Title: 13/14 PSI - Quantum Field Theory II - Lecture 10

Date: Nov 22, 2013 09:00 AM

URL: http://pirsa.org/13110019

Abstract:

Wilson R.G. : For the ϕ^4 QFT

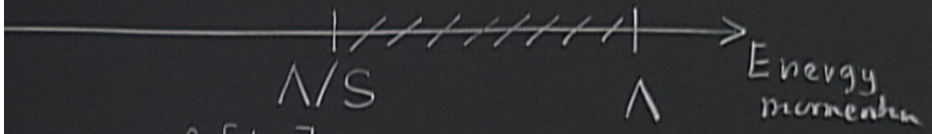
momentum space : Local Potential Approximation

Justified at 1 loop order

Wilson – Polchinsky Renormalized Action method ('84)

Wetterich " Effective action method (~90)

integrate out the high energy modes : $\frac{\Lambda}{S} < |k| < \Lambda$

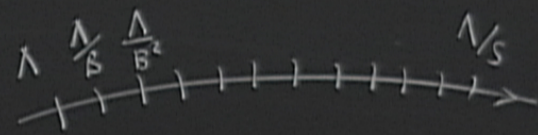


$\mathcal{A}_S[\phi_S] \longleftarrow \mathcal{A}[\phi]$

renormalized action
renormalized Field ϕ_S

Action

S rescaling Factor : S as a continuous parameter



write these RG transformation as a flow equation in the space of couplings

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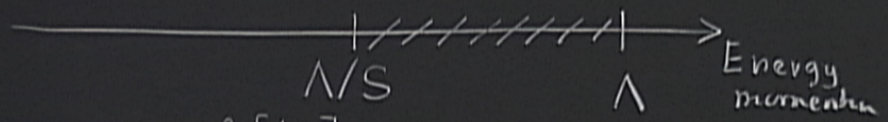
$$\mathcal{A}[\phi] = \int d^D x \sum_\alpha \kappa_\alpha \, O_\alpha(\phi, \nabla\phi, \cdots)$$

↑ coupling ↖ local operators

integrate out the high energy modes : $\frac{\Lambda}{S} < |k| < \Lambda$



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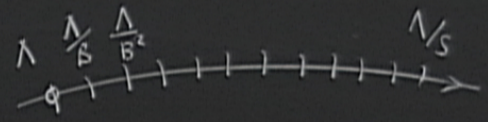
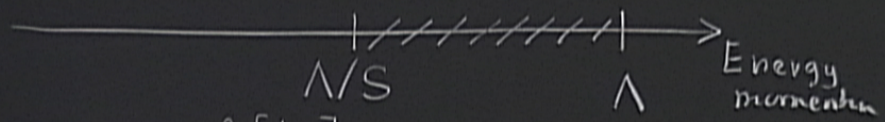


write these RG transformation as a flow equation in the space of couplings

$\{K_\alpha\} \longrightarrow \{K_\alpha(s)\}$

integrate out the high energy modes : $\frac{\Lambda}{S} < |k| < \Lambda$

$$\boxed{\log S = s}$$



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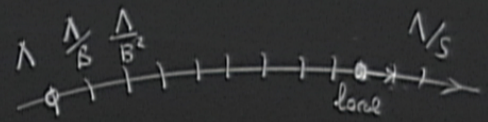

write these RG transformation as a flow equation in the space of couplings

$$\{K_\alpha\} \longrightarrow \{K_\alpha(s)\} \qquad s\frac{\partial K_\alpha(s)}{\partial s} = W[\{K_\beta(s)\}]$$

integrate out the high energy modes : $\frac{\Lambda}{S} < |k| < \Lambda$

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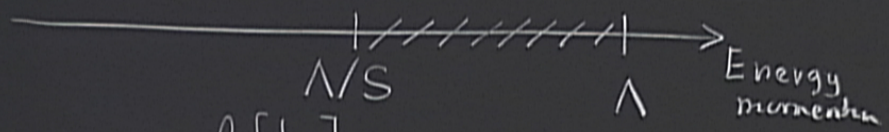

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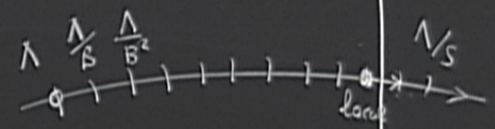


write the RG transformation as a flow equation in the space of couplings

$$\{K_\alpha\} \longrightarrow \{K_\alpha(S)\}$$

$\vec{K}$

$$\boxed{S\frac{\partial K_\alpha(S)}{\partial S} = W_\alpha[\{K_\beta(S)\}]}$$

Wilson Functions

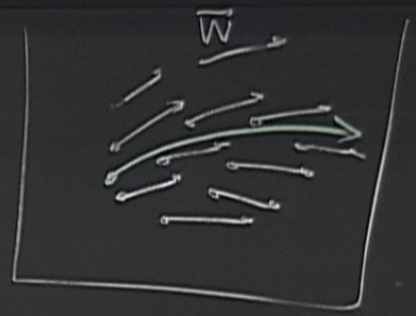
$\{W_\alpha\} = \vec{W}$ vector field

coupling local operators

$$s \frac{\partial}{\partial s} \vec{K}(s) = \vec{W}(\vec{K}(s))$$

Flow equation

RG flow generated by the vector field $\vec{W}$



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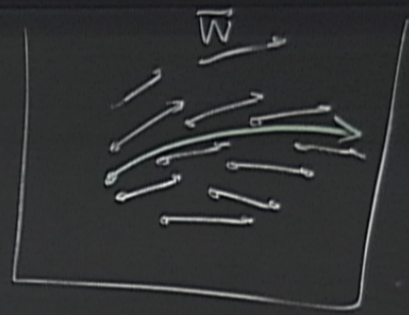

Statement: $W_\alpha \simeq -\beta$ Functions

$V(\phi)$ local potential (Even $\phi \leftrightarrow -\phi$ for simplicity)

$$V(\phi) = \sum_{\alpha=0}^{\infty} K_\alpha \phi^\alpha = K_0 + \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6 + \cdots$$

vacuum energy density (classical)

$t \simeq m^2$

Local Potential Approximation.

$$\mathcal{A}[\phi] = \int d^D x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$

(Euclidean)

$$\int \mathcal{D}[\phi] \exp(-\mathcal{A}[\phi])$$

Sharp cut-off

$V(\phi)$ local potential (Eu

$$V(\phi) = \sum_{\alpha=0}^{\infty} K_\alpha \phi^\alpha = K_0 +$$

$\uparrow$

vacuum e

density

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$$\hat{\phi}(k) \quad |k| < \Lambda$$

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FT of $\phi(x)$

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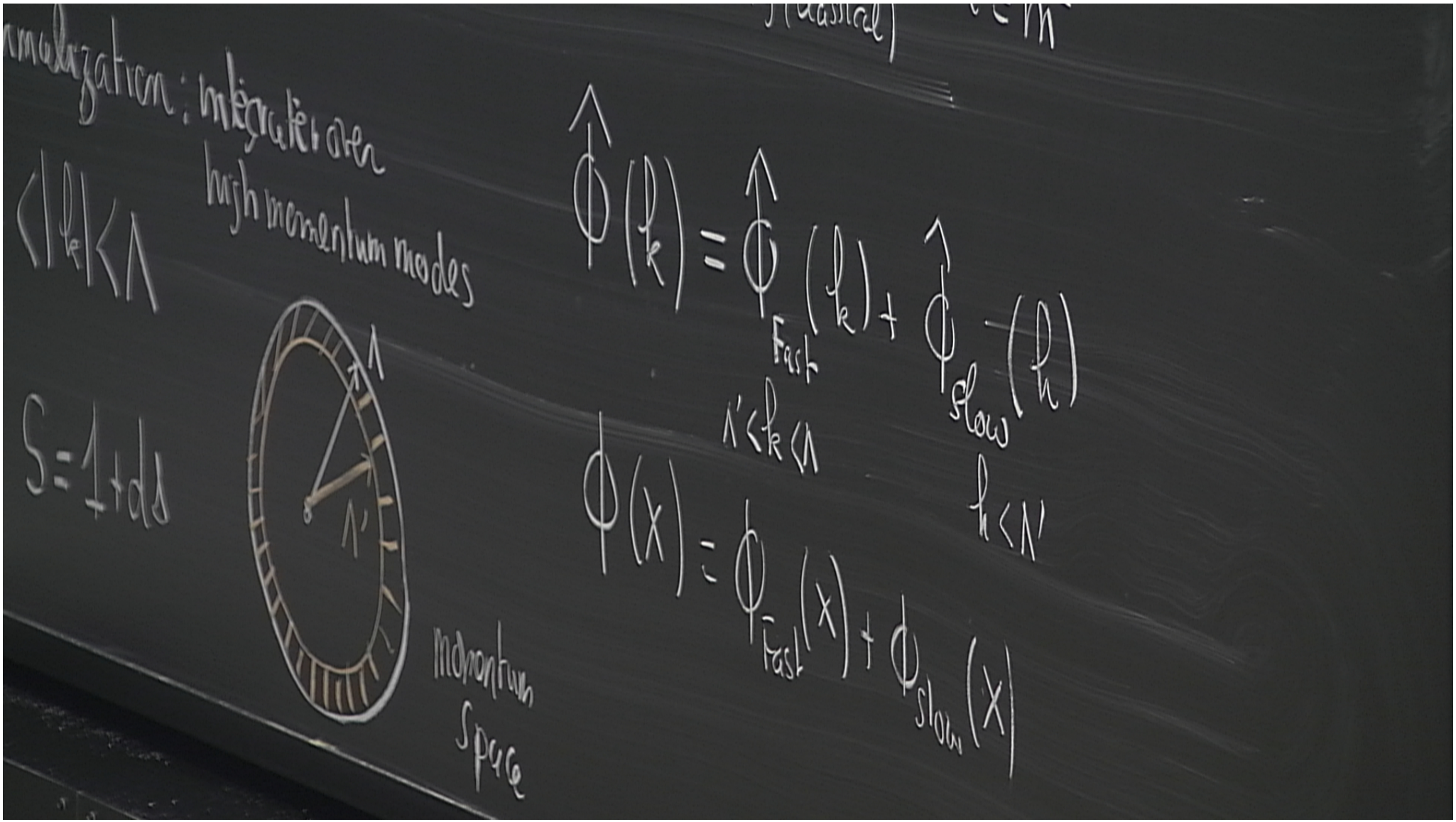
FT of $\phi(x)$

Renormalization: integrate over high momentum modes

$$\frac{\Lambda}{S} < |k| < \Lambda$$

$$S = 1 + ds$$

momentum space


Renormalization: integrate over
high momentum modes
$\Lambda' < |k| < \Lambda$
$S = 1 + ds$
Λ
Λ'
momentum space
$\hat{\phi}(k) = \hat{\phi}_{Fast}(k) + \hat{\phi}_{slow}(k)$
$\Lambda' < k < \Lambda$
$k < \Lambda'$
$\phi(x) = \phi_{Fast}(x) + \phi_{slow}(x)$

operators

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$\{W_\alpha\} = \vec{W}$ Wilson vector

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K_0: vacuum energy density (classical)

$t \simeq m^2$

$$\int D[\phi] \exp(-\mathcal{A}[\phi])$$

Sharp cut-off Λ

$\hat{\phi}(k)$ $|k| < \Lambda$

FT of $\phi(x)$

Renormalization: integrate over high momentum modes

$\Lambda' = \frac{\Lambda}{S} < |k| < \Lambda$

$S = 1 + d\ell$

momentum space

$$\hat{\phi}(k) = \hat{\phi}_{Fast}(k) + \hat{\phi}_{slow}(k)$$

$\Lambda' < k < \Lambda$ $\quad$ $k < \Lambda'$

$$\phi(x) = \phi_{Fast}(x) + \phi_{slow}(x)$$

$$\int D[\phi_{Fast}] \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}_{eff}[\phi_{slow}])$$

effective action for the slow modes

$$\int D[\phi_{Fast}]\, \exp(-\mathcal{A}[\phi]) = \exp(-\mathcal{A}_{eff}[\phi_{slow}])$$

renormalized effective action slow modes

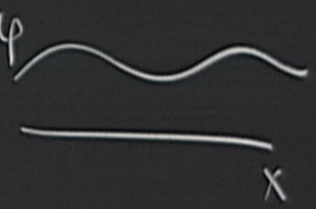
effective action at 1 loop $\Gamma_{eff}[\varphi] = \mathcal{A}[\varphi] + \frac{1}{2} \mathrm{Tr}\left[\mathrm{Log}\left(\mathcal{A}''[\varphi]\right)\right] + \cdots$

$\mathcal{A}''[\varphi] = \left(-\Delta + V''[\varphi]\right)$ Hessian differential operator

$$\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + V''(\varphi)\right)\right] = \int d^D x \langle x | \mathrm{Log}\left(-\Delta + V''(\varphi)\right) | x \rangle \simeq \int d^D x \int \frac{d^D k}{(2\pi)^D} \log\left(k^2 + V''(\varphi)\right)$$

IF you treat $V''(\varphi)$ as independent of x

$\varphi(x)$ (φ slowly varying)



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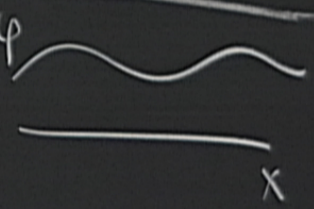

space

assuming ϕ_{slow} slow variation w.r.t ϕ_{fast}

$$\mathcal{A}_{eff}[\phi_{slow}] = \mathcal{A}[\phi_{slow}] + \frac{1}{2}\int d^D x \int \frac{d^D k}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow}(x))\right]$$

assuming ϕ_{slow} slow variation w.r.t ϕ_{fast}

$$\mathcal{A}_{eff}[\phi_{slow}] = \mathcal{A}[\phi_{slow}] + \frac{1}{2}\int d^D x \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow}(x))\right]$$

high momentum shell

space

alized
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$+ \cdots$

$\Lambda' < |k| < \Lambda$
high momentum shell

$\mathcal{A}[\phi_{fast} + \phi_{slow}]$, cut-off $\Lambda \to \Gamma_{eff}[\varphi]$

$V''(\varphi))$

space

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+ - - -

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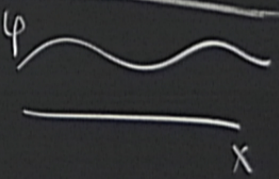

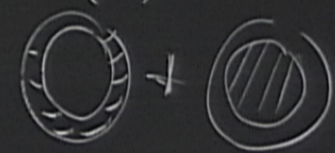

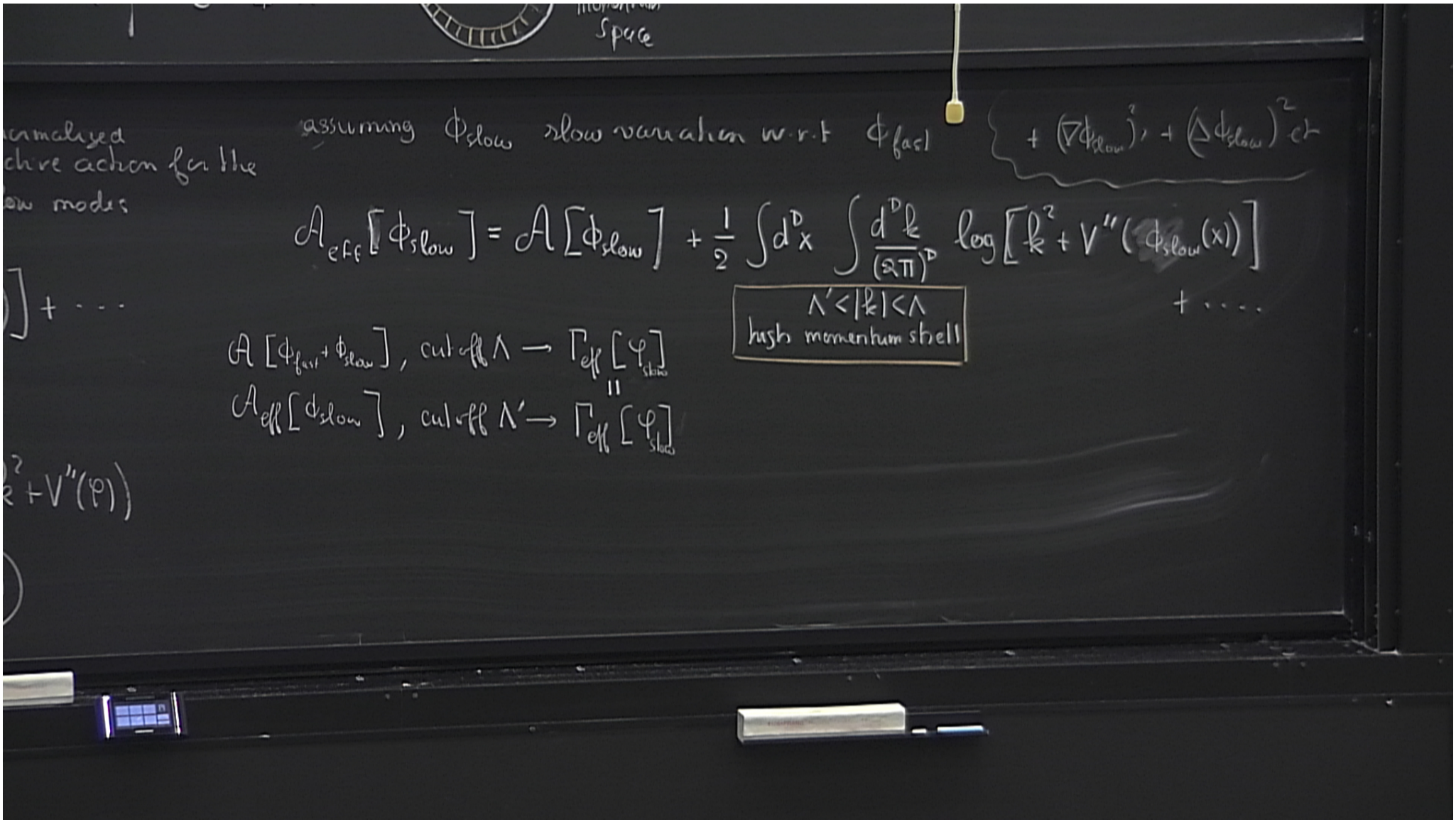
Space
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assuming ϕ_{slow} slow variation w.r.t ϕ_{fast}
$+ (\nabla\phi_{slow})^2 + (\Delta\phi_{slow})^2 + \dots$
$\mathcal{A}_{eff}[\phi_{slow}] = \mathcal{A}[\phi_{slow}] + \frac{1}{2}\int d^Dx \int \frac{d^Dk}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow}(x))\right] + \dots$
$\Lambda' < |k| < \Lambda$
high momentum shell
$\mathcal{A}[\phi_{fast} + \phi_{slow}]$, cutoff $\Lambda \to \Gamma_{eff}[\varphi_{slow}]$
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$]+\dots$
$k^2 + V''(\varphi))$

assuming ϕ_{slow} slow variation w.r.t ϕ_{fast}

$$\left\{ + (\nabla\phi_{slow})^2 + (\Delta\phi_{slow})^2 \right\} + \dots$$

$$\boxed{\mathcal{A}_{EFF}[\phi_{slow}] = \mathcal{A}[\phi_{slow}] + \frac{1}{2}\int d^D x \int \frac{d^D k}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow}(x))\right]} + \dots$$

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$\mathcal{A}[\phi_{fast} + \phi_{slow}]$, cut-off $\Lambda \longrightarrow \Gamma_{eff}[\varphi_{slow}]$

$$\parallel$$

$\mathcal{A}_{eff}[\phi_{slow}]$, cut-off $\Lambda' \longrightarrow \Gamma_{eff}[\varphi_{slow}]$

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<u>Step 1</u>

<u>Step 2</u>: Rescale the distances/momenta and the fields

$$V_{eff}[\phi_{slow}] = V[\phi_{slow}] + \frac{1}{2}\int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \log\left(k^2 + V''(\phi_{slow})\right)$$

<u>Step 1</u>

<u>Step 2</u>: Rescale the distances/momenta and the fields

$$\phi_{slow}(x) \qquad x' = x \cdot S^{-1} \; ; \; k' = k \cdot S \; ; \; \Lambda$$

$$V_{eff}[\phi_{slow}] = V[\phi_{slow}] + \frac{1}{2}\int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \log\left(k^2 + V''(\phi_{slow})\right)$$

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original cut-off

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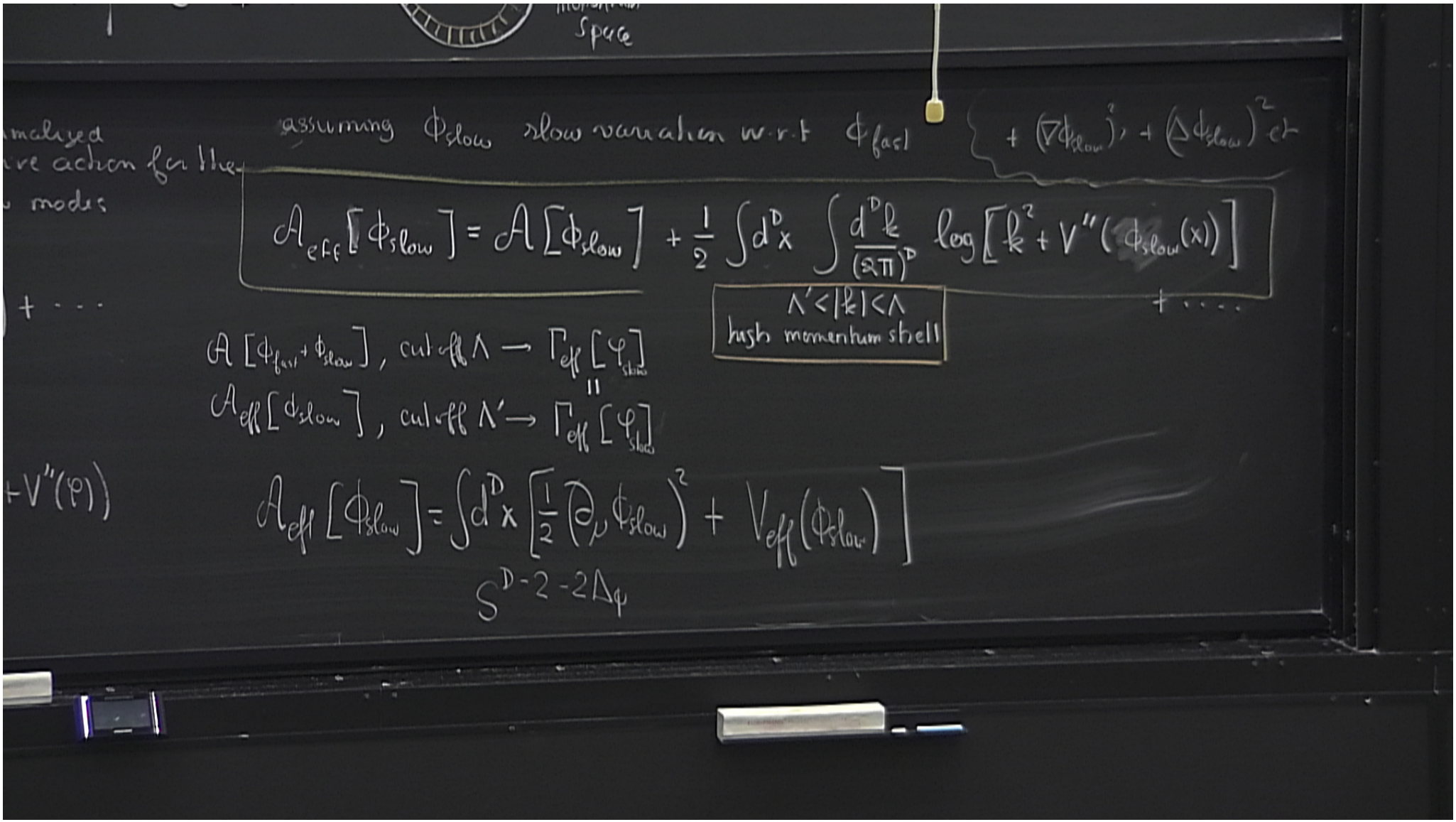
original cut-off

$$= S^{-\Delta_\phi} \phi_{ren}(x')$$

$$\mathcal{A}_{ren}(\phi_{ren}) = \mathcal{A}_{eff}(\phi_{eff}) = \int d^D x' \left[\frac{1}{2}\left(\partial_\mu \phi_{ren}\right)^2 + V_{ren}(\phi_{ren})\right]$$


Space
assuming ϕ_{slow} slow variation w.r.t ϕ_{fast}
$+ (\nabla\phi_{slow})^2 + (\Delta\phi_{slow})^2$ etc
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$S^{D-2-2\Delta_\varphi}$

$''(\phi_{slow}))$

I must choose $\Delta_\phi = \frac{D-2}{2}$
scaling dimension
of the free field

$; \Lambda' \to \Lambda' \cdot S = \frac{\Delta}{S} \cdot S = \Lambda$
original cut-off

$(\phi))$

$''(\phi_{slow}))$

I must choose $\Delta_\phi = \frac{D-2}{2}$

Scaling dimension of the free field

$; \Lambda' \to \Lambda' \cdot S = \frac{\Delta}{S} \cdot S = \Lambda$

original cut-off

$V_{Ren}(\phi_{Ren}) = S^D$

$(\phi))]$

I must choose $\Delta_\phi = \frac{D-2}{2}$

Scaling dimension of the free field

$$= \frac{\Delta}{s} \cdot s = \Lambda$$

$$V_{Ren}(\phi_{Ren}) = s^D V_{eff}(s^{-\Delta} \phi_{Ren})$$

$$S = 1 + \delta s \qquad \Lambda' = \Lambda(1-\delta s) < |k| < \Lambda$$

$$\int \frac{d^D k}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow})\right]$$

$$\Lambda' < |k| < \Lambda$$

$$= d s \, \frac{2}{\Gamma(D/2)(4\pi)^{D-2}}$$

"volume" of the unit sphere

$$\left(S^{-\Delta} \phi_{Ren}\right)$$

Eff

ical

I must choose $\Delta_\phi = \frac{D-2}{2}$

scaling dimension of the free field

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$S = \Lambda$

Rescaling on the potential

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$$\int \frac{d^D k}{(2\pi)^D} \log\left[k^2 + V''(\phi_{slow})\right]$$

$\Lambda' < |k| < \Lambda$

$$= d s \frac{2}{\Gamma(D/2)(4\pi)^{D-2}} \Lambda^D \log\left(\Lambda^2 + V''(\phi)\right)$$

"volume" of the unit sphere

Final Result : Transformation of the Potential $V(\phi)$
under such a small RG transformation

$$V(\phi) \xrightarrow{s} V_s(\phi)$$

$$s\frac{\partial}{\partial s} V_s(\phi) = D\, V_s[\phi] - \frac{D-2}{2} V_s'[\phi]$$

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scaling on the classical part

quantum fluctuation at 1 loop, neglecting $\partial\phi$ terms

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Flow
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$$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6 + \cdots$$

$$V'(\phi) = t\phi + \frac{g}{3!}\phi^3 + \frac{h}{5!}\phi^5$$

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set $\Lambda = 1$

the cut-off is the unit of momentum

non linear term

$$\mathrm{Log}\left(1 + V''(\phi)\right) = \log\left(1 + t + \frac{g}{2}\phi^2 + \frac{h}{4!}\phi^4 + \cdots\right)$$

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expand in ϕ^2

$$= \log(1+t) + \left(\frac{1}{1+t}\left(\frac{g}{2}\phi^2 + \frac{h}{4!}\phi^4 + \cdots\right)\right) - \frac{1}{2}\left(\frac{g}{2}\right)^2\phi^4 + \cdots$$

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t, g, h are couplings

Exercise: h

non linear term

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non linear terms

non linear, 2nd order in ϕ

are couplings

Exercise: Write the RG flow equation for the couplings (t, g, h are small)

coupling & mass are small

$D = 4$

$$s \frac{d}{ds} t(s) = W_t = 2t + g - tg$$

$$s \frac{d}{ds} g(s) = W_g =$$

$$s \frac{d}{ds} h(s) = W_h$$

ngs

Exercise: Write the RG flow equation for the couplings (t, g, h are small)

$$s\frac{d}{ds}t(s) = W_t = 2t + g - tg + \dots$$

$$s\frac{d}{ds}g(s) = W_g = -3g^2 - th + h + \dots$$

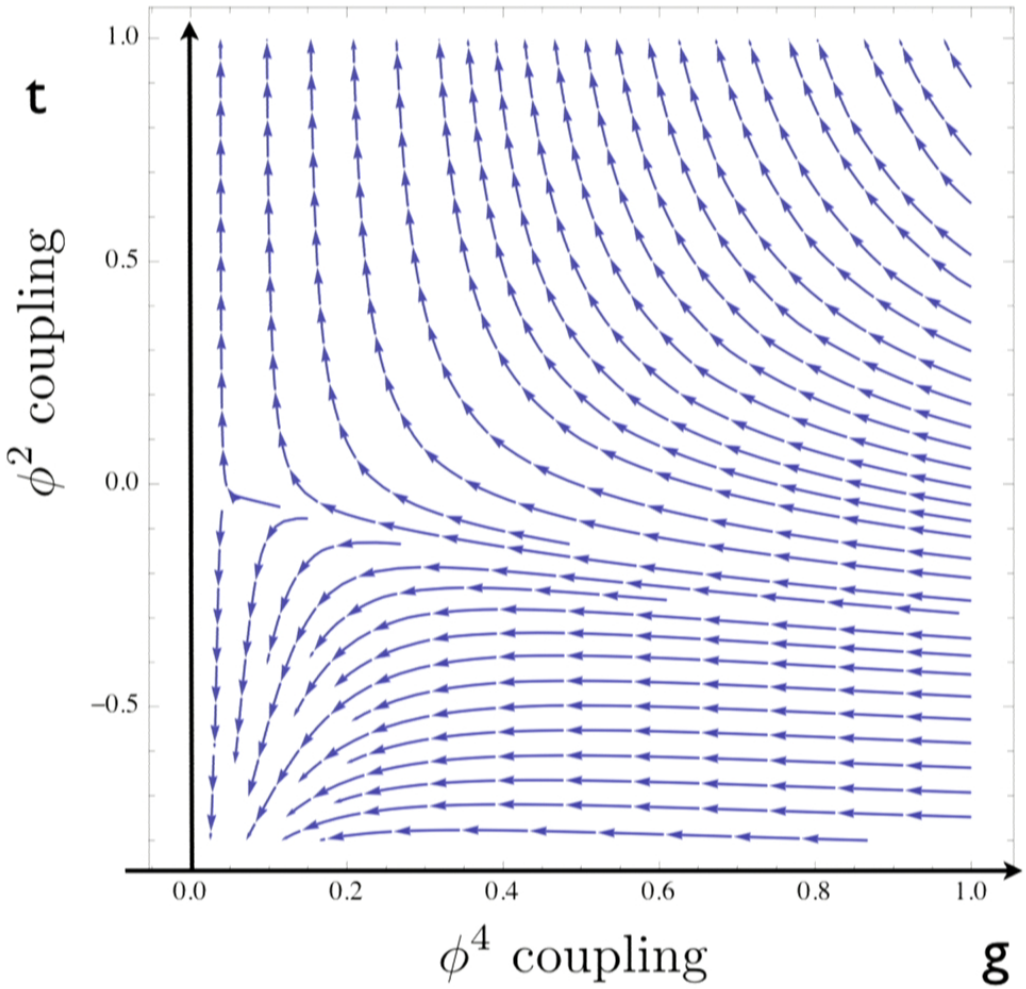
$$s\frac{d}{ds}h(s) = W_h = -2h - 15gh + \dots$$

coupling & mass are small

$\boxed{D=4}$

order 2 in the couplings

RG flows in the (t,g) plane



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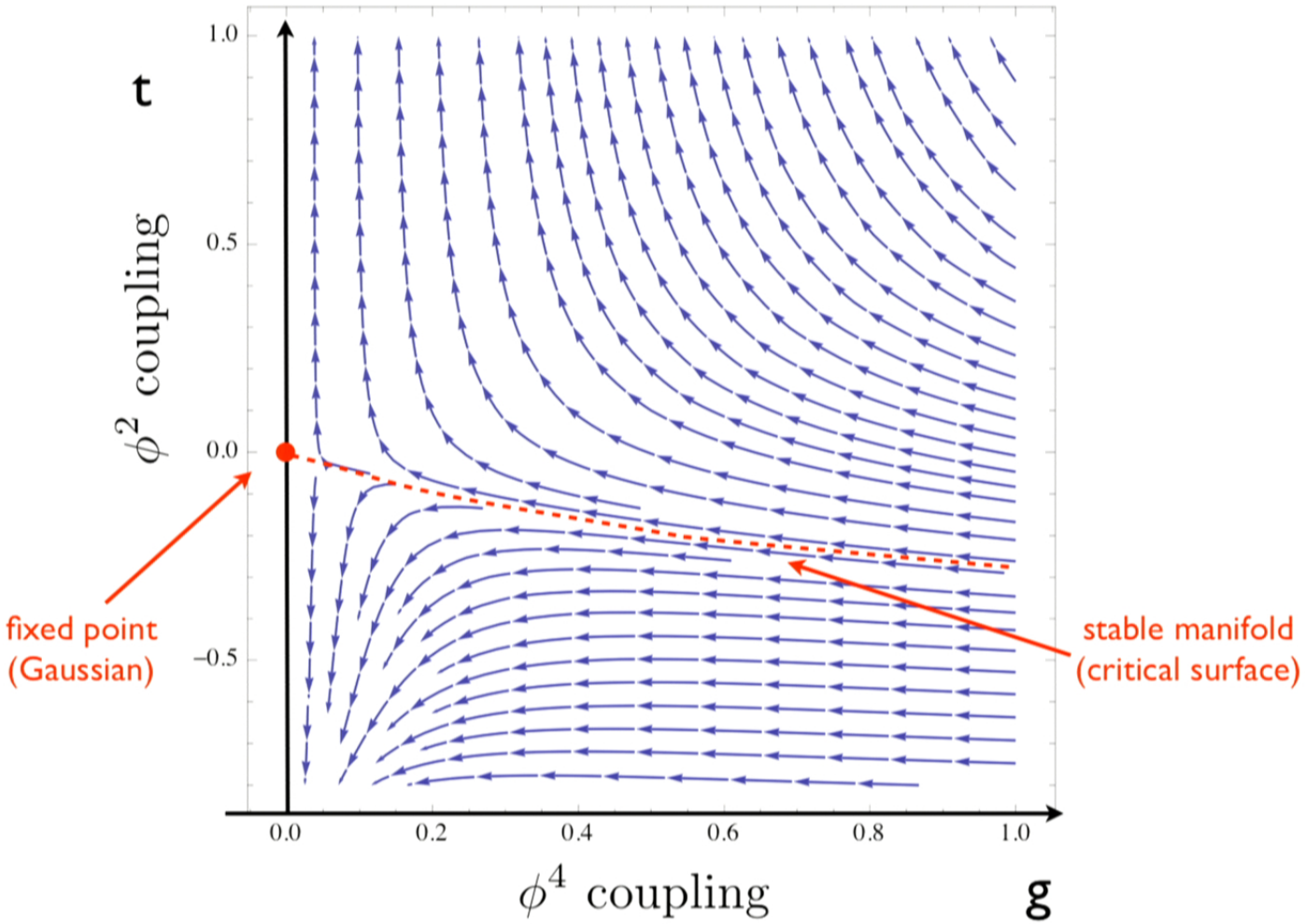
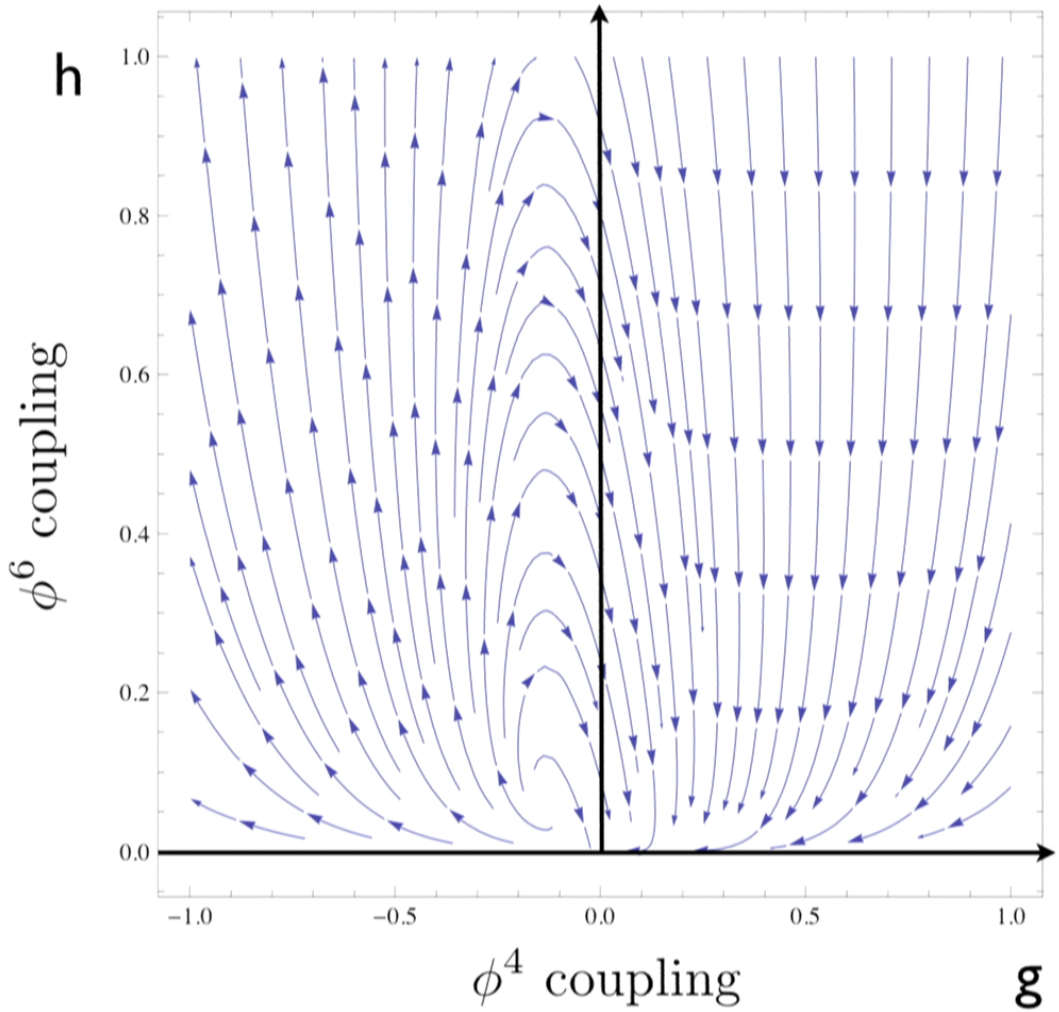


RG flows on the critical surface



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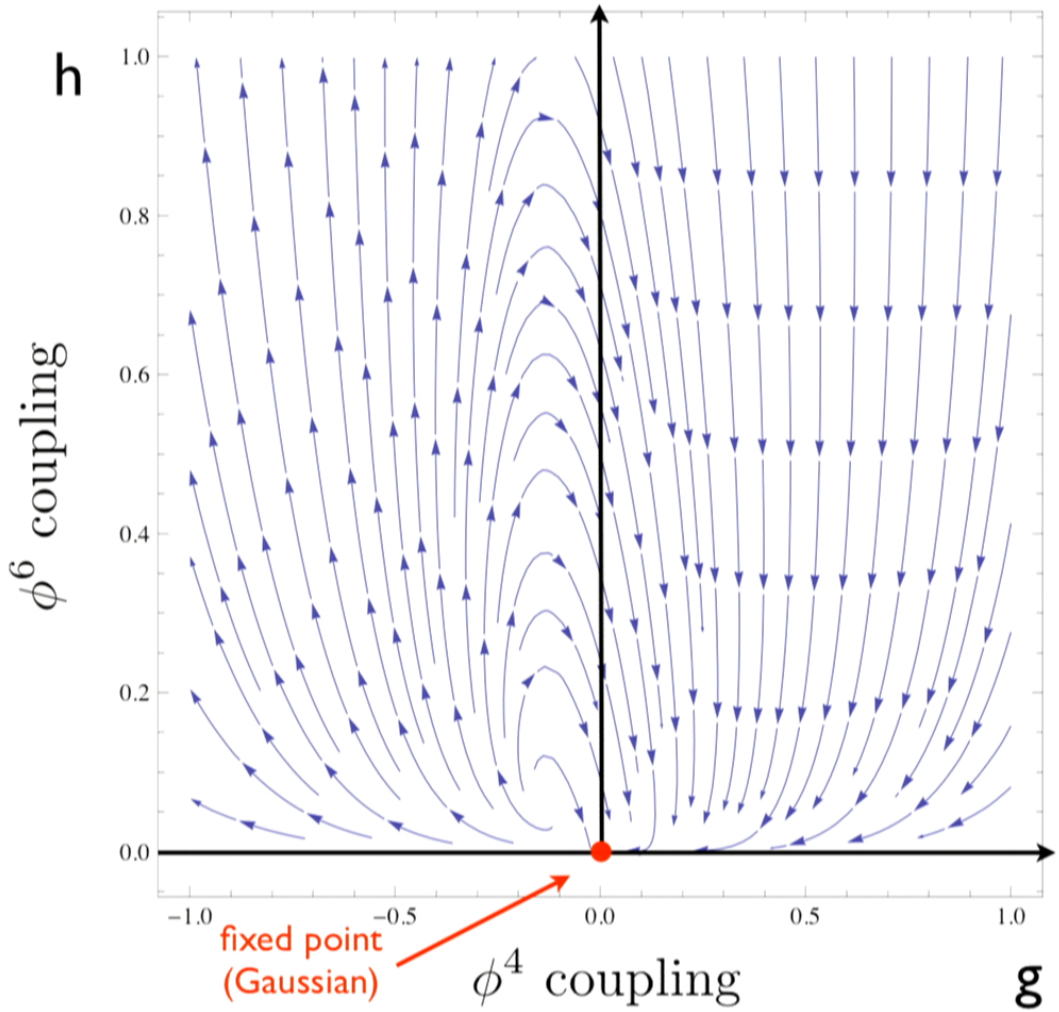

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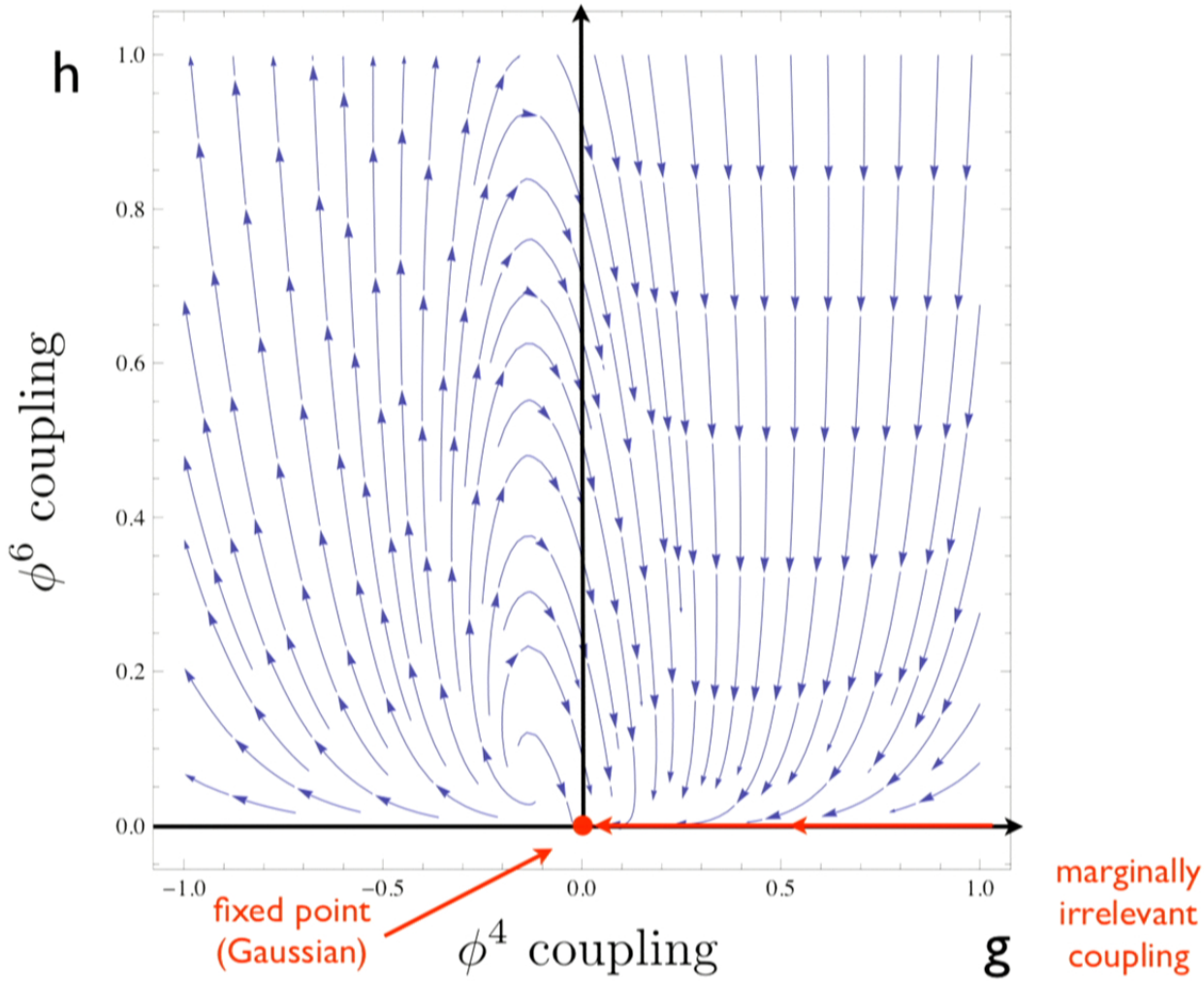
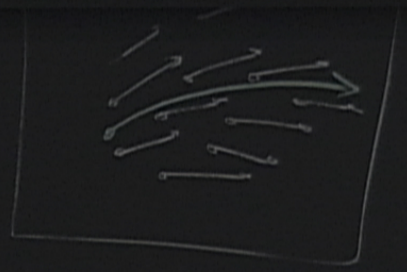


coupling — local operators

$$s\frac{\partial}{\partial s}\vec{K}(s) = \vec{W}(\vec{K}(s))$$

Flow equation

RG flow generated by the vector field $\vec{W}$



Statement: $W_\alpha \simeq -\beta$ Functions

At energy scales $\mu \ll \Lambda$; RG $s = \frac{\Lambda}{\mu} \gg 1$

Local Potential Approxim

$$\mathcal{A}[\phi] = \int d^D x \, \frac{1}{2}(\partial_\mu \phi)^2 +$$

(Euclidean)

$$\int D[\phi] \exp(-\mathcal{A}[\phi])$$

Sharp cut-off Λ

$\hat{\phi}(k)$ $|k| < \Lambda$

FT of $\phi(x)$

Renorm

$\Lambda' = \frac{\Lambda}{s} <$

S

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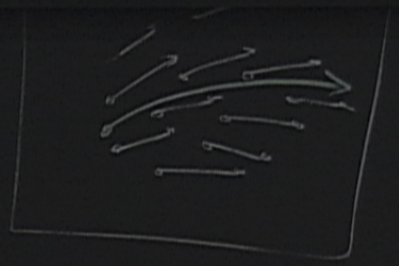

Statement: $W_\alpha \simeq -\beta$ Functions

At energy scales $\mu \ll \Lambda$; RG $s = \frac{\Lambda}{\mu} \gg 1$

one can forget about the $h\phi^6$ coupling ; h irrelevant

You cannot forget about the $g\phi^4$ coupling g marginal

Local Potential Approxim

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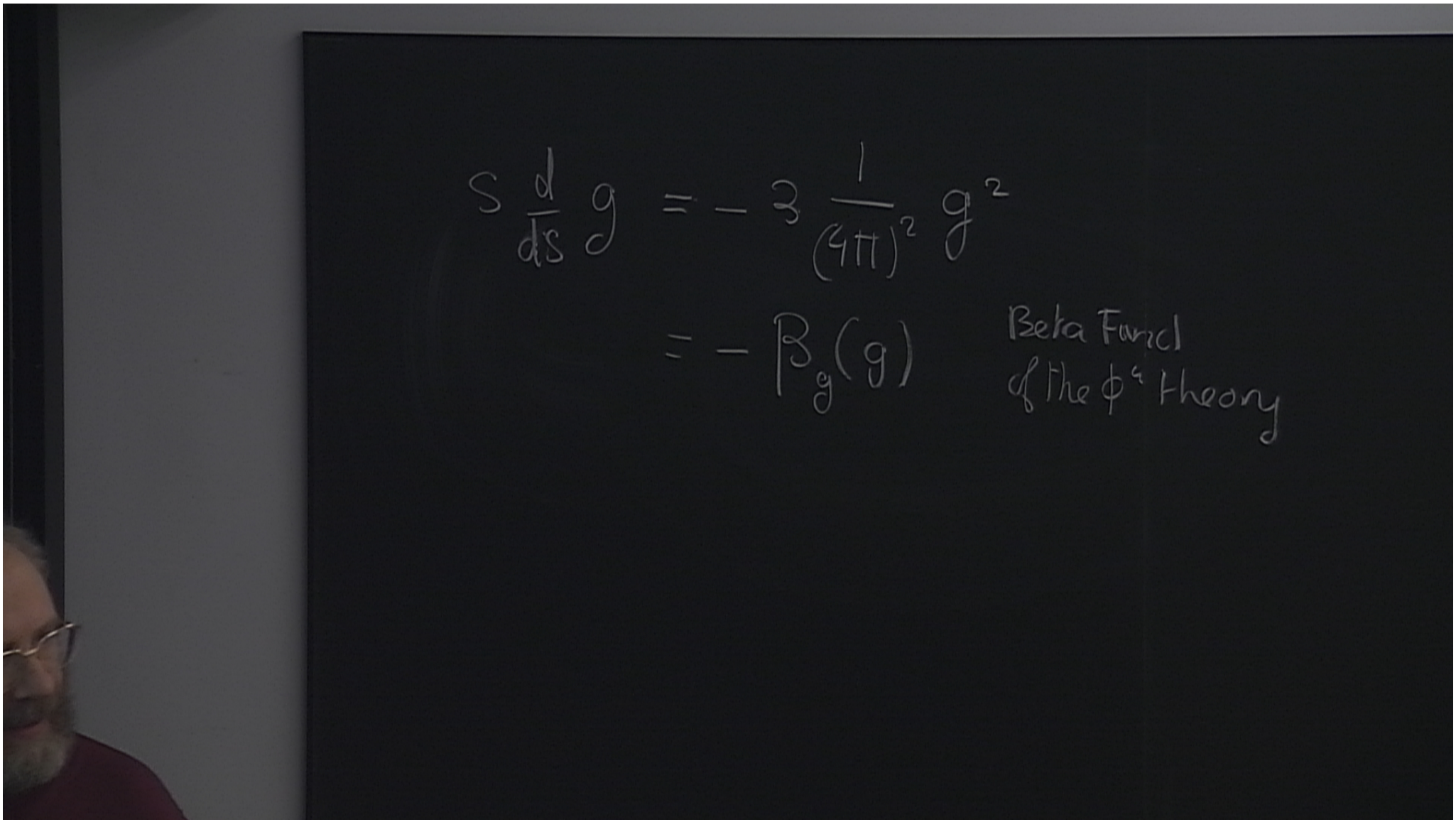
$s \frac{d}{ds} g = -3 \frac{1}{(4\pi)^2} g^2$
$= -\beta_g(g)$
Beta Funct
of the ϕ^4 theory

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$$= -\beta_g(g)$$

Beta Funct of the ϕ^4 theory

$$\mu \frac{\partial}{\partial \mu} g = \beta_g(g)$$

$\mu \nearrow$