Title: Fractional statistics in two-dimensions: Anyon there?

Date: Nov 06, 2013 02:00 PM

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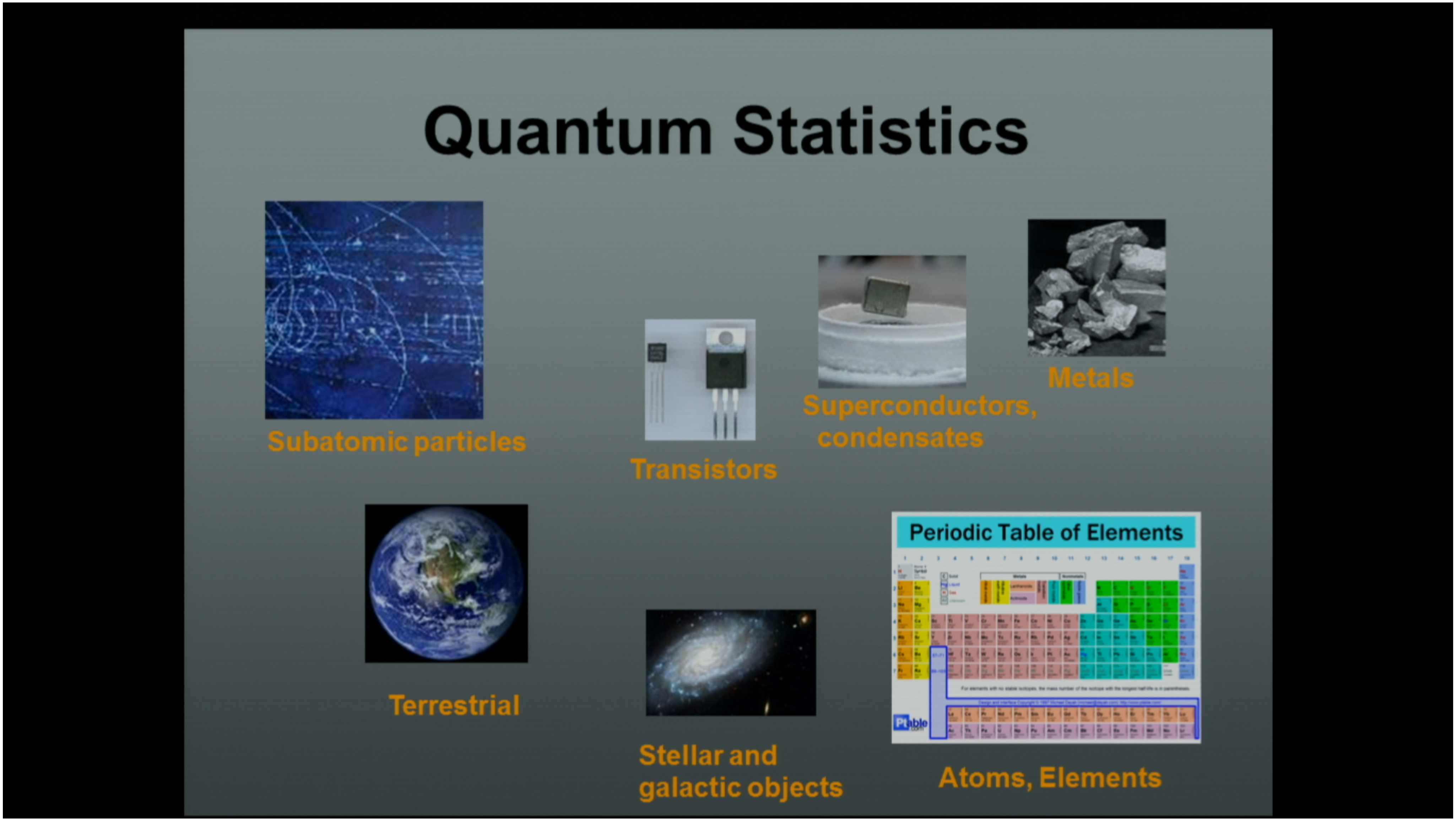
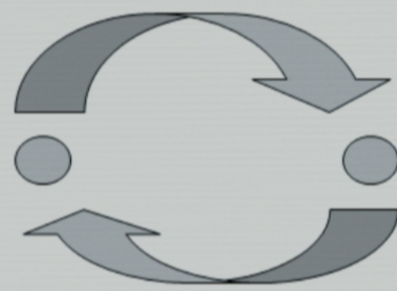
Abstract: A fascinating aspect of the two dimensional world is the possible existence of anyons, particles which obey 'fractional' statistics different from fermionic and bosonic statistics. In this colloquium, following an introduction to fractional particles in the context of quantum Hall systems, some of the tantalizing experiments for detecting the fractional charge of these particles will be described.
Probes of fractional statistics in these systems will be discussed, drawing from analogies with the bosonic behavior of light studied by Hanbury Brown and Twiss in the 1950's as well as in the more recent beam-splitter settings in quantum optics. Finally, the exciting prospect of detecting fractional statistics in the context of superconductors will be explored.


Quantum Statistics
Subatomic particles
Transistors
Superconductors, condensates
Metals
Terrestrial
Stellar and galactic objects
Periodic Table of Elements
Atoms, Elements

Quantum Statistics - Bosons and Fermions

3 spatial dimensions: Only identical particles that are symmetric (bosons) or anti-symmetric (fermions) under exchange allowed.



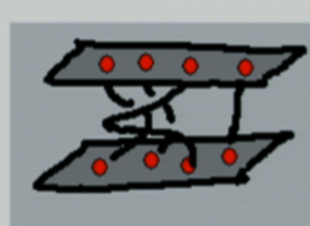
$$\psi(\vec{r}_1, \vec{r}_2) = \pm\psi(\vec{r}_2, \vec{r}_1)$$

(+ bosons ; - fermions)

Two-dimensions: Anyons

Abelian

$$\psi(\vec{r}_1, \vec{r}_2) = e^{\pm i\pi\alpha}\psi(\vec{r}_2, \vec{r}_1)$$



Non-Abelian

Focus

Previous Story / Next Story / July - December 2005 Archive

Phys. Rev. B 72, 075342
(issue of August 2005)
Phys. Rev. Lett. 95, 176402
(issue of 21 October 2005)
Titles and Authors

2 November 2005

Anyon There?

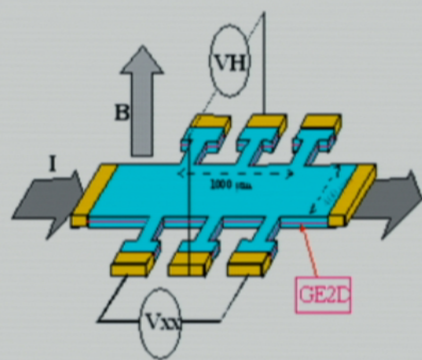
Ordinarily, every particle in quantum theory is neatly classified as either a boson—a particle happy to fraternize with any number of identical particles in a single quantum state—or a fermion, which insists on sole occupancy of its state. Now, in the August *Physical Review B*, a team shows that certain "quasiparticles" associated with electrons are somewhere in between. In the 21 October PRL, another team proposes a different experimental technique that should reveal these properties in an independent way. The new data confirm previous suspicions that states can exist in this quantum no-man's-land, and the proposed experiments could make the case even stronger.

Hall of Quantum Effects. Four voltage "gates" on this semiconductor surface created a central disk with "quasiparticles" having one-fifth of an electron's charge (red) surrounded by a ring of one-third charge quasiparticles (blue). Measurements revealed that the quasiparticles are neither fermions nor bosons.

Fermions must be alone in their quantum states, while bosons can be among an unlimited number of peers. Almost 30 years ago researchers proposed a third category, "anyons," where a limited number of particles could inhabit a single state [1]. No one has observed this property directly, but many researchers believed that the strange state of electrons observed in the 1980s in the so-called fractional quantum Hall (FQH) effect qualifies as an anyon.

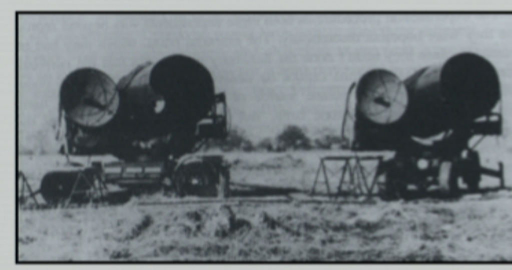
Quantum Hall System: Playground for anyons?

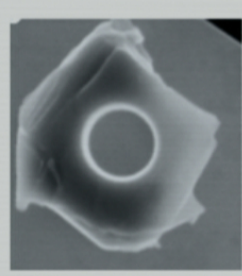
In this talk….



Fractional particles in Quantum Hall systems

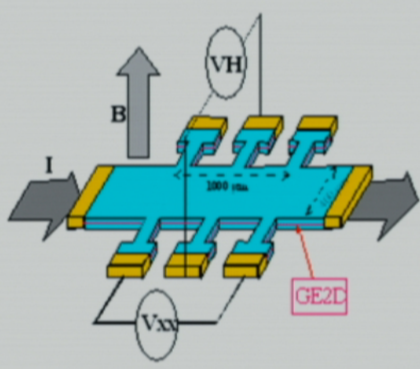
Signatures of quantum statistics HBT correlators; Beam splitter

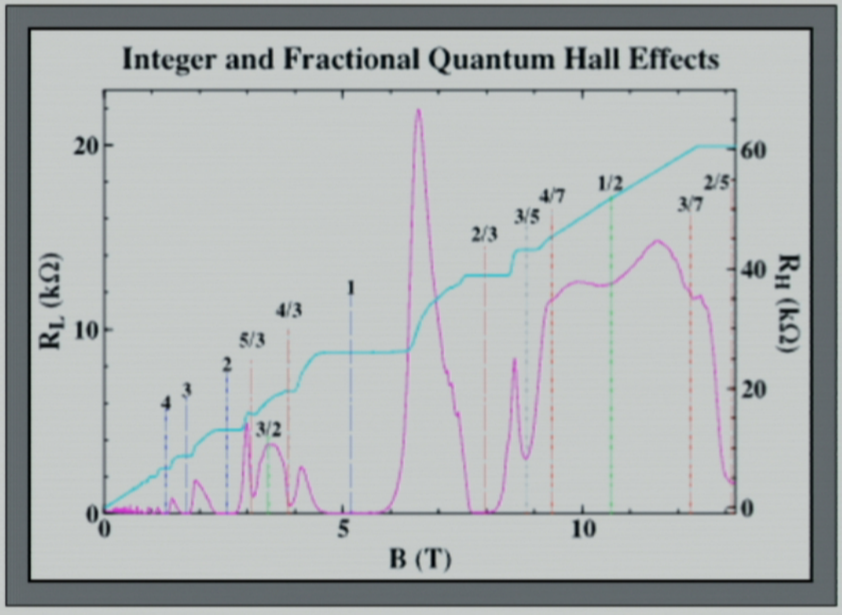




Anyons in superconducting rings

Fractional quantum Hall system



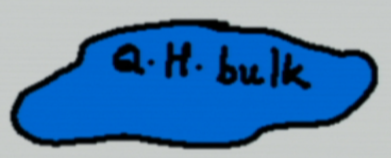


For e.g.,
D. C. Tsui et al, 1982

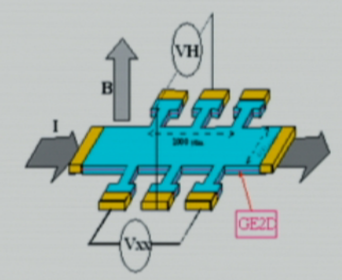
Fractional quantum Hall system

2-dimensional electron gas in strong magnetic field; Interactions + disorder

Incompressible dissipationless bulk fluid



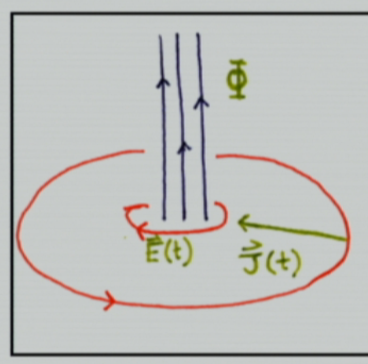
Laughlin state: Filling $\nu = 1/(2n+1)$
Quantized Conductance – $\nu e^2/h$



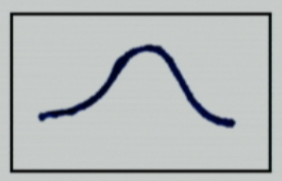
Breakdown of Fermi-liquid theory: Absence of Landau quasiparticles

Fractional quasiholes/particles

Threading extra flux quantum h/e



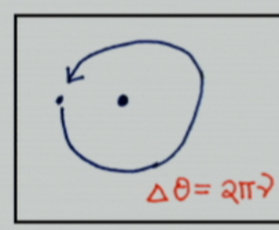
Fractional Charge



$$e^{*} = \nu e$$

Fractional Statistics

$e^{\pm i\pi\nu}$
under exchange

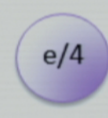


B. I. Halperin, 1984; D. Arovas et al, 1984
F. D. M. Haldane, 1991; G. S. Jeon et al 2003

Non-Abelian quasiparticles

Quantum Hall (ν=5/2)

e/4 fractional quasiparticles



Majorana mode

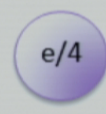
Upon exchange

$$\nu_1 \longrightarrow \nu_2$$
$$\nu_2 \longrightarrow -\nu_1$$

Non-Abelian quasiparticles

Quantum Hall (ν=5/2)

e/4 fractional quasiparticles

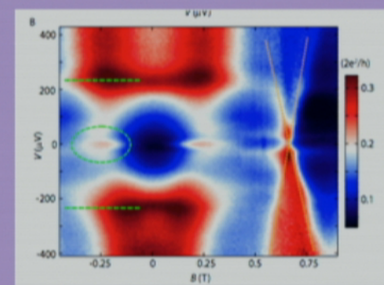


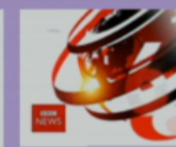
Majorana mode

Upon exchange

$$\nu_1 \longrightarrow \nu_2$$
$$\nu_2 \longrightarrow -\nu_1$$

Majorana particle glimpsed in lab
April 2012





Mourik et al.,
Science (2010)
and more…..

Topological Superconductor

Quantum Hall (ν=5/2)

e/4 fractional quasiparticles

e/4

Majorana mode

Upon exchange

$$\nu_1 \longrightarrow \nu_2$$
$$\nu_2 \longrightarrow -\nu_1$$

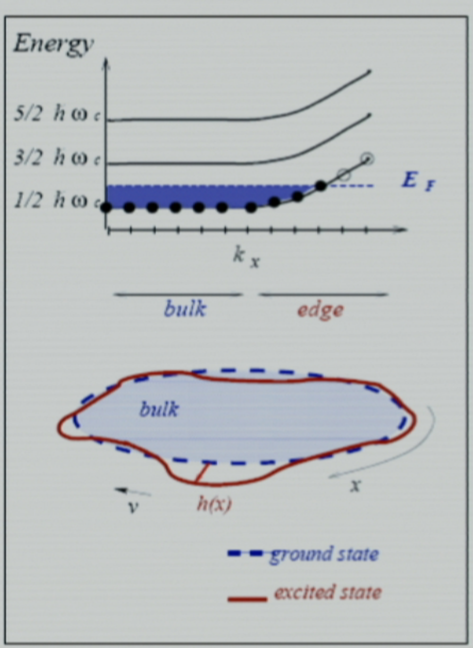
Quantum Hall edge states

Gapless Edge Excitation

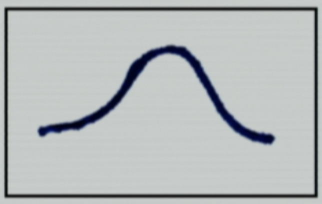
Chiral Luttinger liquid

$$L = -\frac{1}{4\pi\nu}\partial_x\varphi(\partial_t\varphi - \partial_x\varphi)$$

$$\delta\rho \sim \partial_x\phi$$



Edge-state Quasiparticles

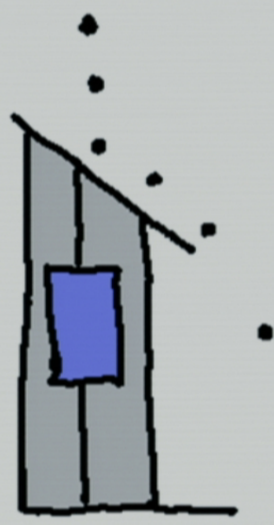


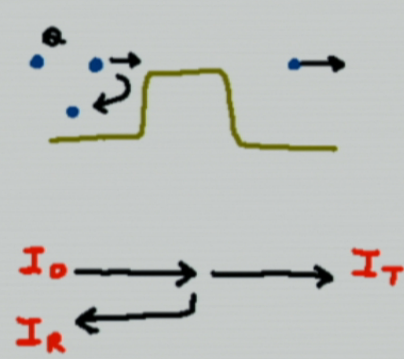
Charge νe, fractional statistics

$$\psi^{\dagger}(x) = \kappa e^{-i\phi(x)}$$

For e.g., X. G. Wen, PRB 41, 12838 (1990) ; Adv. Phys. 44, 405 (1995)

Measuring fractional charge: Shot noise



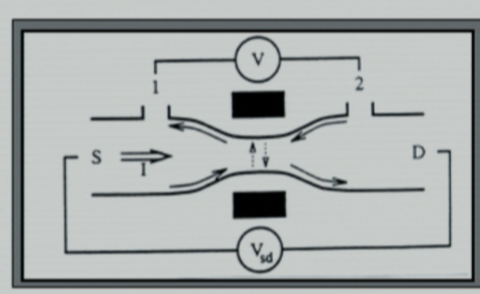


Noise:
Current-current correlation

$$S_I(\omega \to 0) = Q\langle I_B \rangle$$

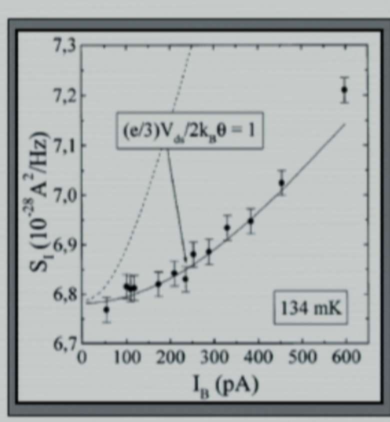
Q – Particle quantum (m,e, etc)
(Weak backscattering, T=0)

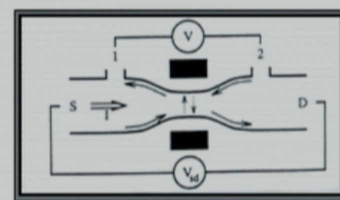
In the Hall system:



C. Kane and M. P. A. Fisher, PRL 72, 724, 1994

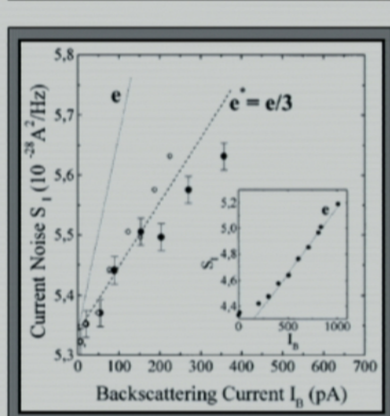
Shot Noise: Experiment

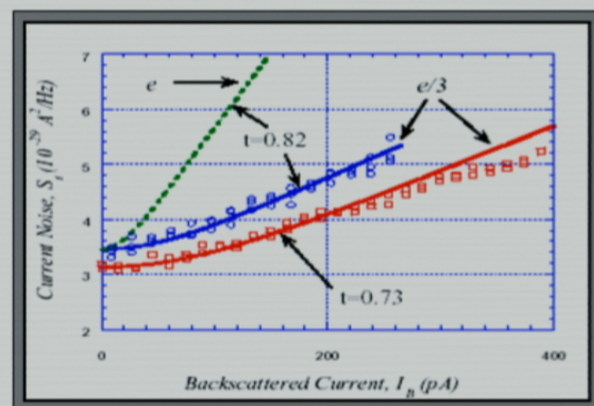




$$S_I(\omega \to 0) = e^* \coth(\beta e^* V/2)\langle I_B \rangle$$

$$e^* = \nu e$$

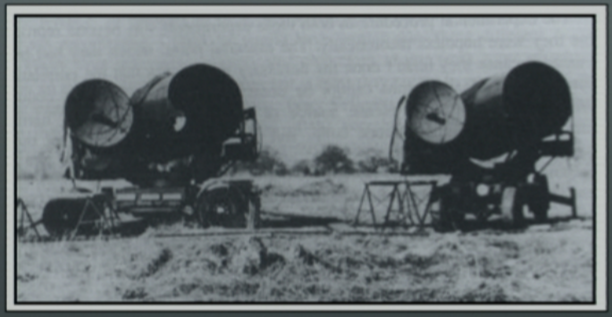




M. Reznikov et al., Nature 399, 238, 1999

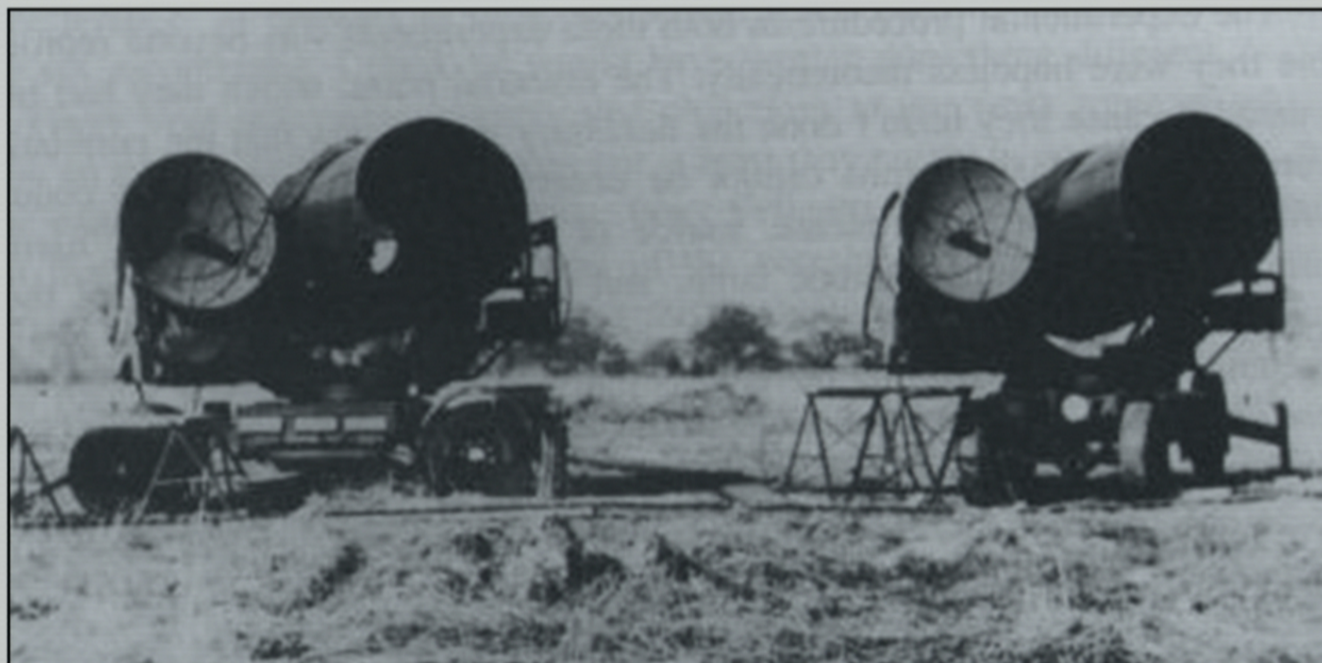
L. Saminadayar et al., PRL 79, 2526, 1997

Signatures of Quantum Statistics



Hanbury-Brown Twiss ideas

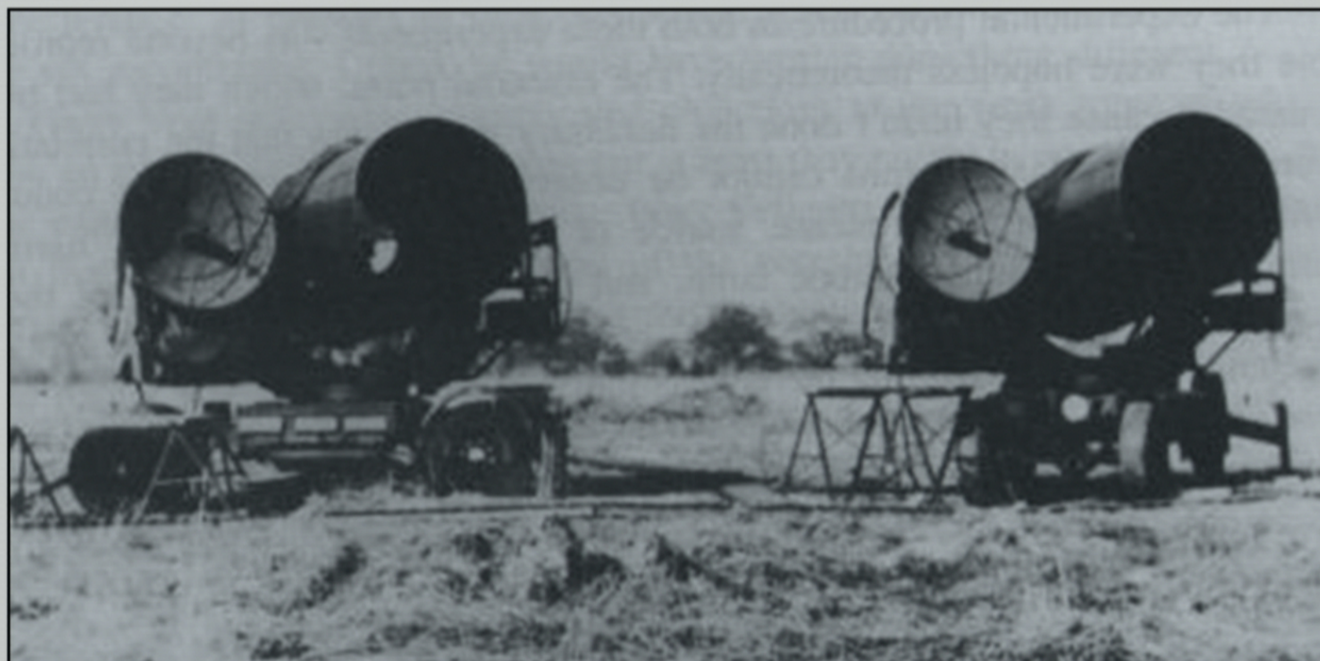
Radio Interferometry : Intensity- intensity correlations



Boffin – Hanbury Brown

Hanbury-Brown Twiss ideas

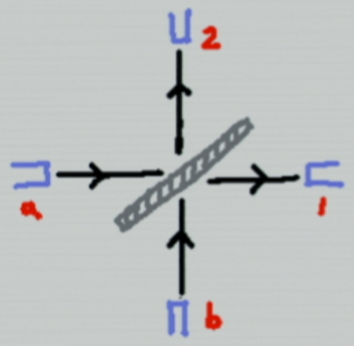
Radio Interferometry : **Intensity- intensity correlations**



Boffin – Hanbury Brown

The Hanbury Brown-Twiss set-up - Quantum Optics

Prob. of 1 particle at detector 1:



$P(1,i)$

(HBT, Nature 177, 27 (1956); E. Purcell, Nature 178, 1449; G. Baym, Act.Phys. Pol B 29,1,etc)

The Hanbury Brown-Twiss set-up - Quantum Optics

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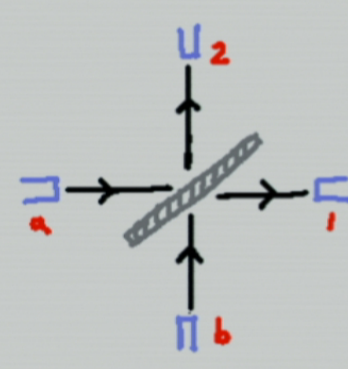


$$P(1,i)$$

Prob. of 1 particle in detector 1, and one in detector 2

$$P(1,2;i,j) = P(1,i)P(2,j) + P(2,i)P(1,j) + exchange$$

Two-particle correlation function (propagator)

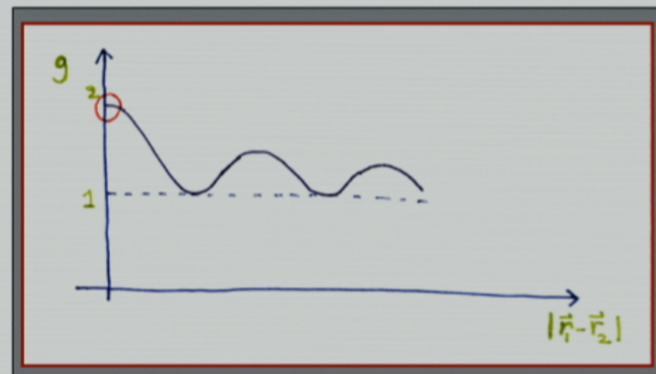
$$\langle \psi_i^\dagger(x)\psi_j^\dagger(x'')\psi_j(x''')\psi_i(x')\rangle$$

(HBT, Nature 177, 27 (1956); E. Purcell, Nature 178, 1449; G. Baym, Act.Phys. Pol B 29,1,etc)

2-particle correlation function

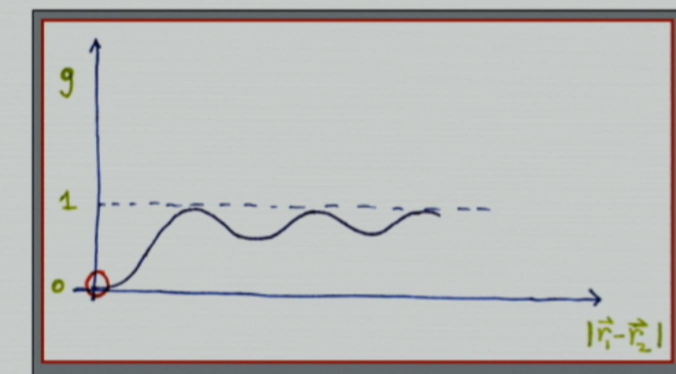
$$g(r_1, r_2) = N(N-1)\int dr_3 \ldots dr_N \,|\, \Psi(r_1, r_2, \ldots . r_N)\,|^2$$

Statistics



Bosons

E.g. Photons from different sources



Fermions

E.g. Spinless non-interacting electrons

2-particle correlation function

$$g(r_1, r_2) = N(N-1)\int dr_3 \ldots dr_N \,|\, \Psi(r_1, r_2, \ldots r_N) \,|^2$$

Statistics

Characteristics of System

Oscillations

Position/Momentum

Energy/Time

Decay

E.g. Spinless non-interacting electrons

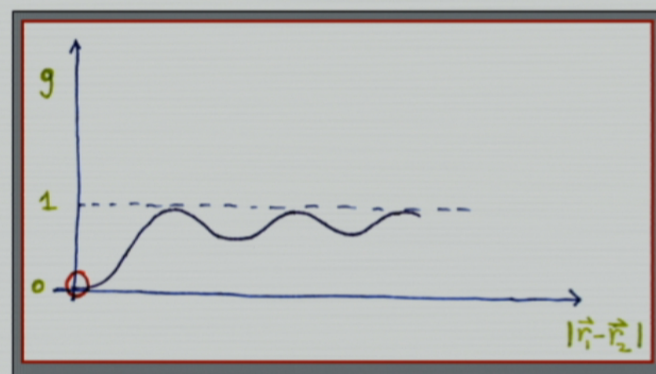
$$g \sim [1-(\frac{\sin k_F r}{k_F r})^2]$$ (for one-dimension)

HBT Correlation Regime

$$\Delta x << (\Delta k)^{-1}$$

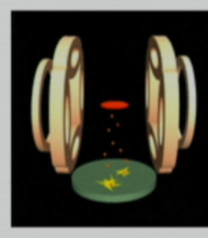
$$\Delta t << \omega^{-1}$$

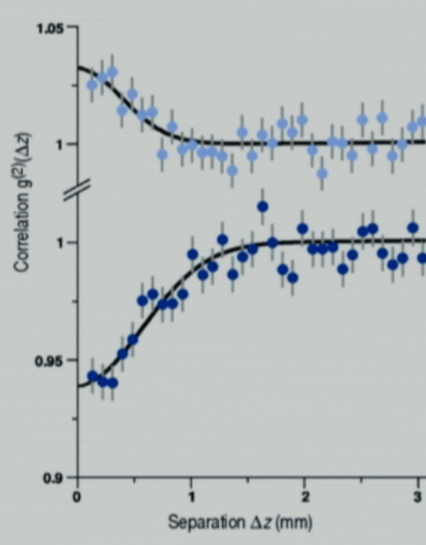
$$kT << \Delta E$$



Manifestations

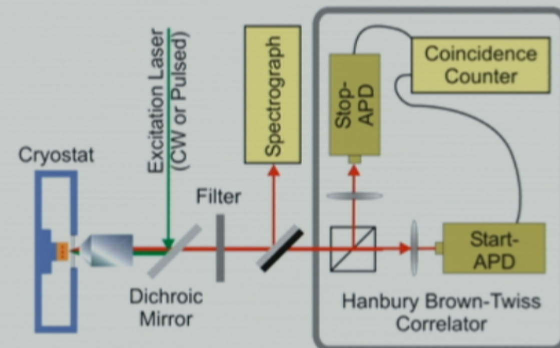
Correlations in trapped helium





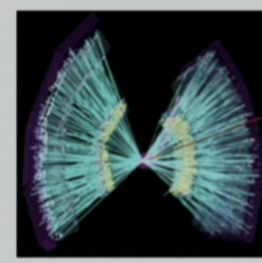
Groupe d'Optique Atomique, France

Quantum Optics



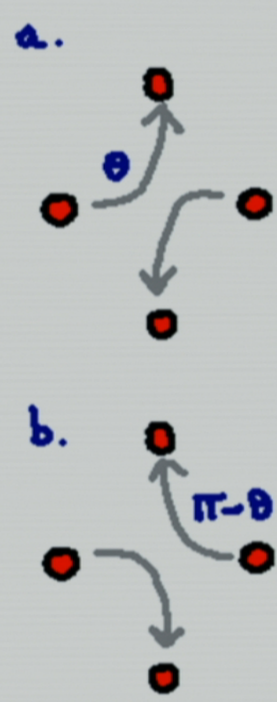
Nano Optics, Humboldt Univ., Germany

Scattering cross-sections



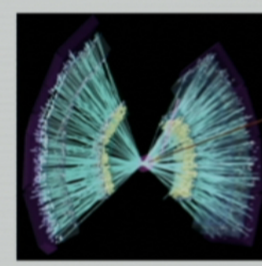
Nuclear physics group, UIUC

Scattering cross-sections

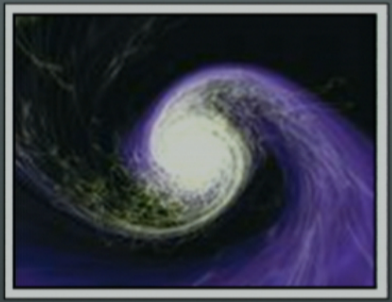


Indistinguishable Particles:
Paths a and b are both present.
Cannot be distinguished

Identical fermions:
Scattering cross-section $|f(\theta) - f(\pi - \theta)|^2$
Vanishes at 90°

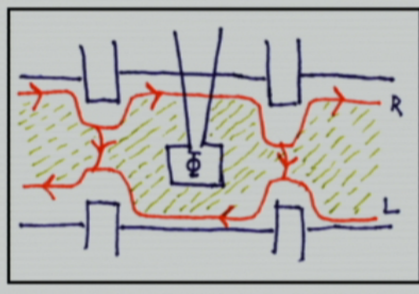


Quantum Statistics - Anyon there?



Proposals for measuring fractional statistics

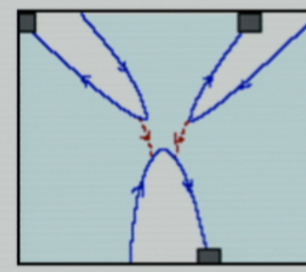
Anomalous Aharonov-Bohm period



Chamon et al., 1997

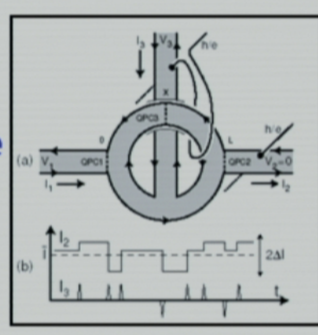
Partitioning Noise

Safi et al., 2001; SV, 2003
E.Kim et al., 2005;

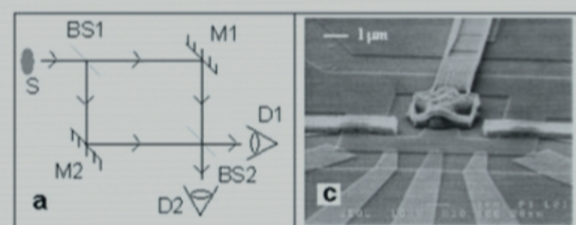


Telegraph Noise

C. L. Kane, 2003



Mach-Zehnder Interferometry



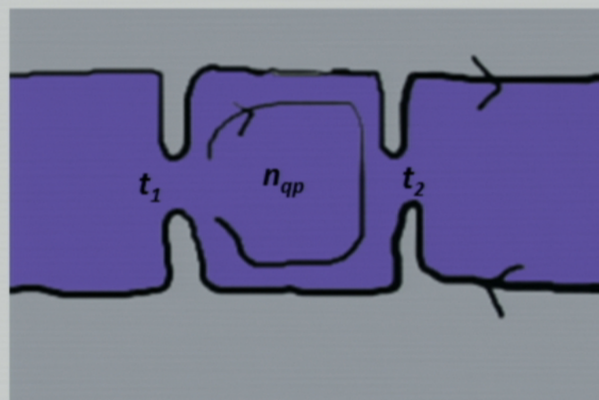
Ji et al.,2003

Some related experiments

Y. C. Chung et al., 2003; F. E. Camino et al., 2005

Proposal for detecting non-Abelian statistics

Quantum Hall ν=5/2



Interferometry

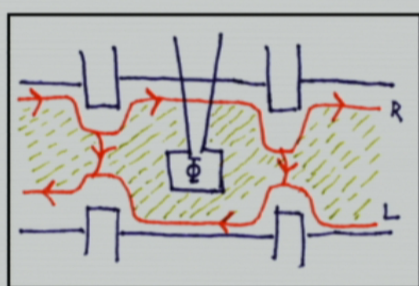
$$\nu_1 \longrightarrow \nu_2$$
$$\nu_2 \longrightarrow -\nu_1$$

$$\sigma_{xx}^{int} \propto Re[t_1^* t_2 e^{i\phi} \langle \psi_0 | U_1^{-1} U_2 | \psi_0 \rangle]$$

For e.g., E. Fradkin et al., 1998; S. Das Sarma et al, 2005;

Proposals for measuring fractional statistics

Anomalous Aharonov-Bohm period



Chamon et al., 1997

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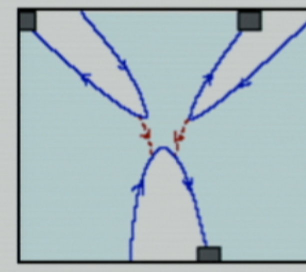
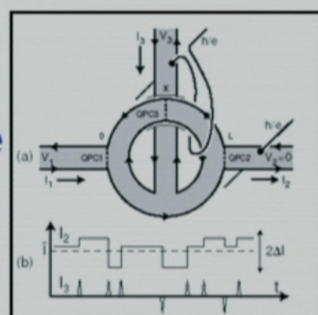
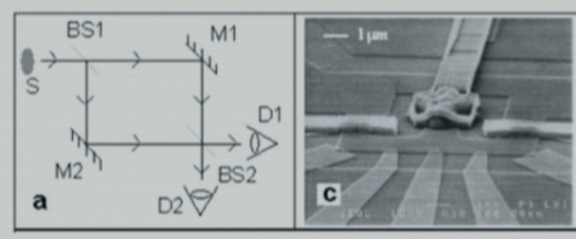


Telegraph Noise

C. L. Kane, 2003



Mach-Zehnder Interferometry



Ji et al.,2003

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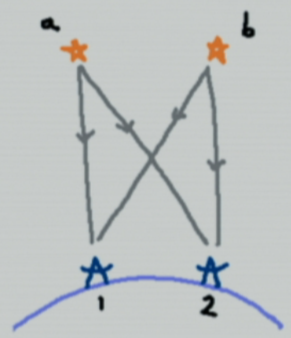
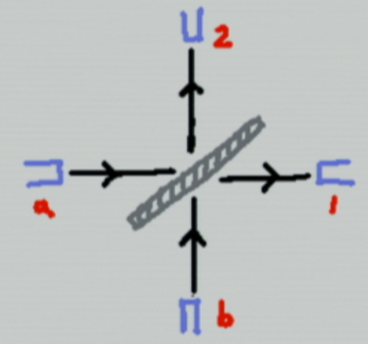
Quantum statistics: two-particle properties

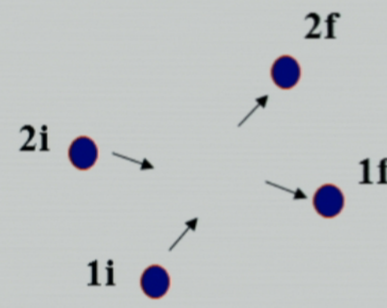
Hanbury-Brown Twiss



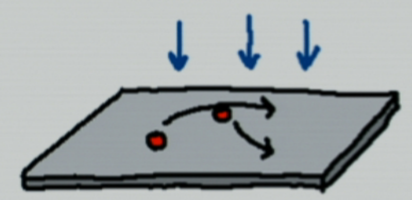
Beam-splitters



Scattering processes

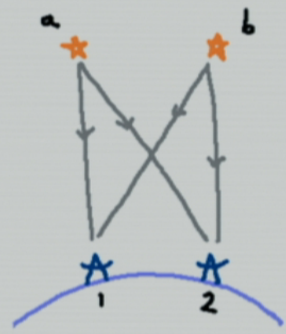


Signatures of statistics in bunching behavior, interference and angular dependence

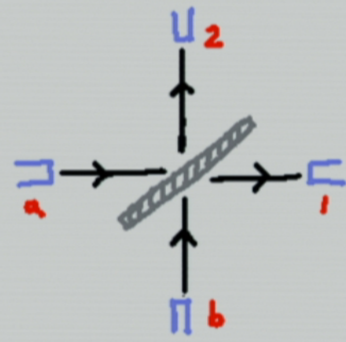


Quantum statistics: two-particle properties

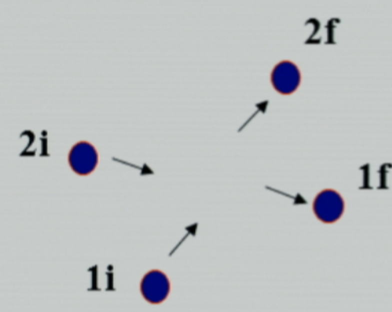
Hanbury-Brown Twiss



Beam-splitters

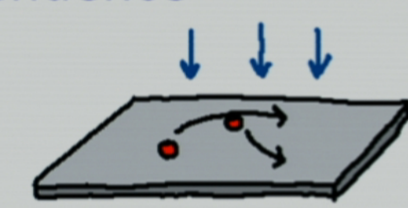


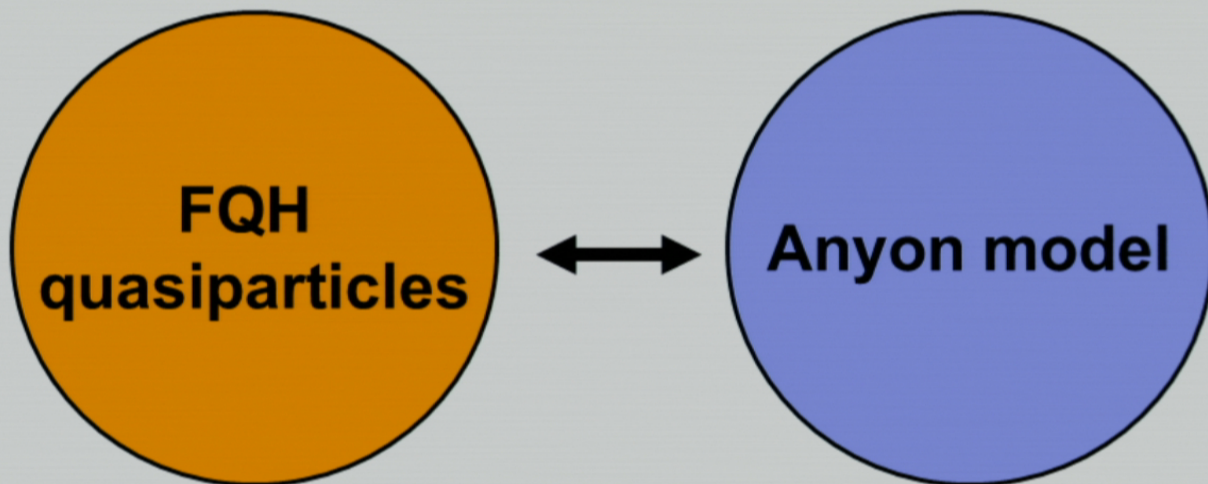
Scattering processes



Signatures of statistics in bunching behavior, interference and angular dependence

Manifestations in QH bulk?





B. I. Halperin, 1984; R. B. Laughlin, 1987
J. Myerheim in Les Houches series (1999);
J. Jain, Composite Fermions (2007)

Two-anyon model

$$H = \frac{1}{4\mu}\left(P_x + \frac{qB}{c}Y\right)^2 + \frac{1}{4\mu}\left(P_y - \frac{qB}{c}X\right)^2$$
$$+ \frac{1}{\mu}\left(p_x + \frac{qB}{4c}y\right)^2 + \frac{1}{\mu}\left(p_y - \frac{qB}{4c}x\right)^2$$

Center of mass: $(\vec{R}, \vec{P})$

Relative co-ordinates: $(\vec{r}, \vec{p})$

Magnetic field B perpendicular to plane

Anyonic feature

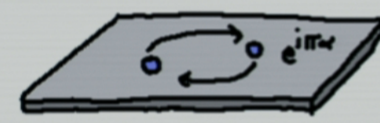
$$\psi(-\vec{r}) = e^{\pm i\pi\alpha}\psi(\vec{r})$$

Two-anyon model

Wavefunctions

$$\psi_p(\vec{r}) = \frac{(4\pi m)^{-1/2}}{\sqrt{\Gamma(2p+\nu+1)}l} \left(\frac{z}{2\sqrt{m}}\right)^{2p+\nu} \exp\,[-\frac{|z|^2}{8ml^2}];$$

$$\psi_n(\vec{R}) = \frac{1}{l\sqrt{m\pi n!}} \left(\frac{Z}{\sqrt{m}}\right)^{n} \exp[-\frac{|Z|^2}{2m}]$$

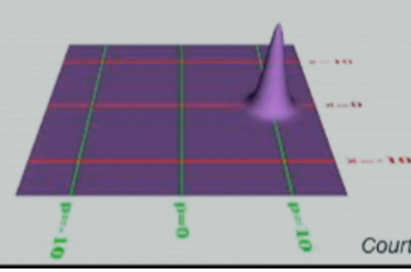


(Symmetric gauge; lowest Landau level; α = ν = 1/m)

Center of mass: $(\vec{R}, \vec{P})$ Relative co-ordinates: $(\vec{r}, \vec{p})$

Analogy: Quantum Optics

Coherent States in Phase Space



Courtesy Denker

Two-anyon LLL Hilbert space

Center of mass:

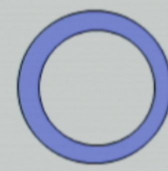
Guiding center coordinates

$$\hat{X} = l(\hat{A} + \hat{A}^{\dagger})/2, \quad \hat{Y} = il(\hat{A} - \hat{A}^{\dagger})/2$$

$$[A, A^{\dagger}] = 1$$

Angular momentum eigenstates

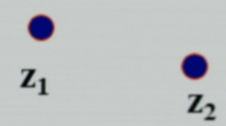
$$\hbar A^{\dagger} A|n\rangle_c = n\hbar|n\rangle_c \qquad \psi_n(\vec{R}) \propto \frac{Z^n}{\sqrt{n!}}$$



Localized (coherent) states

$$|Z\rangle_c = e^{-|Z|^2/2} \sum_{n=0}^{\infty} \frac{(Z^*)^n}{\sqrt{n!}} |n\rangle_c$$

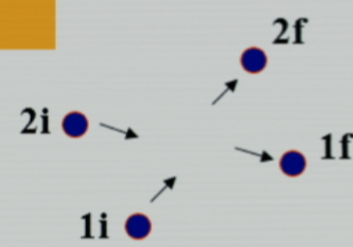
$Z=(z_1+z_2)/2$



Objects of attention

Single particle

$$K_1(\vec{R}_f;\vec{R}_i) = \sum_n \ \psi_n(\vec{R}_f)\ \psi_n^*(\vec{R}_i)\ e^{-E_n\tau/\hbar}$$

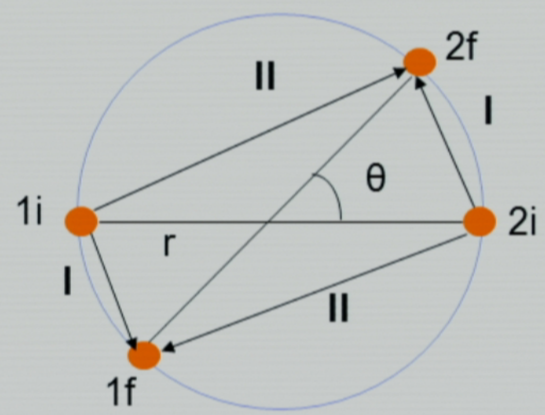


Two particles

$$K_2(\vec{r}_{1f},\vec{r}_{2f};\vec{r}_{1i},\vec{r}_{2i}) = \sum_n \ \psi_n(\vec{R}_f)\ \psi_n^*(\vec{R}_i)\ e^{-E_n\tau/\hbar} \times \sum_p \ \psi_p(\vec{r}_f)\ \psi_p^*(\vec{r}_i)\ e^{-E_p\tau/\hbar}$$

SV et al., 2007

Two-particle propagator in LLL* - fermions and bosons



Destructive Interference

Magnitudes of I and II equal:

$\theta = \pi/2$

Relative phase 0/π:

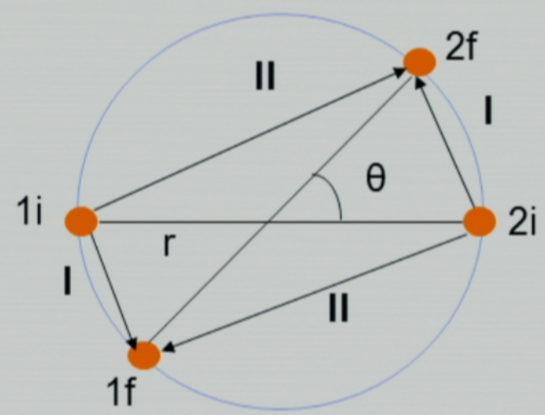
$2Br^2/\Phi_0 = (2n+0/1)/2$

Two-particle Aharonov-Bohm + quantum statistics

$$K_2(z_{1f}, z_{2f}; z_{1i}, z_{2i}) = K_1(z_{1f}; z_{1i})K_1(z_{2f}, z_{2i}) \mp K_1(z_{2f}; z_{1i})K_1(z_{1f}, z_{2i})$$

* LLL - Lowest Landau level

Two-particle propagator in LLL* - fermions and bosons



Destructive Interference

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* LLL - Lowest Landau level

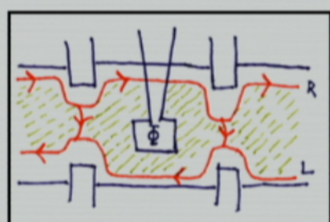
Two-particle propagator - anyons (A ν angle on fractional statistics)

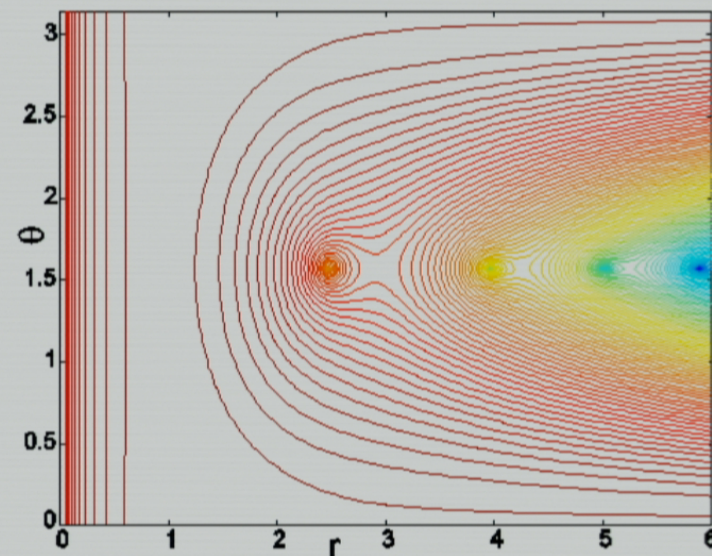
Destructive Interference

Similar geometric picture:

Also at $\theta = \pi/2$

$2Br^2/\Phi_0 \approx (2n+1+\nu)/2$





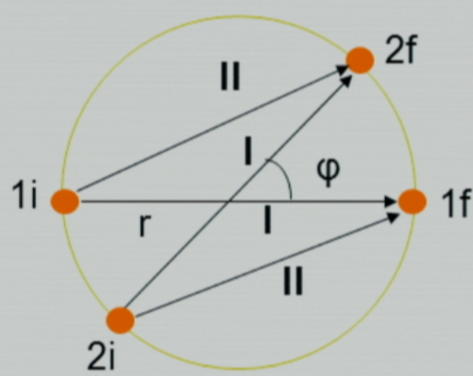
Two-particle Aharonov-Bohm + Fractional statistics

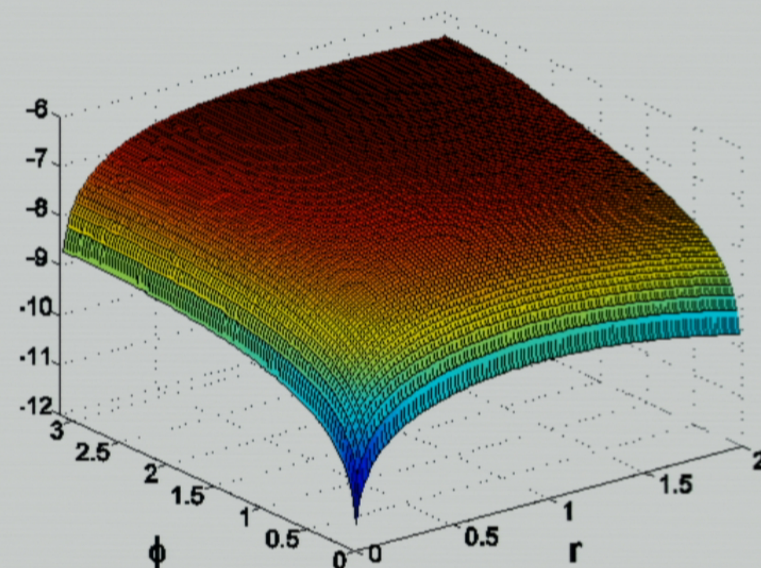
Two-particle propagator – anyons
(A ν angle on fractional statistics)

Exclusion behavior

Haldane, 1991

$$K_2 \sim |\varphi|^{2\nu},\ \varphi \to 0$$

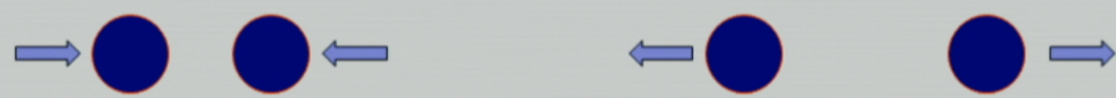




For 1i = 2i = 0, 1f = 2f = r, $K_2 \sim |r|^{2\nu}$, $r \to 0$

Bunching properties

Bosons Fermions

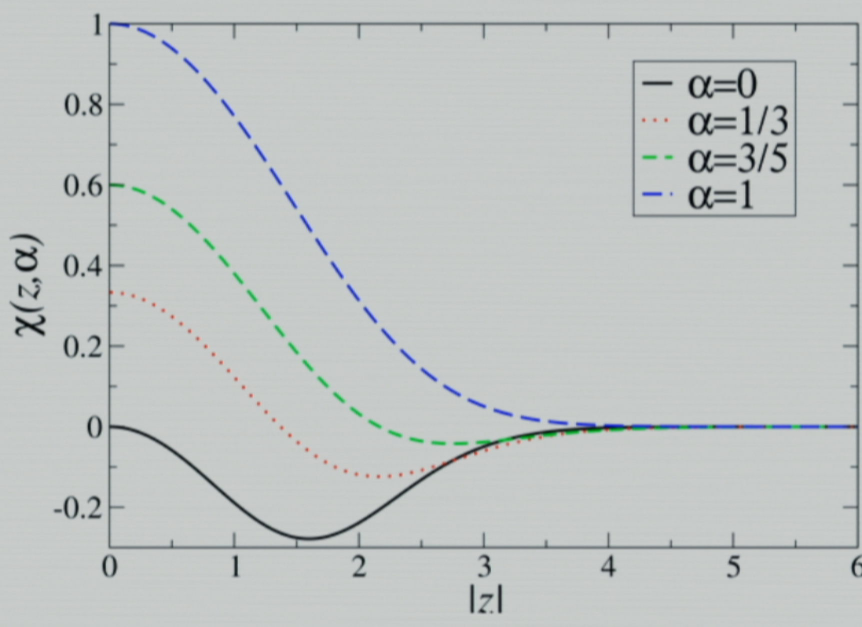


Bunching parameter

$$\chi(|z|,\alpha) \equiv \frac{1}{4l^2}[\langle z|\hat{r}^2|z\rangle_\alpha - \langle z|\hat{r}^2|z\rangle_d]$$

$$\langle \hat{r}^2\rangle \equiv \langle \hat{x}^2 + \hat{y}^2\rangle$$

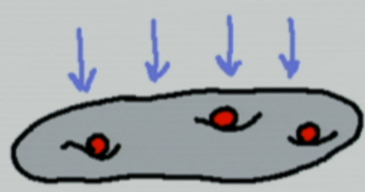
Bunching parameter



$$\chi = (|z|^2/4)[\coth(|z|^2/4) - 1] \quad \text{Fermions}$$

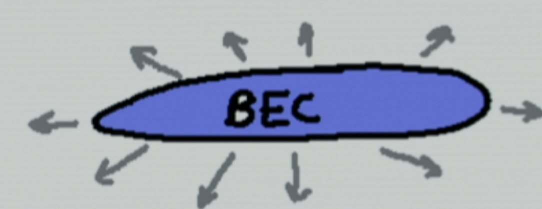
$$= (|z|^2/4)[\tanh(|z|^2/4) - 1] \quad \text{Bosons}$$

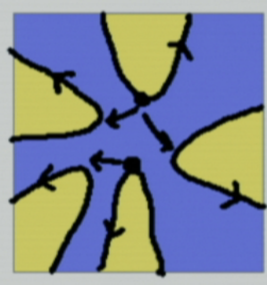
Probing Correlations



Localized quantum Hall bulk quasiparticles

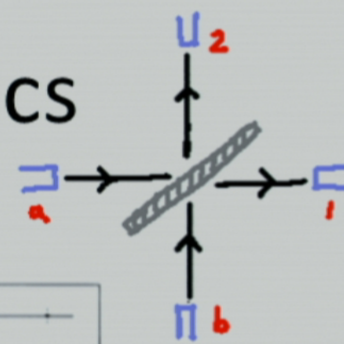
Correlations in ultracold gases



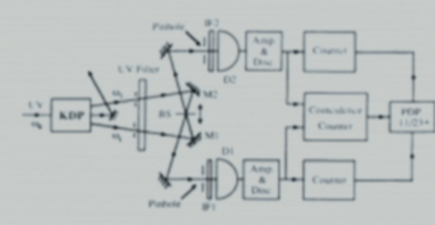


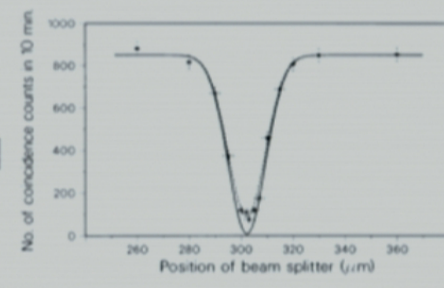
Tunneling into multiple edge states

Beam Splitters and Statistics



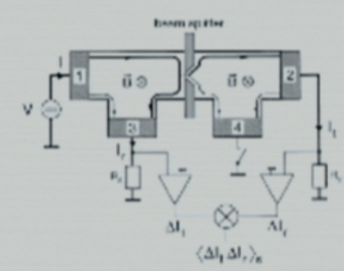
Coincidence measurements for photons

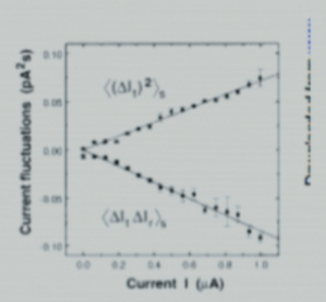




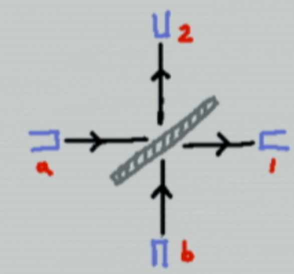
C. K. Hong at al., 1987

Current correlation measurements for electrons

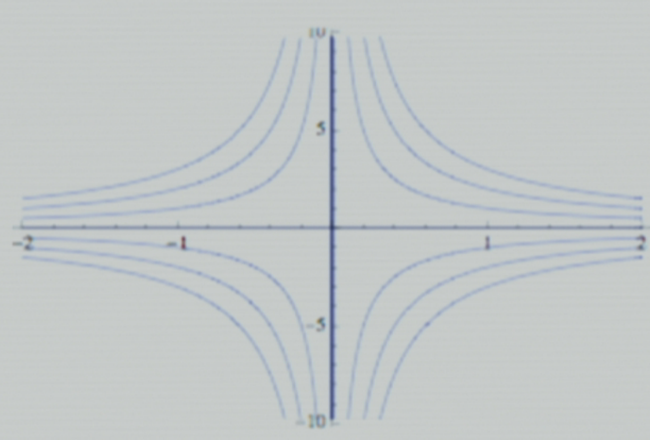




M. Henny et al; W. D. Oliver at al. 1999

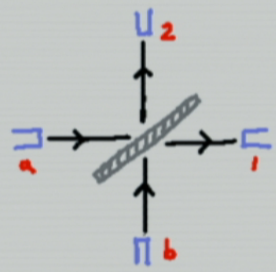


QH beam splitter – Saddle potential

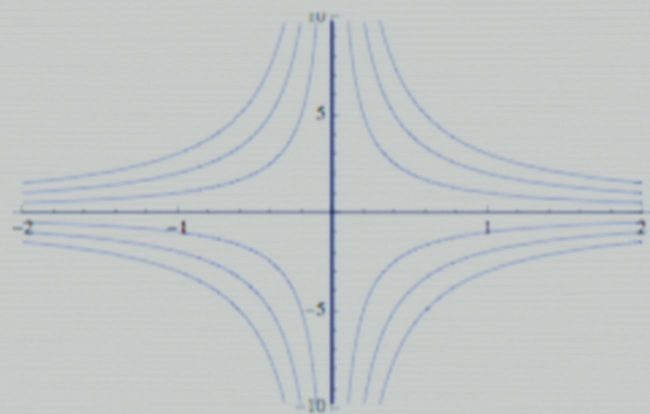


$$\hat{H}_s = \sum_{\gamma=1,2} U \hat{x}_\gamma \hat{y}_\gamma$$

Separable,
Preserves anyon b.c.



QH beam splitter – Saddle potential



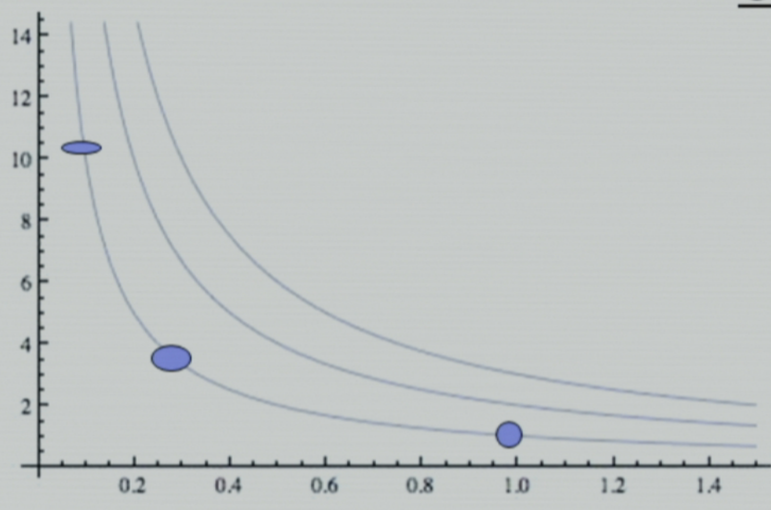
$$\hat{H}_s = \sum_{\gamma=1,2} U \hat{x}_\gamma \hat{y}_\gamma$$

Separable,
Preserves anyon b.c.

LLL projected Hamiltonian

$$\hat{H}_s^P = \frac{1}{2} i U l^2 [\hat{A}^2 - (\hat{A}^\dagger)^2] + 2Ul^2 \hat{c}$$

Saddle dynamics



C.o.m motion

$$|Z\rangle_c = \exp(Z\hat{A}^\dagger - Z^*\hat{A})|0\rangle_c$$
$$|Z(t)\rangle_c = e^{-i\hat{H}_s^P t/\hbar}|Z\rangle_c$$

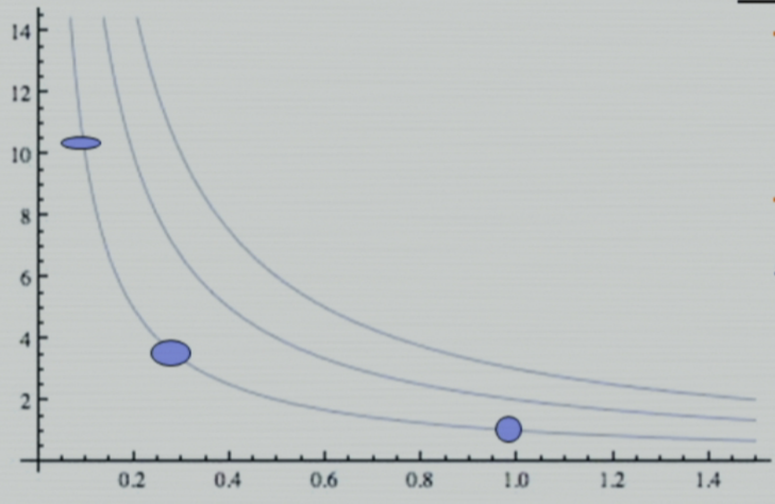
Analogy from qtm optics

Squeeze along $re^{i\phi} \equiv Utl^2/\hbar$

Follow

$$(Xe^{-Utl^2/\hbar}, Ye^{Utl^2/\hbar})$$

Saddle dynamics



C.o.m motion

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Analogy from qtm optics

Squeeze along $re^{i\phi} \equiv Utl^2/\hbar$

Follow

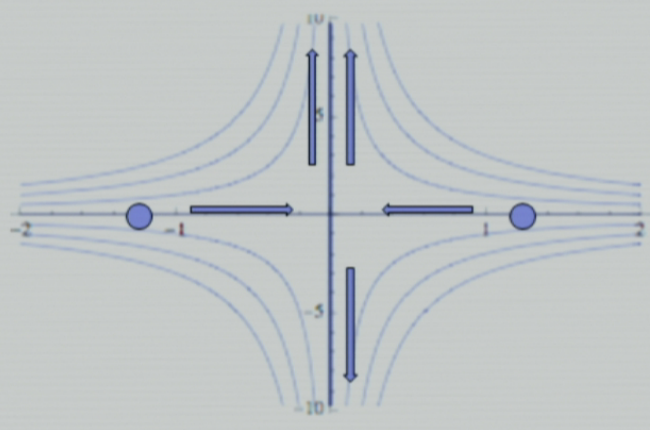
$$(Xe^{-Utl^2/\hbar}, Ye^{Utl^2/\hbar})$$

Relative motion

(Initial amplitude peaked at +z and -z)

$$\hat{x}^2(t) = e^{-2Utl^2/\hbar}\hat{x}^2(0), \hat{y}^2(t) = e^{2Utl^2/\hbar}\hat{y}^2(0)$$

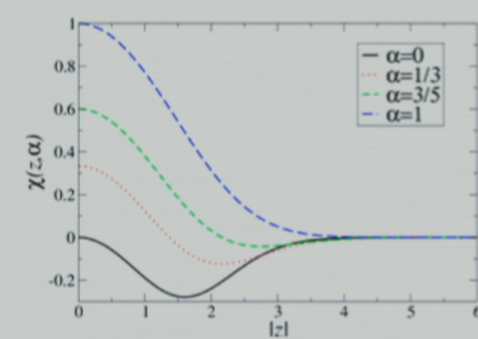
Beam splitter properties

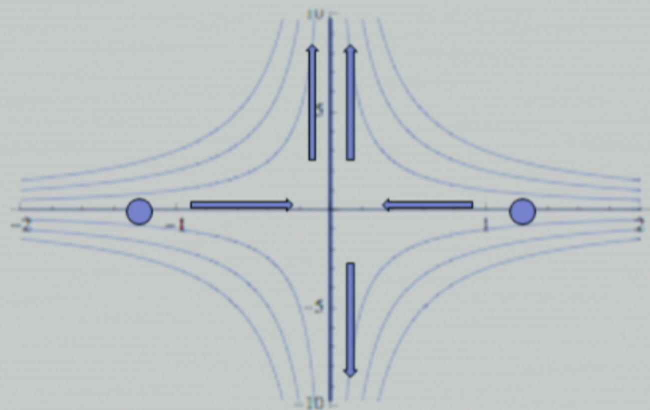


Did the particles go in the same direction or different ones?

Behavior of $\langle y_1 y_2 \rangle$

Beam splitter properties





Did the particles go in the same direction or different ones?

Depends on statistics and bunching parameter

Behavior of <$y_1 y_2$>

$$\langle \hat{y}_1 \hat{y}_2 \rangle = l^2 e^{2Utl^2/\hbar} \left[\mathrm{Im}[Z]^2 - \frac{1}{4}\mathrm{Im}[z]^2 - \frac{1}{2}\chi + \delta \right]$$

QH bulk non-Abelian quasiparticles?

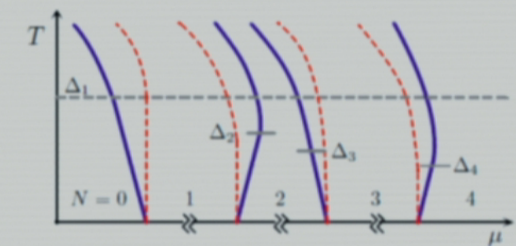
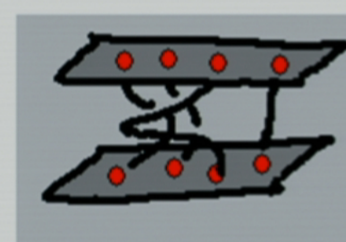
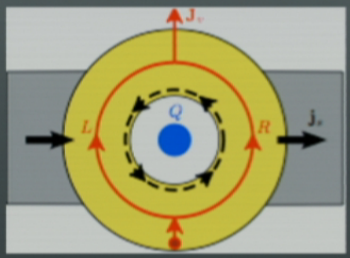


FIG. 1. Cartoon charge stability diagram, showing only peak centres (no broadening). Vertical axis is temperature T; horizontal axis μ is the chemical potential for charged QPs, controlled in experiment by a gate potential. Red dashed lines are for Abelian particles. Blue solid lines correspond to non-Abelian QPs in a tightly confining well. Δ_N is the gap to excited states for N particles, and sets scale for other entropic effects. Notice even-odd effect for non-Abelian anyons.

Ben-Shach et al., 2013



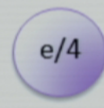
Non-Abelian statistics in superconductor rings



Non-Abelian quasiparticles

Quantum Hall (ν=5/2)

e/4 fractional quasiparticles



Chiral p-wave superconductors

Zero energy state in vortex core

h/(2e) or h/(4e) (One or two component sc)

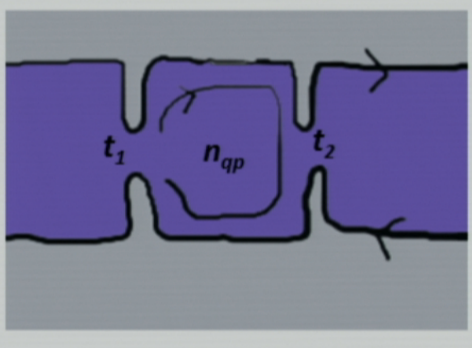
Majorana mode

Upon exchange

$$\nu_1 \longrightarrow \nu_2$$
$$\nu_2 \longrightarrow -\nu_1$$

Non-Abelian quasiparticles: detection

Quantum Hall (ν=5/2)



Interferometry

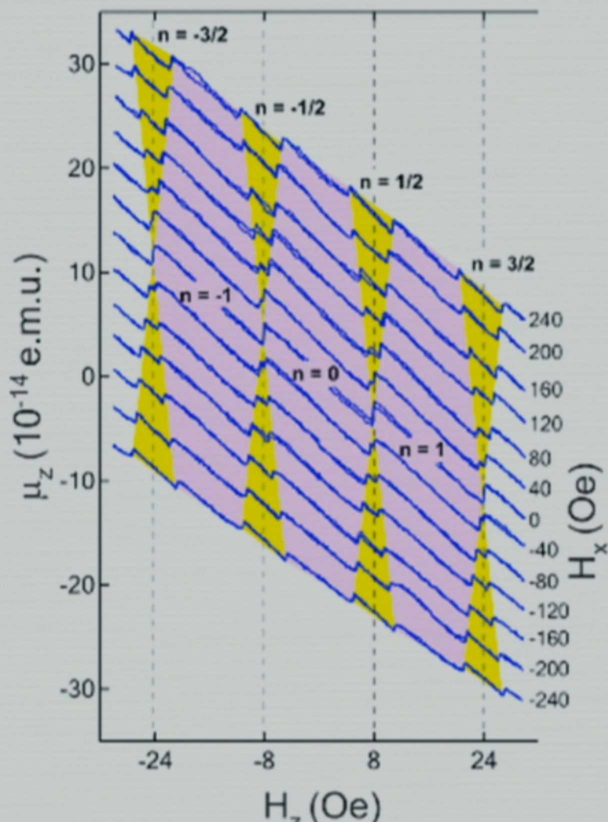
$$\sigma_{xx}^{int} \propto Re[t_1^* t_2 e^{i\phi} \langle \psi_0 | U_1^{-1} U_2 | \psi_0 \rangle]$$

For e.g., E. Fradkin et al., 1998; S. Das Sarma et al, 2005; Stern & Halperin, 2006; P. Bonderson et al., 2006

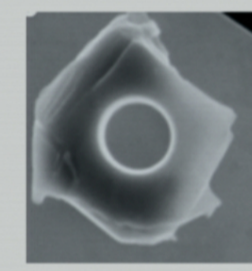
Chiral p-wave superconductors (CpSC)

DUALITY!!?

Measurements from the lab of Raffi Budakian



Strontium Ruthenate (SRO):
Candidate CpSc material



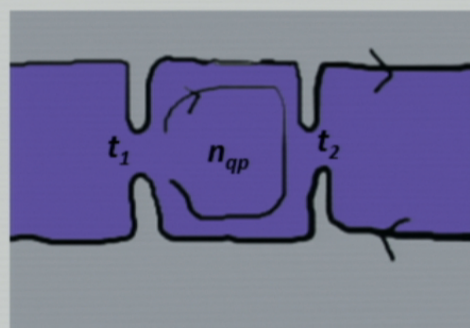
SRO sample –
Mesoscopic ring
Order 1500nm

Magnetic response: Splitting as a function of H_x indicates **Half Quantum Vortices (HQV)**

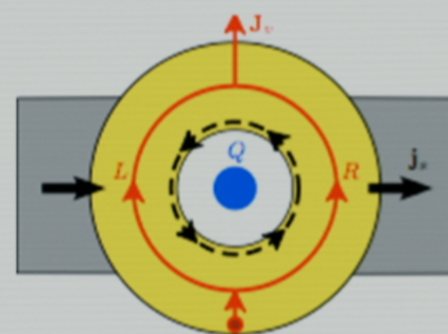
Courtesy R. Budakian
J. Jang et al, 2011

Non-Abelian quasiparticles: detection

Quantum Hall (ν=5/2)



Chiral p-wave superconductors



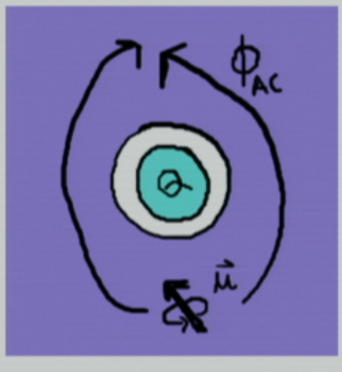
Interferometry

$$\sigma_{xx}^{int} \propto Re[t_1^* t_2 e^{i\phi} \langle \psi_0 | U_1^{-1} U_2 | \psi_0 \rangle]$$

$$J_v^{int} \propto |t_L||t_R| \left[e^{i\varphi} \langle \psi_L | \psi_R \rangle \right]$$

E. Grosfeld et al., 2011

Aharonov-Casher effect



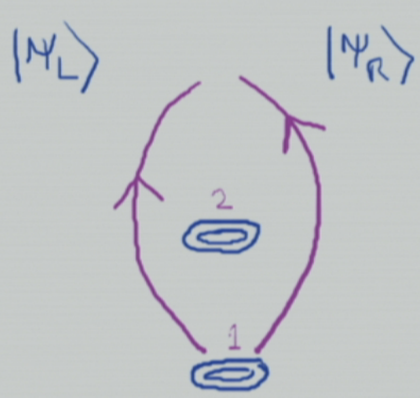
Magnetic moment moving in static electric field – geometric phase:

$$\phi_{AC} = \frac{1}{\hbar c^2} \oint \vec{\mu} \times \vec{E} \cdot d\vec{l}$$

2d vortex encircling charge Q: $\phi_{AC} = \Phi_V Q/\hbar = 2\pi Q/e^*$

Aharonov and Casher, '84; Reznik and Aharonov, '89

Encircling Majorana vortices



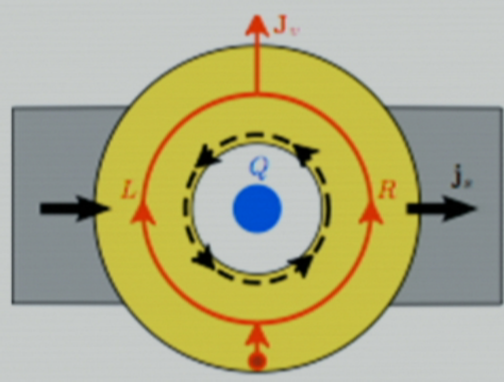
Majorana structure:

$$\begin{array}{l} \nu_1 \rightarrow \nu_2 \\ \nu_2 \rightarrow -\nu_1 \end{array} \longrightarrow \langle \psi_L | \psi_R \rangle = 0$$

Two-component CpSc:

Holds for HQV around HQV as well as around FQV (FQV: can consider one Majorana in each spin component)

Proposed Experiment



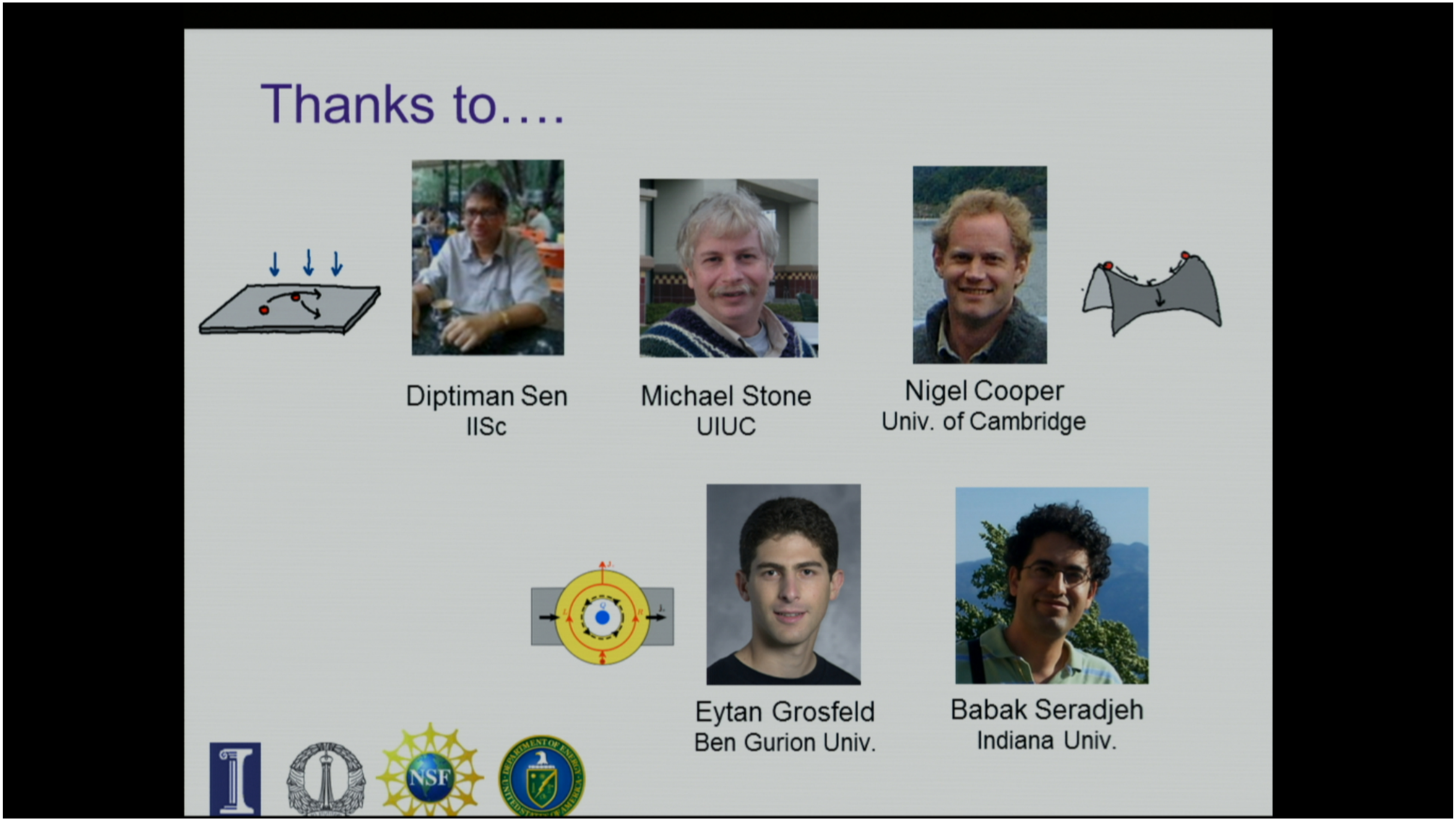
$$J_v^{int} \propto |t_L||t_R| \left[e^{i\varphi} \langle \psi_L | \psi_R \rangle \right]$$

- In zero field, drive j_s
- Vary charge Q, see dI/dV oscillation of period 2e or 4e
- Turn on field to nucleate HQV
- Repeat AC procedure
- Oscillations should disappear

Many thanks to R. Budakian and D. VanHarlingen
E. Grosfeld et al, 2011

Alternative: Josephson vortices

Also: E. Grosfeld and Stern, 2011
Beenakker 2011; Alicea 2012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In summary,

- Quantum Hall systems are expected to display the fascinating phenomenon of fractional statistics

-Two-particle correlations exhibit signatures of quantum statistics – interference, bunching, spatial profiles

- Non-Abelian statistics in quantum Hall systems and topological superconductors


Anyon there!!?