Title: The Geometry of Supersymmetric Partition Functions

Date: Oct 29, 2013 02:00 PM

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Abstract: Three-dimensional N=2
theories with a U(1)_R symmetry, can be placed on a compact three manifold M preserving some supersymmetry if and only if M admits a transversely holomorphic foliation (THF). I will show that the partition function of the resulting theory is independent of the metric and depends holomorphically on the moduli of the THF. When applied to supersymmetric field theories on manifolds diffeomorphic to S^3 and S^2 x S^1, this result explains many of the properties observed in explicit determinations of their partition functions.

Closset, Dumitrescu, Komargodski, GF 1309.5876 (1212.3388)

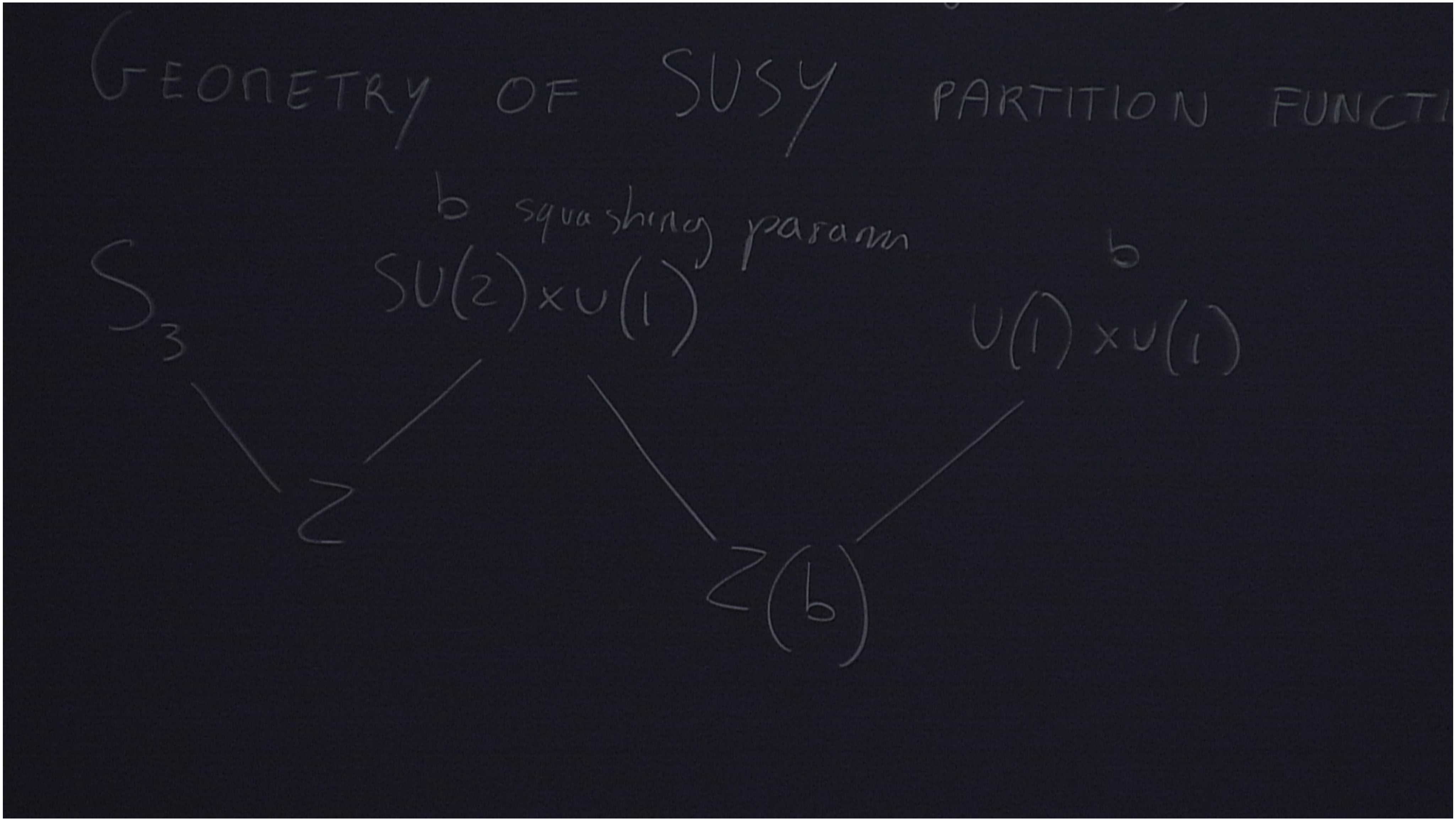
GEOMETRY OF SUSY PARTITION FUNCTIONS.

GEOMETRY OF SUSY PARTITION FUNCTI

b squashing param

S_3 $SU(2)\times U(1)$ $U(1)\times U(1)$

Z $Z(b)$


GEOMETRY OF SUSY PARTITION FUNCT
b squashing param
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$SU(2) \times U(1)$
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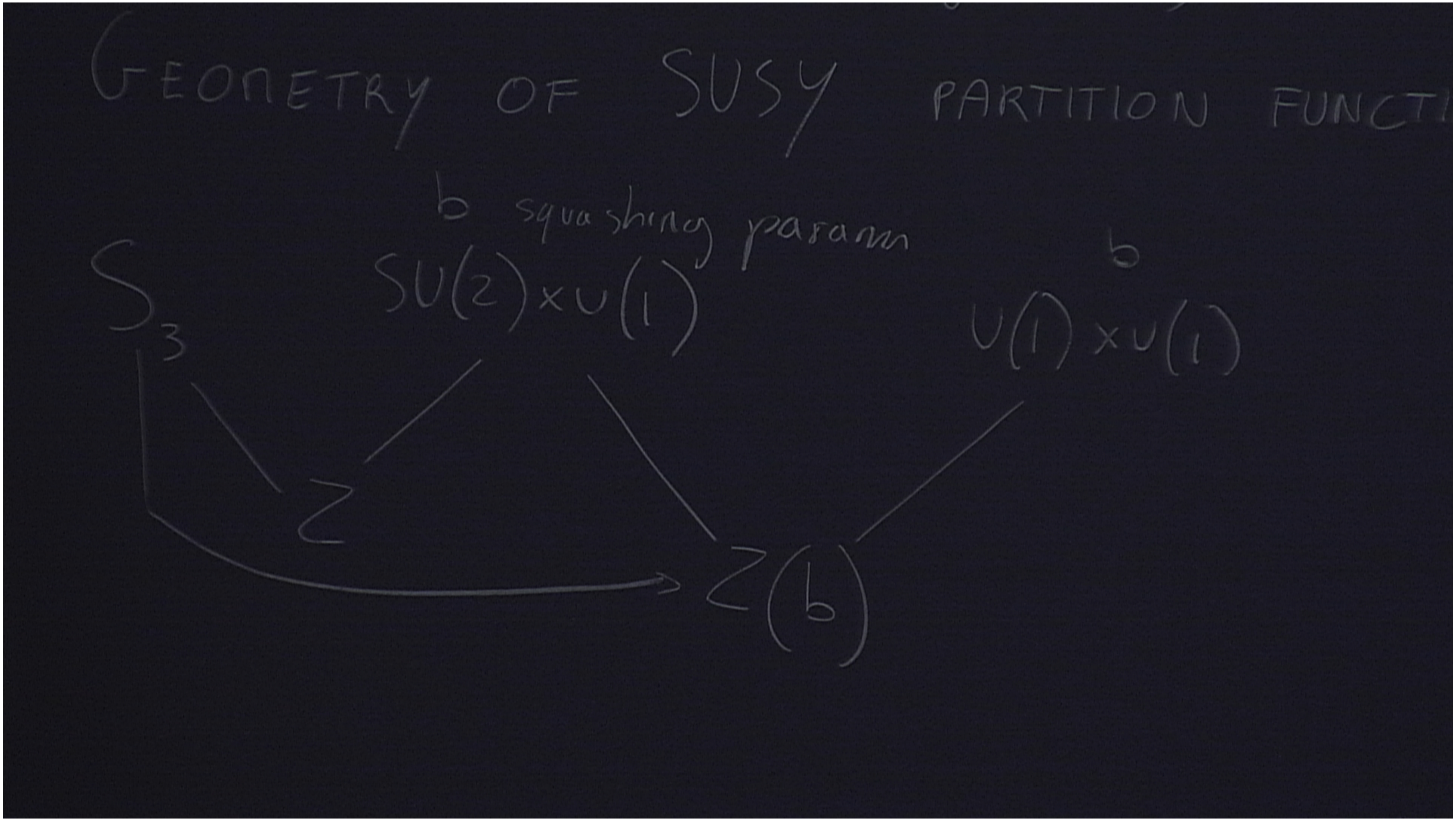
$U(1) \times U(1)$
Z
Z(b)


GEOMETRY OF SUSY PARTITION FUNCT
b squashing param
S3
SU(2)×U(1)
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Z(b)

KI, GF 1309.5876 (1212.3388)

$$\{\varphi_\alpha, \tilde{\varphi}_\beta\} = 2\,\gamma^N_{\alpha\beta} P_N + i\varepsilon_{\alpha\beta} Z$$

TION FUNCTIONS.

b

$\times u(1)$

What is b ?

K I, GF 1309.5876 (1212.3388)

TION FUNCTIONS.

b

) × U(1)

What is b ?

$$\{ \varphi_\alpha, \tilde{\varphi}_\beta \} = 2 \gamma^\nu_{\alpha\beta} P_\nu + i \varepsilon_{\alpha\beta} Z$$

* SUSY IN CURVED SPACE

THF

K1, GF 1309.5876 (1212.3388)

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TION FUNCTIONS.

b

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What is b ?

* SUSY IN CURVED SPACE

THF

* Z, DEPENDENCE ON GEO.

Closset, Dumitrescu, Komargodski, GF 1309.5876 (1212.33

GEOMETRY OF SUSY PARTITION FUNCTIONS.

b squashing param

S_3 $\quad SU(2)\times U(1) \quad$ b $\; U(1)\times U(1)$

What is b ?

Z

$Z(b)$

COUPLE TO SUGRA. | SET GRAVITY FIELDS TO BCK VALUES. $M_p \to \infty$

dsK I, GF 1309.5876 (1212.3388)

$$\{ \varphi_\alpha , \tilde{\varphi}_\beta \} = 2 \gamma^\mu_{\alpha\beta} P_\mu + i \epsilon_{\alpha\beta} Z$$

ARTITION FUNCTIONS.

* SUSY IN CURVED SPACE.
* THF
* Z, DEPENDENCE ON GEO

b

$U(1) \times U(1)$

What is b ?

SUSY

$\Downarrow$

ALITY FIELDS TO BCK VALUES. $M_p \to \infty$ $\quad \psi_{\mu\alpha}, \tilde{\psi}_{\mu\alpha}$. $\quad \delta_\zeta \psi_{\mu\alpha} = 0$

R - multiplet. $j_\mu^{(R)}$ R-current. $j_\mu^{(Z)}$

$h^{(2)}_{\mu\nu}$, $T_{\mu\nu}$, J , $S_{\alpha\mu}$ $\tilde{S}_{\alpha\mu}$

$g_{\mu\nu}$

$\ ^{(2)}_{\mu}$, $T_{\mu\nu}$, J , $S_{\alpha\mu}$ $\tilde{S}_{\alpha\mu}$

$g_{\mu\nu}$ H $\Psi_{\mu\alpha}$, $\bar{\Psi}_{\mu\alpha}$

et. $j^{(R)}_\mu$ R-current. $j^{(Z)}_\mu$, $T_{\mu\nu}$,

$A^{(R)}_\mu$ C_μ $g_{\mu\nu}$

$V^\mu = -i\,\varepsilon^{\mu\nu\rho}\partial_\nu C_\rho$

et.

$j^{(R)}_\mu$ R-current. $j^{(Z)}_\mu$, $T_{\mu\nu}$,

$A^{(R)}_\mu$ C_μ $g_{\mu\nu}$

$$V^\mu = -i\,\varepsilon^{\mu\nu\rho}\partial_\nu C_\rho$$

$$\nabla^\mu V_\mu = 0$$

R - multiplet. $\quad j^{(R)}_{\mu} \quad$ R-current. $\quad j^{(Z)}_{\mu}$

SUGRA. $\quad A^{(R)}_{\mu} \quad C_{\mu}$

$V^{\mu} = -i\,\varepsilon^{\mu}$

$\nabla^{\mu} V_{\mu} = 0$

R-multiplet. $j_\mu^{(R)}$ R-current. $j_\mu^{(Z)}$

SUGRA. $A_\mu^{(R)}$ C_μ

$V^\mu = -i\,\varepsilon^{\mu}$

$\nabla^\mu V_\mu = $

LINEAR MULTIPLET $\left(J_R ,\ K ,\ j_\mu ,\ j_\alpha ,\ \bar{j}_{\dot\alpha} \right)$

R-multiplet: j^μ R-current: j^μ

SUGRA. $A^{(R)}_\mu$ C_μ

$V^\mu = -i\,\varepsilon^{\mu}$

$\nabla^\mu V_\mu =$

LINEAR MULTIPLET $(J_R, K, j_\mu, j_\alpha, \tilde{j}_{\dot\alpha})$

$(D, \sigma, a_\mu, \lambda_\alpha, \tilde{\lambda}_{\dot\alpha})$

R-multiplet. $j_\mu^{(R)}$ R-current. $j_\mu^{(Z)}$, $T_{\mu\nu}$

SUGRA. $A_\mu^{(R)}$ C_μ $\Delta g_{\mu\nu}$

$$V^\mu = -i\,\varepsilon^{\mu\nu\rho}\partial_\nu C_\rho$$

$$\nabla^\mu V_\mu = 0$$

LINEAR MULTIPLET $(J_R, K, j_\mu, j_\alpha, \tilde{j}_\alpha)$

$(D, \sigma, a_\mu, \lambda_\alpha, \tilde{\lambda}_\alpha)$

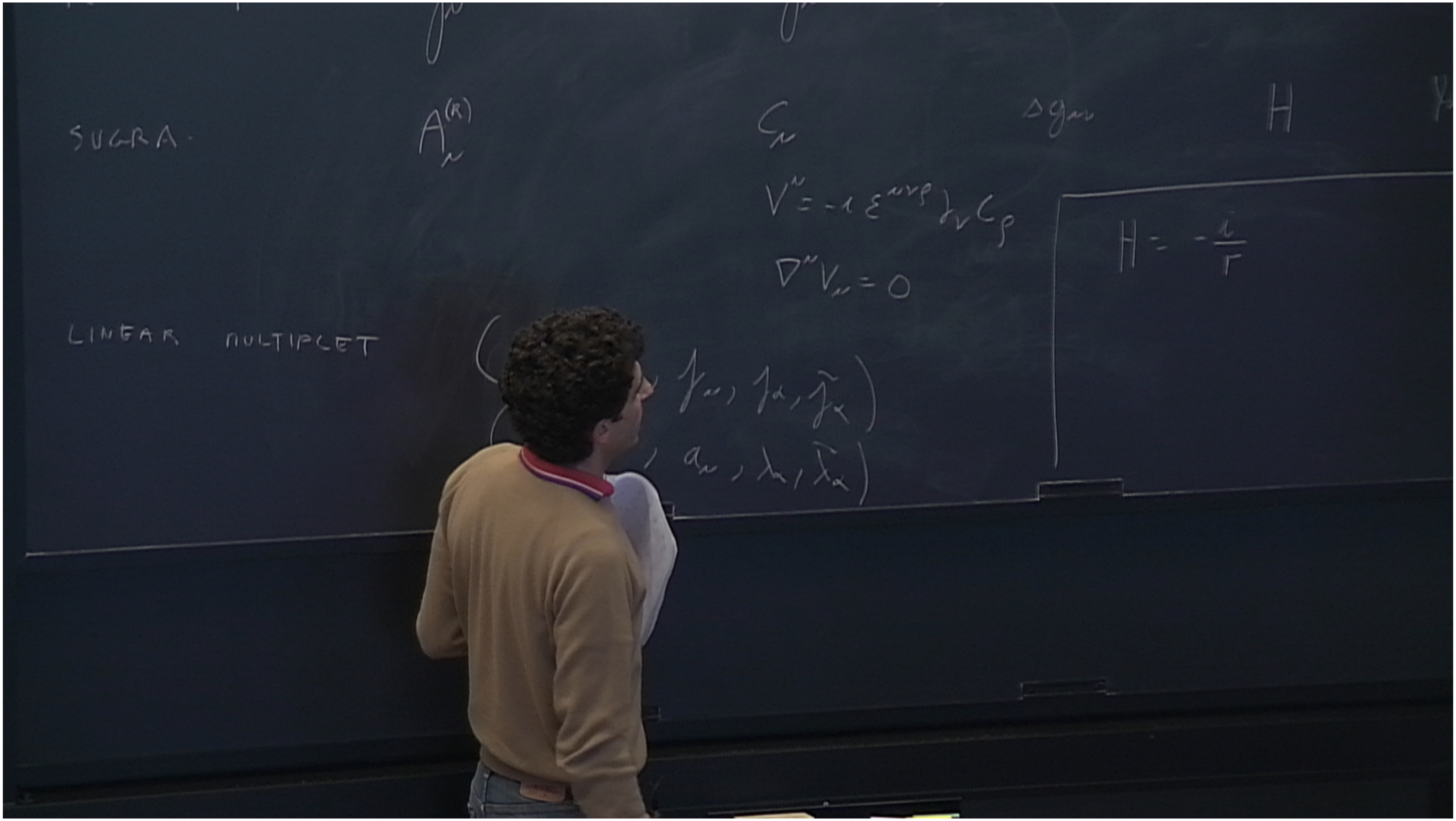
$$\ldots, a_\mu, \lambda_\alpha, \tilde{\lambda}_\alpha)$$

$$\delta \psi_{\mu i} = 0 \Rightarrow (\nabla_\mu - i A_\mu)\zeta = -\frac{1}{2} H \gamma_\mu \zeta + \frac{i}{2} V_\mu \zeta - \frac{1}{2} \varepsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$(D, \sigma, a_\mu, \lambda_\alpha, \tilde{\lambda}_\alpha)$$

$$\delta \psi_{\mu\alpha} = 0 \implies \left(\nabla_\mu - i A_\mu^{(R)}\right)\zeta = -\frac{1}{2} H \gamma_\mu \zeta + \frac{i}{2} V_\mu \zeta - \frac{1}{2}\varepsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$\delta\lambda = 0 \qquad i\zeta\left(D + \sigma H\right) - \frac{i}{2}\gamma_\mu \zeta\, \varepsilon^{\mu\nu\rho} F_{\nu\rho} - i\gamma^\mu \zeta \left(\partial_\mu \sigma + i V_\mu \sigma\right) = 0$$


SUGRA.
LINEAR MULTIPLET
H

$$\left(\xi^{\mu},\ \eta_{\mu},\ \phi^{\mu}{}_{\nu} \right)$$

Almost Contact Structure

$$\xi^{\mu}\eta_{\mu} = 1$$

$$\left(\xi^\mu, \; \eta_\mu, \; \phi^\mu{}_\nu\right)$$

Almost Contact Structure

$$\xi^\mu \eta_\mu = 1 \qquad \phi^\mu{}_\nu \, \phi^\nu{}_\rho = -\delta^\mu{}_\rho + \xi^\mu \eta_\rho$$

$$\left(\xi^{\mu},\ \eta_{\mu},\ \phi^{\mu}{}_{\nu} \right)$$

Almost Contact Structure

$$\xi^{\mu} \eta_{\mu} = 1 \qquad \phi^{\mu}{}_{\nu} \phi^{\nu}{}_{\rho} = -\delta^{\mu}{}_{\rho} + \xi^{\mu} \eta_{\rho}$$

Almost Contact Structure

$$\phi^{\mu}{}_{\nu}\,\phi^{\nu}{}_{\rho} = -\delta^{\mu}{}_{\rho} + \xi^{\mu}\eta_{\rho} \qquad \eta\phi = \phi\xi = 0$$

ρ

$$\eta\phi = \phi\xi = 0$$

$\eta_\mu) \quad \phi^\mu{}_\nu)$

Almost Contact Structure

$= 1 \qquad \phi^\mu{}_\nu \phi^\nu{}_\rho = -\delta^\mu{}_\rho + \xi^\mu \eta_\rho \qquad \eta\phi = \phi\xi = 0$

$0 \qquad \phi|_D = J$

THF leaves are ξ ORBITS

$\phi)^\nu{}_\rho = 0$

$\phi^{\mu}{}_{\nu})$

Almost Contact Structure

$$\phi^{\mu}{}_{\nu}\,\phi^{\nu}{}_{\rho} = -\delta^{\mu}{}_{\rho} + \xi^{\mu}\eta_{\rho} \qquad \eta\phi = \phi\xi = 0$$

$$\phi\big|_{D} = J$$

$= 0$

THF	leaves are ξ ORBITS.	
$(\tau, z, \bar{z})$	$\xi = \partial_{\tau}$	$\eta = d\tau + h(\tau, z, \bar{z})\,dz + \bar{h}\,d\bar{z}$

$\phi^\mu{}_\nu \Big)$

Almost Contact Structure

$$\phi^\mu{}_\nu \, \phi^\nu{}_\rho = -\delta^\mu{}_\rho + \xi^\mu \eta_\rho \qquad \eta\phi = \phi\xi = 0$$

$$\phi\big|_D = J$$

THF leaves are ξ ORBITS

$(\tau, z, \bar{z}) \qquad \xi = \partial_\tau \qquad \eta = d\tau + h(\tau, z, \bar{z})\,dz + \bar{h}\,d\bar{z}$

$$\phi^\mu{}_\nu = \begin{pmatrix} 0 & -ih & i\bar{h} \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$$

$= 0$

$\phi^{\mu}{}_{\nu})$

Almost Contact Structure

$$\phi^{\mu}{}_{\nu}\,\phi^{\nu}{}_{\rho} = -\delta^{\mu}{}_{\rho} + \xi^{\mu}\eta_{\rho} \qquad \eta\phi = \phi\xi = 0$$

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THF leaves are ξ ORBITS

$(\tau, z, \bar z)$ $\quad \xi = \partial_\tau \quad \eta = d\tau + h(\tau, z, \bar z)\,dz + \bar h\, d\bar z$

$$\phi^{\mu}{}_{\nu} = \begin{pmatrix} 0 & -ih & i\bar h \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$$

$\tau' = \tau + t(z, \bar z)$

$z' = f(z)$

$\phi^{\mu}{}_{\nu}\Big)$

Almost Contact Structure

$$\phi^{\mu}{}_{\nu}\,\phi^{\nu}{}_{\rho} = -\delta^{\mu}{}_{\rho} + \xi^{\mu}\eta_{\rho} \qquad \eta\,\phi = \phi\,\xi = 0$$

$$\phi\big|_{D} = J$$

THF leaves are ξ ORBITS

$(\tau, z, \bar{z})$ $\quad \xi = \partial_\tau \quad \eta = d\tau + h(\tau, z, \bar{z})\,dz + \bar{h}\,d\bar{z}$

$= 0$

$$\phi^{\mu}{}_{\nu} = \begin{pmatrix} 0 & -ih & i\bar{h} \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$$

$$\tau' = \tau + t(z, \bar{z})$$
$$z' = f(z)$$

$\mathcal{M}$ $\quad (\xi^\mu, \eta_\mu, \phi^\mu{}_\nu)$ $\quad$ Almost Co

I $\quad \xi^\mu \eta_\mu = 1 \qquad \phi^\mu{}_\nu \phi^\nu{}_\rho = -\delta^\mu{}_\rho$

D $\quad X \cdot \eta = 0 \qquad \phi|_D = J$

$$\phi^\mu{}_\nu \left(\mathcal{L}_\xi \phi\right)^\nu{}_\rho = 0$$

$$g_{\mu\nu} \phi^\mu{}_\alpha \phi^\nu{}_\beta = g_{\alpha\beta} - \eta_\alpha \eta_\beta$$

THF

$(T, z, \bar{z})$

$\phi^\mu{}_\nu = ($

$(\xi^\mu, \eta_\mu, \phi^\mu{}_\nu)$ Almost Contact Structure

I $\quad \xi^\mu \eta_\mu = 1 \qquad \phi^\mu{}_\nu \phi^\nu{}_\rho = -\delta^\mu{}_\rho + \xi^\mu \eta_\rho \qquad \eta \cdot \phi =$

D $\quad X \cdot \eta = 0 \qquad \phi|_D = J$

$$\phi^\mu{}_\nu \left(\mathcal{L}_\xi \phi\right)^\nu{}_\rho = 0$$

$$\phi^\nu{}_\beta = g_{\alpha\beta} - \eta_\alpha \eta_\beta$$

$$ds^2 = \left(d\tau + h\,dz + \bar h\,d\bar z\right)^2 + c^2(\tau, z, \bar z)\,dz\,d\bar z$$

THF $\qquad$ leaves are ξ ORBITS

$(\tau, z, \bar z) \qquad \xi = \partial_\tau \qquad \eta = d\tau + h($

$$\phi^\mu{}_\nu = \begin{pmatrix} 0 & -ih & i\bar h \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$$

$\tau' = \tau +$

$z' = f($

$$\eta = d\tau + \frac{\lambda}{2(1+|z|^2)}\left(\bar{z}\,dz - z\,d\bar{z}\right) \qquad ds^2 = \eta^2 + \frac{dz\,d\bar{z}}{(1+|z|^2)^2}$$

$-z\,d\bar{z})$

$$ds^2 = \eta^2 + \frac{dz\,d\bar{z}}{(1+|z|^2)^2}$$

$\tau \sim \tau + 2\pi$

$$z - z\, d\bar{z})$$

$$ds^2 = \eta^2 + \frac{dz\, d\bar{z}}{(1+|z|^2)^2}$$

$$T \sim T + 2\pi$$

$$z' = 1/z$$

$$T' = T - \frac{i}{2} \log \bar{z}/z$$

$$(D, \sigma, a_\mu, \lambda_\alpha, \tilde{\lambda}_\alpha)$$

$$\delta\psi_{\mu\alpha} = 0 \implies (\nabla_\mu - i A^{(R)}_\mu)\zeta = -\frac{1}{2} H \gamma_\mu \zeta + \frac{i}{2} V_\mu \zeta - \frac{1}{2}\varepsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$\delta\lambda = 0 \qquad i\zeta(D + \sigma H) - \frac{i}{2}\gamma_\mu \zeta \varepsilon^{\mu\nu\rho} F_{\nu\rho} - i\gamma^\mu \zeta(\partial_\mu \sigma + i V_\mu$$

$\zeta \neq 0$ EVERYWHERE.

$$\xi^\mu = \frac{1}{|\zeta|^2}\zeta^\dagger \gamma^\mu \zeta$$

$$\Rightarrow \quad \left(\nabla_\mu - i A^{(R)}_\mu\right)\zeta = -\frac{1}{2} H \gamma_\mu \zeta + \frac{i}{2} V_\mu \zeta - \frac{1}{2}\varepsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$0 \qquad i\zeta\left(D + \sigma H\right) - \frac{i}{2}\gamma_\mu \zeta\, \varepsilon^{\mu\nu\rho} F_{\nu\rho} - i\gamma^\mu \zeta\left(\partial_\mu \sigma + i V_\mu \sigma\right.$$

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$\neq 0$ EVERYWHERE.

$$\xi^\mu = \frac{1}{|\zeta|^2}\, \zeta^\dagger \gamma^\mu \zeta \qquad \phi^\mu{}_\nu = -\varepsilon^\mu{}_{\nu\rho}\, \xi^\rho$$

$$= \varepsilon^{\mu\nu\rho}\, \partial_\nu \eta_\rho + \kappa\, \eta^\mu + U^\mu \qquad \phi^\mu{}_\nu U^\nu = -i U^\mu$$

$$\nabla^\mu\left(U_\mu + \kappa \eta_\mu\right) = 0$$

$= \ldots$

$= \ldots$

$$) \zeta = -\frac{1}{2} H \gamma_\mu \zeta + \frac{i}{2} V_\mu \zeta - \frac{1}{2} \varepsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$+ \sigma - H) - \frac{i}{2} \gamma_\mu \zeta \, \varepsilon^{\mu\nu\rho} F_{\nu\rho} - i \gamma^\mu \zeta (\partial_\mu \sigma + i V_\mu \sigma) = 0$$

$$\xi^\mu = \frac{1}{|\zeta|^2} \zeta^\dagger \gamma^\mu \zeta \qquad \phi^\mu{}_\nu = -\varepsilon^\mu{}_{\nu\rho} \xi^\rho$$

ACAS

$$\zeta \to \left(\frac{f'(z)}{\bar{f}'(\bar{z})} \right)^{1/4} \zeta$$

$$K \eta^\mu + U^\mu \qquad \phi^\mu{}_\nu U^\nu = -i U^\mu$$

$$\nabla^\mu (U_\mu + K \eta_\mu) = 0$$

$$A_\mu = a_\mu + i\sigma\,\eta_\mu$$

$$\frac{i}{2} V_\mu \zeta - \frac{1}{2}\epsilon_{\mu\nu\rho} V^\nu \gamma^\rho \zeta$$

$$\epsilon^{\mu\nu\rho} F_{\nu\rho} - i\gamma^\mu \zeta\left(\partial_\mu \sigma + i V_\mu \sigma\right) = 0$$

$$\varepsilon^{\mu\nu\rho}\partial_\nu C_\rho$$

$$= 0$$

$$H = -\frac{i}{r}$$

$$A_\mu = a_\mu + i\sigma\eta_\mu$$

$$F_{rz} = 0 \qquad D = \ldots$$

$\varepsilon^{\mu\nu\rho}\partial_\nu C_\rho$

$= 0$

$$H = -\frac{i}{r}$$

$A_\mu = a_\mu + i\sigma\eta_\mu \qquad a_\mu = -i\sigma\eta_\mu$

$F_{\tau z} = 0 \qquad D = \dots$

$$\ldots + h\,d\bar z\,) + C(\tau, z, \bar z)\,dz\,d\bar z$$

$$\eta = d\tau + \frac{i}{2(1+|z|^2)}\left(\bar z\,dz - z\,d\bar z\right) \qquad ds^2 = \eta^2 + \frac{dz\,d\bar z}{(1+|z|^2)^2}$$

$$\Pi^\mu{}_\nu = \frac{1}{2}\left(\delta^\mu{}_\nu - i\,\phi^\mu{}_\nu - \xi^\mu \eta_\nu\right) \qquad \Pi\Pi = \Pi \qquad \eta\Pi = \Pi\xi = 0$$

$$X \in T^{(0,1)}(\mathcal{M}) \qquad \Pi^\mu{}_\nu X^\nu = X^\mu$$

$$\ldots + h\, d\bar{z}\,) + C(\tau, z, \bar{z})\, dz\, d\bar{z}$$

$$\eta = d\tau + \frac{i}{2(1+|z|^2)}\left(\bar{z}\, dz - z\, d\bar{z}\right) \qquad ds^2 = \eta^2 + \frac{dz\, d\bar{z}}{(1+|z|^2)^2}$$

$$\Pi^{\mu}{}_{\nu} = \frac{1}{2}\left(\delta^{\mu}{}_{\nu} - i\,\phi^{\mu}{}_{\nu} - \xi^{\mu}\eta_{\nu}\right) \qquad \Pi\Pi = \Pi \qquad \eta\Pi = \Pi\xi = 0$$

$$X \in T^{(1,0)}(\mathcal{M}) \qquad \Pi^{\mu}{}_{\nu}X^{\nu} = X^{\mu} \qquad X^{z}\left(\partial_z - h\,\partial_\tau\right)$$

$$\omega \in \Lambda^{(1,0)}(\mathcal{M}) \qquad \omega_{\mu}\Pi^{\mu}{}_{\nu} = \omega_{\nu}$$

$$\eta = d\tau + \frac{i}{2(1+|z|^2)}\left(\bar{z}\,dz - z\,d\bar{z}\right) \qquad ds^2 = \eta^2 + \frac{dz\,d\bar{z}}{(1+|z|^2)^2}$$

$$\Pi^\mu{}_\nu = \frac{1}{2}\left(\delta^\mu{}_\nu - i\,\phi^\mu{}_\nu - \xi^\mu \eta_\nu\right) \qquad \Pi\Pi = \Pi \qquad \eta\Pi = \Pi\xi = 0$$

$$X \in T^{(1,0)}(\mathcal{M}) \qquad \Pi^\mu{}_\nu X^\nu = X^\mu \qquad X^z\left(\partial_z - h\,\partial_\tau\right)$$

$$\omega \in \Lambda^{(1,0)}(\mathcal{M}) \qquad \omega^{(1,0)}_\mu \Pi^\mu{}_\nu = \omega^{(1,0)}_\nu$$

$$\omega^{(1,0)} = \omega^{(1,0)}_z\,dz.$$

$$\Pi\Pi = \Pi \qquad \eta\Pi = \Pi\xi = 0$$

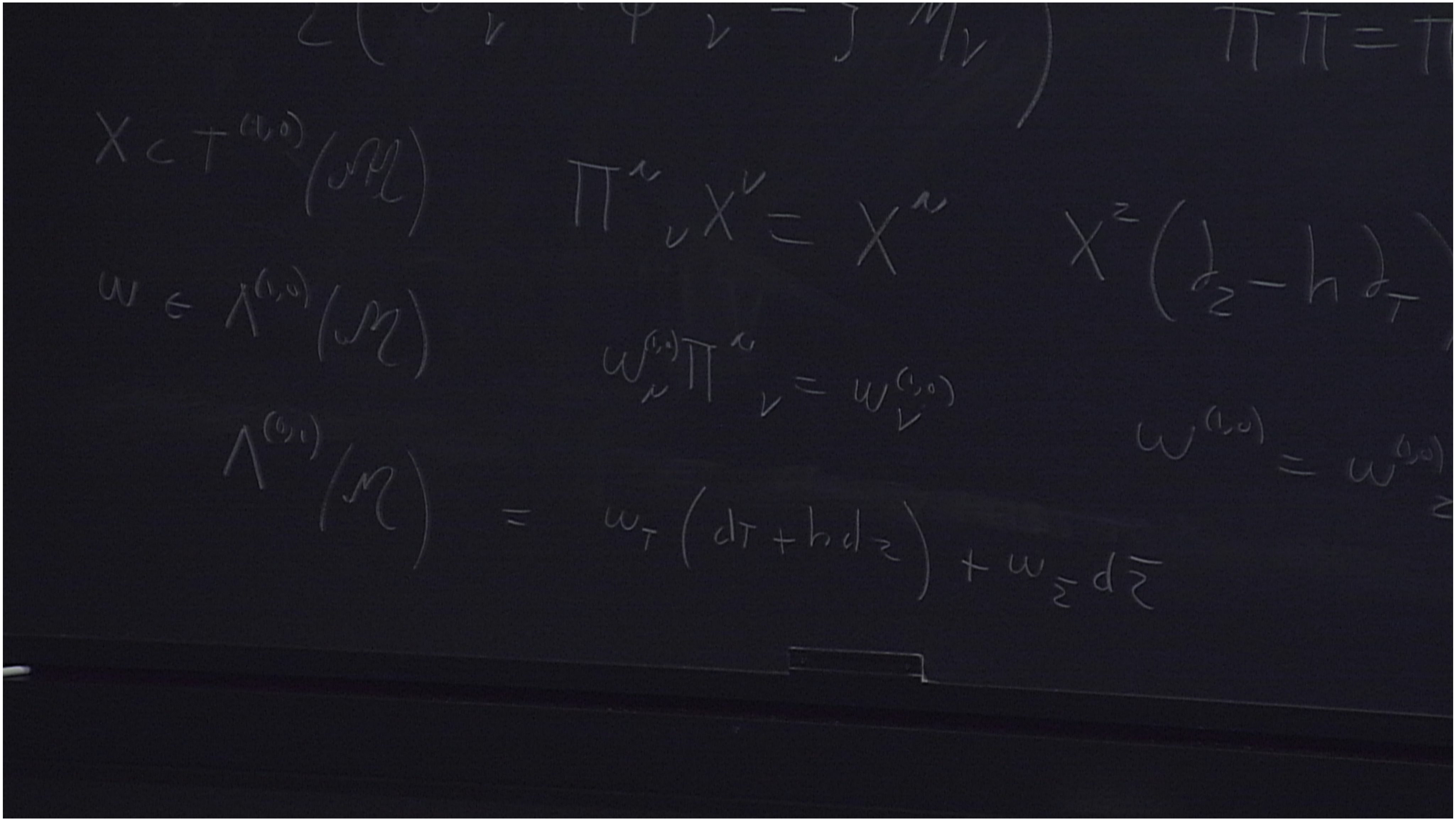
$$T' = T - \frac{i}{2}\log \bar{z}/z$$

$$X^{z}\left(\partial_z - h\,\partial_T\right)$$

$$X'^{z'} = f'(z)\, X^{z}$$

$$\omega'^{(1,0)}_{z} = \frac{1}{f'(z)}\,\omega^{(1,0)}_{z}$$

$$\omega^{(1,0)} = \omega^{(1,0)}_{z}\, dz .$$


$X \in T^{(1,0)}(\mathcal{M})$
$\Pi^{\mu}{}_{\nu} X^{\nu} = X^{\mu}$
$X^{z}\left(\partial_{z} - h\,\partial_{\tau}\right)$
$\omega \in \Lambda^{(1,0)}(\mathcal{M})$
$\omega^{(1,0)}_{\mu}\Pi^{\mu}{}_{\nu} = \omega^{(1,0)}_{\nu}$
$\Lambda^{(0,1)}(\mathcal{M})$
$= \omega_{\tau}\left(d\tau + h\,dz\right) + \omega_{\bar{z}}\,d\bar{z}$
$\omega^{(1,0)} = \omega^{(1,0)}_{z}$
$\Pi\,\Pi = \Pi$

$d \quad (p,q) \quad (p+1,q) \quad (p,q+1)$

$$(1,0) \to (1,1), \quad (1,0) \to (0,2)$$

$$d \qquad (p,q) \longrightarrow (p+1,q) \oplus (p,q+1)$$

$$(1,0) \to (1,1)$$

$$(1,0) \to \cancel{(0,2)}$$

$$\tilde{\partial} \qquad (p,q) \longrightarrow (p,q+1)$$

$$\tilde{\partial}^2 = 0$$

$$d \qquad (p,q) \longrightarrow (p+1,q) \oplus (p,q+1)$$

$$(1,0) \longrightarrow (1,1), \quad \cancel{(0,2)}$$

$$\tilde{\partial} \qquad (p,q) \longrightarrow (p,q+1)$$

$$\tilde{\partial}^2 = 0$$

$$H^{(p,q)} \quad \cdot \ \frac{\tilde{\partial}\ \text{closed}}{\tilde{\partial}\ \text{exact}}$$

$(1,0) \to (1,1)$

$\cancel{\to (0,2)}$

$\tilde{\gamma}^2 = 0$

Choose one THF

$\Delta \xi^\mu$, $\Delta \eta_\mu$, $\Delta \phi^\mu{}_\nu$

(Pr 4+1)

$(1,0) \to (1,1)$

$\cancel{(0,2)}$

$\tilde{\gamma}^2 = 0$

Choose one THF

$\Delta\xi^\nu, \Delta\eta_\nu, \Delta\phi^\nu{}_2$

$\Delta\eta_2, \Delta\eta_{\bar{2}}$

$\Delta\phi^2{}_{\bar{2}}, \Delta\phi^{\bar{2}}{}_2$

$(\mathrm{Pr}\ 4+1)$

$(1,0) \to (1,1)$

$(1,0) \to \cancel{(0,2)}$

$\tilde{\gamma}^2 = 0$

Choose one THF

$\Delta\xi^{\sim}, \Delta\eta_{\sim}, \Delta\phi^{\sim}{}_{z}$

$\Delta\xi^{\sim}$

$\Delta\eta_z, \Delta\eta_{\bar z}$

$\Delta\phi^z{}_{\bar z}, \Delta\phi^{\bar z}{}_z$

$\Theta^2 \in \Lambda^{(1,0)}$ with coeff in $T^{(1,0)}(M)$

$$\Theta^2 = -2i\,\Delta\xi^z(d\tau + h\,dz) + (\Delta\phi^z{}_{\bar z} - i\bar h\,\Delta\xi^z)\,d\bar z$$

(Pr 4+1)

$(1,0) \to$

$\cancel{\to (0,2)}$

$\tilde{\partial}^2 = 0$

Choose one THF

$\Delta \xi^\mu, \Delta \eta_\mu, \Delta \phi^\mu{}_z$

$\Delta \xi^\mu \quad \Delta \eta_z, \Delta \eta_{\bar z}$

$\Delta \phi^z{}_{\bar z}, \Delta \phi^{\bar z}{}_z$

$\Theta^z \in \Lambda^{(1,0)}$ with coeff in $T^{(1,0)}(M)$

$\| $

$-2i \Delta \xi^z (d\tau + h\, dz) + (\Delta \phi^z{}_{\bar z} - i\bar h \Delta \xi^z) d\bar z$

$\tilde{\partial} \Theta^z = 0$

$\varepsilon^\mu \to \Theta^z = \tilde{\partial} \varepsilon^z$

$$d \qquad (p,q) \longrightarrow (p+1,q) \oplus (p,q+1) \qquad (1,0) \longrightarrow \cancel{(0,2)}$$

$$\tilde{\partial} \qquad (p,q) \longrightarrow (p,q+1) \qquad \tilde{\partial}^2 = 0$$

$$H^{(p,q)} : \frac{\tilde{\partial}\ \text{closed}}{\tilde{\partial}\ \text{exact}}$$

$$H^{(1,0)}\left(M, T^{(1,0)}(M)\right)$$

$$\Theta^2 \in \Lambda^{(1,0)} \text{ with coeff in } T^{(1,0)}(\ldots$$

$$\| \quad -2i\,\Delta\xi^2 (d\tau + h\,dz) + (\Delta\phi^2 \ldots$$

$$\tilde{\partial}\,\Theta^2 = 0$$

$$\varepsilon^2 \longrightarrow \Theta^2 = \tilde{\partial}\,\varepsilon^2$$

$$\eta^2 + \frac{dz\,d\bar{z}}{(1+|z|^2)^2}$$

$$\tau \sim \tau + 2\pi$$

$$z' = 1/z$$

$$\tau' = \tau - \frac{i}{2}\log \bar{z}/z$$

$$\Pi = \Pi \qquad \eta\Pi = \Pi\xi = 0$$

$$\Delta \xi^2 = \int \gamma z \partial_z \quad \text{1. min}$$

$$(\gamma z^2 \partial_z)$$

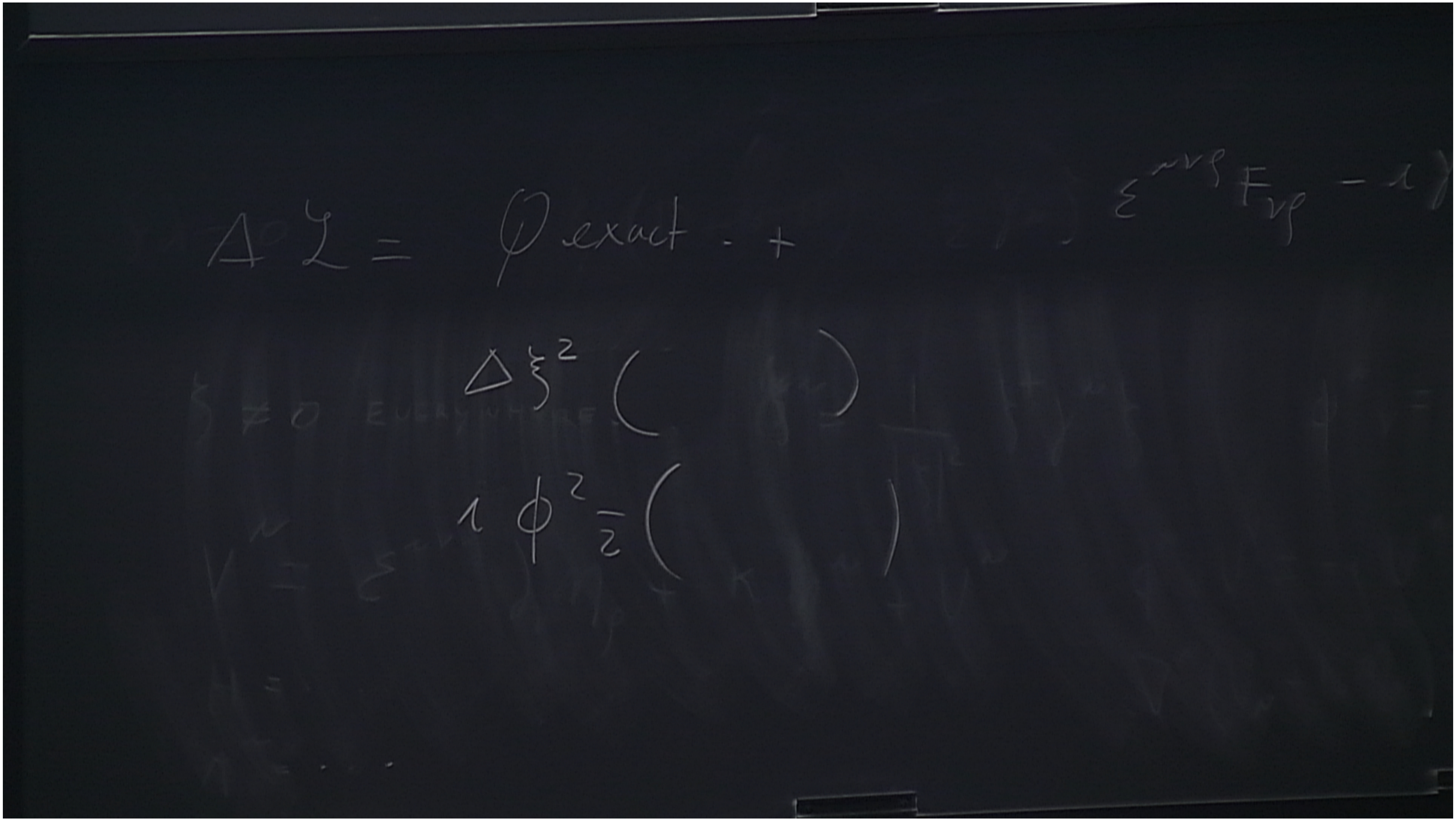
$$(\gamma \partial_z)$$

$$\partial_\tau)$$

$$X'^{z'} = f'(z) X^z$$

$$\omega'^{(1,0)}_{z} = \frac{1}{f'(z)}\,\omega^{(1,0)}_{z}$$

$$\omega^{(1,0)}_{z}\,dz.$$


ΔL = φ exact . +
ε^μνρ F_νρ − 1
Δξ² ( )
1 φ² /2 ( )

$$\Delta^{0} \mathcal{L} = \phi \,\text{exact} \cdot + \qquad \varepsilon^{\mu\nu\rho} F_{\nu\rho} - i$$

$$+ \Delta \xi^{2} \left(\qquad \right)$$

$$+ i\, \phi^{2} \frac{\ }{2} \left(\qquad \right)$$

$$H^{(1,0)}\left(M, T^{(1,0)}(M)\right)$$