Title: Aspects of Modified Gravity

Date: Oct 31, 2013 11:00 AM

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Abstract: In this talk I will give an introduction to some of my research into modified gravity over the last three years. I will begin by describing my implementation of chameleon models into supersymmetry and discuss some of the new features and cosmology that arise in this formalism. I will then change direction and talk about my work using astrophysical effects as novel probes of modified gravity theories and present some new results on modified gravity stellar oscillation theory. I will end by discussing work in progress and the possible future constraints that could be placed.

What I Do (Modified Gravity)

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31/10/2013

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Supersymmetry: Philippe Brax & Anne-Christine Davis
Astrophyics: Bhuvnesh Jain & Vinu Vikram

Outline

Why do People Study Modified Gravity?

Einstein's General Relativity describes gravity perfectly in many situations so why do people study modifications?

Dark Energy:

The universe is accelerating and GR requires ∼69% (Planck) of the universe to be unknown exotic matter in order to account for this.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Any modification of GR necessarily introduces a new degree of freedom Weinberg 1965.

Why do People Study Modified Gravity?

Beyond the Standard Model Physics:

Many theories of *fundamental physics* predict low-energy effective actions where new scalar degrees of freedom couple to matter e.g. string theory

$$\mathcal{L}_{4\text{-D}} \supset e^{-\phi} R.$$

These are a long-range modification of GR.

Why do we need to Screen the Modifications?

General Relativity has been tested to incredibly high precision over the last 100 years or so! So well that any modification is usually rendered irrelevant once the bounds are imposed e.g. Cassini:

$$\frac{F_5}{F_{\rm N}} < 10^{-5} \quad (10^{-13} \quad \text{for WEP violations})$$

The Get-Out Clause

Is this a dead end?

No

All these experiments have been performed locally, in our own solar system.

There is nothing preventing large-scale modifications of GR provided that we satisfy these local bounds.

Theories with this feature are said to posses *screening mechanisms*.

Screening Fifth-Forces

Scalar Field ϕ coupled to matter.

Expand fluctuations about the background $\phi = \phi_0 + \delta\phi$ to second order:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -Z_{\mu\nu}(\phi_0, \partial_\alpha\phi_0, \Box\phi_0, \cdots)\partial^\mu\delta\phi\partial^\nu\delta\phi - m_{\mathrm{eff}}^2(\phi_0)\delta\phi^2 - \beta(\phi_0)\frac{\delta\phi}{M_{\mathrm{pl}}}T + \cdots$$

The coupling to matter gives rise to new or "fifth" forces.

Screening Fifth-Forces

The fifth-force is screened if:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -\underbrace{Z_{\mu\nu}}_{\text{Large}}\partial^\mu\delta\phi\partial^\nu\delta\phi - \underbrace{m_{\text{eff}}^2}_{\text{large}}\delta\phi^2 - \underbrace{\beta(\phi_0)}_{\text{small}}\frac{\delta\phi}{M_{\text{pl}}}T + \cdots$$

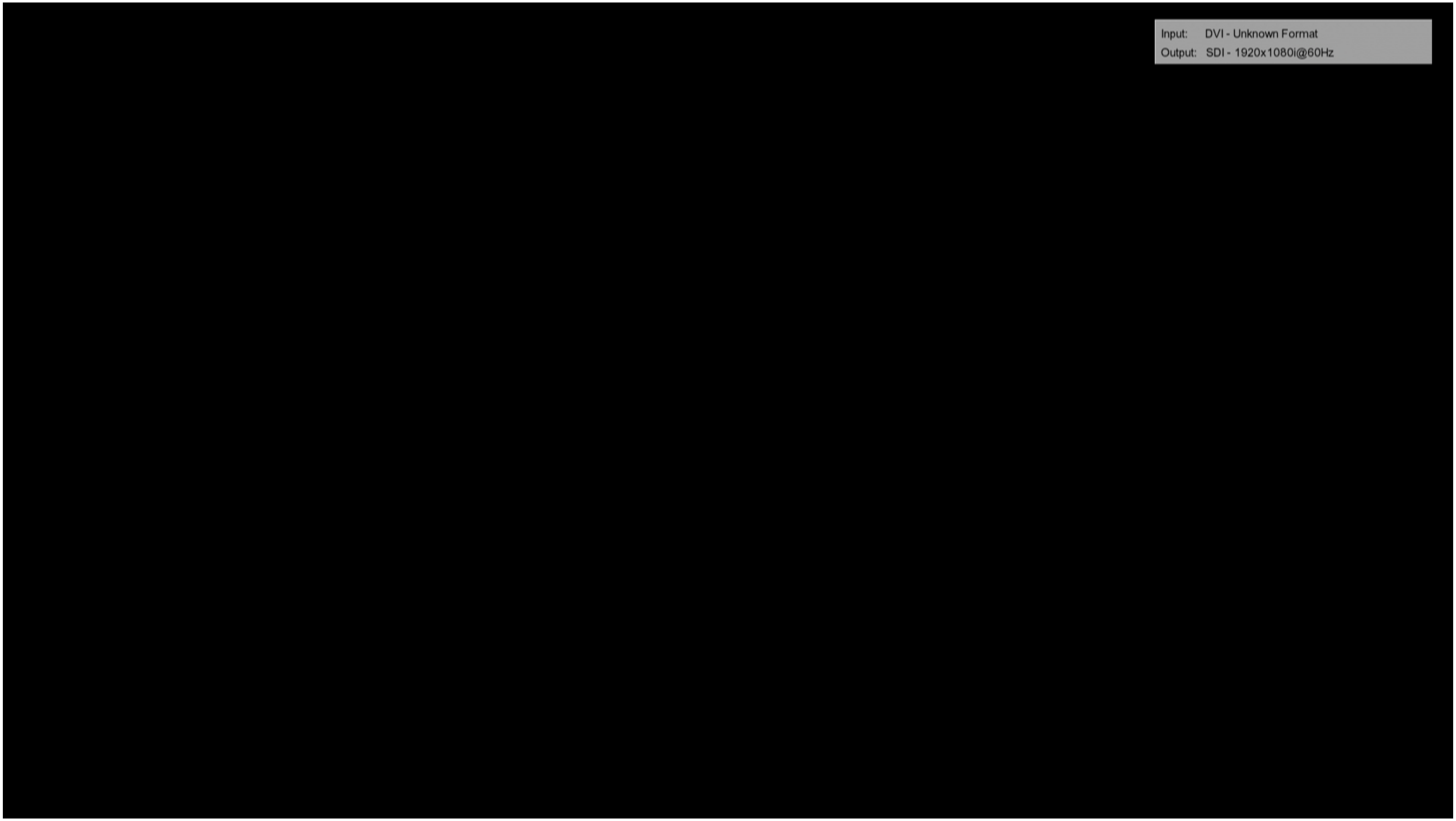
Scalar-Tensor Screening

Scalar field is conformally coupled to matter via the "coupling function" $A(\phi)$ in the Einsten Frame:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2 R}{2} - \nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_{\mathrm{m}}\left[\psi_{\mathrm{i}}; A^2(\phi)g_{\mu\nu}\right]$$

Matter moves on geodesics of the Jordan Frame metric $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} \Rightarrow$ observers in the Einstein frame infer a *fifth-force*

$$\vec{F}_\phi = \frac{\beta(\phi)}{M_{\mathrm{pl}}}\vec{\nabla}\phi \quad \beta(\phi) = M_{\mathrm{pl}}\frac{\mathrm{d}\ln(A)}{\mathrm{d}\phi}$$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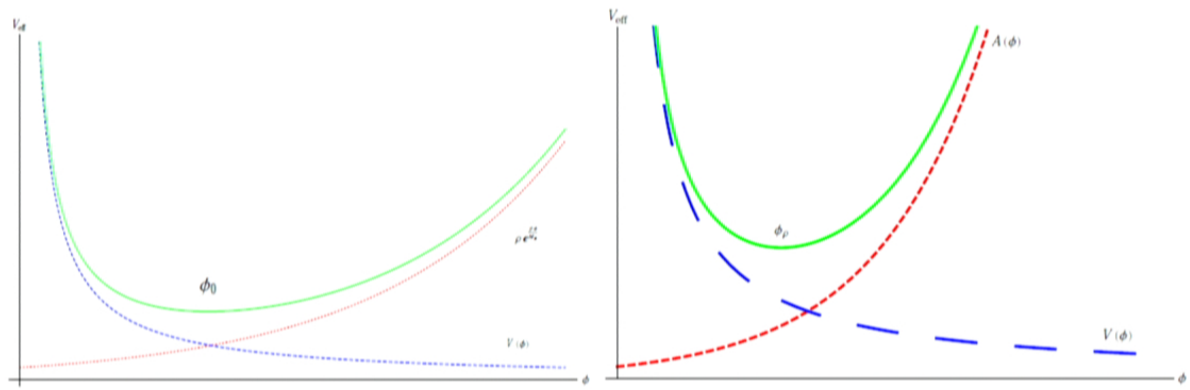
Chameleon Screening

Khoury & Weltman 03

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \quad A(\phi) = e^{\beta \frac{\phi}{M_{\rm pl}}}$$



Low ρ High ρ

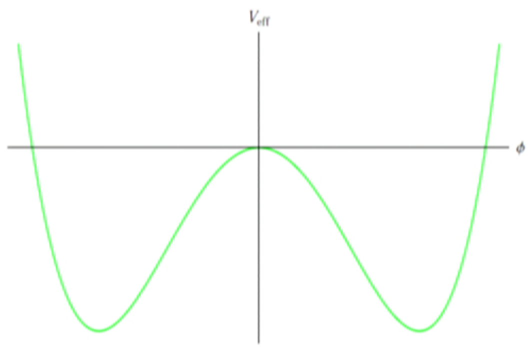
$$m_{\rm eff}(\phi_\rho) \gg m_{\rm eff}(\phi_0)$$

$f(R)$ gravity is a chameleon under some circumstances.

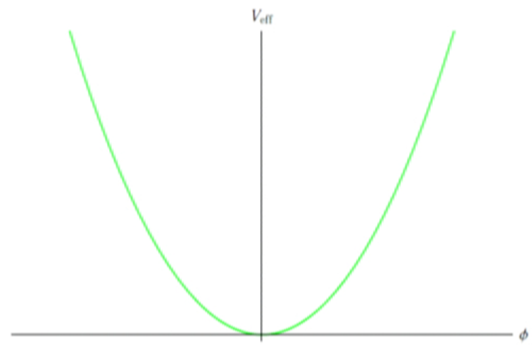
Symmetron Screening

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4, \quad A(\phi) = 1 + \frac{\phi^2}{2M^2}$$

$$V_{\rm eff}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{\lambda}{4}\phi^4$$



$\rho < \mu^2 M2$



$\rho \geq \mu^2 M^2$

$$\beta(\rho \geq \mu^2 M^2) \approx 0 \Rightarrow F_\phi \propto \beta(\phi)\nabla\phi \approx 0$$

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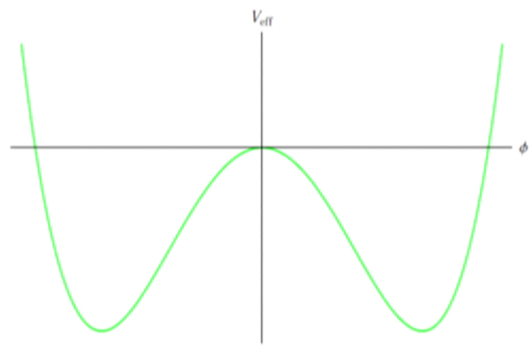

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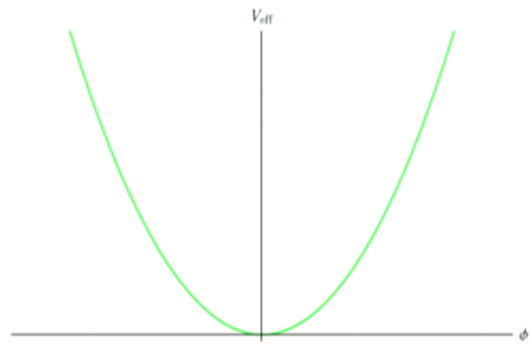

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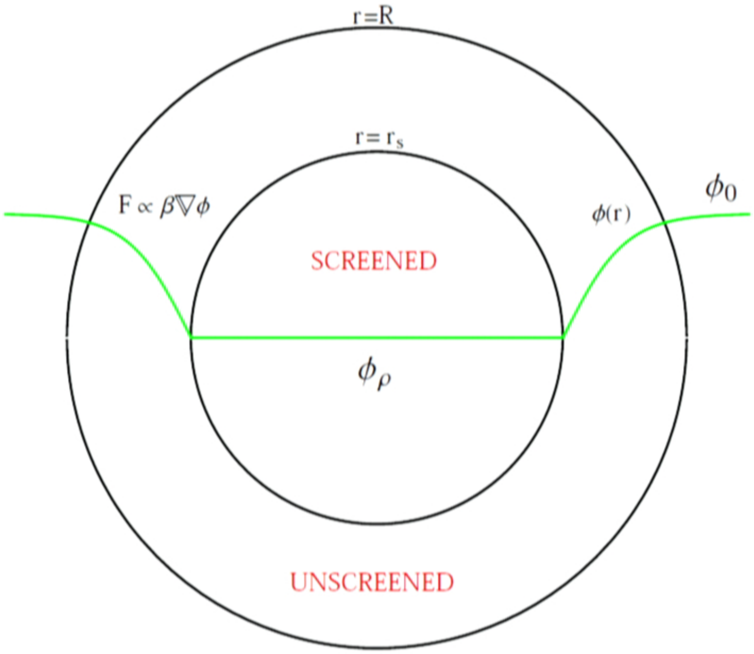
Spherical Screening



Screening Parameterisation

There is a model-independent parameterisation perfect for small-scale tests:

$\alpha \equiv 2\beta(\phi_0)^2 \quad G \to G(1+\alpha)$ when the object is fully unscreened.

$f(R)$ gravity has $\alpha = 1/3$.

$$\chi_0 \equiv \frac{\phi_0}{2M_{\rm pl}\beta(\phi_0)} \text{ - the self-screening parameter.}$$

Rule of thumb: an object is partially unscreened if

$$\chi_0 > \Phi_{\rm N}.$$

$\Phi_{\rm N}$ is the Newtonian Potential.

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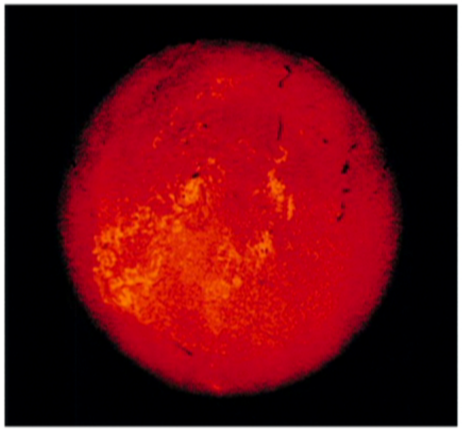
Self-Screening Parameter

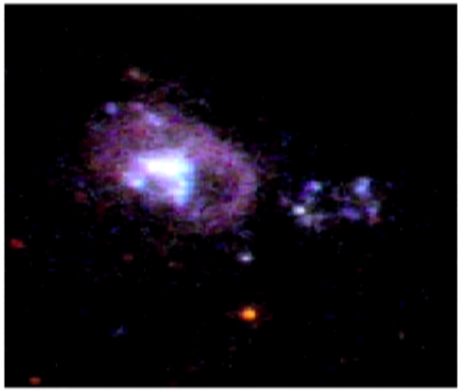
$$\chi_0 \sim 10^{-7}$$



Red giant stars have $\Phi_{\rm N} \sim 10^{-7} - 10^{-8}$ (Jain, Vikram & JS 12).

Self-Screening Parameter

Dwarf galaxies in cosmic voids have $\Phi_{\mathrm{N}} \sim 10^{-8}$.



Current data is not enough for definite constraints (Jain & Vanderplas 11, Vikram, Cabre, Jain & Jake VanderPlas 13).

Dwarf galaxies in voids may host other unscreened astrophysical objects and the data is better for these.

Dwarf Galaxies as Laboratories

Dwarf galaxies in voids have $\Phi_{\rm N} \sim 10^{-8}$: they are the perfect testing ground for scalar-tensor theories.

Any object where $\Phi_{\rm N} < \chi_0$ will be unscreened in a dwarf galaxy located in a cosmic void!

A Screening Map

Screening maps (Cabre et al. 12) for $\chi_0 = 10^{-5}$, 10^{-6} and 10^{-7} have been made using SDSS data and are publicly available:

$$\Phi_{\mathrm{N}}^{\mathrm{self}} > \chi_0 \Rightarrow \text{ self-screened}$$

$$\Phi_{\mathrm{N}}^{\mathrm{ext}} > \chi_0 \Rightarrow \text{ environmentally-screened}$$

The maps have been calibrated and tested against N-body code predictions.

This Talk

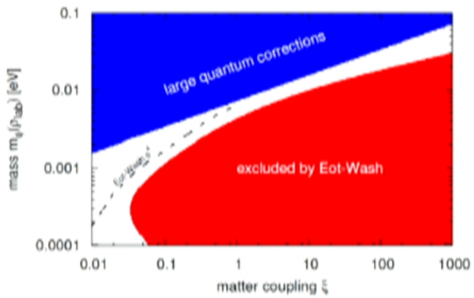
1. Supersymmetric completions.
2. Astrophysical tests (including new stuff, I promise).
3. Things I am working on.

Why Search for UV Completions?

- Make contact with fundamental physics.
- New constraints/observational signatures once we have a unified framework.
- Current models are unstable to quantum corrections.

Upadhye, Khoury & Hu 2012

$$\Delta V_{1-\text{loop}} = m_\phi^4 \ln\left(\frac{m_\phi^2}{\mu^2}\right)$$



A Supersymmetric Framework

Brax, Davis & JS 2012, 2013. See also Brax & Martin (2006, 2007, 2007), Hinterbichler, Khoury & Natase 2011.

Three sectors: Observable, Hidden (SUSY breaking) and Dark. Gravity Superfield Φ, dark matter fields $\Phi_\pm$

$$K = \Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_- + \hat{K}(\Phi^\dagger, \Phi)$$

$$W = \hat{W}(\Phi) + mA(\Phi)\Phi_+\Phi_-$$

The fermions have a mass term

$$\mathcal{L}_\pm = \frac{\partial^2 W}{\partial \Phi_+ \partial \Phi_-}\psi_+\psi_- = mA(\Phi)\psi_+\psi_-; \quad \rho_c = m\langle\psi_+\psi_-\rangle$$

$$V_{\text{eff}}(\Phi) = \frac{1}{K^{\Phi\Phi^\dagger}}\left|\frac{d\hat{W}}{d\Phi}\right|^2 + \rho_c(A(\Phi) - 1)$$

We will generally split $\Phi = \phi e^{i\theta}$ and fix θ at its minimum.

Supersymmetry Breaking

At the minimum of the effective potential

$$\left(\frac{K_{\Phi\Phi^\dagger\Phi^\dagger}}{K^2_{\Phi\Phi^\dagger}} - \frac{1}{K_{\Phi\Phi^\dagger}}\frac{\mathrm{d}^2 W}{\mathrm{d}\Phi^2}\right)\frac{\mathrm{d}W}{\mathrm{d}\Phi} = \rho\frac{\mathrm{d}A(\Phi)}{\mathrm{d}\Phi}.$$

Supersymmetry is broken at finite dark-matter density $\mathrm{d}W/\,\mathrm{d}\Phi \neq 0$.

The scale of breaking is model-dependent but is generally much lower than TeV.

Expect no miraculous cancellations in general.

A No-Go Theorem: Supergravity Corrections

There is a supergravity correction of the form

$$\Delta V_{\not{SUGRA}} = \frac{m_{3/2}^2 |K_\Phi|^2}{K_{\Phi\Phi^\dagger}}$$

$$m_{\text{eff}}(\Phi) = K^{\Phi\Phi^\dagger} \frac{\partial^2 V}{\partial\Phi\partial\Phi^\dagger} \subset m_{3/2}^2$$

The effective mass contains a contribution $\propto m_{3/2}$.

$m_{3/2} \geq 1eV \implies$ the range of the force is less than 10^{-6} m depending on $m_{3/2}$.

Gauge mediated SUSY breaking: $m_{3/2} \sim 1$ eV
Gravity mediated SUSY breaking $m_{3/2} \sim 1$ TeV

Symmetrons Again

This is enough to kill off supersymmetric symmetrons:

$$W(\Phi) = M^2\Phi + \frac{1}{3}g\Phi^3 + m\overbrace{\left(1 - h\frac{\Phi^2}{2m\Lambda_3}\right)}^{A(\phi)}\Phi_+\Phi_-$$

$$V_{\text{eff}}(\phi) = \frac{1}{2}\left(h\frac{\rho}{m\Lambda_3} - [4gM^2 - m_{3/2}^2]\right)\phi^2 + \frac{\tilde{\lambda}}{4}\phi^4$$

Need $M \gtrsim m_{3/2}$ or the mechanism does not exist.

$$\beta(\phi) = M_{\text{pl}}\frac{\mathrm{d}\ln A(\phi)}{\mathrm{d}\phi} \sim \frac{M_{\text{pl}}M}{m\Lambda_3}$$

Need $\beta \sim \mathcal{O}(1) \Rightarrow$ symmetry is only restored when $\rho_c \gtrsim 10^{39}\rho_0$.
The screening mechanism cannot operate in the late-time universe!

SUPER-Screening

What about other mechanisms?

$$-\frac{\beta(\varphi_0)\rho_0}{M_{\rm pl}\varphi_0} = m_{3/2}^2 + \frac{1}{\varphi_0}\frac{\mathrm{d}V_{\rm F}}{\mathrm{d}\varphi} \quad \text{at the minimum}$$

When the mechanism operates $\frac{\beta(\varphi_0)\rho_0}{M_{\rm pl}\varphi_0}$ is at least as large as $m_{3/2}^2$

$$\chi_0 = \frac{\varphi_0}{2M_{\rm pl}\beta(\varphi_0)} \leq \left(\frac{H_0}{m_{3/2}}\right)^2 \leq 10^{-33}$$

No object in the universe is unscreened!

A Get-Out Clause?

Models where $K^{\Phi\Phi^\dagger}|K_\Phi|^2 = 3$ can evade these corrections, they have no soft-mass terms.

These are no-scale models e.g.

$$K(\Phi,\Phi^\dagger) = -3M_{\rm pl}^2 \ln\left[\left(\frac{\Phi+\Phi^\dagger}{M_{\rm pl}}\right).\right]$$

It is historically difficult to find chameleons with these models: the normalised field is $\Phi = M_{\rm pl}\exp(-\sqrt{2/3}\phi/M_{\rm pl})$.

$$W \propto \Phi^n \Rightarrow V_{\rm eff}(\phi) \sim e^{-a\phi} + \rho_c e^{b\phi}$$

It is well-known that this does not screen efficiently.

A SUSY Chameleon

Want a run-away potential so we need K to be of higher degree than W:

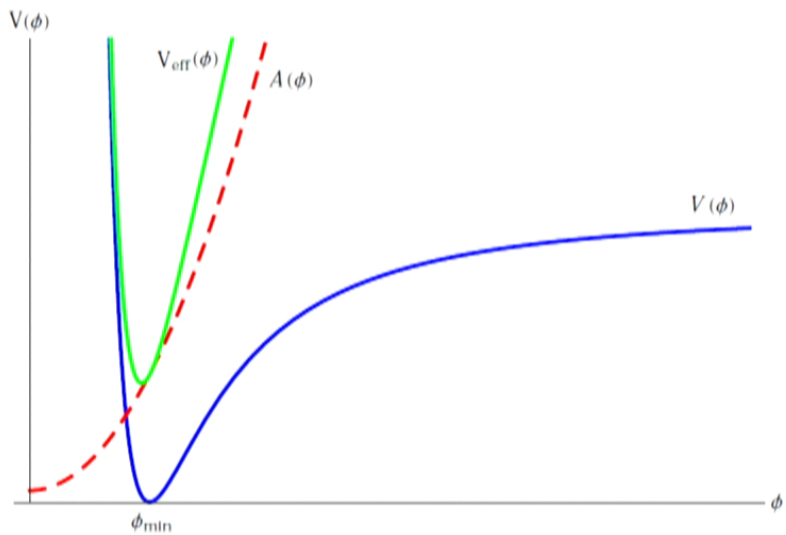
$$K = \frac{\Lambda_1^2}{2}\left(\frac{\Phi^\dagger\Phi}{\Lambda_1^2}\right)^\beta$$

$$W = m\overbrace{\left[1+\frac{g}{m}\left(\frac{\Phi^\delta}{\Lambda_3^{\delta-1}}\right)\right]}^{A(\Phi)}\Phi_+\Phi_- + \frac{\beta}{\sqrt{2}\alpha}\left(\frac{\Phi^\alpha}{\Lambda_0^{\alpha-3}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\Phi^\beta}{\Lambda_2^{\beta-3}}\right)$$

The chameleon is not canonically normalised $\quad \phi = \Lambda_1\left(\frac{\varphi}{\Lambda_1}\right)^{\frac{1}{\beta}}$

SUSY Chameleons

$$V_{\rm eff} = \Lambda^4 \left[1 - \left(\frac{\varphi_{\rm min}}{\varphi}\right)^{\frac{n}{2\beta}}\right]^2 + \rho x \left(\frac{\varphi}{\varphi_{\rm min}}\right)^{\frac{\delta}{\beta}}.$$



Cosmology is the same as regular chameleons.

Linear Perturbations

The theory can still exhibit novel effects at the level of linear perturbations:

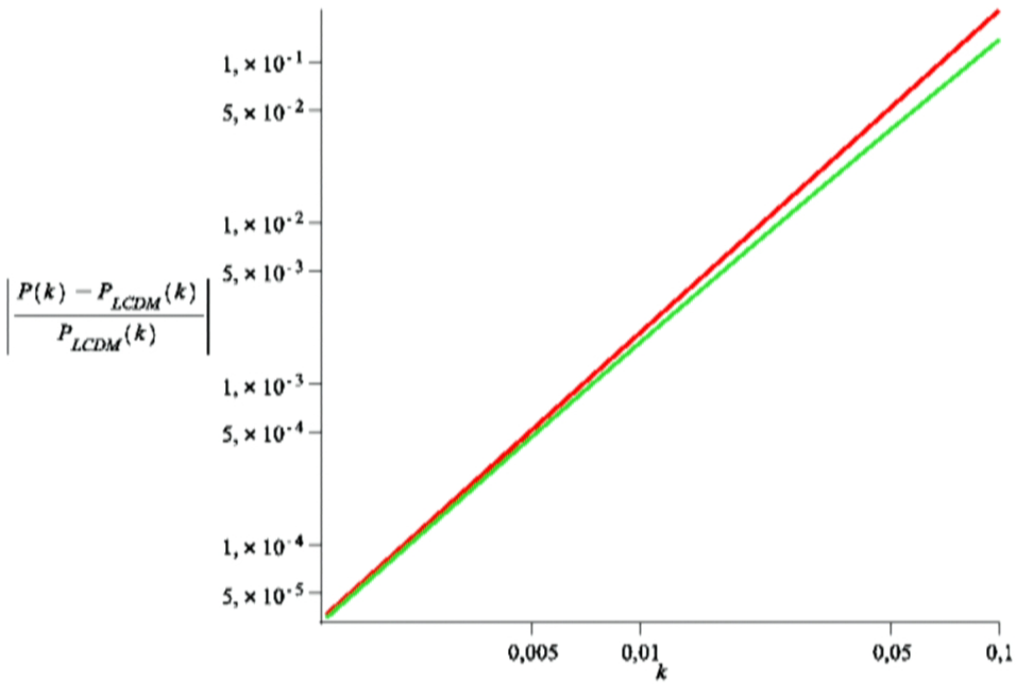
$$\ddot{\delta}_{\rm c}(k,t)+2H\dot{\delta}_{\rm c}(k,t)-\frac{3}{2}\Omega_{\rm c}(a)\underbrace{\left(1+2k^2\frac{\beta(\varphi)^2(\varphi_{\rm min})}{m_\infty^2a^2}\right)}_{G_{\rm eff}(k)}\delta_{\rm c}(k,t)\approx 0$$

$$\delta_{\rm c}(x)=\frac{\delta\rho_{\rm c}}{\rho_{\rm c}}$$

We can solve this in terms of *modified Bessel functions*.

Dark Matter Power Spectrum

This gives novel features in the cold dark matter power spectrum on large scales:



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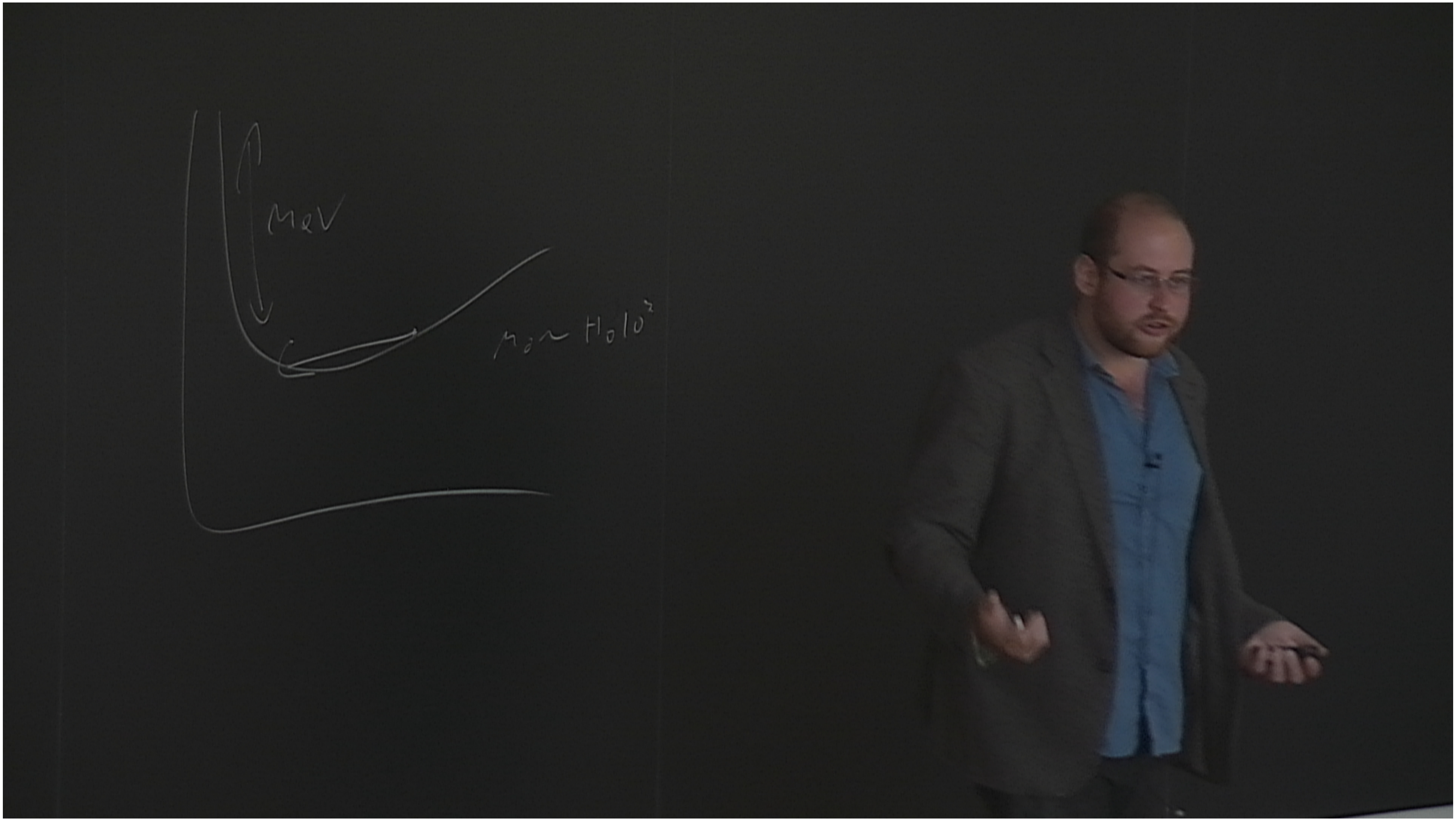
$$\ddot{\delta}_{\rm c}(k,t) + 2H\dot{\delta}_{\rm c}(k,t) - \frac{3}{2}\Omega_{\rm c}(a)\left(1 \ldots\right.$$

$$\ldots \Lambda_3^{\delta-1}\Big)\Big]\Phi_+\Phi_- + \frac{\beta}{\sqrt{2}\alpha}\left(\frac{\Phi^\alpha}{\Lambda_0^{\alpha-3}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\Phi^\beta}{\Lambda_2^{\beta-3}}\right)$$

We ...

Cosmology is the same as regular chameleons.


MeV

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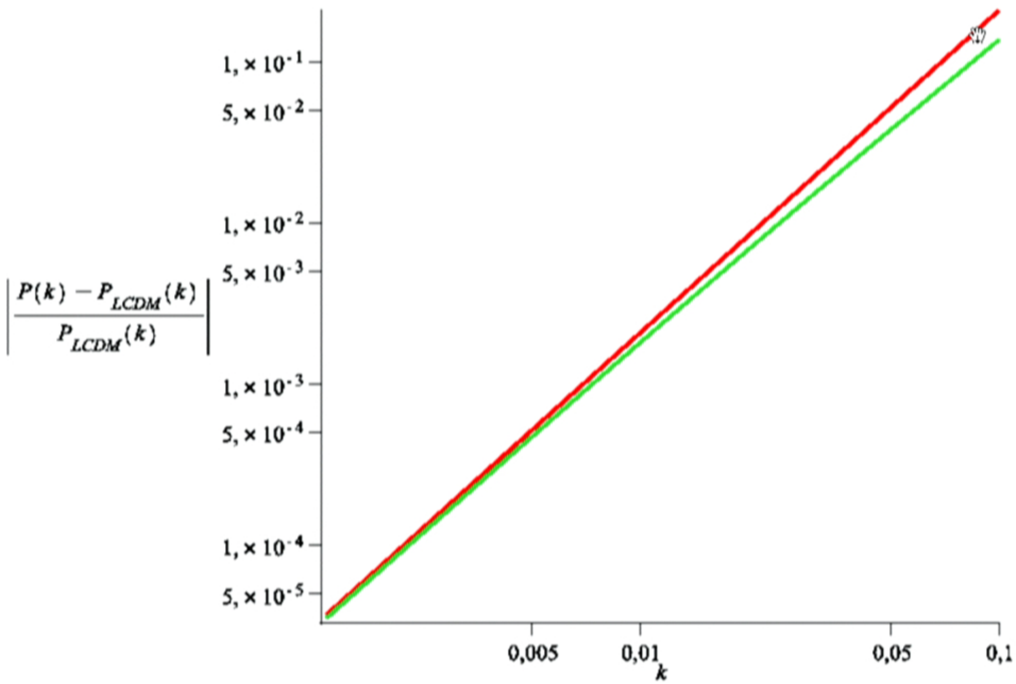

Stars in Modified Gravity

Main Idea:

- Stars are balls of hydrogen (or other elements) gas which support themselves against gravitational collapse by burning fuel in their centre in order to provide an outward pressure.
- An unscreened star feels a stronger gravitational force in its outer layers.
- This means it needs to burn more fuel per unit time in order to stave off collapse.
- This releases more energy per unit time so the star is brighter.

Dark Matter Power Spectrum

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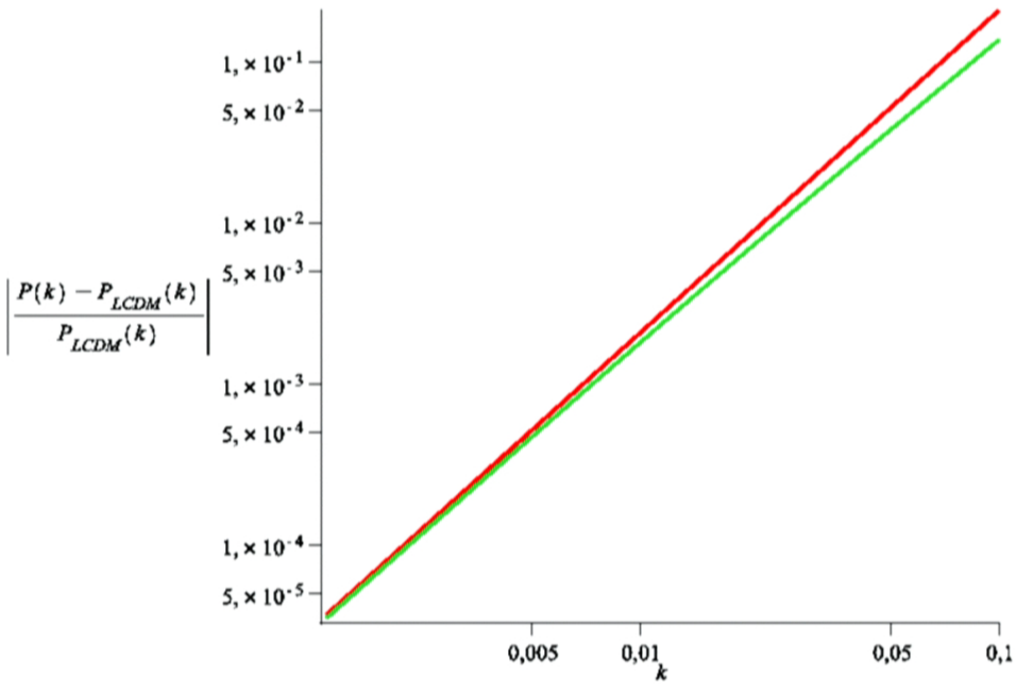

Equations of Stellar Structure

$$\frac{\mathrm{d}\overbrace{P}^{\text{Pressure}}}{\mathrm{d}r} = -\frac{GM(r)\overbrace{\rho(r)}^{\text{Density}}}{r^2}$$ Hydrostatic Equilibrium

+ mass conservation, radiative transfer, energy generation.

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+ mass conservation, radiative transfer, energy generation.

Modified Gravity with MESA

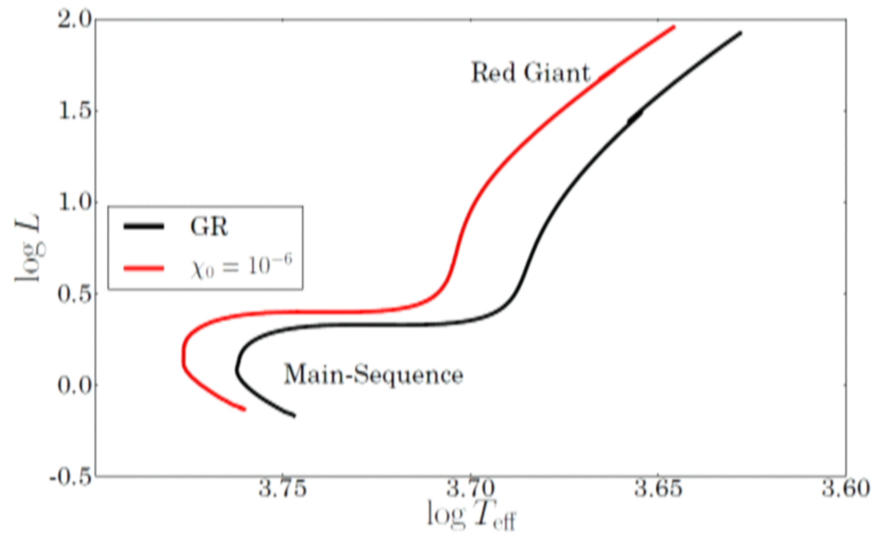
MESA (Paxton et al 11) is a self-consistent stellar evolution code and I have modified it to include modified gravity.

$$G \to G(r) = G\left(1 + \alpha\left[1 - \frac{M(r_{\rm s})}{M(r)}\right]\right).$$

Screening radius is found by solving the field equations for a given χ_0.

Modified Gravity with MESA

$$M = 1M_\odot \quad \alpha = 1/3$$



Davis, Lim, JS & Shaw 2011

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Distance Indicators as Probes of Modified Gravity

Distance to a galaxy depends on the measurement of variables $\{X\}$ and a conversion using some formula $d = F(\{X\})$.

If this formula is different in MG then application of the GR formula will give the wrong distance e.g. luminosity distance:

$$F = \frac{L}{d^2}.$$

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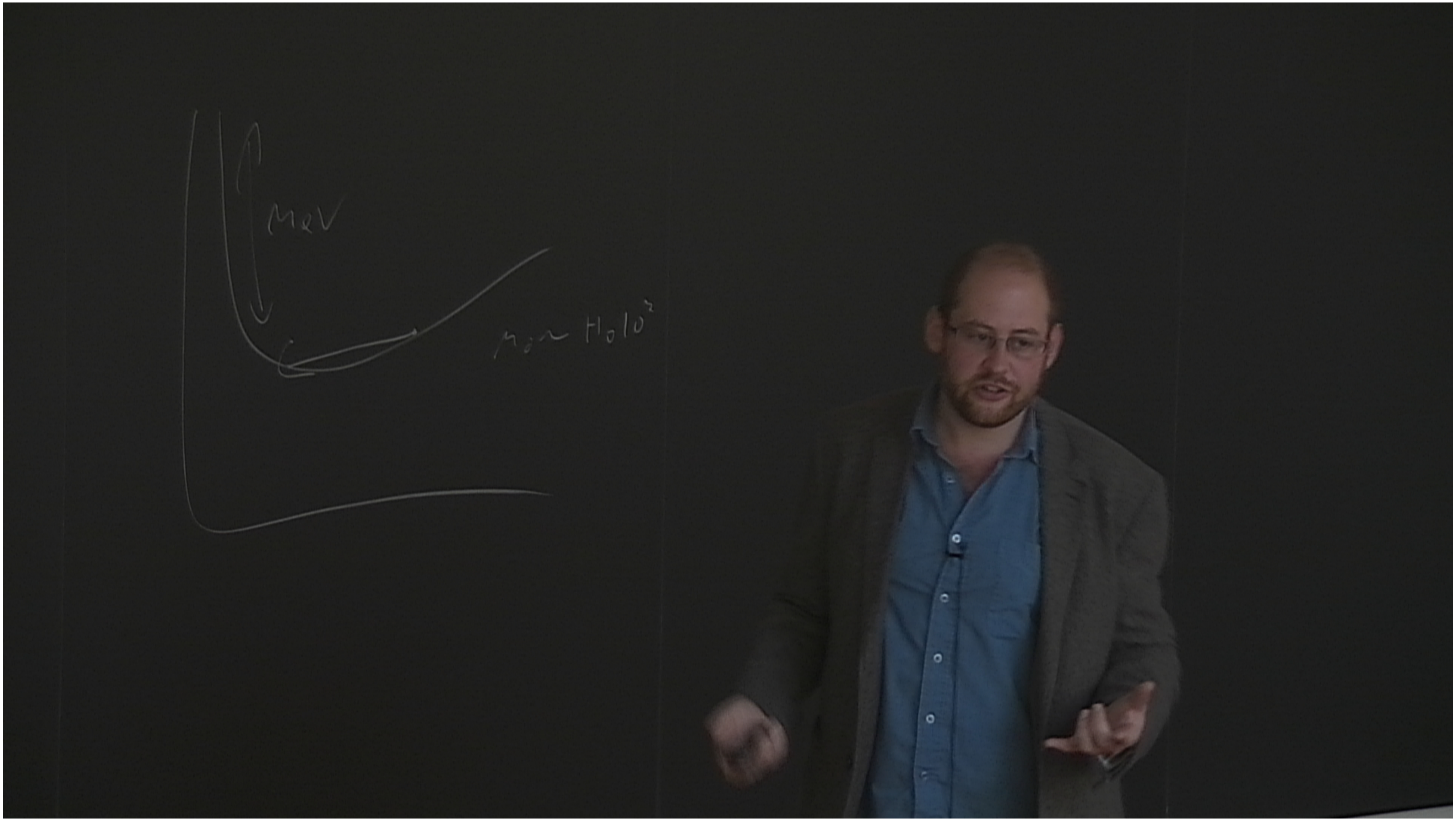

MeV

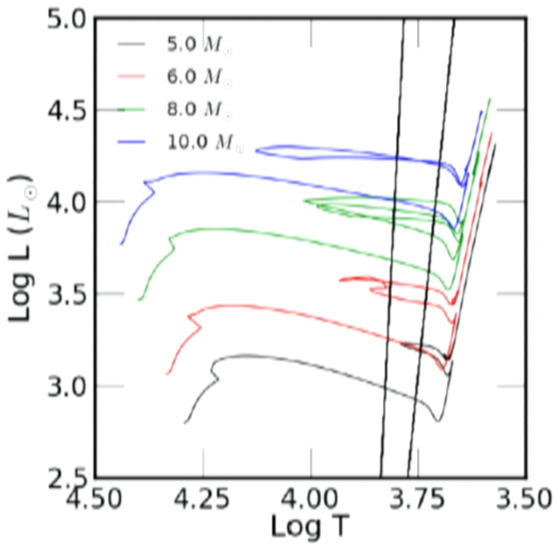
Tip of the Red Giant Branch

$\sim 2M_{\odot}$ stars are large enough to excite Helium burning at $T \sim 10^8$K. They climb the red giant branch until their core temperature is high enough to ignite the central Helium.

This depends only on nuclear physics so the ignition tends to happen at fixed luminosity and temperature.

Cepheid Distances

$5 - 10M_\odot$ stars have short-lived phases during their red giant tracks called loops. Along part of these loops the stars are unstable to *Cepheid Pulsations*.



Cepheid Distances

There is a well-known period-luminosity relation with $\Pi \propto (G\rho)^{-1/2}$.

$$\underbrace{M_{\mathrm{B}}}_{\sim\log(L)} = \mu \underbrace{\log(\Pi)}_{\sim\log(R)} + \nu \underbrace{(B-V)}_{\sim\log(T_{\mathrm{e}})} + \gamma$$

$$L = 4\pi R^2 \sigma T^4$$

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Cepheid Distances

$$\frac{\Delta d}{d} \approx -0.3\frac{\Delta G}{G}$$

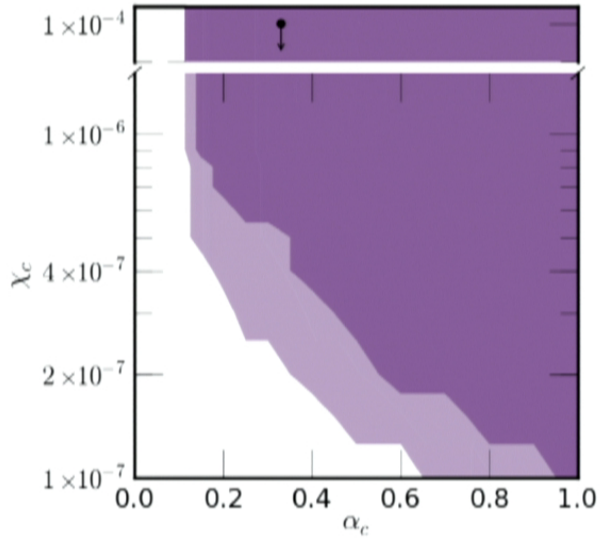
We compute $\langle G \rangle$ using MESA.

TRGB distances are screened but Cepheids are not:

$$\Delta d = d_{\text{Cepheid}} - d_{\text{TRGB}}$$

By comparing Δd in screened and unscreened galaxies we can probe $\chi_0 < 10^{-6}$.

Constraints



Jain, Vikram & JS 2012

$$\chi_0 \lesssim 4 \times 10^{-7} \quad (f(R)\ \text{Gravity})$$

These are currently the strongest constraints in the literature.

Can we do Better?

$$\frac{d_{\rm MG} - d_{\rm GR}}{d_{\rm GR}} \approx -0.3\frac{\Delta G}{G}$$

is an approximation - need full hydrodynamic perturbation theory.

There are two new features in modified gravity hydrodynamics:

1. The modified periods are even smaller than GR.
2. The stars are more stable.

Modified Gravity Hydrodynamics

$$\ddot{\vec{r}} = -\frac{1}{\rho}\nabla P + \underbrace{\vec{f}}_{\text{force per unit mass}}$$

$$\vec{f} = -\frac{GM(r)}{r^2} - \frac{\beta(\phi)}{M_{\rm pl}}\nabla\phi$$

Look at radial perturbations only:

$$r = r_0 + \delta r$$

The rest of the hydrodynamic equations do not depend on gravity.

Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r)e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

General Relativity:

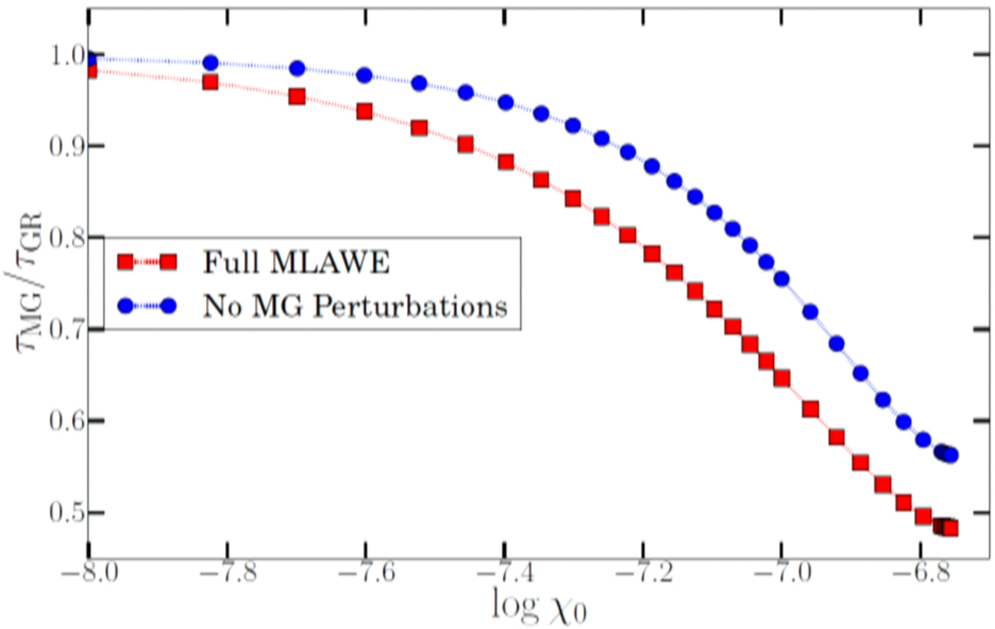
$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4\Gamma_{1,0}P_0\frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0} - 4\right)P_0\right]\xi + r^4\rho_0\omega^2\xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the periods of oscillation.

Example: Lane-Emden Models

Ball of gas with $P = K\rho^{\frac{5}{3}}$:



$$\frac{GM}{R} = 10^{-7}$$

Cepheid Distances

α	χ_0	$\Delta d/d$ (approx)	$\Delta d/d$ (LAWE)	$\Delta d/d$ (MLAWE)
$1/3$	4×10^{-7}	-0.03	-0.04	-0.12
$1/2$	4×10^{-7}	-0.05	-0.06	-0.16
1	2×10^{-7}	-0.06	-0.07	-0.19

Full hydrodynamic prediction is 3 times larger than the equilibrium prediction!

Can improve the constraints using the same data sets.

Future Constraints

Vary χ_0 until $\Delta d/d$ matches the approximation at fixed α:

α	χ_0
$1/3$	9×10^{-8}
$1/2$	7×10^{-8}
1	3×10^{-8}

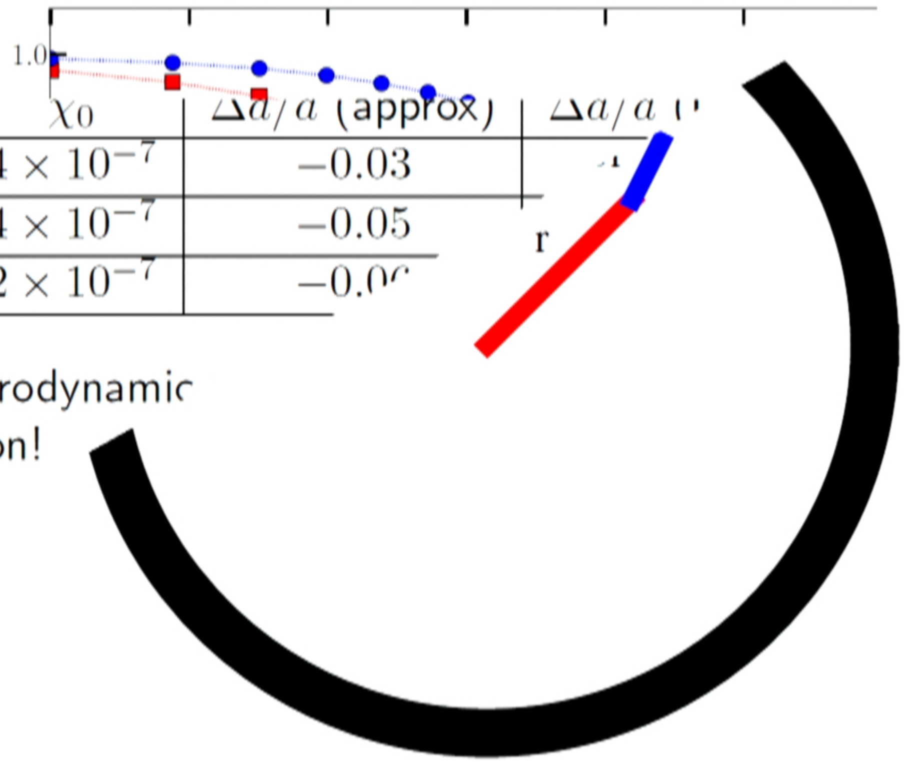
Table: JS, 2013

The constraints can be greatly improved. This is work in progress

Modified Linear Adiabatic Wave Equation

Ball of gas with $P = K\rho^{\frac{4}{3}}$:

α	χ_0	$\Delta a/a$ (approx)	$\Delta a/a$ (
$1/3$	4×10^{-7}	-0.03	[illegible]
$1/2$	4×10^{-7}	-0.05	[illegible]
1	2×10^{-7}	-0.0[illegible]	[illegible]



Full hydrodynamic
prediction!

Ca

Stellar Stability

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

We can use the variational method to find an upper bound on the fundamental frequency.

$$\omega_0^2 \leq F[P_0, \rho_0]$$

The star is unstable if this is imaginary.

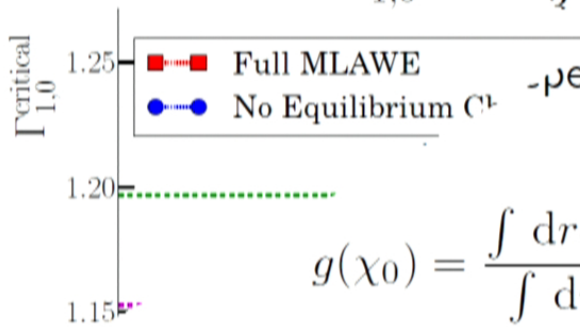
General Relativity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3}$$

Stellar Stability

Modified Gravity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3} - \alpha g(\chi_0)$$



…pendent factor which depends on

$$g(\chi_0) = \frac{\int \mathrm{d}r 4\pi r^4 \rho_0(r)^2}{\int \mathrm{d}r 3r^2 P_0(r)}$$

Stellar Stability

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Stellar Stability

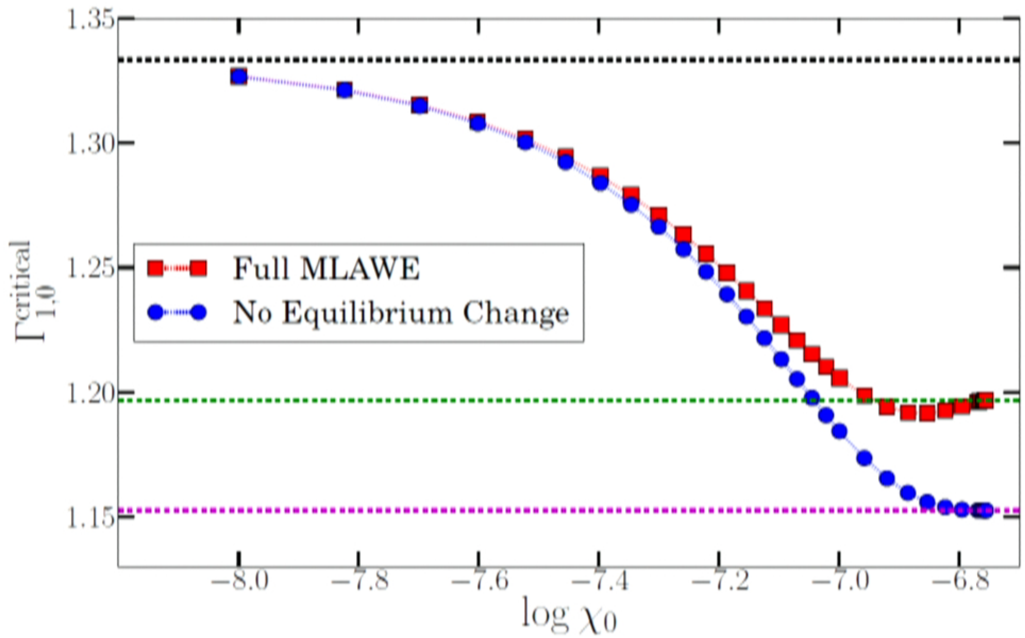
Modified Gravity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3} - \alpha g(\chi_0)$$

$g(\chi_0)$ is an $\mathcal{O}(1)$ composition dependent factor which depends on how unscreened the star is.

$$g(\chi_0) = \frac{\int \mathrm{d}r 4\pi r^4 \rho_0(r)^2}{\int \mathrm{d}r 3r^2 P_0(r)}$$

Stability of Lane-Emden Models

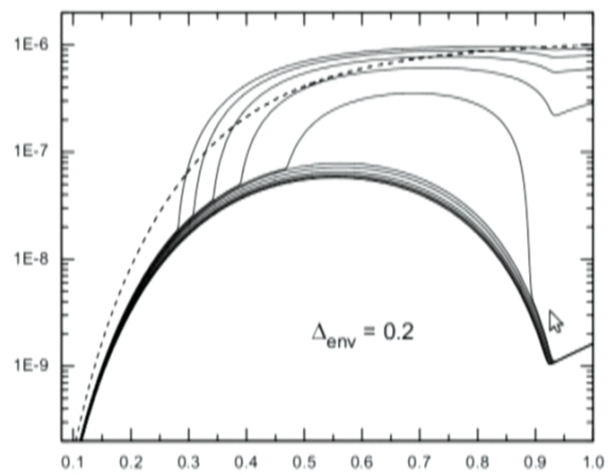


The dip is present because the modified equilibrium structure competes with the perturbations.

Galactic Probes

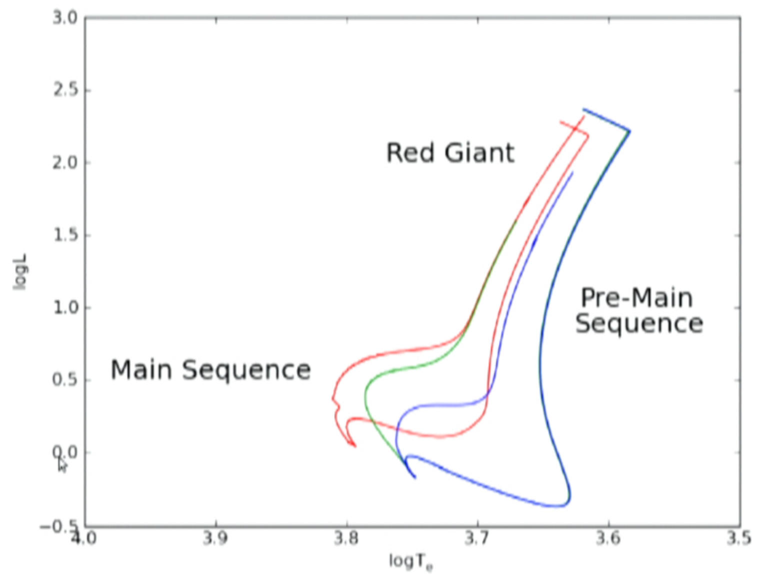
w/ Kazuya Koyama, Baojiu Li & Claudia Maraston Stars are brighter and hotter ⇒ expect changes in the galactic spectra, colour, age...

This requires the cumulative effects and so we need to account for time-dependence: Things are more screened in the past.



Stars with Time-Dependence

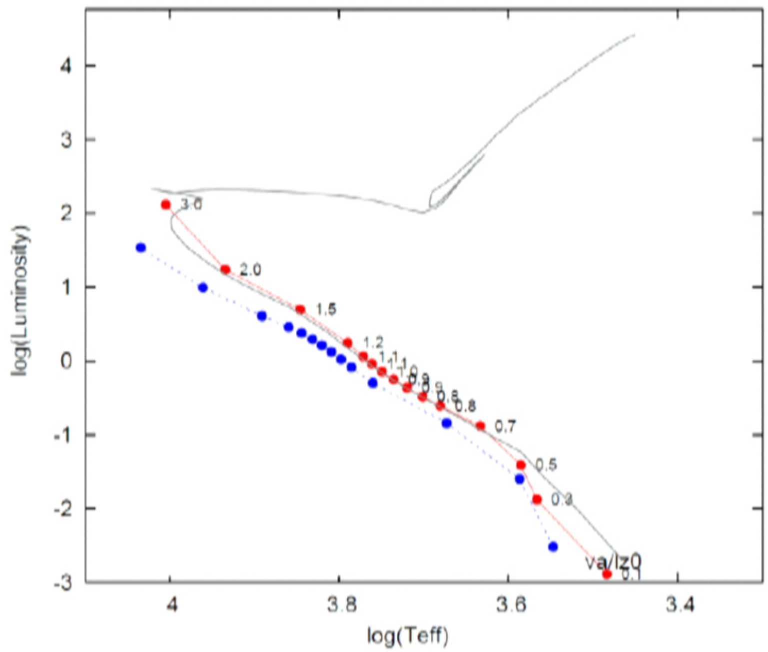
I have updated MESA to include time-dependence:



Next Step: Isochrones

The red-giant branch is found using interpolation

These can be used in galaxy synthesisers to predict the new galactic properties.



Disformal Gravity

Scalar-Tensor modified gravity with

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi; \quad D(\phi) = \frac{1}{M^4} + \mathcal{O}(\frac{\phi}{M^5}) + \cdots$$

$$\Box\phi = \frac{\partial V}{\partial\phi} - \frac{D'}{2}T^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \nabla_\mu\left(T^{\mu\nu}D\nabla_\nu\phi\right)$$

Screening mechanism: static, non-relativistic object $P \ll \rho c^2$:

$$\ddot{\phi} \approx -\frac{D'}{2D}\dot{\phi}^2 \quad \text{Koivisto, Mota \& Zumalacarregui 2012}$$

Large time-derivatives decay to zero quickly $\Rightarrow$ coupling to matter is negligible.

Astrophysical Tests?

$$\Box\phi \sim \rho\dot{\phi}^2 + P\left(\nabla\phi\right)^2$$

No source for non-relativistic objects so most stars are poor probes but relatvistic stars have $P \sim \rho c^2 \Rightarrow$ neutron stars should show novel deviations:

- Write down equations of motion for a perfect fluid in a spherically-symmetric space-time.
- Find and solve new Tolman-Oppenheimer-Volkof equation.
- Resulting structure and properties should show large novel deviations from GR.
- Fun complications e.g. equation of state: $T^{\mu\nu} = \sqrt{1 + D(\partial_\mu\phi)^2}\tilde{T}^{\mu\nu}$.

Other Things

Things I don't have time to tell you about

- Observational tests of the Vainthein mechanism.
- Planetary nebula tests of Chameleon-like theories.
- Scalar radiation in Galileon theories.

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Distance Comparisons