Title: Many-body localization: entanglement, emergent conservation laws, and the structure of eigenstates

Date: Nov 07, 2013 03:00 PM

URL: http://pirsa.org/13110073

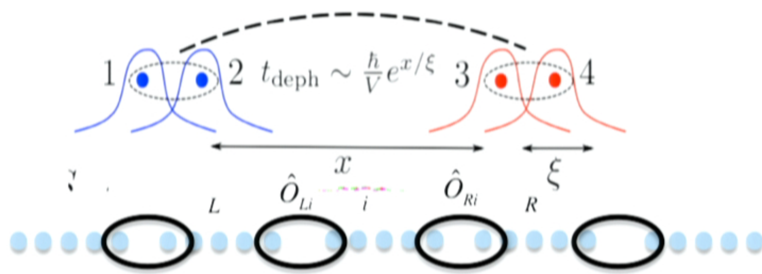
Abstract:

Many-body localization: local conservation laws, dynamics, and entanglement

Collaborators
Max Serbyn
Zlatko Papic

Dima Abanin

Perimeter Institute



Based on: Phys. Rev. Lett. 110, 260601 (2013); Phys. Rev. Lett. 111, 127201 (2013)

Perimeter-Urbana Workshop
Waterloo, November 7, 2013

Many-body localization: local conservation laws, dynamics, and entanglement

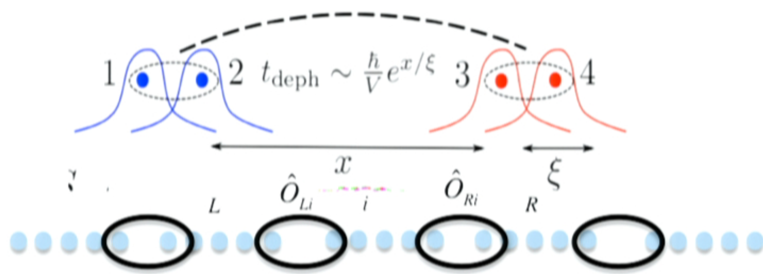
Collaborators

Max Serbyn

Zlatko Papic

Dima Abanin

Perimeter Institute



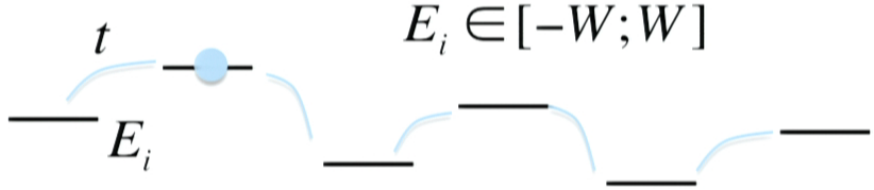
Based on: Phys. Rev. Lett. 110, 260601 (2013); Phys. Rev. Lett. 111, 127201 (2013)

Perimeter-Urbana Workshop

Waterloo, November 7, 2013

Single-particle Anderson localization

One quantum particle in 1D disordered crystal



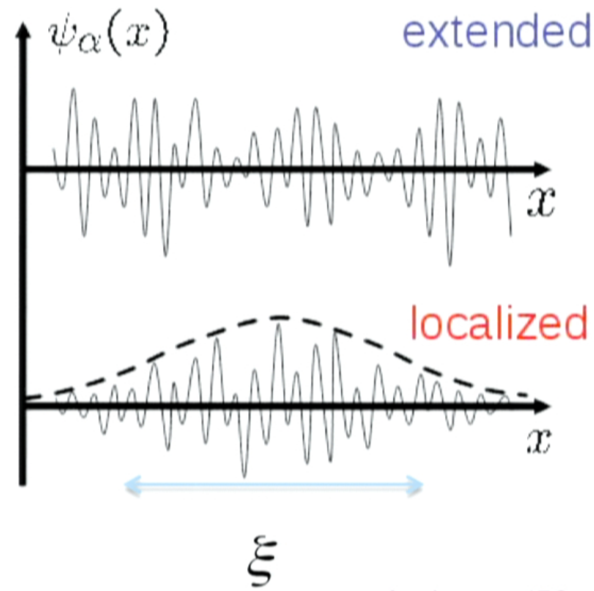
Wave functions become localized

$$\psi(r) \sim \exp(-r/\xi)$$

Contrast to extended Bloch states

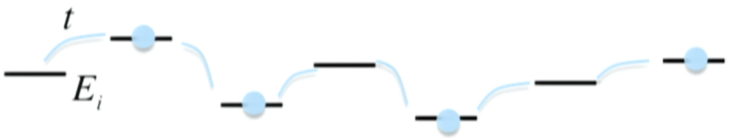
Absence of diffusion → Anderson insulator

Origin: quantum interference enhancement of backscattering



Anderson '58

Many interacting particles in a disorder potential



$$H = \sum_i E_i c_i^+ c_i + t \sum_i c_i^+ c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

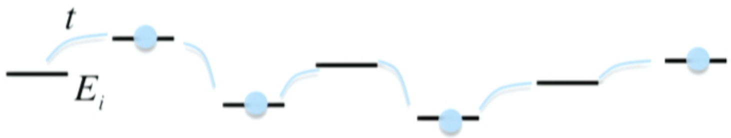
Zero interactions

-Anderson localization

-No diffusion

-Locally, spectrum looks discrete

Many interacting particles in a disorder potential



$$H = \sum_i E_i c_i^+ c_i + t \sum_i c_i^+ c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

Zero

-And

-No d

-Locally, spectrum looks discrete

Strong interactions

-Fast transport of energy and particle number

-Locally continuous spectrum

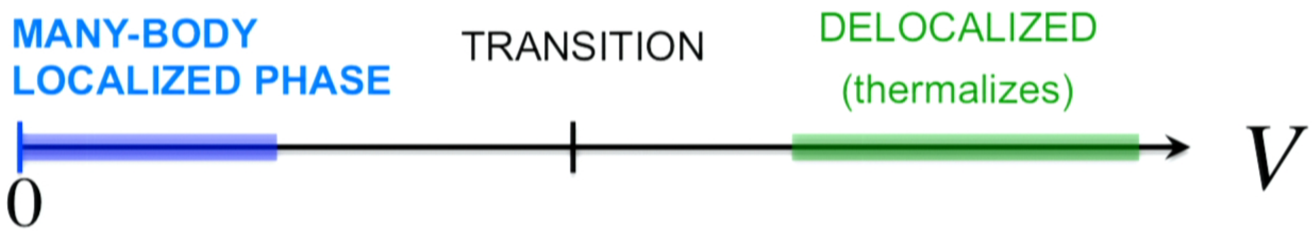
What happens at $V \sim W$?
Does localization survive at small V?
Localization-delocalization phase transition?

Many-body localization

Anderson,Fleishman'80;
Basko,Aleiner,Altshuler'05
Gornyi,Mirlin,Polyakov'05

Hypothesis: localization survives at $V << W$

Basic argument: matrix element for decay of a localized quasipaticle into many particle-hole pairs decays faster than the number of available final configurations. Far resonances absent.

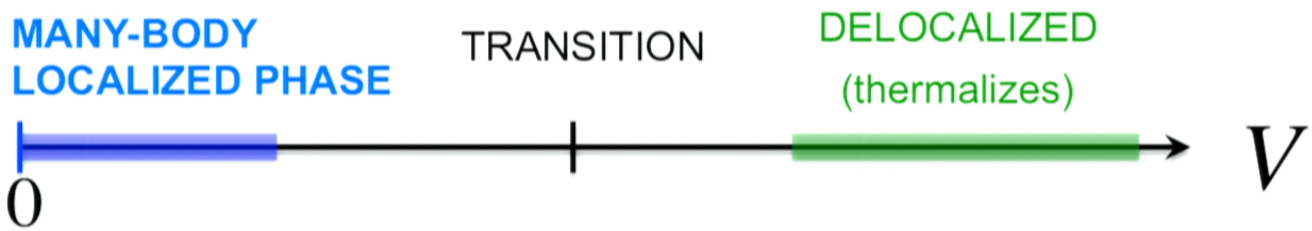


Many-body localization

Anderson,Fleishman'80;
Basko,Aleiner,Altshuler'05
Gornyi,Mirlin,Polyakov'05

Hypothesis: localization survives at $V << W$

Basic argument: matrix element for decay of a localized quasipaticle into many particle-hole pairs decays faster than the number of available final configurations. Far resonances absent.



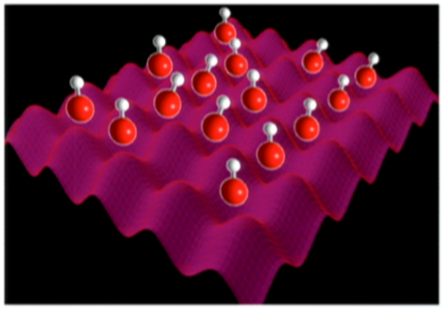
Supported by numerics
(exact diagonalization)

Oganesyan, Huse'07; Znidaric, Prosen'08; Pal, Huse'10;
Monthus, Garel '10, …

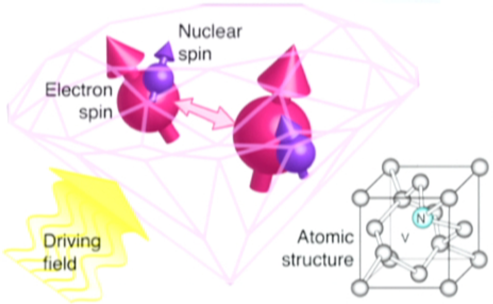
Experimental developments

-In semiconductors, difficult to disentangle effects of e-e interactions from phonons (variable-range hopping) → MBL out of reach

-Recently: Isolated, quantum-coherent systems with controllable disorder available



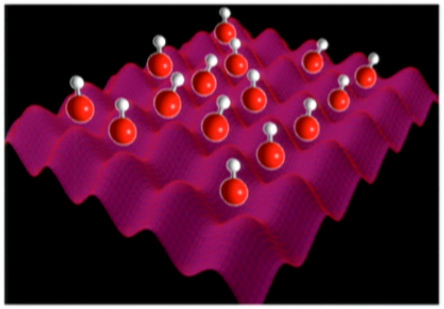
-Cold atoms, polar molecules, spin systems (NV-centers in diamond)



Studying many-body localization experimentally now possible

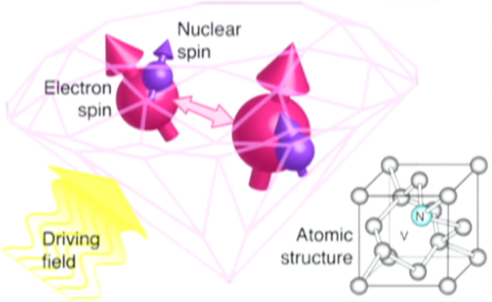
Experimental developments

-In semiconductors, difficult to disentangle effects of e-e interactions from phonons (variable-range hopping) → MBL out of reach



-Recently: Isolated, quantum-coherent systems with controllable disorder available

-Cold atoms, polar molecules, spin systems (NV-centers in diamond)

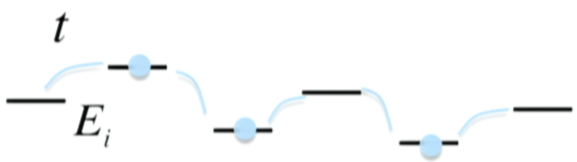


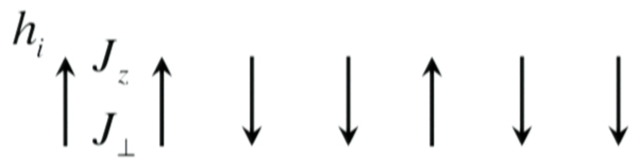
Studying many-body localization experimentally now possible

Model: 1D disordered chain

Jordan-Wigner

Spinless interacting 1D fermions $\approx$ Random-field XXZ spin-1/2 chain





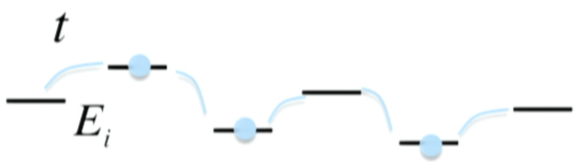
$$H = \sum_i E_i c_i^+ c_i + t \sum_i c_i^+ c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

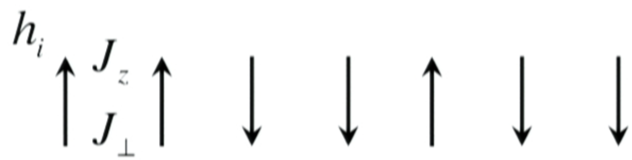
$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

Model: 1D disordered chain

Jordan-Wigner

Spinless interacting 1D fermions ≈ Random-field XXZ spin-1/2 chain





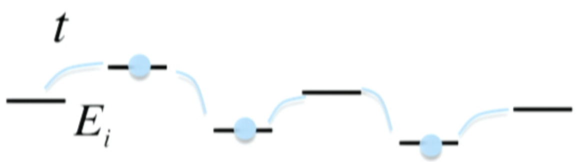
$$H = \sum_i E_i c_i^+ c_i + t \sum_i c_i^+ c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

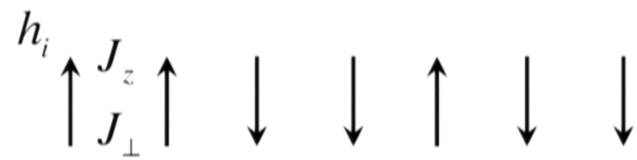
$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

Model: 1D disordered chain

Jordan-Wigner

Spinless interacting 1D fermions ≈ Random-field XXZ spin-1/2 chain





$$H = \sum_i E_i c_i^+ c_i + t\sum_i c_i^+ c_{i+1} + h.c. + V\sum_i n_i n_{i+1}$$

$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

Will use fermionic and spin language interchangeably

Study highly excited states Use disorder as tuning parameter

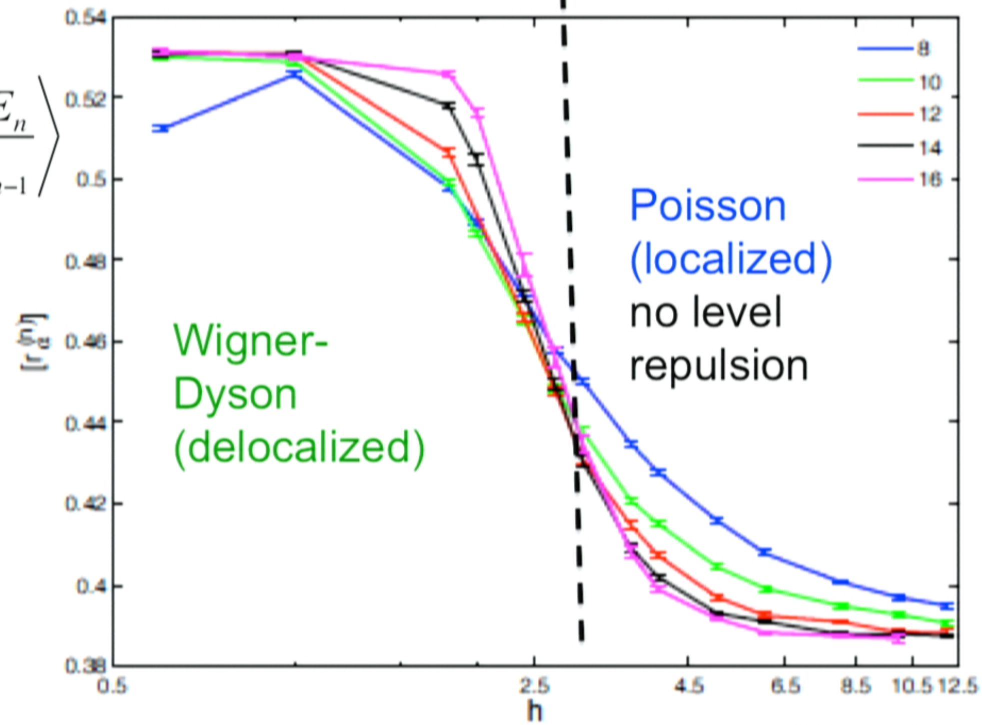
Parameters: disorder $h_i \in [-W;W]$, hopping $J_\perp = 1$, interactions $J_z = 1$

Many-body localization in 1D model

Numerics: many-body localization at $W_* \approx 3$

Level statistics; ratio of consecutive energy spacings (random-matrix theory)

$$\left\langle \frac{E_{n+1}-E_n}{E_n-E_{n-1}} \right\rangle$$



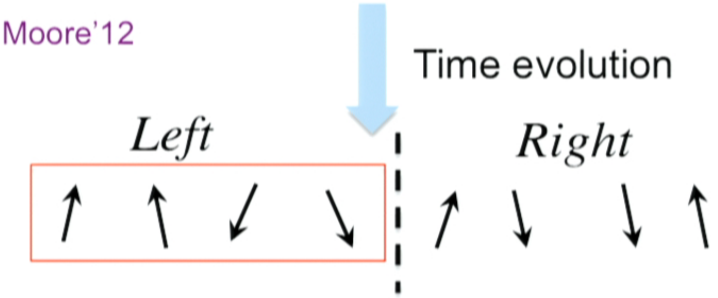
Disorder W

Pal, Huse'10

Dynamics in the many-body localized phase

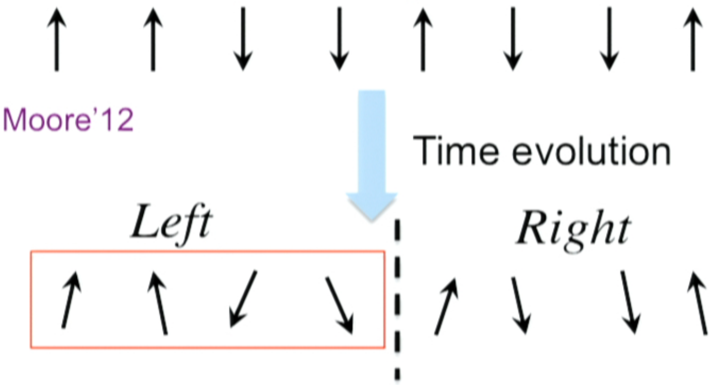
"Global quench":

Bardarson, Pollmann, Moore'12



Dynamics in the many-body localized phase

"Global quench":

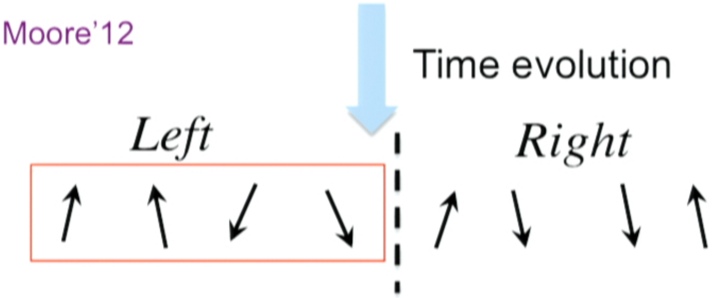


Bardarson, Pollmann, Moore'12

Dynamics in the many-body localized phase

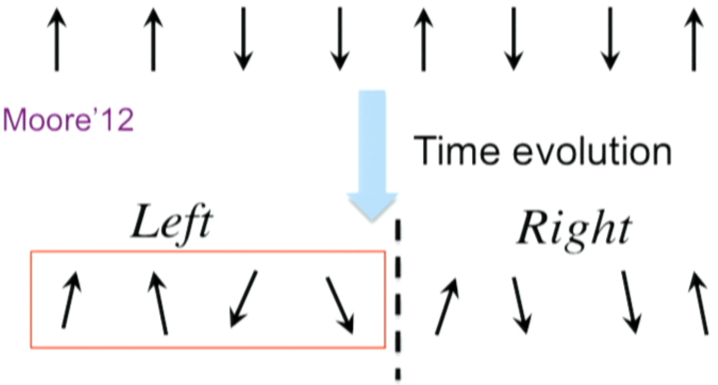
"Global quench": ↑ ↑ ↓ ↓ ↑ ↓ ↓ ↑

Bardarson, Pollmann, Moore'12



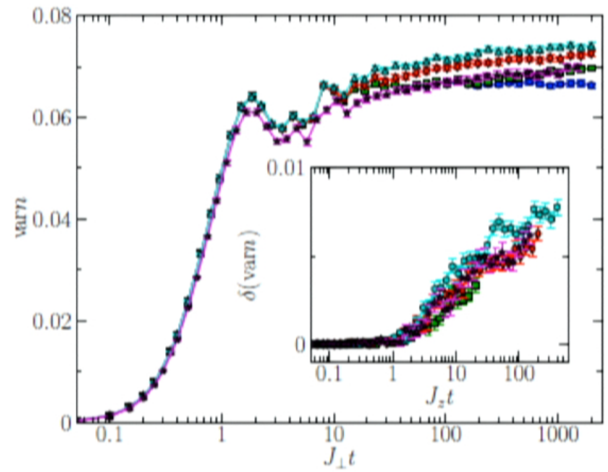
Dynamics in the many-body localized phase

"Global quench":



Bardarson, Pollmann, Moore'12

$W = 5$ (localized)



Fluctuations of S_z (=particle number) in left part

-Weak dependence on interaction strength and system size

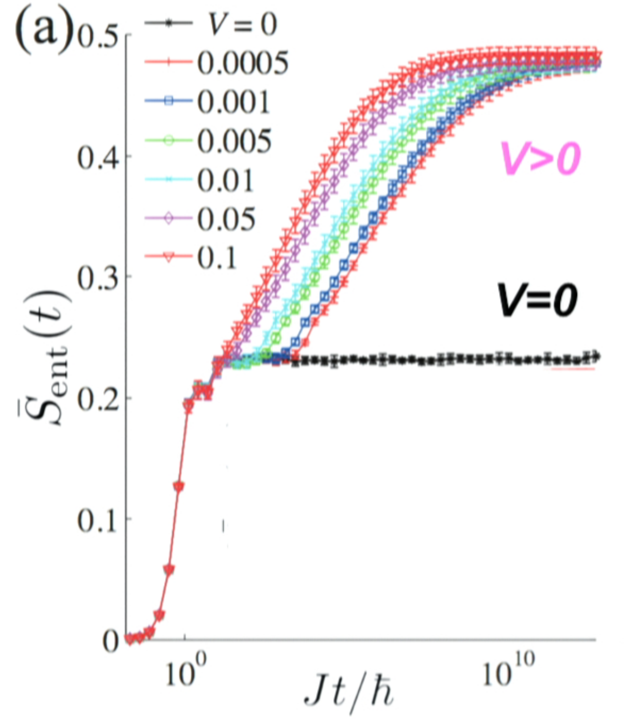
-Particle hops limited to the boundary

Supports existence of localized phase

Entanglement growth in MBL phase

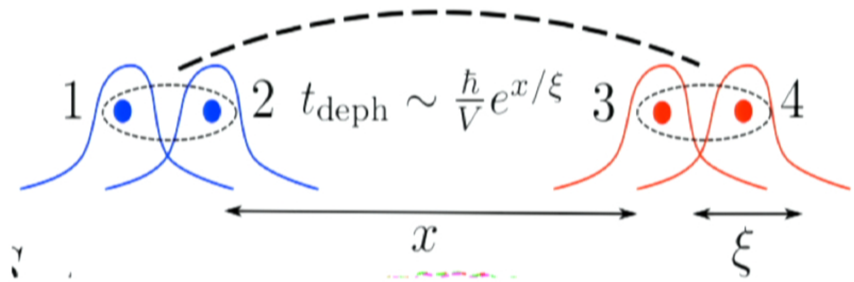
"Global quench" $S_{ent}(t)$ entanglement b/w left & right parts

Logarithmic growth of entanglement $S_{ent}(t) \sim \log(t)$ at nonzero interaction!



The mechanism of entanglement growth: Toy model

Two particles, prepared in a superposition of two localized states

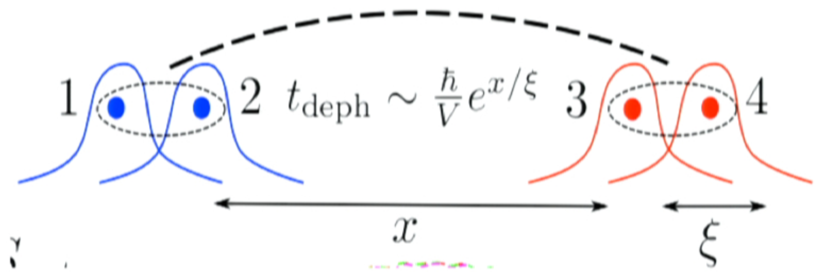


$$|\Psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$

c_i^+ single-particle eigenstates

No interactions: $c_i^+(t) = e^{iE_i t} c_i^+$ → exponentially small entanglement entropy

The mechanism of entanglement growth: Interaction-induced dephasing



$$|\Psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$

Weak interactions: eigenstates same $|\alpha\beta\rangle = c_\alpha^+ c_\beta^+ |0\rangle + O(V)$

But energies change

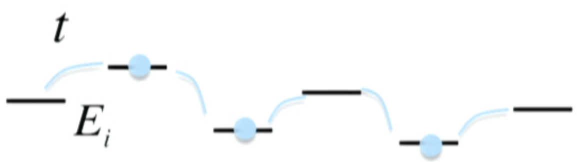
Exponentially small correction

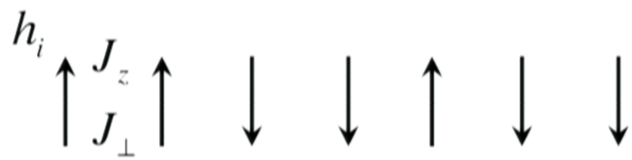
$$E_{\alpha\beta} = E_\alpha + E_\beta + C_{\alpha\beta} V e^{-x/\xi}$$

Model: 1D disordered chain

Jordan-Wigner

Spinless interacting 1D fermions $\approx$ Random-field XXZ spin-1/2 chain





$$H = \sum_i E_i c_i^+ c_i + t \sum_i c_i^+ c_{i+1} + h.c. + V \sum_i n_i n_{i+1}$$

$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

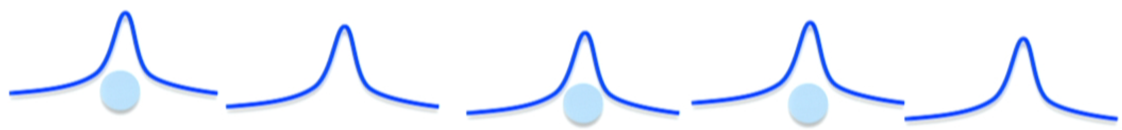
Will use fermionic and spin language interchangeably

Study highly excited states Use disorder as tuning parameter

<u>Parameters</u>: disorder $h_i \in [-W; W]$, hopping $J_\perp = 1$, interactions $J_z = 1$

Case of many particles

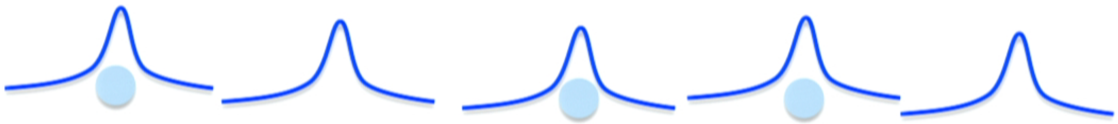
Hypothesis Eigenstates at **small *V*** are "close"* to non-interacting eigenstates (Slater-determinants) $c^+_{\alpha_1} c^+_{\alpha_2} \ldots c^+_{\alpha_i} \ldots c^+_{\alpha_N} |0\rangle$



* "Close" = eigenstates can be obtained from Slater-determinant states by small local unitary rotations

Case of many particles

Hypothesis Eigenstates at **small *V*** are "close"* to non-interacting eigenstates (Slater-determinants) $c^+_{\alpha_1} c^+_{\alpha_2} \ldots c^+_{\alpha_i} \ldots c^+_{\alpha_N} \left|0\right\rangle$



* "Close" = eigenstates can be obtained from Slater-determinant states by small local unitary rotations

Case of many particles: dynamics

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1 ... \alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1 ... \beta_M\rangle$$

Degrees of freedom a distance *x* away get entangled at $t_{deph} \sim \frac{\hbar}{V} e^{x/\xi}$ $\longrightarrow$

$$S_{ent}(t) \propto \xi \log \frac{Vt}{\hbar}$$

Serbyn, Papic, Abanin PRL'13

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1...\alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1...\beta_M\rangle$$

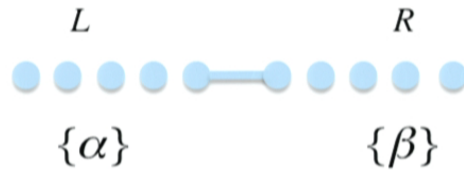
In the long-time limit, local diagonal ensemble emerges

Phases between *A*'s become randomized

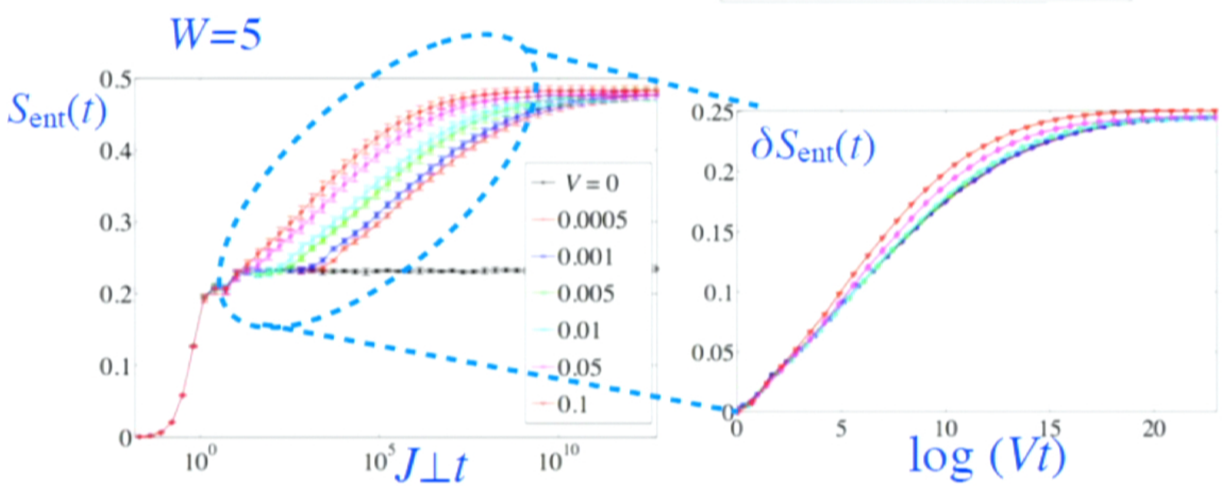
Case of many particles: dynamics

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1...\alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1...\beta_M\rangle$$



Degrees of freedom a distance *x* away get entangled at $t_{deph} \sim \frac{\hbar}{V} e^{x/\xi}$ → $S_{ent}(t) \propto \xi \log \frac{Vt}{\hbar}$



Serbyn, Papic, Abanin PRL'13

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}} |\alpha_1 ... \alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}} |\beta_1 ... \beta_M\rangle$$

In the long-time limit, local diagonal ensemble emerges
Phases between *A*'s become randomized

Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

$$S_{ent}(\infty) \approx S_{diag}$$

$$S_{diag} = -\sum_{\{\alpha\}} \left|A_{\{\alpha\}}\right|^2 \log \left|A_{\{\alpha\}}\right|^2$$

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1 ... \alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1 ... \beta_M\rangle$$

In the long-time limit, local diagonal ensemble emerges
Phases between *A*'s become randomized

Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

$$S_{ent}(\infty) \approx S_{diag}$$

$$S_{diag} = -\sum_{\{\alpha\}} \left|A_{\{\alpha\}}\right|^2 \log\left|A_{\{\alpha\}}\right|^2$$

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1...\alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1...\beta_M\rangle$$

In the long-time limit, local diagonal ensemble emerges
Phases between *A*'s become randomized

Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

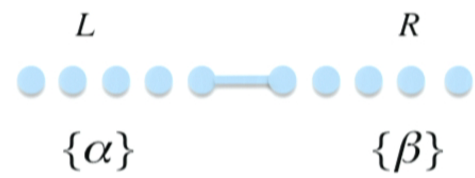
$$S_{ent}(\infty) \approx S_{diag}$$

$$S_{diag} = -\sum_{\{\alpha\}} \left|A_{\{\alpha\}}\right|^2 \log\left|A_{\{\alpha\}}\right|^2$$

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1 ... \alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1 ... \beta_M\rangle$$



In the long-time limit, local diagonal ensemble emerges
Phases between *A*'s become randomized

Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

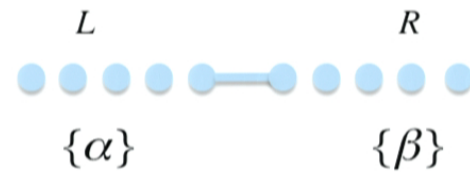
$$S_{ent}(\infty) \approx S_{diag}$$

$$S_{diag} = -\sum_{\{\alpha\}} |A_{\{\alpha\}}|^2 \log |A_{\{\alpha\}}|^2$$

Saturated value of entanglement

Initial product state

$$|\Psi(t=0)\rangle = \sum_{\{\alpha\}\in\mathcal{L}} A_{\{\alpha\}}|\alpha_1 ... \alpha_K\rangle \times \sum_{\{\beta\}\in\mathcal{R}} B_{\{\beta\}}|\beta_1 ... \beta_M\rangle$$



In the long-time limit, local diagonal ensemble emerges
Phases between *A*'s become randomized

Saturated entanglement determined by decomposition of initial state in the basis of non-interacting eigenstates:

$$S_{ent}(\infty) \approx S_{diag}$$

$$S_{diag} = -\sum_{\{\alpha\}} \left|A_{\{\alpha\}}\right|^2 \log \left|A_{\{\alpha\}}\right|^2$$

We considered weak interactions (starting from single-body-localized phase)

Can we describe localized phase at strong interactions?
Is dynamics universal? Entanglement growth?

YES. Show that in MBL phase there are infinitely many local integrals of motion → understand eigenstates

c_i^+

We considered weak interactions (starting from single-body-localized phase)

Can we describe localized phase at strong interactions?
Is dynamics universal? Entanglement growth?

YES. Show that in MBL phase there are infinitely many local integrals of motion → understand eigenstates

Note: at weak interactions, we assumed* $\left[c_i^+, H\right] \approx 0$
and constructed all many-body eigenstates

*in reality, c_i^+'s should be slightly modified to be true integrals of motion

Constructing integrals of motion

Consider a system in a MBL phase, localization length ξ

Key property: A local perturbation acts locally (checked numerically)

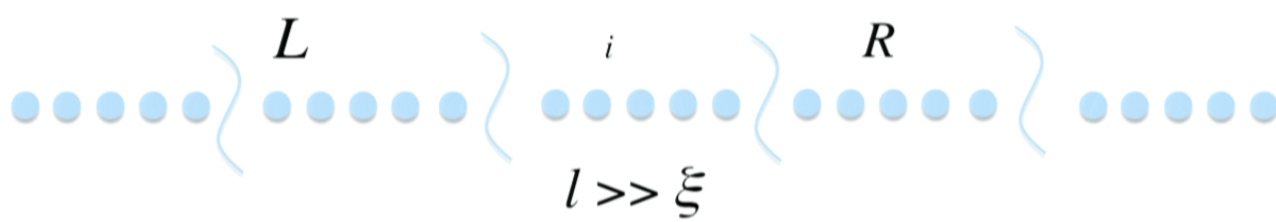
Implication: eigenstates obtained from product states by ~local unitary rotations

Constructing integrals of motion

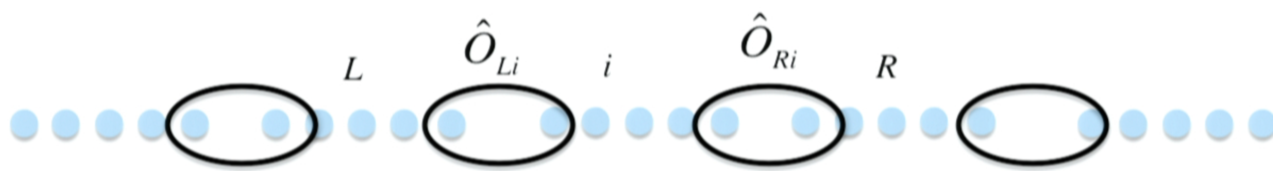
Consider a system in a MBL phase, localization length ξ

Key property: A local perturbation acts locally (checked numerically)

Implication: eigenstates obtained from product states by ~local unitary rotations



Eigenstates of a disconnected system are product states $|\alpha\beta\gamma\rangle_0 = |\alpha\rangle_L \otimes |\beta\rangle_i \otimes |\gamma\rangle_R$



Implications: structure of many-body localized states

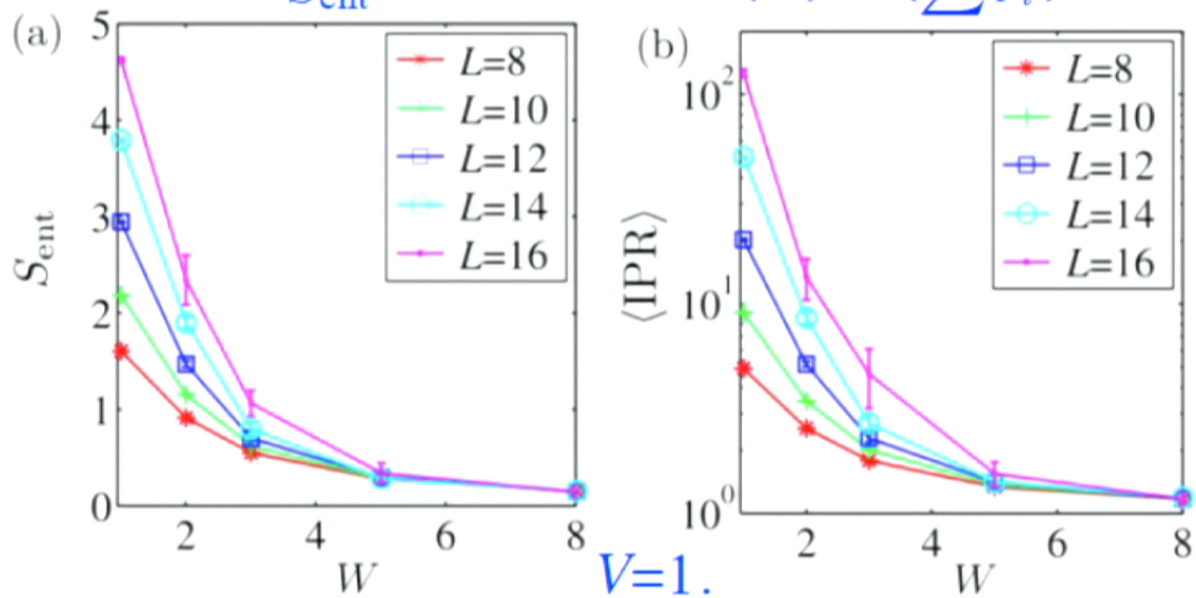
-Eigenstates obtained from product states (or Slater-determinant states of non-interacting particles) by a quantum circuit of finite depth

-Entanglement entropy of excited states in MBL phase obeys area law (similar to ground states of gapped systems!)

Numerical tests

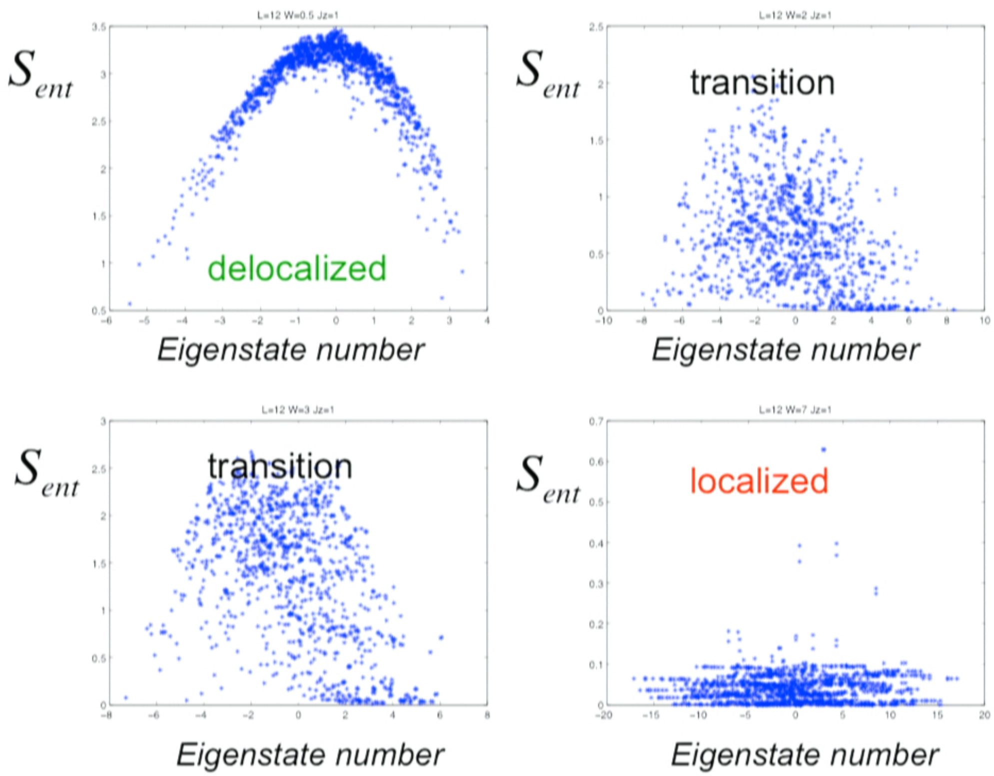
- Entanglement entropy and IPR of product states L⊗R:

S_{ent} $\quad$ $\text{IPR}(\Psi) = (\sum p_i^2)^{-1} = 1 \ldots \mathcal{H}$



Numerical tests

Entanglement distribution across transition



Effective spin-1/2 representation

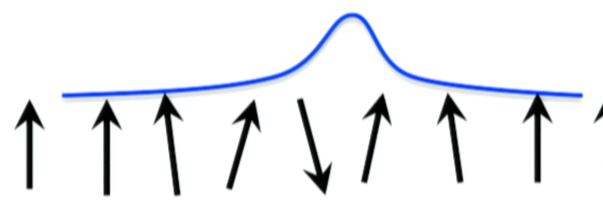
Integrals of motion form a complete set, take values $P_i = 1, 2, \ldots, 2^l$

Change variables:

Introduce "effective" spins ½ τ_z^i with conserved z-projection

$$[\tau_z^i, H] = 0$$

- τ_z^i Have support in a region of size~ ξ



Effective spin-1/2 representation

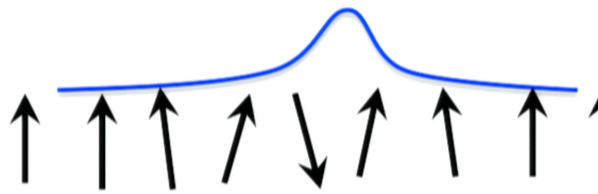
Integrals of motion form a complete set, take values $P_i = 1, 2, \ldots, 2^l$

Change variables:

Introduce "effective" spins ½ τ_z^i with conserved z-projection

$$[\tau_z^i, H] = 0$$

- τ_z^i Have support in a region of size~ ξ



Hamiltonian depends only on τ_z^i 's

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

$$H_{ij} \propto \exp(-|i-j|a/\xi)$$

Exponentially decaying 2-,3-,4-…body interactions between remote spins

Effective spins are quantum bits which cannot relax, but do dephase

Outlook

-Can we *prove* the existence of many-body localized phase?

-Nature of the transition

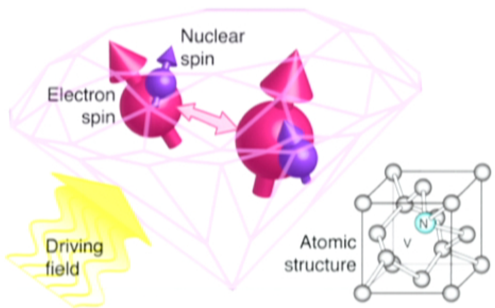
-Long-range interactions

-Strong-disorder RG (Vosk, Altman, Refael, Demler..),

-MBL without quenched disorder? (Muller, Huveneers, de Roeck,..)

-Experiments

-Integrals of motion may be useful to protect coherence, and for quantum information processing



Summary

-MBL phase has infinitely many local conservation laws.
Area-law entanglement of excited states

-Dynamics: slow dephasing with a broad distribution of dephasing time scales

-Universal logarithmic growth of entanglement entropy

DETAILS IN:
Serbyn, Papic, DA, Phys. Rev. Lett. **110**, 260601 (2013); Phys. Rev. Lett. **111**, 127201 (2013)

MESSAGE: UNIVERSAL DYNAMICS AND ENTANGLEMENT IN MANY-BODY LOCALIZED PHASE