Title: Classical Space Times from S Matrices

Date: Nov 26, 2013 01:00 PM

URL: http://pirsa.org/13110053

Abstract: <span>Progress
in calculating S matrix elements have shown that
the malicious redundancies in non-linear

gauge
theories can be circumvented by utilizing  unitarity methods in
conjunction

with BCFW recursion relations.  When calculating in this fashion  all
 of the interaction vertices

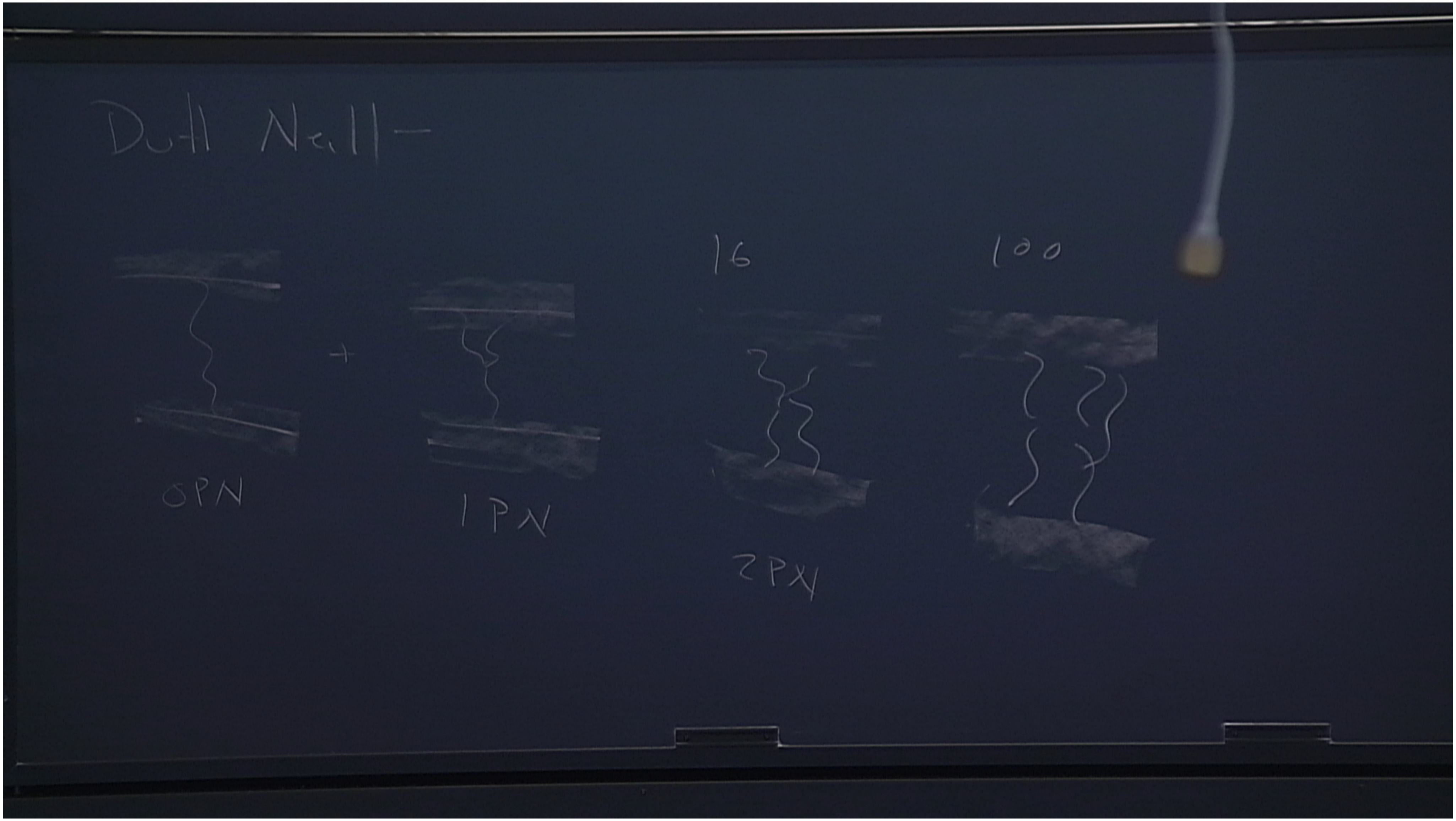
beyond the three point function can be ignored.  This simplification is
especially useful in gravity

which contains an infinite number of such non-linear interactions. It is natural to
ask whether off-shell quantities, such as classical solutions, can also be generated using only the three point
vertex. In this

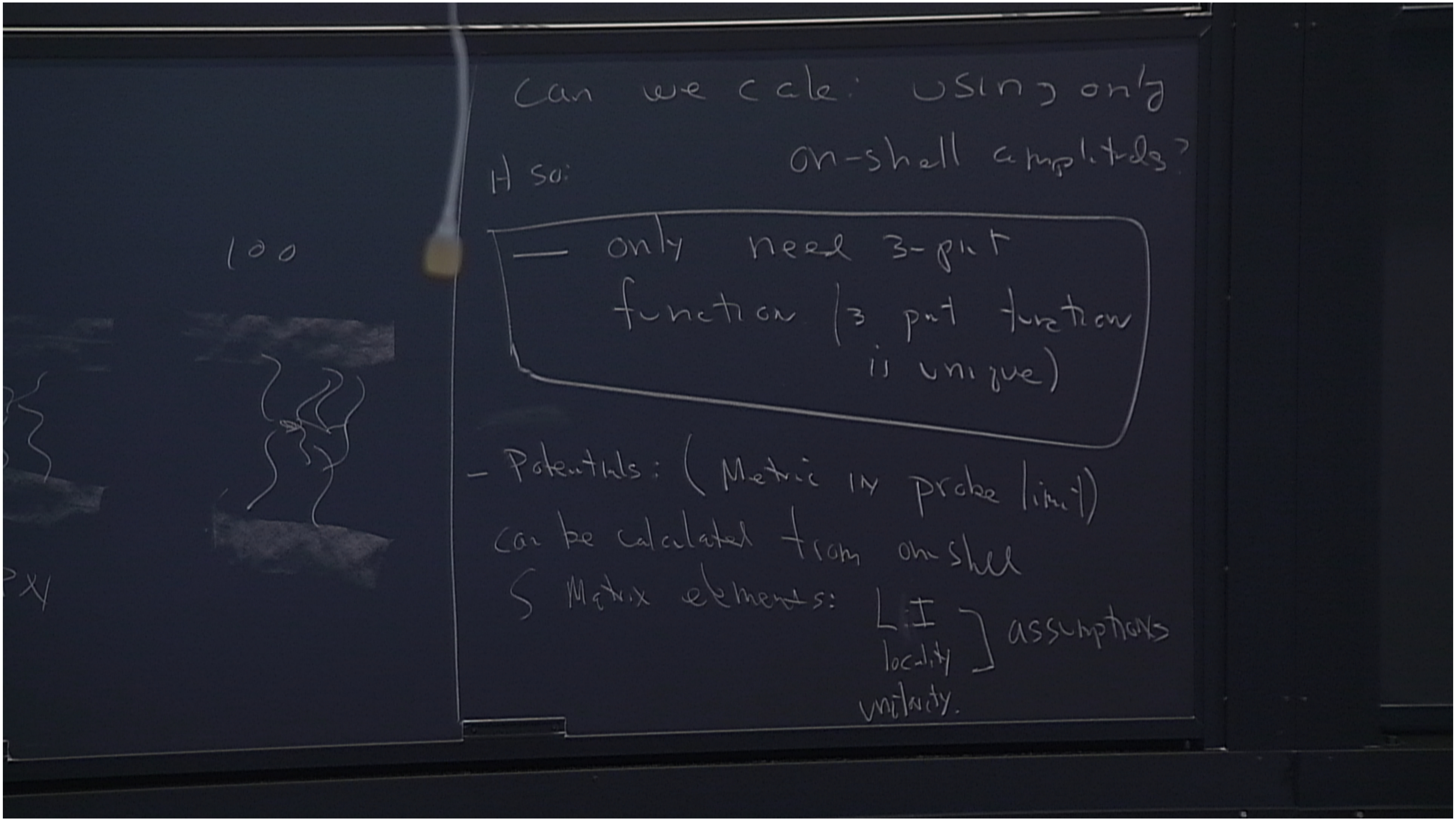
talk
I will show that this is indeed the case by extracting classical solutions to
GR from on-hell two to two scattering S-matrix elements. In
so doing we will completely circumvent the action as well as the equations
of motion. The only inputs will be Lorentz invariance, the existence of a massless spin-two particle and locality. Because of the double copy
relation this implies there exists, a yet to be understood, connection between solutions to Yang-Mills theory and Gravity. I
will also

discuss
how this technique can be used to simplify calculations of higher order post-Newtonian corrections

to
gravitational potentials relevant to the problem of binary inspirals.</span>



Dutt Nell -
16
100
0PN
1PN
2PN


Can we calc: using only
on-shell amplitudes?
If so:
— only need 3-pnt
function (3 pnt function
is unique)
- Potentials: (Metric in probe limit)
can be calculated from on shell
S Matrix elements: L.I.
locality ] assumptions
unitarity.
100

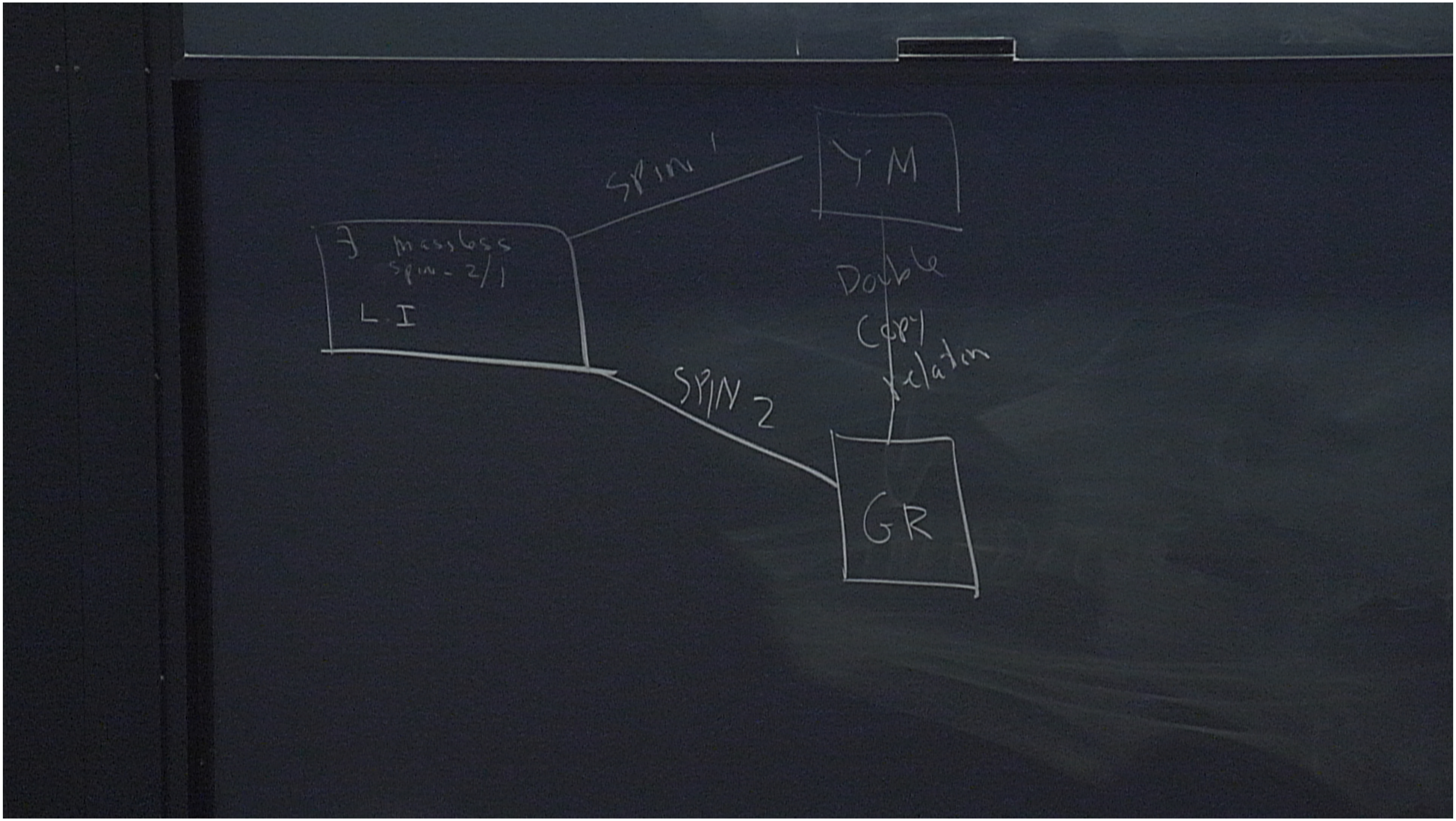
100

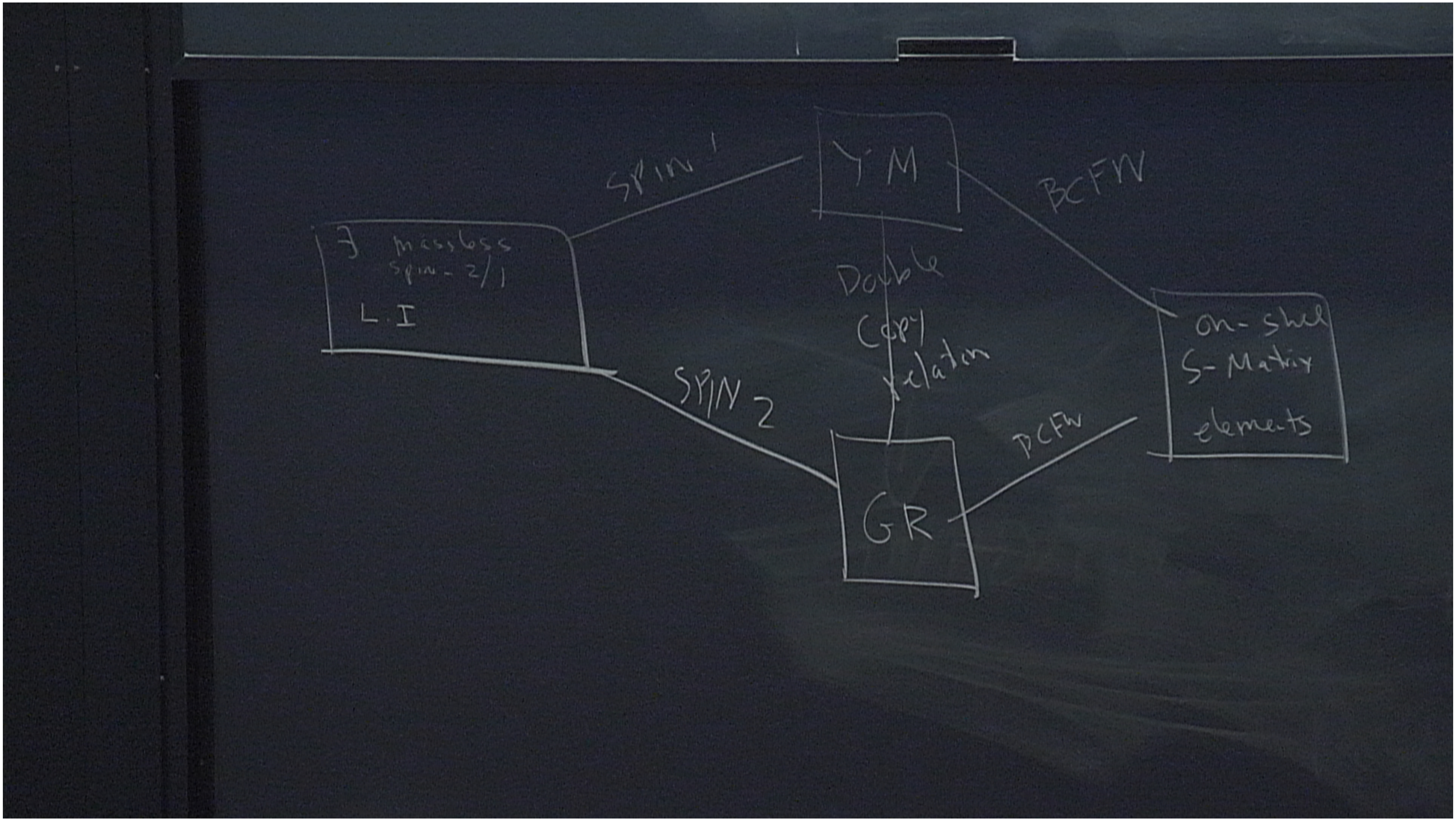
Can we calc: using only on-shell amplitudes?

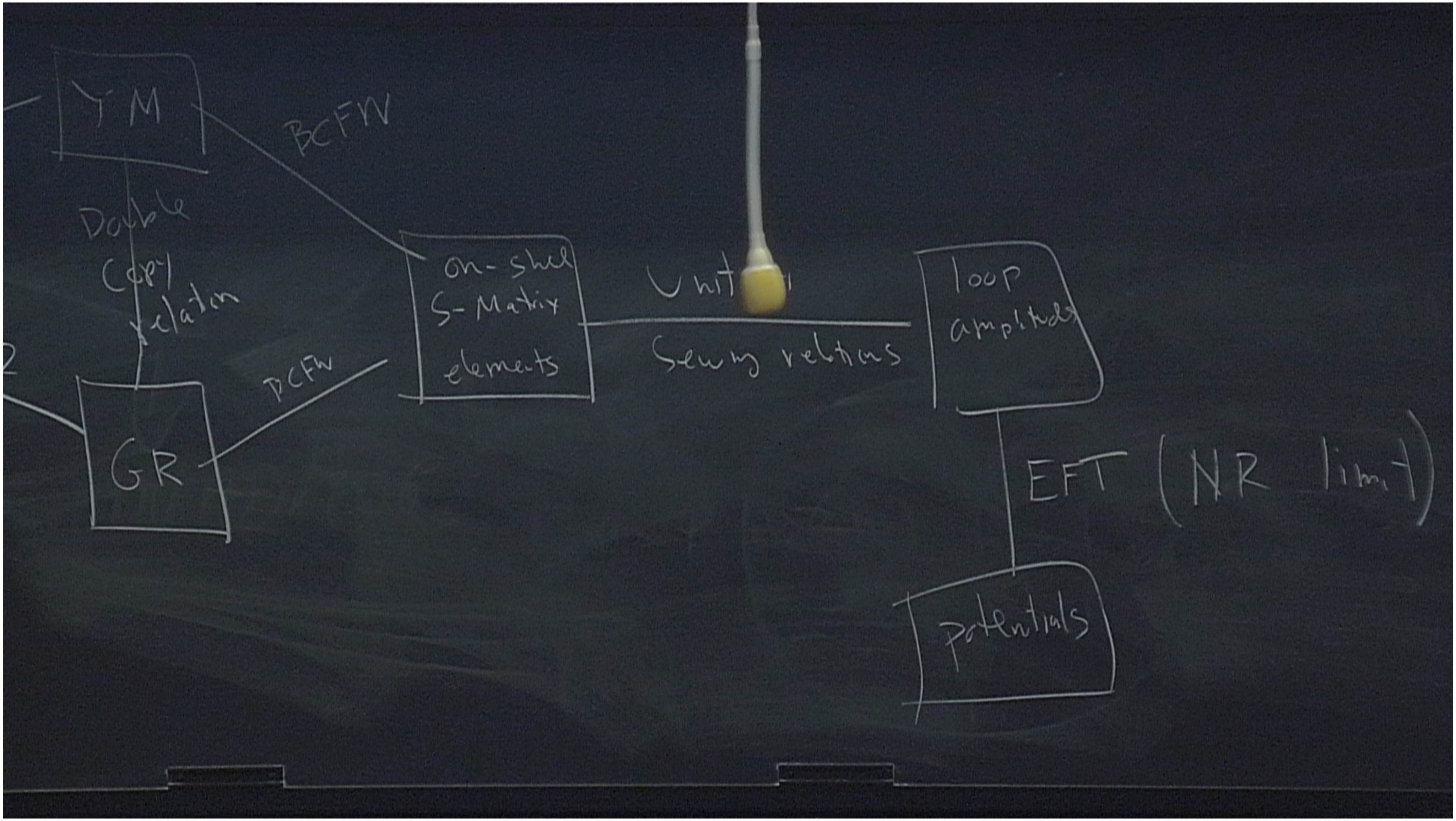
H so:

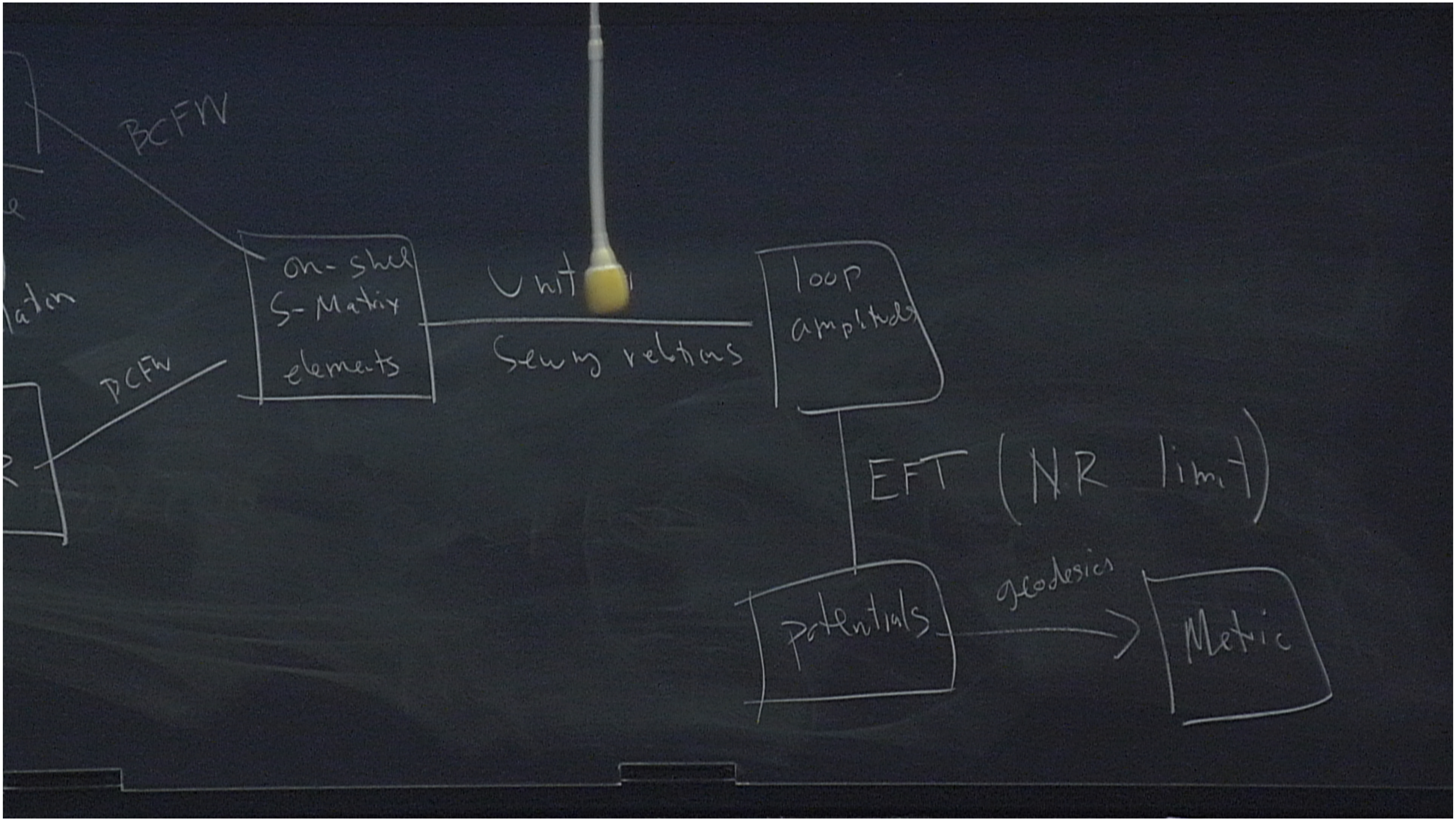
— only need 3-pt function (3 pt function is unique)

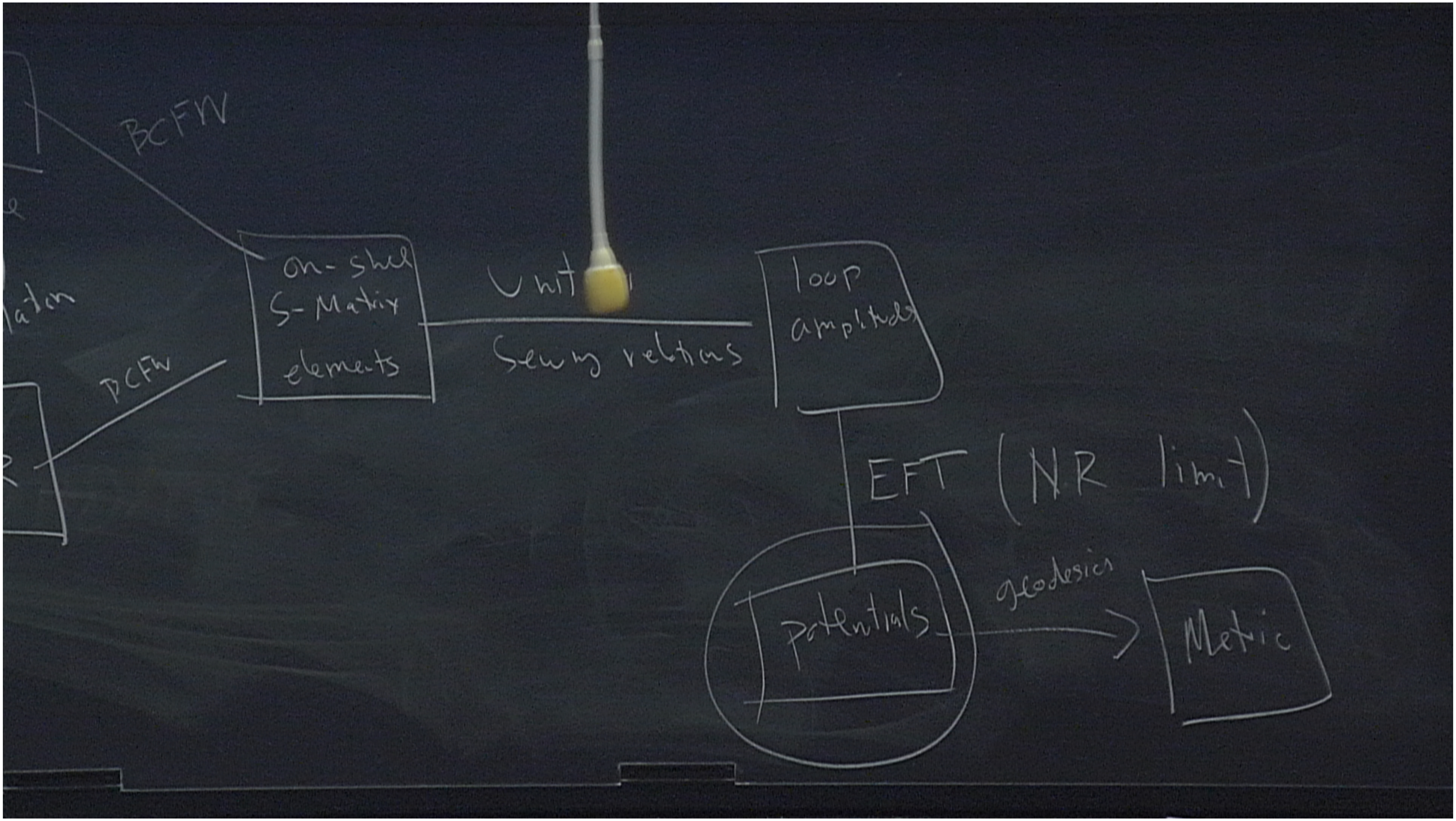
— Potentials: (Metric in probe limit) can be calculated from on shell S Matrix elements: L.I, locality ] assumptions
unitarity.

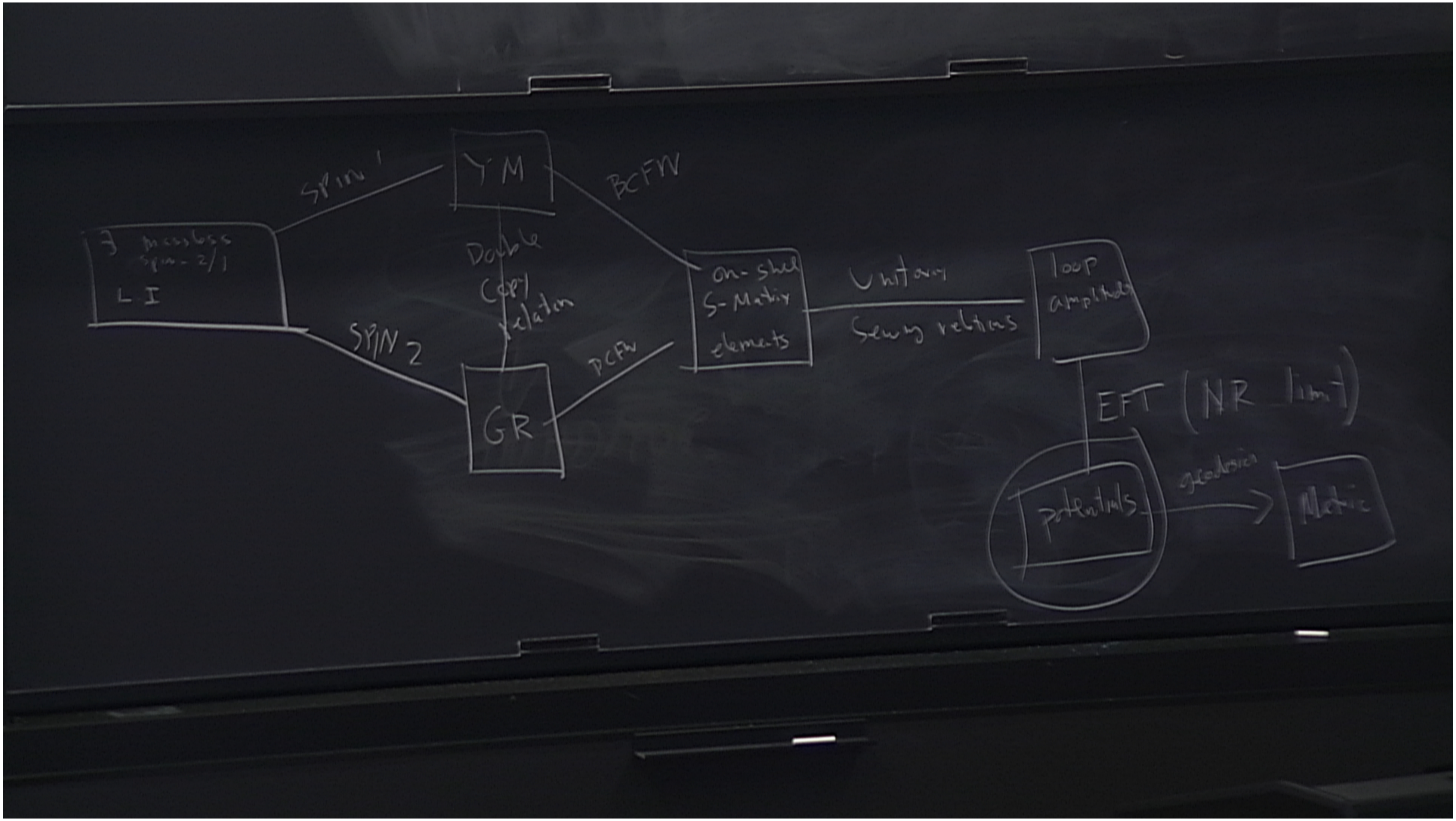

YM
SPIN 1
∃ massless spin-2/1
L.I
Double Copy relation
SPIN 2
GR

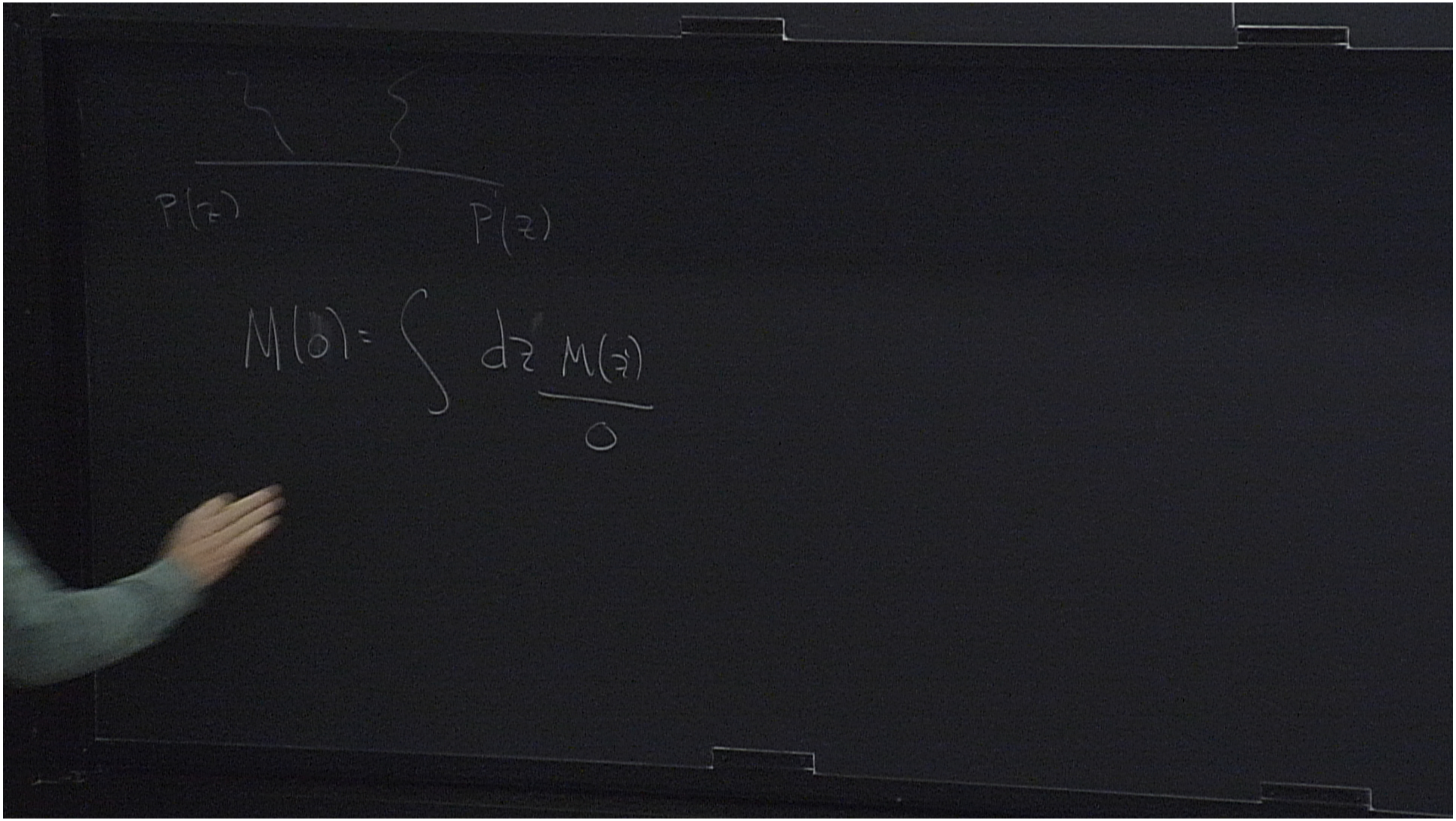

∃ massless spin-2/1
L.I
Spin 1
YM
BCFW
Double Copy relation
on-shell S-Matrix elements
SPIN 2
GR
BCFW

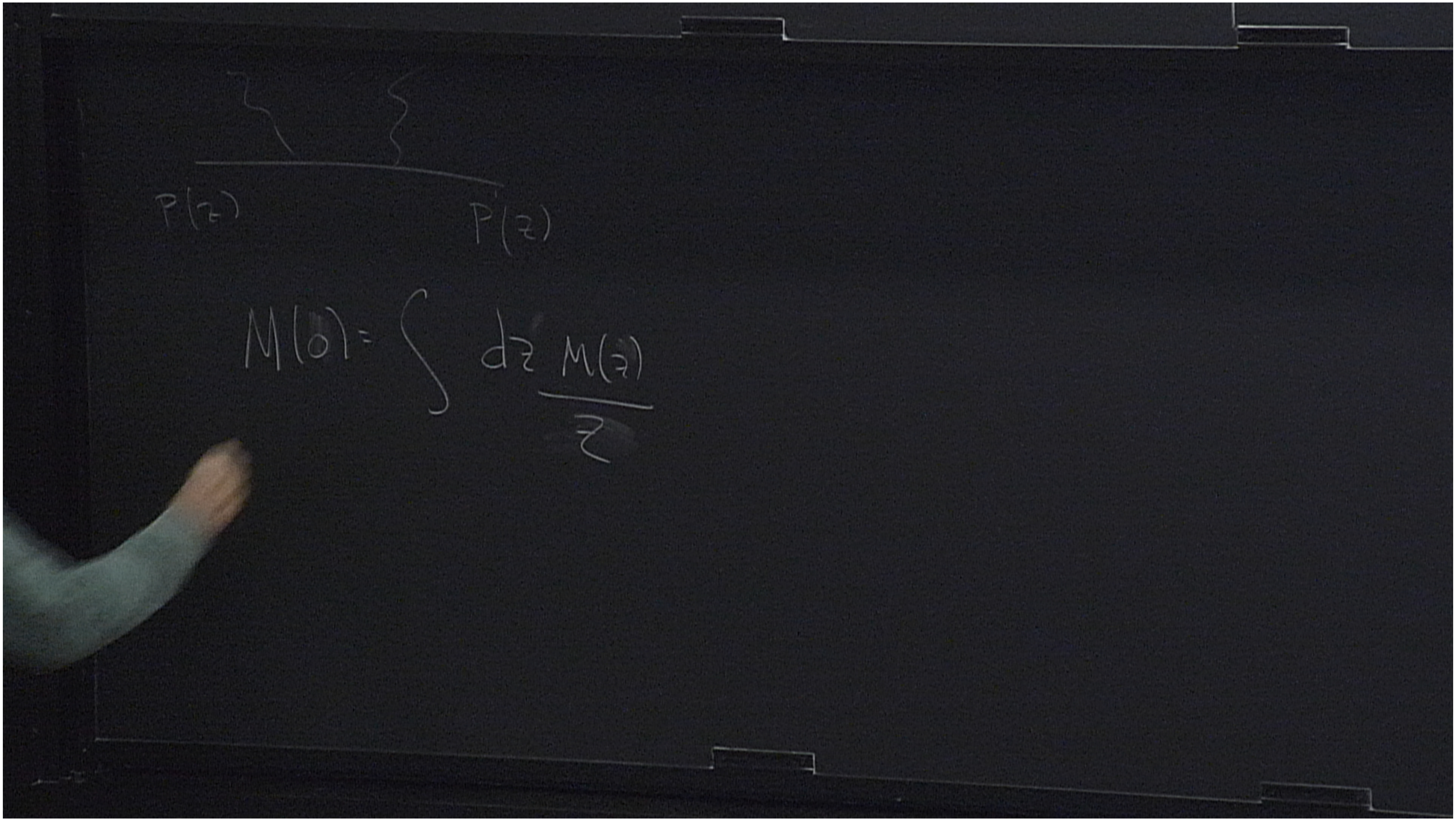

YM
BCFW
Double
copy
relation
on-shell
S-Matrix
elements
Unit
Sewing relations
loop
amplitudes
BCFW
GR
EFT (NR limit)
potentials

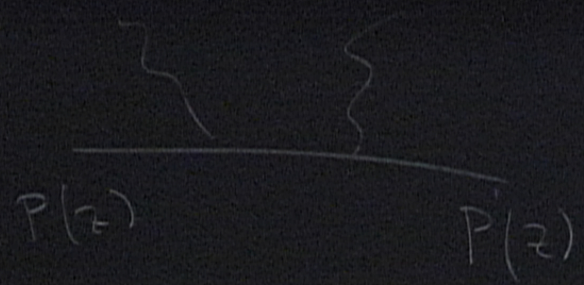

BCFW
on-shell S-Matrix elements
BCFW
Unit
Sewing relations
loop amplitudes
EFT (N.R limit)
potentials
geodesics
Metric


BCFW
on-shell S-Matrix elements
BCFW
Unit
Sewing relations
loop amplitudes
EFT (N.R limit)
potentials
geodesics
Metric

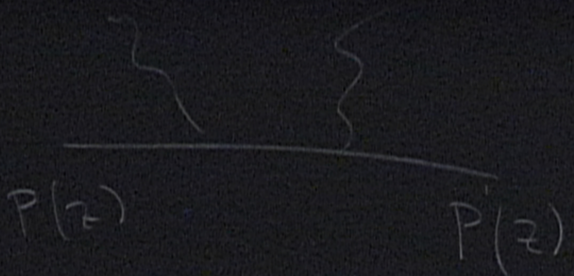

YM
BCFW
LI
Double copy relation
SPIN 2
GR
on-shell S-Matrix elements
Unitarity
Sewing relations
loop amplitudes
EFT (NR limit)
potentials
Metric


P(z)
P'(z)
M(0) = ∫ dz M(z)

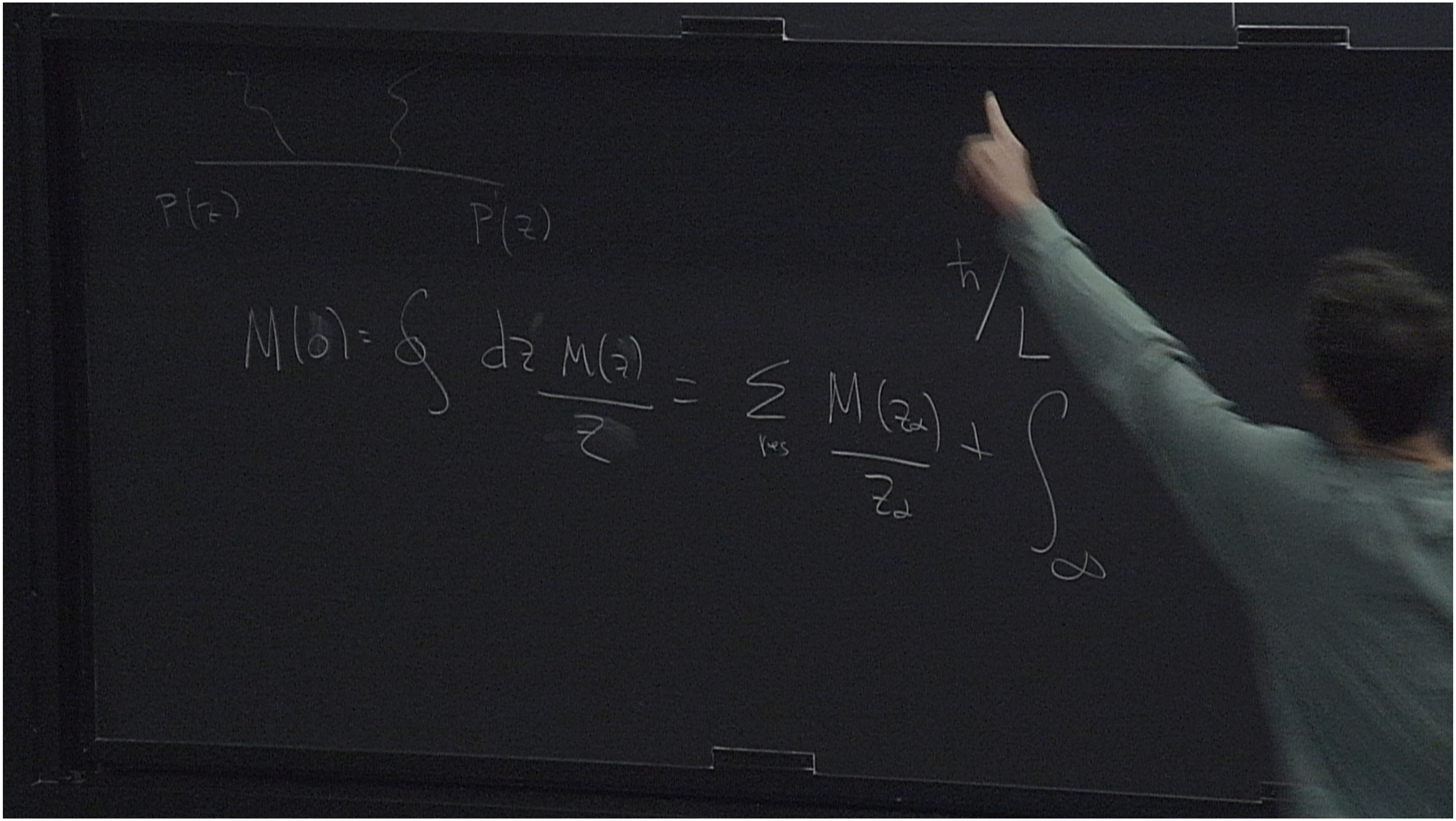

P(z)
P(z)
$M(0) = \int dz \frac{M(z)}{z}$

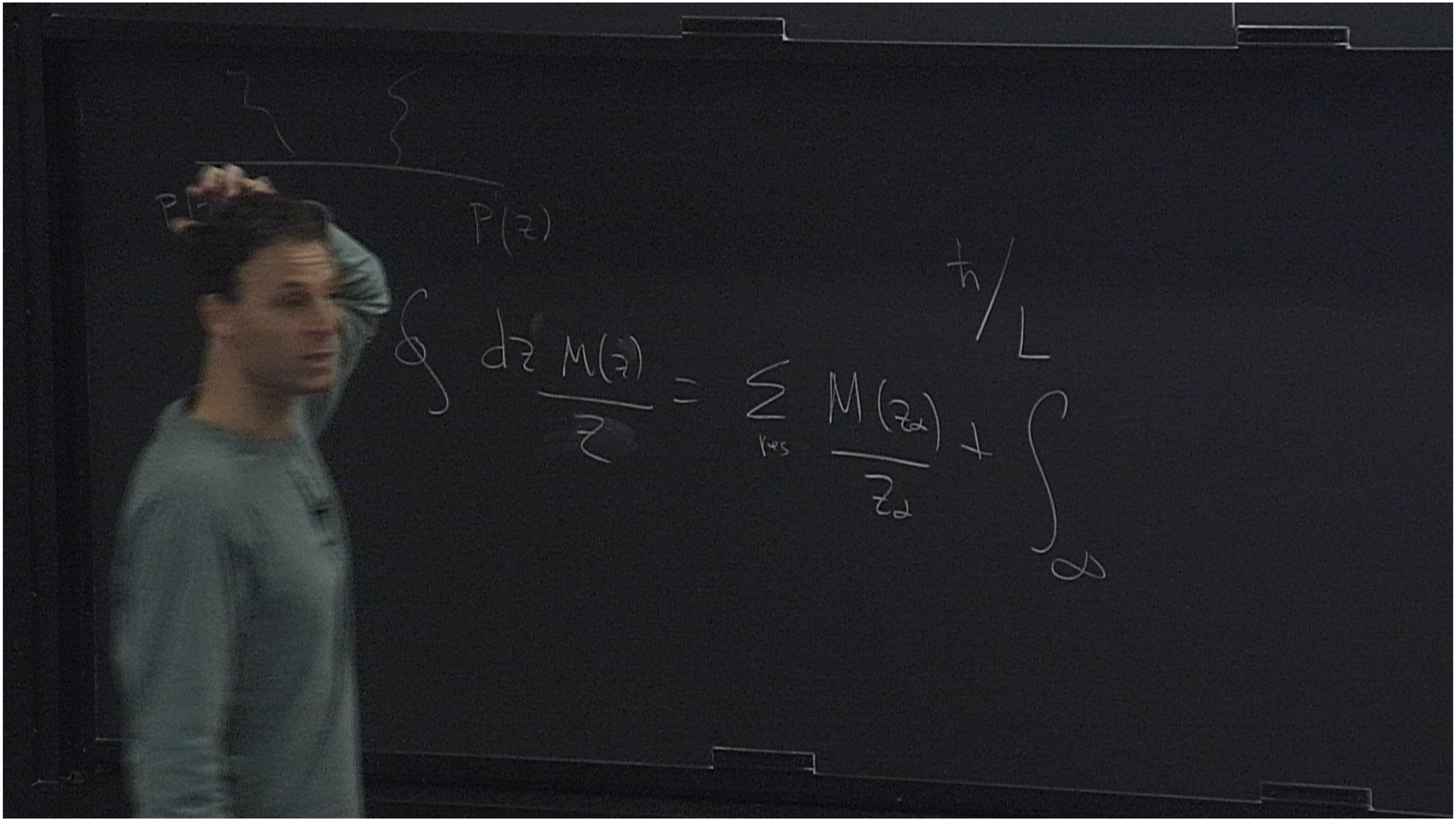


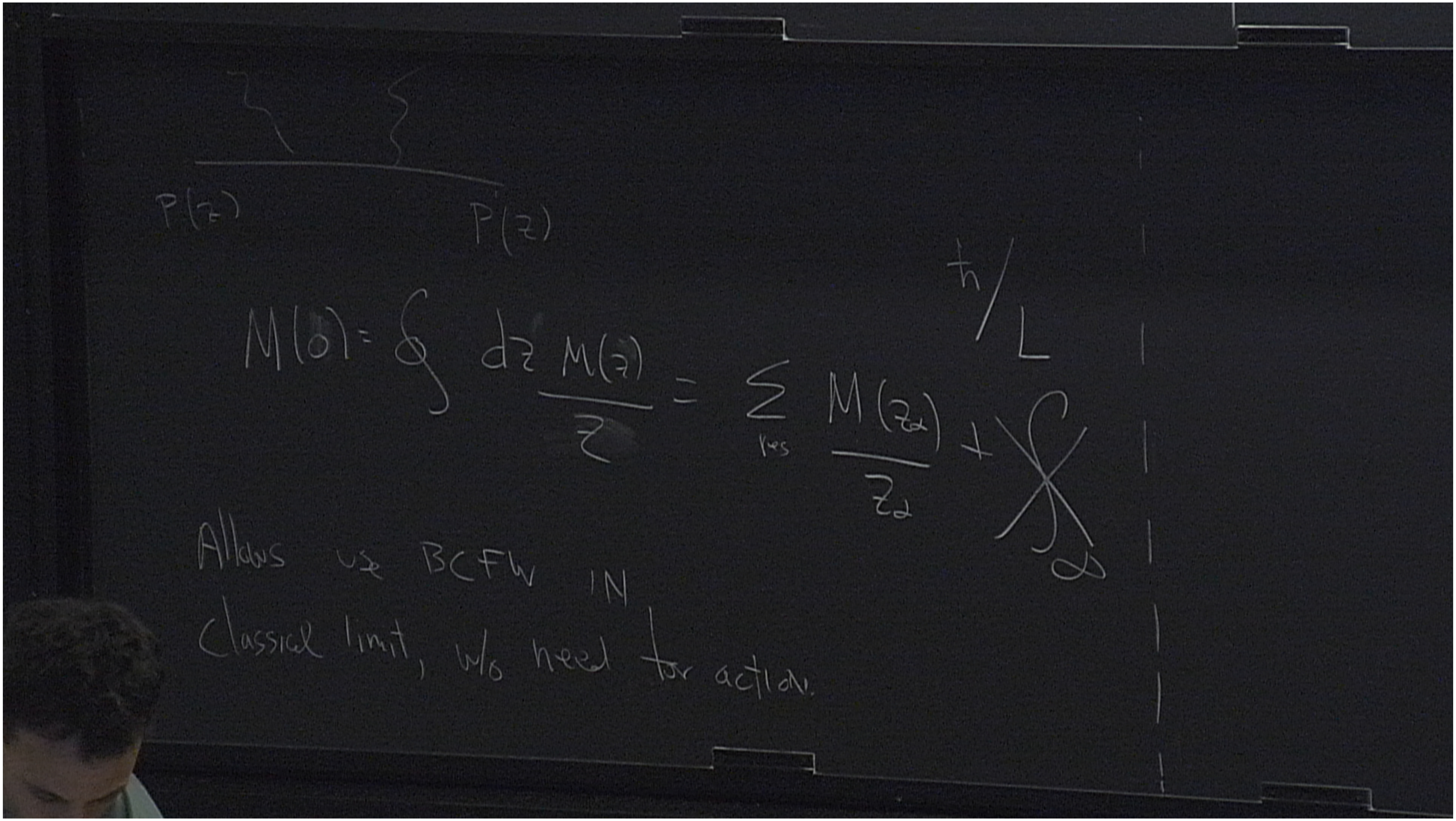

$$M(0) = \oint dz \, \frac{M(z)}{z}$$

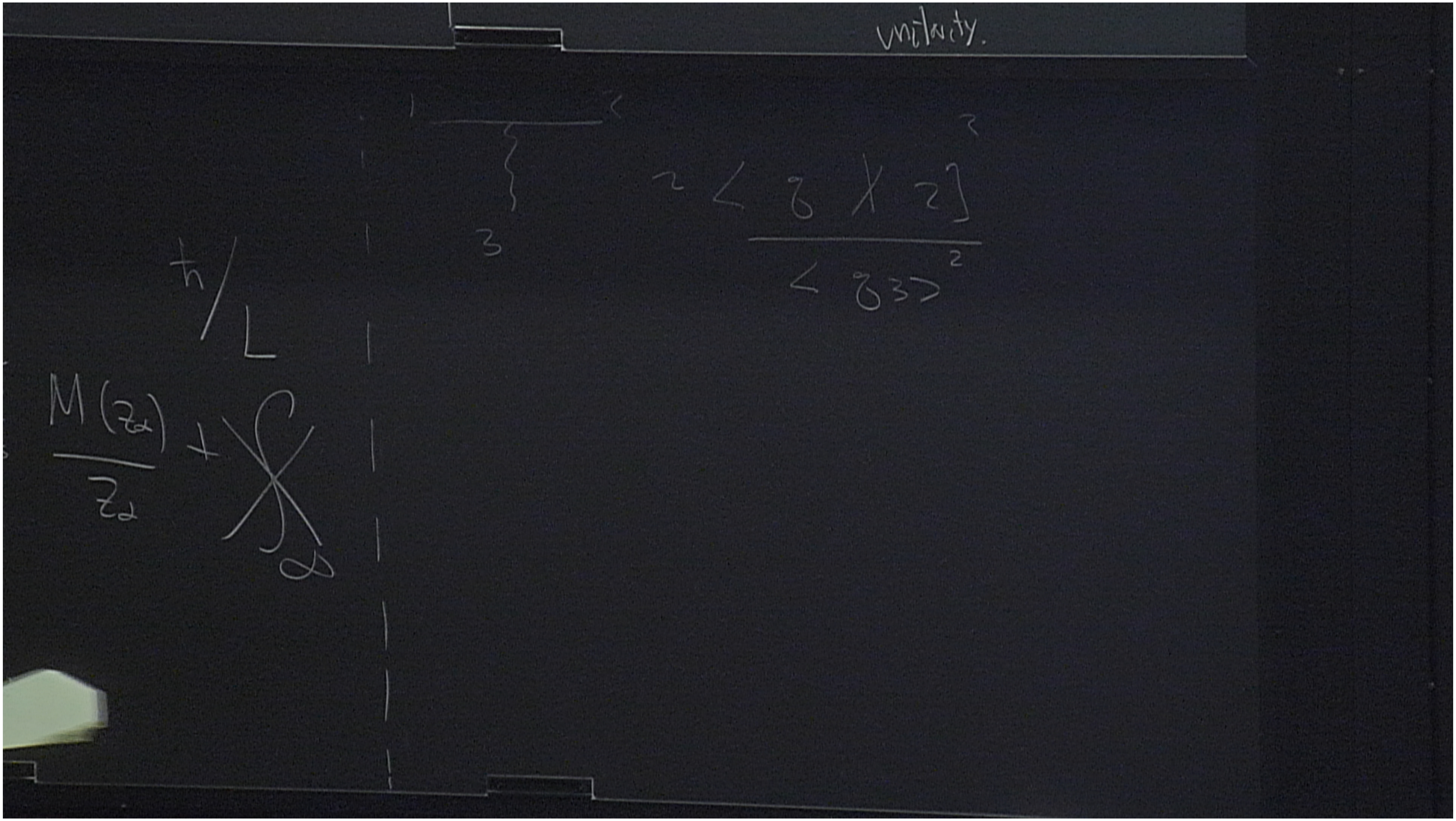


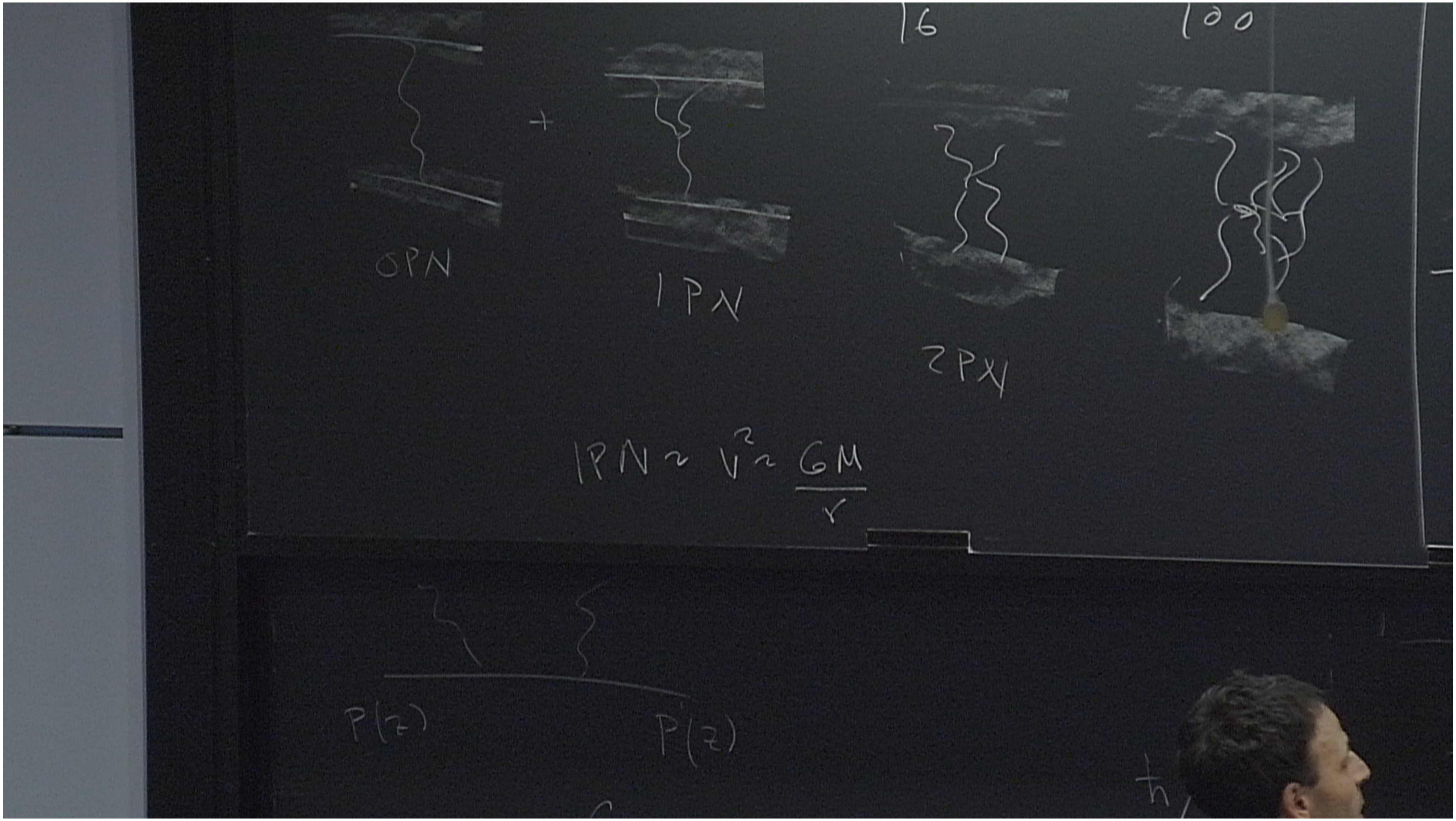

$$M(0) = \oint dz \, \frac{M(z)}{z} = \sum_{\text{res}} \frac{M(z_\alpha)}{z_\alpha} + \int_\infty$$

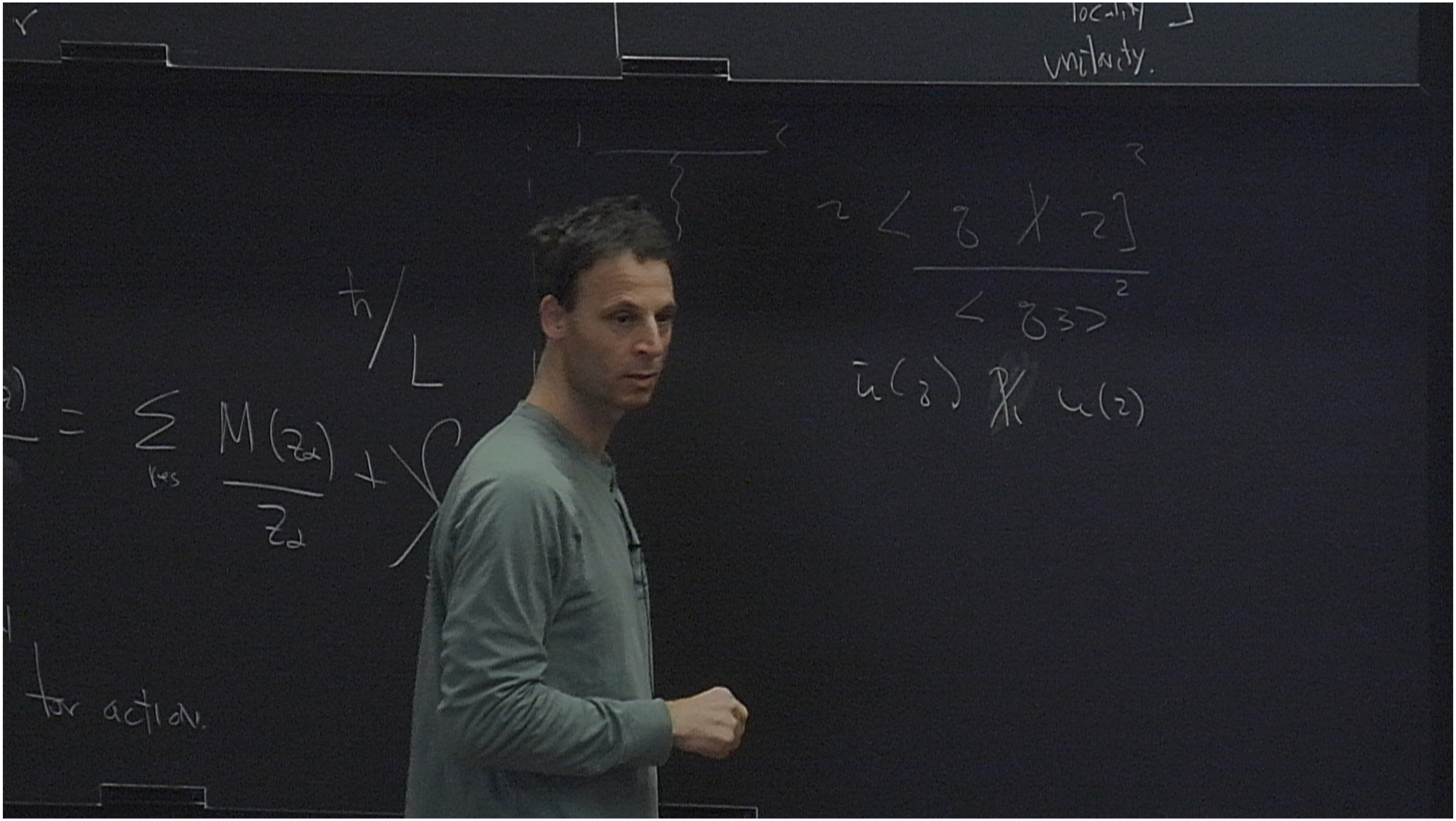

P(z)
P'(z)
ħ/L
$M(0) = \oint dz \frac{M(z)}{z} = \sum_{res} \frac{M(z_\alpha)}{z_\alpha} + \int_\infty$

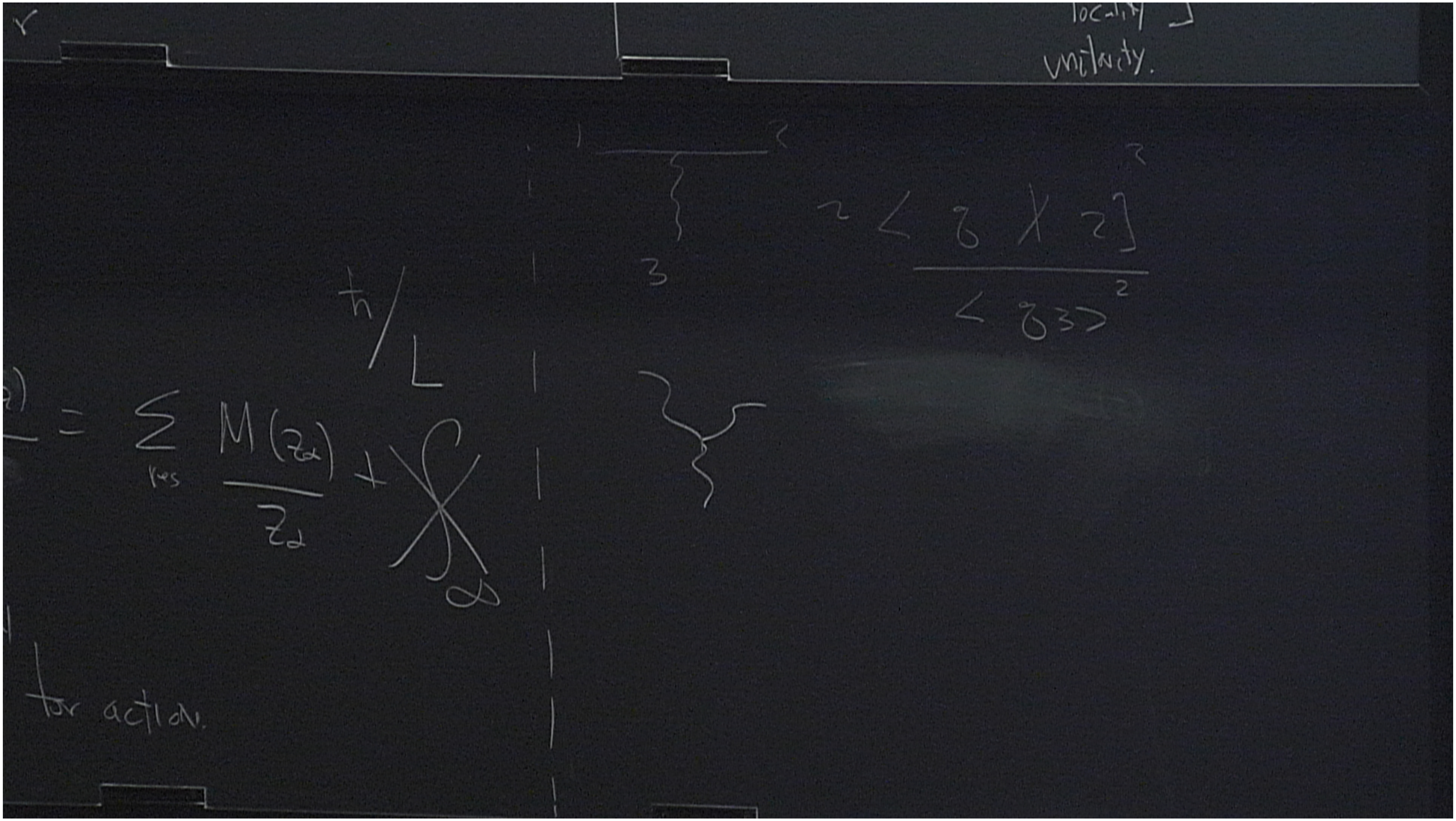

P(z)
ħ/L
$\oint dz \frac{M(z)}{z} = \sum_{res} \frac{M(z_\alpha)}{z_\alpha} + \int_\infty$

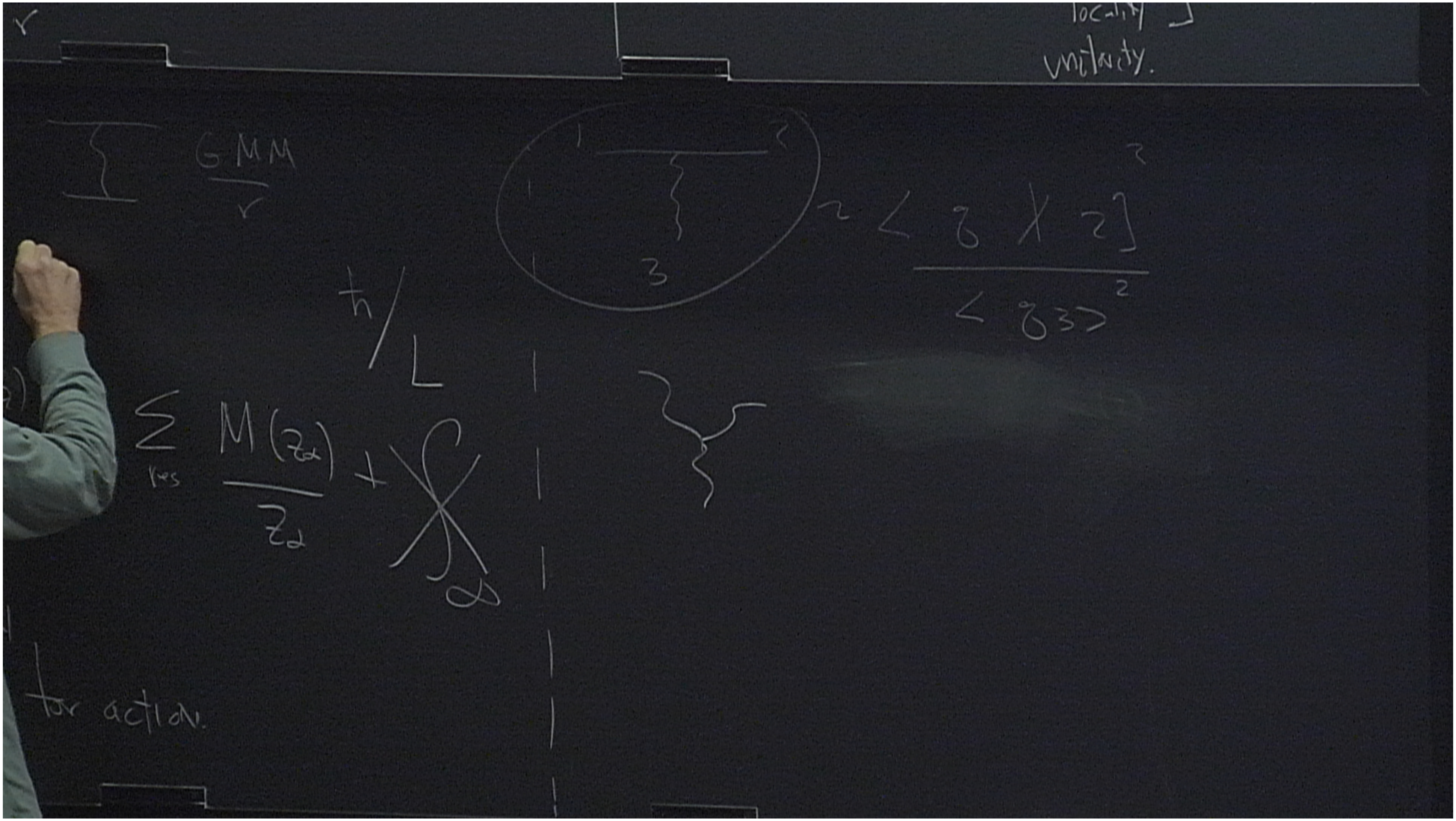

P(z)
P'(z)
$\hbar / L$
$M(0) = \oint dz \frac{M(z)}{z} = \sum_{res} \frac{M(z_\alpha)}{z_\alpha} + \cancel{\oint_\infty}$
Allows use BCFW IN
classical limit, w/o need for action.

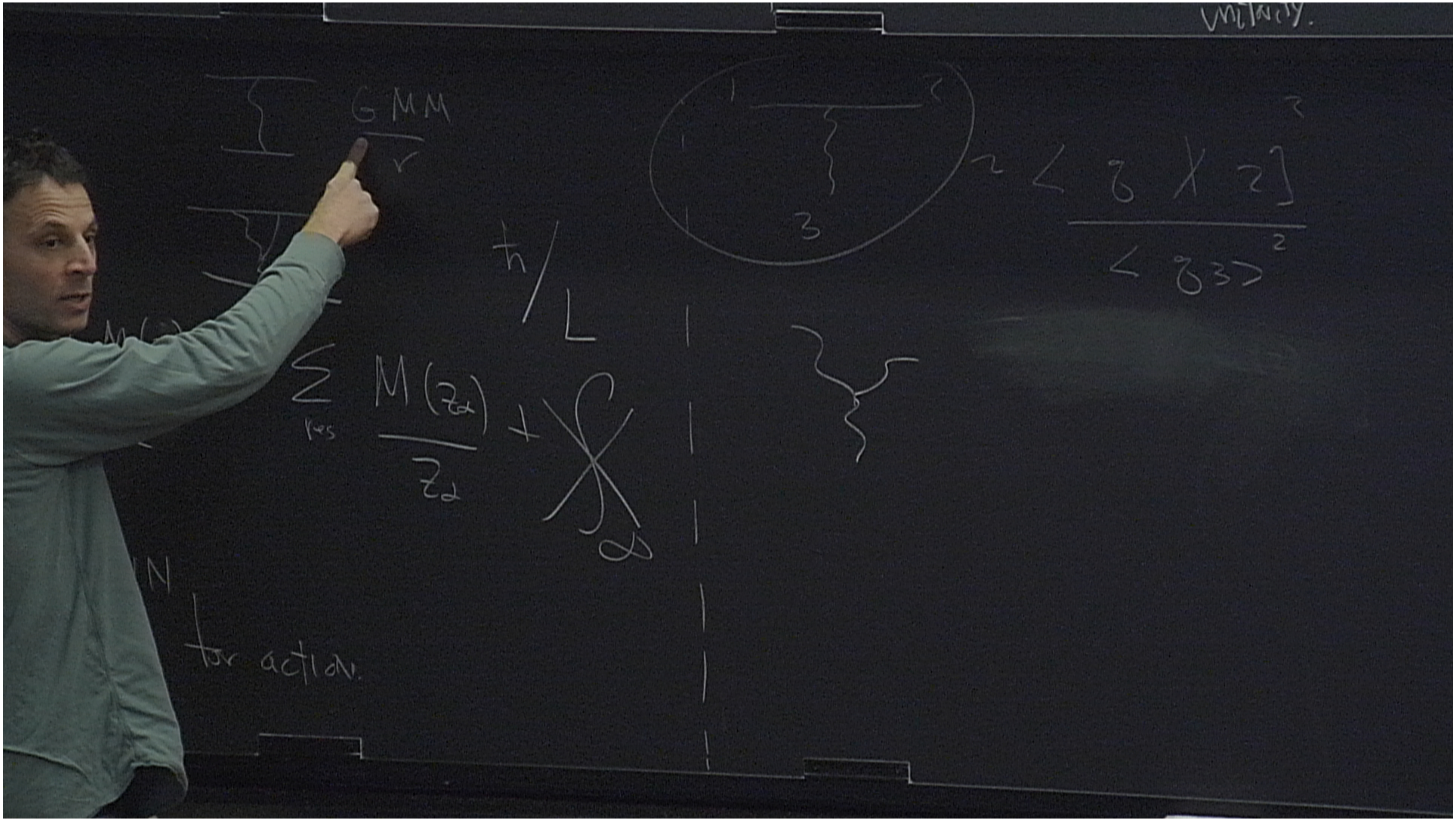

unitarity.

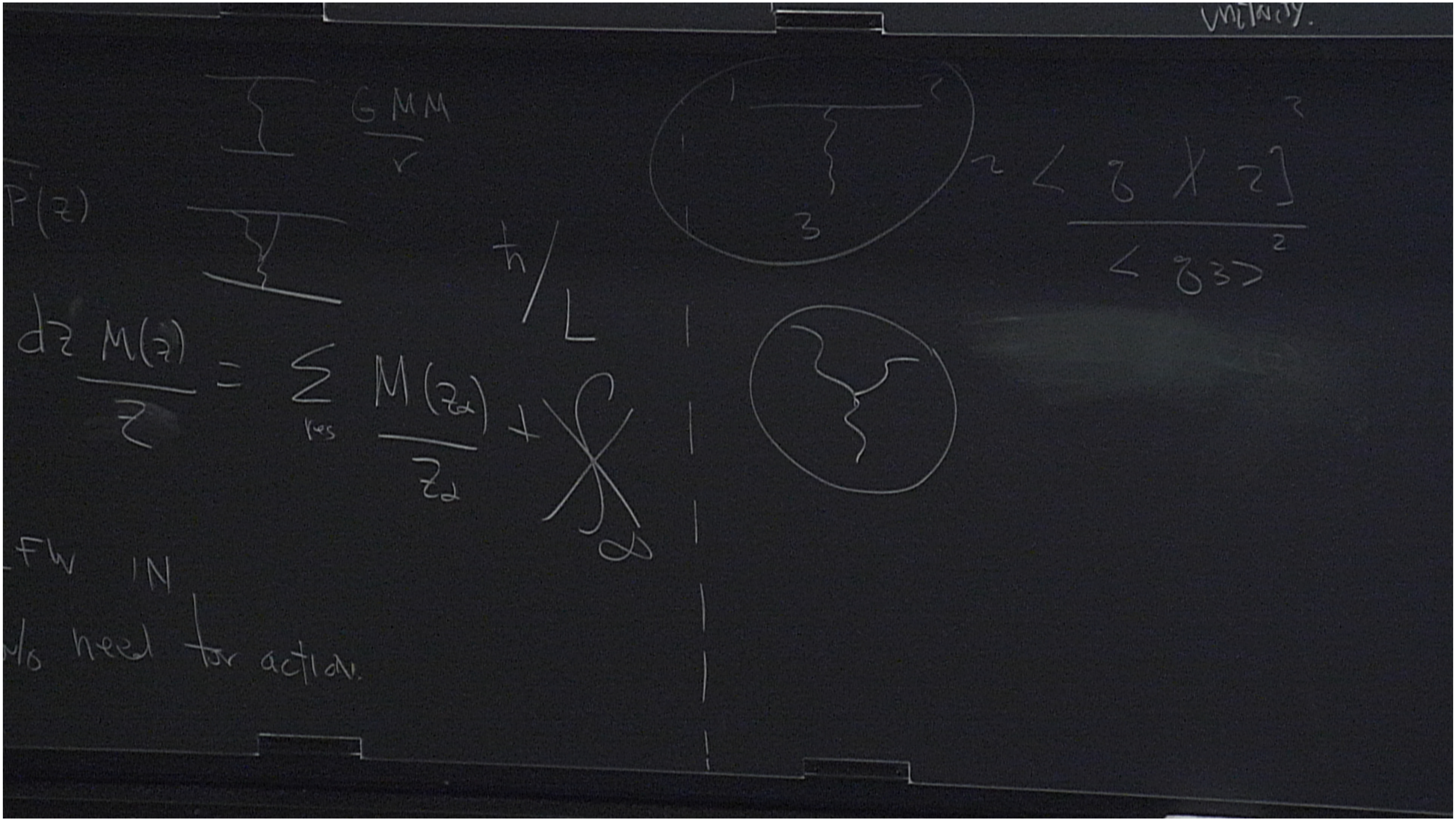

16
100
0PN
1PN
2PN
1PN ~ v² ~ GM/r
P(z)
P(z)

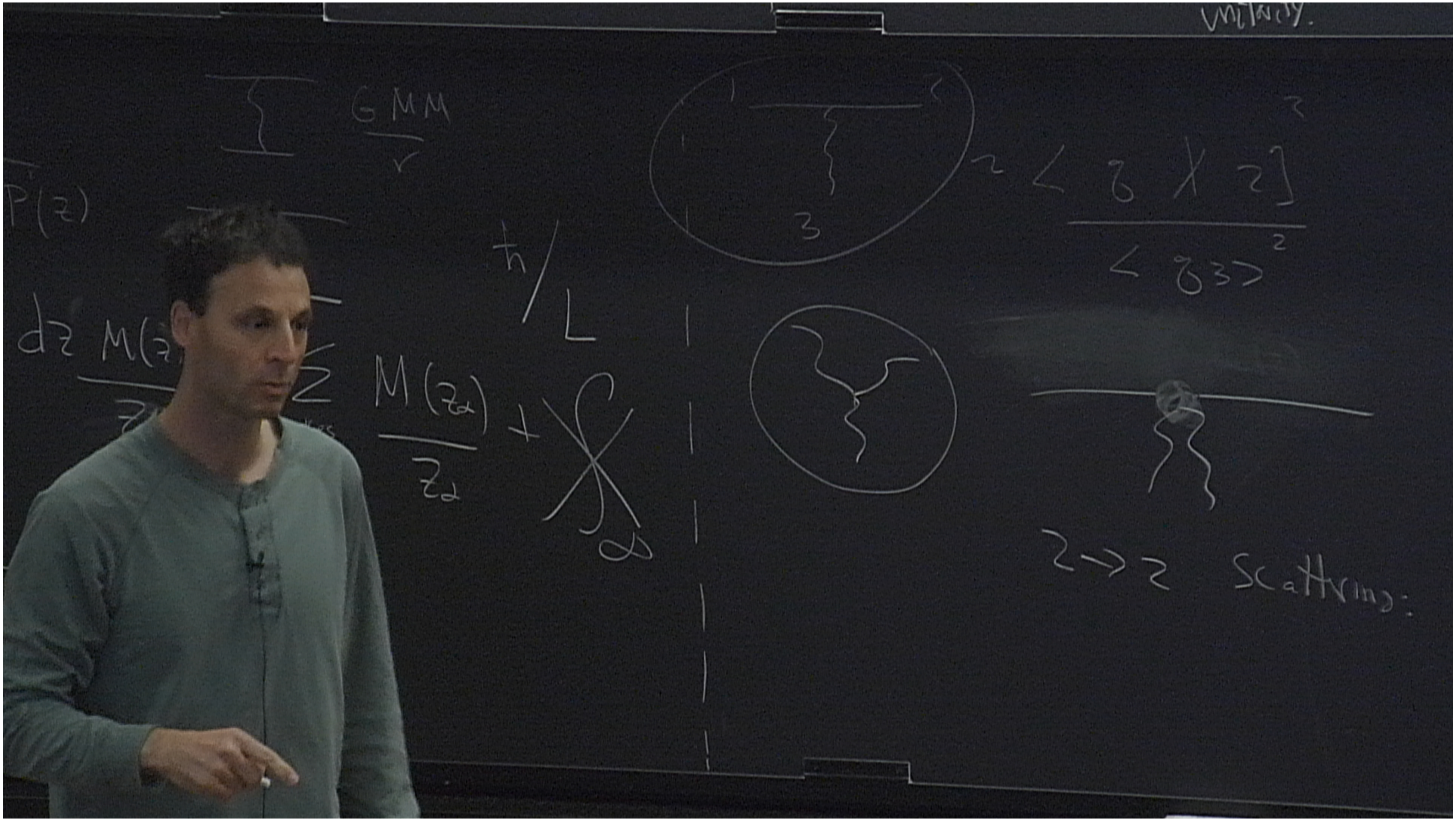

unitarity.
for action.

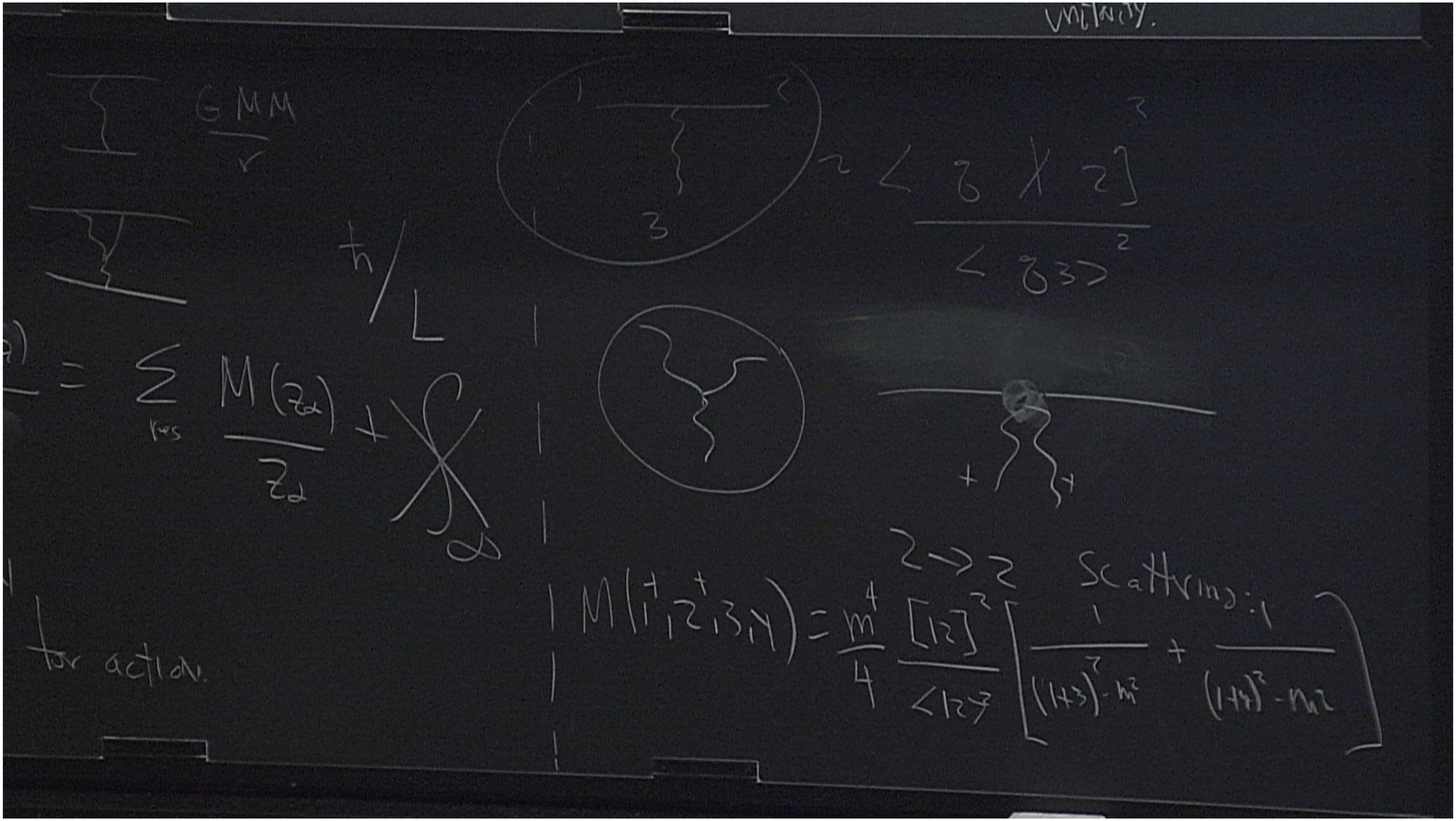

locality
unitarity.
for action.
res

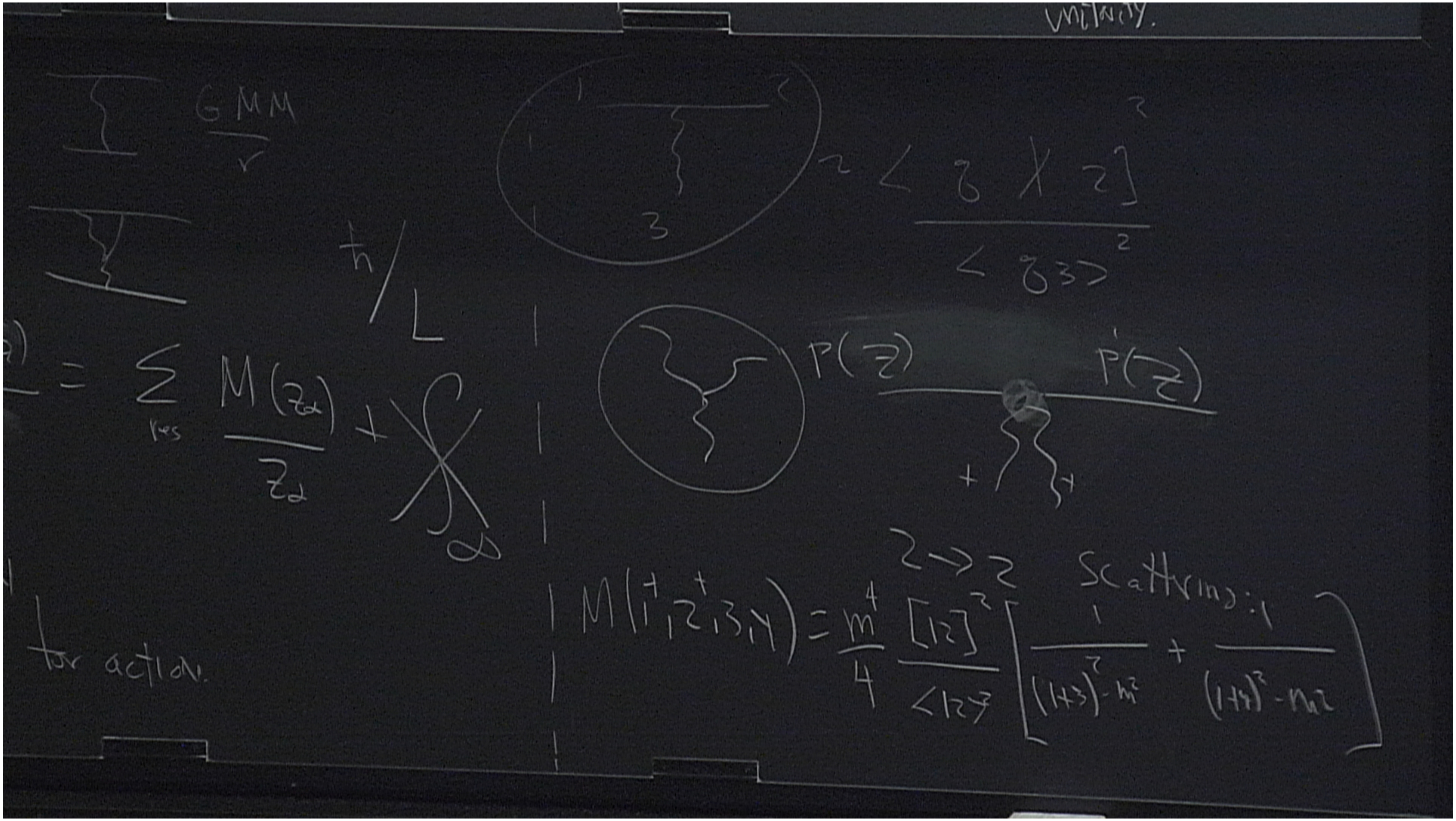

locality ]
unitarity.
GMM / r
ħ / L
Σ res M(zα) / zα +
< g X 2 ] ^2 / < g 3 > ^2
for action.

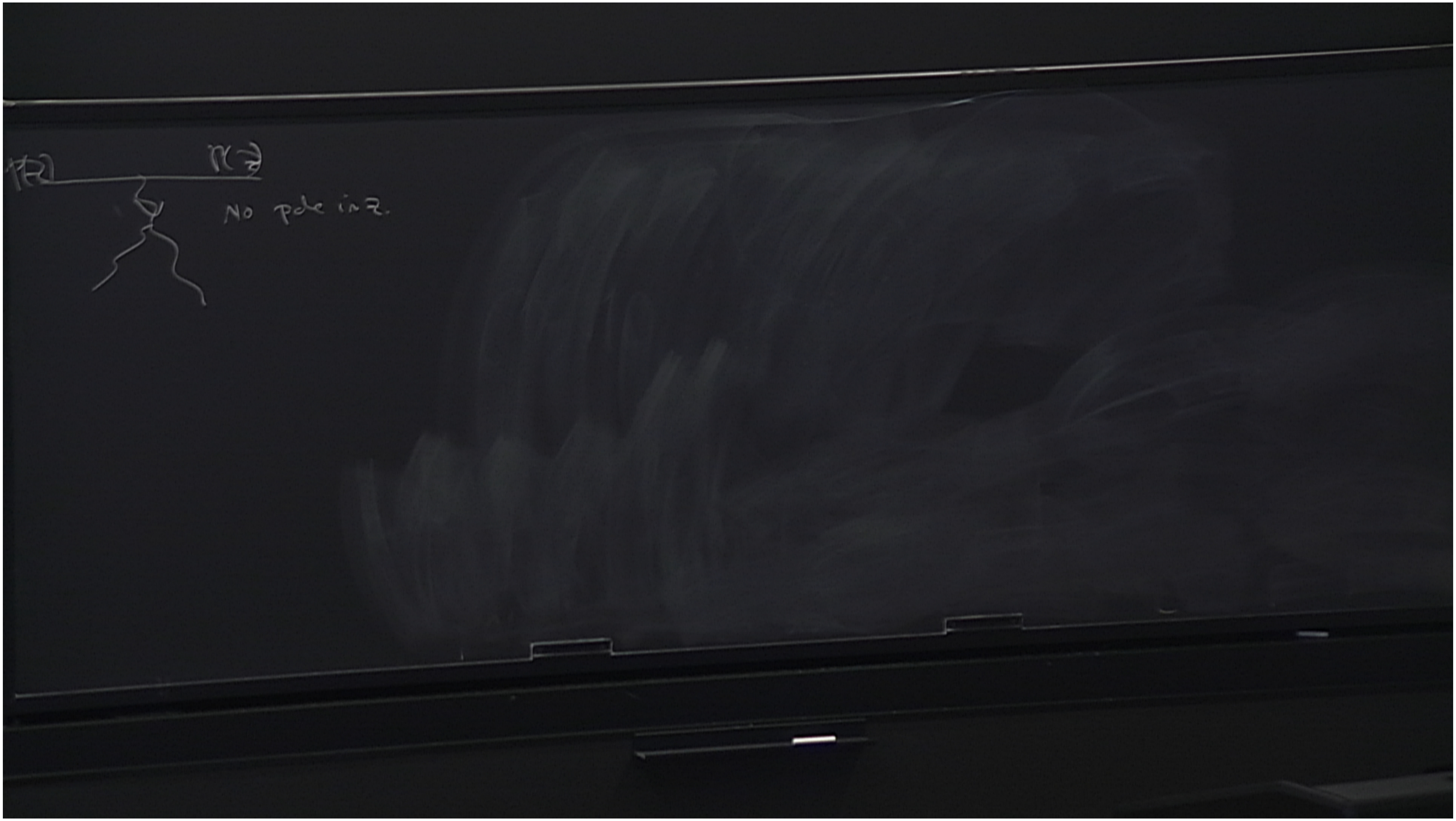

unitarity.
GMM
r
ħ/L
$\sum_{\text{res}} \frac{M(z_\alpha)}{z_\alpha}$ +
for action.
$\langle 3\,X\,2]^3$
$\langle 3\,3\rangle^2$

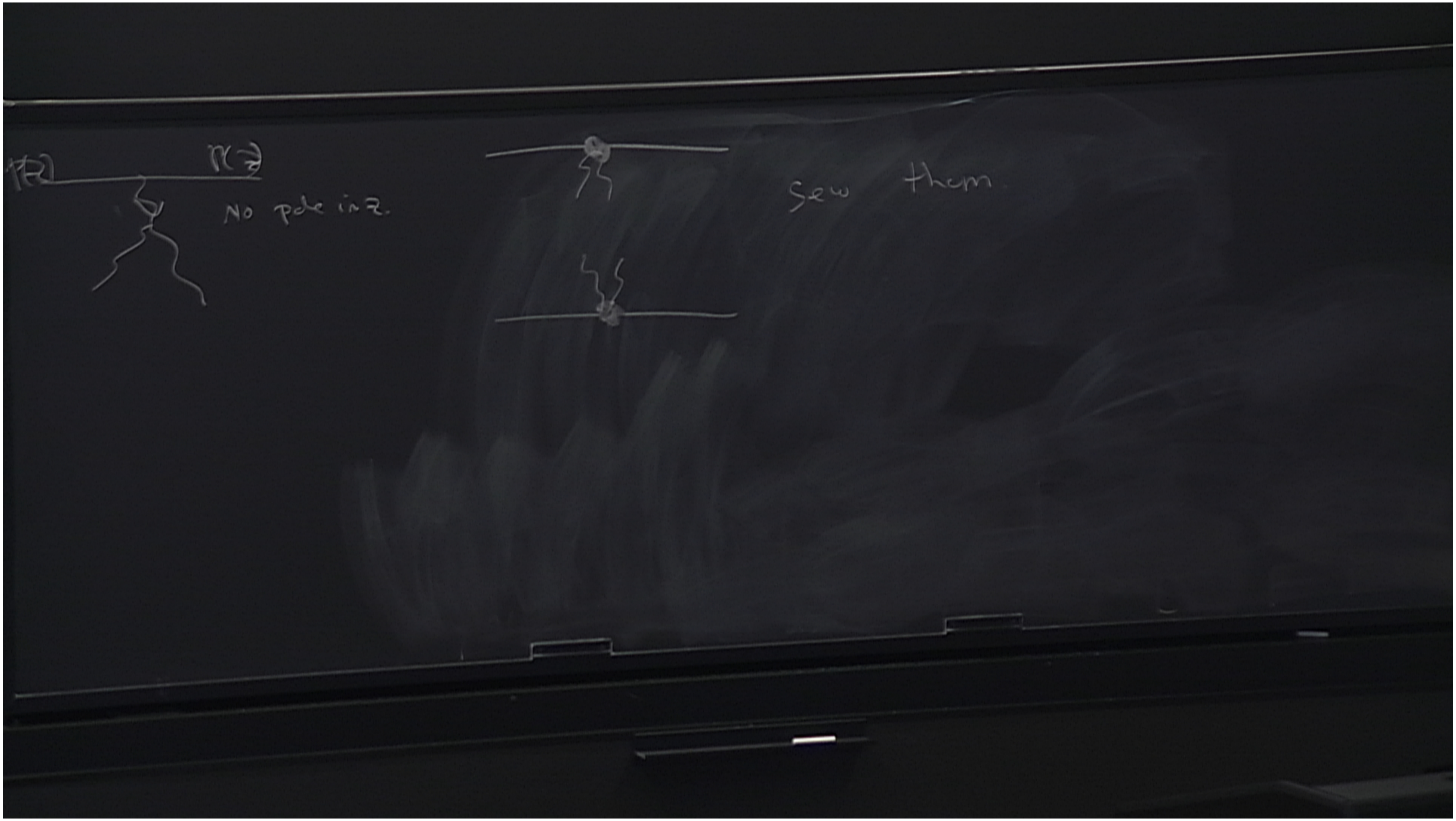

unitarity.
GMM
r
P(z)
ħ/L
FW IN
w/o need for action.

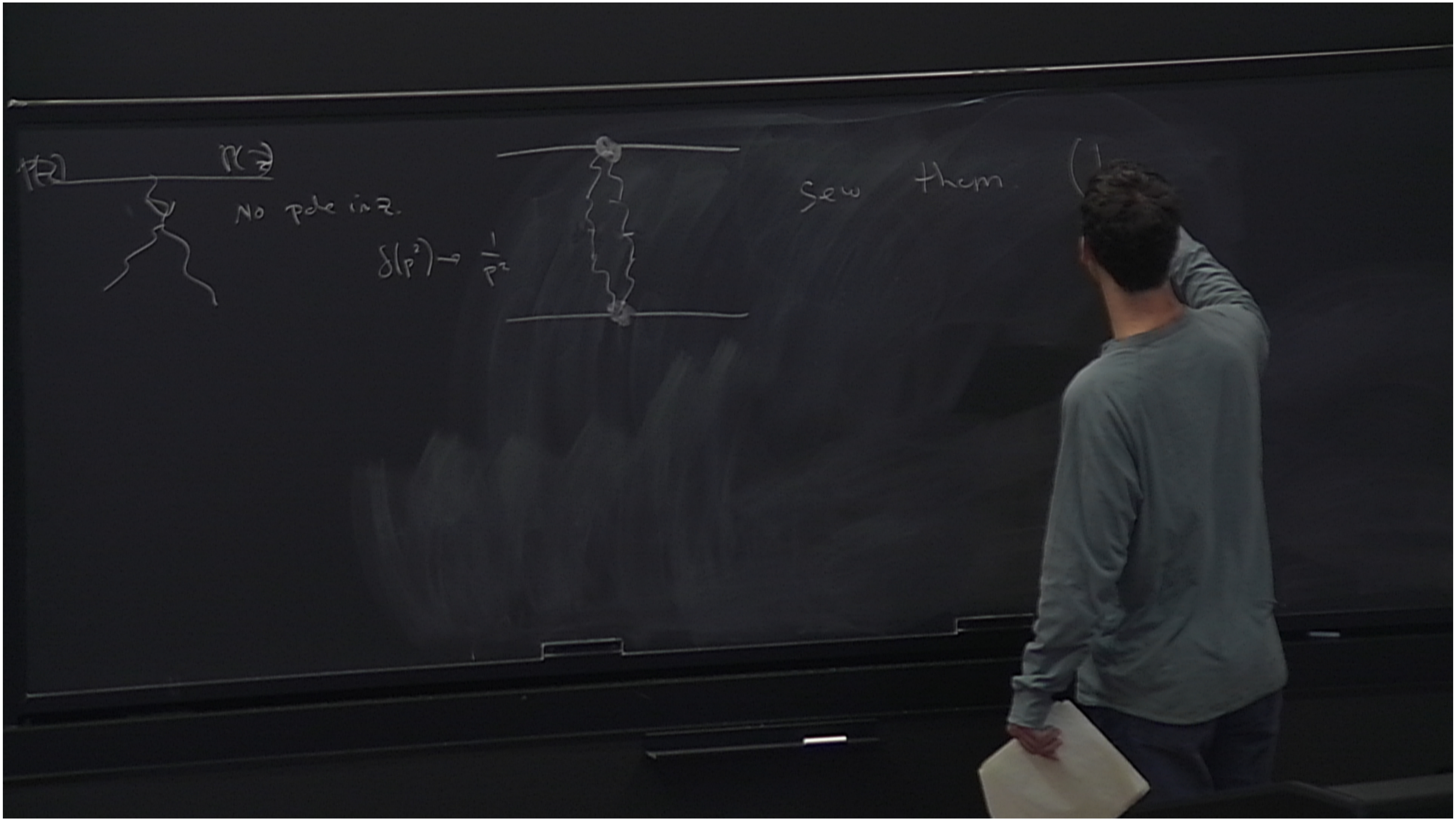

G M M
r
ħ/L
2 → 2 Scattering:

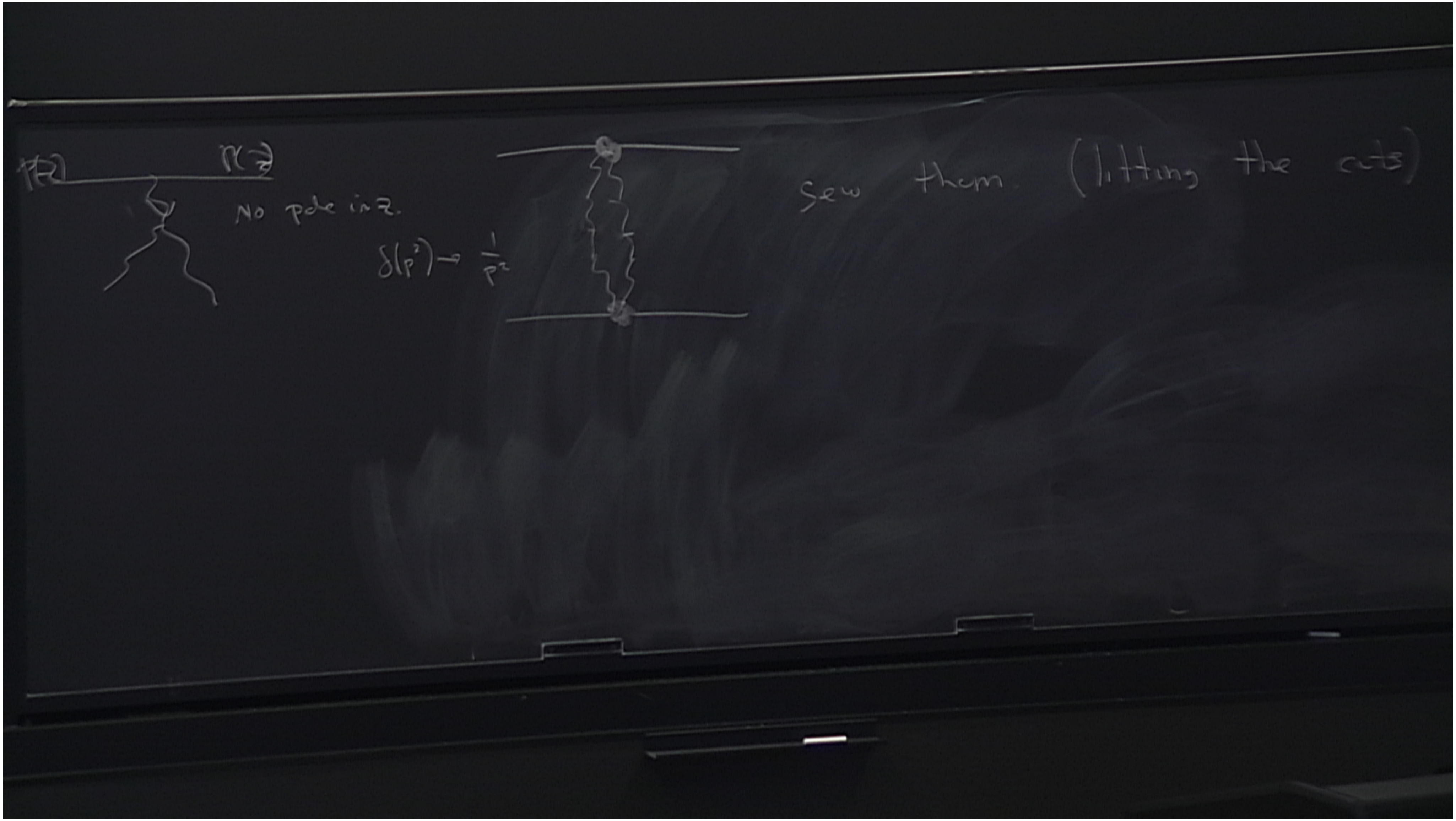

unitarity.
GMM/r
ħ/L
= Σ res M(zα)/zα + ∮∞
for action.
~ ⟨q 3 2]^2 / ⟨q 3⟩^2
2 → 2 Scattering:
M(1+, 2+, 3, 4) = m^4/4 [12]^2/⟨12⟩^2 [ 1/((1+3)^2 − m^2) + 1/((1+4)^2 − m^2) ]

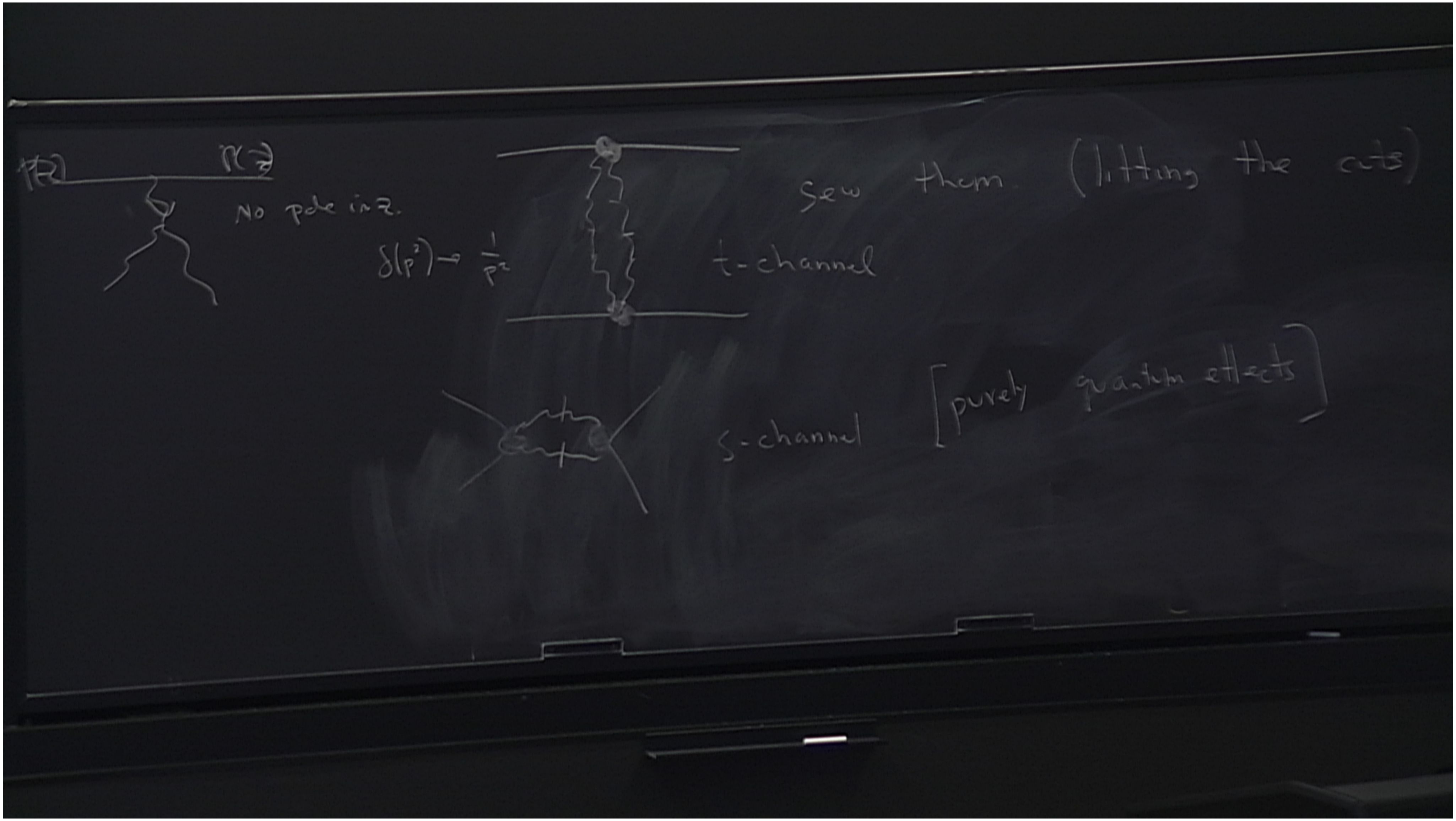

unitarity.
GMM / r
ħ / L
= Σ_res M(z_α) / z_α + ∮_∞
⟨ξ χ 2]^2 / ⟨ξ 3⟩^2
P(z)   P'(z)
2 → 2 Scattering:
M(1^+, 2^+, 3, 4) = m^4/4 [12]^2/⟨12⟩^2 [ 1/((1+3)^2 - m^2) + 1/((1+4)^2 - m^2) ]
for action.

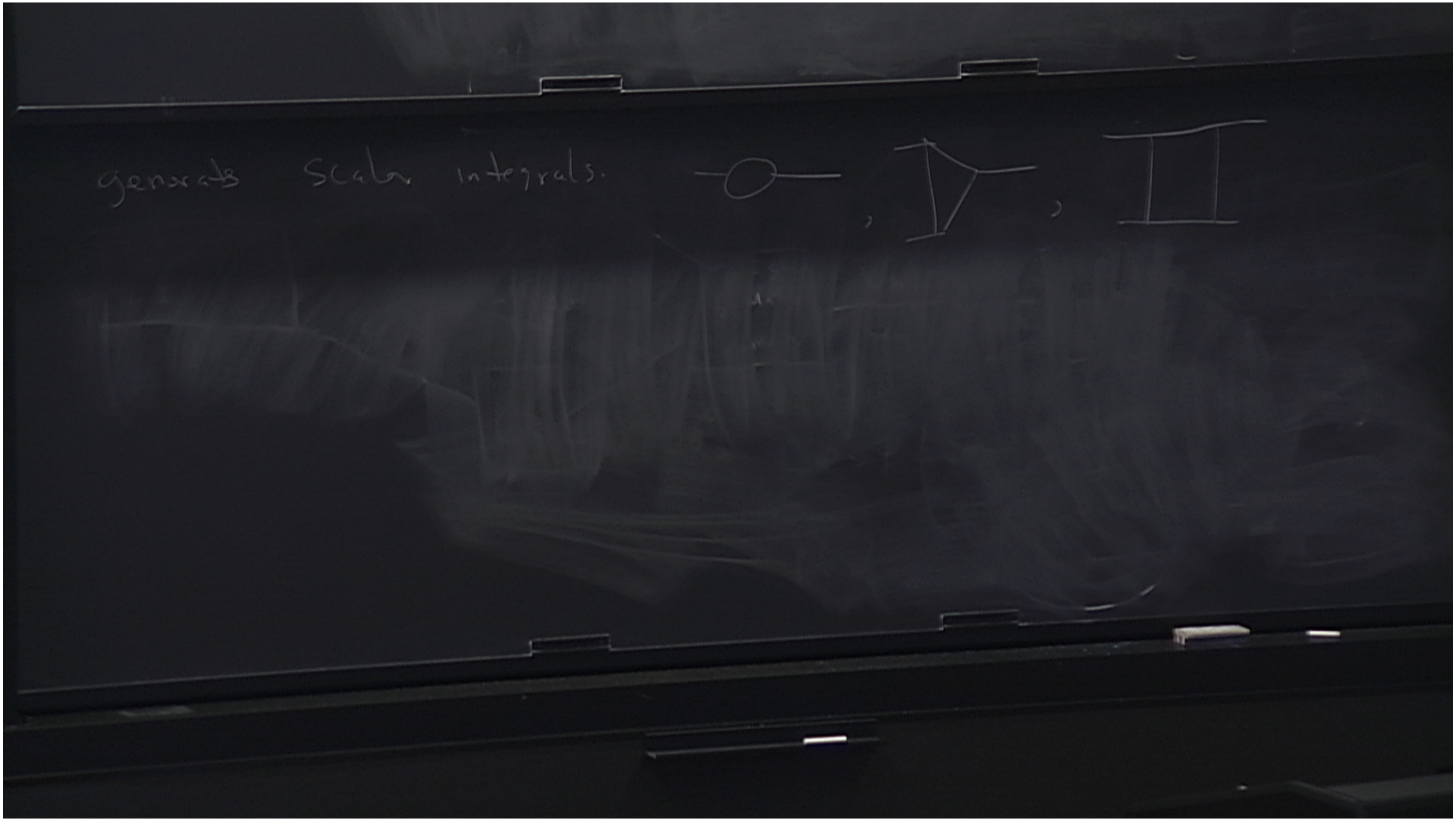

No pole in z.

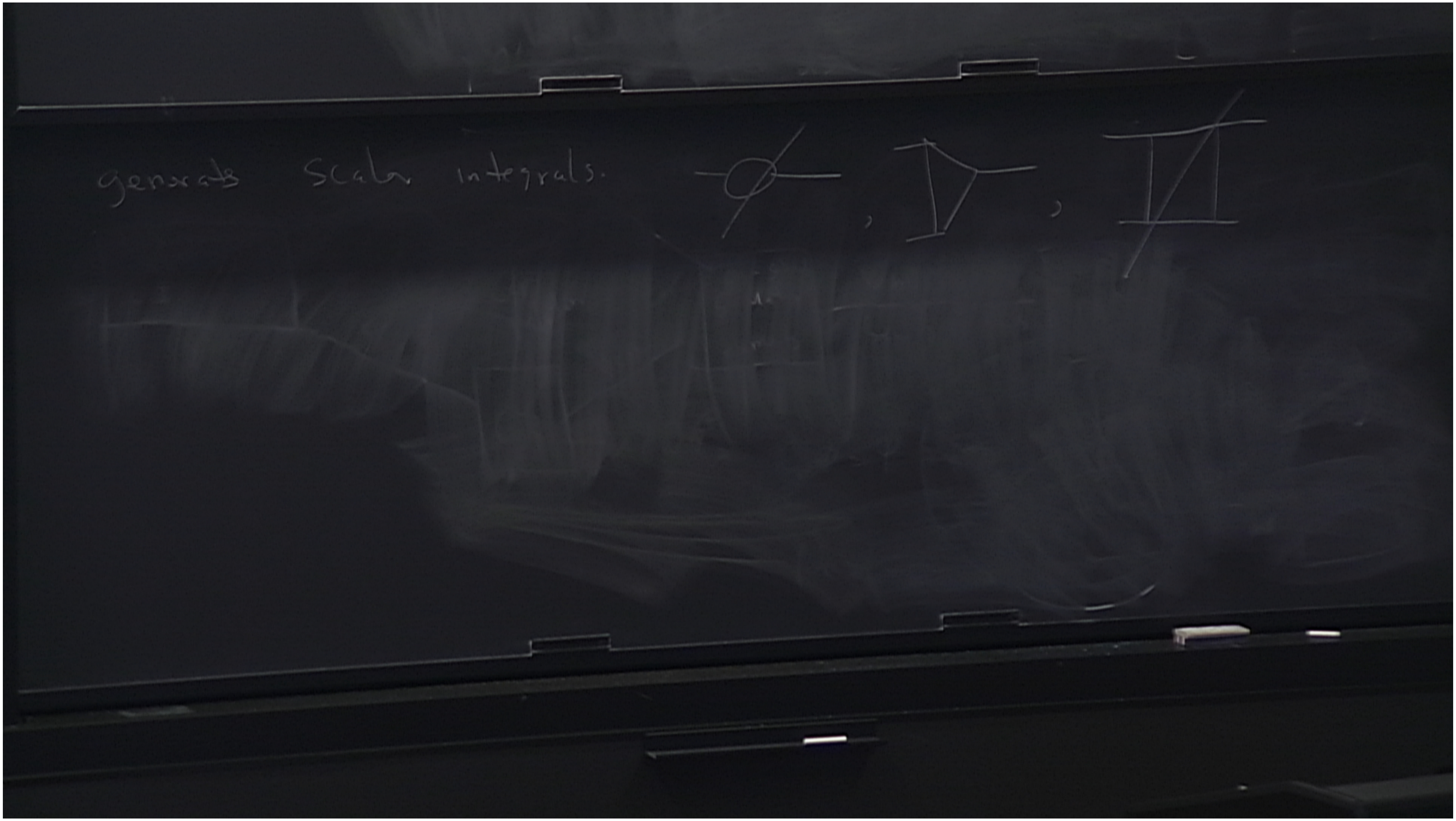

No pole in z.
Sew them.

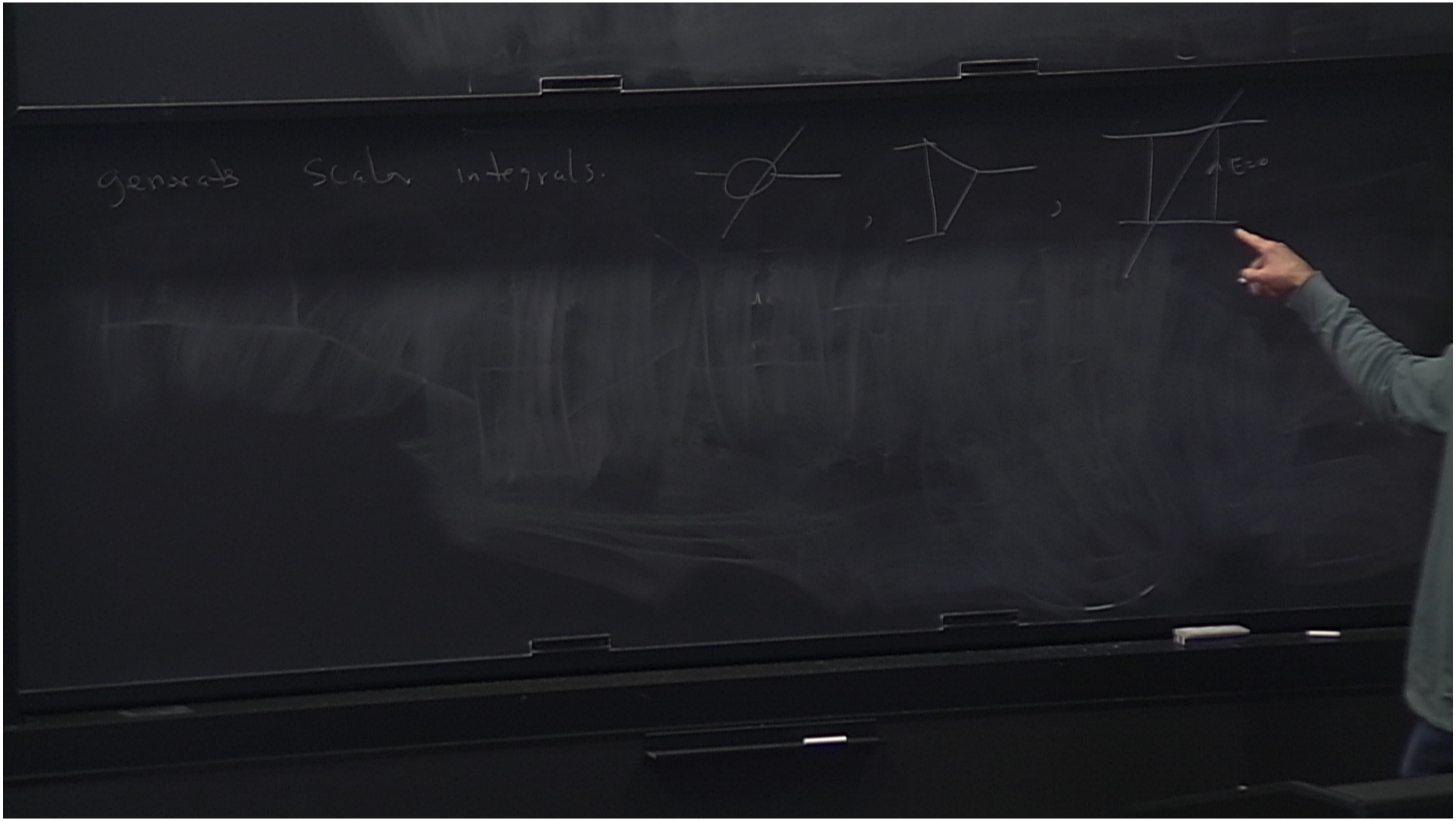

No pole in z.
δ(p²) → 1/p²
sew them

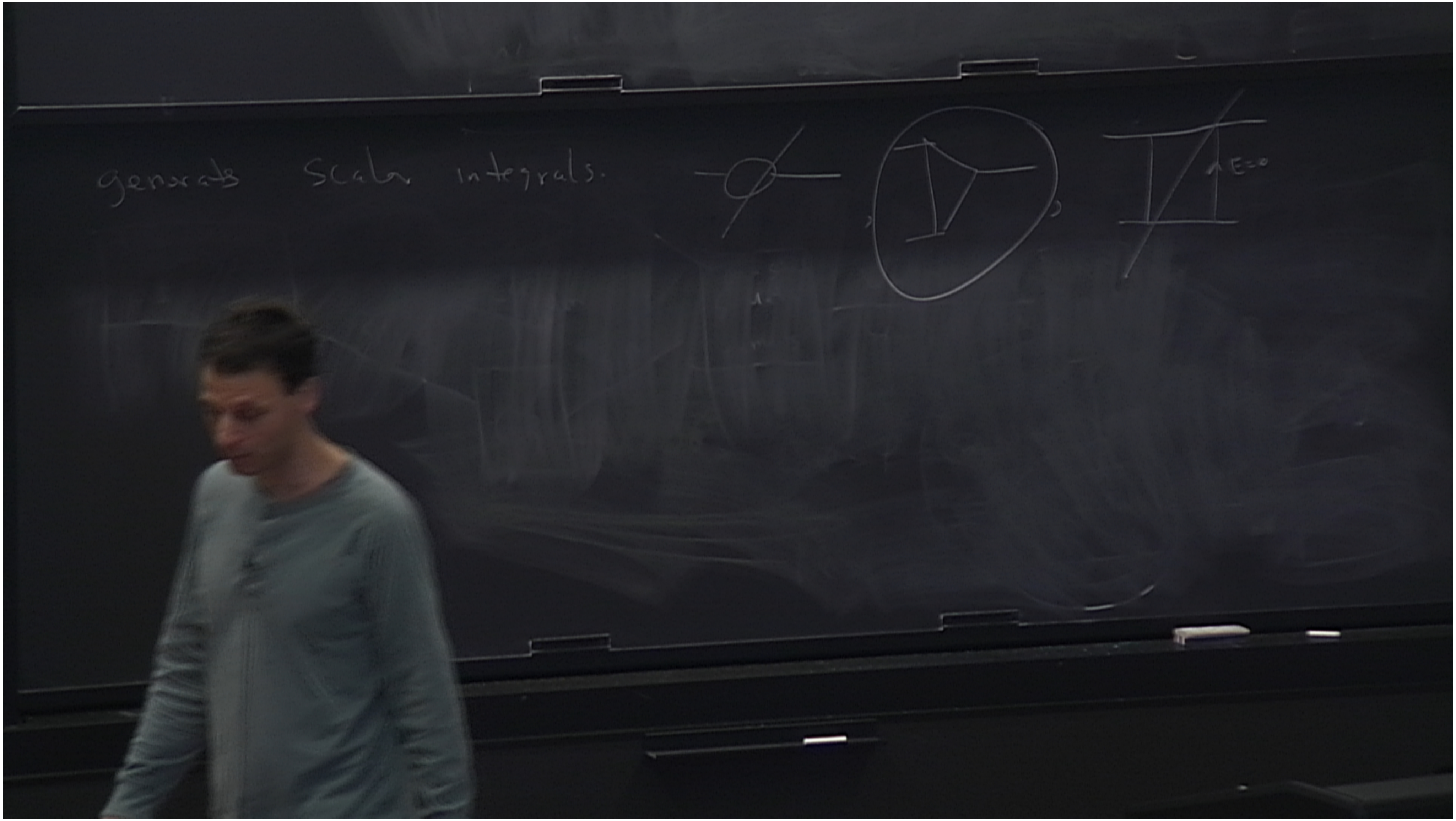

No pole in z.
δ(p²) → 1/p²
Sew them (lifting the cuts)

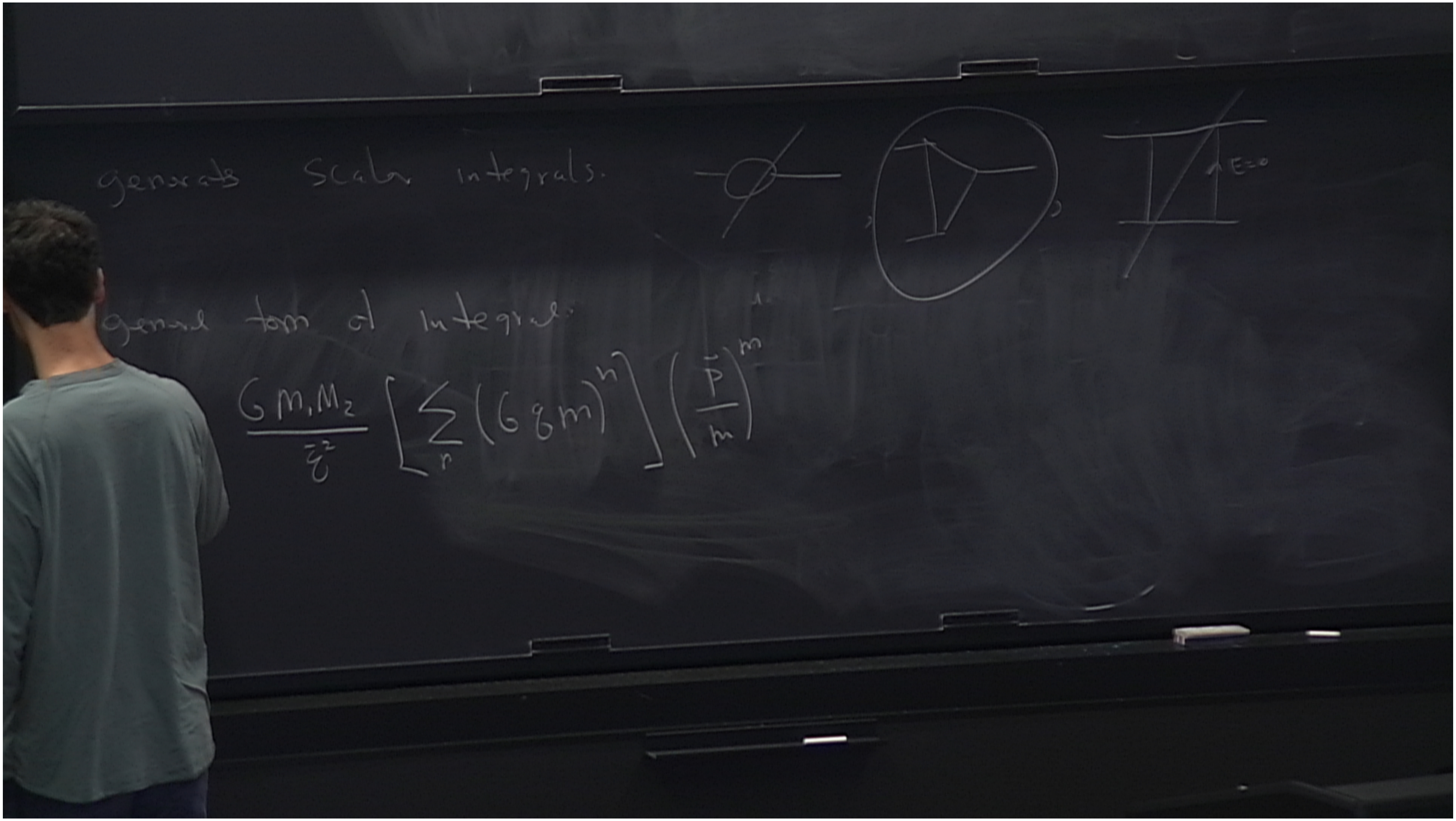

No pole in z.
δ(p²) → 1/p²
Sew them. (lifting the cuts)
t-channel
s-channel [purely quantum effects]

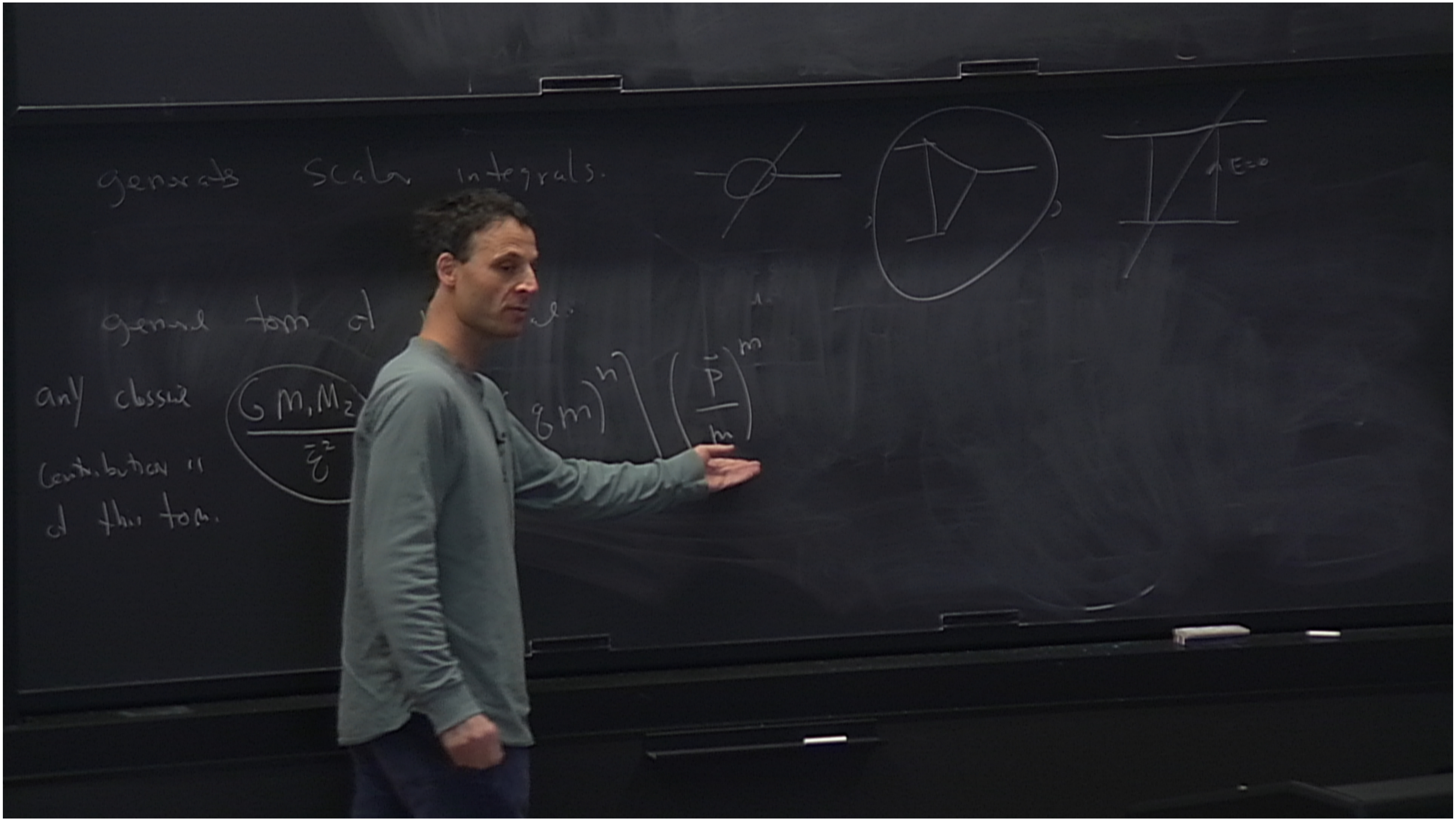

generate scalar integrals.


generate scalar integrals.

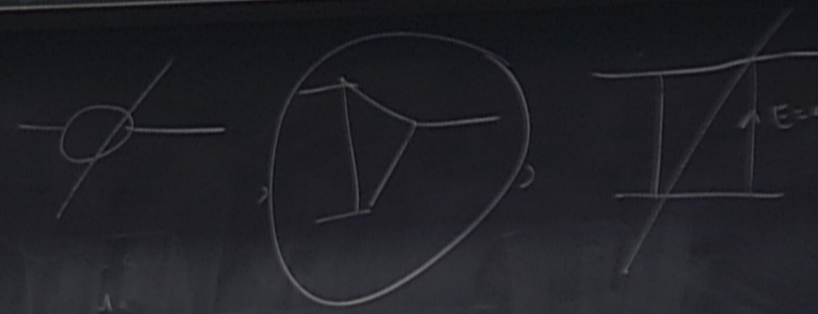

generate scalar integrals.
E=0


generate scalar integrals.
E=0


generate scalar integrals.
general form of integral:
E=0


generate scalar integrals.
any classical contribution is of this form.
E=0

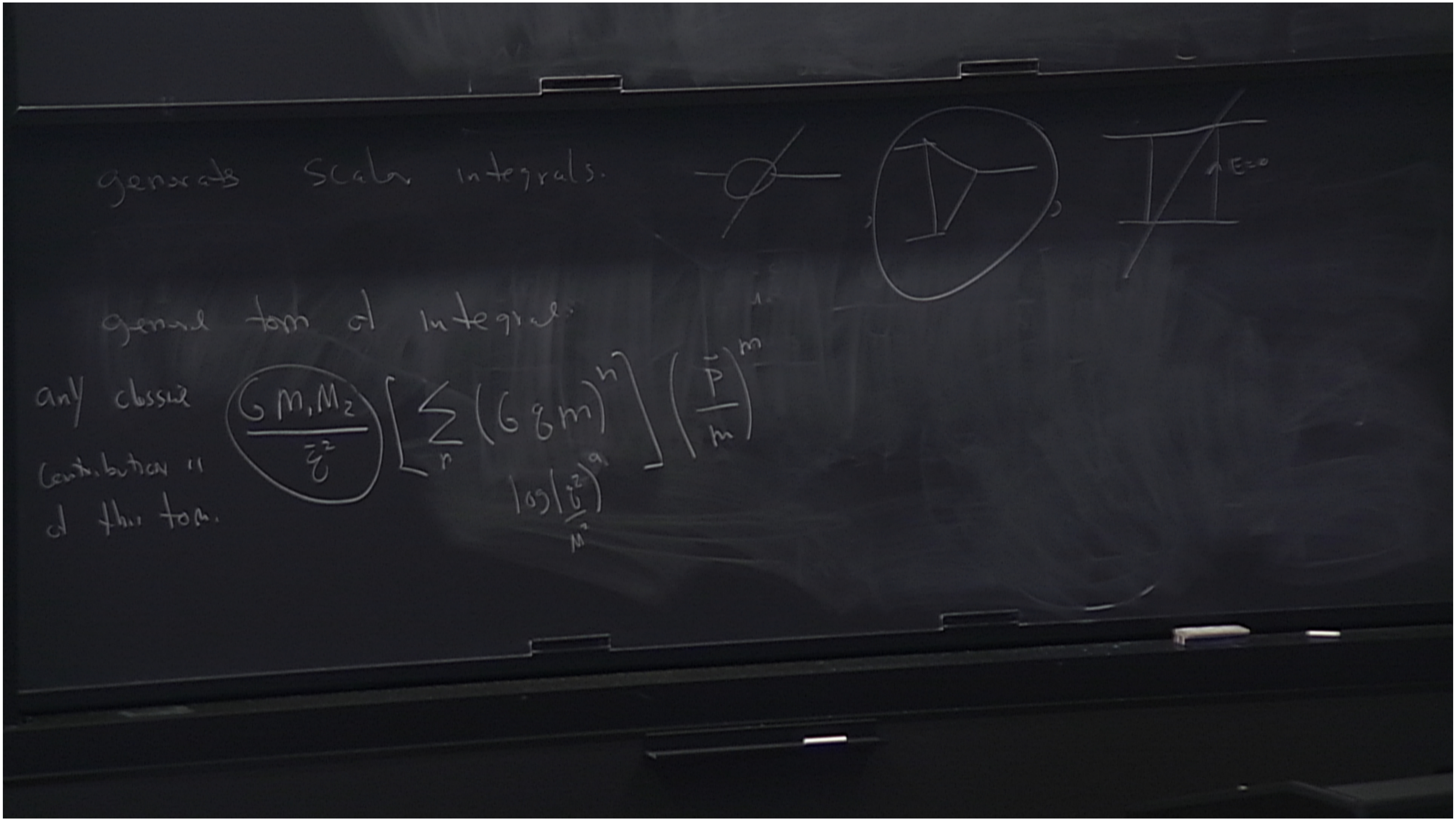
generate scalar integrals.

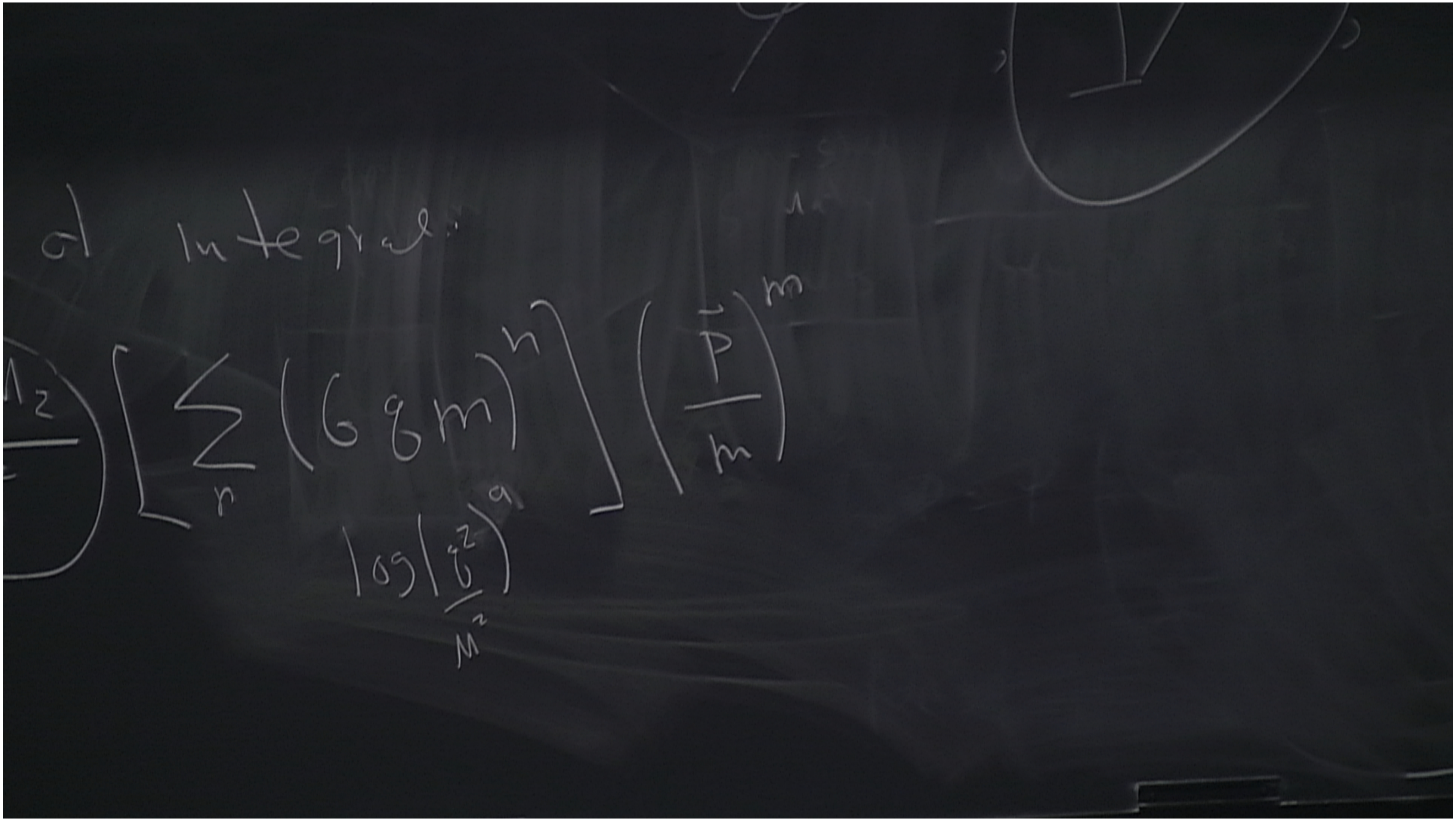


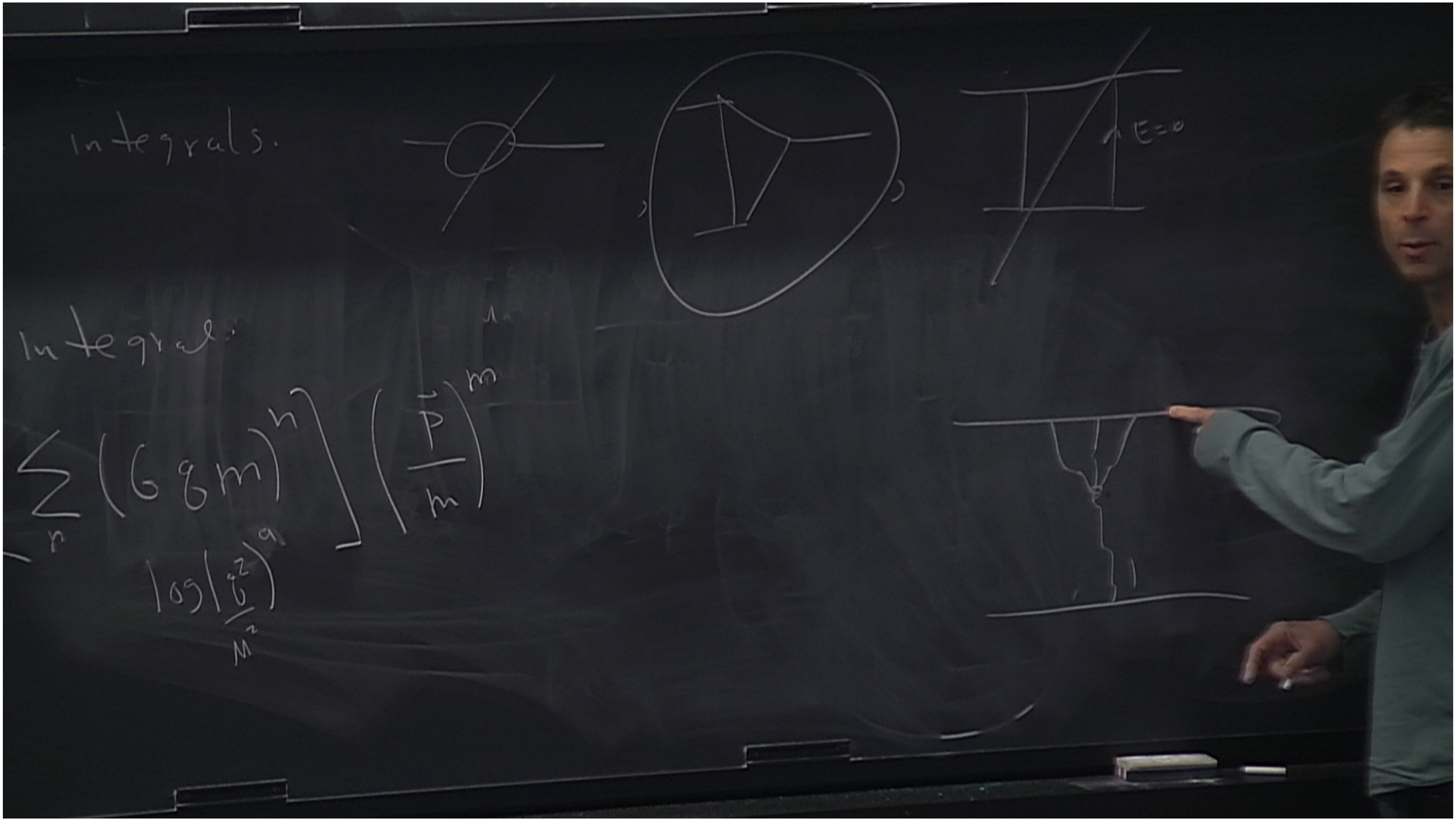

general form of integral:

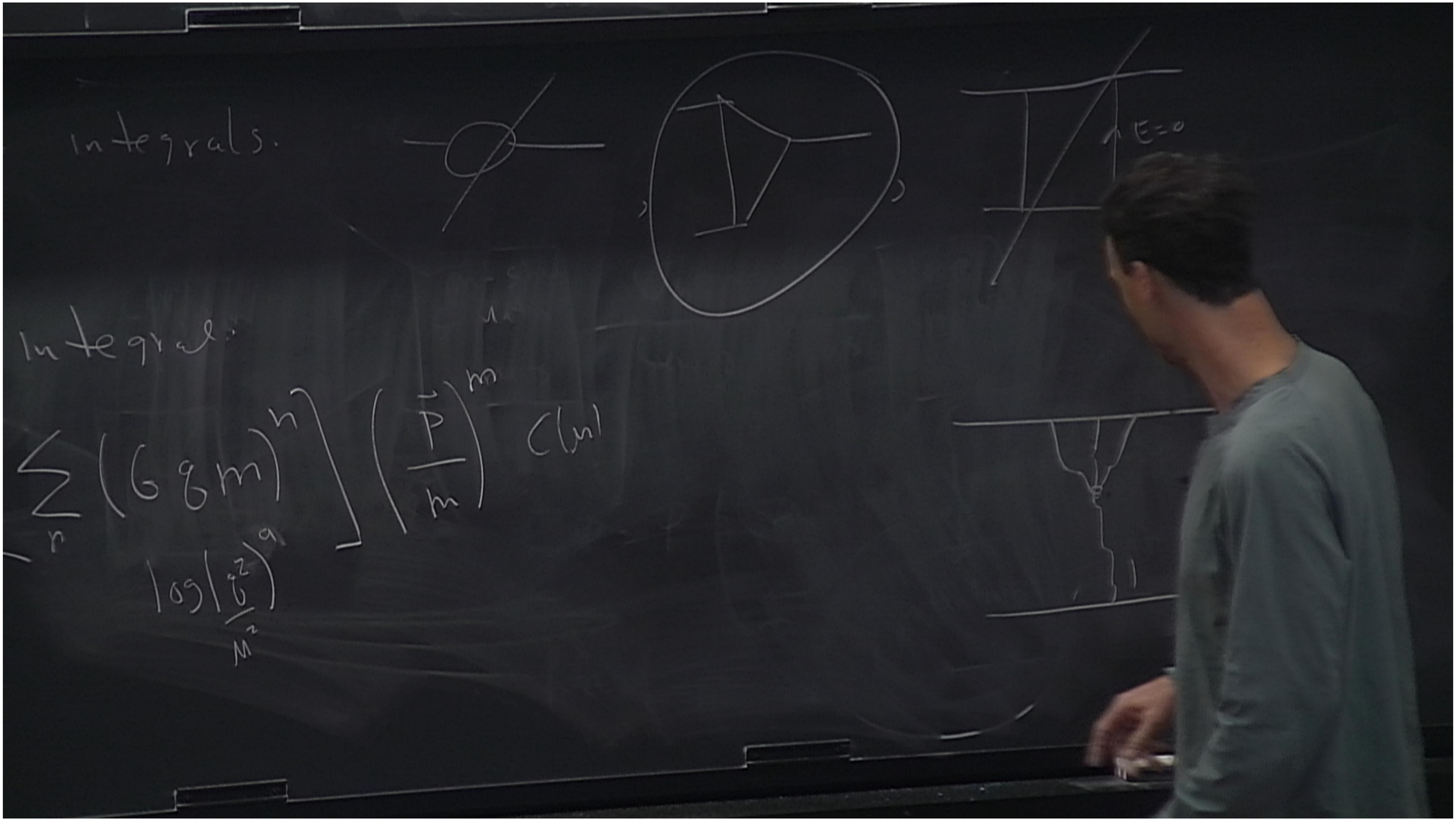
any classical contribution is of this form.

$$\frac{G m_1 M_2}{\bar{q}^2}\left[\sum_n (G q m)^n \log(q^2)^a\right]\left(\frac{\bar{p}}{m}\right)^m$$

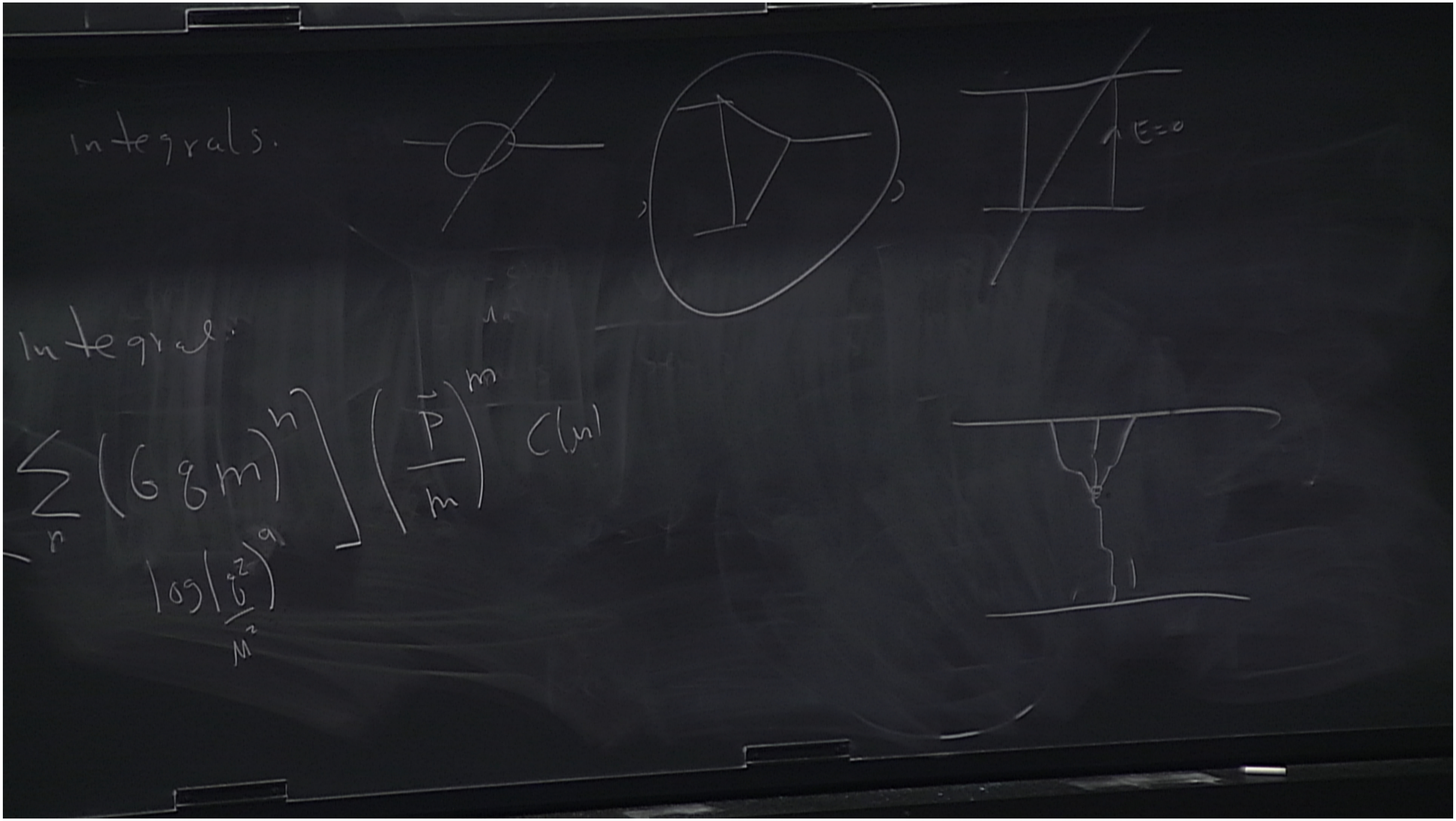

generate scalar integrals.
E=0
general form of integral:
any classical contribution is of this form.

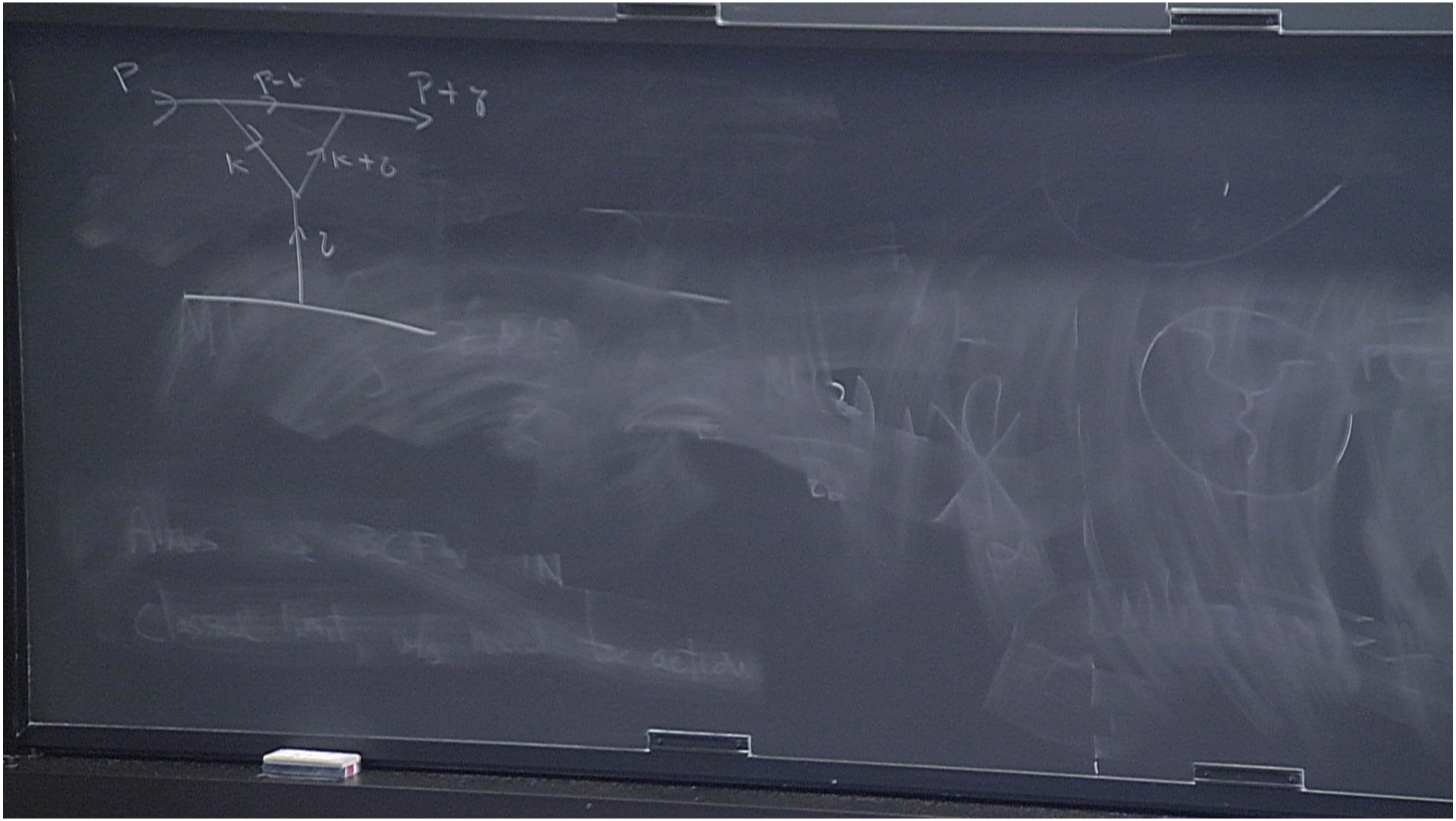

integral:

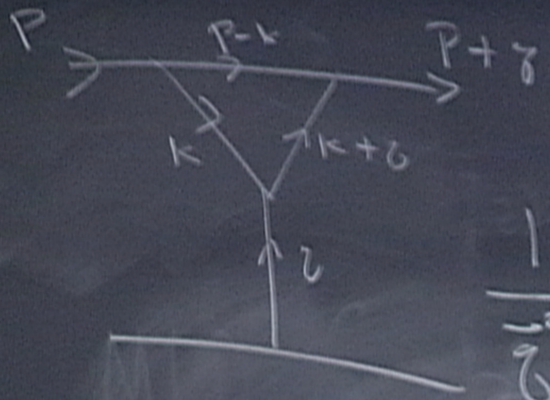

Integrals.
E=0
Integral:
log



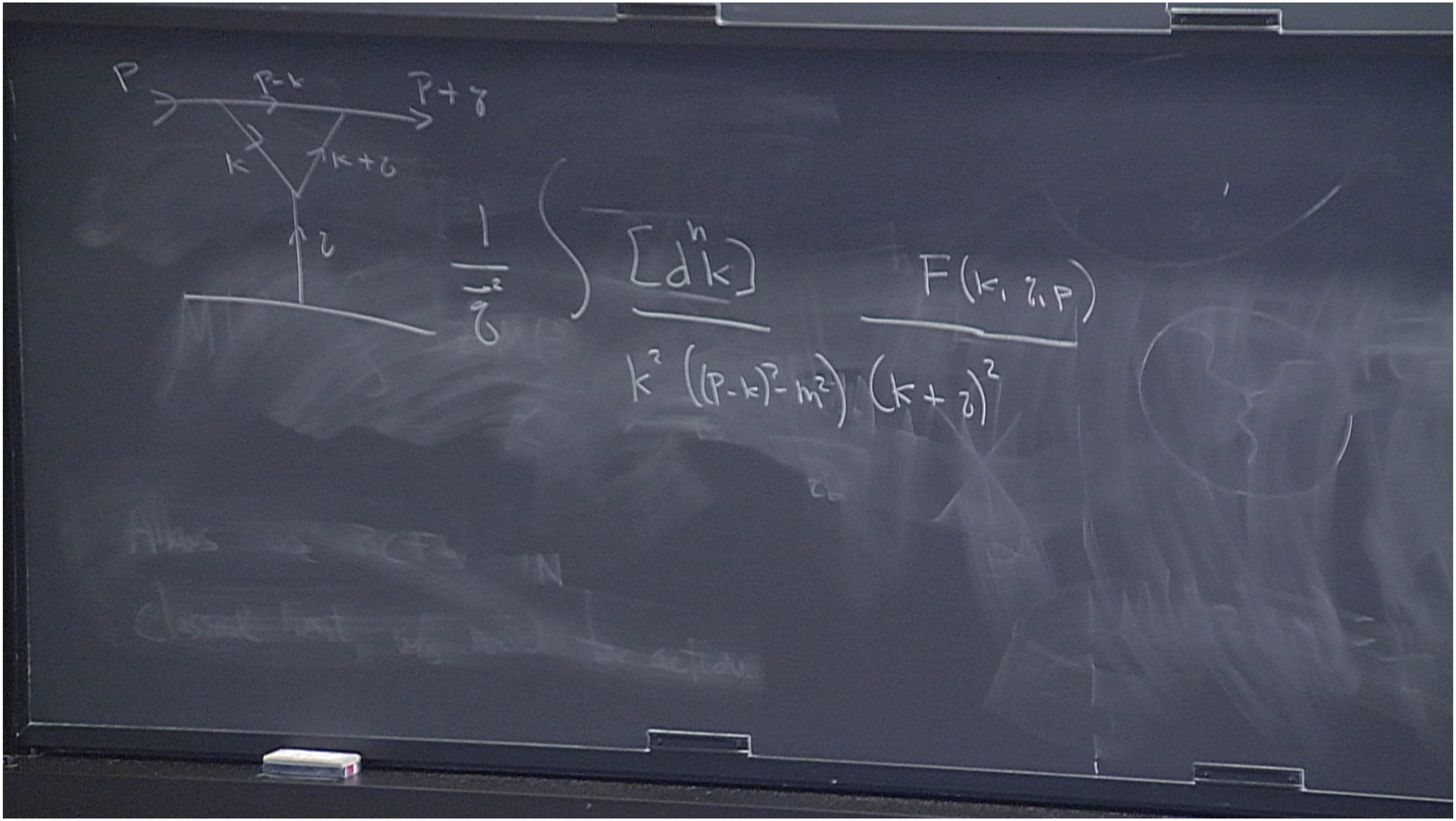
integrals.
E=0
Integral:
log
C(n)

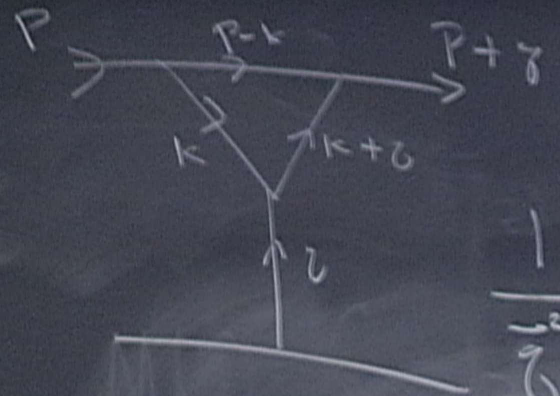

Integrals.
E=0
Integral:


P
P-k
k




$$\frac{1}{\vec{q}^{\,2}} \int \frac{[d^n k]}{k^2\left((P-k)^2-m^2\right)} \frac{1}{(k+q)^2}$$

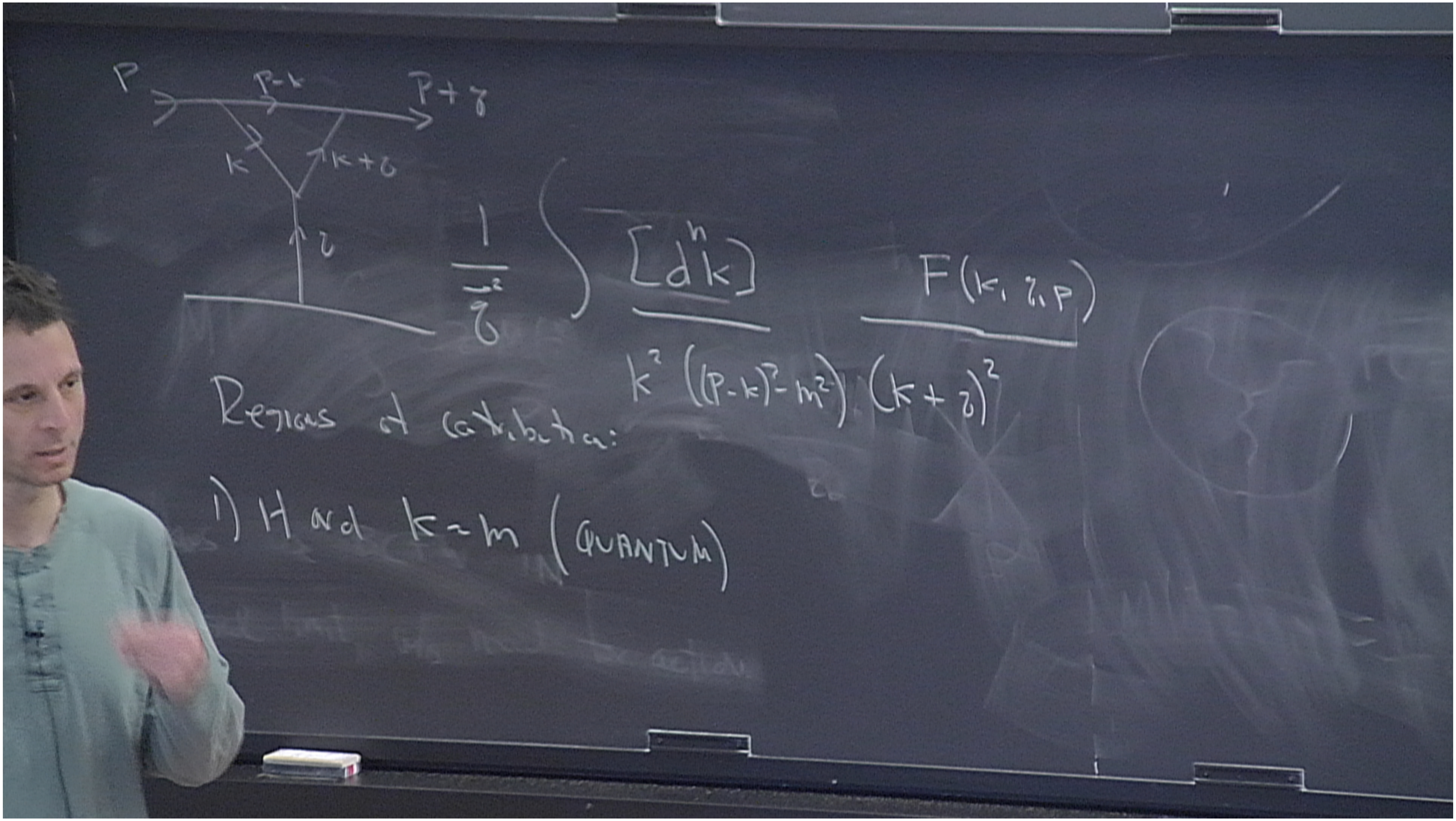

P
P-k
P+q
k
k+q
q
1/q² ∫ [dⁿk] F(k, q, p) / k² ((p-k)² - m²) (k+q)²

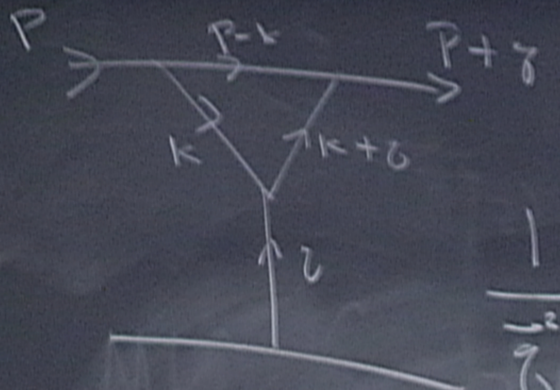



$$\frac{1}{\vec{q}^{\,2}}\int \frac{[d^n k]}{k^2\left((p-k)^2-m^2\right)}\,\frac{F(k,q,p)}{(k+q)^2}$$

Regions of contribution:

1) Hard $k \sim m$


P
p-k
p+q
k
k+q
q
$\frac{1}{q^2}\int \frac{[d^nk]\;F(k,q,p)}{k^2\,((p-k)^2-m^2)\,(k+q)^2}$
Regions of contribution:
1) Hard k ~ m (QUANTUM)

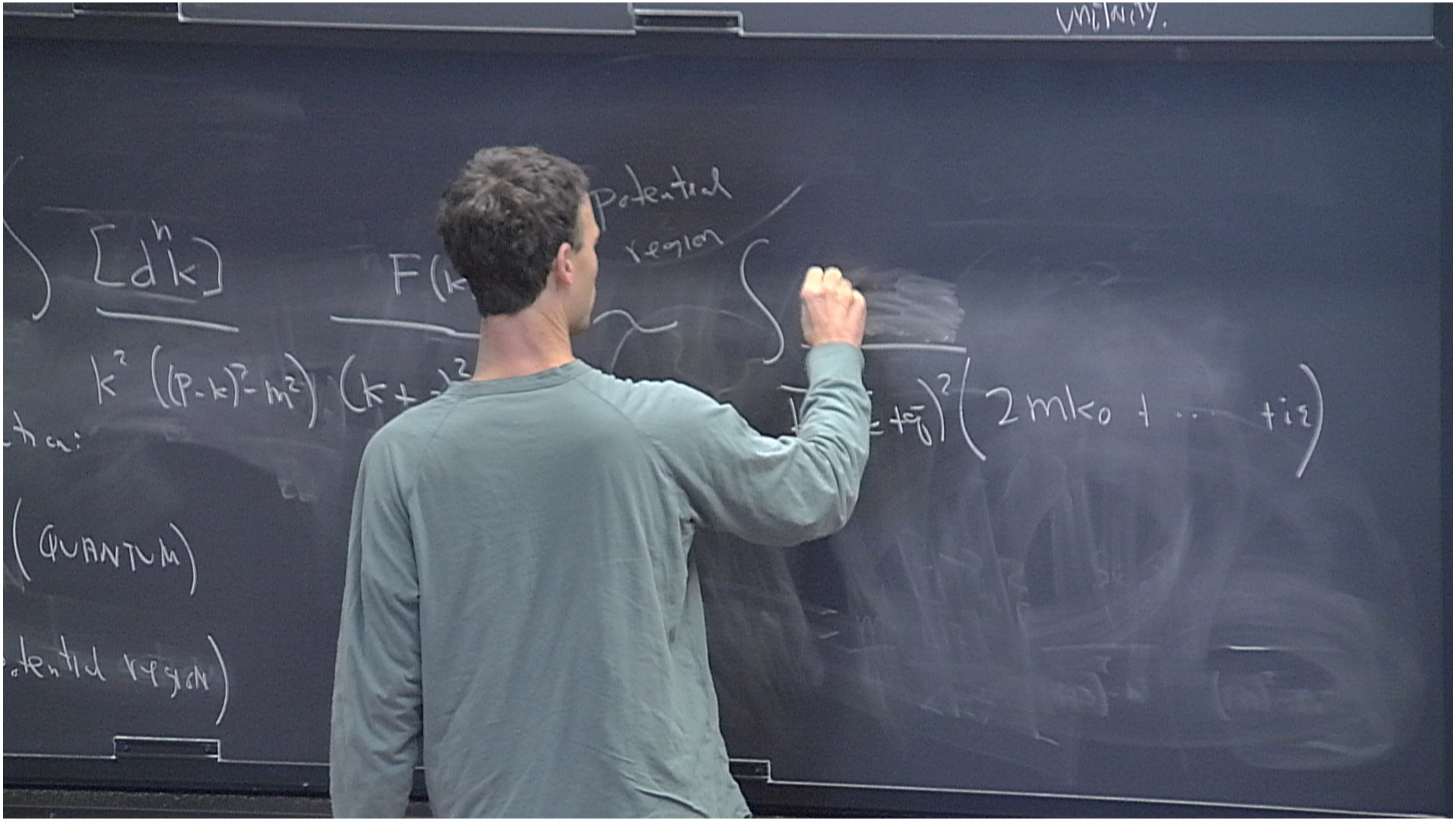



$$\frac{1}{\vec{q}^{\,2}} \int \frac{[d^n k] \quad F(k,q,p)}{k^2\left((p-k)^2-m^2\right)(k+q)^2}$$

Regions of contribution:

1) Hard $k \sim m$ (QUANTUM)

2) $k_0 \sim 0$, $\vec{k} \sim \vec{p}$ (potential region)


unitarity.
potential region
[d^n k]
k^2 ((p-k)^2 - m^2)
2mk_0 + ... + iε
(QUANTUM)
potential region

unitarity.

$$\int \frac{[d^n k]}{k^2\left((p-k)^2-m^2\right)} \frac{F(k, q, p)}{(k+q)^2} \sim \int \frac{dk_0\, d^{n-1}\bar{k}}{\bar{k}^2 (\bar{k}+\bar{q})^2 (2mk_0 + \cdots + i\varepsilon)}$$

potential region

(QUANTUM)

(potential region)

unitarity.

$k_0 \ll \vec{k}$

potential region

$$\int \frac{[d^n k]}{k^2\left((p-k)^2-m^2\right)} \frac{F(k, q, p)}{(k+q)^2} \sim \int \frac{dk_0\, d\vec{k}^{n-1}}{\vec{k}^2 (\vec{k}+\vec{q})^2 \left(2mk_0 + \dots + i\varepsilon\right)}$$

...tion:

(QUANTUM)

(potential region)

unitarity.

$k_0 \ll \vec{k}$

potential region

$$\int \frac{[d^n k]}{k^2\left((p-k)^2-m^2\right)} \frac{F(k,q,p)}{(k+q)^2} \sim \lim_{\eta \to 0} \int \frac{dk_0\, d^{n-1}k\; e^{i\eta k_0}}{\vec{k}^2 (\vec{k}+\vec{q})^2 \left(2mk_0 + \cdots + i\varepsilon\right)}$$

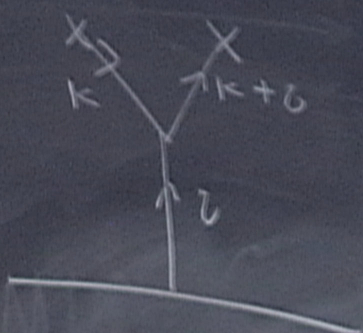
...tion:

(QUANTUM)

$$\int \frac{d^3 k}{\vec{k}^2 (\vec{k}+\vec{q})^2} =$$

(potential region)

$k_0 \ll |\vec{k}|$




$$\frac{1}{\vec{q}^{\,2}} \int \frac{[d^n k]}{k^2\left((p-k)^2-m^2\right)} \frac{F(k,q,p)}{(k+q)^2}$$

Regions of contribution:

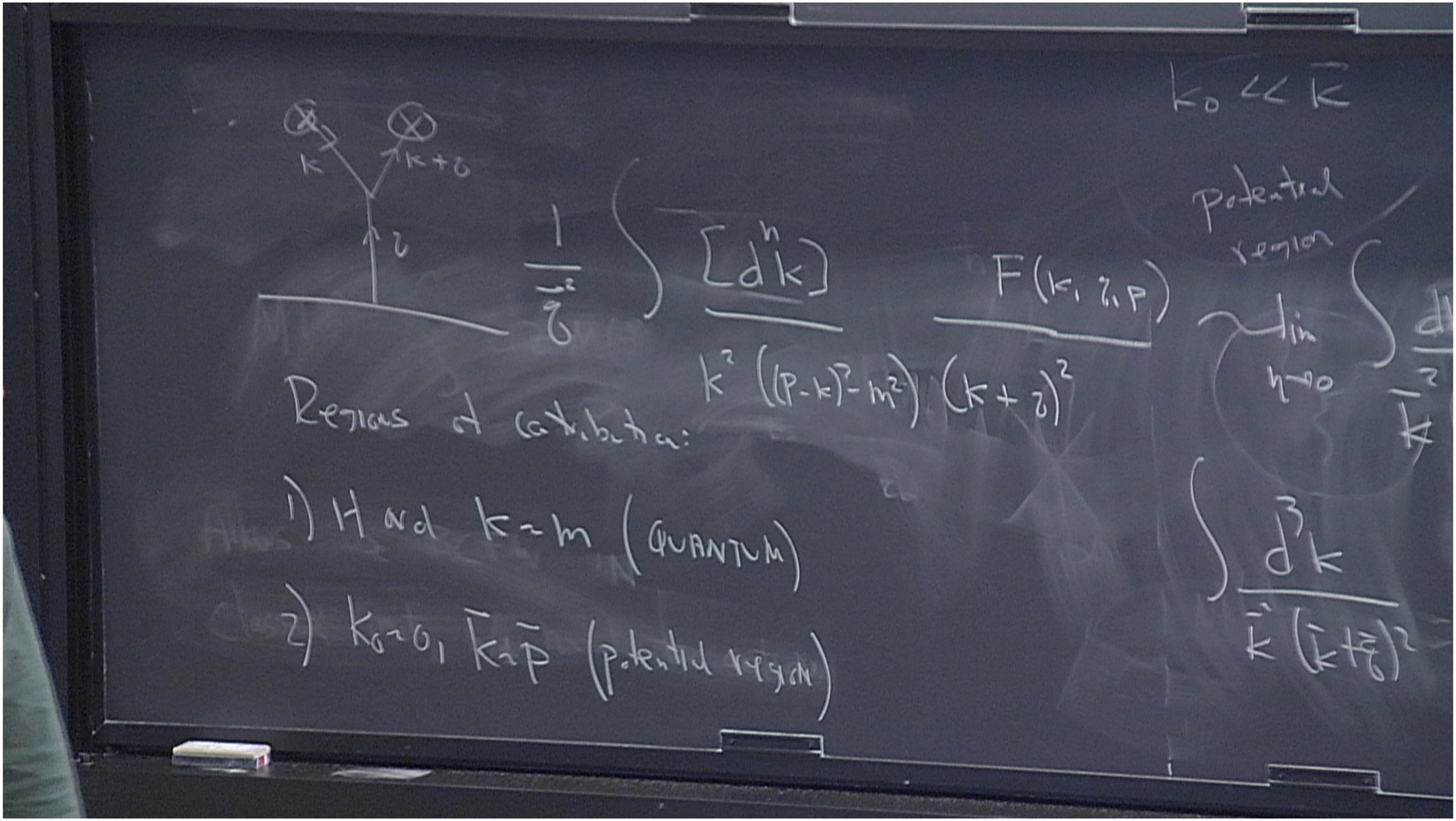
1) Hard $k \sim m$ (QUANTUM)

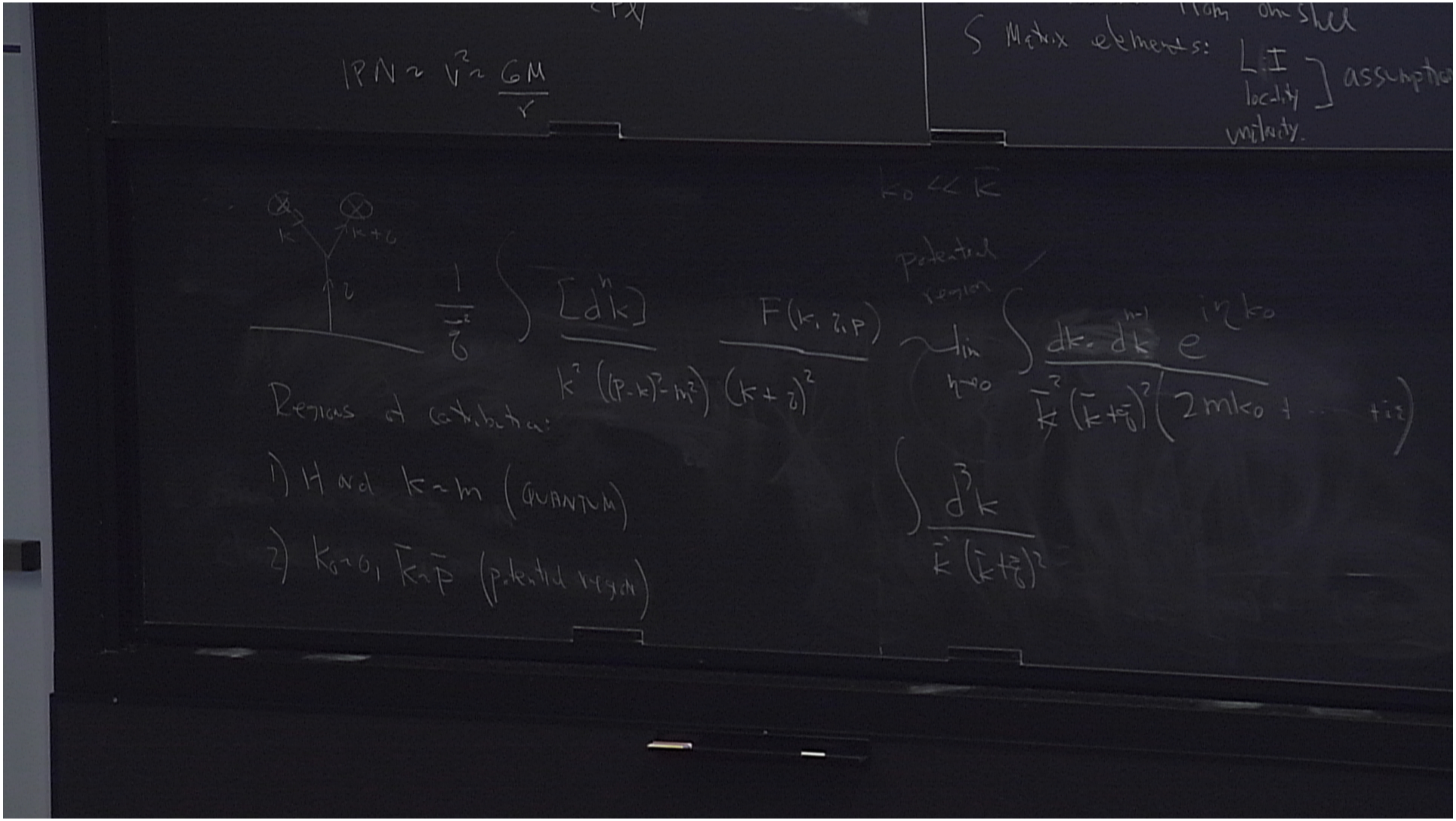
2) $k_0 \sim 0,\ \vec{k} \sim \vec{p}$ (potential region)

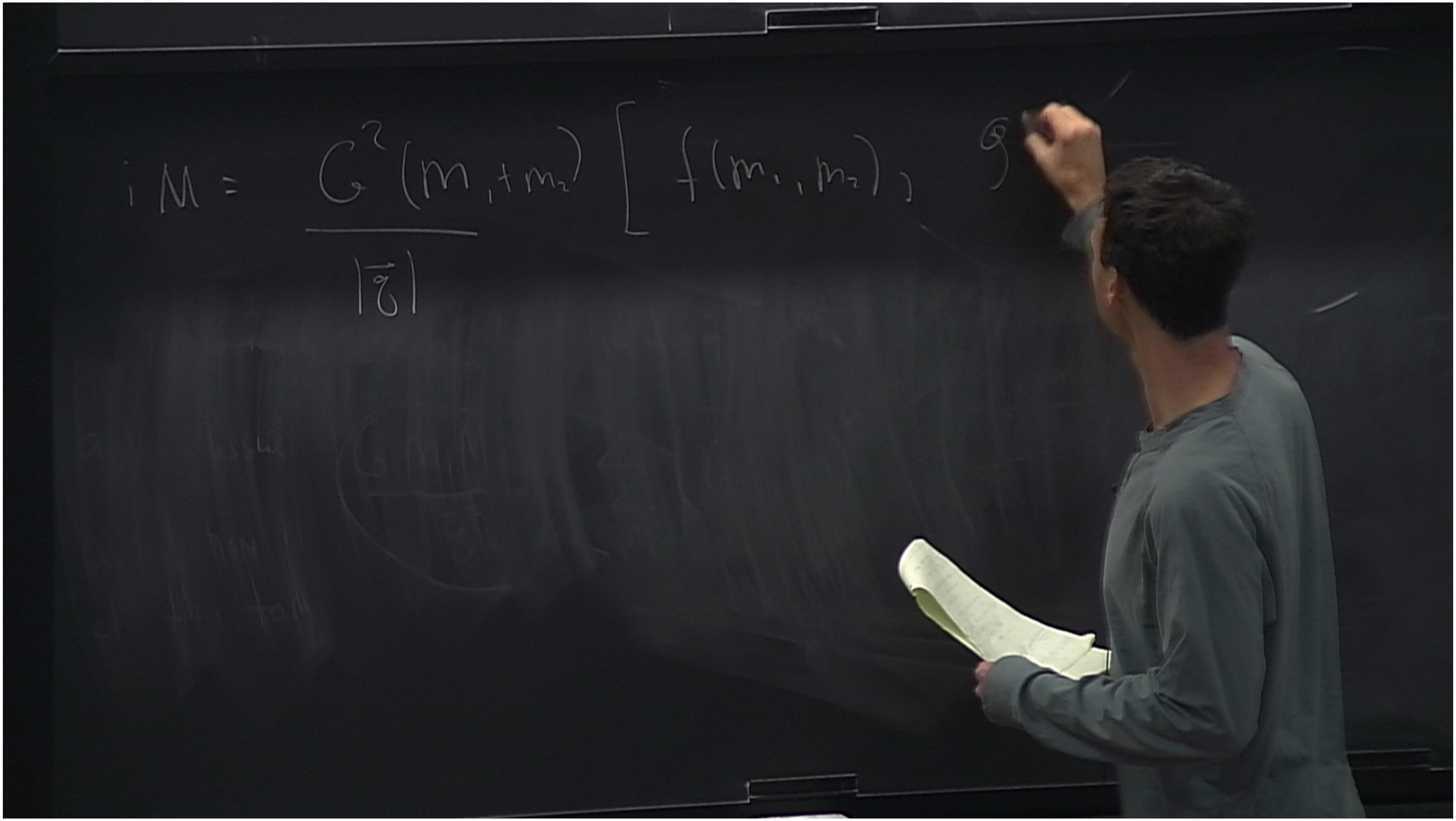
potential region

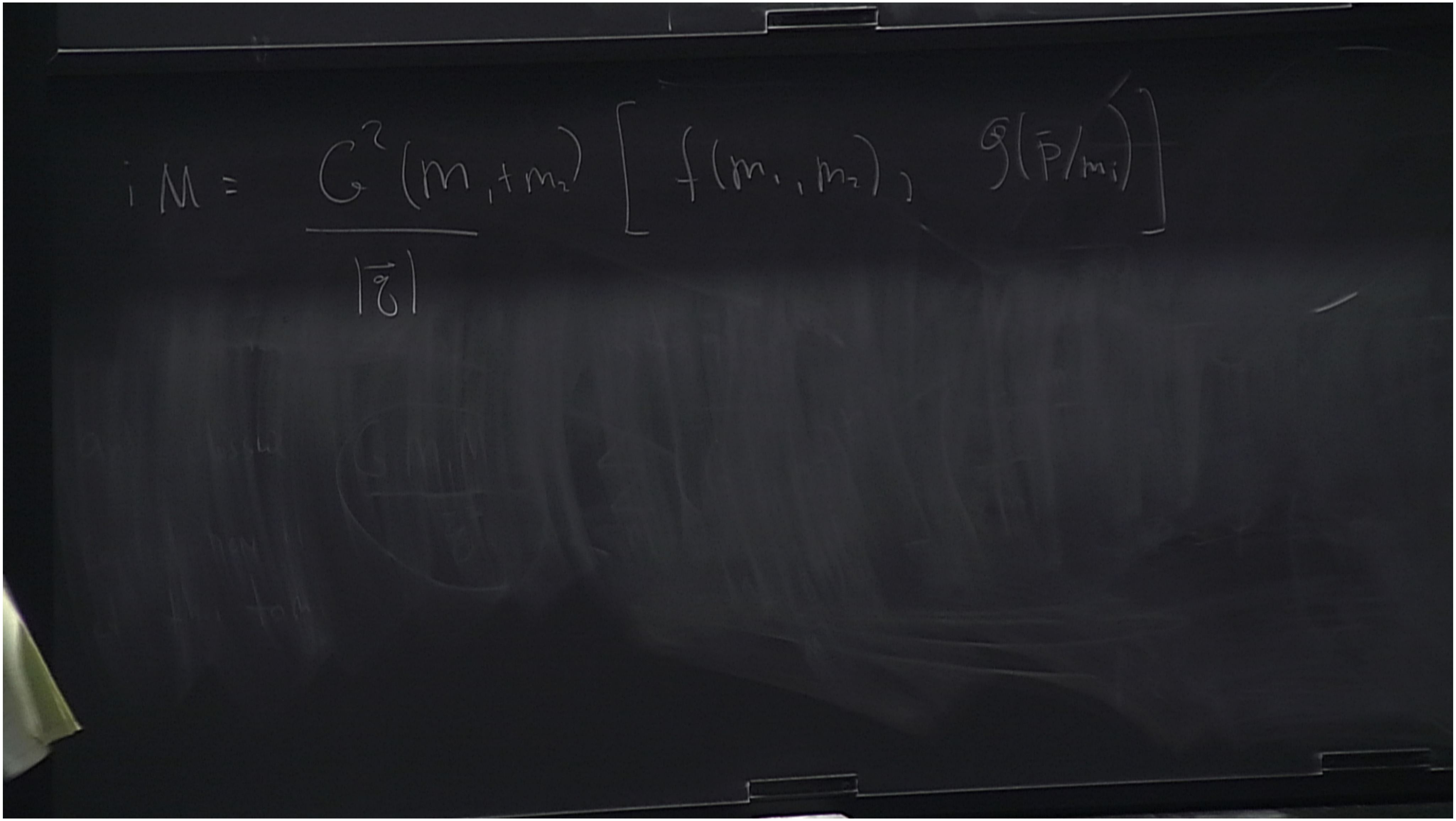
$$\lim_{\eta \to 0} \int \frac{d\ldots}{\vec{k}^2 \ldots}$$

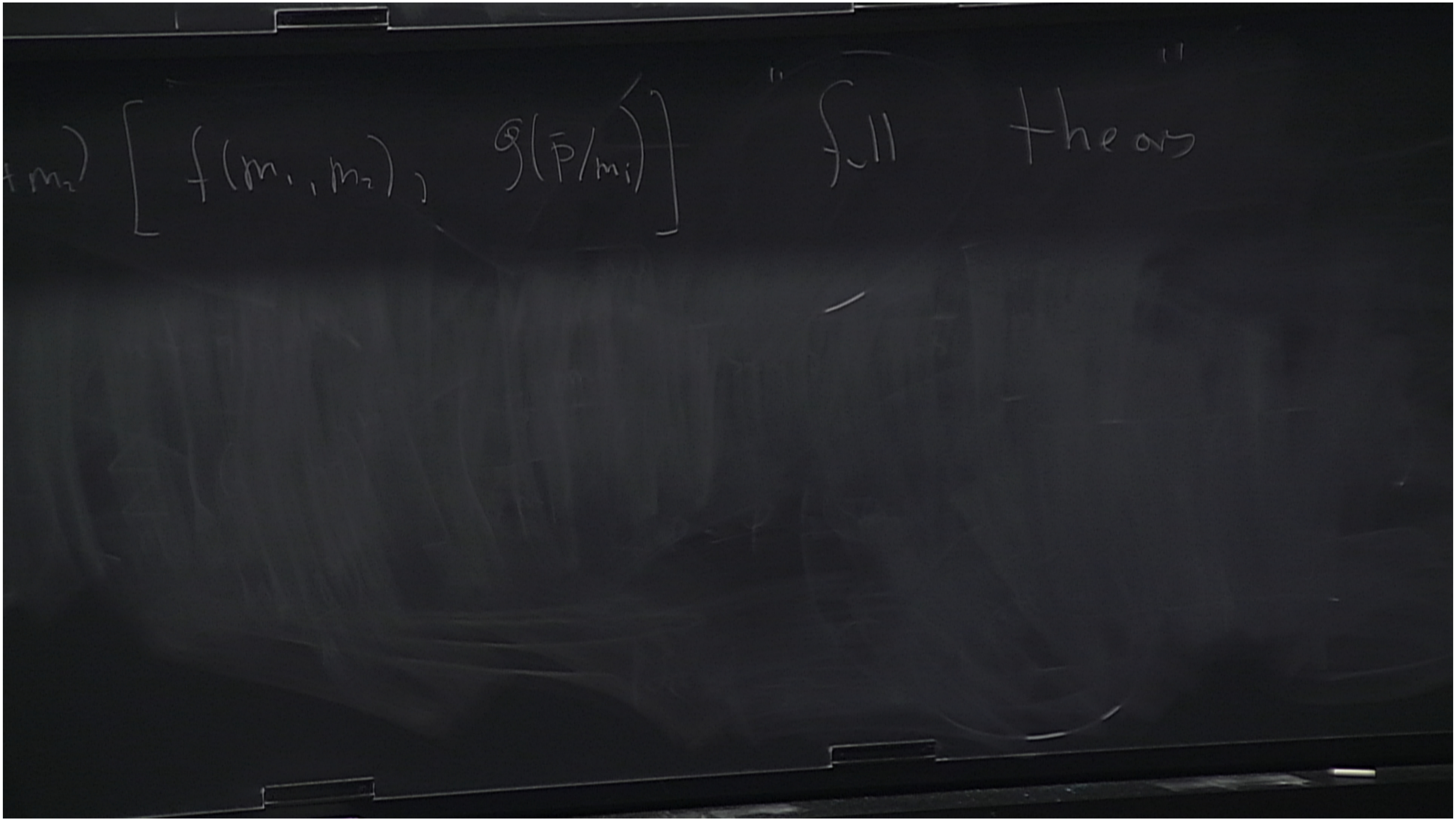
$$\int \frac{d^3 k}{\vec{k}^2 (\vec{k}+\vec{q})^2}$$

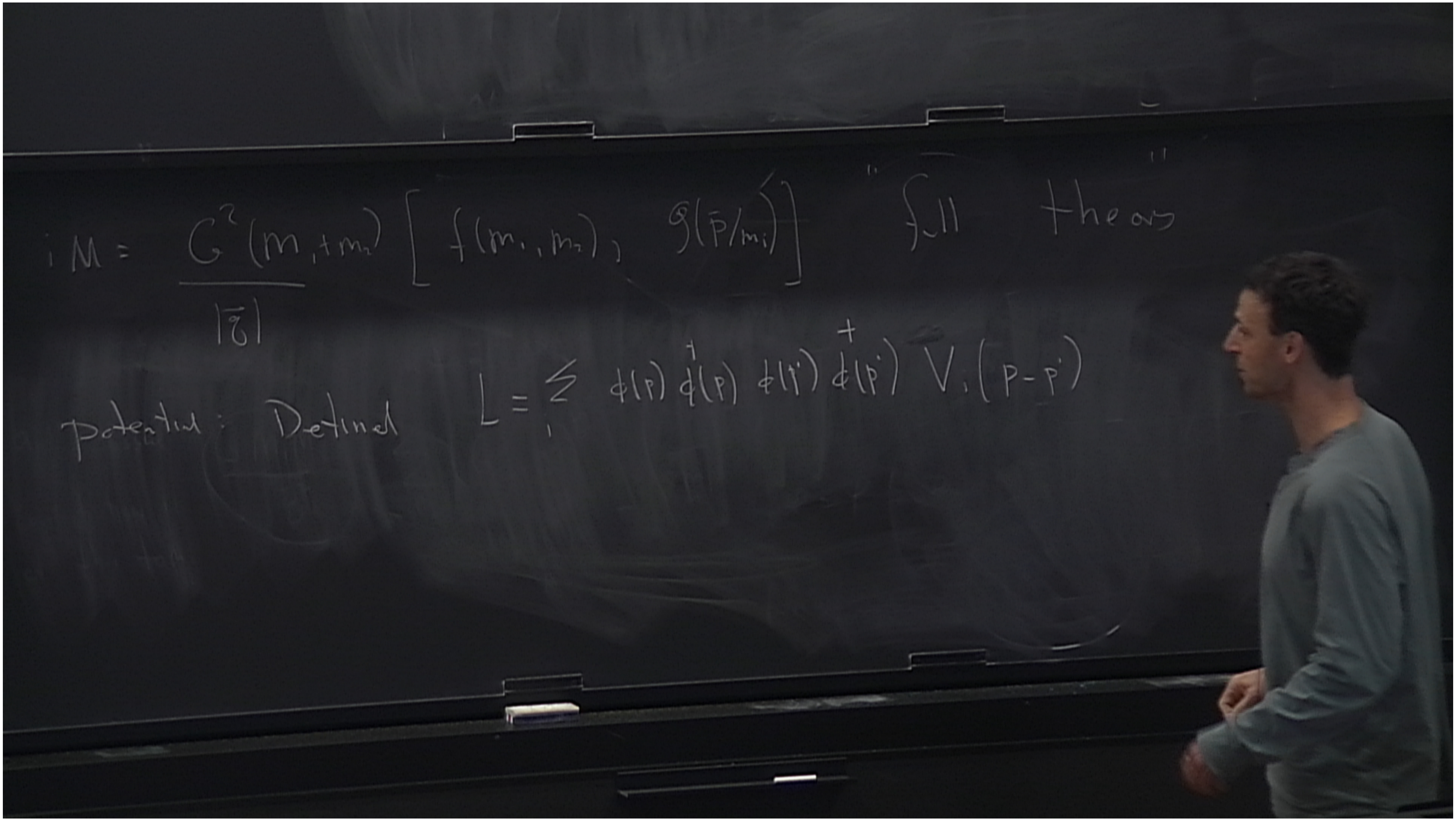

$k_0 \ll \vec{k}$
$k$
$k+q$
$q$
$\frac{1}{\vec{q}^2} \int \frac{[d^n k]}{k^2 \left((p-k)^2 - m^2\right)} \frac{F(k, q, p)}{(k+q)^2}$
potential region
$\lim_{\eta \to 0}$
Regions of contribution:
1) Hard $k \sim m$ (QUANTUM)
2) $k_0 \sim 0$, $\vec{k} \sim \vec{p}$ (potential region)
$\int \frac{d^3 k}{\vec{k}^2 (\vec{k}+\vec{q})^2}$


1PN ~ v² ~ GM/r
S Matrix elements:
L.I
locality
unitarity.
assumptions
k₀ << k̄
potential region
lim η→0
Regions of contribution:
1) Hard k ~ m (QUANTUM)
2) k₀ ~ v, k̄ ~ p̄ (potential region)


$iM = \frac{G^2(m_1+m_2)}{|\vec{q}|}\left[ f(m_1, m_2),\right.$


$$iM = \frac{G^2(m_1 + m_2)}{|\vec{q}|}\left[ f(m_1, m_2),\ g(\bar{p}/m_i) \right]$$

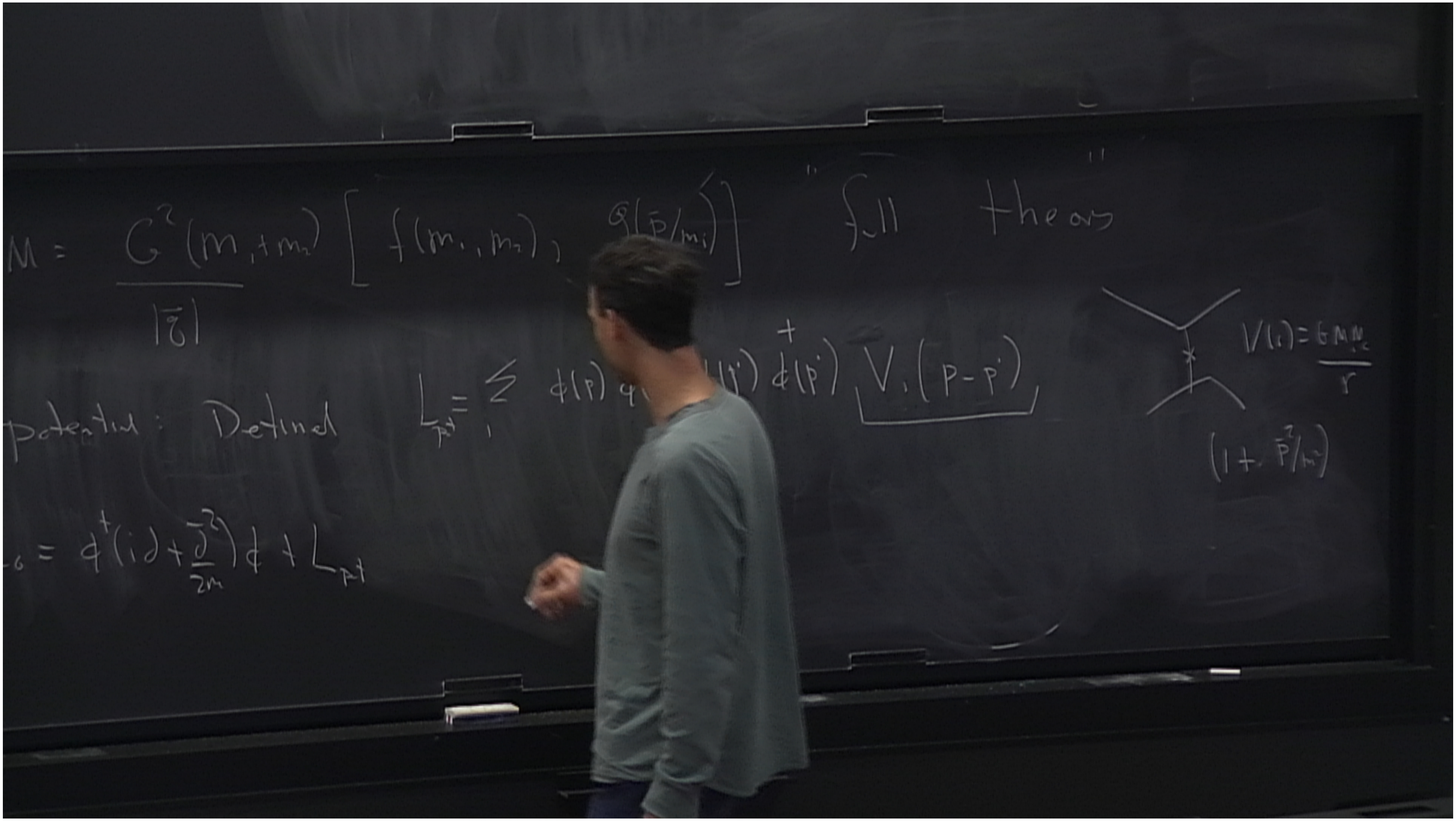

$+m_2)\left[f(m_1, m_2),\ g(\bar{p}/m_i)\right]$
"full theory"

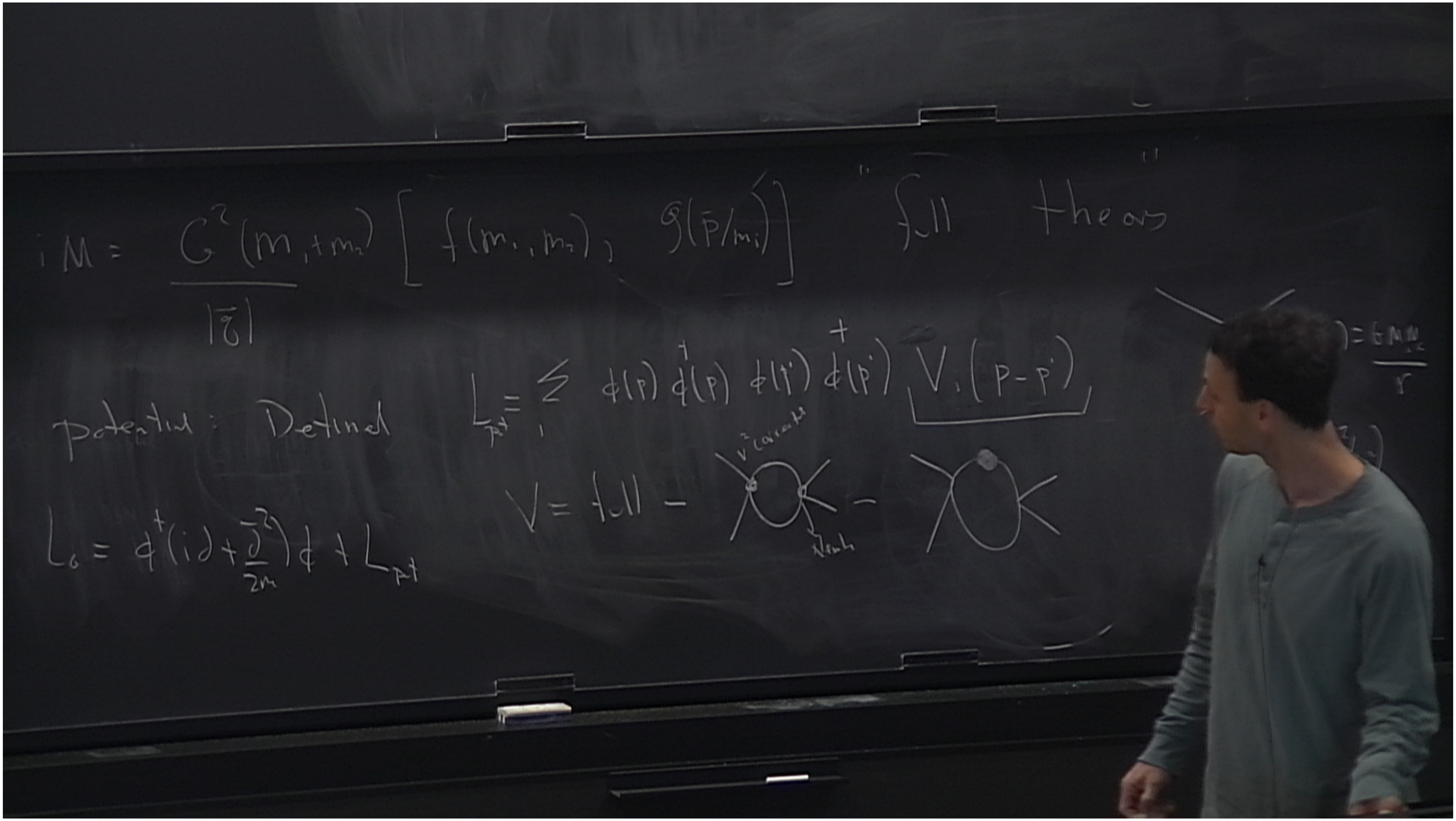

$iM = \frac{G^2(m_1+m_2)}{|\bar{q}|} \left[ f(m_1, m_2),\ g(\bar{p}/m_i) \right]$
"full theory"
potential: Defined
$L = \sum_i \phi(p)\, \phi^\dagger(p)\, \phi(p')\, \phi^\dagger(p')\, V_i(p-p')$

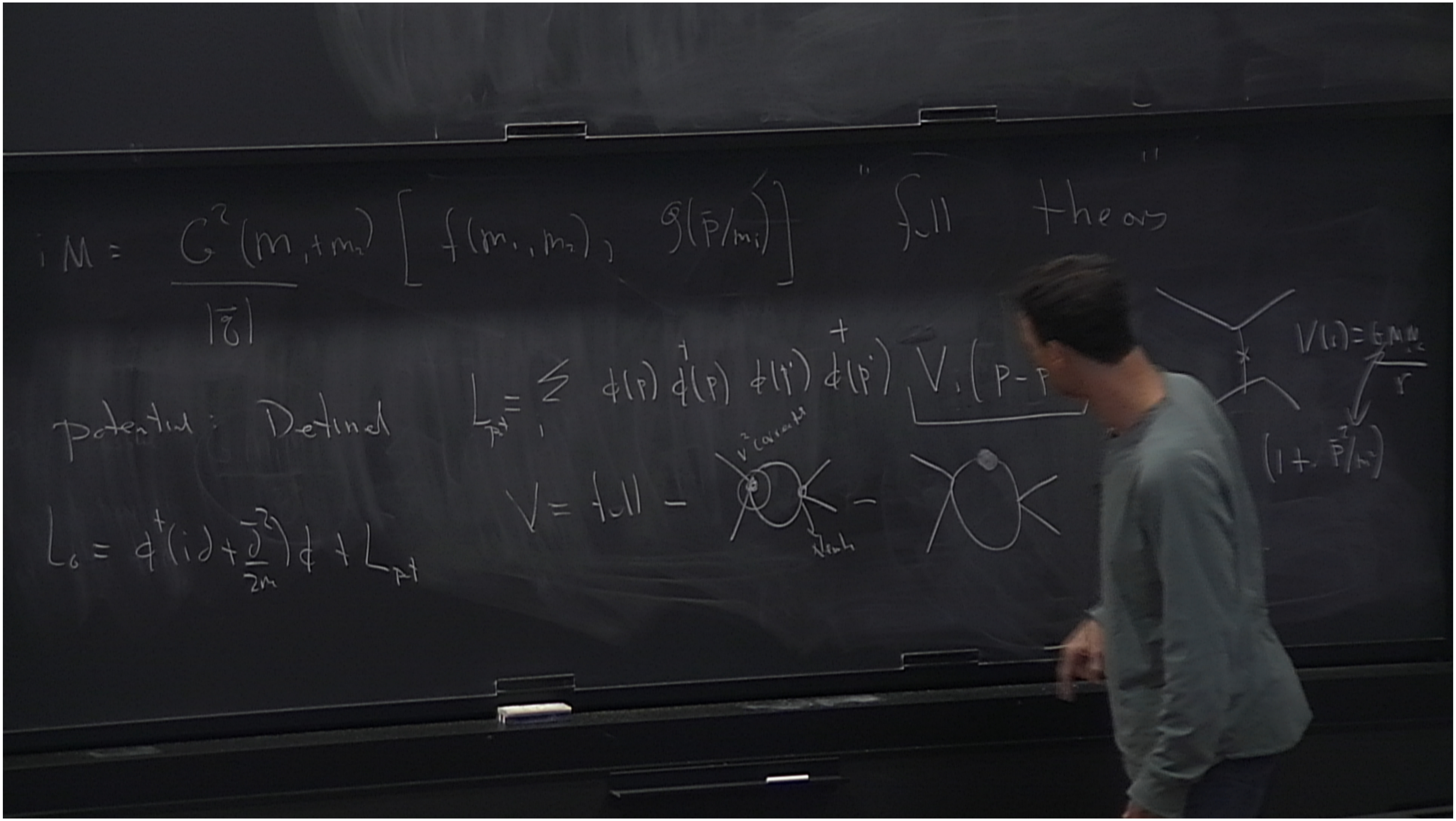
$$iM = \frac{G^2(m_1+m_2)}{|\bar{q}|}\left[ f(m_1, m_2),\ g(\bar{p}/m_i) \right] \quad \text{"full theory"}$$

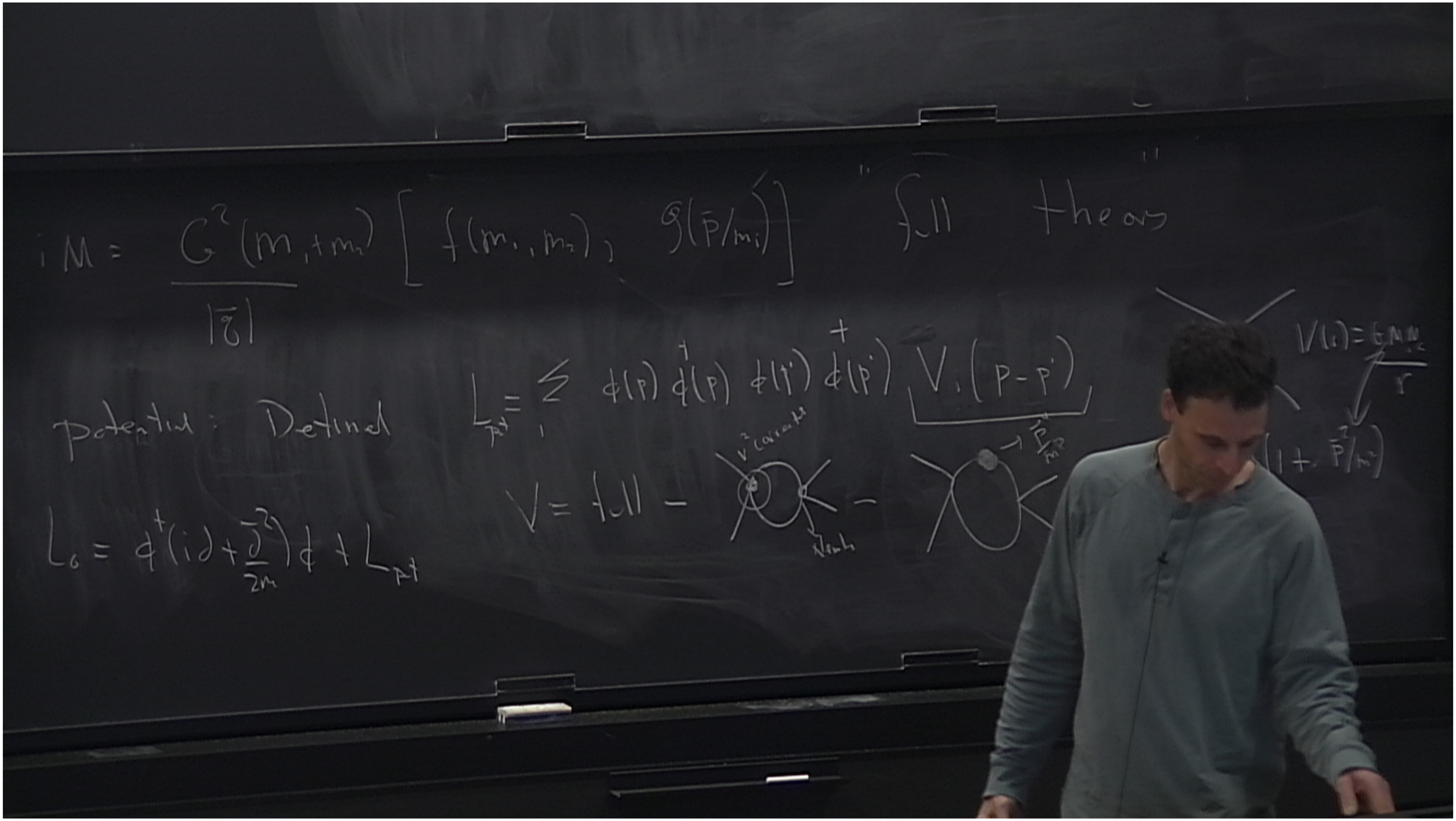
potential: Defined $\quad L_{pot} = \sum_i \phi(p)\,\phi^\dagger(p)\,\phi(p')\,\phi^\dagger(p')\, V_i(p-p')$

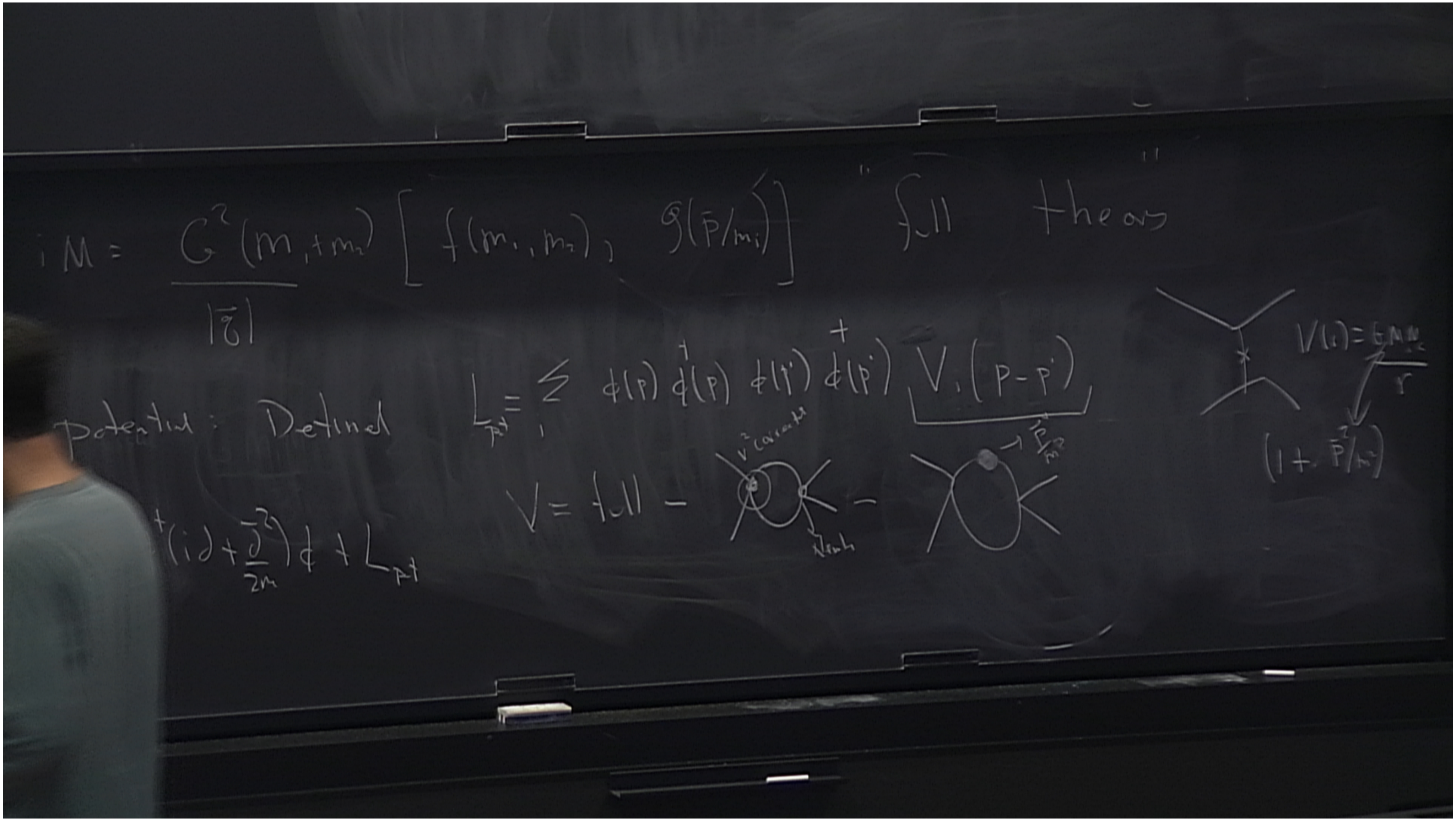
$$L_0 = \phi^\dagger\left(i\partial + \frac{\bar{\partial}^2}{2m}\right)\phi + L_{pot}$$

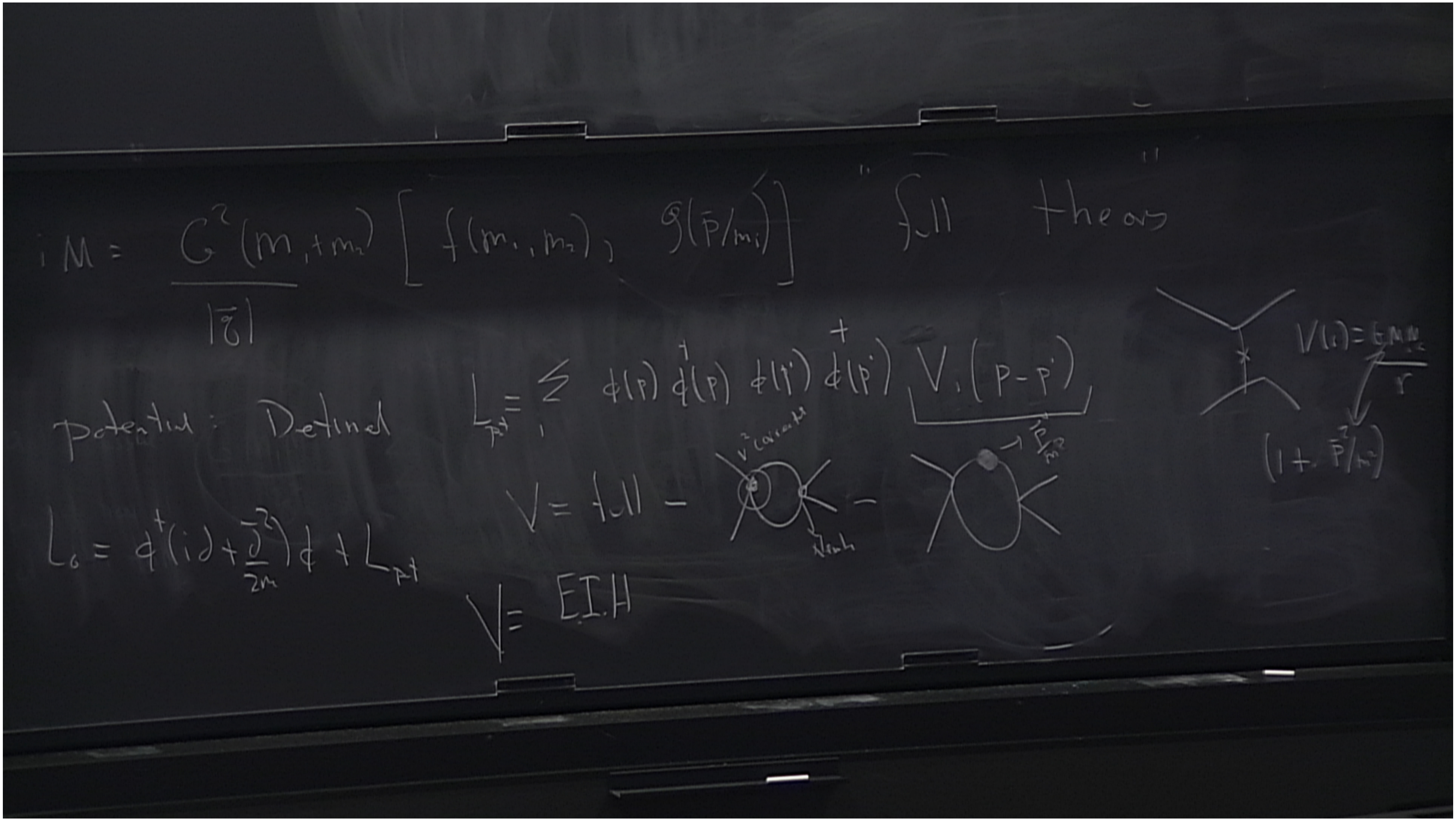

"full theory"
potential: Defined
V(r) = GMm/r


iM = G²(m₁+m₂)/|q| [ f(m₁, m₂), g(p/m₁) ]
"full theory"
potential: Defined
L_pot = Σ φ(p) φ†(p) φ(p') φ†(p') V_i(p − p')
v² corrections
Newton
V = full −
L₀ = φ†(i∂ + ∂²/2m)φ + L_pot

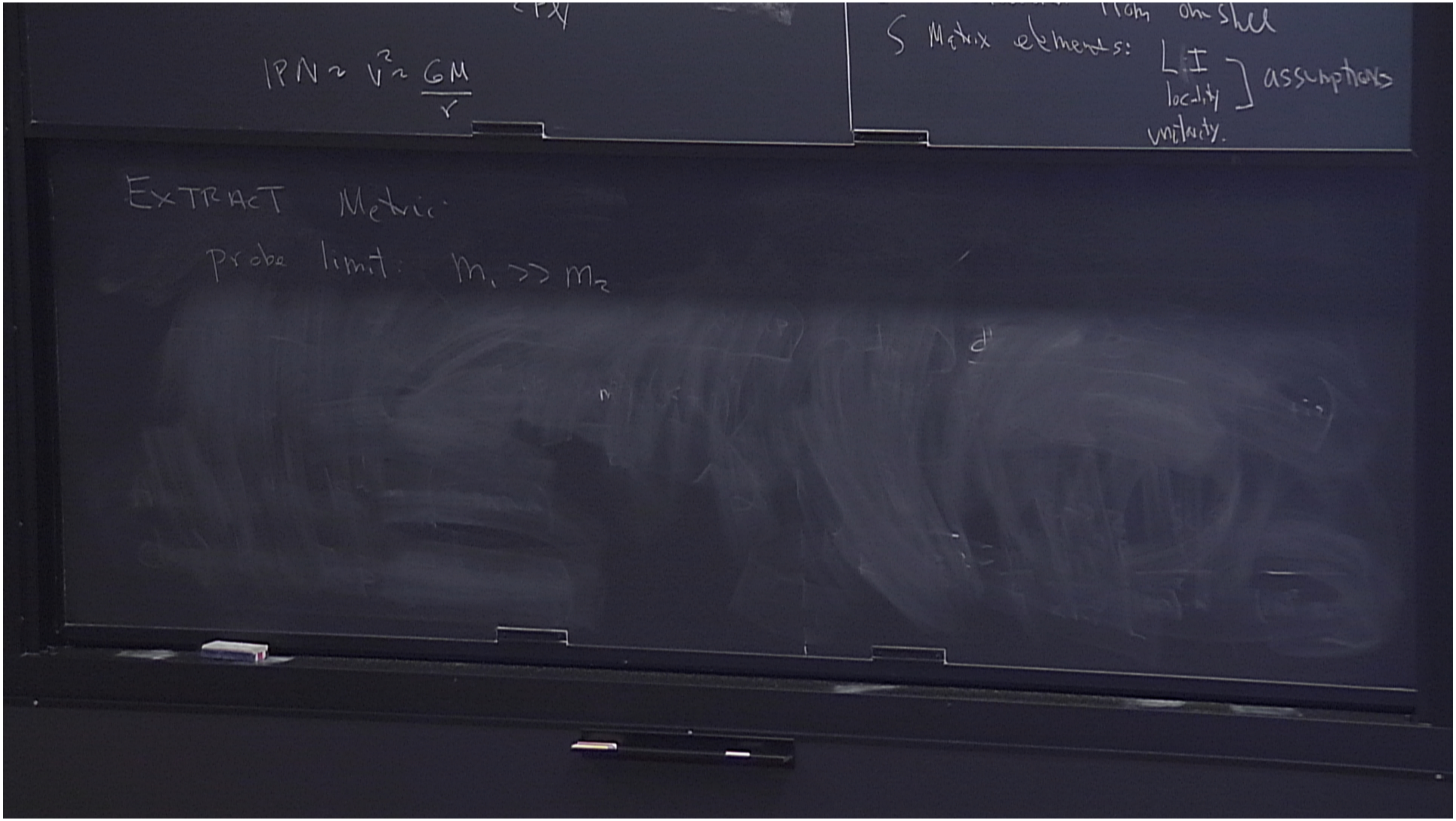

$iM = \frac{G^2(m_1+m_2)}{|\vec{q}|}\left[f(m_1,m_2),\ g(\vec{p}/m_i)\right]$ "full theory"
$V(r) = \frac{G M_1 M_2}{r}$
$(1+ \vec{p}^2/m^2)$
potential: Defined
$L_{pt} = \sum_i \phi(p)\,\phi^\dagger(p)\,\phi(p')\,\phi^\dagger(p')\,V_i(p-p')$
$v^2$ correction
Newton
$V = \text{full} - \ldots - \ldots$
$L_0 = \phi^\dagger\left(i\partial + \frac{\vec{\partial}^2}{2m}\right)\phi + L_{pt}$

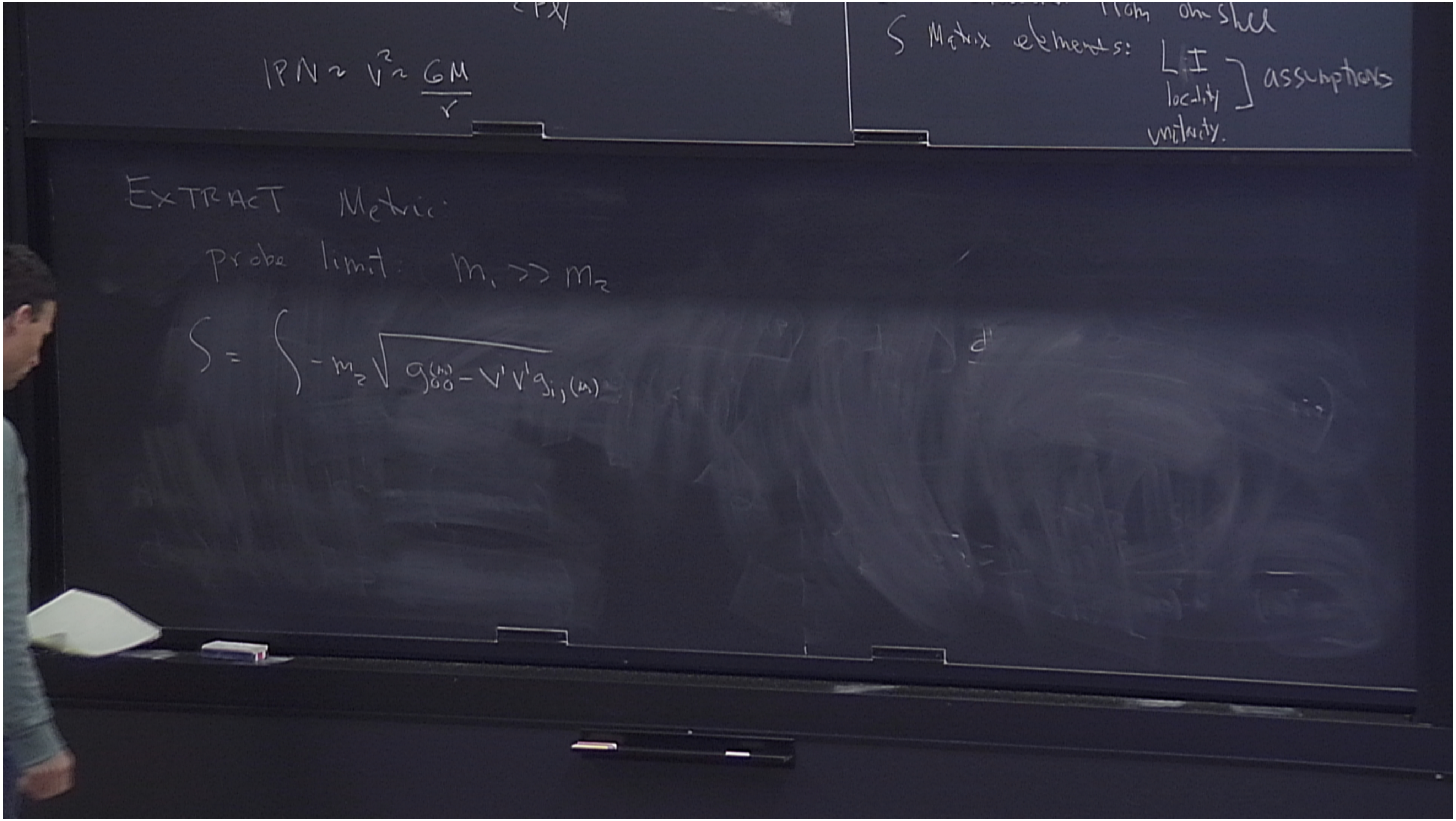

iM = G²(m₁+m₂) [ f(m₁, m₂), g(p̄/m₁) ] / |q̄|
"full theory"
V(r) = G M₁ M₂ / r
potential: Defined
L_pot = Σ ϕ(p) ϕ†(p) ϕ(p') ϕ†(p') V_i(p − p')
V = full −
v² corrections
Newton
p̄²/m²
(1 + p̄²/m²)
L₀ = ϕ†(i∂ + ∂²/2m) ϕ + L_pot

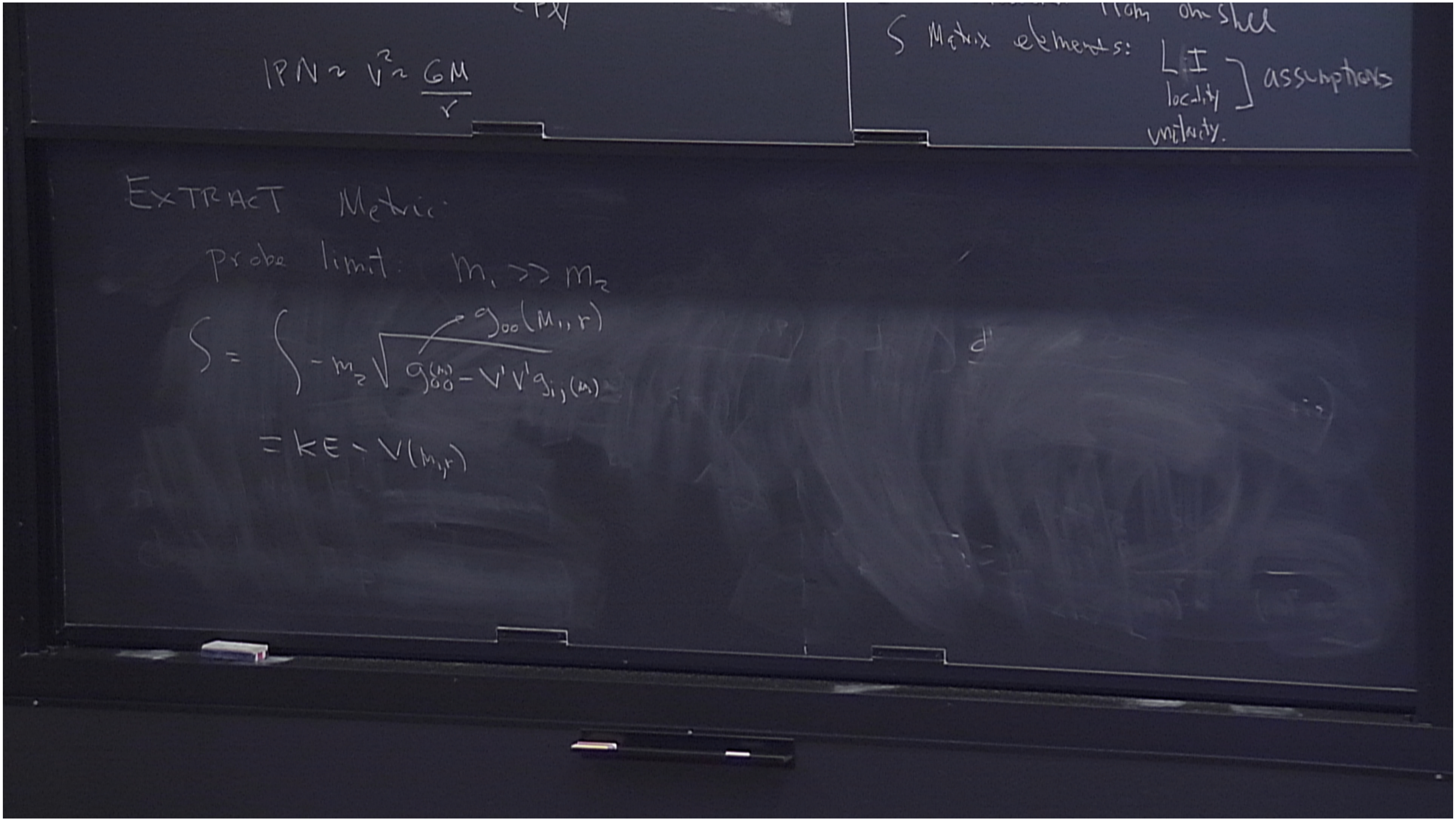

$iM = \frac{G^2(m_1+m_2)}{|\vec{q}|}\left[ f(m_1,m_2),\ g(\vec{p}/m_i) \right]$
"full theory"
$L_{pot} = \sum_i \phi^\dagger(p)\phi(p)\phi^\dagger(p')\phi(p') V_i(p-p')$
potential: Defined
$V = \text{full} - \ldots - \ldots$
$V(r) = \frac{GMm}{r}$
$(1 + \vec{p}^2/m^2)$
$\phi^\dagger(i\partial + \frac{\vec{\partial}^2}{2m})\phi + L_{pot}$

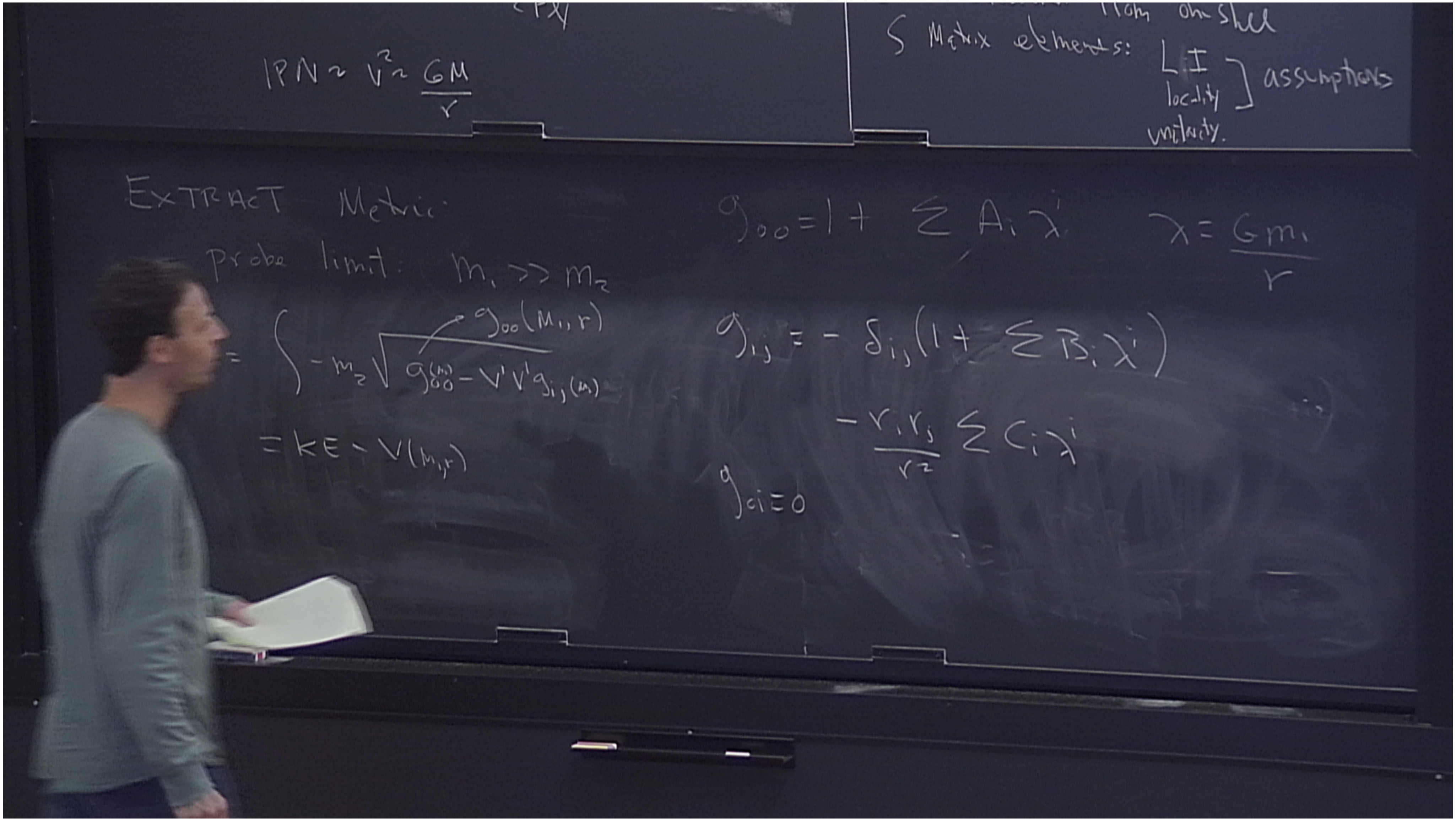


iM = G^2(m1+m2)/|q| [ f(m1, m2), g(p/mi) ]
"full theory"
potential: Defined
L_pot = Σ φ(p) φ†(p) φ(p') φ†(p') V_i(p-p')
V = full − − 
V = E.I.H
L_0 = φ†(i∂ + ∂²/2m) φ + L_pot
V(r) = GMm/r
(1 + p²/m²)


1PN ~ v² ~ GM/r
from on shell
S Matrix elements: L.I
locality
unitarity.
assumptions
EXTRACT Metric
probe limit: m₁ >> m₂


from on shell
S Matrix elements: L.I, locality, unitarity ] assumptions
1PN ~ v^2 ~ GM/r
EXTRACT Metric
probe limit: m_1 >> m_2
S = ∫ -m_2 √(g_00^(m) - v^i v^j g_ij (m))


1PN ~ v^2 ~ GM/r
from on shell
S Matrix elements:
L.I
locality
unitarity.
assumptions
EXTRACT Metric
probe limit: m1 >> m2
g00(M1, r)
S = ∫ -m2 √(g00^(M) - v^i v^j g_ij(M))
= KE - V(M1,r)



$1PN \sim v^2 \sim \frac{GM}{r}$
from on shell
S Matrix elements: L.I
locality
unitarity
assumptions
EXTRACT Metric
probe limit: $m_1 \gg m_2$
$= \int -m_2 \sqrt{g_{00}^{(m)} - v^i v^j g_{ij}(m)}$
$g_{00}(m_1, r)$
$= KE - V(m_1, r)$
$g_{00} = 1 + \sum A_i \lambda^i$
$\lambda = \frac{Gm_1}{r}$
$g_{ij} = -\delta_{ij}\left(1 + \sum B_i \lambda^i\right) - \frac{r_i r_j}{r^2} \sum C_i \lambda^i$
$g_{0i} = 0$

$$iM = \frac{G^2(m_1+m_2)}{|\vec{q}|}\Big[ f(m_1, m_2),\ g(\vec{p}/m_i) \Big] \quad \text{"full theory"}$$

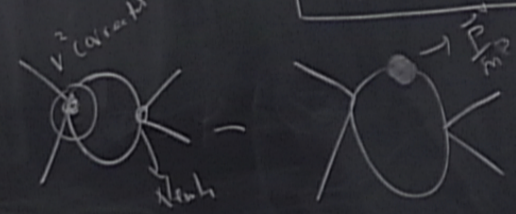
$$V(r) = \frac{G M_1 M_2}{r}$$

$$(1 + \vec{p}^2/m^2)$$

potential: Defined

$$L_{pot} = \sum_i \phi^\dagger(p)\,\phi(p)\,\phi^\dagger(p')\,\phi(p')\,\underbrace{V_i(p-p')}$$

$$V = \text{full} -$$





$$L_0 = \phi^\dagger\left(i\partial + \frac{\vec{\partial}^2}{2m}\right)\phi + L_{pot}$$

$$\boxed{V = \text{E.I.H}}$$

from on shell S Matrix elements:

L.I
locality
unitarity

] assumptions

$$1PN \sim v^2 \sim \frac{GM}{r}$$

EXTRACT Metric

probe limit: $m_1 \gg m_2$

$$S = \int -m_2 \sqrt{g_{00}^{(m)} - v^i v^j g_{ij}(m)}$$

$g_{00}(M_1, r)$

$$= KE - V(m_1, r)$$

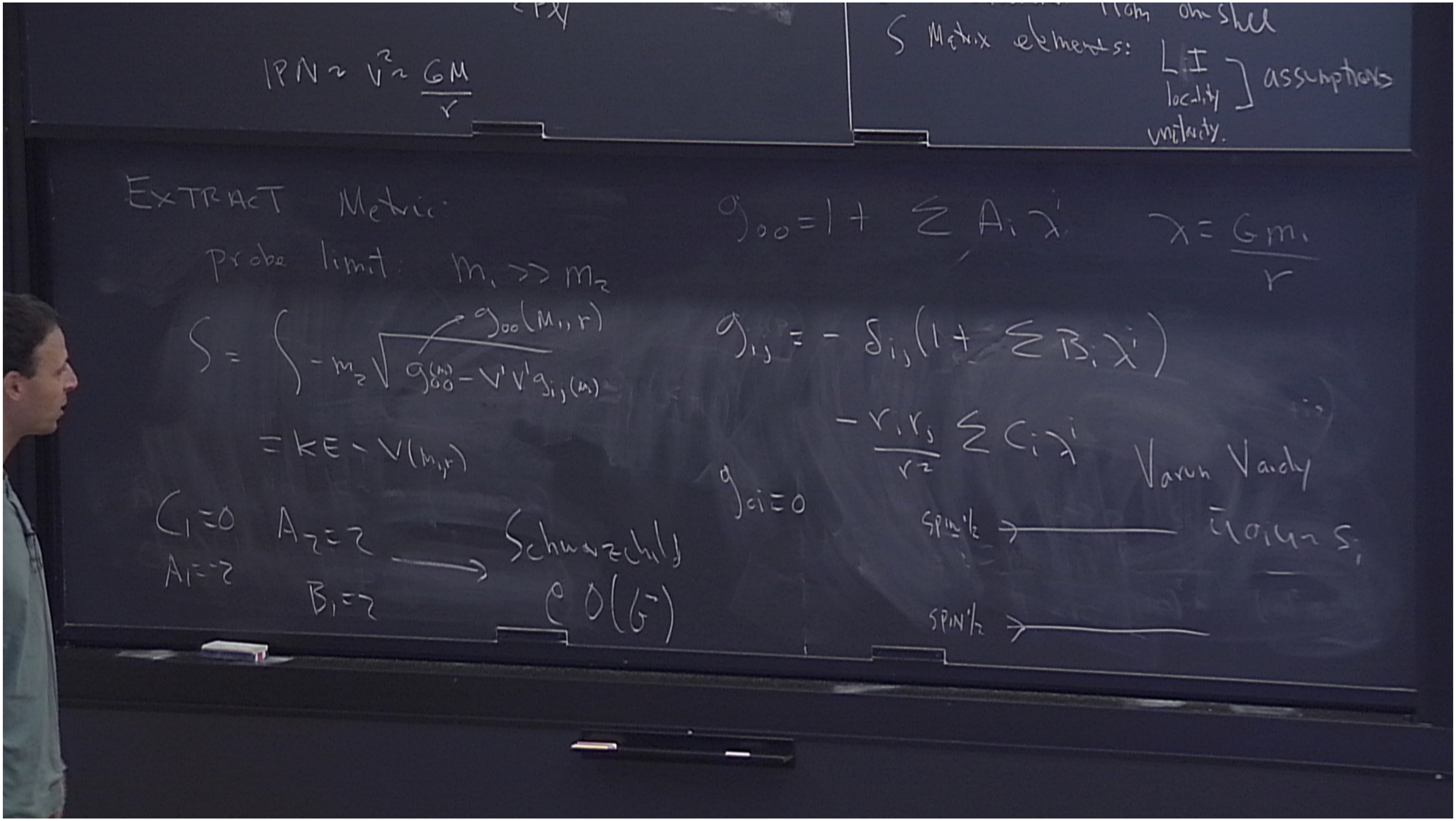
$$g_{00} = 1 + \sum A_i \lambda^i \qquad \lambda = \frac{Gm_1}{r}$$

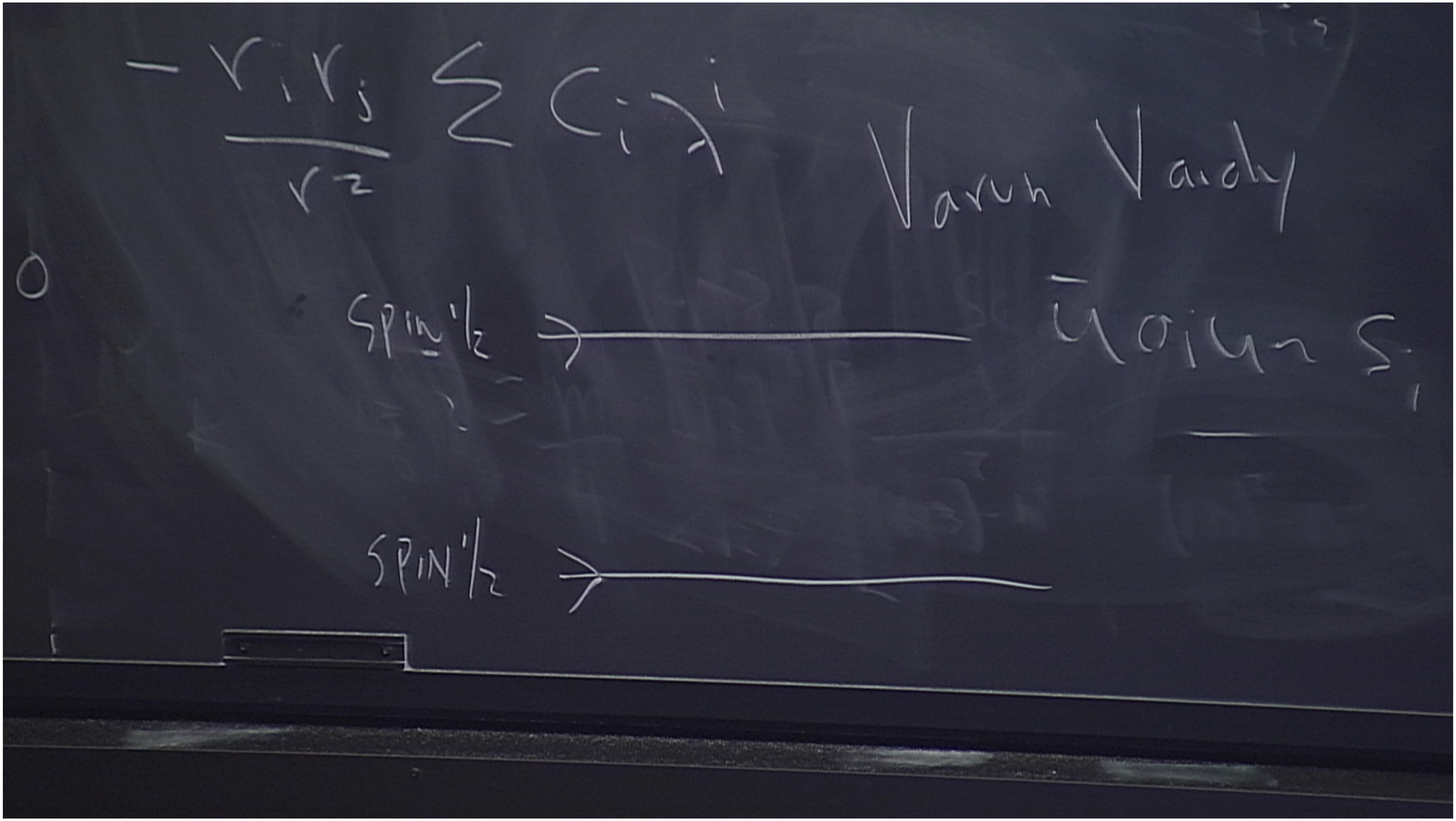
$$g_{ij} = -\delta_{ij}\left(1 + \sum B_i \lambda^i\right) - \frac{r_i r_j}{r^2} \sum C_i \lambda^i$$

$$g_{0i} = 0$$

$C_1 = 0$ $\quad A_2 = 2$ $\longrightarrow$ Schwarzchild

$A_1 = -2$ $\quad B_1 = 2$ $\quad$ @ $O(G^2)$


1PN ~ v^2 ~ GM/r
S Matrix elements:
L.I
locality
unitarity.
assumptions
EXTRACT Metric
probe limit: m1 >> m2
g00 = 1 + Σ Ai λ^i
λ = Gm1/r
S = ∫ -m2 √(g00(m) - v^i v^j gij(m))
g00(M1, r)
gij = - δij (1 + Σ Bi λ^i)
- ri rj / r^2 Σ Ci λ^i
= KE - V(m1, r)
g0i = 0
Varun Vaidya
C1 = 0
A2 = 2
A1 = -2
B1 = 2
Schwarzchild
@ O(G^2)
SPIN ½
SPIN ½


Varun Vaidy
spin ½
SPIN ½