

Title: 13/14 PSI - Quantum Field Theory II - Lecture 15

Date: Nov 29, 2013 09:00 AM

URL: <http://pirsa.org/13110026>

Abstract:

Quantization of non-abelian Gauge theory : Gauge Fixing Lorentz-Landau-Feynman.

$$SU(2) \quad A_\mu^a \quad a=1,2,3$$

$$F[A]=0$$

$$\partial_\mu A_\nu^a = \epsilon_{\mu\nu}^a \quad \text{Fixed Functions}$$

$$\int D[A] \exp(iS[A]) = \int D[A] \delta[F[A]] |\det[F[A]]| \exp(iS[A])$$

$$S[A] = -\frac{1}{2g^2} \int d^4x \text{Tr}[F_{\mu\nu} F^{\mu\nu}] \quad \left(= \int D[A, c, \bar{c}] \delta[F[A]] \exp(iS[A] + S_{\text{ghost}}[A, c, \bar{c}]) \right)$$

orientz-Landau-Feynman.

$$\eta^a = \epsilon^a$$

Fixed Function

))

$$A] + S_{ghost}[A, c, \bar{c}])$$

Ghosts: $C^a(x)$ $\bar{C}^a(x)$

Grassmann anticommuting
field

Spin 0, charge

$$S_{ghost}[A, c, \bar{c}] = \int d^4x [\bar{C}_a(-\Delta) C_a + \epsilon_{abc} (\partial_\mu \bar{C}_a) A_\mu^b \cdot C^c]$$

Landau-Feynman.

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Spin 0, charge

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* The final result should not depend on ϵ (gauge invariance)

$A] + S_{\text{ghost}}[A, c, \bar{c}]$

orient3-Landau-Feynman.

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$\chi^a = \epsilon^a$
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$A] + S_{\text{ghost}}[A, c, \bar{c}])$

$$Z = \int \mathcal{D}[\epsilon] \exp\left(-\frac{i}{2\xi} \int d^4x \epsilon^a(x) \epsilon_a(x)\right)$$

orient3-landau-Feynman.

* Ghosts: $C^a(x)$ $\bar{C}^a(x)$

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Spin 0, charge

$\chi^a = \epsilon^a$
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* The final result should not depend on ϵ (gauge invariance) Trick

$$Z = \int D[\epsilon] \exp\left(-\frac{i}{2\xi} \int d^4x \epsilon^a(x) \epsilon_a(x)\right) \int \delta[\partial A - \epsilon] \dots D[A]$$

$A] + S_{\text{ghost}}[A, c, \bar{c}])$

$$S[A, c, \bar{c}] = S_{\text{YM}}[A] + S_{\text{ghosts}}[A, c, \bar{c}] + S_{\text{additional}}[A]$$

$$S_{\text{additional}}[A] = -\frac{1}{2\xi} \int d^D x \partial_\mu A_a^\mu \partial_\nu A_a^\nu$$

modify the quadratic part
in $A \cdot A$
change the propagator of A

suppress the zero-modes of the quadratic form in S_{YM}

entz-Landau-Feynman.

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Grassmann anticommuting field

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ξ is a real parameter (arbitrary) $\Rightarrow$ the physics should not depend

Wentz-Landau-Feynman.

* Ghosts: $C^a(x)$ $\bar{C}^a(x)$

Grassmann anticommuting field

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Quantization of non-abelian Gauge theory : Gauge Fixing Lorentz-Landau

$$SU(2) \quad A_\mu^a \quad a=1,2,3$$

$$F[A]=0$$

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$$\int D[A] \exp(iS[A]) = \int D[A] \delta[F[A]] |\det[F[A]]| \exp(iS[A])$$

$$S[A] = -\frac{1}{2g^2} \int d^4x \text{Tr}[F_{\mu\nu} F^{\mu\nu}] = \int D[A, c, \bar{c}] \delta[F[A]] \exp(i(S[A] + S_{\text{ghost}}[A, c, \bar{c}]))$$

$$(\partial A - \partial A)^2 + A A (\partial A - \partial A) + A A A A$$

Maxwell

non-abelian

$$S[A, c, \bar{c}] = S_{\text{YM}}[A] + S_{\text{ghosts}}[A, c, \bar{c}] + S_{\text{additional}}[A]$$

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modify the quadratic part
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change the propagator of A

suppresses the zero-modes of the quadratic form in S_{YM}

Feynman Rules out of this action

Gauge propagator: $\leftarrow$ same as QED

$$\begin{array}{c} a \\ \mu \end{array} \text{---} \text{wavy line} \text{---} \begin{array}{c} b \\ \nu \end{array} = \langle A_\mu^a A_\nu^b \rangle_{\text{tree}} = \delta_{ab} \frac{-i}{k^2 - \epsilon_+} \left(\eta^{\mu\nu} + (\xi - 1) \frac{k^\mu k^\nu}{k^2} \right)$$

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Feynman Rules out of this action $k^2 = -k_0^2 + \vec{k}^2$

Gauge propagator: $\Leftarrow$ same as QED

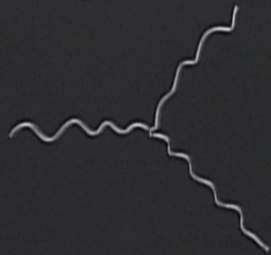
$$\begin{array}{c} a \\ \mu \end{array} \text{---} \text{wavy line} \text{---} \begin{array}{c} b \\ \nu \end{array} = \langle A_\mu^a A_\nu^b \rangle_{\text{tree}} = \delta_{ab} \frac{-i}{k^2 - \epsilon_+} \left(\eta^{\mu\nu} + (\xi - 1) \frac{k^\mu k^\nu}{k^2} \right)$$

Ghost propagator:

$$a \text{ --- } \rightarrow \text{ --- } b = \langle c^a(x) \bar{c}(y) \rangle \approx \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}$$

Interactions

3- gauge field vertex

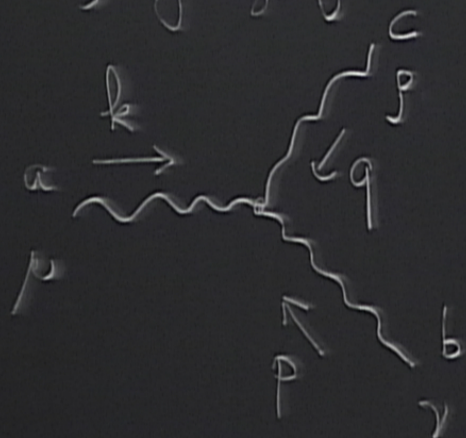


Ghost propagator:

$$a \xrightarrow{x} \text{---} \xrightarrow{y} b = \langle c^a(x) \bar{c}(y) \rangle \approx \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}$$

Interactions

3-gauge field vertex



$$g \left[\epsilon_{abc} h^{\mu\nu} (k-p)^\mu + h^{\nu\rho} (k-p)^\mu + h^{\rho\nu} (k-p)^\nu \right]$$

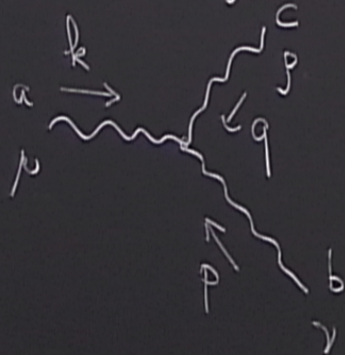
↑
gauge coupling

Ghost propagator:

$$a \text{ --- } x \text{ --- } y \text{ --- } b = \langle c^a(x) \bar{c}(y) \rangle \approx \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}$$

Interactions

3-gauge field vertex

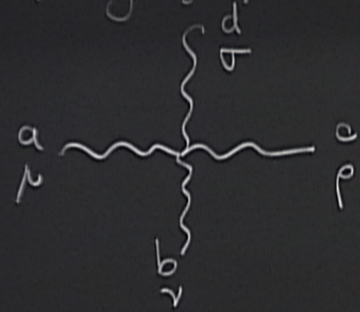


$$g \left[\epsilon_{abc} h^{\mu\nu} (k-p)^\rho + h^{\nu\rho} (k-p)^\mu + h^{\rho\mu} (k-p)^\nu \right]$$

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gauge coupling

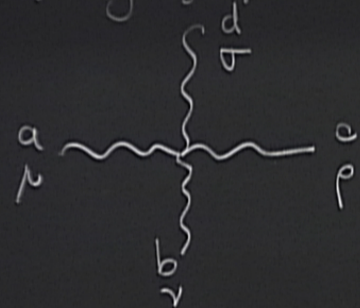
Spin dependent interactions

4 - gauge fields vertex



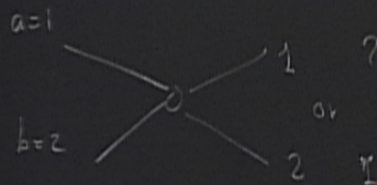
$$(-i) g^2 \left[\epsilon_{abe} \epsilon_{cde} (h^{\mu\rho} h^{\nu\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right. \\ \left. + \epsilon_{ace} \epsilon_{bde} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right. \\ \left. + \epsilon_{ade} \epsilon_{bce} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right]$$

4 - gauge fields vertex

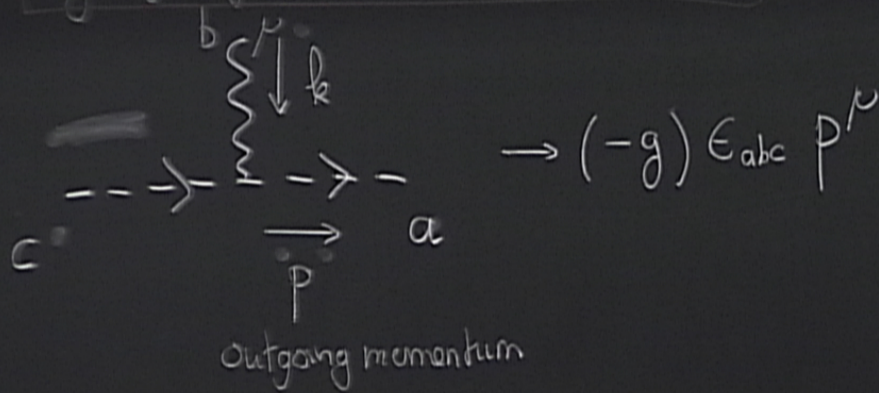


$$(-i) g^2 \left[\epsilon_{abe} \epsilon_{cde} (h^{\mu\rho} h^{\nu\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right. \\ \left. + \epsilon_{ace} \epsilon_{bde} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right. \\ \left. + \epsilon_{ade} \epsilon_{bce} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \right]$$

$$\epsilon_{abe} \epsilon_{cde} \approx \delta_{ac} \delta_{bd} - \delta_{ad} \delta_{bc}$$



gauge-ghost interaction vertex



Tree Level. Gauge invariance

gauge particle momentum $P=(\vec{p})$ and polarization $\epsilon=(\epsilon^\mu)$
quantum number a

$$A_\mu^a(x) \approx \epsilon_\mu \exp(i p \cdot x)$$

Tree Level. Gauge invariance

gauge particle momentum $P=(\vec{p})$ and polarization $\epsilon=(\epsilon^\mu)$
quantum number a

$$A_\mu^a(x) \sim \epsilon_\mu \exp(i P \cdot X)$$

2 physical states, $\epsilon_\mu P^\mu = 0 \Rightarrow \epsilon_\mu \left(h^{\mu\nu} + (3-1) \frac{h^\mu h^\nu}{h^2} \right) \epsilon_\nu$
transverse polarizations

Tree Level. Gauge invariance

gauge particle momentum $P=(P^\mu)$ and polarization $\epsilon=(\epsilon^\mu)$
 quantum number a

$$A_\mu^a(x) \sim \epsilon_\mu \exp(i P \cdot x) V^a$$

2 physical states;

transverse polarizations

if $P^2=0$ on-shell

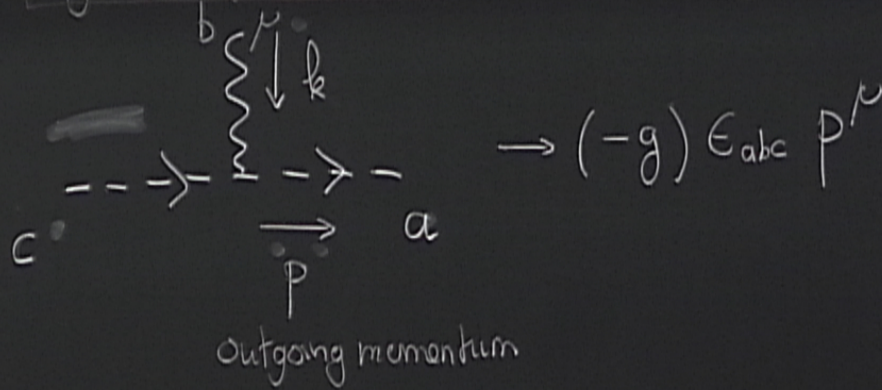
$$\epsilon_\mu P^\mu = 0 \Rightarrow \epsilon_\mu \left(\eta^{\mu\nu} + (\xi-1) \frac{P^\mu P^\nu}{P^2} \right) \epsilon_\nu$$

$$P = \begin{pmatrix} E \\ \mathbf{k} \\ 0 \\ 0 \end{pmatrix}$$

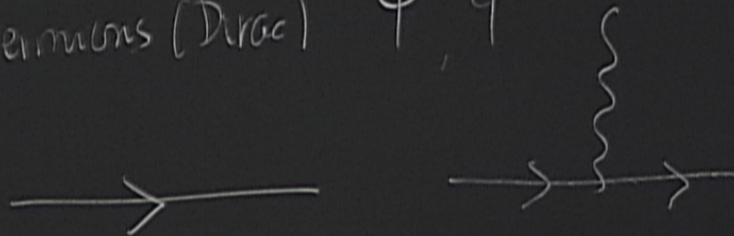
$$E = |\mathbf{k}|$$

$$\left[\begin{array}{l} \epsilon_{abc} \epsilon_{cde} (h^{\mu\rho} h^{\nu\sigma} - h^{\mu\sigma} h^{\nu\rho}) \\ \epsilon_{abc} \epsilon_{bde} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \\ \epsilon_{abc} \epsilon_{bce} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\rho} h^{\nu\sigma}) \end{array} \right]$$

gauge-ghost interaction vertex



Fermions (Dirac) $\psi, \bar{\psi}$



Tree Level

gauge part

$$A_\mu^a(x) \sim \theta$$

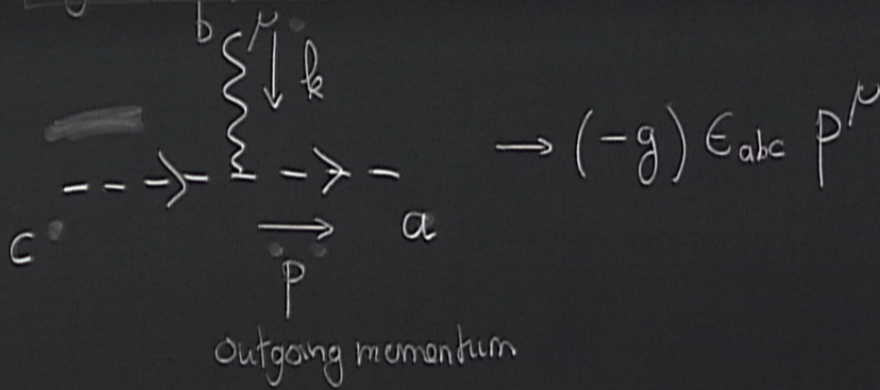
2 physical

transverse po

$$\text{if } p^2 = 0$$

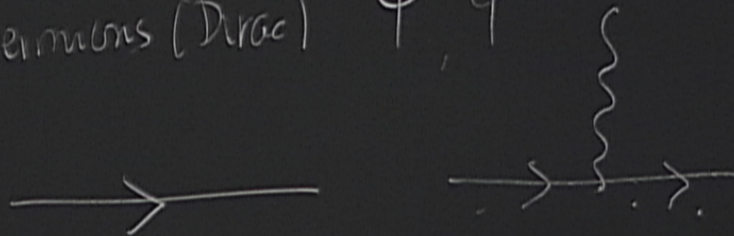
$$\left[\begin{array}{l} \epsilon_{abc} \epsilon_{cde} (h^{\mu\rho} h^{\nu\sigma} - h^{\mu\sigma} h^{\nu\rho}) \\ \epsilon_{ace} \epsilon_{bde} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\sigma} h^{\nu\rho}) \\ \epsilon_{ade} \epsilon_{bce} (h^{\mu\nu} h^{\rho\sigma} - h^{\mu\rho} h^{\nu\sigma}) \end{array} \right]$$

gauge-ghost interaction vertex



Fermions (Dirac) $\psi, \bar{\psi}$

as in QED



Tree Level

gauge part

$$A_\mu^a(x) \sim \theta$$

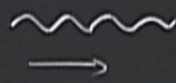
2 physical

transverse pos

$$\text{if } p^2 = 0$$

1 loop diagrams $\Rightarrow$ radiative or quantum corrections (Renormalizable?)

propagator



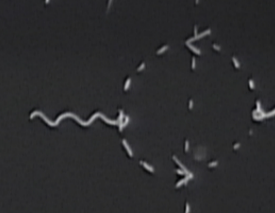
$+\frac{1}{2}$



$+\frac{1}{2}$



$-$



$$G^{\mu\nu}(p)$$

p

gauge independent?

it is gauge independent

$$\epsilon_\mu G^{\mu\nu} \epsilon_\nu \quad \epsilon_\mu p^\mu = 0$$

unphysical states with // polarization

gluon loop

ghosts are fermions

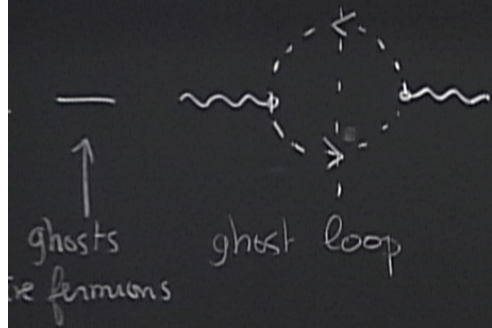
ghost loop

depend

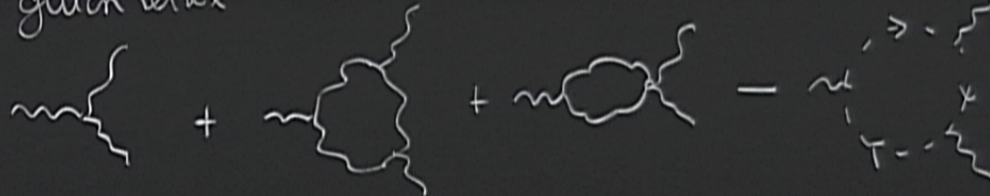
zahl ?)

gluons

$SU(3)$ strong interactions



3 gluon vertex



h ?)

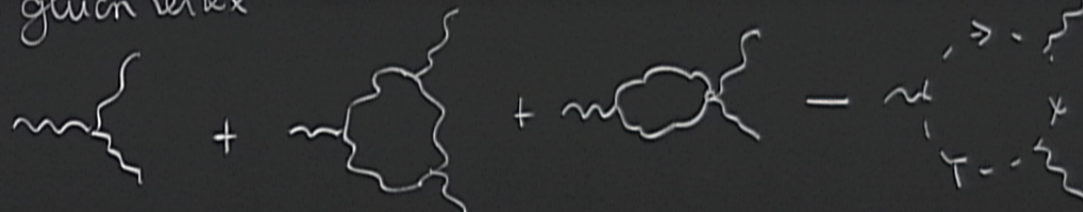
gluons

$SU(3)$ strong interactions

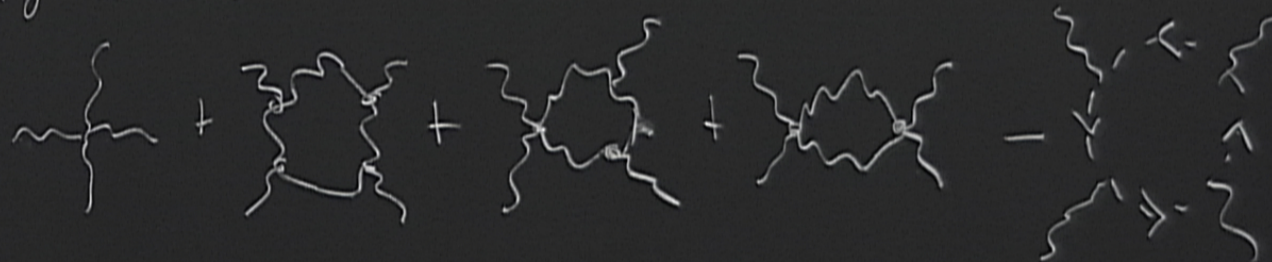


ghost loop

3 gluon vertex



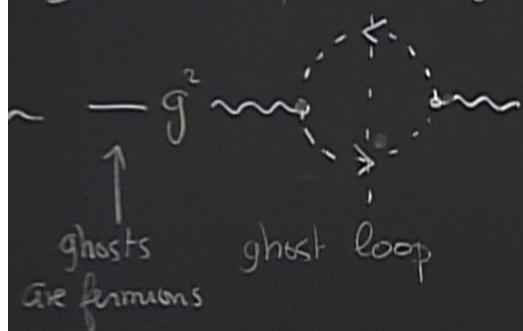
4 gluon vertex



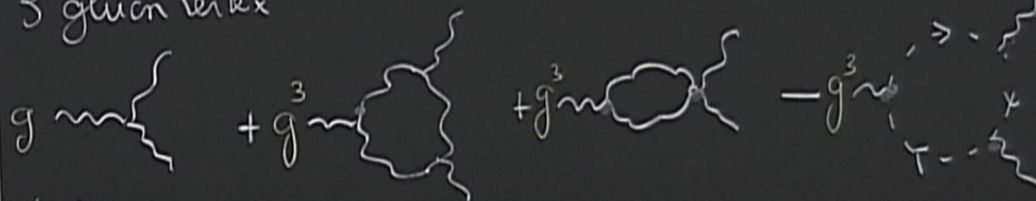
linearizable ?)

gluons

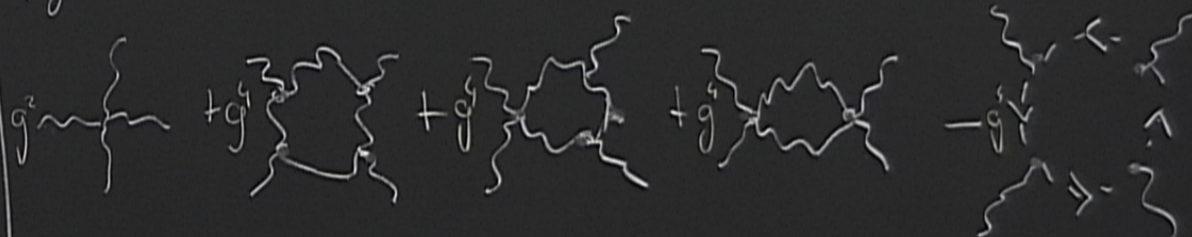
SU(3) strong interactions



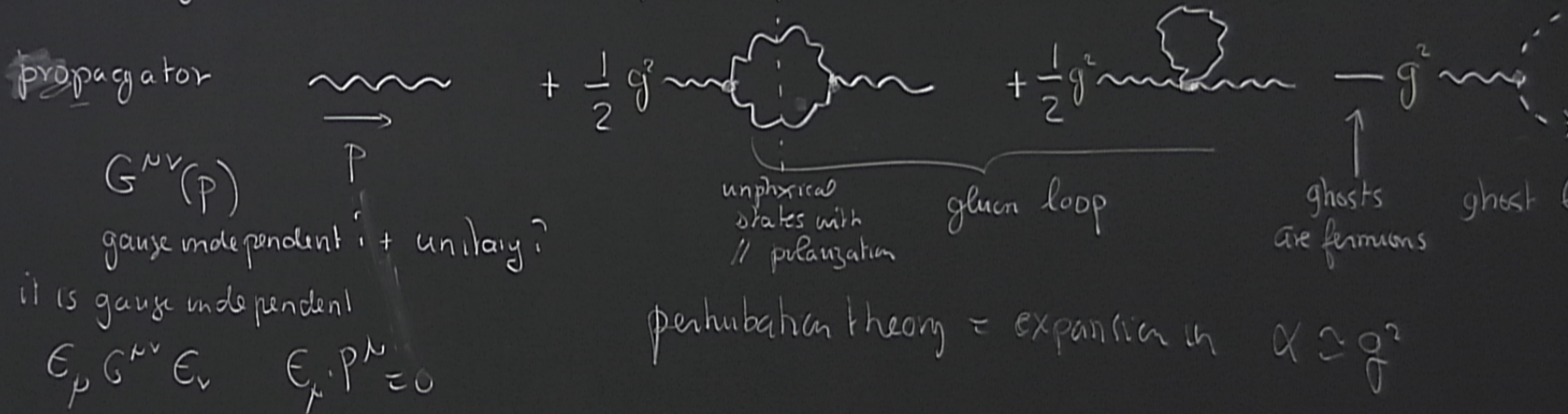
3 gluon vertex



4 gluon vertex



1 loop diagrams $\Rightarrow$ radiative or quantum corrections (Renormalizable?)



λ is a real parameter (arbitrary)

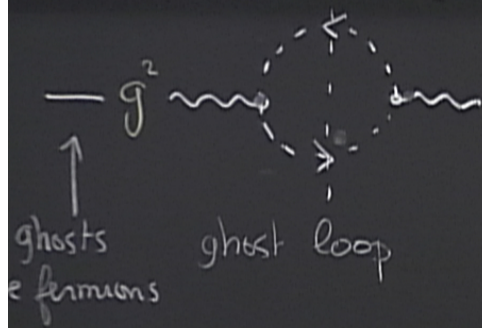
$\Rightarrow$ The physics should not depend

gauge ?)

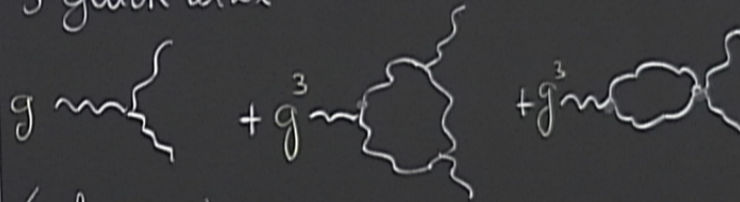
gluons

$SU(3)$ strong interactions

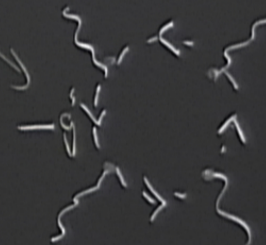
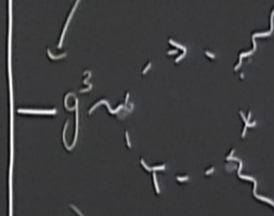
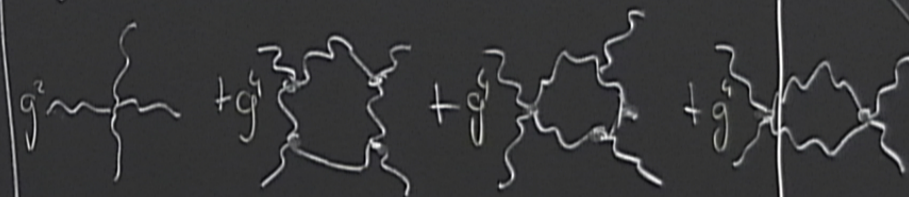
ghosts are fermions



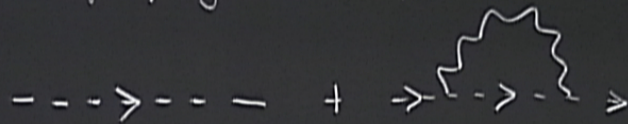
3 gluon vertex



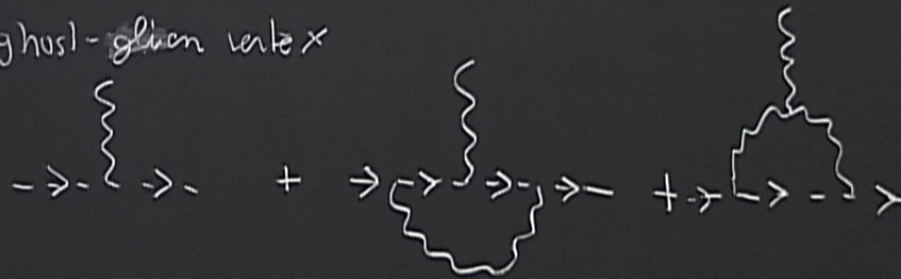
4 gluon vertex



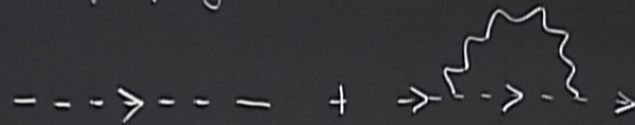
ghost propagator



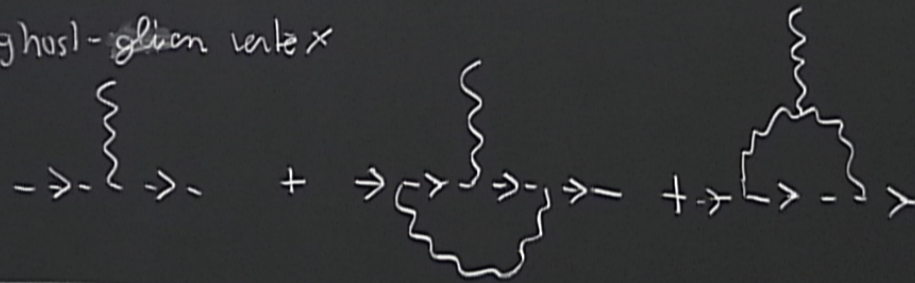
ghost-gluon vertex



ghost propagator



ghost-gluon vertex



Renormalization

$$\text{ghost loop} - \text{ghost loop} \approx \langle A_\mu^a \rangle = 0$$

gauge invariance

ghost propagator

$$1 \dashrightarrow \dashrightarrow + g \rightarrow \text{loop} \rightarrow \dashrightarrow$$

ghost-gluon vertex

$$g \dashrightarrow \text{loop} \dashrightarrow + g^3 \rightarrow \text{loop} \rightarrow \dashrightarrow + \text{loop} \rightarrow g^2$$

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

$$\text{loop} - \text{loop} \approx \langle A_\mu^a \rangle = 0$$

gauge invariance

ghost propagator

$$1 \dashrightarrow \dashrightarrow + g \rightarrow \text{loop} \rightarrow \dashrightarrow$$

ghost-gluon vertex

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$$\text{loop} - \text{loop} \approx \langle A_\mu^a \rangle = 0$$

gauge invariance

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

$$\text{dashed line} \sim \frac{1}{k^2}$$

$$P_1 \rightarrow \text{triangle} \rightarrow P_2 \sim P_1$$

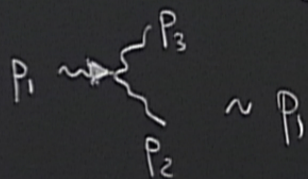
$$\text{wavy line} \sim 1$$

$$\text{triangle} \sim P_2$$

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

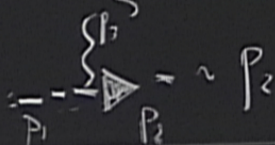
$$\text{dashed line} \sim \frac{1}{k^2}$$



$$\sim p_1$$

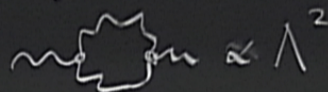


$$\sim 1$$



$$\sim p_2$$

might have



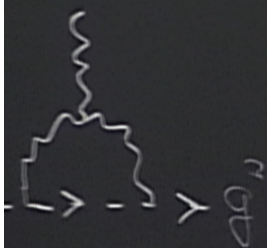
$$\sim \Lambda^2$$



$$\sim \Lambda^2$$



$$\sim \Lambda^2$$

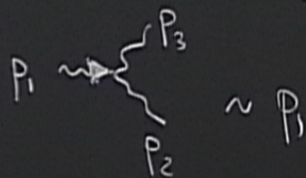


gauge invariance

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

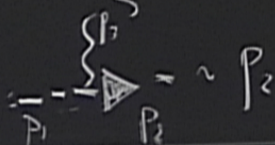
$$\text{dashed line} \sim \frac{1}{k^2}$$



$$\sim p_1$$



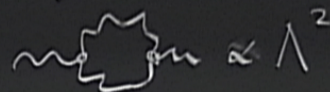
$$\sim p_1$$



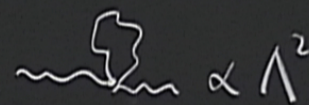
$$\sim p_2$$

gauge invariance

might have



$$\propto \Lambda^2$$



$$\propto \Lambda^2$$



$$\propto \Lambda^2$$

very bad! $\Rightarrow A_\mu^a A_\mu^b \Lambda^2$

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

$$\text{dashed line} \sim \frac{1}{k^2}$$

$$P_1 \rightarrow \begin{array}{c} P_3 \\ \nearrow \\ P_2 \end{array} \sim P_1$$

$$\text{wavy line} \sim 1$$

$$\begin{array}{c} \text{wavy line} \\ \nearrow \\ \text{triangle} \end{array} \sim P_2$$

might have

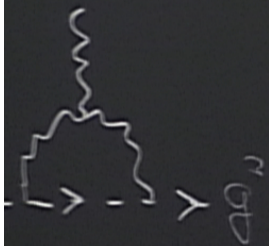
$$+ \text{wavy line} \propto \Lambda^2$$

$$+ \text{wavy line} \propto \Lambda^2$$

$$- \text{wavy line} \propto \Lambda^2$$

$$\text{very bad!} \Rightarrow A_\mu^a A_\mu^b \Lambda^2$$

not-gauge invariant



gauge invariance

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

$$\text{dashed line} \sim \frac{1}{k^2}$$

$$P_1 \rightarrow \begin{array}{c} P_3 \\ \nearrow \\ P_2 \end{array} \sim P_1$$

$$\text{wavy line} \sim 1$$

$$\begin{array}{c} \text{ghost} \\ \nearrow \\ P_2 \end{array} \sim P_2$$

gauge invariance

might have

$$+ \text{wavy loop} \propto \Lambda^2$$

$$+ \text{wavy loop} \propto \Lambda^2$$

$$- \text{ghost loop} \propto \Lambda^2$$

$$\text{very bad!} \Rightarrow A_\mu^a A_\mu^b \Lambda^2$$

not-gauge invariant

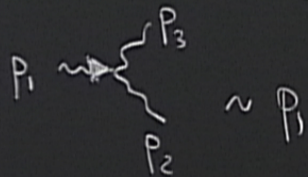
thanks to the ghost!

$$= 0$$

Renormalization

$$\text{wavy line} \sim \frac{1}{k^2}$$

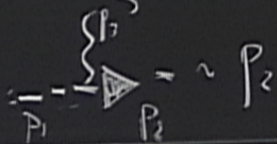
$$\text{dashed line} \sim \frac{1}{k^2}$$



$$\sim p_1$$



$$\sim 1$$



$$\sim p_2$$

gauge invariance

might have quadratic divergences.

$$+ \text{wavy loop} \propto \Lambda^2 \quad (p^\mu p^\nu - \eta^{\mu\nu} p^2) \log \Lambda \quad \text{😊}$$

$$+ \text{wavy loop} \propto \Lambda^2$$

$$- \text{dashed loop} \propto \Lambda^2$$

very bad! $\Rightarrow A_\mu^a A_\mu^b \Lambda^2$

not-gauge invariant

thanks to the ghost!

$$= 0$$

might have quadratically divergent.

 $\propto \Lambda^2$ $(\not{p} \not{p} - h^{\mu\nu} p^{\mu} p^{\nu}) \log \Lambda$ ☺

 $\propto \Lambda^2$

 $\propto \Lambda^2$

very bad! $\Rightarrow A_{\mu}^a A_{\mu}^b \Lambda^2$

not-gauge invariant

thanks to the ghost!

$= 0$

Gauge invariance.

QED: Ward Identities

YM: Taylor-Slavnov Identities

might have quadratically divergent

 $\propto \Lambda^2$ $(p^\mu p^\nu - \eta^{\mu\nu} p^2) \log \Lambda$ ☺

 $\propto \Lambda^2$

 $\propto \Lambda^2$

very bad! $\Rightarrow A_\mu^a A_\mu^b \Lambda^2$

not-gauge invariant

thanks to the ghost!

$$= 0$$

Gauge invariance

QED: Ward Identities

YM: Taylor-Slavnov Identities

↓
Ensure gauge invariance

normalization

$$\sim \frac{1}{k^2}$$

$$\sim \frac{1}{k^2}$$

$$\sim p_1$$

$$\sim 1$$

$$\sim p_2$$

might have quadratic divergences.

$$+ \text{diagram} \propto \Lambda^2 \quad (p^\mu p^\nu - h^{\mu\nu} p^2) \log \Lambda \quad \text{☺}$$

$$+ \text{diagram} \propto \Lambda^2$$

$$- \text{diagram} \propto \Lambda^2$$

$$\text{very bad!} \Rightarrow A_\mu^a A_\mu^b \Lambda^2$$

not-gauge invariant

thanks to the ghost!

$$= 0$$

$$\text{diagram} \approx \phi \times \Lambda + p^\mu \log \Lambda$$

$$\text{diagram} \approx \log \Lambda$$

$$\text{diagram} \approx \phi \Lambda + \log \Lambda$$

$$\downarrow$$

$$\Lambda \bar{C} C \Rightarrow \text{mass for the ghost}$$

Counterterm

Gauge

QED:

YM: T

Ensue

- 't Hooft & Veltman
Dimensional regularization
 $d = 4 - \epsilon$

- 't Hooft & Veltman

Dimensional regularization : respects gauge invariance

$$d = 4 - \epsilon \quad \text{Log} \Lambda \rightarrow \frac{1}{\epsilon} \text{ poles}$$

- T.S. identity $\Rightarrow$ general proof : B. Lee & J. Zinn-Justin

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- B.R.S.T symmetry (Supersymmetric hidden symmetry)

counterterms

$$Z_A = 1 + g^2 \frac{1}{\epsilon} C_A \dots$$

$$A = 1, \dots, 5$$

$$S_R[A, c, \bar{c}] = Z_1 \partial A \cdot \partial A + g Z_2 \partial A \cdot A \cdot A + g^2 Z_3 A \cdot A \cdot A \cdot A + Z_4 \bar{c} c + g Z_5 \partial \bar{c} \cdot c \cdot A$$

- 't Hooft & Veltman

Dimensional regularization: respects gauge invariance
 $d = 4 - \epsilon$ $\log \Lambda \rightarrow \frac{1}{\epsilon}$ poles

- T.S. identity $\Rightarrow$ general proof: B. Lee & J. Zinn-Justin
- B.R.S.T symmetry (Supersymmetric hidden symmetry)

counterterms

$$Z_A = 1 + g^2 \frac{1}{\epsilon} C_A \dots$$

$A = 1, \dots, 5$

$$S[A, c, \bar{c}] = Z_1 \int d^4x \partial_\mu A^\mu \partial_\nu A^\nu + g Z_2 \int d^4x \partial_\mu A^\mu A^\nu A^\nu + g^2 Z_3 \int d^4x A^\mu A^\mu A^\nu A^\nu + Z_4 \int d^4x \bar{c} c + g Z_5 \int d^4x \partial_\mu \bar{c} \cdot c \cdot A^\mu =$$

$\int_R$
 renormalized action $\neq$ Gauge invariant if only 3 Z_A are independent

ance $A \in$
 $A = 1, \dots, 5$

Justin
 (theory)

$$Z_3 A \cdot A \cdot A \cdot A + Z_4 \bar{c} \cdot c + g Z_5 \partial \bar{c} \cdot c \cdot A = S[\tilde{Z}_A, \tilde{Z}_c, \tilde{Z}_{\bar{c}}, Z_g]$$

re independent $\tilde{Z}_A, \tilde{Z}_{ghost}, \tilde{Z}_g$ coupling constant

$$+g Z_5 \partial \bar{c} \cdot c A = S[\tilde{Z}_A A, \tilde{Z}_c c, \tilde{Z}_{\bar{c}} \bar{c}, Z_g g]$$

hence, Z_g coupling constant $\leftarrow$ gauge invariance ensure that this is true

$$\dim [A] = 1, \dim [g] = 0$$

$$\dim [c]$$

$$\dim[A] = 1, \dim[g] = 0$$

$$\dim[c] = 1$$

$$\text{Beta function } \mu \frac{\partial g}{\partial \mu} = \beta(g)$$

Ren scale $\uparrow$

YM is renormalizable !

$$\beta(g)$$

SU(N) gauge theory

$$\dim[A] = 1, \dim[g] = 0$$

$$\dim[c] = 1$$

$$\text{Beta function } \mu \frac{\partial}{\partial \mu} g = \beta(g)$$

Ren scale $\uparrow$

't Hooft

Gross & Wilczek, Politzer 74

YM is renormalizable!

$$\beta(g) = g^3 \frac{1}{(4\pi)^2} \left(-\frac{11}{3} N \right)$$

SU(N) gauge theory

Renormalized action = gauge invariant if only $\angle_A$ are independent $\angle_A / \angle_{ghost}, \angle_g$ coupling

$$\dim[A] = 1, \dim[g] = 0$$

$$\dim[c] = 1$$

$$\text{Beta function } \mu \frac{\partial g}{\partial \mu} = \beta(g)$$

Ren scale $\uparrow$

't Hooft

Gross & Wilczek, Politzer 74

YM is renormalizable!

$$\beta(g) = g^3 \frac{1}{(4\pi)^2} \left(-\frac{11}{3} N + \frac{4}{3} N_{\text{Fermions}} \right)$$

$\uparrow$ minus sign.

SU(N) gauge theory

$N = \dim$ of fundamental representation

Renormalized action = Satisfies if only $\mathcal{L}_A$ are independent $\mathcal{L}_A / \mathcal{L}_{ghost}, \mathcal{L}_g$

$$\dim[A] = 1, \dim[g] = 0$$

$$\dim[c] = 1$$

$$\text{Beta function } \mu \frac{\partial g}{\partial \mu} = \beta(g)$$

Ren scale $\uparrow$

't Hooft

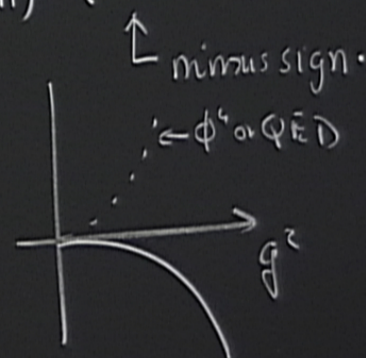
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SU(N) gauge theory

$N = \dim$ of fundamental representation



Running coupling constant is small at high energies
large at small energies

$\hookrightarrow$ coupling constant $\Leftarrow$ gauge invariance

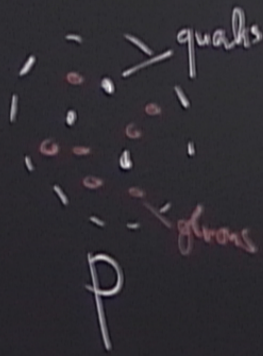
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$SU(3)$ QCD = Strong Interactions

N_{Fermions}

Running coupling constant is small at high energies
large at small energies

$SU(3) \text{ QCD} = \text{Strong Interactions}$

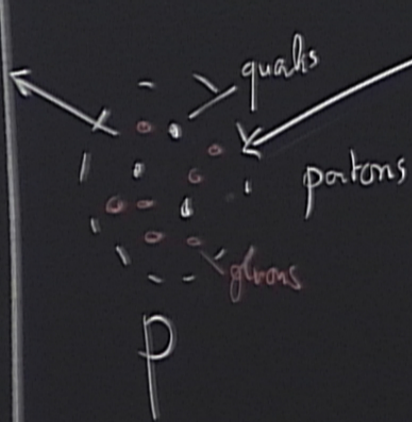


$\hookrightarrow$ coupling constant $\Leftarrow$ gauge invariance

Running coupling constant is small at high energies
large at small energies

$SU(3)$ QCD = Strong Interactions

N_{Fermions}



confinement

