

Title: Entanglement at strongly-interacting quantum critical points

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URL: <http://pirsa.org/13110071>

Abstract: <span>At a quantum critical point (QCP) in two or more spatial dimensions, leading-order contributions to the scaling of entanglement entropy typically follow the "area" law, while sub-leading behavior contains universal physics.&nbsp; Different universal functions can be access through entangling subregions of different geometries.&nbsp; For example, for polygonal shaped subregions, quantum field theories have demonstrated that the sub-leading scaling is logarithmic, with a universal coefficient dependent on the number of vertices in the polygon.&nbsp; Although such universal quantities are routinely studied in non-interacting field theories, it requires numerical simulation to access them in interacting theories.&nbsp; In this talk, we discuss numerical calculations of the Renyi entropies at QCPs in 2D quantum lattice models.&nbsp; We calculate the universal coefficient of the vertex-induced logarithmic scaling term, and compare to non-interacting field theory calculations.&nbsp; Also, we examine the shape dependence of the Renyi entropy for finite-size lattices with smooth subregion boundaries. Such geometries provide a sensitive probe of the gapless wavefunction in the thermodynamic limit, and give new universal quantities that could be examined by field-theoretical studies in 2+1D.</span>



Ann Kallin



Stephen Inglis



Hyejin Ju



Katie Hyatt



Jean-Marie Stéphan

*Entanglement scaling in two-dimensional gapless systems*, **Hyejin Ju, Ann Kallin**, Paul Fendley, Matthew B Hastings, RGM, Phys. Rev. B 85, 165121 (2012)

*Thermodynamic singularities in the entanglement entropy at a two-dimensional quantum critical point*, Rajiv Singh, RGM, Jaan Oitmaa, Phys. Rev. B 86, 075106 (2012)

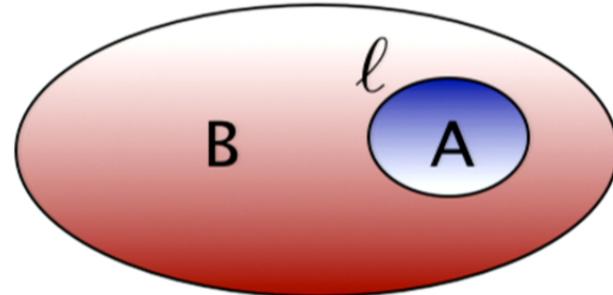
*Entanglement at a Two-Dimensional Quantum Critical Point: A Numerical Linked-Cluster Expansion Study*, **Ann Kallin, Katharine Hyatt**, Rajiv Singh, RGM, Phys. Rev. Lett. 110, 135702 (2013)

*Entanglement in gapless resonating-valence-bond states*, **Jean-Marie Stéphan, Hyejin Ju**, Paul Fendley, RGM, New J. Phys. **15**, 015004 (2013)

*Entanglement at a Two-Dimensional Quantum Critical Point: a T=0 projector QMC study*, **Stephen Inglis** and RGM, New J. Phys. **15** 073048 (2013)

## Entanglement Area Law

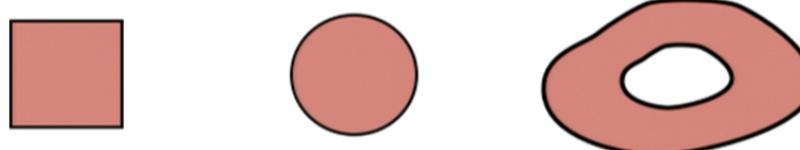
$$S_n(\rho_A) = \frac{1}{1-n} \ln [\text{Tr}(\rho_A^n)]$$



- **Subleading** scaling terms have been successful in characterizing exotic gapped systems in 2D – e.g. Topological entanglement entropy:

$$S_n = A\ell + \gamma_n$$

- Similarly, will it be possible to classify gapless systems (spin liquids, quantum critical points) by subleading scaling terms?
- In gapless systems in  $D > 1$ ,  $\gamma$  has a complicated dependence on subregion shape and topology



# Subleading Scaling in D>1 critical systems

- This shape dependence can identify the Universality class and more...
  - predicted to be a resource in distinguishing conventional from deconfined critical points

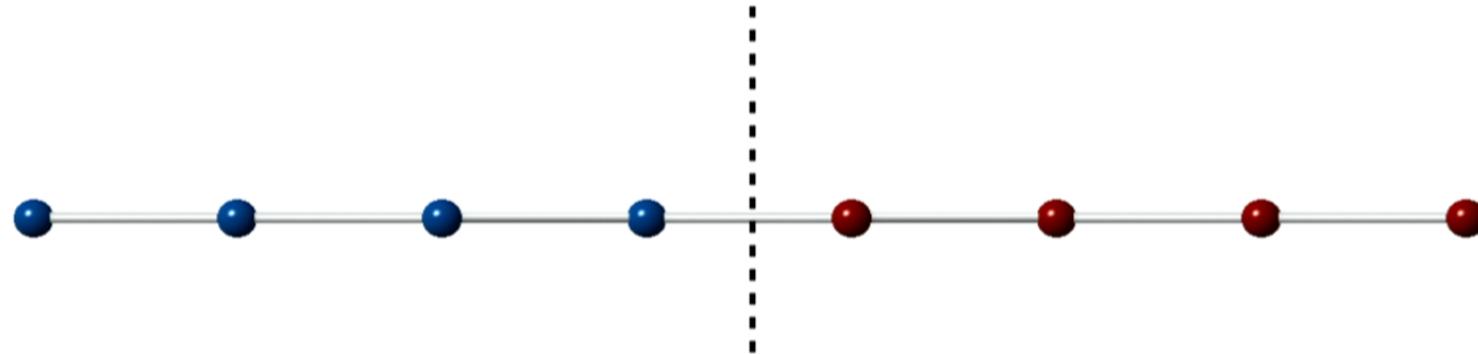
$$\gamma_{XY*} = \gamma_{XY} + \gamma_{Z_2} \quad \text{Swingle, Senthil, Phys. Rev. B 86, 155131 (2012)}$$

- Numerics/field theory/holography has potential to give c,F-theorems working relevance for condensed-matter systems

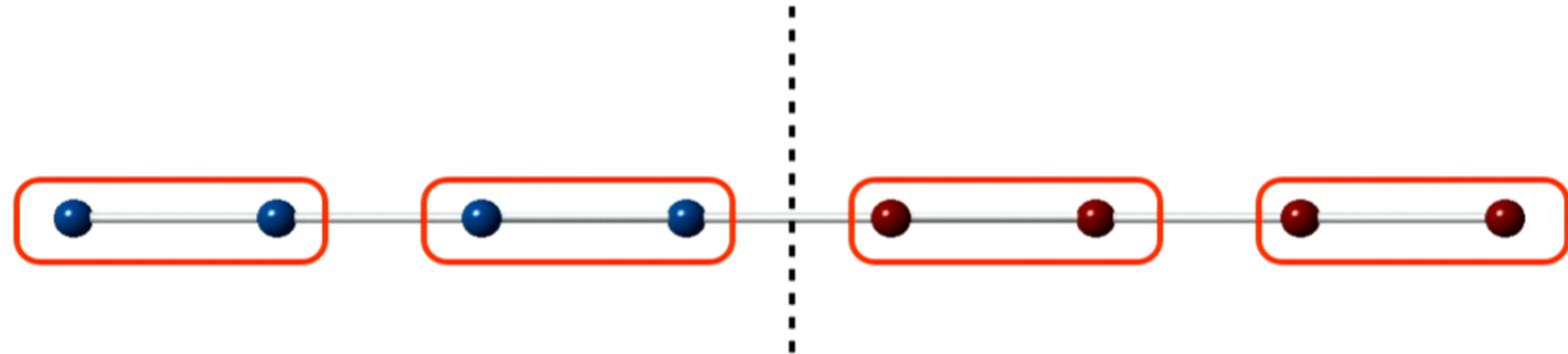
**Chiral Symmetry Breaking, Deconfinement and Entanglement Monotonicity**

Tarun Grover, arXiv:1211.1392

- Critical systems in 1D... Scale invariance: assume  $\mathcal{O}(1)$  unit of entanglement entropy at **each** length scale

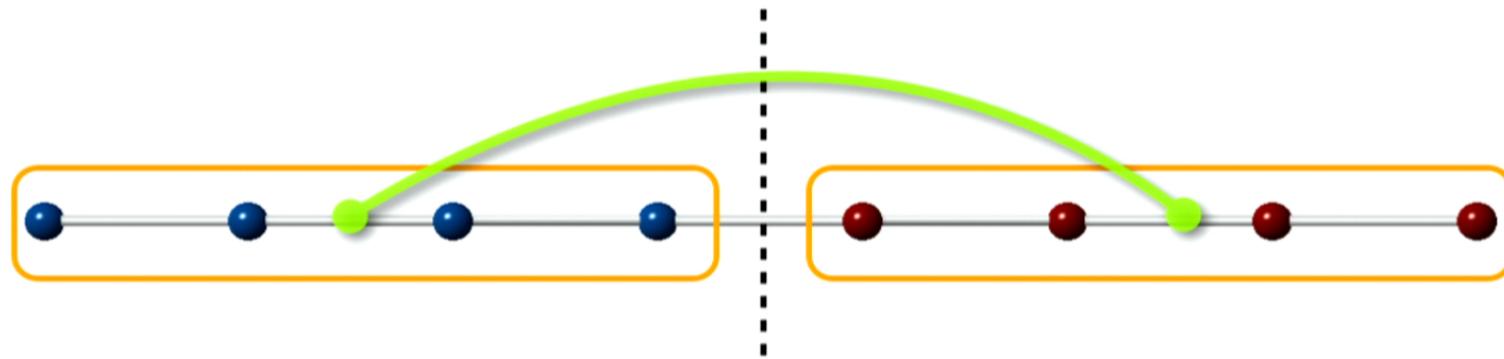


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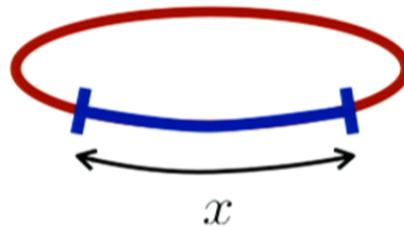
$$\mathcal{O}(1) \times (1 +$$

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$$\mathcal{O}(1) \times (1+1+1) = 3 = \log_2(8) = \log(L)$$

- not the whole story:



$$S_n = c \log(L) + \gamma(x/L)$$

C. Holzhey, F. Larsen, and F. Wilczek, Nucl. Phys. B424, 443 (1994)  
G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. 90, 227902 (2003)  
Calabrese and Cardy, J. Stat. Mech: Theory Exp. P06002 (2004)

- Proper way: conformal mapping from a cylinder to a plane:

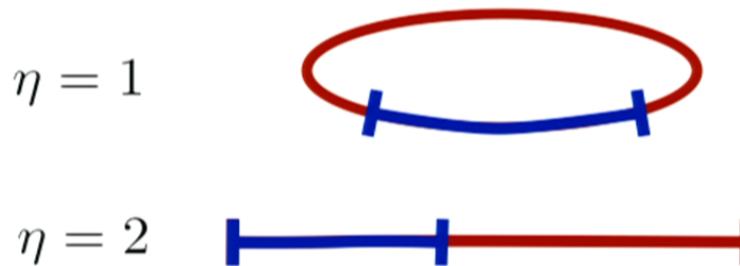
$$S_n = \frac{c}{3\eta} \left(1 + \frac{1}{n}\right) \log \left[ \frac{\eta L}{\pi} \right] + \frac{c}{3\eta} \left(1 + \frac{1}{n}\right) \log \left[ \sin \frac{\pi x}{L} \right] + \dots$$

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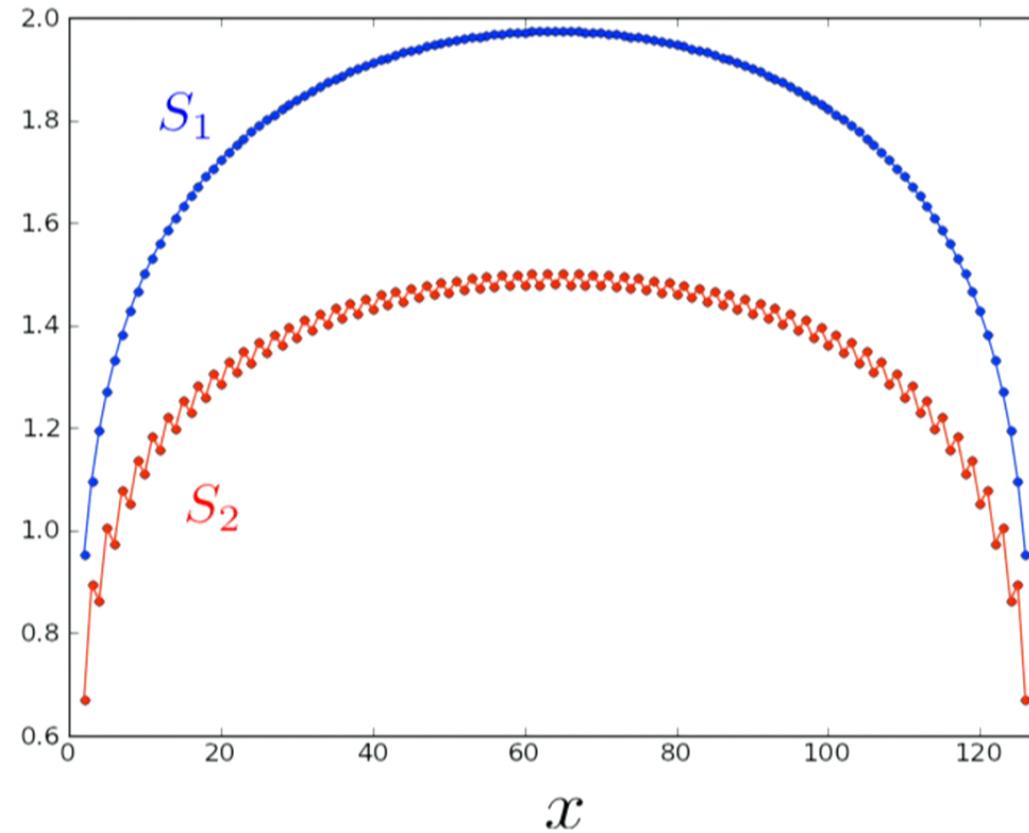
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- Finite-size scaling form allows a detailed comparison with numerics

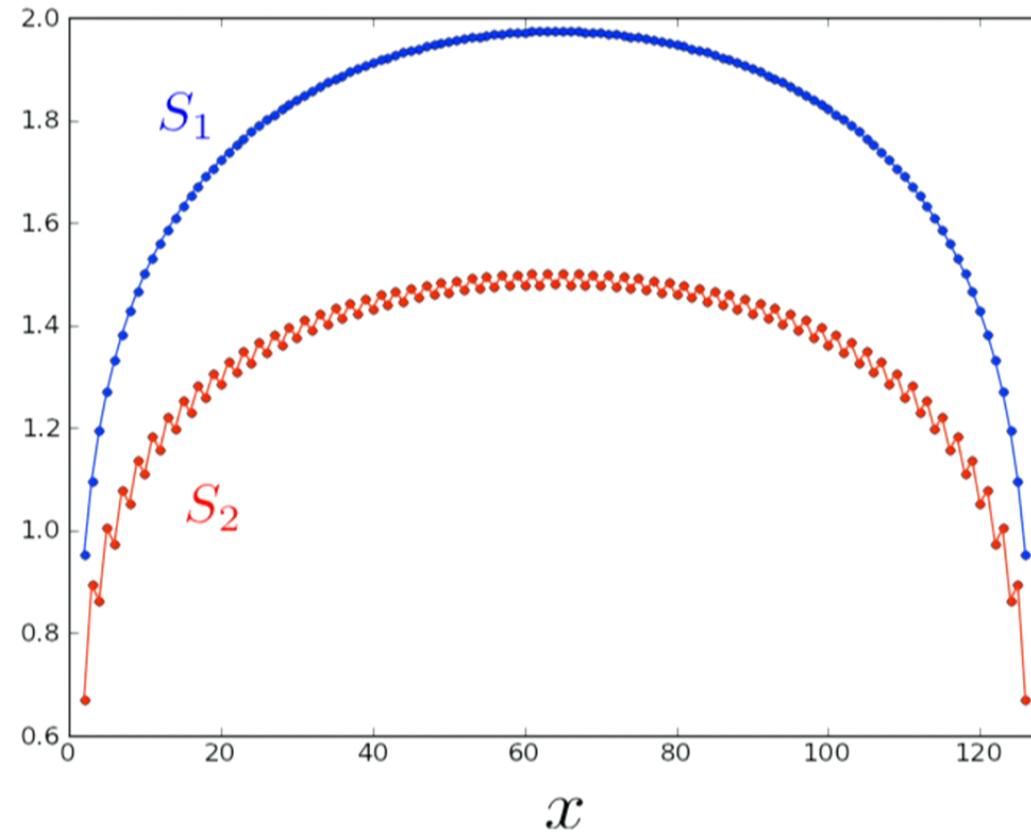


- For example in 1+1, the entanglement entropy can be calculated directly from the reduced density matrix in DMRG

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

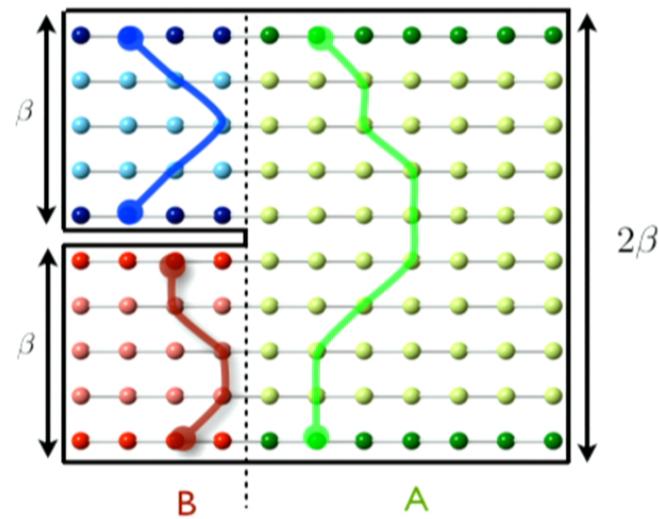


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Calabrese and Cardy, J. Stat. Mech. 0406, P002 (2004).  
 Fradkin and Moore, Phys. Rev. Lett. 97, 050404 (2006)  
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 Buividovich and Polikarpov, Nucl. Phys. B, 802, 458 (2008)  
 M. A. Metlitski, et.al, Phys.Rev. B 80, 115122 (2009).

- In Quantum Monte Carlo: We have access to  $n \geq 2$  without explicitly knowing the reduced density matrix

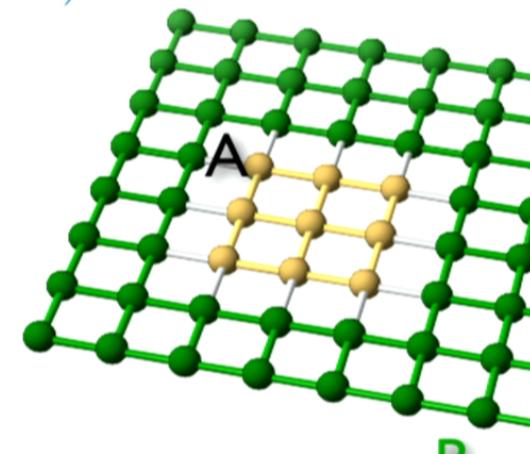
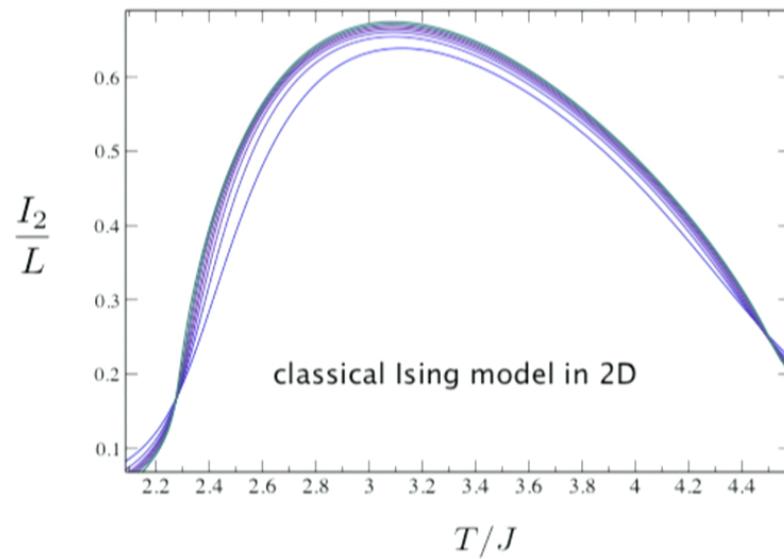


$n$ -sheeted Riemann surface:

$$S_2(A) = -\ln \left( \frac{Z[A, 2, T]}{Z^2} \right)$$

- note: Monte Carlo can access finite-temperature critical points

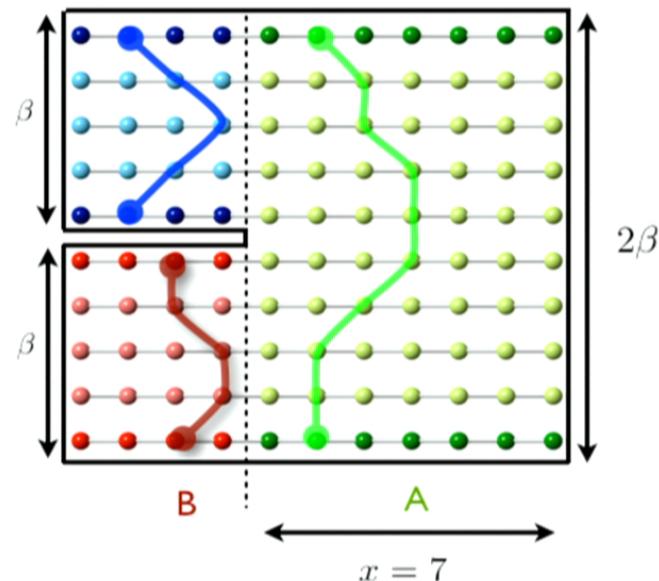
$$I_2 = S_2(A) + S_2(B) - S_2(A \cup B)$$



$$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z$$

Calabrese and Cardy, J. Stat. Mech. 0406, P002 (2004).  
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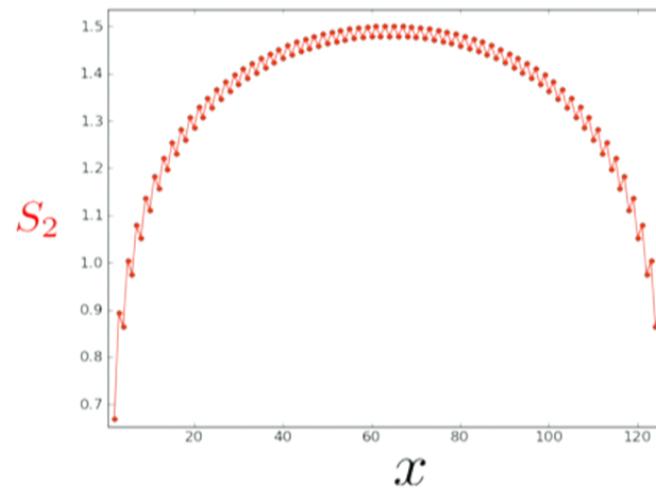
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We can examine the same shape functions, or define new ones

$n$ -sheeted Riemann surface:

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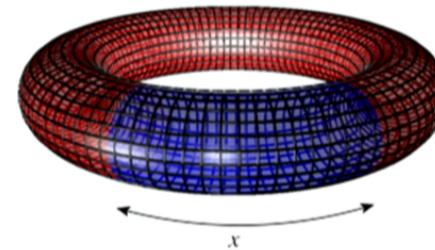
# Outline

- Review of entanglement entropy scaling in 1D

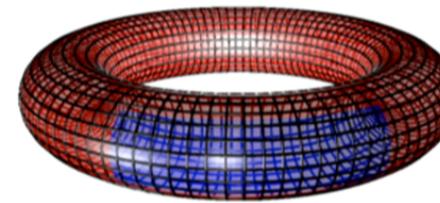


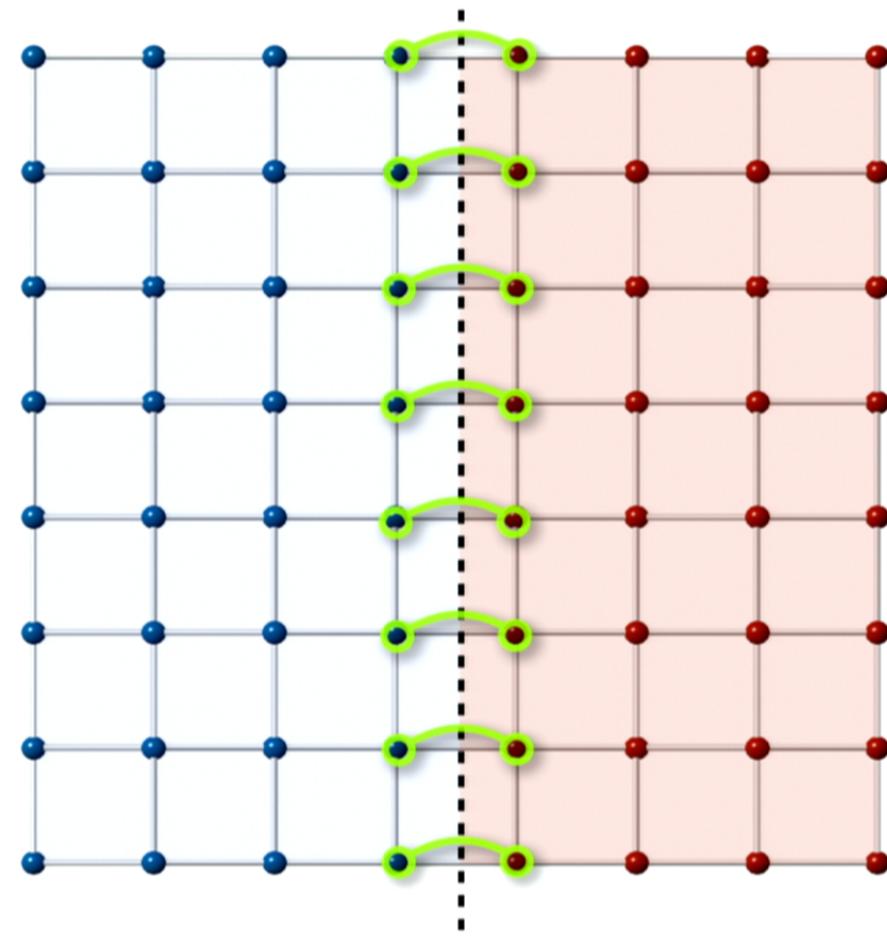
- 2D: area law and beyond

- Two-cylinder entropies

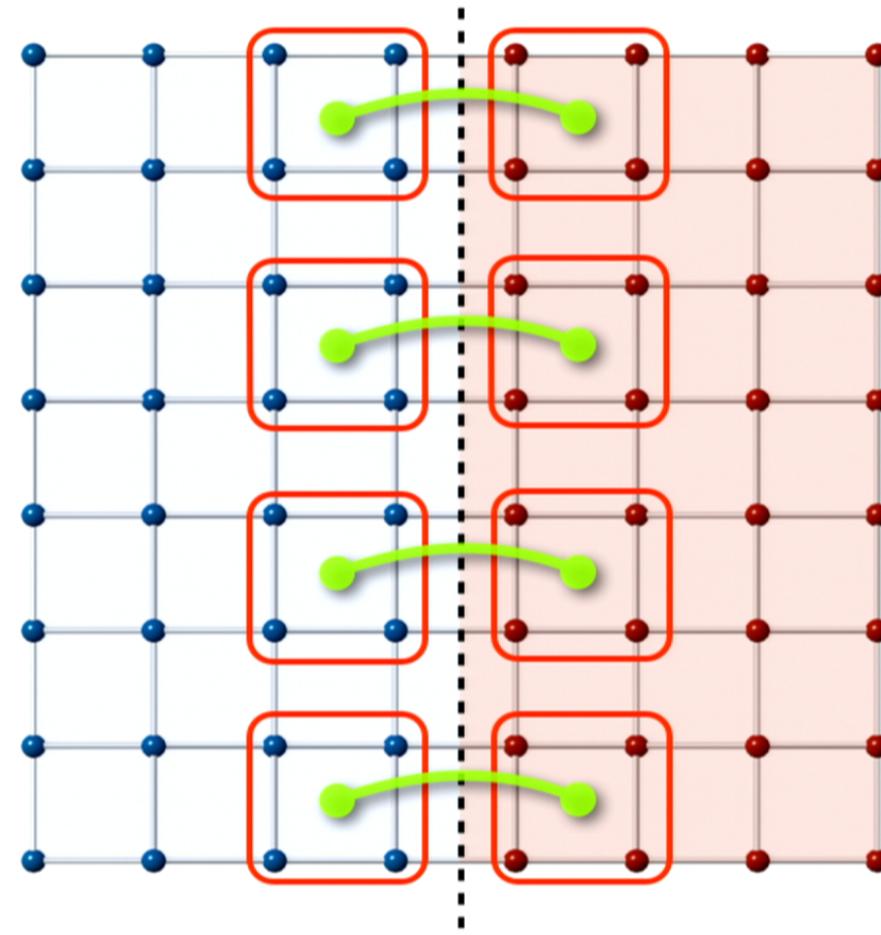


- Universal contributions due to vertices





$$\mathcal{O}(1) \times (L +$$



$$\mathcal{O}(1) \times (L+L/2)$$

- In a critical system in 2D, the leading-order scaling is area-law:

$$S_n = A\ell + \dots$$

(this is confirmed numerically on a variety of quantum models)

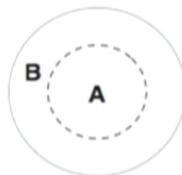
- General expectation: geometry-dependent subleading term

$$S_n = A\ell + \gamma_n(\ell, L, \dots)$$

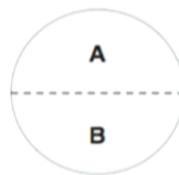
Fradkin, Moore  
Ryu and Takayanagi  
Cardy, Peschel

Zhang, Grover, Vishwanath  
Casini, Huerta, Myers  
Fuertes, Metlitski, Sachdev

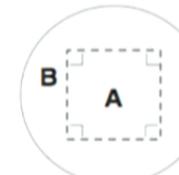
- For some critical theories and geometries,  $\gamma_n$  is well known



$$S_{\log} = 0$$



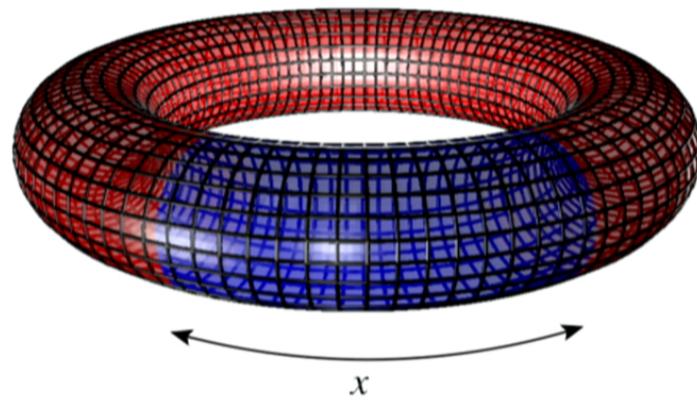
$$S_{\log} = -\frac{c \log R}{4}$$



$$S_{\log} = -\frac{c \log R}{9}$$

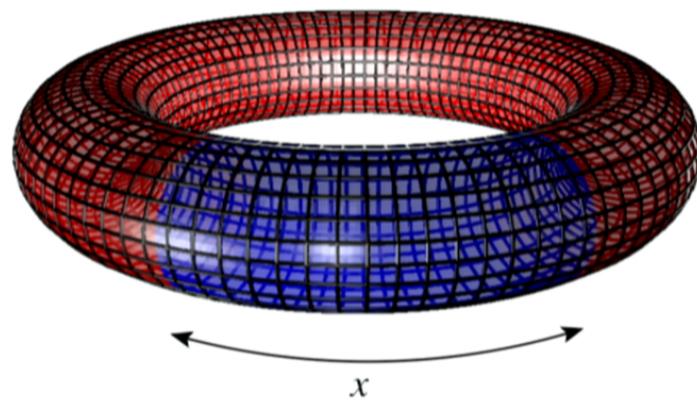
Fradkin, Moore, PRL 97, 050404 (2006)

- Let's examine simplest shape-dependent subleading term for typical QMC lattice geometries: a 2D torus divided into two cylinders

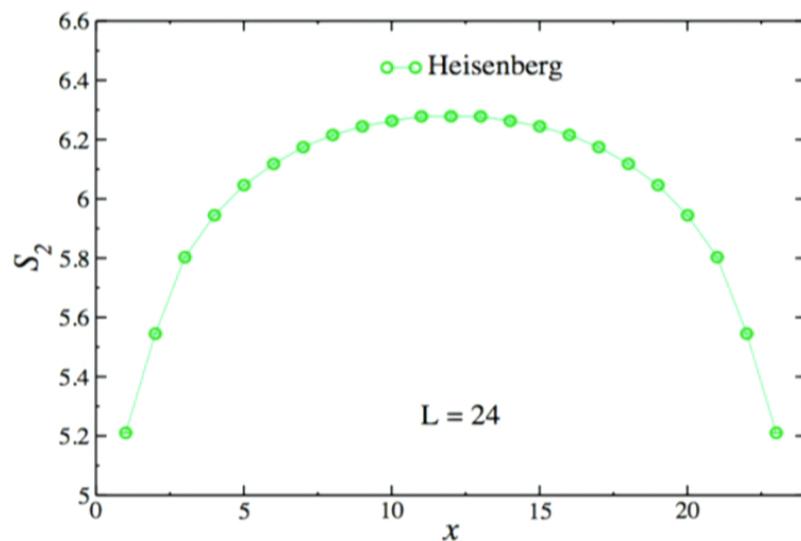


$$S_n = A\ell + \gamma_n(x/L_x, L_x/L_y)$$

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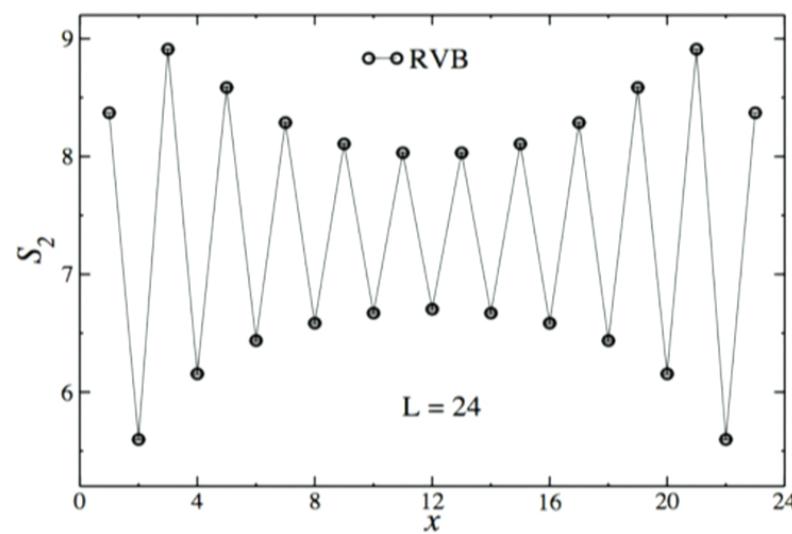
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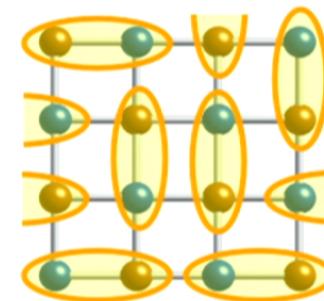
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

+  $\log(L)$  term from Goldstone modes

Kallin, Hastings, RGM, Singh, PRB 84, 165134 (2011)  
Metlitski, Grover arXiv:1112.5166 (2011)



$$|\psi\rangle = \sum_V |V\rangle$$

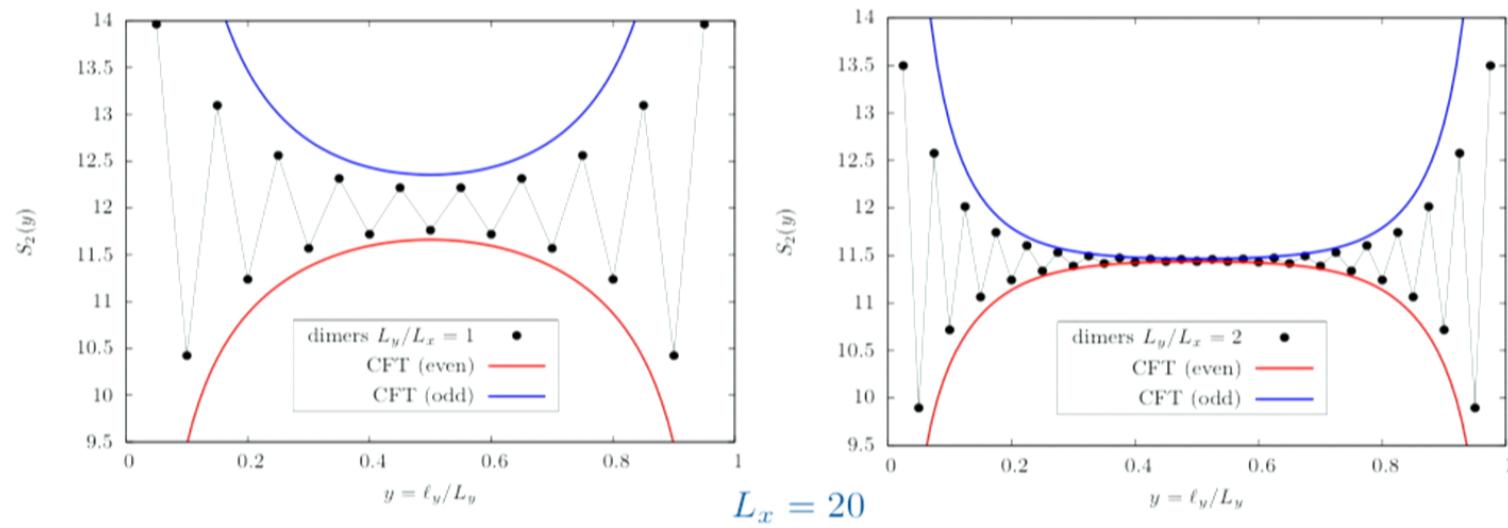


even-odd branching

Ju, Kallin, Fendley, Hastings, RGM, PRB 85, 165121 (2012)

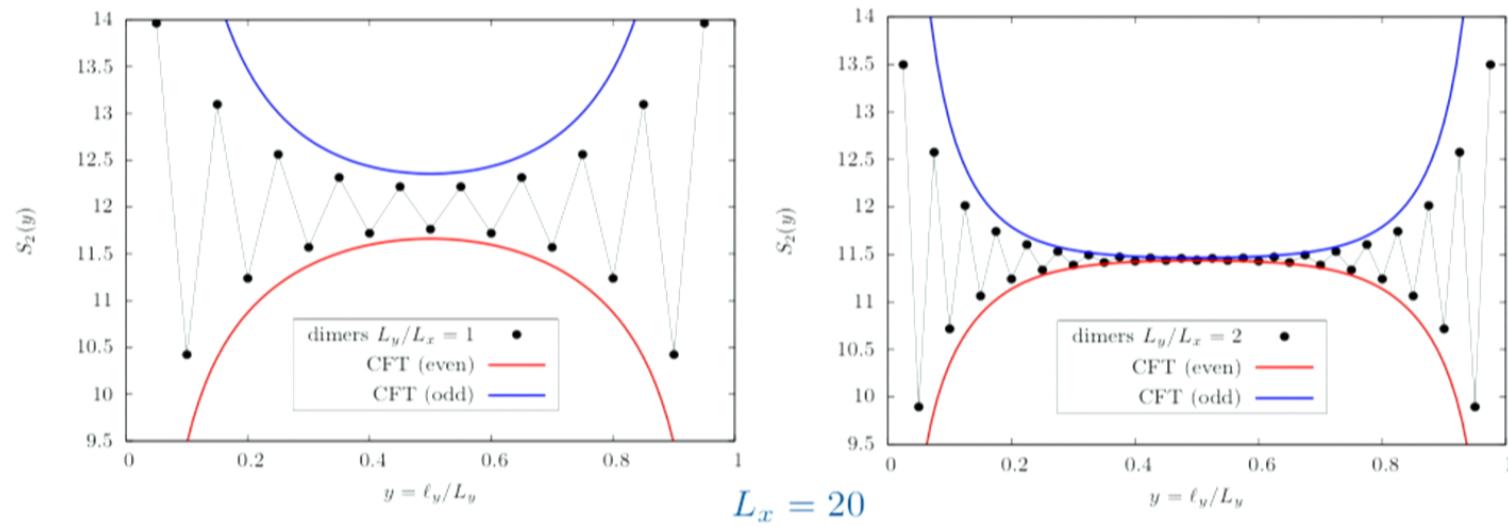
- Jean-Marie Stéphan: “RVB-shape function”, derived in a quantum Lifshitz free scalar field theory

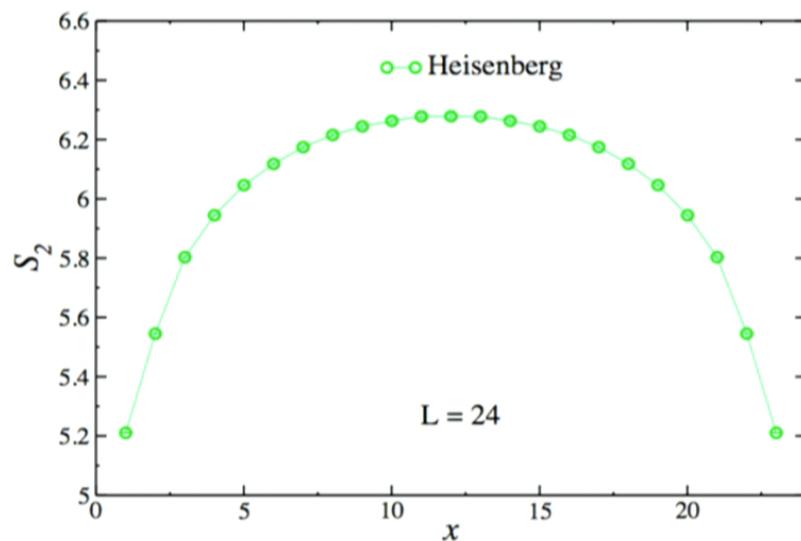
$$J_n(y) = \frac{n}{1-n} \log \left[ \frac{\eta(\tau)^2}{\theta_3(2\tau)\theta_3(\tau/2)} \frac{\theta_3(2y\tau)\theta_3(2(1-y)\tau)}{\eta(2y\tau)\eta(2(1-y)\tau)} \right] \quad \begin{aligned} y &= x/L \\ \tau &= iL_x/L_y \end{aligned}$$



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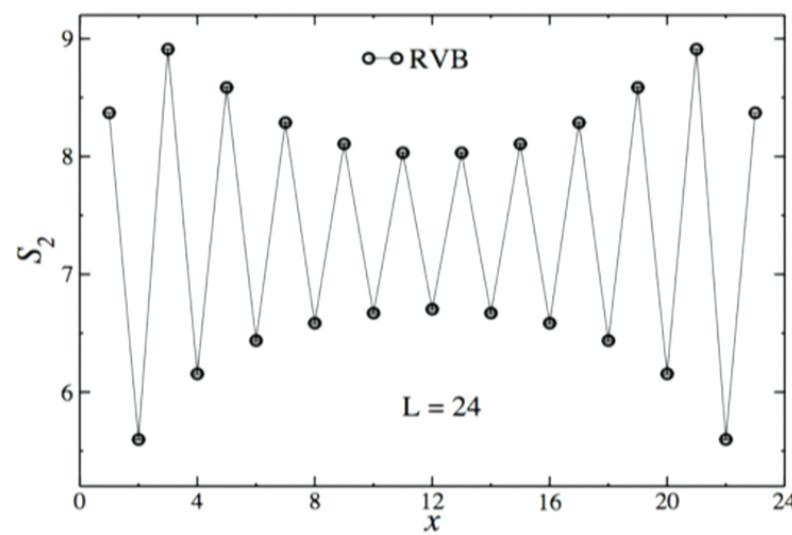




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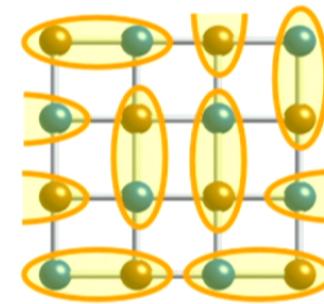
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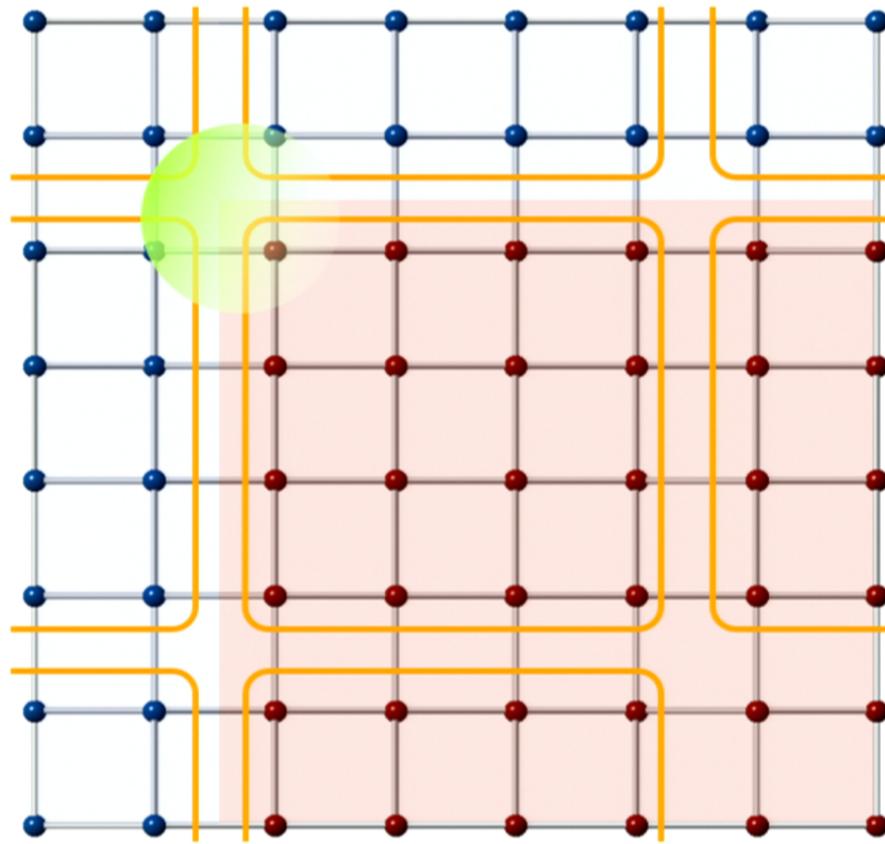


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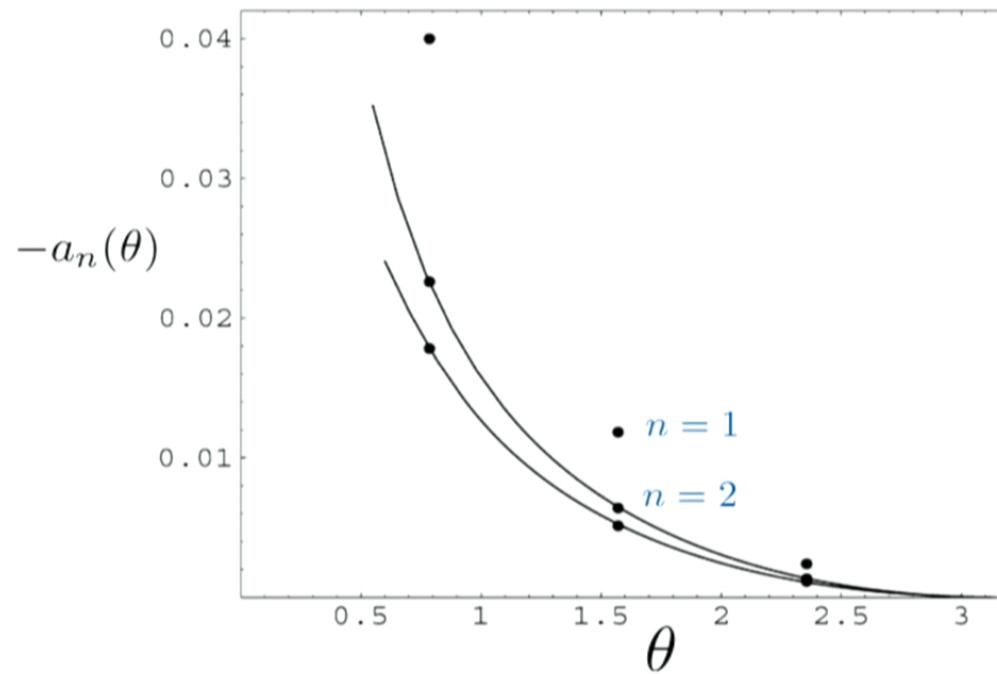
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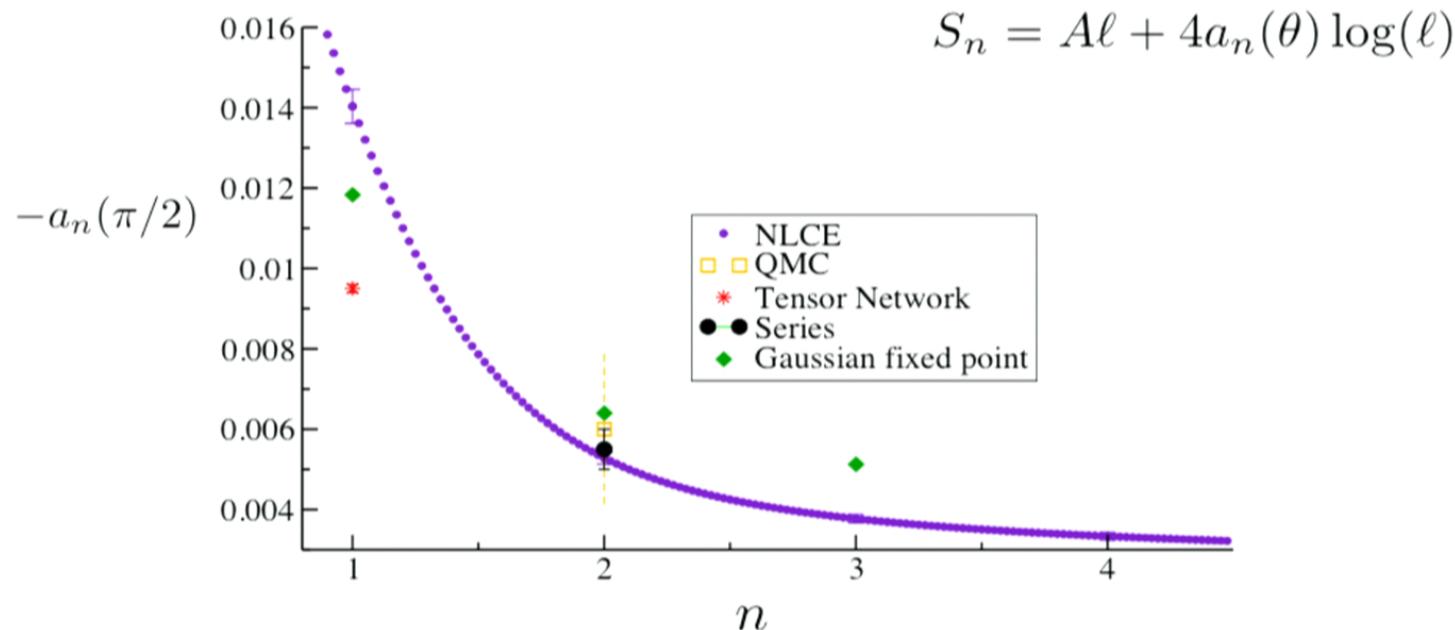
$$\mathcal{O}(1) \times (1+1+1+\dots) = \log(L)$$

H. Casini, M. Huerta:  
slightly more sophisticated analysis in Nuclear Physics B 764, 183 (2007)

- Corner contributions have been calculated at Gaussian fixed point

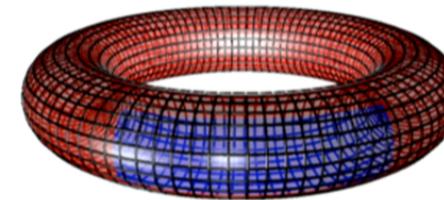


## Results for 90-degree corner coefficient from the literature

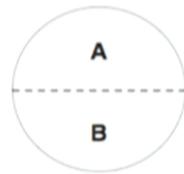


- Kallin, Hyatt, Singh, RGM, Phys. Rev. Lett. 110, 135702 (2013)
- \* Tagliacozzo, Evenbly, Vidal Phys. Rev. B 80, 235127 (2009)
- Singh, RGM, Jaan Oitmaa, Phys. Rev. B 86, 075106 (2012)
- ◆ Casini, Huerta, Nuclear Physics B 764, 183 (2007)

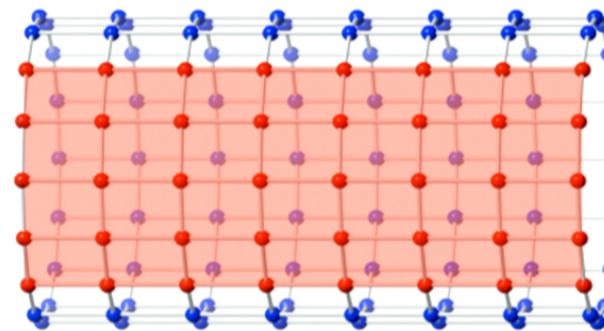
## universal corner coefficient



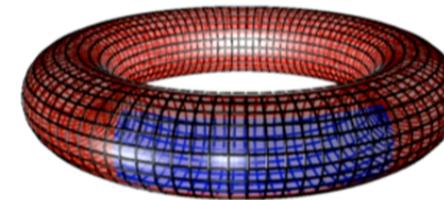
- We're starting to get a handle on these numerical values for the Ising QCP
- Can we use this number to distinguish Gaussian from Ising? Which numerical method is working best?
- Calculation in progress: corner term from disconnecting the original system



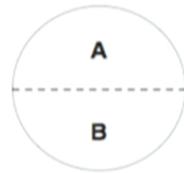
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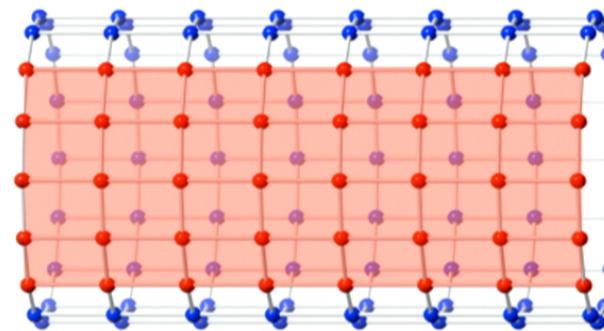
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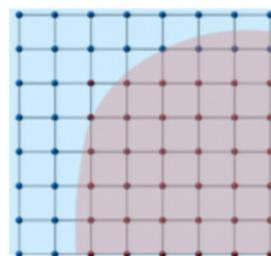
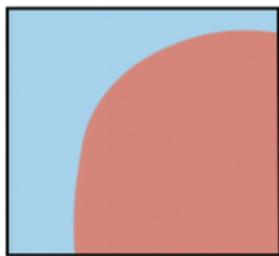


## Future work

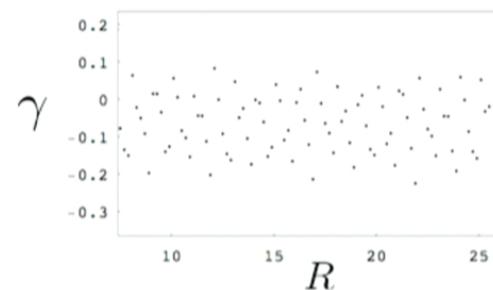
- We are looking at other interacting critical points, in particular corner contributions in a Heisenberg bilayer (2+1D, O(3) universality).

(talk to Ann Kallin)

- There is an interesting “open” problem about whether the subleading terms that occurs in a disk geometry in 2+1 converges on a lattice:



Casini, Huerta, J. Phys.A 42, 504007 (2009)



## Discussion

- We have the simulation technology to take a more serious look at entanglement in gapless systems. Much more field theory work to be done.
- It would be nice to look at exceptions to the area law also: e.g. bosonic systems with a Bose surface

$$S_n \propto \ell \log(\ell)$$

Violation of Entanglement Area Law in Bosonic Systems with Bose surfaces,  
Lai, Yang and Bonesteel, arXiv:1306.2698

- Renyi entropies are accessible through QMC and field theory; also will be the quantities eventually measured in experiments

Cardy, PRL 106, 150404 (2011)  
Abanin, Demler, PRL 109, 020504 (2012)  
Pichler, Bonnes, Daley, Läuchli, Zoller arXiv:1302.1187