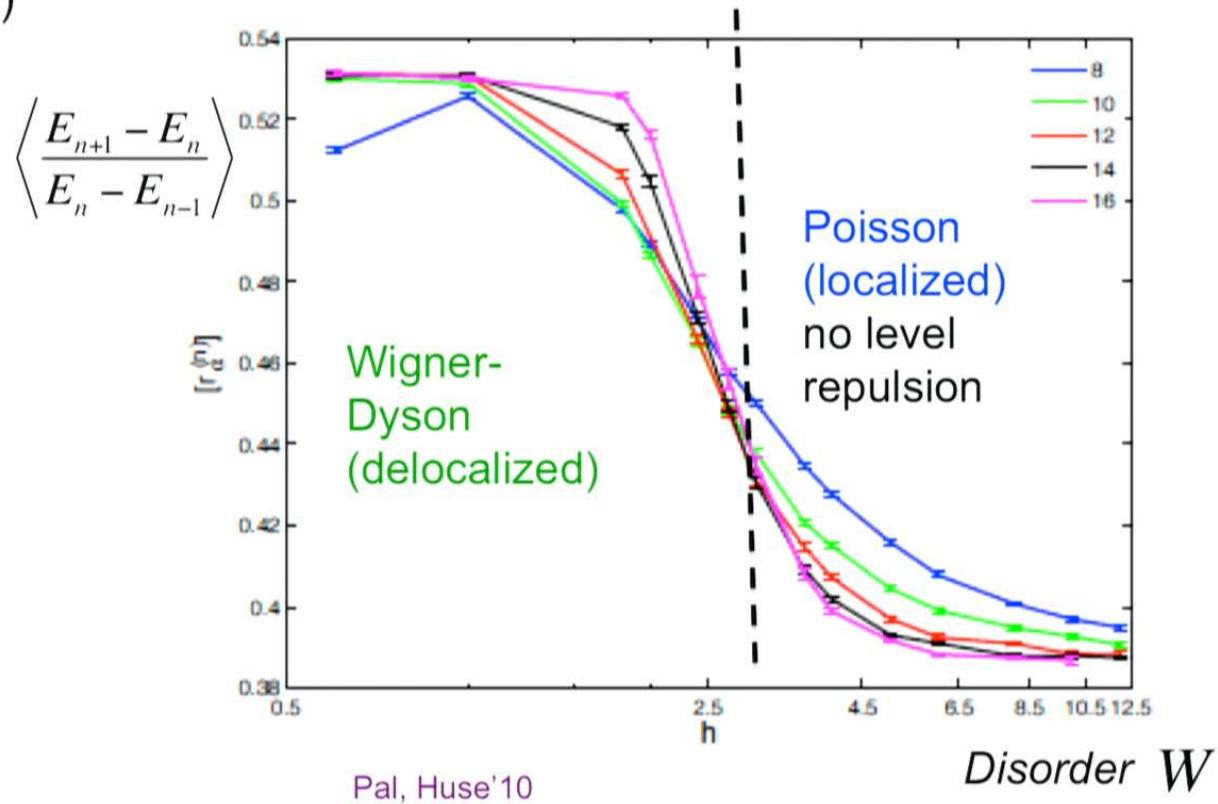
Study highly excited states Use disorder as tuning parameter

Parameters: disorder $h_i \in [-W; W]$, hopping $J_\perp = 1$, interactions $J_z = 1$

Many-body localization in 1D model

Numerics: many-body localization at $W_* \approx 3$

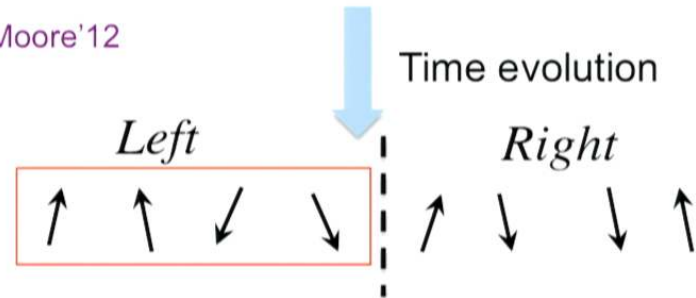
Level statistics; ratio of consecutive energy spacings (random-matrix theory)



Dynamics in the many-body localized phase

“Global quench”: $\uparrow \uparrow \downarrow \downarrow \uparrow \downarrow \downarrow \uparrow$

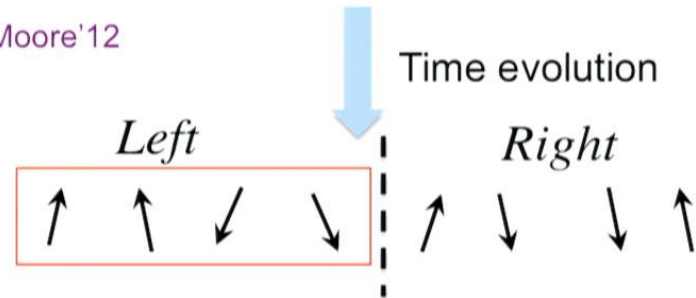
Bardarson, Pollmann, Moore'12



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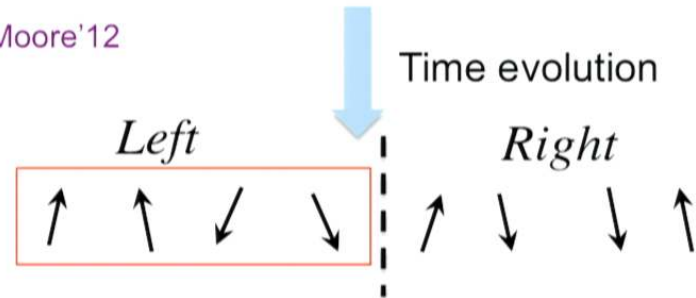
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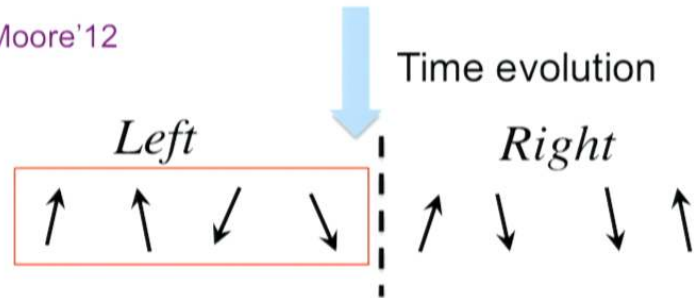
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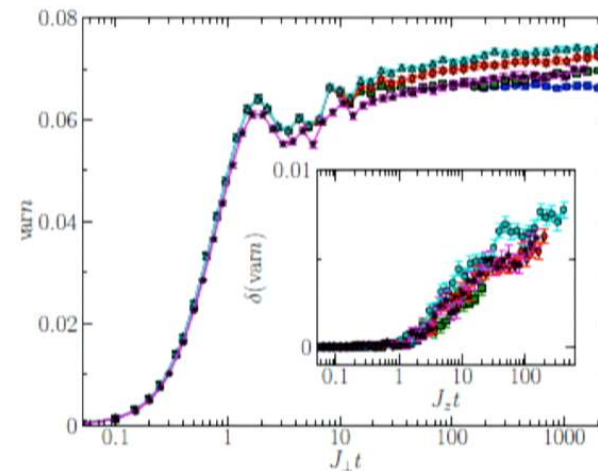
$W = 5$ (localized)

Fluctuations of S_z (=particle number) in left part

-Weak dependence on interaction strength and system size

-Particle hops limited to the boundary

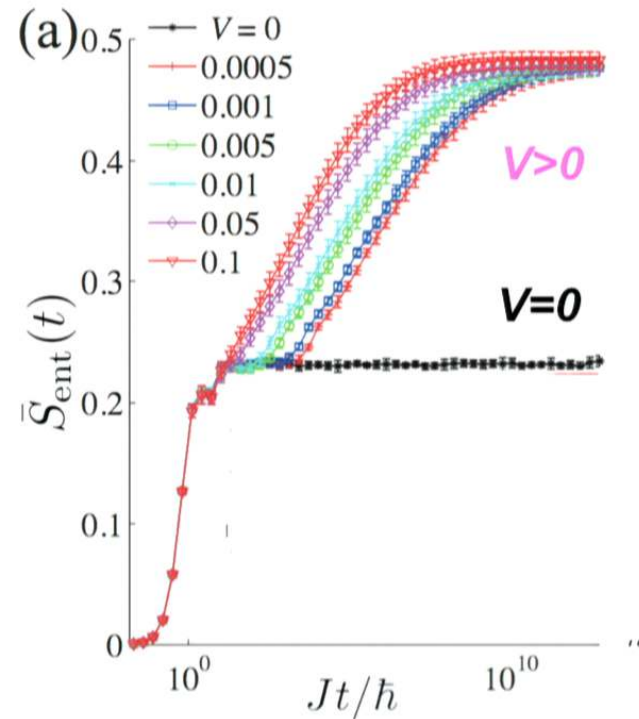
Supports existence of localized phase



Entanglement growth in MBL phase

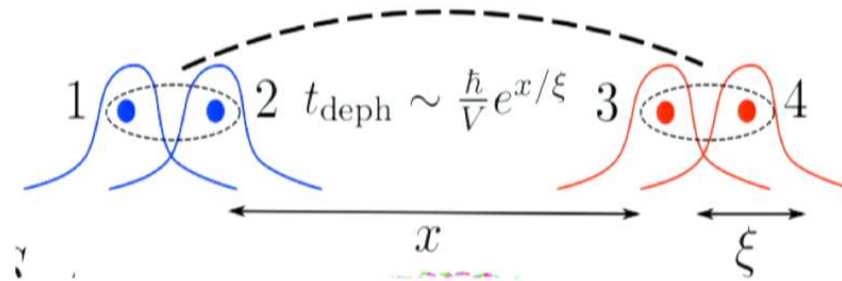
“Global quench” $S_{ent}(t)$ entanglement b/w left & right parts

Logarithmic growth of entanglement $S_{ent}(t) \sim \log(t)$ at nonzero interaction!



The mechanism of entanglement growth: Toy model

Two particles, prepared in a superposition of two localized states

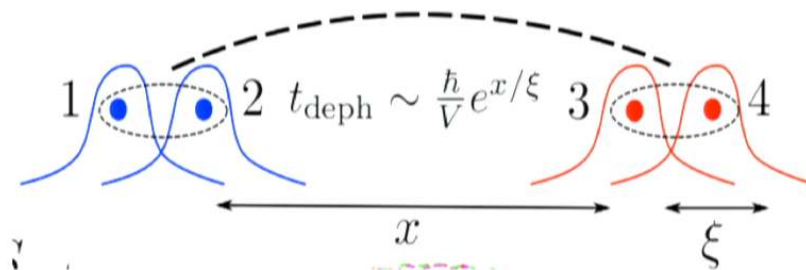


$$|\Psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$

c_i^+ single-particle eigenstates

No interactions: $c_i^+(t) = e^{iE_i t} c_i^+ \rightarrow$ exponentially small entanglement entropy

The mechanism of entanglement growth: Interaction-induced dephasing



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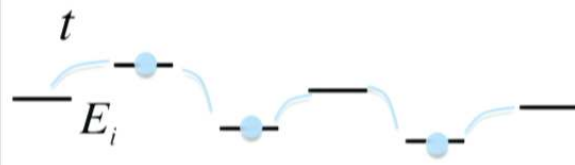
Weak interactions: eigenstates same $|\alpha\beta\rangle = c_\alpha^+ c_\beta^+ |0\rangle + O(V)$
But energies change

$$E_{\alpha\beta} = E_\alpha + E_\beta + C_{\alpha\beta} V e^{-x/\xi} \quad \leftarrow \text{Exponentially small correction}$$

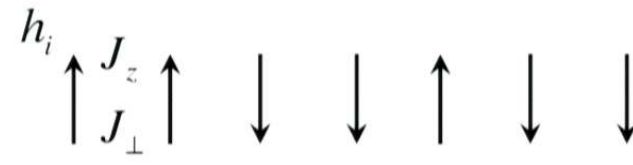
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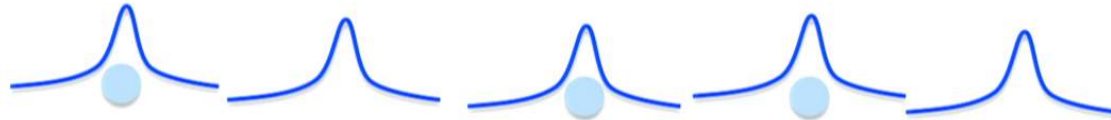
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Case of many particles

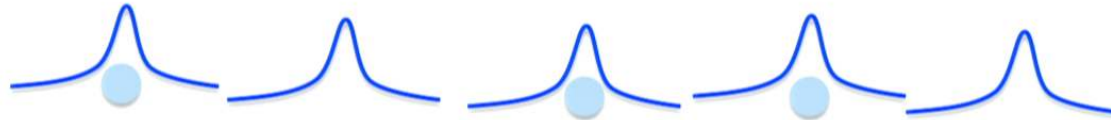
Hypothesis Eigenstates at **small V** are “close”* to non-interacting eigenstates (Slater-determinants) $c_{\alpha_1}^+ c_{\alpha_2}^+ \dots c_{\alpha_i}^+ \dots c_{\alpha_N}^+ |0\rangle$



* “Close” = eigenstates can be obtained from Slater-determinant states by small local unitary rotations

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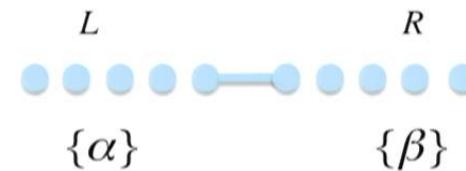


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Case of many particles: dynamics

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\} \in \mathcal{L}} A_{\{\alpha\}} |\alpha_1 \dots \alpha_K\rangle \times \sum_{\{\beta\} \in \mathcal{R}} B_{\{\beta\}} |\beta_1 \dots \beta_M\rangle$$



Degrees of freedom a distance x away get entangled at

$$t_{deph} \sim \frac{\hbar}{V} e^{x/\xi}$$



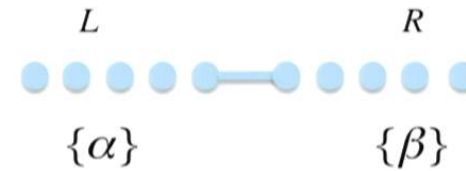
$$S_{ent}(t) \propto \xi \log \frac{Vt}{\hbar}$$

Serbyn, Papić, Abanin PRL'13

Saturated value of entanglement

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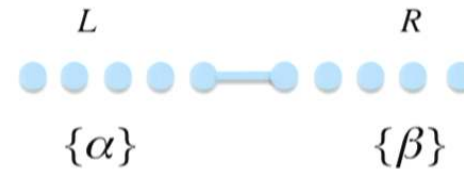
In the long-time limit, local diagonal ensemble emerges

Phases between A 's become randomized

Case of many particles: dynamics

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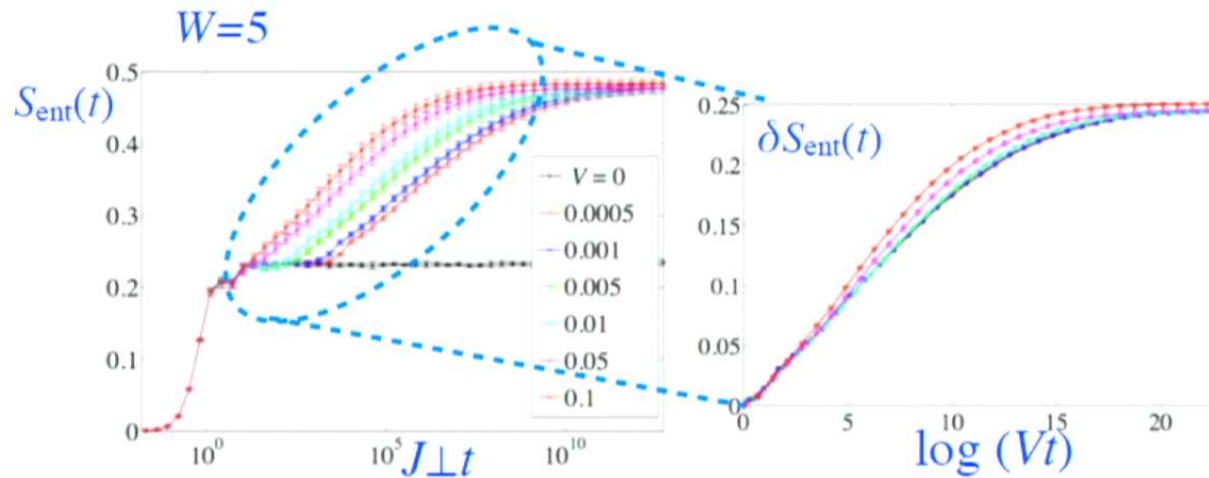
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Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

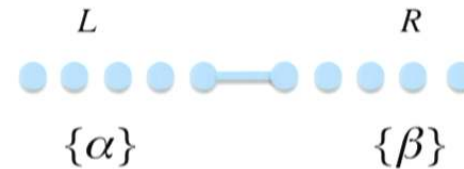
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$$S_{diag} = - \sum_{\{\alpha\}} |A_{\{\alpha\}}|^2 \log |A_{\{\alpha\}}|^2$$

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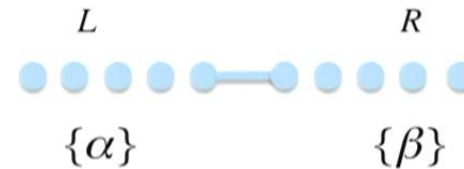
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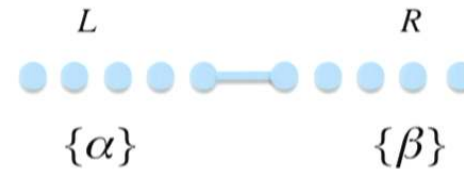
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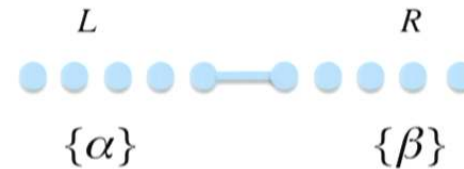
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We considered weak interactions (starting from single-body-localized phase)

Can we describe localized phase at strong interactions?
Is dynamics universal? Entanglement growth?

YES. Show that in MBL phase there are infinitely many local integrals of motion \rightarrow understand eigenstates

c_i^+

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Note: at weak interactions, we assumed* $[c_i^+, H] \approx 0$
and constructed all many-body eigenstates

*in reality, c_i^+ 's should be slightly modified to be true integrals of motion

Constructing integrals of motion

Consider a system in a MBL phase, localization length ξ

Key property: A local perturbation acts locally (checked numerically)

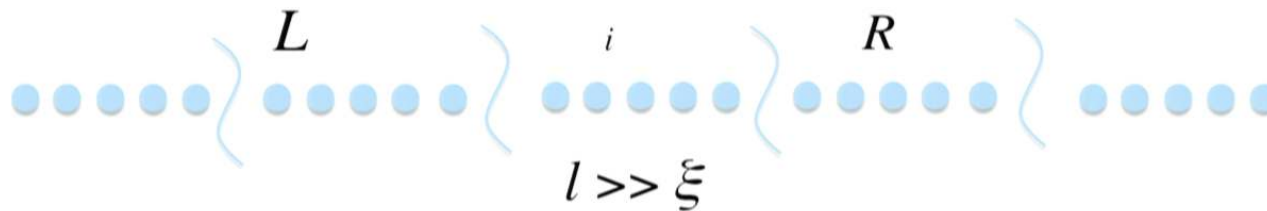
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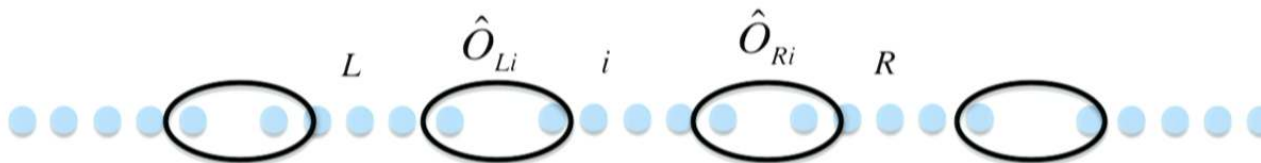
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Eigenstates of a disconnected system are product states $|\alpha\beta\gamma\rangle_0 = |\alpha\rangle_L \otimes |\beta\rangle_i \otimes |\gamma\rangle_R$



Implications: structure of many-body localized states

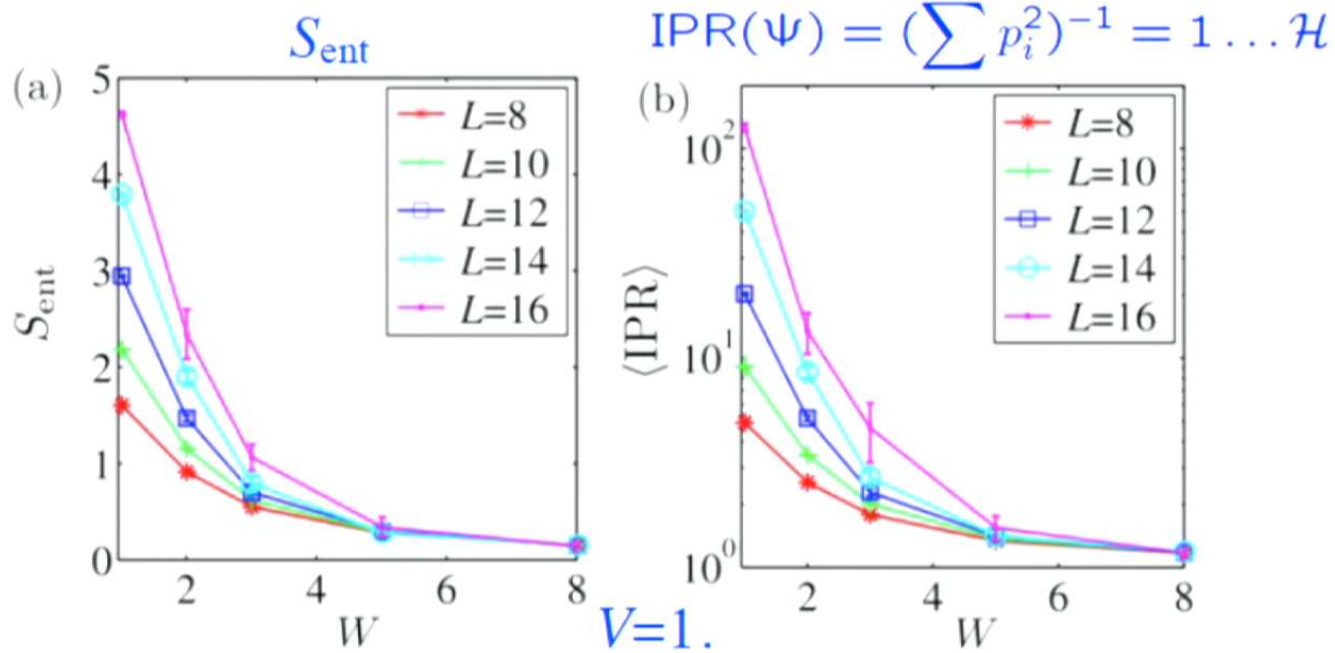
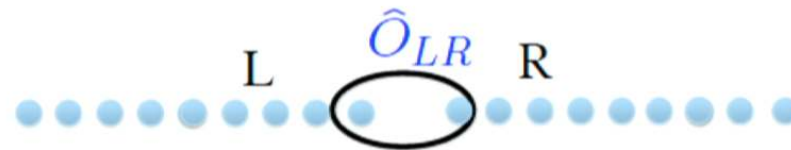


-Eigenstates obtained from product states (or Slater-determinant states of non-interacting particles) by a **quantum circuit of finite depth**

-Entanglement entropy of excited states in MBL phase obeys **area law** (similar to ground states of gapped systems!)

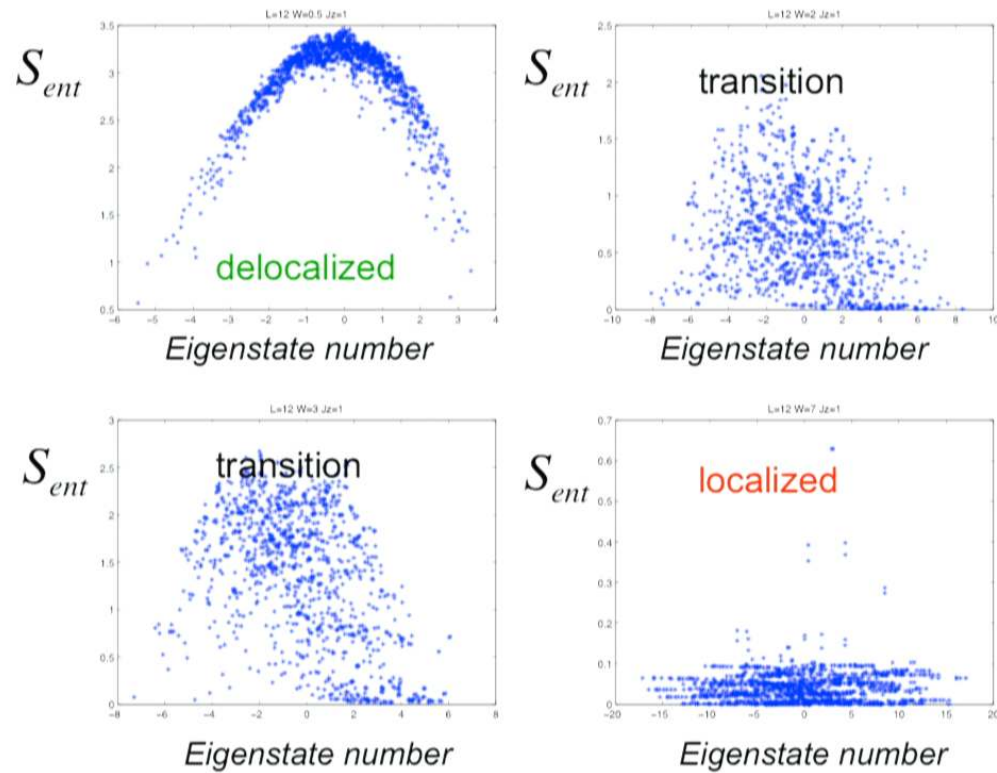
Numerical tests

- Entanglement entropy and IPR of product states $L \otimes R$:



Numerical tests

Entanglement distribution across transition



Effective spin-1/2 representation

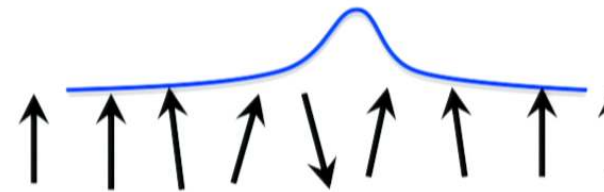
Integrals of motion form a complete set, take values $P_i = 1, 2, \dots, 2^l$

Change variables:

Introduce "effective" spins $\frac{1}{2} \tau_z^i$ with conserved z-projection

$$[\tau_z^i, H] = 0$$

- τ_z^i Have support in a region of size $\sim \xi$



Effective spin-1/2 representation

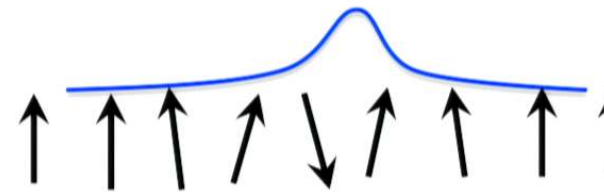
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Change variables:

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$$[\tau_z^i, H] = 0$$

- τ_z^i Have support in a region of size $\sim \xi$



Hamiltonian depends only on τ_z^i 's

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + \dots$$

$$H_{ij} \propto \exp(-|i - j|a/\xi)$$

Exponentially decaying 2-,3-,4-...body interactions between remote spins

Effective spins are quantum bits which cannot relax, but do dephase

Outlook

-Can we *prove* the existence of many-body localized phase?

-Nature of the transition

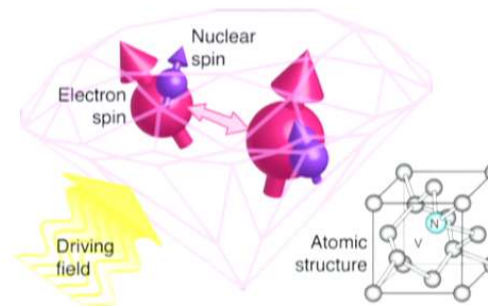
-Long-range interactions

-Strong-disorder RG (Vosk, Altman, Refael, Demler..),

-MBL without quenched disorder? (Muller, Huveneers, de Roeck,..)

-Experiments

-Integrals of motion may be useful to **protect coherence**, and for quantum information processing



Summary

-MBL phase has infinitely many local conservation laws.

Area-law entanglement of excited states

-Dynamics: slow dephasing with a broad distribution of dephasing time scales

-Universal logarithmic growth of entanglement entropy

DETAILS IN:

Serbyn, Papic, DA, Phys. Rev. Lett. **110**, 260601 (2013); Phys. Rev. Lett. **111**, 127201 (2013)

*MESSAGE: UNIVERSAL DYNAMICS AND ENTANGLEMENT
IN MANY-BODY LOCALIZED PHASE*