Title: Matter matters in asymptotically safe quantum gravity

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Abstract: The Functional Renormalisation Group technique has received great attention in recent times proving itself as a powerful tool to describe the high energy behaviour of gravitational interactions.
Its key ingredient is a nontrivial fixed point of the theory renormalization group flow which controls the behavior of the coupling constants in the ultraviolet regime and ensures that physical quantities are safe from divergences. I will briefly review the main ingredients of the gravitational asymptotic safety framework before focusing on the effect of massless minimally coupled scalars, fermions and vector fields on the gravitational fixed point. I will then set bounds on the type and number of such fields requiring the existence of a UV attractive fixed point. I will also discuss the dynamically generated quantum-gravity scale in asymptotic safety, which is the transition scale to the fixed-point regime, and its variation as a function of matter degrees of freedom. To conclude I will also consider the case of higher dimensional models.

Matter matters in Asymptotically Safe gravity

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Matters matters in asymptotically safe gravity,
P.D., A. Eichhorn, R. Percacci
arXiv:very soon

Motivation

Matter in quantum gravity approaches

- Ignored or Not dynamical

  non interacting matter in the UV

- Emergent (topologically non trivial configuration)

  SM with all its quantum numbers?

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Our approach

- Straightforward inclusion of matter in the Asymptotic Safety (AS) scenario

  local continuum QFT

- All observed matter is fundamental

  stong indication of fully interactive matter [A. Eichhorn, G.P. Vacca, O. Zanusso]

Introduction to AS

Perturbative Quantum Gravity is non renormalizable

- at one loop level with matter ['t Hooft and Veltman, 1974]
- at two loop level pure gravity [Goroff and Sagnotti, 1986]

Maybe gravity is Asymptotically Safe! [Weinberg, 1976]

- Exist a nontrivial fixed point of the underlying renormalization group (RG) flow for gravity
- Calculability improved in the early '90s with the use of *e*ffective average action [Reuter and Wetterich, 1994]
- More than two decades of work contributed to find evidences on the existence of a NGFP in a lot of truncations.[1]

[1] Shhh! We still miss a final proof!

The basics of Asymptotic Safety program:

- Specify the theory
  - *field content*
  - *symmetries*

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- Specify the theory
  - *field content*
  - *symmetries*
- Choose the action
  - *a specific combination of monomials of the fields*
  - *with the required symmetries*
- Look at the theory space
  - *all the possible actions*
  - *parametrized by the couplings* g_i
- Renormalization group flow
  - *connects physics at different scales*
  - *for coupling constant* $k\partial_k g_i = \beta_i(g_i)$

Fixed Points

- Definition

  - *a point* (g_i^*) *in theory space where the running of all couplings stops running*

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- Properties

  - *well defined continuum limit (no divergences)*

  - *two classes of RG trajectories (relevant/irrelevant)*

  - *predictivity = finite number of relevant directions*

- Recover GR in IR Limit

  - *RG-trajectories have part where GR is good approximation*

Functional Renormalization Group Approach

- Start from a generic action $S[\Phi]$
- Introduce a scale (k) dependent mass term $\Delta S_k[\Phi]$

  - $\Delta S_k[\Phi] = \frac{1}{2}\int \mathrm{d}^d x \; \mathcal{R}_k(\Delta)\Phi^2$

  - *$\mathcal{R}_k(z)$ is a monotonically decreasing (cutoff) function ($\mathcal{R}_k \approx z$ if $z \ll k$ and $\mathcal{R}_k \approx 0$ if $z \gg k$)*

  - *this discriminate between high momentum modes (integrated out) and low momentum modes (suppressed by mass term)*

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- as in standard QFT W_k scale dependent generating functional for connected Green functions
- its “modified” Lagrange transform is what we call Effective average action Γ_k

  $k \to 0$ *standard effective action,* $k \to \infty$ *bare starting action*

- it satisfy the Wetterich equation:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k$$

Why matter matters in AS?

Our truncation is given by

$$\Gamma_k = \Gamma_{\text{EH}} + S_{\text{gf}} + S_{\text{gh}} + \Gamma_{\text{matter}} + \Gamma_{\text{abelian}}$$

- Einstein-Hilbert Action
- Massless minimally coupled matter and gauge fields (N_s, N_f, N_v)
- Background Field Method

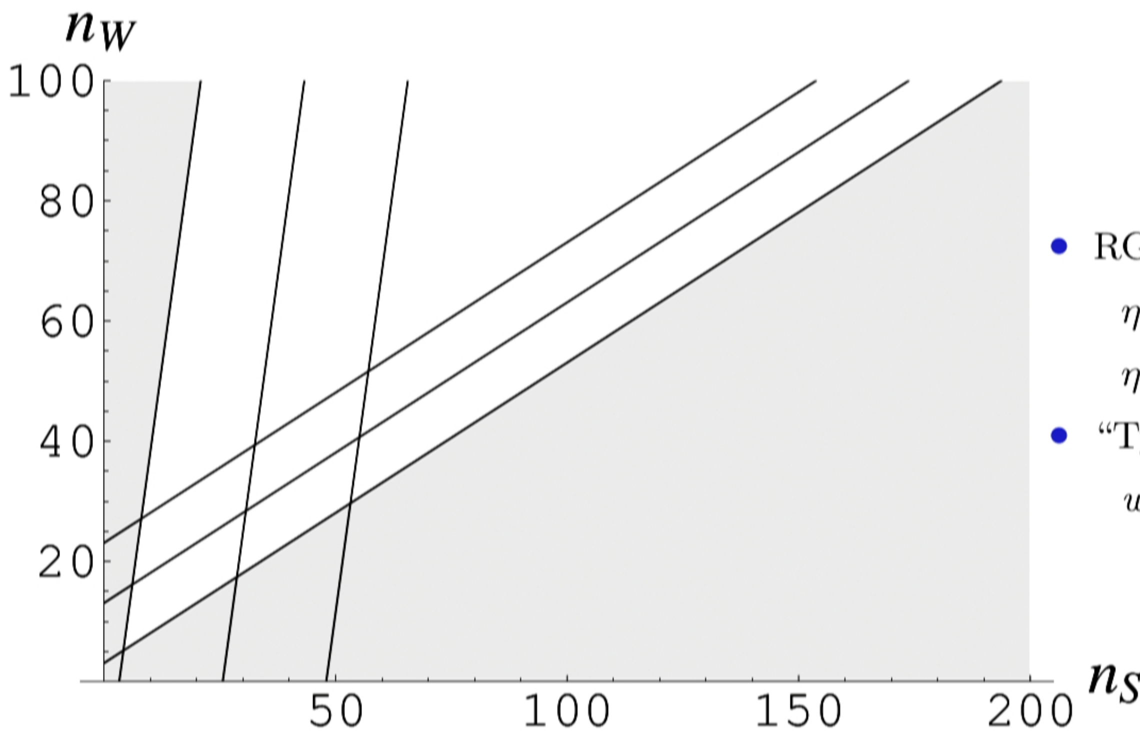
  - *gauge invariance of the* Γ

  - *meaningful distinction between high- and low-momentum quantum fluctuation*

  - *background independence achieved by keeping the background field general*

R. Percacci and D. Perini, *"Constraints on matter from asymptotic safety"* Phys. Rev. D **67**, 081503 (2003) [hep-th/0207033].

R. Percacci and D. Perini, *"Asymptotic safety of gravity coupled to matter"* Phys. Rev. D **68**, 044018 (2003) [hep-th/0304222].



- RG flow closure

  $\eta_h \equiv \eta_G$

  $\eta_c = \eta_{\text{matter}} = 0$

- "Type I" regulator

  wrong for fermions[2]

[2] P. D. and R. Percacci, "Functional renormalization with fermions and tetrads," Phys. Rev. D **87**, no. 4, 045002 (2013)

$$\eta_h = -\partial_t \ln Z_h, \qquad \eta_\psi = -\partial_t \ln Z_\psi,$$
$$\eta_c = -\partial_t \ln Z_c, \qquad \eta_\phi = -\partial_t \ln Z_\phi,$$
$$\eta_F = -\partial_t \ln Z_F,$$

- computing the running of the wave-function renormalization
  - *expansion of the exact flow equation up to second order*
  - *graviton and ghost contribution to their own wave function renormalization*[3]
  - *matter fluctuation contributes to graviton wave function renormalization*
  - *graviton fluctuation contributes to matter wave function renormalization*

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[3] A. Codello, G. D'Odorico and C. Pagani, *"Consistent closure of RG flow equations in quantum gravity"* arXiv:1304.4777 [gr-qc].

Computing β functions

$$\beta_G(G,\ \Lambda,\ \eta_h,\ \eta_c,\ \eta_\phi,\ \eta_\psi,\ \eta_F,\ N_s,\ N_f,\ N_v)$$
$$\beta_\Lambda(G,\ \Lambda,\ \eta_h,\ \eta_c,\ \eta_\phi,\ \eta_\psi,\ \eta_F,\ N_s,\ N_f,\ N_v)$$

- Solve a linear system in the ηs
- Look for fixed points in G, Λ

Compatibility with matter: (N_s, N_f, N_v)

- Positive Newton coupling at the fixed point
- Looking for fixed point with two relevant directions
- Moderate critical exponent

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One loop analysis

$$\beta_g|_{1\text{loop}} = 2g + \frac{g^2}{6\pi}(N_s + 2N_f - 4N_V - 22)$$

$$\beta_\lambda|_{1\text{loop}} = -2\lambda - 2\frac{g}{15\pi^2}(-N_s + 4N_f - 3N_v - 5) + \frac{g\lambda}{45\pi^2}(5N_s + 10N_f - 15N_v + 61)$$

One loop analysis

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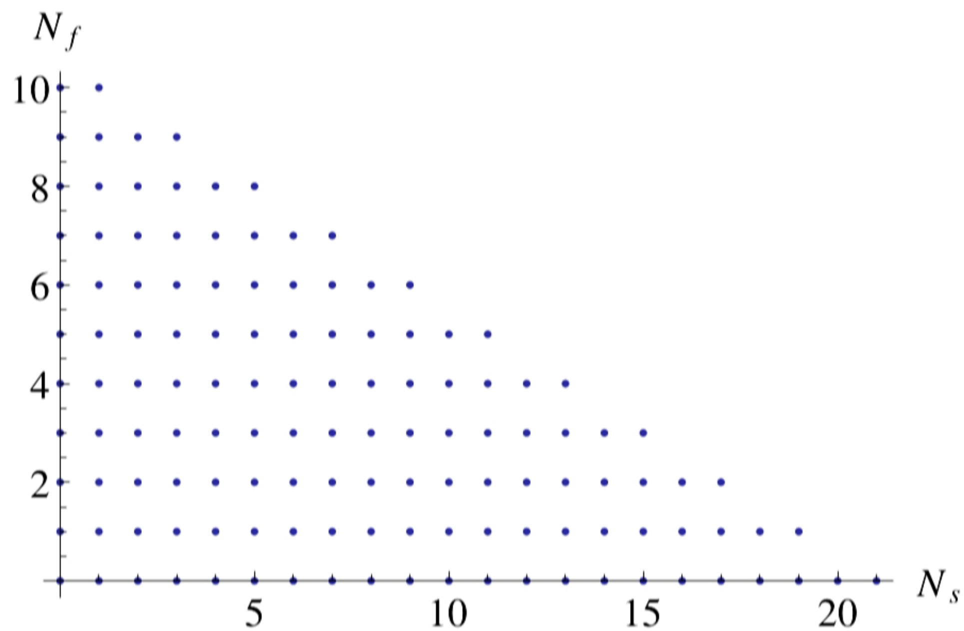


Anomalous dimension and predictivity:

$$\mathcal{O} = \Phi^n \longrightarrow d_{\bar{g_{\mathcal{O}}}} = d - n\, d_\Phi$$

passing to the dimensionless quantities

$$g_{\mathcal{O}} = \bar{g}_{\mathcal{O}} \frac{k^{-d+nd_\Phi}}{Z_\Phi^{\frac{n}{2}}} \longrightarrow \beta_{g_{\mathcal{O}}} = \left(-d + nd_\Phi + \frac{n}{2}\eta_\Phi\right) g_{\mathcal{O}} + \dots$$

$$\theta_{\mathcal{O}} = -\frac{\partial \beta_{g_{\mathcal{O}}}}{\partial g_{\mathcal{O}}}|_{g_{\mathcal{O}} = g_{\mathcal{O}*}} = -\left(-d + nd_\Phi + \frac{n}{2}\eta_\Phi\right) + \dots .$$

Requiring just a finite number of operator will be shifted to relevance

$$\eta_\Phi > -2d_\Phi + \frac{2d}{n} \xrightarrow{n\to\infty} -2d_\Phi$$

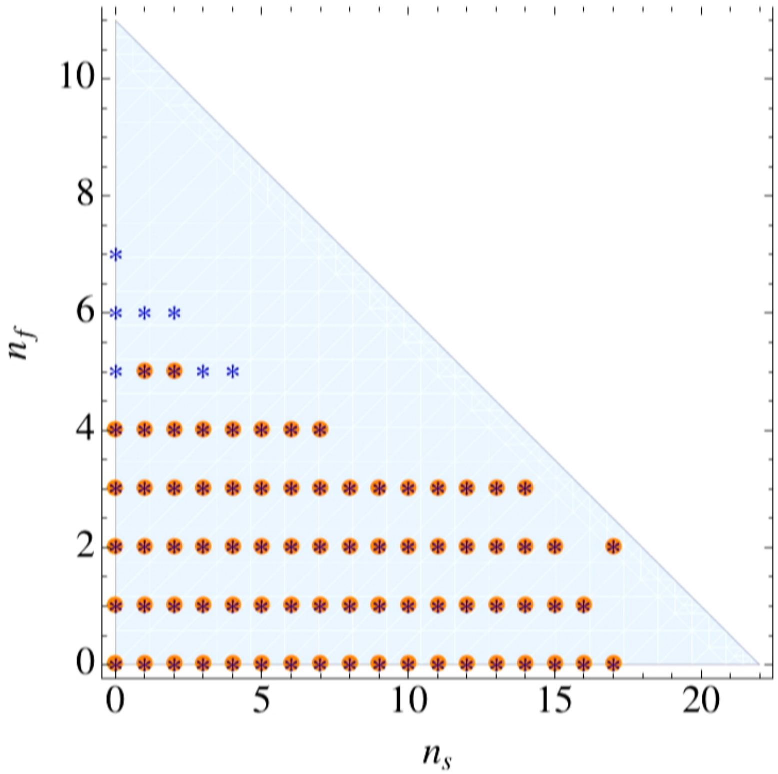
For the graviton $\eta_h > -2d + 4$ is an additional requirement on the fixed point!

Effects of matter

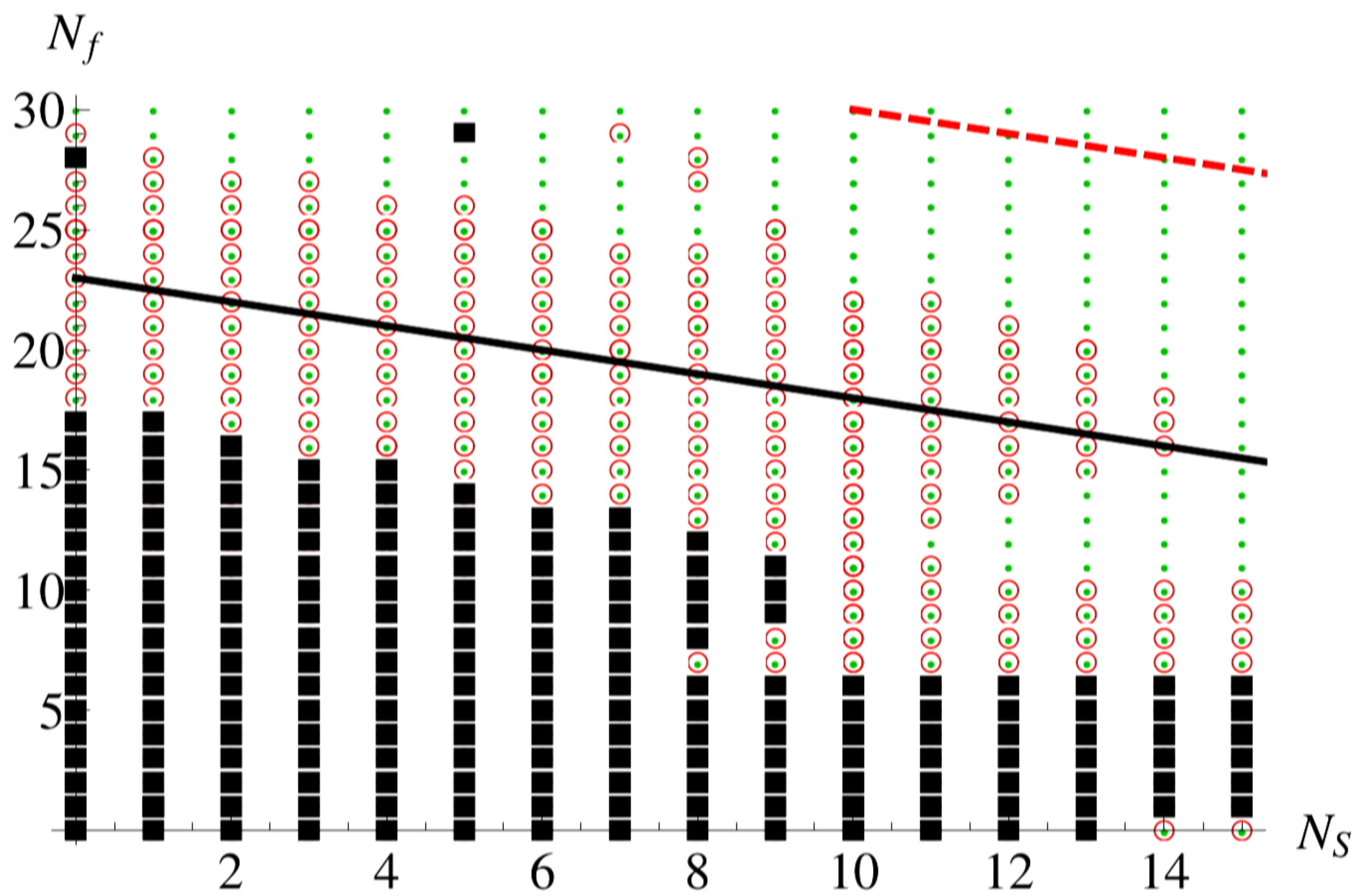
- Scalar fields $\rightarrow g_*$ to smaller values and λ_* to larger positive values
- Fermion fields $\rightarrow g_*$ to larger values and λ_* to more negative values
- Vector fields $\rightarrow g_*$ to smaller values and λ_* to larger positive values

Effects of matter

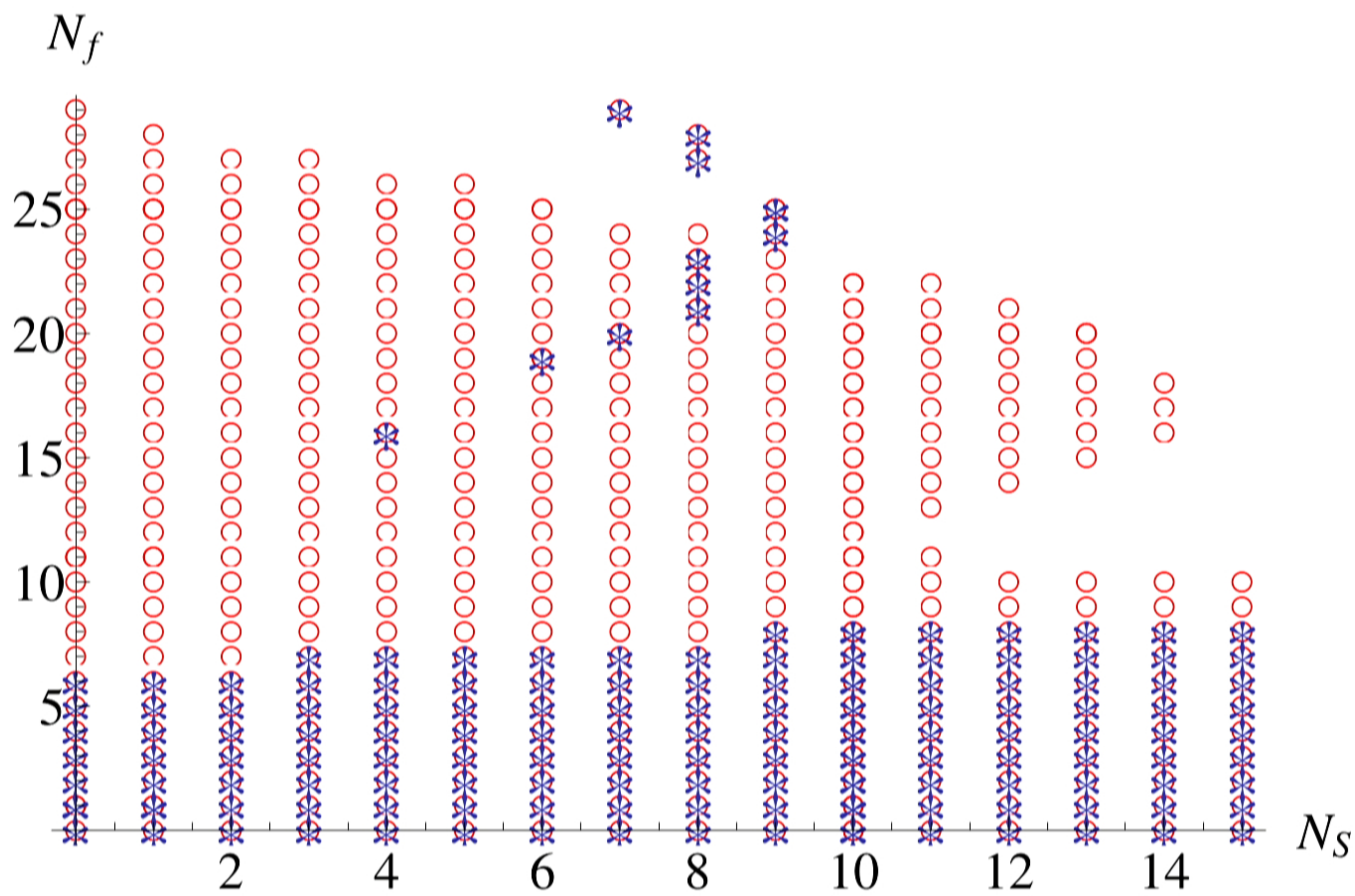
- Scalar fields $\to g_*$ to smaller values and λ_* to larger positive values
- Fermion fields $\to g_*$ to larger values and λ_* to more negative values
- Vector fields $\to g_*$ to smaller values and λ_* to larger positive values



- inclusion anomalous dimensions $\to$ more complicated shape of the boundary
- the region shrinks considerably
- higher-derivative operators can have a significant effect on η_h, refine truncation



N_f
30
25
20
15
10
5
2
4
6
8
10
12
14
N_S


N_f
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N_S

Specific matter models and the AS scenario:

model	N_s	N_f	N_V	g_*	λ_*	θ_1	θ_2	η_h
no matter	0	0	0	1.45	-0.008	.08	1.55	0.07
SM	4	45/2	12	5.34	-7.03	3.90	1.95	-34.90
MSSM	4	61/2	12	-	-	-	-	-
SM +dm scalar	5	45/2	12	6.32	-8.19	3.90	1.95	-40.87

- Big η_h in the SM $\rightarrow$ predictivity needs to be examined carefully

  extended truncation, momentum-dependent graviton-matter-interaction and selfinteractions
- MSSM not compatible

- A quantum-gravity scale will emerge dynamically

  - transition scale to the fixed-point regime

  - close to the Planck scale in previous studies of the Einstein-Hilbert truncation

  - the dimensionful Newton coupling pass from constant to a scale-free regime in which $G_N(k^2) \sim \frac{1}{k^2}$
- In QCD quantum fluctuations lead to the dynamical generation of Λ_{QCD}
- Study if matter fluctuations change that scale and in which direction

  - scalars seem to have little effect on the transition scale

  - fermions shift this scale towards larger values

  - in the SM the effect is less pronounced than in the case with fermions only

  - we found a shift of the transition scale to the fixed-point regime by a factor of approximately 10

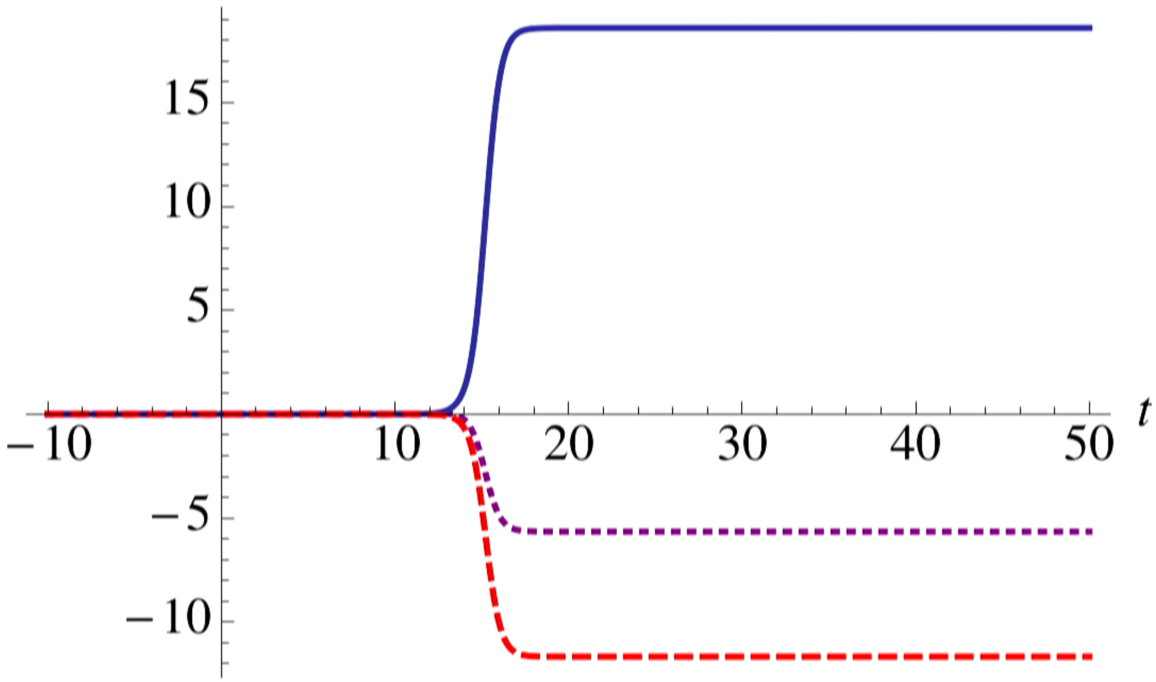
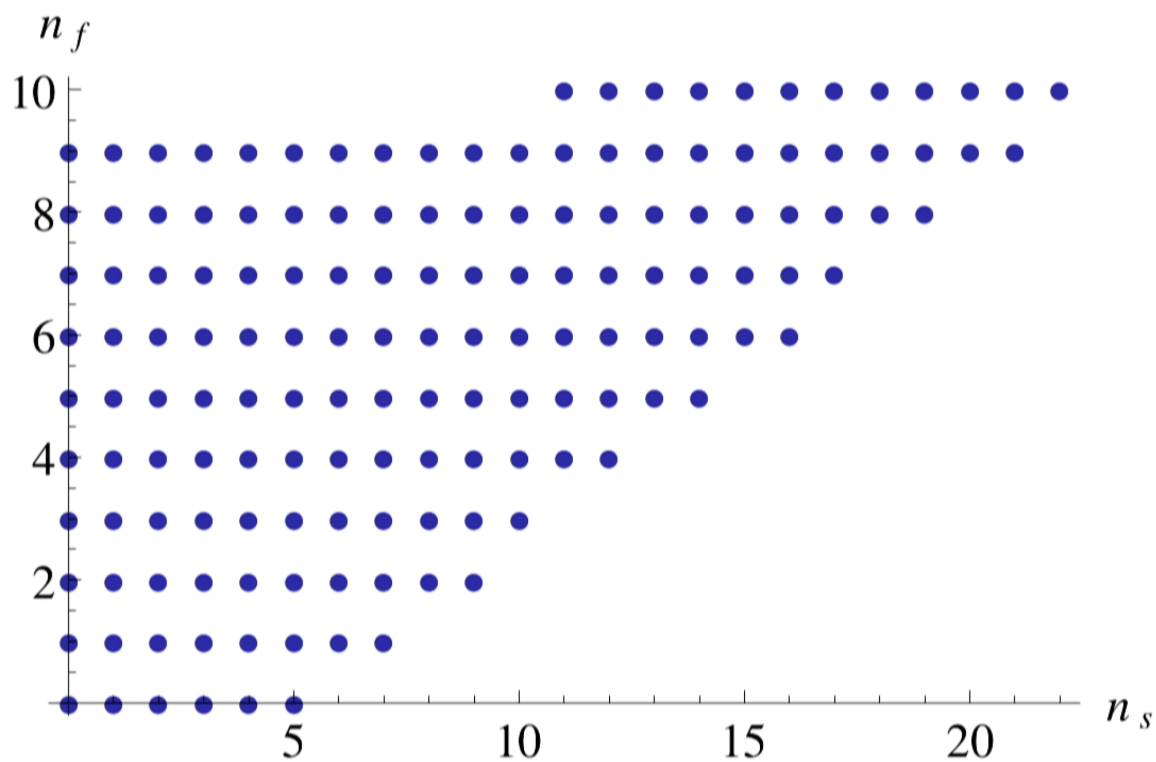


Figure: We show the dimensionless Newton coupling (blue thick line), the dimensionless cosmological constant (red dashed line) and the graviton anomalous dimension (purple dotted line) as a function of "RG-time" $t = ln(k/k_0)$ for the case $N_s = 0$, $N_f = 6$, transiting from near-GFP to near-NGFP behavior.

Higher dimensional case

- Extra dimension are not required in AS scenario of pure gravity but compatible
- For $d = 5, 6$ the Standard Model matter degrees of freedom are incompatible with a viable gravitational fixed point



Conclusions

- Compatibility of matter degrees of freedom with the asymptotic safety scenario for gravity

  - studied the effect of scalar, fermionic and abelian vector field fluctuations on the existence of an interacting fixed point

  - resolved the approximation of equating background field and fluctuations field anomalous dimensions

  - new criterion that exploits predictability and the anomalous dimension

  - upper limits on the allowed number of scalar, fermionic and vector degrees of freedom

- Focusing on particular models
  - *standard Model matter content is compatible with an appropriate fixed point*
  - *restrictions on its extensions*
- Going to larger dimensions
  - *we find that the allowed region for the matter content shrinks*
  - *no more compatibility with the SM*
- Effect of matter degrees of freedom on the quantum gravity scale
  - *fermionic matter moves the scale to higher regimes*

Outlook - Is it possible to define a RG in LQG?

- Maybe using FRGE on GFT [Thomas Krajewski, Daniele Oriti, Dario Beenedetti]
- A la Wilson by defining a coarse grain [Bianca Dittrich and coll.]
- In the twistor space [Etera R. Livine]
- Geometrically [P.D. and Simone Speziale]

Still no definitive result but a lot of work to do!