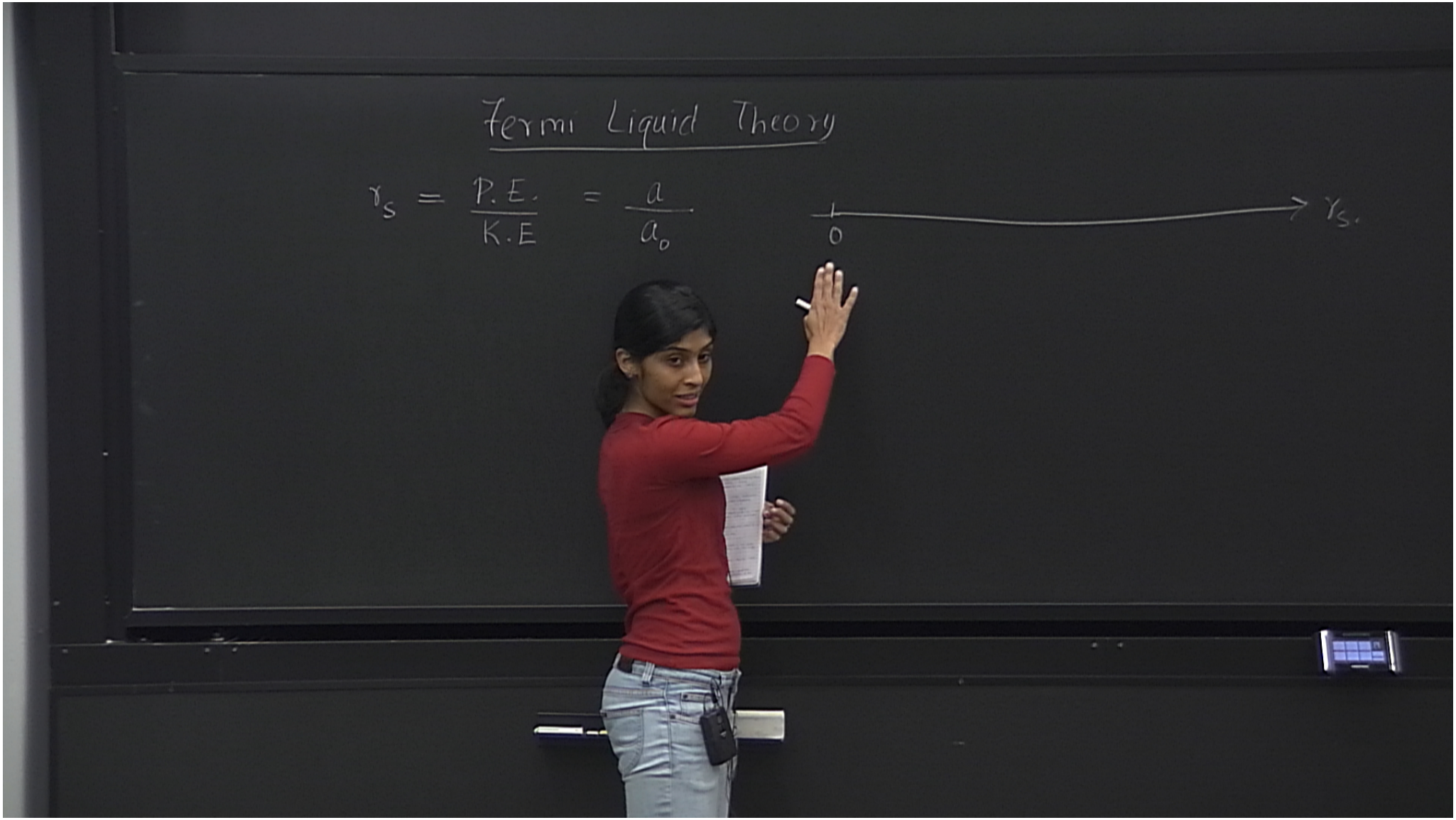
Title: 13/14 PSI - Condensed Matter - Lecture 12

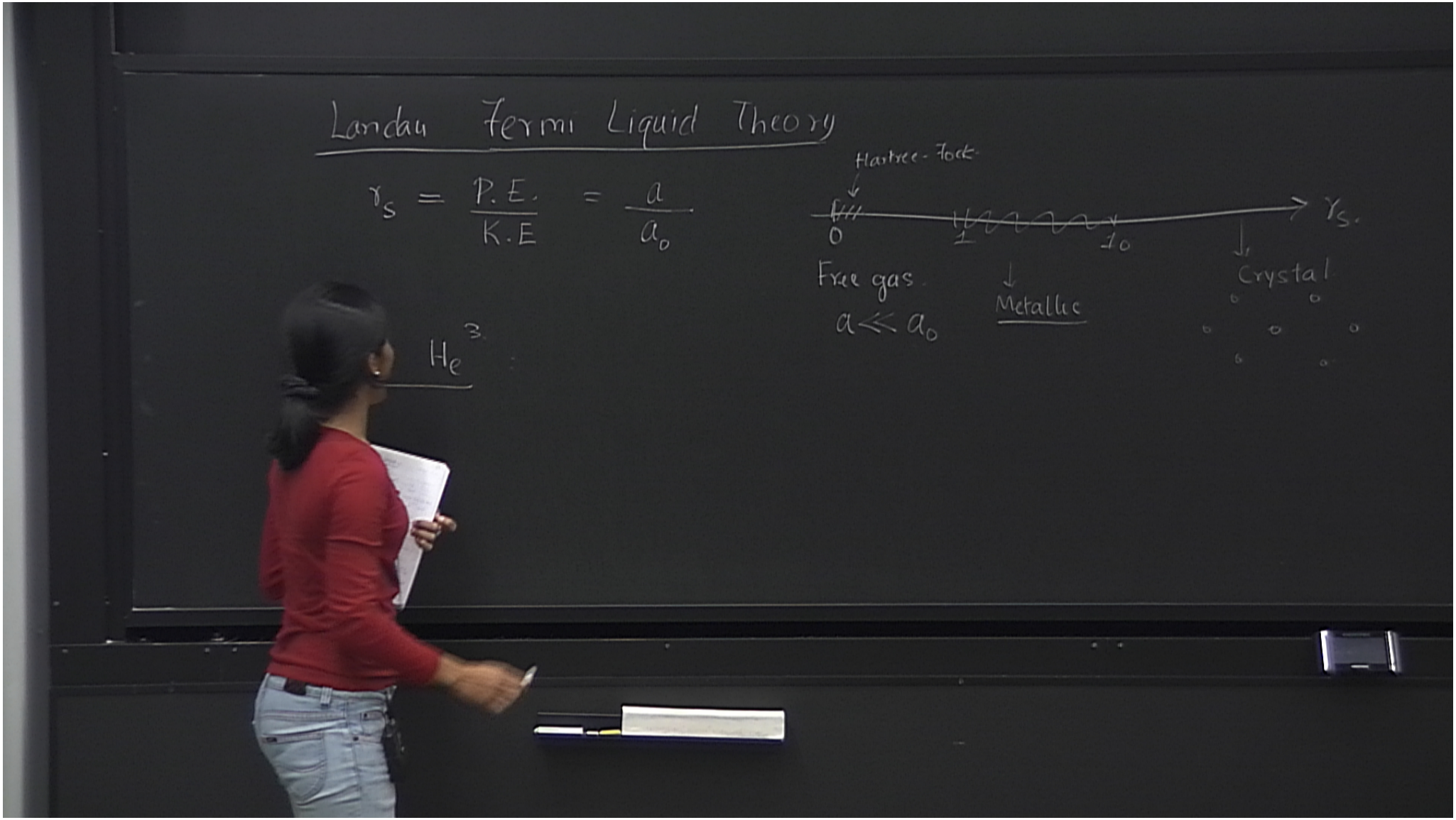
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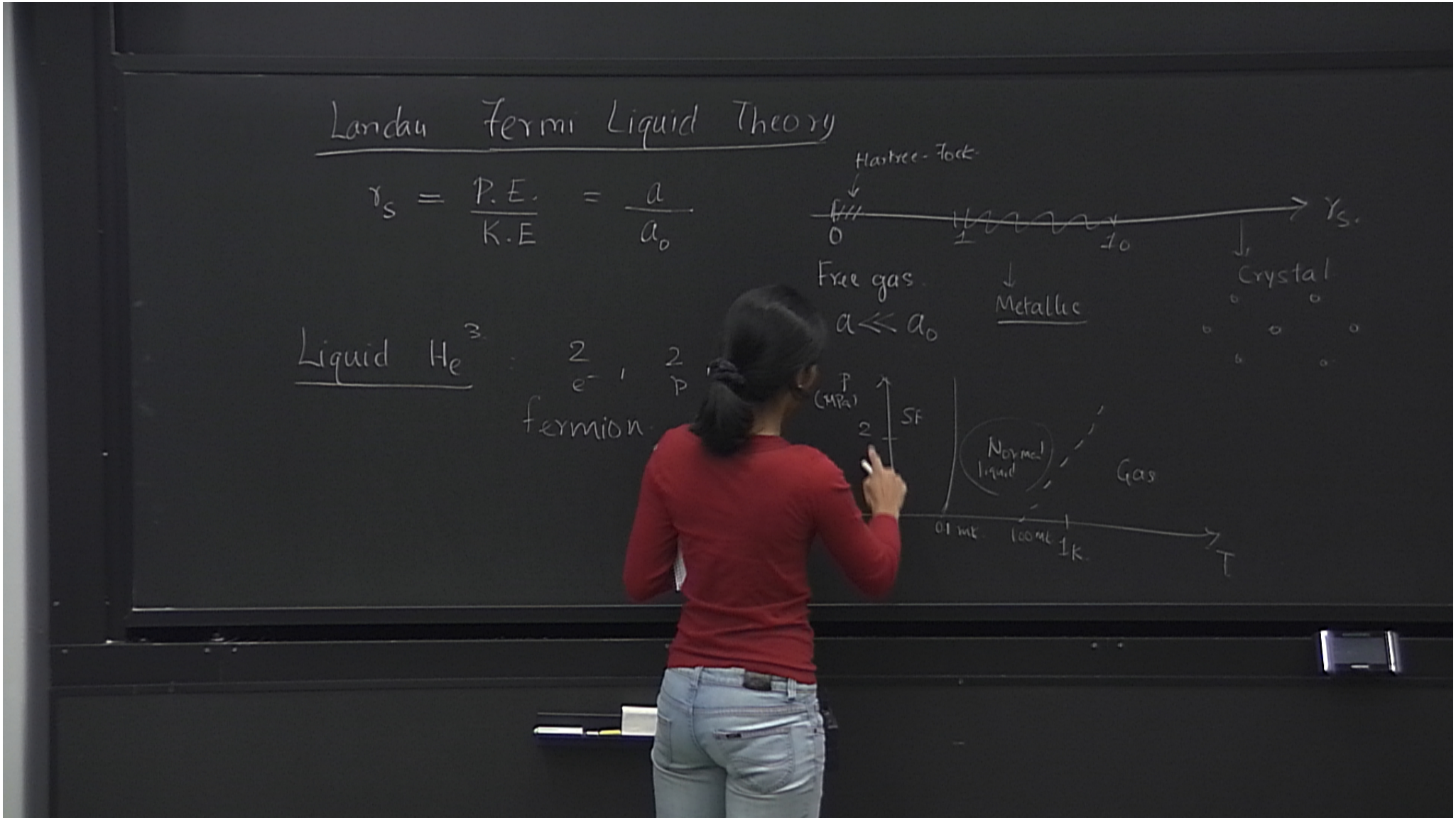
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Abstract:


Fermi Liquid Theory

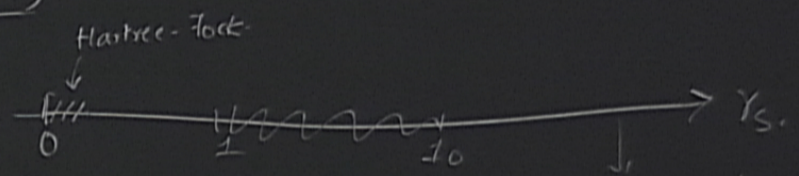

Fermi Liquid Theory
$r_s = \frac{P.E.}{K.E} = \frac{a}{a_0}$
0
r_s


Landau Fermi Liquid Theory
$r_s = \frac{P.E.}{K.E} = \frac{a}{a_0}$
Hartree-Fock
0
1
r_s
Free gas
$a \ll a_0$
Metallic
Crystal
He^3


Landau Fermi Liquid Theory
$r_s = \frac{P.E.}{K.E} = \frac{a}{a_0}$
Hartree-Fock
0
1
r_s
Free gas
$a \ll a_0$
Metallic
Crystal
Liquid He^3
2 e^-, 2 P
fermion
P (MPa)
2
SF
Normal liquid
Gas
0.1 mK
100 mK
1 K
T

Landau Fermi Liquid Theory

$$r_s = \frac{P.E.}{K.E} = \frac{a}{a_0}$$

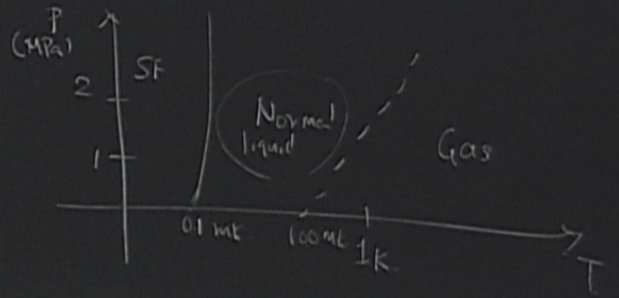


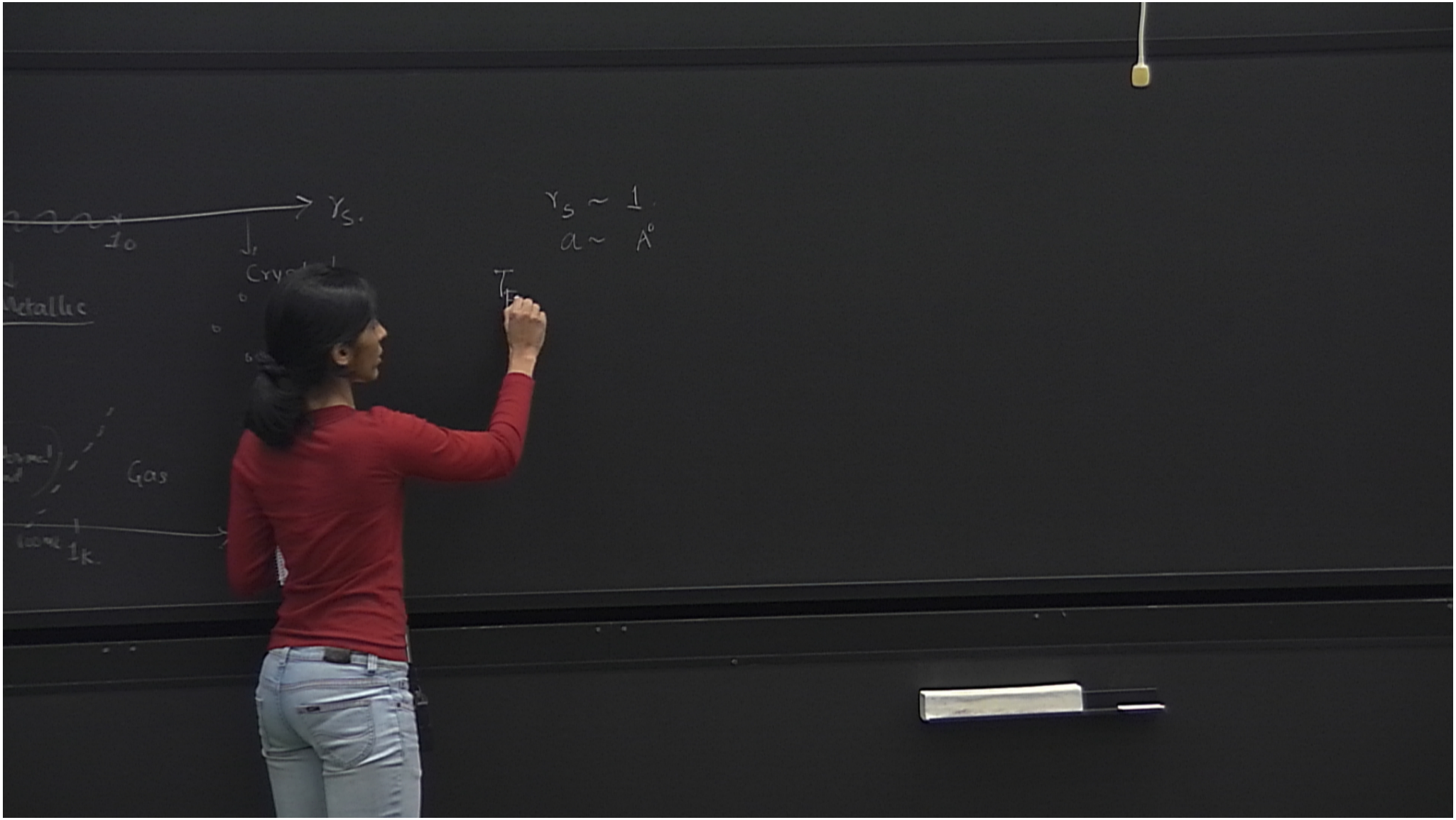
Free gas $a \ll a_0$ — Metallic — Crystal

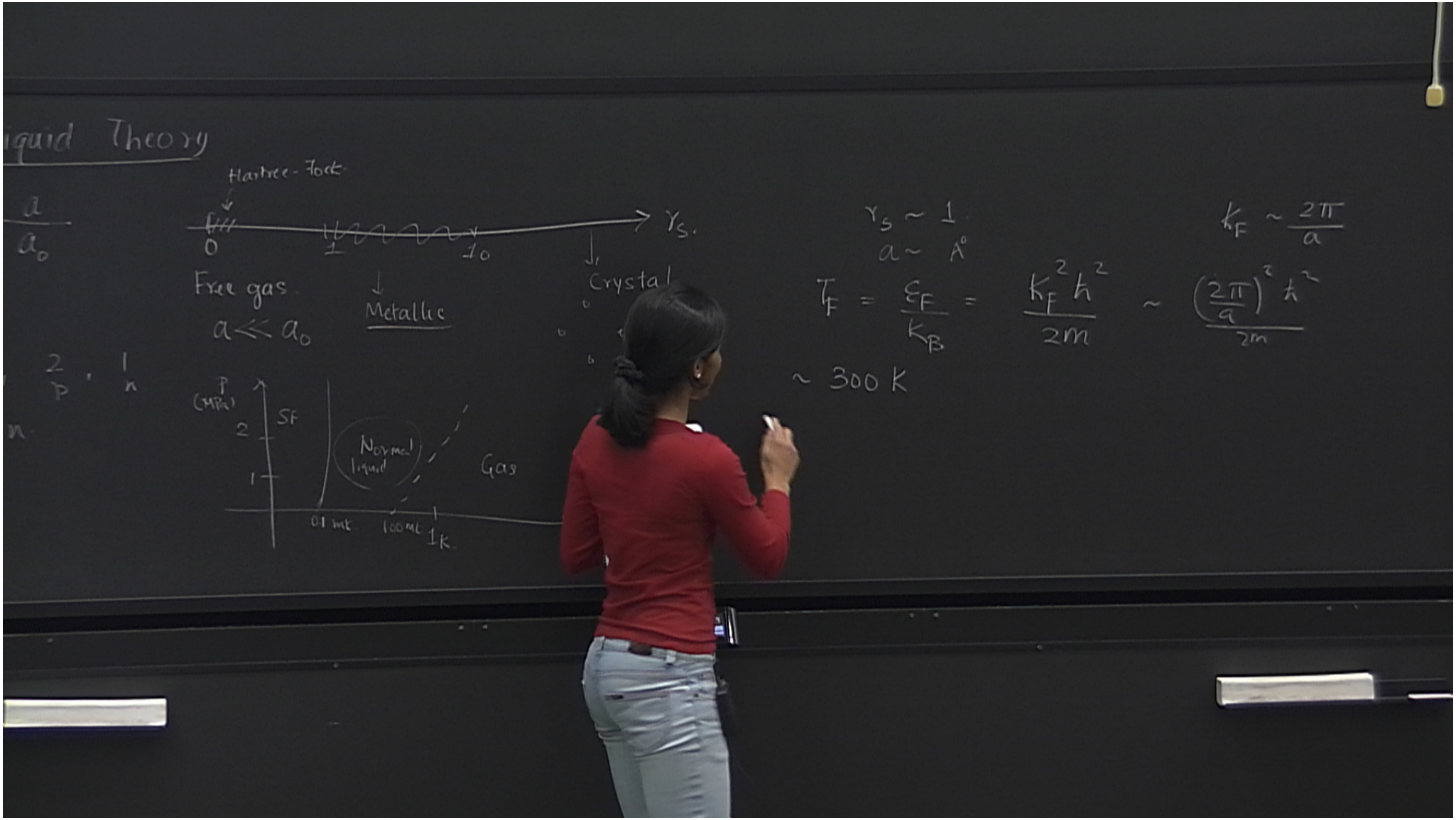
Liquid He^3

$2\ e^-,\ 2\ p,\ 1\ n$

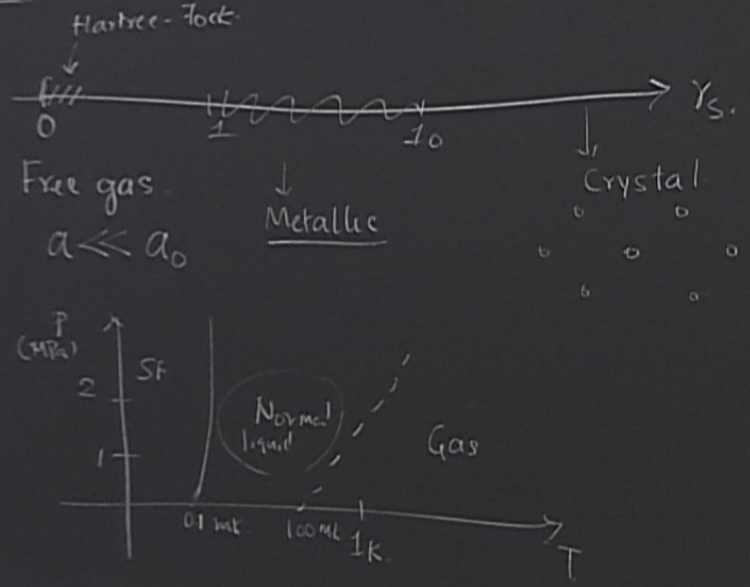
fermion




r_s.
$r_s \sim 1$.
$a \sim$ Å
Gas
1K.


iquid Theory
Hartree-Fock
r_s
0
1
10
Free gas
$a \ll a_0$
Metallic
Crystal
$r_s \sim 1$
$a \sim$ Å
$k_F \sim \frac{2\pi}{a}$
$T_F = \frac{\varepsilon_F}{k_B} = \frac{k_F^2 \hbar^2}{2m} \sim \frac{\left(\frac{2\pi}{a}\right)^2 \hbar^2}{2m}$
~ 300 K
P (MPa)
2
1
SF
Normal liquid
Gas
0.1 mK
100 mK
1 K

d Theory



$r_s \sim 1$

$a \sim \text{Å}$

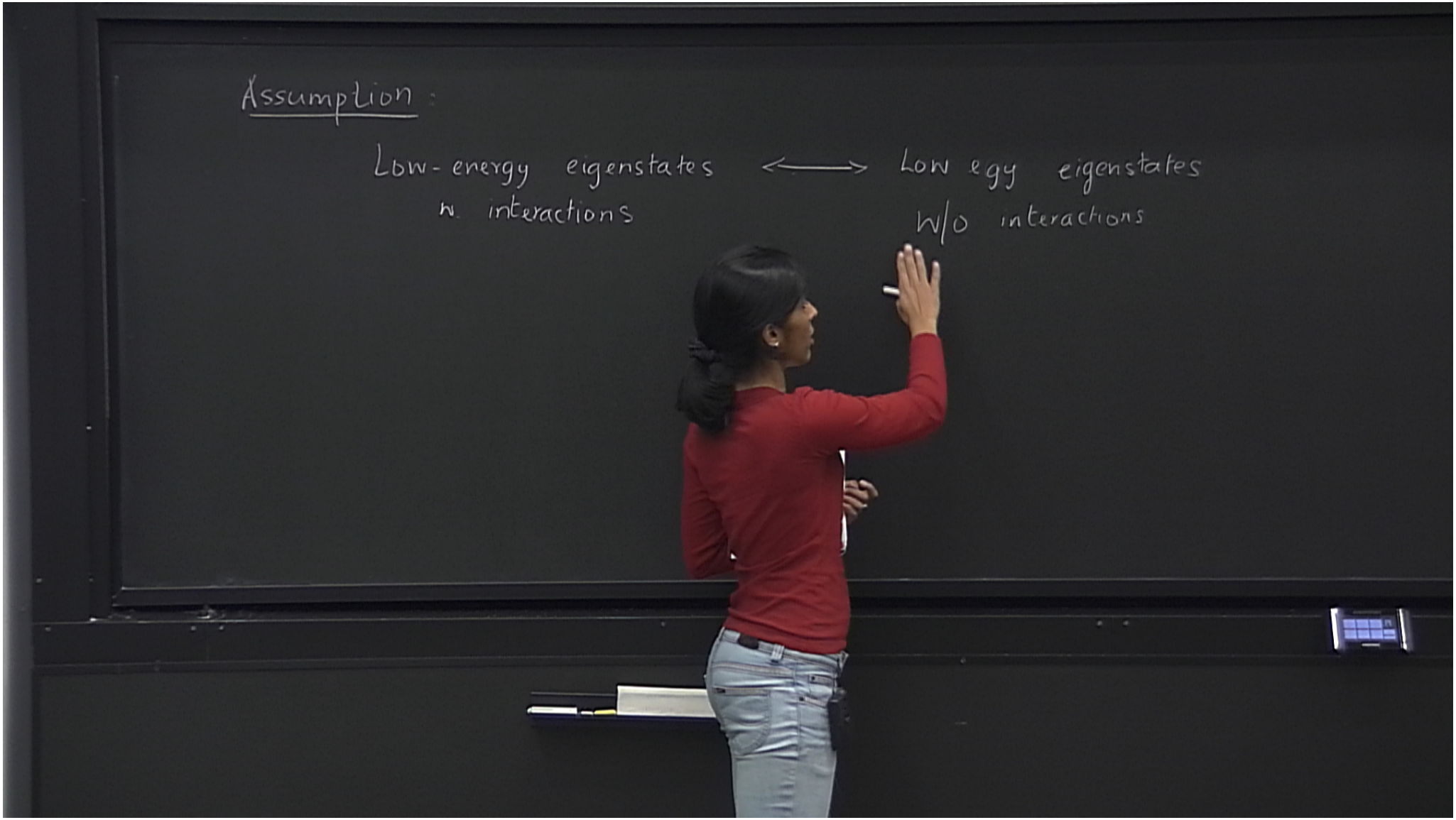
$k_F \sim \frac{2\pi}{a}$

$T_F = \frac{\varepsilon_F}{k_B} = \frac{k_F^2 \hbar^2}{2m} \sim \frac{\left(\frac{2\pi}{a}\right)^2 \hbar^2}{2m}$

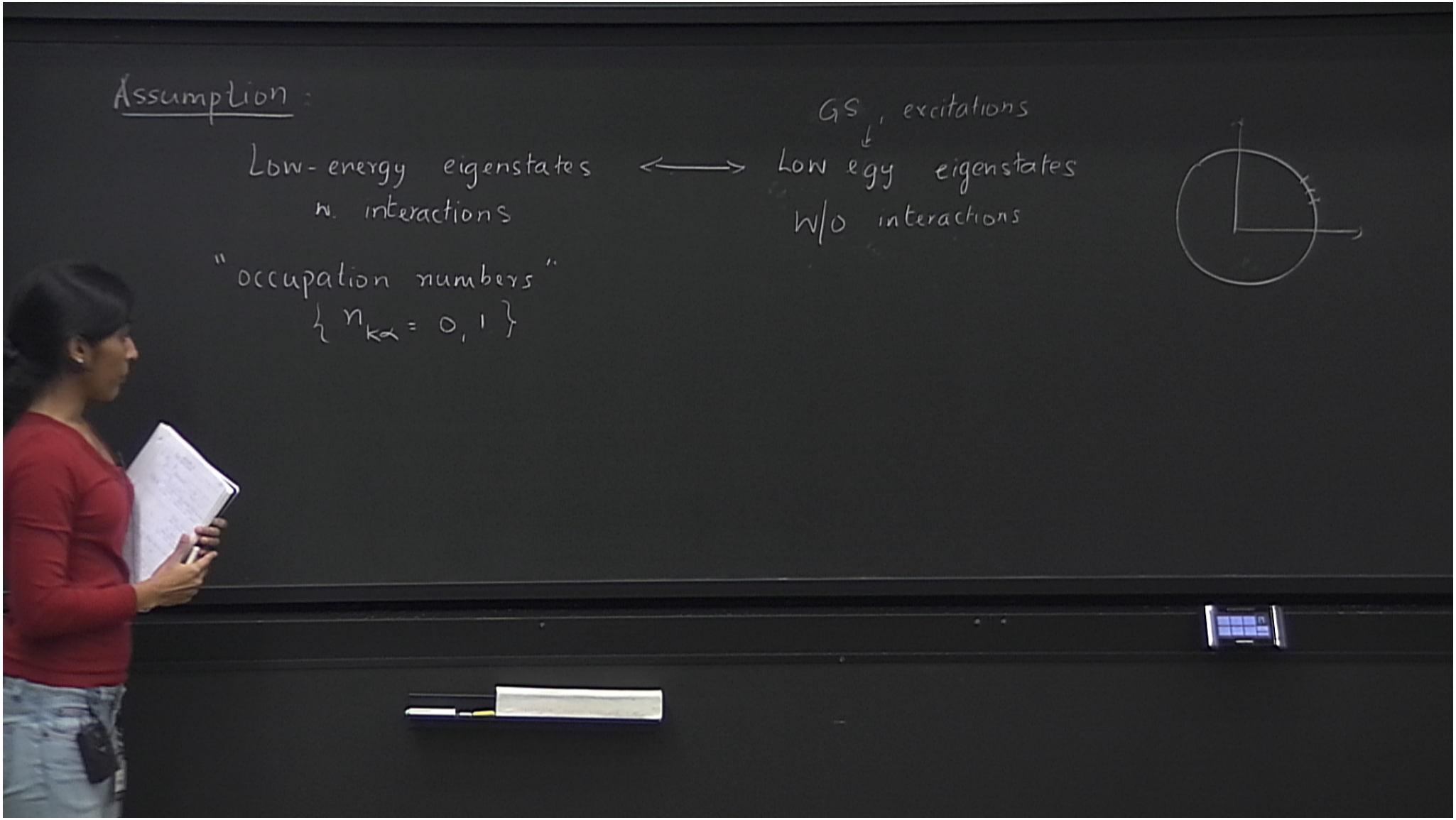
$\sim 300\ K$

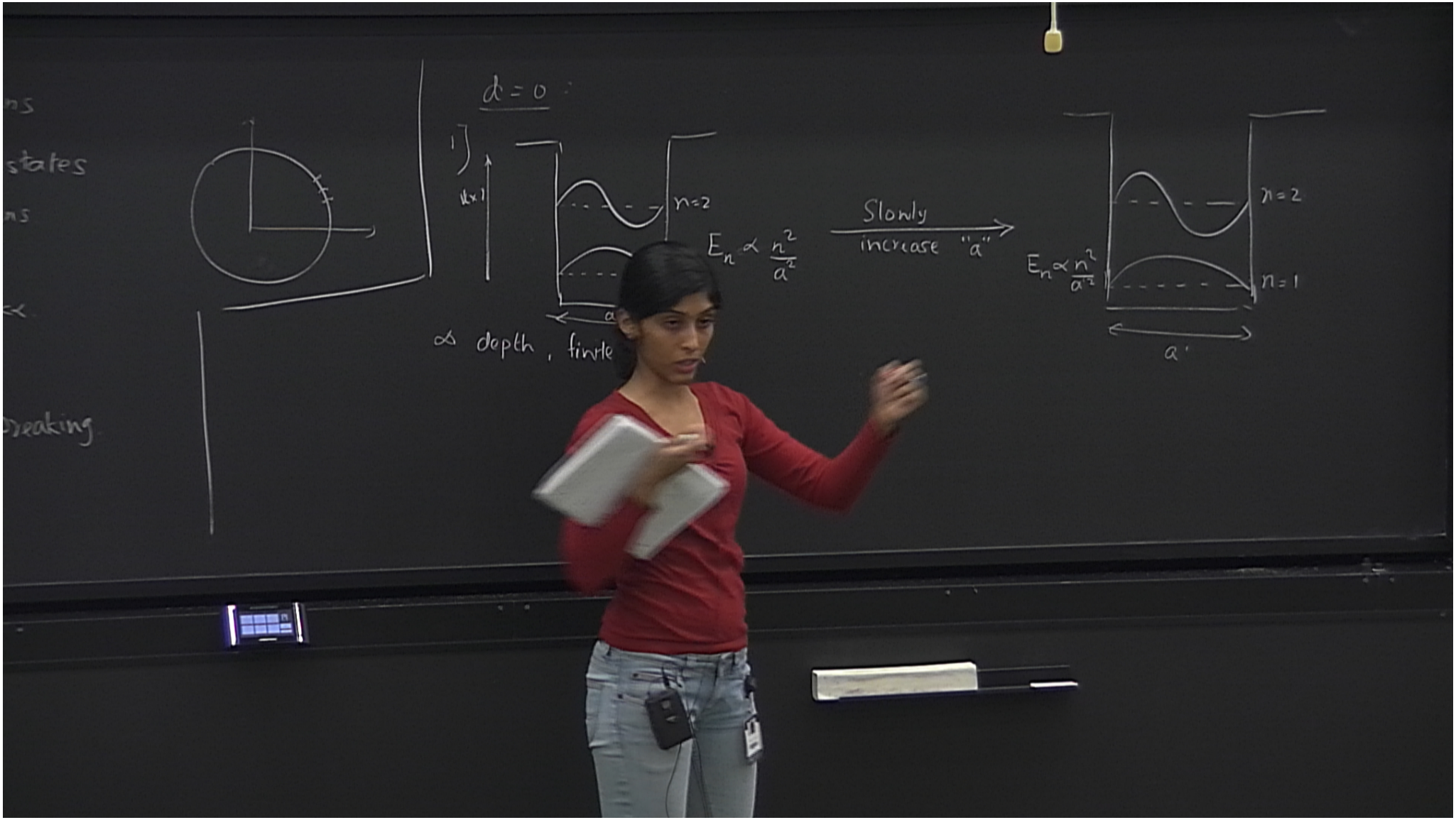
$\times\ T_F \sim 3\ K$

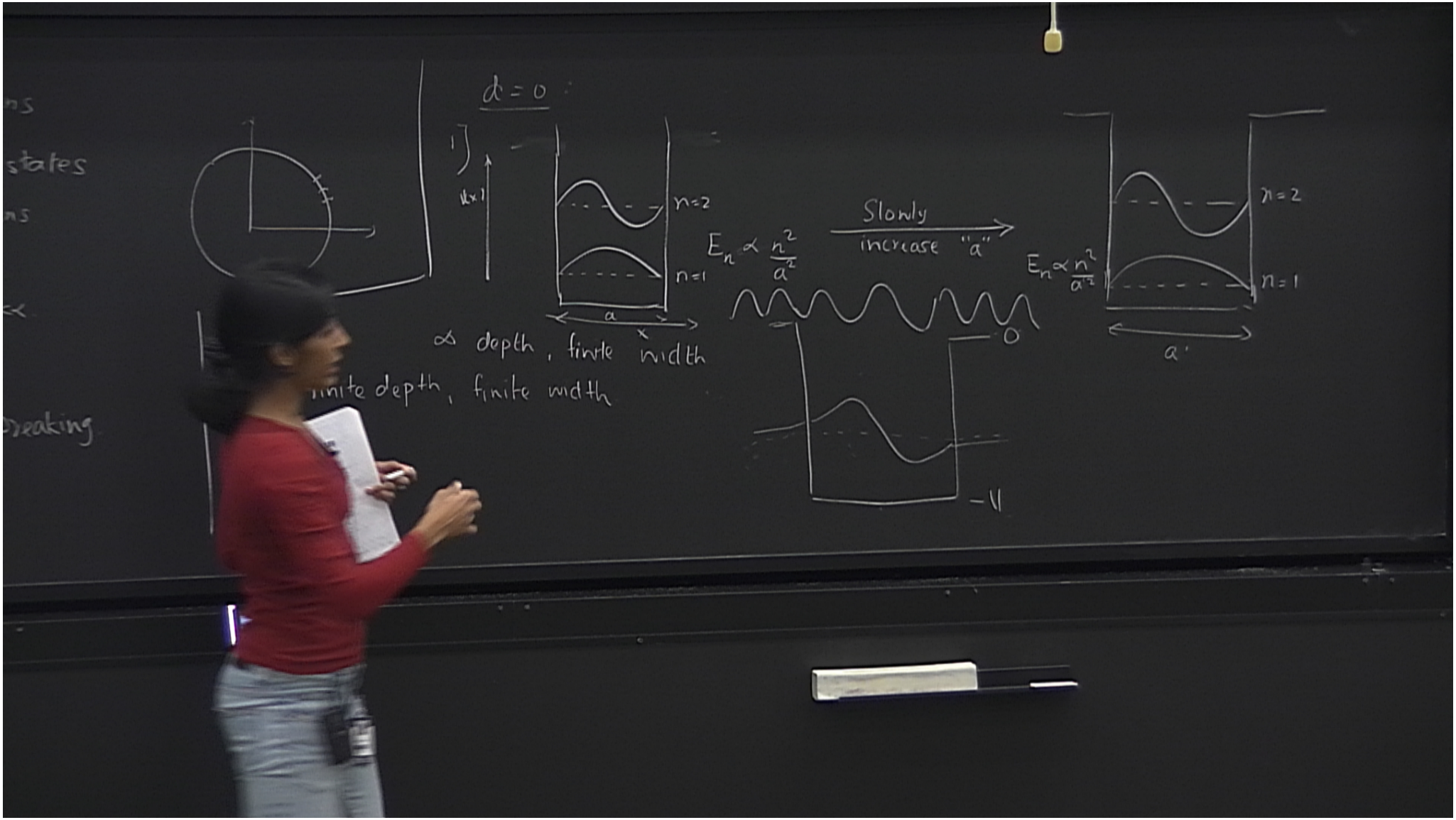
$a \sim 1\ nm$

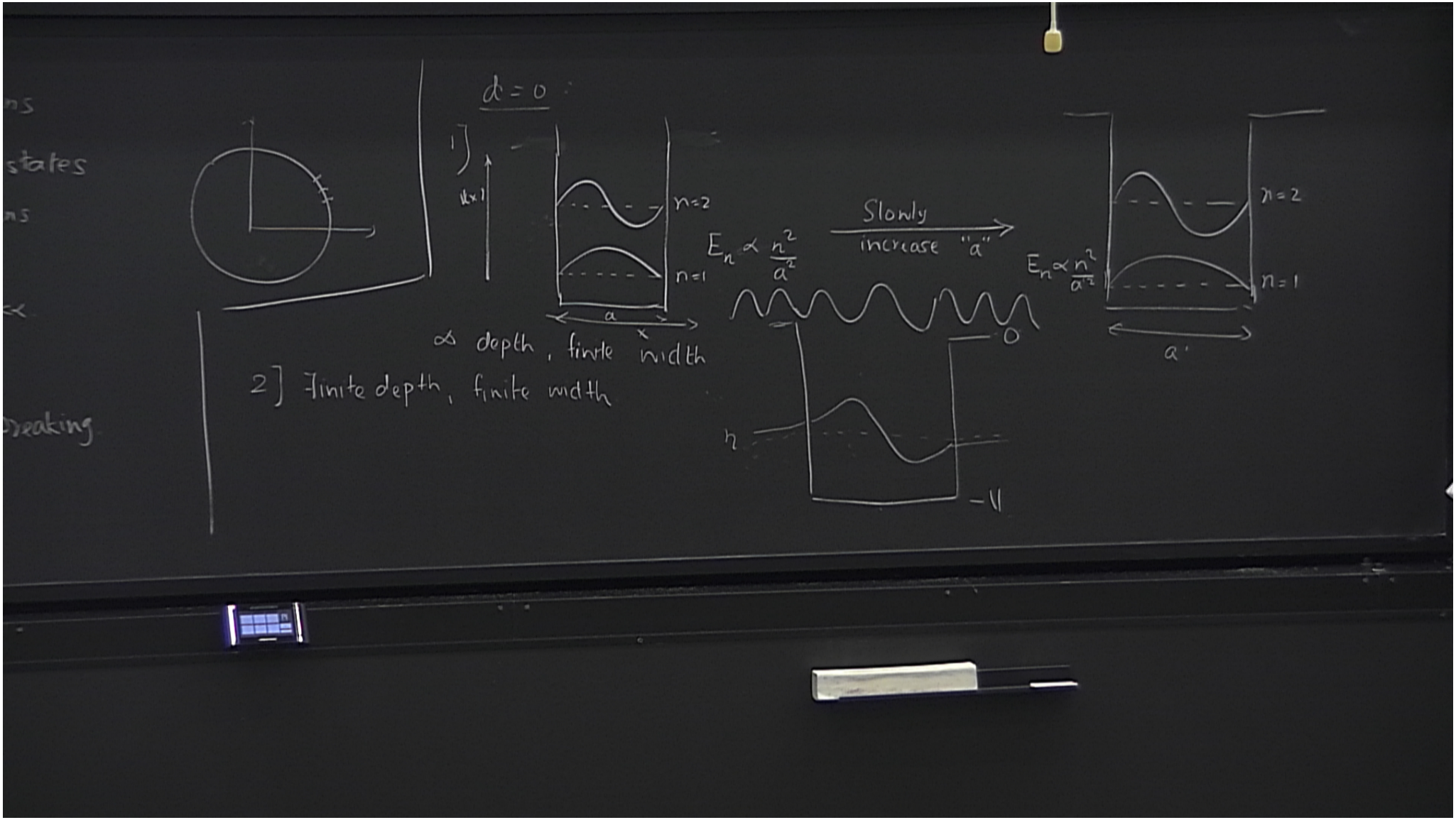

Assumption :
Low-energy eigenstates ⟷ Low egy eigenstates
w. interactions
W/O interactions


tion:
Low-energy eigenstates
w. interactions
GS, excitations
Low egy eigenstates
W/O interactions

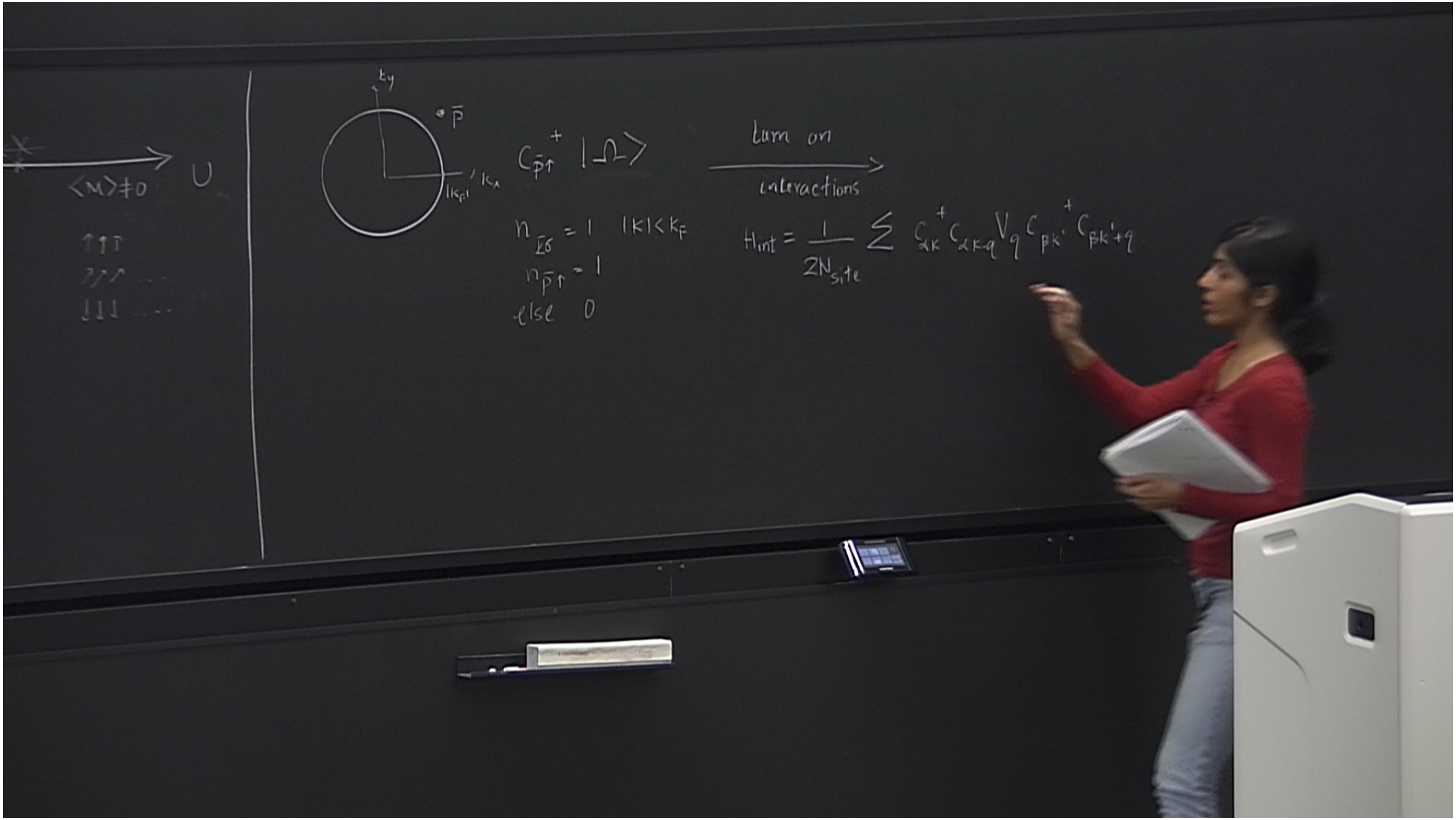

Assumption:
Low-energy eigenstates
w. interactions
<->
GS, excitations
Low egy eigenstates
W/O interactions
"occupation numbers"
$\{n_{k\alpha} = 0, 1\}$

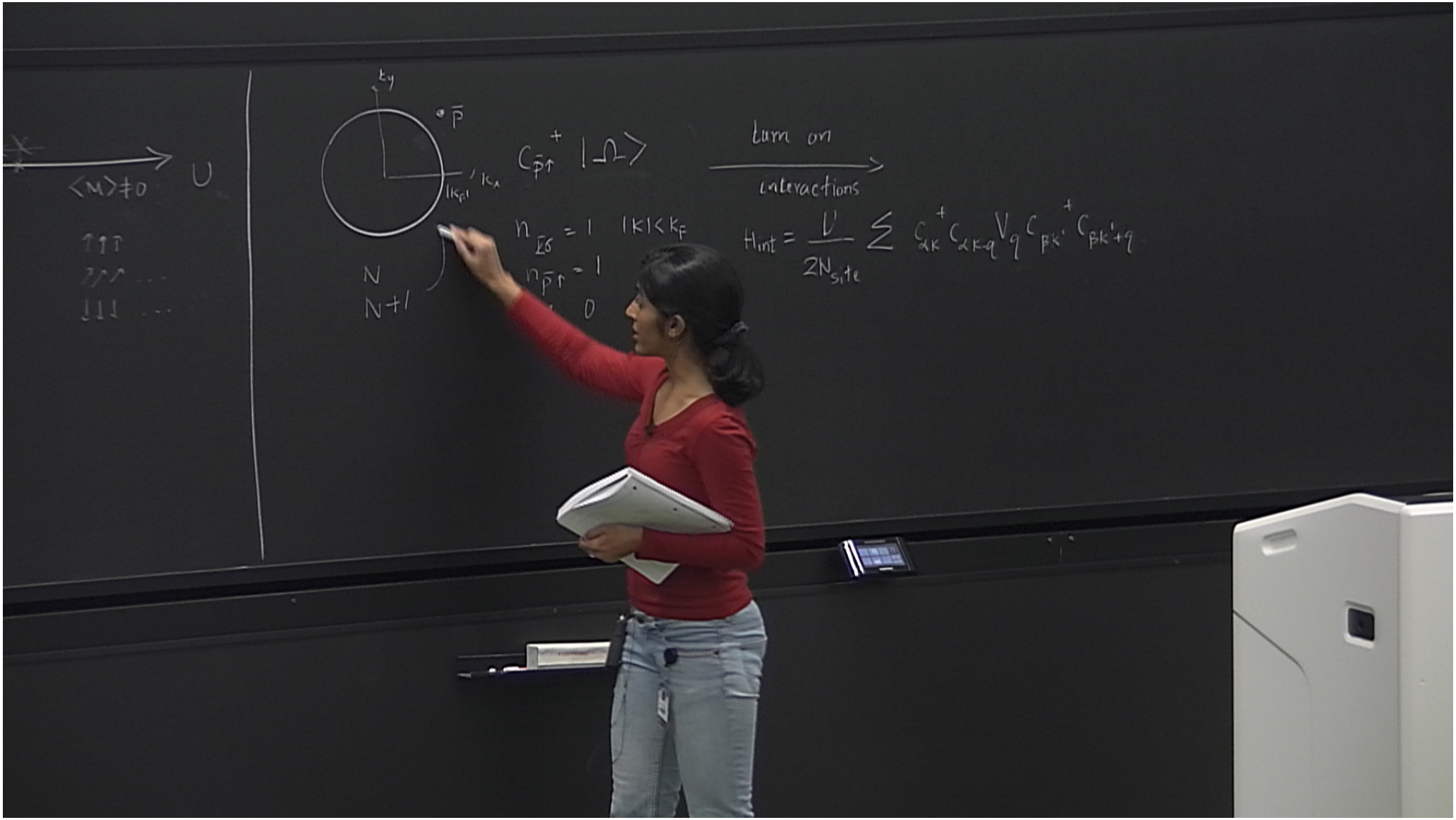

a = 0
n=2
$E_n \propto \frac{n^2}{a^2}$
Slowly
increase "a"
$E_n \propto \frac{n^2}{a^2}$
n=2
n=1
a'
∝ depth, finite

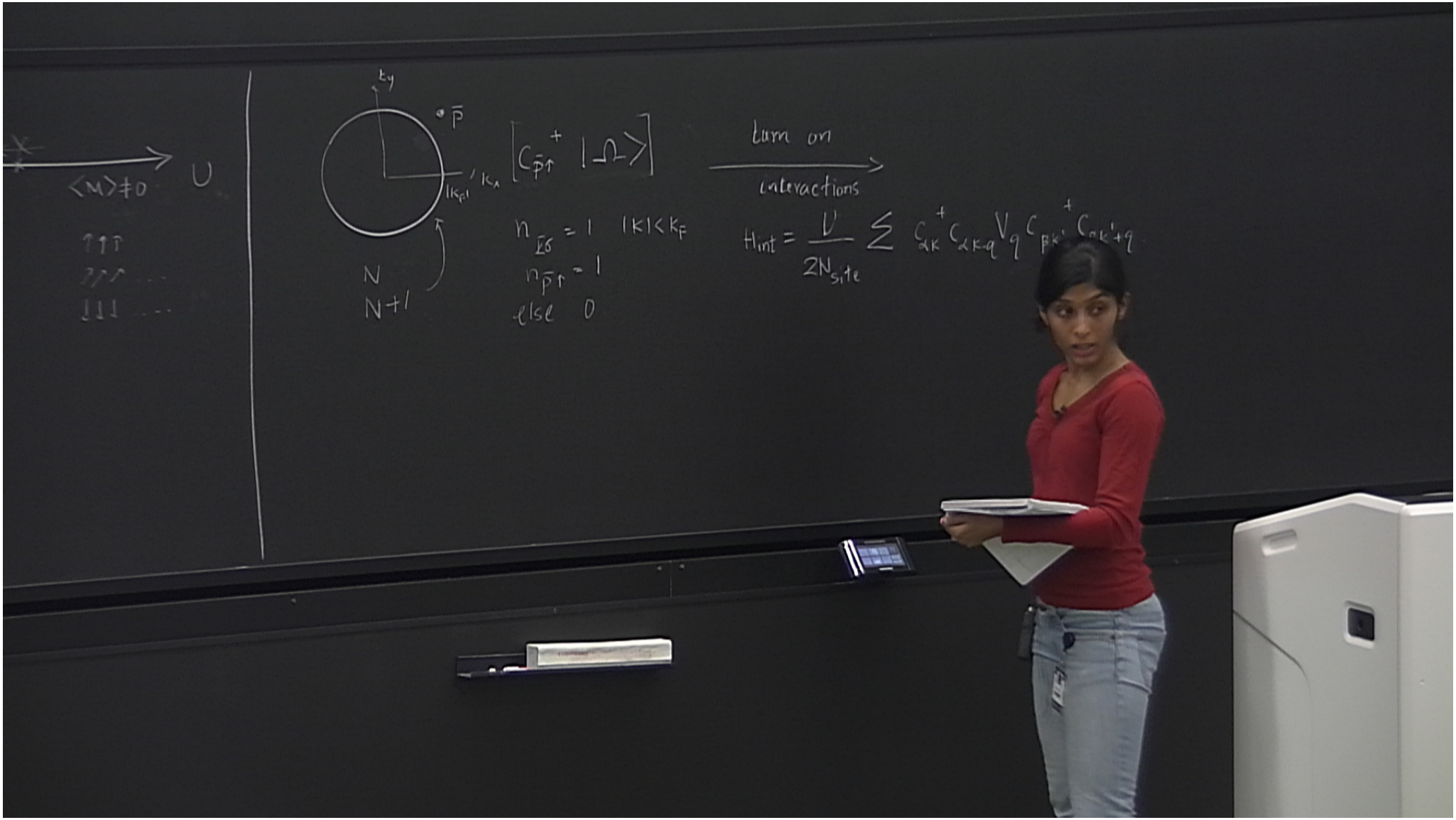

d = 0
n=2
n=1
a
x
Slowly
increase "a"
$E_n \propto \frac{n^2}{a^2}$
$E_n \propto \frac{n^2}{a^2}$
n=2
n=1
a'
0
-V
depth, finite width
finite depth, finite width

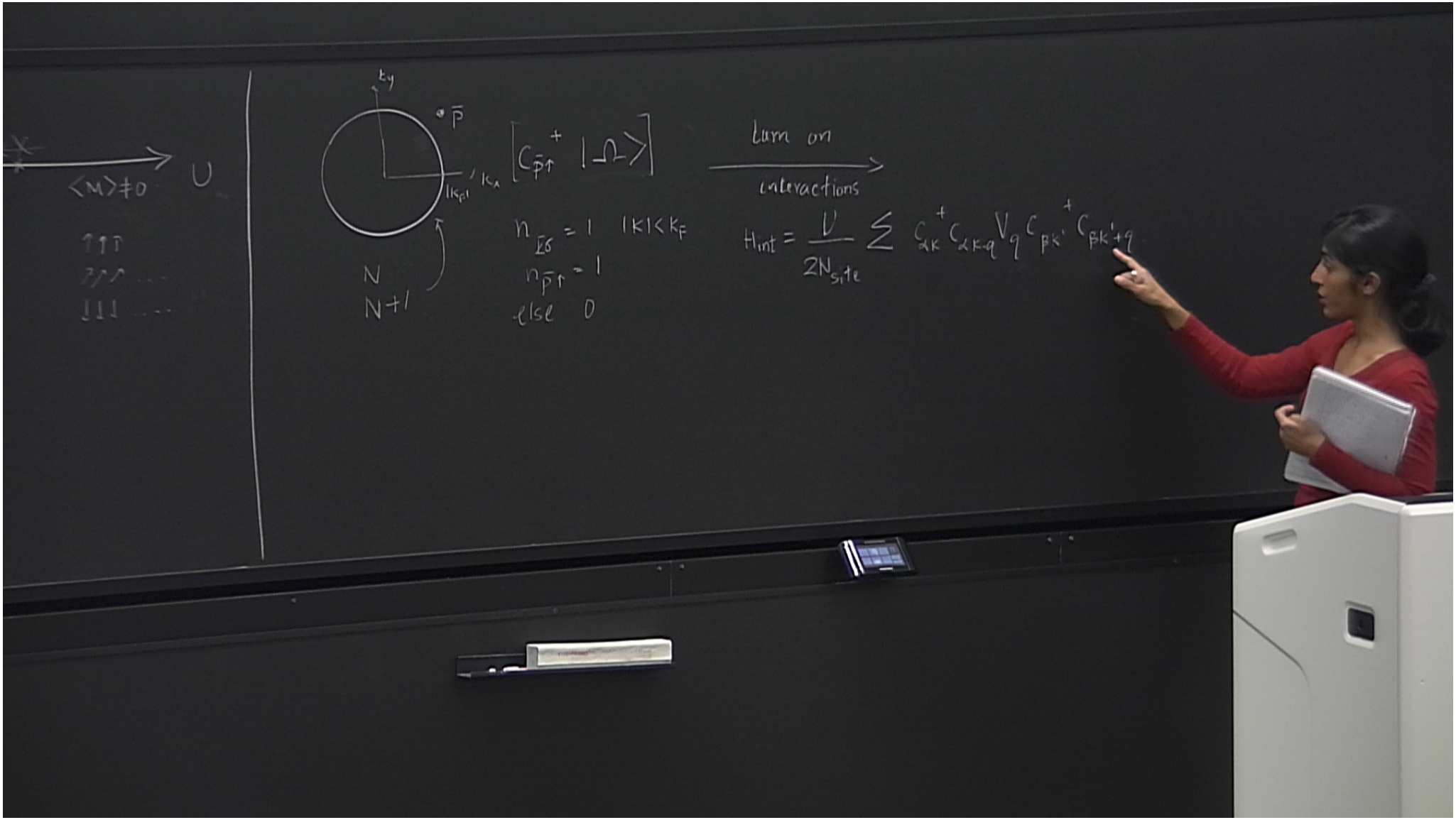

d = 0
1]
n=2
n=1
a
x
∞ depth, finite width
2] finite depth, finite width
$E_n \propto \frac{n^2}{a^2}$
Slowly
increase "a"
$E_n \propto \frac{n^2}{a^2}$
n=2
n=1
a'
0
-V

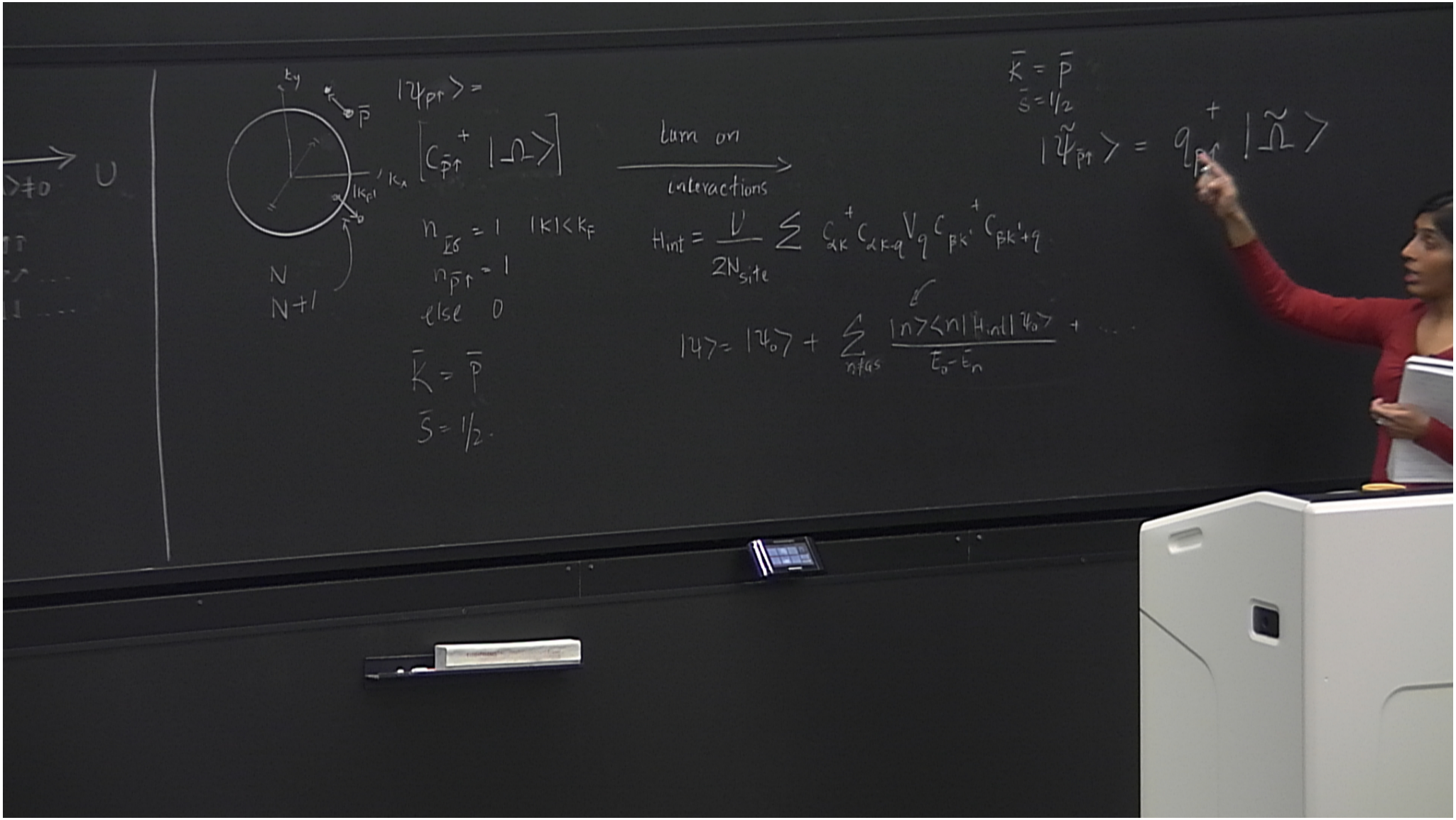

⟨M⟩ = 0
⟨M⟩ ≠ 0
U
p̄
$c^{+}_{\bar{p}\uparrow}|\Omega\rangle$

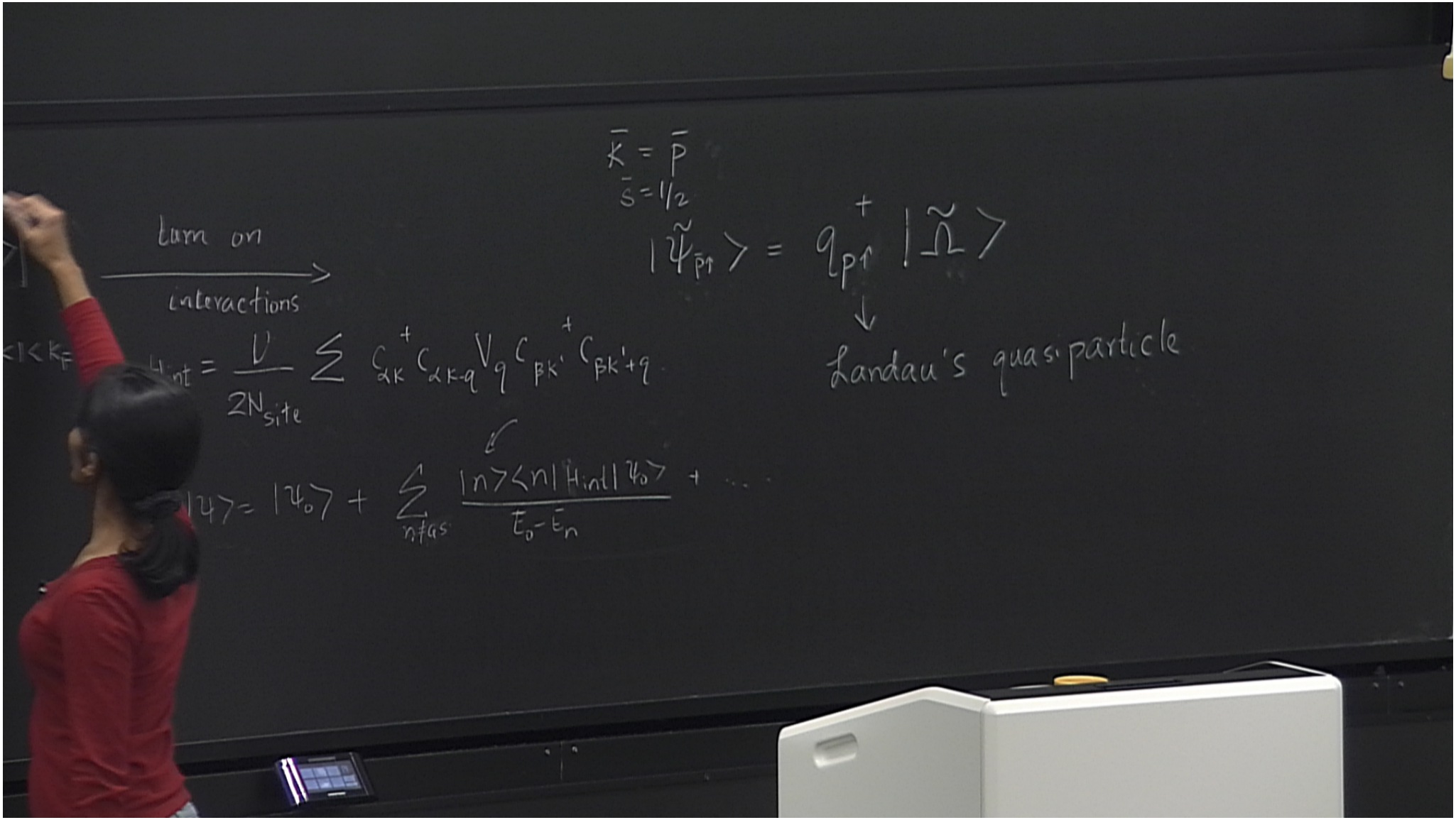

$\langle M \rangle \neq 0$
U
k_y
$\bar{p}$
k_x
$|k_F|$
$c^\dagger_{\bar{p}\uparrow} |\Omega\rangle$
turn on
interactions
$n_{\bar{p}\uparrow} = 1$
else 0
$H_{int} = \frac{1}{2N_{site}} \sum c^\dagger_{\alpha k} c_{\alpha k-q} V_q c^\dagger_{\beta k'} c_{\beta k'+q}$

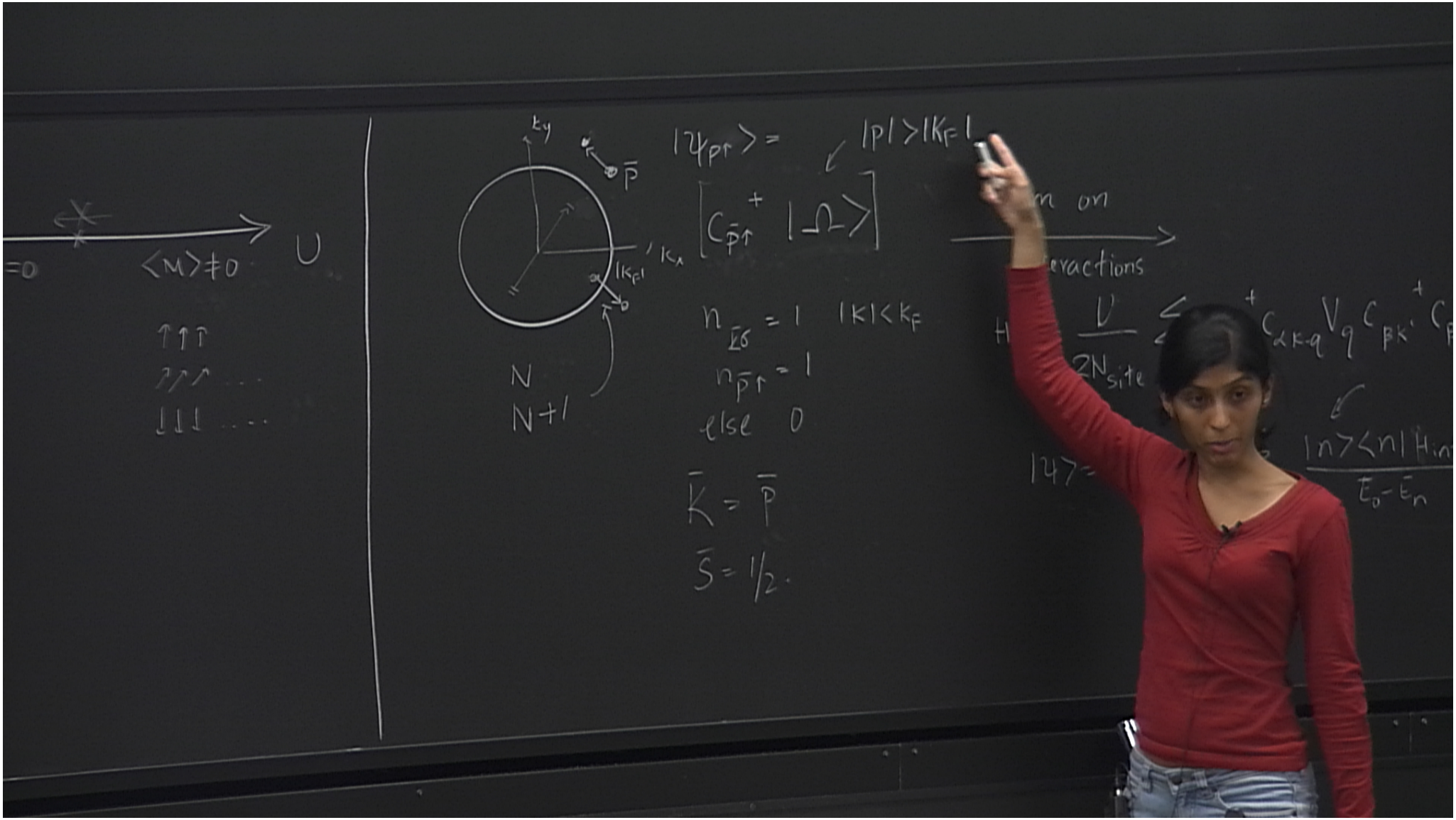

k_y
$\bar{p}$
k_x
$|k_{F\uparrow}|$
$c^{\dagger}_{\bar{p}\uparrow}|\Omega\rangle$
turn on
interactions
$n_{E_G} = 1 \quad |k| < k_F$
$n_{\bar{p}\uparrow} = 1$
$H_{int} = \frac{U}{2N_{site}} \sum c^{\dagger}_{\alpha k} c_{\alpha k-q} V_q c^{\dagger}_{\beta k'} c_{\beta k'+q}$
$\langle M \rangle \neq 0$
U
N
N+1

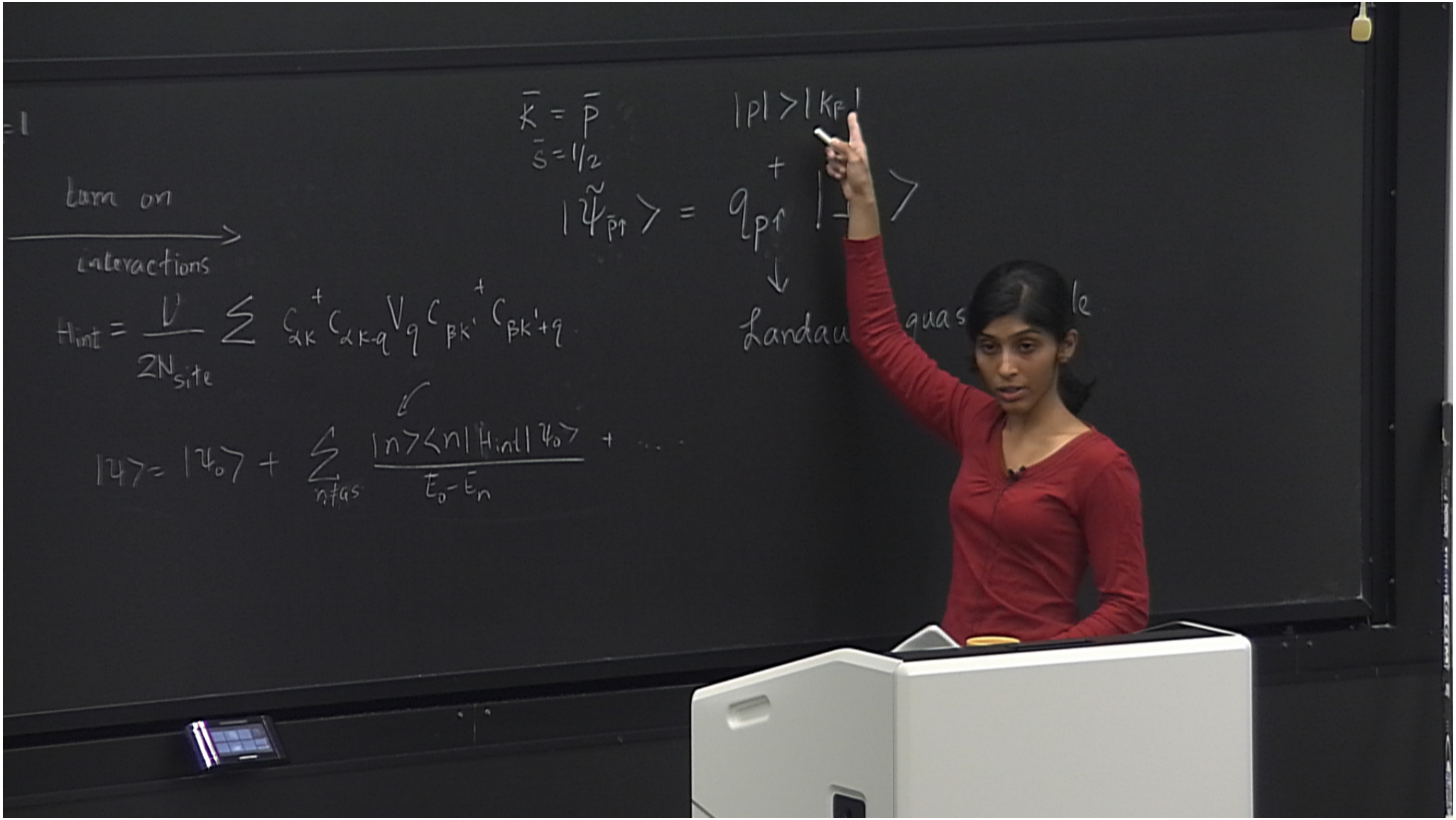

$\langle M \rangle \neq 0$
U
k_y
$\bar{p}$
$|k_{p\uparrow}|$
k_x
N
$N+1$
$\left[c_{\bar{p}\uparrow}^{+} |\Omega\rangle \right]$
$n_{E_G} = 1 \quad |k| < k_F$
$n_{\bar{p}\uparrow} = 1$
else 0
turn on
interactions
$H_{int} = \frac{V}{2N_{site}} \sum c_{\alpha k}^{+} c_{\alpha k-q} V_q c_{\beta k'}^{+} c_{\alpha k'+q}$

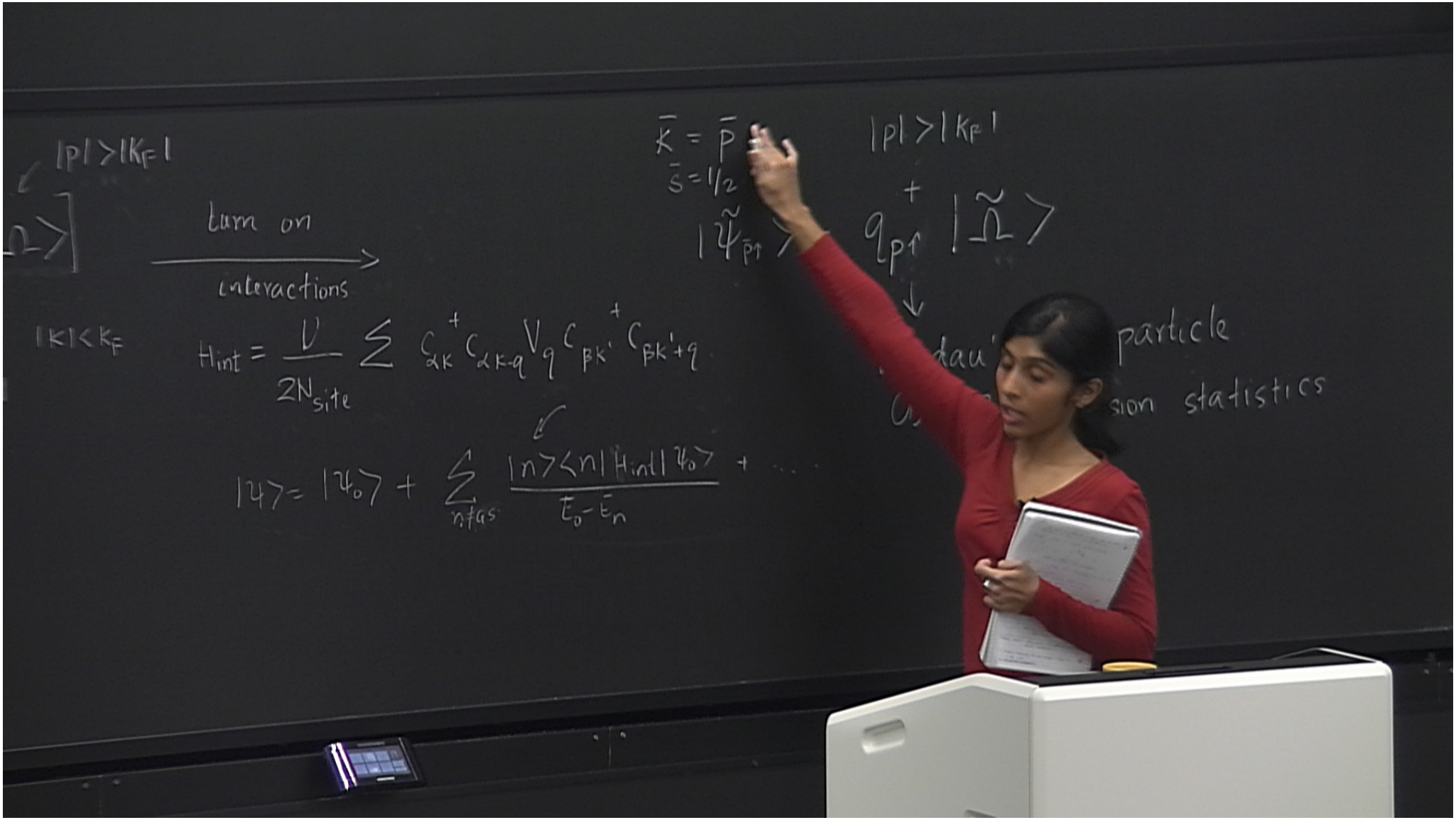

$\langle M \rangle \neq 0$
U
$\uparrow\uparrow\uparrow \ldots$
$\nearrow\nearrow\nearrow \ldots$
$\downarrow\downarrow\downarrow \ldots$
k_y
$\bar{p}$
$|k_{F\uparrow}|$
k_x
N
N+1
$\left[c_{\bar{p}\uparrow}^{+} |\Omega\rangle \right]$
$n_{\bar{p}\uparrow} = 1$
else 0
turn on
interactions
$H_{int} = \frac{V}{2N_{site}} \sum c_{\alpha k}^{+} c_{\alpha k-q} V_q c_{\beta k'}^{+} c_{\beta k'+q}$
$n_{EG} = 1 \quad |k| < k_F$

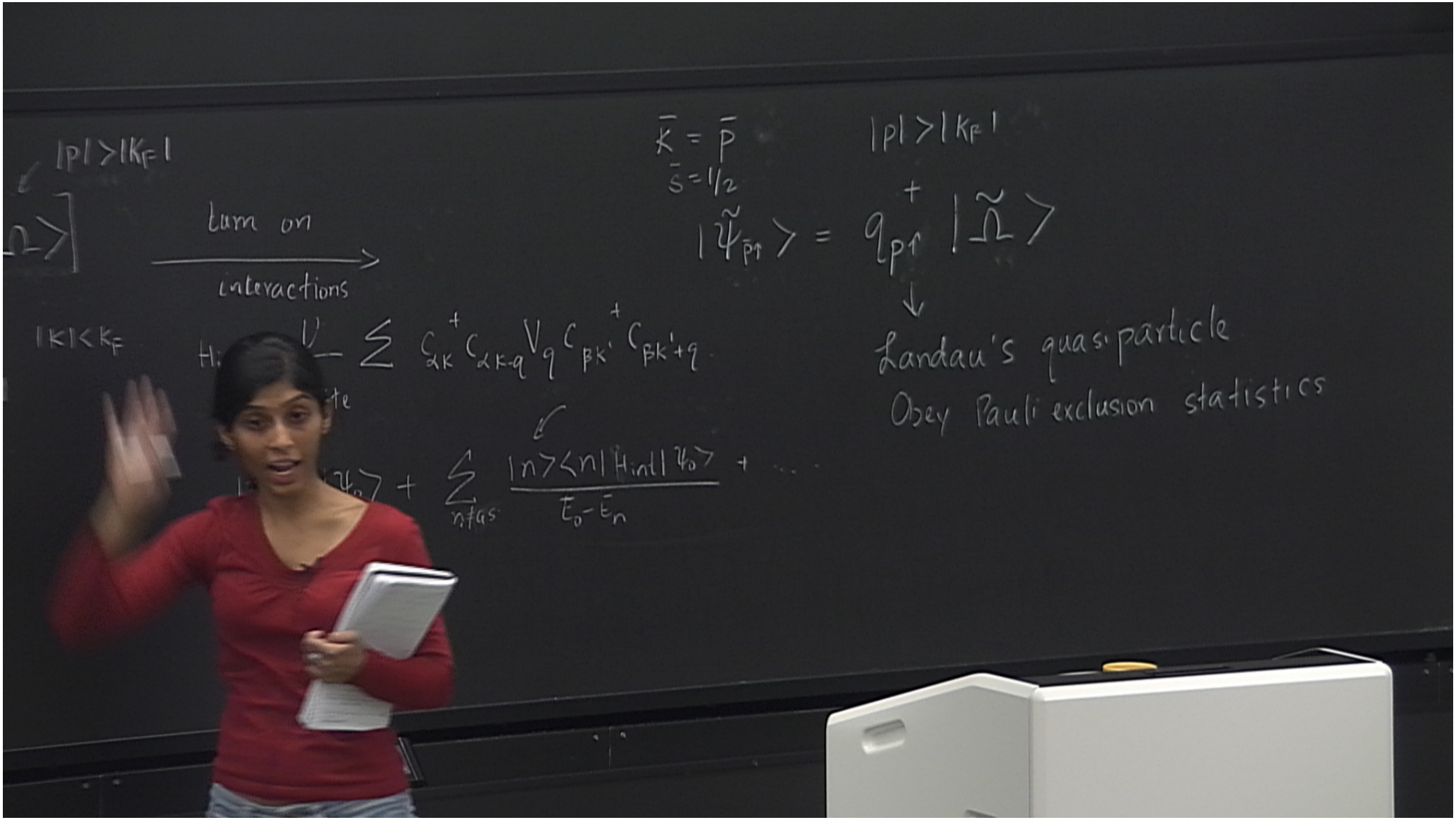

$|\psi_{\bar{P}\uparrow}\rangle =$
$\left[c^{+}_{\bar{P}\uparrow}|\Omega\rangle\right]$
$\bar{P}$
k_y
k_x
$|k_F|$
N
N+1
$n_{\bar{k}\sigma} = 1 \quad |k| < k_F$
$n_{\bar{P}\uparrow} = 1$
else 0
$\bar{K} = \bar{P}$
$\bar{S} = 1/2$
turn on
interactions
$H_{int} = \frac{V}{2N_{site}} \sum c^{+}_{\alpha k} c_{\alpha k-q} V_q c^{+}_{\beta k'} c_{\beta k'+q}$
$|\Psi\rangle = |\Psi_0\rangle + \sum_{n \neq 0} \frac{|n\rangle\langle n|H_{int}|\Psi_0\rangle}{E_0 - E_n} + \ldots$
$\bar{K} = \bar{P}$
$\bar{S} = 1/2$
$|\tilde{\Psi}_{\bar{P}\uparrow}\rangle = q^{+}_{\bar{P}\uparrow}|\tilde{\Omega}\rangle$


$\bar{k} = \bar{p}$
$\bar{s} = 1/2$
$|\tilde{\psi}_{\bar{p}\uparrow}\rangle = q^{\dagger}_{p\uparrow}|\tilde{\Omega}\rangle$
Landau's quasiparticle
turn on
interactions
$H_{int} = \frac{U}{2N_{site}} \sum c^{\dagger}_{\alpha k} c_{\alpha k-q} V_q c^{\dagger}_{\beta k'} c_{\beta k'+q}$
$|\psi\rangle = |\psi_0\rangle + \sum_{n \neq 0} \frac{|n\rangle\langle n|H_{int}|\psi_0\rangle}{E_0 - E_n} + \ldots$


$\langle M \rangle \neq 0$
U
k_y
$\bar{p}$
$|K_F|$
N
N+1
$|\psi_{p\uparrow}\rangle =$
$|p| > |K_F|$
$|\Omega\rangle$
$n_{\bar{k}\sigma} = 1 \quad |k| < k_F$
$n_{\bar{p}\uparrow} = 1$
else 0
$\bar{K} = \bar{p}$
$\bar{S} = 1/2$
$2N_{site}$
$E_0 - E_n$

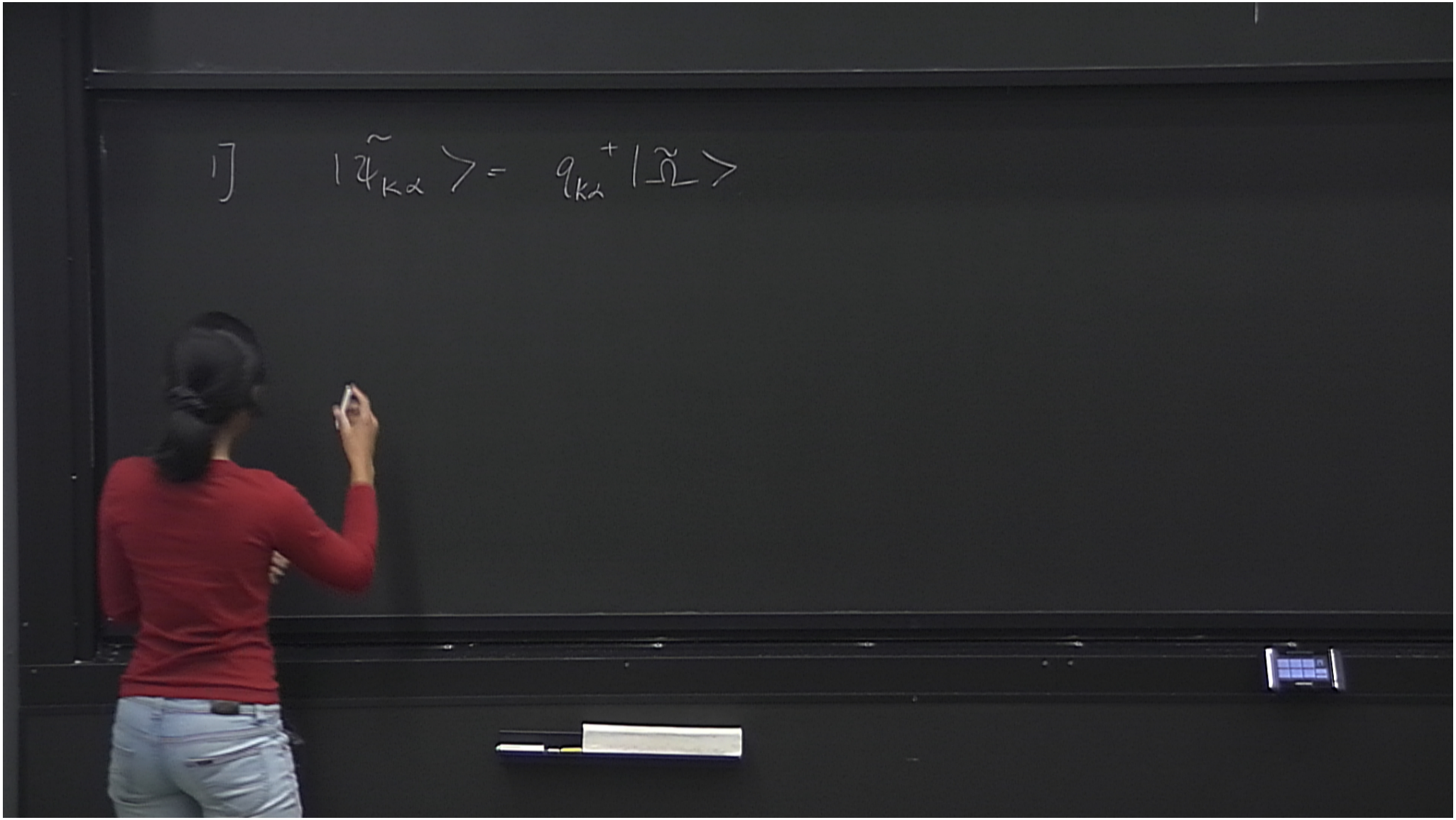

$\bar{K} = \bar{P}$
$\bar{S} = 1/2$
$|p| > |k_F|$
+
$|\tilde{\Psi}_{\bar{p}\uparrow}\rangle = q_{p\uparrow}$
turn on
interactions
$H_{int} = \frac{U}{2N_{site}} \sum c^{+}_{\alpha k} c_{\alpha k-q} V_q c^{+}_{\beta k'} c_{\beta k'+q}$
Landau
$|\Psi\rangle = |\Psi_0\rangle + \sum_{n \neq 0} \frac{|n\rangle\langle n| H_{int} |\Psi_0\rangle}{E_0 - E_n} + \ldots$

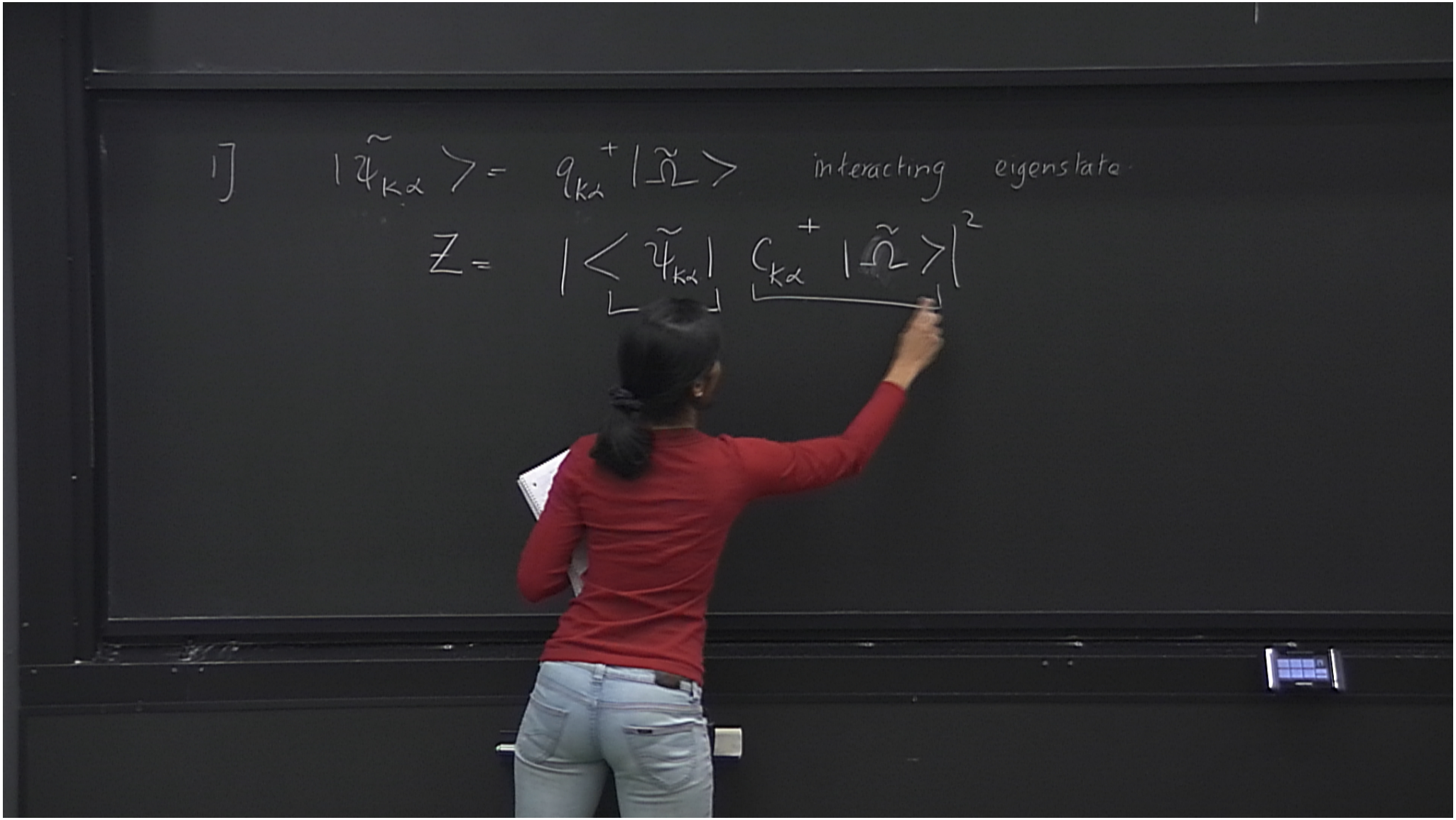

|p| > |k_F|
turn on
interactions
|k| < k_F
particle
statistics

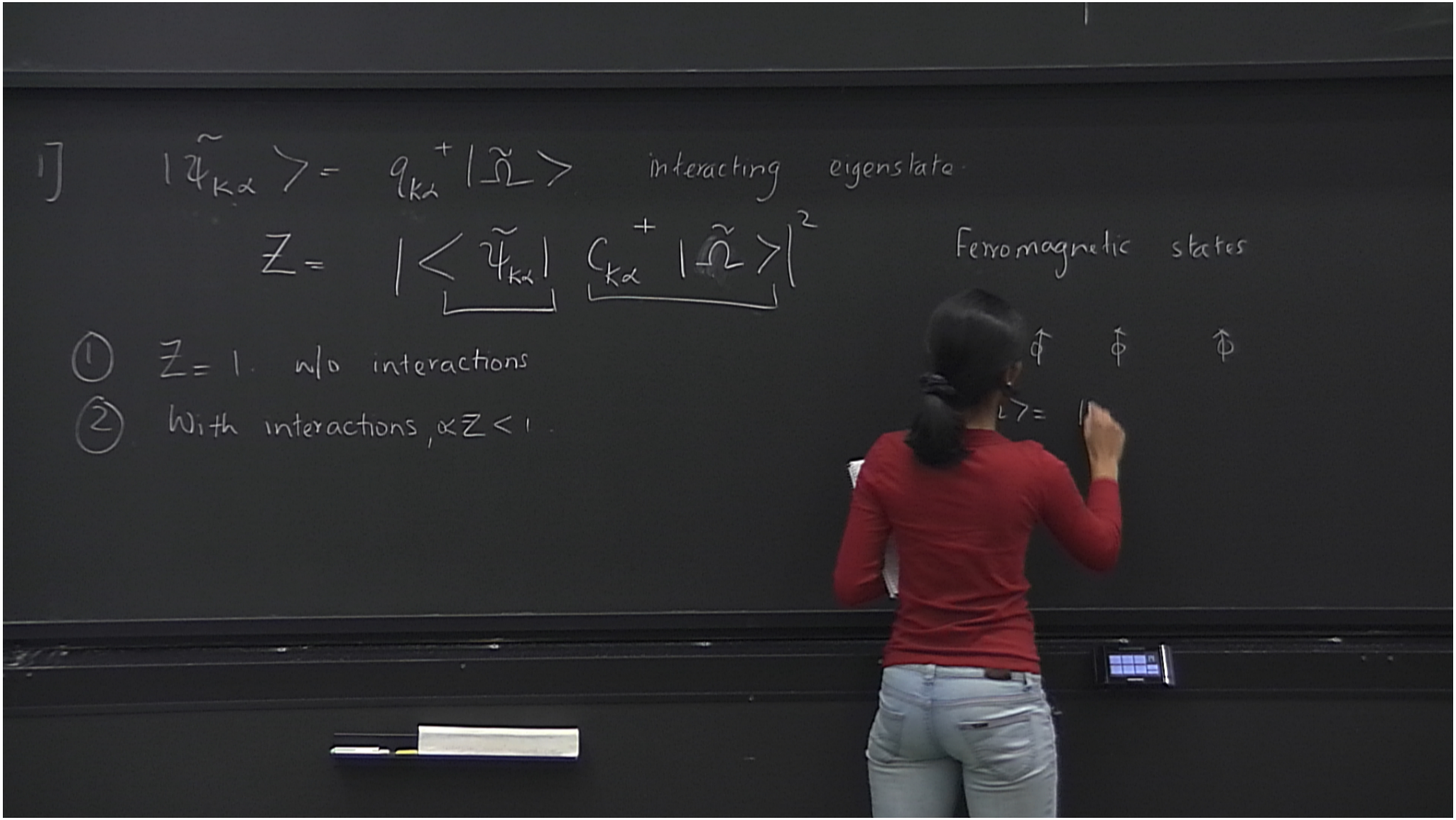

|p| > |k_F|
turn on
interactions
|k| < k_F
Landau's quasi-particle
Obey Pauli exclusion statistics


⟨M⟩ = 0
⟨M⟩ ≠ 0
U
N
N+1
else 0
K̄ = P̄
S̄ = 1/2




1] $|\tilde{q}_{k\alpha}\rangle = q_{k\alpha}^{+}|\tilde{\Omega}\rangle$


1] $|\tilde{\psi}_{k\alpha}\rangle = q_{k\alpha}^{+}|\tilde{\Omega}\rangle$ interacting eigenstate
$Z = |\langle \tilde{\psi}_{k\alpha}| \; c_{k\alpha}^{+}|\tilde{\Omega}\rangle|^2$


1] $|\tilde{\psi}_{k\alpha}\rangle = q_{k\alpha}^{+}|\tilde{\Omega}\rangle$ interacting eigenstate
$Z = |\langle \tilde{\psi}_{k\alpha}| \; c_{k\alpha}^{+} |\tilde{\Omega}\rangle|^2$
Ferromagnetic states
① $Z = 1$ w/o interactions
② With interactions, $\propto Z < 1$

cting eigenstate

$|\tilde{2}\rangle|^2$

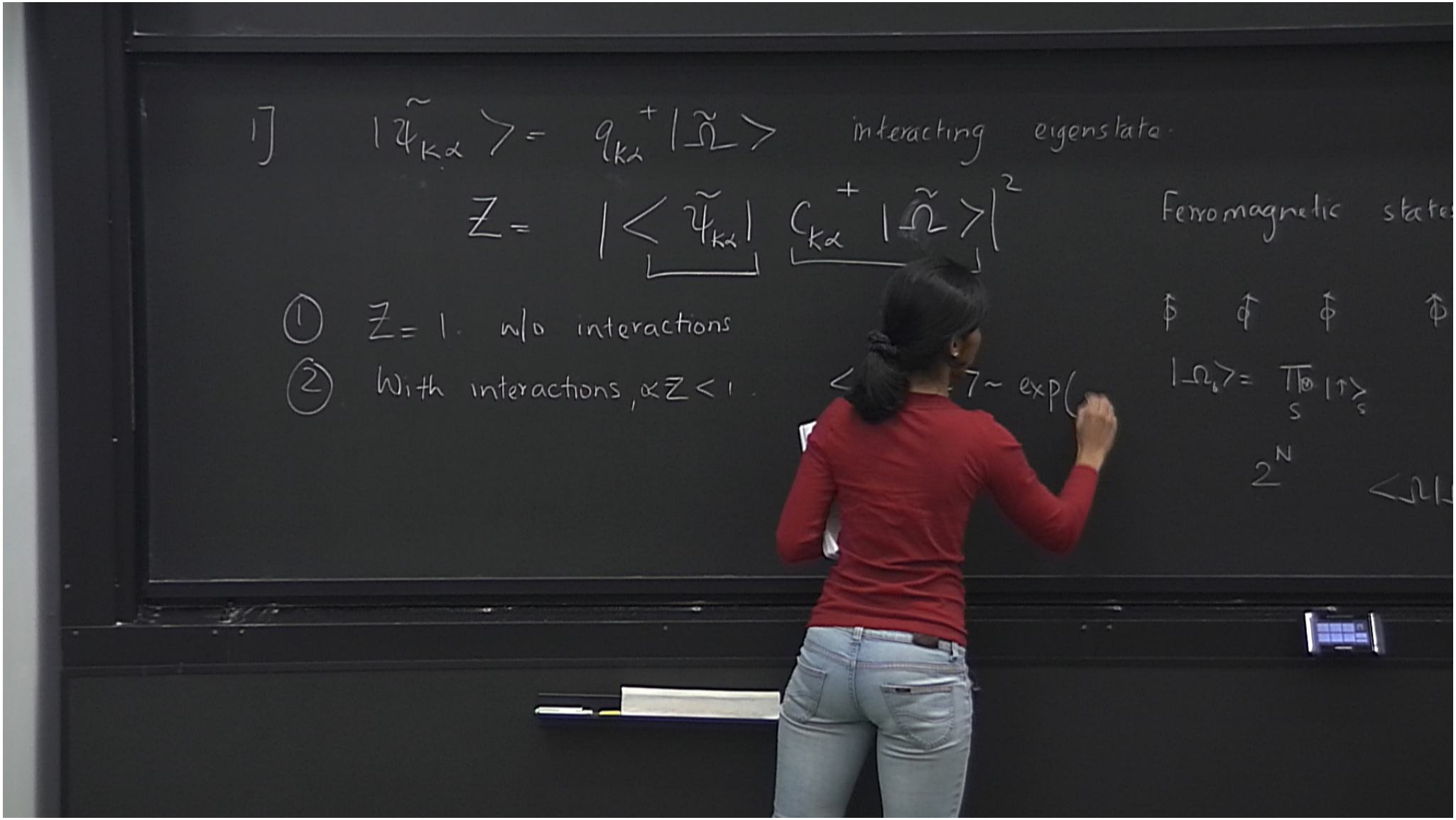
Ferromagnetic states

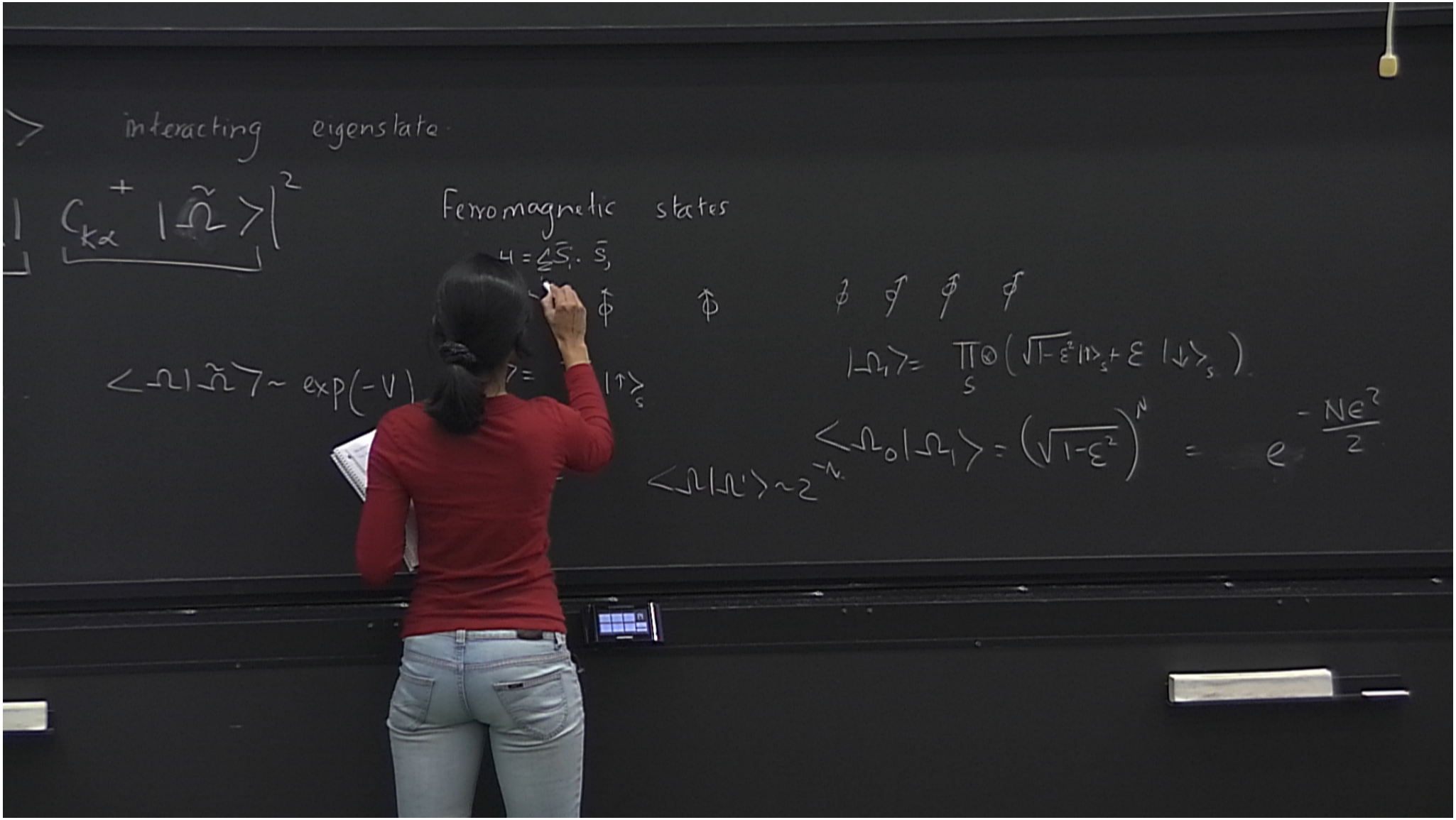
$|\Omega_0\rangle = \prod_s \otimes |\uparrow\rangle_s$

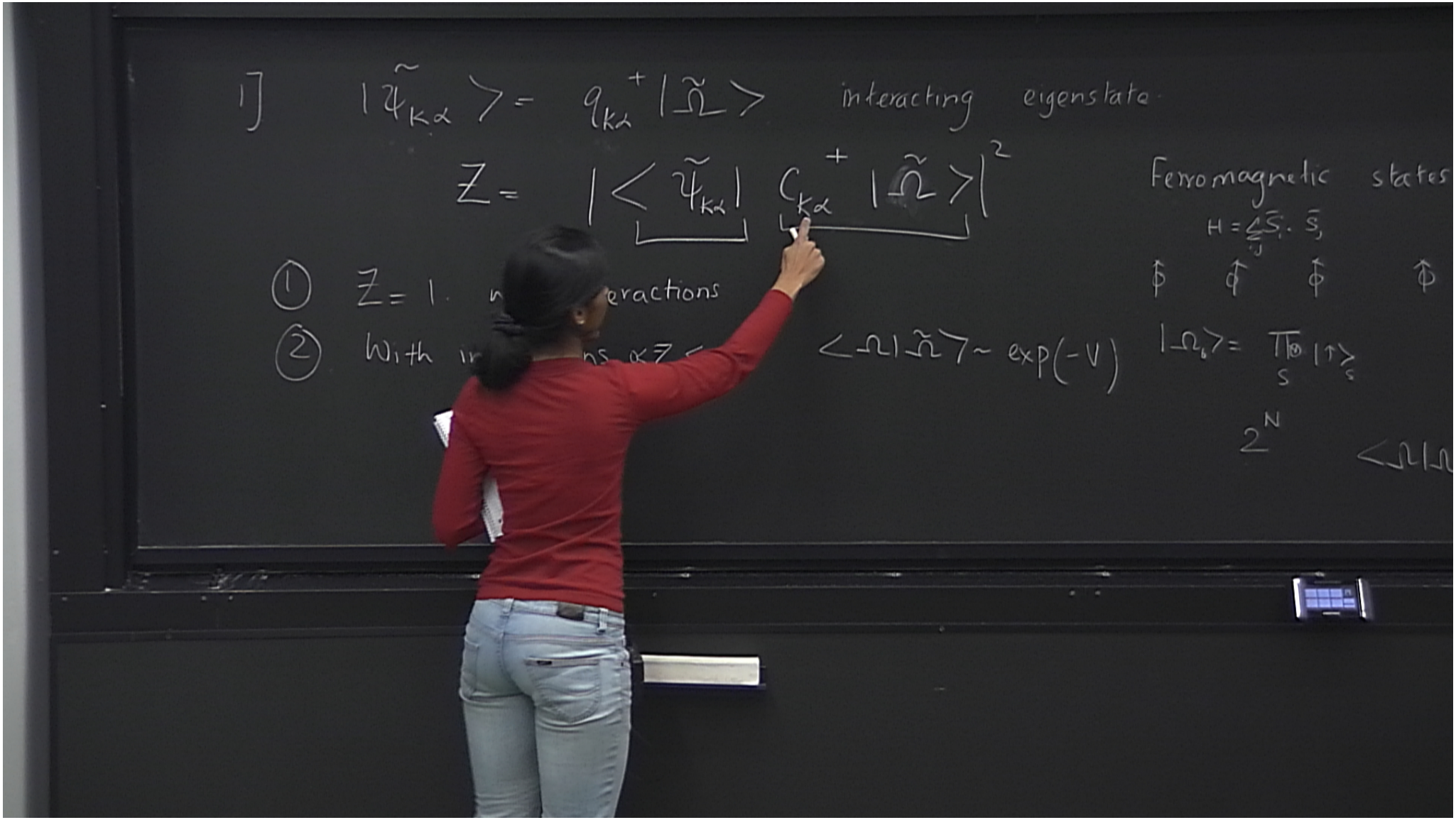
$|\Omega_1\rangle = \prod_s \otimes \left(\sqrt{1-\varepsilon^2}\,|\uparrow\rangle_s + \varepsilon\,|\downarrow\rangle_s\right)$ $\quad \varepsilon \ll 1$

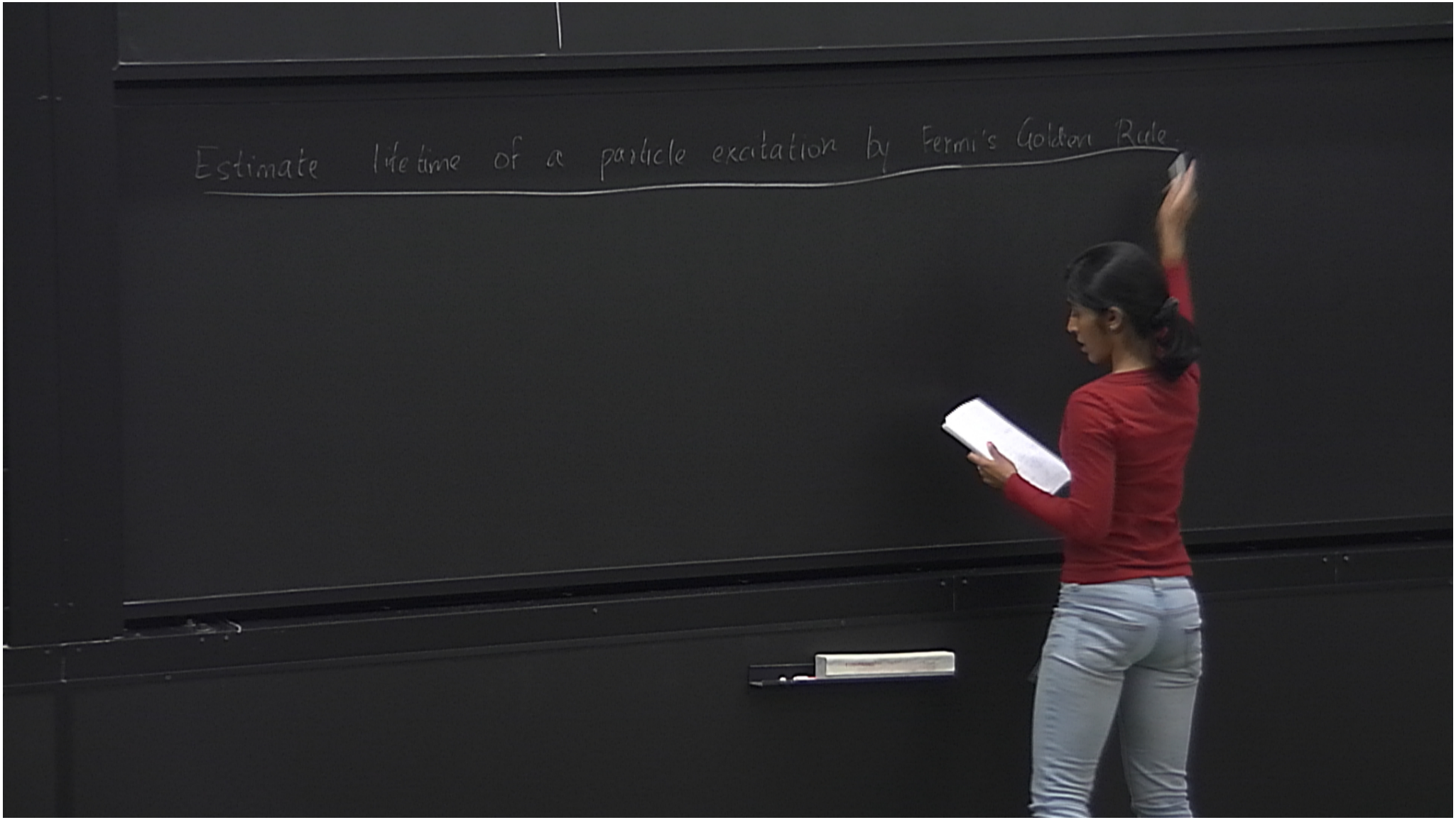
2^N $\quad \langle\Omega|\Omega'\rangle \sim 2^{-N}$

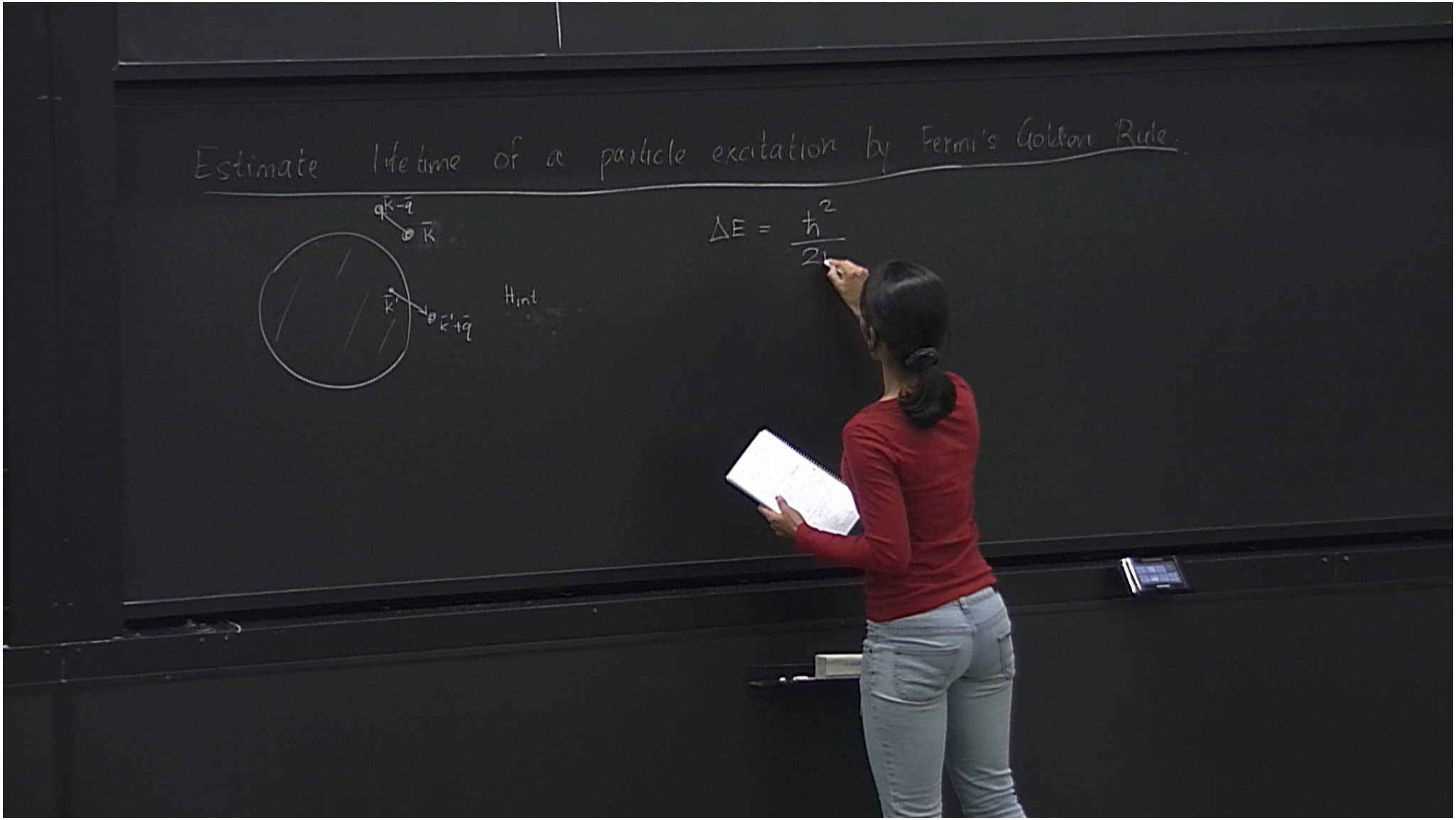
$\langle\Omega_0|\Omega_1\rangle = \left(\sqrt{1-\varepsilon^2}\right)^N = e^{-\frac{N\varepsilon^2}{2}}$


1] $|\tilde{\psi}_{k\alpha}\rangle = q_{k\alpha}^{+}|\tilde{\Omega}\rangle$ interacting eigenstate
$Z = |\langle \tilde{\psi}_{k\alpha}| \; c_{k\alpha}^{+} |\tilde{\Omega}\rangle|^2$
Ferromagnetic state
① $Z = 1$ w/o interactions
② With interactions, $\propto Z < 1$
$|\Omega_0\rangle = T_S^{\otimes} |\uparrow\rangle_S$
2^N

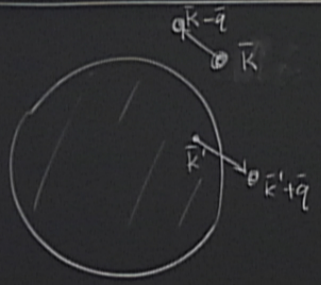

interacting eigenstate
Ferromagnetic states
$H = \sum \vec{S}_i \cdot \vec{S}_j$
$|\Omega_1\rangle = \prod_s \otimes \left(\sqrt{1-\varepsilon^2}\,|\uparrow\rangle_s + \varepsilon\,|\downarrow\rangle_s\right)$
$\langle\Omega_0|\Omega_1\rangle = \left(\sqrt{1-\varepsilon^2}\right)^N = e^{-\frac{N\varepsilon^2}{2}}$
$\langle\Omega|\Omega'\rangle \sim z^{-N}$


1] $|\tilde{\psi}_{k\alpha}\rangle = q_{k\alpha}^{+}|\tilde{\Omega}\rangle$ interacting eigenstate
$Z = |\langle \tilde{\psi}_{k\alpha}| c_{k\alpha}^{+} |\tilde{\Omega}\rangle|^2$
Ferromagnetic states
$H = \sum_{ij} \vec{S}_i \cdot \vec{S}_j$
① $Z = 1$ ... eractions
② With ir... ns $\propto Z$ ...
$\langle \Omega | \tilde{\Omega} \rangle \sim \exp(-V)$
$|\Omega_0\rangle = \prod_s |\uparrow\rangle_s$
2^N
$\langle \Omega | \Omega$


Estimate lifetime of a particle excitation by Fermi's Golden Rule


Estimate lifetime of a particle excitation by Fermi's Golden Rule.
$\vec{k}-\vec{q}$
$\vec{k}$
$\vec{k}'$
$\vec{k}'+\vec{q}$
H_{int}
$\Delta E = \frac{\hbar^2}{2}$

Estimate lifetime of a particle excitation by Fermi's Golden Rule.

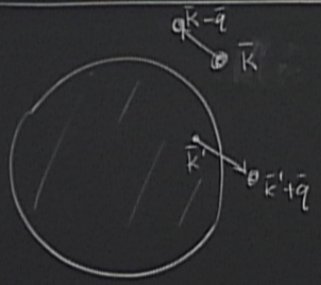


H_{int}

$$\Delta E = \frac{\hbar^2}{2m}\left[\,|\vec{k}|^2 - |k_F|^2\,\right].$$

$$\vec{k} = \vec{k}_F + \vec{a} \quad ; \quad \Delta E$$

Estimate lifetime of a particle excitation by Fermi's Golden Rule.

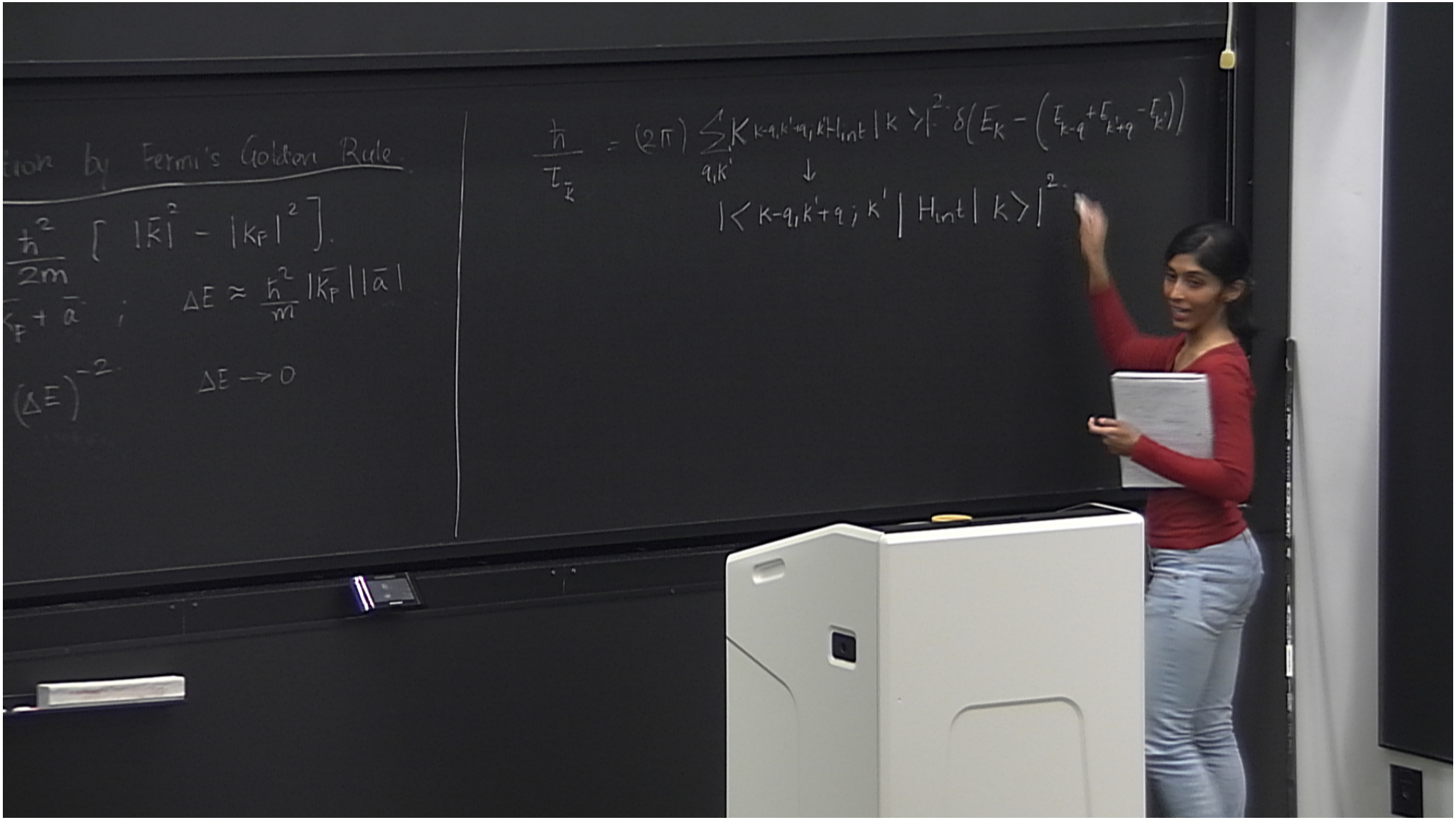


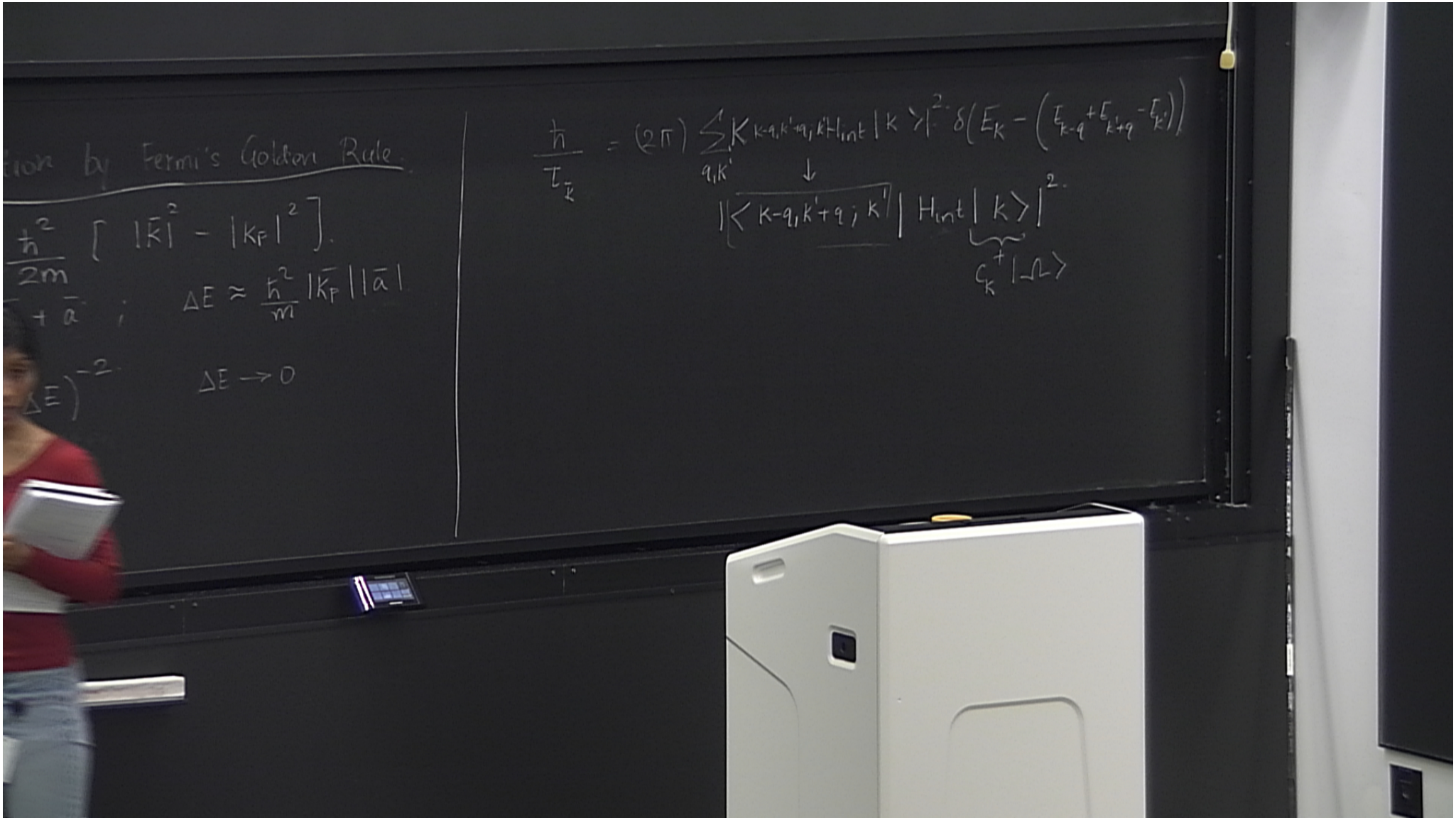
H_{int}

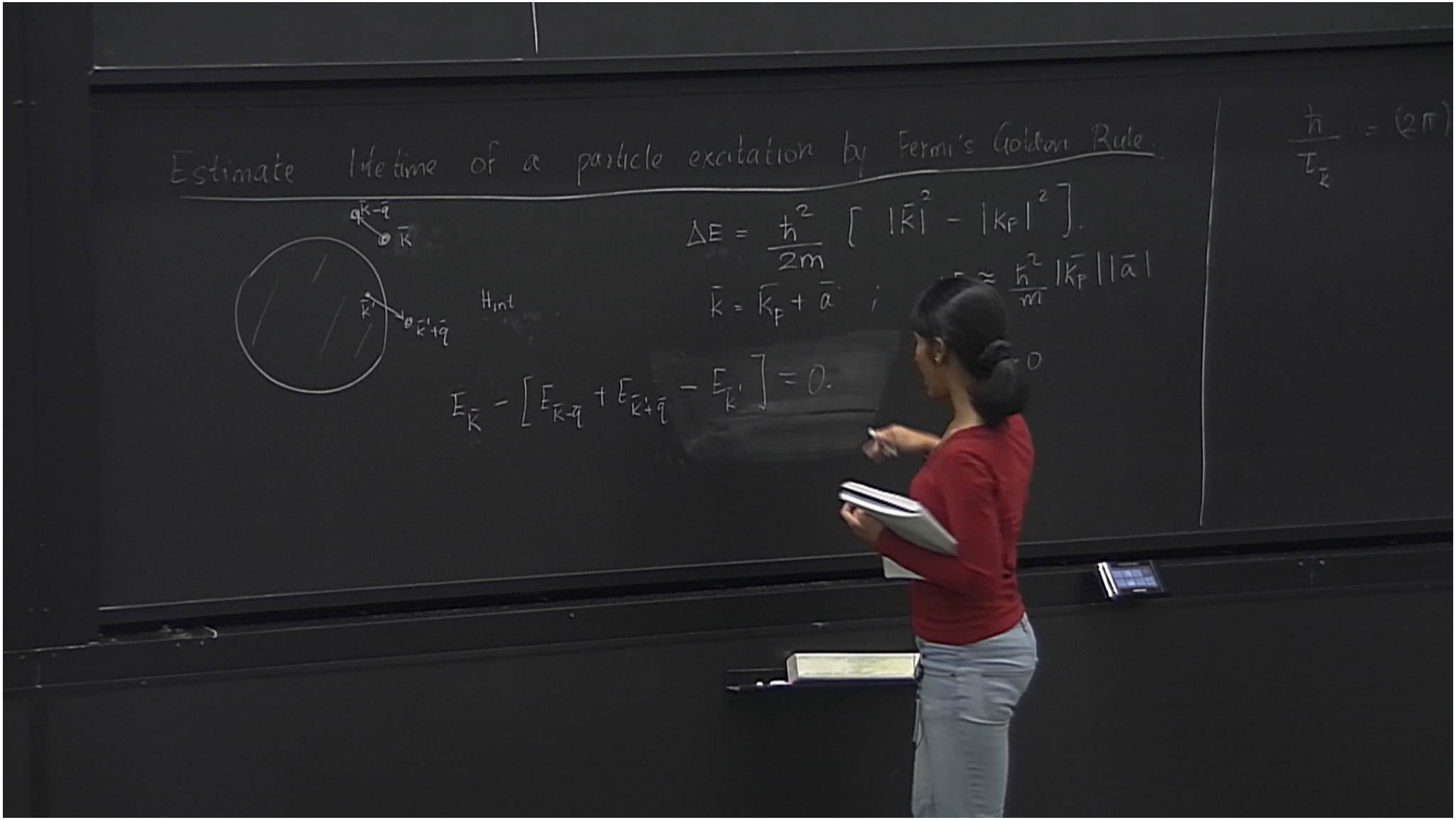
$$\Delta E = \frac{\hbar^2}{2m}\left[\,|\vec{k}|^2 - |k_F|^2\,\right].$$

$$\vec{k} = \vec{k}_F + \vec{a} \quad ; \quad \Delta E \approx \frac{\hbar^2}{m}|\vec{k}_F||\vec{a}|$$

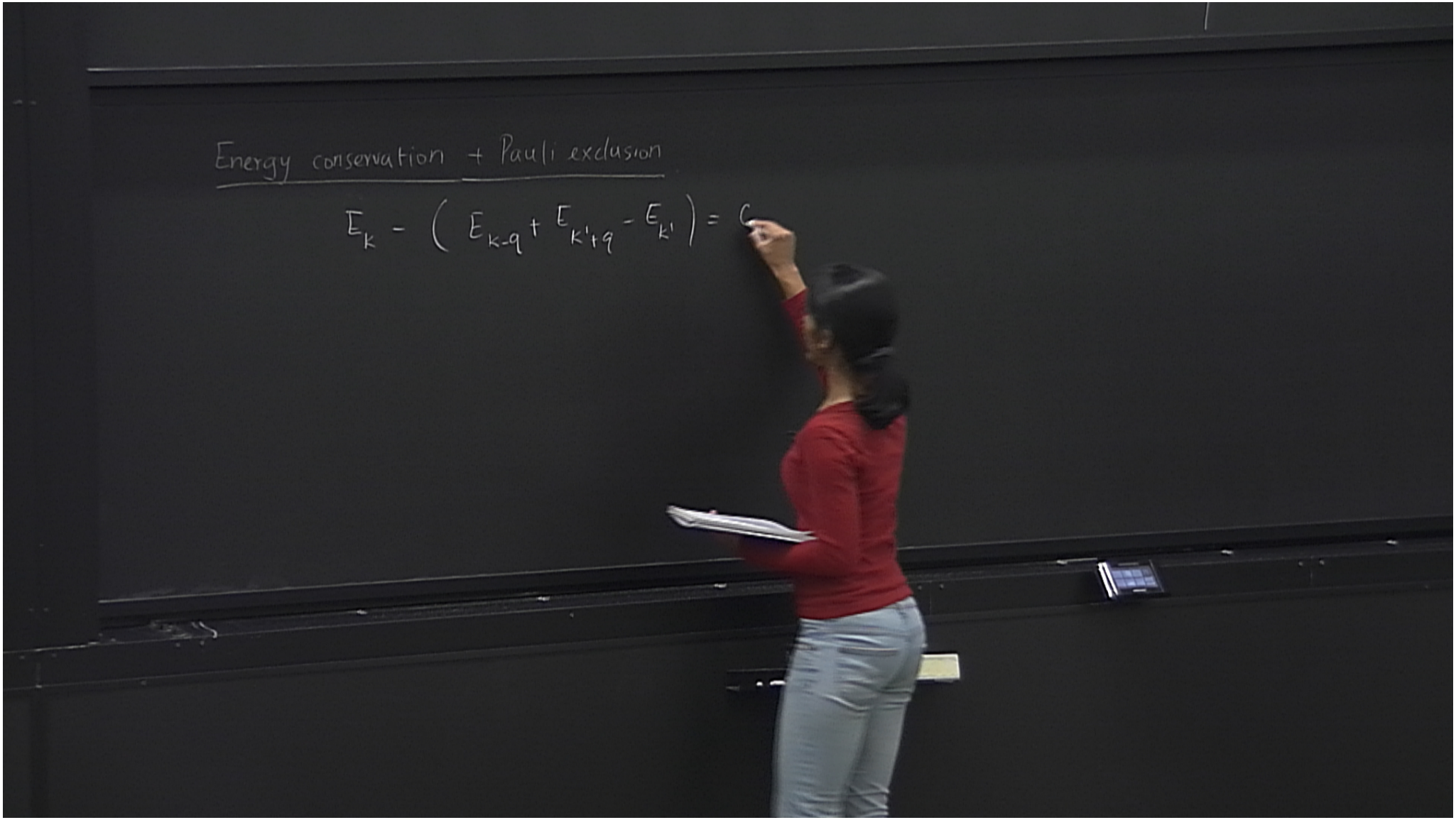
$$\tau_{\vec{k}} \propto (\Delta E)^{-2} \qquad \Delta E \to 0$$

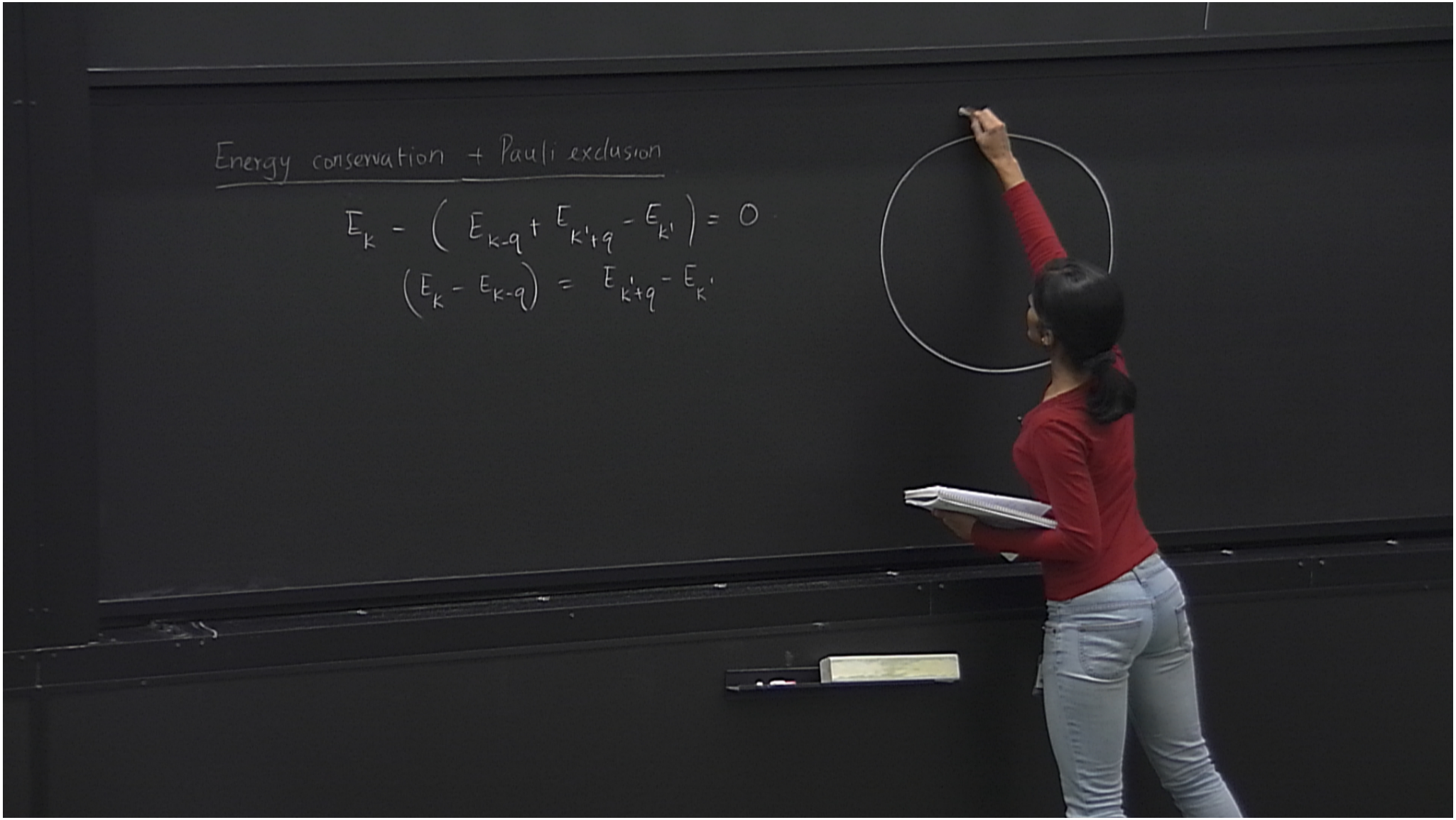

tion by Fermi's Golden Rule
$\frac{\hbar^2}{2m}\left[\,|\bar{k}|^2 - |k_F|^2\,\right]$.
$\bar{k}_F + \bar{a}$;
$\Delta E \approx \frac{\hbar^2}{m}|\bar{k}_F||\bar{a}|$
$(\Delta E)^{-2}$
$\Delta E \to 0$
$\frac{\hbar}{\tau_{\bar{k}}} = (2\pi)\sum_{q,k'} |\langle k-q, k'+q, k'|H_{int}|k\rangle|^2 \cdot \delta(E_k - (E_{k-q} + E_{k'+q} - E_{k'}))$
$|\langle k-q, k'+q; k'|H_{int}|k\rangle|^2$

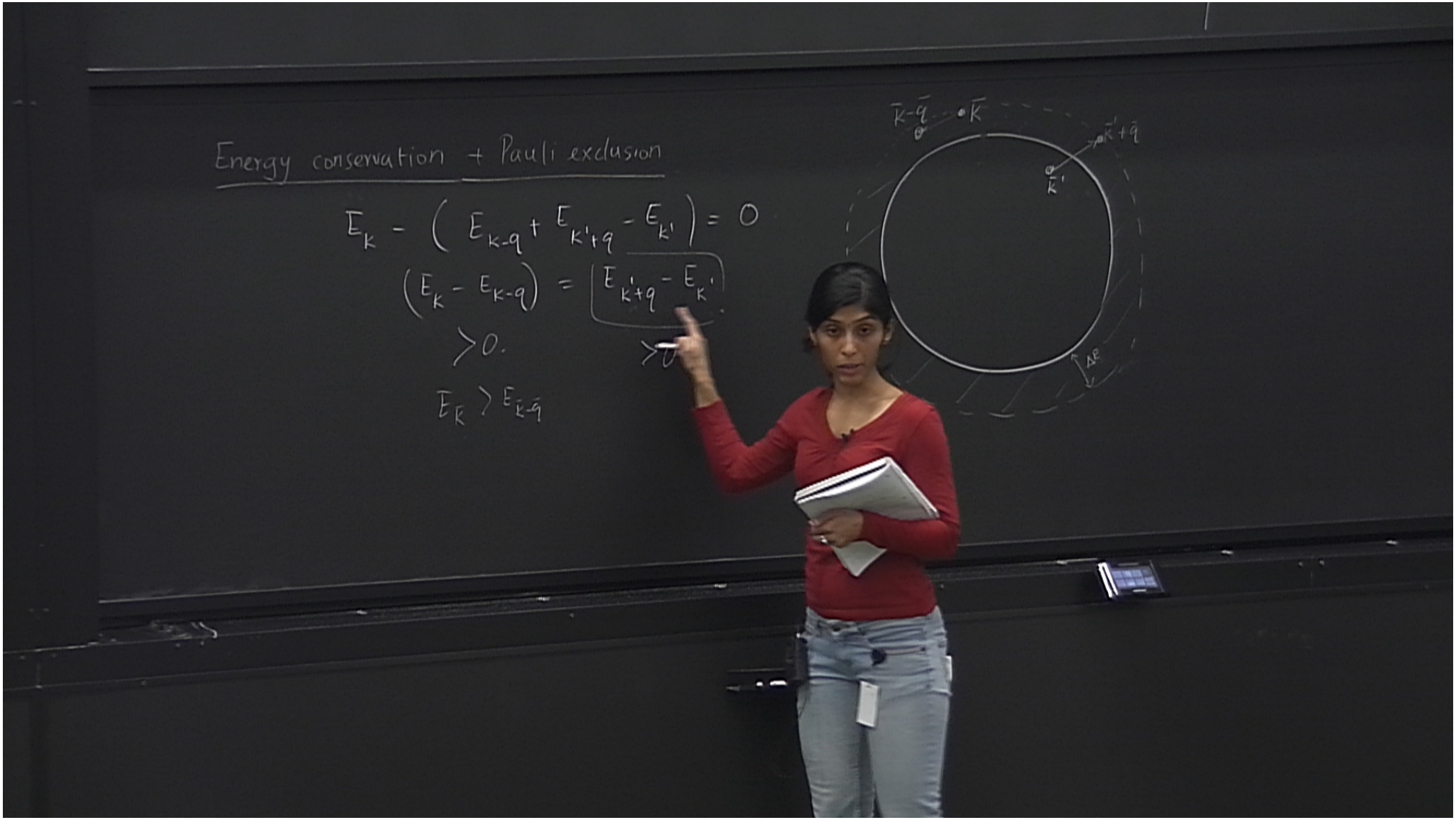

Fermi's Golden Rule
$\frac{\hbar^2}{2m}\left[|\bar{k}|^2 - |k_F|^2\right]$.
$\Delta E \approx \frac{\hbar^2}{m}|\bar{k}_F||\bar{a}|$
$\Delta E \to 0$
$\frac{\hbar}{\tau_{\bar{k}}} = (2\pi)\sum_{q,k'} |\langle k-q, k'+q; k'|H_{int}|k\rangle|^2 \cdot \delta(E_k - (E_{k-q} + E_{k'+q} - E_{k'}))$
$|\langle k-q, k'+q; k'|H_{int}\underbrace{|k\rangle}_{c_k^\dagger|\Omega\rangle}|^2$

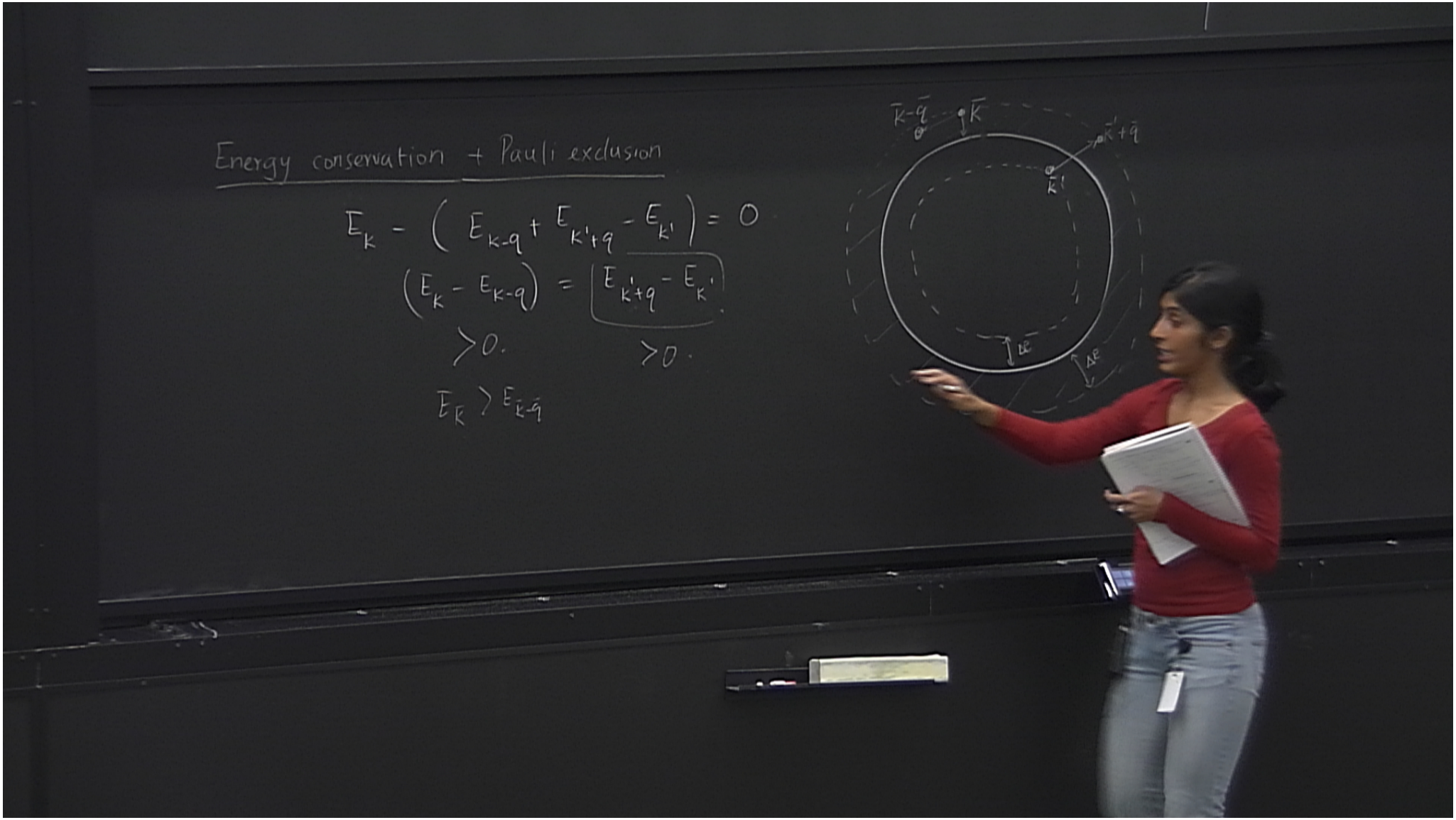

Estimate lifetime of a particle excitation by Fermi's Golden Rule
$\frac{\hbar}{\tau_{\bar{k}}} = (2\pi)$
$\Delta E = \frac{\hbar^2}{2m}\left[\,|\bar{k}|^2 - |k_F|^2\,\right]$.
$\bar{k} = \bar{k}_F + \bar{a}$;
$\approx \frac{\hbar^2}{m} |\bar{k}_F||\bar{a}|$
H_{int}
$E_{\bar{k}} - \left[E_{\bar{k}-\bar{q}} + E_{\bar{k}'+\bar{q}} - E_{\bar{k}'}\right] = 0.$
$= 0$

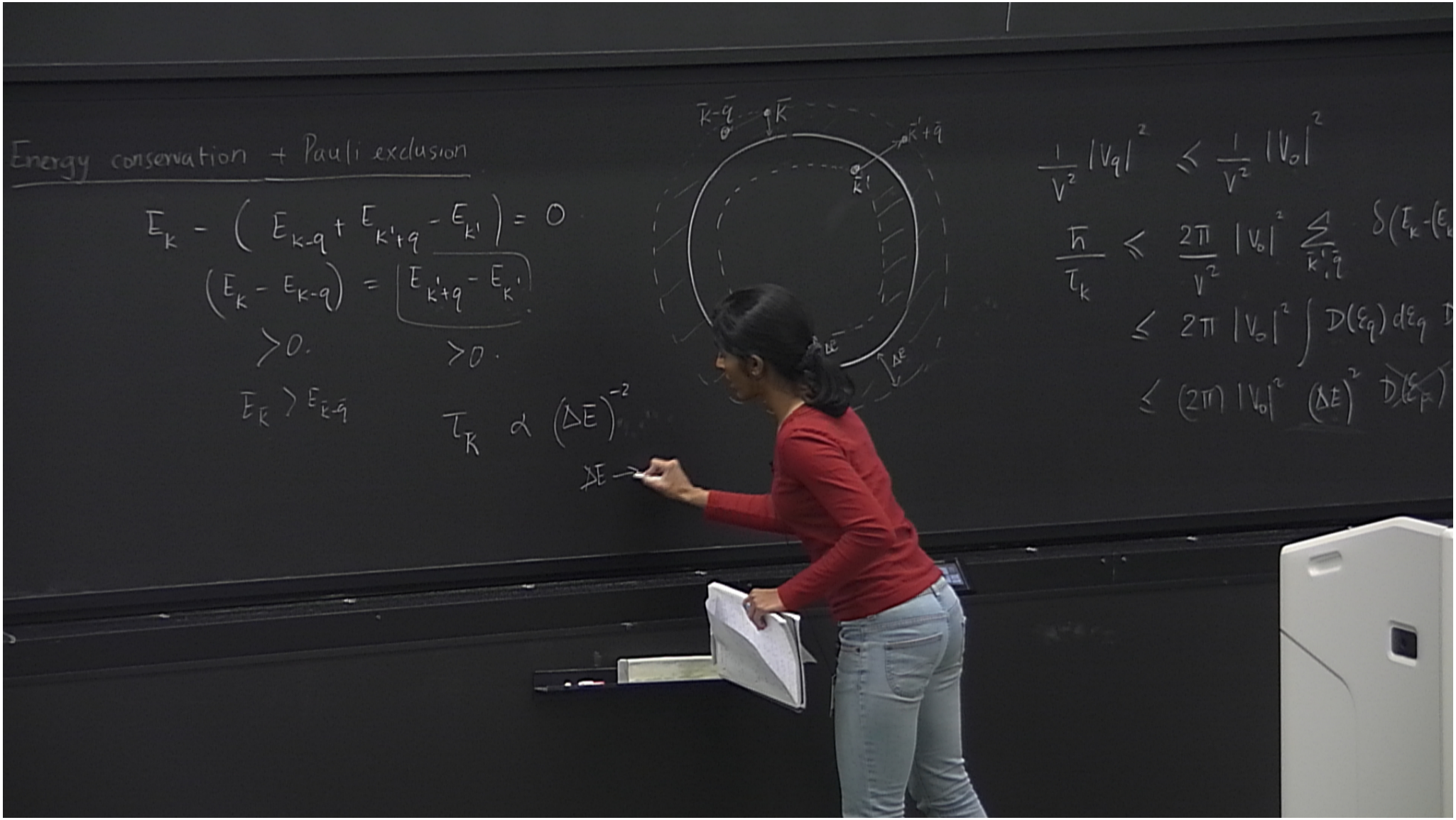

Energy conservation + Pauli exclusion


Energy conservation + Pauli exclusion
$E_k - \left(E_{k-q} + E_{k'+q} - E_{k'} \right) = 0$


Energy conservation + Pauli exclusion
$E_k - (E_{k-q} + E_{k'+q} - E_{k'}) = 0$
$(E_k - E_{k-q}) = E_{k'+q} - E_{k'}$


Energy conservation + Pauli exclusion
$E_k - (E_{k-q} + E_{k'+q} - E_{k'}) = 0$
$(E_k - E_{k-q}) = E_{k'+q} - E_{k'}$
$> 0.$
> 0
$E_k > E_{k-q}$
$\vec{k}-\vec{q}$
$\vec{k}$
$\vec{k}'+\vec{q}$
$\vec{k}'$
$\Delta\theta$


Energy conservation + Pauli exclusion
$E_k - (E_{k-q} + E_{k'+q} - E_{k'}) = 0$
$(E_k - E_{k-q}) = E_{k'+q} - E_{k'}$
$> 0.$
$> 0.$
$E_k > E_{k-q}$
$\vec{k}-\vec{q}$
$\vec{k}$
$\vec{k}'+\vec{q}$
$\vec{k}'$
ΔE
ΔE


Energy conservation + Pauli exclusion
$E_k - (E_{k-q} + E_{k'+q} - E_{k'}) = 0$
$(E_k - E_{k-q}) = E_{k'+q} - E_{k'}$
$> 0.$ $> 0.$
$E_k > E_{k-q}$
$\tau_k \propto (\Delta E)^{-2}$
$\Delta E \to$
$\frac{1}{V^2}|V_q|^2 \leq \frac{1}{V^2}|V_0|^2$
$\frac{\hbar}{\tau_k} \leq \frac{2\pi}{V^2}|V_0|^2 \sum_{k',q} \delta(E_k - (E$
$\leq 2\pi |V_0|^2 \int D(\varepsilon_q)\, d\varepsilon_q$
$\leq (2\pi)|V_0|^2 (\Delta E)^2 D(\varepsilon_F)$