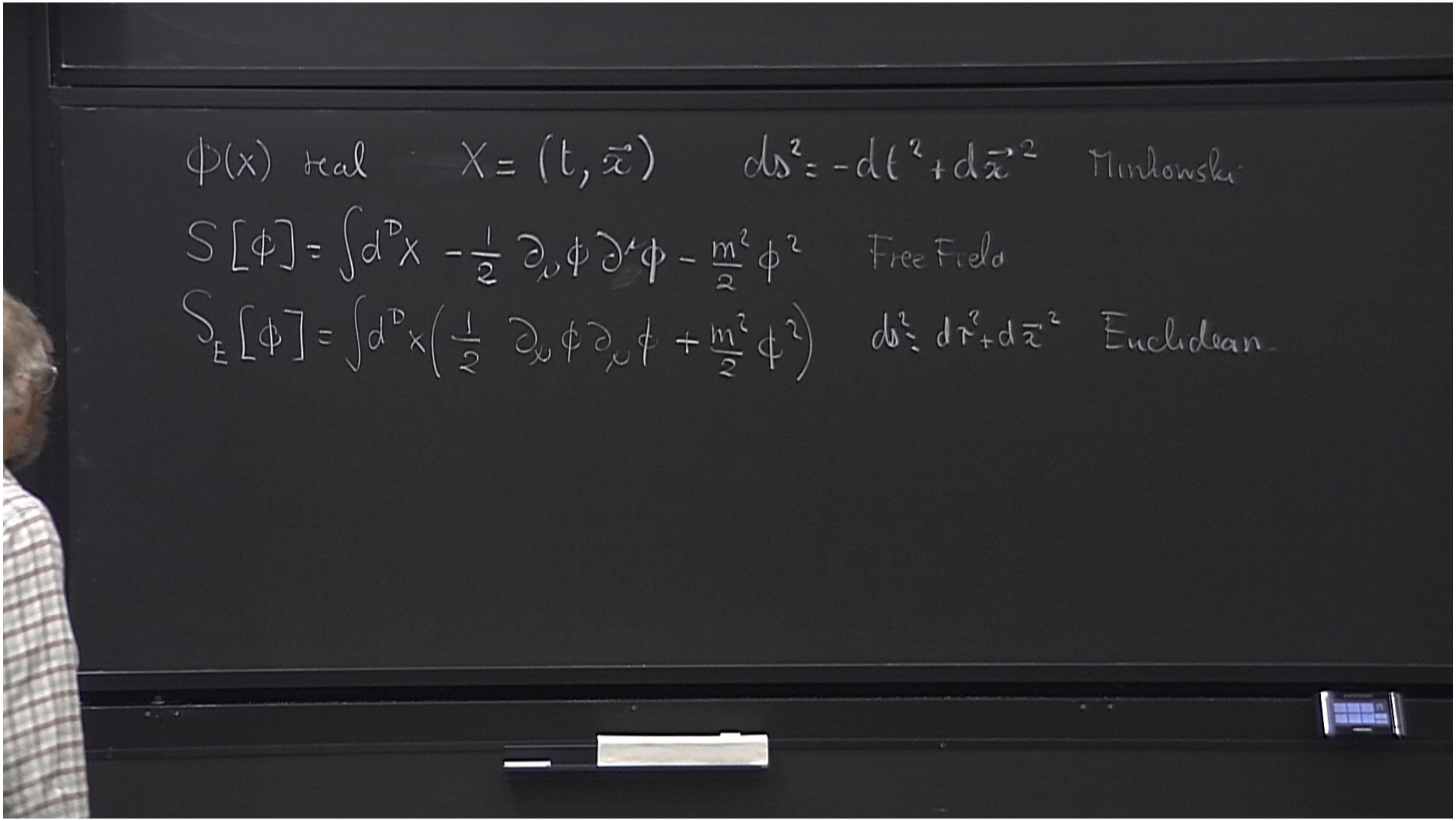
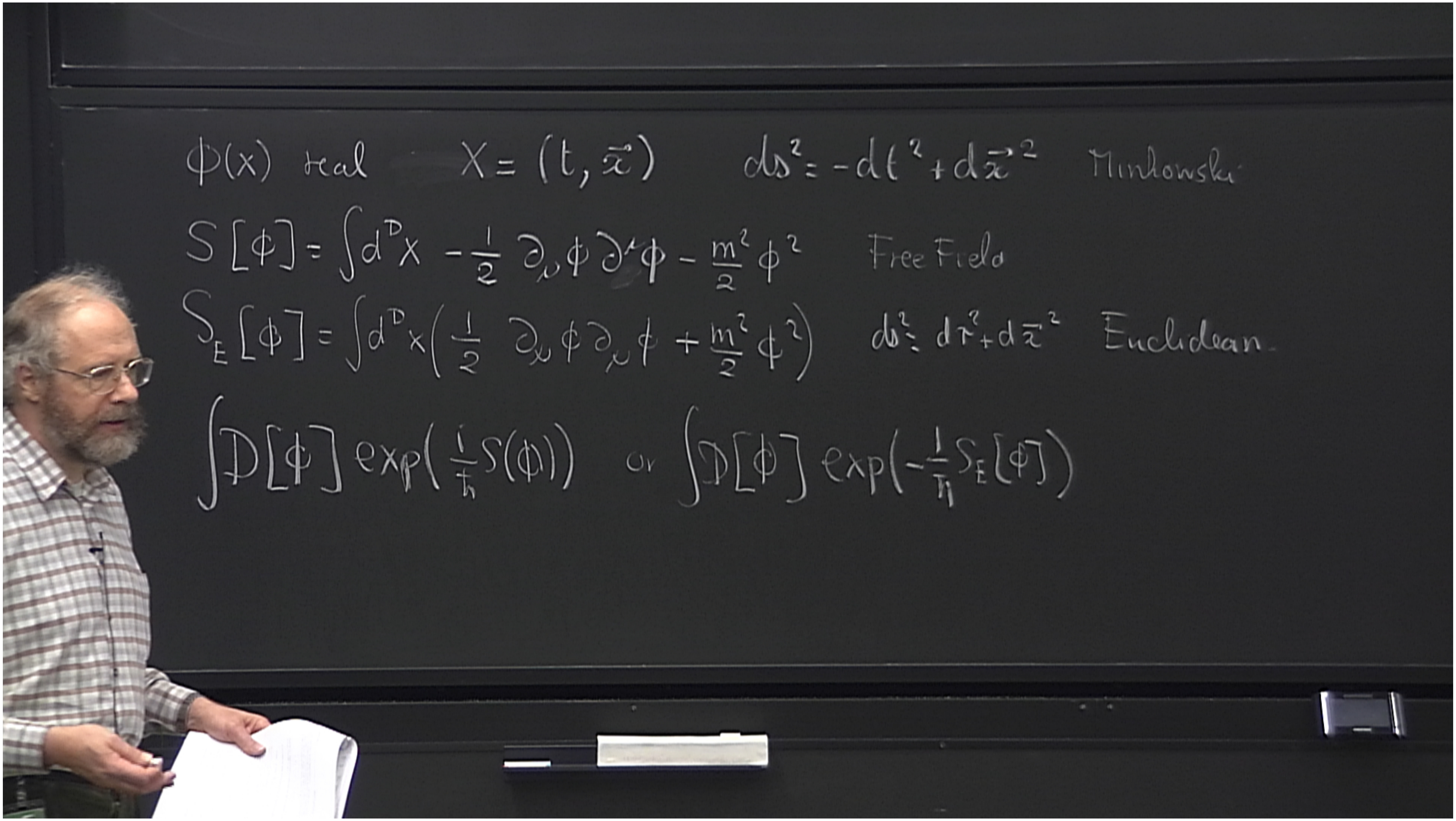
Title: 13/14 PSI - Quantum Field Theory II - Lecture 4

Date: Nov 14, 2013 09:00 AM

URL: http://pirsa.org/13110011

Abstract:


$\phi(x)$ real $X = (t, \vec{x})$ $ds^2 = -dt^2 + d\vec{x}^2$ Minkowski
$S[\phi] = \int d^D x \, -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2$ Free Field
$S_E[\phi] = \int d^D x \left(\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{m^2}{2} \phi^2 \right)$ $ds^2 = d\tau^2 + d\vec{x}^2$ Euclidean


$\phi(x)$ real $\quad X = (t, \vec{x}) \quad ds^2 = -dt^2 + d\vec{x}^2$ Minkowski
$S[\phi] = \int d^D x \; -\frac{1}{2}\partial_\nu \phi \partial^\nu \phi - \frac{m^2}{2}\phi^2$ Free Field
$S_E[\phi] = \int d^D x \left(\frac{1}{2}\partial_\nu \phi \partial_\nu \phi + \frac{m^2}{2}\phi^2\right) \quad ds^2 = d\tau^2 + d\vec{x}^2$ Euclidean
$\int \mathcal{D}[\phi] \exp\left(\frac{i}{\hbar} S(\phi)\right)$ or $\int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} S_E[\phi]\right)$

wski

$$\langle 0 | T(\Phi(x_1)\,\Phi(x_2)) | 0 \rangle = \frac{\int D[\phi]\, \exp\left(\frac{i}{\hbar} S[\phi]\right) \phi(x_1)\, \phi(x_2)}{\int D[\phi]\, \exp\left(\frac{i}{\hbar} S[\phi]\right)}$$

Gaussian measure $\Rightarrow$

$$= \langle x_1 | \frac{-\hbar i}{\Delta^2 - m^2} | x_2 \rangle = G(x_1, x_2)$$

lidean.

Fourier transform.

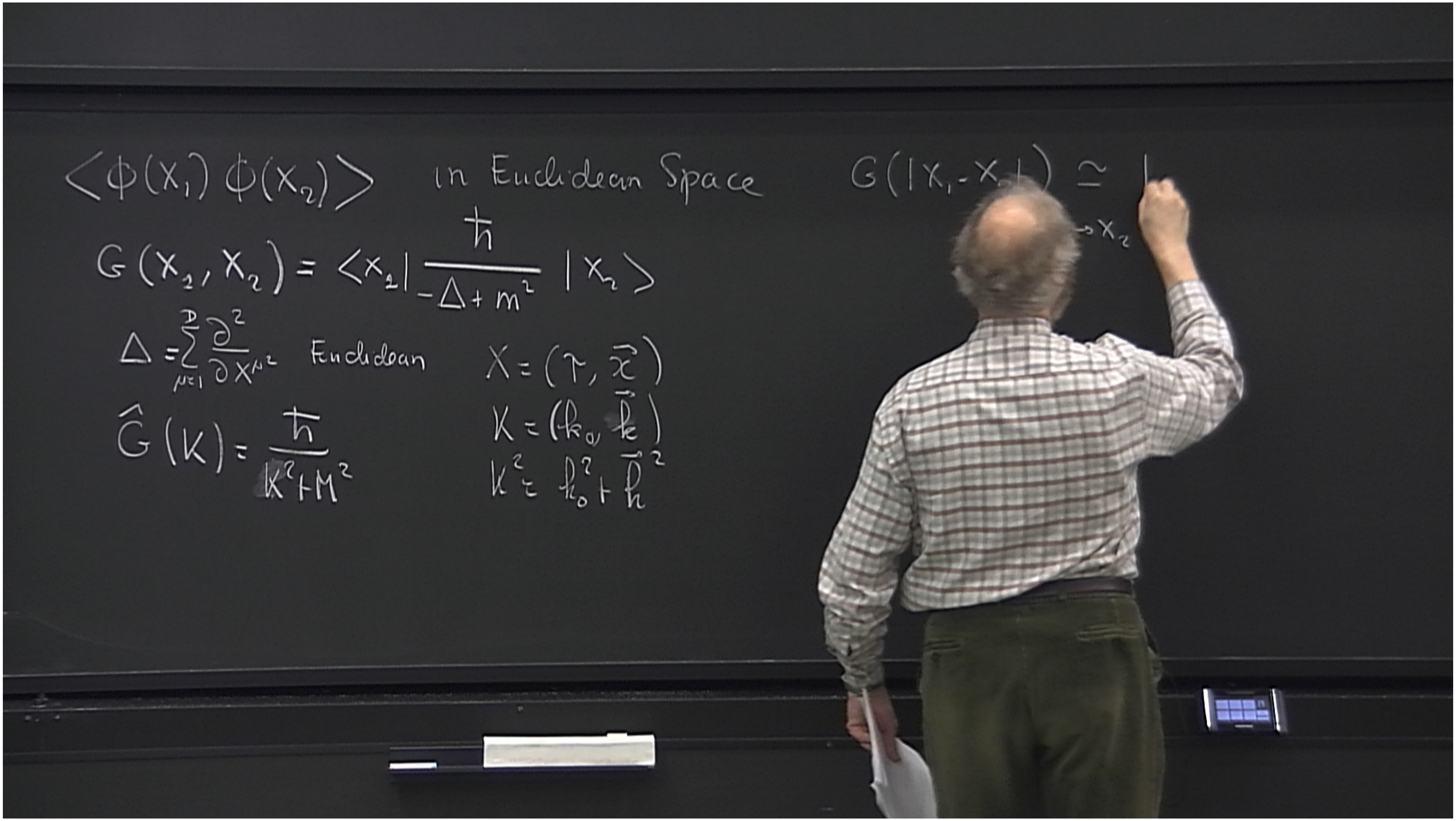
$$\underset{\text{Feynman}}{\hat{G}}(k) = \frac{-i}{k^2 + m^2 - i\epsilon_+}$$

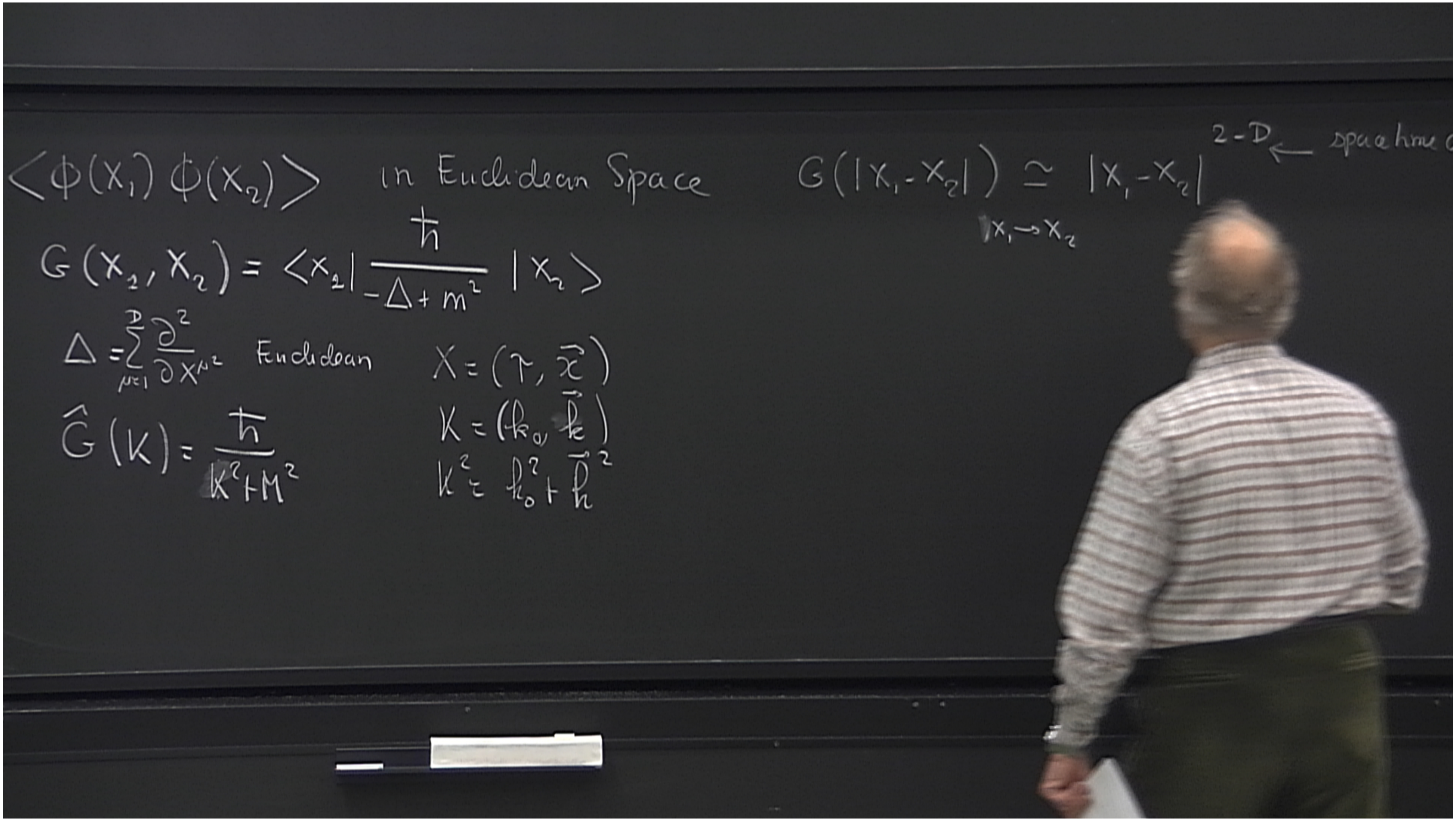
$$k = (\omega, \vec{k}) = (k_\mu)$$

$$\langle \phi(x_1)\, \phi(x_2) \rangle \quad \text{in Euclidean Space}$$

$$G(x_1, x_2) = \langle x_1 | \frac{\hbar}{-\Delta + m^2} | x_2 \rangle$$

$$\Delta = \sum_{\mu=1}^{D} \frac{\partial^2}{\partial x^{\mu 2}} \quad \text{Euclidean}$$


$\langle \phi(x_1)\, \phi(x_2) \rangle$ in Euclidean Space
$G(|x_1 - x_2|) \simeq$
$G(x_1, x_2) = \langle x_1 | \frac{\hbar}{-\Delta + m^2} | x_2 \rangle$
$\Delta = \sum_{\mu=1}^{D} \frac{\partial^2}{\partial x^{\mu 2}}$ Euclidean
$X = (\tau, \vec{x})$
$K = (k_0, \vec{k})$
$\hat{G}(K) = \frac{\hbar}{K^2 + M^2}$
$K^2 = k_0^2 + \vec{k}^2$


$\langle \phi(x_1)\,\phi(x_2) \rangle$ in Euclidean Space
$G(x_1, x_2) = \langle x_2 | \frac{\hbar}{-\Delta + m^2} | x_2 \rangle$
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$X = (\tau, \vec{x})$
$K = (k_0, \vec{k})$
$K^2 = k_0^2 + \vec{k}^2$
$\hat{G}(K) = \frac{\hbar}{K^2 + M^2}$
$G(|x_1 - x_2|) \simeq |x_1 - x_2|^{2-D}$
spacetime
$|x_1 \to x_2$

Space

$$G(|x_1 - x_2|) \simeq |x_1 - x_2|^{2-D} \leftarrow \text{spacetime dimension}$$

diverges: $D \geq 2$

$x_1 \to x_2$

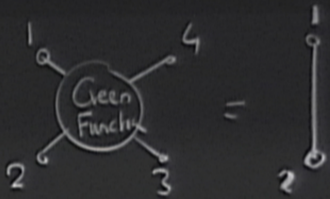
"Ultra-violet" divergences $\Leftarrow$ important feature of QFT

...lean Space

$$G(|x_1 - x_2|) \simeq |x_1 - x_2|^{2-D} \leftarrow \text{spacetime dimension}$$

$x_1 \to x_2$ diverges: $D \geq 2$

"Ultra-violet" divergences $\Leftarrow$ important feature of QFT



...$x_2\rangle$

$..., \vec{x})$

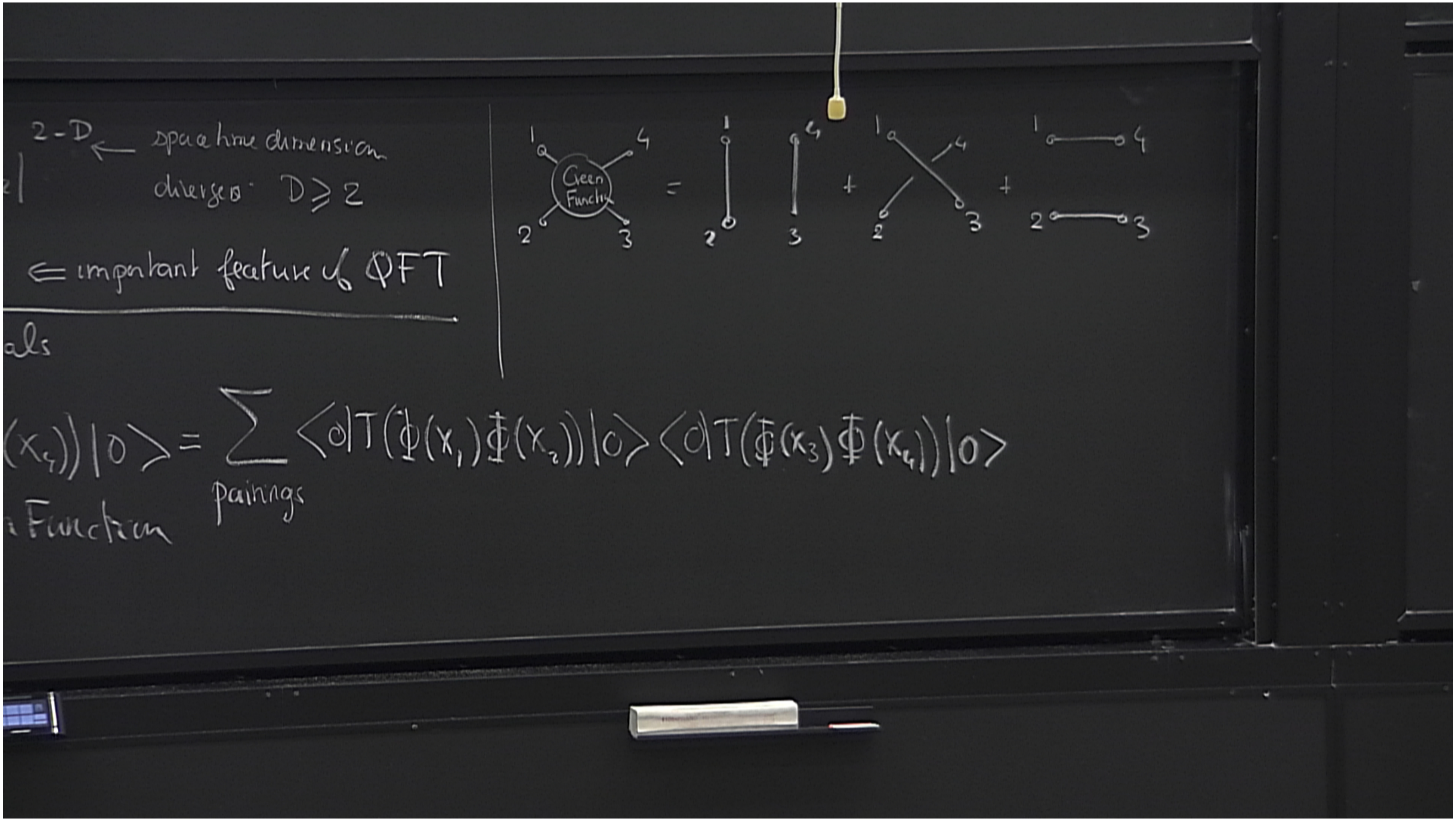
$\vec{k})$

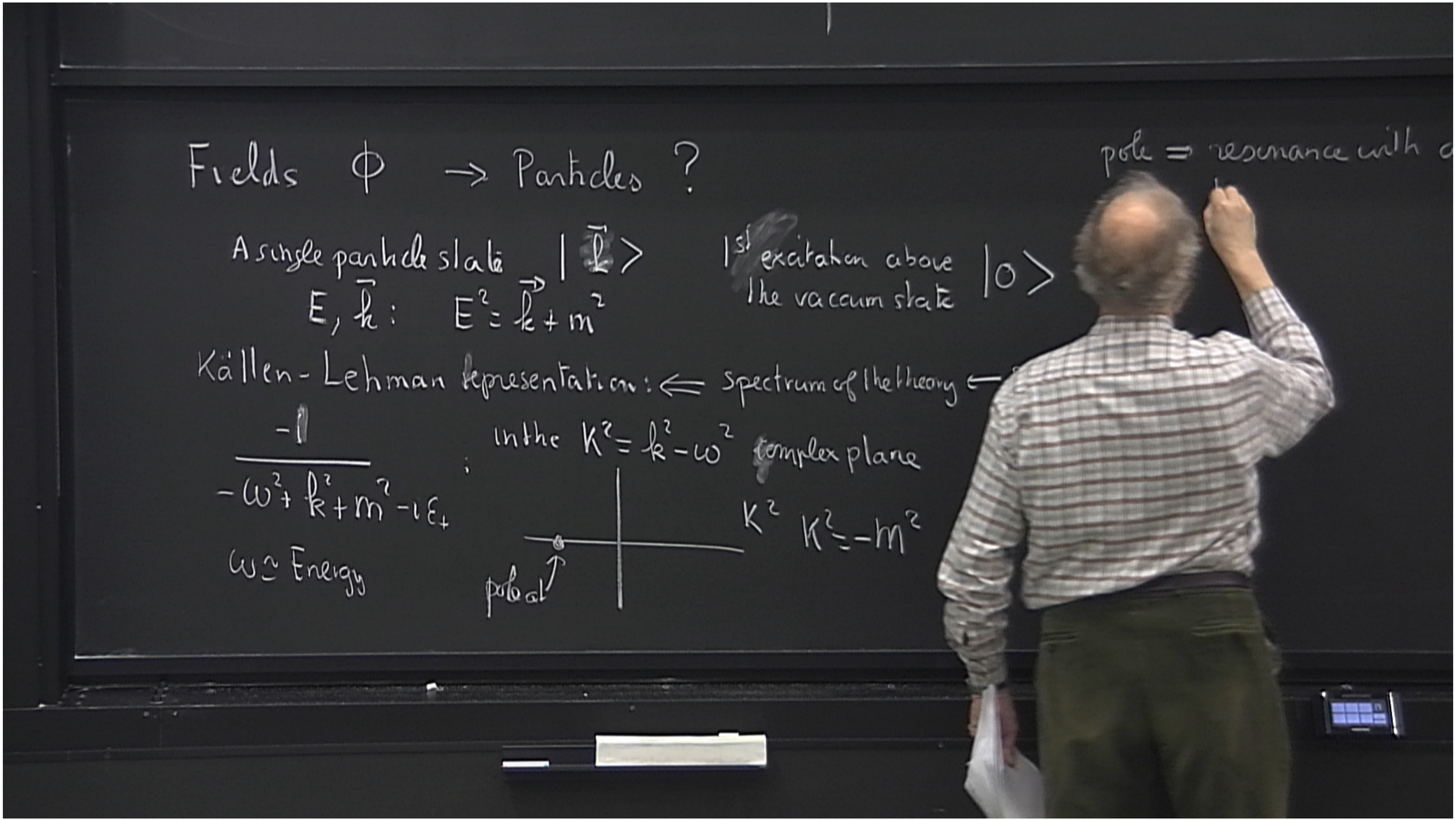
$... + \vec{h}^2$

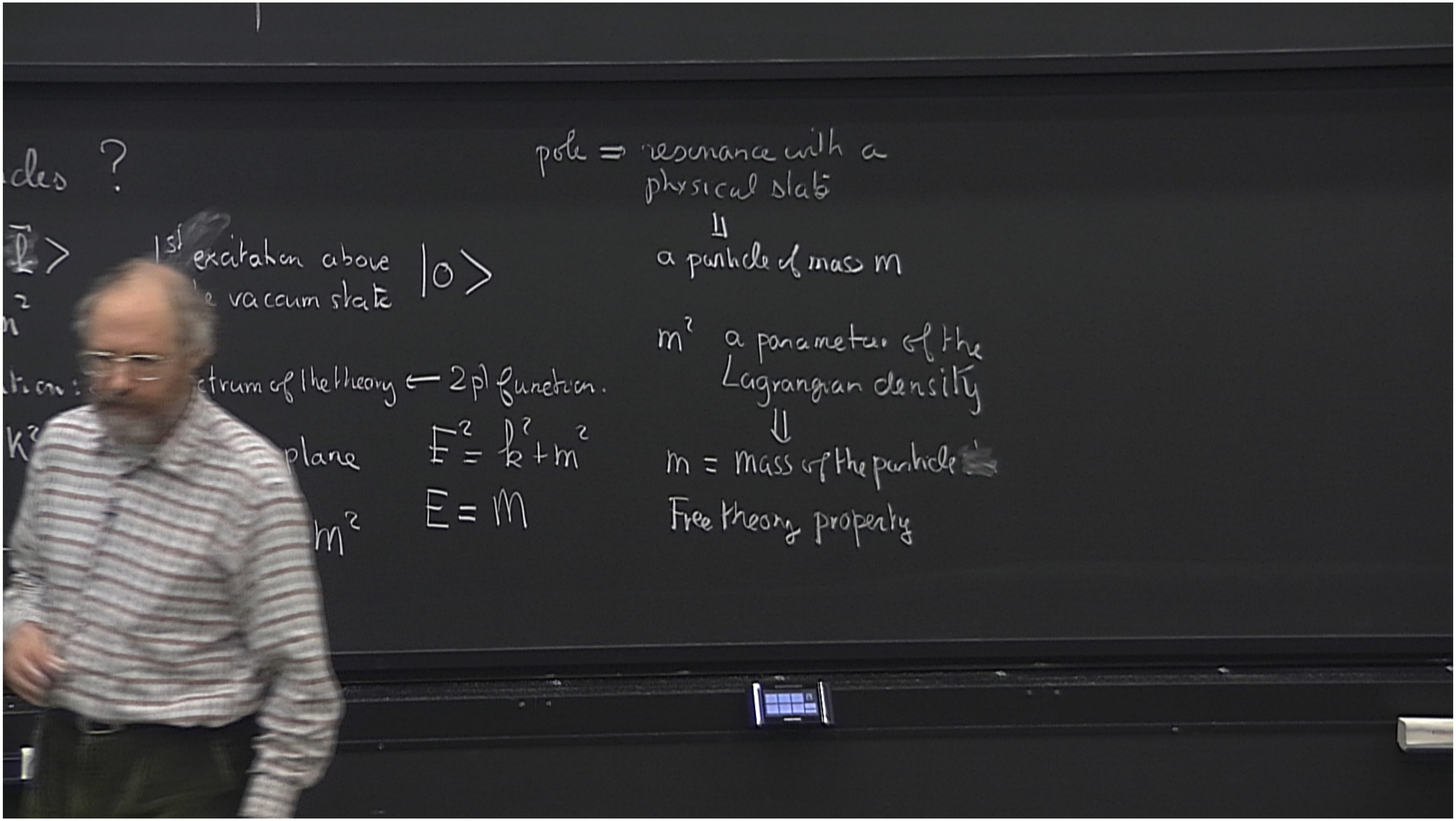
Wick Theorem $\Leftarrow$ Gaussian integrals

$$\langle 0|T(\Phi(x_1)\Phi(x_2)\Phi(x_3)\Phi(x_4))|0\rangle = \sum_{\text{pairings}} \langle 0|T(\Phi(x_1)\Phi(x_2))|0\rangle \langle 0|T(\Phi(x...$$

$N=4$ point function or Green Function


2-D ← spacetime dimension
diverges D ≥ 2
⇐ important feature of ΦFT
als
Green Function
(x4))|0> = Σ pairings <0|T(Φ(x1)Φ(x2))|0><0|T(Φ(x3)Φ(x4))|0>
Function

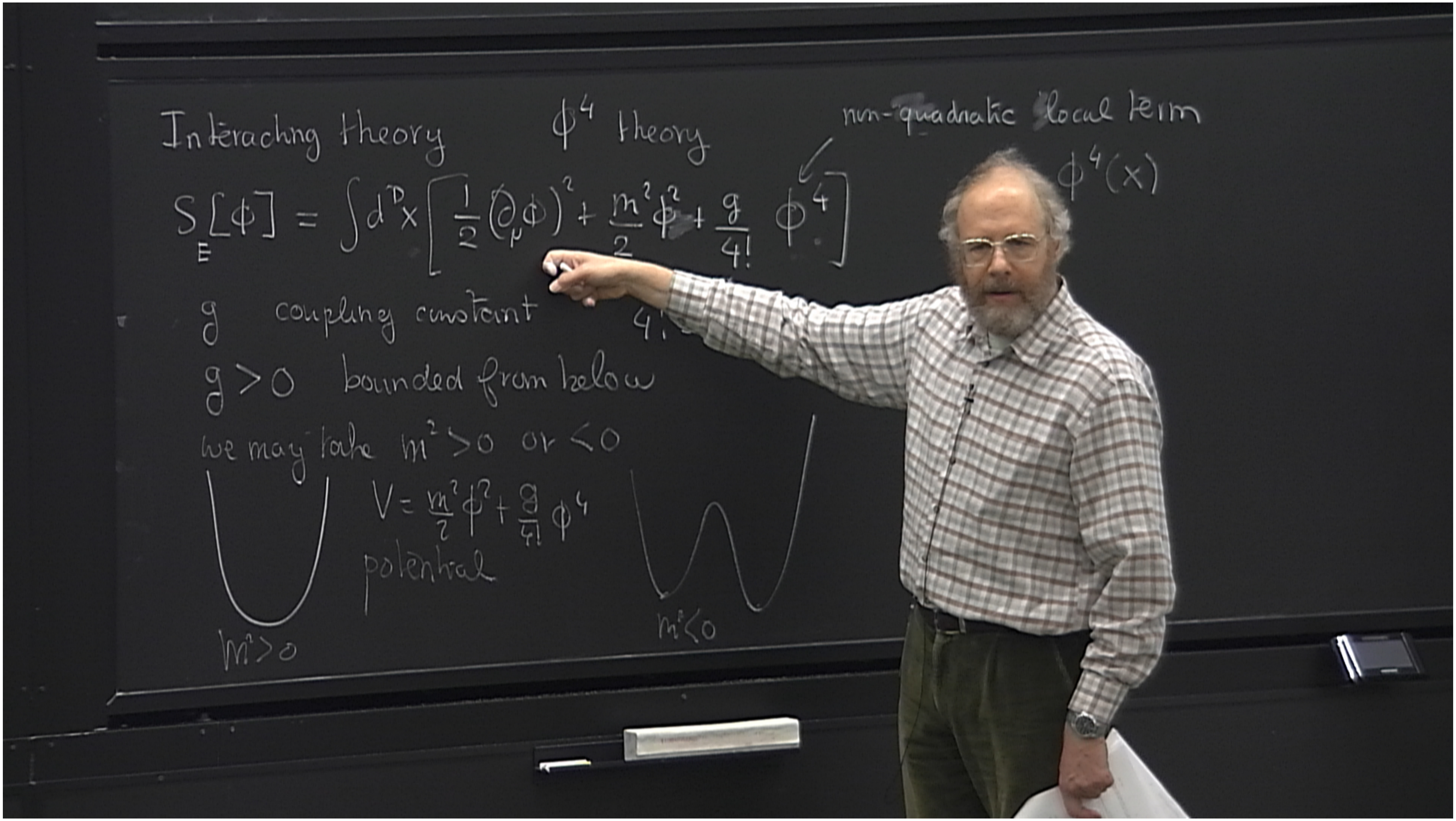

Fields φ → Particles ?
A single particle state | k >
1st excitation above the vacuum state |0>
E, k : E² = k² + m²
Källen - Lehman representation : ⇐ spectrum of the theory ←
-1
-ω² + k² + m² - iε₊
ω ≃ Energy
in the K² = k² - ω² complex plane
K²
K² = -m²
pole at
pole ⇒ resonance with


pole ⟹ resonance with a
physical state
a particle of mass m
m² a parameter of the
Lagrangian density
m = mass of the particle
Free theory property
excitation above
vacuum state |0>
trum of the theory ⟵ 2pt function.
plane
E² = k² + m²
E = m

Interacting theory $\qquad \phi^4$ theory $\qquad$ non-quadratic local term

$$S_E[\phi] = \int d^Dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right] \qquad \propto \phi^4(x)$$

g coupling constant


Interacting theory
ϕ^4 theory
non-quadratic local term
$\phi^4(x)$
$S_E[\phi] = \int d^Dx \left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$
g coupling constant
$g > 0$ bounded from below
we may take $m^2 > 0$ or < 0
$V = \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$
potential
$m^2 > 0$
$m^2 < 0$

Interacting theory $\qquad \phi^4$ theory $\qquad$ non-quadratic local term $\propto \phi^4(x)$ $\qquad$ Real time S

$$S_E[\phi] = \int d^Dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$$

g coupling constant $\qquad 4! = 24$ for convenience

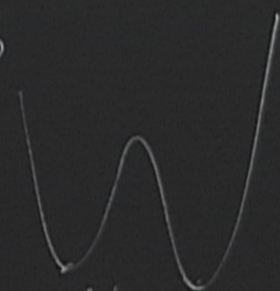
$g > 0$ bounded from below

we may take $m^2 > 0$ or < 0

$m^2 > 0$

$$V = \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$$

potential



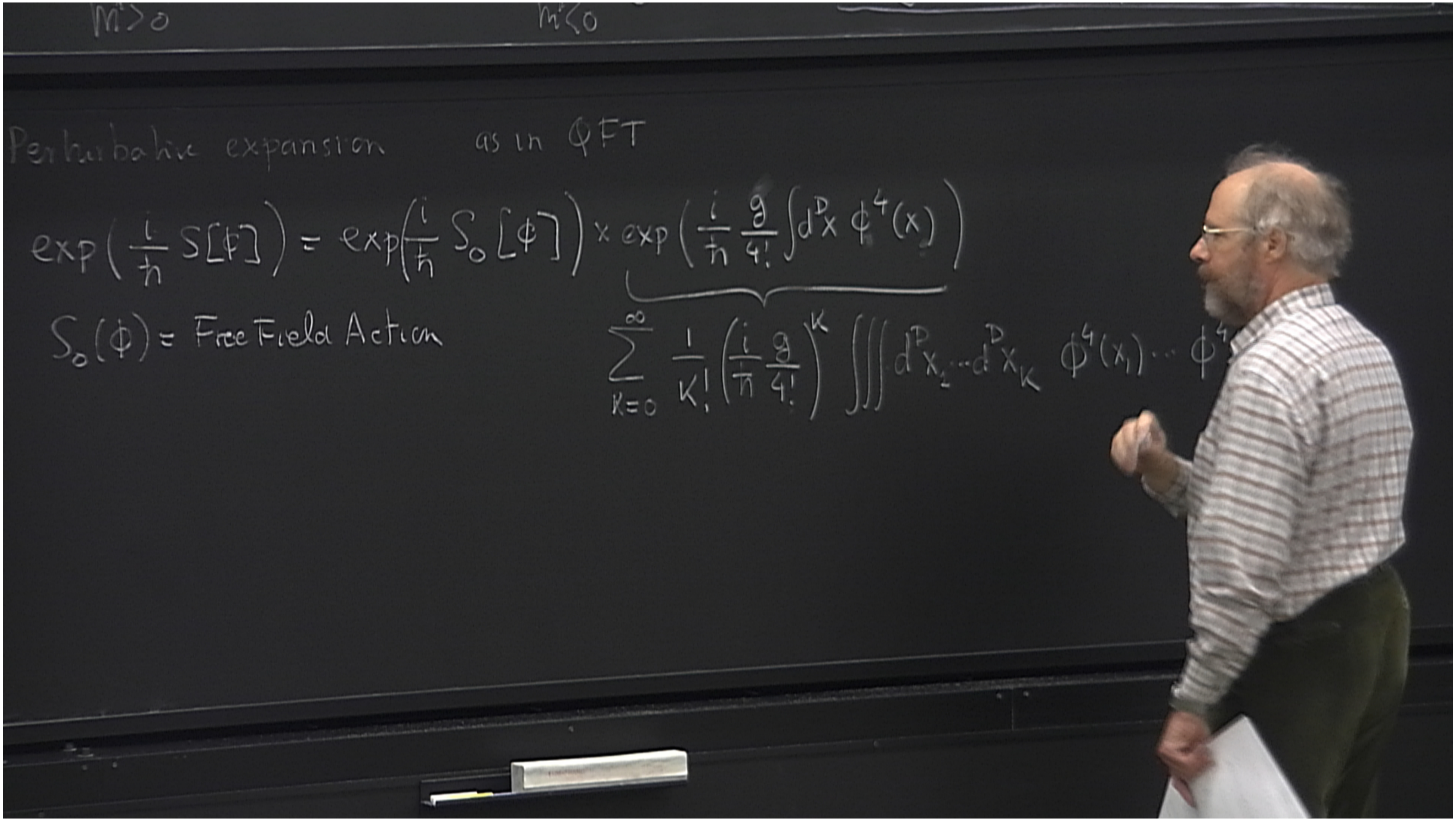
$m^2 < 0$

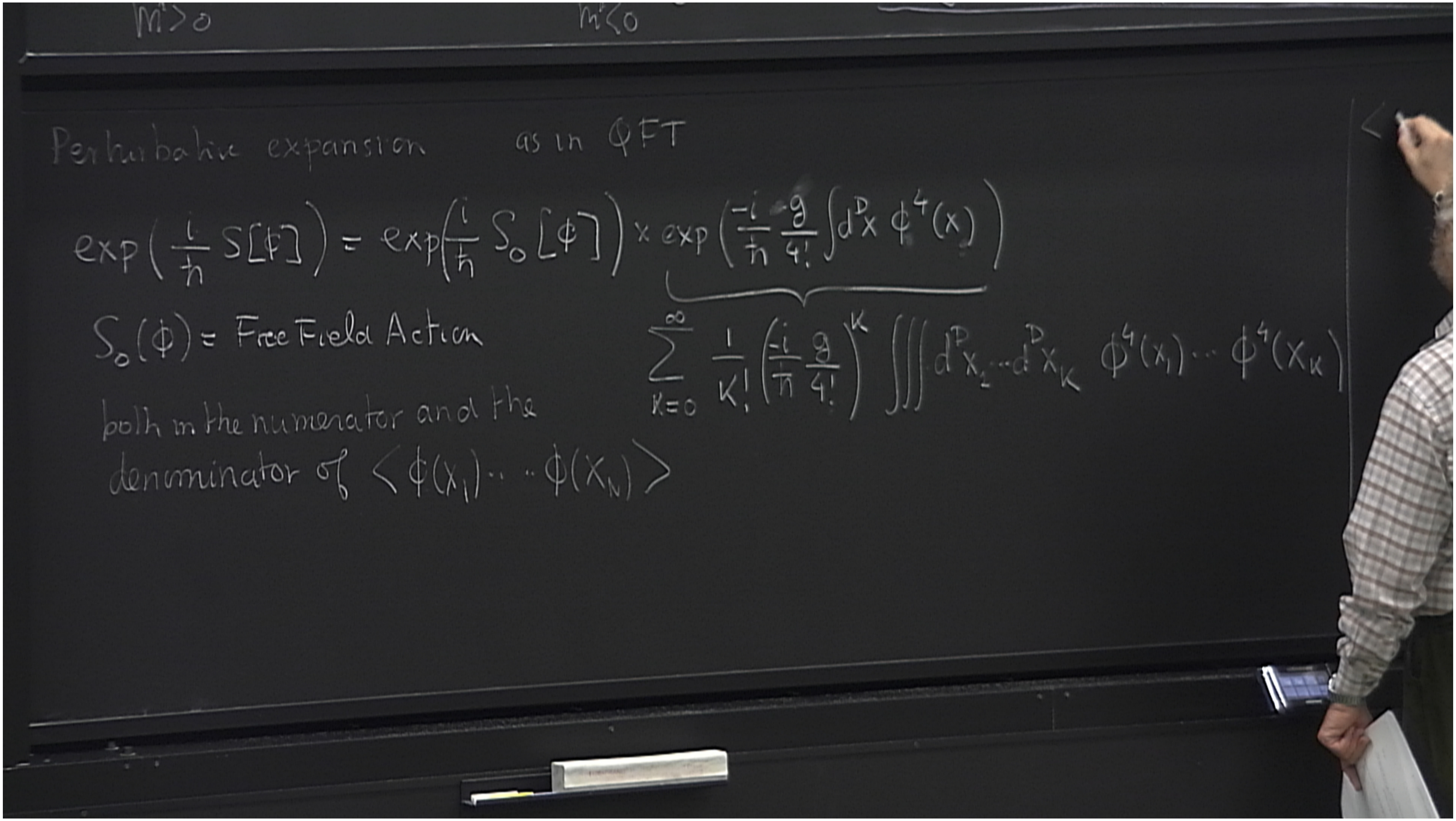
$m^2 > 0$ $m^2 < 0$

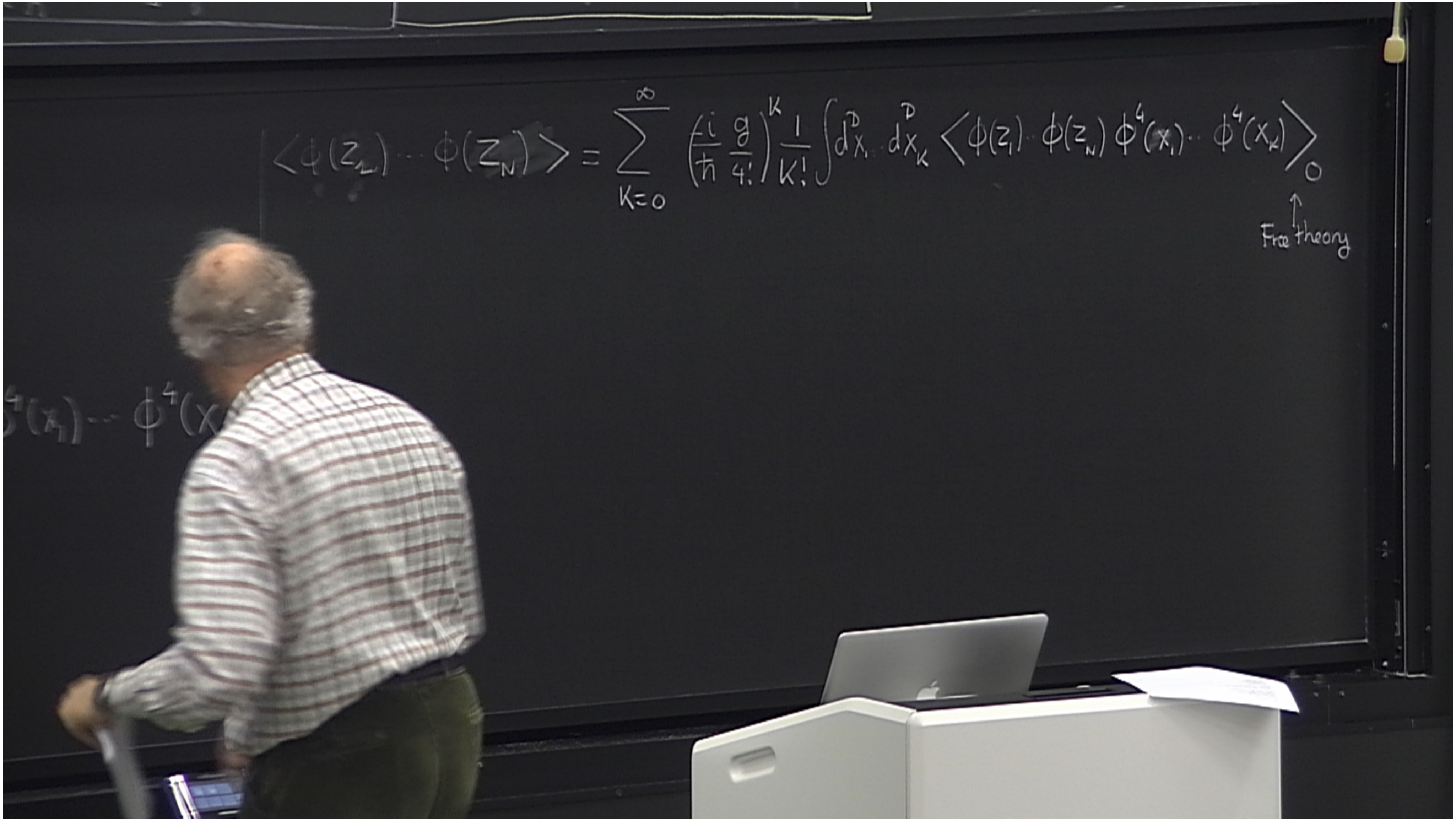
Perturbative expansion as in ΦFT

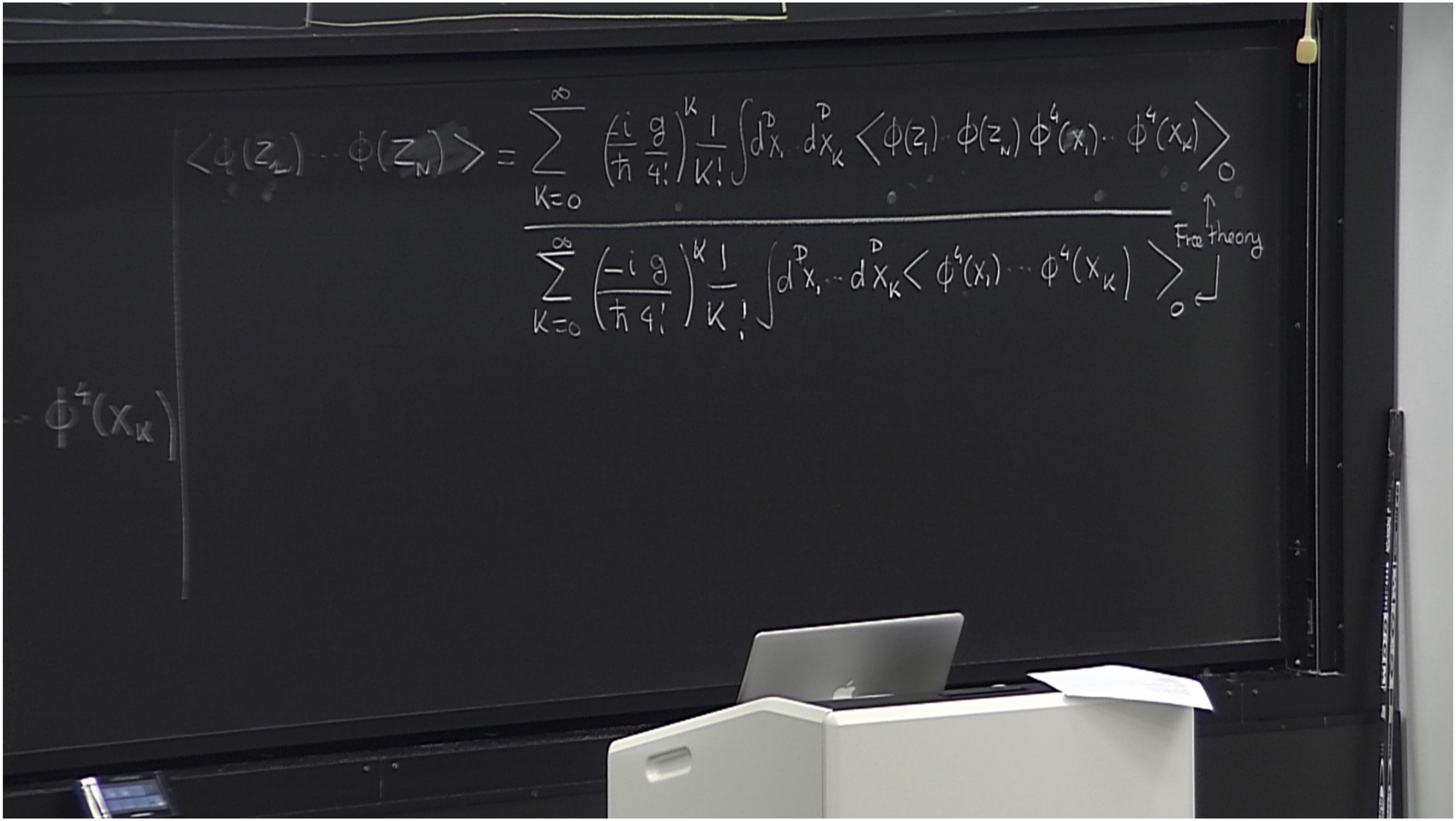
$$\exp\left(\frac{i}{\hbar} S[\phi]\right) = \exp\left(\frac{i}{\hbar} S_0[\phi]\right)$$

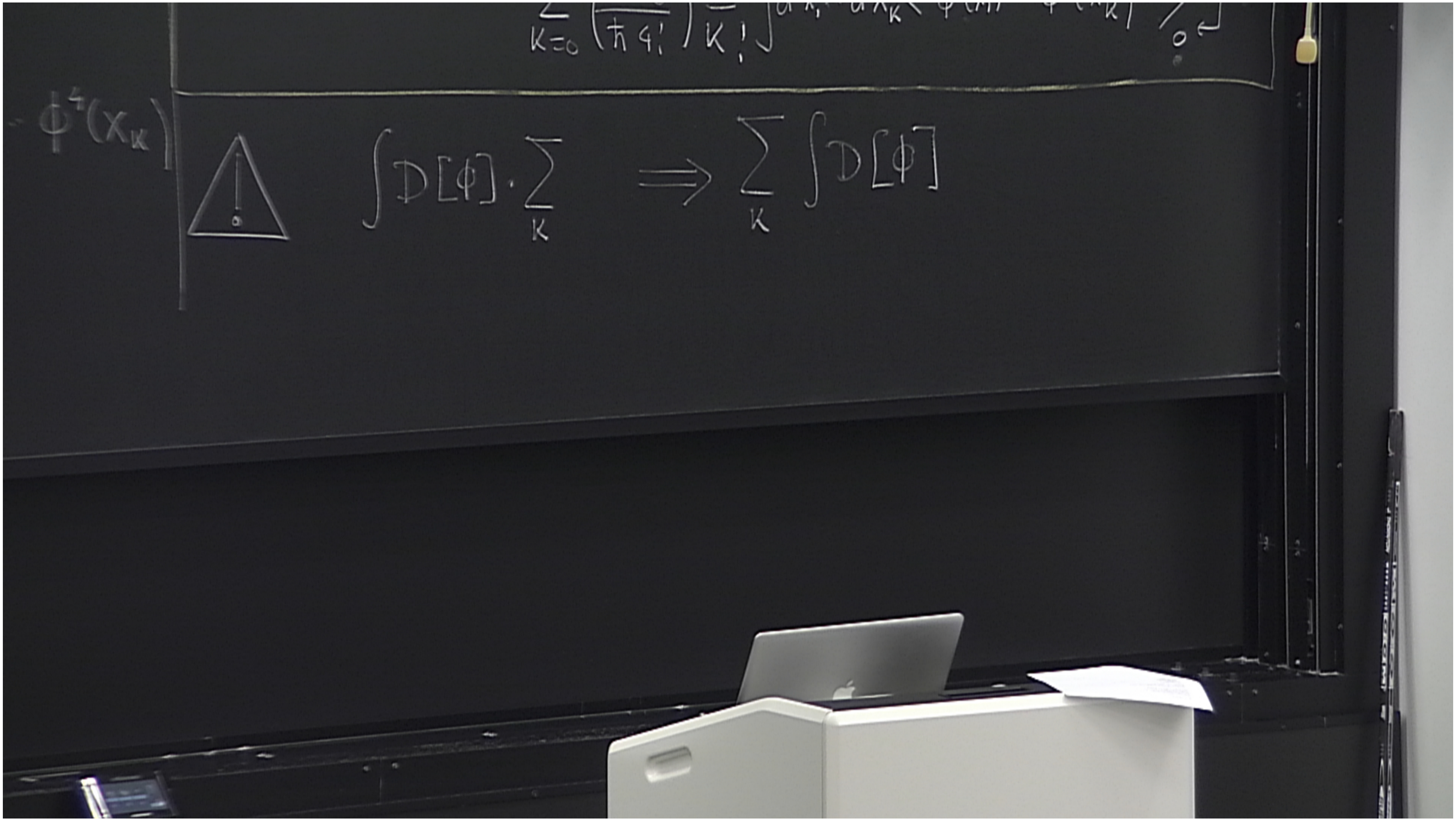
$S_0(\phi)$ = Free Field Action

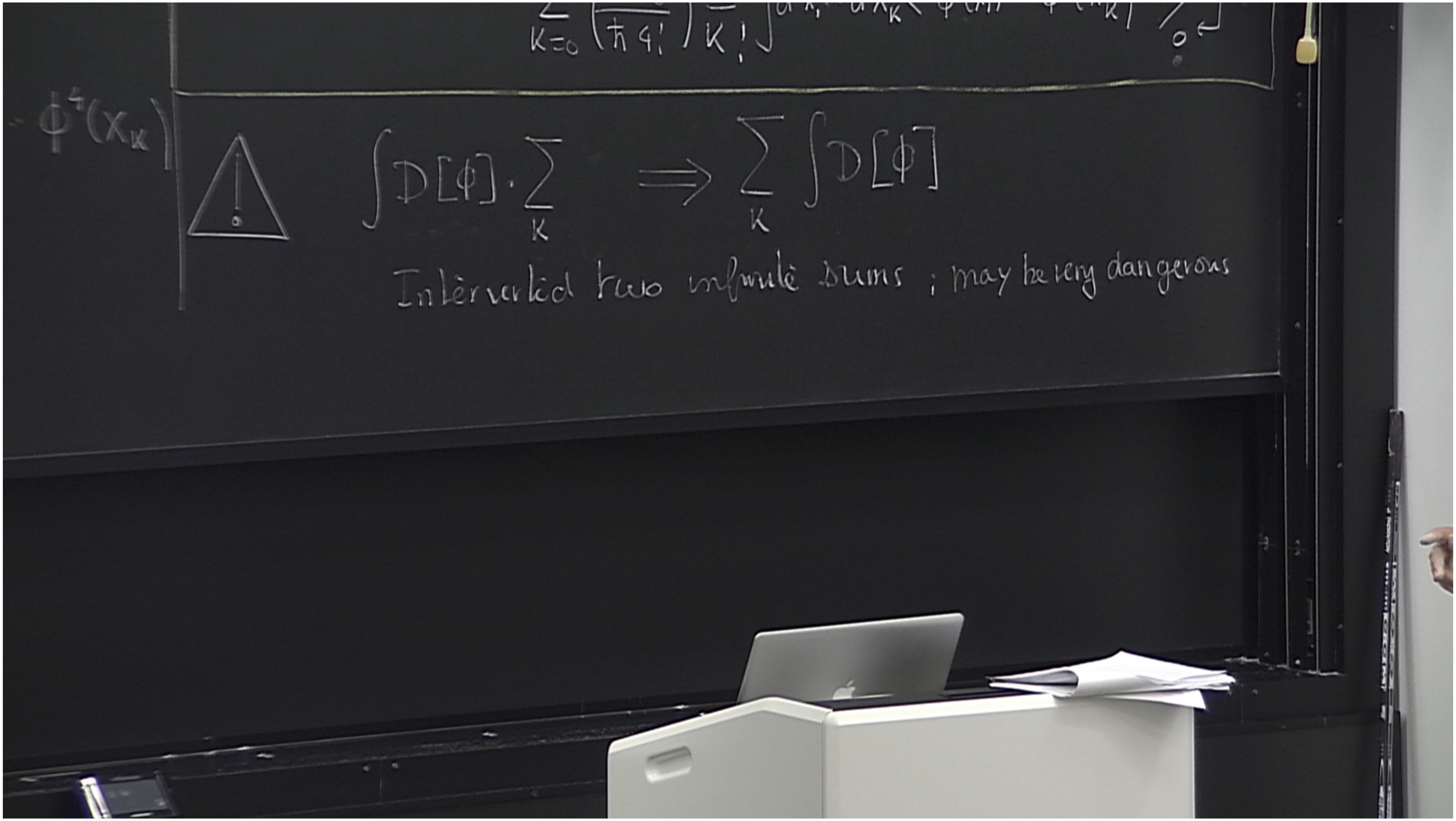

$m^2>0$
$m^2<0$
Perturbative expansion as in QFT
$\exp\left(\frac{i}{\hbar}S[\phi]\right) = \exp\left(\frac{i}{\hbar}S_0[\phi]\right) \times \exp\left(\frac{i}{\hbar}\frac{g}{4!}\int d^Dx\, \phi^4(x)\right)$
$S_0(\phi)$ = Free Field Action
$\sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{i}{\hbar}\frac{g}{4!}\right)^K \iiint d^Dx_1 \cdots d^Dx_K\ \phi^4(x_1)\cdots \phi^4$

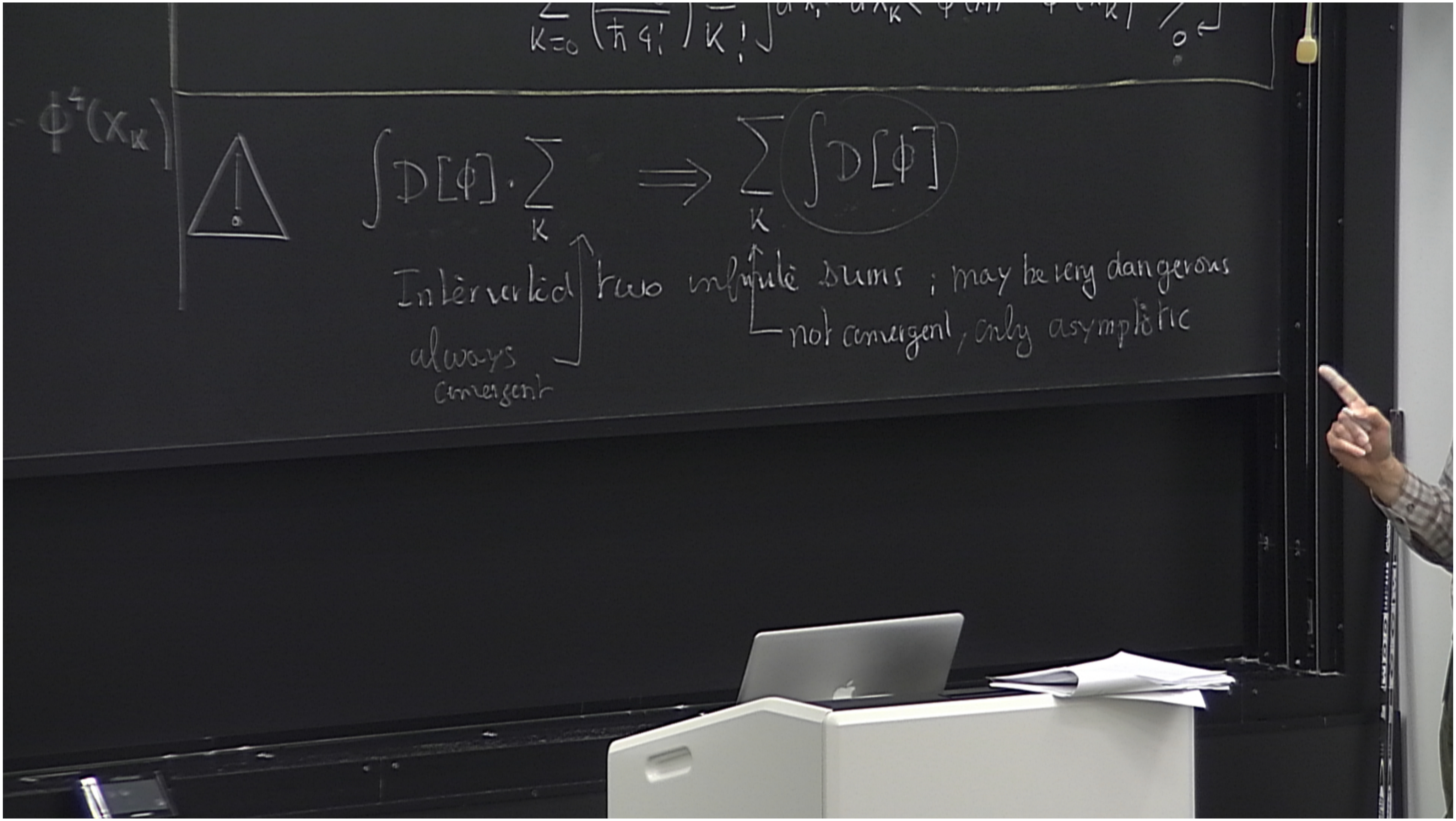

$m^2>0$
$m^2<0$
Perturbative expansion as in ϕFT
$\exp\left(\frac{i}{\hbar}S[\phi]\right) = \exp\left(\frac{i}{\hbar}S_0[\phi]\right) \times \underbrace{\exp\left(\frac{-i}{\hbar}\frac{g}{4!}\int d^Dx\, \phi^4(x)\right)}_{\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{-i}{\hbar}\frac{g}{4!}\right)^k \int\int\int d^Dx_1 \cdots d^Dx_k\, \phi^4(x_1)\cdots\phi^4(x_k)}$
$S_0(\phi)$ = Free Field Action
both in the numerator and the
denominator of $\langle \phi(x_1)\cdots\phi(x_N)\rangle$


$\langle \phi(z_1) \cdots \phi(z_N) \rangle = \sum_{k=0}^{\infty} \left(\frac{-i}{\hbar} \frac{g}{4!} \right)^k \frac{1}{k!} \int d^D x_1 \cdots d^D x_k \langle \phi(z_1) \cdots \phi(z_N) \phi^4(x_1) \cdots \phi^4(x_k) \rangle_0$
Free theory


$\cdots \phi^4(x_K))$
$$\langle \phi(z_1)\cdots\phi(z_N)\rangle = \frac{\sum_{K=0}^{\infty}\left(\frac{-i}{\hbar}\frac{g}{4!}\right)^K \frac{1}{K!}\int d^Dx_1\cdots d^Dx_K \langle \phi(z_1)\cdots\phi(z_N)\,\phi^4(x_1)\cdots\phi^4(x_K)\rangle_0}{\sum_{K=0}^{\infty}\left(\frac{-ig}{\hbar\, 4!}\right)^K \frac{1}{K!}\int d^Dx_1\cdots d^Dx_K \langle \phi^4(x_1)\cdots\phi^4(x_K)\rangle_0}$$
Free theory


$\int D[\phi]\cdot\sum_K \Rightarrow \sum_K \int D[\phi]$


∫D[φ]·Σ_K ⟹ Σ_K ∫D[φ]
Interverted two infinite sums ; may be very dangerous



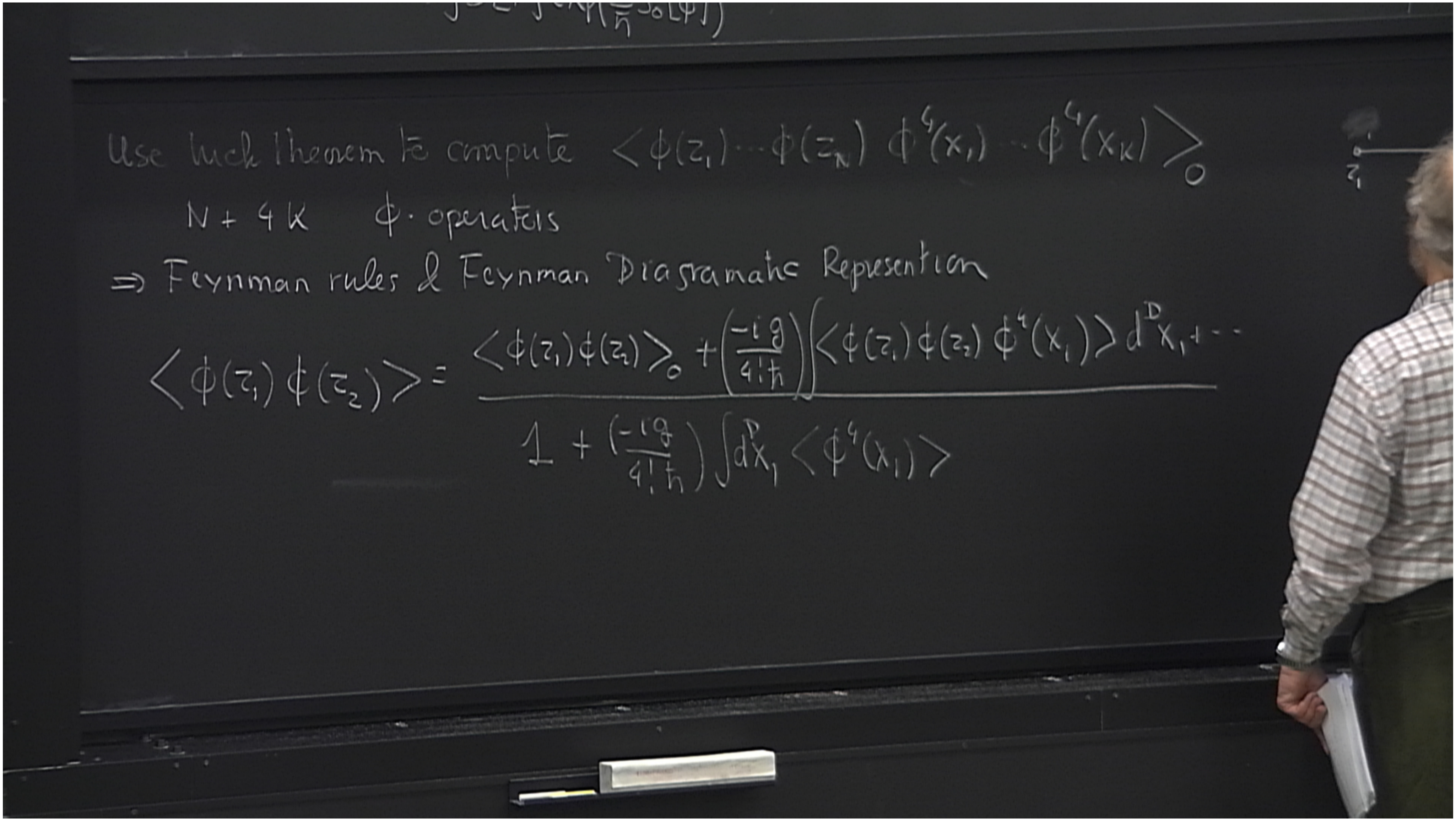
$\int D[\phi]\cdot\sum_K \Longrightarrow \sum_K \int D[\phi]$
Interverted two infinite sums ; may be very dangerous
always convergent
not convergent, only asymptotic

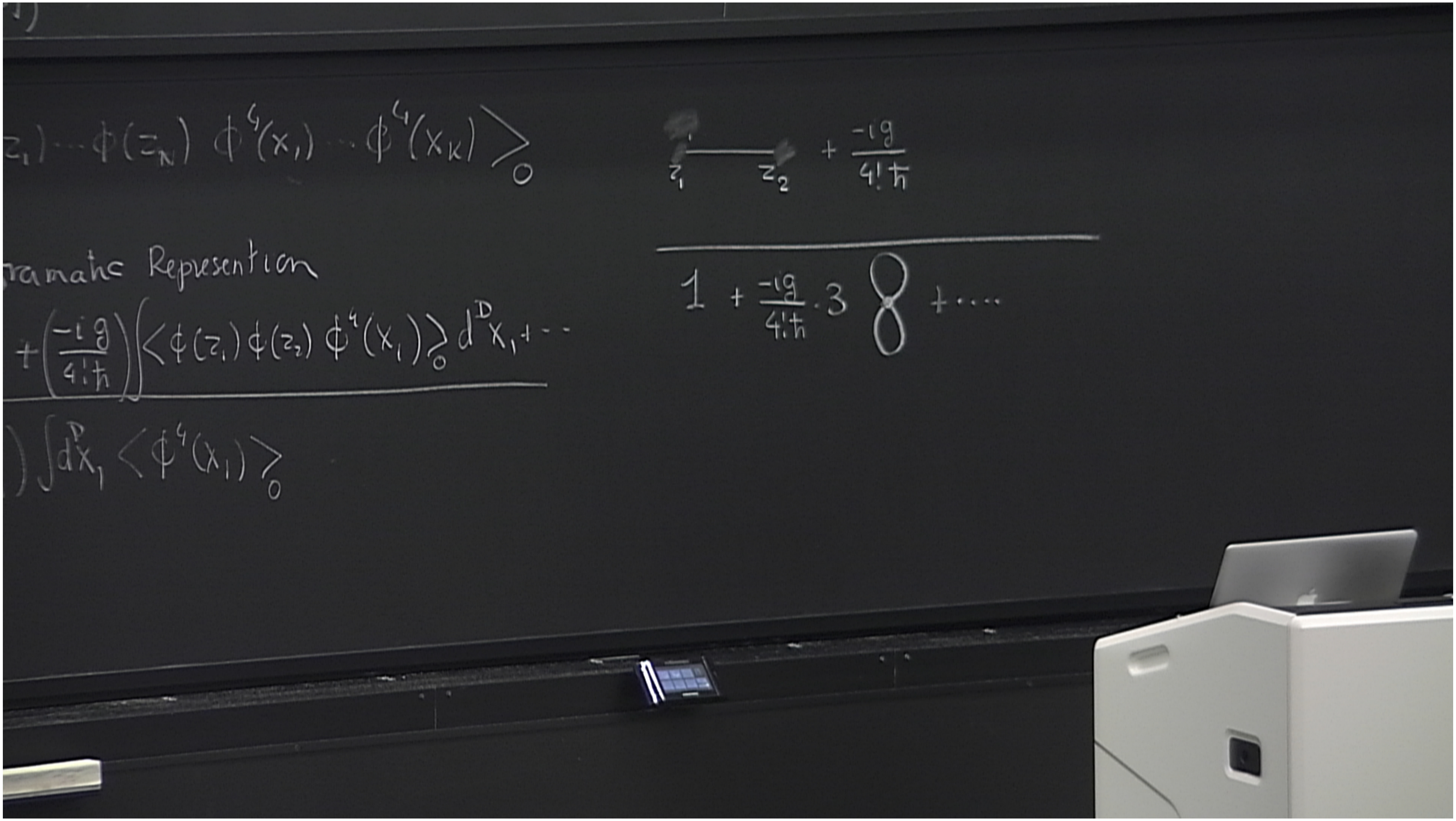
$\int \mathcal{D}[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$

Use Wick theorem to compute $\langle \phi(z_1) \cdots \phi(z_N)\, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0$

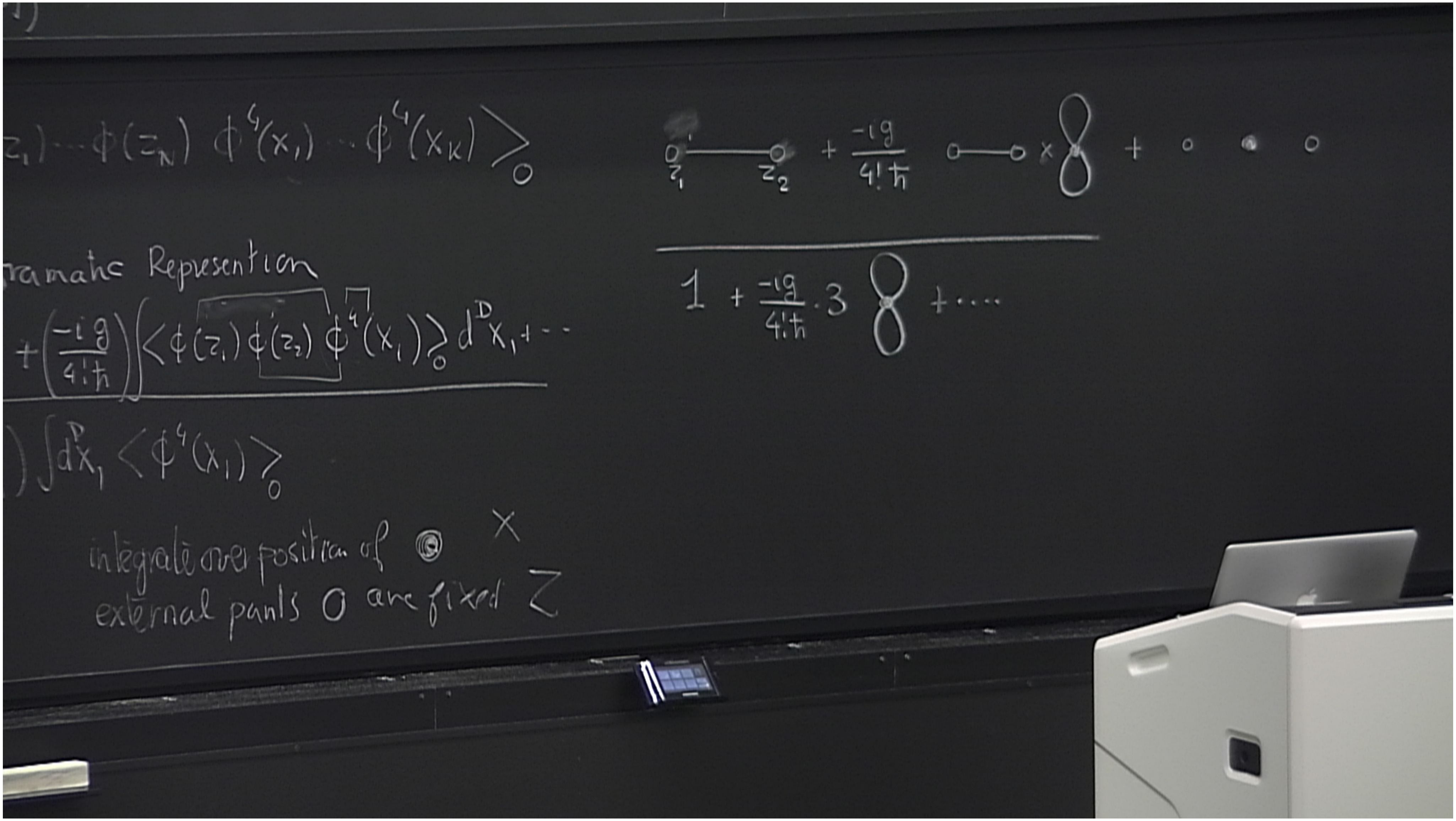
$N + 4K$ ϕ-operators

$\Rightarrow$ Feynman rules & Feynman Diagramatic Represention

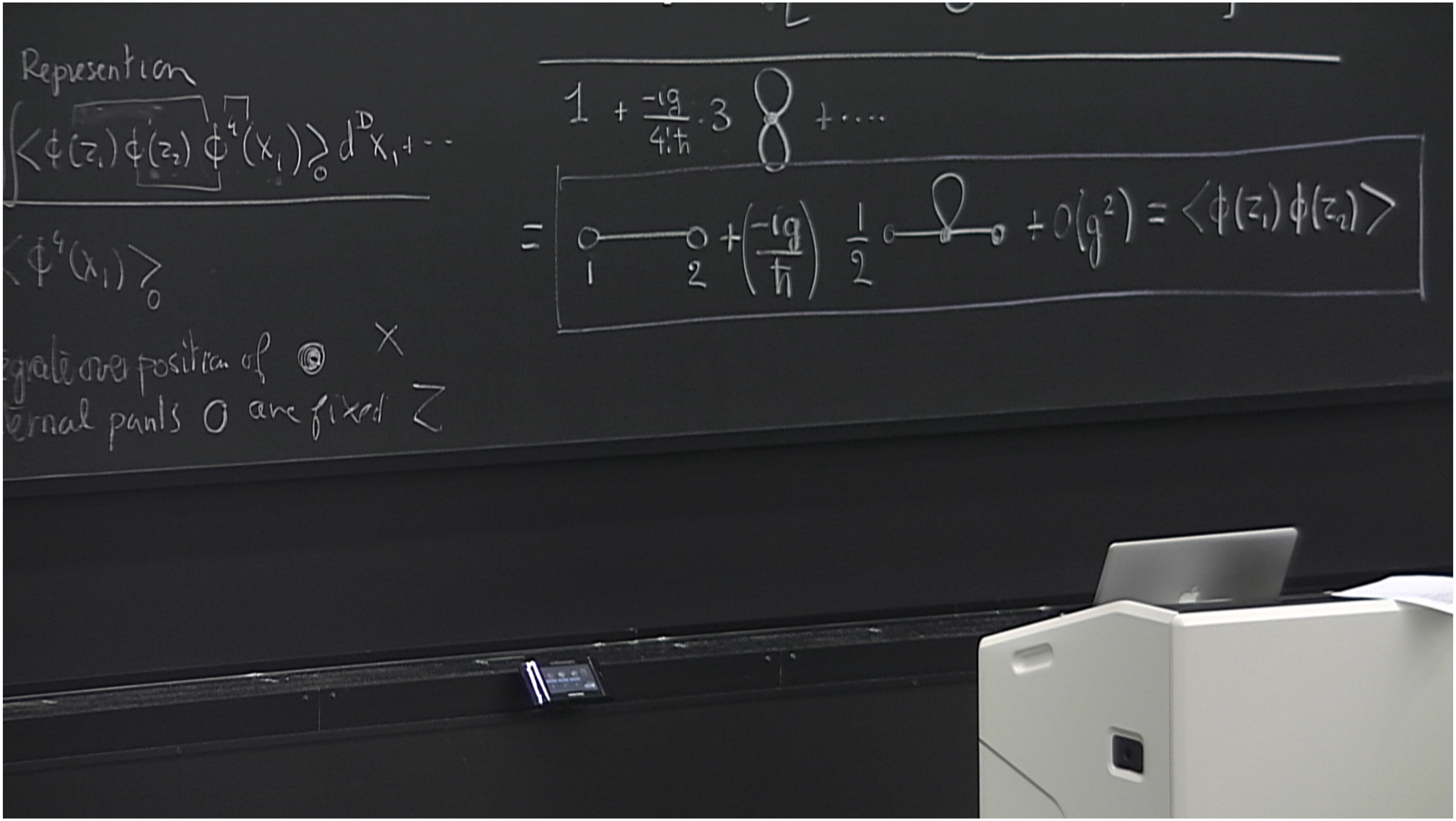

Use Wick theorem to compute $\langle \phi(z_1)\cdots\phi(z_N)\,\phi^4(x_1)\cdots\phi^4(x_K)\rangle_0$
$N+4K$ ϕ· operators
⇒ Feynman rules & Feynman Diagramatic Represention
$\langle \phi(z_1)\phi(z_2)\rangle = \dfrac{\langle\phi(z_1)\phi(z_2)\rangle_0 + \left(\frac{-ig}{4!\hbar}\right)\int\langle\phi(z_1)\phi(z_2)\,\phi^4(x_1)\rangle\, d^Dx_1 + \cdots}{1+\left(\frac{-ig}{4!\hbar}\right)\int d^Dx_1\,\langle\phi^4(x_1)\rangle}$

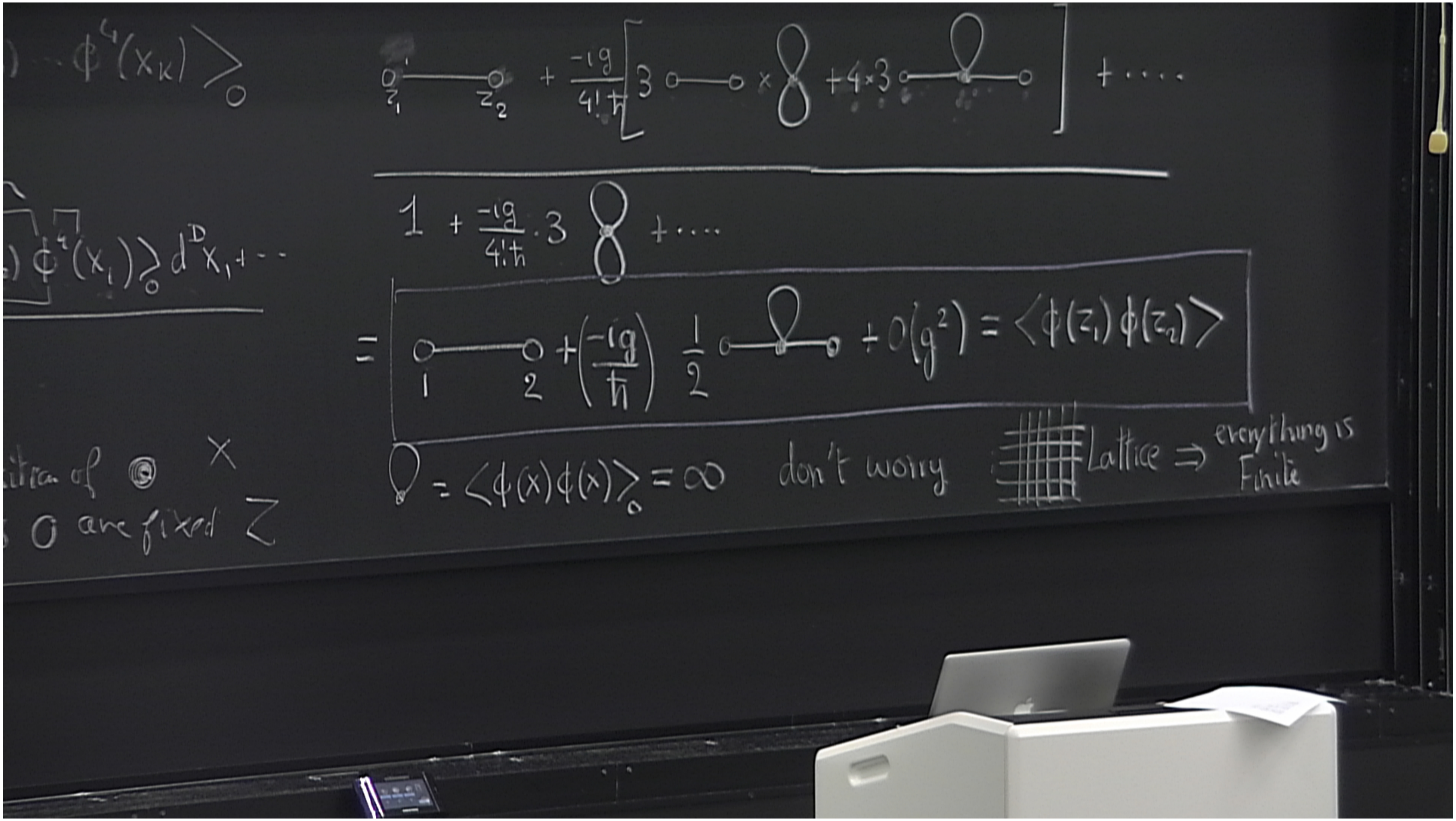


$z_1) \cdots \phi(z_N)\, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0$
ramatic Representation
$+ \left(\frac{-ig}{4!\hbar}\right) \int \langle \phi(z_1)\phi(z_2)\, \phi^4(x_1) \rangle_0\, d^D x_1 + \cdots$
$) \int d^D x_1 \langle \phi^4(x_1) \rangle_0$
z_1 z_2 $+ \frac{-ig}{4!\hbar}$
$1 + \frac{-ig}{4!\hbar} \cdot 3$ $+ \cdots$



$z_1) \cdots \phi(z_N)\ \phi^4(x_1) \cdots \phi^4(x_K) \rangle_0$
z_1 z_2 $+ \frac{-ig}{4!\hbar}$ $\times$ $+$
ramatic Represention
$+ \left(\frac{-ig}{4!\hbar}\right) \int \langle \phi(z_1)\phi(z_2)\phi^4(x_1) \rangle_0 d^D x_1 + \cdots$
$1 + \frac{-ig}{4!\hbar} \cdot 3$ $+ \cdots$
$) \int d^D x_1 \langle \phi^4(x_1) \rangle_0$
integrate over position of $\times$
external points O are fixed Z


Represention
$\int \langle \phi(z_1)\phi(z_2)\phi^4(x_1)\rangle_0 \, d^D x_1 + \cdots$
$\langle \phi^4(x_1)\rangle_0$
...grate over position of
...ernal points O are fixed Z
$1 + \frac{-ig}{4!\hbar} \cdot 3$ + ...
= 1 2 $+\left(\frac{-ig}{\hbar}\right)\frac{1}{2}$ $+ O(g^2) = \langle \phi(z_1)\phi(z_2)\rangle$


= ⟨φ(z1) φ(z2)⟩
= ⟨φ(x)φ(x)⟩0 = ∞
don't worry
Lattice ⇒ everything is Finite
are fixed

Interacting theory

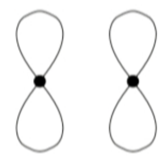
vacuum diagrams

$N = 0,\ K = 1$

$$\frac{1}{4!}\int_{x_1}\langle\Phi^4(x_1)\rangle_0 \; = \; \frac{1}{8}$$

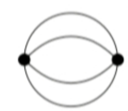
$N = 0,\ K = 2$

$$\frac{1}{2!\,(4!)^2}\int_{x_1}\int_{x_2}\langle\Phi^4(x_1)\Phi^4(x_2)\rangle_0 \; = \; \frac{1}{128}$$



$+\frac{1}{16}$

$+\frac{1}{48}$



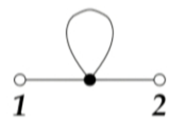
2 points diagrams

$N = 2,\ K = 0$

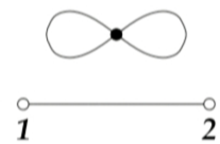
$$\langle \Phi(z_1)\Phi(z_2)\rangle_0 \ = $$

$N = 2,\ K = 1$

$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1)\rangle_0 \ = \ \tfrac{1}{2}$$



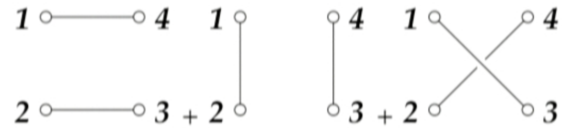
$+\ \tfrac{1}{8}$



4 points diagrams

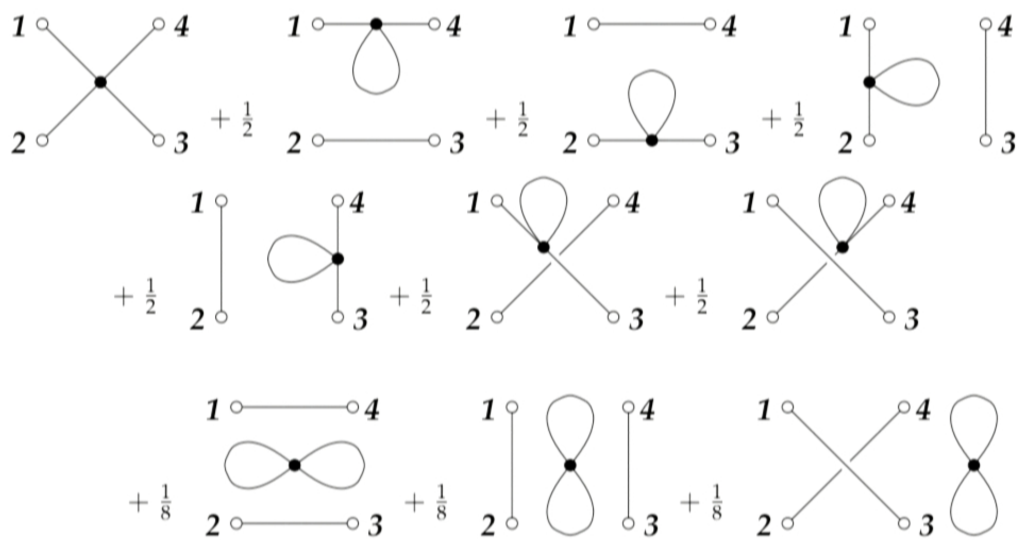
$N = 4,\ K = 0$

$$\langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\rangle_0 \ =$$

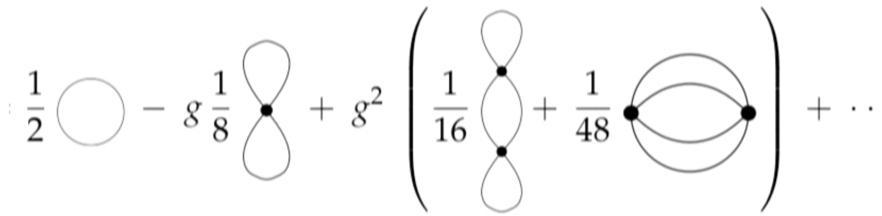


$N = 4,\ K = 1$

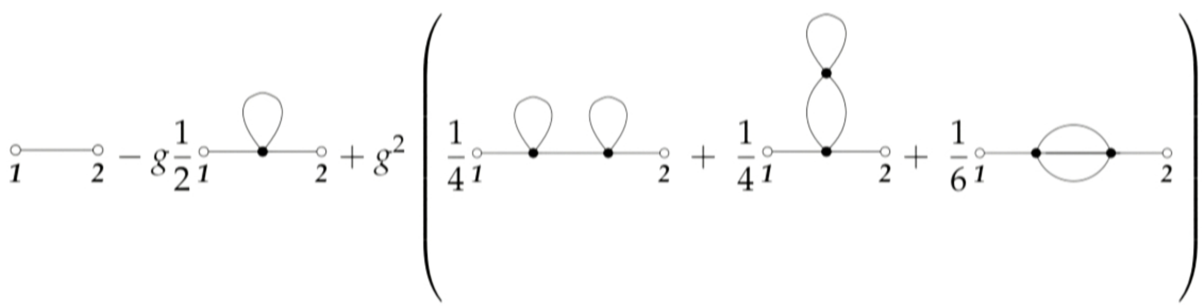
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi(z_3)\Phi(z_4)\Phi^4(x_1)\rangle_0 \ =$$



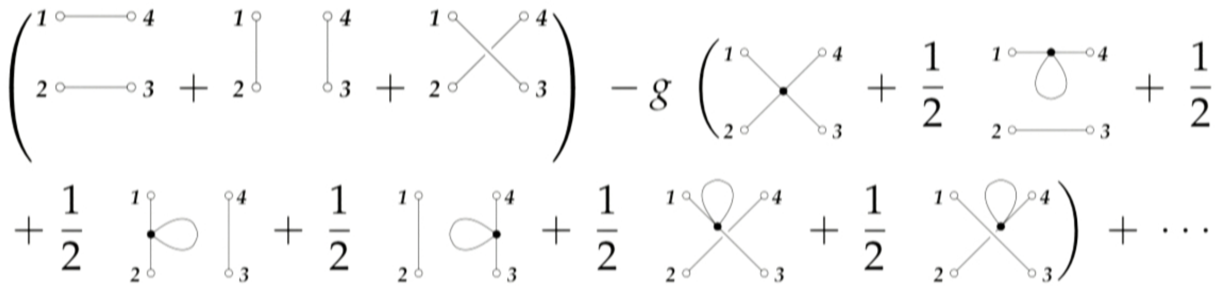
Connected vacuum diagrams



2 points function = connected 2 points function

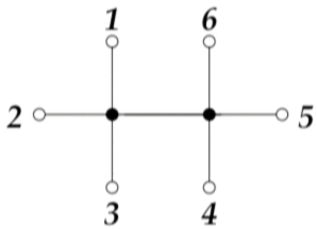


4 points function (up to order 1)



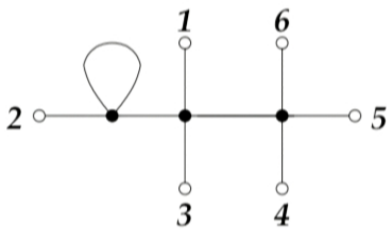
Connected 6 points function (up to order 3)

g^2



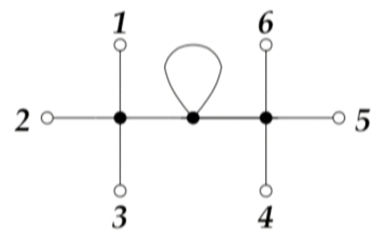
$+\cdots$

$-g^3\frac{1}{2}$



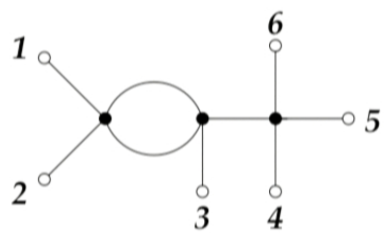
$+\cdots$

$-g^3\frac{1}{2}$



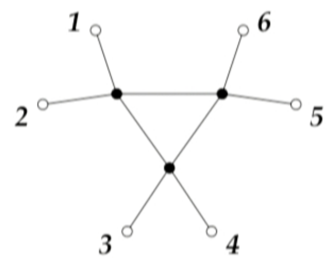
$+\cdots$

$-g^3\frac{1}{2}$



$+\cdots$

$-g^3$



$+\cdots$