

Title: 13/14 PSI - Condensed Matter - Lecture 8

Date: Nov 20, 2013 10:45 AM

URL: <http://pirsa.org/13110036>

Abstract:

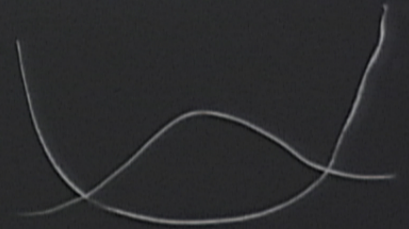
$$L(\psi, \dot{\psi}) = i \langle \psi | \frac{d}{dt} | \psi \rangle - \langle \psi | H | \psi \rangle$$

$$L(\psi_i, \dot{\psi}_i) = i \langle \psi_{\underline{3}_i} | \frac{d}{dt} | \psi_{\underline{3}_i} \rangle - \langle \psi_{\underline{3}_i} | H | \psi_{\underline{3}_i} \rangle$$

$$L(\psi, \dot{\psi}) = i \langle \psi | \frac{d}{dt} | \psi \rangle - \langle \psi | H | \psi \rangle$$

$$L(\psi_i, \dot{\psi}_i) = i \langle \psi_{3i} | \frac{d}{dt} | \psi_{3i} \rangle - \langle \psi_{3i} | H | \psi_{3i} \rangle$$

$$H = \frac{p^2}{2} + \frac{1}{2} \alpha x^2 + \frac{1}{4} x^4$$



$$|\psi_{3_1, 3_2}\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{e^{i \tilde{3}_2 x}}{\tilde{3}_1} e^{-\frac{1}{2} \alpha (x - \tilde{3}_1)^2}$$

$\tilde{3}_1$

$$L(\psi, \dot{\psi}) = i \langle \psi | \frac{d}{dt} | \psi \rangle - \langle \psi | H | \psi \rangle$$

$$\hat{H} =$$

$$L(\psi_i, \dot{\psi}_i) = i \langle \psi_{3i} | \frac{d}{dt} | \psi_{3i(t)} \rangle - \langle \psi_{3i} | H | \psi_{3i} \rangle$$

$$|\psi_{3i}\rangle$$

$$= a_i(\psi_i) \dot{\psi}_i - \bar{H}(\psi_i)$$

$$a_i = i \langle \psi_{3i} | \frac{\partial}{\partial \dot{\psi}_i} | \psi_{3i} \rangle$$

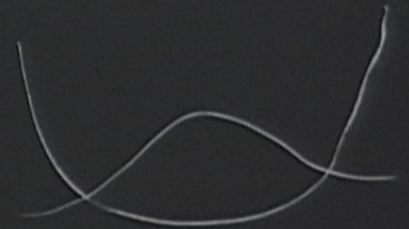
$$\langle \psi_{3_i} | \psi_{3_i} + \delta \psi_i \rangle = e^{-i\delta\phi}$$

$$= 1 - i\delta\phi$$

$$\delta\phi = i \langle \psi_{3_i} | \frac{\partial}{\partial \psi_i} | \psi_i \rangle \delta \psi_i$$

$$= a_i \delta \psi_i$$

$$H = \frac{p^2}{2} + \frac{1}{2} \alpha x^2 + \frac{1}{4} x^4$$



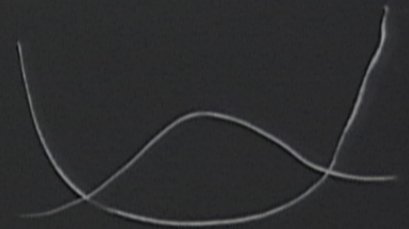
$$|\psi_{3_1, 3_2}\rangle = \left(\frac{2}{\pi}\right)^{1/4} \underline{e^{i \frac{3_2}{2} x}} \underline{e^{-\frac{1}{2} \alpha (x - \frac{3_1}{2})^2}}$$

$$a_1 = 0 \quad a_2 = -\frac{3_1}{2}$$

$$\psi_{\frac{3_1}{2}}$$

$$e^{-\frac{1}{2} \alpha (x - \frac{3_1}{2})^2} \underline{(x - \frac{3_1}{2})} e^{-\frac{1}{2} \alpha (x - \frac{3_1}{2})^2}$$

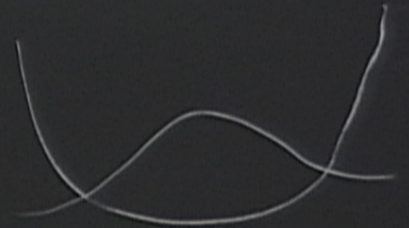
$$H = \frac{p^2}{2} + \frac{1}{2} \alpha x^2 + \frac{1}{4} x^4$$



$$|\psi_{3,3_2}\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} \underbrace{e^{i\frac{3_2}{3_1}x}}_{\psi_{3_1}} e^{-\frac{1}{2}\alpha(x-\frac{3_1}{3_2})^2} = e^{-\frac{1}{2}\alpha(x-\frac{3_1}{3_2})^2} \times e^{i\frac{3_2}{3_1}x}$$

$$a_1 = 0 \quad a_2 = -\frac{3_1}{3_2}$$

$$H = \frac{p^2}{2} + \frac{1}{2} \alpha x^2 + \frac{1}{4} x^4$$



$$|\psi_{3_1, 3_2}\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{i \frac{3_2}{\alpha} x} e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2}$$

$\psi_{\frac{3_1}{\alpha}}$
 $e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2}$
 $\times e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2}$

$$a_1 = 0 \quad a_2 = -\frac{3_1}{\alpha}$$

$$H = \frac{1}{2} \left(\frac{3_2}{\alpha}\right)^2 + \frac{3\alpha + 2\alpha^2 \alpha}{4\alpha^2} \left(\frac{3_1}{\alpha}\right)^2 + \frac{1}{4} \left(\frac{3_1}{\alpha}\right)^4 + \frac{3 + 4\alpha^3 + 4\alpha^2 \alpha}{16\alpha^2}$$

$$L(\psi, \dot{\psi}) = i \langle \psi | \frac{d}{dt} | \psi \rangle - \langle \psi | H | \psi \rangle$$

$$L(\psi_i, \dot{\psi}_i) = i \langle \psi_{3i} | \frac{d}{dt} | \psi_{3i(t)} \rangle - \langle \psi_{3i} | H | \psi_{3i} \rangle$$

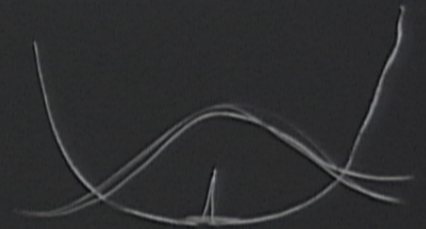
$$= \boxed{a_i(\psi_i) \dot{\psi}_i - H(\psi_i)}$$

$$\boxed{a_i = i \langle \psi_{3i} | \frac{\partial}{\partial \dot{\psi}_i} | \psi_{3i} \rangle}$$

$$-\dot{\psi}_1 \dot{\psi}_2$$

$$= \dot{\psi}_2 \dot{\psi}_1 - \frac{d}{dt}(\psi_1 \psi_2)$$

$$H = \frac{p^2}{2} + \left[\frac{1}{2} x^2 + \frac{1}{4} x^4 \right]$$



$$|\psi_{3_1, 3_2}\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{i \frac{3_2}{\hbar} x} e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2}$$

$$e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2} \times e^{-\frac{1}{2} \alpha (x - \frac{3_1}{\alpha})^2}$$

$$a_1 = 0 \quad a_2 = -\frac{3_1}{\hbar}$$

$$H = \frac{1}{2} \left(\frac{3_2}{\hbar}\right)^2 + \left[\frac{3\alpha + 2\alpha^2 \hbar^2}{4\alpha^2} \left(\frac{3_1}{\hbar}\right)^2 + \frac{1}{4} \left(\frac{3_1}{\hbar}\right)^4 + \frac{3 + 4\alpha^3 \hbar^3 + 4\alpha^2 \hbar^2}{16\alpha^2} \right]$$

$$H = - \sum t_{ij} |i\rangle\langle j|$$

$$\epsilon(\vec{k}) = - \sum_{\vec{n}} t_{\vec{n}} e^{-i\vec{k}\cdot\vec{n}}$$

$$H = - \sum_{ij} t_{ij} |i\rangle\langle j|$$

$$\epsilon(\vec{k}) = - \sum_{\vec{n}} t_{\vec{n}} e^{-i\vec{k}\cdot\vec{n}}$$

$$|\vec{k}, \vec{x}\rangle = C \sum_i e^{-i(\vec{i}-\vec{x})^2/2z^2} e^{i\vec{k}\cdot\vec{i}} |\vec{i}\rangle$$

$$H = - \sum_{i,j} t_{ij} |i\rangle\langle j|$$

$$\epsilon(\vec{k}) = - \sum_{\vec{n}} t_{\vec{n}} e^{-i\vec{k}\cdot\vec{n}}$$

$$\leftarrow s_k \sim \frac{1}{3}$$

$$|\vec{k}, \vec{x}\rangle = C \sum_i \underbrace{e^{-(\vec{i}-\vec{x})^2/2\zeta^2}} e^{i\vec{k}\cdot\vec{i}} |\vec{i}\rangle$$

$$H = - \sum_{ij} t_{ij} |\vec{i}\rangle \langle \vec{j}| \\ + \sum_i \phi_i |\vec{i}\rangle \langle \vec{i}|$$

$$\epsilon(\vec{k}) = - \sum_{\vec{n}} t_{\vec{n}} e^{-i\vec{k}\cdot\vec{n}}$$

$$\leftarrow s_k \sim \frac{1}{3}$$

$$|\vec{k}, \vec{x}\rangle = \left(\sum_i \underbrace{e^{-(\vec{i}-\vec{x})^2/2\zeta^2}} e^{i\vec{k}\cdot\vec{i}} |\vec{i}\rangle \right)$$

$$e^{-i\vec{k}\vec{n}}$$

$$t_{\vec{n}} = t_{\vec{i}+\vec{n}, \vec{i}}$$

$$e^{-\frac{1}{2\vec{z}^2}(\vec{i}-\vec{x})^2} \underset{\psi^*}{\vec{i}} e^{-\frac{1}{2\vec{z}^2}(\vec{i}-\vec{x})^2} = \vec{x}$$

$\psi \quad \quad \psi$

$$e^{-i\vec{k}\vec{n}}$$

$$t_{\vec{n}} = t_{\vec{i}+\vec{n}, \vec{i}}$$

$$e^{-\frac{1}{2\vec{z}^2}(\vec{i}-\vec{x})^2}(\vec{i}-\vec{x})e^{-\frac{1}{2\vec{z}^2}(\vec{i}-\vec{x})^2} = 0$$

$$\psi^*(\vec{i}-\vec{x})\psi = 0$$

$$\langle \vec{k}(t), \vec{x}(t) | i \frac{d}{dt} | \vec{k}(t), \vec{x}(t) \rangle = -\vec{x} \cdot \dot{\vec{k}} + \cancel{\vec{x} \cdot \dot{\vec{x}}}$$

$$\langle \vec{k}, \vec{x} | \hat{H}_{\text{hop}} | \vec{k}, \vec{x} \rangle$$

$$\langle \vec{k}(t), \vec{x}(t) | i \frac{d}{dt} | \vec{k}(t), \vec{x}(t) \rangle = -\vec{x} \cdot \dot{\vec{k}} + \cancel{\vec{x} \cdot \dot{\vec{x}}}$$

$$\langle \vec{k}, \vec{x} | \hat{H}_{\text{hop}} | \vec{k}, \vec{x} \rangle = \epsilon(\vec{k}) + \left(\frac{\partial \epsilon}{\partial \vec{k}} \cdot \vec{\delta k} \right) \left(\frac{1}{3} \right)$$

$$\langle \vec{k}(t), \vec{x}(t) | i \frac{d}{dt} | \vec{k}(t), \vec{x}(t) \rangle = -\vec{x} \cdot \dot{\vec{k}} + \cancel{\dot{\vec{x}}}$$

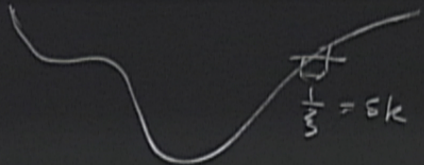
$$\langle \vec{k}, \vec{x} | \hat{H}_{hop} | \vec{k}, \vec{x} \rangle \approx \epsilon(\vec{k}) + \cancel{\left(\frac{\partial \epsilon}{\partial \vec{k}} \cdot \delta \vec{k} \right)}$$

$$\langle \vec{k}, \vec{x} | \hat{H}_f | \vec{k}, \vec{x} \rangle$$

$$\langle \vec{k}(t), \vec{x}(t) | i \frac{d}{dt} | \vec{k}(t), \vec{x}(t) \rangle = -\vec{x} \cdot \vec{k} + \cancel{\vec{x} \cdot \vec{k}}$$

$$\langle \vec{k}, \vec{x} | \hat{H}_{hop} | \vec{k}, \vec{x} \rangle \approx \epsilon(\vec{k}) + \cancel{\left(\frac{\partial \epsilon}{\partial \vec{k}} \cdot \delta \vec{k} \right)}$$

$$\langle \vec{k}, \vec{x} | \hat{H}_\phi | \vec{k}, \vec{x} \rangle = \phi(\vec{x})$$

$$\epsilon(k)$$


$$\frac{1}{3} = 5/k$$

$$L = \vec{k} \cdot \vec{x} - [\epsilon(\vec{k}) + \phi(\vec{x})] - \frac{d}{dt}(\vec{x} \cdot \vec{k})$$

$$\epsilon(k)$$

$\frac{1}{3} = \epsilon k$

$$L = \vec{k} \cdot \dot{\vec{x}} - [\epsilon(\vec{k}) + \phi(\vec{x})] - \frac{d}{dt}(\vec{x} \cdot \vec{k})$$

$$\dot{\vec{k}} = -\frac{\partial \phi}{\partial \vec{x}}, \quad \dot{\vec{x}} = \frac{\partial \epsilon}{\partial \vec{k}}$$

$$\dot{\vec{x}} = \frac{\vec{p}}{m}$$

$$\epsilon(k)$$

$\frac{1}{3} = \epsilon k$

$$\epsilon = \frac{1}{2m} \vec{k}^2$$

$$L = \vec{k} \cdot \dot{\vec{x}} - [\epsilon(\vec{k}) + \phi(\vec{x})] - \frac{d}{dt}(\vec{x} \cdot \vec{k})$$

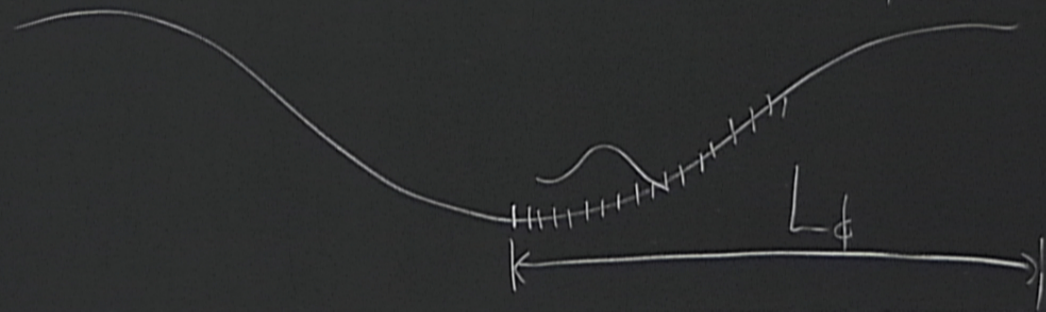
$$\dot{\vec{k}} = -\frac{\partial \phi}{\partial \vec{x}}, \quad \dot{\vec{x}} = \frac{\partial \epsilon}{\partial \vec{k}}$$

$$\dot{\vec{x}} = \frac{\vec{k}}{m}$$

$$\langle \vec{k}(t), \vec{x}(t) | i \frac{d}{dt} | \vec{k}(t), \vec{x}(t) \rangle = -\vec{x} \cdot \vec{k} + \cancel{\vec{x} \cdot \vec{k}}$$

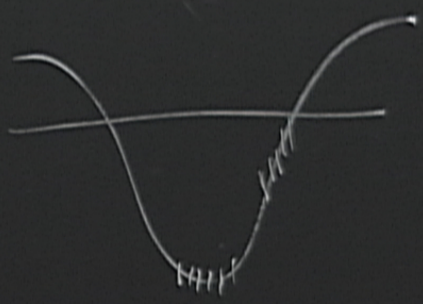
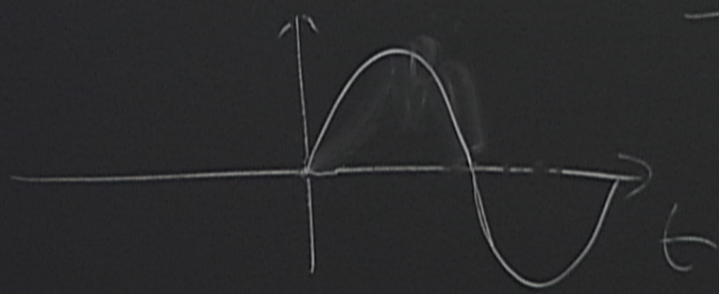
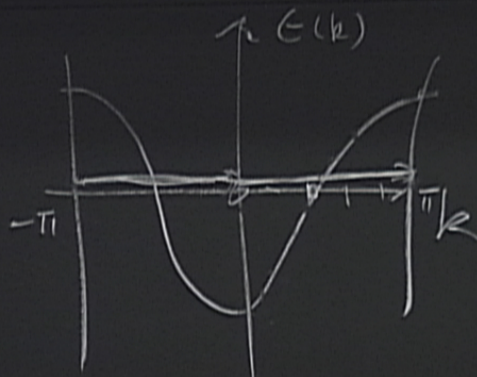
$$\langle \vec{k}, \vec{x} | \hat{H}_{hop} | \vec{k}, \vec{x} \rangle \approx \epsilon(\vec{k}) + \cancel{\left(\frac{\partial \epsilon}{\partial \vec{k}} \cdot \delta \vec{k} \right)}$$

$$\langle \vec{k}, \vec{x} | H_{\phi} | \vec{k}, \vec{x} \rangle = \phi(\vec{x})$$

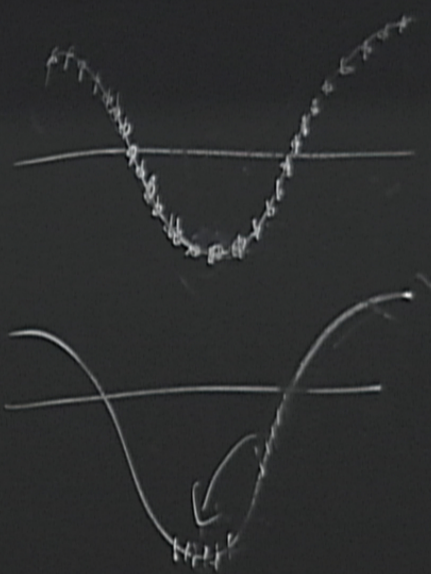
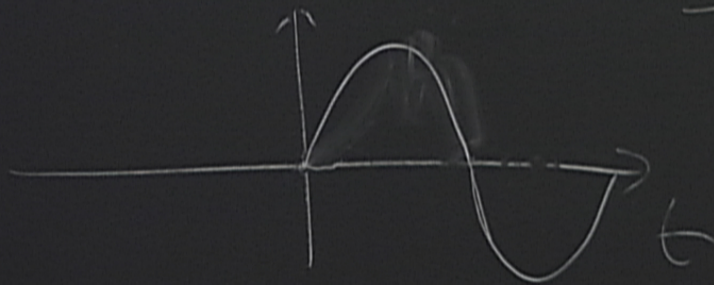
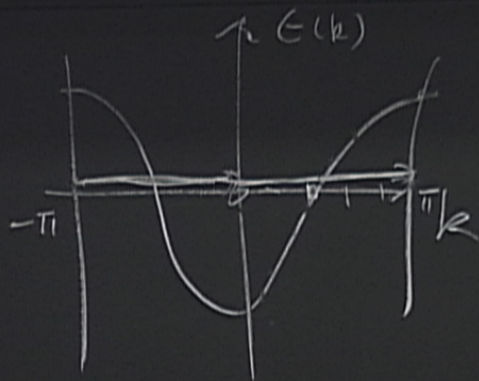


$$\lambda \gg a$$

$$\lambda \ll L_{\phi}$$



of levels = N



of levels = N

$$\frac{1}{3} = \epsilon k$$

$$\epsilon(k)$$

$$\epsilon = \frac{1}{2m} \vec{k}^2$$

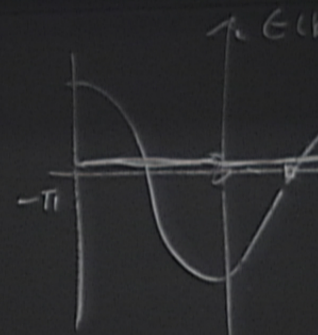
$$\epsilon = \sqrt{mc^2 + c^2 p^2}$$

$$\vec{k} \cdot \vec{x} = [\epsilon(\vec{k}) + \phi(\vec{x})] - \frac{d}{dt}(\vec{x} \cdot \vec{k})$$

$$\frac{\partial \phi}{\partial \vec{x}}$$

$$\vec{x} = \frac{\partial \epsilon}{\partial \vec{k}}$$

$$\vec{x} = \frac{\vec{k}}{m}$$



$$H_{\text{hop}} = \sum_{\vec{i}, \vec{j}} t_{\vec{i}, \vec{j}}^{ab} |\vec{i}, a\rangle \langle \vec{j}, b|$$

$$(M(\vec{k}))^{ab} = \sum_{\vec{n}} t_{\vec{n}}^{ab} e^{-i\vec{n} \cdot \vec{k}}$$

$$H_{\text{hop}} = \sum_{\vec{k}} M(\vec{k})^{ab} |\vec{k}, a\rangle \langle \vec{k}, b|$$

$$|\vec{k}, a\rangle = \sum_{\vec{j}} \frac{1}{\sqrt{N}} e^{i\vec{j} \cdot \vec{k}} |\vec{j}, a\rangle$$

$$|k, \gamma_\alpha\rangle = \sum_{j, \alpha} \frac{1}{\sqrt{N}} e^{i\vec{j} \cdot \vec{k}} |j, \alpha\rangle$$

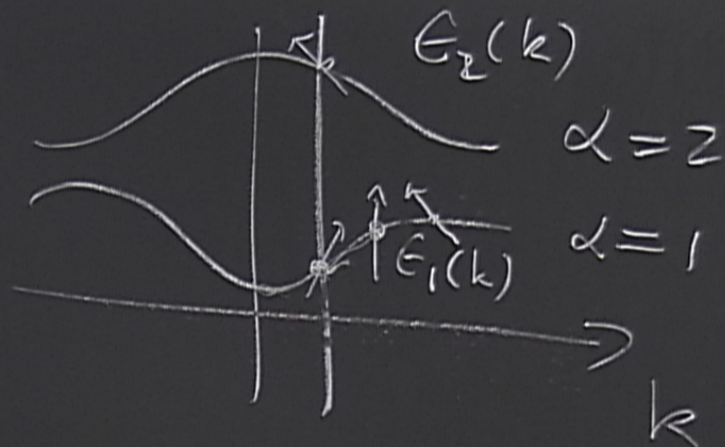
$$|\vec{k}, a\rangle = \sum_{\vec{j}} \frac{1}{\sqrt{N}} e^{i\vec{k} \cdot \vec{j}} |\vec{j}, a\rangle$$

$$|k, \psi_\alpha\rangle = \sum_{j,a} \frac{1}{\sqrt{N}} e^{i\vec{k} \cdot \vec{j}} (\psi_\alpha(\vec{k}))_a |\vec{j}, a\rangle$$

$$M(k)^{ab} (\psi_\alpha(\vec{k}))_b = \epsilon_\alpha(\vec{k}) (\psi_\alpha(k))_a$$

$$(\vec{k})_a | j, a \rangle$$

$$e))_a$$

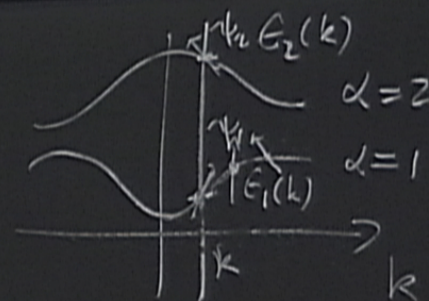


$$|\vec{k}, a\rangle = \sum_j \frac{1}{\sqrt{N}} e^{i\vec{k}\cdot\vec{j}} |j, a\rangle$$

$$|k, \psi_\alpha\rangle = \sum_{j,a} \frac{1}{\sqrt{N}} e^{i\vec{k}\cdot\vec{j}} (\psi_\alpha(\vec{k}))_a |j, a\rangle$$

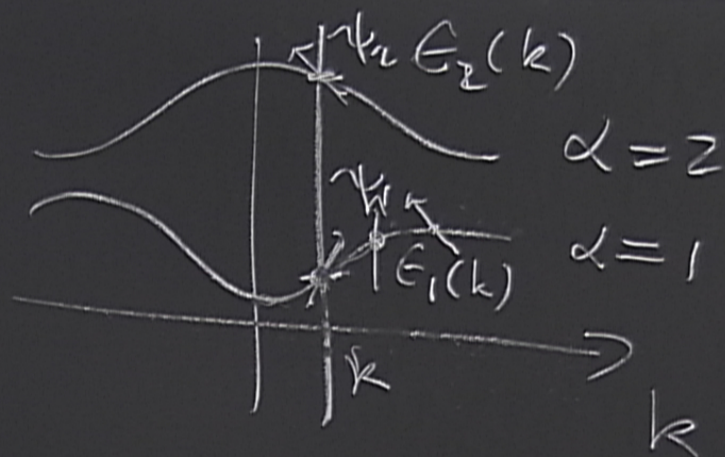
$$M(\vec{k})^{ab} (\psi_\alpha(\vec{k}))_b = \epsilon_\alpha(\vec{k}) (\psi_\alpha(\vec{k}))_a$$

$$M(\vec{k}) \psi_\alpha(\vec{k}) = \epsilon_\alpha(\vec{k}) \psi_\alpha(\vec{k})$$



$$\psi_{\alpha=1} = \begin{pmatrix} \psi_{\alpha=1} & \underline{a=1} \\ \psi_{\alpha=1} & \underline{a=2} \end{pmatrix}$$

$$(\vec{k})_a | j, a \rangle$$



$$\psi_{\alpha=1} = \begin{pmatrix} \psi_{\alpha=1}^{\alpha=1} \\ \psi_{\alpha=1}^{\alpha=2} \end{pmatrix}$$

$$|\vec{k}, x, \alpha\rangle = C \bar{z} e^{-(i-\vec{x})^2/2z^2} e^{i\vec{k}\cdot\vec{r}} \underbrace{\psi_{\alpha,a}(\vec{k})}_{\sim} |\vec{i}, a\rangle$$

$$|\vec{k}, \vec{x}, \alpha\rangle = C \bar{z} e^{-(i-\vec{x})^2/2z^2} e^{i\vec{k}\cdot\vec{r}} \underbrace{\psi_{\alpha, a}(\vec{k})}_{\sim} |\vec{i}, a\rangle$$

$$\langle \vec{k}, \vec{x}, \alpha | i \frac{d}{dt} | \vec{k}, \vec{x}, \alpha \rangle = -\vec{x} \cdot \vec{k} + \underbrace{\vec{c}(\vec{k}) \cdot \vec{k}}_{\sim}$$

$$\underbrace{\langle \vec{k}, \vec{x}, \alpha | H | \vec{k}, \vec{x}, \alpha \rangle}_{\sim} = E_{\alpha}(\vec{k}) + \phi(x)$$

$$(\vec{k}) \quad |i, a\rangle$$

$$c^i(\vec{k}) = \psi_a^\dagger(\vec{k}) \frac{\partial}{\partial \vec{k}} \psi_a(\vec{k})$$

$$\psi_a(\vec{k}) = \begin{pmatrix} \psi_{a1}(\vec{k}) \\ \psi_{a2}(\vec{k}) \\ \vdots \\ 1 \end{pmatrix}$$

$$|\vec{k}, \vec{x}, \alpha\rangle = C \bar{z} e^{-(i-\vec{x})^2/2z^2} e^{i\vec{k}\vec{r}} \underbrace{\psi_{\alpha,a}(\vec{k})}_{\sim} |\vec{r}, a\rangle$$

$$\langle \vec{k}, \vec{x}, \alpha | i \frac{d}{dt} | \vec{k}, \vec{x}, \alpha \rangle = -\vec{x} \cdot \vec{k} + \underbrace{\vec{c}(\vec{k}) \cdot \vec{k}}_a$$

$$\underbrace{\langle \vec{k}, \vec{x}, \alpha | H | \vec{k}, \vec{x}, \alpha \rangle}_{\sim} = \underbrace{E_q(\vec{k}) + \phi(x)}_a$$

$$|\vec{k}, \vec{x}, \alpha\rangle = C \bar{z} e^{-\frac{(\vec{i} - \vec{x})^2}{2z^2}} e^{i\vec{k} \cdot \vec{i}} \psi_{\alpha, a}(\vec{k}) |\vec{i}, a\rangle$$

$$\langle \vec{k}, \vec{x}, \alpha | i \frac{d}{dt} | \vec{k}, \vec{x}, \alpha \rangle = \underbrace{\left(-\vec{x} \cdot \vec{k} \right)}_{\vec{k} \cdot \vec{x} + \frac{d}{dt} \#} + \underbrace{\vec{C}(\vec{k}) \cdot \vec{k}}_{\vec{k} \cdot \vec{x} + \frac{d}{dt} \#} + O(\vec{x})$$

$$\langle \vec{k}, \vec{x}, \alpha | H | \vec{k}, \vec{x}, \alpha \rangle = \underline{E_{\alpha}(\vec{k}) + \phi(x)}$$

$$L = \vec{k} \cdot \vec{x} + \vec{C} \cdot \vec{k} - (E(\vec{k}) + \phi(\vec{x}))$$

$$\vec{k}_i = -\frac{\partial \phi}{\partial x^i} + \cancel{B^{ij} \dot{x}^j} \quad \dot{x}^i = \frac{\partial \mathcal{L}}{\partial k^i} + \underbrace{D^{ij} k_j}$$

$$D^{ij}(\vec{k}) = \frac{\partial C^j}{\partial k^i} - \frac{\partial C^i}{\partial k^j}$$