Title: Bulk-boundary correspondence in PEPS

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Abstract: TBA

Bulk-boundary correspondence with PEPS

J. Ignacio Cirac
Max-Planck Institute of Quantum Optics
DRC, Perimeter Institute

Perimeter Institute, November 13th, 2013


Bulk-boundary correspondence
with PEPS
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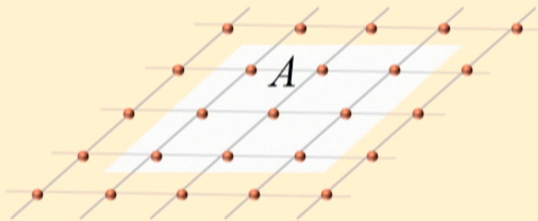
OUTLINE

- Bulk-boundary correspondence in lattice systems at T=0:

  N. Schuch (Aachen), D. Perez-Garcia (Madrid), D. Poilblanc (Tolouse)

  Yang (MPQ), Lehman (Aachen), Acoley,F. Verstraete (Vienna)



$|\Psi\rangle$: certain GS of local Hamiltonians

- Map: $A \rightarrow \partial A$

  $X_A \rightarrow x_{\partial A}$

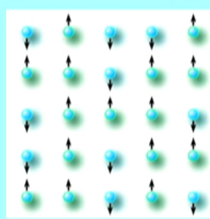
- Theory at the boundary:

  $\sigma_{\partial A} = e^{-h}$

  $h = h_{UNIV} + h_{LOCAL}$

- Quantum Memories: Robustness to noise and decoherence

  L. Mazza (Pisa), M. Rizzi (Mainz), M. Lukin (Harvard)



- Kitaev chain
- Local time-dependent perturbations

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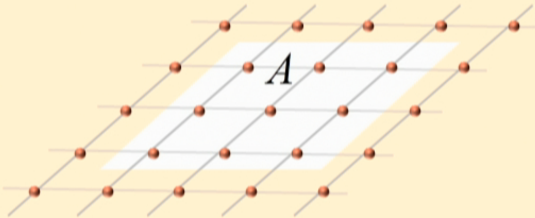

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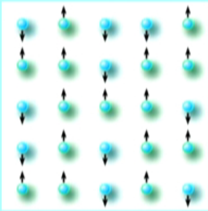

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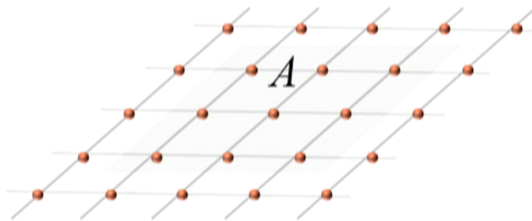
BULK-BOUNDARY CORRESPONDENCE

Schuch, Poilblanc, IC, Perez-Garcia, PRL **111**, 090501 (2013)
Yang, Lehman, Poilblanc, Acoleyen, Vesrtraete, IC, Schuch,arxiv:13094596

Related work: Dubail, Read, Rezayi
Qi, Katsura, and Ludwig
Vidal
Chen, Gu, Wen

SPIN LATTICES

- Spins on a lattice in 2D at zero temperature:



- Many-body state: $|\Psi\rangle$
- Parent Hamiltonian (local)

$$H|\Psi\rangle = E_0|\Psi\rangle$$

- Reduced state in region A:

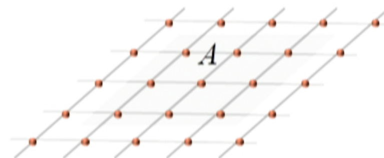
$$\rho_A = \mathrm{tr}\left[|\Psi\rangle\langle\Psi|\right]$$

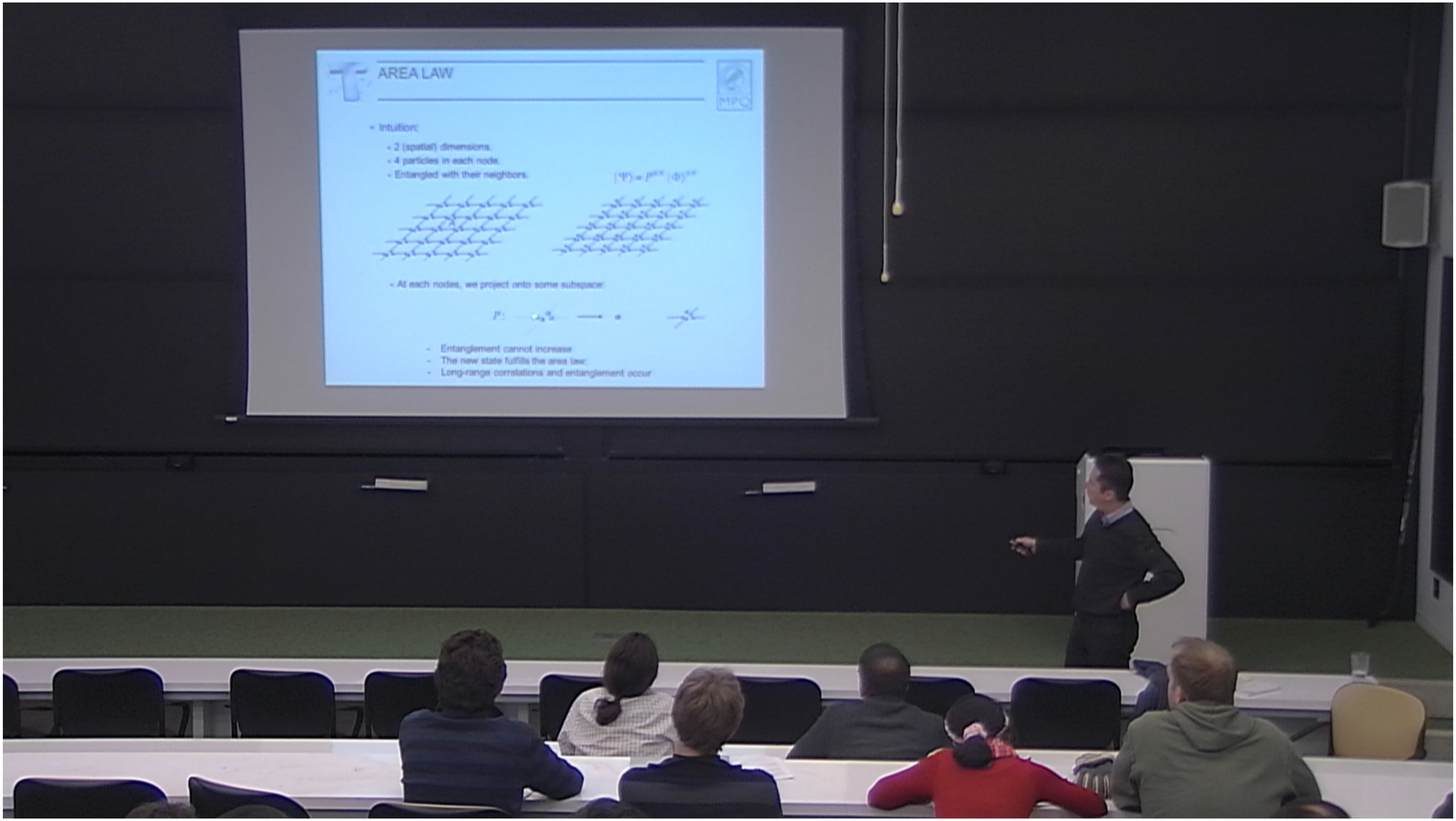
SPIN LATTICES

- Area law: (Sredniky 93):

$$S(\rho_A) \sim N_{\partial A}$$

degrees of freedom $\prec$ # particles at boundary




AREA LAW
MPQ
• Intuition:
• 2 (spatial) dimensions.
• 4 particles in each node.
• Entangled with their neighbors.
• At each nodes, we project onto some subspace:
P :
- Entanglement cannot increase
- The new state fulfills the area law:
- Long-range correlations and entanglement occur

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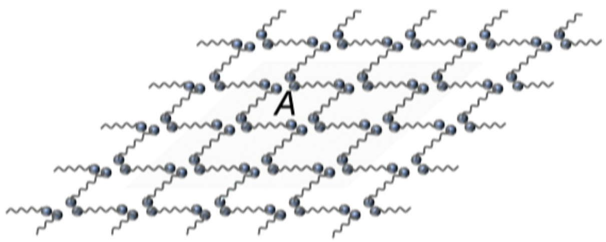
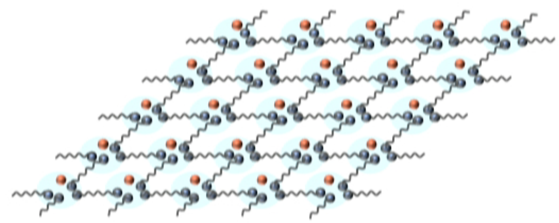
$$|\Psi\rangle = P^{\otimes N}|\Phi\rangle^{\otimes N}$$





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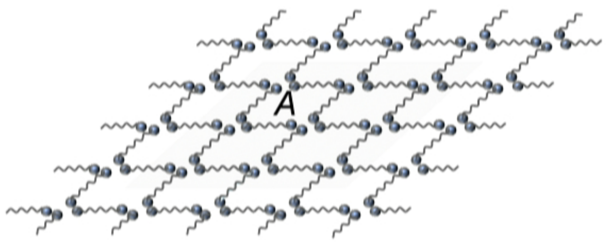

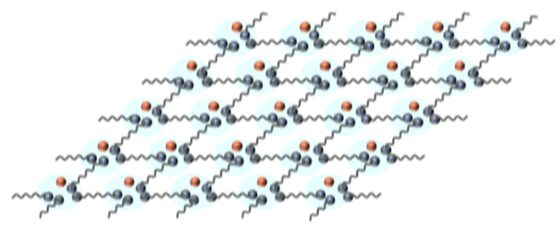

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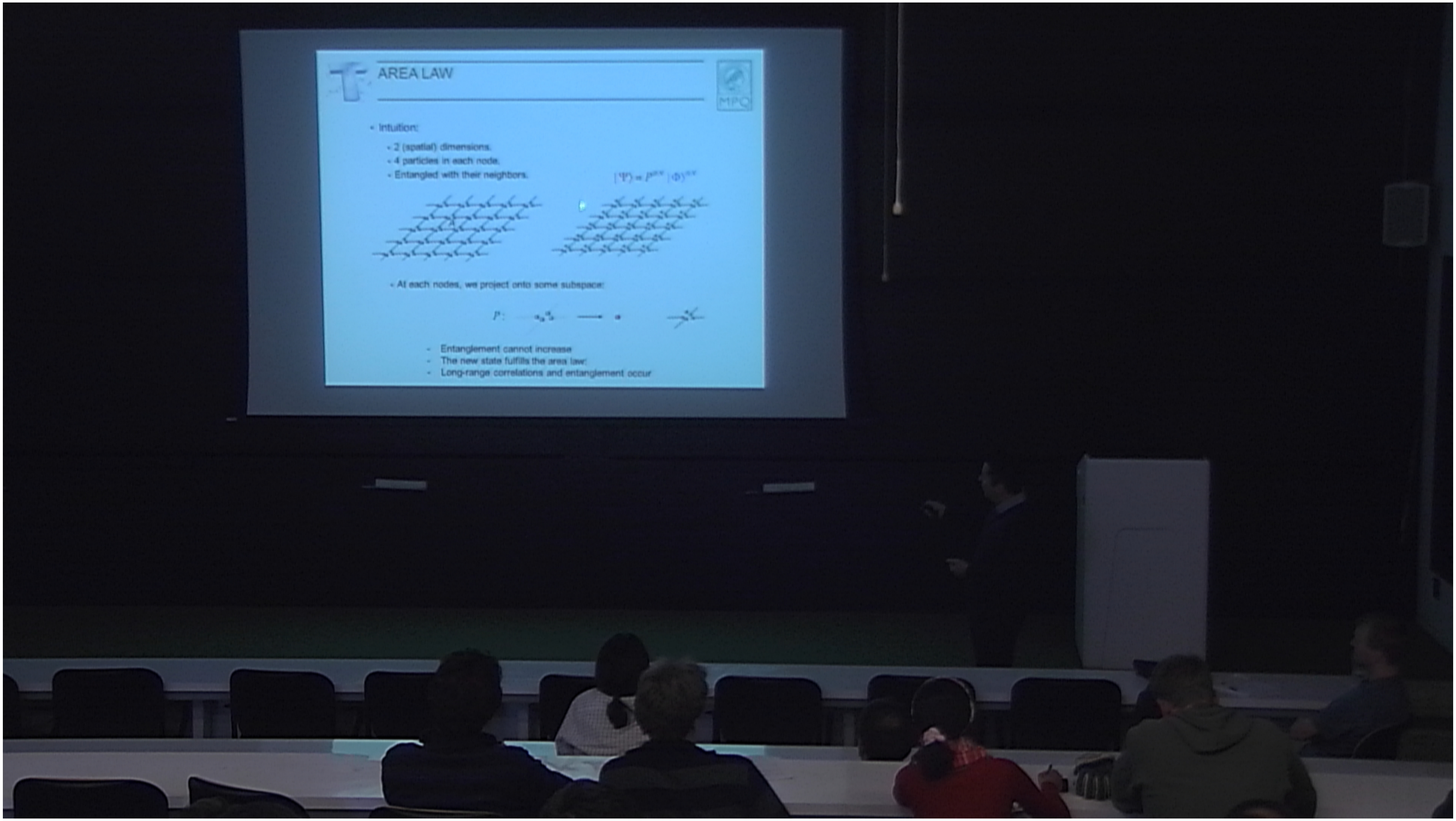
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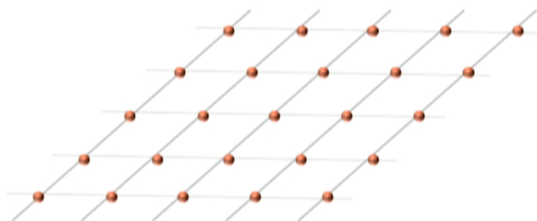
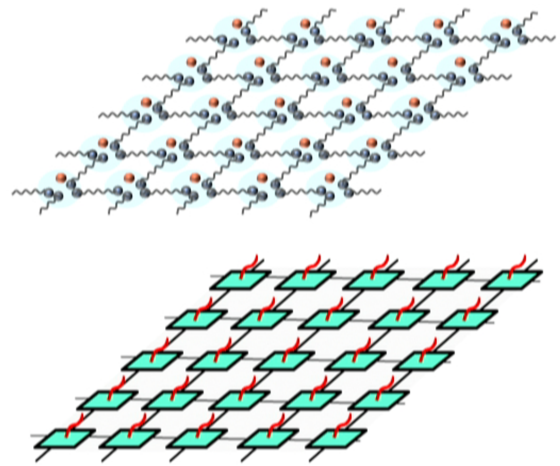
PROJECTED ENTANGLED-PAIR STATES

(Verstraete and IC, 2004)

$|\Psi\rangle$

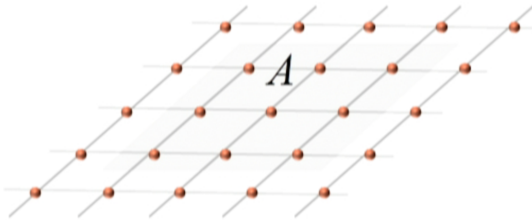


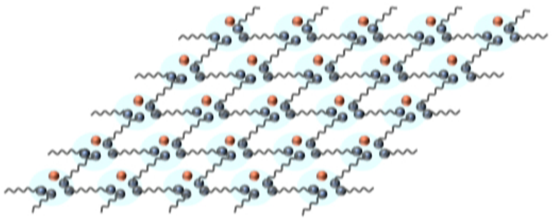


$$|\Psi\rangle \in H^{\otimes N} \quad \Longleftrightarrow \quad A^{i_n}_{\alpha\beta\gamma\delta}$$

PEPS give a natural playground to investigate this subject

PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE





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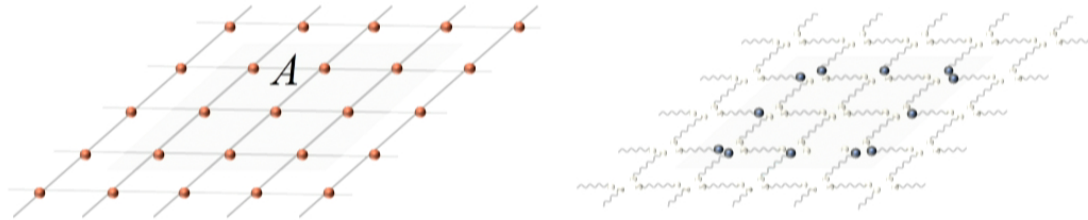


- The theory corresponds to the auxiliary particles living in the boundary

- Isommetry between the spins in the bulk and the auxiliary ones in the boundary

$$\sigma_{\partial A} = U \rho_A U^\dagger$$

↖ isommetry

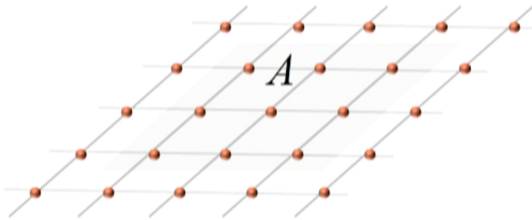
- It „compresses" the degrees of freedom
- Conserves the spectrum
- Allows to determine expectation values

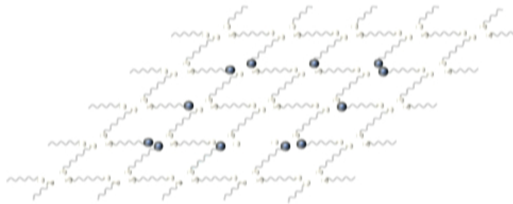
- It defines a BOUNDARY HAMILTONIAN

$$\sigma_{\partial A} = e^{-H_{\partial A}}$$

- Has the same entanglement spectrum $\sigma(H_{\partial A}) = \sigma(H_A)$
- It can be easily determined (exactly or approximately)

PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE





What can we say starting from the boundary Hamiltonian?
(beyond the entanglement spectrum)

- Is the Hamiltonian local?
- What are its symmetries, and how are they related to those of H?
- How do topological properties manifest themselves?
- What happens in quantum phase transitions?
- How general are those predictions?

Other approaches: Qi, Katsura, and Ludwig, 2012, Dubail, Read, and Rezayi, 2012

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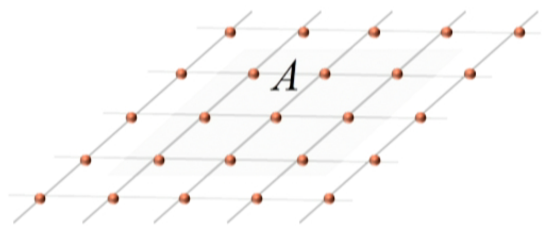

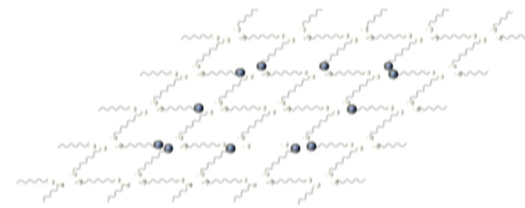

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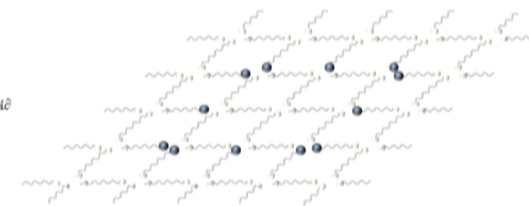
PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE

- Results:

$$\sigma_{\partial A} = e^{-H_{A\partial}}$$



- Symmetries: The boundary Hamiltonian inherits the symmetries

$$u_g \,|\Psi\rangle = e^{i\theta_g}\,|\Psi\rangle \quad \Rightarrow \quad U_g H_{\partial A} U_g^{\dagger} = H_{\partial A}$$

- Locality:

  - For gapped systems, it is local
  - For critical systems, it becomes non-local

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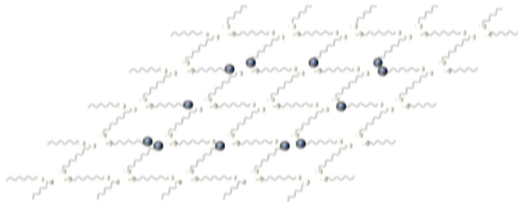

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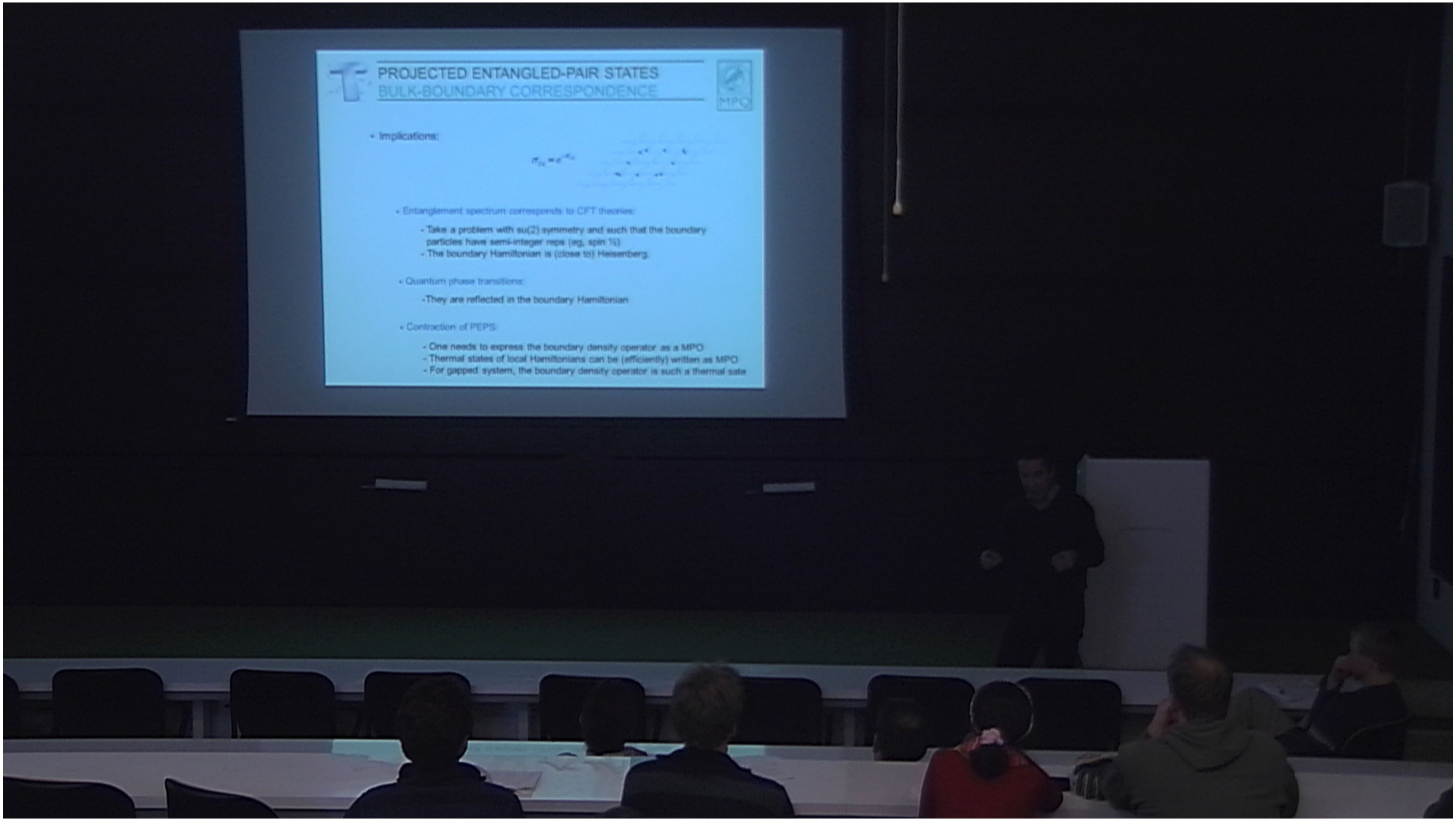
PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE
MPQ
• Implications:
• Entanglement spectrum corresponds to CFT theories:
- Take a problem with su(2) symmetry and such that the boundary particles have semi-integer reps (eg, spin ½)
- The boundary Hamiltonian is (close to) Heisenberg.
• Quantum phase transitions:
-They are reflected in the boundary Hamiltonian
• Contraction of PEPS:
- One needs to express the boundary density operator as a MPO
- Thermal states of local Hamiltonians can be (efficiently) written as MPO
- For gapped system, the boundary density operator is such a thermal sate

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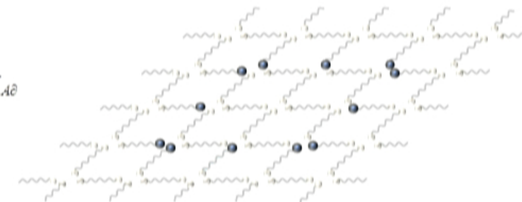

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BOUNDARY THEORY
TOPOLOGICAL PHASES

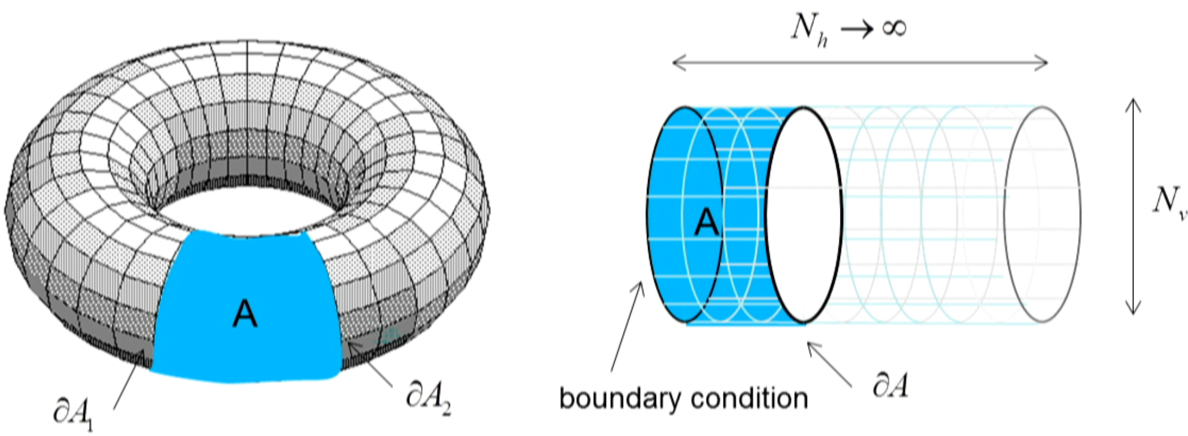
- Results:
  - The boundary theory develops an extra symmetry

$$\sigma_{\partial A} = U_g \sigma_{\partial A} U_g^{\dagger}$$

- In general, the boundary operator is block diagonal $\sigma_{\partial A} = \sigma_{\partial A}^{1} \oplus \sigma_{\partial A}^{2} \oplus \ldots$
- The projector, P_i, on each subspace is higly non-local

BOUNDARY THEORY
TOPOLOGICAL PHASES

- Setup:
  - Cylinder or Torus

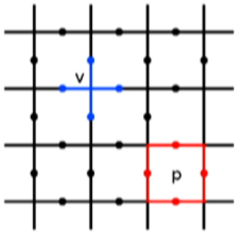


$$\sigma_{\partial A_1 \partial A_2}$$

$$\sigma_{\partial A} = \mathrm{tr}\left[X_{\partial A_1} \sigma_{\partial A_1 \partial A_2} \right]$$

BOUNDARY THEORY
TORIC CODE

- Ground state of: $H = -\sum_p H_p - \sum_v H_v$
  - Take one of the four ground states
    (the other can be easily obtained from that)
  - Auxiliary particles: qubits (D=2)



BOUNDARY THEORY
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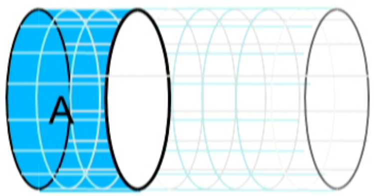


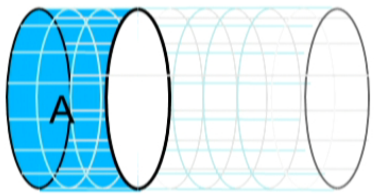
$$\sigma_{\partial A} = \mathrm{tr}\left[X_{\partial A_1} \sigma_{\partial A_1 \partial A_2} \right] = p_e P_e + p_o P_o$$

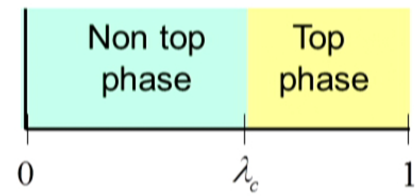
- Symmetry: $\sigma_{\partial A} = Z^{\otimes N_v} \sigma_{\partial A} Z^{\otimes N_v}$
- Projectors: $P_e = \frac{1}{2}\left(1 + Z^{\otimes N_v}\right)$ even parity
  $P_o = \frac{1}{2}\left(1 - Z^{\otimes N_v}\right)$ odd parity
- Boundary Hamiltonian: $H_{\partial A} = H_{\partial A}^{\mathrm{topo}} = -\log(p_e) P_e - \log(p_o) P_o$

  - The values of $p_{e,o}$ depend on the chosen boundaries
  - It is highly non-local (like the projectors)
  - There is no non-universal part (it is a fixed point of the RG flow)

BOUNDARY THEORY
PHASE TRANSITIONS

- Deformed Kitaev model:





- Conjecture: $H_{\partial A} = H_{\partial A}^{\text{topo}} + H_{\partial A}^{\text{non-universal}}$

  where $H_{\partial A}^{\text{topo}} = -\log(p_e)P_e - \log(p_o)P_o$ is the one calculated for λ=1 (toric code)

- Calculation:

  1. Fix the boundary condition (determines $p_{e,o}$).
  2. Determine the boundary state: $\sigma_{\partial A}$
  3. Compute the boundary Hamiltonian: $H_{\partial A} = -\log(\sigma_{\partial A})$
  4. Substract the universal topo Hamiltonian: $H_{\partial A}^{\text{non-universal}} = H_{\partial A} - H_{\partial A}^{\text{topo}}$
  5. Determine the „interaction lenght of the non-universal part“

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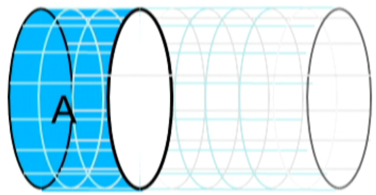

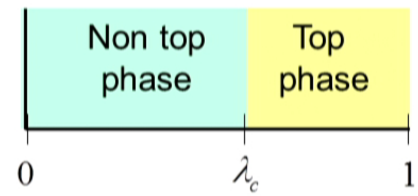

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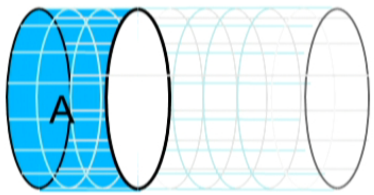
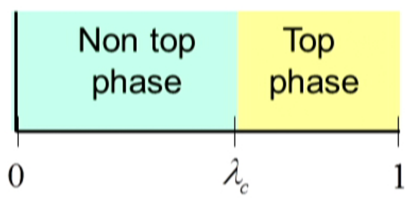
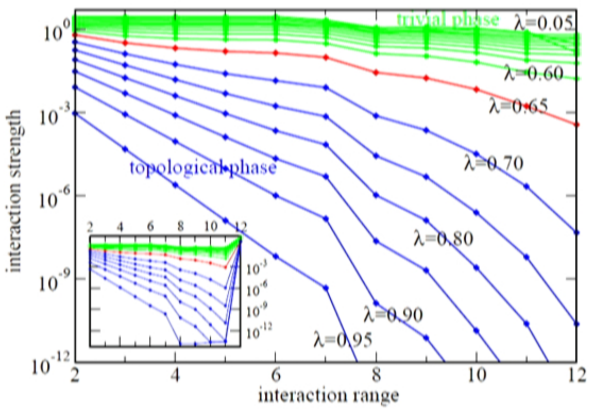
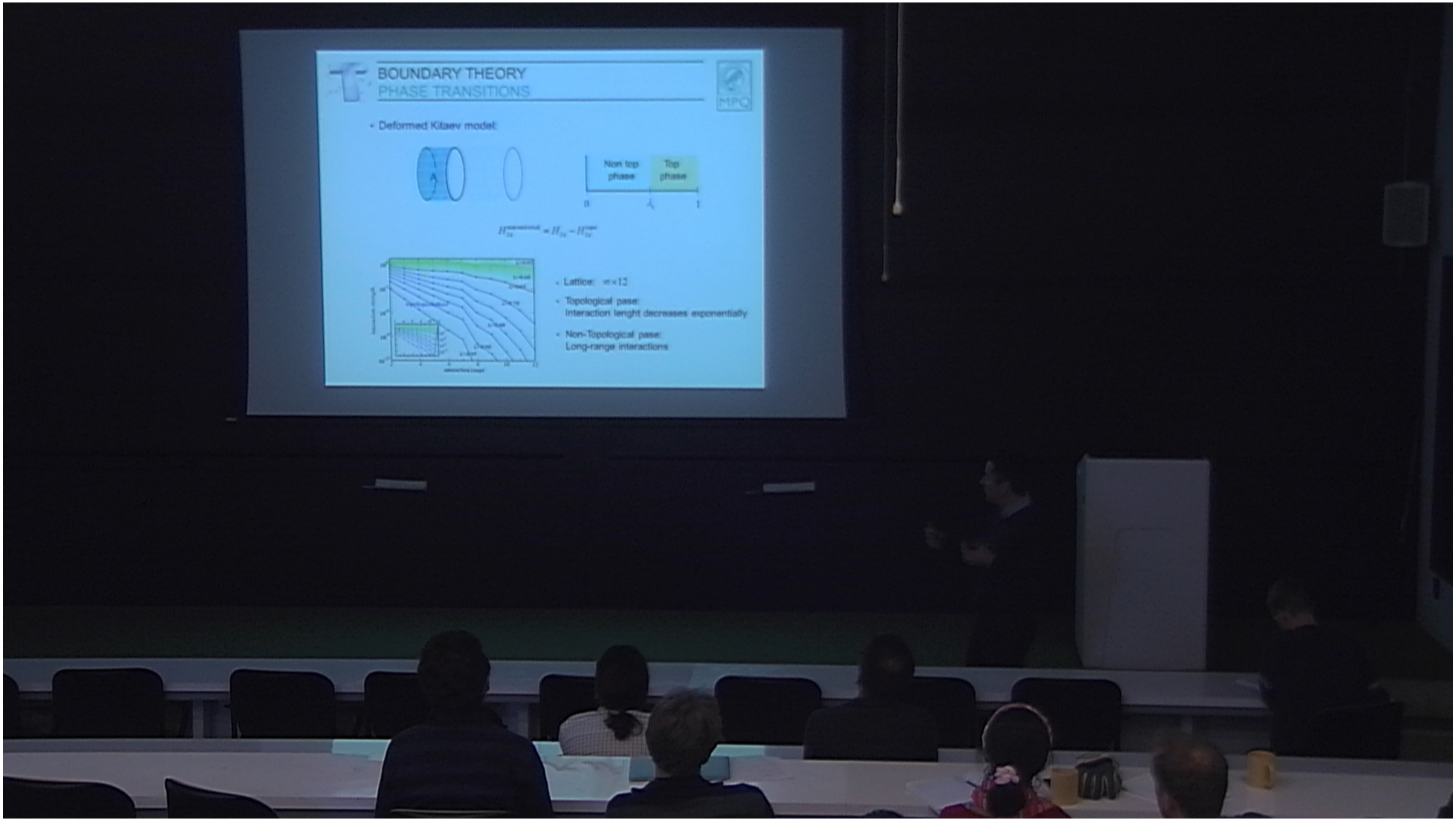




$$H_{\partial A}^{\text{non-universal}} = H_{\partial A} - H_{\partial A}^{\text{topo}}$$



- Lattice: $\infty \times 12$
- Topological pase:
  Interaction lenght decreases exponentially
- Non-Topological pase:
  Long-range interactions



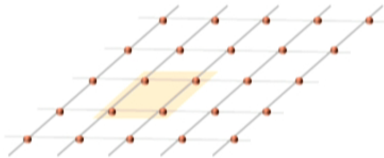
BOUNDARY THEORY
PHASE TRANSITIONS
Deformed Kitaev model:
Non top phase
Top phase
Lattice:
Topological pase:
Interaction lenght decreases exponentially
Non-Topological pase:
Long-range interactions
MPQ

BOUNDARY THEORY
EDGES OF A SYSTEM

- PEPS: Parent Hamiltonians

$$H|\Psi\rangle = 0$$

$$H = \sum h_n$$ ← Finite range interaction $h_n \geq 0$



$$h_n|\Psi\rangle = 0$$

BOUNDARY THEORY
EDGES OF A SYSTEM

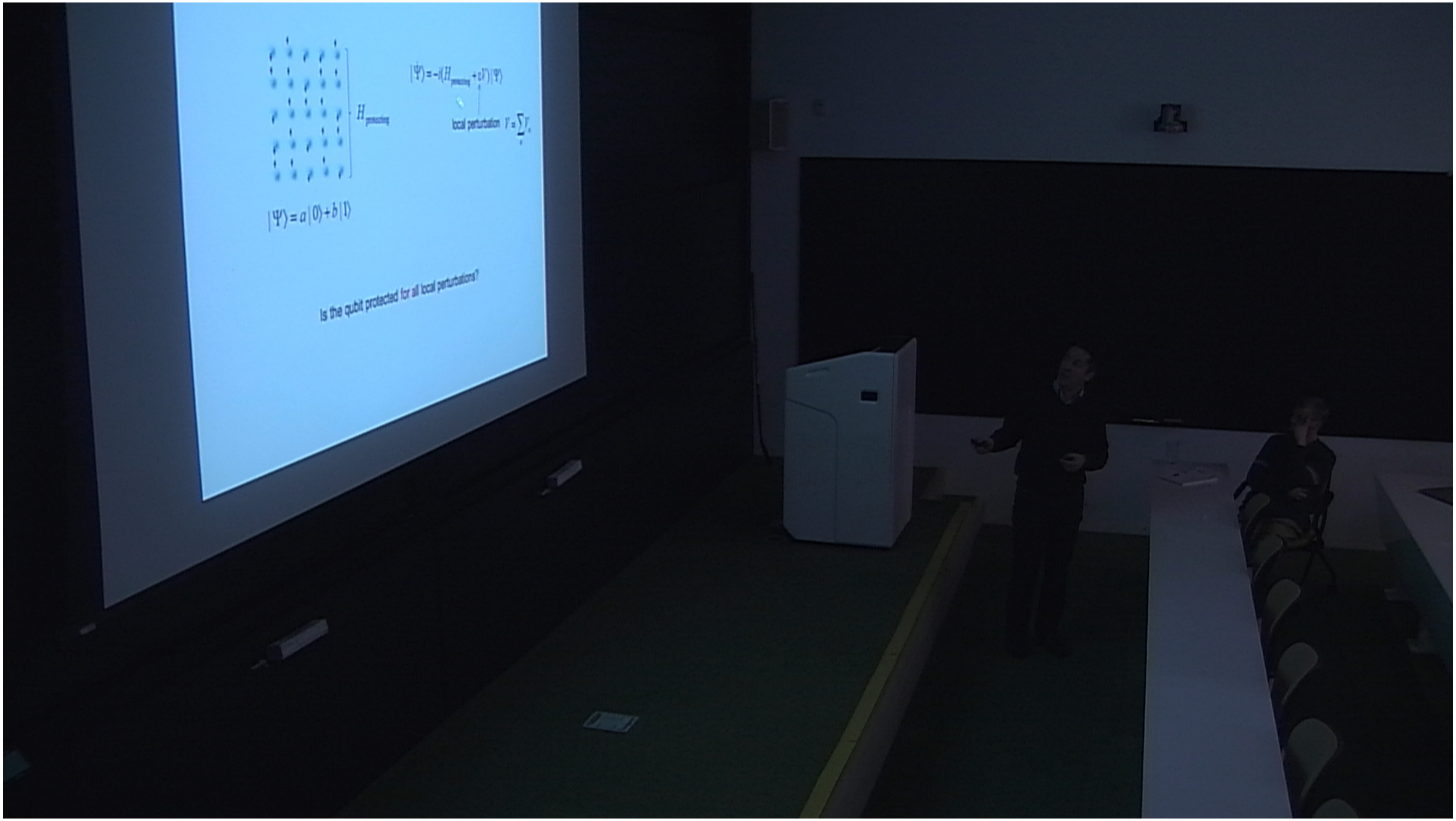
- Topological models
  - Edge theory: non-conventional 1D Hamiltonian
  - Example: Kitaev-model ⟶ Ising model without symmetry breaking

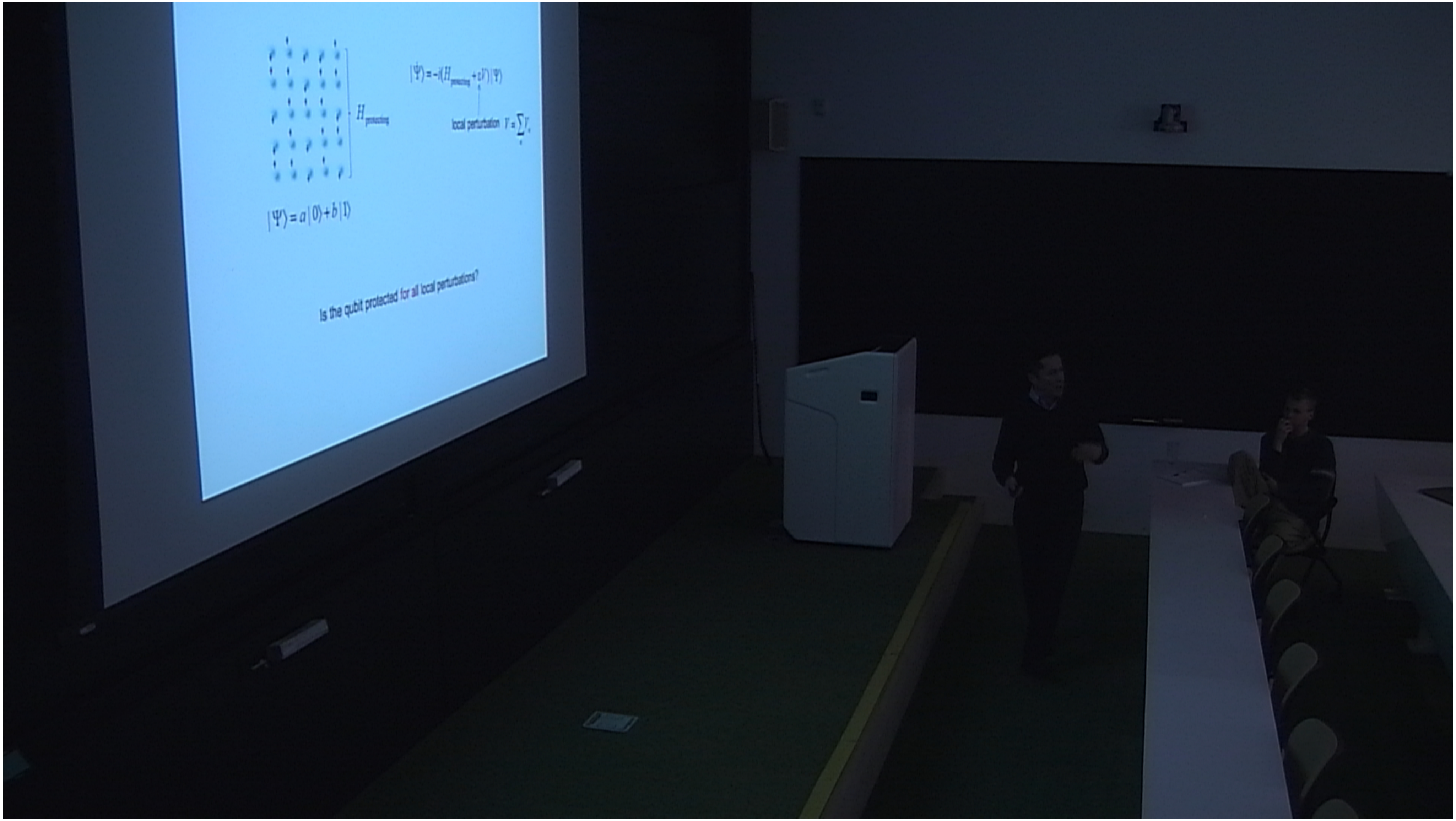
S.Yang,L. Lehman, D.Poilblanc, Acoleyen, F. Vesrtraete,IC, N. Schuch,arxiv:13094596

See also: X. Chen, ZC Gu, ZX Liu, and XG Wen, *Science* **338** 1604-1606 (2012)

QUANTUM MEMORIES:
ROBUSTNESS TO NOISE AND DECOHERENCE

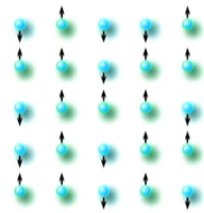
L. Mazza, M. Rizzi, M. Lukin, JIC, arxiv:1212.4778


$H_{protecting}$
$|\dot{\Psi}\rangle = -i(H_{protecting} + \varepsilon V)|\Psi\rangle$
local perturbation $V = \sum_i V_i$
$|\Psi\rangle = a|0\rangle + b|1\rangle$
Is the qubit protected for all local perturbations?



H protecting
|Ψ̇⟩ = −i(H protecting + εV)|Ψ⟩
local perturbation V = ∑ V_i
|Ψ⟩ = a|0⟩ + b|1⟩
Is the qubit protected for all local perturbations?

QUANTUM MEMORIES
2. HAMILTONIAN NOISE



PROBLEM

- Initial state: $\rho_0 = |\varphi\rangle\langle\varphi|$
- Evolution:

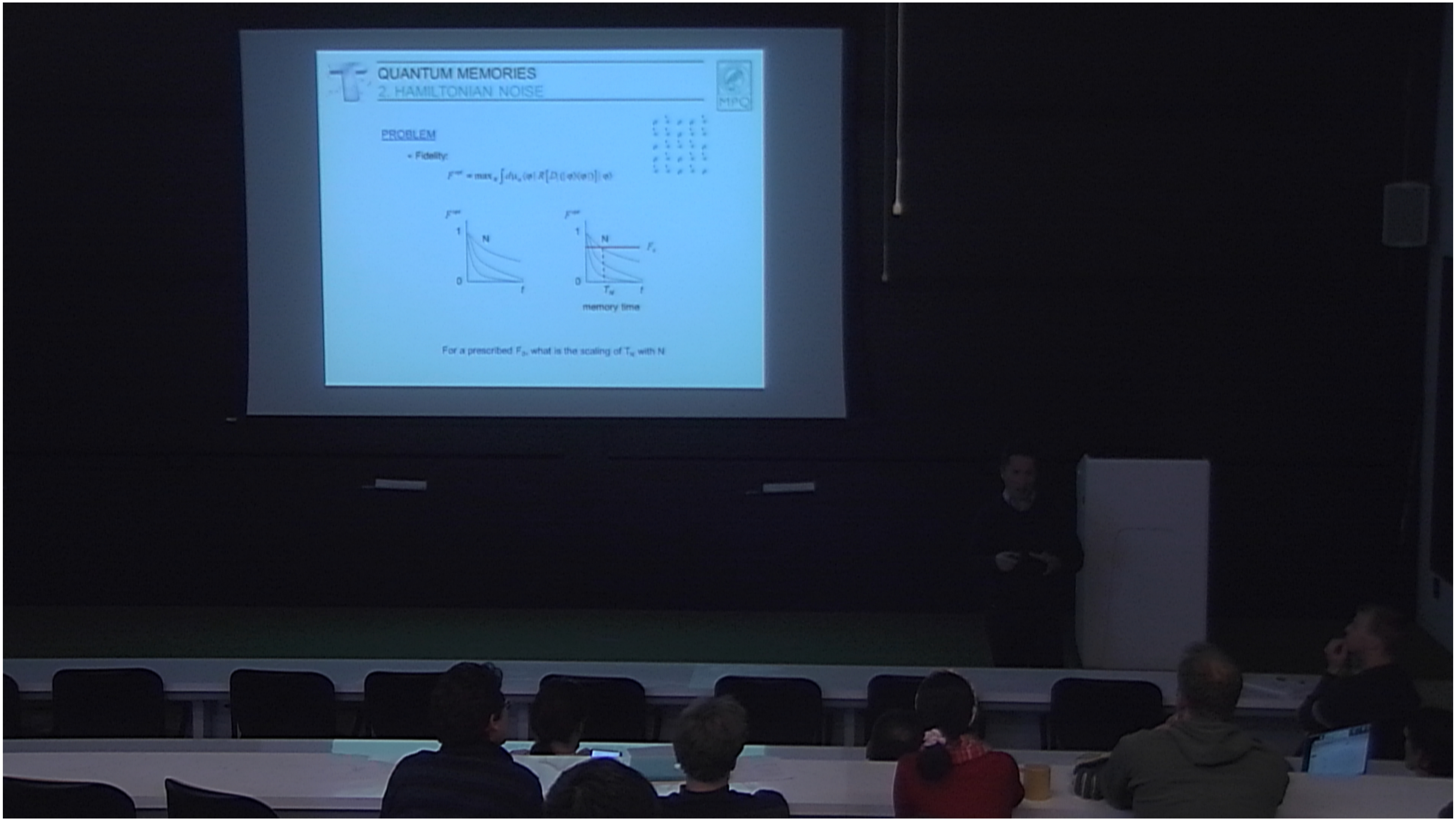
$$\rho(t) = \int d\mu_V e^{-i(H_{prot}+\varepsilon V)t} \rho_0 e^{i(H_{prot}+\varepsilon V)t} = D_t(\rho)$$

- Recovery operation:

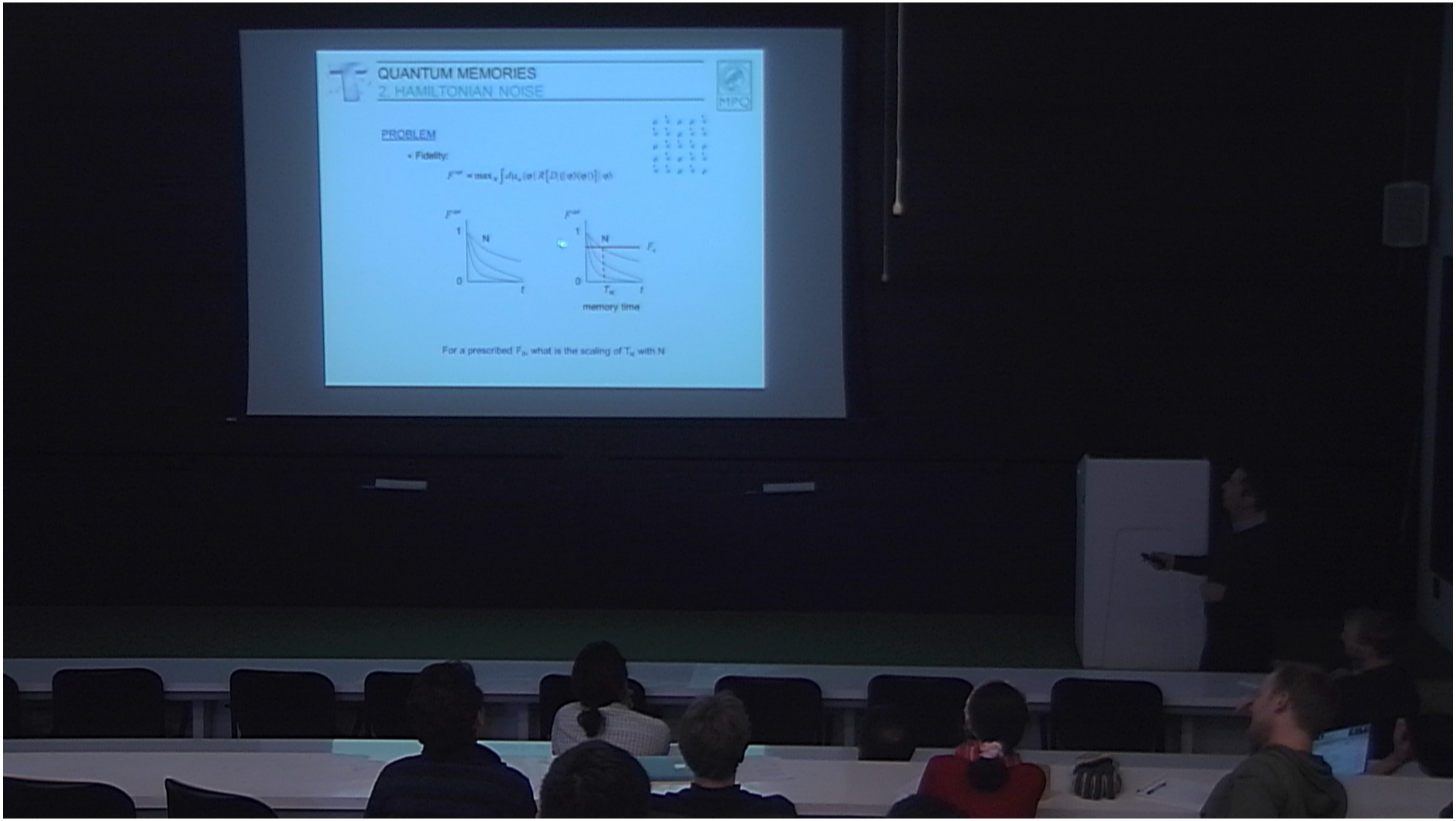
$$\rho_F = R[\rho(t)]$$

- Fidelity:

$$F^{opt} = \max_R \int d\mu_\varphi \langle\varphi| R[D_t(|\varphi\rangle\langle\varphi|)] |\varphi\rangle$$



QUANTUM MEMORIES
2. HAMILTONIAN NOISE
MPQ
PROBLEM
- Fidelity:
1
N
0
t
1
N
0
T_M
t
memory time
For a prescribed F_0, what is the scaling of T_M with N


QUANTUM MEMORIES
2. HAMILTONIAN NOISE
PROBLEM
• Fidelity:
N
memory time
For a prescribed F, what is the scaling of T with N
MPQ

QUANTUM MEMORIES
2. HAMILTONIAN NOISE

KITAEV's CHAIN

- Use two separated chains to store a qubit
- Hamiltonian perturbations conserve parity (SSR)
- Problem is Gaussian: Can be solved
- Optimal Fidelity:

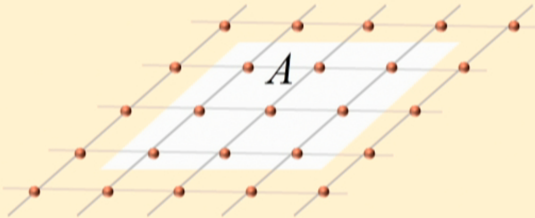
$$F^{opt} = \frac{2}{3} + \frac{1}{6} \| D_t(|\varphi_+\rangle\langle\varphi_+|) - D_t(|\varphi_-\rangle\langle\varphi_-|) \|_{tr}$$

CONCLUSIONS

- Bulk-boundary correspondence in lattice systems at T=0:

  N. Schuch (Aachen), D. Perez-Garcia (Madrid), D. Poilblanc (Tolouse)

  Yang (MPQ), Lehman (Aachen), Acoley,F. Verstraete (Vienna)



$|\Psi\rangle$: certain GS of local Hamiltonians

- Map: $A \rightarrow \partial A$

  $X_A \rightarrow x_{\partial A}$

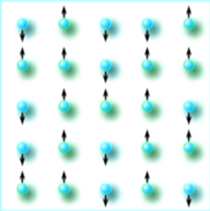
- Theory at the boundary:

  $\sigma_{\partial A} = e^{-h}$

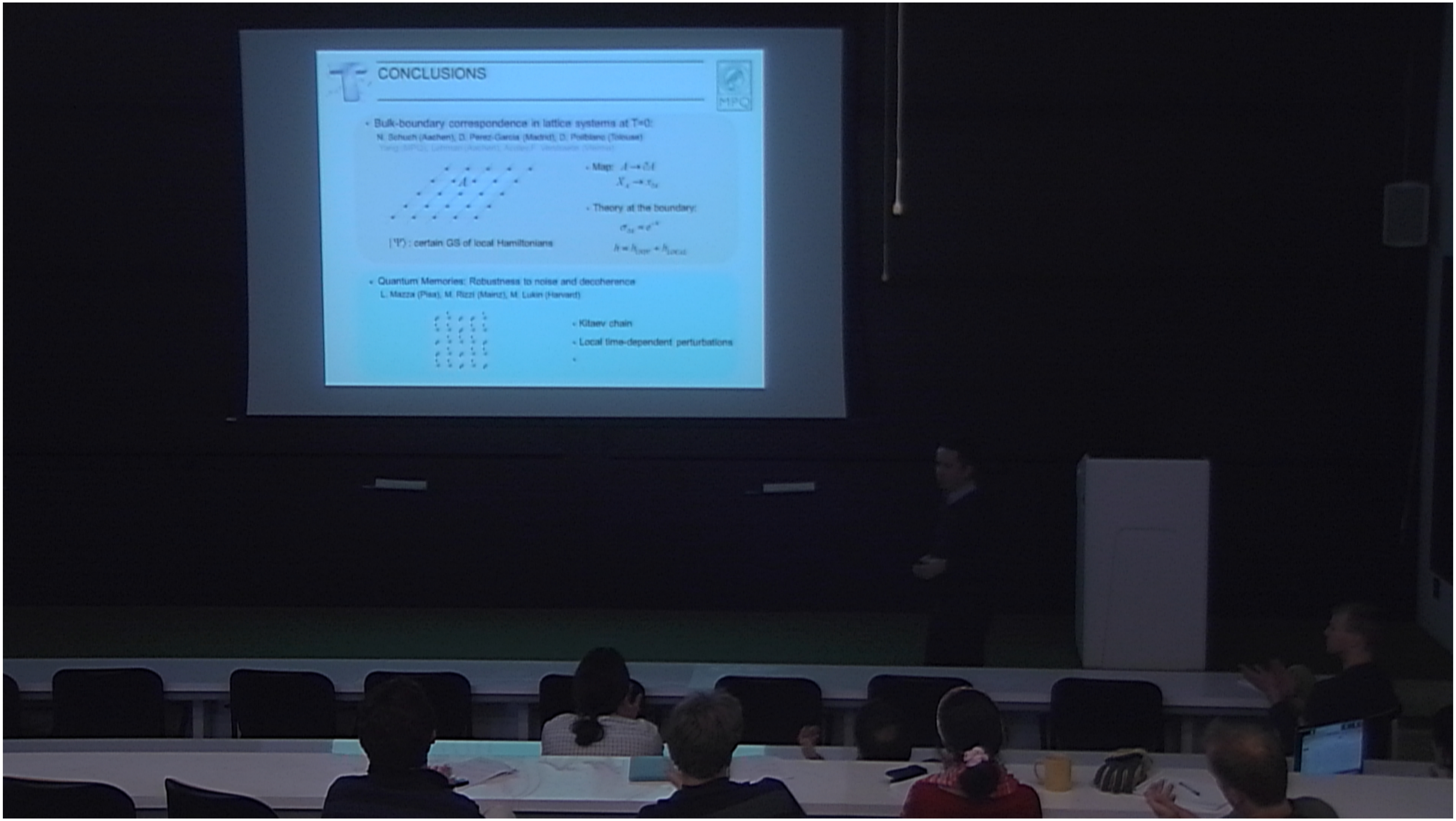
  $h = h_{UNIV} + h_{LOCAL}$

- Quantum Memories: Robustness to noise and decoherence

  L. Mazza (Pisa), M. Rizzi (Mainz), M. Lukin (Harvard)



- Kitaev chain
- Local time-dependent perturbations
-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