Title: Hofstadterâ€™s Butterfly and interaction driven quantum Hall ferromagnetism in graphene

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Abstract: Electrons moving in a periodic electric potential form Bloch energy bands where the mass of electrons are effectively changed. In a strong magnetic field, the cyclotron orbits of free electrons are quantized and Landau levels forms with a massive degeneracy within. In 1976, Hofstadter showed that for 2-dimensional electronic system, the intriguing interplay between these two quantization effects can lead into a self-similar fractal set of energy spectrum known as â€œHofstadterâ€™s Butterfly.â€• Experimental efforts to demonstrate this fascinating electron energy spectrum have continued ever since. Recent advent of graphene, where its Bloch electrons can be described by Dirac feremions, provides a new opportunity to investigate this half century old problem experimentally. In this presentation, I will discuss the experimental realization Hofstadterâ€™s Butterfly via substrate engineered graphene under extremely high magnetic fields controlling two competing length scales governing Dirac-Bloch states and Landau orbits, respectively. In addition, the strong Coulomb interactions and approximate spin-pseudo spin symmetry are predicted to lead to a variety of integer quantum Hall ferromagnetic and fractional quantum Hall states and the quantum phase transition between them in graphene. I will discuss several recent experimental evidences to demonstrate the role of the electron interaction in single and bilayer graphene.

Hofstadter's Butterfly in Graphene

and electron interaction driven quantum Hall ferromagnetism

Philip Kim

Physics Department, Columbia University


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Prof. Ken Shepard
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Hofstadter's Butterfly

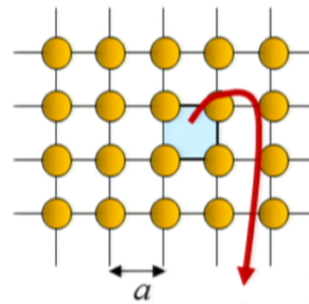
PHYSICAL REVIEW B VOLUME 14, NUMBER 6 15 SEPTEMBER 1976

Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields*

Douglas R. Hofstadter†
Physics Department, University of Oregon, Eugene, Oregon 97403
(Received 9 February 1976)

Harper's Equation

$$2\psi_l \cos\left(2\pi l b - \kappa\right) + \psi_{l+1} + \psi_{l-1} = E\psi_l$$



$$b = \frac{Ba^2}{\phi_0} = \frac{\phi}{\phi_0}$$

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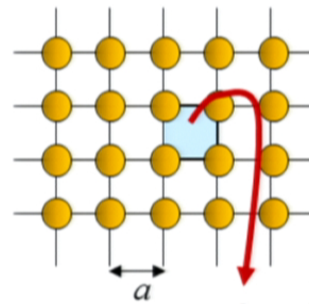

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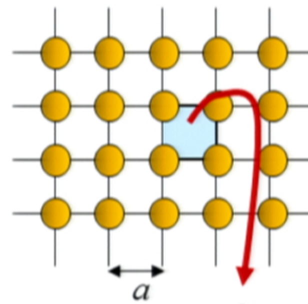

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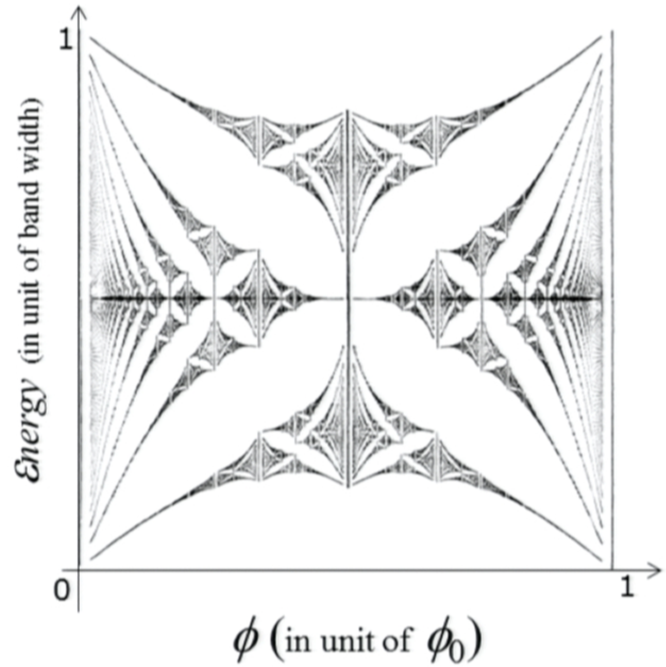
Energy bands develop ***fractal structure*** when magnetic length is of order the periodic unit cell



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TKNN (1982); Stredar (1982)

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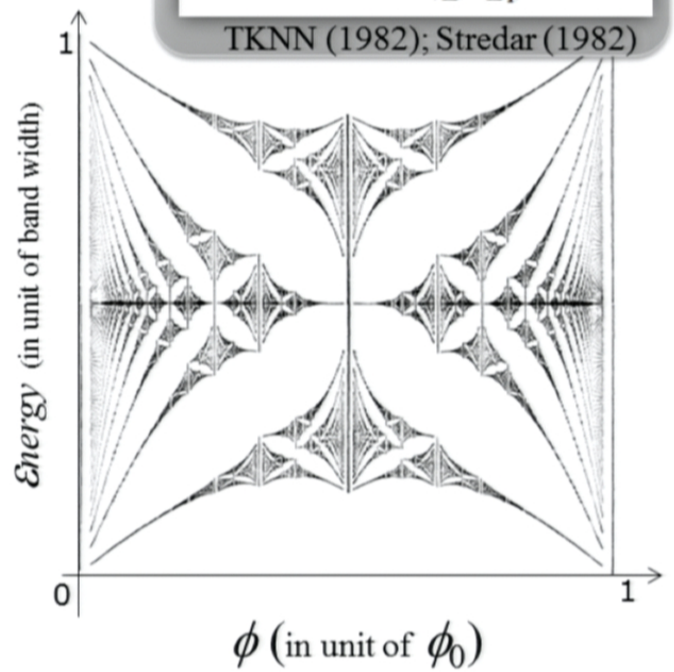

Diophantine equation for gaps

$$(n/n_0) = t(\phi/\phi_0) + s \qquad t, s \in \mathbb{Z}$$

Wannier (1978)

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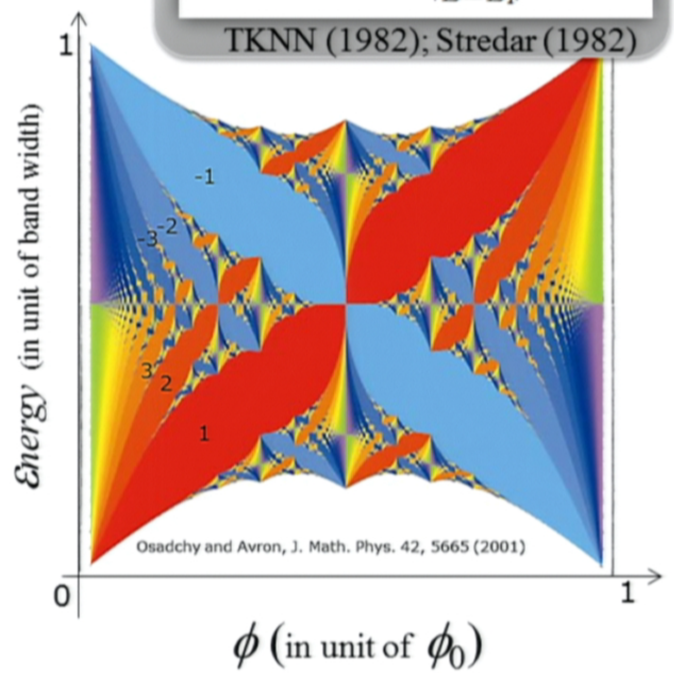

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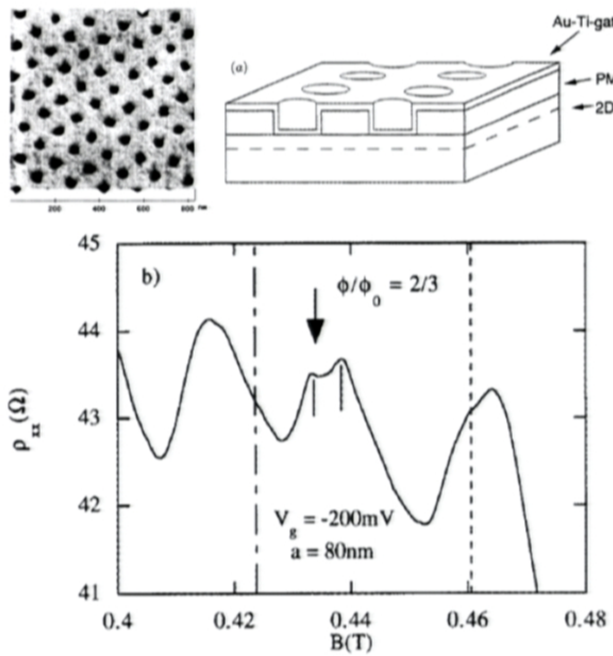
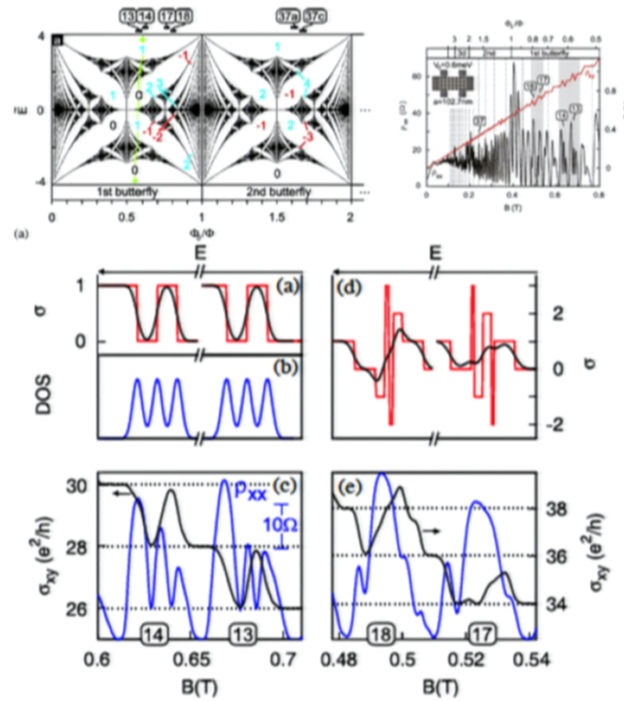
Experimental Search For Butterfly



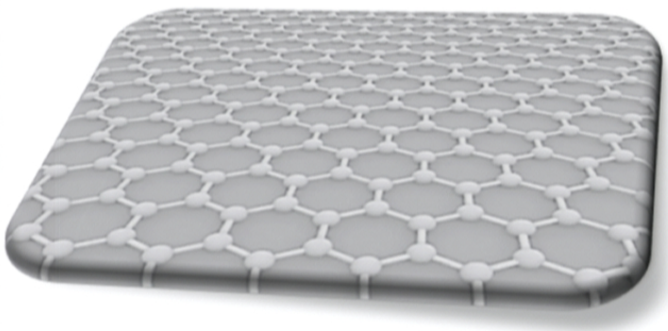
-Schlosser et al, Semicond. Sci. Technol. (1996)



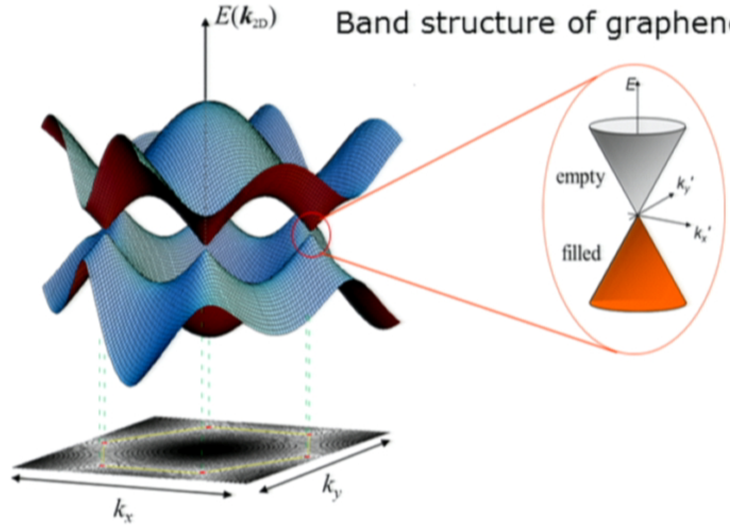
Albrecht et al, PRL. (2001);
Geisler et al, PRL (2004)

Electrons in Graphene: Effective Dirac Fermions

Graphene,
ultimate 2-d conducting system



Band structure of graphene



Effective Dirac Equations

$$H_{eff} = \pm \hbar v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} = \pm \hbar v_F \vec{\sigma} \cdot \vec{k}_\perp$$

DiVincenzo and Mele, PRB (1984); Semenov, PRL (1984)

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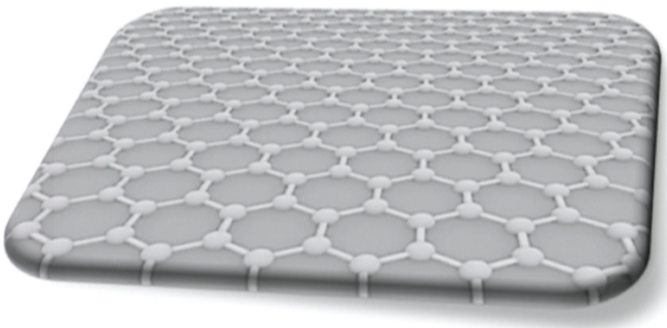

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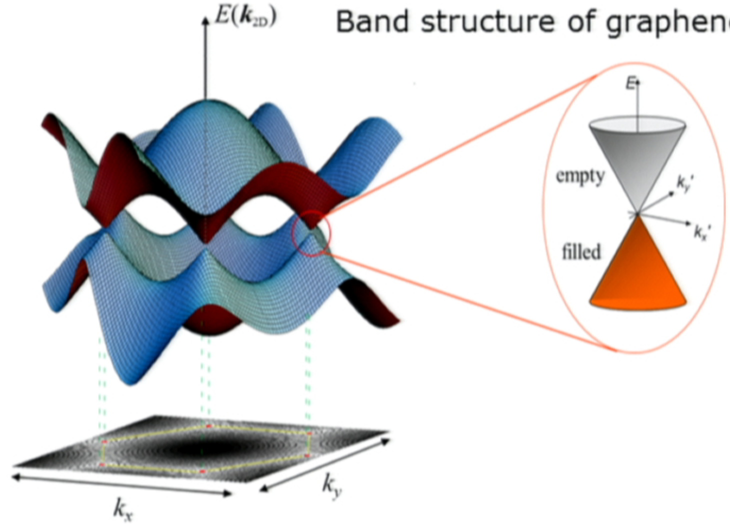

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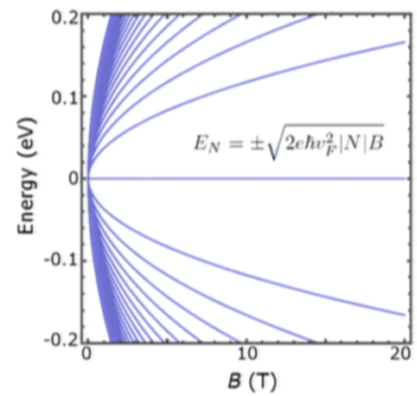
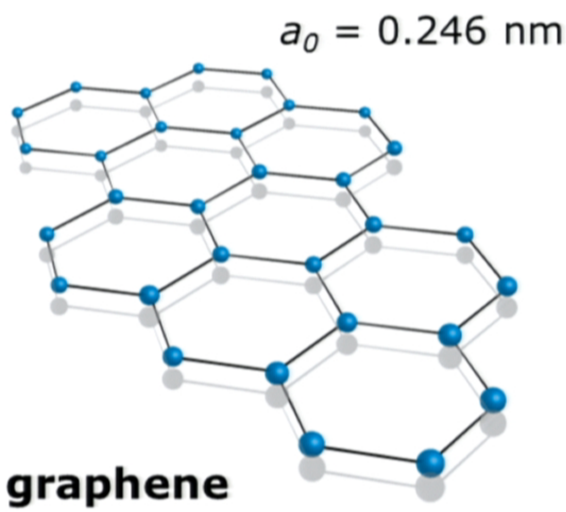
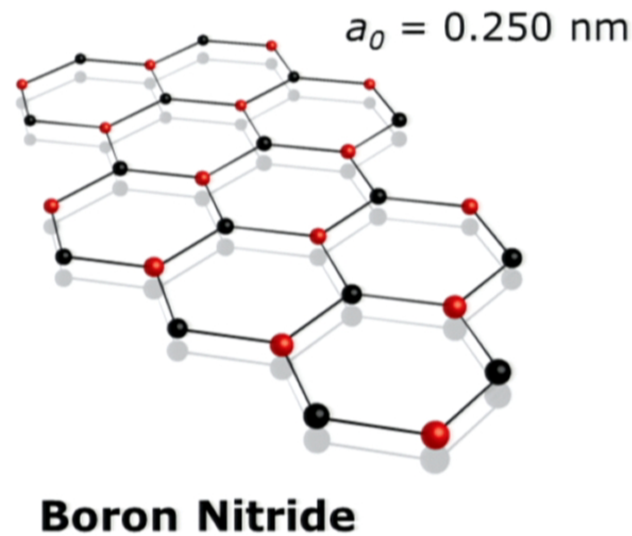
Landau Levels in Graphene



Hexa Boron Nitride: Polymorphic Graphene





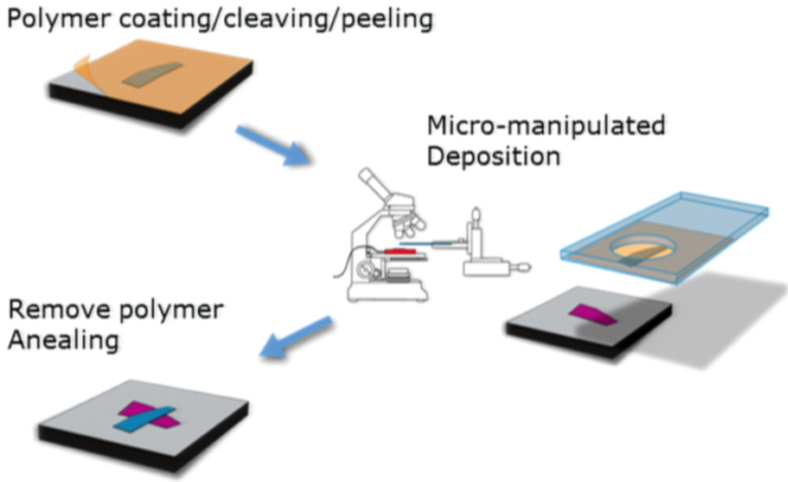
Comparison of h-BN and SiO_2

	Band Gap	Dielectric Constant	Optical Phonon Energy	Structure
BN	5.5 eV	~4	>150 meV	Layered crystal
SiO2	8.9 eV	3.9	59 meV	Amorphous

Stacking graphene on hBN

Dean et al. Nature Nano (2009)

(Hone, Kim and Shepard groups @ Columbia)



- Co-lamination techniques
- Submicron size precision
- Atomically smooth interface

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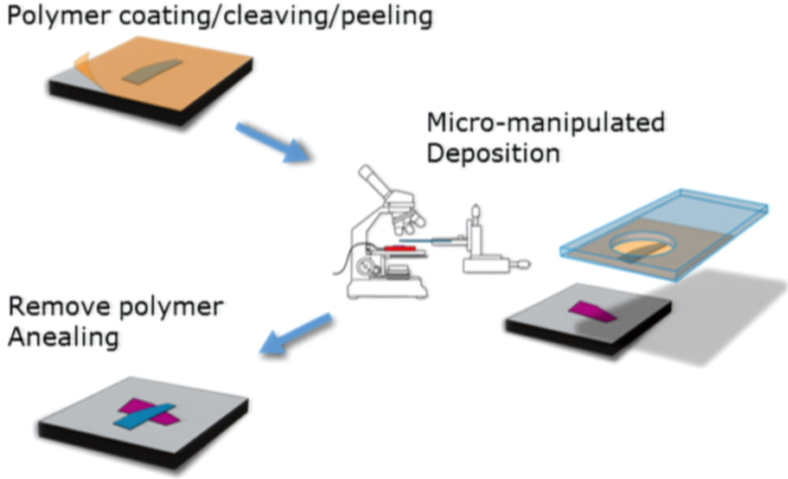

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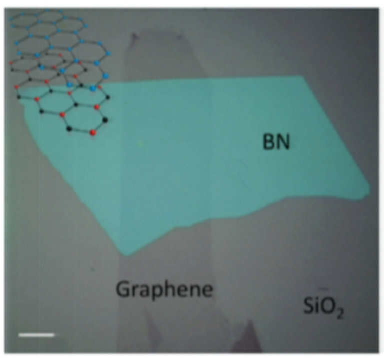

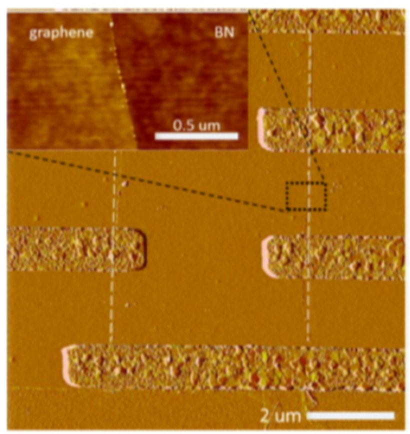

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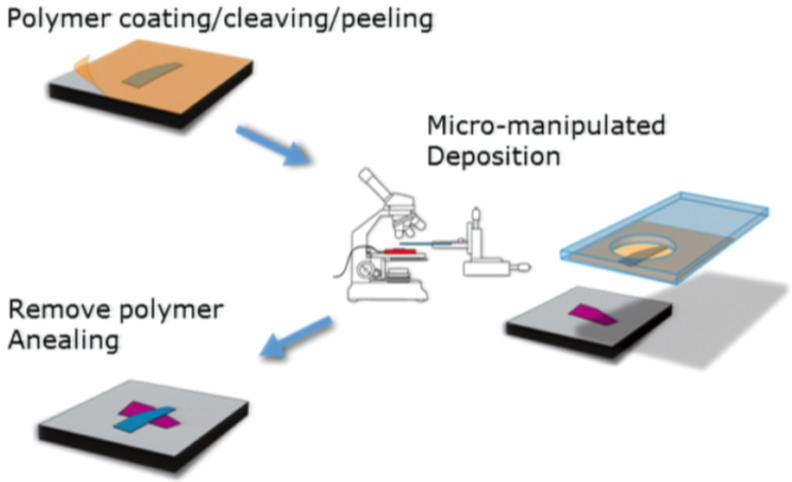

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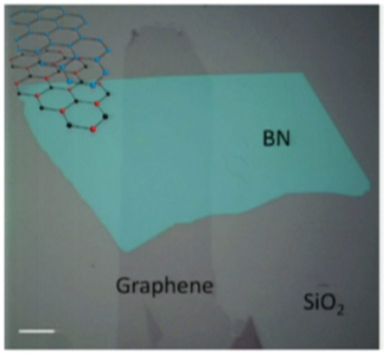
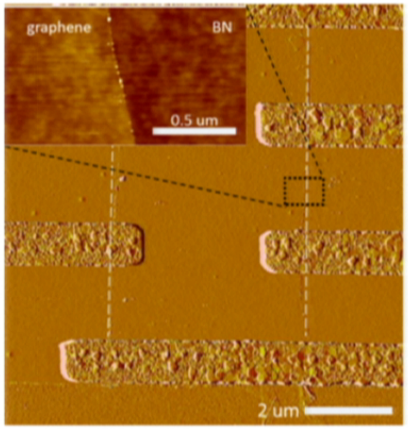
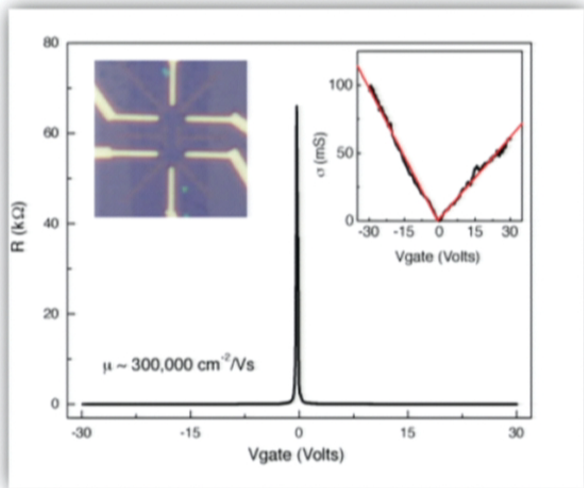


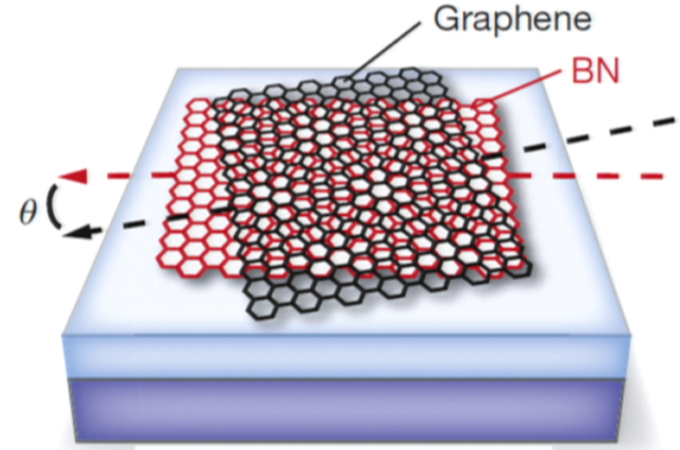




Mobility > 100,000 cm^{2}V^{-1}s^{-1}

Bilayer Graphene on hBN: Moire Pattern

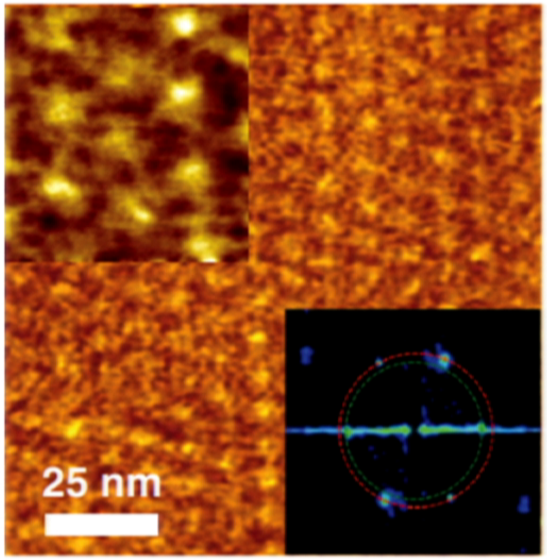
Misaligned graphene on hBN: Moire pattern

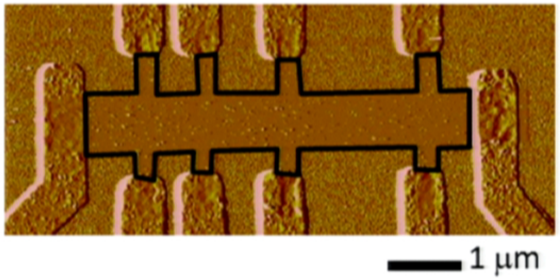


Moire wavelength $\lambda = \frac{(1+\delta)a}{\sqrt{2(1+\delta)(1-\cos(\phi))+\delta^2}}$

Bilayer Graphene on hBN: Moire Pattern

UHV AFM (Ishigami group)





Moiré λ_{AFM} = 15.5 nm

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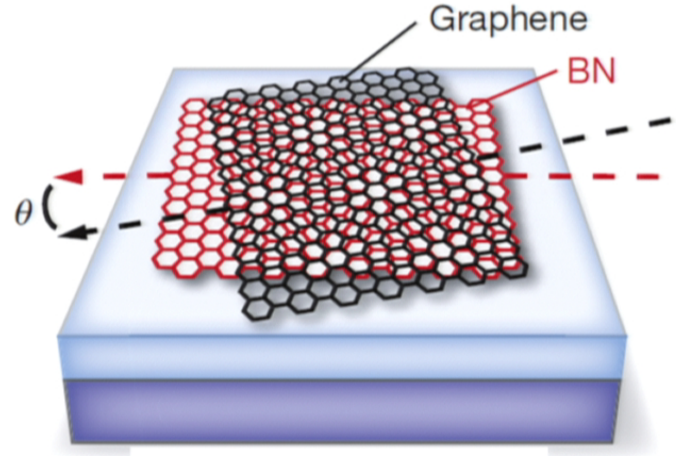

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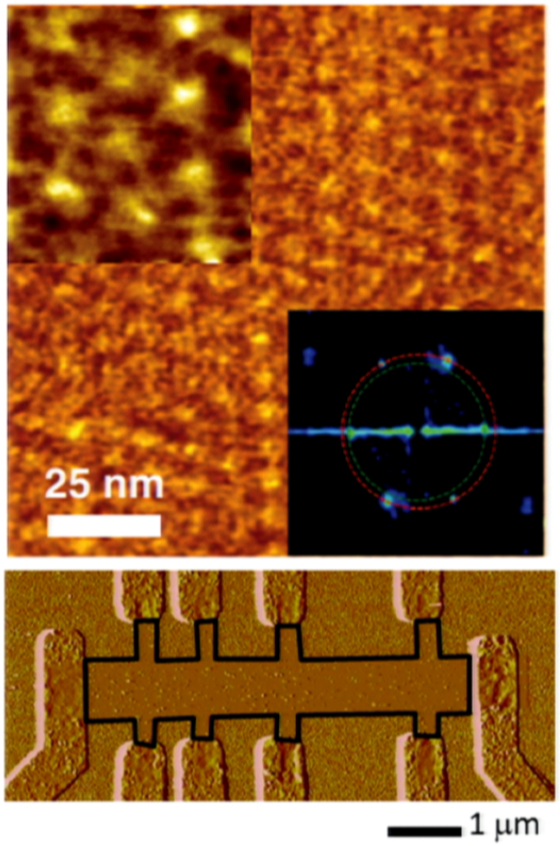

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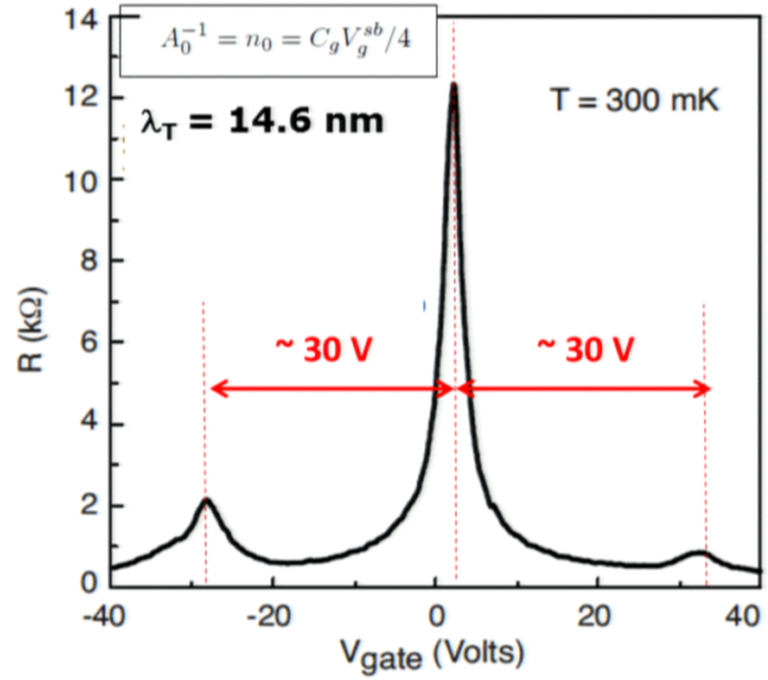
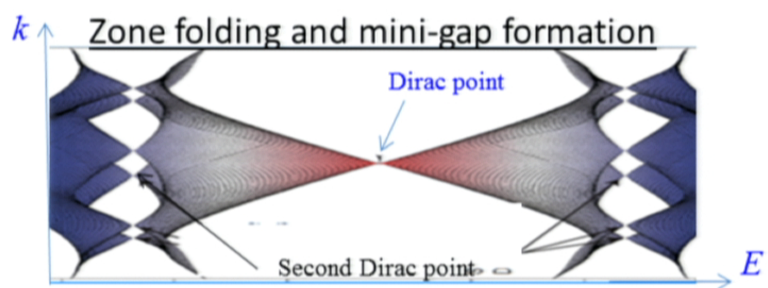
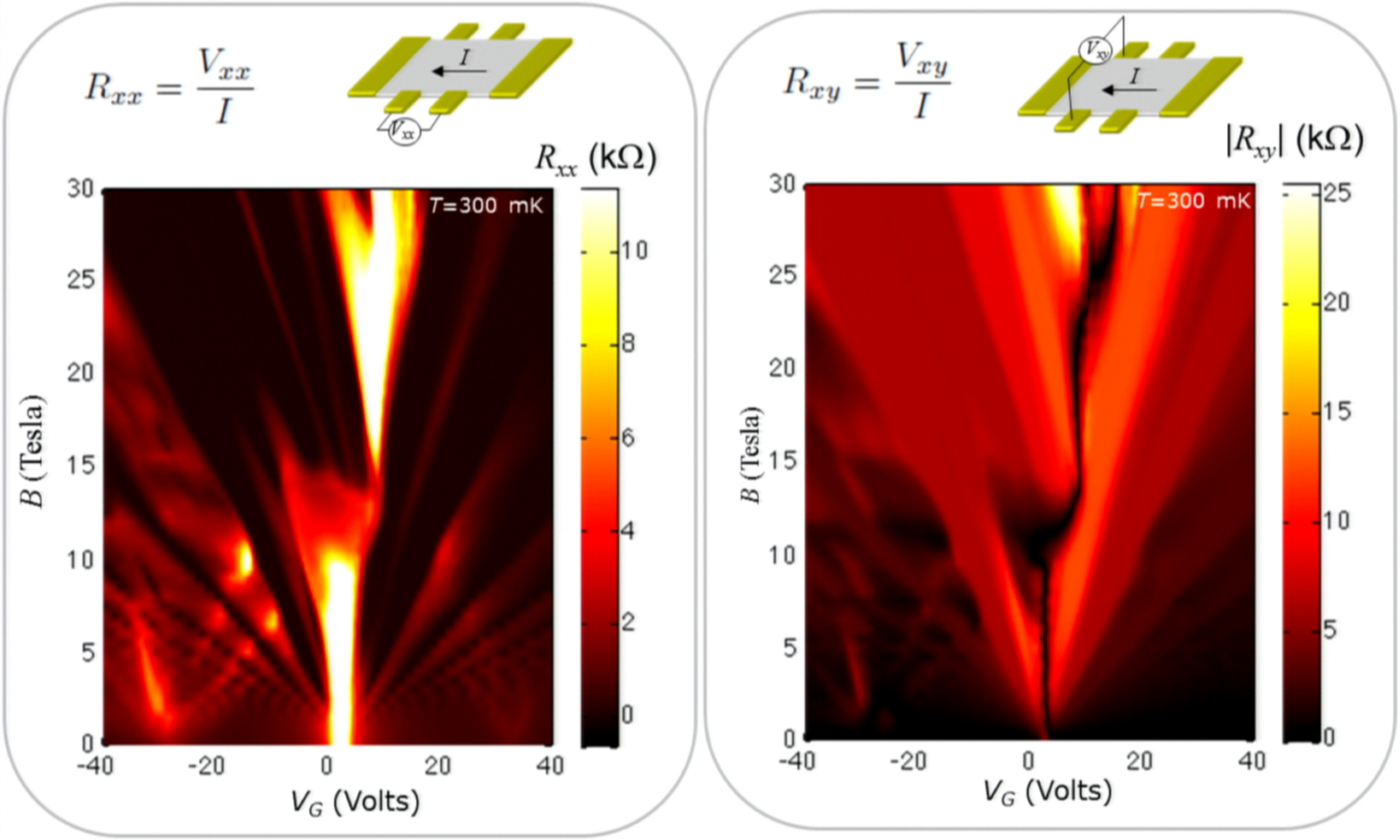




Abnormal Landau Fan Diagram in Graphene on hBN

Bilayer Graphene on hBN: Dean *et al*, *Nature* (2013)



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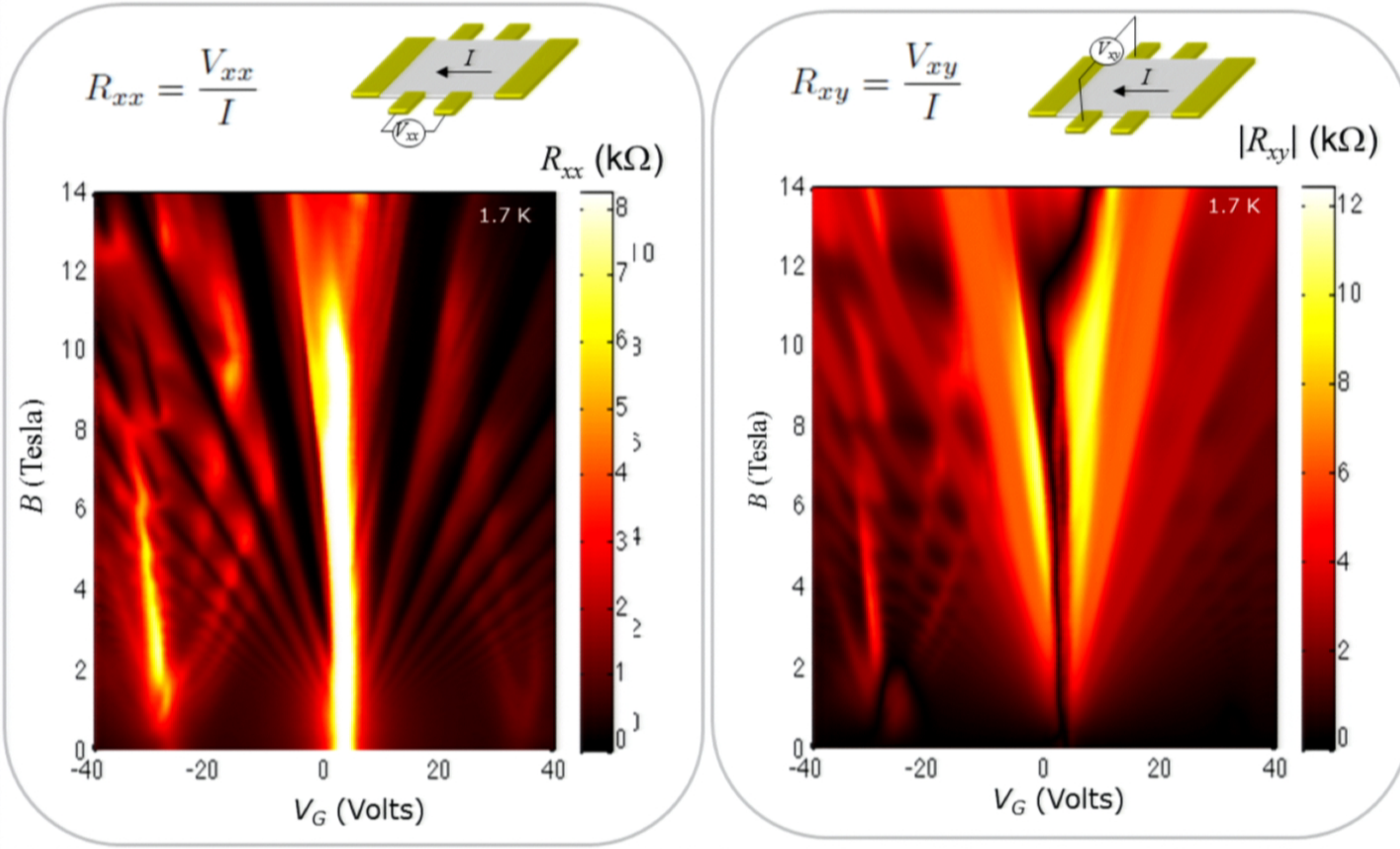
Low Magnetic field regime



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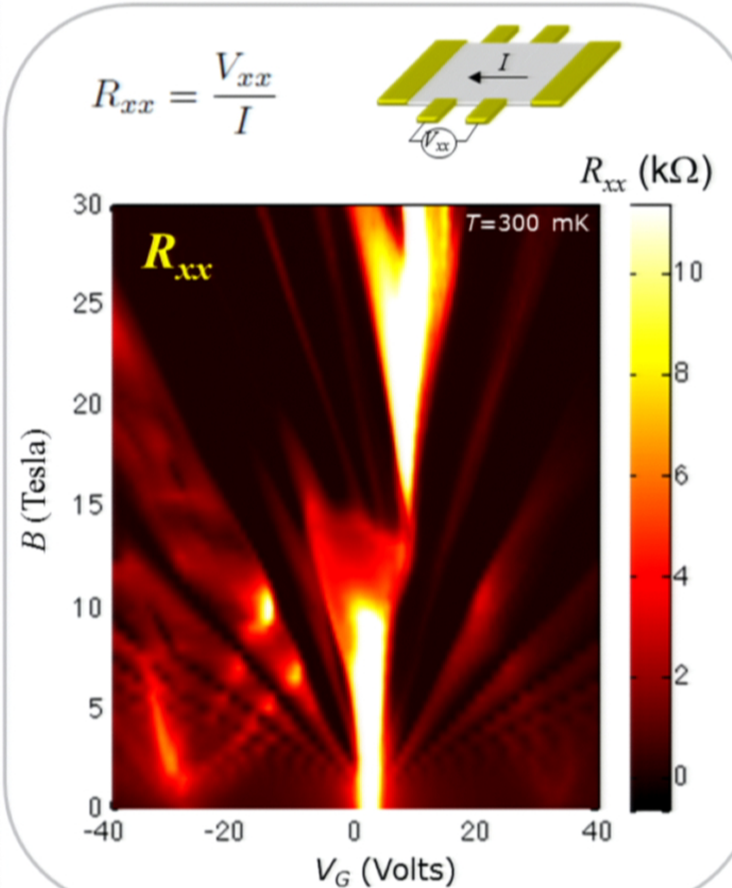
Quantum Hall Effect in Graphene Moire

Quantum Hall-like Transport

$$R_{xx} = \frac{V_{xx}}{I}$$



Landau level filling factor

$$\nu = \frac{\phi_0}{B} n$$

Quantum Hall conductance

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$$\nu = t \quad \in \mathbb{Z}$$

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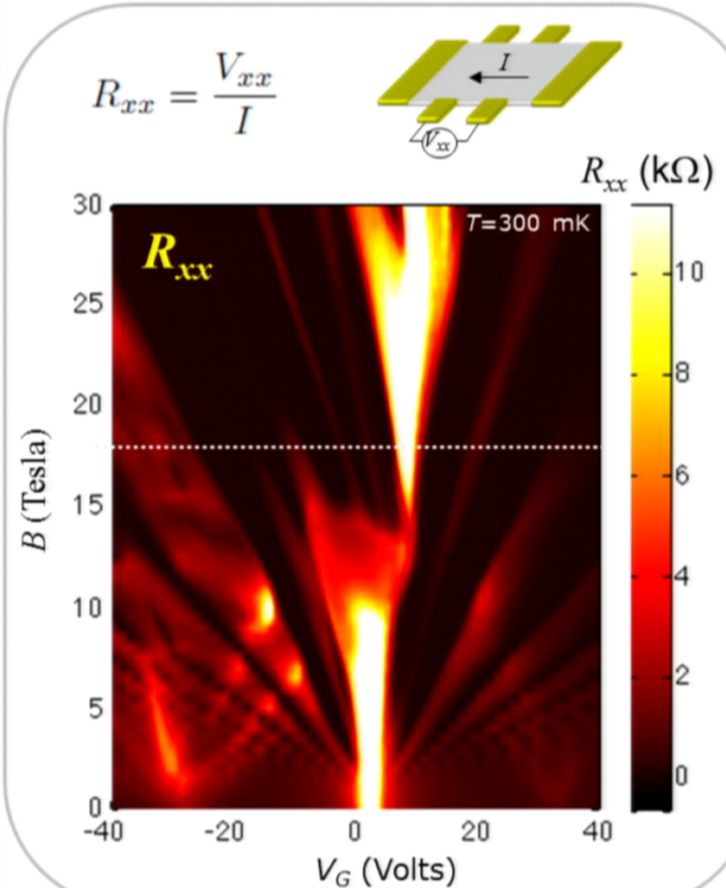

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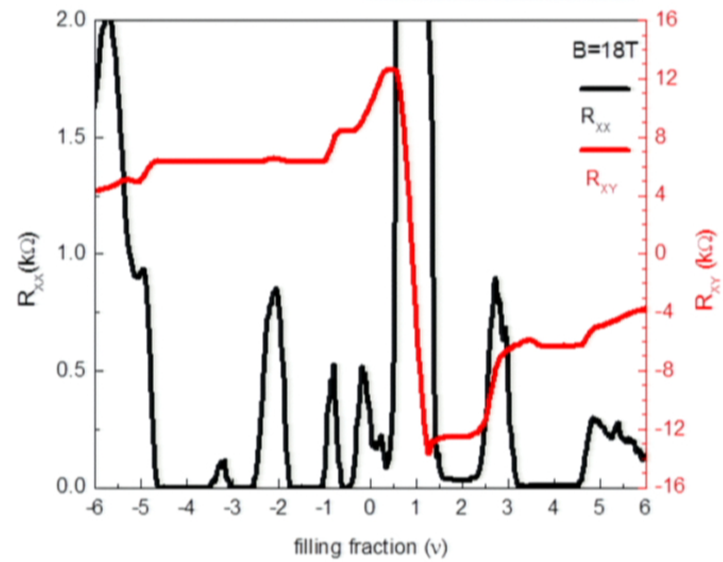

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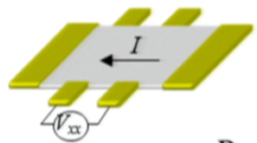

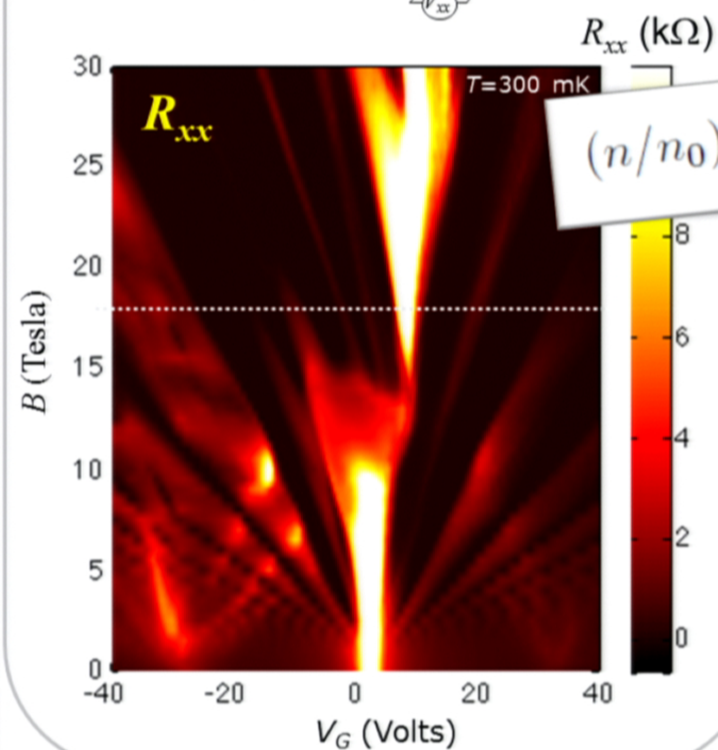

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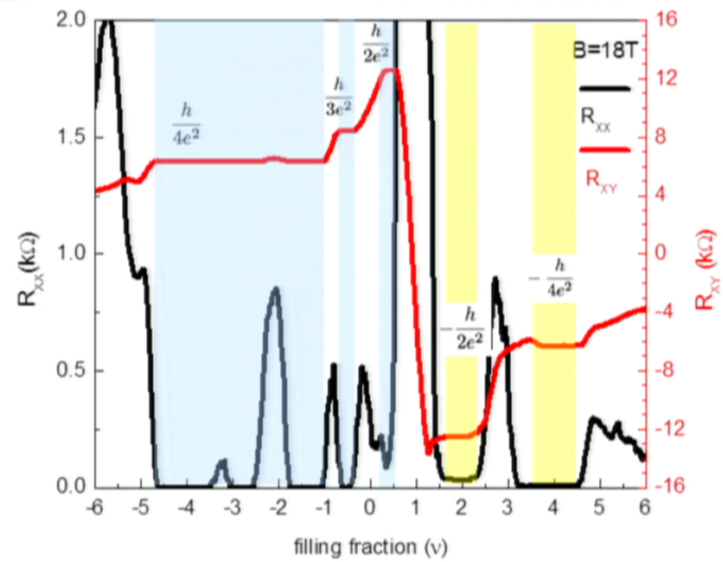
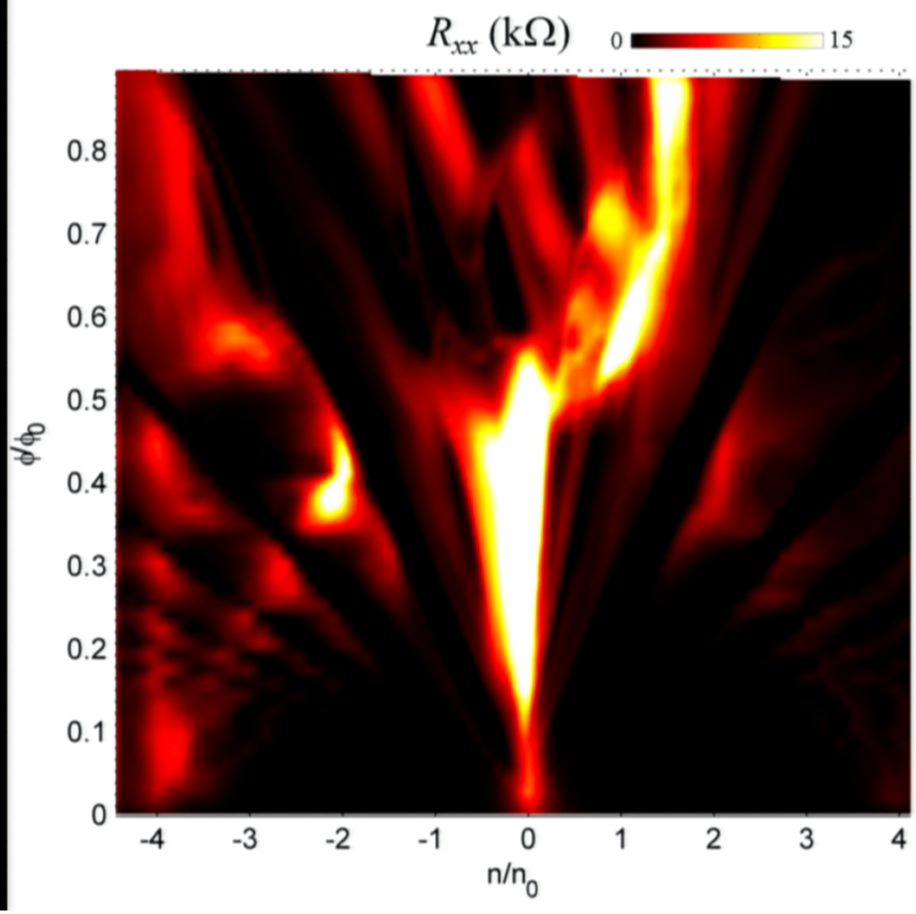


Quantum Hall Effect with Two Integer Numbers

n/n_0: density per unit cell; ϕ : flux per unit cell



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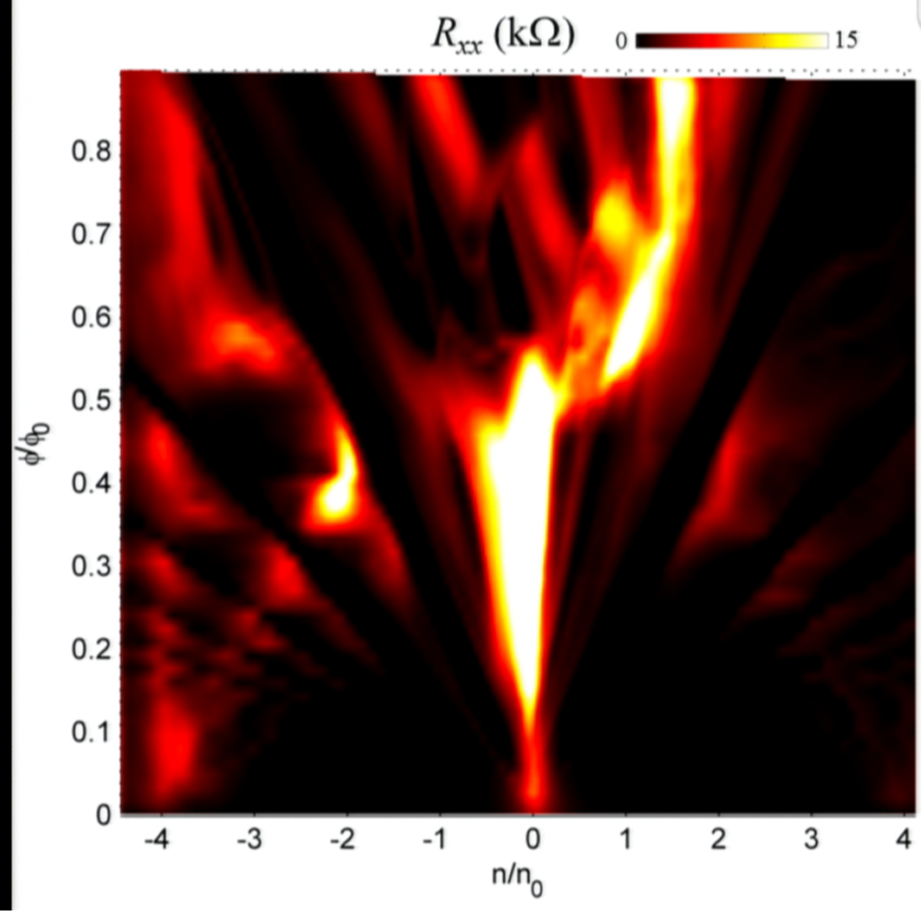

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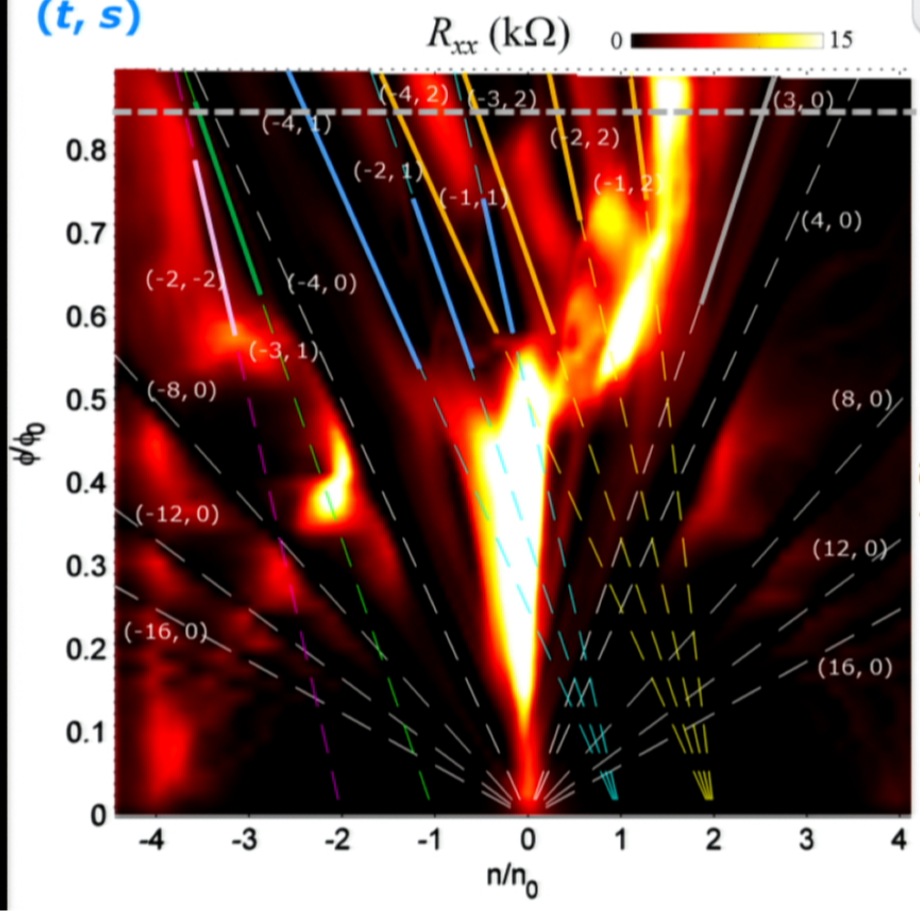
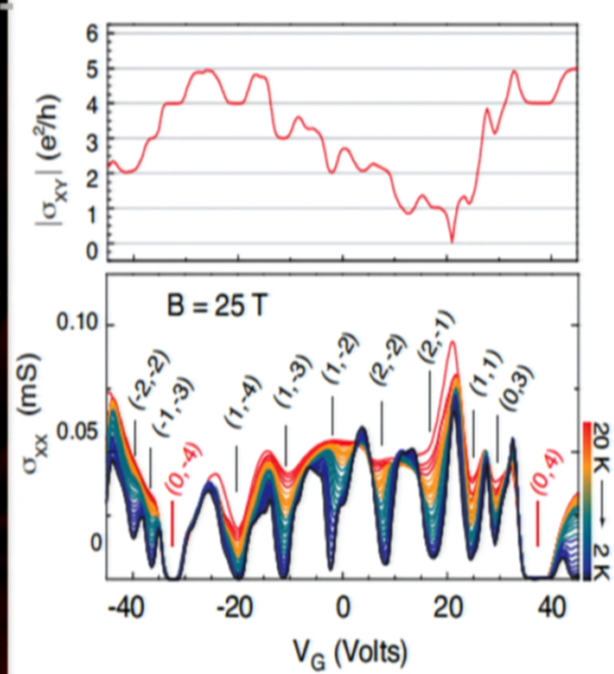
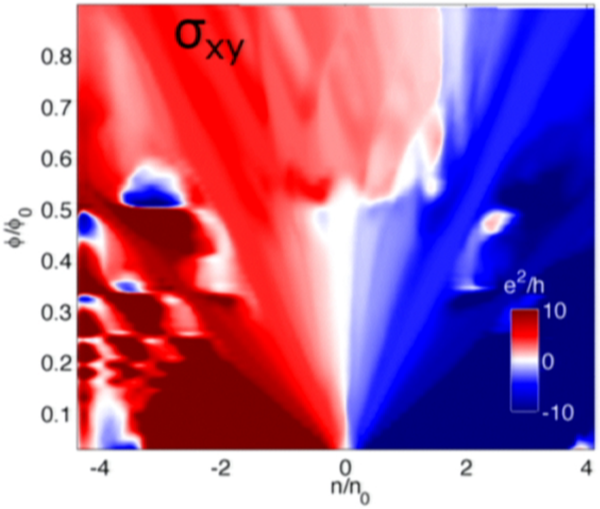


Quantization of σ_{xx} and σ_{xy}



Recursive QHE near the Fractal Bands

Higher quality sample with lower disorder



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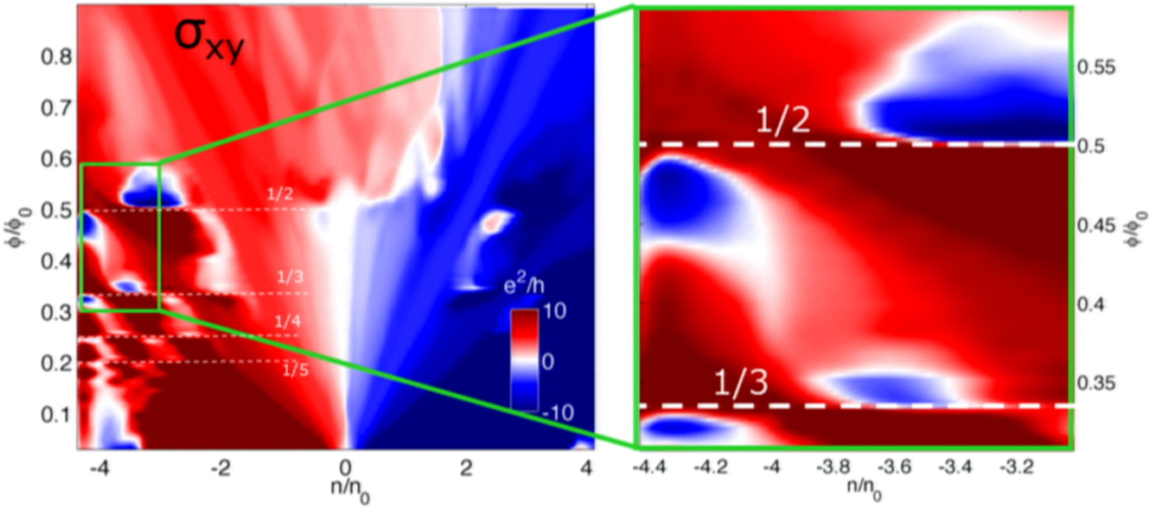
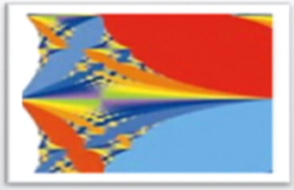


Hall conductivity across Fractal Band



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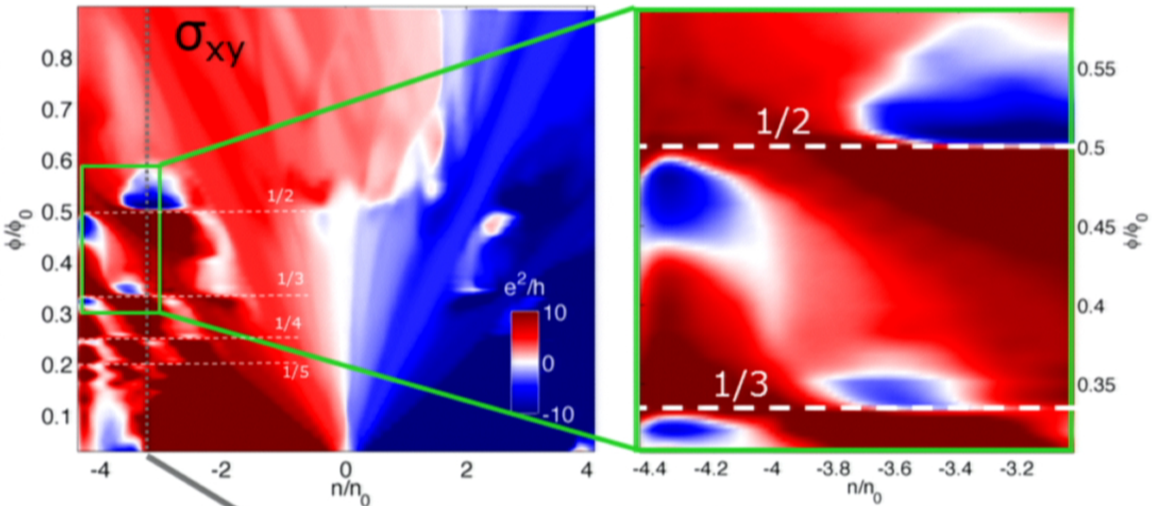
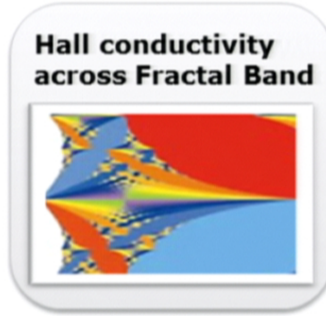
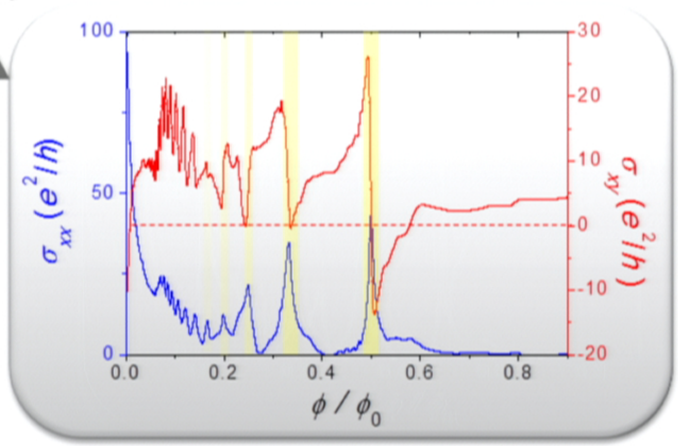




At the Fractal Bands

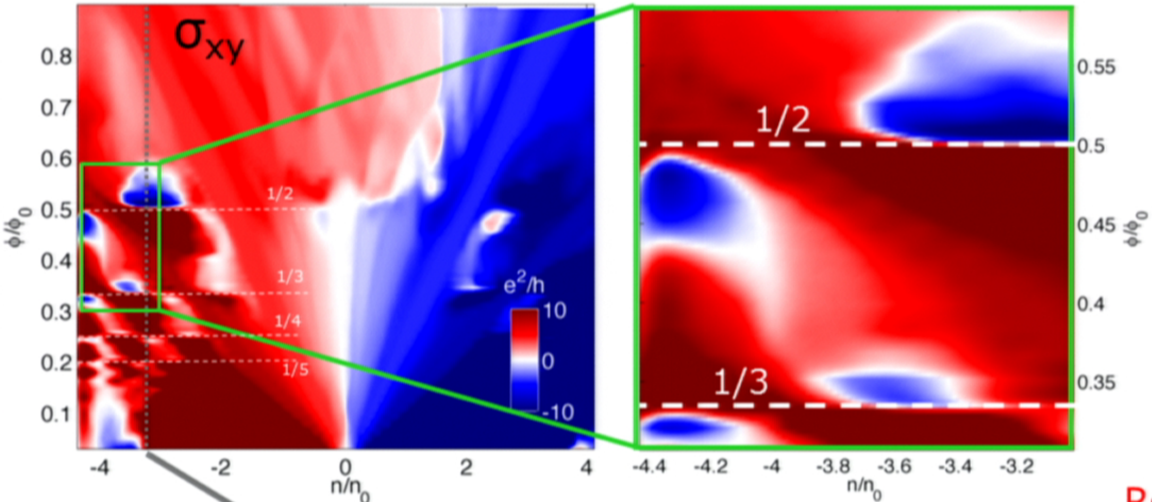
Sign reversal of σ_{xy}

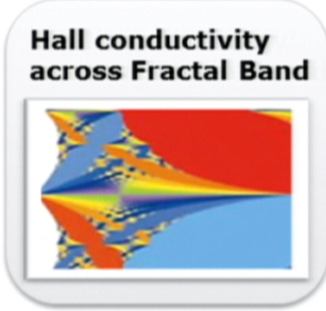
Large enhancement of σ_{xx}



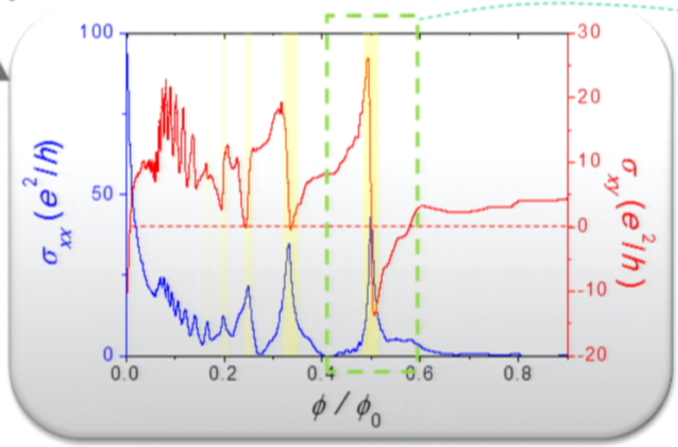
Recursive QHE near the Fractal Bands

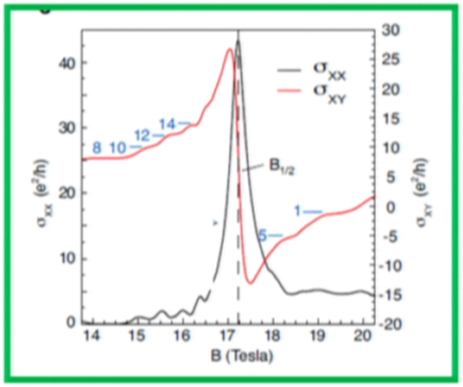
Higher quality sample with lower disorder





Recursive QHE!





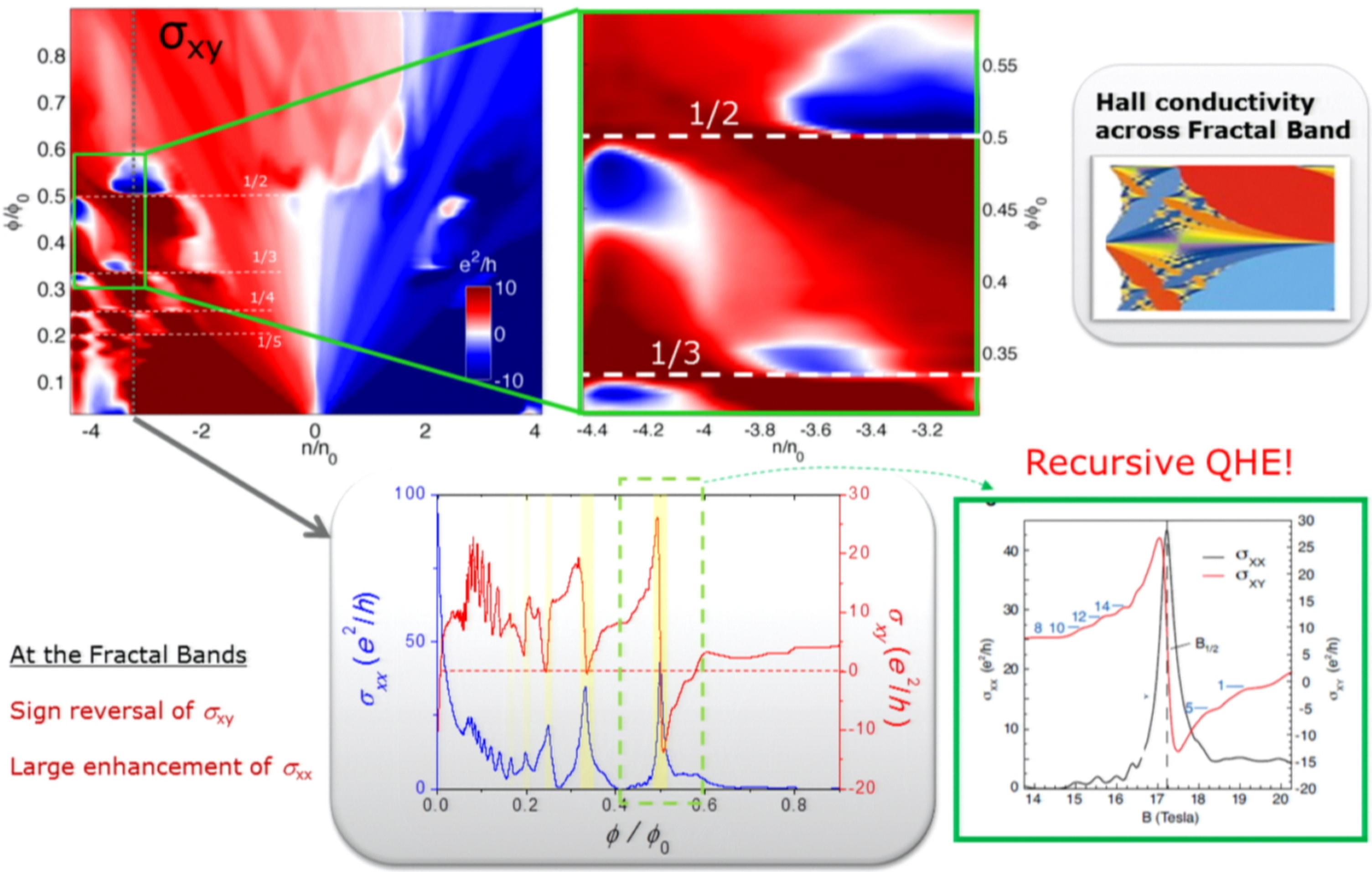
At the Fractal Bands

Sign reversal of σ_{xy}

Large enhancement of σ_{xx}

Recursive QHE near the Fractal Bands

Higher quality sample with lower disorder

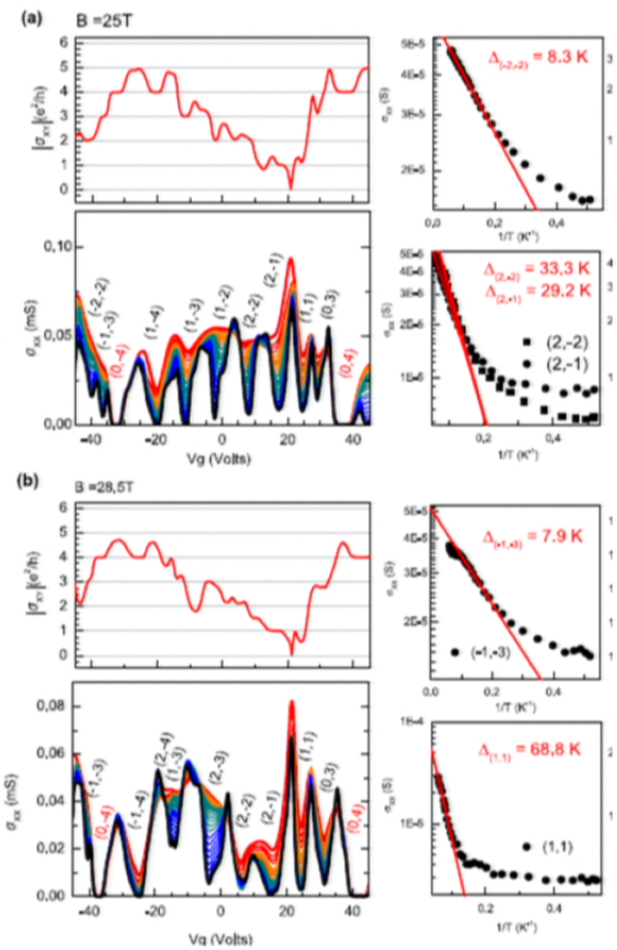


At the Fractal Bands

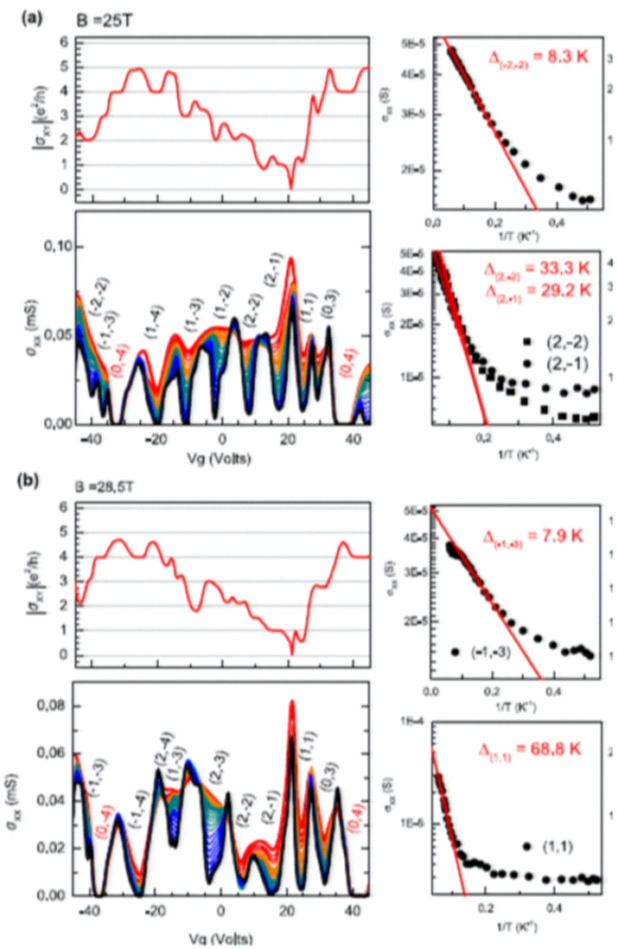
Sign reversal of σ_{xy}

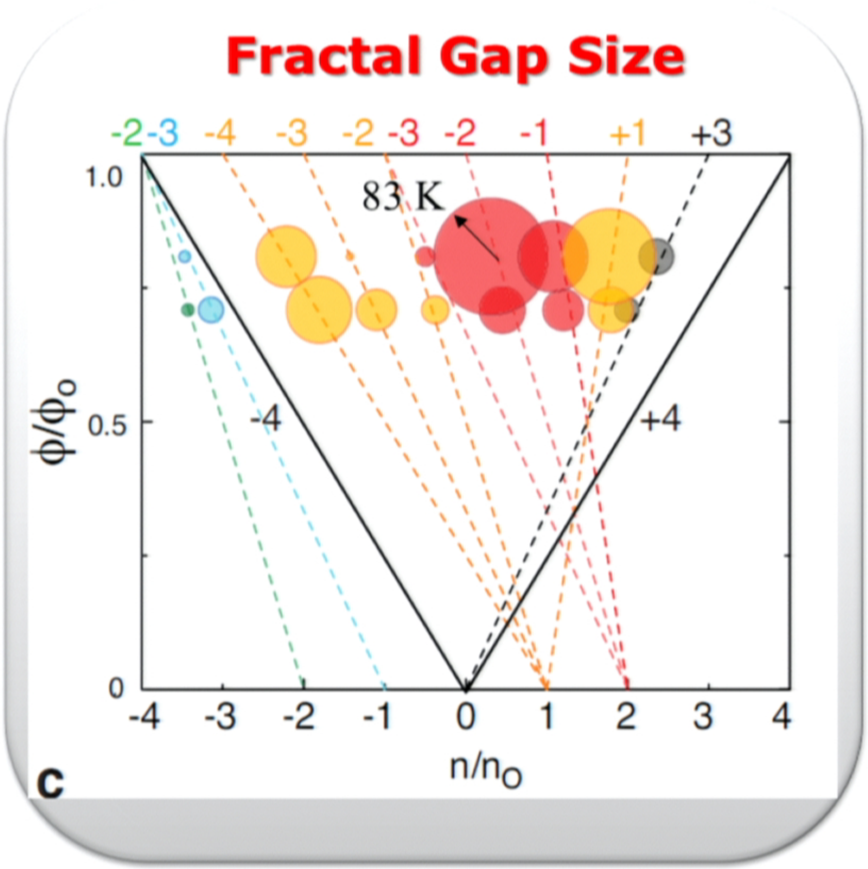
Large enhancement of σ_{xx}

Fractal Gaps: Energy Scales



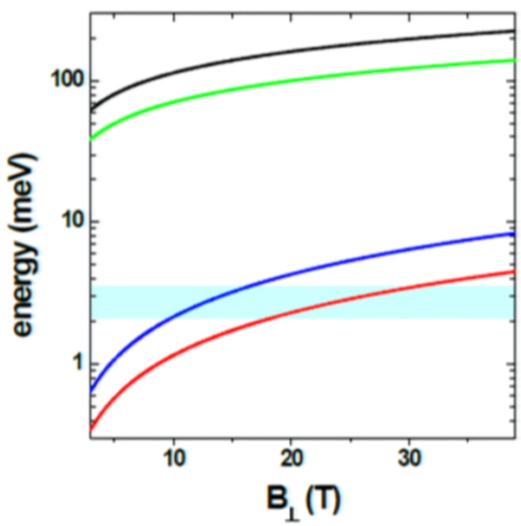
Fractal Gaps: Energy Scales





Graphene SU(4): Energy Scales

Monolayer Graphene



—	E_0	$\frac{\hbar v_F \sqrt{2}}{\ell_B}$
—	E_C	$\frac{e^2}{\varepsilon \ell_B}$
—	E_{SR}	$\frac{a}{\ell_B} \frac{e^2}{\varepsilon \ell_B}$
—	E_Z	$g \mu_B B_T$
—	Γ	*exp.*

Cyclotron Energy

Coulomb Energy

Short Range Lattice Interaction
(Pseudo/valley spin anisotropy)

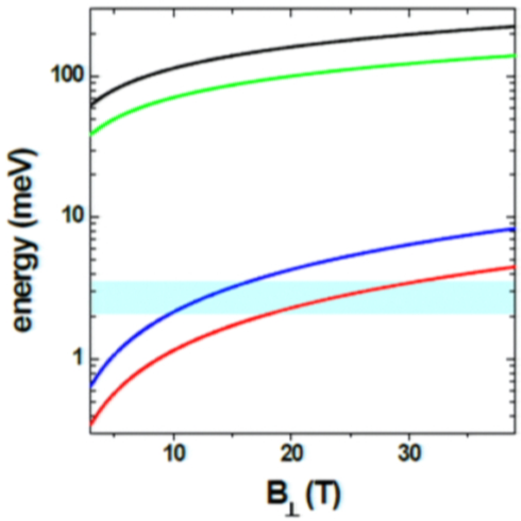
Zeeman Energy
(Spin anisotropy)

Disorder

Goerbig, RMP (2011)

Graphene SU(4): Energy Scales

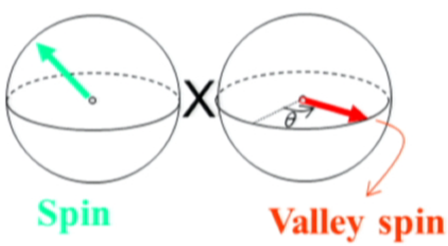
Monolayer Graphene



E_0	$\frac{\hbar v_F \sqrt{2}}{\ell_B}$	Cyclotron Energy
E_C	$\frac{e^2}{\varepsilon \ell_B}$	Coulomb Energy
E_{SR}	$\frac{a}{\ell_B}\frac{e^2}{\varepsilon \ell_B}$	Short Range Lattice Interaction (Pseudo/valley spin anisotropy)
E_Z	$g\mu_B B_T$	Zeeman Energy (Spin anisotropy)
Γ	*exp.*	**Disorder**

Goerbig, RMP (2011)

$$SU(2) \otimes SU(2) \quad < \quad SU(4)$$



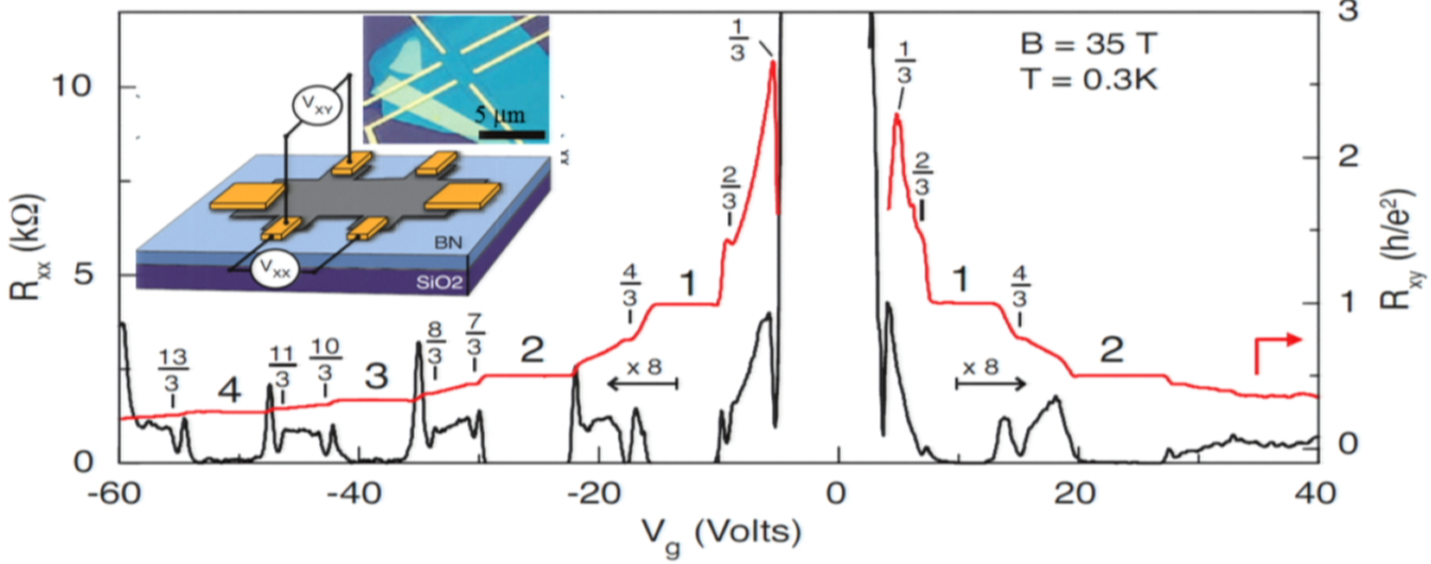
$$\begin{bmatrix} \psi_{K\Uparrow} \\ \psi_{K\Downarrow} \\ \psi_{K'\Uparrow} \\ \psi_{K'\Downarrow} \end{bmatrix}$$

***SU(4)* symmetry is conjectured to produce rich new set of ground state wavefunctions in graphene**

Yang, Das Sarma and MacDonal, PRB (2006);

Spin & pseudo spins: many body physics in graphene

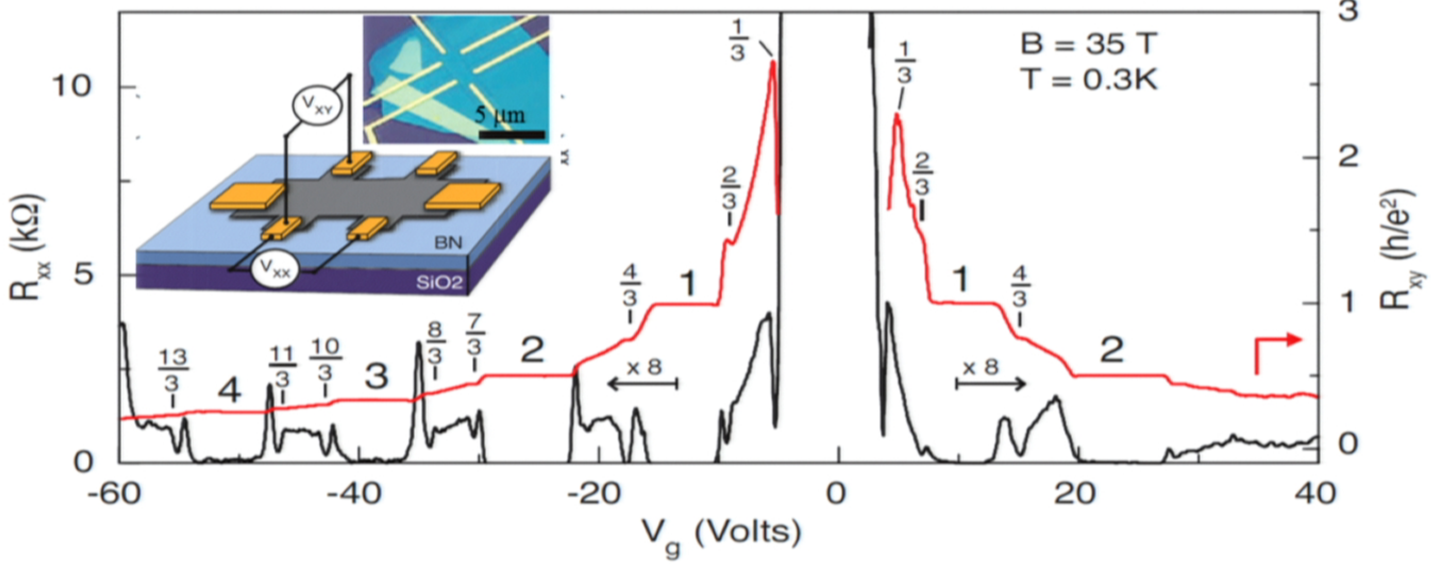
Dean *et al.* Nature Physics (2011)
Young *et al.*, Nature Physics (2012)



Integer quantum Hall Ferromagnetism and Fractional Quantum Hall Effect in single layer graphene

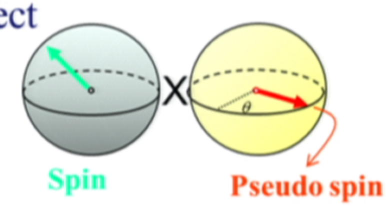
Spin & pseudo spins: many body physics in graphene

Dean *et al*. Nature Physics (2011)
Young *et al*., Nature Physics (2012)

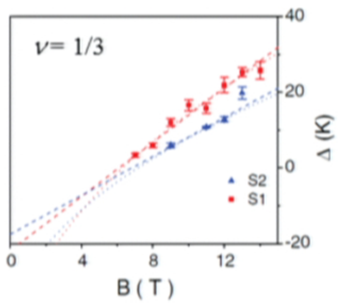


Integer quantum Hall Ferromagnetism and Fractional Quantum Hall Effect in single layer graphene

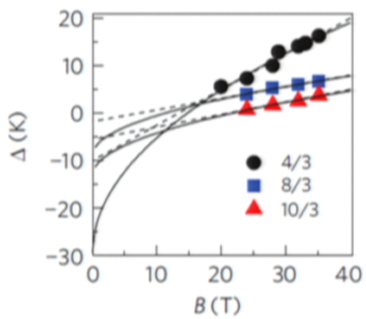
- SU(4) hierarchical Integer and Fractional Quantum Hall Effect
- Spin and Pseudospin Ferromagnetic Quantum Hall Effect
- Spin Skyrmion and Valley Skyrmions



Hierachical Structure in Fractional Quantum Hall Gaps

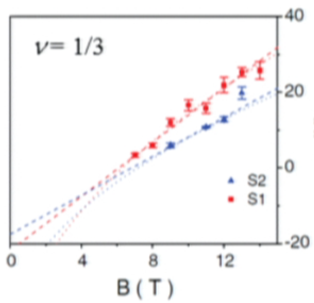


Ghahari et al. PRL (2011)

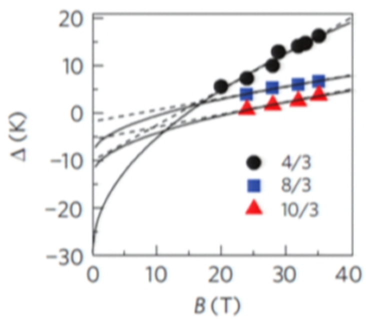


Young et al. Nature Phys. (2012)

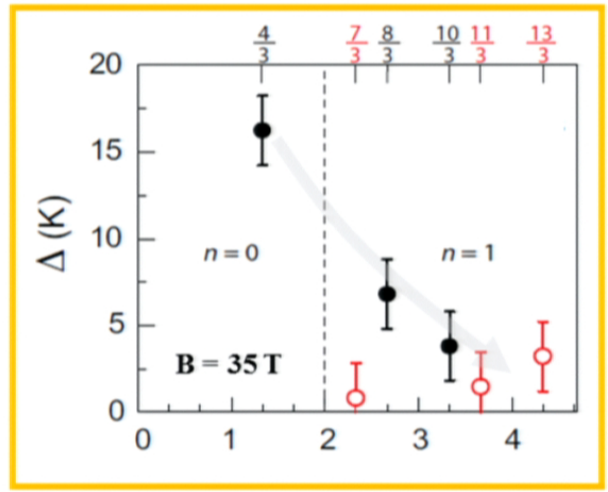
Hierachical Structure in Fractional Quantum Hall Gaps



Ghahari et al. PRL (2011)

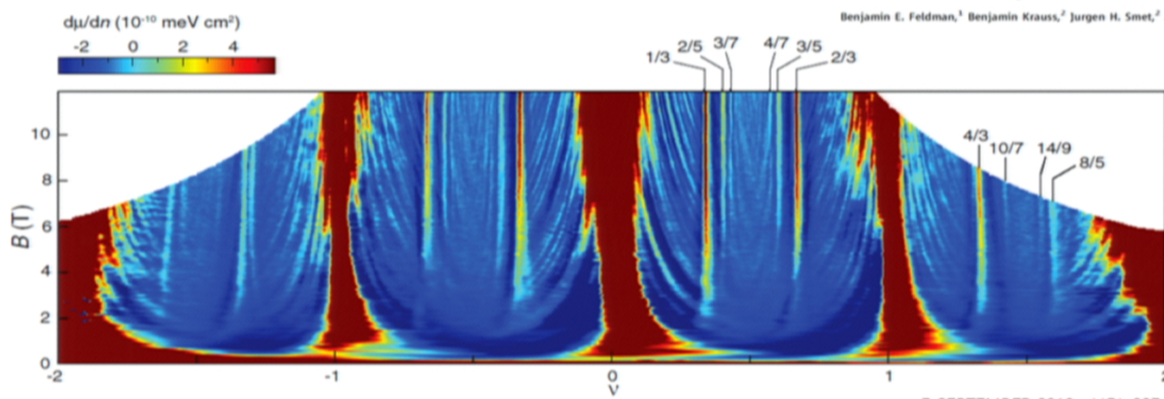


Young et al. Nature Phys. (2012)



Unconventional Sequence of Fractional Quantum Hall States in Suspended Graphene

Benjamin E. Feldman,[1] Benjamin Krauss,[2] Jurgen H. Smet,[2] Amir Yacoby[1]*



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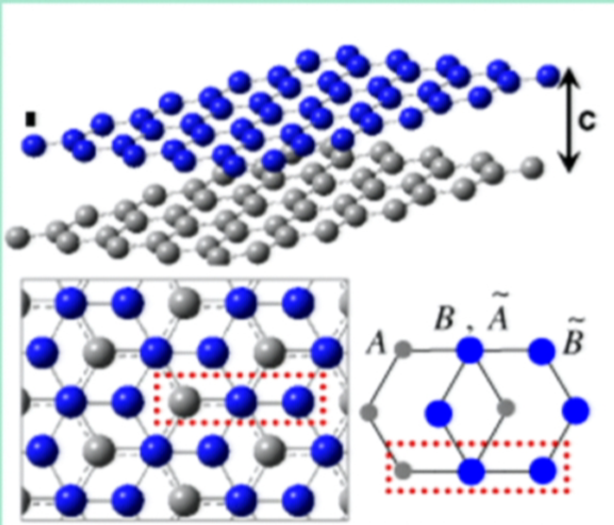
Pseudo Spin/ Valley Spin in Bilayer Graphene

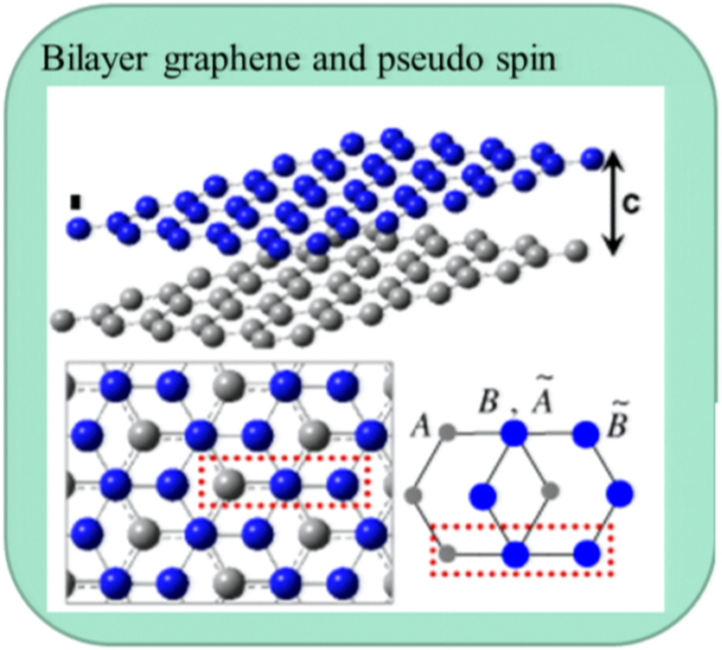
Bilayer graphene and pseudo spin



F. Guinea, A. H. C. Neto, & N. M. R. Peres, Phys. Rev. B 73, 245426 (2006)

E. McCann, Phys. Rev. B 74, 161403 (2006)

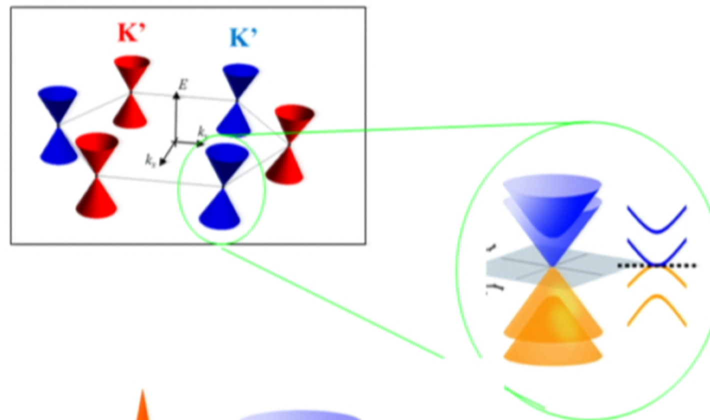
Pseudo Spin/ Valley Spin in Bilayer Graphene

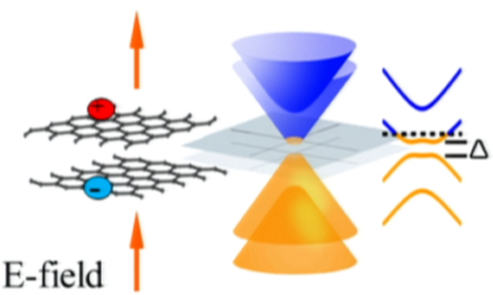


F. Guinea, A. H. C. Neto, & N. M. R. Peres, Phys. Rev. B 73, 245426 (2006)

E. McCann, Phys. Rev. B 74, 161403 (2006)

Low energy approximation in 1st BZ

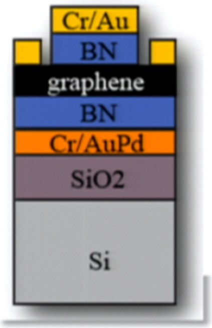


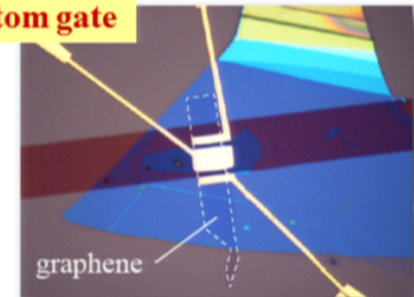


Perpendicular electric field breaks layer symmetry, polarize pseudo spin, and open up a gap

Gap Opening in Bilayer Graphene

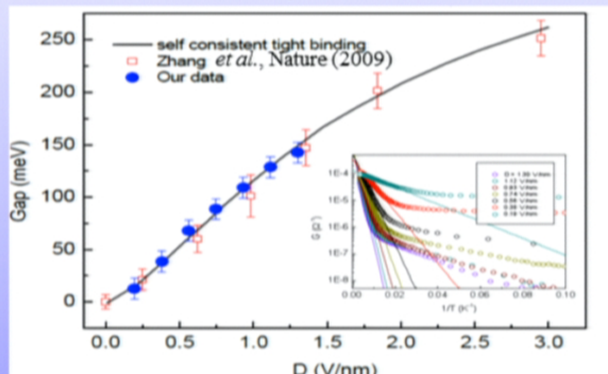
BN top and bottom gate





Wang et al. (Hone, Kim, Shepard) (2012)

Electric Field Induced Layer Polarization Induced Gaps

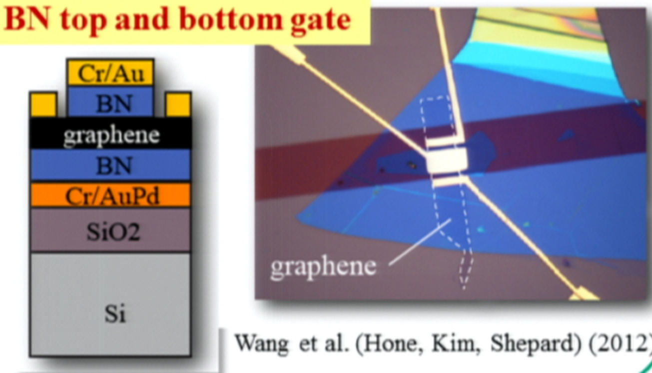


>200 meV transport gaps may be possible!

see also Yan et al (2010), Cheng et al (2010), Taychatanapat (2010)

Gap Opening in Bilayer Graphene

BN top and bottom gate



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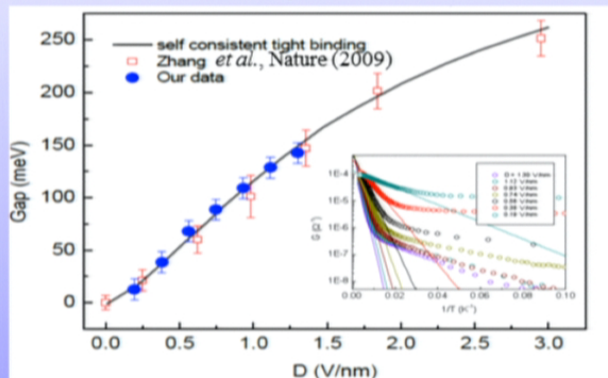
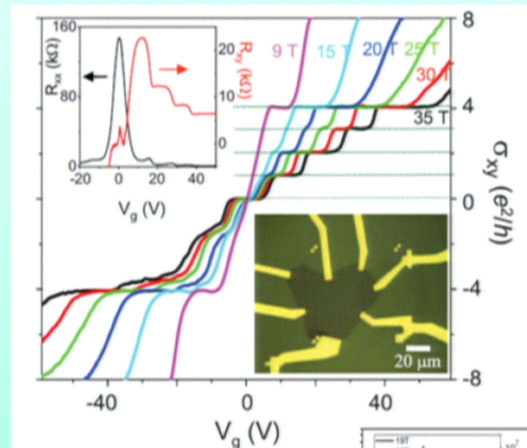
Electric Field Induced Layer Polarization Induced Gaps



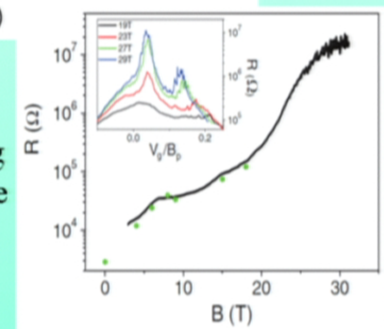
>200 meV transport gaps may be possible!

see also Yan et al (2010), Cheng et al (2010), Taychatanapat (2010)

Broken Symmetry QHE in Zero Energy LL in Bilyaer Graphene:
magneticticaly induced insulating state



Magnetic Field Induced Insulating State at the Charge Neutrality



Zhao et al PRL (2010);
See also Feldman et al (2009)

Nature of Magnetically Induced Insulating States

Partial list of references

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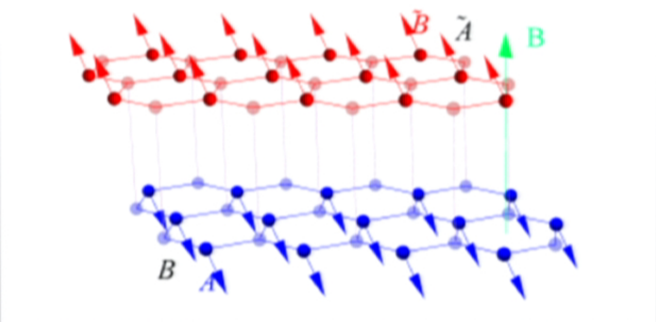
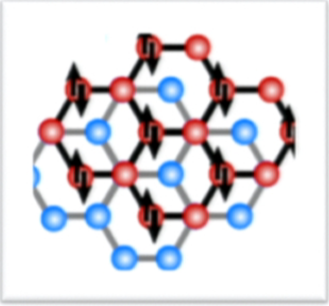
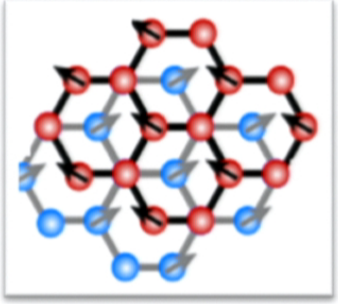
Nature of Magnetically Induced Insulating States

Half filled Bilayer Graphene Degree of freedom:
Layer-sublattice Pseudospin / Spin

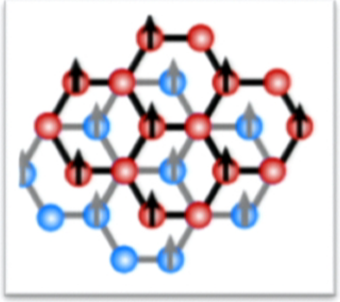




Layer Polarized



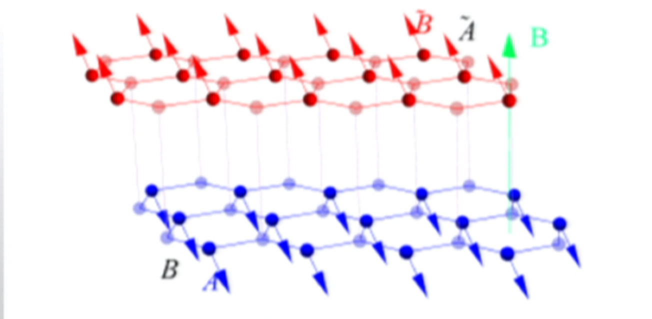
(canted)
Anti Ferromagnet

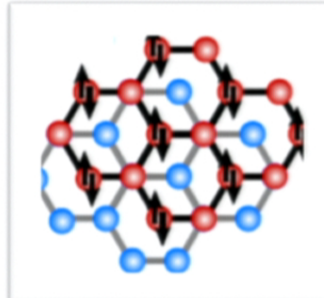


Ferromagnet

Nature of Magnetically Induced Insulating States

Half filled Bilayer Graphene Degree of freedom:
Layer-sublattice Pseudospin / Spin





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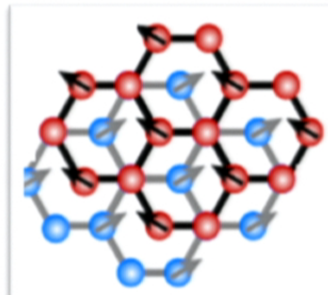

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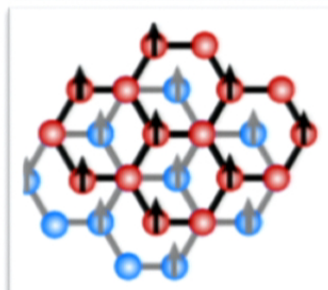
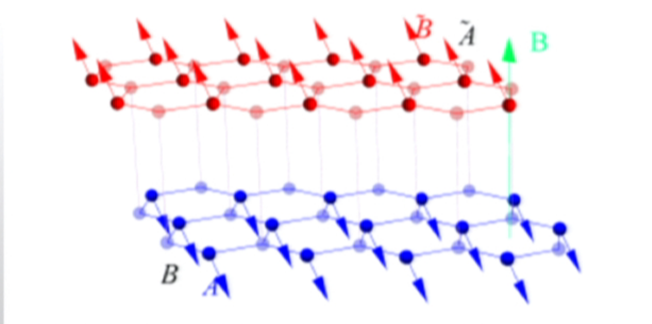
Anti Ferromagnet



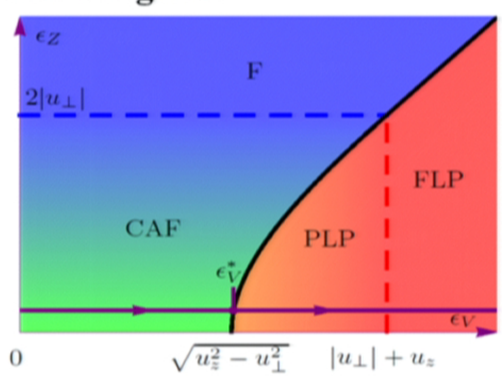
Ferromagnet

Nature of Magnetically Induced Insulating States

Half filled Bilayer Graphene Degree of freedom:
Layer-sublattice Pseudospin / Spin

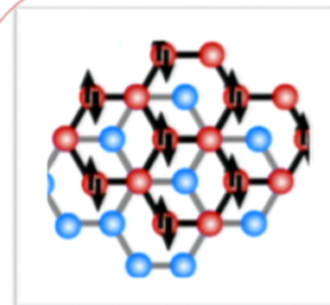


Phase Diagram

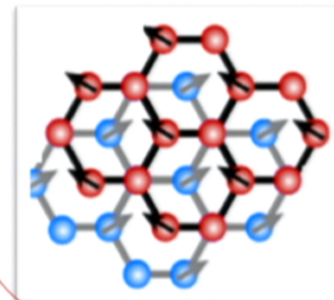


Kharitonov, arXiv:1105.5386 (2011)

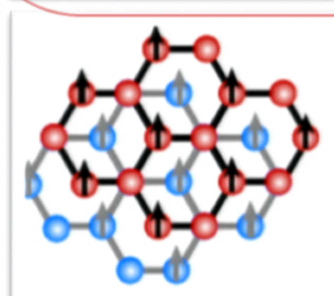
Spin-upolarized



Layer Polarized



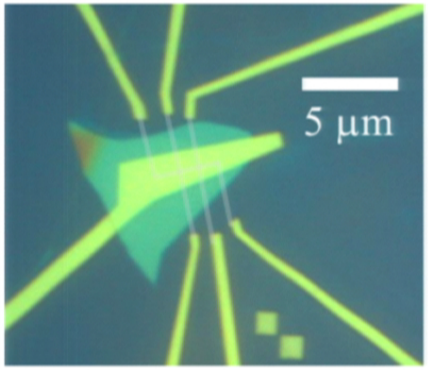
(canted) Anti Ferromagnet

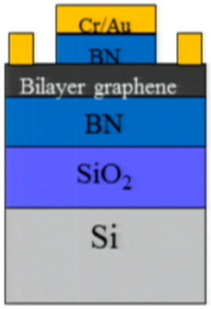


Ferromagnet

Controlling Spin and Pseudo Spin in Bilayer

Doubly gated Bilayer graphene with BN dielectric

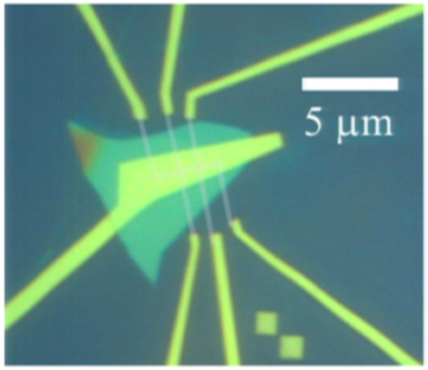


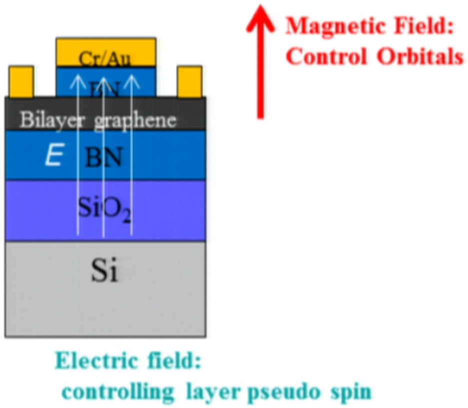


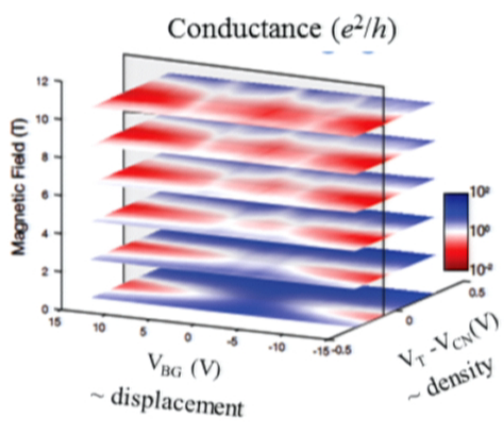
Maher *et al*. (2012)

Controlling Spin and Pseudo Spin in Bilayer

Doubly gated Bilayer graphene with BN dielectric





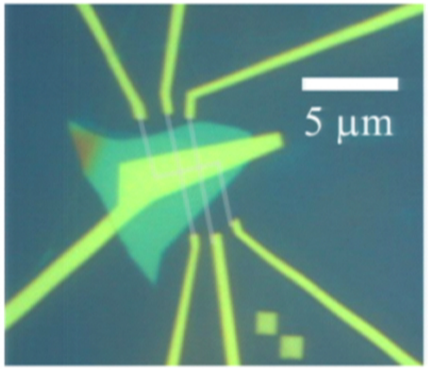


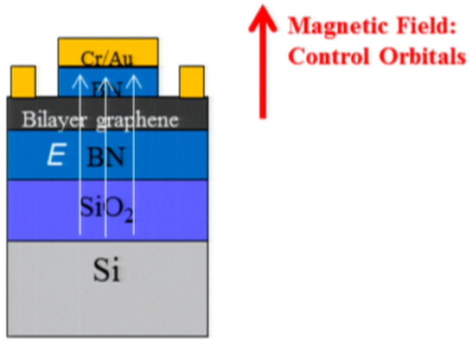
T = 1.5 K

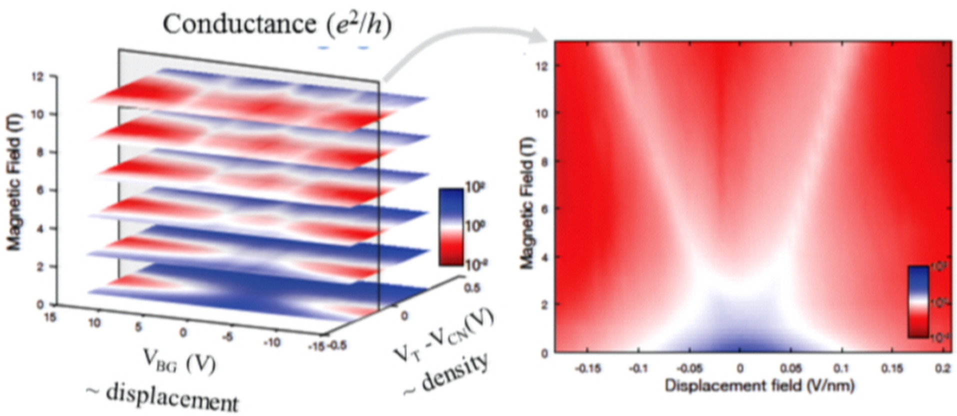
Maher *et al.* (2012)

Controlling Spin and Pseudo Spin in Bilayer

Doubly gated Bilayer graphene with BN dielectric







T = 1.5 K

See also Previous work: Weitz et al (2010) and also work from Manchester & UCR

Maher *et al*. (2012)

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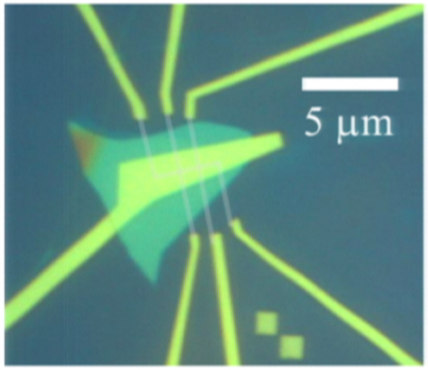
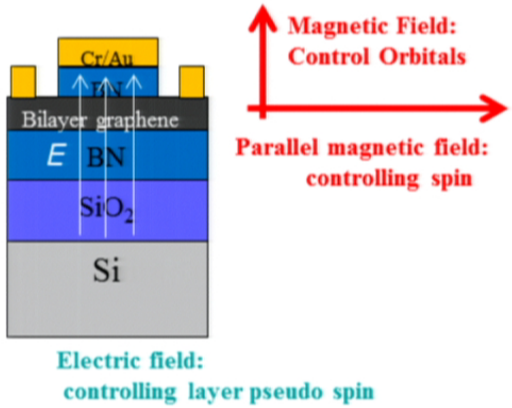
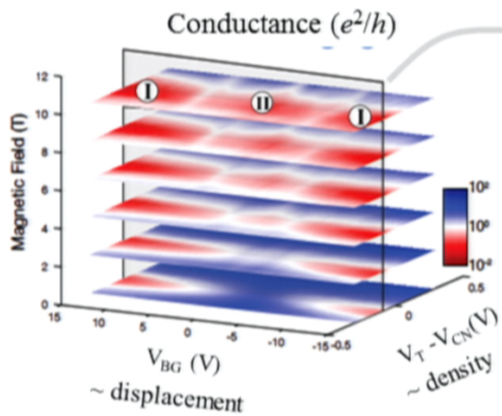
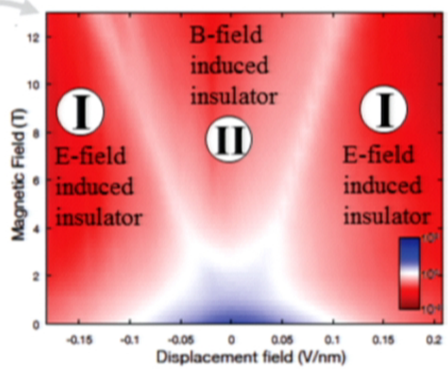






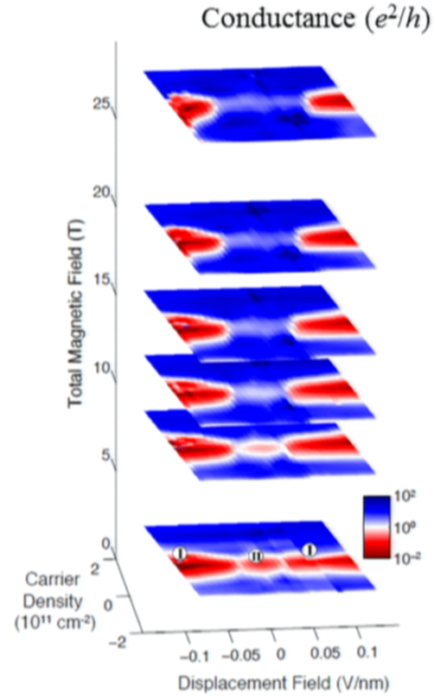


T = 1.5 K

See also Previous work: Weitz et al (2010) and also work from Manchester & UCR

Parallel Magnetic Field

$B_{\perp}$ =1.75 T (fixed)



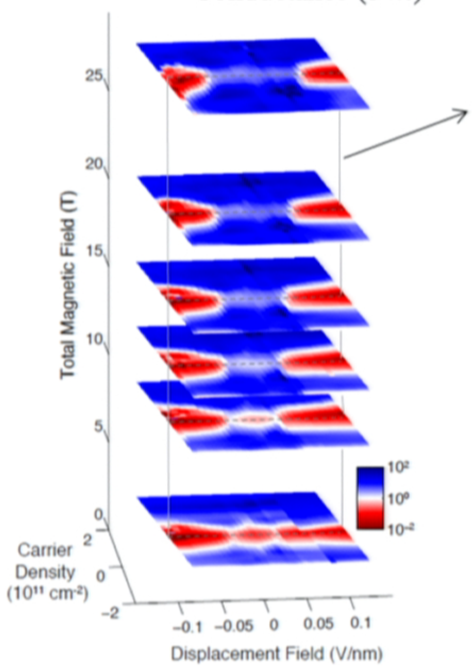
T = 0.35 K

Maher *et al.* (2012)

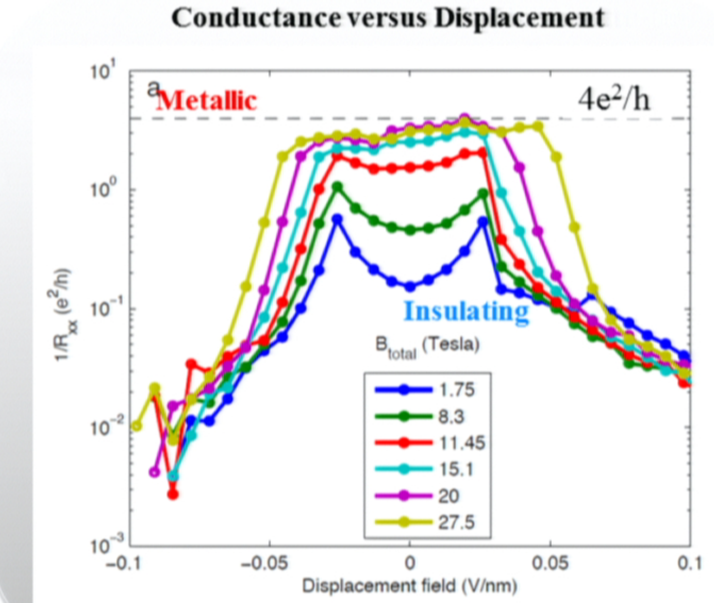
Transition from Insulating State to Metallic State

Metal to Insulator Transition

$B_\perp = 1.75$ T (fixed)



$T = 0.35$ K



- Low B_{tot} : insulating
- High B_{tot} : metallic ($\sim 4e^2/h$)

Maher *et al.* (2012)

Transition from Insulating State to Metallic State

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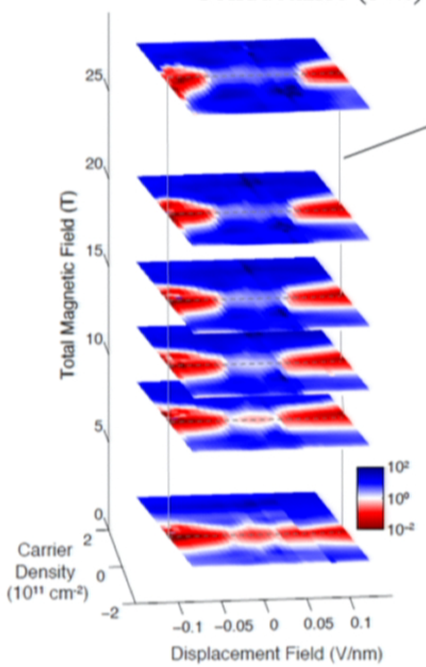
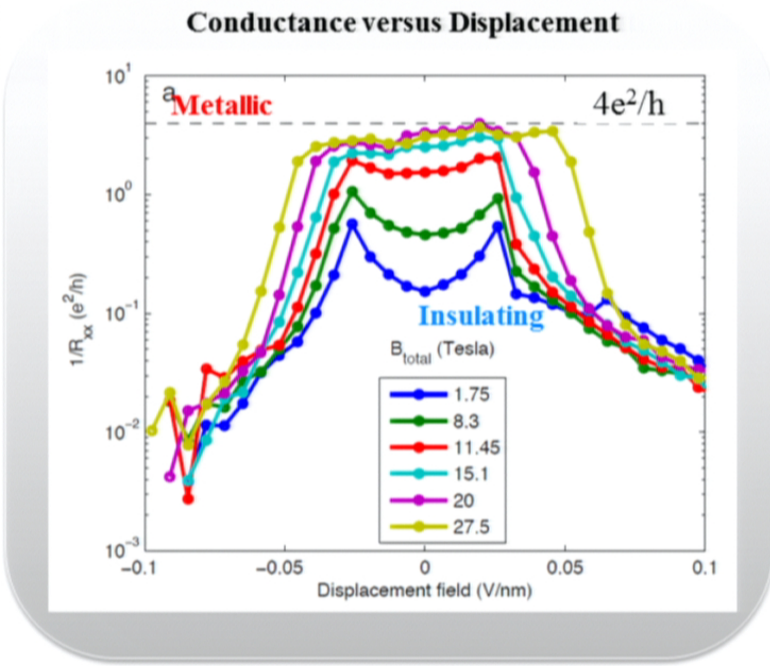
$B_\perp$ =1.75 T (fixed)



T = 0.35 K



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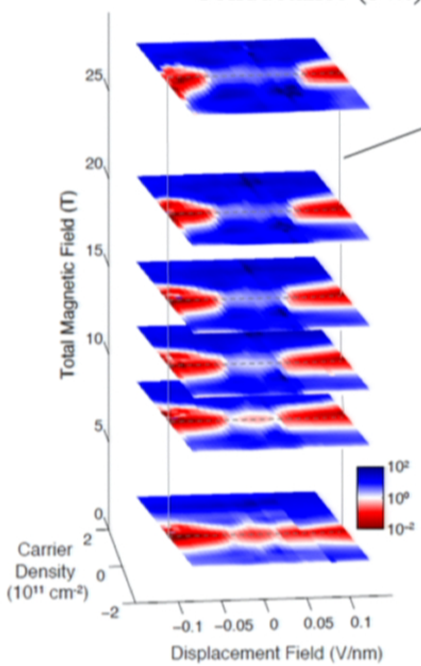
Maher *et al*. (2012)

Transition from Insulating State to Metallic State

Metal to Insulator Transition

$B_\perp = 1.75$ T (fixed)

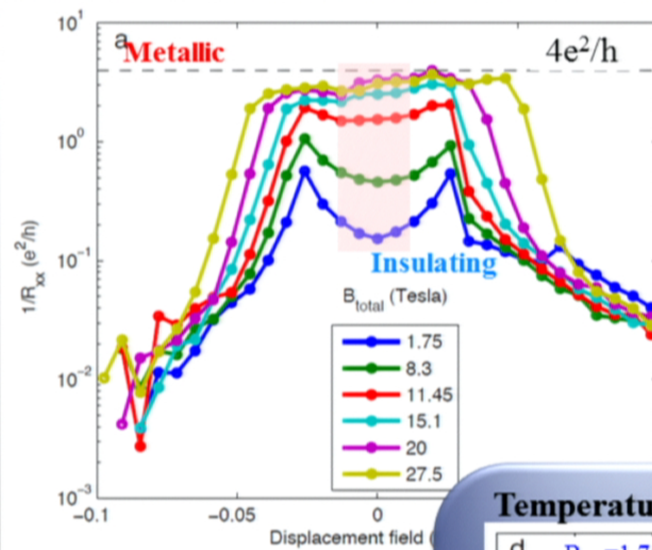
Conductance (e^2/h)



T = 0.35 K

Maher *et al.* (2012)

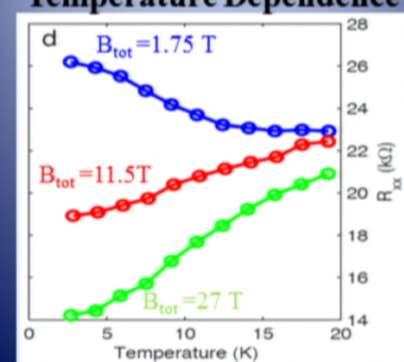
Conductance versus Displacement



- Low B_{tot} : insulating
- High B_{tot} : metallic

($\sim 4e^2/h$)

Temperature Dependence

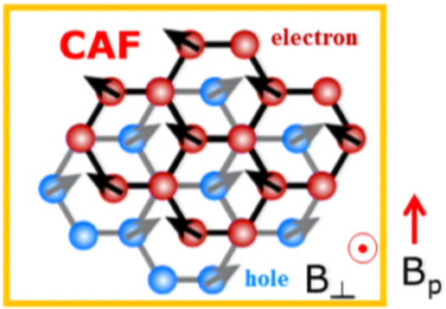


Insulator to Metallic: Edge State Configuration

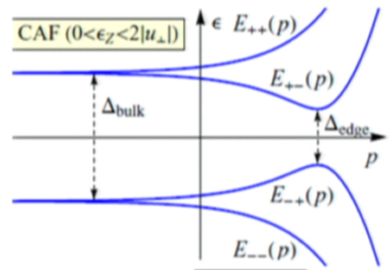
M. Kharitonov (2012) and others

$\varepsilon_z = -\mu_B B_{tot}$

Canted Anti-Ferromagnetic

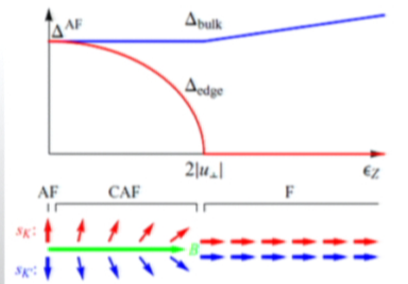


Edge Configuration

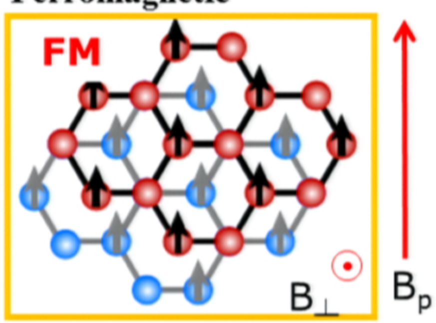


$\Delta_{edge} = \Delta^{AF}\sqrt{1-(\frac{\epsilon_Z}{2|u_\perp|})^2}$

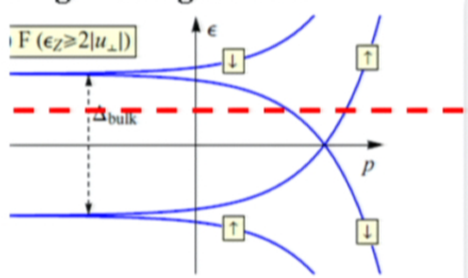
Energy Gap



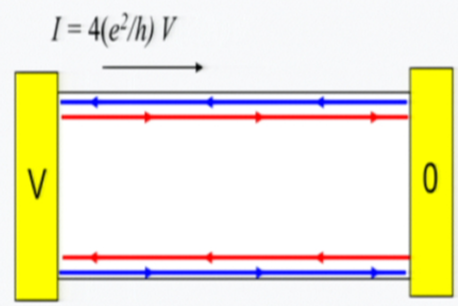
Ferromagnetic



Edge Configuration

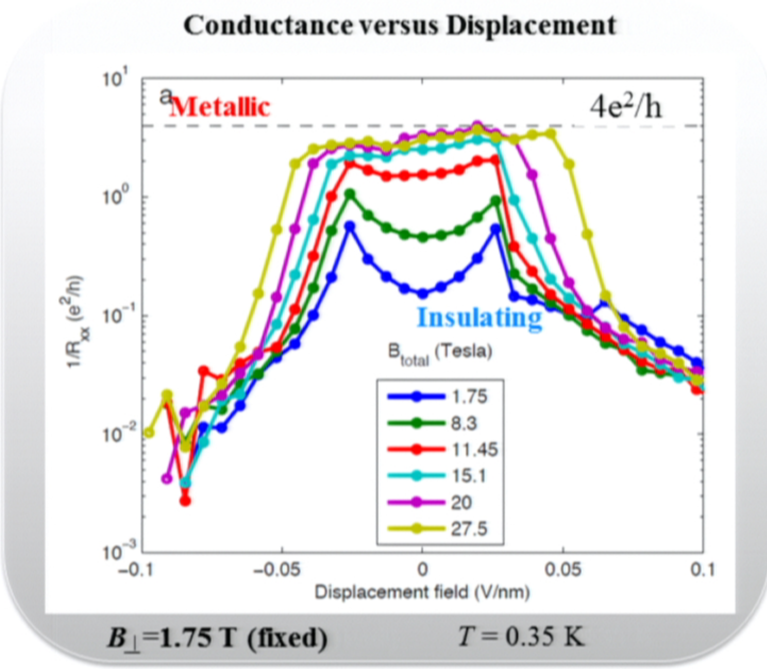


Counter Propagating Spin polarized edge states



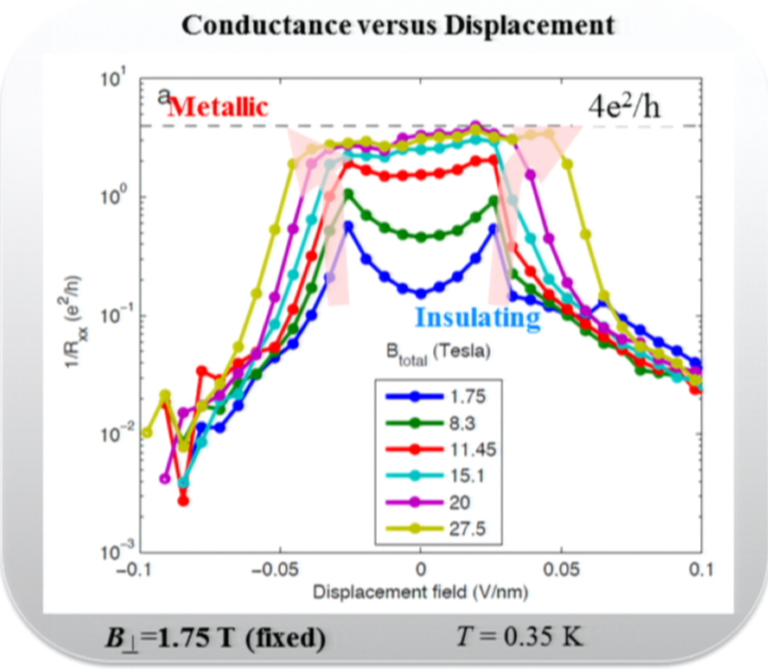
Experimental results are reported by Hunt *et al.* *Science* (2013) in SLG

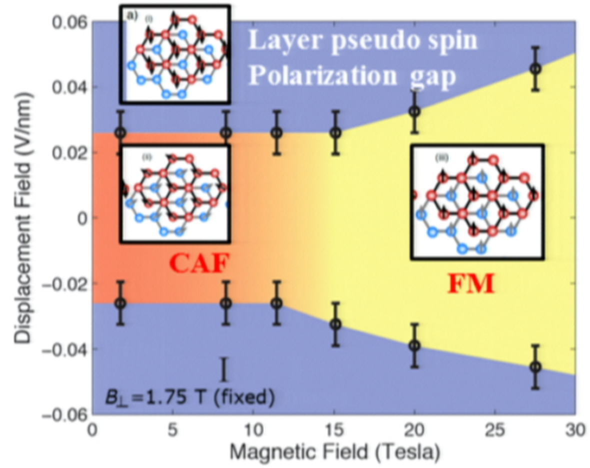
Energy Scale of Phase Boundaries



Maher *et al*. (2013)

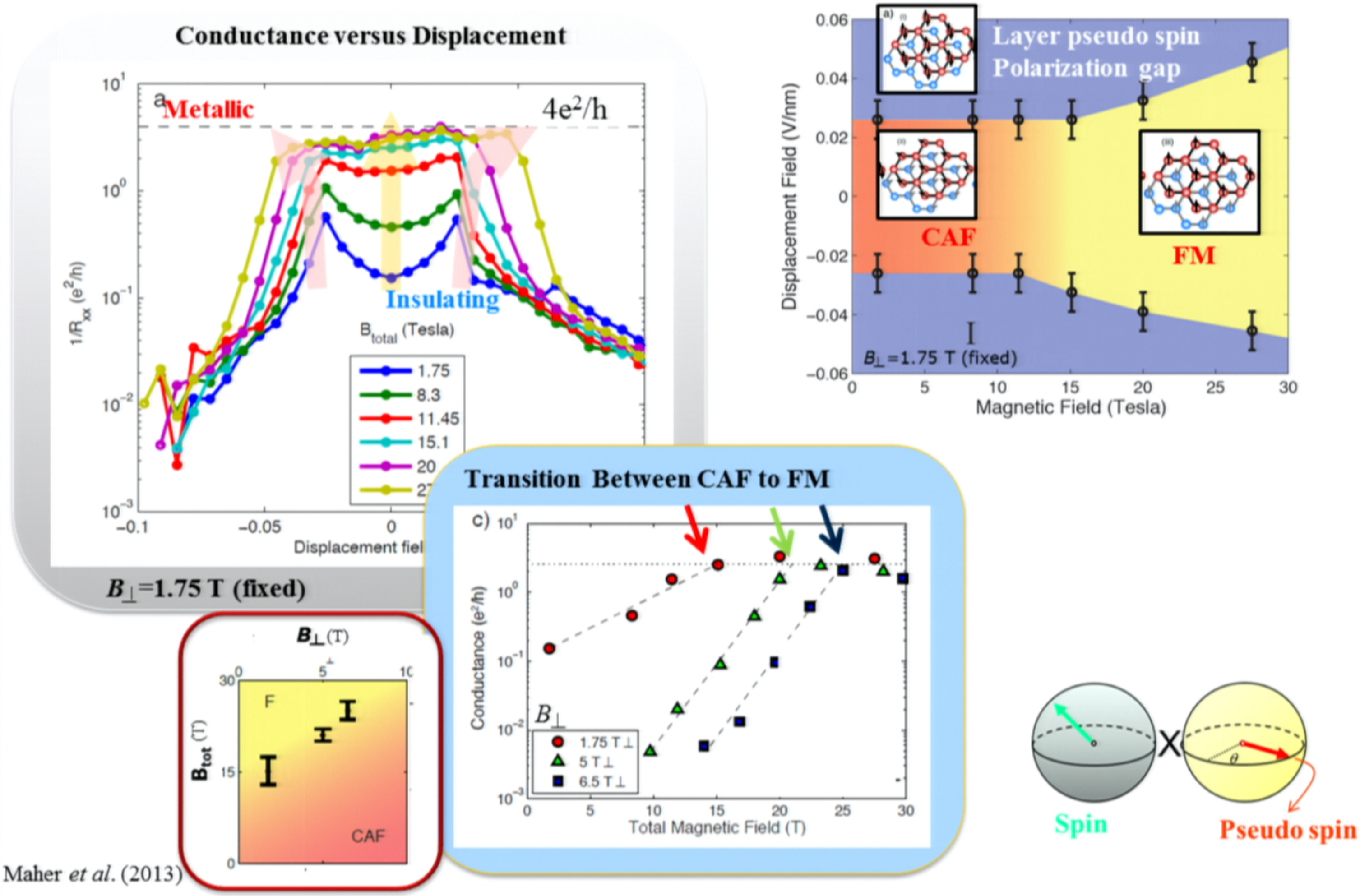
Energy Scale of Phase Boundaries





Maher *et al*. (2013)

Energy Scale of Phase Boundaries



Maher *et al.* (2013)

PHYSICAL REVIEW B VOLUME 34, NUMBER 4 15 AUGUST 1986

Thermodynamic properties of an interacting two-dimensional electron gas in a strong magnetic field

A. H. MacDonald, H. C. A. Oji, and K. L. Liu*
National Research Council of Canada, Ottawa, Canada K1A 0R6
(Received 29 January 1986)

Hartree Fock calculation of

Entropy

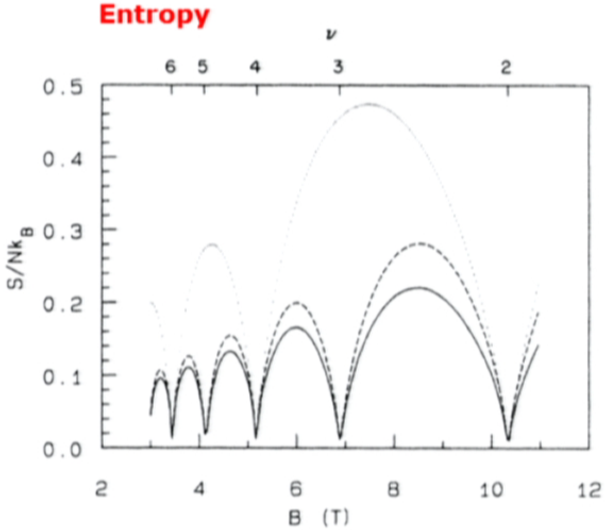


FIG. 2. Entropy versus magnetic field for ideal electron gas (dotted curve) interacting electron gas without disorder (dashed curve), and with disorder ($\Gamma[\text{meV}]=0.34\sqrt{B[\text{T}]}$). $n=5.0\times10^{11}$ cm^{-2} and $T=4.2$ K. The main effect of interaction is to split the peak values through the gigantic exchange enhancement as discussed in the text.

Chemical Potential

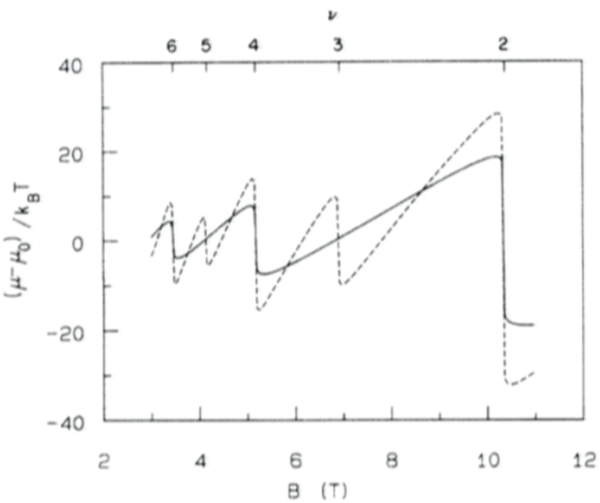
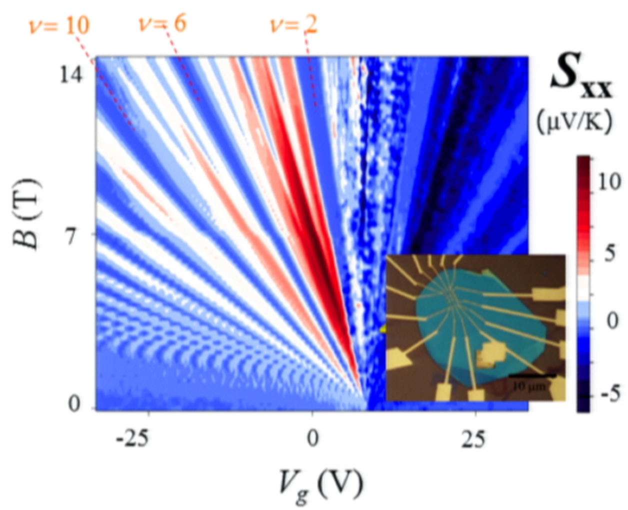
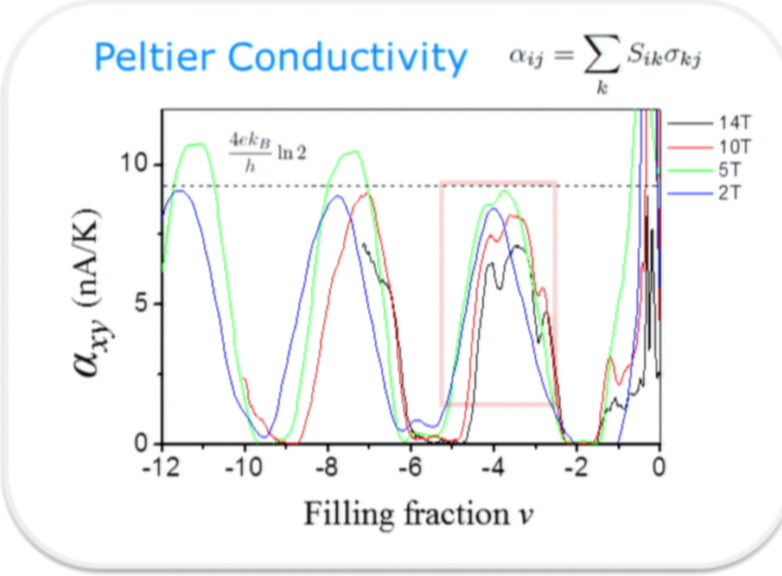


FIG. 3. Shift of the chemical potential from the zero-field value μ_0. $\Gamma[\text{meV}]=0.34\sqrt{B[\text{T}]}$, $n=5\times10^{11}$ cm^{-2}, and $T=4.2$ K. The dashed curve shows the effect of interaction. The discontinuities in μ which occur at integer filling factors is a measure of the gaps in the quasiparticle energy spectrum (see text).

Magneto Thermoelectric Power in Broken Symmetry Quantum Hall States



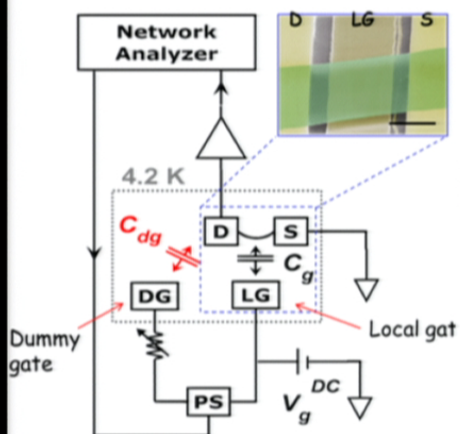


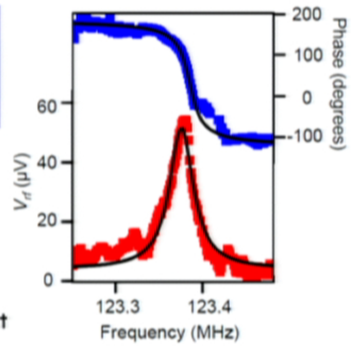
F Ghahari et al (2013)

Mechanical Electromagnetic Measurements

(Deshpande et al, with Hone group collaboration)

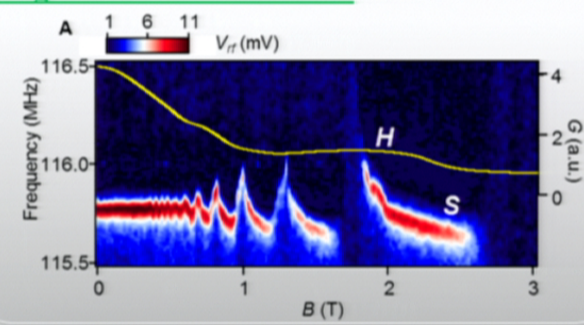
Electromechanical Devices:
suspended graphene

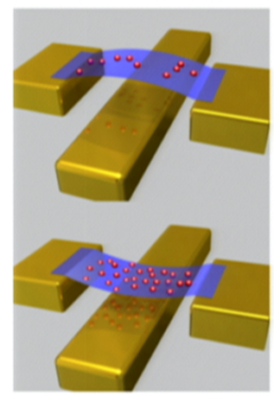




Chen *at al* (2011)
Vikram *et al* (2013)

Magnetization measurement



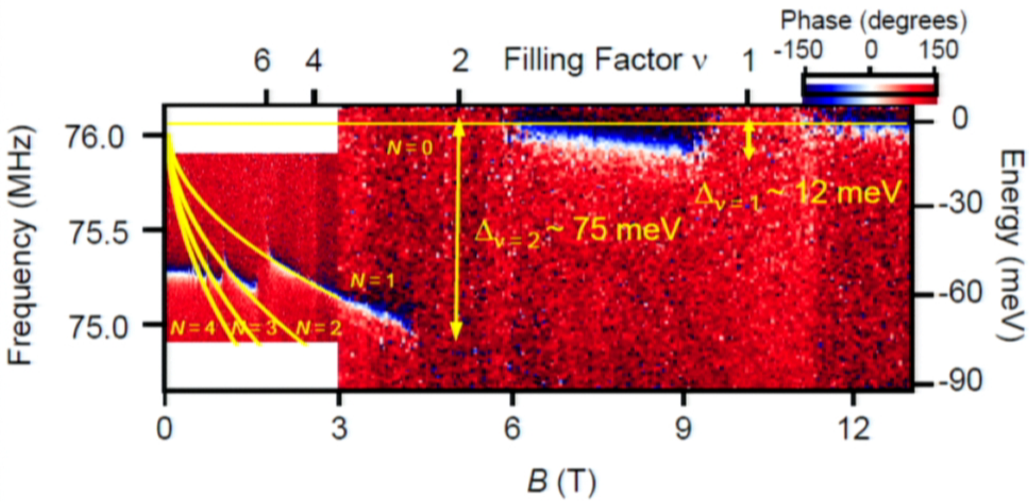


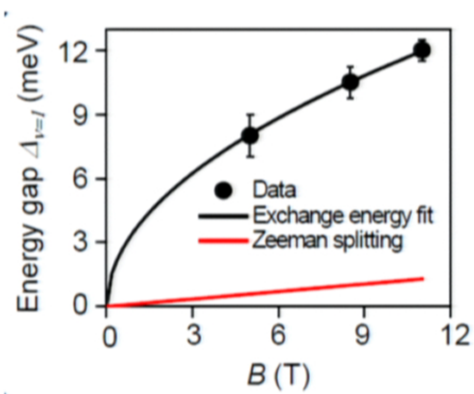
$$F_{mag} = -\frac{\partial U}{\partial z} = -\frac{\partial U(n)}{\partial n}\frac{\partial n}{\partial z} = -\mu V_g \frac{\partial C_g}{\partial z}$$

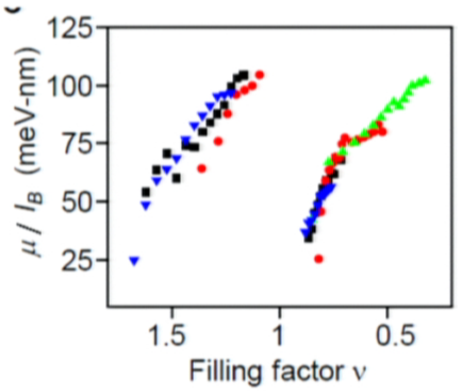
Direct experimental access of chemical potential!

Chemical Potential Measurement of Broken Symmetry QH States

(Deshpande *et al* (2013), with Hone group collaboration)







- Broken Symmetry State (ν=1) are observed
- $\Delta_{\nu=1}$ is measured to be scale as $B^{1/2}$
- Partially filled broken symmetry LL provides chemical potential universal scaling of

$$\mu = \hbar v_F / \ell_B \, F(\nu)$$

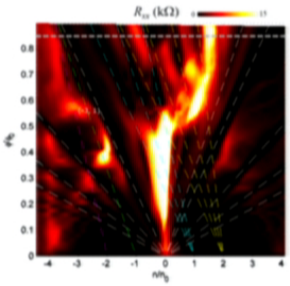
Kravchenko et al. PRB (1990)

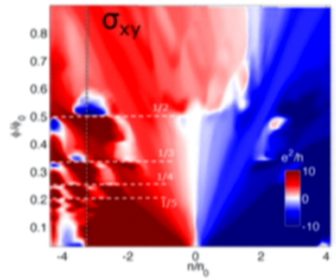
Summary

- **Graphene/ hBN heterostructures:**

  Observation of Hofstadter's Butterfly

  Dean *et al*, *Nature* (2013);
  similar results were reported by Gorbachev *et al. Nature* (2013) and Hunt *et al. Science* (2013)

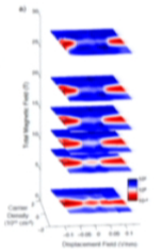


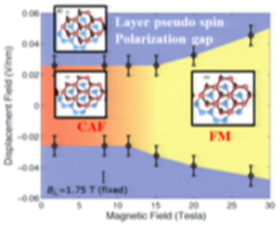


- **Bilayer graphene:**

  e-e interaction and spin and pseudo-spin SU(4) phase transition

  Maher *et al*, *Nature Phys.* (2013);
  similar results were reported by Hunt *et al. Science* (2013)





- **Magneto-thermoelectric effect and Magentoelectromechanical effect:**

  Reduction of transport entropy in broken symmetry integer quantum Hall effect
  Universal scaling of chemical potential in QHF