Title: Charged particle motion in magnetized black holes

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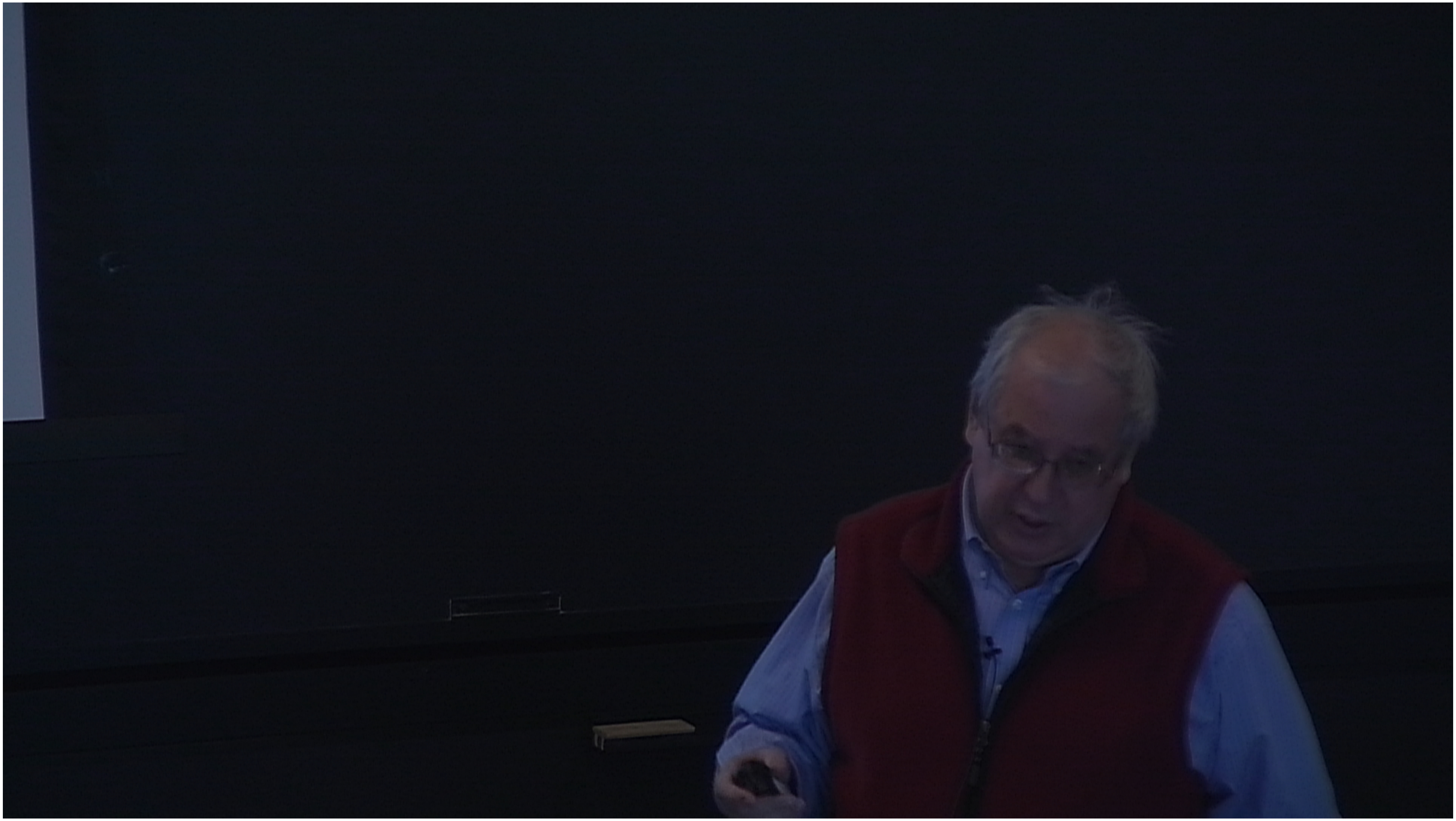
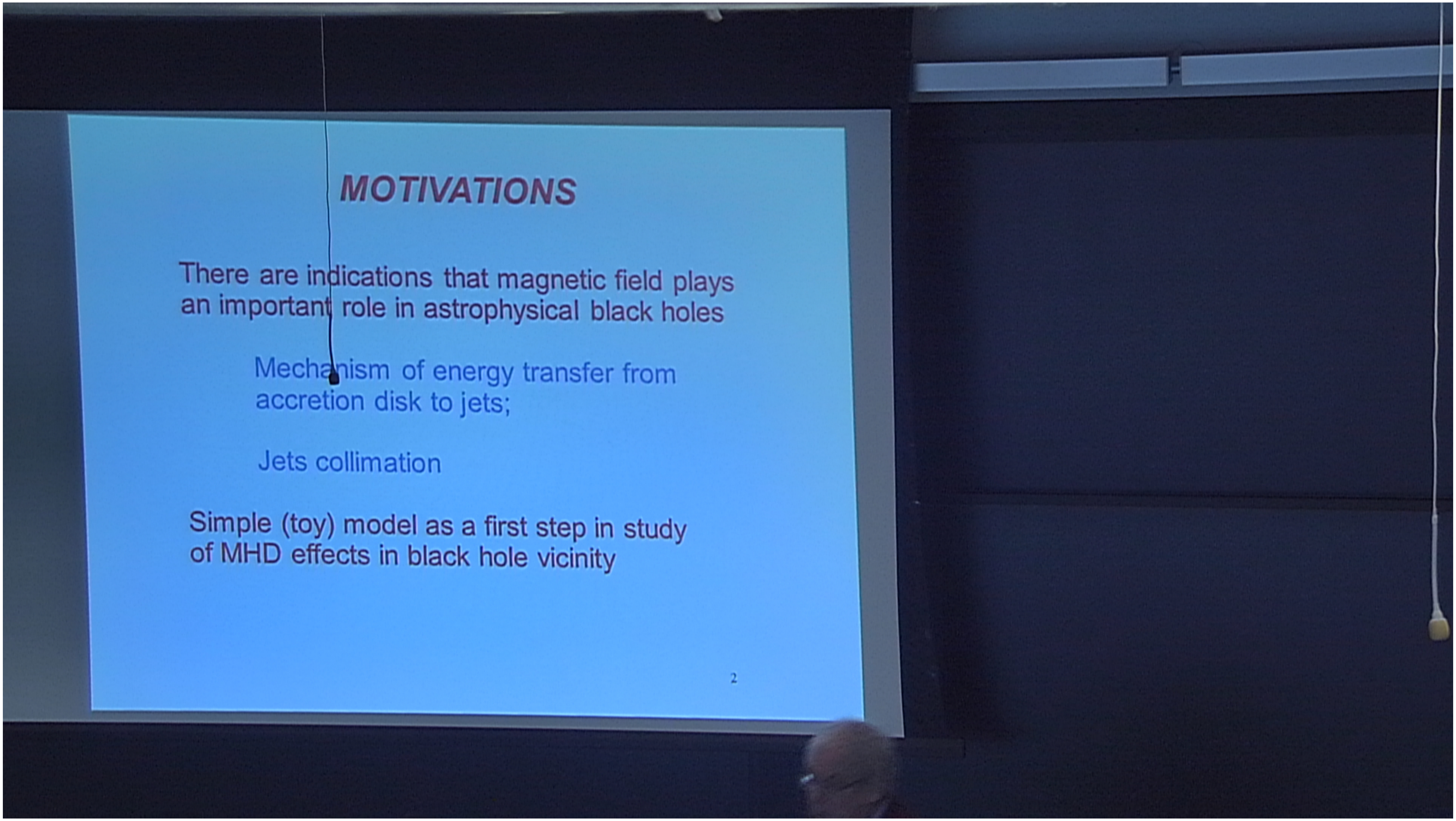
Abstract: There exist evidences that magnetic field in the vicinity of astrophysical black holes plays an important role. In particular it is required for explanation of such phenomenon as jet formation. Study of such problems in all their complexity requires 3D numerical simulations of the magnetohydrodynamics in a strong gravitational field. Quite often when dealing with such a complicated problem it is instructive to consider first its simplifications, which can be treated either analytically, or by integrating ordinary differential equations. Motion of a charged particle in a weakly magnetized black hole is an important example. We consider a non-rotating black hole in the weak magnetic field which is homogeneous at infinity. In the talk I discuss the following problems: How does such a magnetic field affect charged particle motion in the equatorial plane? How does it change the radius of the innermost stable circular orbits (ISCO) and period of rotation? I shall demonstratethat the magnetic field increases the efficiency of the energy extraction from the black hole and that magnetized black holes can be used as "particle accelerators". Finally, I shall discuss out-of-equatorial-plane motion and demonstrate that it is chaotic. Possible applications of these results to astrophysics are briefly discussed.




MOTIVATIONS
There are indications that magnetic field plays an important role in astrophysical black holes
Mechanism of energy transfer from accretion disk to jets;
Jets collimation
Simple (toy) model as a first step in study of MHD effects in black hole vicinity
2

MOTIVATIONS

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- Jets collimation

Simple (toy) model as a first step in study of MHD effects in black hole vicinity

2

Black holes are formed when large mass M is in a compact region $r_g = 2GM/c^2$

Black hole candidates are selected by observing their masses:

(i) Stellar mass black holes in binary systems $(M > 3M_{\odot})$

(ii) Supermassive black holes $M \sim 10^6 - 10^9 M_{\odot}$

Black hole identification is based on GR effects

3

Examples:

(i) Radius of the innermost stable circular orbit (ISCO)

(ii) Period of Keplerian motion on ISCO

(iii) Energy released by a particle before its fall into BH

These parameters are specific for GR and,
in particular, depend on the rotation parameter a/M.

4

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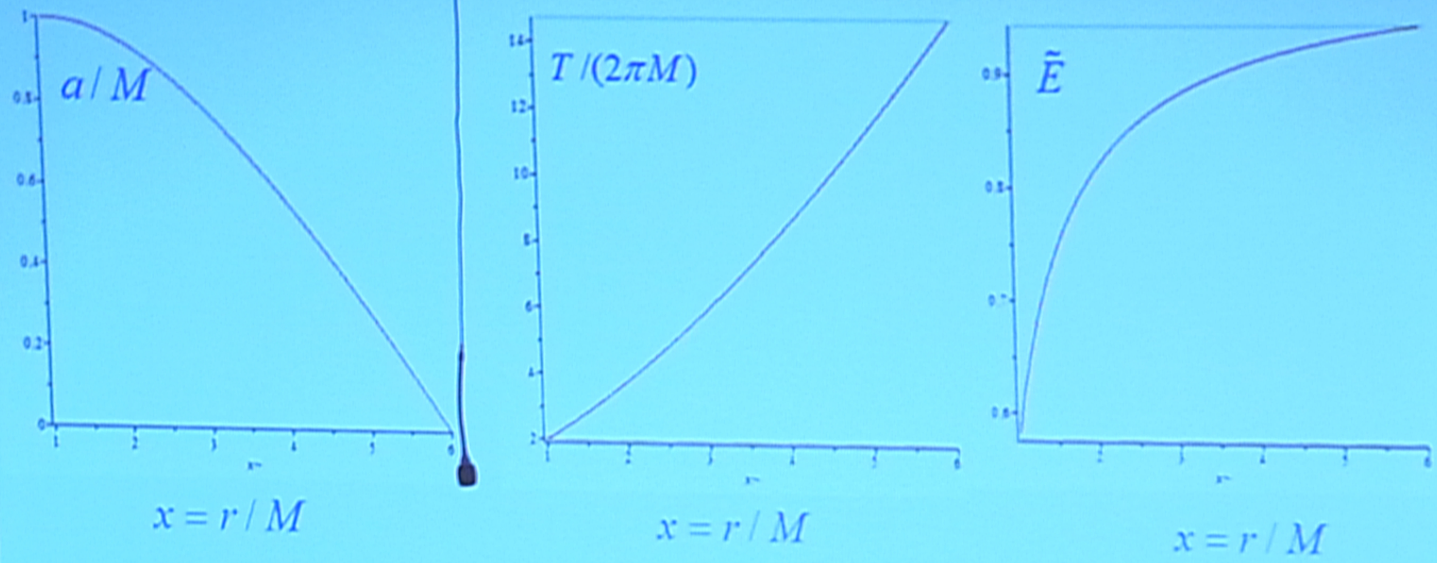
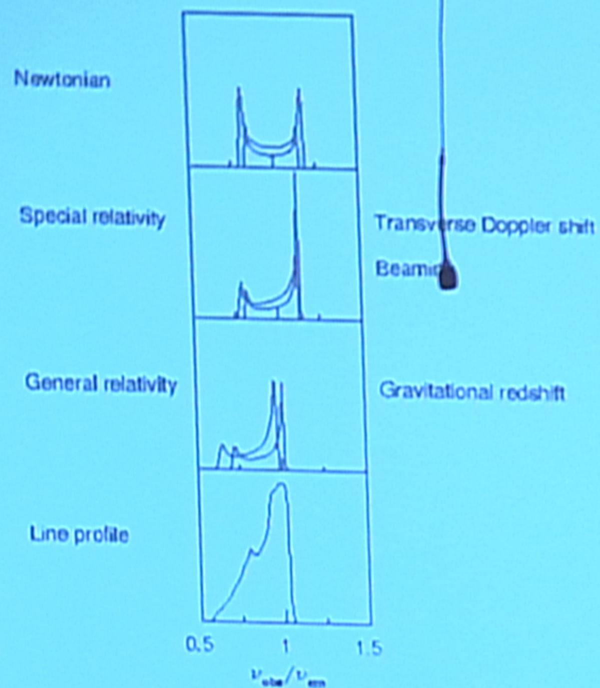


Rotation parameter a/M, period $T/(2\pi M)$, and specific energy $\tilde{E}$ as functions of the radius r/M of ISCO for a rotating BH ($l > 0$)

5

	$a=0$	$a=M$	
		$\mathcal{L}>0$	$\mathcal{L}<0$
$\mathcal{E}$	$\sqrt{8/9}$	$\sqrt{1/3}$	$\sqrt{25/27}$
$1-\mathcal{E}$	0.0572	0.4236	0.0377
$\lvert\mathcal{L}\rvert/M$	$2\sqrt{3}$	$2/\sqrt{3}$	$22/3\sqrt{3}$

Kα line broadening



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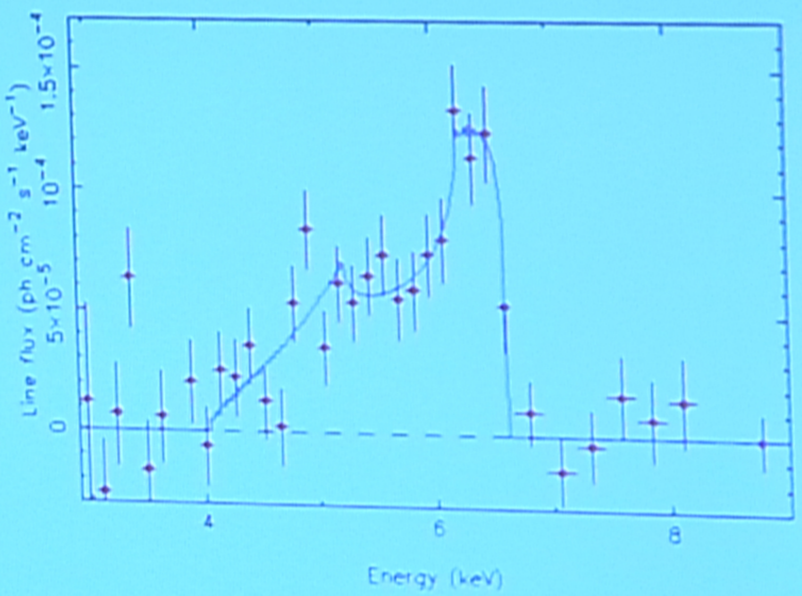
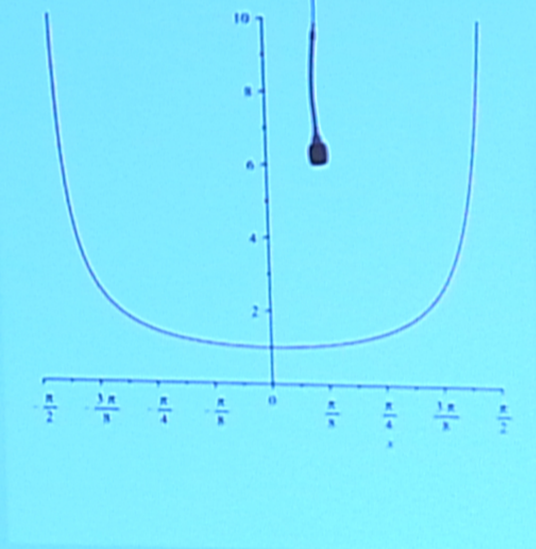


The profile of Fe Kα line from Seyfert galaxy MCG-6-30-15

(From P.Jovanovic, New Astronomy Review **56**, 37 (2012)) 7

Newtonian limit



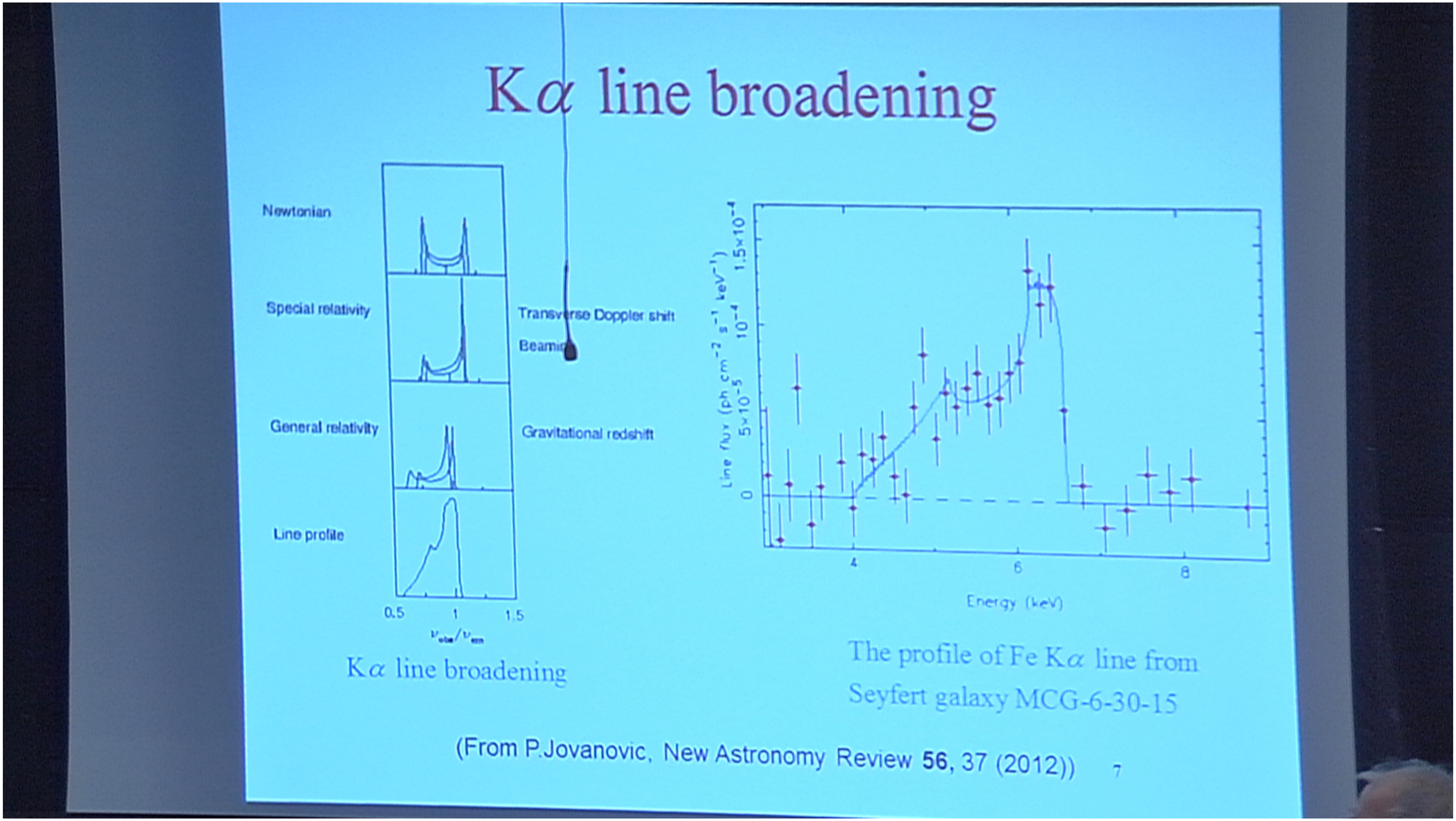
$$v_n = -v_n^0 \cos(\Omega t)$$

$$\omega \sim \omega_0 (1 + v_n)$$

$$\frac{dN}{d\omega} \sim (dN/dt)/(d\omega/dt)$$

$$\sim \frac{1}{\sqrt{(v_n^0)^2 - v_n^2}}$$

8


Kα line broadening
Newtonian
Special relativity
General relativity
Line profile
Transverse Doppler shift
Beaming
Gravitational redshift
0.5
1
1.5
Energy (keV)
Kα line broadening
The profile of Fe Kα line from
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(From P.Jovanovic, New Astronomy Review 56, 37 (2012))
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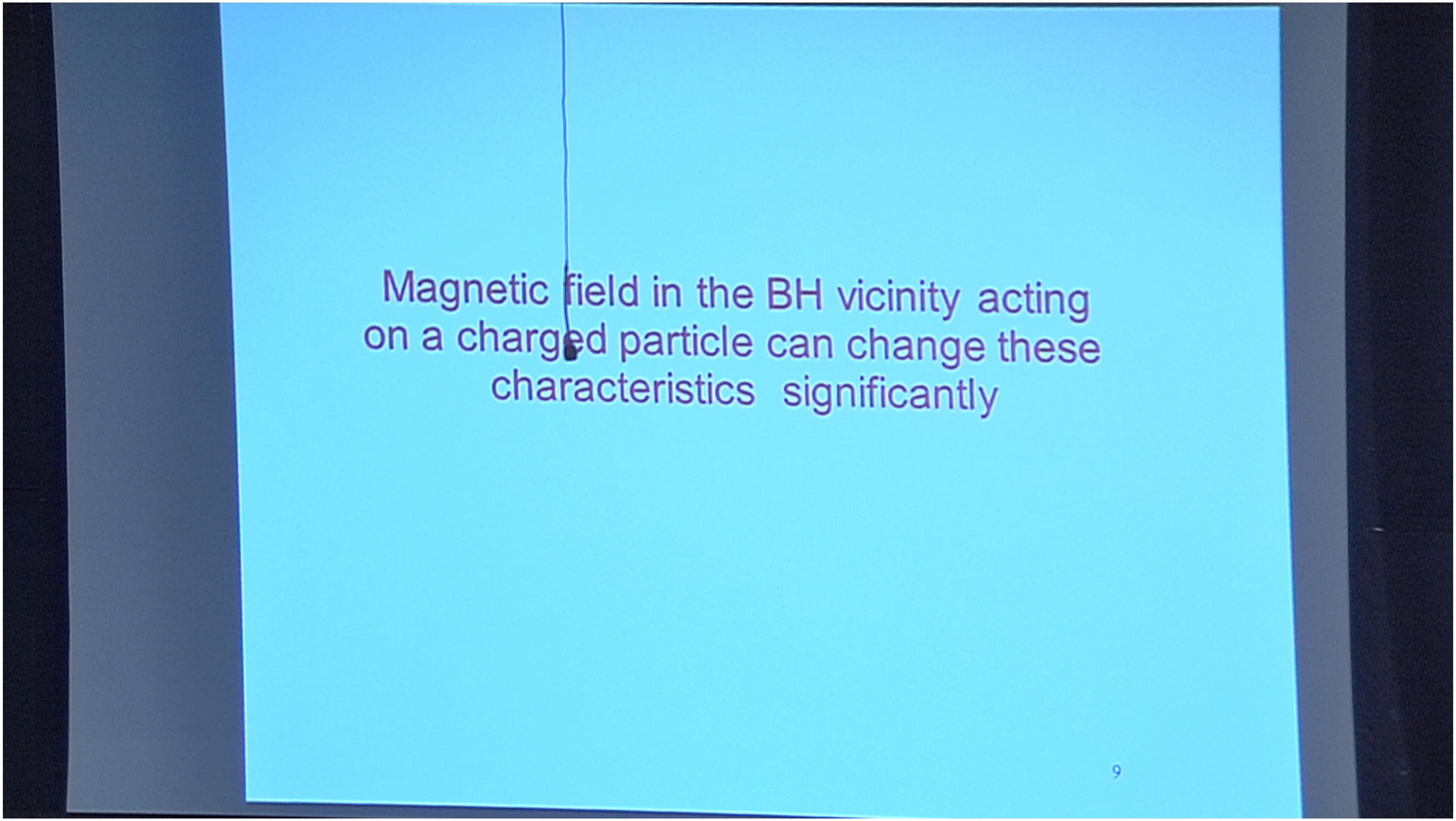
Magnetic field in the BH vicinity acting
on a charged particle can change these
characteristics significantly
9

Magnetic field in the vicinity of black holes

J. Larmor, "Aether and Matter", (Cambridge University Press, Cambridge, England, 1900), Ch. VI

- Original magnetic field of the collapsed progenitor star.
- The dynamo mechanism in the accretion disc of a black hole. Disc structure and properties (Penna et al, MNRAS **408** (2010))

Larmor's theorem: for a system of charged particles, all having the same ratio of charge to mass, moving in a central field of force, the motion in a uniform magnetic induction *B* is, to first order in *B*, the same as a possible motion in the absence of *B* except for the superposition of a common precession of angular frequency equal to the Larmor frequency.

Magnetic field in the vicinity of black holes

- Original magnetic field of the collapsed progenitor star.
- The dynamo mechanism in the accretion disc of a black hole. Disc structure and properties (Penna et al, MNRAS,**408** (2010)
- Transfer of frozen magnetic field to inner part of accretion disk

Observational evidences, e.g.:

Observations of Faraday rotation of radiation from pulsar in the vicinity of BH in the center of Milky Way (Sgr A*): several hundred gauss at few Schw. radius

(Eatough et al, Nature, 501, 391 (2013))

- Numerical simulations of jet formation in strong magnetic and gravitational fields,
  see, e.g., S. Koide, et al., Science **295**, 1688 (2002)

- Extraction of the rotational energy form black holes: Blandford-Znajek and Penrose mechanism.

In BZ mechanism:

$B \sim 10^4 G$ is required to produce power $\sim 10^{45} erg/\sec$ seen in jets from supermassive ($M \sim 10^9 M_\odot$) BH;

$B \sim 10^{15} G$ is required to produce power $\sim 4\times 10^{52} erg/\sec$ seen in GRB (for BH with $M \sim 10 M_\odot$).

M. Yu. Piotrovich, et. al, arXiv:1002.4948

Weak or Strong?

Gravitational effect of a magnetic field:

Spacetime curvature $\sim GB^2/c^4$ generated by magnetic field B is comparable with at the horizon curvature $\sim r_g^{-2}$ if

$$\frac{GB^2}{c^4} \sim \frac{1}{r_g^2} \sim \frac{c^4}{G^2M^2} \quad \Rightarrow \quad B \sim \frac{c^4}{G^{3/2}M_\odot}\left(\frac{M_\odot}{M}\right) \times 10^{19}\,(M_\odot/M)\,\mathrm{G}\,.$$

Charged particle motion:

$$m\frac{du^a}{d\tau} = qF^a_{\ b}u^b \qquad e/m_e \approx 5.2728\times10^{17}\ \mathrm{g}^{-1/2}\mathrm{cm}^{3/2}\mathrm{s}^{-1}$$

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Effect of magnetic field on charged particles

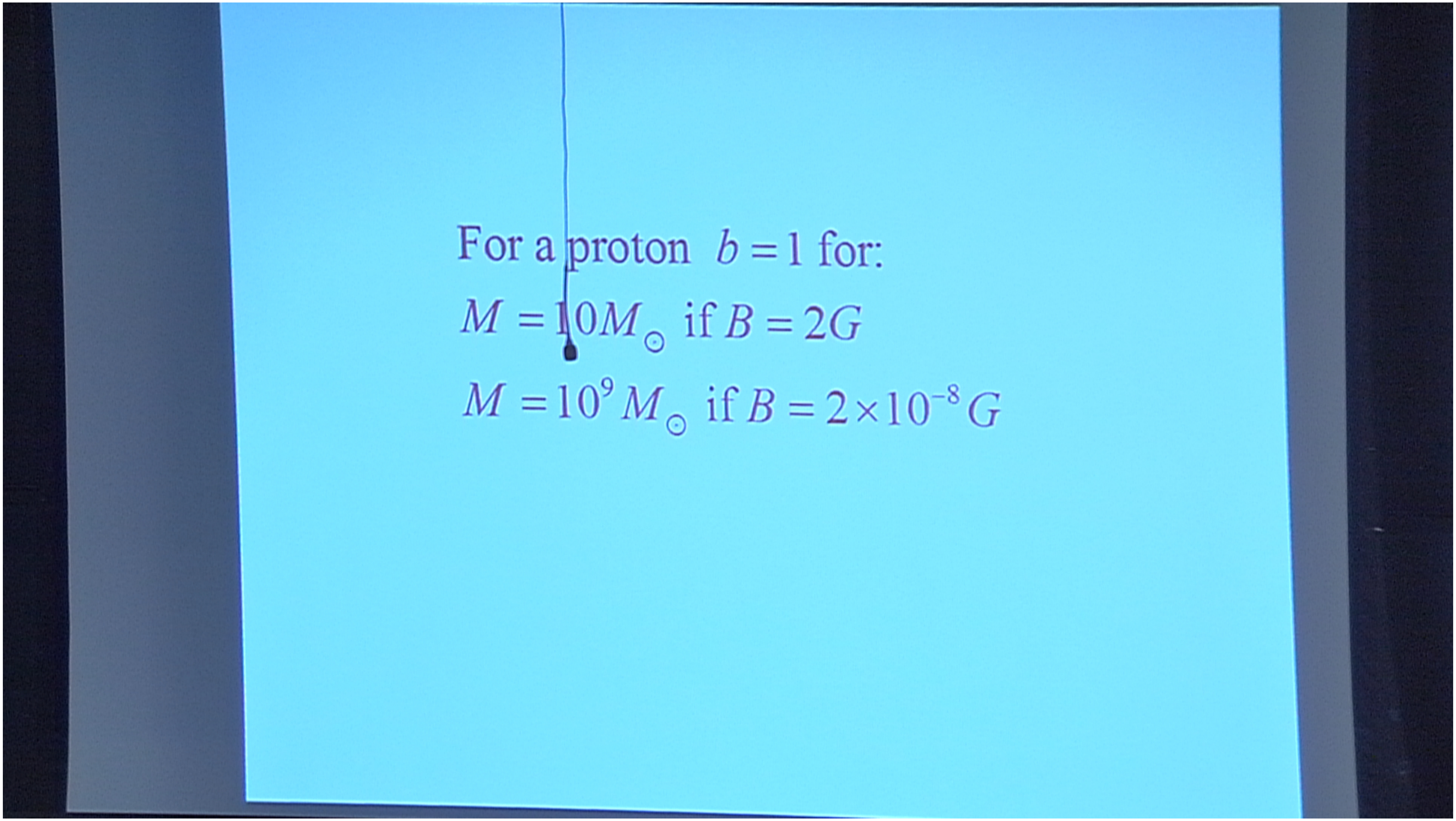
Cyclotron frequency: $\Omega_c = \dfrac{|qB|c}{E}, E = \gamma mc^2$

Keplerian frequency: $\Omega_K = \dfrac{r_g^{1/2} c}{r^{3/2}\sqrt{2}}$ $\qquad r_g = \dfrac{2MG}{c^2}$

$$E \sim mc^2, r_{ISCO} = 3r_g = 6GM / c^2$$

$$\Omega_c / \Omega_K = 6^{3/2} b, \quad b = \frac{qBMG}{mc^4}$$

For large b magnetic field essentially modifies motion of a charged particle


For a proton $b = 1$ for:
$M = 10M_{\odot}$ if $B = 2G$
$M = 10^{9}M_{\odot}$ if $B = 2\times10^{-8}G$

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Magnetized black hole

The Schwarzschild space-time:

$$ds^2 = -\left(1-\frac{r_g}{r}\right)dt^2 + \left(1-\frac{r_g}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Killing vectors:

$$\xi_{(t)} = \frac{\partial}{\partial t}, \quad \xi_{(\phi)} = \frac{\partial}{\partial \phi}, \quad \xi^{a;b}{}_{;b} = 0, \quad (R_{ab} = 0)$$

The electromagnetic 4-potential: $A^a = \frac{1}{2} B\, \xi^a_{(\phi)}$

Static, axisymmetric, uniform at infinity magnetic field:

$$B^a\partial_a = B\left(1-\frac{r_g}{r}\right)^{1/2}\left[\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right]$$

Dynamical equations

Motion of a charged particle:

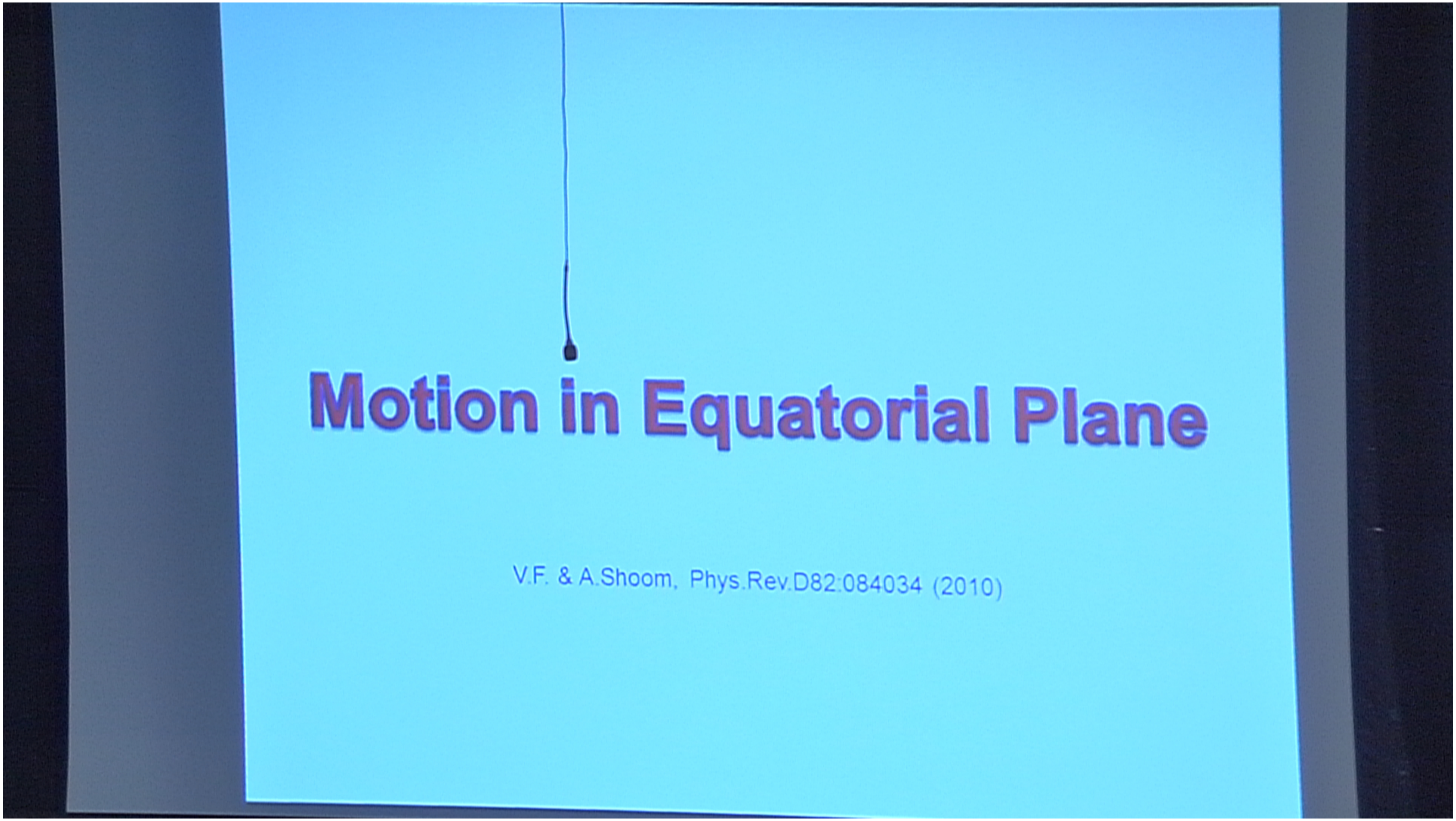
$$m\frac{du^a}{d\tau} = qF^a{}_b u^b, \qquad u^a u_a = -1$$

Generalized momentum: $P_\mu = mu_\mu + qA_\mu$.

Integrals of motion:

$$E \equiv -\xi^a_{(t)} P_a = m\frac{dt}{d\tau}\left(1 - \frac{r_g}{r}\right),$$

$$L \equiv \xi^a_{(\phi)} P_a = m\frac{d\phi}{d\tau} r^2 \sin^2\theta + \frac{qB}{2} r^2 \sin^2\theta$$


Motion in Equatorial Plane
V.F. & A.Shoom, Phys.Rev.D82:084034 (2010)

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3 conserved quantities: $m, E, L \rightarrow$ complete integrability

$$\left(\frac{dr}{d\tau}\right)^2 = \tilde{E}^2 - \left(1 - \frac{r_g}{r}\right)\left[1 + r^2\left(\frac{\tilde{L}}{r^2} - \tilde{B}\right)^2\right],$$

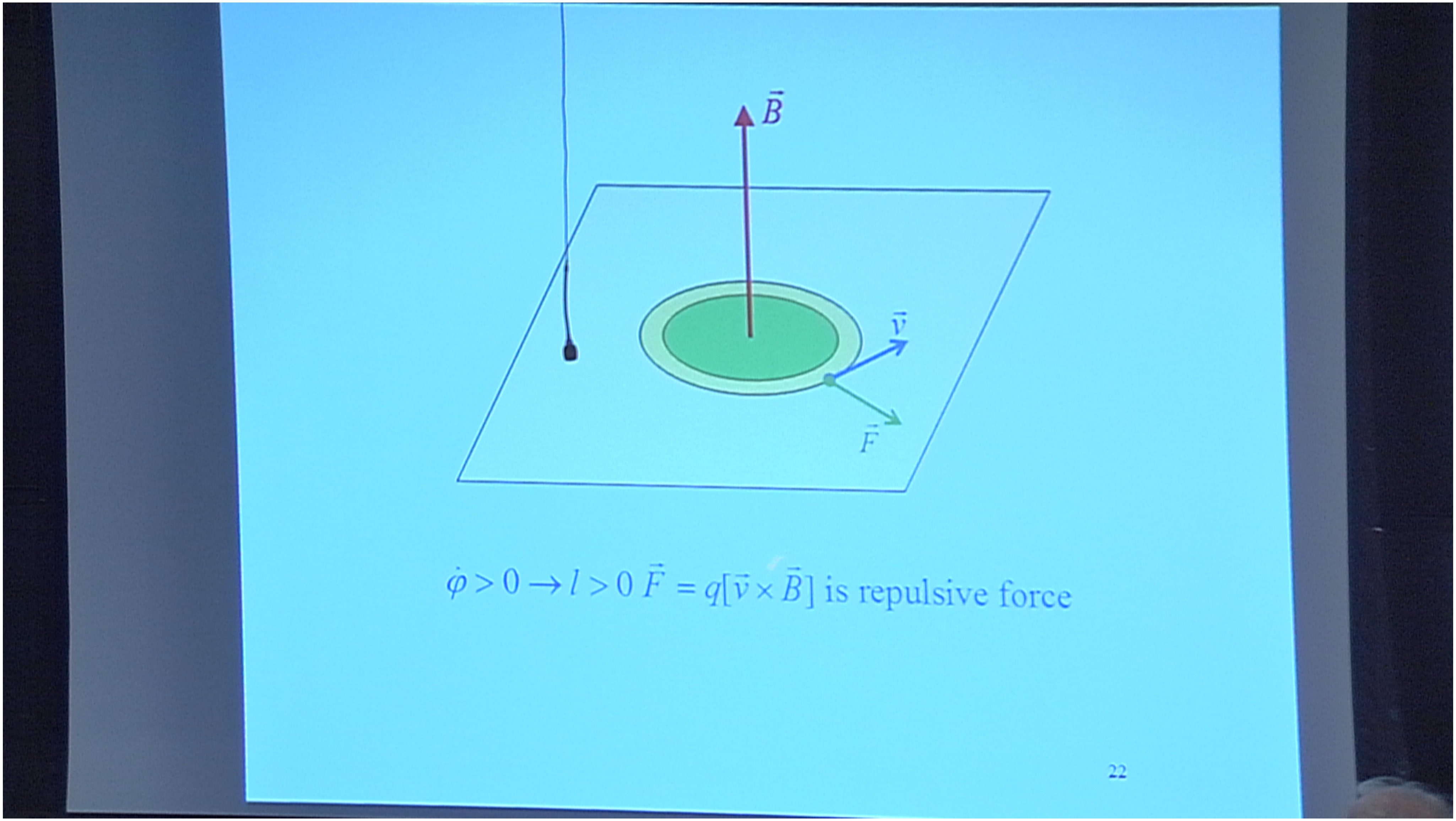
$$\frac{d\phi}{d\tau} = \frac{\tilde{L}}{r^2} - \tilde{B}, \qquad \frac{dt}{d\tau} = \tilde{E}\left(1 - \frac{r_g}{r}\right)^{-1}$$

$$\tilde{E} \equiv \frac{E}{m}, \quad \tilde{L} \equiv \frac{L}{m}, \quad \tilde{B} \equiv \frac{qB}{2m},$$

Discrete symmetry: $B \rightarrow -B, \quad L \rightarrow -L, \quad \phi \rightarrow -\phi$

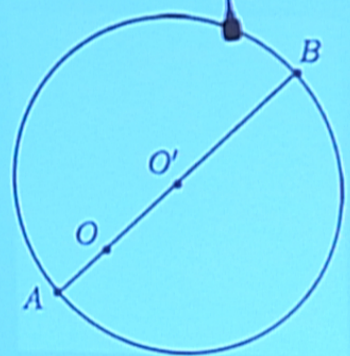
We assume $\tilde{B} > 0$

21

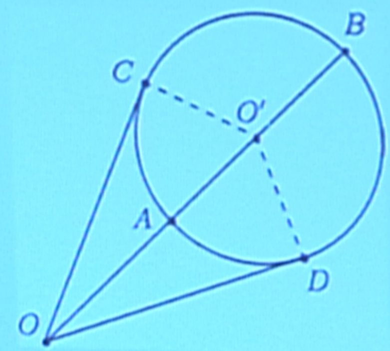

$\vec{B}$
$\vec{v}$
$\vec{F}$
$\dot{\varphi} > 0 \rightarrow l > 0$ $\vec{F} = q[\vec{v} \times \vec{B}]$ is repulsive force
22

Flat space-time limit

$$\left(\frac{dr}{d\tau}\right)^2 = \tilde{E}^2 - 1 - r^2\left(\frac{\tilde{L}}{r^2} - \tilde{B}\right)^2, \quad \frac{d\phi}{d\tau} = \frac{\tilde{L}}{r^2} - \tilde{B}, \quad \frac{dt}{d\tau} = \tilde{E}$$



$\tilde{L} < 0$



$\tilde{L} > 0$

$$r_c = \sqrt{|\tilde{L}|/\tilde{B}}, \quad \Omega_c = \frac{2\tilde{B}}{\tilde{E}}, \quad \tilde{E} = \sqrt{1 + 4\tilde{B}|\tilde{L}|}.$$

Weak Gravitational Field

Far away from the black hole:

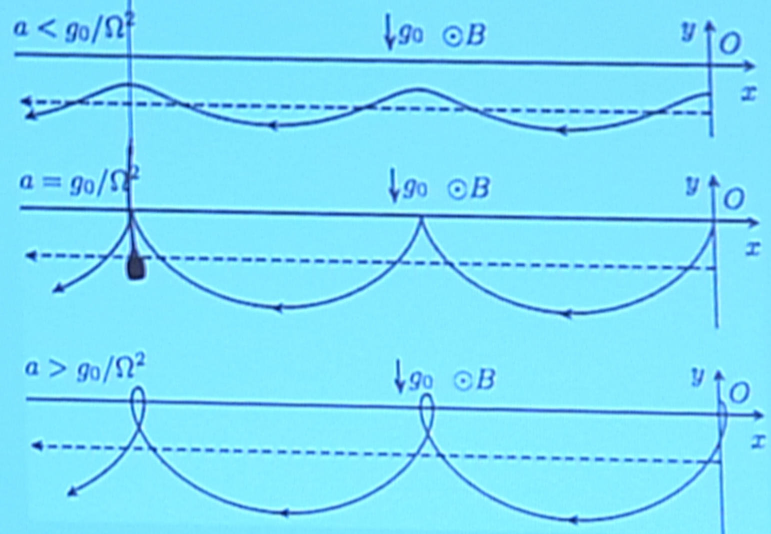
$$r_g / r \quad \text{small parameter}$$

Dynamical equations:

$$\frac{d^2 r}{d\tau^2} = \frac{\tilde{L}^2}{r^3} - \tilde{B}^2 r - g\,, \qquad \dot{\phi} = \frac{\tilde{L}}{r^2} - \tilde{B}\,, \qquad \dot{t} \approx \tilde{E}$$

Newtonian gravitational acceleration: $g = \dfrac{r_g}{2r^2}$

Particle trajectory in the mutually orthogonal uniform magnetic and gravitational fields



Gravitational drift velocity in the rest frame: $V = \dfrac{g_0}{\Omega_c \tilde{E}^2}$

Frame moving with the velocity V: $B \rightarrow E_y = \gamma V B$

Motion of a Charged Particle: Strong Field Case

Dimensionless quantities:

$$T = t / r_g, \quad \rho = r / r_g, \quad \sigma = \tau / r_g, \quad \ell = \tilde{L} / r_g, \quad b = \tilde{B} r_g$$

Dynamical equations:

$$\left(\frac{d\rho}{d\sigma}\right)^2 = \tilde{E}^2 - U, \qquad \frac{d\phi}{d\sigma} = \frac{\ell}{\rho^2} - b, \quad \frac{dT}{d\sigma} = \frac{\tilde{E}\rho}{\rho - 1}$$

Attractive Lorentz force: $\ell = -|\ell|$

Repulsive Lorentz force: $\ell = +|\ell|$

Effective potential

$$U = \left(1 - \frac{1}{\rho}\right)\left[1 + \frac{(\ell - b\rho^2)^2}{\rho^2}\right]$$

At horizon: $U|_{\rho \to 1} \to 0;$ At infinity: $U|_{\rho \to +\infty} \to b^2 \rho^2$

$\# \max = \# \min \Rightarrow$ even number of extremal points in $\rho \in (1, \infty)$

Extremum: $U_{,\rho} = 0 \leftrightarrow P(\rho) = Q(\rho)$

$$P(\rho) = b^2 \rho^4 (2\rho - 1) + \rho^2$$

$$Q(\rho) = 2\ell b \rho^2 + 2\ell^2 \rho - 3\ell^2$$

29

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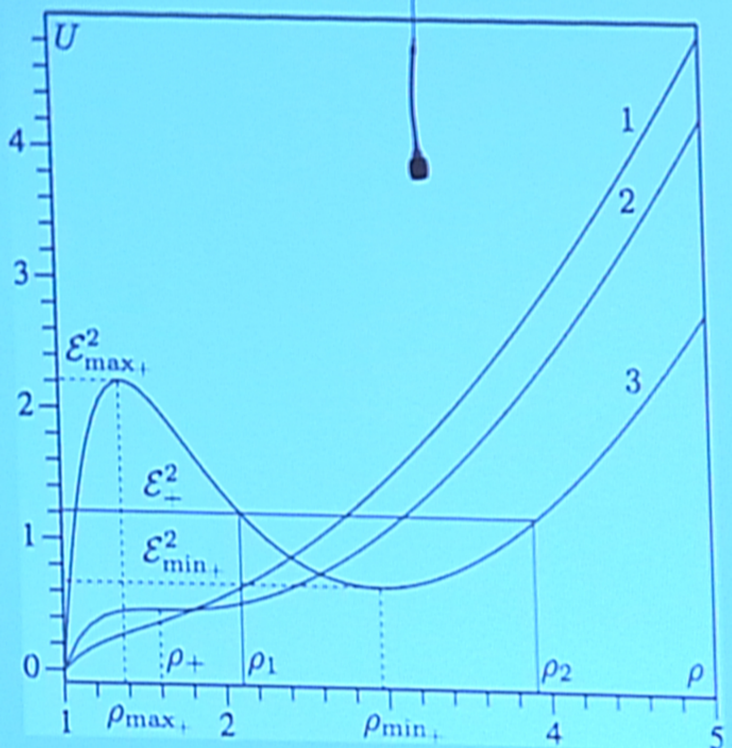
29

Effective potential has either 1 maximum and 1 minimum, or no extrema. Critical case: when maximum and minimum coincide. At this ISCO: $U_{,\rho} = U_{,\rho\rho} = 0 \leftrightarrow P_{,\rho} = Q_{,\rho}$



(a) Stable sircular orbit

(b) Unstable circular orbit

(c) Innermost stable circular orbit

$\rho_* \equiv \sqrt{\ell / b}, \qquad U_{,\rho}(\rho_*) = b / \ell$

$(\ell > 0)$

$\rho_* > \rho_{\min} \geq \rho_{\max}$

Circular orbit: $u^{\mu} = \gamma(\xi_{(t)} + \Omega\, r_g^{-1} \xi_{(\phi)})$

$\gamma = (1 - \rho^{-1} - \Omega^2 \rho^2), \quad \rho = r / r_g$

Innermost stable circular orbits

$$U_{,\rho} = 0, \qquad U_{,\rho\rho} = 0$$

32

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Innermost stable circular orbits

$$U_{,\rho} = 0, \qquad U_{,\rho\rho} = 0$$

32

$$b = \frac{\sqrt{2}(3-\rho_{\pm})^{1/2}}{2\rho_{\pm}\left(4\rho_{\pm}^{2} - 9\rho_{\pm} + 3 \pm \sqrt{(3\rho_{\pm}-1)(3-\rho_{\pm})}\right)^{1/2}},$$

$$\ell_{\pm} = \pm\frac{\rho_{\pm}(3\rho_{\pm}-1)^{1/2}}{\sqrt{2}\left(4\rho_{\pm}^{2} - 9\rho_{\pm} + 3 \pm \sqrt{(3\rho_{\pm}-1)(3-\rho_{\pm})}\right)^{1/2}},$$

$$\rho_+|_{b>>1} = 1 + \frac{1}{b\sqrt{3}} + O(b^{-2}),$$

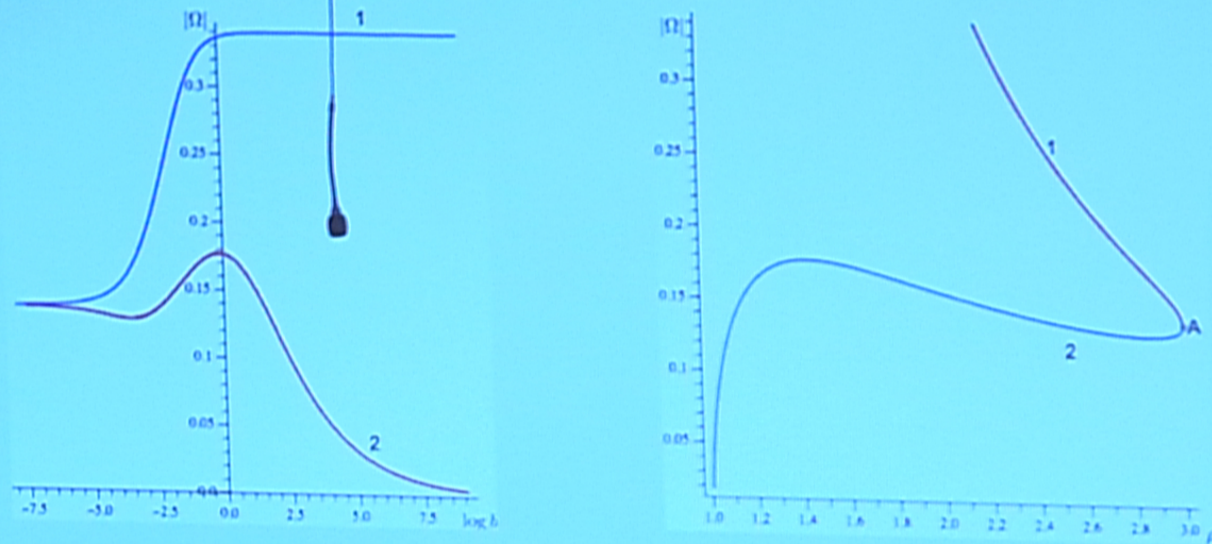
$$\rho_-|_{b>>1} = \frac{5+\sqrt{13}}{4} + \frac{41-11\sqrt{13}}{36\sqrt{13}b^2} + O(b^{-4})$$

$$\ell_+|_{b>>1} = b + \sqrt{3} + O(b^{-1}),$$

$$\ell_-|_{b>>1} = -\frac{47+13\sqrt{13}}{8}b + O(b^{-1})$$

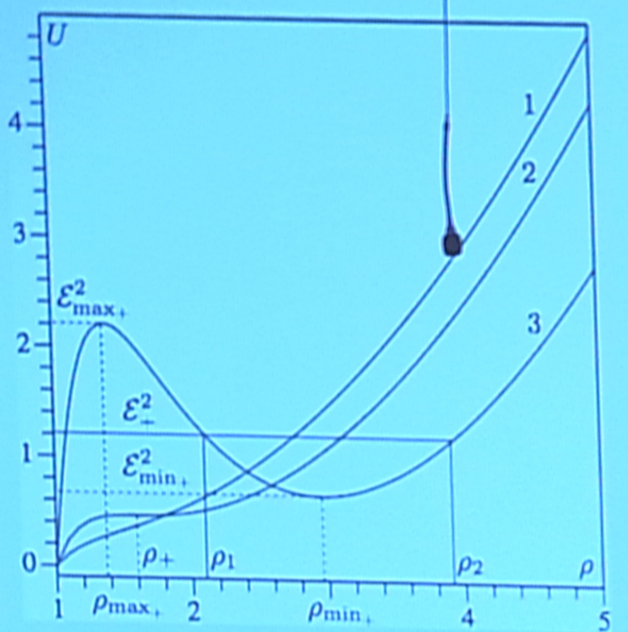
$$\Omega_+|_{b>>1} = \frac{3^{3/4}}{6b^{1/2}} + O(b^{-3/2}),$$

$$\Omega_-|_{b>>1} = -0.34 + \frac{0.41}{b^2} + O(b^{-4})$$



Lines 1 for $-\Omega_-$ and lines 2 for Ω_+

Types of Trajectories: General Case



(1)Trapped orbits

(2) Bounded orbits

$$\rho_{\max} \le \rho_1 \le \rho_{\min} \le \rho_2,$$
$$E_{\min} \le E \le E_{\max}$$

Radial motion is periodic

`Angular' equation

Angular velocity: $$\frac{d\phi}{d\sigma} = \frac{\ell}{\rho^2} - b$$

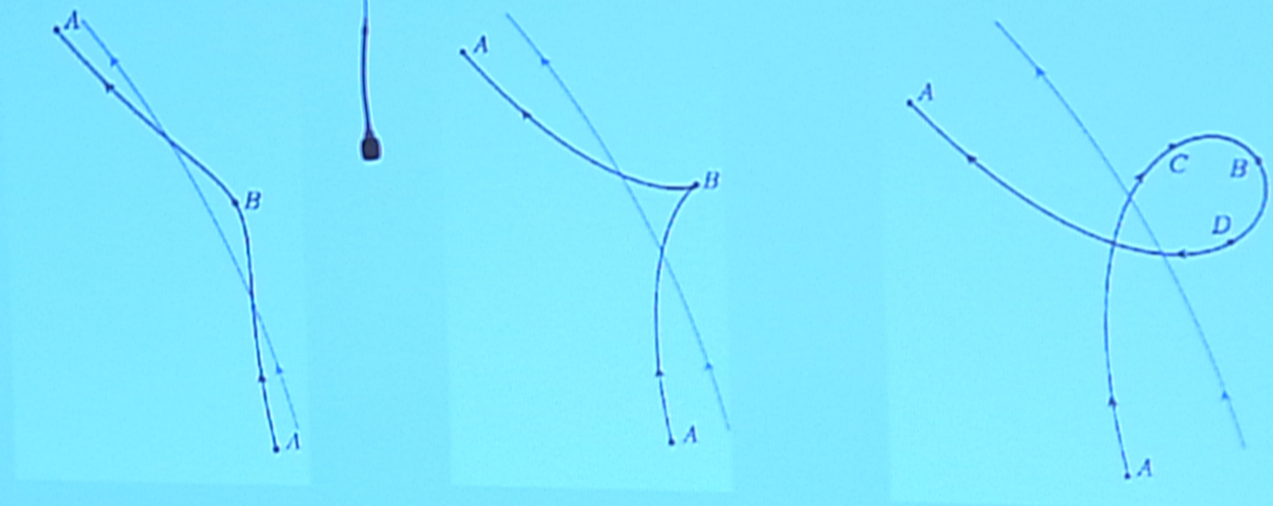
Attractive Lorentz force: $\ell < 0 \quad \Rightarrow \quad \dot{\phi} < 0$:

Clockwise motion modulated by radial oscillations

Repusive Lorentz force: $\ell > 0$: 2 types of orbits:

(i) `smooth' and (ii) `curly'

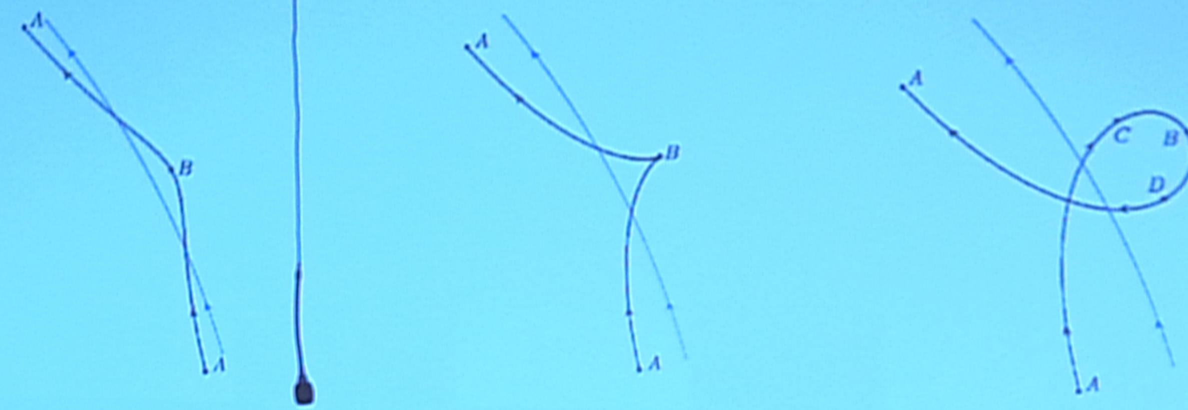
Types of orbits $\ell > 0$

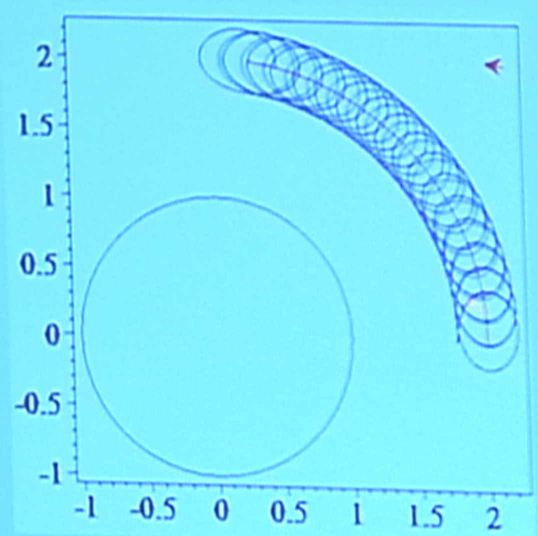


$\rho_2 < \rho_*, \quad E_- \in [E_{\min}, E_*)$

$\rho_2 = \rho_*, \quad E_- = E_*$

$\rho_2 > \rho_*, \quad E_- > E_*$





$b \approx 8.7 \times 10^{14}, \quad \rho_* = 2.0,$

$A \approx 0.2, \quad E_{\min +} \approx 0.7,$

$\omega_o \approx 0.7, \quad N \approx 74$

42

Lessons

(i) Radius of ISCO $\rho_+ |_{b>>1} \sim 1 + \frac{1}{b\sqrt{3}}$

(ii) Energy release $\Delta E_{ISCO+} \sim mc^2(1 - \frac{2}{3^{3/4} b^{1/2}})$

(iii) Angular velocity $\Omega_+ |_{b>>1} \sim \frac{3^{3/4}}{6b^{1/2}}$

43

Work in progress:
$K\alpha$ lines broadening in magnetized black holes

(1) Ray tracing in Schwarzschild geometry

(2) Image of circular orbit at the impact plane

(3) Time delay

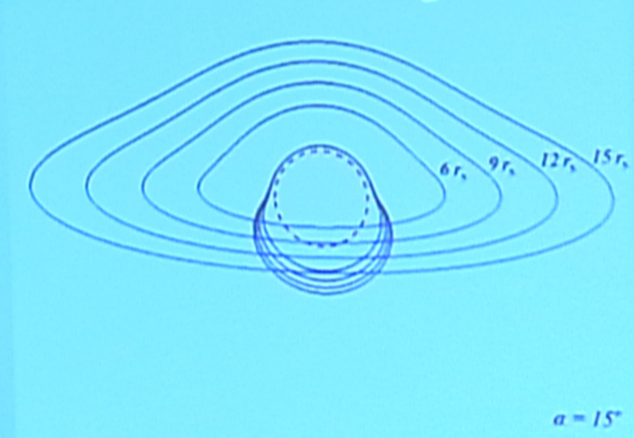
(4) Frequency shift

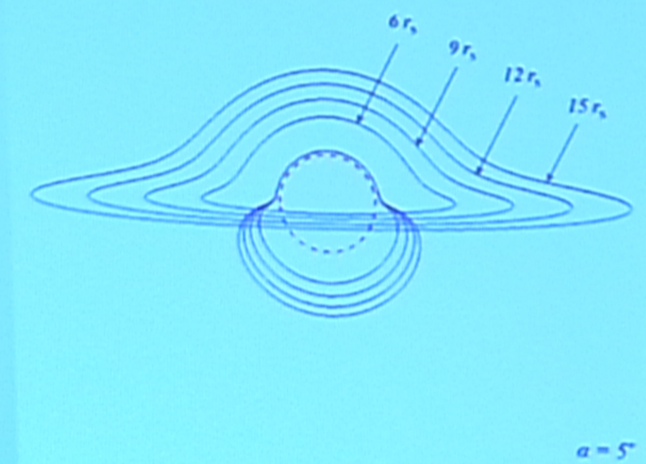
(5) Brightness

New features: close to horizon orbits and non-Keplerian frequency

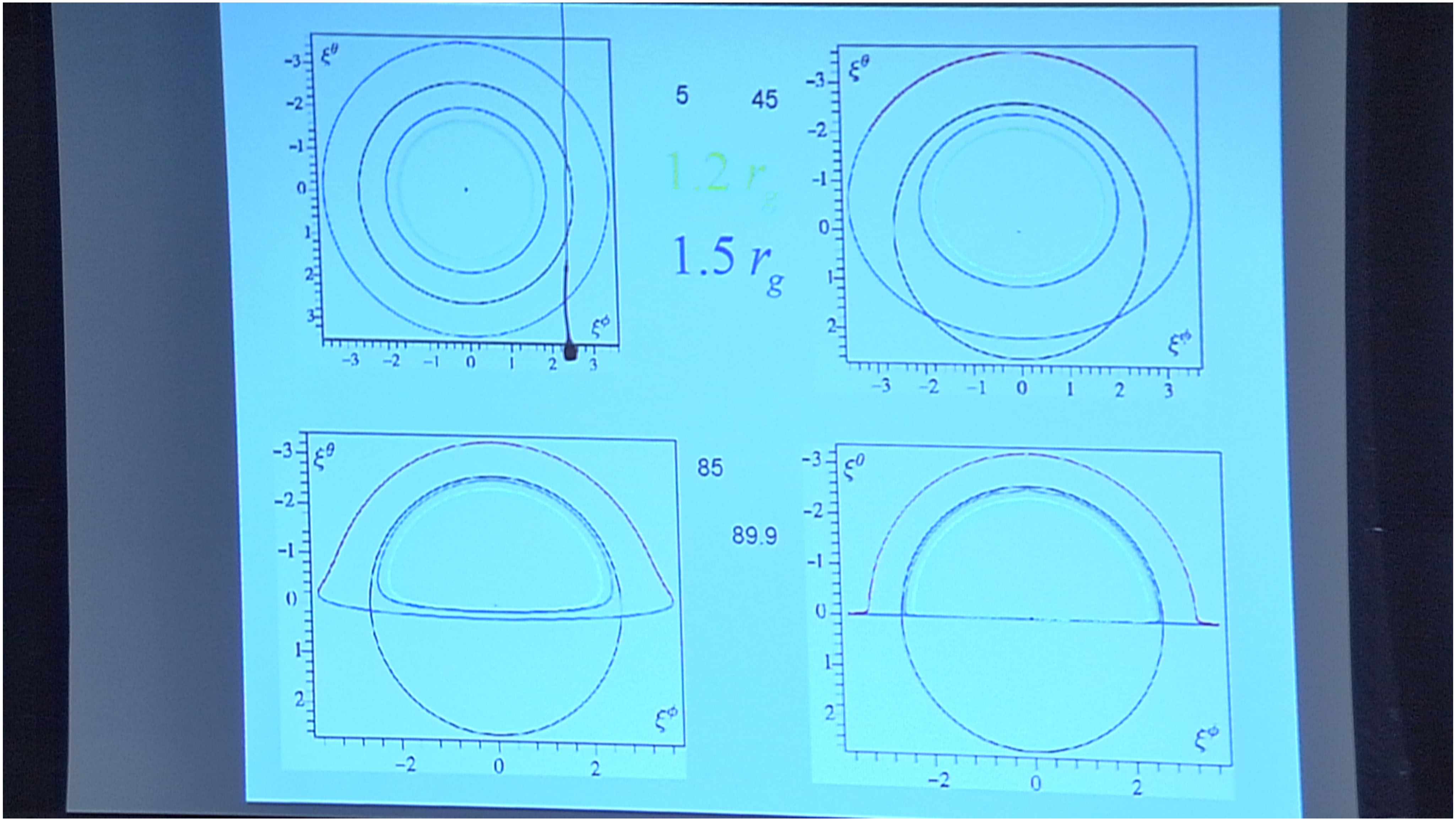
44

Orbits images





45



5
45
1.2 r_g
1.5 r_g
85
89.9
ξ^θ
ξ^ϕ

Magnetized BH as "Particle Accelerator"

V.F., Phys.Rev. D85: 024020 (2012);

Banados, Silk & West PRL 103, 111102 (2009): Center of mass energy for collision of 2 particles near the horizon of a rotating black hole can be arbitrary large for special (fine tuned) choice of their angular momenta and $\alpha \equiv a/M \to 1$. The effect is propotional to $(1-\alpha)^{-1/4}$.

Similar effect occurs in magnetised (even non-rotating) black holes. The effect is proportional to $b^{1/4}$.

Consider collision of 2 particles in the vicinity of magnetized BH.

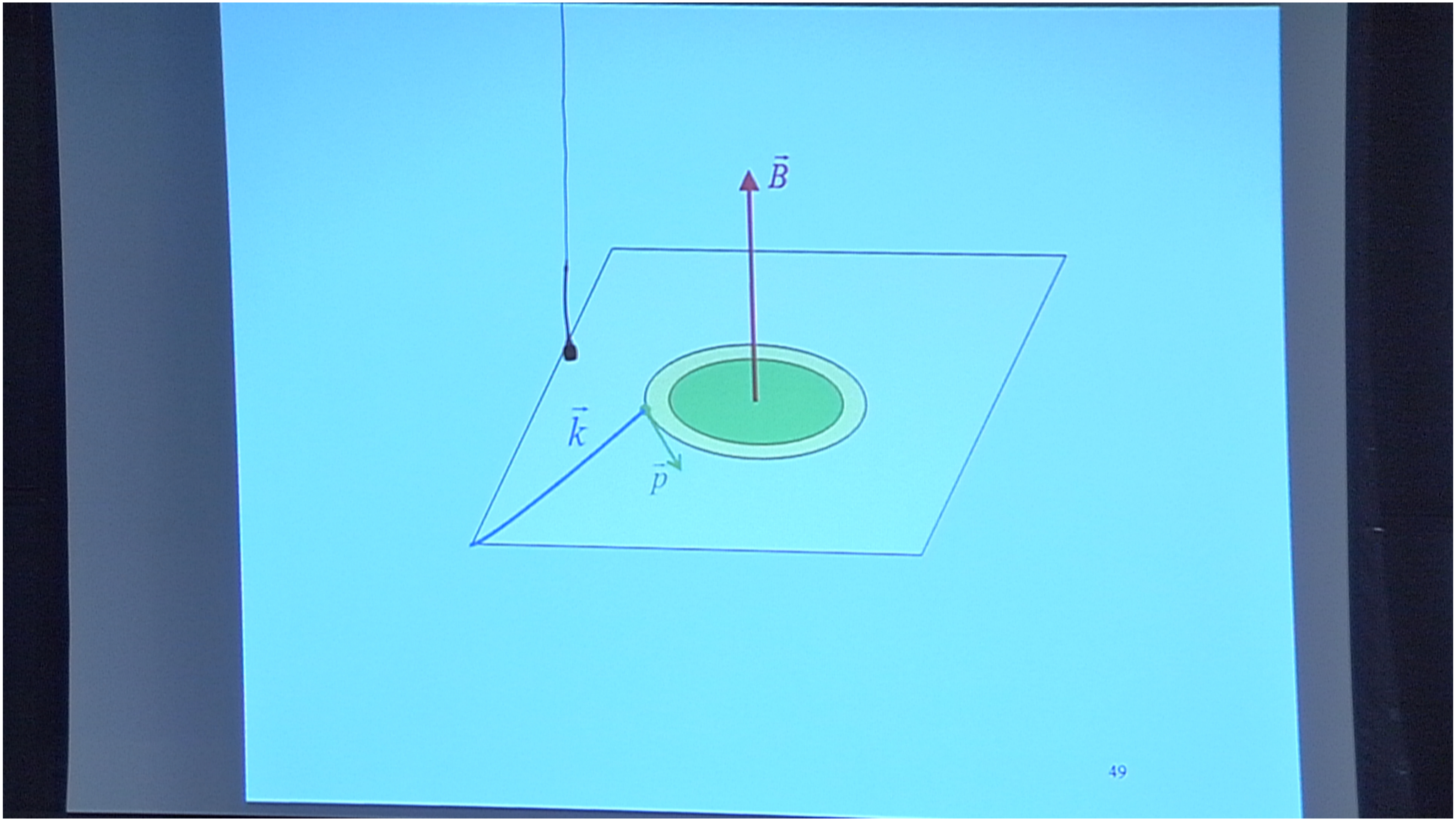
(1) First charged particle (with charge q and mass m) is at ISCO.

(2) Second (neutral) particle of mass μ is freely falling.

At the moment of collision the four momentum is $P^{\lambda} = p^{\lambda} + k^{\lambda}$,

and the center-of-mass energy M is: $M^2 = m^2 + \mu^2 - 2(p,k)$

48


$\vec{B}$
$\vec{k}$
$\vec{p}$
49

Charged particle: ISCO motion in magnetic field

$$p^{\mu} = m\gamma(e^{\mu}_{(t)} + v e^{\mu}_{(\varphi)}), \quad v\gamma = \beta;$$

$$e^{\mu}_{(t)} = f^{-1/2}\xi^{\mu}_{(t)} = f^{-1/2}\delta^{\mu}_{t}, \quad e^{\mu}_{(\varphi)} = r^{-1}\xi^{\mu}_{(\varphi)} = r^{-}\delta^{\mu}_{\varphi}$$

Freely falling particle

$$k^{\mu} = (E/f, \dot{r}, \dot{\theta}, L_z/r^2)$$

50

$$M^2 = m^2 + \mu^2 + 2m\gamma E\left(f^{-1/2} - \frac{v l_z}{\rho}\right), \quad l_z = L_z/(E r_g);$$

$$\left|\frac{v l_z}{\rho}\right| \le \frac{3\sqrt{3}}{2}, \quad \text{(max for photons in equatorial plane)}$$

$$\mathrm{M} \sim \frac{(2\mathrm{m}\gamma \mathrm{E})^{1/2}}{(\rho_{\mathrm{ISCO}} - 1)^{1/4}} \quad \Rightarrow \quad M \sim 1.74\, b^{1/4} \sqrt{mE}$$

$$1 - a \Leftrightarrow b^{-1}$$

Generalization to magnetized rotating BHs:
Igata, Harada & Kimura, PRD 85, 104028 (2012)

51

Motion out of Equatorial Plane

A.M. Al Zahrani, V.F. & A.Shoom, D87: 084043 (2013)

52

We consider again a charged particle (with charge q and mass m) revolving in the equatorial plane around a magnetized non-rotating black hole at the ISCO. We suppose now that it is `kicked' out of this orbit by some other particle or photon and gets an orthogonal to the plane velocity $v = -r\dot{\theta}$. What is the critical escape velocity v_* and what are properties of the near critical motion?

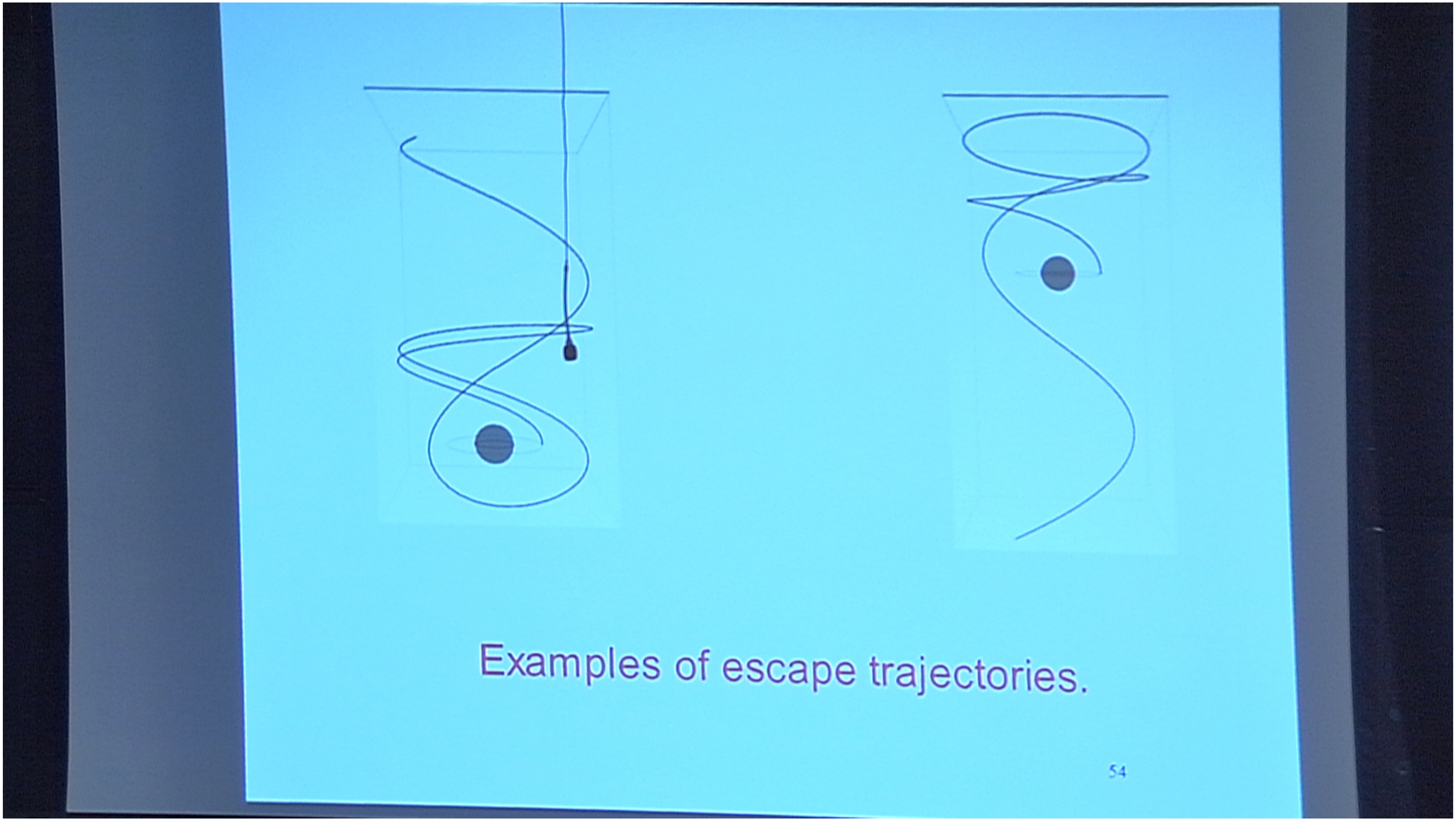
Three possible asymptotic types of motion:

(1) Capture (red);

(2) Escape in the direction of $\mathbf{B}$ (green);

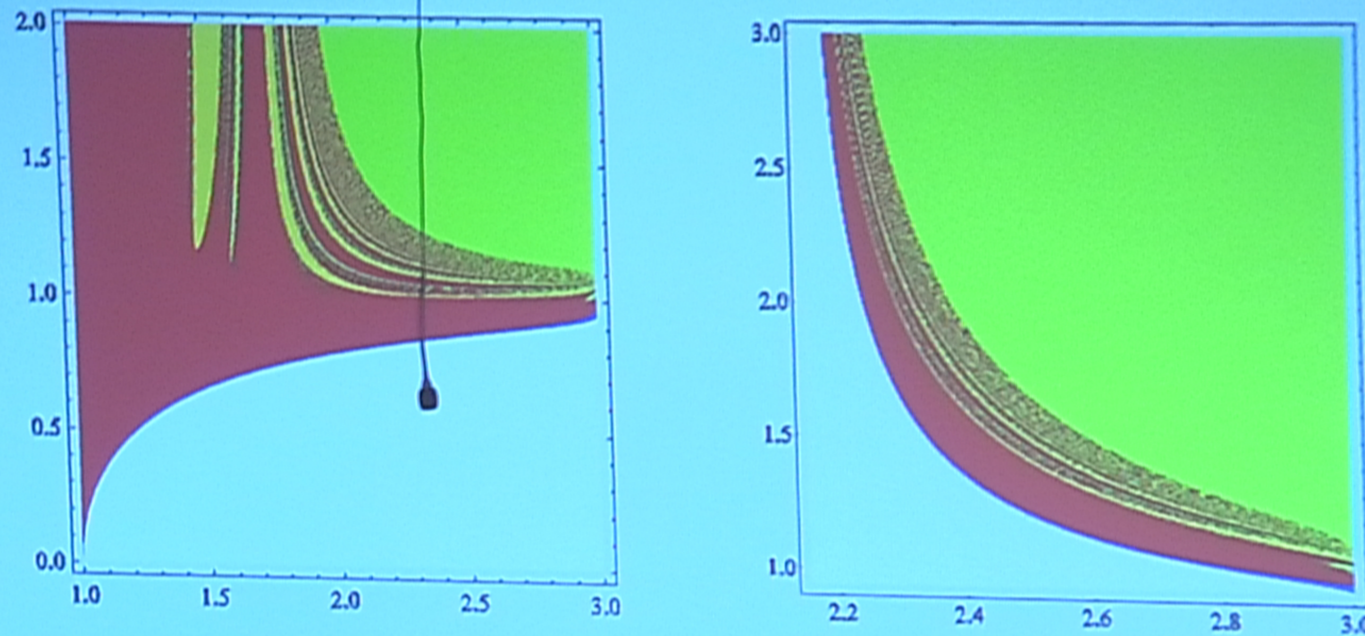
(3) Escape in the direction opposite to $-\mathbf{B}$ (yellow).

53



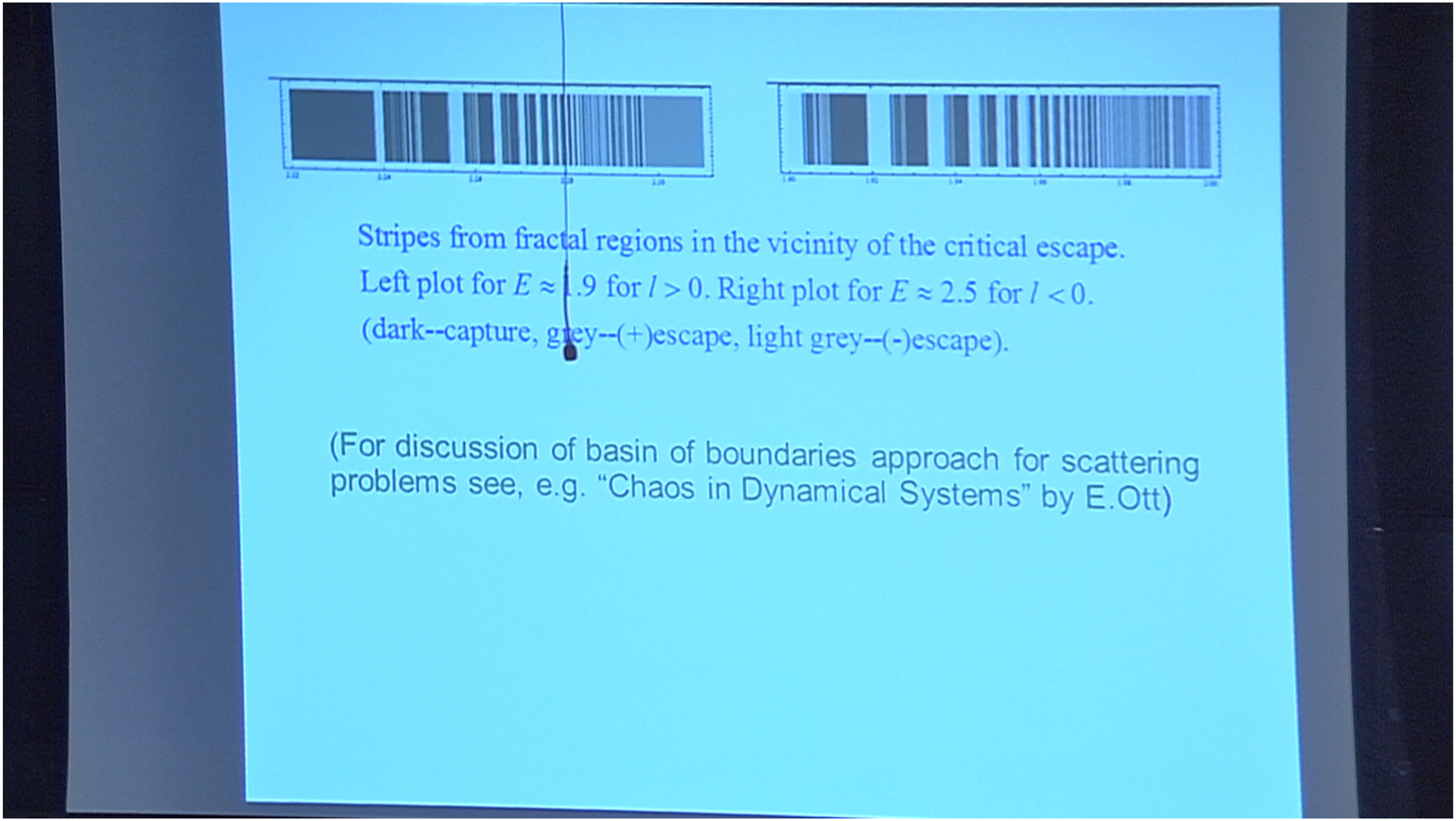
Examples of escape trajectories.
54

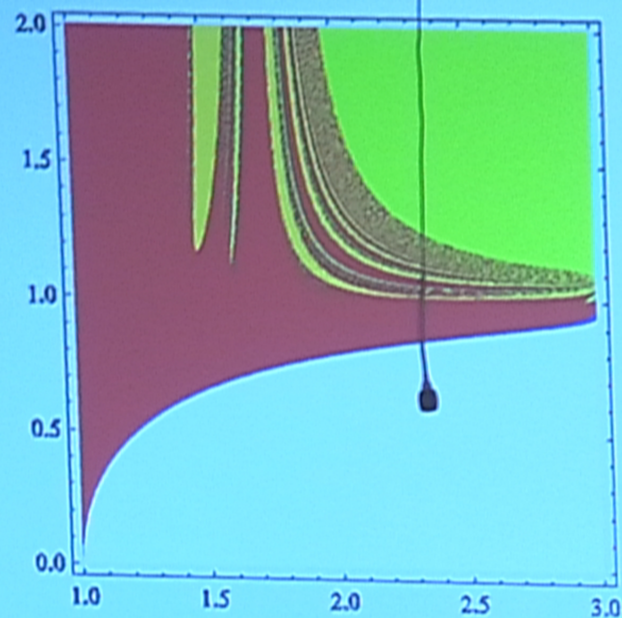

55

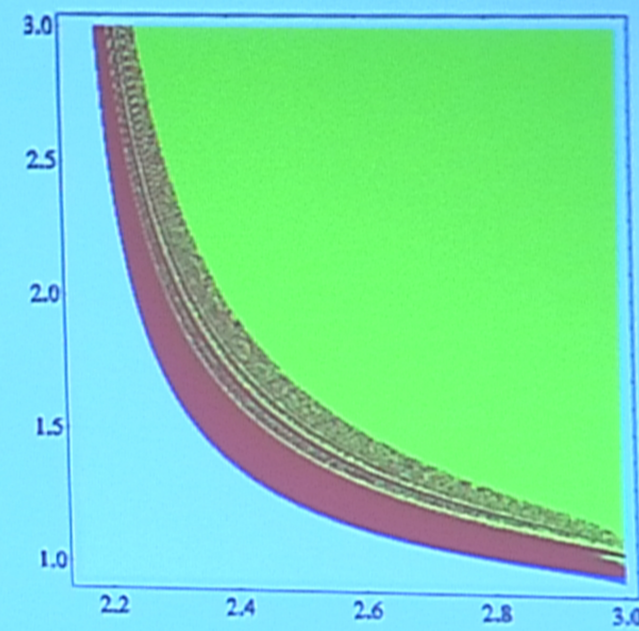


Basin of boundaries plots for a charged particle kicked from the last stable circular orbits at different radii $r_{ISCO}/2M$ defined by the magnetic field b (horizontal axis) with different kicking energies (vertical axis). Left plot for $l > 0$, right plot for $l < 0$.

Capture (red); escape along **B** (green); escape opposite to $-\mathbf{B}$ (yellow)


Stripes from fractal regions in the vicinity of the critical escape.
Left plot for $E \approx 1.9$ for $l > 0$. Right plot for $E \approx 2.5$ for $l < 0$.
(dark--capture, grey--(+)escape, light grey--(-)escape).
(For discussion of basin of boundaries approach for scattering problems see, e.g. “Chaos in Dynamical Systems” by E.Ott)





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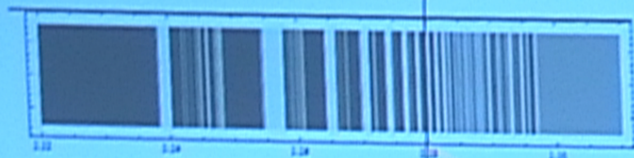
56



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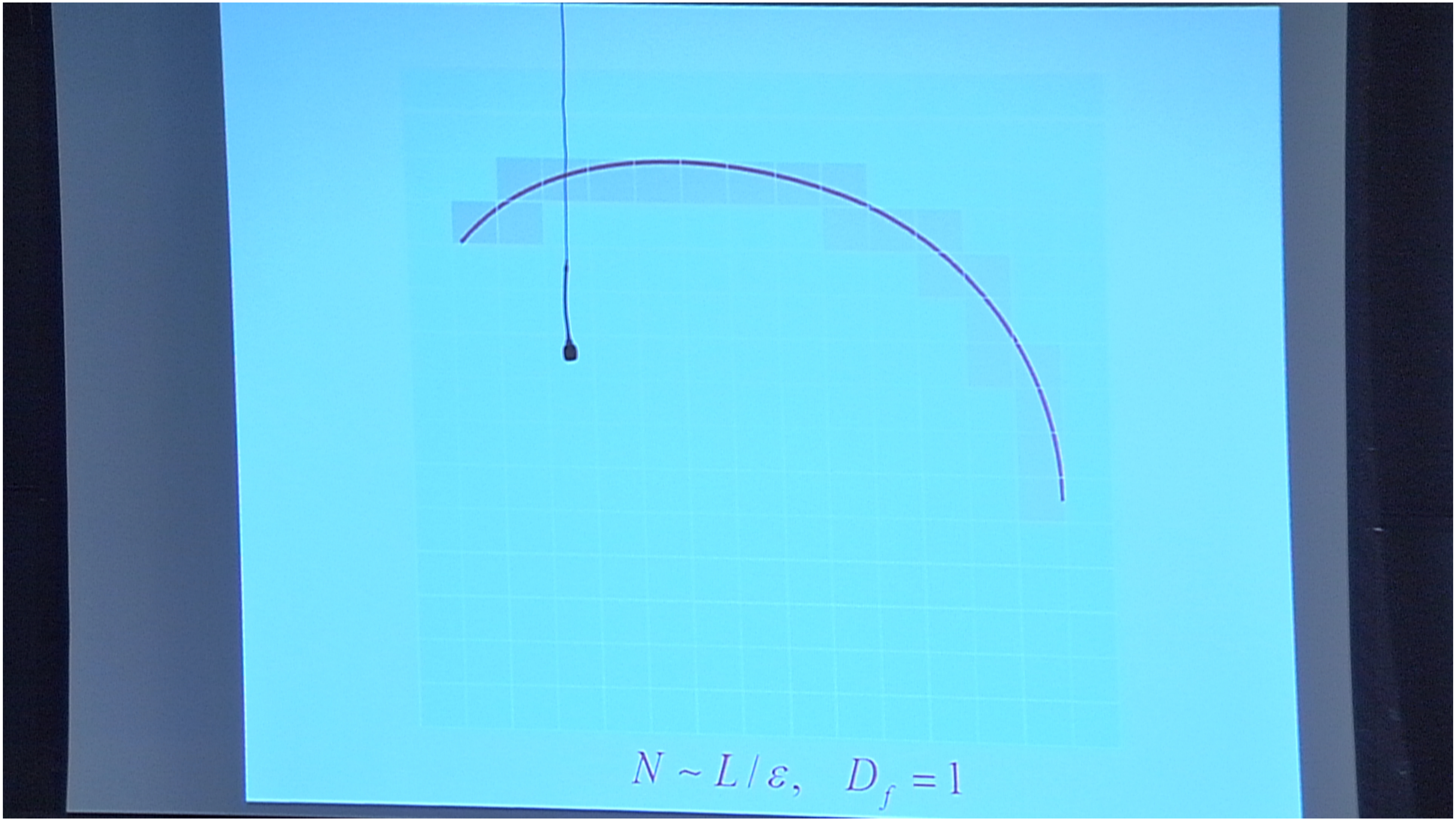
(For discussion of basin of boundaries approach for scattering problems see, e.g. "Chaos in Dynamical Systems" by E.Ott)

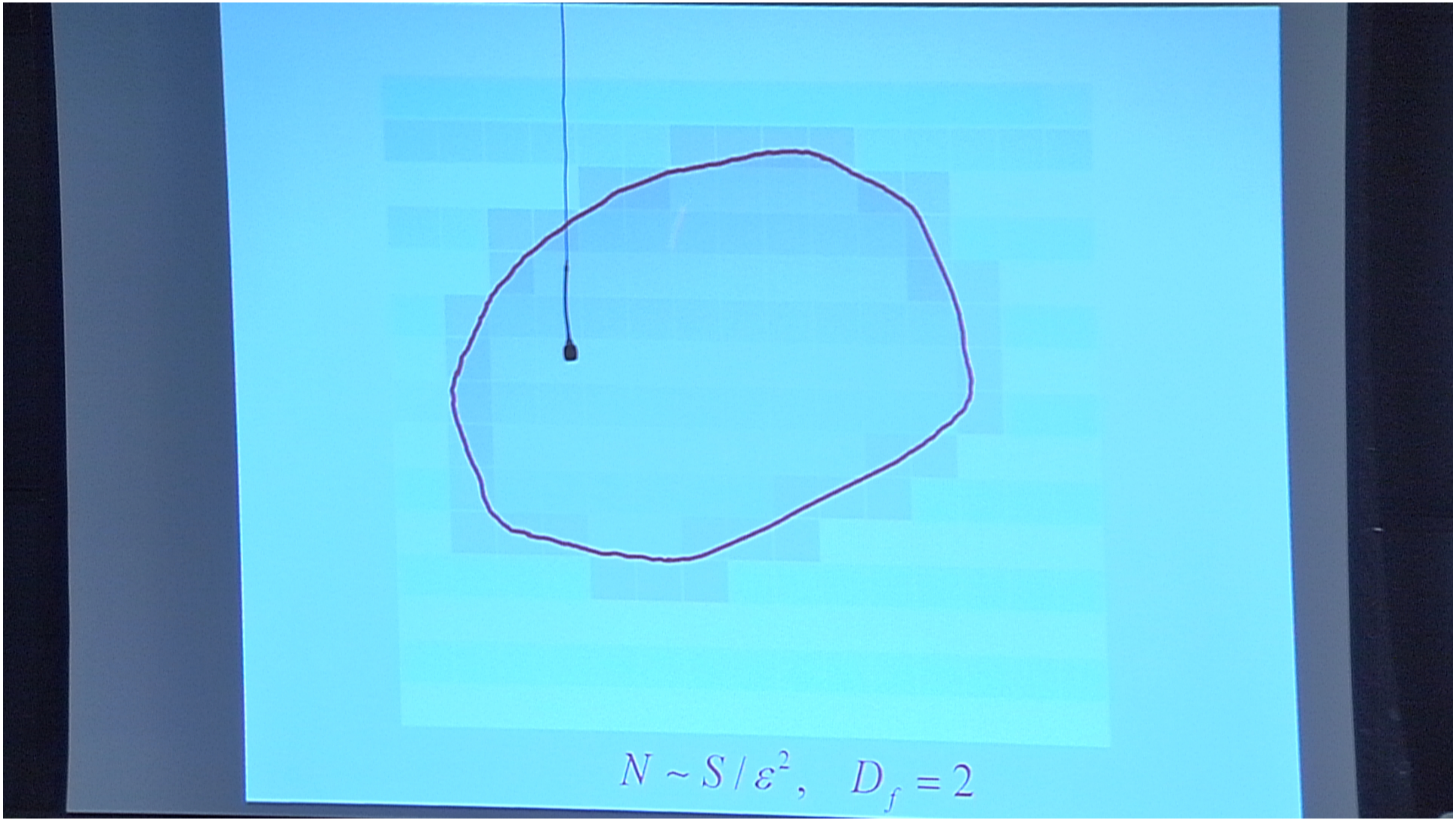
Denote by $N(\varepsilon)$ a number of square stripes of size ε, which is required to cover a basin boundary.

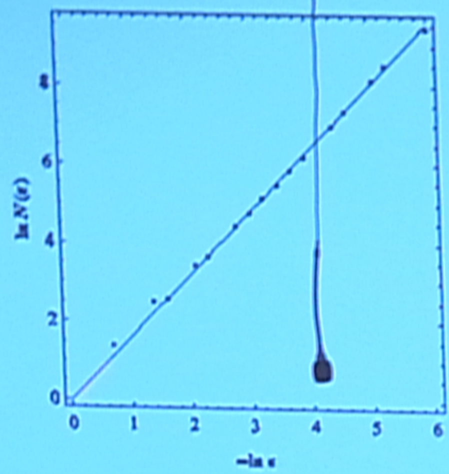
Each of the stripes must contain at least 2 different colors. The box-counting fractal dimension D_f is

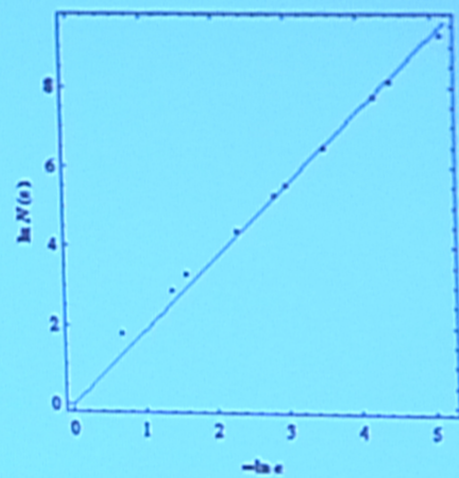
$$D_f = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon^{-1}}, \quad 1 \le D_f < 2$$

58


$N \sim L / \varepsilon, \quad D_f = 1$

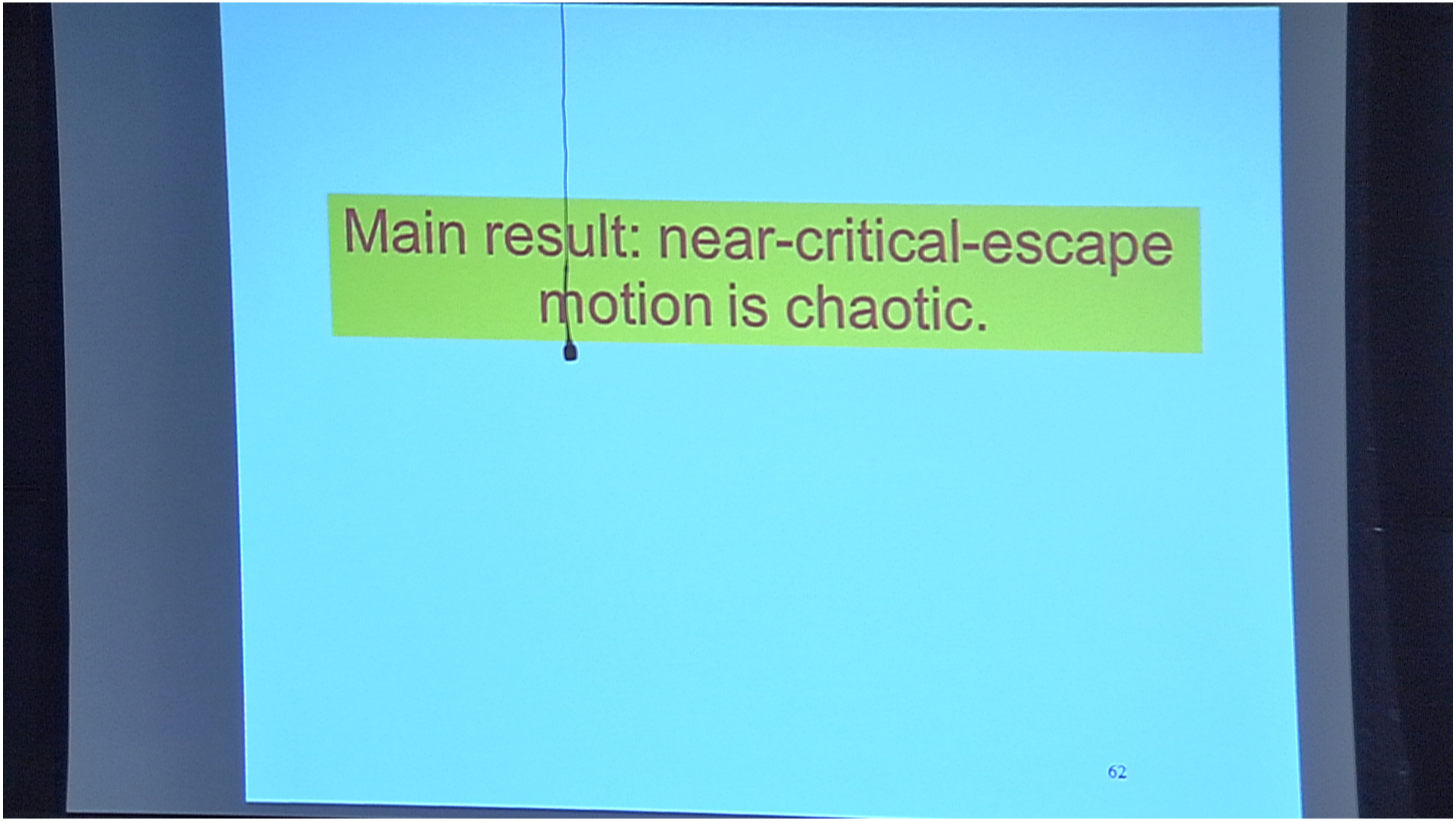

$N \sim S/\varepsilon^2, \quad D_f = 2$





The box counting dimension. Plots of $\ln \varepsilon$ vs. $\ln(1/\varepsilon)$.
Left plot for $l > 0$, right one for $l < 0$.

$$D_f \approx 1.60, l > 0; \quad D_f \approx 1.85, l < 0$$


Main result: near-critical-escape motion is chaotic.
62

(1) For a black hole of mass M in a magnetic field B its effect on charged particle motion is strong when the dimensionless parameter $b = \frac{qGMB}{mc^4}$ is large. This seams to be the case for realistic astrophysical black holes.

(2) Magnetic field makes position of the ISCO closer to the gravitational radius. Efficiency of energy release and period for ISCO particles strongly depend of the parameter b.

(3) Center-of-mass energy for collision of a free falling particle (photon) and a charged particle revolving near a magnetized black hole can be (at least formally) large ($\sim b^{1/4}$).

(4) Near-critical-escape motion out of the equatorial plane is chaotic and basin of boundaries plots have fractal structure.

(5) Kα line broadening as a possible test of magnetic field near BH

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