Title: Quantum mechanics as an operationally time symmetric probabilistic theory

Date: Nov 12, 2013 03:30 PM

URL: http://pirsa.org/13110057

Abstract: <span>The standard formulation of quantum mechanics is operationally asymmetric with respect to time reversal---in the language of compositions of tests, tests in the past can influence the outcomes of test in the future but not the other way around. The question of whether this represents a fundamental asymmetry or it is an artifact of the formulation is not a new one, but even though various arguments in favor of an inherent symmetry have been made, no complete time-symmetric formulation expressed in rigorous operational terms has been proposed. Here, we discuss such a possible formulation based on a generalization of the usual notion of test. We propose to regard as a test any set of events between an input and an output system which can be obtained by an autonomously defined laboratory procedure. This includes standard tests, as well as proper subsets of the complete set of outcomes of standard tests, whose realization may require post-selection in addition to pre-selection. In this approach, tests are not expected to be operations that are up to the choices of agents---the theory simply says what circuits of tests may occur and what the probabilities for their outcomes would be, given that they occur. By virtue of the definition of test, the probabilities for the outcomes of past tests can depend on tests that take place in the future.

Such theories have been previously called non-causal, but here we revisit that notion of causality. Using the Choi-Jamiolkowski isomorphism, every test in that formulation, commonly regarded as inducing transformations from an input to an output system, becomes equivalent to a passive detection measurement applied jointly on two input systems---one from the past and one from the future. This is closely related to the two-state vector formalism, but it comes with a conceptual revision: every measurement is a joint measurement on two separate systems and not on one system described by states in the usual Hilbert space and its dual. We thus obtain a static picture of quantum mechanics in space-time or more general structures, in which every experiment is a local measurement on a global quantum state that generalizes the recently proposed quantum process matrix. The existence of two types of

systems in the proposed formalism allows us to define causation in terms of correlations without invoking the idea of intervention, offering a possible answer to the problem of the meaning of causation. The framework is naturally compatible with closed time-like curves and other exotic causal structures.

Quantum mechanics as a time-symmetric operational probabilistic theory

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In what sense is the standard formulation asymmetric?

A system is described by $\rho(t)$. $\rho \in \mathcal{L}(\mathcal{H}),\ \rho \geq 0,\ \mathrm{Tr}(\rho) = 1$

Operational meaning of $\rho(t)$: probabilities for the outcomes of all possible measurements one could perform on the system at time t , ***conditional on events in the past*** (the 'preparation' of the state).

to be made precise later

The probabilities are given by the Born rule:

$$p(i|\{E_j\}, r) = \mathrm{Tr}(E_i \rho) \qquad \sum_i E_j = \mathbb{1}$$

past events defining the preparation (can be the trivial event)

The two-state vector formalism

Watanabe, Rev. Mod. Phys. 27, 179 (1955).

Aharonov, Bergmann, Lebowitz, PRB 134, 1410 (1964):

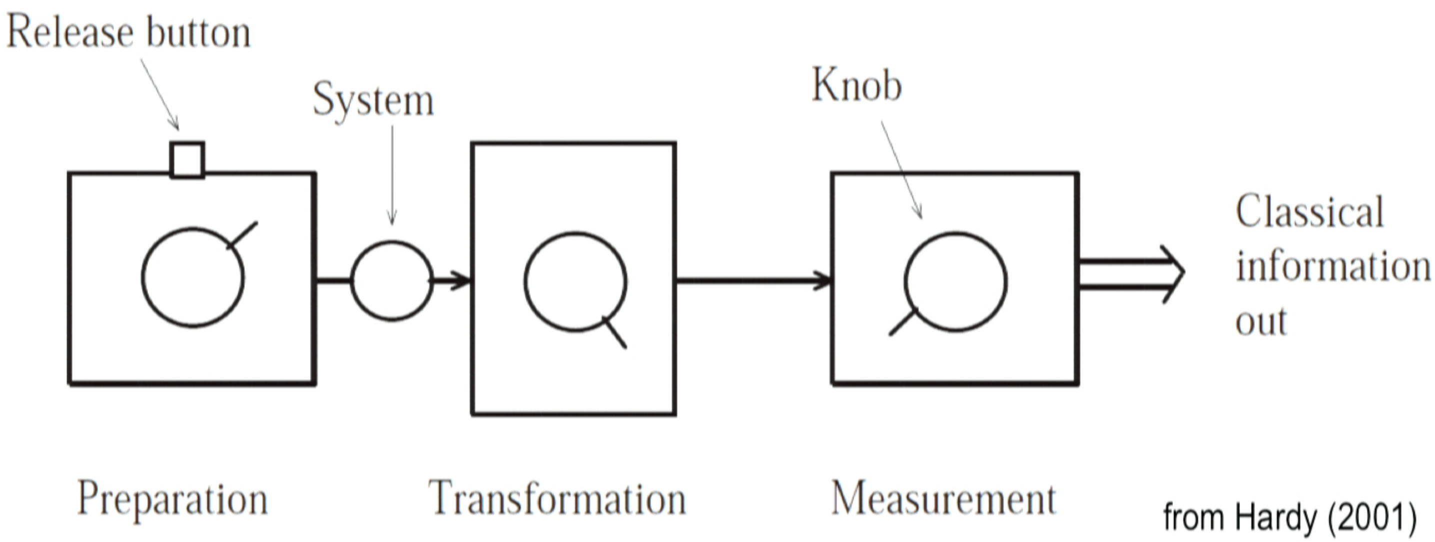
$$p(j|\psi,\phi) = \frac{|\langle\phi|P_j|\psi\rangle|^2}{\sum_i |\langle\phi|P_j|\psi\rangle|^2} \longrightarrow \langle\phi| \quad |\psi\rangle$$

(two-state vector)

Why are the probabilities nonlinear in the state?

Why aren't the probabilities noncontextual functions of P_j?

Operational Approach



from Hardy (2001)

Significant progress in understanding QM from **operational** perspective, with primitive laboratory procedures as basic ingredients.

Hardy (2001), Barrett (2005), Dakic and Brukner (2009), Massanes and Mülelr (2010), Chiribella, D'Ariano, and Perinotti (2010) , Hardy ....

Sketch

- Time-symmetric reformulation of QM in the circuit framework
- Time-symmetric process matrix framework
  (extension of Oreshkov, Costa, Brukner, Nat. Comm. 3, 1092 (2012).)
- Relation to the two-state vector formalism
- Defining causation from local time
- Towards a field picture without predefined causal structure

The circuit framework

Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010), PRA 84, 012311 (2011);
Hardy, arXiv:1005.5164, arXiv:1104.2066…

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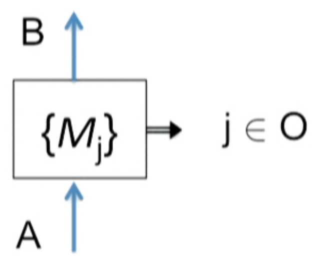
Operation (test): one use of a device with an input and an output system



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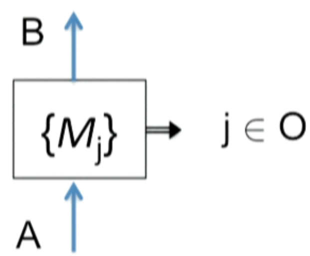

In quantum mechanics: A → H^A , B → H^B (Hilbert spaces)

$\{M_j\}$ → CP maps from $L(H^A)$ to $L(H^B)$,

such that $\sum_{j\in O} M_j = M$ is CPTP.

The circuit framework

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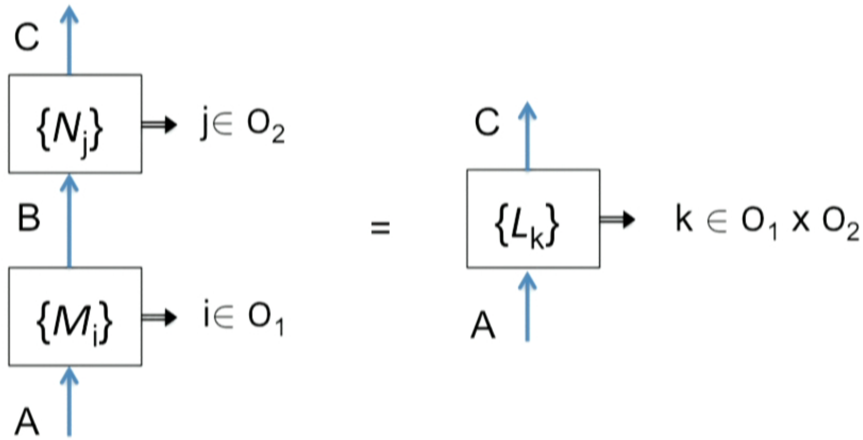
Sequential composition:



For foundations of compositional theories: see, e.g., Abramsky and Coecke, Quantum Logic and Quantum Structures, vol II (2008). Coecke, Contemporary Physics 51, 59 (2010).

The circuit framework

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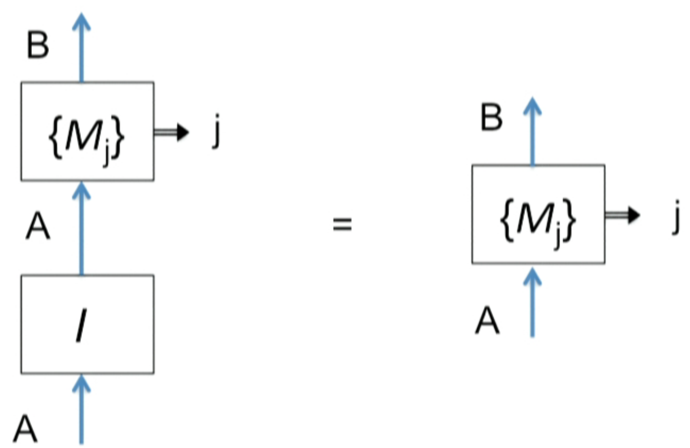
Identity operation:



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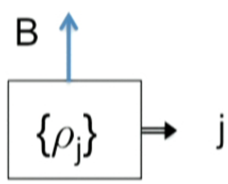
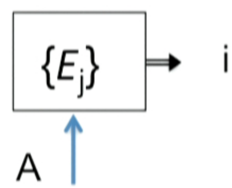
Preparation operations (the input system is the trivial system):



Detection operations (the output system is the trivial system):



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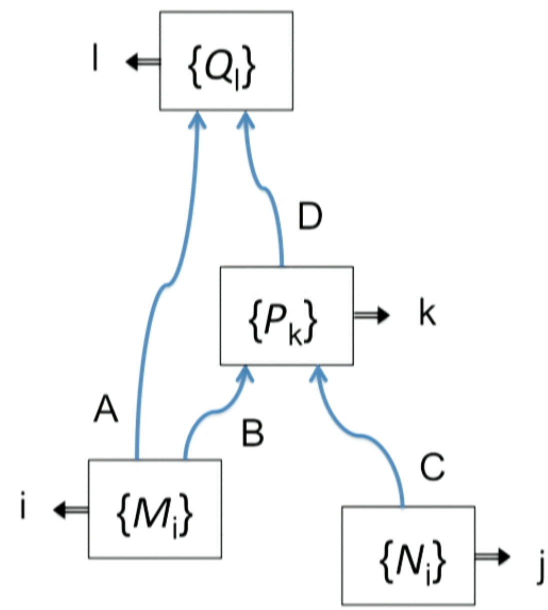
Circuit (directed acyclic graphs (DAG) of operations with no open wires):



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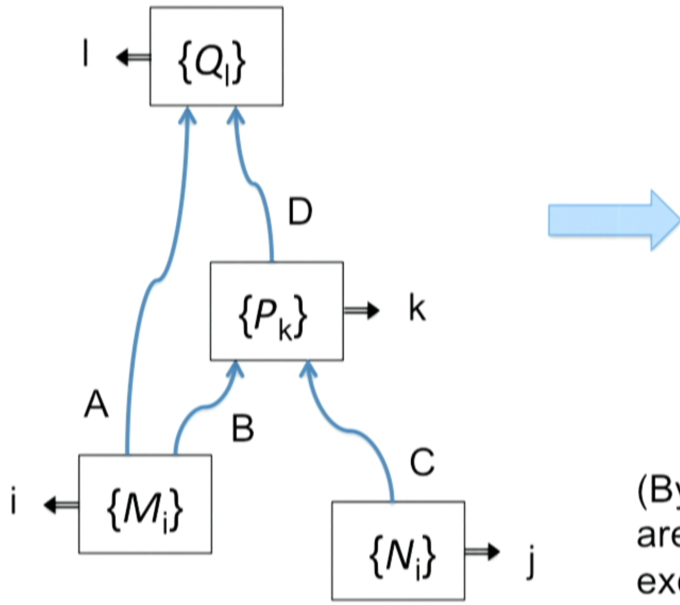
Probabilistic structure:

The theory prescribes probabilities for the outcomes of any given circuit:



Joint probabilities

p(i, j, k, l)

$p(i, j, k, l) \geq 0, \;\; \sum_{ijkl} p(i, j, k, l) = 1$

(By definition, the wires in a circuit are the only means of information exchange between the operations.)

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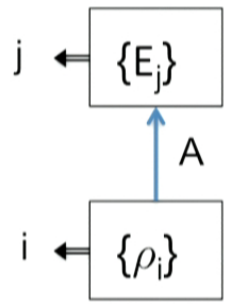
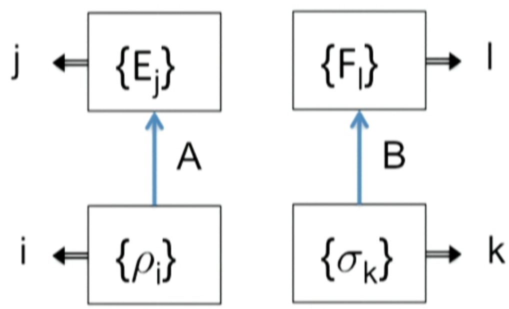
Equivalently,



Joint probabilities

$p(i, j)$

with the property



$$p(i, j, k, l) = p(i, j)\, p(k, l)$$

(factorizability)

The circuit framework

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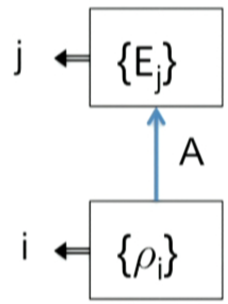

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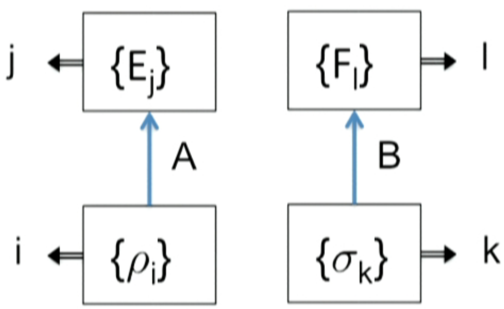

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(factorizability)

The circuit framework

A theory is completely defined by specifying the possible operations and the probabilities for the outcomes of all circuits!

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A theory is completely defined by specifying the possible operations and the probabilities for the outcomes of all circuits!

The description of a theory can be simplified significantly by grouping events into equivalence classes of indistinguishable events.

If two events yield the same probabilities for all possible circuits they may be part of, they are equivalent.

States**:** equivalence classes of preparation events

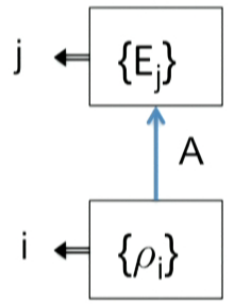
Effects**:** equivalence classes of detection events

Transformations from A to B**:** equivalence classes of events from A to B

The asymmetry

Axiom (Chiribella, D'Ariano, Perinotti):

PRA 81, 062348 (2010)
PRA 84, 012311 (2011)



$\Rightarrow$ $p(i, j) \equiv p(\rho_i, E_j)$

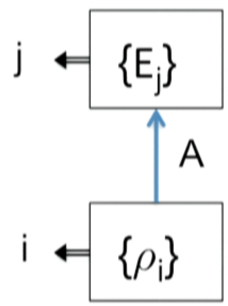
The marginal probabilities of the preparation, $p(\rho_i | \{E_j\}) \equiv \sum_j p(\rho_i, E_j)$, are independent of the detection:

$$p(\rho_i|\{E_j\}) = p(\rho_i|\{F_k\}) \qquad \forall\ \{E_j\}, \{F_j\}, \{\rho_i\}.$$

Called *causality* or '*no signalling from the future*'.

The asymmetry

In quantum mechanics:



$$p(\rho_i, E_j) = \mathrm{Tr}(\rho_i\, E_j)$$

$$\rho_i \in L(H^A);\ \ \rho_i \geq 0,\ \ \mathrm{Tr}(\textstyle\sum_i \rho_i) = 1$$

$$E_j \in L(H^A);\ \ E_j \geq 0,\ \ \textstyle\sum_j E_j = I$$

The preparation probabilities are

$$p(\rho_i |\ \{E_j\}) \equiv \textstyle\sum_j \mathrm{Tr}(\rho_i\, E_j) = \mathrm{Tr}(\rho_i) \equiv p_i, \quad \forall\ \{E_j\}.$$

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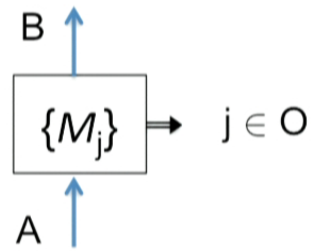
Reconsidering the basics

Operation (test): one use of a device with an input and an output system



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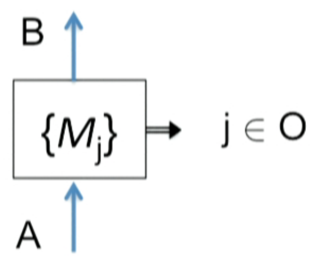
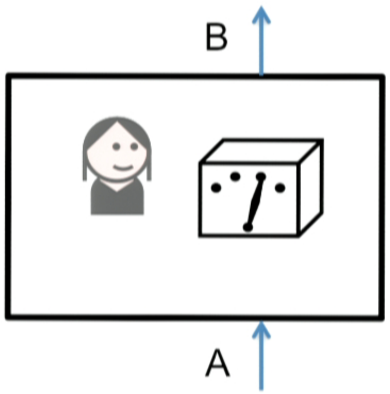


What is one use of a device?

Reconsidering the basics

Consider the following scenario:



Alice can choose to use different devices $\{M^{\alpha}_{j_\alpha}\}$, each selected at random with probability $p(\alpha)$.

The whole experiment is equivalent to a big operation $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$.

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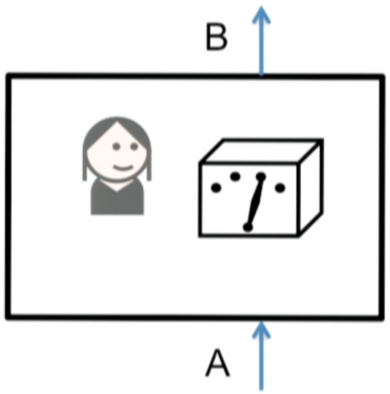

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Is this fundamentally different from the application of a single physical device $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$?

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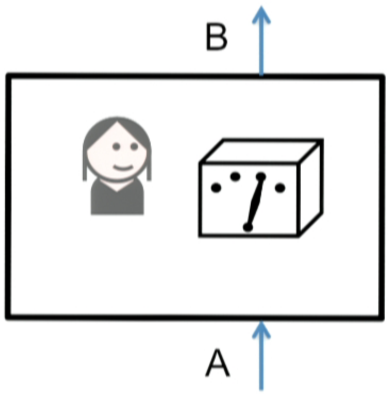

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Is this fundamentally different from the application of a single physical device $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$?

I think NO, because from the outside these cases are indistinguishable.

Reconsidering the basics

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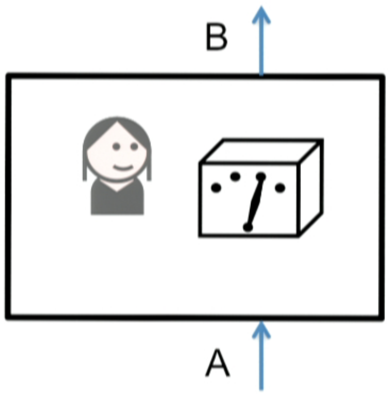

Alice can choose to use different devices $\{M^{\alpha}_{j_\alpha}\}$, each selected at random with probability $p(\alpha)$.

The whole experiment is equivalent to a big operation $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$.

If a single device $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$ is applied and we only obtain information about the subset α to which the outcome belongs, is this fundamentally different from learning that Alice applied $\{M^{\alpha}_{j_\alpha}\}$?

Reconsidering the basics

→ Subsets of the outcomes of 'devices' also can be called operations. But according to the standard formulation, only special subsets of the outcomes of operations correspond again to operations – those for which the sum of the CP maps of the outcomes is proportional to a CPTP map.

Reconsidering the basics

→ Subsets of the outcomes of ‘devices’ also can be called operations. But according to the standard formulation, only special subsets of the outcomes of operations correspond again to operations – those for which the sum of the CP maps of the outcomes is proportional to a CPTP map.

Proposal: Regard any subset of the outcomes of an operation as an operation.

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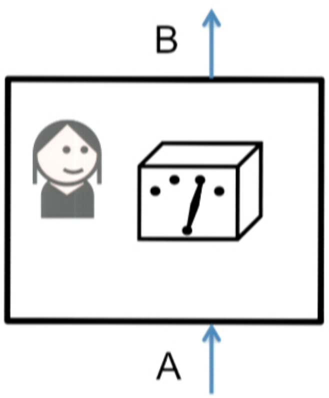
Note 1: Every operation defined as above corresponds to a local laboratory procedure (may require post-selection in addition to pre-selection.)



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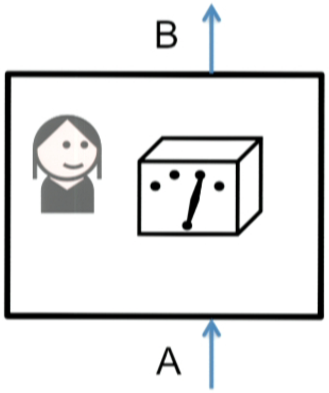

Note 2: The idea that the correlations between the events in different regions is due solely to information exchange via the input/output systems remains.

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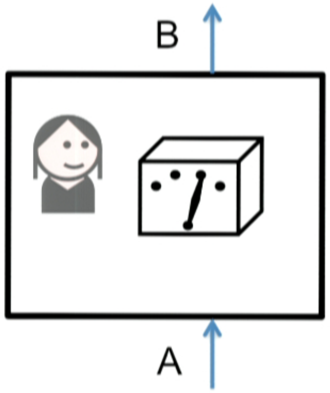
! Operations are not up to the choices of agents.



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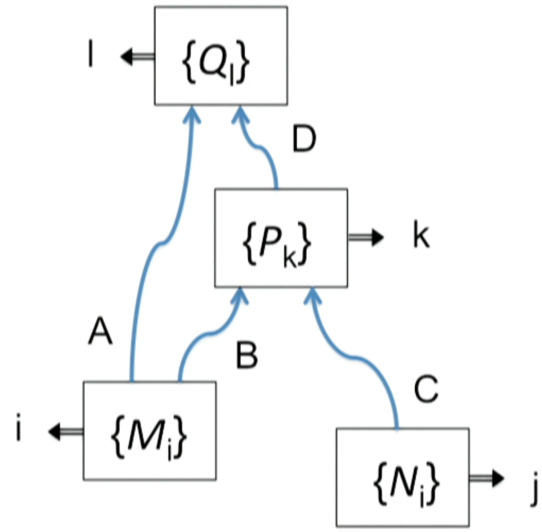
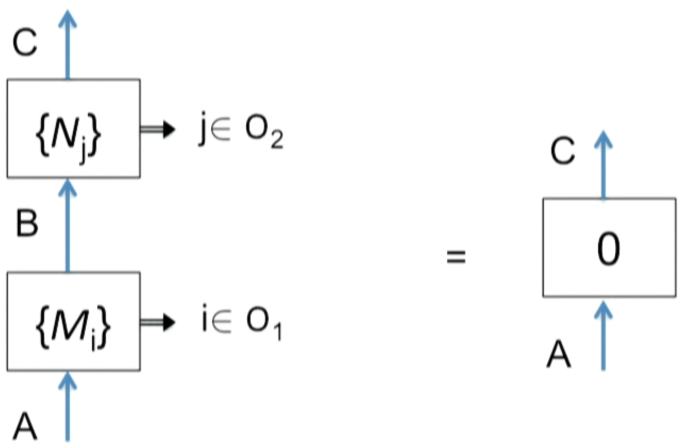
Can think of circuits of such operations:



An addition to the rules

Some operations are *incompatible* even if they have the same input-output systems:



(the null operation)

Probabilities and equivalence classes

Consider a standard preparation operations $\{\rho_i\}$, $i \in O_1$ and a standard detection operation $\{E_j\}$, $j \in O_2$. We require that any subset of events of *any* operation defines an operation. Consider $Q_1 \subset O_1$ and $Q_2 \subset O_2$.

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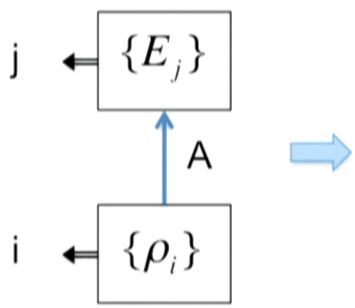
Joint probabilities:



$$p(i, j \mid i \in Q_1, j \in Q_2) = \frac{Tr(\rho_i E_j)}{\sum_{i \in Q_1, j \in Q_2} Tr(\rho_i E_j)}$$

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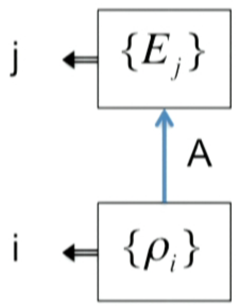

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$\{\rho_i\}$, $i \in Q_1$ and $\{\alpha\rho_i\}$, $i \in Q_1$, $\forall\ \alpha \geq 0$, are equivalent operations.

$\{E_j\}$, $j \in Q_2$ and $\{\alpha E_j\}$, $j \in Q_2$, $\forall\ \alpha \geq 0$, are equivalent operations.

Probabilities and equivalence classes

Preparation operations are still described by $\{\rho_i\}$, $\rho_i \geq 0$, $\mathrm{Tr}(\sum_I \rho_i)=1$.

Detection operations are now described by $\{E_j\}$, $E_j \geq 0$, $\mathrm{Tr}(\sum_j E_j) = d$.

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Any subset of the outcomes of an operation defines a new operation, with the new elements related to the old ones via a renormalization factor:

Start with a given $\{E_j\}$, $j \in O$. Select only events within a subset, $j \in Q \subset O$.

The new operation is described by $\{E'_j\}$, $j \in Q$, where $E'_j = E_j\, d/\mathrm{Tr}(\sum_{j\in Q} E_j)$.

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Note: This says in particular how to realize any operation from a standard operation using post-selection. But the starting operation can be arbitrary!

There is no claim that operations such as $\sum_j E_j = I$ are more 'complete' !!!

Probabilities and equivalence classes

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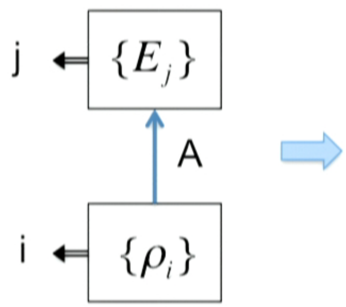
Joint probabilities:



$$p(i,j) = \frac{Tr(\rho_i E_j)}{\sum_{ij} Tr(\rho_i E_j)}$$

Probabilities and equivalence classes

States (equivalent preparation events): $(\rho, \overline{\rho})$, where $0 \le \rho \le \overline{\rho}$, $Tr(\overline{\rho}) = 1$.

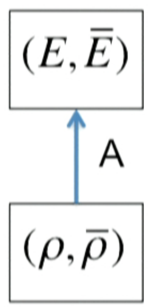
Effects (equivalent detection events): $(E, \overline{E})$, where $0 \le E \le \overline{E}$, $Tr(\overline{E}) = d$.

Probabilities and equivalence classes

States (equivalent preparation events): $(\rho, \bar{\rho})$, where $0 \le \rho \le \bar{\rho}, \quad Tr(\bar{\rho}) = 1$.

Effects (equivalent detection events): $(E, \bar{E})$, where $0 \le E \le \bar{E}, \quad Tr(\bar{E}) = d$.

Joint probabilities:



$$p[(\rho, \bar{\rho}), (E, \bar{E})] = \frac{Tr(\rho E)}{Tr(\bar{\rho} \bar{E})}$$

States can be thought of as functions on effects and vice versa.

Probabilities and equivalence classes

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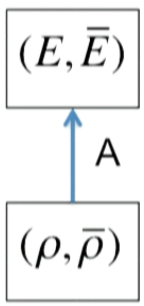
Effects (equivalent detection events): $(E, \overline{E})$, where $0 \le E \le \overline{E}$, $Tr(\overline{E}) = d$.

Joint probabilities:

The set of states (effects), however, is not closed under convex combinations!

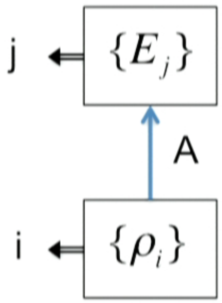


$$p[(\rho, \overline{\rho}), (E, \overline{E})] = \frac{Tr(\rho E)}{Tr(\overline{\rho}\,\overline{E})}$$

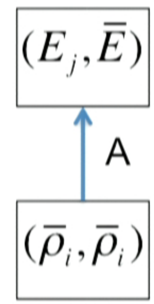
States can be thought of as functions on effects and vice versa.

Probabilities and equivalence classes

Conditional states:



$$p(j\,|\,i) = \frac{Tr(\rho_i E_j)}{Tr(\rho_i \bar{E})} = \frac{Tr(\bar{\rho}_i E_j)}{Tr(\bar{\rho}_i \bar{E})} =$$



Every conditional states can be described by a single normalized density matrix $\bar{\rho}$, just like in the standard formulation.

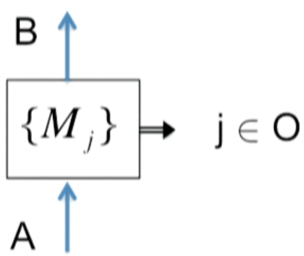
The probabilities for the outcomes of detections on a given conditional state are:

$$p[(E,\bar{E})\,|\,\bar{\rho}] = \frac{Tr(E\bar{\rho})}{Tr(\bar{E}\bar{\rho})}$$

(Born's rule is the case $\bar{E} = I$.)

Probabilities and equivalence classes

General operations: collections of CP maps $\{M_j\}$, s.t. $Tr(\sum_j M_j(\frac{I}{d_A}))=1.$



Equivalent outcome events: $(M,\bar{M})$, where $0 \le M \le \bar{M}$, $Tr(\bar{M}(\frac{I}{d_A}))=1$.

Time symmetry

The set of operations from A to B is isomorphic that from B to A.

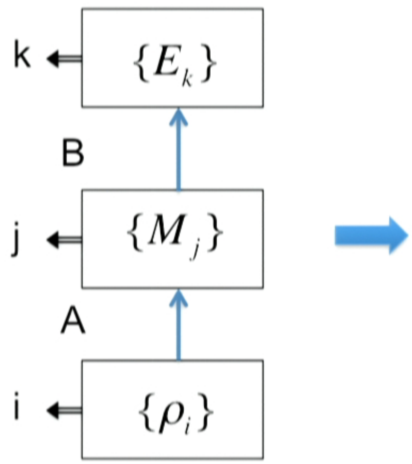
Time symmetry

The set of operations from A to B is isomorphic that from B to A.

Every circuit can be equivalently read in the opposite direction by replacing every CP map by its transpose:

$M(\ ) = \sum_\alpha K_\alpha (\ ) K_\alpha^\dagger \rightarrow M^T(\ ) = d_B/d_A \sum_\alpha K_\alpha^\dagger (\ ) K_\alpha$

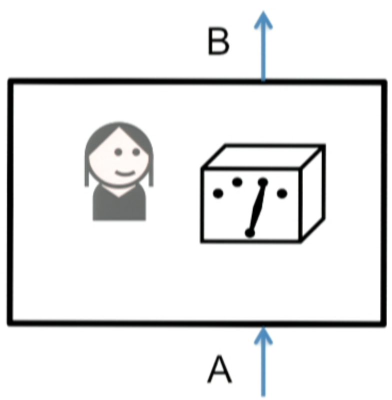
Example:



$$p(i,j,k) = \frac{Tr(E_k M_j(\rho_i))}{Tr(\bar{E}\bar{M}(\bar{\rho}))}$$

But we would like more

A time-neutral description?



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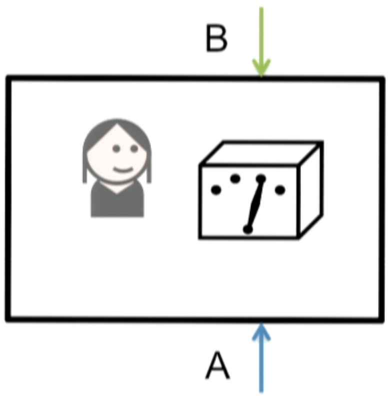
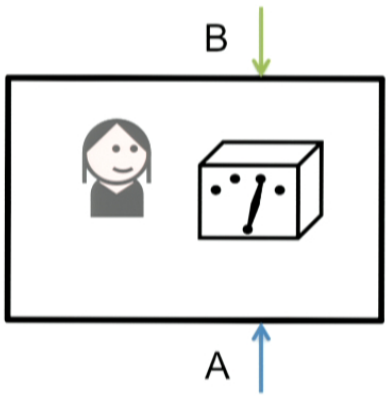


Can we view Alice's operation as applied on two input systems – one from the past and one from the future?

But we would like more

A time-neutral description?

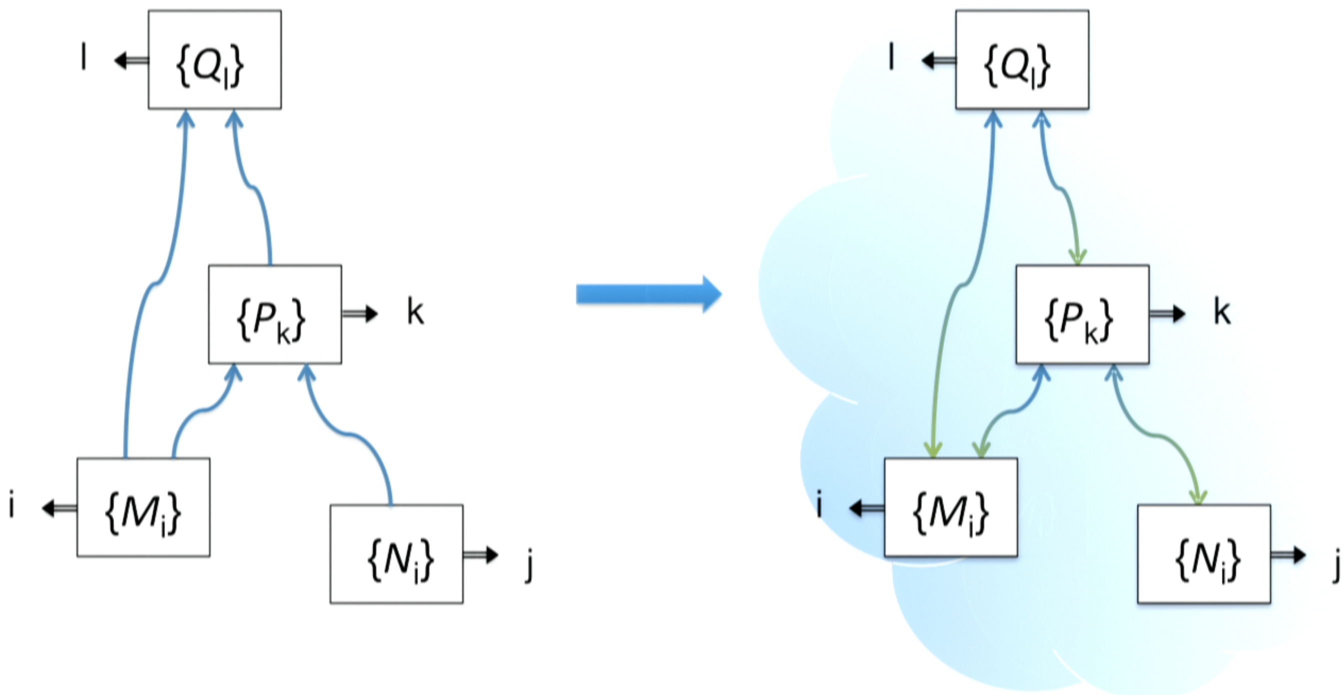


Can we view Alice's operation as applied on two input systems – one from the past and one from the future?

In other words, given knowledge about the rest of the circuit, can we associate a mathematical object (state) with Alice's experiment from which the probabilities for the outcomes of her operations can be calculated?

But we would like more

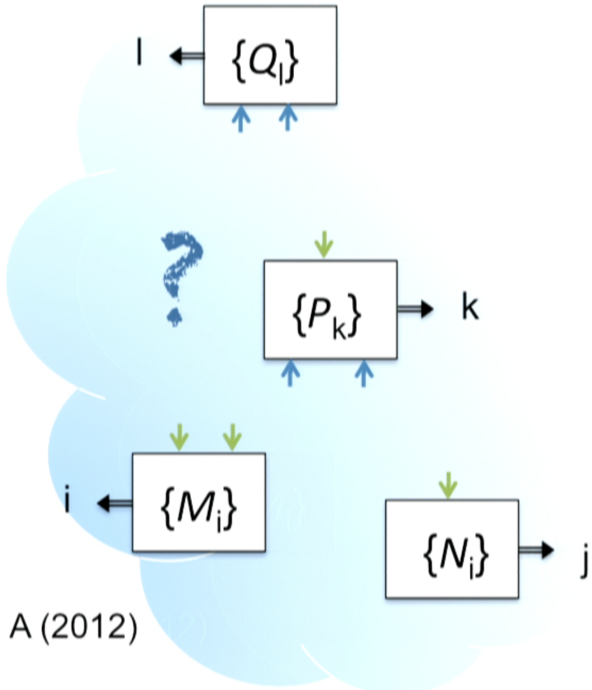
Local operations on a global quantum state?



But we would like more

Local operations on a global quantum state?

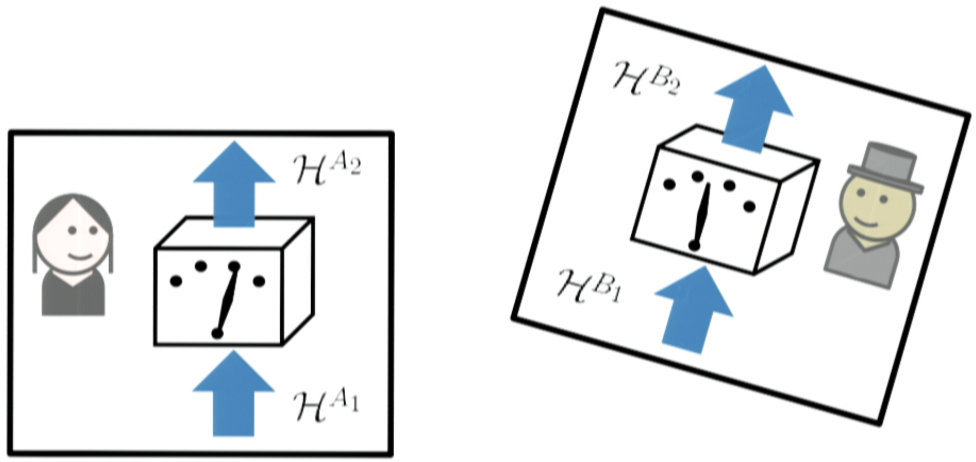
Beyond definite connections:



Hardy, arXiv:0509120, arXiv:0804.0054
Chiribella, D'Ariano, Perinotti, arXiv: 0912.0195, PRA 88 (2013)
Oreshkov, Costa, Brukner, Nat. Comm. 3 (2012)
Chiribella, PRA(R) 86 (2012)
Colnaghi, D'Ariano, Perinotti, Facchini, Phys. Lett. A (2012)

The original process matrix formalism

Local experiments without pre-defined causal order:



Oreshkov, Costa, Brukner, Nat. Comm. 3, 1092 (2012), arXiv:1105.4464.

Choi-Jamiołkowski isomorphism

CP maps		Positive semidefinite matrices
$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

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$$M^{12} := d_1 d_2 [\mathcal{I} \otimes \mathcal{M}(|\Phi^+\rangle\langle\Phi^+|)]^{\mathrm{T}}$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{d_1}} \sum_{i=1}^{d_1} |i\rangle|i\rangle$$

$$|i\rangle \in \mathcal{H}^1$$

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("Channel-effect duality")

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$$|i\rangle \in \mathcal{H}^1$$

The original process matrix formalism

Assuming *noncontextual linear* probabilities for the outcomes of *standard* local operations (quantum instruments):

Representation

$$p(\mathcal{M}_i^A, \mathcal{M}_j^B, \cdots) = \mathrm{Tr}\left[W^{A_1A_2B_1B_2\cdots}\left(M_i^{A_1A_2} \otimes M_j^{A_1A_2} \otimes \cdots\right)\right]$$

Process Matrix

The original process matrix formalism

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Process Matrix

Captures all scenarios obtained without post-selection in a definite causal structure (where the operations are part of a circuit).

Captures probabilistic mixtures of such scenarios, as well as *indefinite* causal order!

The original process matrix formalism

$$p(\mathcal{M}_i^A, \mathcal{M}_j^B, \cdots) = \mathrm{Tr}\left[W^{A_1A_2B_1B_2\cdots}\left(M_i^{A_1A_2} \otimes M_j^{A_1A_2} \otimes \cdots\right)\right]$$

Conditions on W:

1. Non-negative probabilities (with shared ancillas): $W^{A_1A_2B_1B_2} \geq 0$

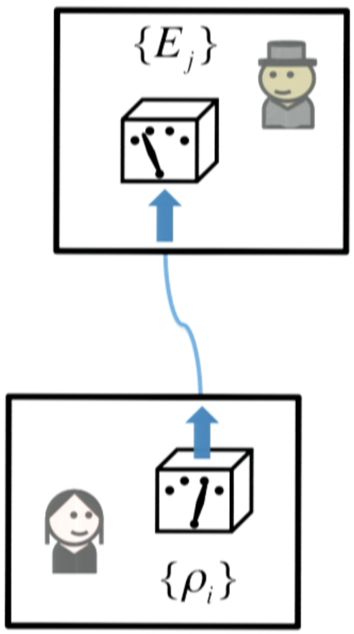
2. Probabilities sum up to 1:

$$\mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(M^{A_1A_2} \otimes M^{B_1B_2}\right)\right] = 1$$

$\forall$ CPTP $M^{A_1A_2}, M^{B_1B_2}$

The original process matrix formalism

Example (simple quantum circuit):



The original process matrix formalism

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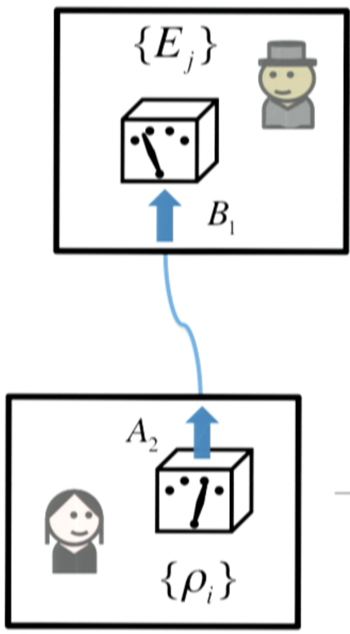

$\longrightarrow E_j = M_j^{B_1}$

$\longrightarrow d_{A_2}\rho_i^T = M_i^{A_2}$

$$p(i,j) = Tr[W^{A_2B_1}(M_i^{A_2} \otimes M_j^{B_1})]$$

$$W^{A_2B_1} = |\Phi^+\rangle\langle\Phi^+|^{A_2B_1}$$

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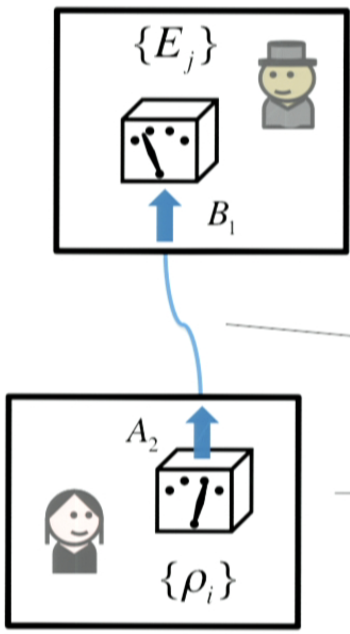

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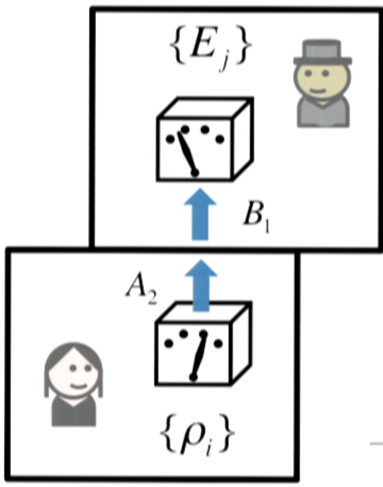
The Choi state of
the identity channel

$\rightarrow d_{A_2}\rho_i^T = M_i^{A_2}$

The original process matrix formalism

Example (simple quantum circuit):

Note: A_2 and B_1 are two separate systems even if the channel is instantaneous.



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The Choi state of
the identity channel

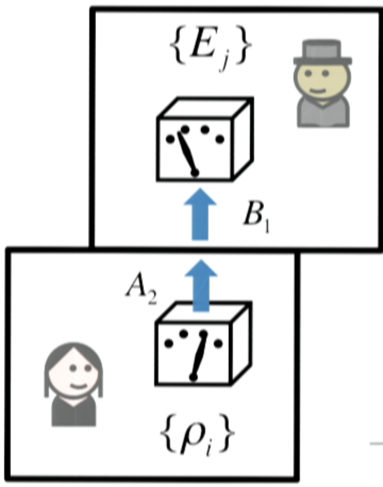
$\rightarrow d_{A_2}\rho_i^T = M_i^{A_2}$

A_2 is NOT the output system of Alice, B_1 is!

Time-symmetric process matrix formalism

Example (simple quantum circuit):

Allowing the more general notion of measurement:



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The Choi state of the identity channel

Time-symmetric process matrix formalism

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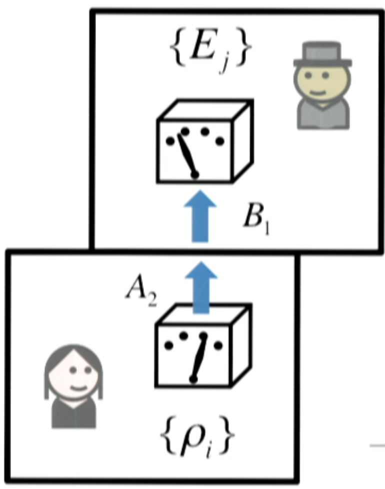


$\longrightarrow E_j = M_j^{B_1}$

$\longrightarrow d_{A_2}\rho_i^T = M_i^{A_2}$

Conditional probabilities for Bob:

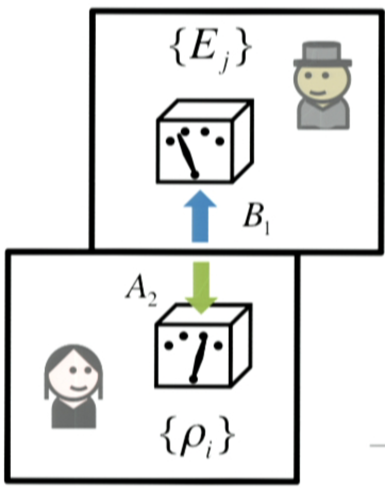
$$p(j\,|\,i) = \frac{Tr[W_i^{B_1} M_j^{B_1}]}{Tr[W_i^{B_1} \bar{M}^{B_1}]}$$

$$W_i^{B_1} = \rho_i^{B_1}$$

The conditional process matrix of Bob
(‘prepared’ by the event in Alice’s lab)

Time-symmetric process matrix formalism

Example (simple quantum circuit):



$\longrightarrow E_j = M_j^{B_1}$

$\longrightarrow d_{A_2}\rho_i^T = M_i^{A_2}$

Conditional probabilities for Alice:

$$p(i\,|\,j) = \frac{Tr[W_j^{A_2} M_i^{A_2}]}{Tr[W_j^{A_2} \bar{M}^{A_2}]}$$

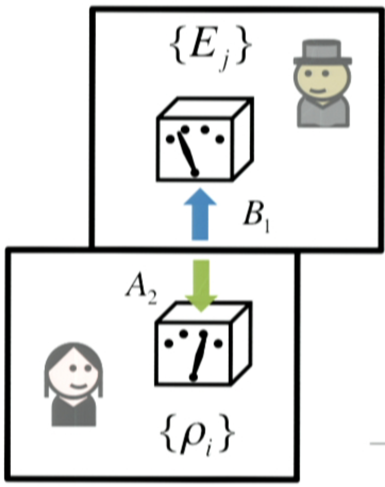
$$W_j^{A_2} = (E_j^T)^{A_2} / d_{B_1}$$

The conditional process matrix of Alice (`prepared' by the event in Bob's lab)

A_2 is like an **input from the future**.

Time-symmetric process matrix formalism

Example (simple quantum circuit):



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Conditional probabilities for Alice:

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$$W_j^{A_2} = (E_j^T)^{A_2} / d_{B_1}$$

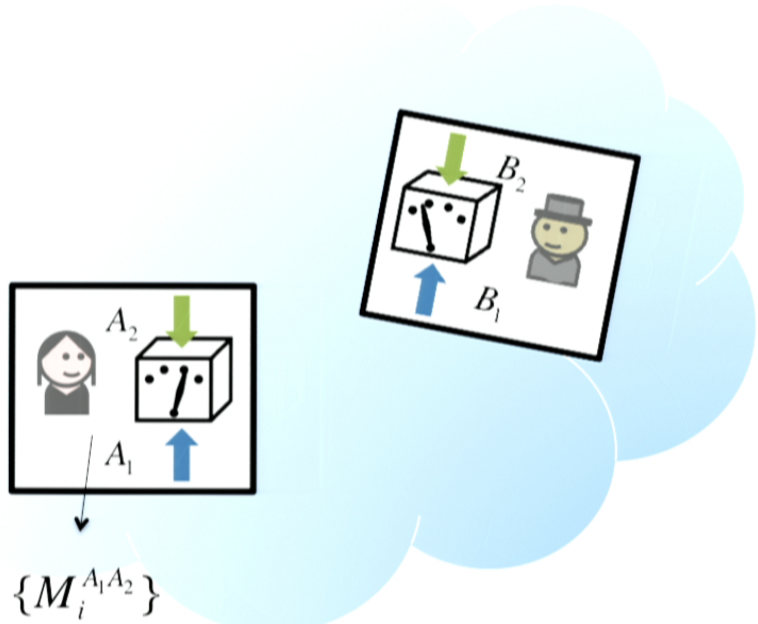
↑ 'retrodictive state'

Barnett, Pegg, Jeffers, J. M. Opt. 47 (2000).
Leifer, Spekkens, arXiv:1107.5849

Our definition differs by a transposition.

Time-symmetric process matrix formalism

The general case:



$\{M_i^{A_1A_2}\}$

$$M_i^{A_1A_2} \geq 0, \quad Tr(\sum_i M_i^{A_1A_2}) \equiv Tr(\bar{M}^{A_1A_2}) = d_{A_1}d_{A_2}$$

Time-symmetric process matrix formalism

The general case:

variables that define the setup (may involve post-selection)

$$p(i, j, \cdots | \{M_i^{A_1A_2}\}, \{M_j^{B_1B_2}\}, \cdots, W) = \frac{Tr[W^{A_1A_2B_1B_2\cdots}(M_i^{A_1A_2} \otimes M_j^{B_1B_2} \otimes \cdots)]}{Tr[W^{A_1A_2B_1B_2\cdots}(\bar{M}^{A_1A_2} \otimes \bar{M}^{B_1B_2} \otimes \cdots)]}$$

The process matrix:

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Time-symmetric process matrix formalism

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Note: Any process matrix can be created using post-selection

Time-symmetric process matrix formalism

Conditional reduced process matrix (un update rule):

$$M^{B_1B_2} : W^{A_1A_2B_1B_2} \rightarrow W^{A_1A_2} = \frac{Tr_{B_1B_2}[W^{A_1A_2B_1B_2}(I^{A_1A_2} \otimes M^{B_1B_2})]}{Tr[W^{A_1A_2B_1B_2}(I^{A_1A_2} \otimes M^{B_1B_2})]}$$

Any PM can be created using post-selection in a circuit simply by teleporting a suitable preselected state onto the respective systems.

But PMs can also describe situations that can be obtained without post-selection and yet go beyond definite circuits, such as mixtures or 'superpositions' of connections!

Time-symmetric process matrix formalism

Even more generally:

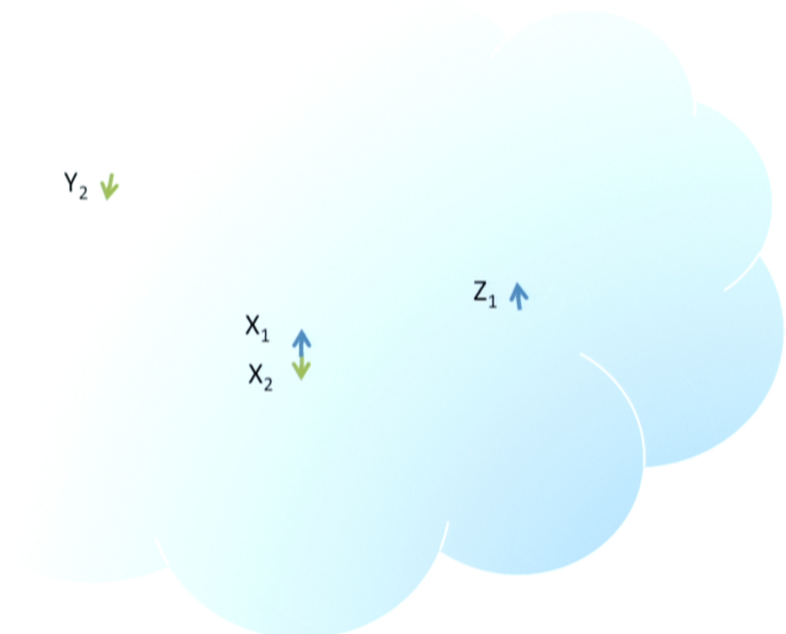
$$p(i \mid \{M_i^{A_1A_2B_1B_2\cdots}\}, W) = \frac{Tr[W^{A_1A_2B_1B_2\cdots} M_i^{A_1A_2B_1B_2\cdots}]}{Tr[W^{A_1A_2B_1B_2\cdots} \bar{M}^{A_1A_2B_1B_2\cdots}]}$$

Alice, Bob, etc., could perform ***non-local operations*** too, with the help of entanglement and post-selection.

→ We do not need to label the systems by the name of the laboratory.

Time-symmetric process matrix formalism

Even more generally:



If we have a notion of point-like locality (such as points in the space-time manifold), may be more natural to label the systems according to it.

Relation to the two-state vector formalism

The Aharonov-Bergmann-Lebowitz (ABL) rule [PRB 134, 1410 (1964)]:

$$p(j|\psi,\phi) = \frac{|\langle\phi|P_j|\psi\rangle|^2}{\sum_i |\langle\phi|P_j|\psi\rangle|^2} \longrightarrow \langle\phi| \quad |\psi\rangle$$

(two-state vector)

The 'backward evolving' state lives in the dual Hilbert space of the 'forward evolving' state.

In contrast, in our formalism forward and backward oriented states are associated with two separate systems. There is no notion of multiplication between them that yields a number – numbers arise by contracting states with effects.

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Relation to the two-state vector formalism

The TSVF has been generalized to include superpositions, mixtures, generalized measurements, and multi-time states times:
E.g. Aharonov, Popescu, Tollaksen, Vaidman [PRA (2009), arXiv:0712.0320]:

$$\mathrm{Prob}(\mu,\nu,\xi) = \frac{1}{N}\left|\left|\sum_{ijkl} \alpha_{ijkl} C_\xi |l\rangle_{t_4 t_3} \langle k|B_\nu|j\rangle_{t_2 t_1} \langle i|A_\mu\right|\right|^2$$

$$= \frac{1}{N} \sum_{ijkli'j'k'l'} \alpha^*_{i'j'k'l'} \alpha_{ijkl}\ {}_{t_4}\langle l'|C^\dagger_\xi C_\xi|l\rangle_{t_4 t_2} \langle j'|B^\dagger_\nu|k'\rangle_{t_3} \times$$

$$\times\ {}_{t_3}\langle k|B_\nu|j\rangle_{t_2 t_1} \langle i|A_\mu A^\dagger_\mu|i'\rangle_{t_1}, \qquad (37)$$

where N is such that $\sum_{\mu,\nu,\xi} \mathrm{Prob}(\mu,\nu,\xi) = 1$.

Relation to the two-state vector formalism

Recent work: Silva, Guryanova, Brunner, Linden, Short, Popecu, arXiv:1308.2089

$$\underline{\Psi} = \sum_{ij} \alpha_{ij} \; {}_{t_2}\langle i| \otimes |j\rangle_{t_1}$$

$$\underline{\eta} = \sum_r p^r (\underline{\Psi}^r \otimes \underline{\Psi}^{r\dagger})$$

density vector

$$\underline{A}^\mu \in \mathcal{H}_{t_1}^{\downarrow} \otimes \mathcal{H}_{t_2}^{\uparrow} \,, \quad \underline{A}^\mu = \sum_{ij} A_{ij}^\mu \, |i\rangle_{t_2} \otimes {}_{t_1}\langle j|$$

Kraus density vector $\underline{K}^\mu = \sum_\chi \underline{A}_\chi^\mu \otimes \underline{A}_\chi^{\mu\dagger}$

They find the isomorphism that maps the TSVF to the one presented here, as well as point the equivalence to detection obtained with postselection!

$$\underline{K}^\mu \bullet \underline{\eta} = tr(\tilde{E}^\mu \rho)$$

$$P(\mu) = \frac{tr(\tilde{E}^\mu \rho)}{\sum_\nu tr(\tilde{E}^\nu \rho)}$$

Definition of causation

"If correlation doesn't imply causation, then what does?"

R. Spekkens, talk at Causal Structure in Quantum Theory, Benasque, 2013.

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Can we have an intervention-free notion of causation?

Definition of causation

In the language of standard quantum operations (in a circuit or process):

If the choice of Alice's operation is correlated with the outcome of Bob:
→ signalling (causal influence) from Alice to Bob.

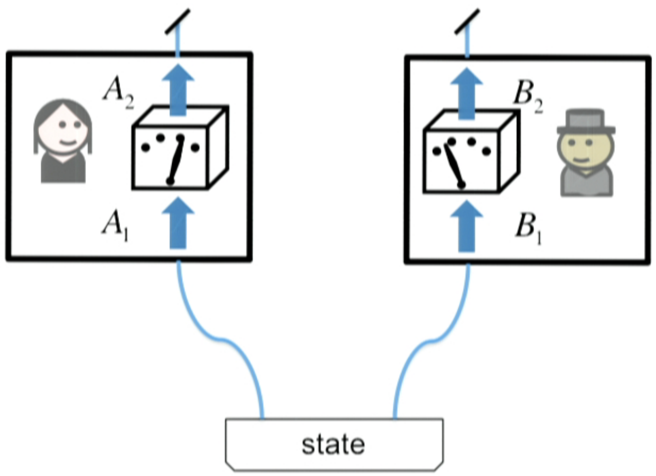
But this formulation fails in the time-symmetric framework due to the more general notion of operation (e.g., it would mean "signalling" between space-like separated measurements).

Can we have an intervention-free notion of causation?

Definition of causation

Consider two experiments in a causal structure in circumstances defined without post-selection.

No signalling b/w Alice to Bob:



$$W^{A_1A_2B_1B_2} = \rho^{A_1B_1} \otimes \frac{I^{A_2}}{d_{A_2}} \otimes \frac{I^{B_2}}{d_{B_2}}$$

Definition of causation

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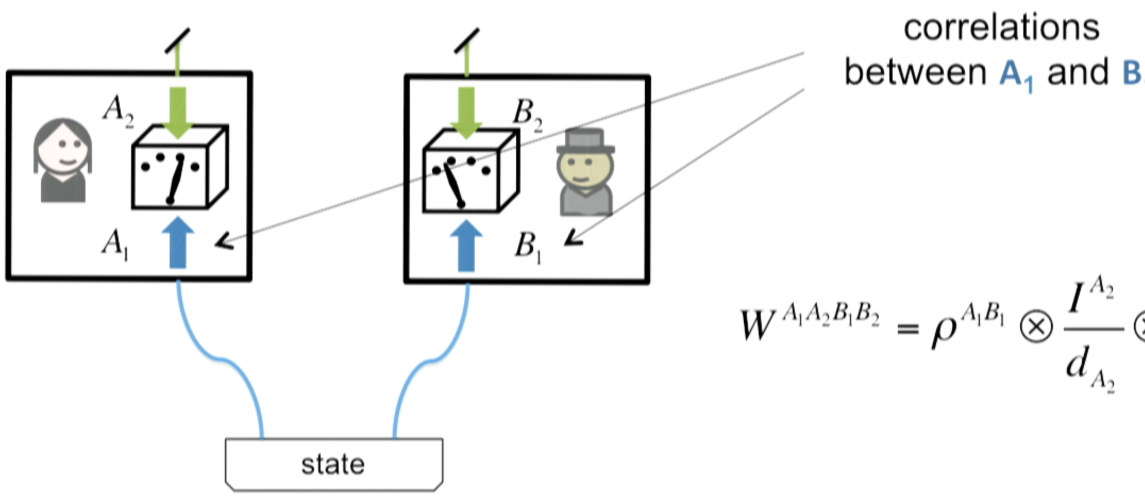

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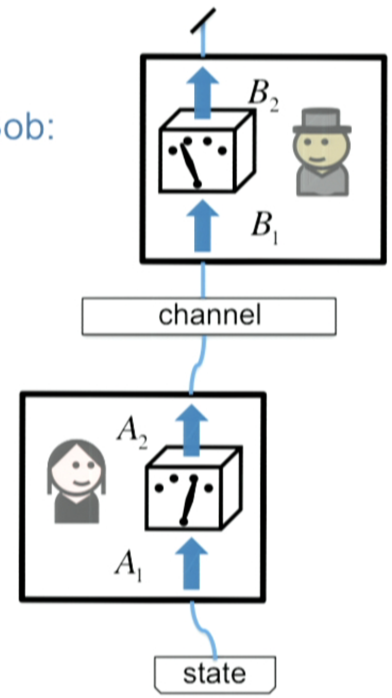
Channel from Alice to Bob:

(when $A_2 \prec B_1$)



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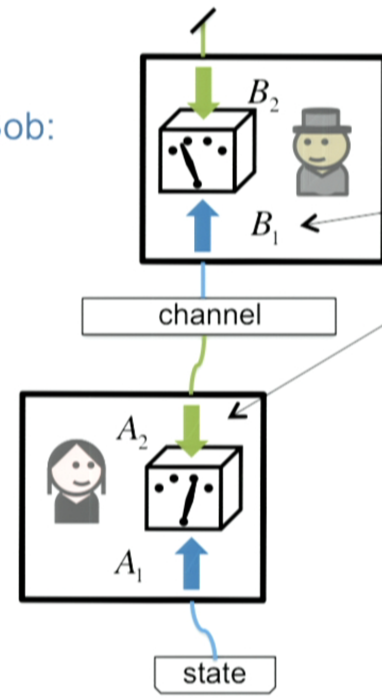


correlations between A_2 and B_1

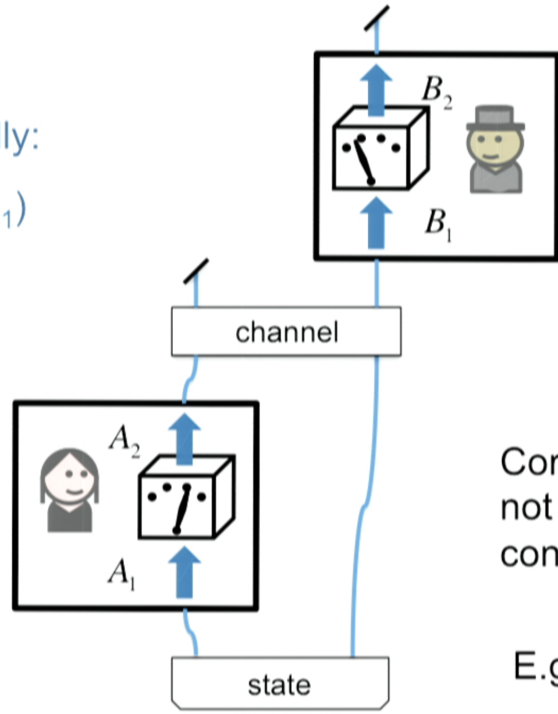
$$W^{A_1A_2B_1B_2} = \rho^{A_1} \otimes W^{A_2B_1} \otimes \frac{I^{B_2}}{d_{B_2}}$$

Definition of causation

Consider two experiments in a causal structure in circumstances defined without post-selection.

More generally:

(when $B_2 \not\prec A_1$)



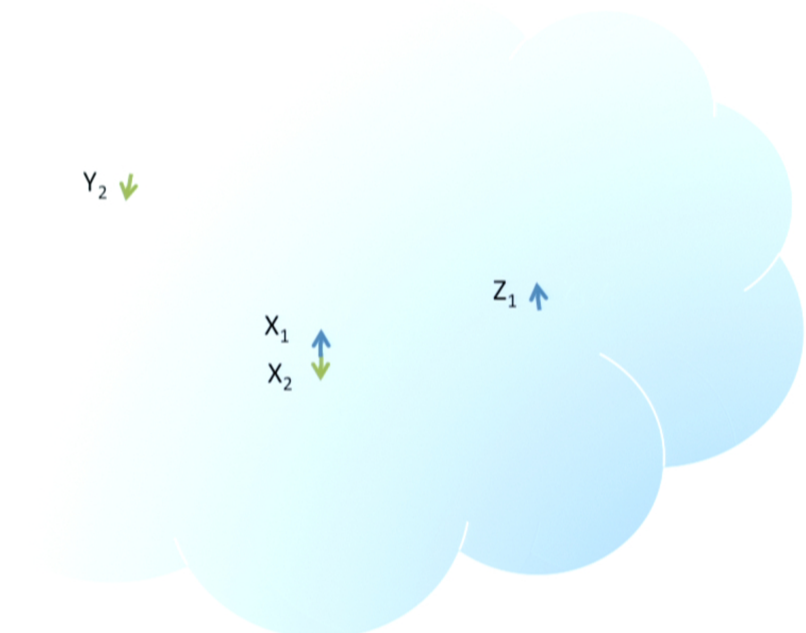
$$W^{A_1A_2B_1B_2} = W^{A_1A_2B_1} \otimes \frac{I^{B_2}}{d_{B_2}}$$

Correlations between A_2 and B_1 may not be *explicit*, but can be revealed conditionally on measurement on A_1.

E.g. $$W^{A_1A_2B_1} = \frac{I^{A_1A_2B_1B_2} + \sigma_Z^{A_1}\sigma_Z^{A_2}\sigma_Z^{B_1}}{d_{A_1}d_{A_2}d_{B_1}}$$

Definition of causation

Proposal: Define signalling (causation) as correlations between system of type **2** and system of type **1**.



Definition of causation

Proposal: Define signalling (causation) as correlations between system of type **2** and system of type **1**.

Definition: There is signalling (causation) from observable $\{M_i^{X_2}\}$ to observable $\{N_j^{Y_1}\}$ in circumstances defined by W, if the joint distribution

$$p(i, j| \{M_i^{X_2}\}, \{N_j^{Y_1}\}, W)$$

is correlated.

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Remark 1: *By definition*, signalling is defined between observables of two different types and goes from type 2 (the cause) to type 1 (the effect).

Remark 2: This notion of signalling is symmetric under time reversal and exchanging cause and effect.

Definition of causation

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Does it make sense in cases obtained through post-selection?

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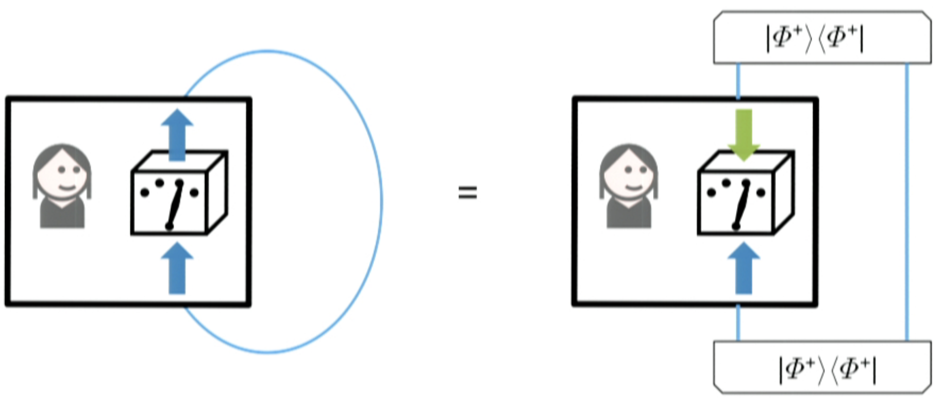
Does it make sense in cases obtained through post-selection?

E.g., are post-selected closed timelike curves true closed timelike curves?

Bennett, Schumacher (2004); Svetlichny (2009); Lloyd et al. (2010); Brun, Wilde (2010), da Silva, Galvao, Kashefi (2010), Genkina, Chiribella, Hardy (2012)...



Quantum field picture

A discrete (lattice) model:

One possible approach

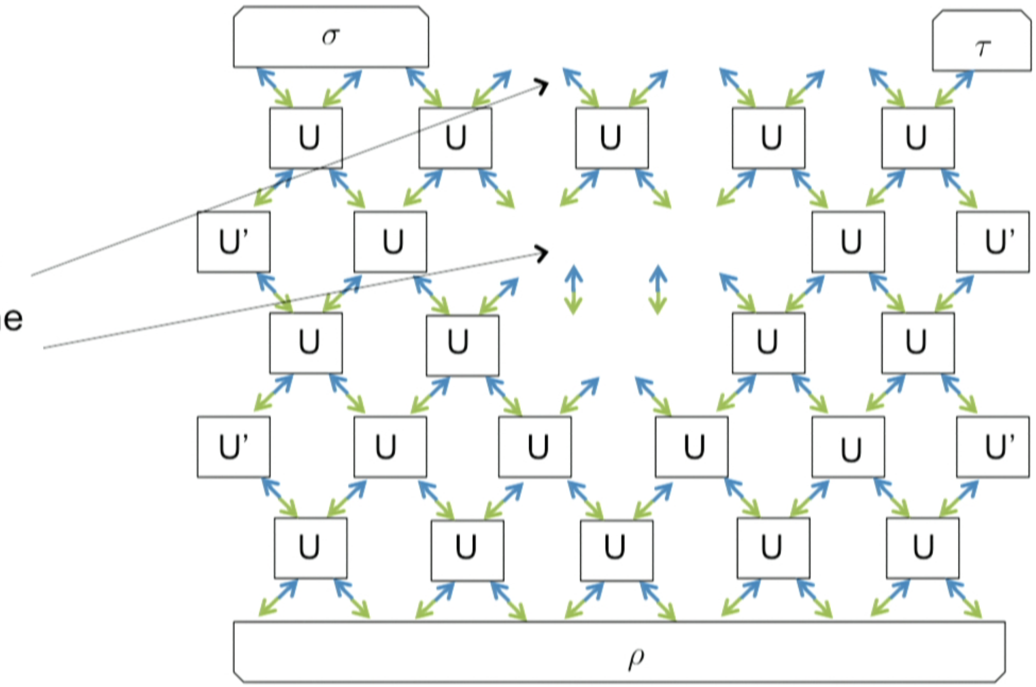
Prior to specifying the dynamics: a “master” process matrix on points in the ‘manifold’:

$$W_{Master} = \bigotimes_x \left| \Phi^+ \right\rangle\left\langle \Phi^+ \right|_x$$

Quantum field picture

A discrete (lattice) model:

Process matrix W associated with the free wires.



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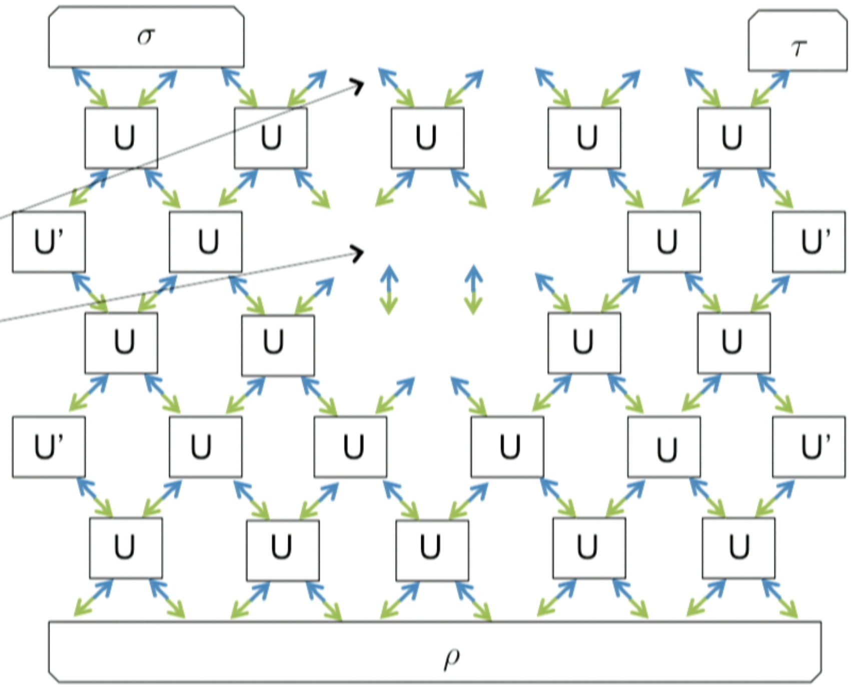
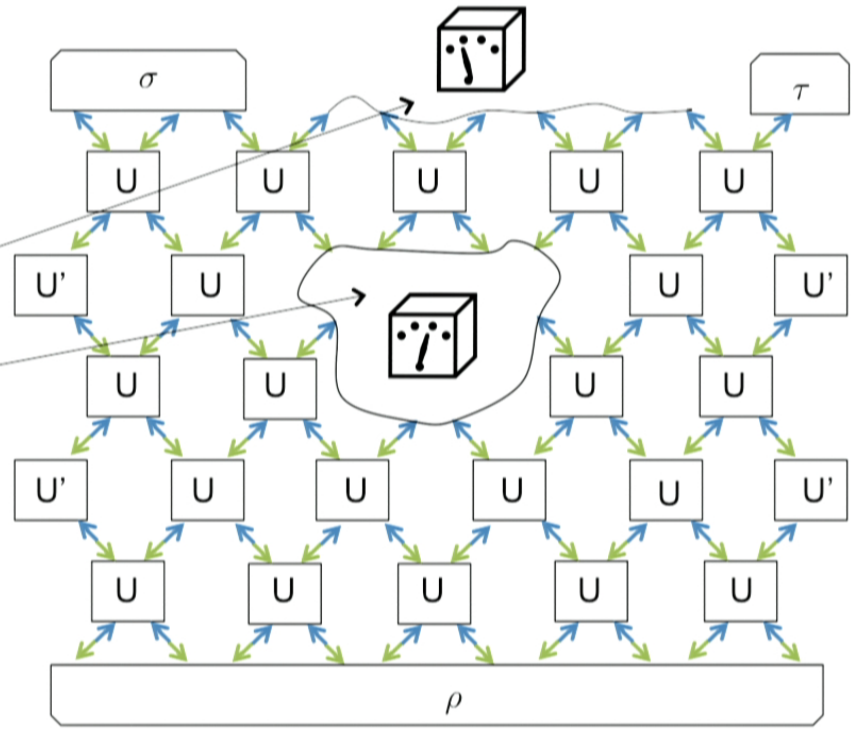


Quantum field picture

A discrete (lattice) model:

Process matrix W associated with the free wires (or the boundary of a cut).

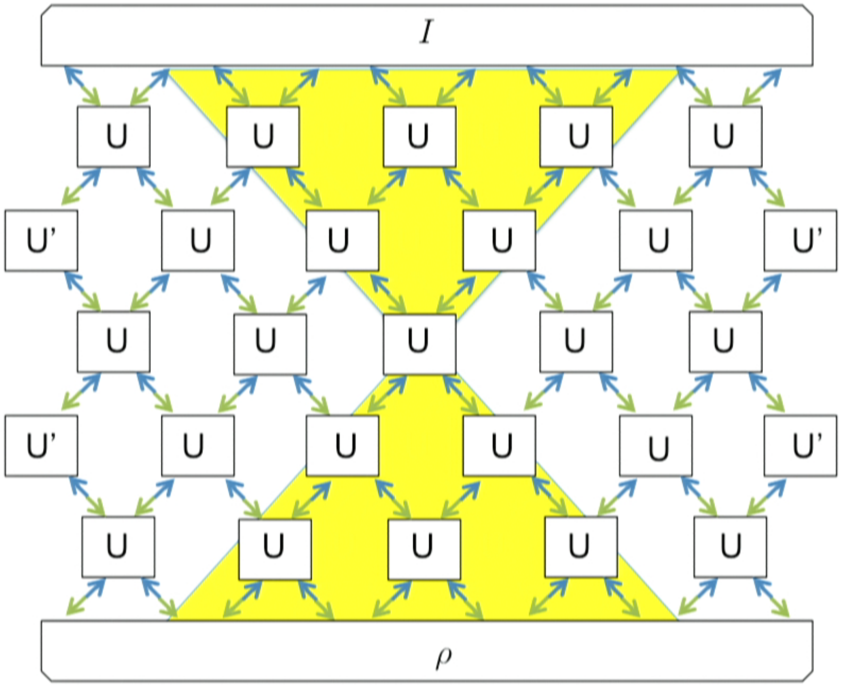


R. Oeckl, General boundary formulation of QFT,
Phys. Lett. B 575 (2003),…
… Found. Phys. 43 (2013).

Quantum field picture

A discrete (lattice) model:

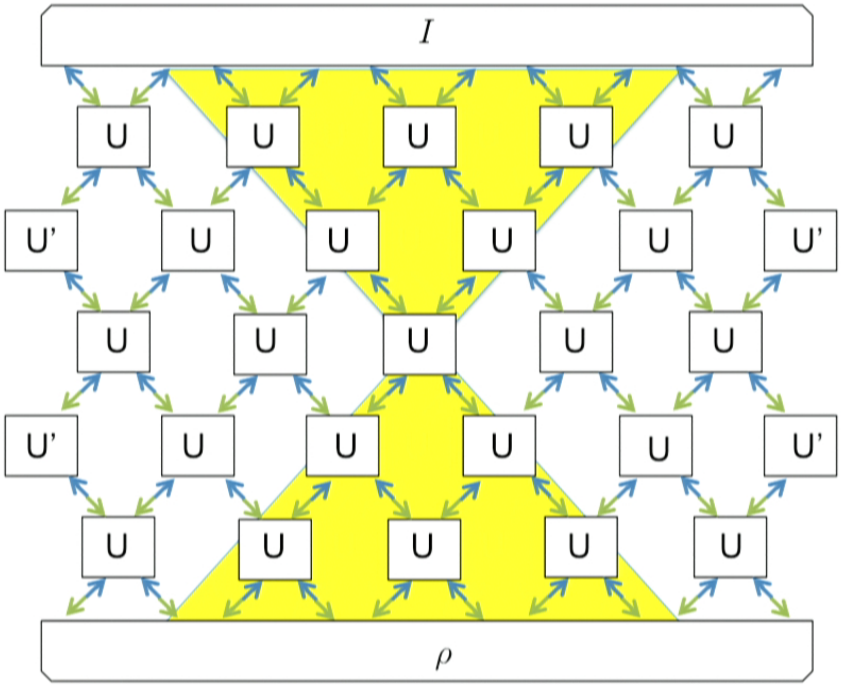
The dynamics defines a causal structure over the underlying 'manifold'.



Quantum field picture

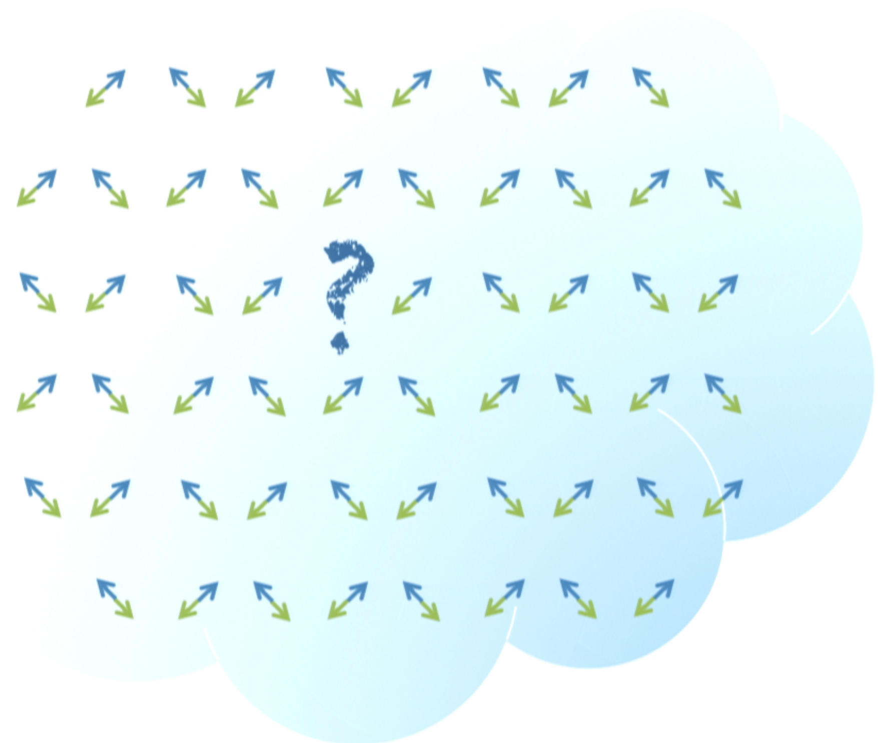
A discrete (lattice) model:

Remark: the claim that without post-selection an operation satisfies the *standard completeness relation* is a statement about the form of the dynamics and the final condition, not a rule of the theory.



Quantum field picture

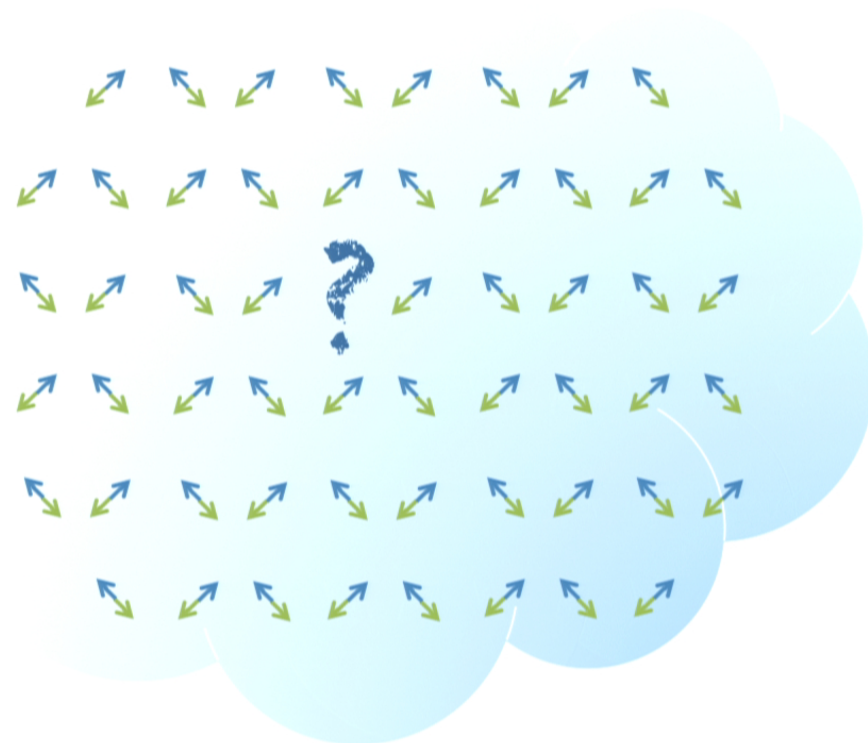
Can this picture help us go beyond a fixed background causal structure?



Quantum field picture

Can this picture help us go beyond a fixed background causal structure?

Is the distinction between ↓ and ↑ fundamental or emergent?



Conclusion

- QM can be seen as a time-symmetric operational probabilistic theory if we drop the assumption that there are special 'complete' operations.

- The CJ isomorphism and the requirement for a local theory lead to the idea of two types of systems at each point, which allows us to treat QM in space-time as 'static' QM on a larger number of systems.

- The formalism is directly related to the two-state (or multitime-state) vector formalism, but it differs in that it postulates two separate systems at each point, which yields an elegant mathematical formalism. It also captures situations beyond definite causal structure.

- Assuming a local distinction between `forward' and `backward' systems, we can give an intervention-independent definition of causation which agrees with the usual notion in cases without post-selection.

- The framework generalizes the process matrix formalism, including post-selection, CTCs and other exotic structures. It may suggest new ways of thinking about quantum gravity.

Outlook

- What are the process matrices that can be created without post-selection?
- Is the distinction between 'forward' and 'backward' systems fundamental or emergent?
- Could the postulate of two types of systems suggest new physical effects?
- Can we still have a convex time-symmetric theory?
- Insights into the foundations of QM?
- Insights for quantum information?
- Continuous quantum field theory in the process matrix framework?
- …..