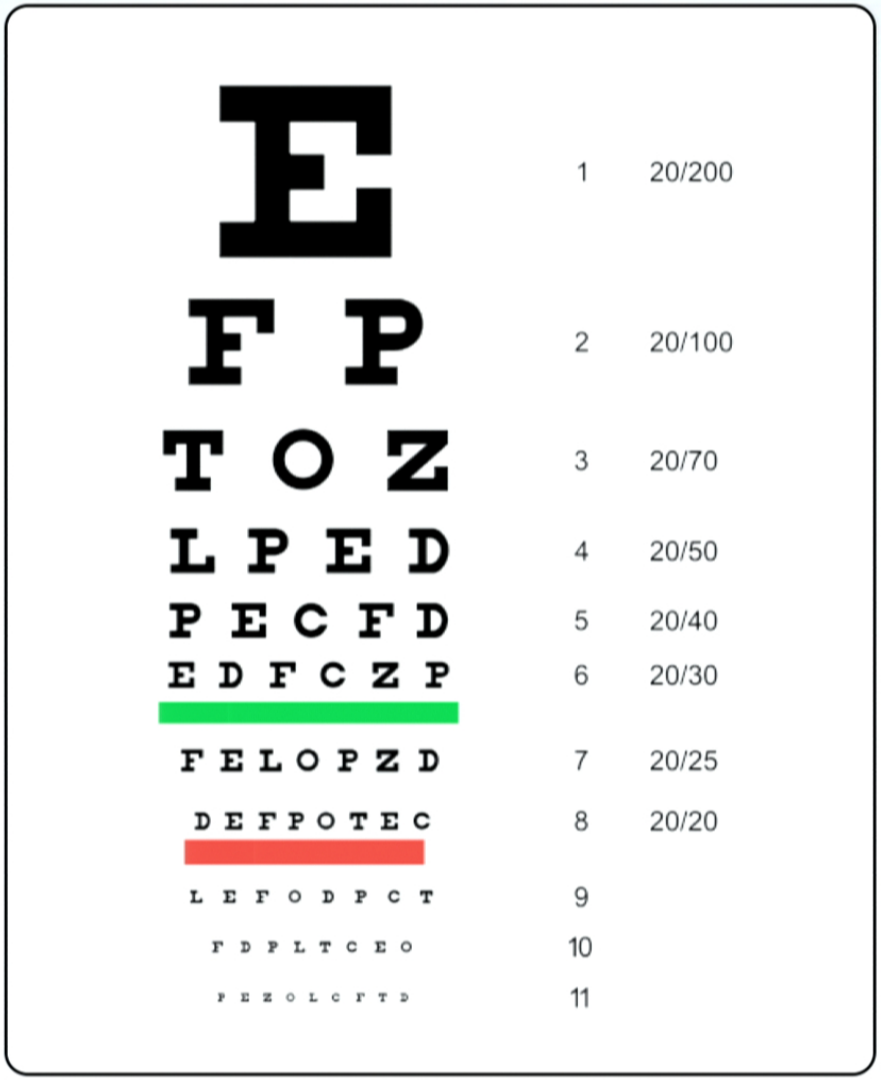
Title: The Amplitude Mode in Condensed Matter : Higgs Hunting on a Budget

Date: Nov 19, 2013 03:30 PM

URL: http://pirsa.org/13110054

Abstract: The amplitude mode is a ubiquitous phenomenon in systems with broken continuous symmetry and effective relativistic dynamics, and has been observed in magnets, charge density waves, cold atom systems, and superconductors. It is a simple analog of the Higgs boson of particle physics. I will discuss the properties of the amplitude mode and its somewhat surprising visibility in two-dimensional systems, recently confirmed in cold atom experiments. The behavior in the vicinity of a quantum critical point will be stressed, comparing theoretical, numerical, and experimental results.


E 1 20/200
F P 2 20/100
T O Z 3 20/70
L P E D 4 20/50
P E C F D 5 20/40
E D F C Z P 6 20/30
F E L O P Z D 7 20/25
D E F P O T E C 8 20/20
L E F O D P C T 9
F D P L T C E O 10
F E Z O L C F T D 11

San Diego

Toronto

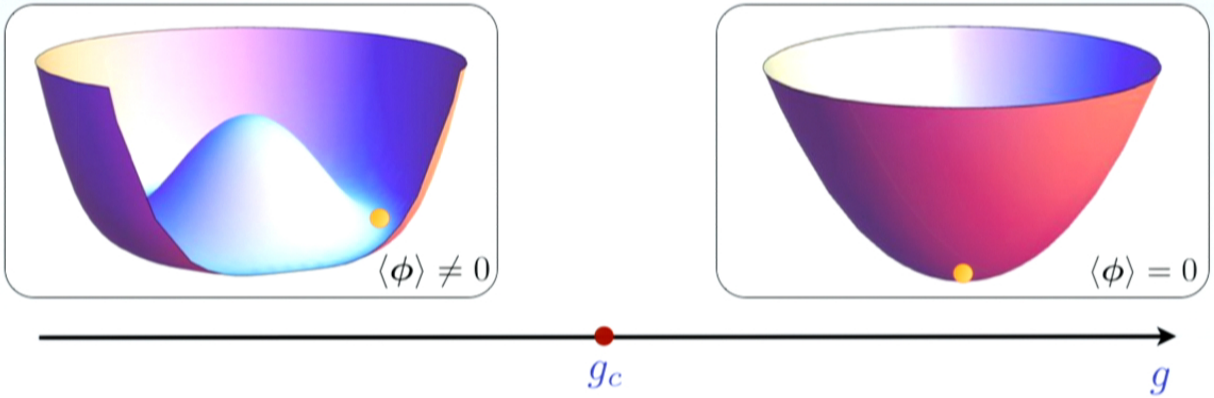

Seminar
BINGO!
To play, simply print out this bingo sheet and attend a departmental seminar.
Mark over each square that occurs throughout the course of the lecture.
The first one to form a straight line (or all four corners) must yell out BINGO!! to win!
BINGO!!
SEMINAR
B I N G O
Speaker bashes previous work
Repeated use of "um..."
Speaker sucks up to host professor
Host Professor falls asleep
Speaker wastes 5 minutes explaining outline
Laptop malfunction
Work ties in to Cancer/HIV or War on Terror
"...et al."
You're the only one in your lab that bothered to show up
Blatant typo
Entire slide filled with equations
"The data clearly shows..."
FREE
Speaker runs out of time
Use of Powerpoint template with blue background
References Advisor (past or present)
There's a Grad Student wearing same clothes as yesterday
Bitter Post-doc asks question
"That's an interesting question"
"Beyond the scope of this work"
Master's student bobs head fighting sleep
Speaker forgets to thank collaborators
Cell phone goes off
You've no idea what's going on
"Future work will..."
Results conveniently show improvement
JORGE CHAM © 2007
WWW.PHDCOMICS.COM

Visibility of the amplitude mode in condensed matter and cold atom systems

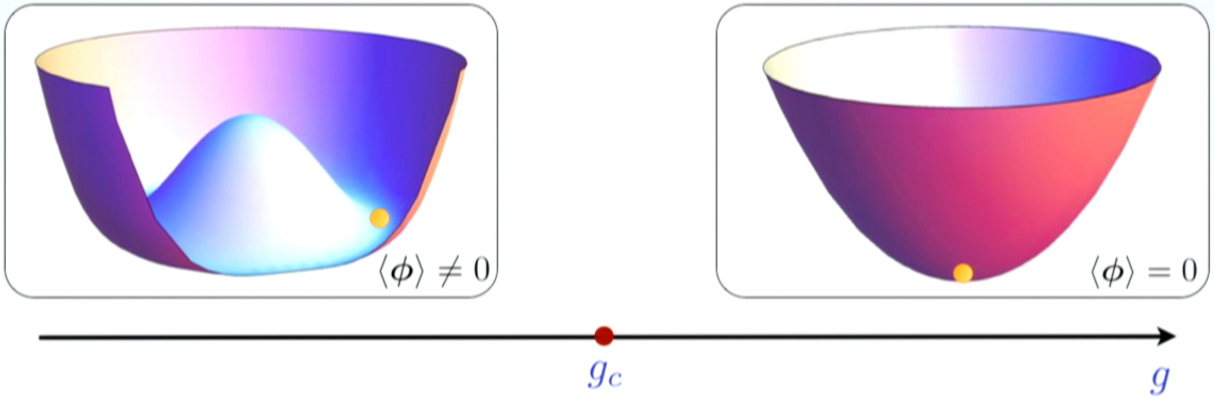
- Higgs hunting on a budget -

Snir Gazit, Assa Auerbach, Daniel Podolsky, DPA

Spontaneous Symmetry Breaking



Spontaneous Symmetry Breaking



$N-$component real scalar field : $\boldsymbol{\phi}^{\mathrm{t}} = (\phi_1, \ldots, \phi_N)$

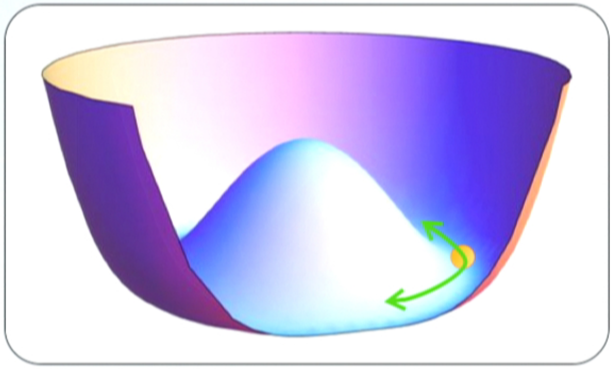
Model action ($T=0$) : $S[\boldsymbol{\phi}] = \frac{1}{g}\int d^d x \int dt \left[\frac{1}{2}(\partial_t \boldsymbol{\phi})^2 - \frac{1}{2}(\boldsymbol{\nabla}\boldsymbol{\phi})^2 - V(\boldsymbol{\phi}) \right]$

"Mexican hat" potential : $V(\boldsymbol{\phi}) = \frac{m_0^2}{8N}(\boldsymbol{\phi}^2 - N)^2$ O(N) symmetry

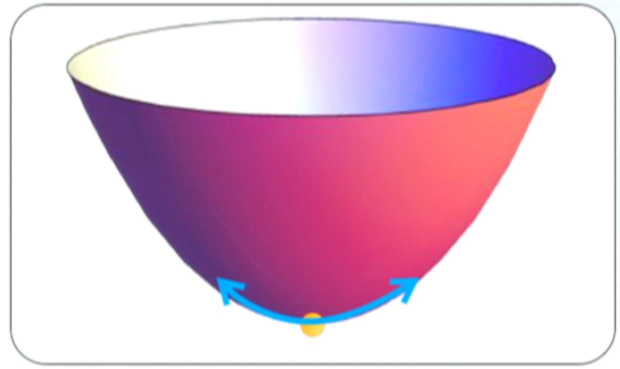
Basic physics : if g small, $\boldsymbol{\phi}$ is forced to local minimum of V, and $\langle \boldsymbol{\phi} \rangle \neq 0$

if g large, fluctuations overwhelm tendency to order, $\langle \boldsymbol{\phi} \rangle = 0$

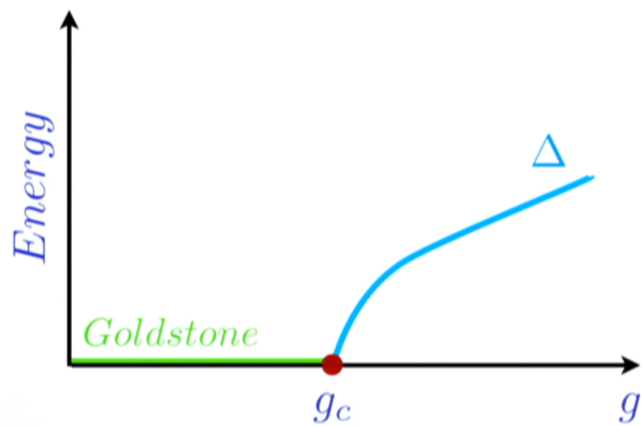
Collective excitations in broken and unbroken phases



broken phase



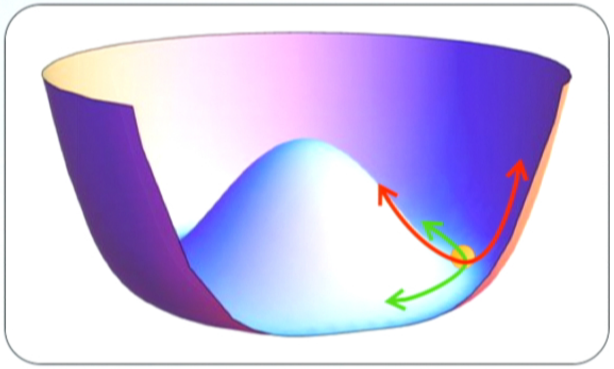
unbroken phase



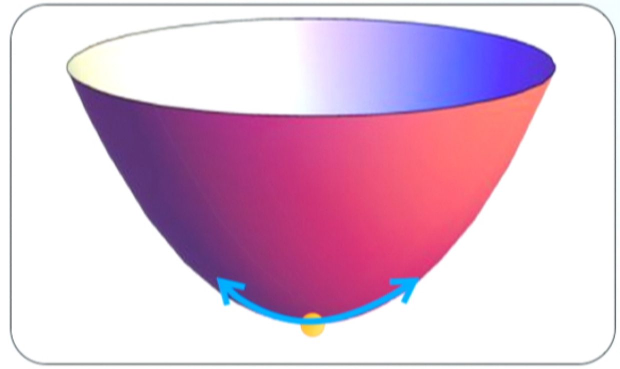
Mean field:

$\Delta \sim A|g - g_c|^{0.5}$

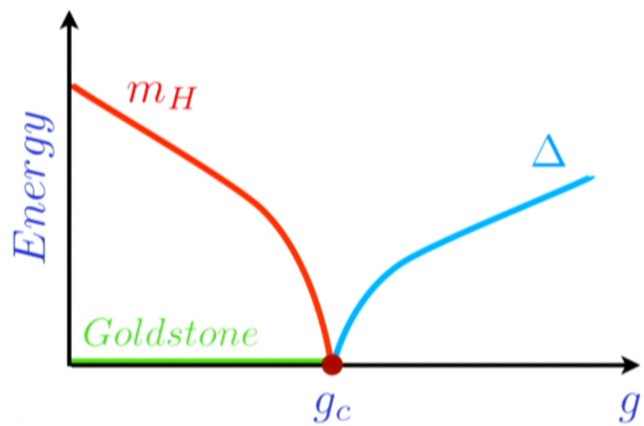
Collective excitations in broken and unbroken phases



broken phase



unbroken phase



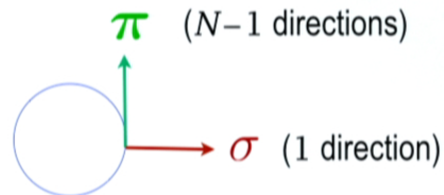
Mean field:

$$\Delta \sim A|g - g_c|^{0.5}$$

$$m_H \sim \sqrt{2}A|g - g_c|^{0.5}$$

Broken symmetry and collective modes

Two ways to parameterize deviations from the ordered state :



1) Cartesian : $\phi = (\sqrt{N} + \sigma, \boldsymbol{\pi})$

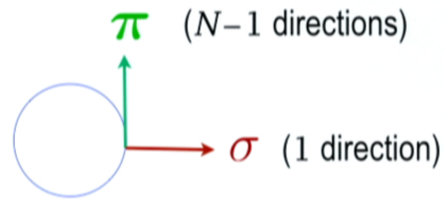
$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$

$$\mathcal{L}_1 = \frac{m^2}{2g}\left[\tfrac{1}{\sqrt{N}}\,\sigma\boldsymbol{\pi}^2 + \tfrac{1}{\sqrt{N}}\,\sigma^3 + \tfrac{1}{4N}\,\sigma^4 + \tfrac{2}{N}\,\sigma^2\boldsymbol{\pi}^2 + \tfrac{1}{4N}(\boldsymbol{\pi}^2)^2\right]$$

Broken symmetry and collective modes

Two ways to parameterize deviations from the ordered state :

1) Cartesian : $\phi = \left(\sqrt{N} + \sigma, \boldsymbol{\pi}\right)$



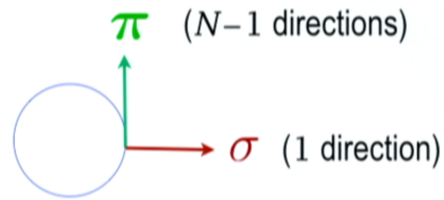
$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$

$$\mathcal{L}_1 = \frac{m^2}{2g}\left[\frac{1}{\sqrt{N}}\,\sigma\boldsymbol{\pi}^2 + \frac{1}{\sqrt{N}}\,\sigma^3 + \frac{1}{4N}\,\sigma^4 + \frac{2}{N}\,\sigma^2\boldsymbol{\pi}^2 + \frac{1}{4N}(\boldsymbol{\pi}^2)^2\right]$$

Broken symmetry and collective modes

Two ways to parameterize deviations from the ordered state :

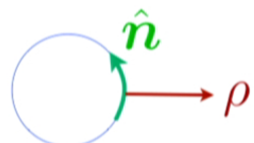
1) Cartesian : $\phi = (\sqrt{N} + \sigma, \boldsymbol{\pi})$



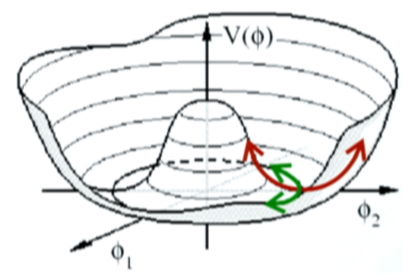
$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$

$$\mathcal{L}_1 = \frac{m^2}{2g}\left[\tfrac{1}{\sqrt{N}}\,\sigma\boldsymbol{\pi}^2 + \tfrac{1}{\sqrt{N}}\,\sigma^3 + \tfrac{1}{4N}\,\sigma^4 + \tfrac{2}{N}\,\sigma^2\boldsymbol{\pi}^2 + \tfrac{1}{4N}(\boldsymbol{\pi}^2)^2\right]$$

2) Polar : $\phi = \sqrt{N}\,(1+\rho)^{1/2}\,\hat{\boldsymbol{n}}$



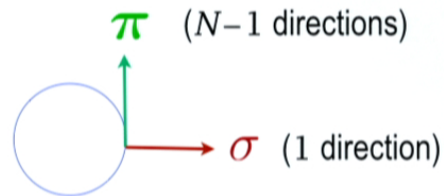
$$\mathcal{L} = \frac{1}{2g}\left[N(1+\rho)(\partial_\mu \hat{\boldsymbol{n}})^2 + \frac{(\partial_\mu \rho)^2}{4(N+\rho)} + \frac{m^2\rho^2}{4N}\right]$$



Broken symmetry and collective modes

Two ways to parameterize deviations from the ordered state :

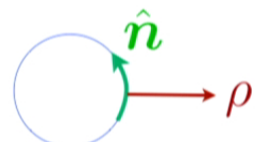
1) Cartesian : $\phi = (\sqrt{N} + \sigma, \boldsymbol{\pi})$



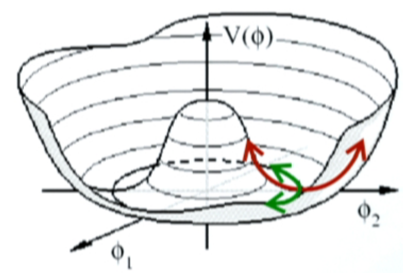
$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$

$$\mathcal{L}_1 = \frac{m^2}{2g}\left[\boxed{\frac{1}{\sqrt{N}}\,\sigma\boldsymbol{\pi}^2} + \frac{1}{\sqrt{N}}\,\sigma^3 + \frac{1}{4N}\,\sigma^4 + \frac{2}{N}\,\sigma^2\boldsymbol{\pi}^2 + \frac{1}{4N}(\boldsymbol{\pi}^2)^2\right]$$

2) Polar : $\phi = \sqrt{N}\,(1+\rho)^{1/2}\,\hat{\boldsymbol{n}}$



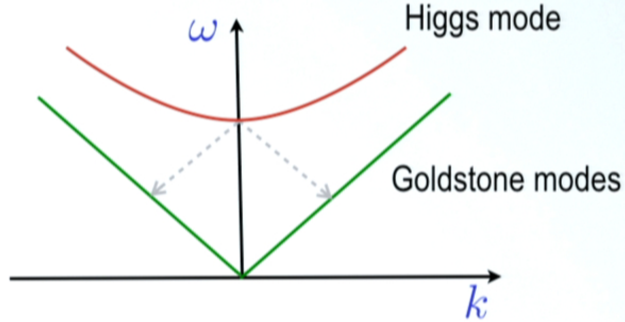
$$\mathcal{L} = \frac{1}{2g}\left[N(1+\boxed{\rho)(\partial_\mu \hat{\boldsymbol{n}})^2} + \frac{(\partial_\mu \rho)^2}{4(N+\rho)} + \frac{m^2\rho^2}{4N}\right]$$



The Higgs mode is unstable

It can decay into a pair of Goldstone bosons :

$$\mathcal{L}_{\rm int} \propto \begin{cases} \sigma \boldsymbol{\pi}^2 & \text{(Cartesian)} \\ \rho\,(\partial_\mu \hat{\boldsymbol{n}})^2 & \text{(polar)} \end{cases}$$



What is the lifetime of the Higgs?

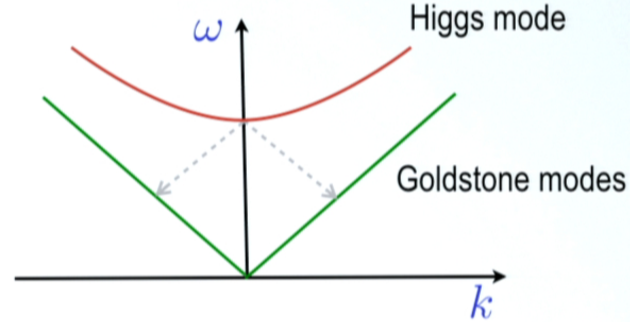
longitudinal response : $\chi_{\sigma\sigma}(k) = \int d^{d+1}k\, \langle \sigma(x)\, \sigma(0)\rangle\, e^{ikx} = \frac{g}{k^2 + m^2 - g\Sigma_\sigma(k)}$

***d*=3 : finite *N* model has D=4 as its UCD. As *g*→*g*c, longitudinal mode becomes sharp.**

The Higgs mode is unstable

It can decay into a pair of Goldstone bosons :

$$\mathcal{L}_{\text{int}} \propto \begin{cases} \sigma \boldsymbol{\pi}^2 & \text{(Cartesian)} \\ \rho\, (\partial_\mu \hat{\boldsymbol{n}})^2 & \text{(polar)} \end{cases}$$



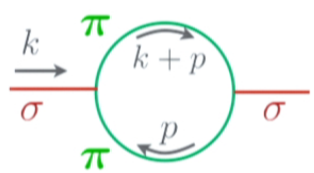
What is the lifetime of the Higgs?

longitudinal response : $\chi_{\sigma\sigma}(k) = \int d^{d+1}k \, \langle \sigma(x)\, \sigma(0) \rangle \, e^{ikx} = \dfrac{g}{k^2 + m^2 - g\Sigma_\sigma(k)}$

***d*=3 : finite *N* model has D=4 as its UCD. As $g \to g_c$, longitudinal mode becomes sharp.**

d*=2 : self-energy to lowest order in *g

$$\Sigma_\sigma(k) =$$



$$\propto \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2(p+k)^2} = \frac{1}{8|k|}$$

infrared divergent!

Analytic continuation from Euclidean result : $\text{Im}\, \Sigma_\sigma(\boldsymbol{k} = 0, \omega) \propto \dfrac{1}{\omega}$

Higgs is overdamped?

(Nepomnyaschii)[2] (1978)
Sachdev (1999), Zwerger (2004)

"The pole at the position of the Higgs energy $\omega = \pm(c^2k^2+2|r|)^{1/2}$ has disappeared, and we only get a branch cut having its onset at the spin-wave energy $\omega = \pm ck$. Thus, for $d<3$ and $N>1$, there is no Higgs particle, and only a broad continuum of multiple spin-wave excitations in the longitudinal response."

S. Sachdev, "Quantum Phase Transitions" (ch. 7)

“The pole at the position of the Higgs energy $\omega = \pm(c^2k^2+2|r|)^{1/2}$ has disappeared, and we only get a branch cut having its onset at the spin-wave energy $\omega = \pm ck$. Thus, for $d<3$ and $N>1$, there is no Higgs particle, and only a broad continuum of multiple spin-wave excitations in the longitudinal response.”

S. Sachdev, “Quantum Phase Transitions” (ch. 7)

"The pole at the position of the Higgs energy $\omega = \pm(c^2k^2+2|r|)^{1/2}$ has disappeared, and we only get a branch cut having its onset at the spin-wave energy $\omega = \pm ck$. Thus, for $d<3$ and $N>1$, there is no Higgs particle, and only a broad continuum of multiple spin-wave excitations in the longitudinal response."

S. Sachdev, "Quantum Phase Transitions" (ch. 7)

Longitudinal versus scalar measurements

Longitudinal versus scalar measurements

The scalar response function is measured by an experiment where the probe couples directly to the magnitude of the order parameter field:

$$S_{\text{probe}} = \int d^d x \int dt \, u(\boldsymbol{x}, t) \left| \boldsymbol{\phi}(\boldsymbol{x}, t) \right|^2 \quad ; \quad |\boldsymbol{\phi}|^2 = N(1 + \rho)$$

Linear response theory : $\langle \rho(\boldsymbol{k}, \omega) \rangle = N \chi_{\rho\rho}(\boldsymbol{k}, \omega) \, u(\boldsymbol{k}, \omega)$

Correlation function: $\chi^{ab}(\boldsymbol{k}, \omega_n) = \int_0^{\hbar\beta} d\tau \int d^d x \, \langle \rho(\boldsymbol{x}, \tau) \, \rho(0, 0) \rangle \, e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_n \tau)}$

Longitudinal versus scalar measurements

The scalar response function is measured by an experiment where the probe couples directly to the magnitude of the order parameter field:

$$S_{\mathrm{probe}} = \int d^d x \int dt \; u(\boldsymbol{x}, t) \left| \boldsymbol{\phi}(\boldsymbol{x}, t) \right|^2 \quad ; \quad |\boldsymbol{\phi}|^2 = N(1 + \rho)$$

Linear response theory : $\left\langle \rho(\boldsymbol{k}, \omega) \right\rangle = N \chi_{\rho\rho}(\boldsymbol{k}, \omega) \, u(\boldsymbol{k}, \omega)$

Correlation function: $\chi^{ab}(\boldsymbol{k}, \omega_n) = \int_0^{\hbar\beta} d\tau \int d^d x \left\langle \rho(\boldsymbol{x}, \tau) \, \rho(0, 0) \right\rangle e^{i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega_n \tau)}$

Examples : two-magnon Raman scattering in antiferromagnet insulators
pump-probe spectroscopy in charge density wave systems

Fleury-Loudon-Elliott form of the effective Raman Hamiltonian :

$$\mathcal{H}_{\mathrm{R}} = D \sum_{\langle ij \rangle} \left(\boldsymbol{\epsilon}_{\mathrm{in}} \cdot \hat{\boldsymbol{r}}_{ij} \right) \left(\boldsymbol{\epsilon}_{\mathrm{out}} \cdot \hat{\boldsymbol{r}}_{ij} \right) \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\propto \boldsymbol{\epsilon}_{\mathrm{in}} \cdot \boldsymbol{\epsilon}_{\mathrm{out}} \int d^d x \left(\left| \boldsymbol{\phi}(\boldsymbol{x}) \right|^2 + \ldots \right)$$

direct coupling of light to pairs of spin waves

Fleury (1969), Freitas and Singh (2000)

Longitudinal versus scalar measurements

The scalar response function is measured by an experiment where the probe couples directly to the magnitude of the order parameter field:

$$S_{\rm probe} = \int d^d x \int dt\, u(\boldsymbol{x},t) \left|\boldsymbol{\phi}(\boldsymbol{x},t)\right|^2 \quad ; \quad |\boldsymbol{\phi}|^2 = N(1+\rho)$$

Linear response theory : $\big\langle \rho(\boldsymbol{k},\omega) \big\rangle = N \chi_{\rho\rho}(\boldsymbol{k},\omega)\, u(\boldsymbol{k},\omega)$

Correlation function: $\chi^{ab}(\boldsymbol{k},\omega_n) = \int_0^{\hbar\beta} d\tau \int d^d x \,\big\langle \rho(\boldsymbol{x},\tau)\,\rho(0,0)\big\rangle\, e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_n \tau)}$

Examples : two-magnon Raman scattering in antiferromagnet insulators
pump-probe spectroscopy in charge density wave systems

Fleury-Loudon-Elliott form of the effective Raman Hamiltonian :

$$\mathcal{H}_{\rm R} = D \sum_{\langle ij \rangle} \left(\boldsymbol{\epsilon}_{\rm in} \cdot \hat{\boldsymbol{r}}_{ij}\right)\left(\boldsymbol{\epsilon}_{\rm out} \cdot \hat{\boldsymbol{r}}_{ij}\right) \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\propto \boldsymbol{\epsilon}_{\rm in} \cdot \boldsymbol{\epsilon}_{\rm out} \int d^d x \left(\left|\boldsymbol{\phi}(\boldsymbol{x})\right|^2 + \ldots \right)$$

direct coupling of light to pairs of spin waves

Fleury (1969), Freitas and Singh (2000)

Relation between longitudinal and scalar susceptibilities

The radial field is expressed in terms of the longitudinal field σ and the transverse field π :

$$\rho = \frac{\phi^2}{N} - 1 = \frac{2\sigma}{\sqrt{N}} + \frac{\sigma^2 + \pi^2}{N}$$

Relation between longitudinal and scalar susceptibilities

The radial field is expressed in terms of the longitudinal field σ and the transverse field π :

$$\rho = \frac{\phi^2}{N} - 1 = \frac{2\sigma}{\sqrt{N}} + \frac{\sigma^2 + \pi^2}{N}$$

Therefore, one finds $\chi_{\rho\rho} = \frac{1}{N}\left(4\,\chi_{\sigma\sigma} + \chi_{\text{sing}} + \chi_{\text{reg}}\right)$ with

$$\chi_{\text{sing}} = \frac{4}{\sqrt{N}}\,\chi_{\sigma\pi^2} + \frac{1}{N}\,\chi_{\pi^2\pi^2}$$

$$\chi_{\text{reg}} = \frac{1}{N}\,\chi_{\sigma^2\sigma^2} + \frac{4}{\sqrt{N}}\,\chi_{\sigma\sigma^2} + \frac{2}{N}\,\chi_{\sigma^2\pi^2}$$

Large N theory

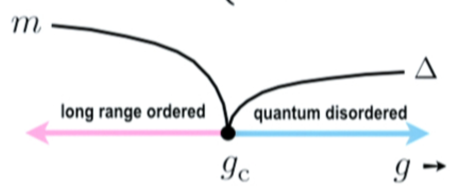
Large N theory

$$\langle \boldsymbol{\phi} \rangle = r\sqrt{N}\hat{\mathbf{e}}_1$$
$$m = r m_0$$

Controlled expansion in powers of $1/N$ about $N \to \infty$ limit.

$N \to \infty$ renormalization of order parameter : $r^2(g, \Lambda) = 1 - g \int^{\Lambda} \frac{d^{d+1}k}{(2\pi)^{d+1}} \frac{1}{k^2} \equiv 1 - \frac{g}{g_c}$

Critical coupling : $g_c = \begin{cases} 4\pi/\Lambda & d = 2 \\ 8\pi^2/\Lambda^2 & d = 3 \end{cases}$



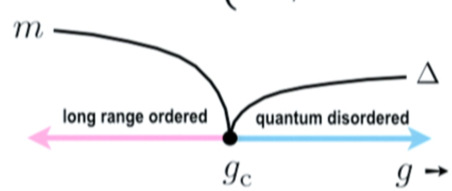
Large N theory

$$\langle \boldsymbol{\phi} \rangle = r\sqrt{N}\hat{\mathbf{e}}_1$$
$$m = r m_0$$

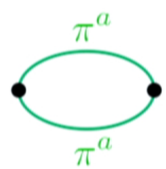
Controlled expansion in powers of $1/N$ about $N \to \infty$ limit.

$N \to \infty$ renormalization of order parameter : $$r^2(g,\Lambda) = 1 - g\int^{\Lambda} \frac{d^{d+1}k}{(2\pi)^{d+1}}\frac{1}{k^2} \equiv 1 - \frac{g}{g_c}$$

Critical coupling : $$g_c = \begin{cases} 4\pi/\Lambda & d=2 \\ 8\pi^2/\Lambda^2 & d=3 \end{cases}$$



$$\Sigma_\sigma(k) = \frac{\Pi^0(k)}{1 + g\,\Pi^0(k)/m^2} \qquad \Pi^0(k) =$$

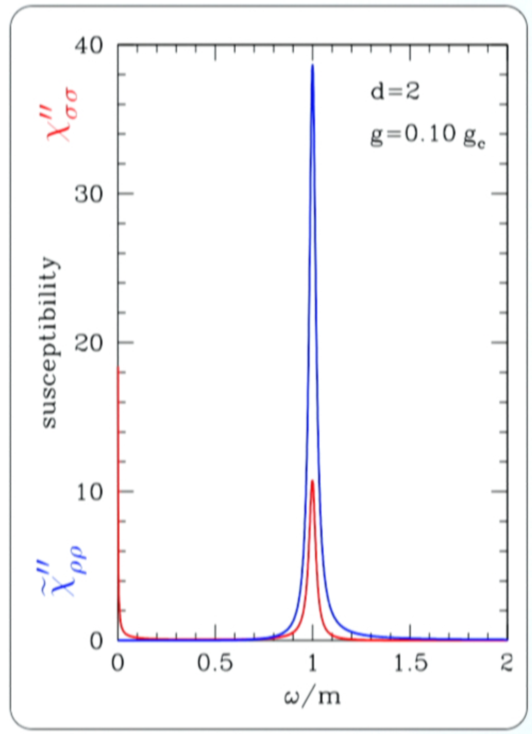


$$\chi_{\sigma\sigma}(\boldsymbol{k}=0,\omega) = \frac{g}{\omega^2 - m^2 - g\Sigma_\sigma(\omega)}$$

$\Sigma_\sigma(0) = -m^2/g$
(Ward identity)

$$\widetilde{\chi}_{\rho\rho}(\boldsymbol{k}=0,\omega) = \frac{4g(m/m_0)^2}{\omega^2 - m^2}\left(1 + \frac{\omega^4}{m^4}\,\frac{g\Sigma_\sigma(\omega)}{\omega^2 - m^2 - g\Sigma_\sigma(\omega)}\right)$$

$\widetilde{\chi}_{\rho\rho} = N\chi_{\rho\rho}$

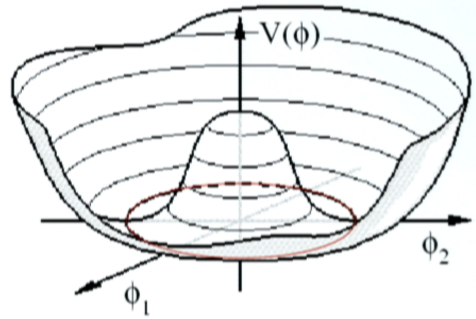


Why are scalar measurements sharper?

Longitudinal *versus* scalar perturbations :

$$S_{\mathrm{long}} = \int d^d x \int dt \, h(\boldsymbol{x}, t)\, \sigma(\boldsymbol{x}, t)$$

$$S_{\mathrm{scalar}} = \int d^d x \int dt \, u(\boldsymbol{x}, t)\, |\boldsymbol{\phi}(\boldsymbol{x}, t)|^2$$



$$\boldsymbol{\phi} = \left(\sqrt{N} + \sigma, \boldsymbol{\pi}\right)$$

Higgs lifetime in scalar measurements

The scalar peak is sharpest deep inside the ordered phase, where $g/g_c \ll 1$

Close to the quantum critical point, the peak broadens. Investigate for $N \rightarrow \infty$:

Higgs lifetime in scalar measurements

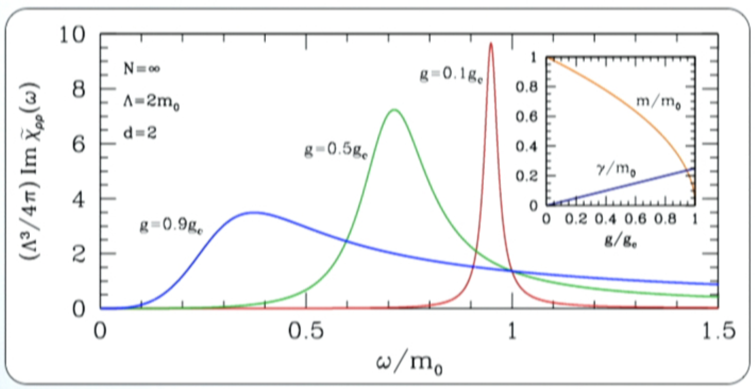
The scalar peak is sharpest deep inside the ordered phase, where $g/g_c \ll 1$

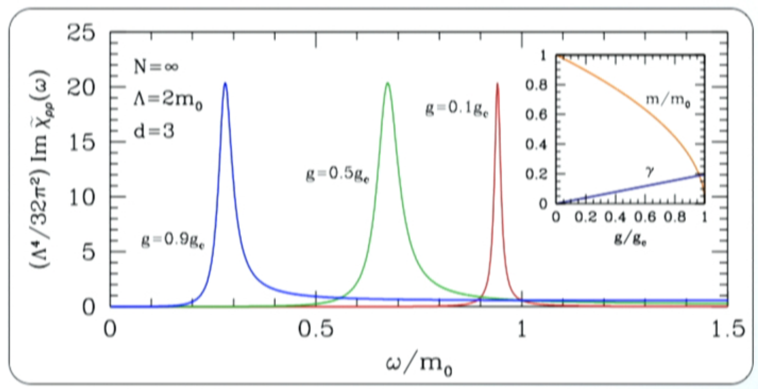
Close to the quantum critical point, the peak broadens. Investigate for $N \to \infty$:

$$d=2: \quad \tilde{\chi}''_{\rho\rho}(\boldsymbol{k}=0,\omega) = \frac{4g}{m_0^2}\frac{2\gamma\omega^3}{(\omega^2-m^2)^2+4\gamma^2\omega^2} \quad , \quad \gamma = \frac{gm_0^2}{32}$$

$$d=3: \quad \tilde{\chi}''_{\rho\rho}(\boldsymbol{k}=0,\omega) = \frac{4g}{m_0^2}\frac{\pi\gamma\omega^4 \mathrm{sgn}(\omega)}{\left[\omega^2-m^2+\gamma^2\omega^2\left(1+\ln\frac{\Lambda^2}{\omega^2}\right)\right]^2+\pi^2\gamma^4\omega^4} \quad , \quad \gamma = \left(\frac{gm_0^2}{32\pi^2}\right)^{1/2}$$

In d=3, a peak is observable right up to the critical point. What about d=2 ?





Higgs lifetime in scalar measurements

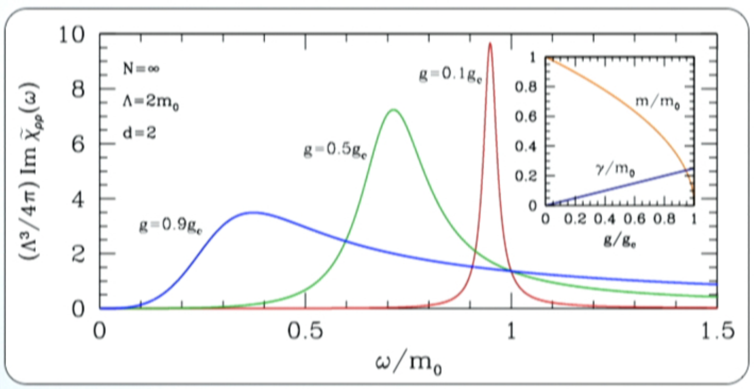
The scalar peak is sharpest deep inside the ordered phase, where $g/g_c \ll 1$

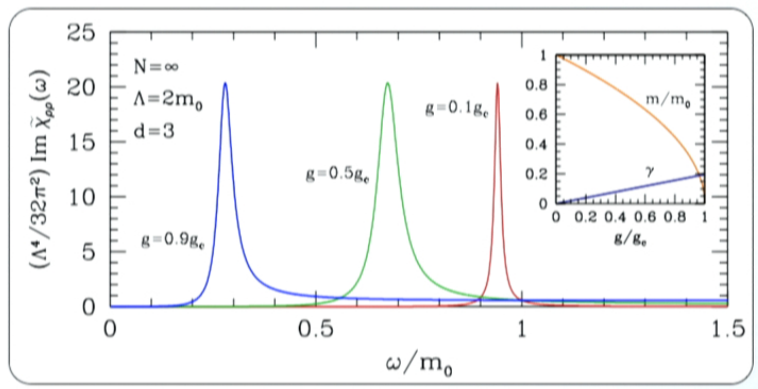
Close to the quantum critical point, the peak broadens. Investigate for $N \to \infty$:

$$d=2: \quad \widetilde{\chi}''_{\rho\rho}(\boldsymbol{k}=0,\omega) = \frac{4g}{m_0^2}\frac{2\gamma\omega^3}{(\omega^2-m^2)^2+4\gamma^2\omega^2} \quad , \quad \gamma = \frac{gm_0^2}{32}$$

$$d=3: \quad \widetilde{\chi}''_{\rho\rho}(\boldsymbol{k}=0,\omega) = \frac{4g}{m_0^2}\frac{\pi\gamma\omega^4 \mathrm{sgn}(\omega)}{\left[\omega^2-m^2+\gamma^2\omega^2\left(1+\ln\frac{\Lambda^2}{\omega^2}\right)\right]^2+\pi^2\gamma^4\omega^4} \quad , \quad \gamma = \left(\frac{gm_0^2}{32\pi^2}\right)^{1/2}$$

In d=3, a peak is observable right up to the critical point. What about d=2 ?





$O(1/N)$ calculations

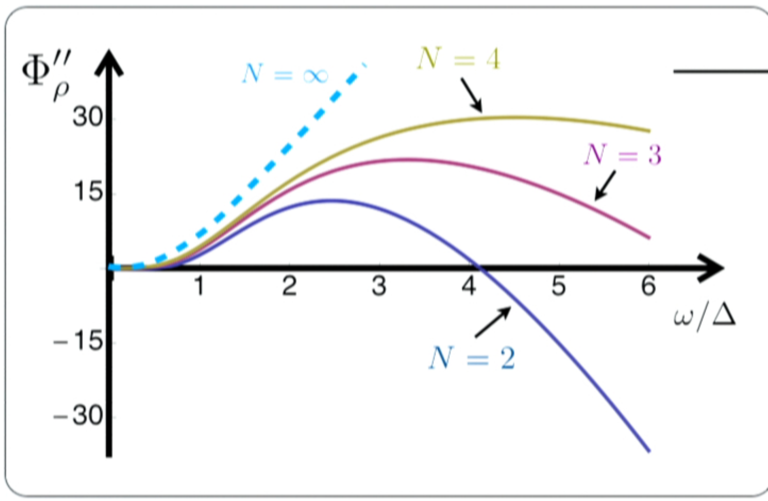
Podolsky and Sachdev, 2012

Near g_c, the scalar response function satisfies a scaling relation (d=2):

$$\chi_{\rho\rho}(\omega) = \Delta^{3-2/\nu}\Phi_\rho\left(\frac{\omega}{\Delta}\right)$$

where $\Delta \sim |g - g_c|^\nu$ is the Higgs mass, and $\nu = 0.6717(1)$ (for N=2)

Recall that for $N \to \infty$, the peak in $\chi_{\rho\rho}$ lies at $\omega \propto (g - g_c)^{1/2}$



$N \to \infty$ peak lies at parametrically larger ω

Lattice boson models

O(2) symmetry

Bose-Hubbard model:

$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{1}{2} U \sum_i n_i (n_i - 1) + \sum_i (V_i - \mu) n_i$$

Small t/U : system is a Mott insulator, with $\langle n_i \rangle \in \mathbb{Z}$

Large t/U : system is a superfluid, with $\langle b_i \rangle = \psi = |\psi| \, e^{i\theta}$

Lattice boson models   O(2) symmetry

Bose-Hubbard model:

$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \tfrac{1}{2} U \sum_i n_i(n_i - 1) + \sum_i (V_i - \mu) n_i$$

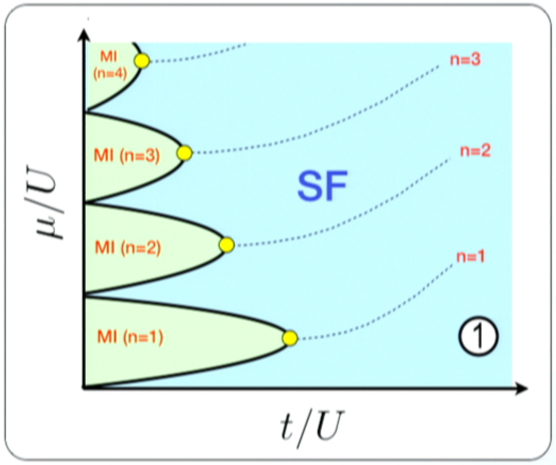
Small t/U: system is a Mott insulator, with $\langle n_i \rangle \in \mathbb{Z}$

Large t/U: system is a superfluid, with $\langle b_i \rangle = \psi = |\psi| \, e^{i\theta}$

1) Far from the Mott lobe, Gross-Pitaevskii model,

$$\mathcal{L} = -i\psi^* \partial_t \psi - \frac{\hbar^2}{2m^*} |\nabla \psi|^2 + \mu |\psi|^2 - g|\psi|^4$$

Gapless Goldstone mode, but no Higgs.



Lattice boson models

O(2) symmetry

Bose-Hubbard model:

$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \tfrac{1}{2} U \sum_i n_i(n_i - 1) + \sum_i (V_i - \mu) n_i$$

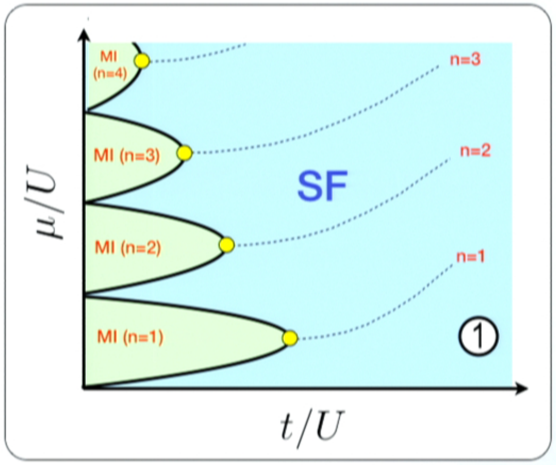
Small t/U : system is a Mott insulator, with $\langle n_i \rangle \in \mathbb{Z}$

Large t/U : system is a superfluid, with $\langle b_i \rangle = \psi = |\psi|\, e^{i\theta}$

1) **Far from the Mott lobe**, Gross-Pitaevskii model,

$$\mathcal{L} = -i\psi^* \partial_t \psi - \frac{\hbar^2}{2m^*} |\nabla \psi|^2 + \mu |\psi|^2 - g|\psi|^4$$

Gapless Goldstone mode, but no Higgs.



Lattice boson models   O(2) symmetry

Bose-Hubbard model:

$$H = -t\sum_{\langle ij\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) + \tfrac{1}{2}U\sum_i n_i(n_i-1) + \sum_i (V_i - \mu)n_i$$

Small t/U : system is a Mott insulator, with $\langle n_i\rangle \in \mathbb{Z}$

Large t/U : system is a superfluid, with $\langle b_i\rangle = \psi = |\psi|\, e^{i\theta}$

1) **Far from the Mott lobe**, Gross-Pitaevskii model,

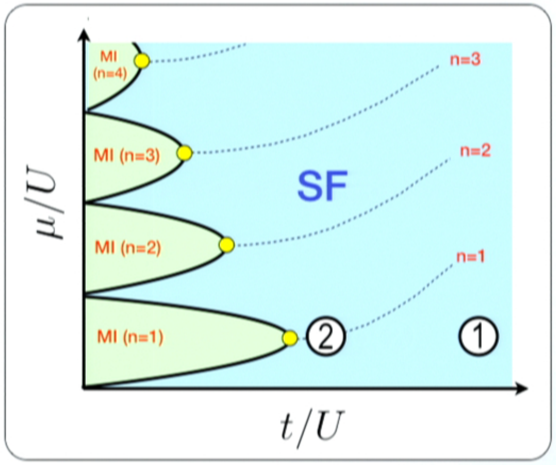
$$\mathcal{L} = -i\psi^*\partial_t\psi - \frac{\hbar^2}{2m^*}|\nabla\psi|^2 + \mu|\psi|^2 - g|\psi|^4$$

Gapless Goldstone mode, but no Higgs.

2) **Close to the Mott lobe**, relativistic model,

$$\mathcal{L} = |\partial_t\psi|^2 - c^2|\nabla\psi|^2 + r|\psi|^2 - u|\psi|^4$$

Both Goldstone mode and Higgs present.



Experiments

- Energy absorption of periodically modulated lattice $\propto \omega \chi''_{\rho\rho}(\omega)$

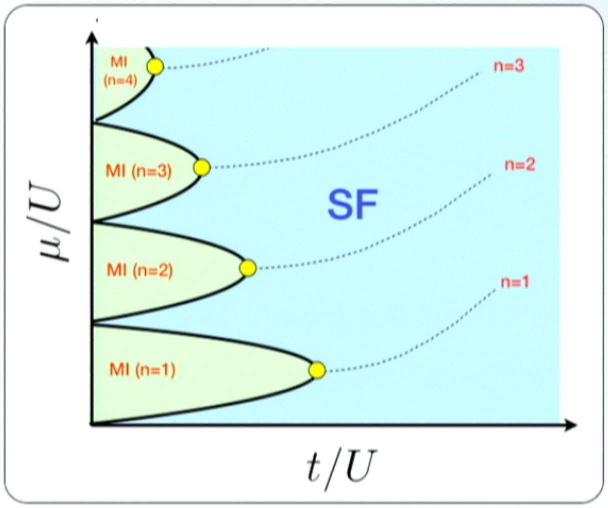
- Schori *et al.* (2004): modulate depth of 3d optical lattice. Far from linear regime.

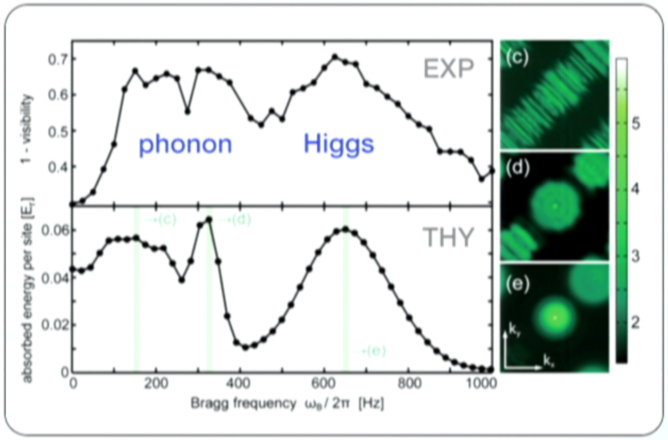
- Density variations from confining potential can destroy signature (Huber *et al.*, Pekker *et al.*, Bernier *et al.*)

- Bragg spectroscopy :

$$\widehat{\mathcal{B}}(t) = \tfrac{1}{2} V \left(e^{-i\omega_B t} \rho^\dagger_{\boldsymbol{\kappa}} + e^{i\omega_B t} \rho_{\boldsymbol{\kappa}} \right)$$

Bissbort et al. (2011) : highly nonlinear response at large wavevector, “Higgs” energy close to U





U. Bissbort *et al.*, *Phys. Rev. Lett.* **106**, 205303 (2011)

Experiments

- Energy absorption of periodically modulated lattice $\propto \omega \chi''_{\rho\rho}(\omega)$

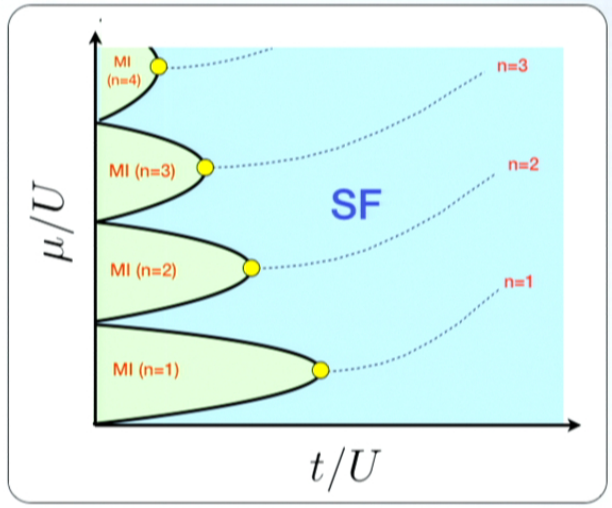
- Schori *et al.* (2004): modulate depth of 3d optical lattice. Far from linear regime.

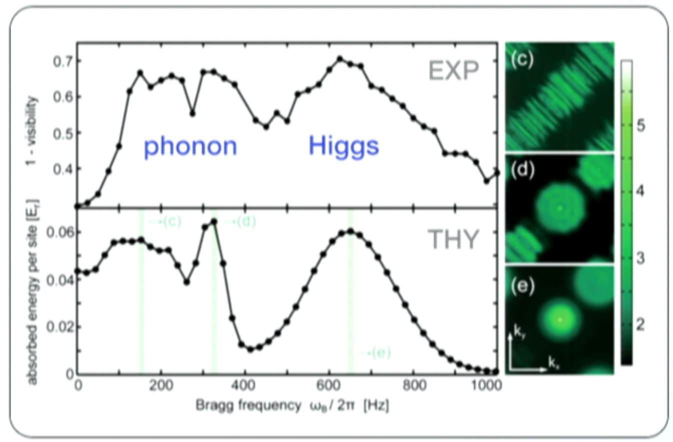
- Density variations from confining potential can destroy signature (Huber *et al.*, Pekker *et al.*, Bernier *et al.*)

- Bragg spectroscopy :

$$\widehat{\mathcal{B}}(t) = \tfrac{1}{2} V \left(e^{-i\omega_{\mathrm{B}} t} \rho^{\dagger}_{\boldsymbol{\kappa}} + e^{i\omega_{\mathrm{B}} t} \rho_{\boldsymbol{\kappa}} \right)$$

Bissbort et al. (2011) : highly nonlinear response at large wavevector, "Higgs" energy close to U





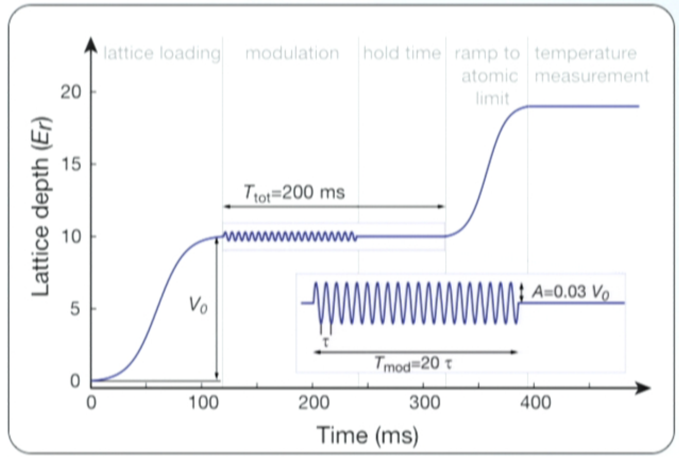
U. Bissbort *et al.*, *Phys. Rev. Lett.* **106**, 205303 (2011)

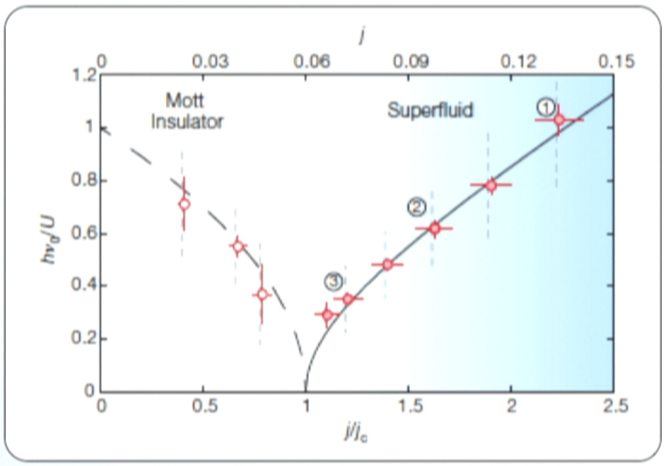
Two-dimensional condensates of ^{87}Rb

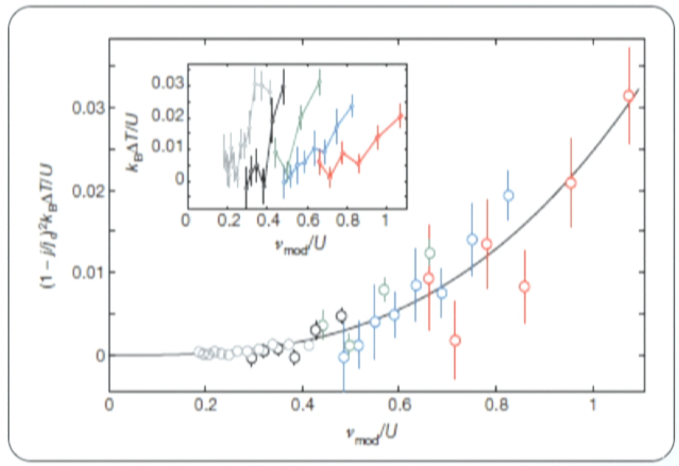
M. Endres *et al.*, *Nature* **487**, 454 (2012)

- Similar principle to Schori *et al.* work, but performed with higher resolution

- Energy absorption rate of periodically modulated lattice $\propto \omega \chi''_{\rho\rho}(\omega)$





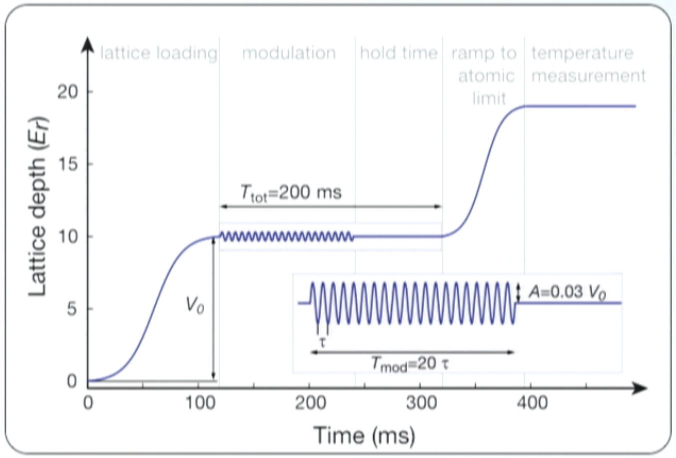


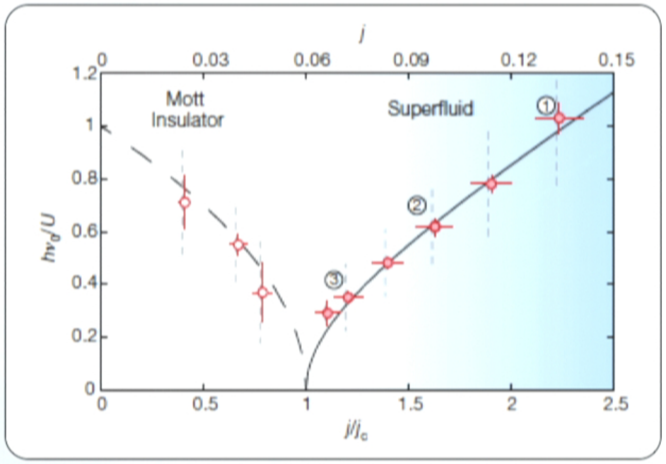
Two-dimensional condensates of ^{87}Rb

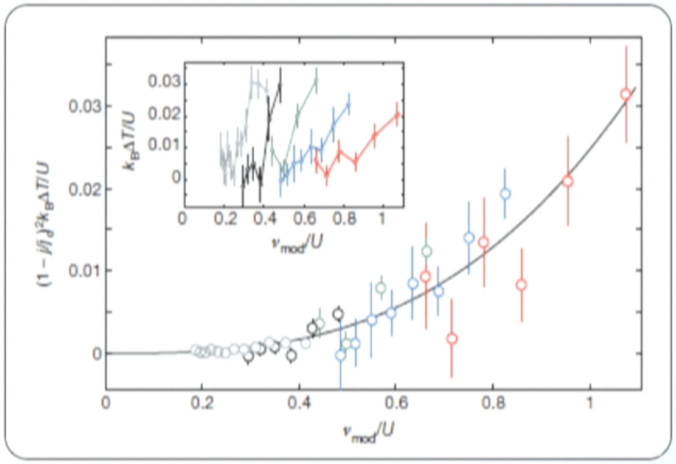
M. Endres *et al.*, *Nature* **487**, 454 (2012)

- Similar principle to Schori *et al.* work, but performed with higher resolution

- Energy absorption rate of periodically modulated lattice $\propto \omega\chi''_{\rho\rho}(\omega)$





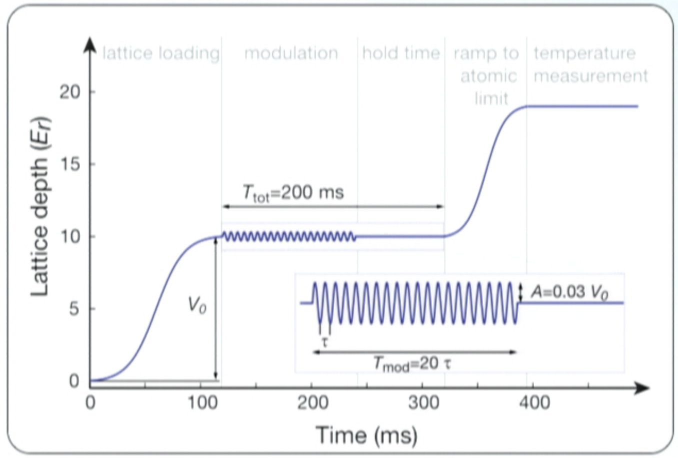


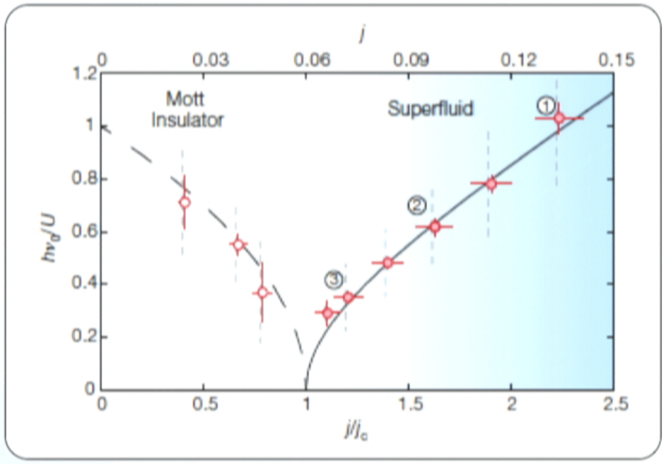
Two-dimensional condensates of ^{87}Rb

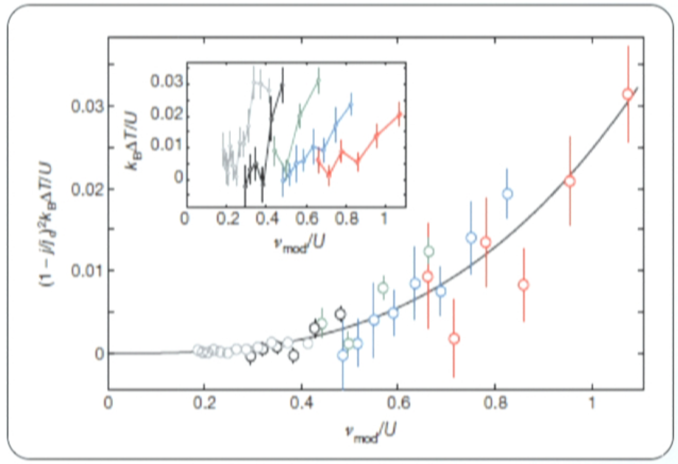
M. Endres *et al.*, *Nature* **487**, 454 (2012)

- Similar principle to Schori *et al.* work, but performed with higher resolution

- Energy absorption rate of periodically modulated lattice $\propto \omega\chi''_{\rho\rho}(\omega)$







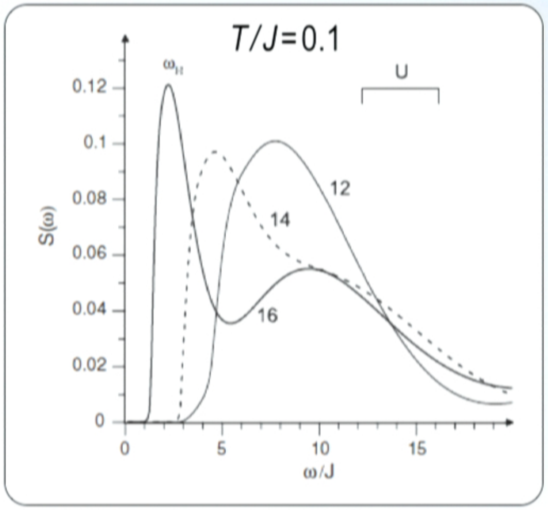
Numerical simulations

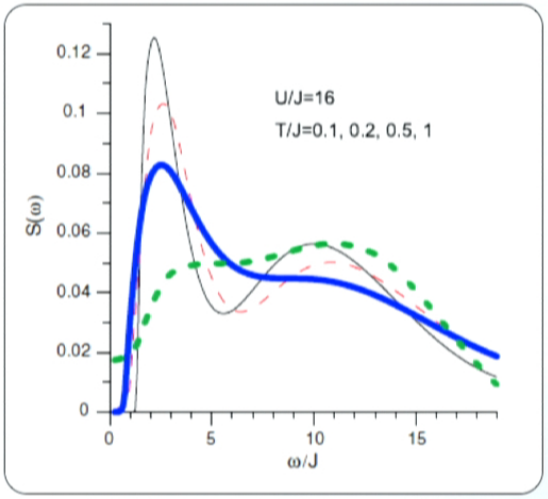
Pollet and Prokof'ev, 2012 :

- Bose Hubbard model

$$H = -J\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{1}{2}U\sum_i n_i(n_i-1) + \sum_i (V_i - \mu)n_i$$

- Superfluid-insulator transition at $U_c \approx 16.7$ J (Mott lobe tip)





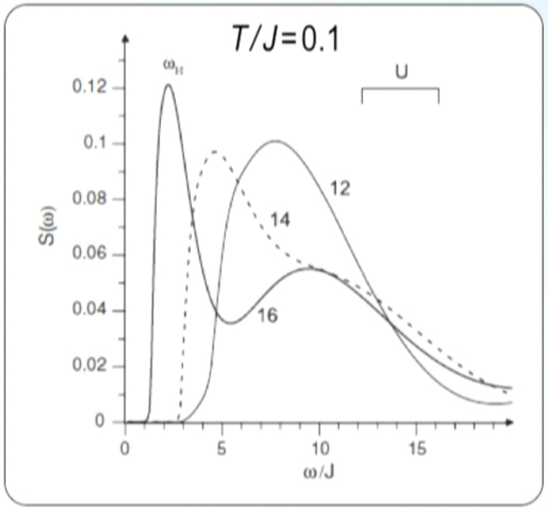
Numerical simulations

Pollet and Prokof'ev, 2012 :

- Bose Hubbard model

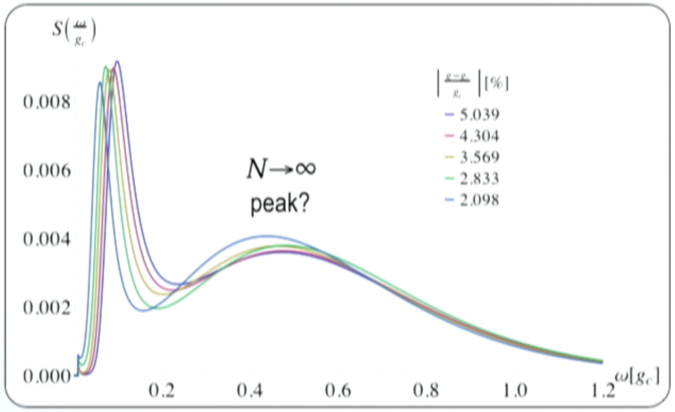
$$H = -J\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{1}{2}U\sum_i n_i(n_i-1) + \sum_i (V_i - \mu) n_i$$

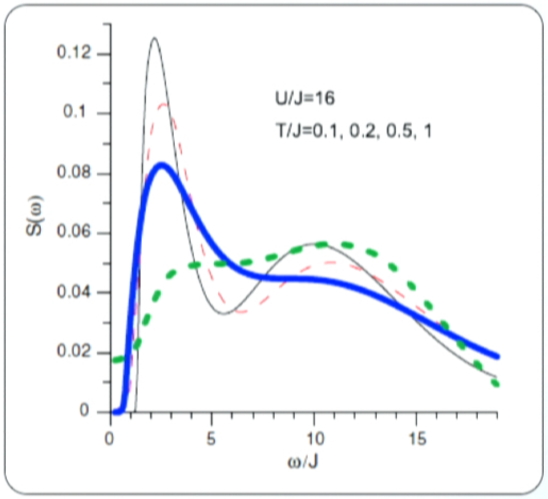
- Superfluid-insulator transition at $U_c \approx 16.7$ J (Mott lobe tip)



Gazit, Auerbach, Podolsky, 2013

- Discretized relativistic O(N) model





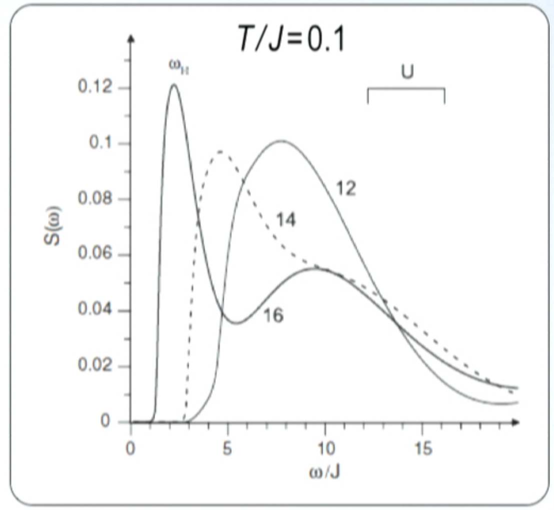
Numerical simulations

Pollet and Prokof'ev, 2012 :

- Bose Hubbard model

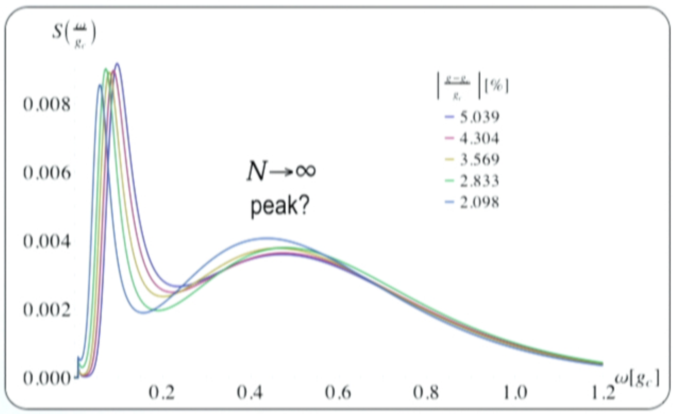
$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \tfrac{1}{2} U \sum_i n_i(n_i - 1) + \sum_i (V_i - \mu) n_i$$

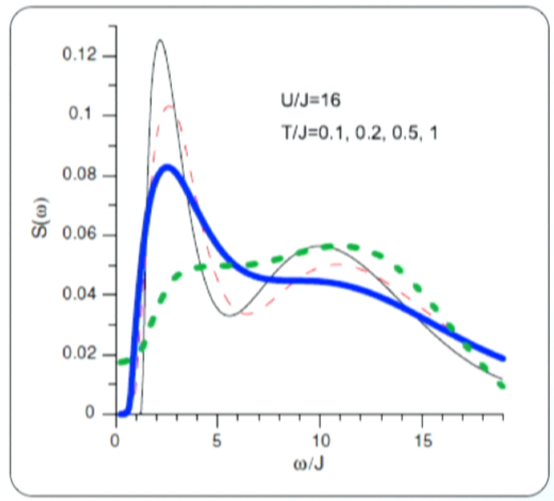
- Superfluid-insulator transition at $U_c \approx 16.7$ J (Mott lobe tip)



Gazit, Auerbach, Podolsky, 2013

- Discretized relativistic O(N) model



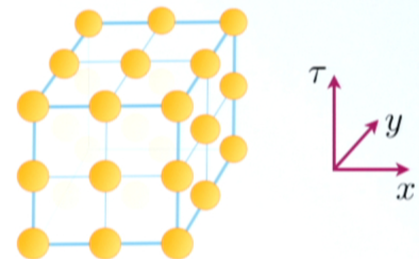


Monte Carlo Simulations (Gazit, Podolsky, Auerbach, 2013)

Lattice model:

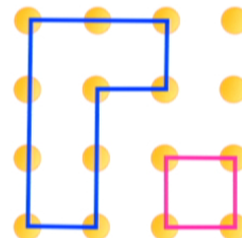
$$\mathcal{Z} = \int \mathcal{D}\vec{\phi}\, e^{-S[\vec{\phi}]}$$

$$S = -\sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \mu \sum_i |\vec{\phi}_i|^2 + g \sum_i |\vec{\phi}_i|^4$$



Worm algorithm (Prokof'ev and Svistunov, 2001) **:**

Loop model with N flavors dual to original :



System size: $1 \ll \xi \ll L$ $(1 \ll 30 \ll 200)$

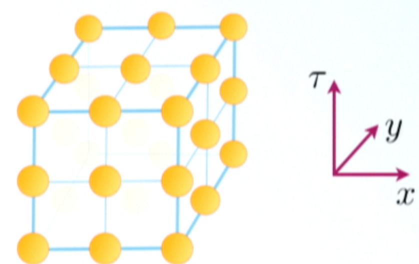
Numerical analytical continuation from Matsubara to real frequencies

Monte Carlo Simulations (Gazit, Podolsky, Auerbach, 2013)

Lattice model:

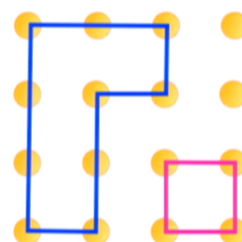
$$\mathcal{Z} = \int \mathcal{D}\vec{\phi}\, e^{-S[\vec{\phi}]}$$

$$S = -\sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \mu \sum_i |\vec{\phi}_i|^2 + g \sum_i |\vec{\phi}_i|^4$$



Worm algorithm (Prokof'ev and Svistunov, 2001) **:**

Loop model with N flavors dual to original :



System size: $1 \ll \xi \ll L$ $\quad (1 \ll 30 \ll 200)$

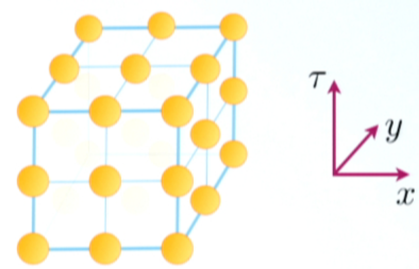
Numerical analytical continuation from Matsubara to real frequencies

Monte Carlo Simulations (Gazit, Podolsky, Auerbach, 2013)

Lattice model:

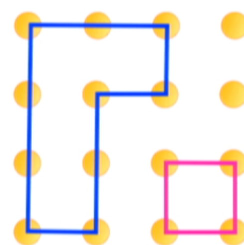
$$\mathcal{Z} = \int \mathcal{D}\vec{\phi}\, e^{-S[\vec{\phi}]}$$

$$S = -\sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \mu \sum_i |\vec{\phi}_i|^2 + g \sum_i |\vec{\phi}_i|^4$$



Worm algorithm (Prokof'ev and Svistunov, 2001) **:**

Loop model with N flavors dual to original :

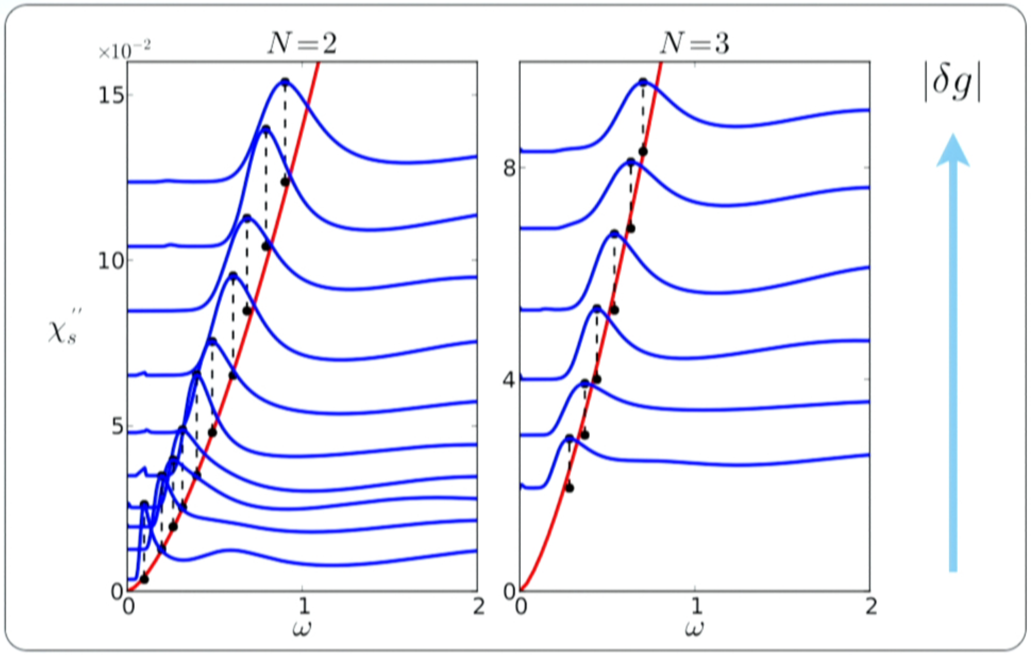


System size: $1 \ll \xi \ll L \qquad (1 \ll 30 \ll 200)$

Numerical analytical continuation from Matsubara to real frequencies

Tracking the Higgs peak (Gazit, Podolsky, Auerbach, 2013)

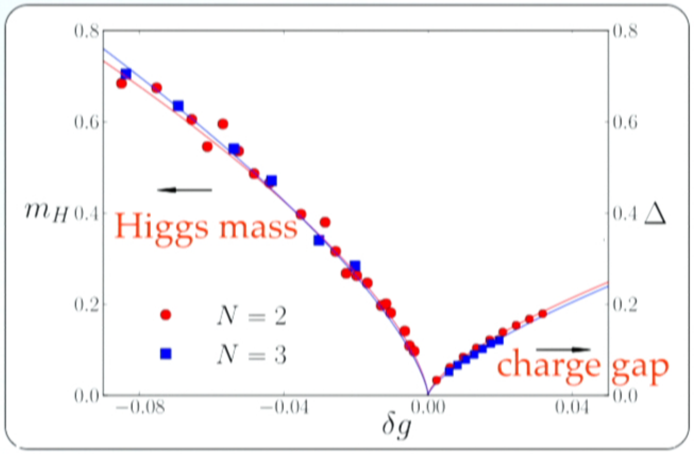
Scalar susceptibility in ordered phase:

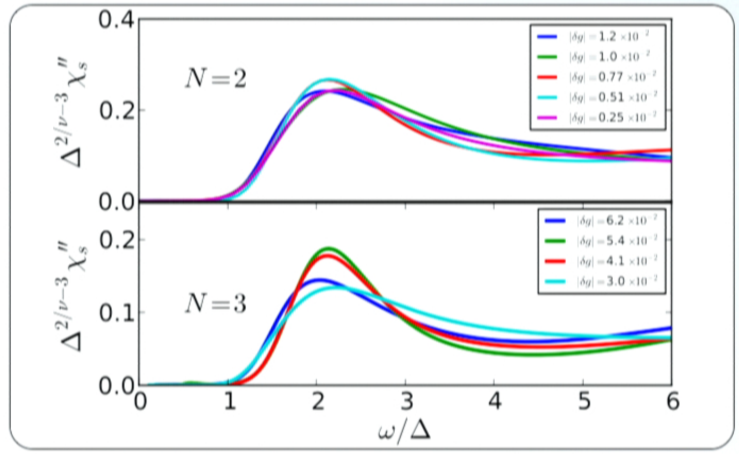


$$m_H \sim B|\delta g|^{\nu}$$

$$\frac{12\%}{0.25\%} = 48$$

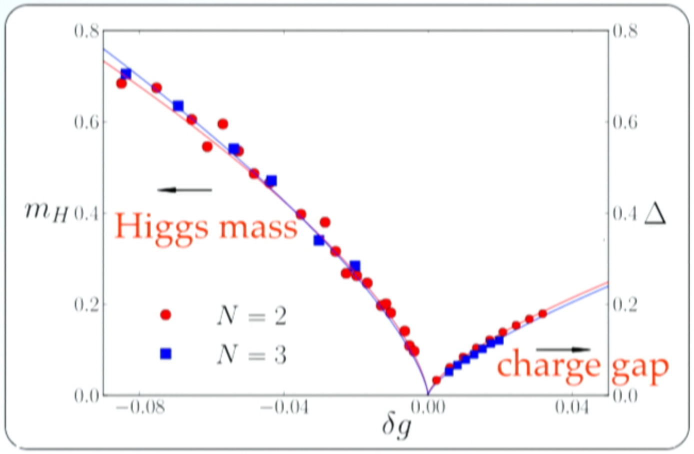
Fate of the Higgs in two dimensions

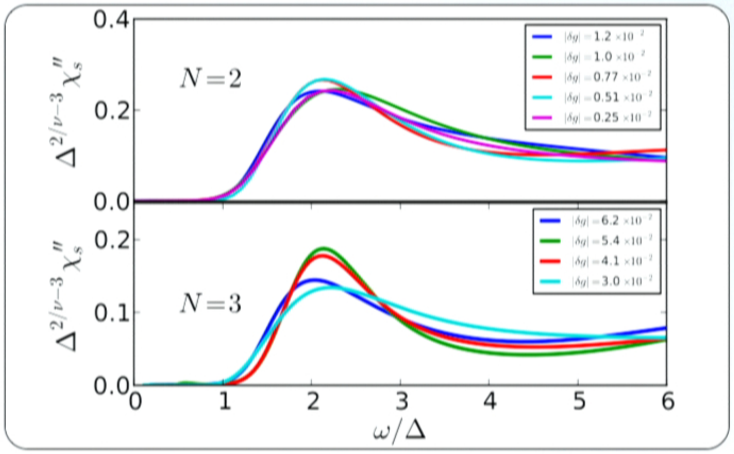




universal Higgs spectral function

Fate of the Higgs in two dimensions





universal Higgs spectral function

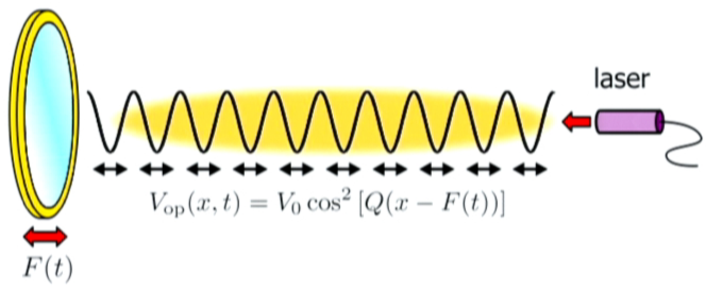
Conclusion: Higgs resonance ***survives*** *close to criticality in d=2*

Prediction: $\frac{m_H}{\Delta} = 2.1(3)$ Chen *et al*, PRL (2013): $\frac{m_H}{\Delta} = 3.3(8)$

Dynamical conductivity

Higgs peak can be seen in optical conductivity of charged bosons (Lindner *et al.*, 2010)

Optical conductivity is relevant to cold atoms experiments in a phase-fluctuating optical lattice (Tokuno and Giamarchi, PRL 2011):

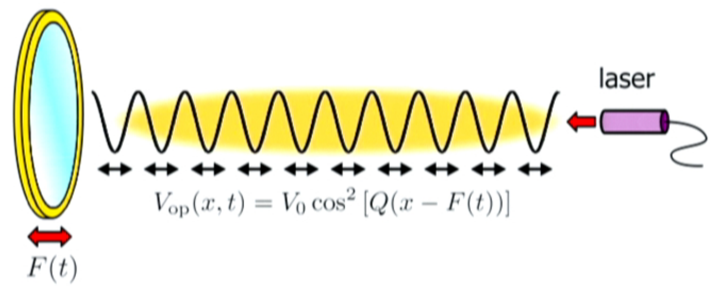


For $F(t) = F_0 \cos(\omega t)$ the energy absorption rate is $\propto \sigma(\omega)$

Dynamical conductivity

Higgs peak can be seen in optical conductivity of charged bosons (Lindner *et al.*, 2010)

Optical conductivity is relevant to cold atoms experiments in a phase-fluctuating optical lattice (Tokuno and Giamarchi, PRL 2011):



For $F(t) = F_0 \cos(\omega t)$ the energy absorption rate is $\propto \sigma(\omega)$

General formalism for $\mathrm{O}(N)$ conductivity

For $N = 2$ we can write

$$\boldsymbol{\phi} = \sqrt{N}\,(1 + \rho)(\cos\varphi\,,\, \sin\varphi)$$

and compute the Kubo formula with $\boldsymbol{j} = (1 + \rho)\boldsymbol{\nabla}\varphi$. How does one define the conductivity for a general $\mathrm{O}(N)$ model?

General formalism for O(N) conductivity

For $N=2$ we can write

$$\boldsymbol{\phi} = \sqrt{N}\,(1+\rho)(\cos\varphi\,,\,\sin\varphi)$$

and compute the Kubo formula with $\boldsymbol{j} = (1+\rho)\boldsymbol{\nabla}\varphi$. How does one define the conductivity for a general O(N) model? Start by writing

$$\mathcal{L}_{\mathsf{E}} = \frac{1}{2g}(\partial_\mu\boldsymbol{\phi} + A_\mu\boldsymbol{\phi})^2 + \frac{m_0^2}{8Ng}\left(|\boldsymbol{\phi}|^2 - N\right)^2$$

Where $A_\mu = A_{a\mu}T^a$ is an antisymmetric tensor vector potential expanded in terms of the $N(N-1)/2$ generators of O(N).

Generalized Kubo formula for O(N) models :

$$\langle I_{a\mu}(x)\rangle = -\int d^{d+1}x\; K^{ab}_{\mu\nu}(x,x')\; A_{b\nu}(x') + \mathcal{O}(A^2)$$

O(N) current — response function — vector potential

The electromagnetic response tensor is

$$K^{ab}_{\mu\nu}(x,x') = \langle I^{\mathsf{P}}_{a\mu}(x)\, I^{\mathsf{P}}_{b\nu}(x')\rangle - g^{-1}\delta_{\mu\nu}\,\delta(x-x')\,\langle T^a\boldsymbol{\phi}(x)\cdot T^b\boldsymbol{\phi}(x')\rangle = K^{ab\,\mathsf{P}}_{\mu\nu} + K^{ab\,\mathsf{D}}_{\mu\nu}$$

paramagnetic response — *diamagnetic response*

<u>Symmetric phase</u> ($g > g_c$): K^{ab} is diagonal in O(N) indices a,b

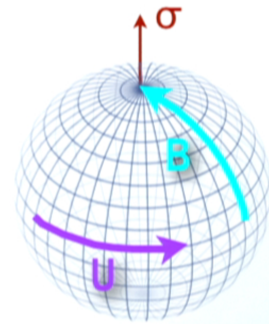
<u>Broken symmetry phase</u> ($g < g_c$): generators T^a fall into two classes, B and U

class B (broken) : rotate between $\Phi^1 = \sigma$ and $\Phi^{1+j} = \pi^j$ $\quad j \in \{1,\ldots,N-1\}$

$(N-1)$ generators total

class U (unbroken) : rotate between $\Phi^{1+j} = \pi^j$ and $\Phi^{1+k} = \pi^k$

$(N-1)(N-2)/2$ generators total



Generalizing from the N=2 case, we take

$$\sigma(\omega) = \frac{1}{\omega}\,\mathrm{Im}\,K^{\mathsf{BB}}_{xx}(\boldsymbol{k}=0,\omega+i0^+)$$

Generalized Kubo formula for O(*N*) models :

$$\langle I_{a\mu}(x)\rangle = -\int d^{d+1}x\, K^{ab}_{\mu\nu}(x,x')\, A_{b\nu}(x') + \mathcal{O}(A^2)$$

O(*N*) current — response function — vector potential

The electromagnetic response tensor is

$$K^{ab}_{\mu\nu}(x,x') = \langle I^{\mathsf{P}}_{a\mu}(x)\, I^{\mathsf{P}}_{b\nu}(x')\rangle - g^{-1}\delta_{\mu\nu}\,\delta(x-x')\,\langle T^a\boldsymbol{\phi}(x)\cdot T^b\boldsymbol{\phi}(x')\rangle = K^{ab\,\mathsf{P}}_{\mu\nu} + K^{ab\,\mathsf{D}}_{\mu\nu}$$

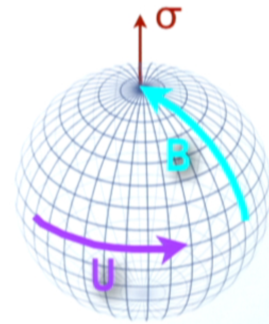
paramagnetic response — *diamagnetic response*

Symmetric phase ($g>g_c$): K^{ab} is diagonal in O(N) indices a,b

Broken symmetry phase ($g<g_c$): generators T^a fall into two classes, B and U

class B (broken) : rotate between $\Phi^1 = \sigma$ and $\Phi^{1+j} = \pi^j$ $j\in\{1,\ldots,N-1\}$
$(N-1)$ generators total

class U (unbroken) : rotate between $\Phi^{1+j} = \pi^j$ and $\Phi^{1+k} = \pi^k$
$(N-1)(N-2)/2$ generators total



Generalizing from the N=2 case, we take

$$\sigma(\omega) = \frac{1}{\omega}\,\mathrm{Im}\, K^{\mathsf{BB}}_{xx}(\boldsymbol{k}=0,\omega+i0^+)$$

Generalized Kubo formula for O(N) models :

$$\langle I_{a\mu}(x)\rangle = -\int d^{d+1}x\, K^{ab}_{\mu\nu}(x,x')\, A_{b\nu}(x') + \mathcal{O}(A^2)$$

O(N) current — response function — vector potential

The electromagnetic response tensor is

$$K^{ab}_{\mu\nu}(x,x') = \langle I^{\mathsf{P}}_{a\mu}(x)\, I^{\mathsf{P}}_{b\nu}(x')\rangle - g^{-1}\delta_{\mu\nu}\,\delta(x-x')\,\langle T^a\boldsymbol{\phi}(x)\cdot T^b\boldsymbol{\phi}(x')\rangle = K^{ab\,\mathsf{P}}_{\mu\nu} + K^{ab\,\mathsf{D}}_{\mu\nu}$$

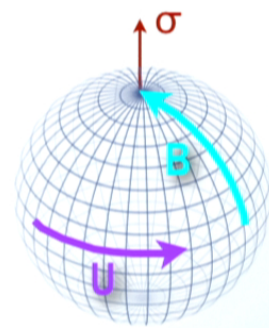
paramagnetic response — *diamagnetic response*

<u>Symmetric phase</u> ($g > g_c$): K^{ab} is diagonal in O(N) indices a,b

<u>Broken symmetry phase</u> ($g < g_c$): generators T^a fall into two classes, B and U

class B (broken) : rotate between $\Phi^1 = \sigma$ and $\Phi^{1+j} = \pi^j$ $\quad j \in \{1,\ldots,N-1\}$
$(N-1)$ generators total

class U (unbroken) : rotate between $\Phi^{1+j} = \pi^j$ and $\Phi^{1+k} = \pi^k$
$(N-1)(N-2)/2$ generators total



Generalizing from the N=2 case, we take

$$\sigma(\omega) = \frac{1}{\omega}\,\text{Im}\,K^{\mathsf{BB}}_{xx}(\boldsymbol{k}=0,\omega+i0^+)$$

We find $\sigma(\omega) = A\,\delta(\omega) + \widetilde{\sigma}(\omega)$ with $A = Ne^2 g^{-1} + \mathcal{O}\big(g^0\big)$.

The finite frequency part is computed from

$$\widehat{K}^{\mathsf{P}}_{\mu\nu}(k) = \frac{1}{(N-1)g^2} \int d^{d+1}x\, e^{ik\cdot(x-x')} \big\langle \big(\sigma\,\partial_\mu \boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\mu \sigma\big)_x \cdot \big(\sigma\,\partial_\nu \boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\nu \sigma\big)_{x'} \big\rangle$$

We find $\sigma(\omega) = A\,\delta(\omega) + \widetilde{\sigma}(\omega)$ with $A = Ne^2 g^{-1} + \mathcal{O}\big(g^0\big)$.

The finite frequency part is computed from

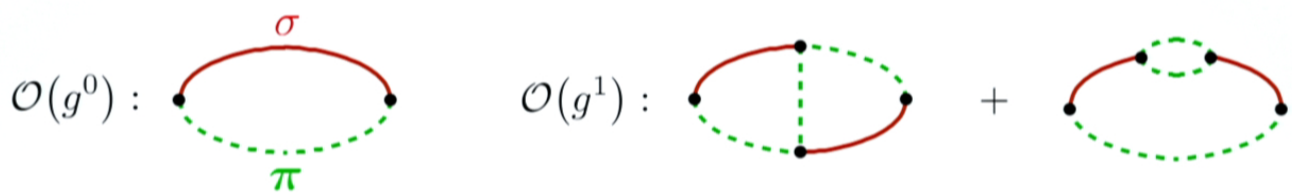
$$\widehat{K}^{\mathsf{P}}_{\mu\nu}(k) = \frac{1}{(N-1)g^2}\int d^{d+1}x\; e^{ik\cdot(x-x')}\big\langle\big(\sigma\,\partial_\mu\boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\mu\sigma\big)_x\cdot\big(\sigma\,\partial_\nu\boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\nu\sigma\big)_{x'}\big\rangle$$

We find $\sigma(\omega) = A\,\delta(\omega) + \widetilde{\sigma}(\omega)$ with $A = Ne^2 g^{-1} + \mathcal{O}(g^0)$.

The finite frequency part is computed from

$$\widehat{K}^{\mathsf{P}}_{\mu\nu}(k) = \frac{1}{(N-1)g^2}\int d^{d+1}x\; e^{ik\cdot(x-x')}\big\langle \big(\sigma\,\partial_\mu \boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\mu \sigma\big)_x \cdot \big(\sigma\,\partial_\nu \boldsymbol{\pi} - \boldsymbol{\pi}\,\partial_\nu \sigma\big)_{x'}\big\rangle$$

We evaluate this perturbatively in the coupling *g*. Conductivity diagrams :

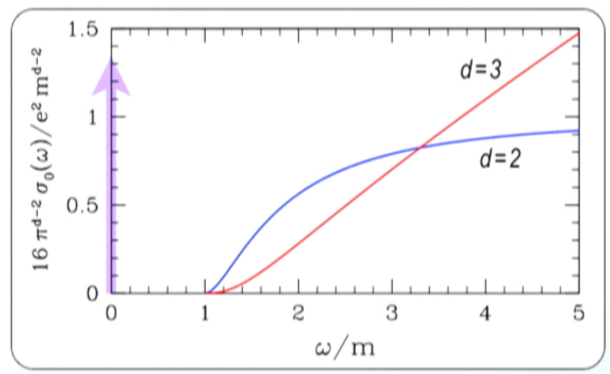


To lowest order, $\widetilde{\sigma}_0(\omega) = \dfrac{\pi \mathcal{S}_d e^2}{d\omega^2}\left(\dfrac{\omega^2 - m^2}{4\pi\omega}\right)^d \Theta(\omega^2 - m^2)$

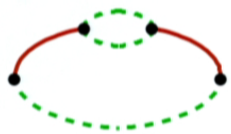
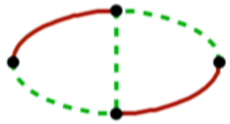
This yields a threshold at the Higgs mass, with

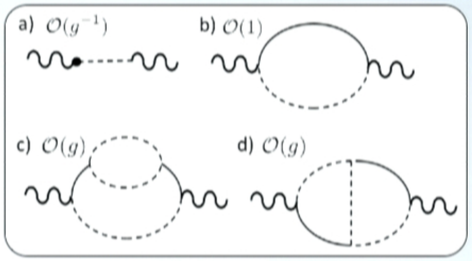
$$\sigma(\omega) \propto (\omega - m)^d$$

Does this change qualitatively at higher orders?

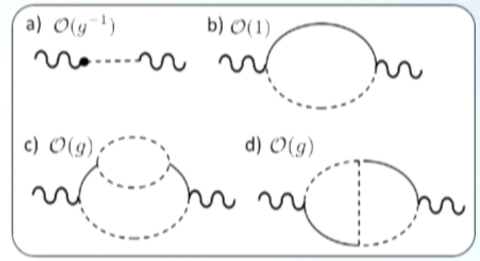


Subleading, subthreshold corrections to $\mathcal{O}(g)$:

$\mathcal{O}(g^1)$:  $+$ 



Subleading, subthreshold corrections to $\mathcal{O}(g)$:

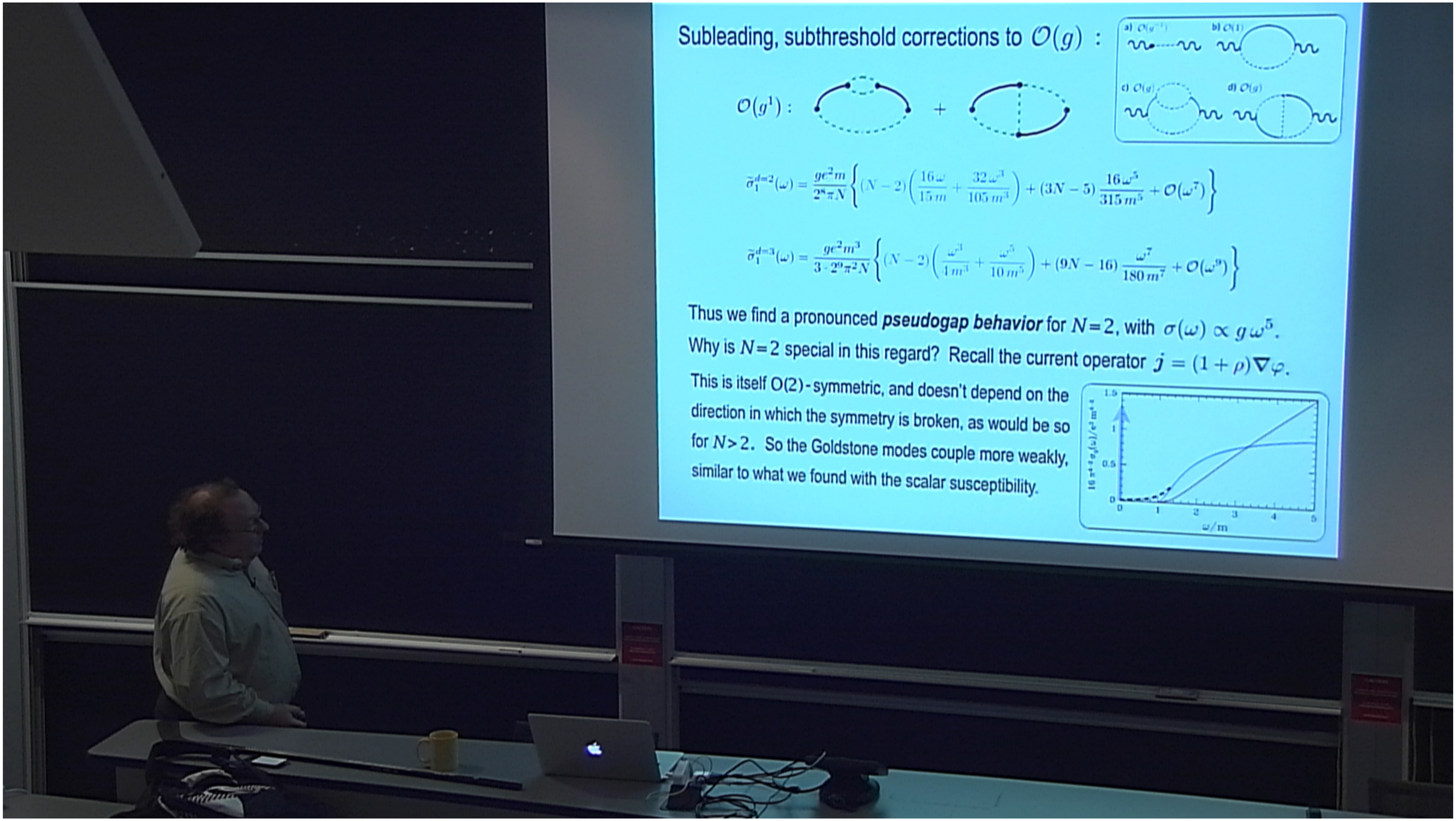


$\mathcal{O}(g^1)$:

$$\tilde{\sigma}_1^{d=2}(\omega) = \frac{ge^2 m}{2^8 \pi N}\left\{(N-2)\left(\frac{16\,\omega}{15\,m} + \frac{32\,\omega^3}{105\,m^3}\right) + (3N-5)\,\frac{16\,\omega^5}{315\,m^5} + \mathcal{O}(\omega^7)\right\}$$

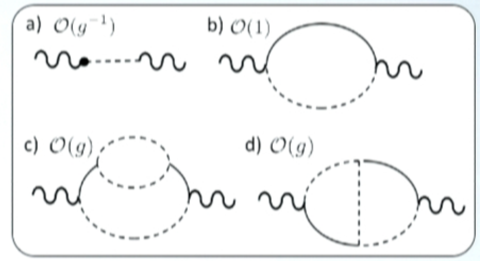
$$\tilde{\sigma}_1^{d=3}(\omega) = \frac{ge^2 m^3}{3\cdot 2^9 \pi^2 N}\left\{(N-2)\left(\frac{\omega^3}{4\,m^3} + \frac{\omega^5}{10\,m^5}\right) + (9N-16)\,\frac{\omega^7}{180\,m^7} + \mathcal{O}(\omega^9)\right\}$$

Thus we find a pronounced ***pseudogap behavior*** for $N=2$, with $\sigma(\omega) \propto g\,\omega^5$.



Subleading, subthreshold corrections to $\mathcal{O}(g)$:
$\mathcal{O}(g^1)$:
+
$\tilde{\sigma}_1^{d=2}(\omega) = \frac{g e^2 m}{2^8 \pi N} \left\{ (N-2)\left(\frac{16\,\omega}{15\,m} + \frac{32\,\omega^3}{105\,m^3}\right) + (3N-5)\,\frac{16\,\omega^5}{315\,m^5} + \mathcal{O}(\omega^7) \right\}$
$\tilde{\sigma}_1^{d=3}(\omega) = \frac{g e^2 m^3}{3 \cdot 2^9 \pi^2 N} \left\{ (N-2)\left(\frac{\omega^3}{4\,m^3} + \frac{\omega^5}{10\,m^5}\right) + (9N-16)\,\frac{\omega^7}{180\,m^7} + \mathcal{O}(\omega^9) \right\}$
Thus we find a pronounced ***pseudogap behavior*** for $N=2$, with $\sigma(\omega) \propto g\,\omega^5$.
Why is $N=2$ special in this regard? Recall the current operator $j = (1+\rho)\nabla\varphi$.
This is itself O(2)-symmetric, and doesn't depend on the direction in which the symmetry is broken, as would be so for $N>2$. So the Goldstone modes couple more weakly, similar to what we found with the scalar susceptibility.

Subleading, subthreshold corrections to $\mathcal{O}(g)$:



$\mathcal{O}(g^1)$:

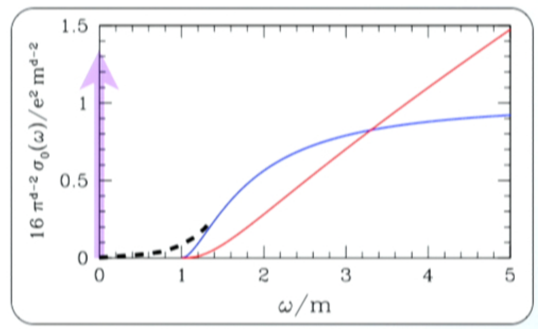
$$\tilde{\sigma}_1^{d=2}(\omega) = \frac{ge^2 m}{2^8 \pi N} \left\{ (N-2)\left(\frac{16\,\omega}{15\,m} + \frac{32\,\omega^3}{105\,m^3} \right) + (3N-5)\,\frac{16\,\omega^5}{315\,m^5} + \mathcal{O}(\omega^7) \right\}$$

$$\tilde{\sigma}_1^{d=3}(\omega) = \frac{ge^2 m^3}{3\cdot 2^9 \pi^2 N} \left\{ (N-2)\left(\frac{\omega^3}{4\,m^3} + \frac{\omega^5}{10\,m^5} \right) + (9N-16)\,\frac{\omega^7}{180\,m^7} + \mathcal{O}(\omega^9) \right\}$$

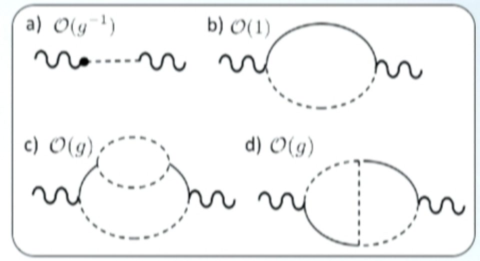
Thus we find a pronounced ***pseudogap behavior*** for $N=2$, with $\sigma(\omega) \propto g\,\omega^5$.

Why is $N=2$ special in this regard? Recall the current operator $\boldsymbol{j} = (1+\rho)\boldsymbol{\nabla}\varphi$.

This is itself O(2) - symmetric, and doesn't depend on the direction in which the symmetry is broken, as would be so for $N>2$. So the Goldstone modes couple more weakly, similar to what we found with the scalar susceptibility.



Subleading, subthreshold corrections to $\mathcal{O}(g)$:



$\mathcal{O}(g^1)$:

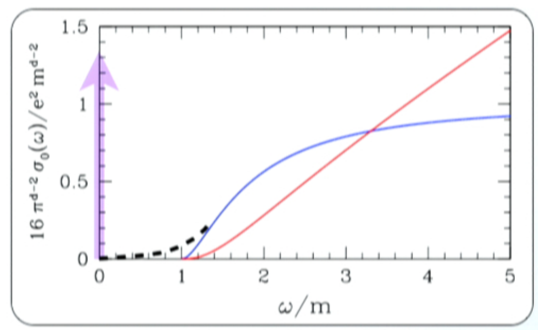
$$\tilde{\sigma}_1^{d=2}(\omega) = \frac{ge^2 m}{2^8 \pi N} \left\{ (N-2)\left(\frac{16\,\omega}{15\,m} + \frac{32\,\omega^3}{105\,m^3}\right) + (3N-5)\,\frac{16\,\omega^5}{315\,m^5} + \mathcal{O}(\omega^7) \right\}$$

$$\tilde{\sigma}_1^{d=3}(\omega) = \frac{ge^2 m^3}{3\cdot 2^9 \pi^2 N} \left\{ (N-2)\left(\frac{\omega^3}{4\,m^3} + \frac{\omega^5}{10\,m^5}\right) + (9N-16)\,\frac{\omega^7}{180\,m^7} + \mathcal{O}(\omega^9) \right\}$$

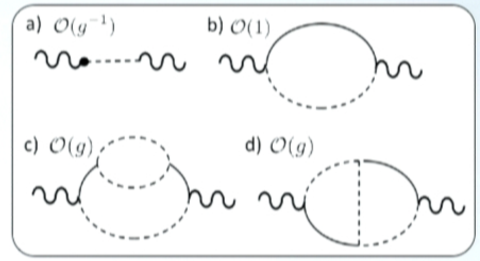
Thus we find a pronounced ***pseudogap behavior*** for $N=2$, with $\sigma(\omega) \propto g\,\omega^5$.

Why is $N=2$ special in this regard? Recall the current operator $\boldsymbol{j} = (1+\rho)\boldsymbol{\nabla}\varphi$.

This is itself O(2) - symmetric, and doesn't depend on the direction in which the symmetry is broken, as would be so for $N>2$. So the Goldstone modes couple more weakly, similar to what we found with the scalar susceptibility.



Subleading, subthreshold corrections to $\mathcal{O}(g)$:



$\mathcal{O}(g^1)$:

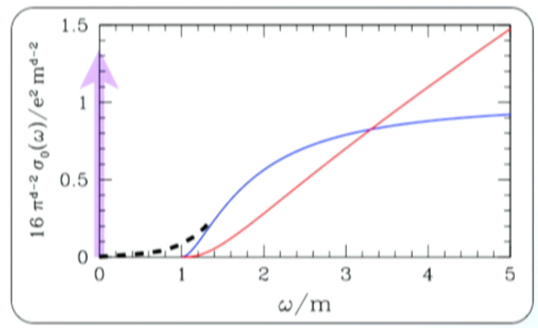
$$\tilde{\sigma}_1^{d=2}(\omega) = \frac{ge^2 m}{2^8 \pi N}\left\{(N-2)\left(\frac{16\,\omega}{15\,m} + \frac{32\,\omega^3}{105\,m^3}\right) + (3N-5)\,\frac{16\,\omega^5}{315\,m^5} + \mathcal{O}(\omega^7)\right\}$$

$$\tilde{\sigma}_1^{d=3}(\omega) = \frac{ge^2 m^3}{3\cdot 2^9 \pi^2 N}\left\{(N-2)\left(\frac{\omega^3}{4\,m^3} + \frac{\omega^5}{10\,m^5}\right) + (9N-16)\,\frac{\omega^7}{180\,m^7} + \mathcal{O}(\omega^9)\right\}$$

Thus we find a pronounced ***pseudogap behavior*** for $N=2$, with $\sigma(\omega) \propto g\,\omega^5$.

Why is $N=2$ special in this regard? Recall the current operator $\boldsymbol{j} = (1+\rho)\boldsymbol{\nabla}\varphi$.

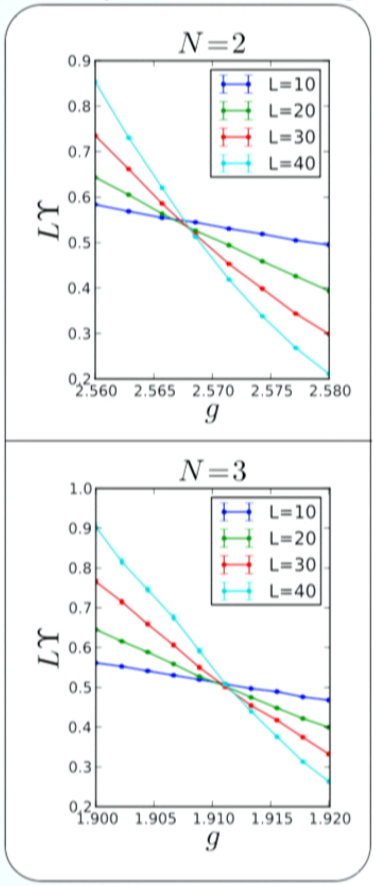
This is itself O(2) - symmetric, and doesn't depend on the direction in which the symmetry is broken, as would be so for $N>2$. So the Goldstone modes couple more weakly, similar to what we found with the scalar susceptibility.



Numerical evaluation of O(2) model conductivity

Gazit, Podolsky, Auerbach, DPA (PRB in press 2013)

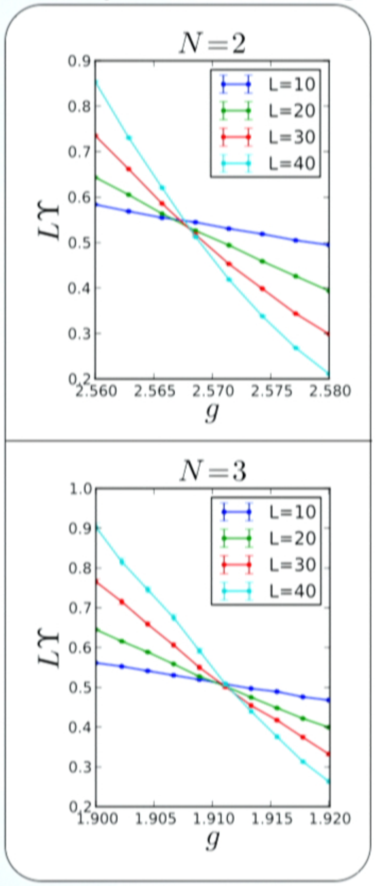
helicity modulus scaling



Numerical evaluation of O(2) model conductivity

Gazit, Podolsky, Auerbach, DPA (PRB in press 2013)

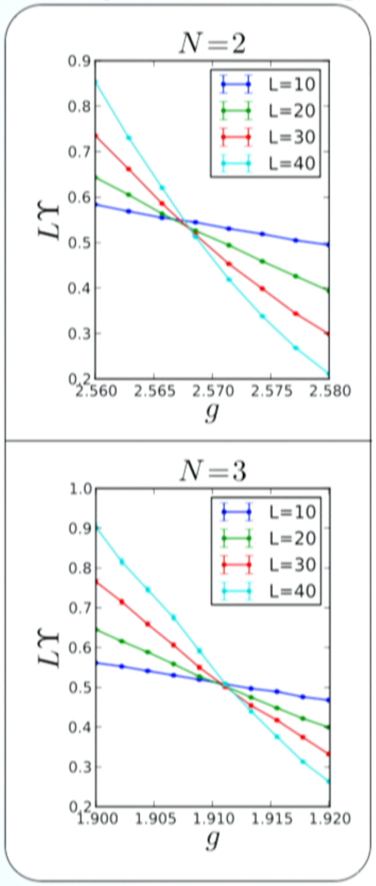
helicity modulus scaling



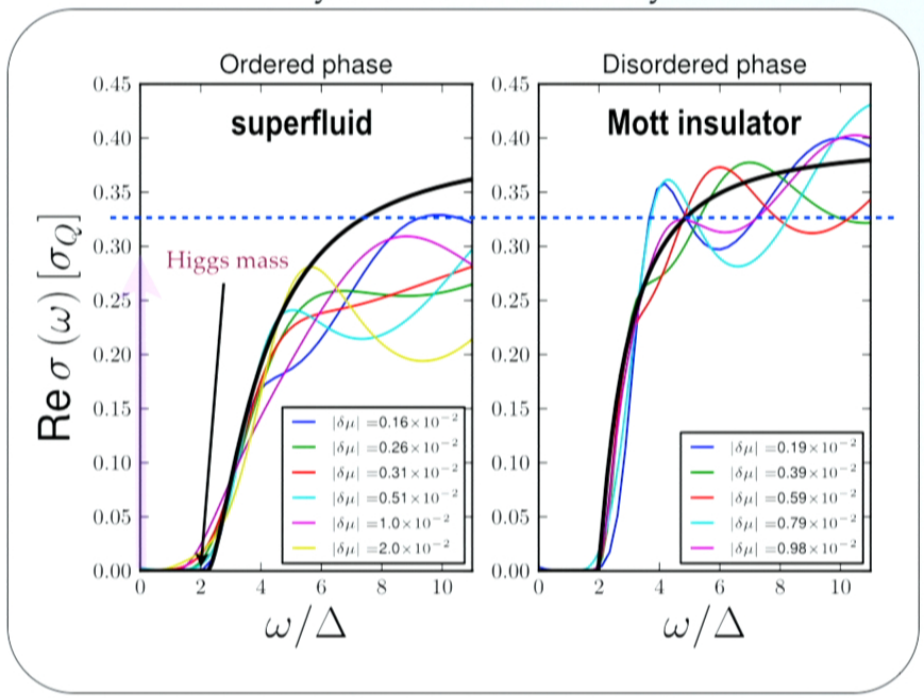
Numerical evaluation of O(2) model conductivity

Gazit, Podolsky, Auerbach, DPA (PRB in press 2013)

helicity modulus scaling



dynamical conductivity



$$\sigma_{\rm SF}(\omega) = 2\pi \left(\frac{\omega^2 - m_H^2}{4\omega^2}\right)^2 \Theta(\omega - m_H) \qquad \sigma_{\rm Mott}(\omega) = 2\pi \left(\frac{\omega^2 - 4\Delta^2}{16\omega^2}\right) \Theta(\omega - 2\Delta)$$

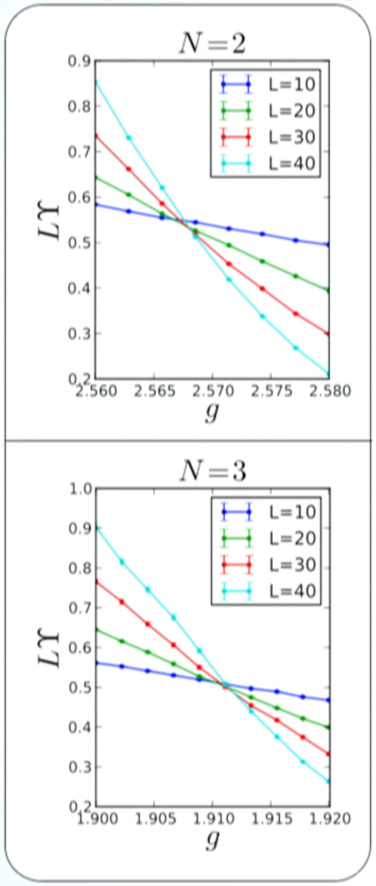
Estimated universal value of $\sigma(\omega \to \infty) = 0.33(7)\,\sigma_Q$

See also: W. Witczak-Krempa, E. Sorensen, and S. Sachdev, *arXiv 1309.2941*
K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, *arXiv 1309.5635*

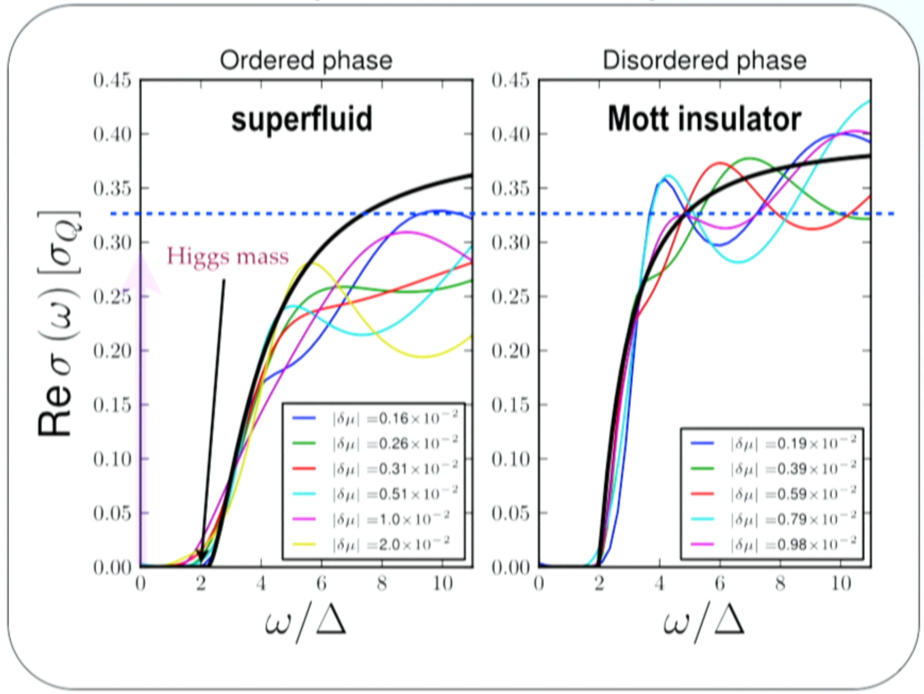
Numerical evaluation of O(2) model conductivity

Gazit, Podolsky, Auerbach, DPA (PRB in press 2013)

helicity modulus scaling



dynamical conductivity



$$\sigma_{\rm SF}(\omega) = 2\pi \left(\frac{\omega^2 - m_H^2}{4\omega^2}\right)^2 \Theta(\omega - m_H) \qquad \sigma_{\rm Mott}(\omega) = 2\pi \left(\frac{\omega^2 - 4\Delta^2}{16\omega^2}\right) \Theta(\omega - 2\Delta)$$

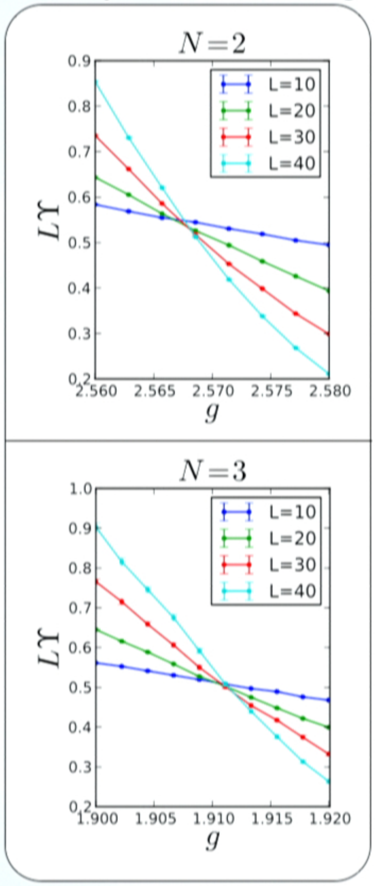
Estimated universal value of $\sigma(\omega \to \infty) = 0.33(7)\,\sigma_Q$

See also: W. Witczak-Krempa, E. Sorensen, and S. Sachdev, *arXiv 1309.2941*
K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, *arXiv 1309.5635*

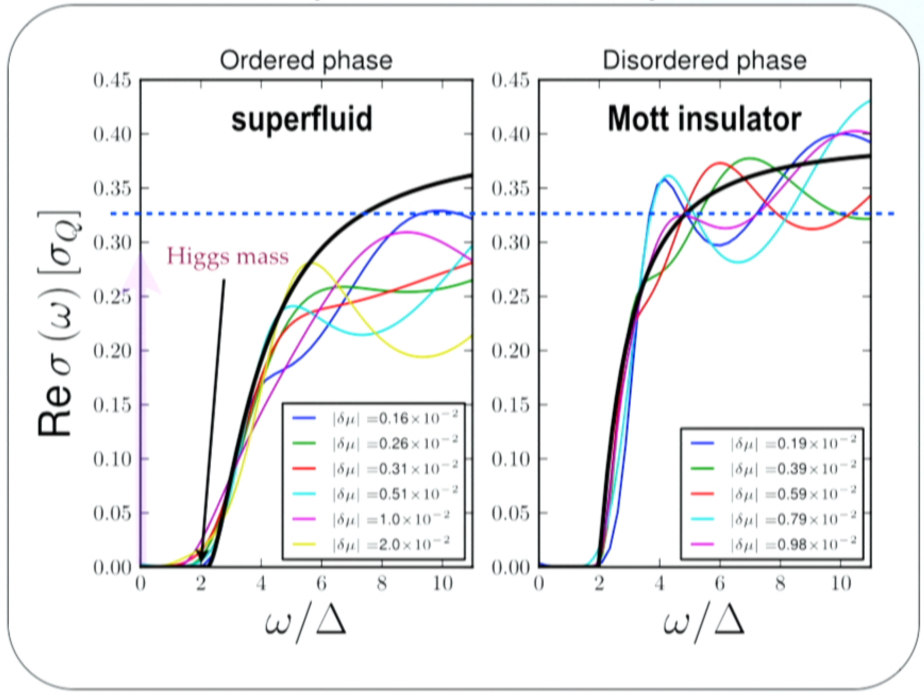
Numerical evaluation of O(2) model conductivity

Gazit, Podolsky, Auerbach, DPA (PRB in press 2013)

helicity modulus scaling



dynamical conductivity



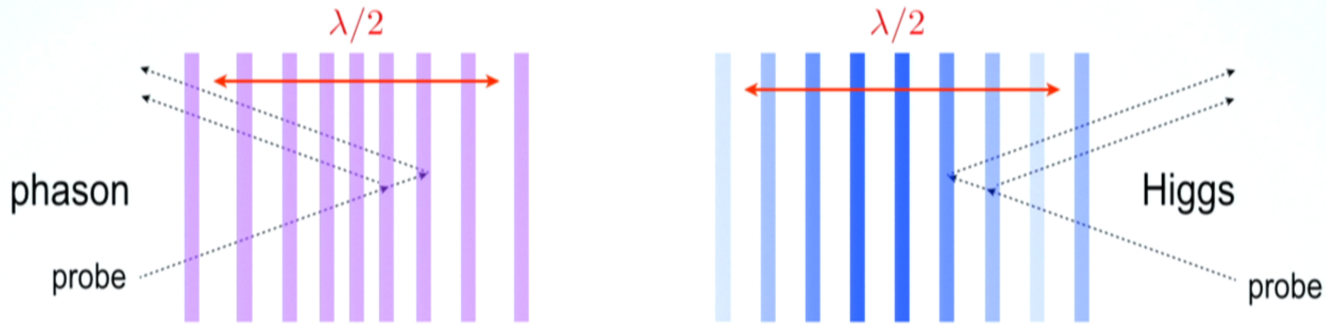
$$\sigma_{\rm SF}(\omega) = 2\pi\left(\frac{\omega^2 - m_H^2}{4\omega^2}\right)^2 \Theta(\omega - m_H) \qquad \sigma_{\rm Mott}(\omega) = 2\pi\left(\frac{\omega^2 - 4\Delta^2}{16\omega^2}\right)\Theta(\omega - 2\Delta)$$

Estimated universal value of $\sigma(\omega \to \infty) = 0.33(7)\,\sigma_Q$

See also: W. Witczak-Krempa, E. Sorensen, and S. Sachdev, *arXiv 1309.2941*
K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, *arXiv 1309.5635*

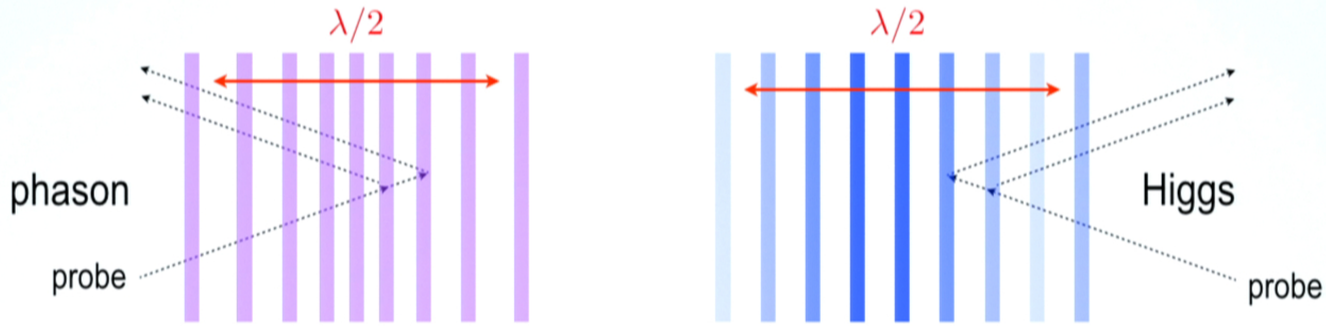
More Experiments

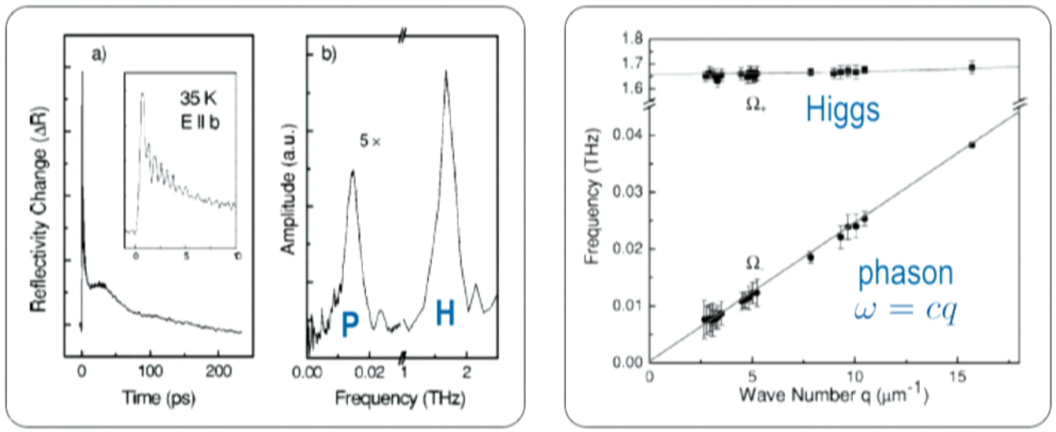
1) Phasons and amplitude (Higgs) modes observed by fs pump-probe spectroscopy in the one-dimensional CDW conductor $K_{0.3}MoO_3$:



More Experiments

1) Phasons and amplitude (Higgs) modes observed by fs pump-probe spectroscopy in the one-dimensional CDW conductor $K_{0.3}MoO_3$:



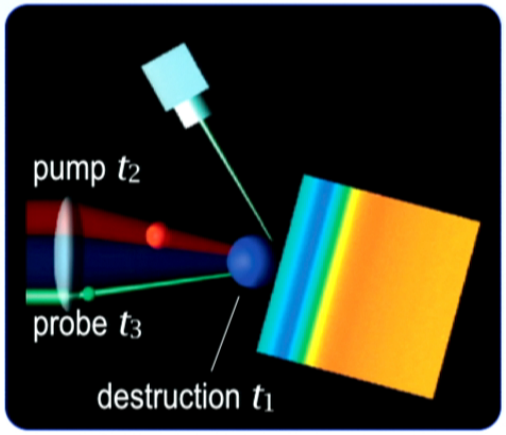


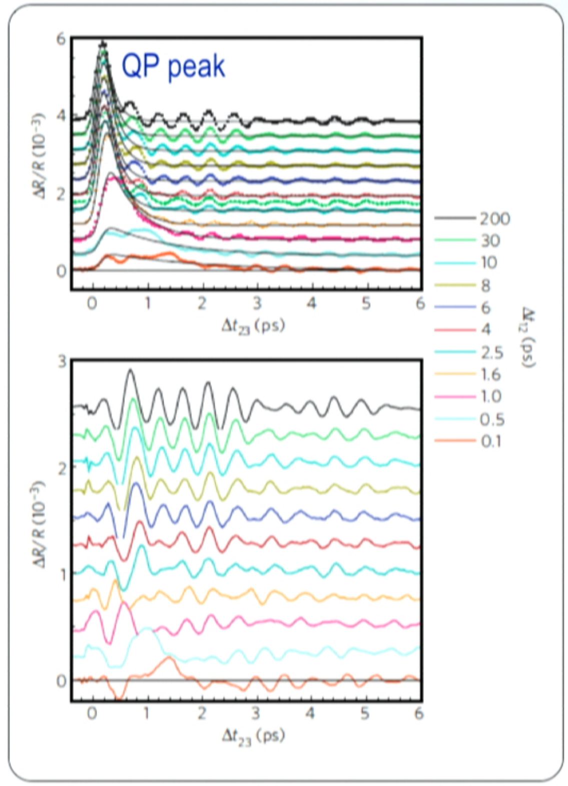
Y. Ren, Z. Xu, and G. Lüpke, J. Chem. Phys. 120, 4755 (2004)

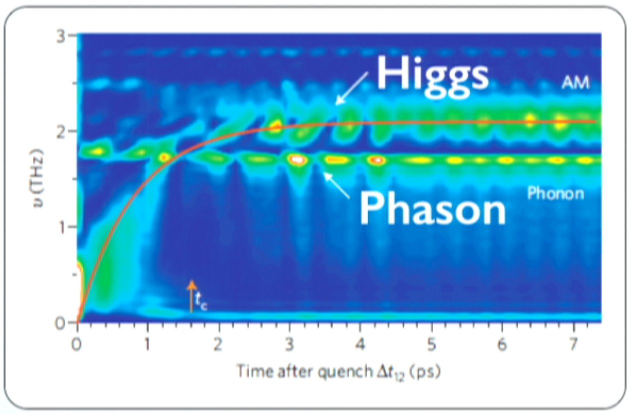
2) Other CDW systems ($TbTe_3$, $DyTe_3$, $2H$-$TaSe_2$) :

2) Other CDW systems ($TbTe_3$, $DyTe_3$, *2H*-$TaSe_2$) :

R. Yusupov *et al.*, *Nature Phys.* **6**, 681 (2010)







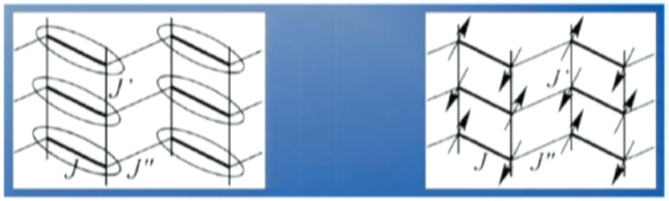
Fourier transform

3) Neutron scattering in antiferromagnets

Ch. Rüegg *et al.*, *Phys. Rev. Lett.* **100**, 205701 (2008)

pressure-induced quantum phase transition in $TlCuCl_3$ (d=3 system)

longitudinal susceptibility



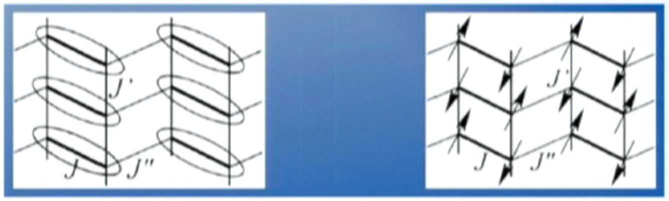
$p < p_c$ $p > p_c$

3) Neutron scattering in antiferromagnets

Ch. Rüegg *et al.*, *Phys. Rev. Lett.* **100**, 205701 (2008)

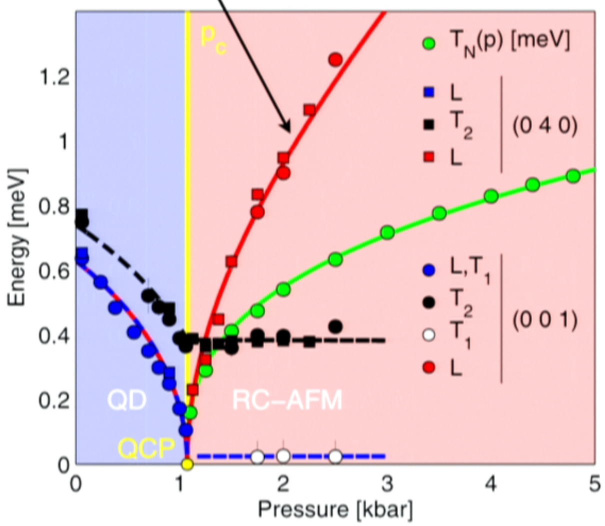
pressure-induced quantum phase transition in $TlCuCl_3$ (d=3 system)

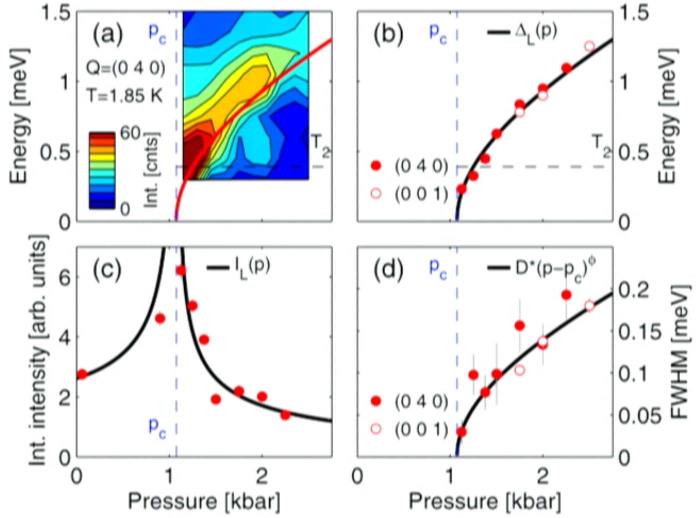
longitudinal susceptibility



$p < p_c$ $\qquad$ $p > p_c$

Higgs



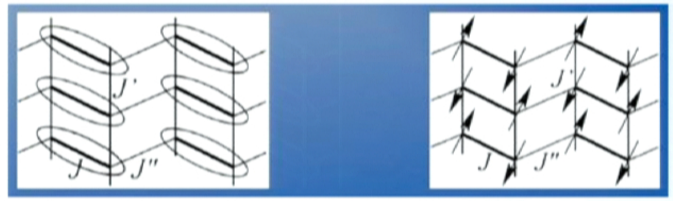


3) Neutron scattering in antiferromagnets

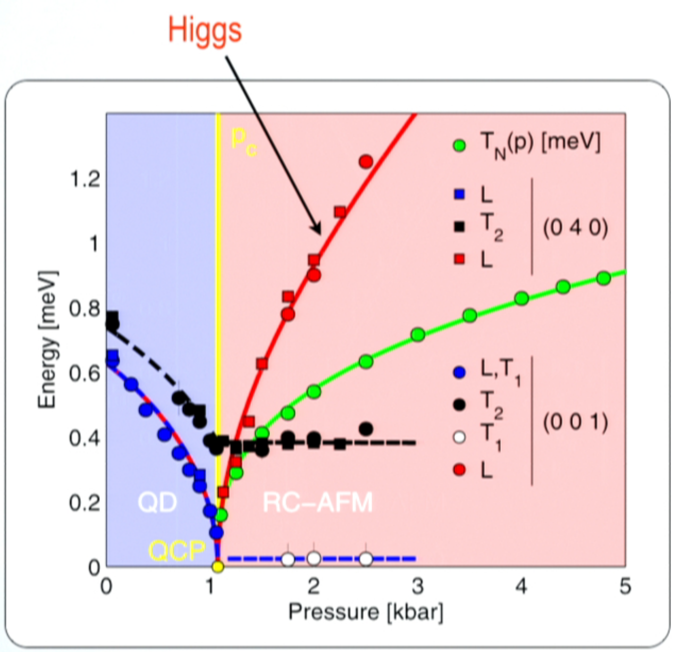
Ch. Rüegg *et al.*, *Phys. Rev. Lett.* **100**, 205701 (2008)

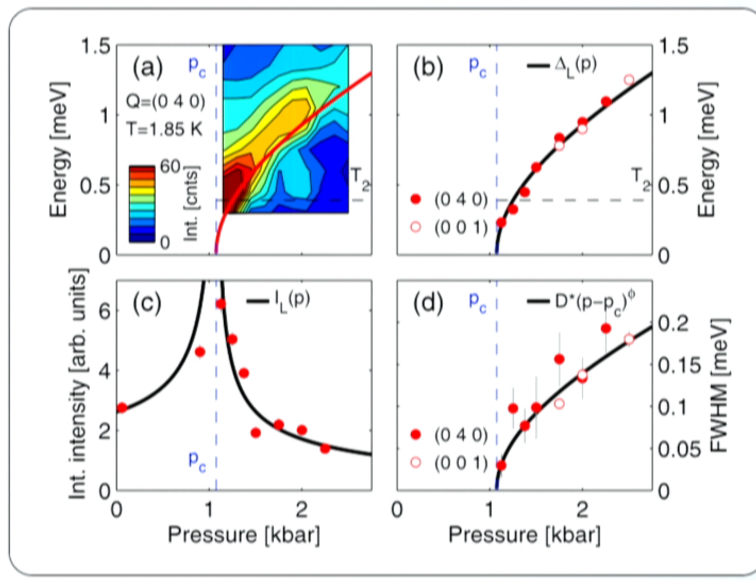
pressure-induced quantum phase transition in $TlCuCl_3$ (d=3 system)

longitudinal susceptibility



$p < p_c$ $p > p_c$





4) Raman spectra in *2H*-$NbSe_2$

Neutron diffraction shows incommensurate CDW order sets in at $T_c^{CDW} = 33\,\mathrm{K}$

A superconducting energy gap develops at $T_c^{SC} = 7.2\,\mathrm{K}$

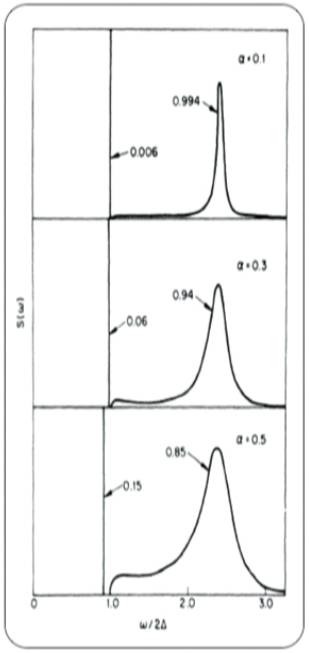
Usually in superconductors, the Higgs mode is invisible, because there is no coupling to charge fluctuations. When CDW order sets in, the Higgs becomes visible. It appears as a pole in the phonon self energy :

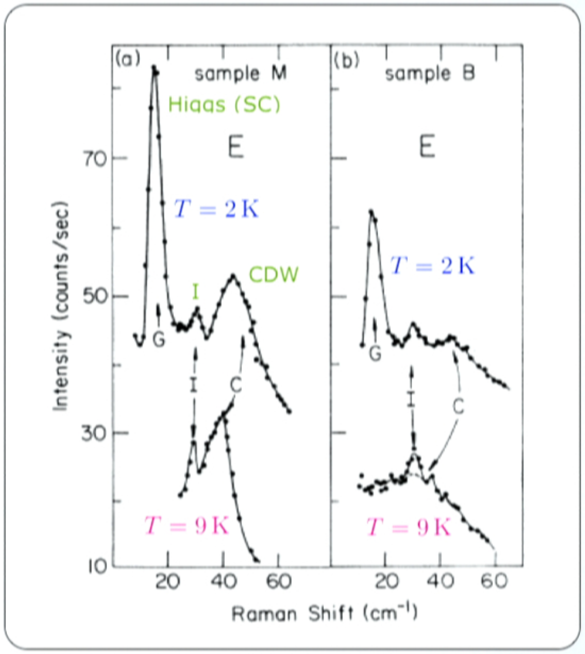
$$\nu^2 = \omega_0^2 + 2\omega_0 \Pi(\nu)$$

$$\nu \approx 2\Delta \left\{ 1 - \frac{2\alpha^2}{\pi^2} \frac{\omega_0^2}{\omega_0^2 - 4\Delta^2} \right\}$$

For $\nu > 2\Delta$ one obtains a broadened peak near the bare CDW amplitude mode ω_0 .

P. B. Littlewood and C. M. Varma, *Phys. Rev. B* **26**, 4883 (1982)





R. Sooryakumar *et al.*, *Phys. Rev. B* **23**, 3222 (1981)

4) Raman spectra in *2H*-$NbSe_2$

Neutron diffraction shows incommensurate CDW order sets in at $T_c^{\rm CDW} = 33\,{\rm K}$

A superconducting energy gap develops at $T_c^{\rm SC} = 7.2\,{\rm K}$

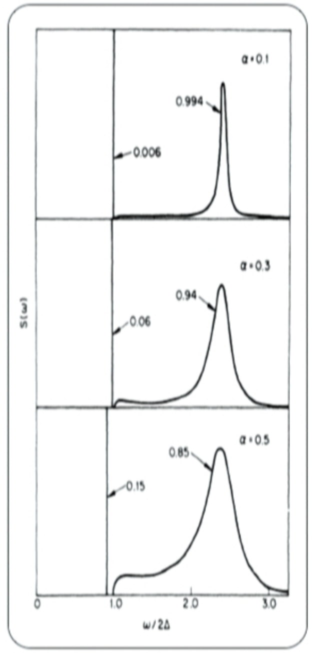
Usually in superconductors, the Higgs mode is invisible, because there is no coupling to charge fluctuations. When CDW order sets in, the Higgs becomes visible. It appears as a pole in the phonon self energy :

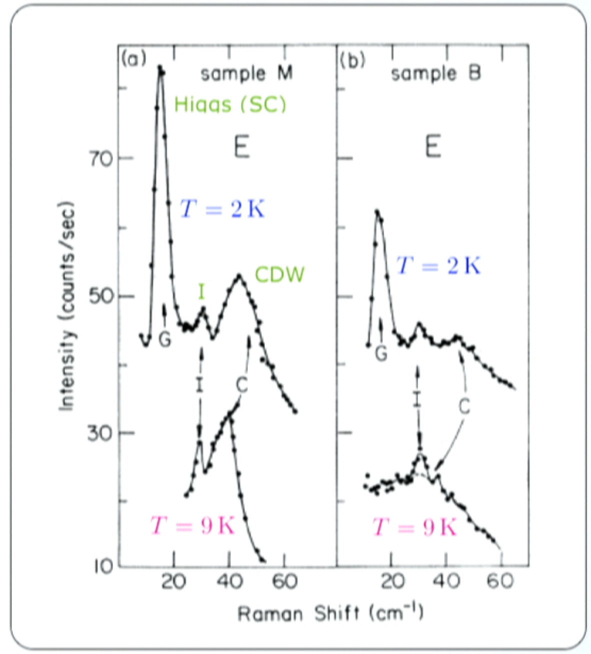
$$\nu^2 = \omega_0^2 + 2\omega_0 \Pi(\nu)$$

$$\nu \approx 2\Delta \left\{ 1 - \frac{2\alpha^2}{\pi^2} \frac{\omega_0^2}{\omega_0^2 - 4\Delta^2} \right\}$$

For $\nu > 2\Delta$ one obtains a broadened peak near the bare CDW amplitude mode ω_0 .

P. B. Littlewood and C. M. Varma, *Phys. Rev. B* **26**, 4883 (1982)





R. Sooryakumar *et al.*, *Phys. Rev. B* **23**, 3222 (1981)

Higgs mechanism

E. Abers and B. W. Lee, *Phys. Rep.* **9**, 1 (1973)

Start with gauged relativistic O(2) theory :

$$\mathcal{L} = \frac{1}{2g}(\partial_\mu + ieA_\mu)\psi^* (\partial^\mu - ieA^\mu)\psi - \frac{m_0^2}{2g}(\psi^*\psi - 1)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Rewrite the OP in terms of radial and angular fields : $\psi = (1+\eta)\, e^{i\xi}$

We know that the Lagrangian is gauge invariant :

$$\psi \to e^{-i\alpha}\,\psi \qquad , \qquad A_\mu \to A_\mu - e^{-1}\partial_\mu\alpha$$

Choose $\alpha = \xi$ as gauge function and define $\widetilde{A}_\mu \equiv A_\mu - e^{-1}\partial_\mu\xi$. Then

$$\mathcal{L} = \frac{1}{2g}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{e^2}{2g}(1+\eta)^2\widetilde{A}_\mu\widetilde{A}^\mu - \frac{m_0^2}{2g}(4\eta^2 + 4\eta^3 + \eta^4) - \tfrac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}$$

- ⋆ The photon becomes a massive triplet $\widetilde{A}_\mu$ with $m_{\tilde{A}} = eg^{-1/2}$
- ⋆ There is no ξ particle ! (It is 'eaten' by the photon field.)
- ⋆ The Higgs (η) mass is independent of e, with $m_{\rm H} = 2m_0$

Higgs mechanism

E. Abers and B. W. Lee, *Phys. Rep.* **9**, 1 (1973)

Start with gauged relativistic O(2) theory :

$$\mathcal{L} = \frac{1}{2g}(\partial_\mu + ieA_\mu)\psi^* \, (\partial^\mu - ieA^\mu)\psi - \frac{m_0^2}{2g}(\psi^*\psi - 1)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Rewrite the OP in terms of radial and angular fields : $\psi = (1+\eta)\, e^{i\xi}$

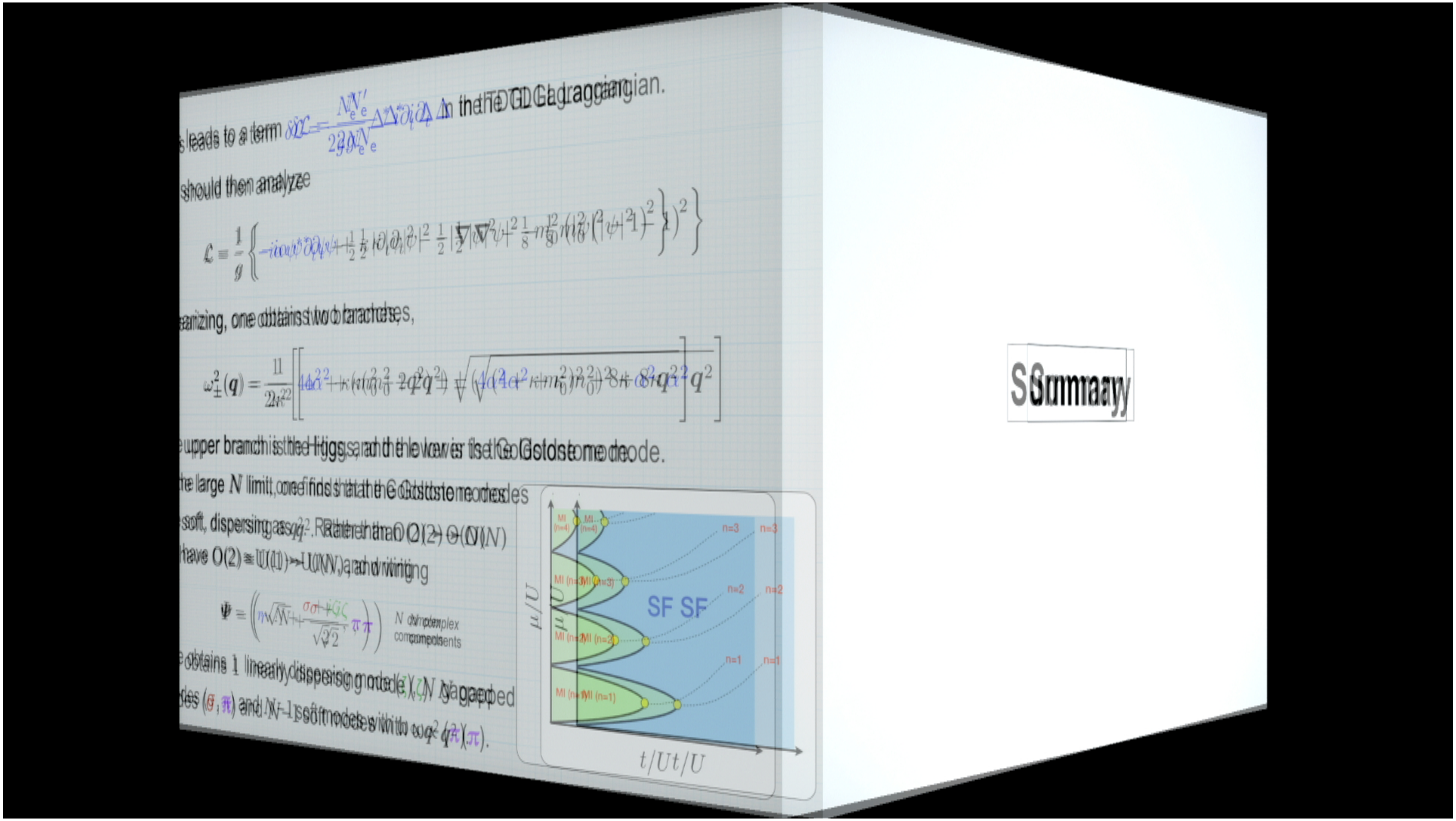
We know that the Lagrangian is gauge invariant :

$$\psi \to e^{-i\alpha}\,\psi \qquad , \qquad A_\mu \to A_\mu - e^{-1}\partial_\mu \alpha$$

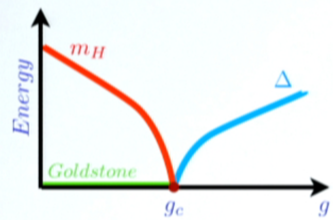
Choose $\alpha = \xi$ as gauge function and define $\widetilde{A}_\mu \equiv A_\mu - e^{-1}\partial_\mu \xi$. Then

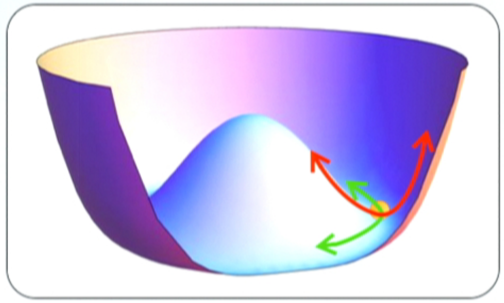
$$\mathcal{L} = \frac{1}{2g}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{e^2}{2g}(1+\eta)^2 \widetilde{A}_\mu \widetilde{A}^\mu - \frac{m_0^2}{2g}(4\eta^2 + 4\eta^3 + \eta^4) - \tfrac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}$$

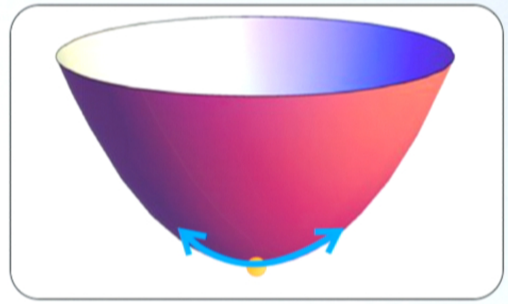
- ⋆ The photon becomes a massive triplet $\widetilde{A}_\mu$ with $m_{\tilde{A}} = eg^{-1/2}$
- ⋆ There is no ξ particle ! (It is 'eaten' by the photon field.)
- ⋆ The Higgs (η) mass is independent of e, with $m_{\mathrm{H}} = 2m_0$


should then analyze
upper branch is the Higgs, and the lower is the Goldstone mode.
N complex components
SF
n=3
n=2
n=1
MI (n=1)
MI (n=2)
MI (n=3)
Summary

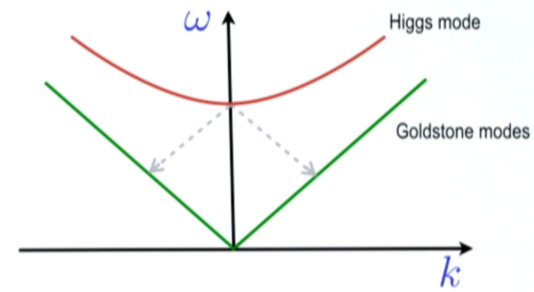
O(N) model

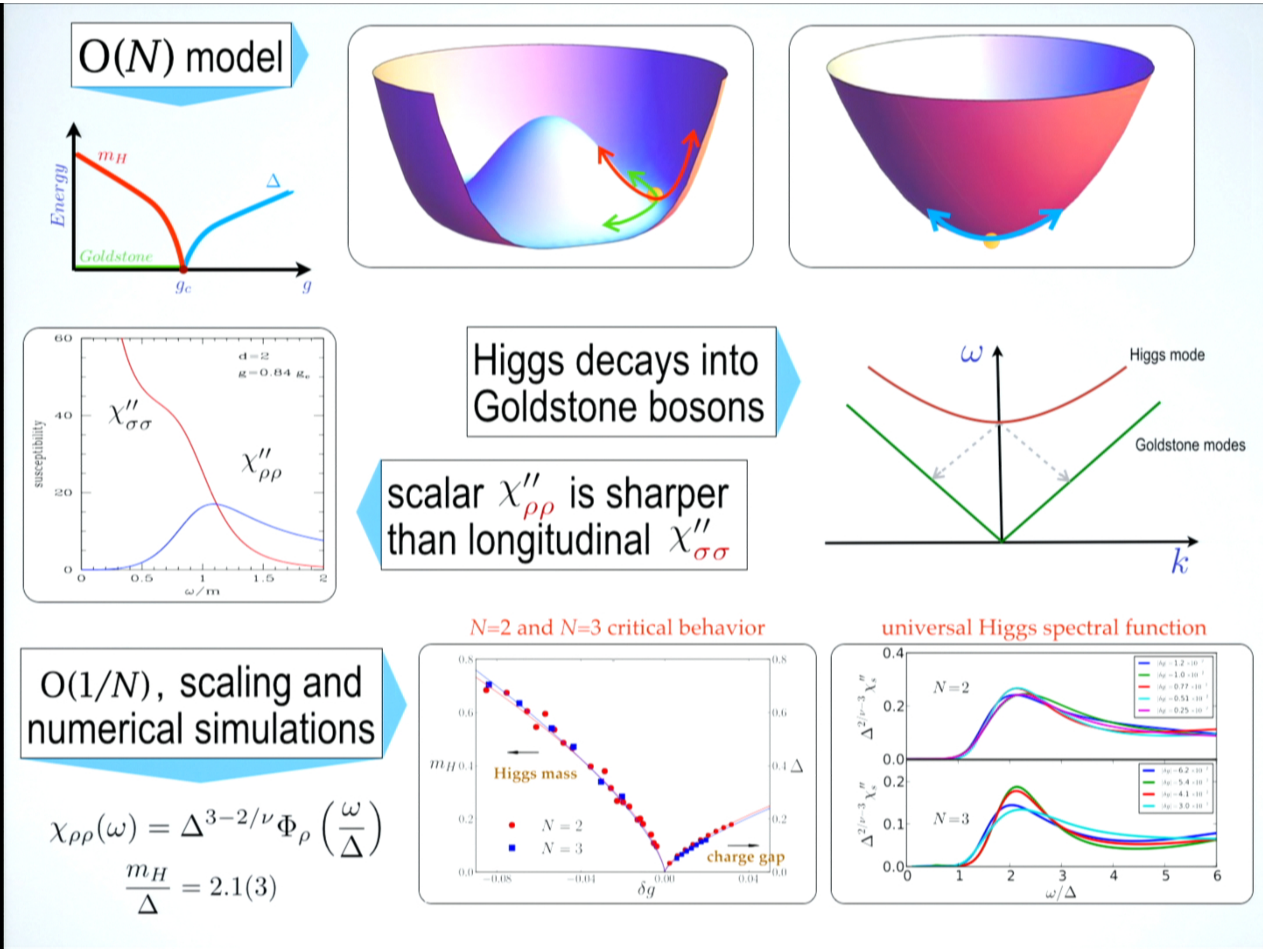






Higgs decays into Goldstone bosons

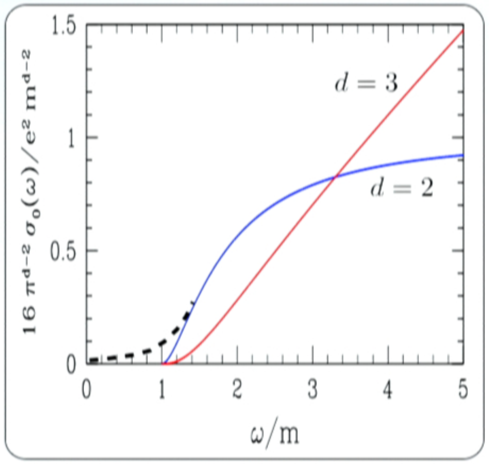


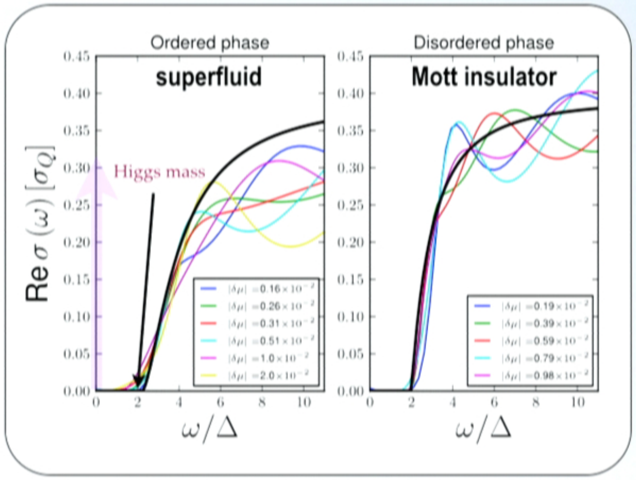


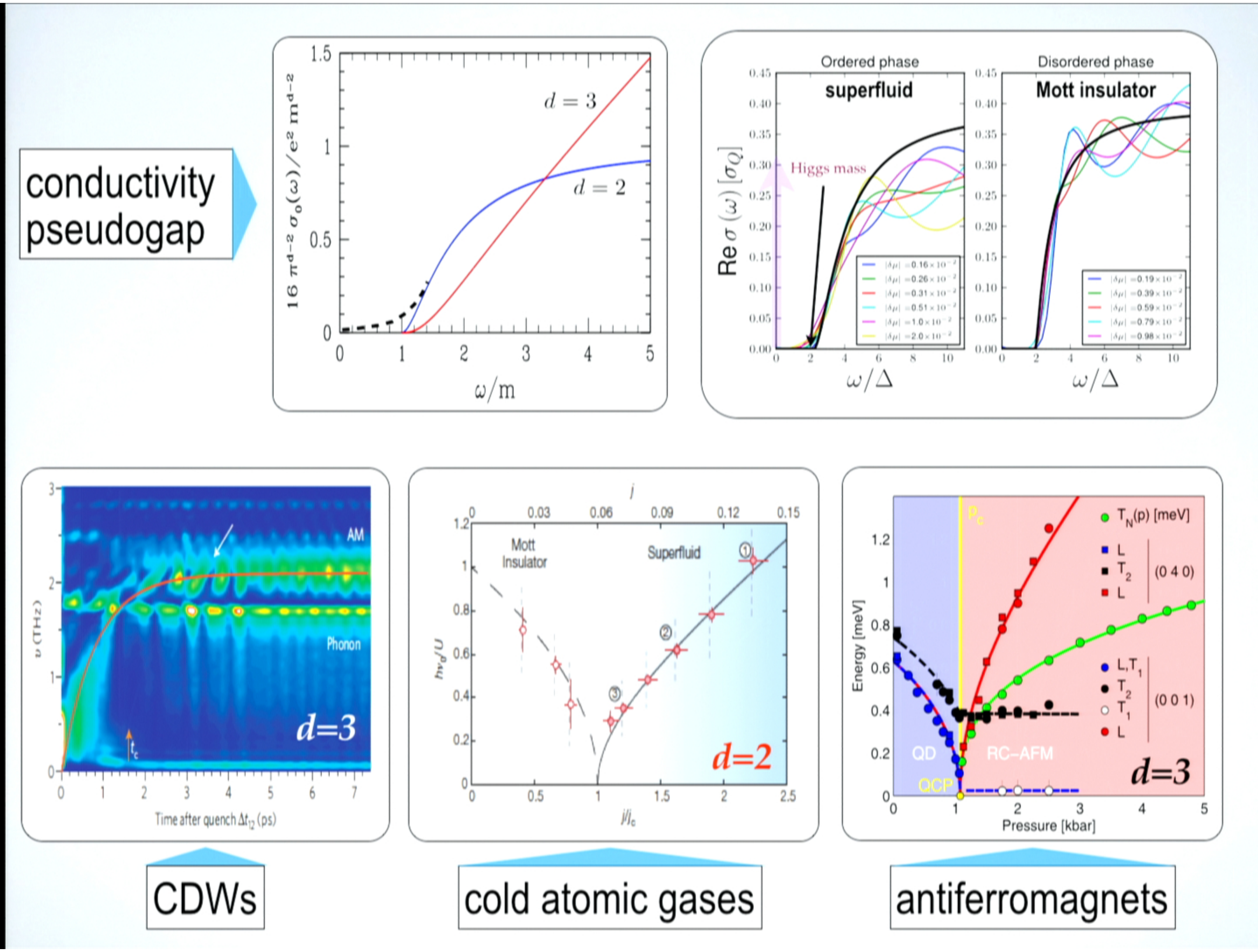
$$\chi_{\rho\rho}(\omega) = \Delta^{3-2/\nu}\Phi_\rho\left(\frac{\omega}{\Delta}\right)$$

$$\frac{m_H}{\Delta} = 2.1(3)$$

conductivity
pseudogap






conductivity
pseudogap
$d=3$
$d=2$
ω/m
Ordered phase
superfluid
Disordered phase
Mott insulator
Higgs mass
ω/Δ
ω/Δ
AM
Phonon
$d=3$
Time after quench
Mott Insulator
Superfluid
$d=2$
$T_N(p)$ [meV]
(0 4 0)
(0 0 1)
QD
RC-AFM
QCP
$d=3$
Energy [meV]
Pressure [kbar]
CDWs
cold atomic gases
antiferromagnets

Open questions

- Perturbative results found a pronounced conductivity pseudo gap for $N = 2$.
  Is the low-frequency conductivity qualitatively different for $N > 2$ for all $g \lesssim g_c$?

Open questions

- Perturbative results found a pronounced conductivity pseudo gap for $N = 2$.
  Is the low-frequency conductivity qualitatively different for $N > 2$ for all $g \lesssim g_c$?

- Theories of deconfined quantum criticality are described by the NCCP^N action,

$$\mathcal{L}_{\mathrm{E}} = \left|(\partial_\mu - i a_\mu) z\right|^2 + s\, z^\dagger z + u\, (z^\dagger z)^2 + \frac{1}{e^2}\, (\epsilon_{\mu\nu\lambda} \partial_\nu\, a_\lambda)^2$$

  with a dynamical gauge field, giving rise to the full-fledged Higgs mechanism.
  In the Néel phase there are spin waves, a Higgs, and massive vector bosons.
  Whether or not H can decay to MVBs depends on the charge e. What is the fate of the Higgs near a DQCP?

Open questions

- Perturbative results found a pronounced conductivity pseudo gap for $N=2$.
  Is the low-frequency conductivity qualitatively different for $N>2$ for all $g \lesssim g_c$?

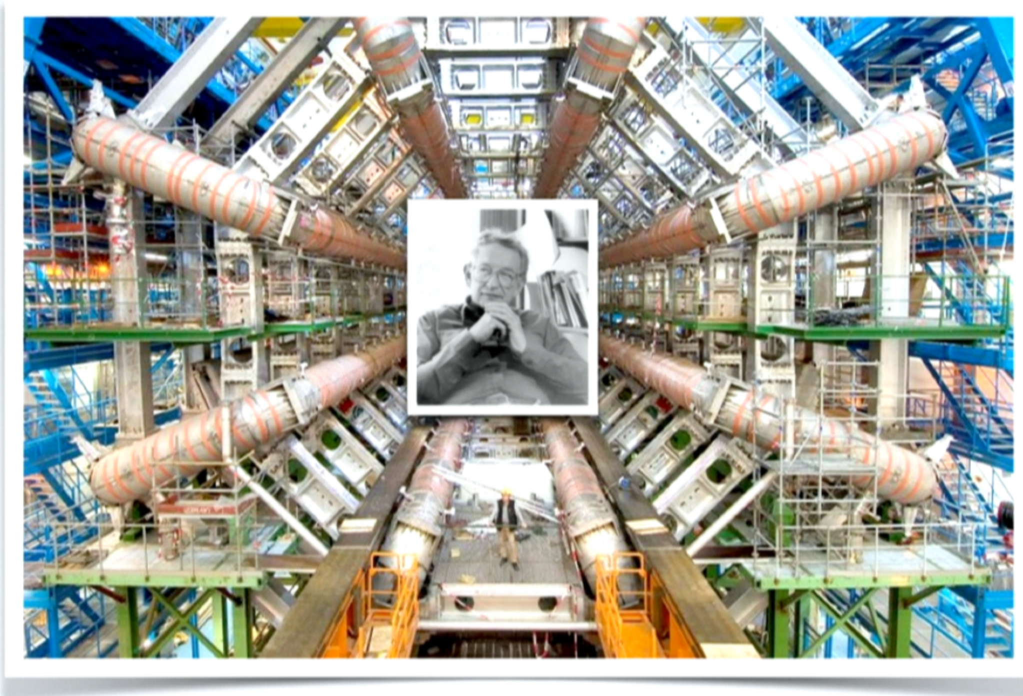
- Theories of deconfined quantum criticality are described by the NCCP^N action,

$$\mathcal{L}_{\mathrm{E}} = \left|(\partial_\mu - i a_\mu) z\right|^2 + s\, z^\dagger z + u\,(z^\dagger z)^2 + \frac{1}{e^2}\,(\epsilon_{\mu\nu\lambda}\partial_\nu\, a_\lambda)^2$$

  with a dynamical gauge field, giving rise to the full-fledged Higgs mechanism. In the Néel phase there are spin waves, a Higgs, and massive vector bosons. Whether or not H can decay to MVBs depends on the charge e. What is the fate of the Higgs near a DQCP?

- Lots of interesting work on diluted AFMs shows differences between site and dimer bond dilution. Interplay of geometric criticality and quantum fluctuations leads to two transitions: an $\mathrm{O}(N)$ transition at weak dilution as a function of g and a percolation transition as a function of p when g is small, with a multi critical point at (p_c, g^*). What is going on with Higgs?

THE SEARCH FOR THE HIGGS



BCS and TDGL Theory

A. van Otterlo *et al.*, *Phys. Rev. Lett.* **75**, 3736 (1995)

Generic s-wave BCS instability :

$$\mathcal{L}_\mathsf{E} = \bar{\psi}_\sigma \Big[\partial_\tau - ieA_0 + \varepsilon(-i\boldsymbol{\nabla} - e\boldsymbol{A}) \Big] \psi_\sigma - g\, \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow + ie\, n_{\text{ion}}\, A_0 + \tfrac{1}{8\pi}\big(\boldsymbol{E}^2 + \boldsymbol{B}^2\big)$$

Introduce Hubbard-Stratonovich field Δ :

$$\widetilde{\mathcal{L}}_\mathsf{E} = \frac{|\Delta|^2}{g} + \frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{8\pi} + ie\, n_{\text{ion}}\, A_0 - \mathsf{Tr} \ln \mathcal{G}^{-1}$$

$$\mathcal{G}^{-1} = \begin{pmatrix} \partial_\tau - ieA_0 + \varepsilon(-i\boldsymbol{\nabla} - e\boldsymbol{A}) & \Delta \\ \Delta^* & \partial_\tau + ieA_0 + \varepsilon(i\boldsymbol{\nabla} - e\boldsymbol{A}) \end{pmatrix}$$

The electron density is

$$n_\mathsf{e} = -\,\mathsf{Tr}\,\big[\mathcal{G}\,\sigma^3\big] = \tfrac{1}{2}\int\limits_{-\infty}^{\infty} d\xi\; N_\mathsf{e}(\xi + \mu + ieA_0) \left[1 - \tfrac{\xi}{\sqrt{\xi^2 + |\Delta|^2}} \right] = n_{\text{ion}} + ieN_\mathsf{e}A_0 + n_\Delta + \ldots$$

where $n_\Delta = \dfrac{|\Delta|^2 N_\mathsf{e}'}{g N_\mathsf{e}}$. This is nonzero only when PH symmetry is broken.

BCS and TDGL Theory

A. van Otterlo *et al.*, *Phys. Rev. Lett.* **75**, 3736 (1995)

Generic *s*-wave BCS instability :

$$\mathcal{L}_{\mathsf{E}} = \bar{\psi}_\sigma \Big[\partial_\tau - ieA_0 + \varepsilon(-i\boldsymbol{\nabla} - e\boldsymbol{A})\Big]\psi_\sigma - g\,\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow + ie\,n_{\text{ion}}\,A_0 + \tfrac{1}{8\pi}\big(\boldsymbol{E}^2 + \boldsymbol{B}^2\big)$$

Introduce Hubbard-Stratonovich field Δ :

$$\widetilde{\mathcal{L}}_{\mathsf{E}} = \frac{|\Delta|^2}{g} + \frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{8\pi} + ie\,n_{\text{ion}}\,A_0 - \mathsf{Tr}\ln\mathcal{G}^{-1}$$

$$\mathcal{G}^{-1} = \begin{pmatrix} \partial_\tau - ieA_0 + \varepsilon(-i\boldsymbol{\nabla} - e\boldsymbol{A}) & \Delta \\ \Delta^* & \partial_\tau + ieA_0 + \varepsilon(i\boldsymbol{\nabla} - e\boldsymbol{A}) \end{pmatrix}$$

The electron density is

$$n_{\mathsf{e}} = -\,\mathsf{Tr}\left[\mathcal{G}\,\sigma^3\right] = \tfrac{1}{2}\int\limits_{-\infty}^{\infty} d\xi\; N_{\mathsf{e}}(\xi + \mu + ieA_0)\left[1 - \tfrac{\xi}{\sqrt{\xi^2 + |\Delta|^2}}\right] = n_{\text{ion}} + ieN_{\mathsf{e}}A_0 + n_\Delta + \ldots$$

where $n_\Delta = \dfrac{|\Delta|^2 N_{\mathsf{e}}'}{gN_{\mathsf{e}}}$. This is nonzero only when PH symmetry is broken.

This leads to a term $\delta\mathcal{L} = \frac{N_e'}{2gN_e}\,\Delta^*\,i\partial_t\,\Delta$ in the TDGL Lagrangian.

We should then analyze

$$\mathcal{L} = \frac{1}{g}\left\{-i\alpha\,\psi^*\partial_t\psi + \tfrac{1}{2}\,\kappa\,|\partial_t\psi|^2 - \tfrac{1}{2}\,|\boldsymbol{\nabla}\psi|^2 - \tfrac{1}{8}\,m_0^2\,\big(|\psi|^2-1\big)^2\right\}$$

This leads to a term $\delta\mathcal{L} = \dfrac{N'_{\mathsf{e}}}{2gN_{\mathsf{e}}}\,\Delta^*\, i\partial_t\,\Delta$ in the TDGL Lagrangian.

We should then analyze

$$\mathcal{L} = \frac{1}{g}\left\{-i\alpha\,\psi^*\partial_t\psi + \tfrac{1}{2}\,\kappa\,|\partial_t\psi|^2 - \tfrac{1}{2}\,|\boldsymbol{\nabla}\psi|^2 - \tfrac{1}{8}\,m_0^2\,\big(|\psi|^2 - 1\big)^2\right\}$$

This leads to a term $\delta\mathcal{L} = \dfrac{N'_{\rm e}}{2gN_{\rm e}}\,\Delta^*\, i\partial_t\,\Delta$ in the TDGL Lagrangian.

We should then analyze

$$\mathcal{L} = \frac{1}{g}\left\{ -i\alpha\,\psi^*\partial_t\psi + \tfrac{1}{2}\,\kappa\,|\partial_t\psi|^2 - \tfrac{1}{2}\,|\boldsymbol{\nabla}\psi|^2 - \tfrac{1}{8}\,m_0^2\,\big(|\psi|^2 - 1\big)^2 \right\}$$

Linearizing, one obtains two branches,

$$\omega_\pm^2(\boldsymbol{q}) = \frac{1}{2\kappa^2}\left[4\alpha^2 + \kappa(m_0^2 + 2\boldsymbol{q}^2) \pm \sqrt{(4\alpha^2 + \kappa\,m_0^2)^2 + 8\kappa\,\alpha^2\boldsymbol{q}^2}\,\right]$$

The upper branch is the Higgs, and the lower is the Goldstone mode.

This leads to a term $\delta\mathcal{L} = \frac{N'_{\rm e}}{2gN_{\rm e}}\,\Delta^*\, i\partial_t\,\Delta$ in the TDGL Lagrangian.

We should then analyze

$$\mathcal{L} = \frac{1}{g}\left\{-i\alpha\,\psi^*\partial_t\psi + \tfrac{1}{2}\,\kappa\,|\partial_t\psi|^2 - \tfrac{1}{2}\,|\boldsymbol{\nabla}\psi|^2 - \tfrac{1}{8}\,m_0^2\,\big(|\psi|^2-1\big)^2\right\}$$

Linearizing, one obtains two branches,

$$\omega_\pm^2(\boldsymbol{q}) = \frac{1}{2\kappa^2}\left[4\alpha^2 + \kappa(m_0^2+2\boldsymbol{q}^2) \pm \sqrt{(4\alpha^2+\kappa\,m_0^2)^2 + 8\kappa\,\alpha^2\boldsymbol{q}^2}\,\right]$$

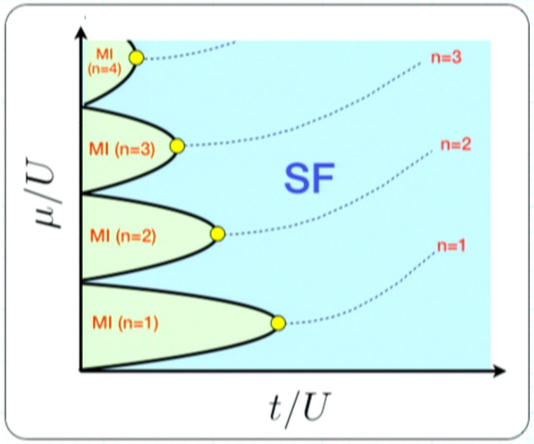
The upper branch is the Higgs, and the lower is the Goldstone mode.

In the large N limit, one finds that the Goldstone modes are soft, dispersing as $\boldsymbol{q}^2$. Rather than $\mathrm{O}(2)\rightarrow\mathrm{O}(N)$ we have $\mathrm{O}(2)\cong\mathrm{U}(1)\rightarrow\mathrm{U}(N)$, and writing

$$\boldsymbol{\Psi} = \left(r\sqrt{N} + \frac{\sigma+i\zeta}{\sqrt{2}}\,,\,\boldsymbol{\pi}\right)$$

N complex components

one obtains 1 linearly dispersing mode (ζ), N gapped modes (σ, π) and $N-1$ soft modes with $\omega \propto \boldsymbol{q}^2$ (π).



This leads to a term $\delta\mathcal{L} = \frac{N'_e}{2gN_e}\,\Delta^*\, i\partial_t\,\Delta$ in the TDGL Lagrangian.

We should then analyze

$$\mathcal{L} = \frac{1}{g}\left\{ -i\alpha\,\psi^*\partial_t\psi + \tfrac{1}{2}\,\kappa\,|\partial_t\psi|^2 - \tfrac{1}{2}\,|\boldsymbol{\nabla}\psi|^2 - \tfrac{1}{8}\,m_0^2\,\big(|\psi|^2-1\big)^2 \right\}$$

Linearizing, one obtains two branches,

$$\omega_\pm^2(\boldsymbol{q}) = \frac{1}{2\kappa^2}\left[4\alpha^2 + \kappa(m_0^2 + 2\boldsymbol{q}^2) \pm \sqrt{(4\alpha^2 + \kappa\,m_0^2)^2 + 8\kappa\,\alpha^2\boldsymbol{q}^2}\right]$$

The upper branch is the Higgs, and the lower is the Goldstone mode.

In the large N limit, one finds that the Goldstone modes are soft, dispersing as $\boldsymbol{q}^2$. Rather than $\mathrm{O}(2) \rightarrow \mathrm{O}(N)$ we have $\mathrm{O}(2) \cong \mathrm{U}(1) \rightarrow \mathrm{U}(N)$, and writing

$$\boldsymbol{\Psi} = \left(r\sqrt{N} + \frac{\sigma + i\zeta}{\sqrt{2}}\,,\,\boldsymbol{\pi} \right)$$

N complex components

one obtains 1 linearly dispersing mode (ζ), N gapped modes (σ, π) and $N-1$ soft modes with $\omega \propto \boldsymbol{q}^2$ (π).