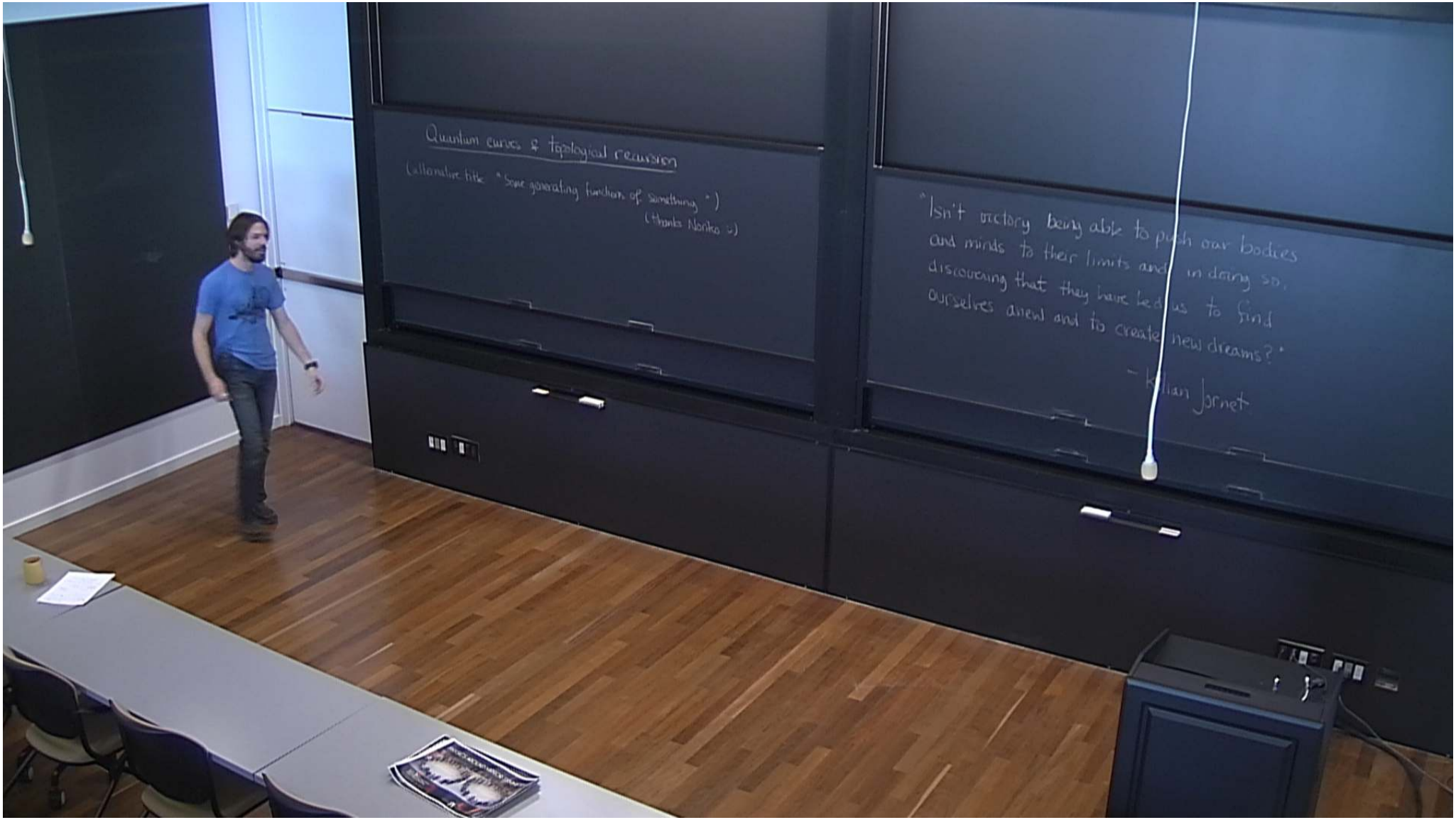
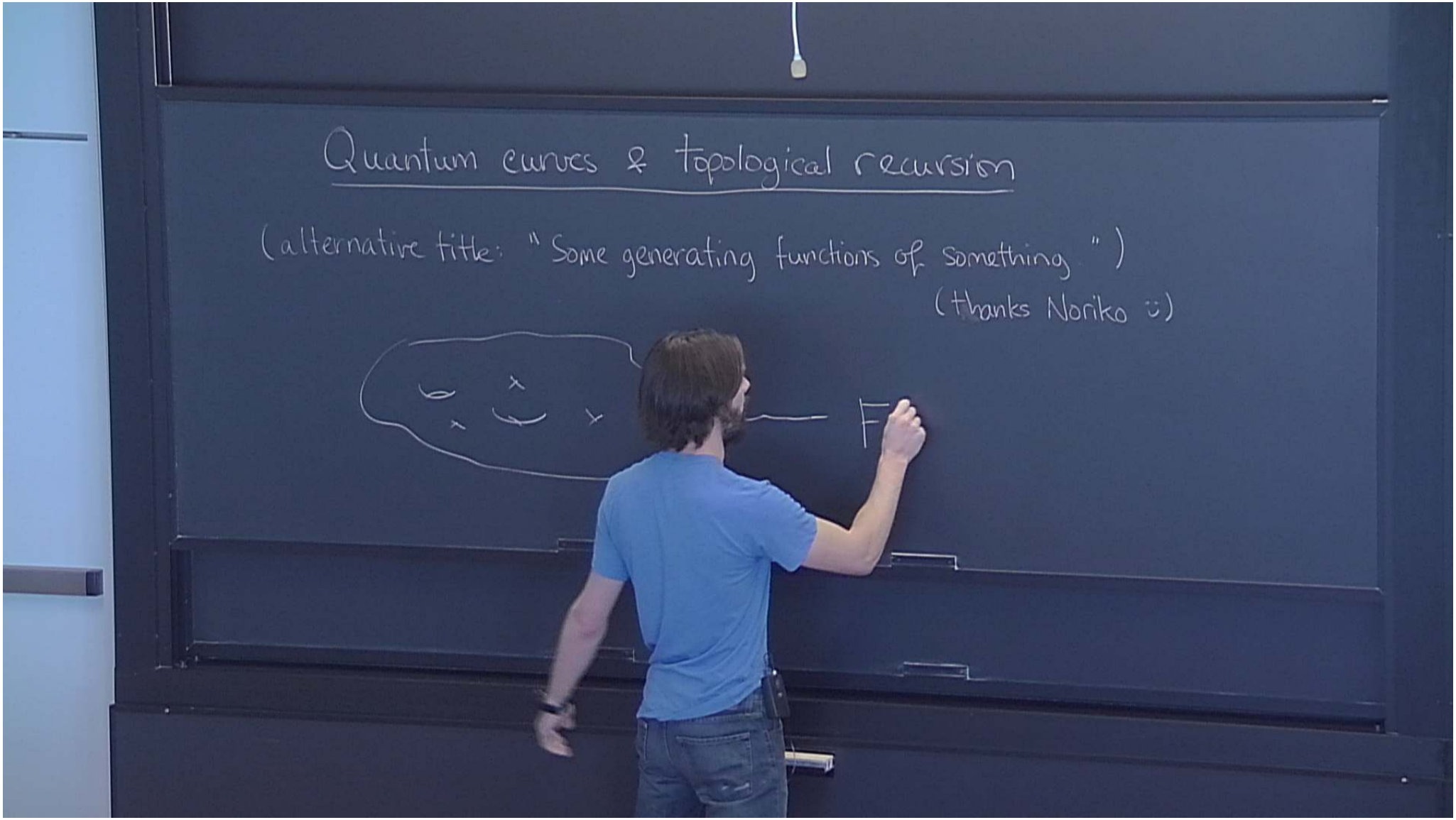
Title: TBA

Date: Oct 24, 2013 10:00 AM

URL: http://pirsa.org/13100122

Abstract:


Quantum curves & topological recursion
(alternative title: "Some generating functions of something")
(thanks Nonko :)
"Isn't victory being able to push our bodies
and minds to their limits and in doing so,
discovering that they have led us to find
ourselves anew and to create new dreams?"
- Lilian Jornet


Quantum curves & topological recursion
(alternative title: "Some generating functions of something")
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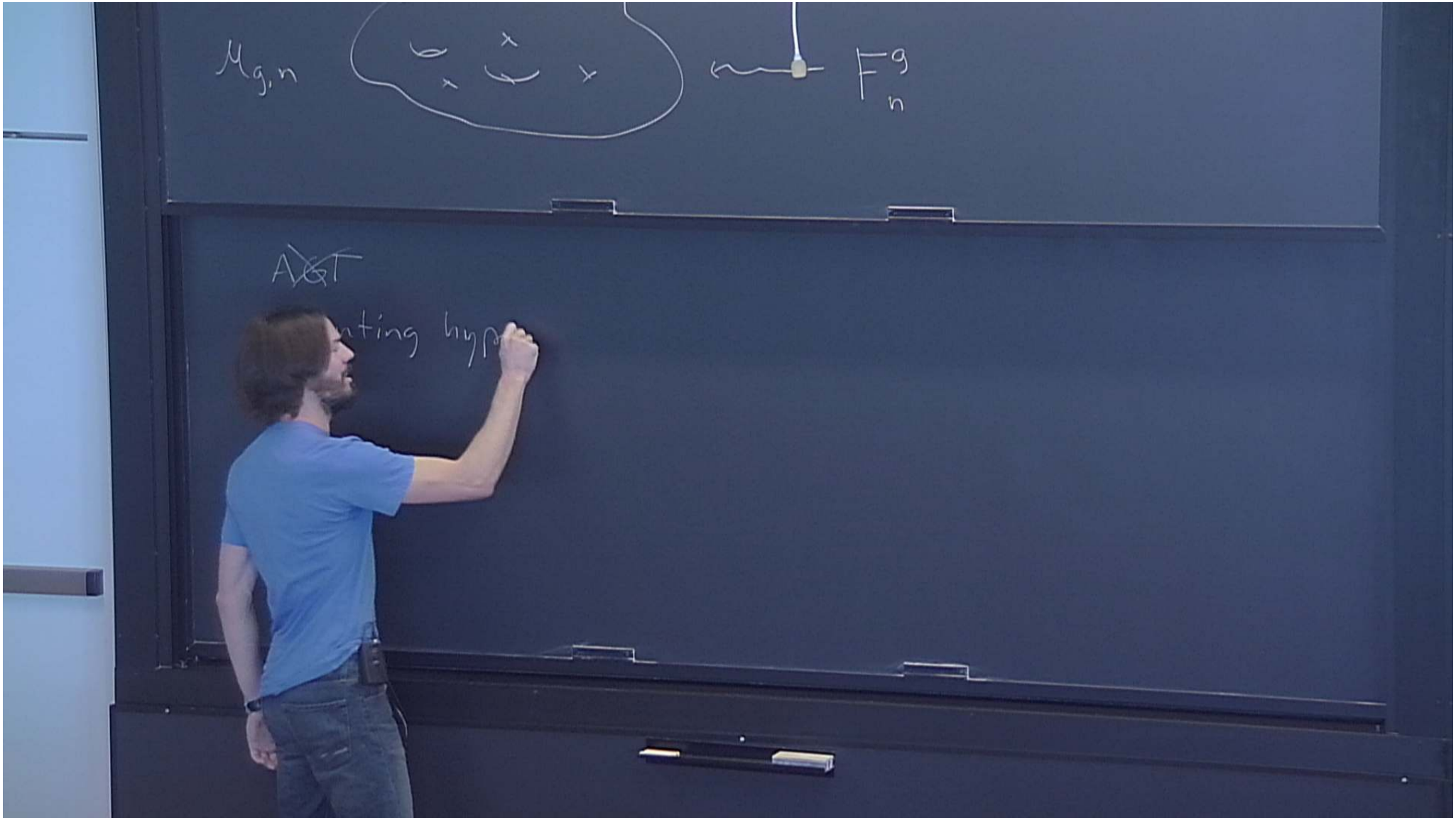
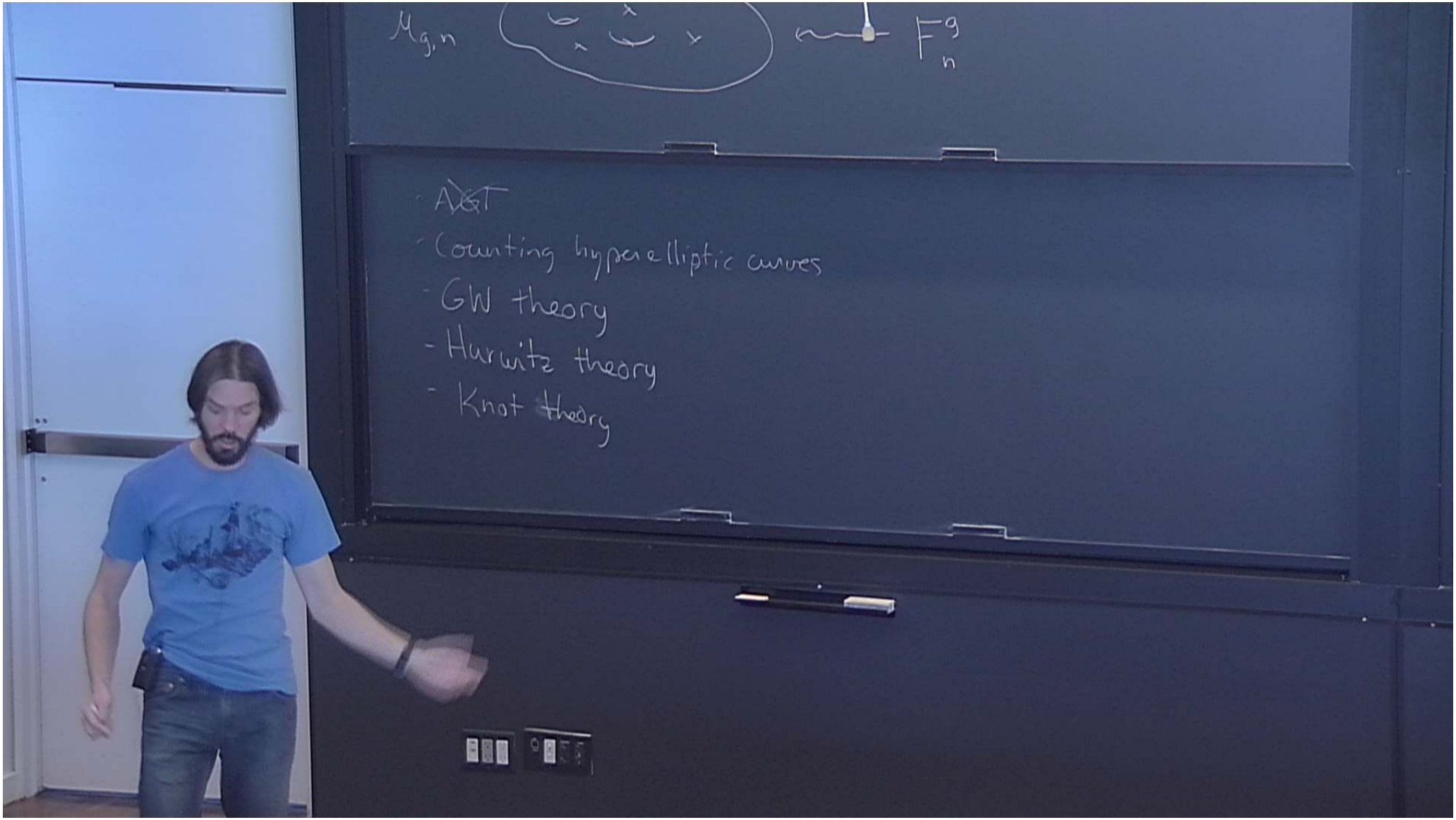
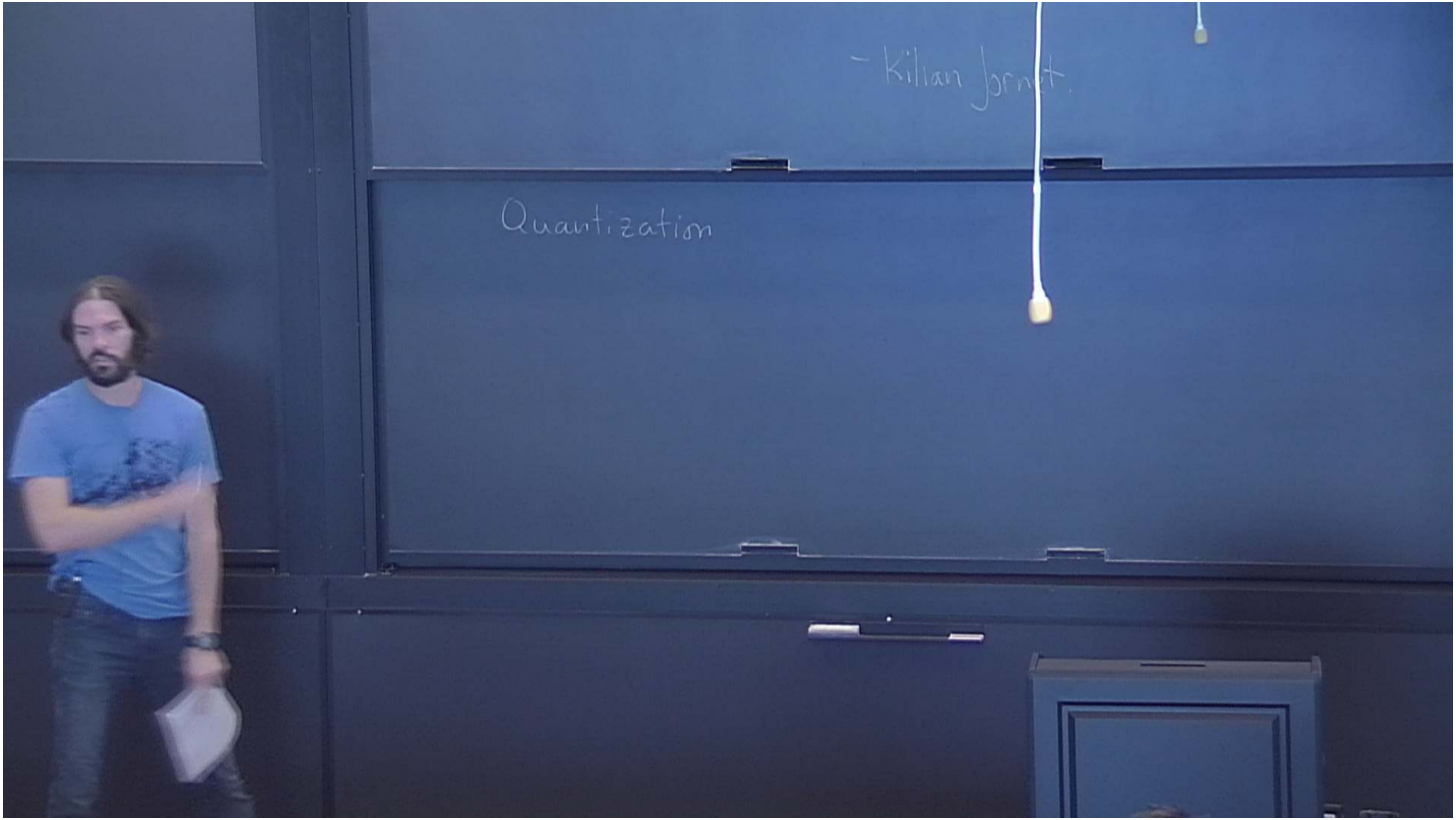
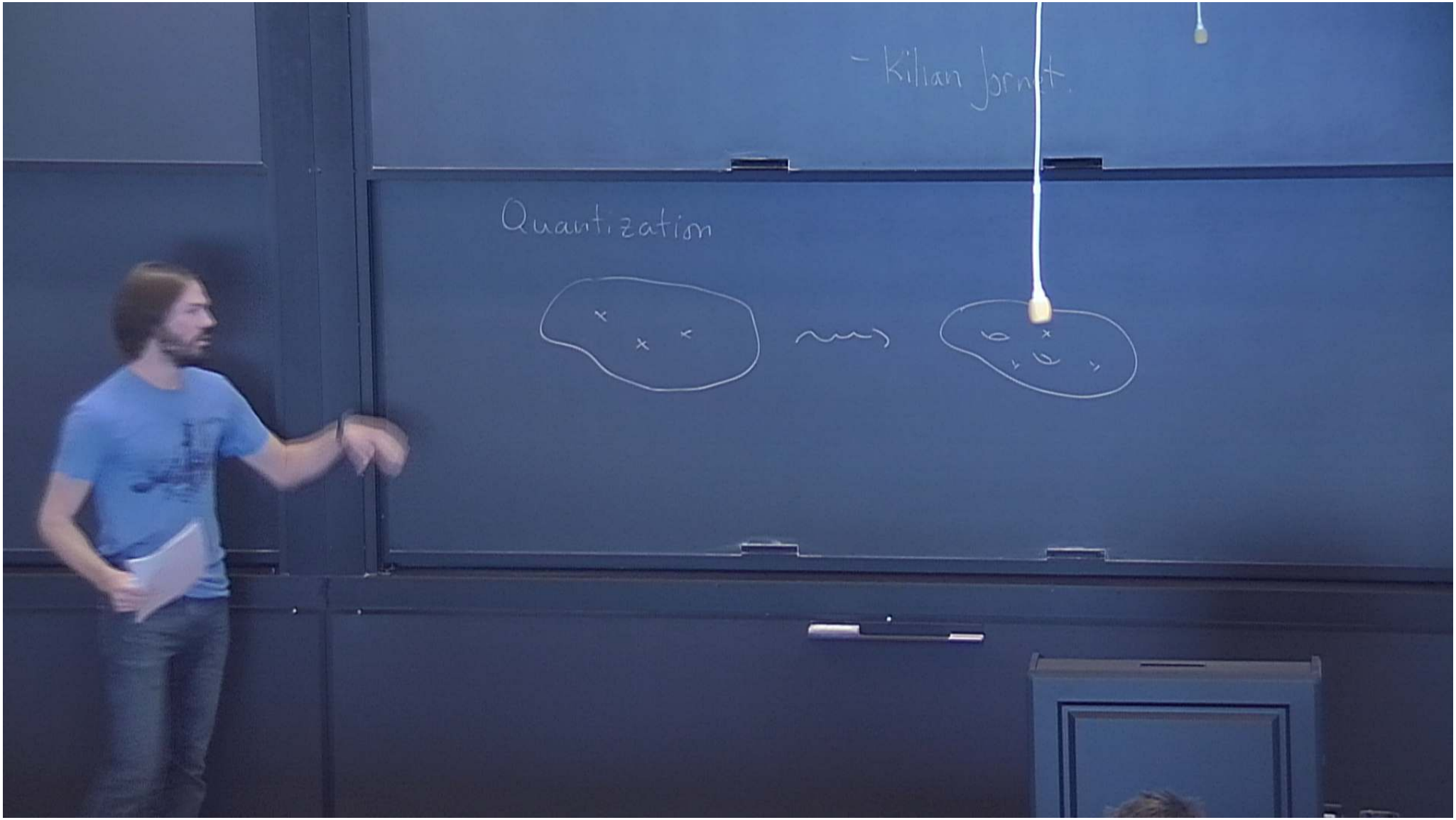
F


$\mathcal{M}_{g,n}$
F_n^g
AGT


$\mathcal{M}_{g,n}$
F_n^g
- ~~AGT~~
- Counting hyperelliptic curves
- GW theory
- Hurwitz theory
- Knot theory


- Kilian Jornet.
Quantization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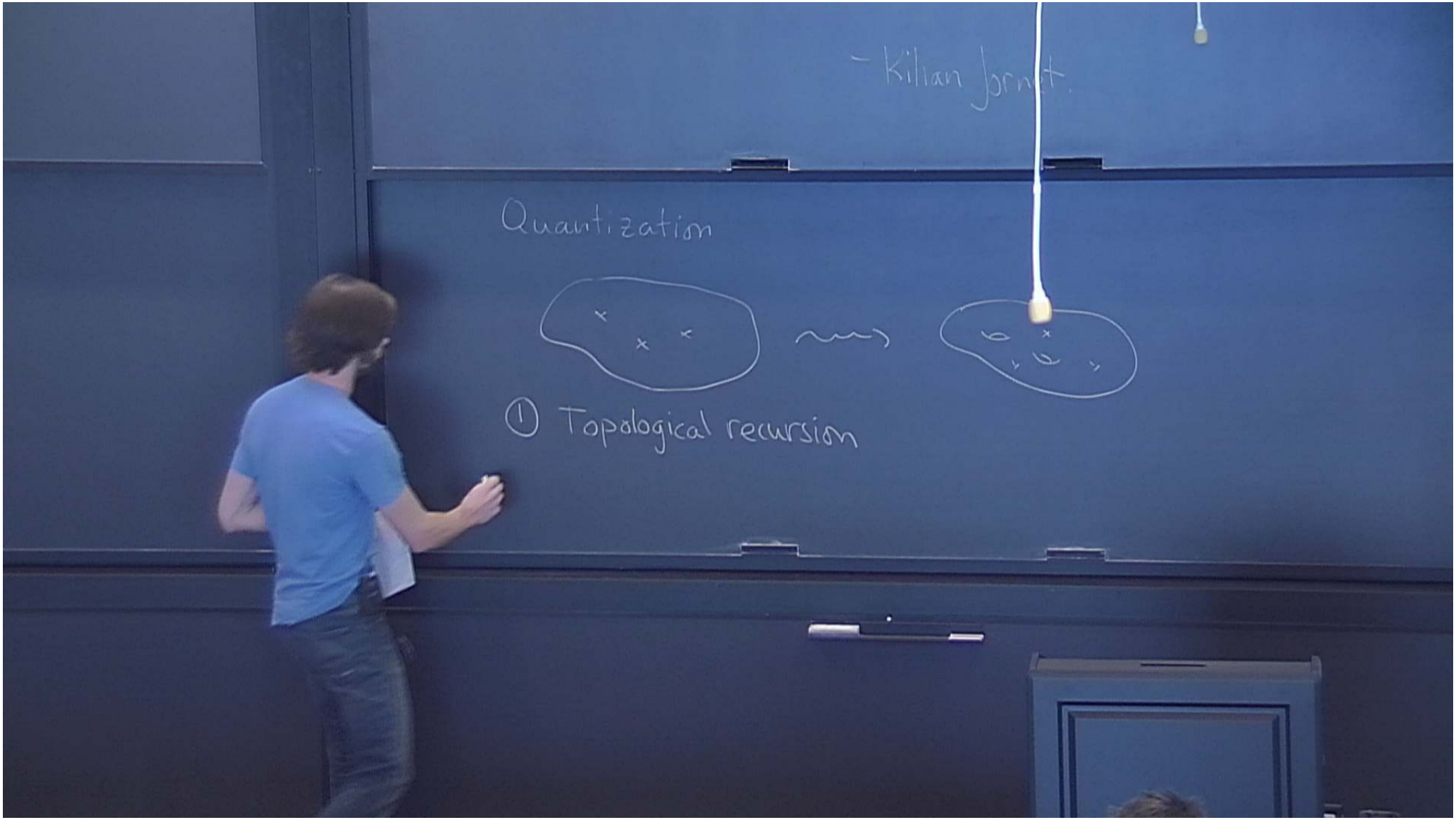
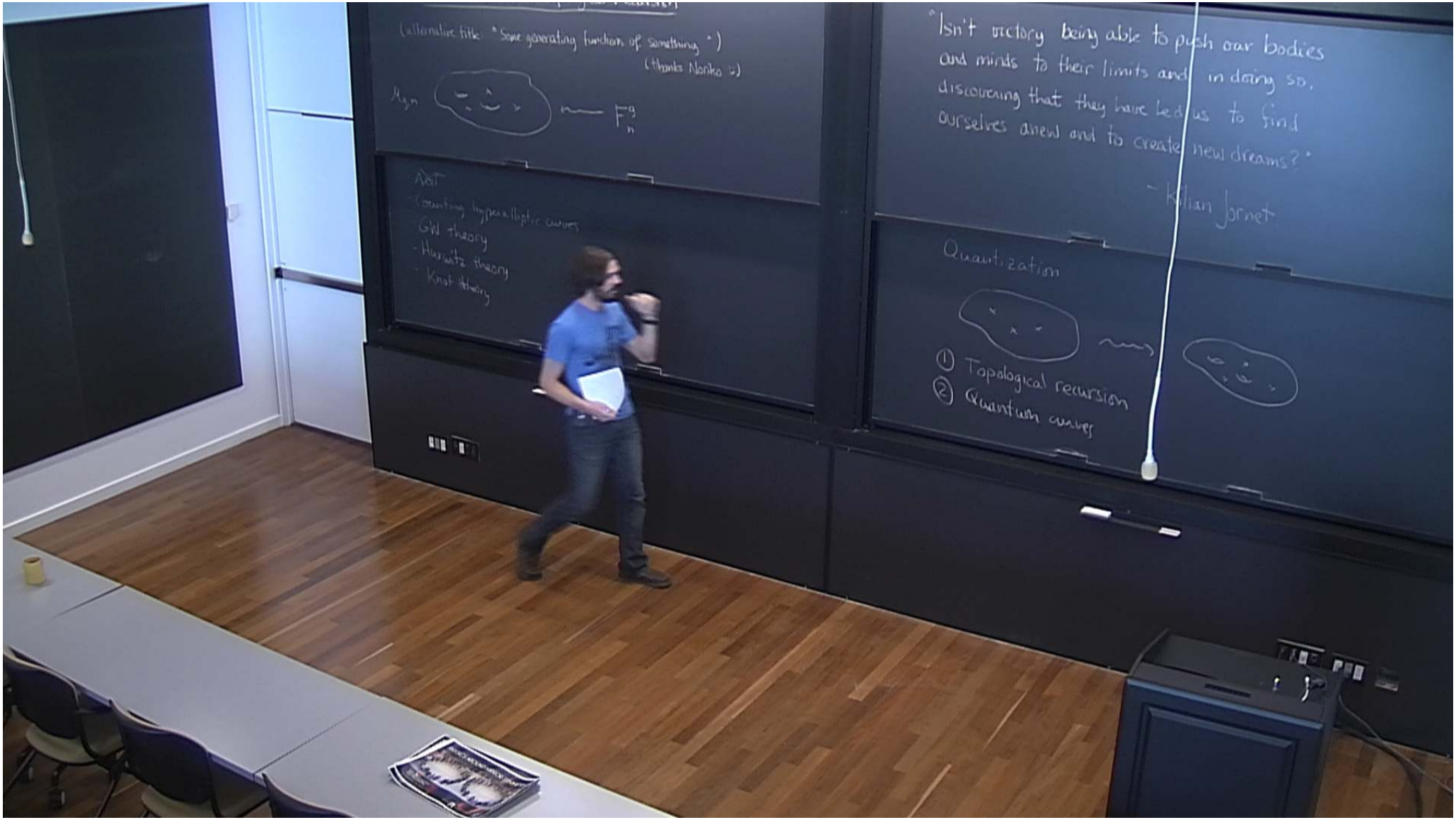
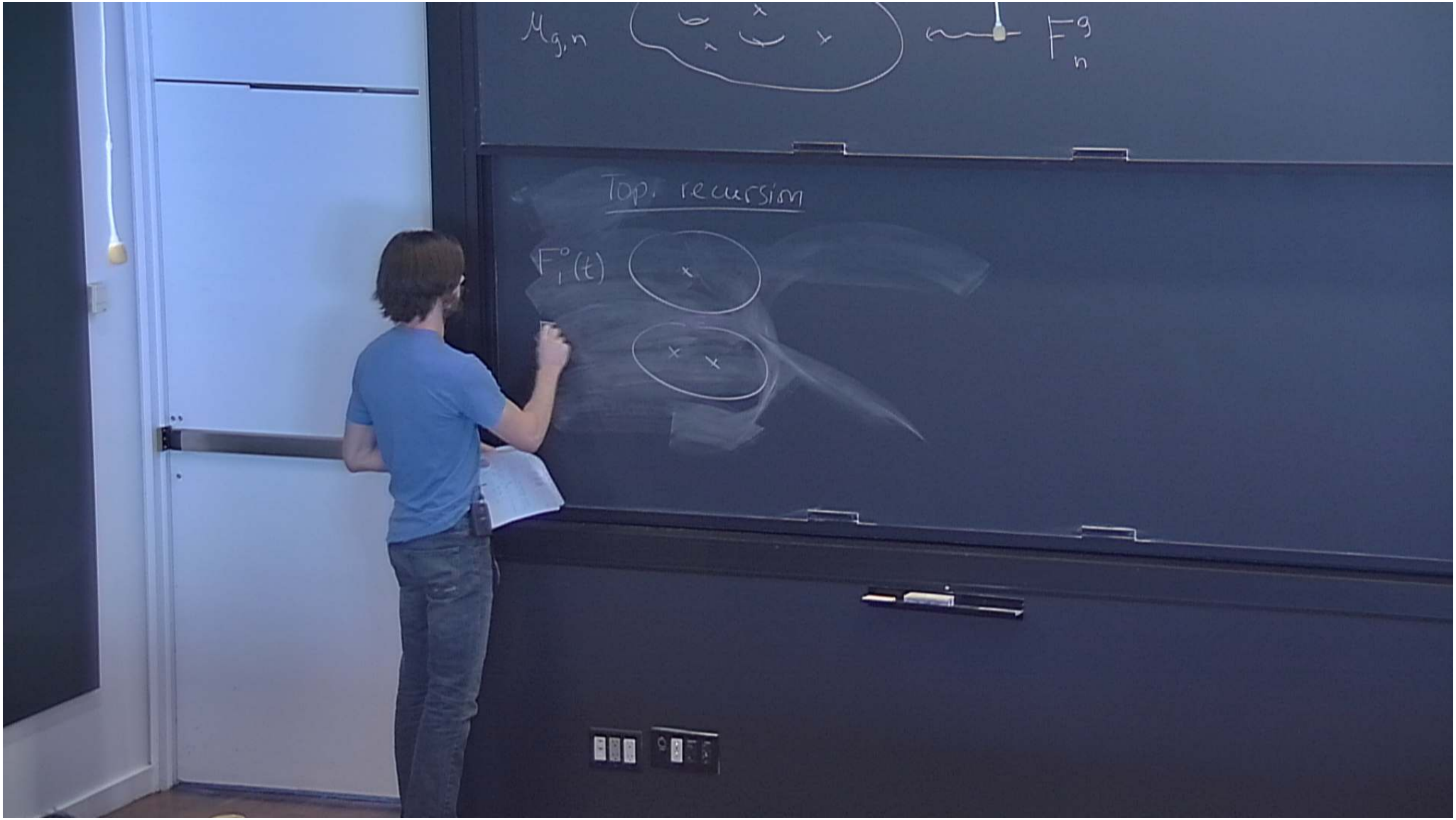
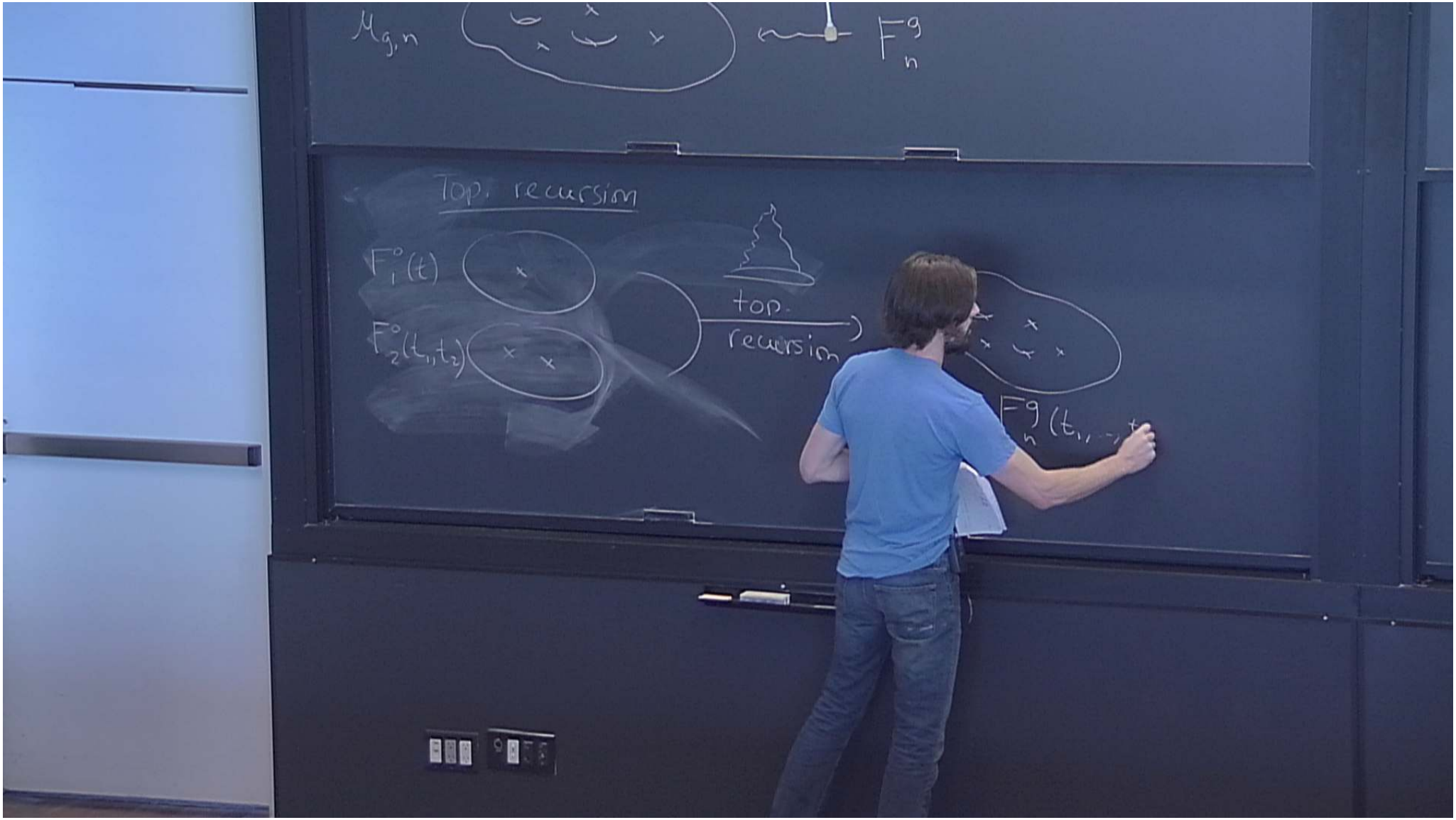
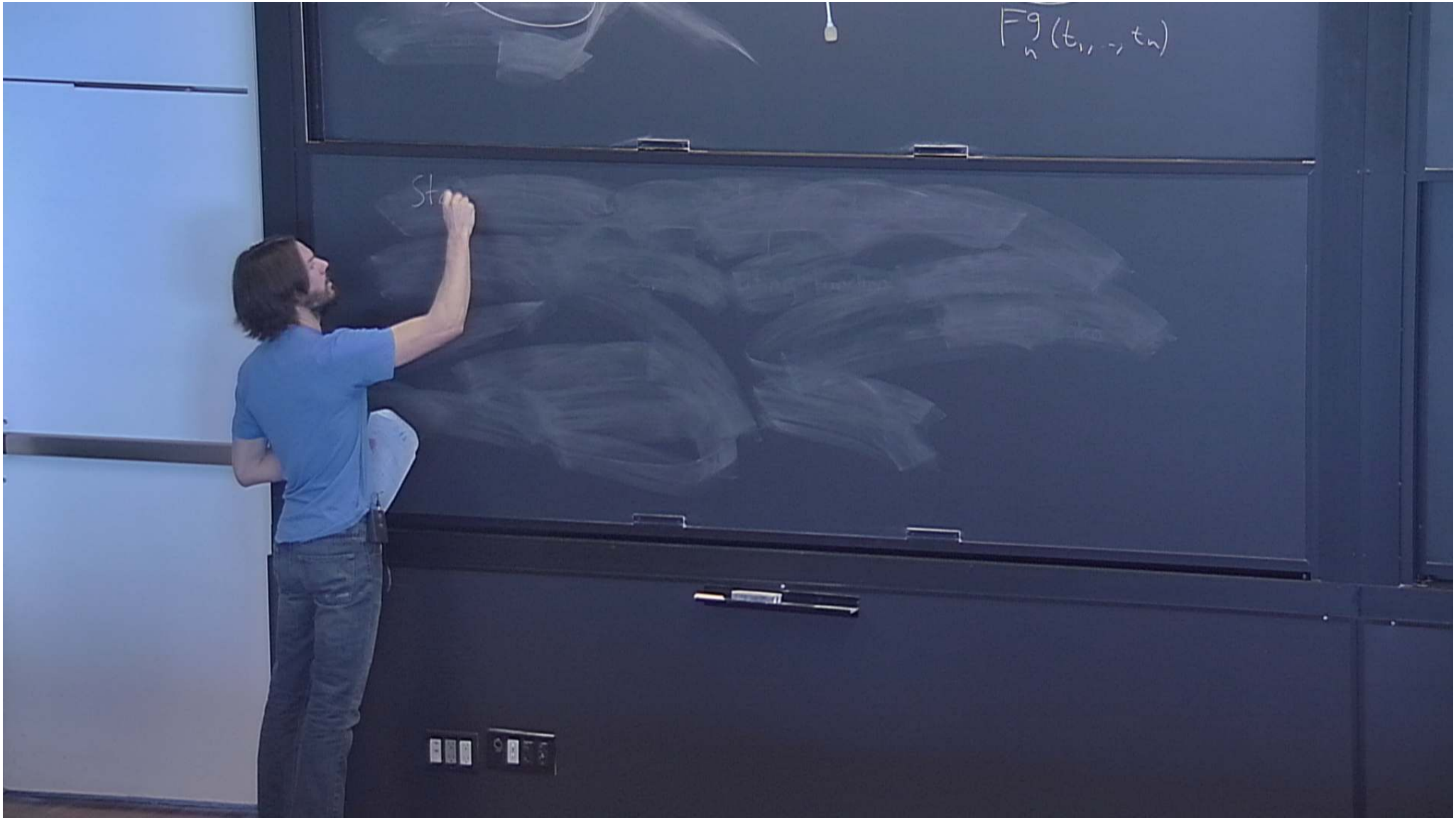
Quantization

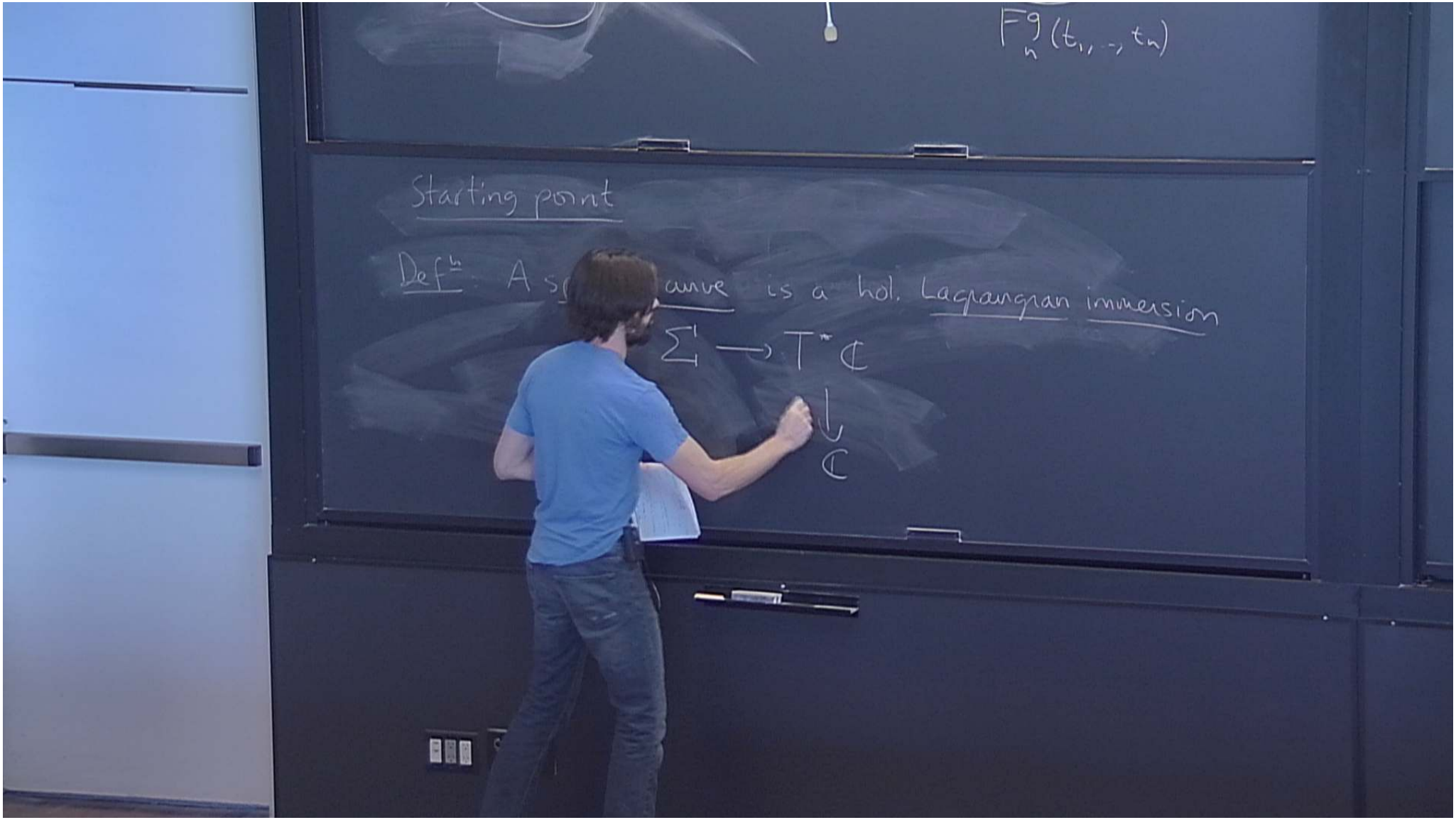

- Kilian Jornet.
Quantization
① Topological recursion

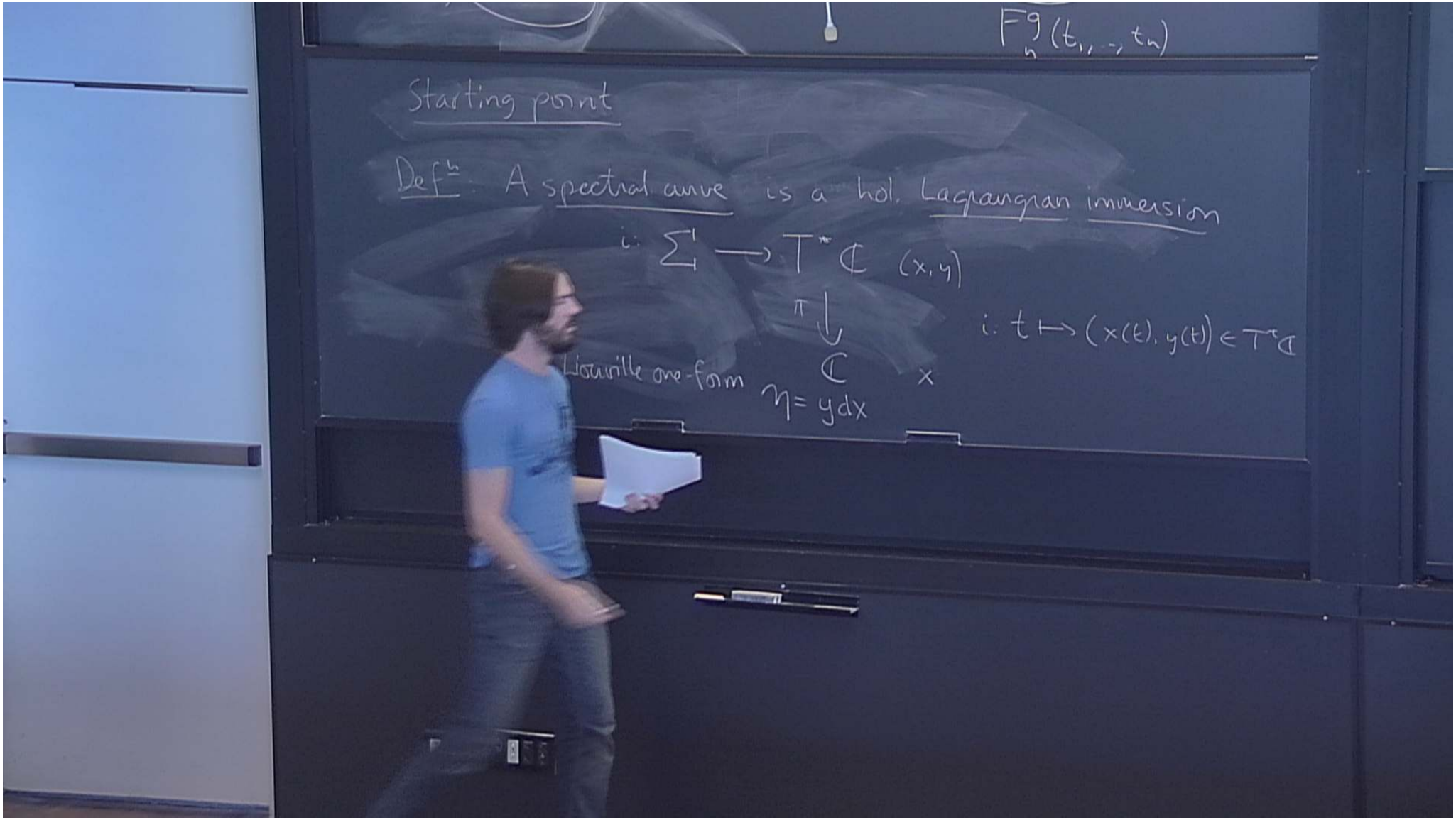

(alternative title: "Some generating function of something")
(thanks Marko :)
$\Delta_{g,n}$
F_n^g
"Isn't victory being able to push our bodies and minds to their limits and in doing so, discovering that they have led us to find ourselves anew and to create new dreams?"
- Killian Jornet
Quantization
① Topological recursion
② Quantum curves

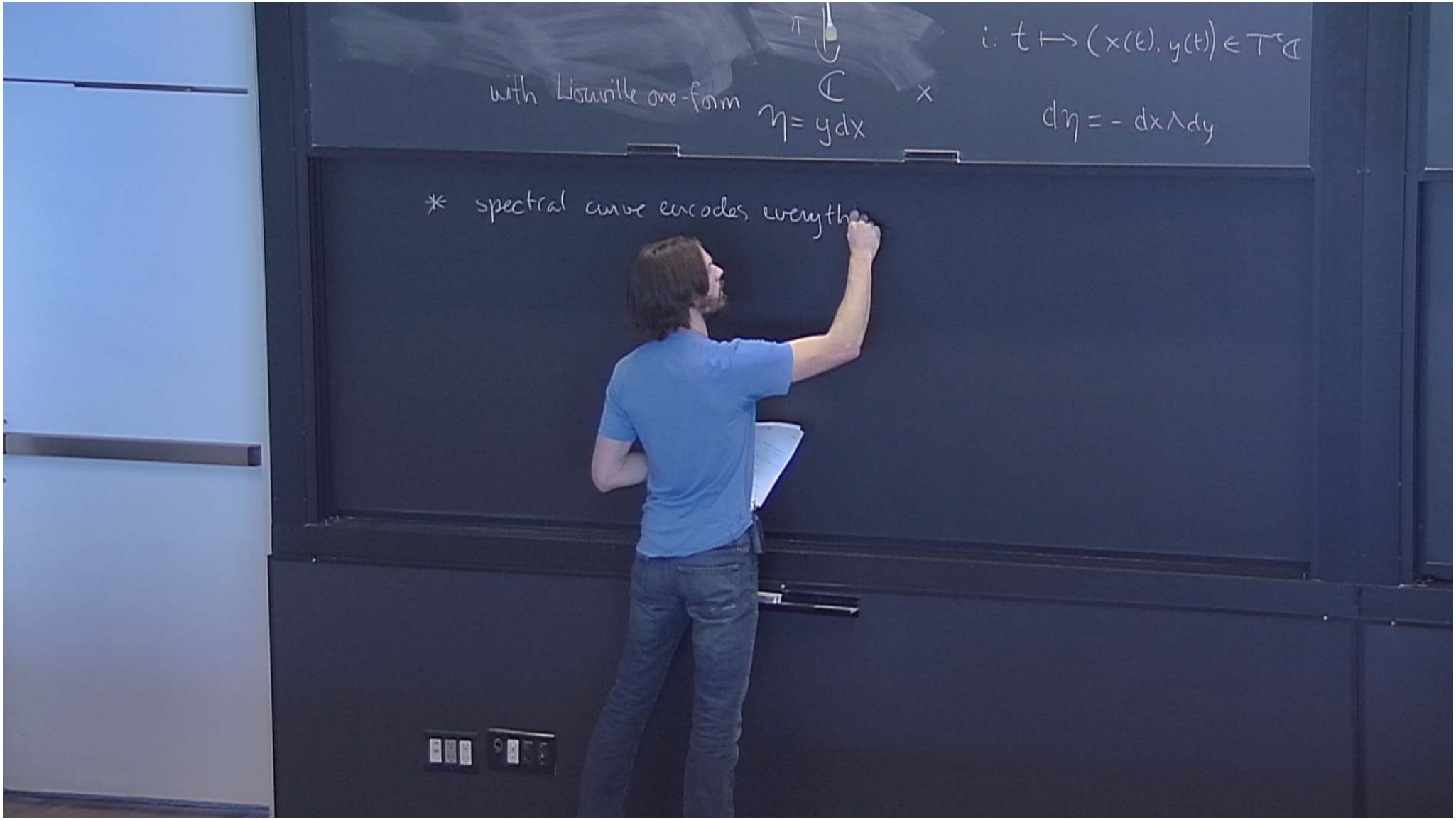

$\mathcal{M}_{g,n}$
F_n^g
Top. recursion
$F_i^\circ(t)$


$\mathcal{M}_{g,n}$
F_n^g
Top. recursion
$F_1^\circ(t)$
$F_2^\circ(t_1,t_2)$
top. recursion
$F_n^g(t_1,\dots$


$F^g_n(t_1, \ldots, t_n)$


$F^g_n(t_1, \ldots, t_n)$
Starting point
Def^n A s curve is a hol. Lagrangian immersion
$\Sigma^1 \to T^*\mathbb{C}$
$\downarrow$
$\mathbb{C}$


$F^g_n(t_1, \ldots, t_n)$
Starting point
Def: A spectral curve is a hol. Lagrangian immersion
$i: \Sigma' \to T^*\mathbb{C}$ (x, y)
$\pi \downarrow$
$\mathbb{C}$ x
$i: t \mapsto (x(t), y(t)) \in T^*\mathbb{C}$
Liouville one-form $\eta = y\,dx$


with Liouville one-form
$\eta = y\,dx$
$i: t \mapsto (x(t), y(t)) \in T^*\mathcal{C}$
$d\eta = -\,dx \wedge dy$
* spectral curve encodes

Def: A spectral curve is a hol. Lagrangian immersion
$\Sigma \to T^*\mathbb{C}_{(x,y)}$
$\pi \downarrow$
$\mathbb{C}_x$
with Liouville one form $\eta = ydx$
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* spectral curve encodes everything about
"Isn't victory being able to push
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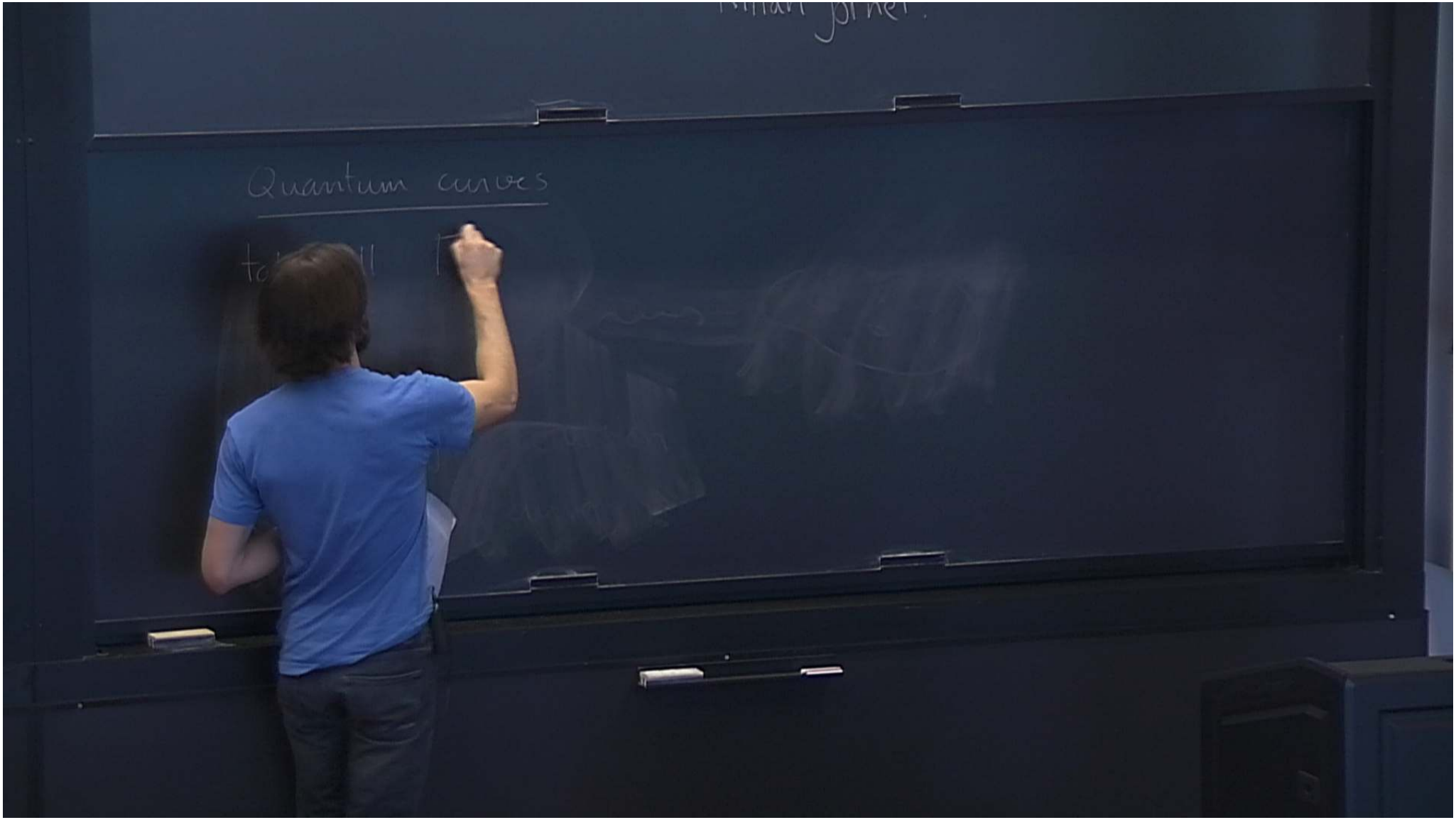
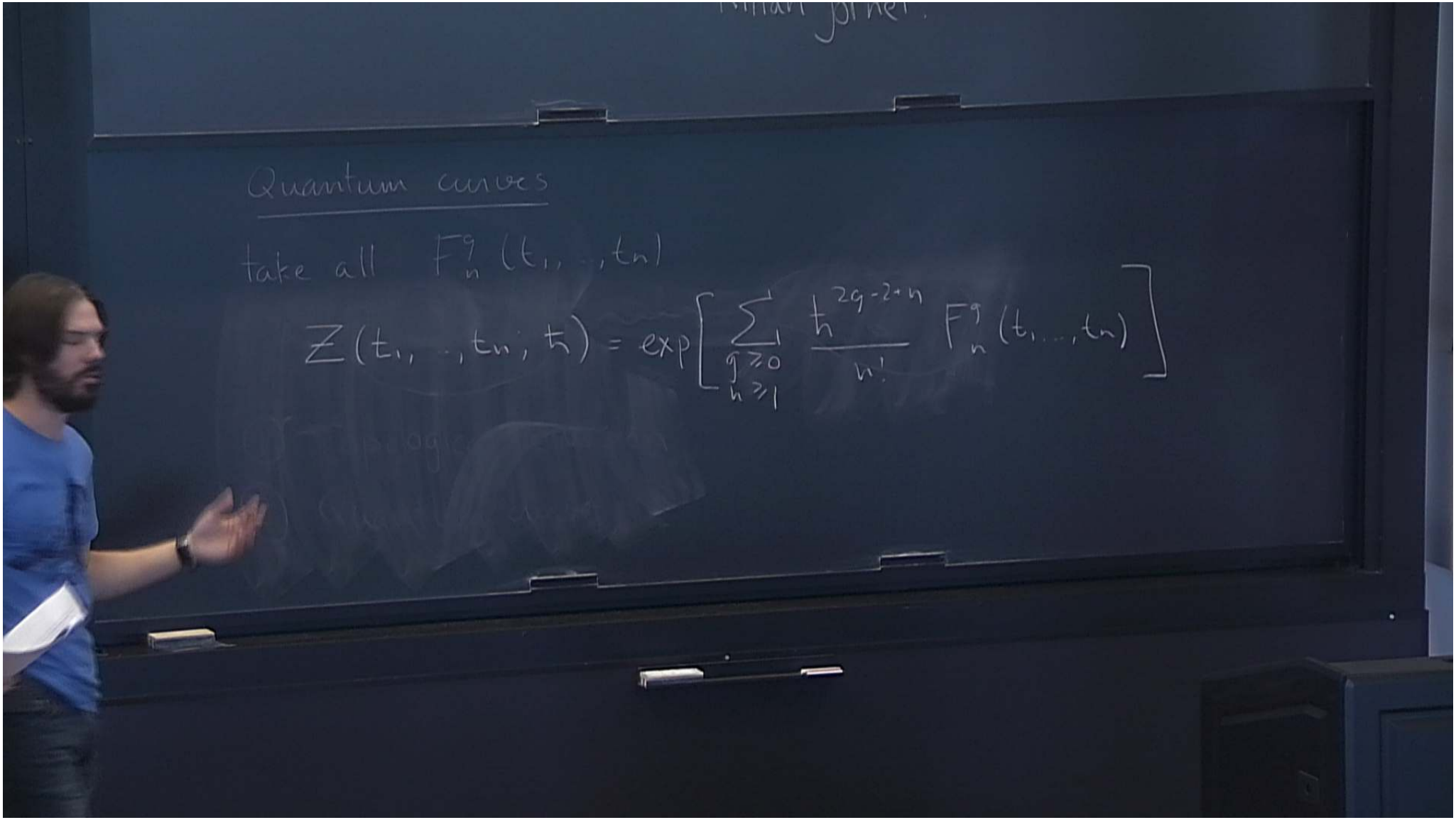
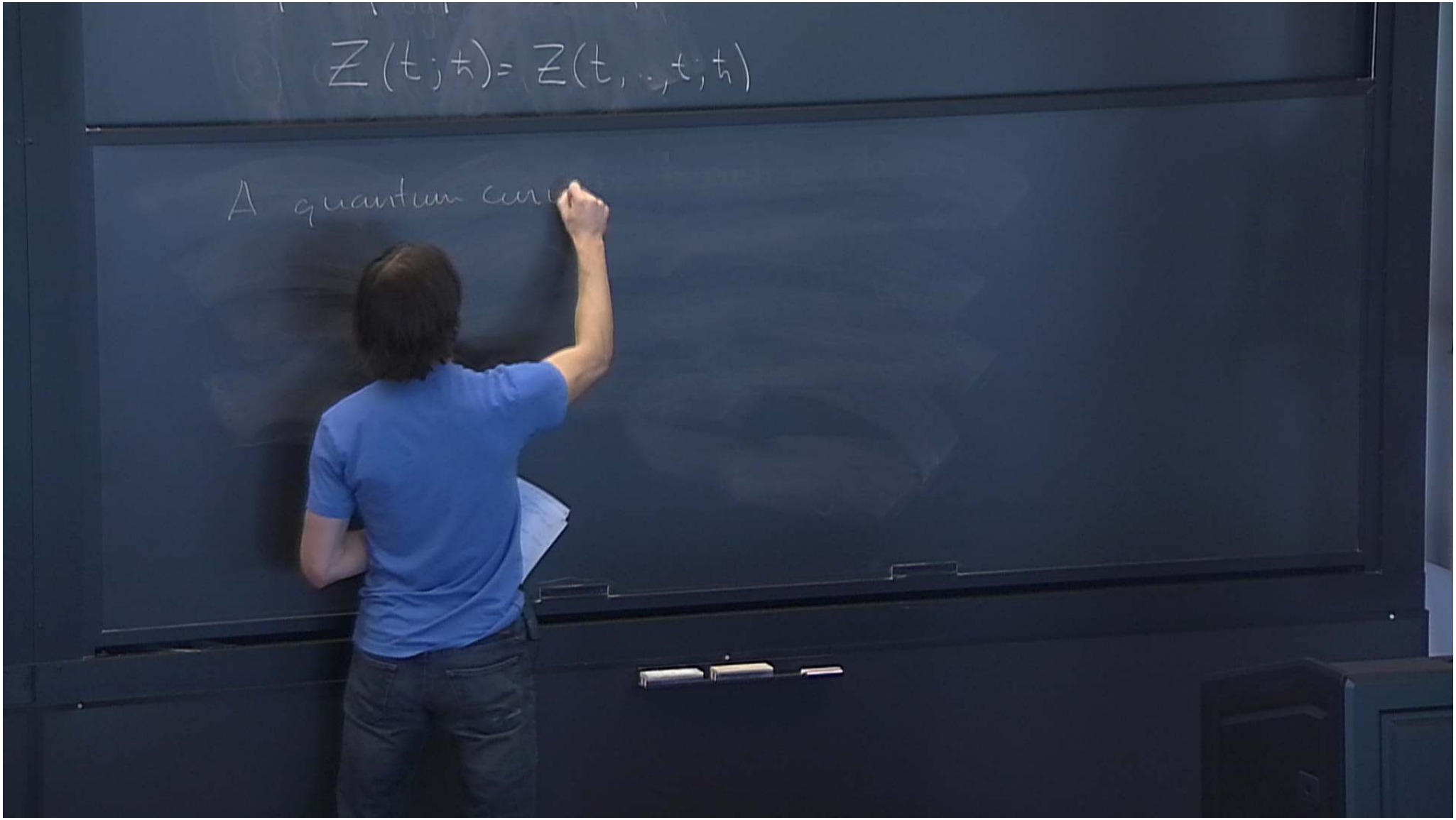
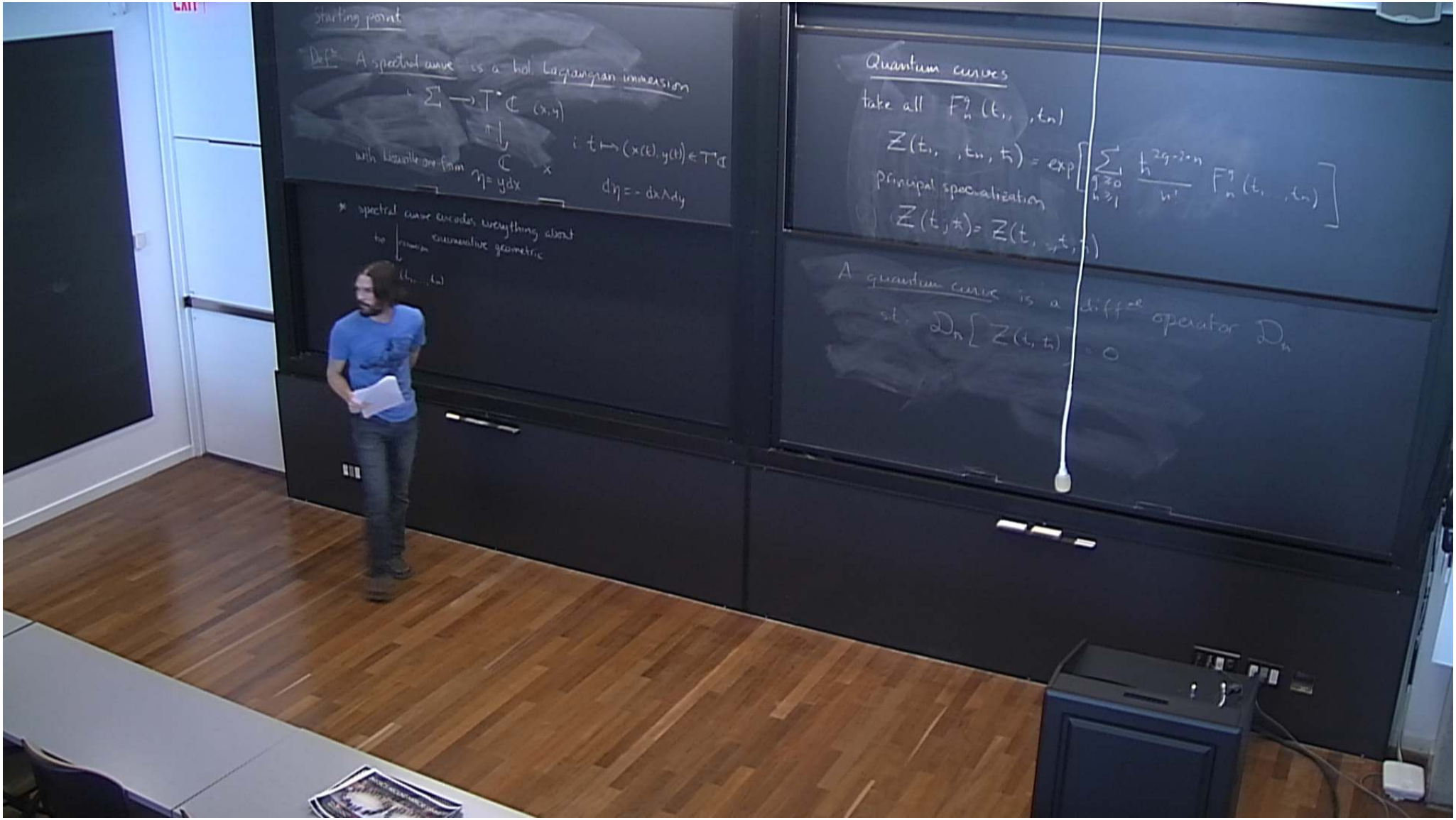
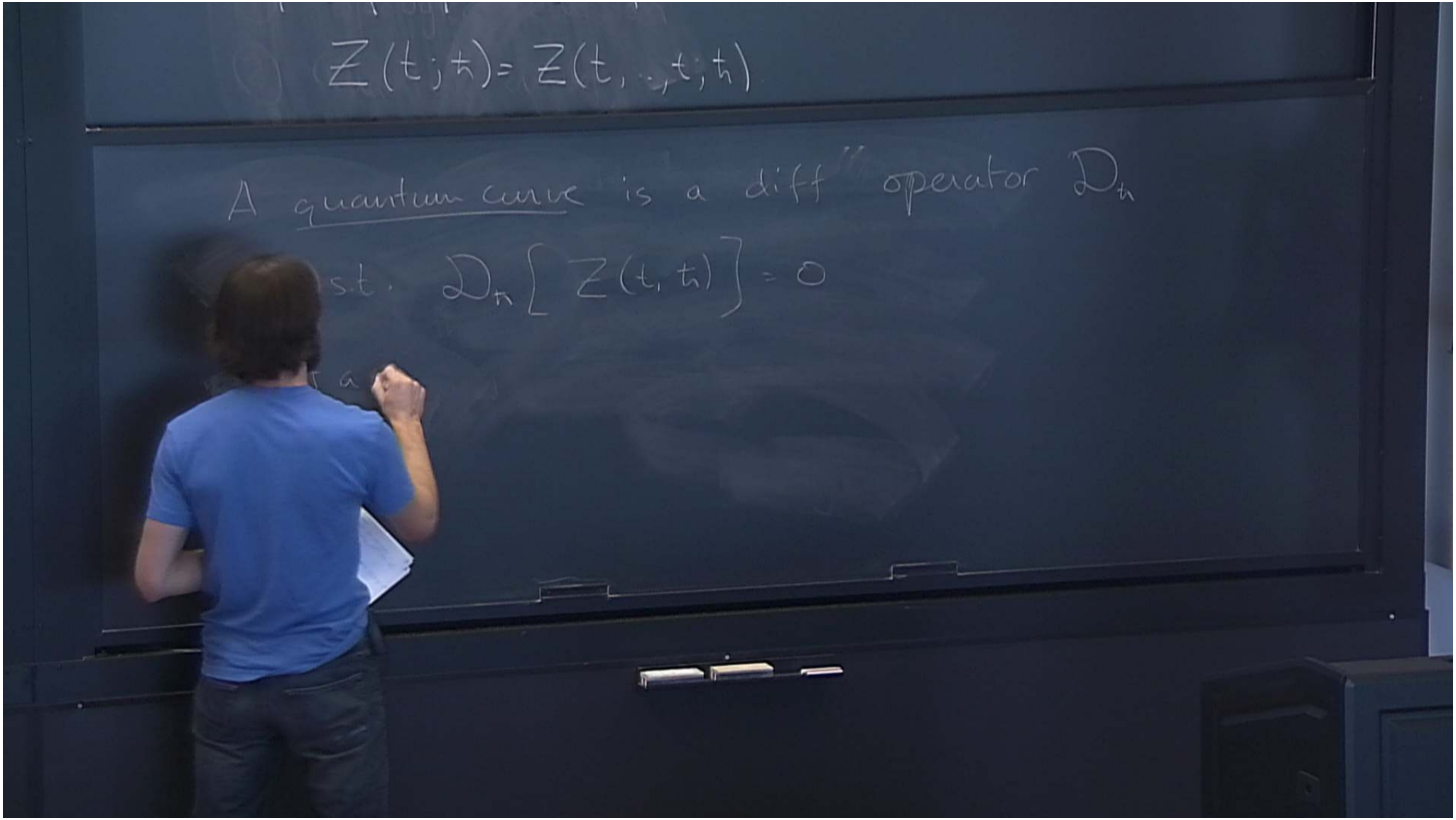
- Kilian
Quantization
① Topological recursion
② Quantum curves

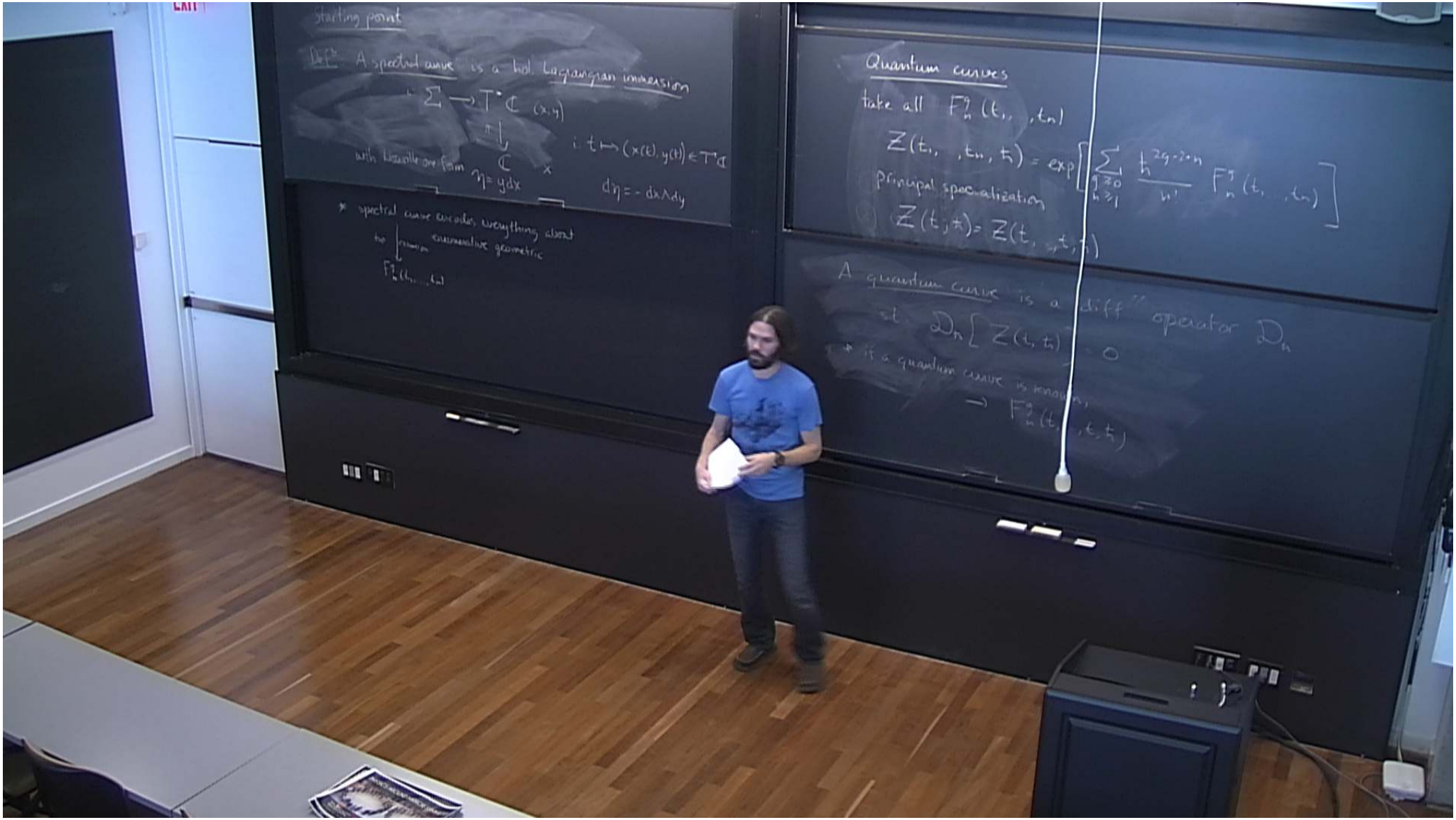

Quantum curves


Quantum curves
take all $F_n^g(t_1, \dots, t_n)$
$Z(t_1, \dots, t_n, \hbar) = \exp\left[\sum_{\substack{g \geq 0 \\ n \geq 1}} \hbar^{2g-2+n} \frac{}{n!} F_n^g(t_1, \dots, t_n) \right]$


Z(t;ħ) = Z(t, ..., t; ħ)
A quantum


Starting point
Def: A spectral curve is a hol. Lagrangian immersion
with Liouville one-form
Quantum curves
take all
principal specialization
A quantum curve is a
operator


$Z(t;\hbar) = Z(t_1, \ldots, t;\hbar)$
A quantum curve is a diff" operator $\mathcal{D}_\hbar$
s.t. $\mathcal{D}_\hbar \left[Z(t,\hbar) \right] = 0$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Quantum curves
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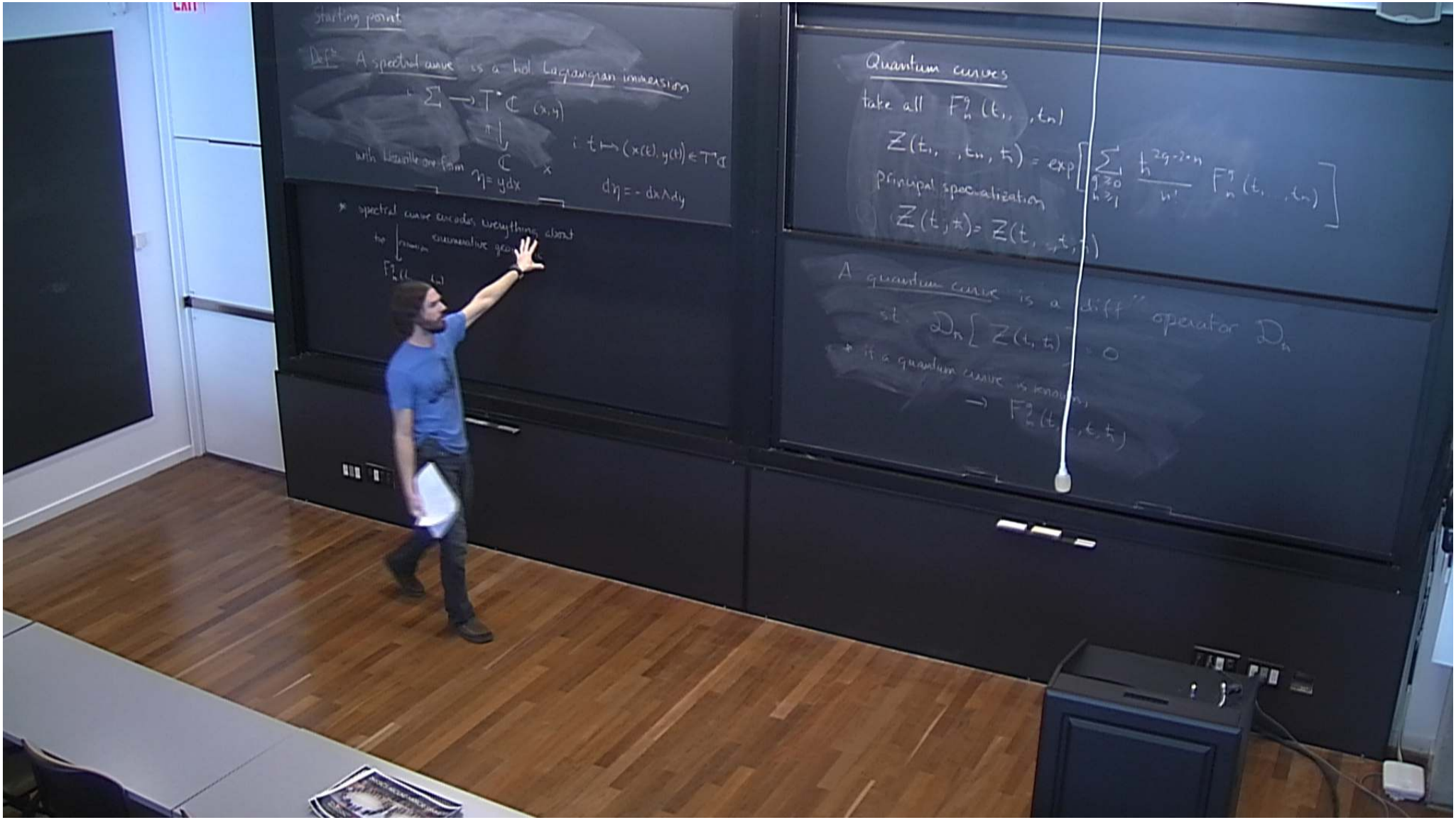
A quantum curve is a "diff" operator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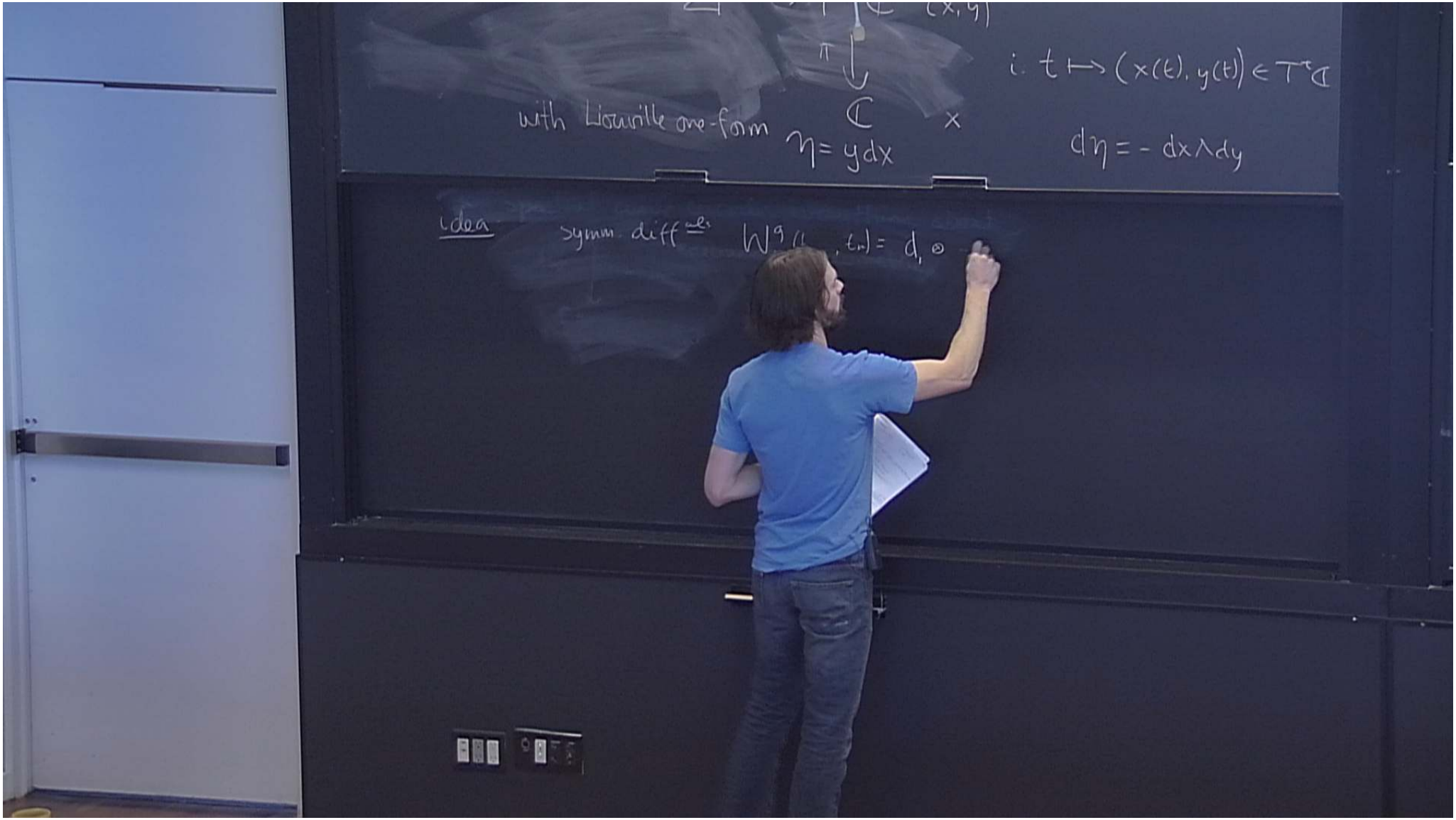
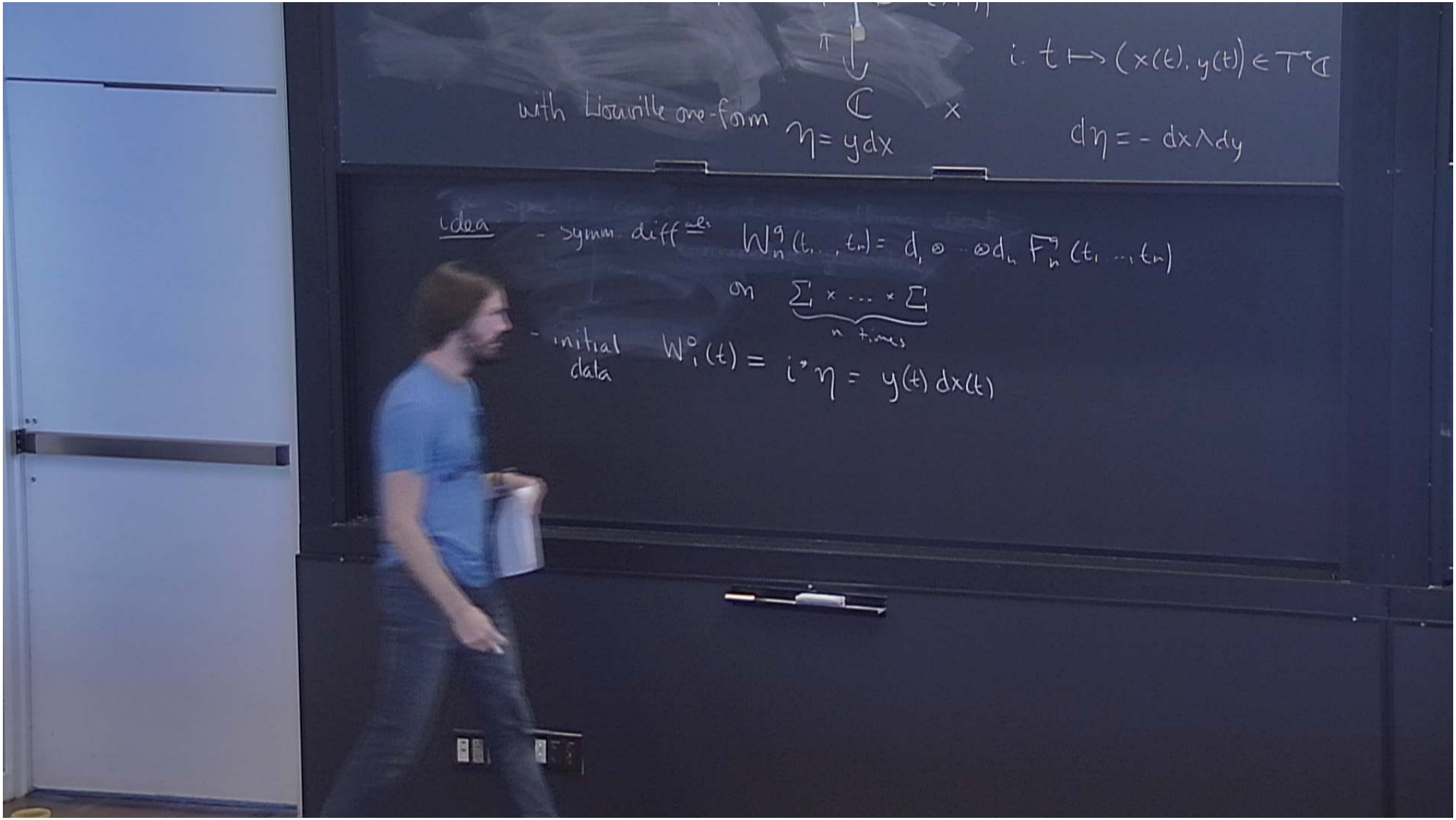
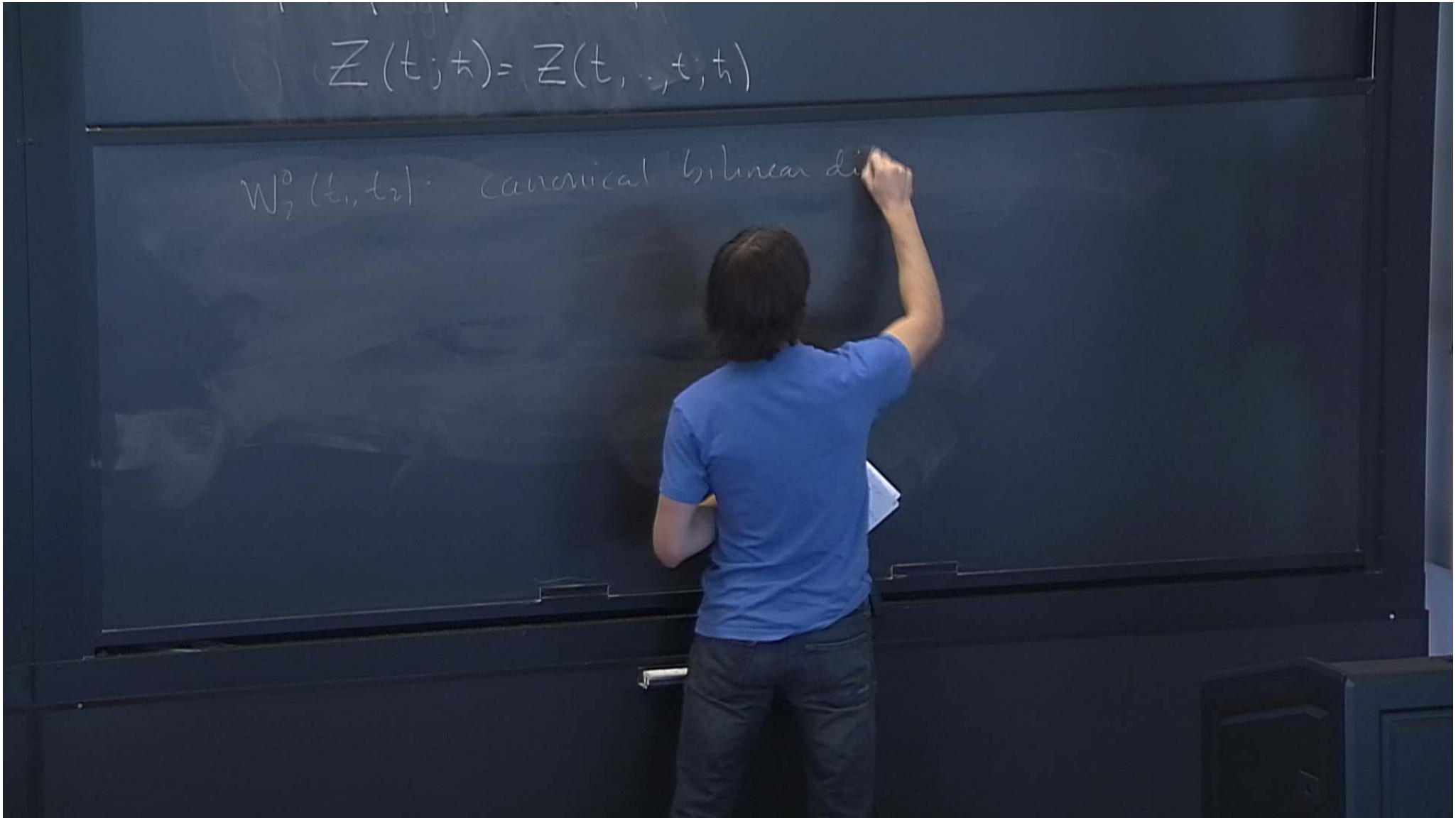
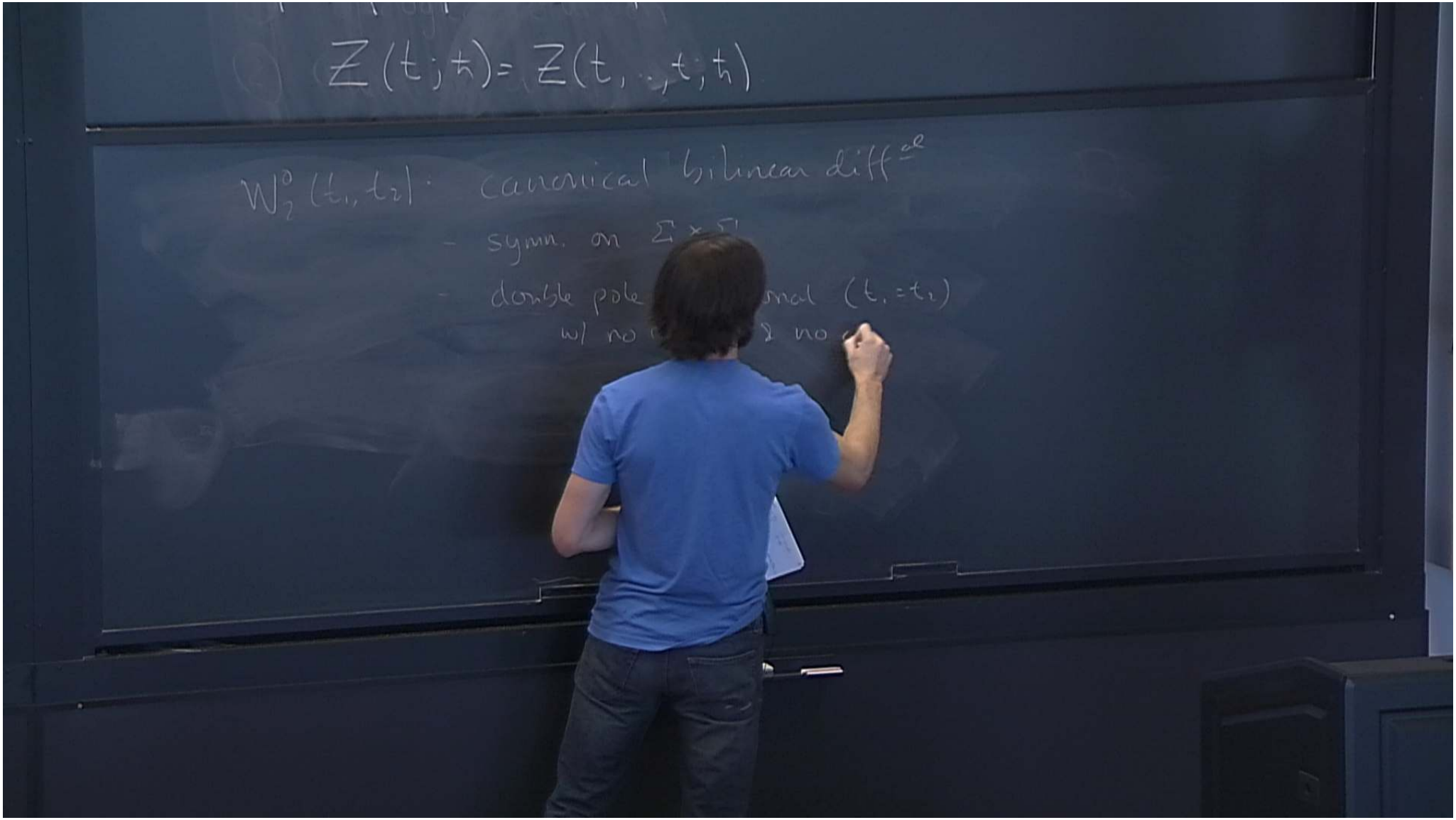
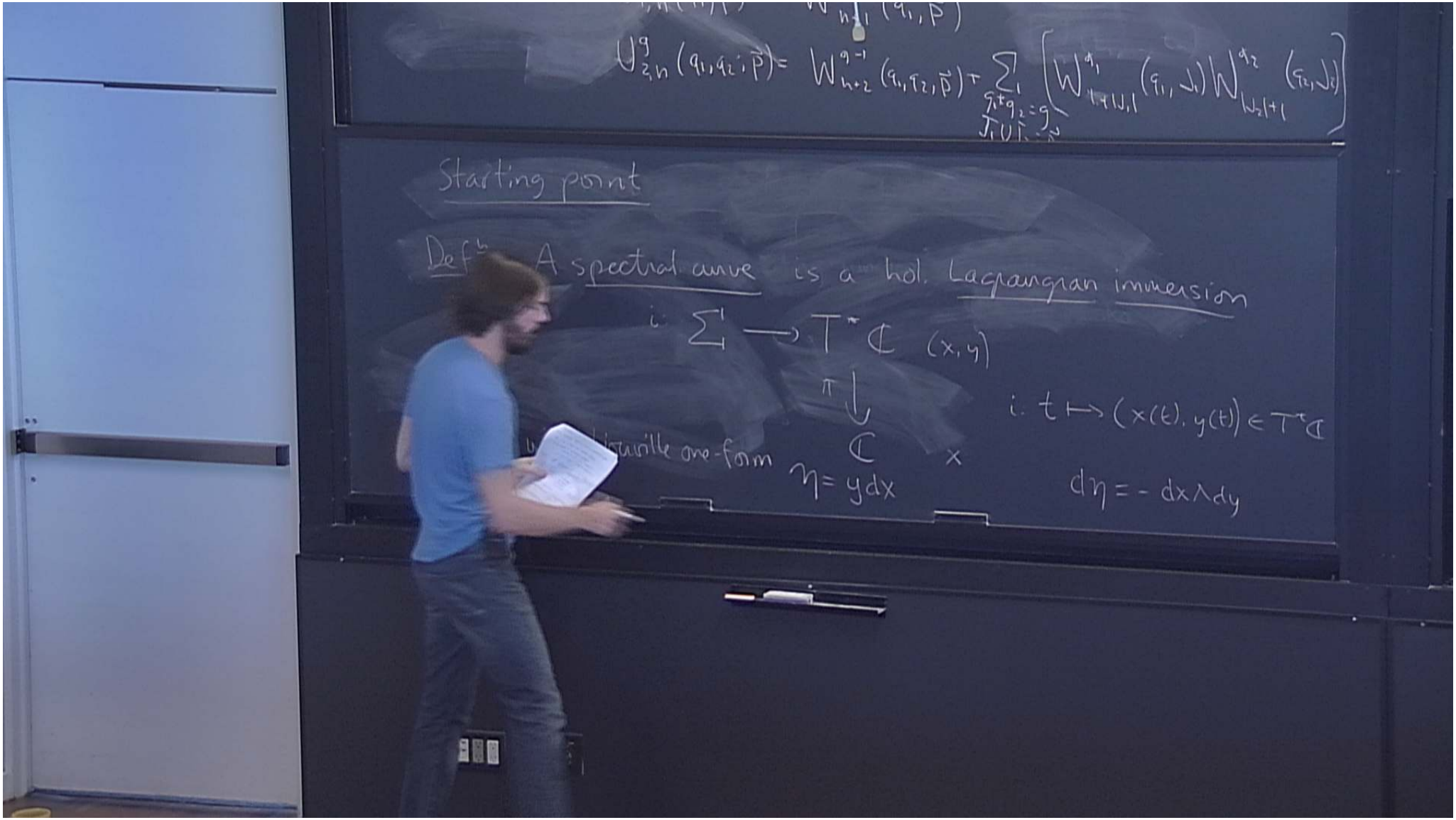
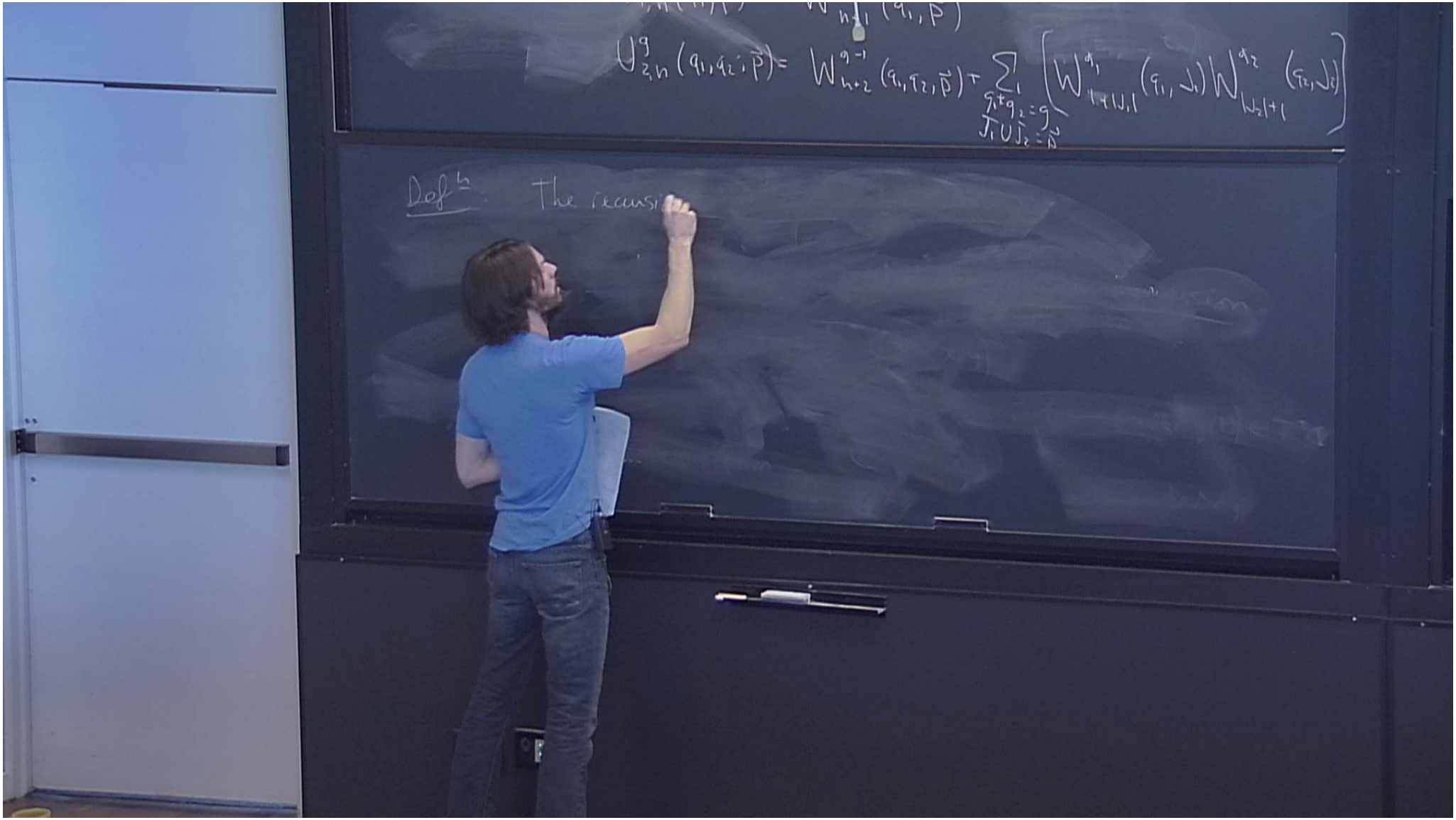

π
$\mathbb{C}$
$i: t \mapsto (x(t), y(t)) \in T^*\mathbb{C}$
with Liouville one-form $\eta = y\,dx$
$d\eta = -\,dx \wedge dy$
Idea
symm. diff^{es}

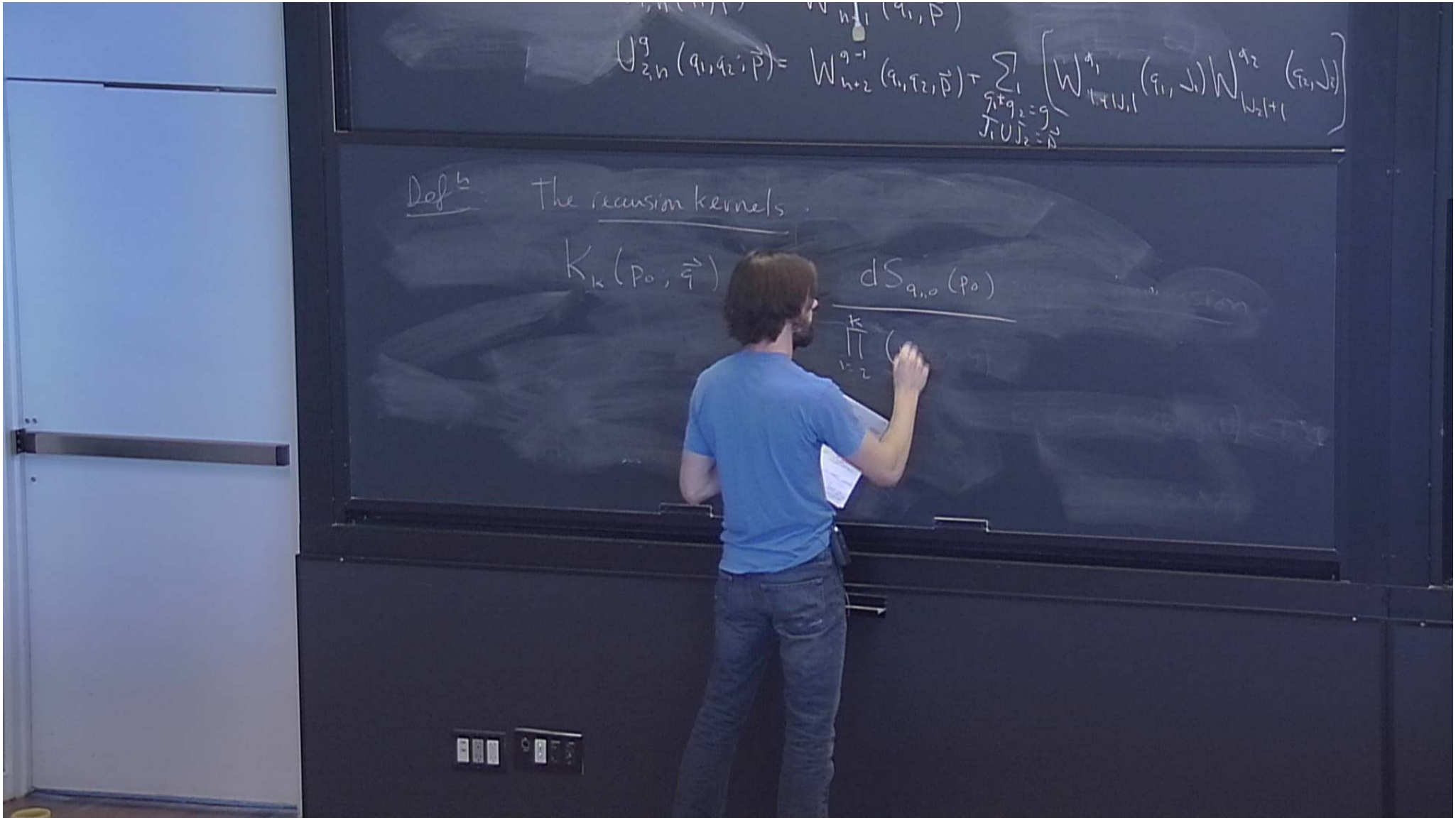

$i: t \mapsto (x(t), y(t)) \in T^*\mathbb{C}$
π
$\mathbb{C}$
$\times$
with Liouville one-form $\eta = y\,dx$
$d\eta = -\,dx \wedge dy$
Idea
- symm. diff^{ble} $W_n^q(t_1, \dots, t_n) = d_1 \otimes \cdots \otimes d_n F_n^q(t_1, \dots, t_n)$
on $\underbrace{\Sigma \times \cdots \times \Sigma}_{n \text{ times}}$
- initial data $W_1^\circ(t) = i^*\eta = y(t)\,dx(t)$

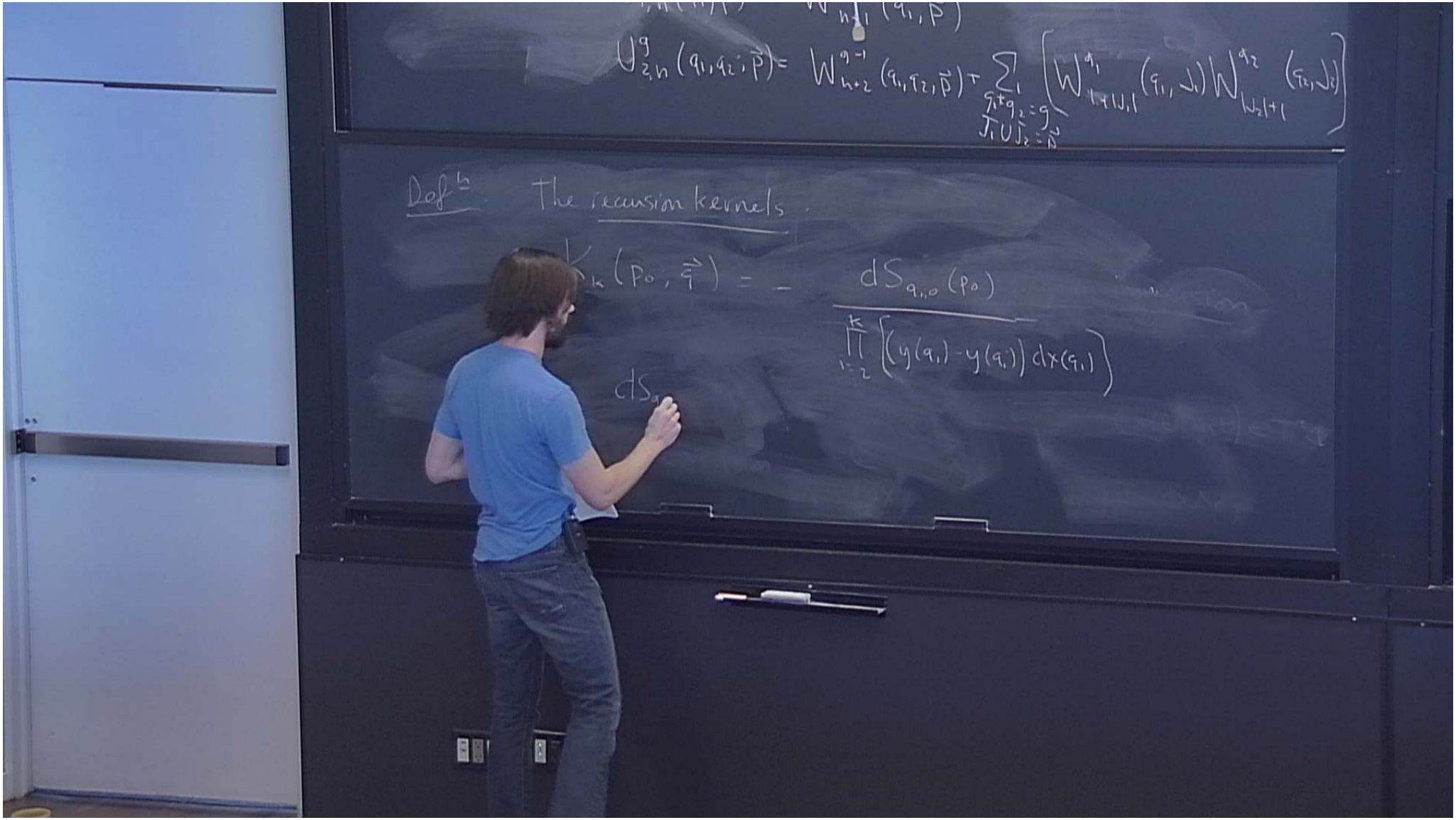

$Z(t;\hbar) = Z(t_1, \ldots, t;\hbar)$
$W_2^0(t_1, t_2)$ · canonical bilinear

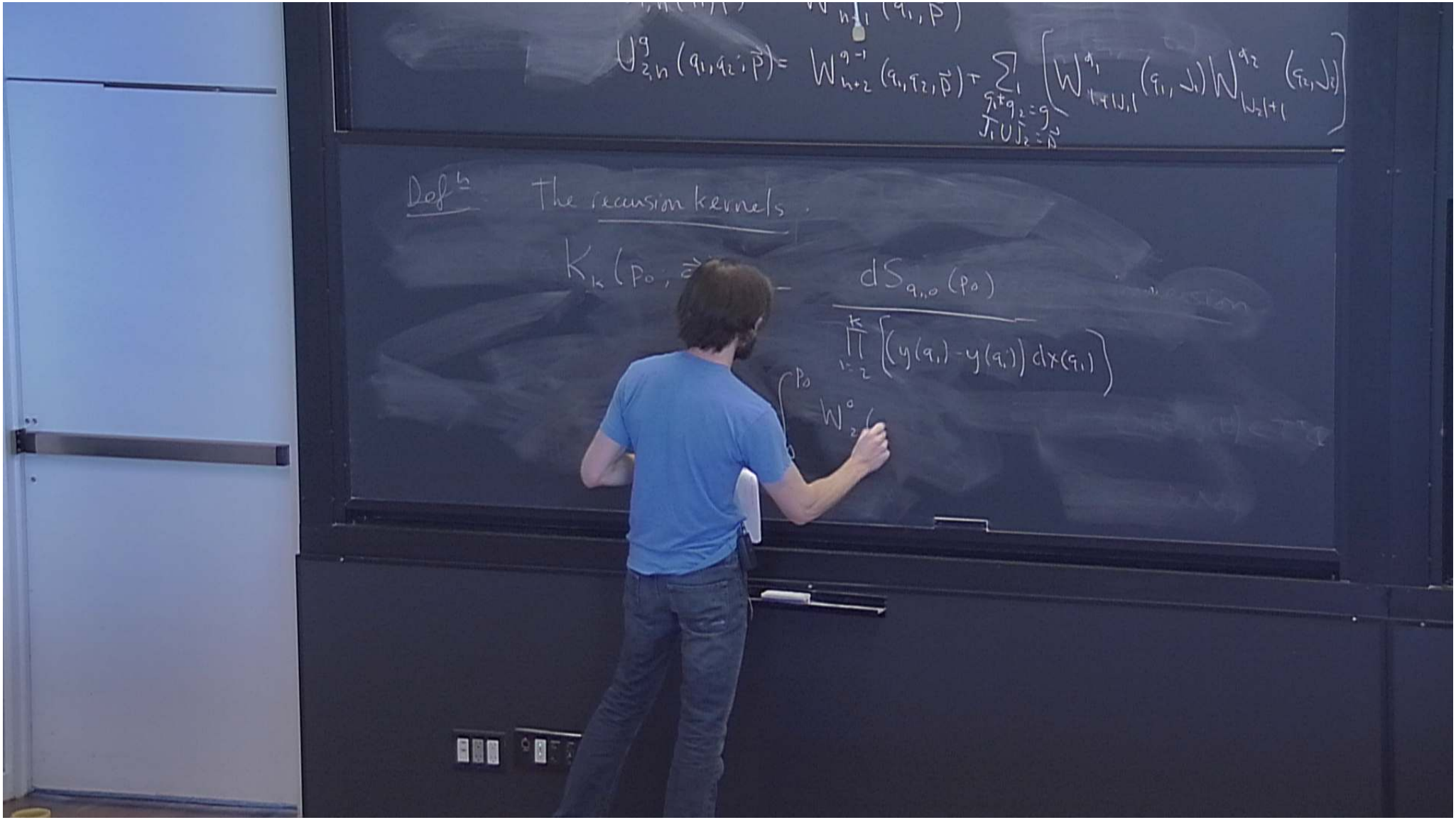

$Z(t;\hbar) = Z(t_1, \ldots, t; \hbar)$
$W_2^0(t_1, t_2)$: canonical bilinear diff^{n}
- symm. on $\Sigma \times \Sigma$
- double pole
w/ no
no

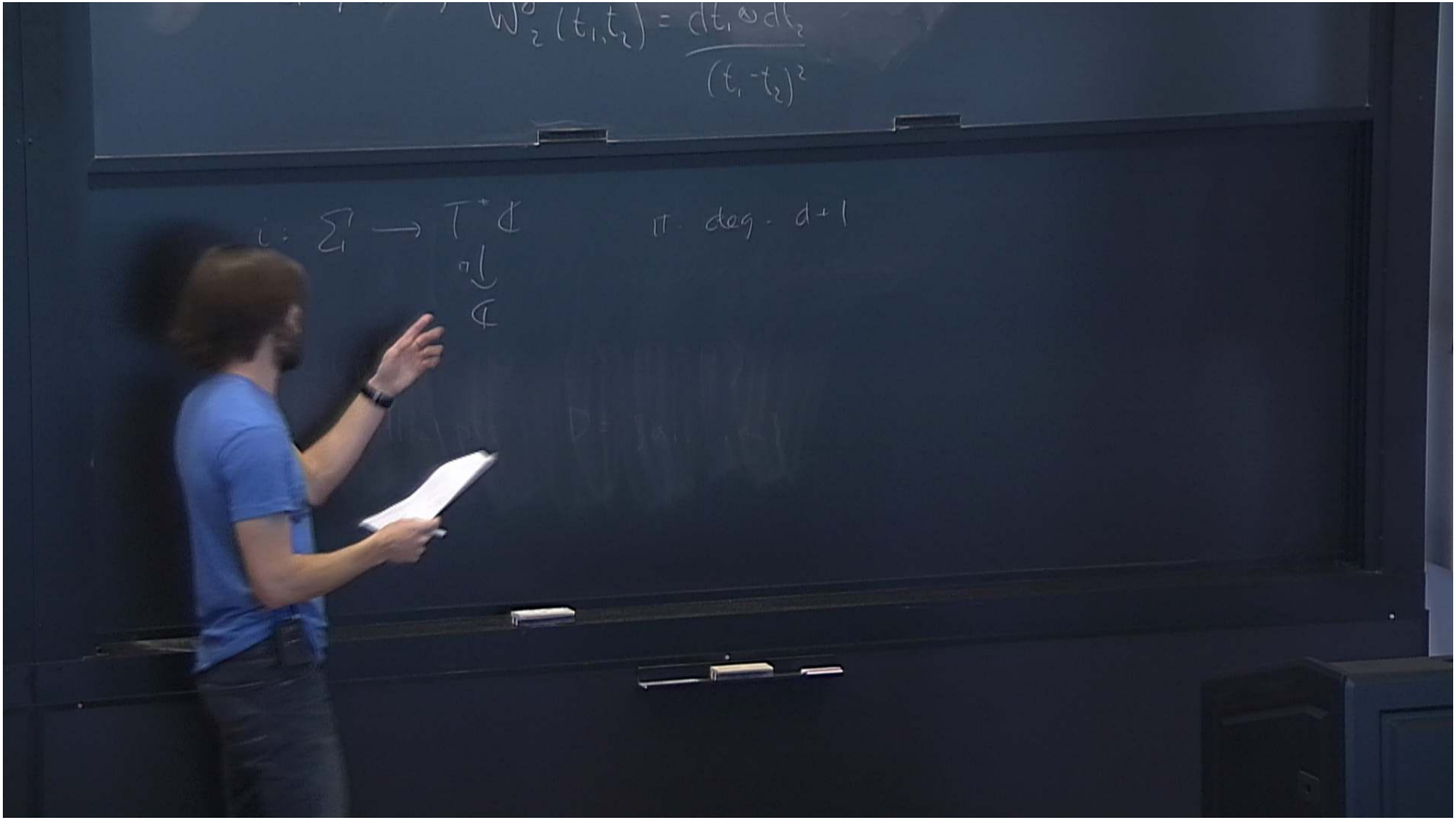

Starting point
Def^n A spectral curve is a hol. Lagrangian immersion
i: Σ' → T*ℂ (x,y)
π
ℂ x
i: t ↦ (x(t), y(t)) ∈ T*ℂ
η = ydx
dη = -dx∧dy

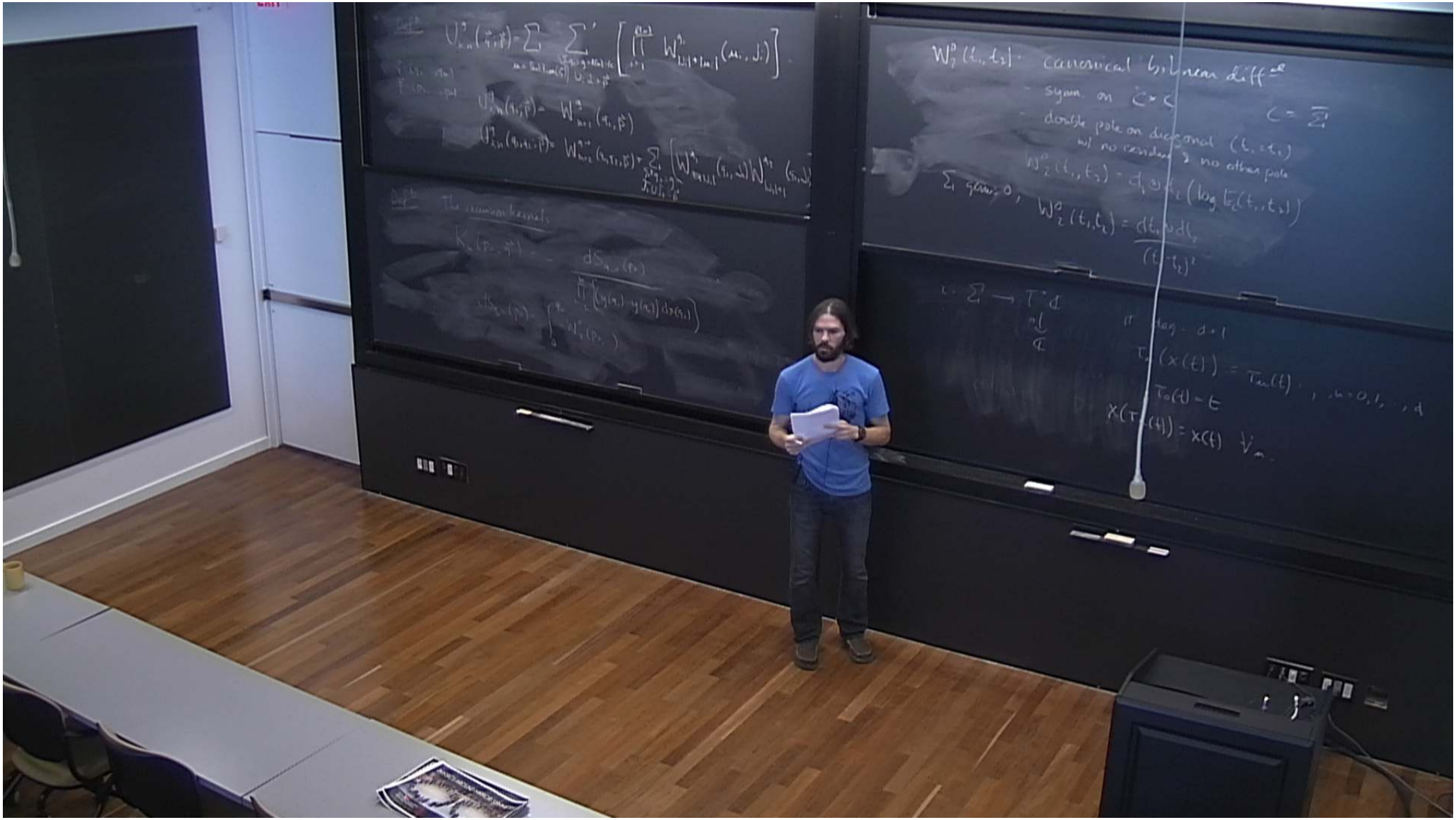

Def^n

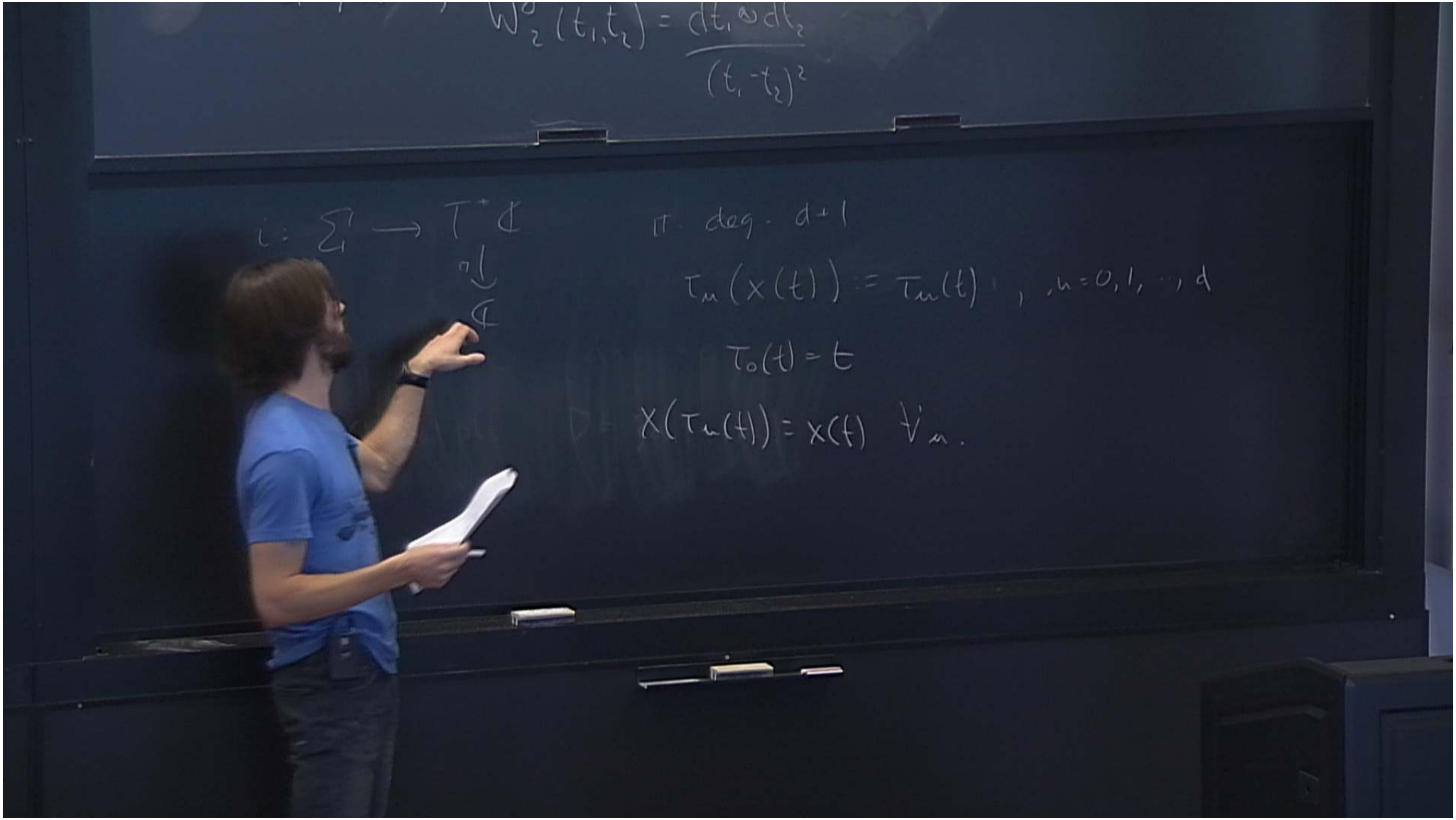

Def
The recursion kernels.

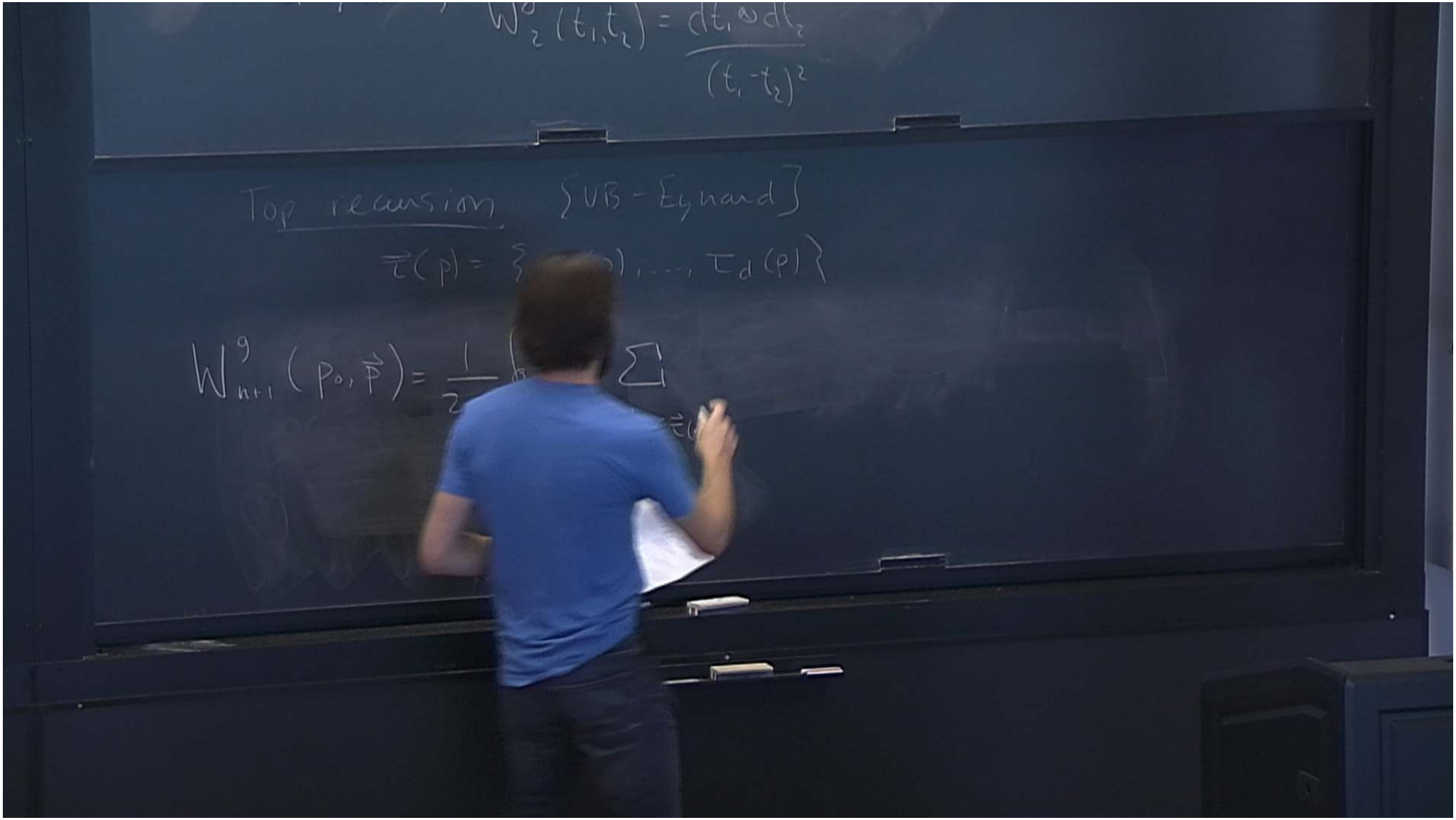

Def^n
The recursion kernels

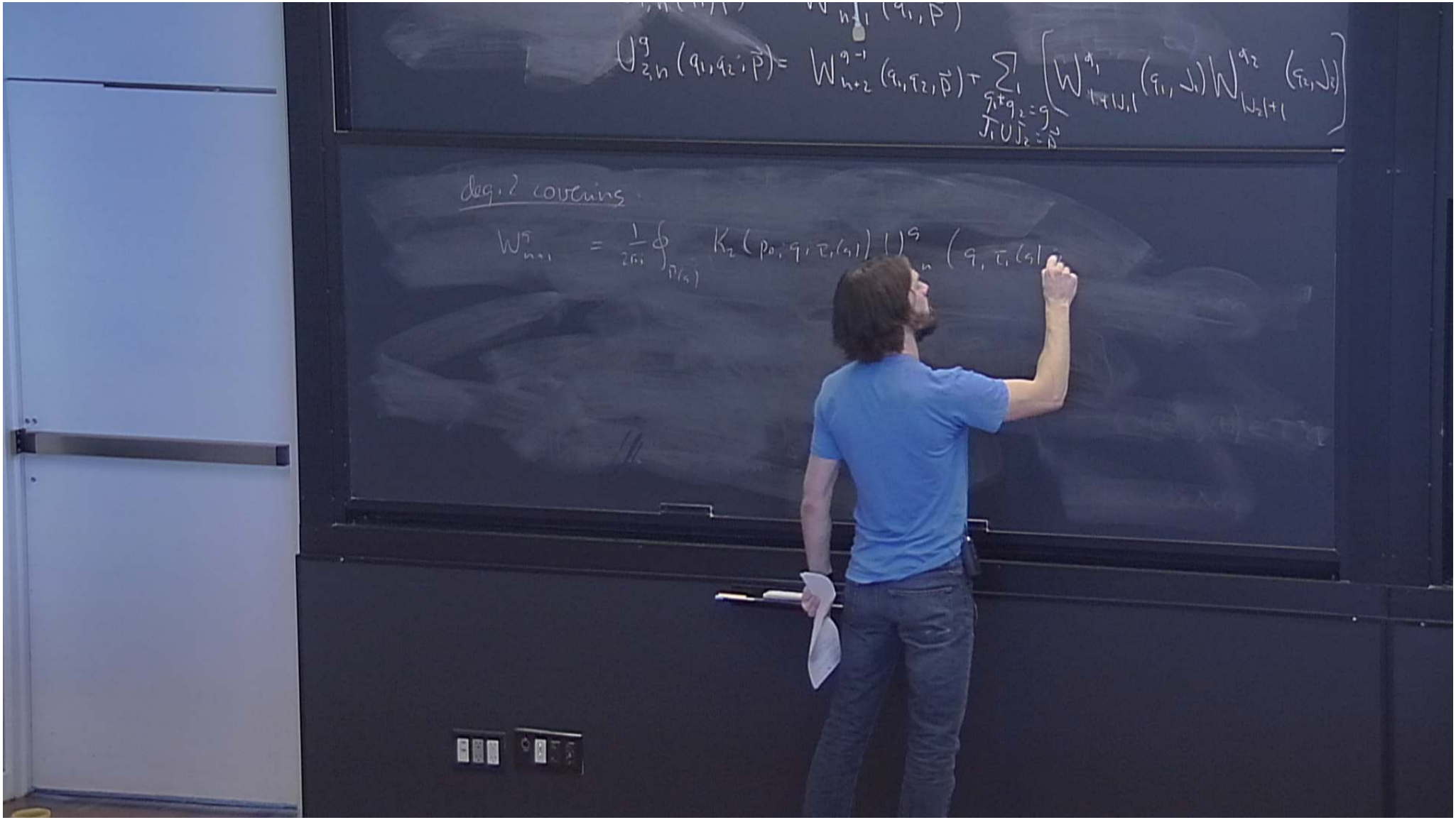

Def^n
The recursion kernels.

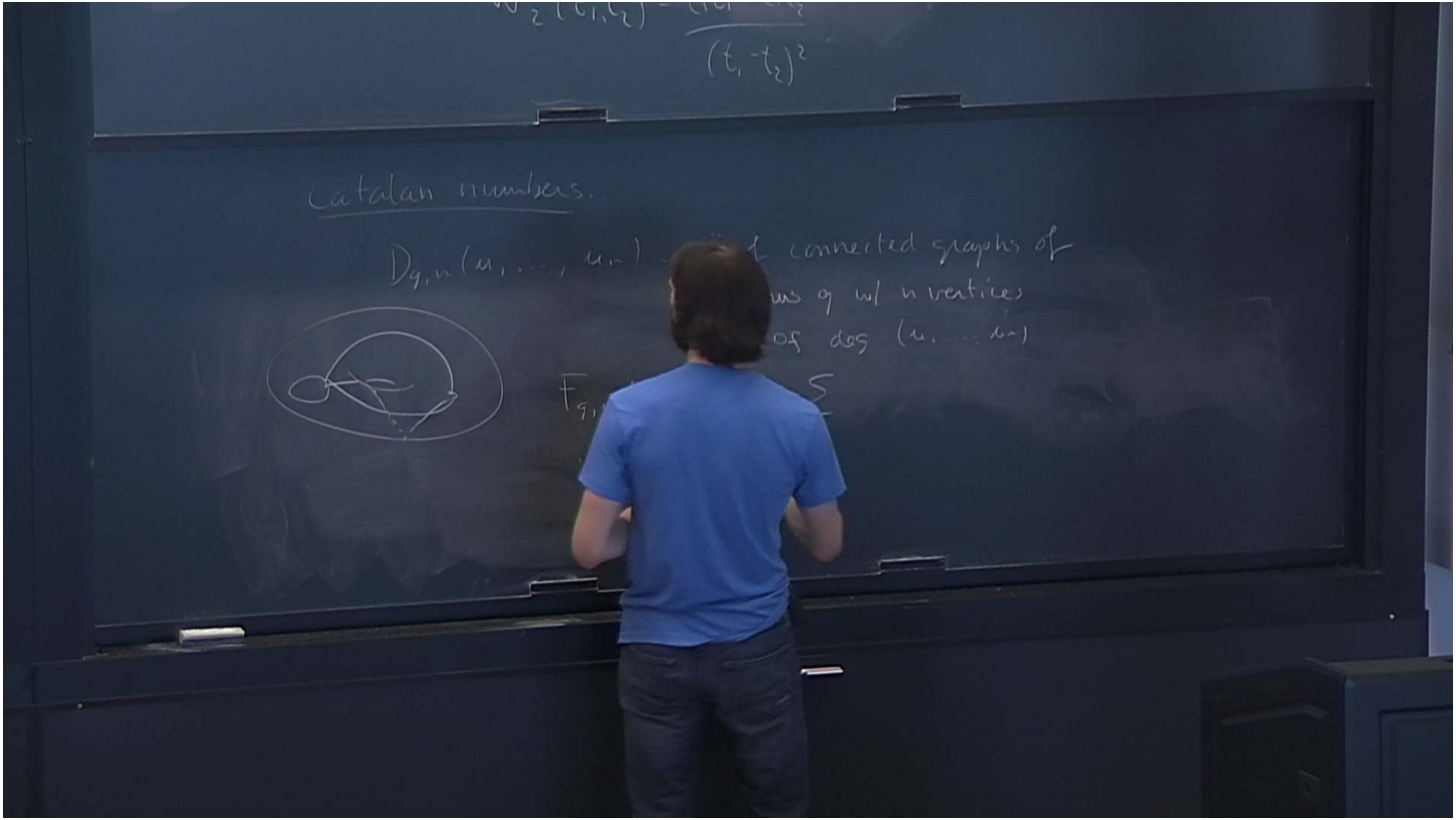

$W_2^0(t_1,t_2) = \frac{dt_1 \otimes dt_2}{(t_1-t_2)^2}$
$i: \Sigma_1 \to T^*\mathcal{C}$
$\pi \cdot \deg \cdot d+1$

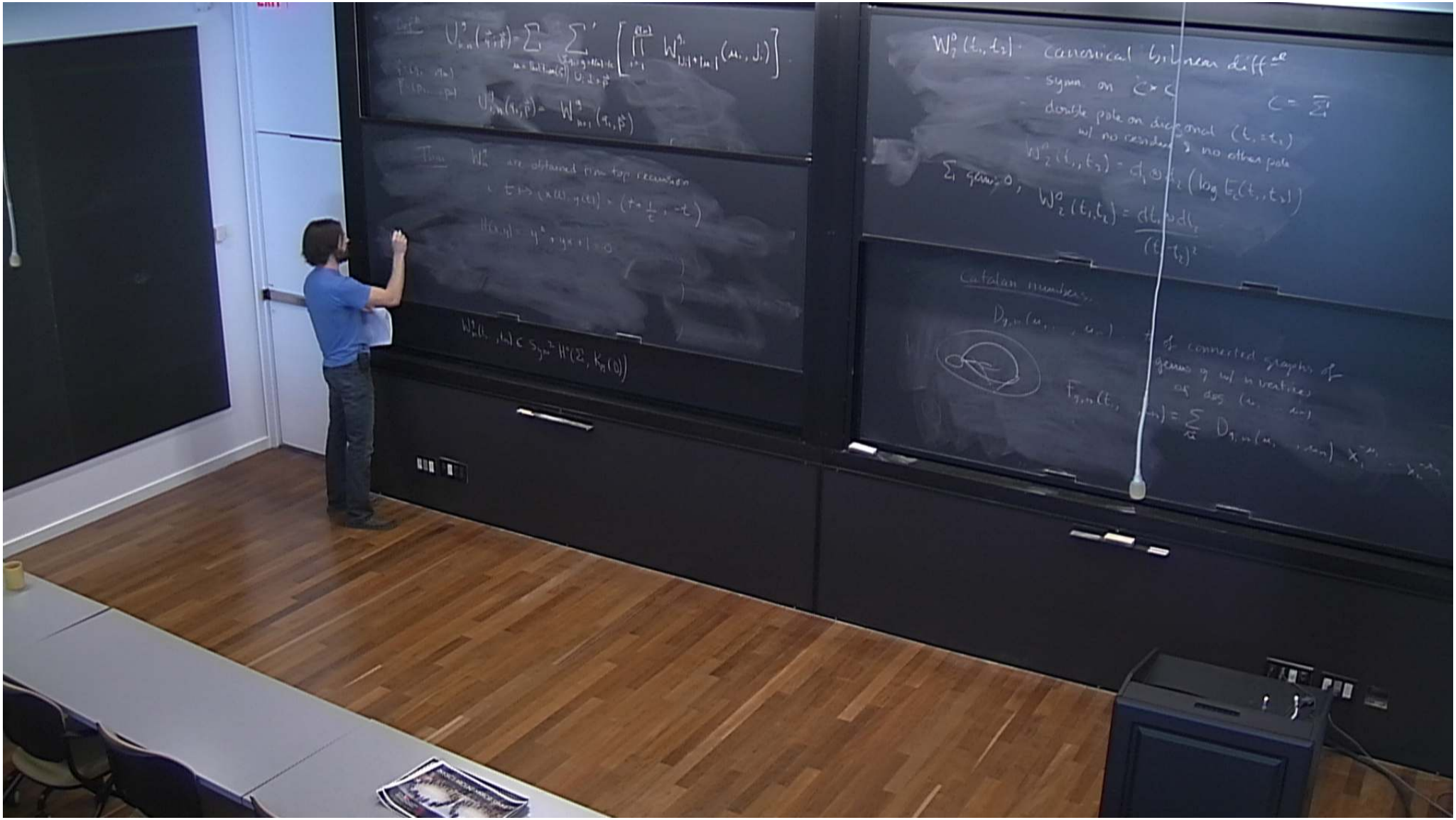


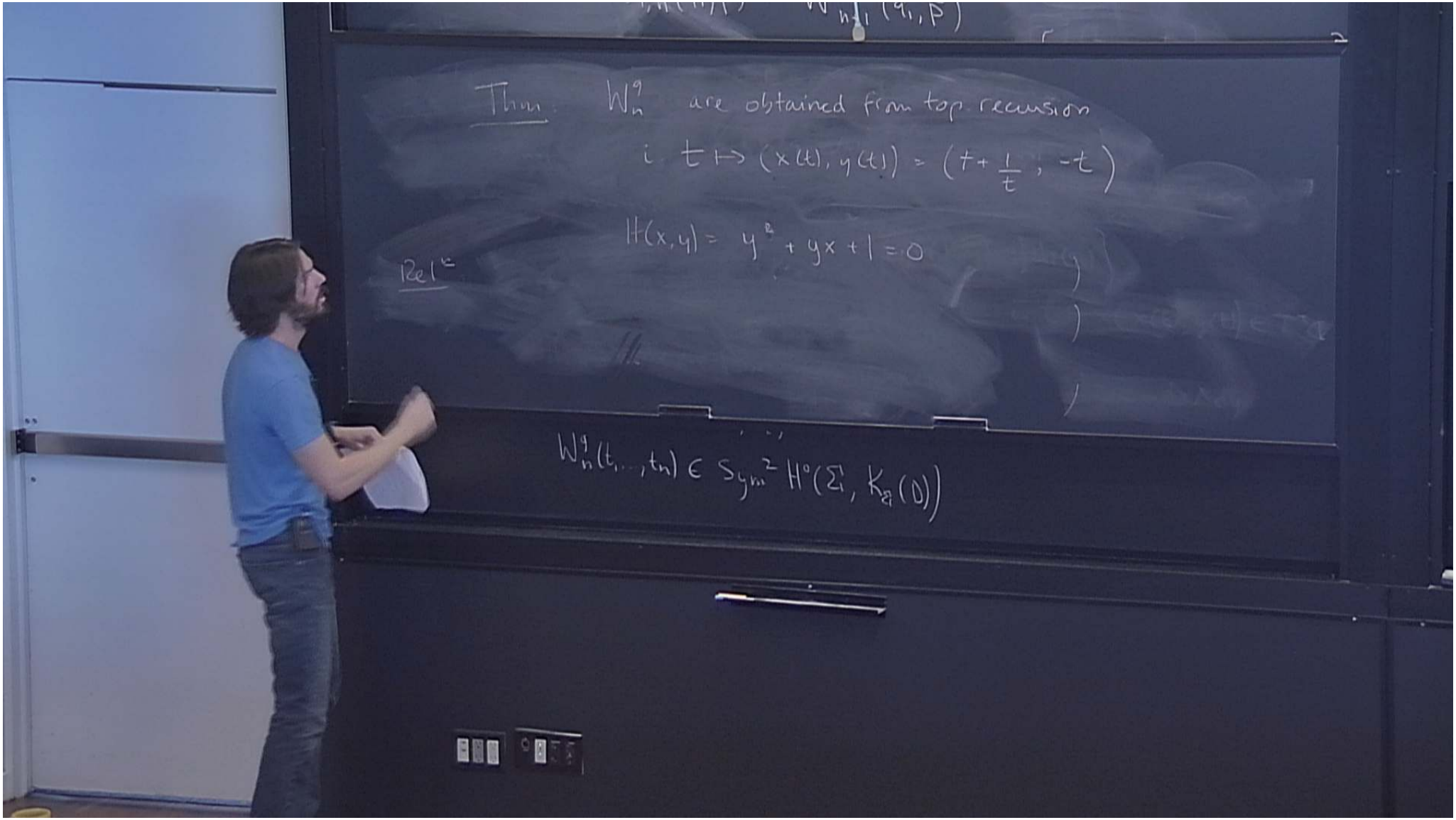

$W_2^0(t_1,t_2) = \frac{dt_1 \otimes dt_2}{(t_1-t_2)^2}$
$i : \Sigma' \to T^* \mathcal{C}$
π
deg $d+1$
$\tau_n(x(t)) = \tau_n(t)$, $n = 0, 1, \dots, d$
$\tau_0(t) = t$
$X(\tau_n(t)) = x(t) \quad \forall n.$

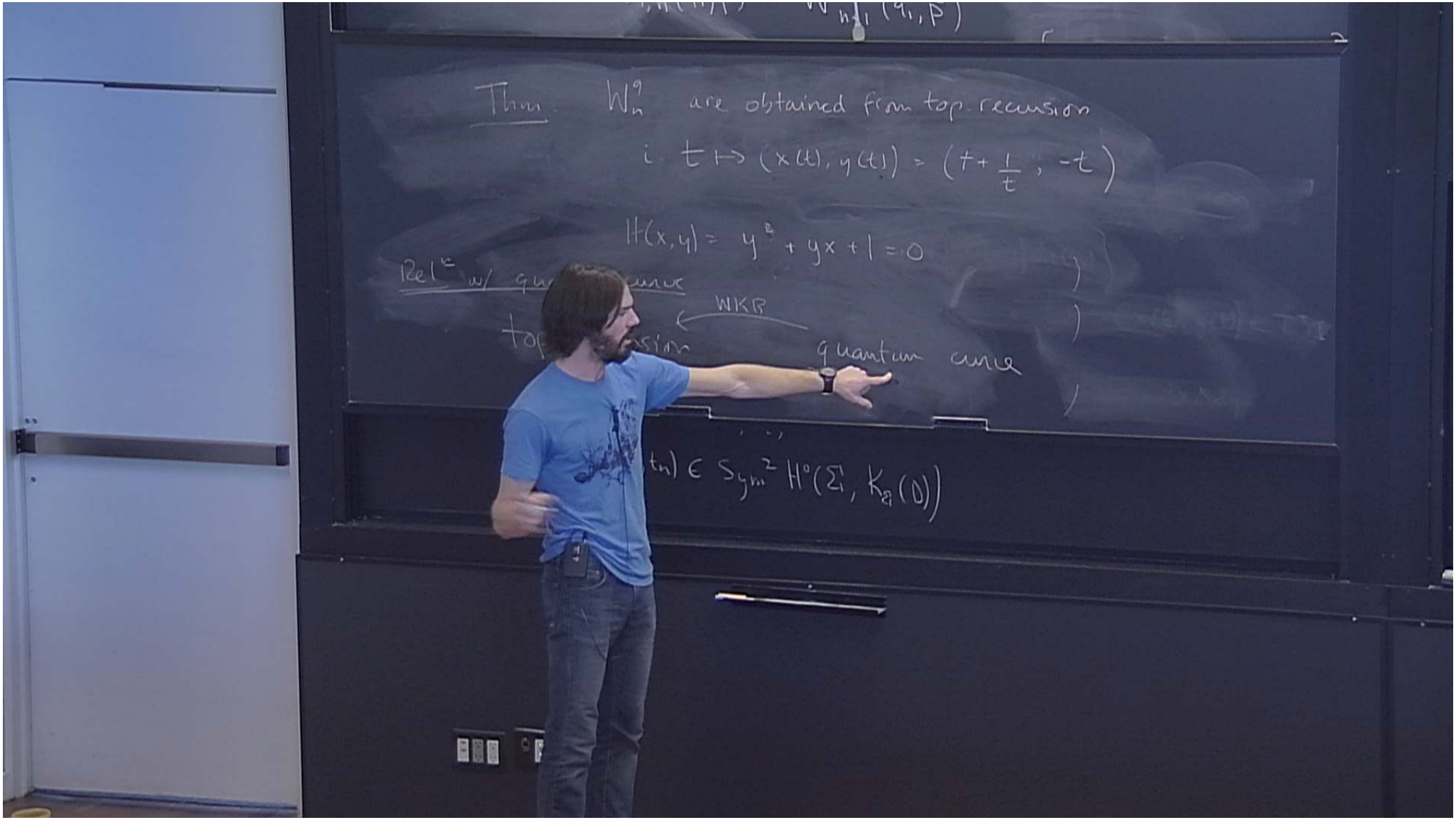

$W_2^0(t_1,t_2) = \frac{dt_1 \otimes dt_2}{(t_1-t_2)^2}$
Top recursion {VB - Eynard}
$\vec{\tau}(p) = \{\tau_1(p), \ldots, \tau_d(p)\}$
$W_{n+1}^g(p_0,\vec{p}) = \frac{1}{2}$
$\sum$

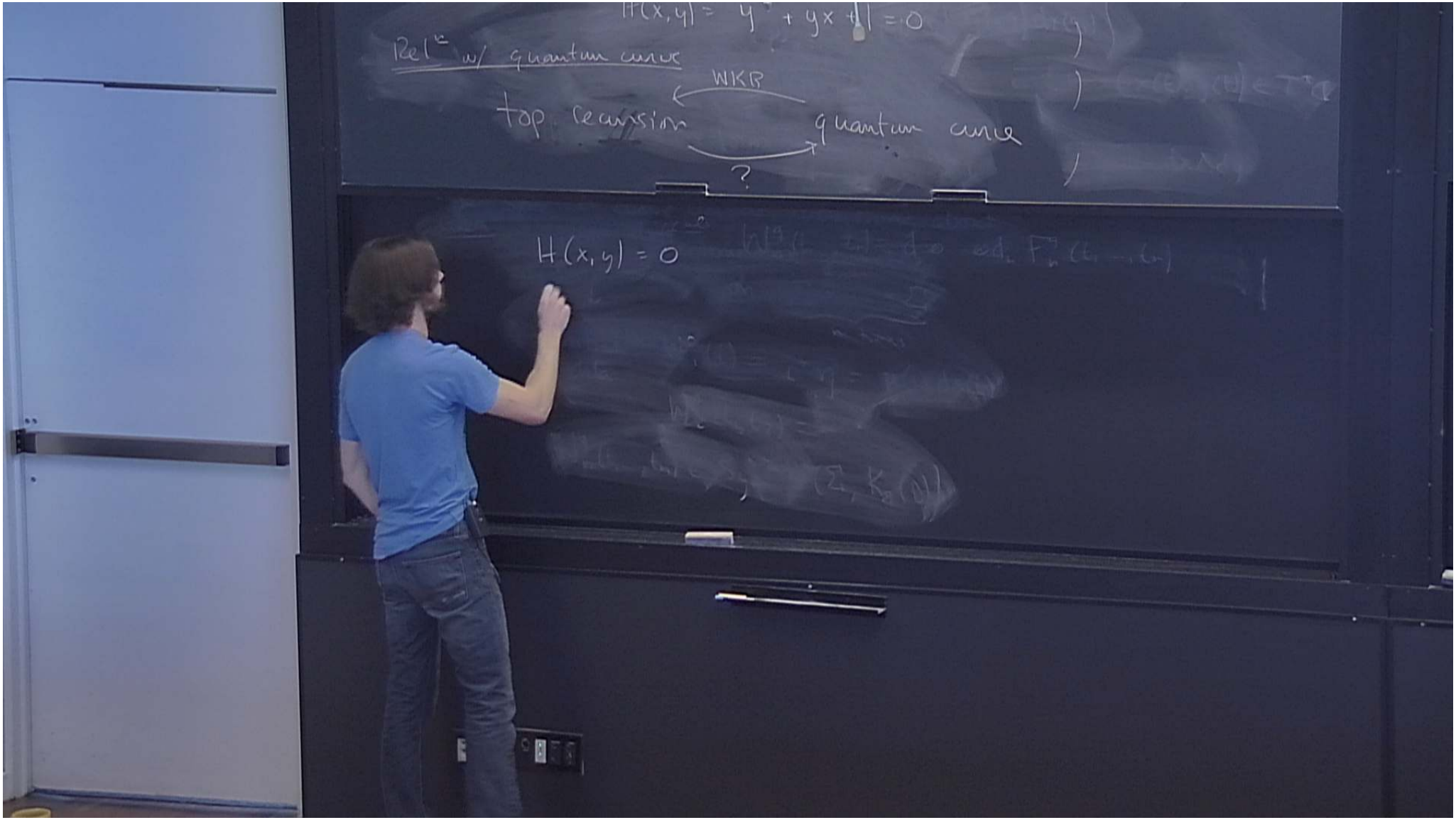

deg. 2 cousins

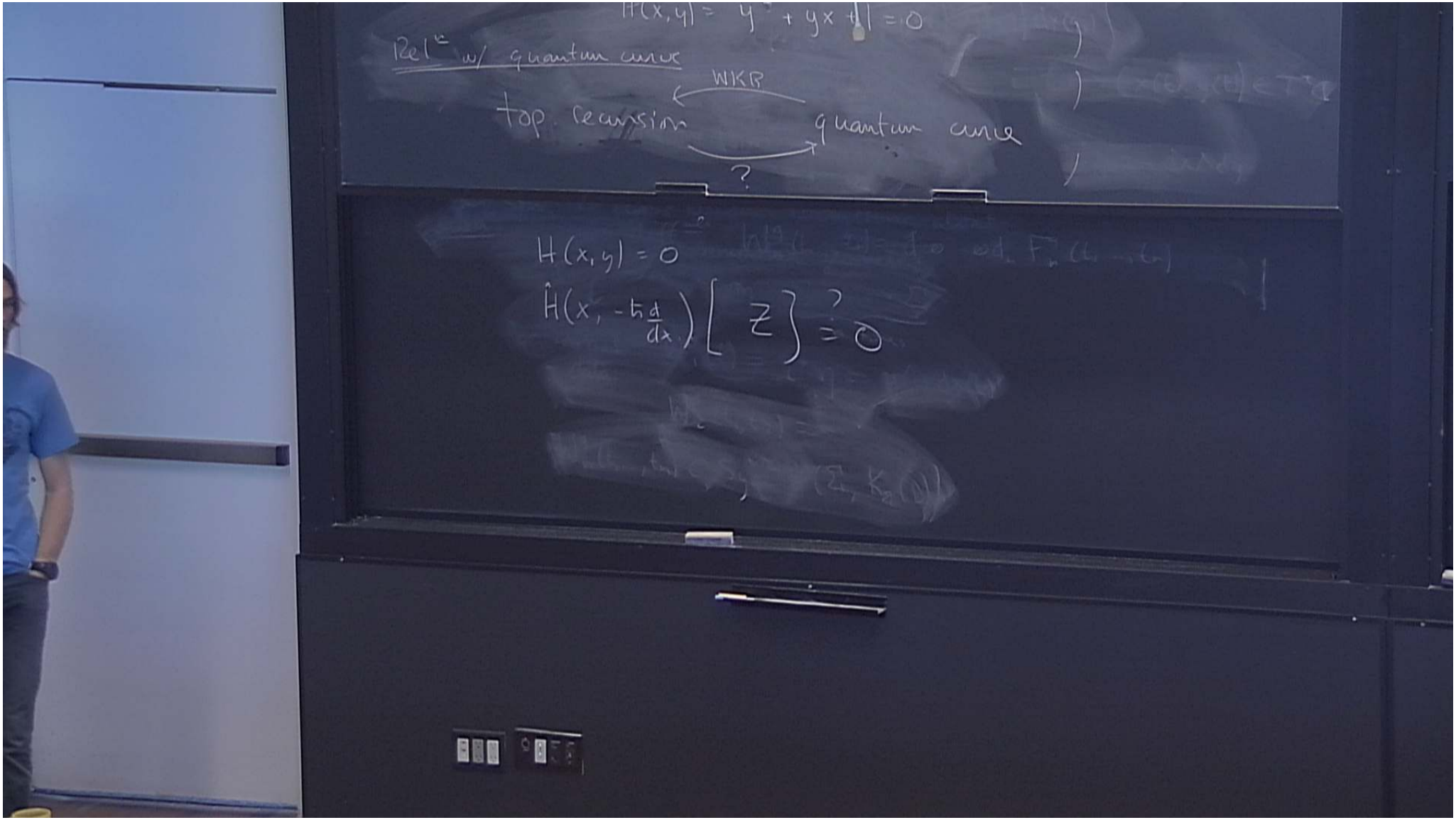

$(t_1 - t_2)^2$
Catalan numbers.
$D_{g,n}(u_1, \ldots, u_n)$
connected graphs of
of deg $(u_1 \ldots u_n)$

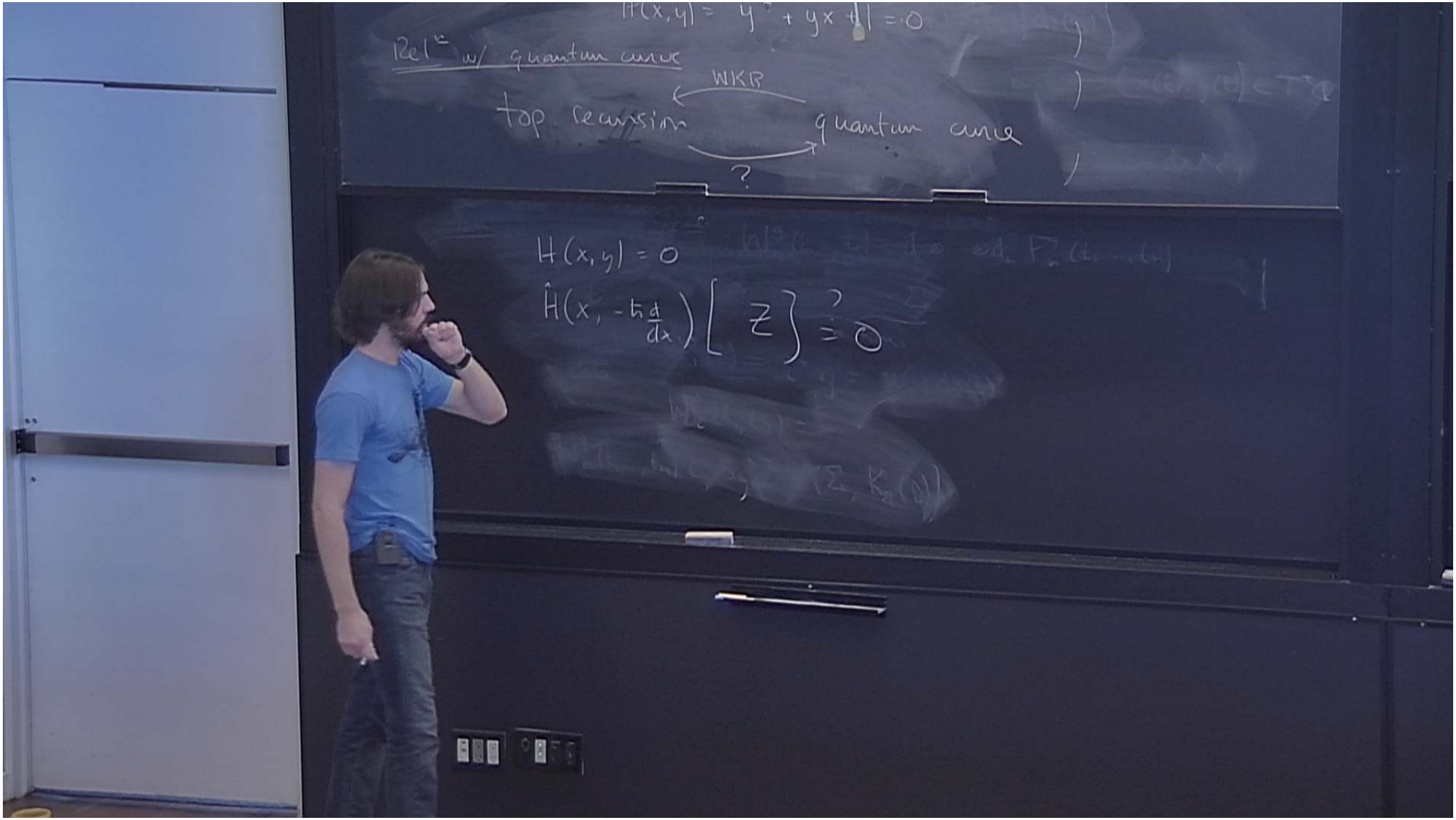

Thm
canonical
Catalan numbers

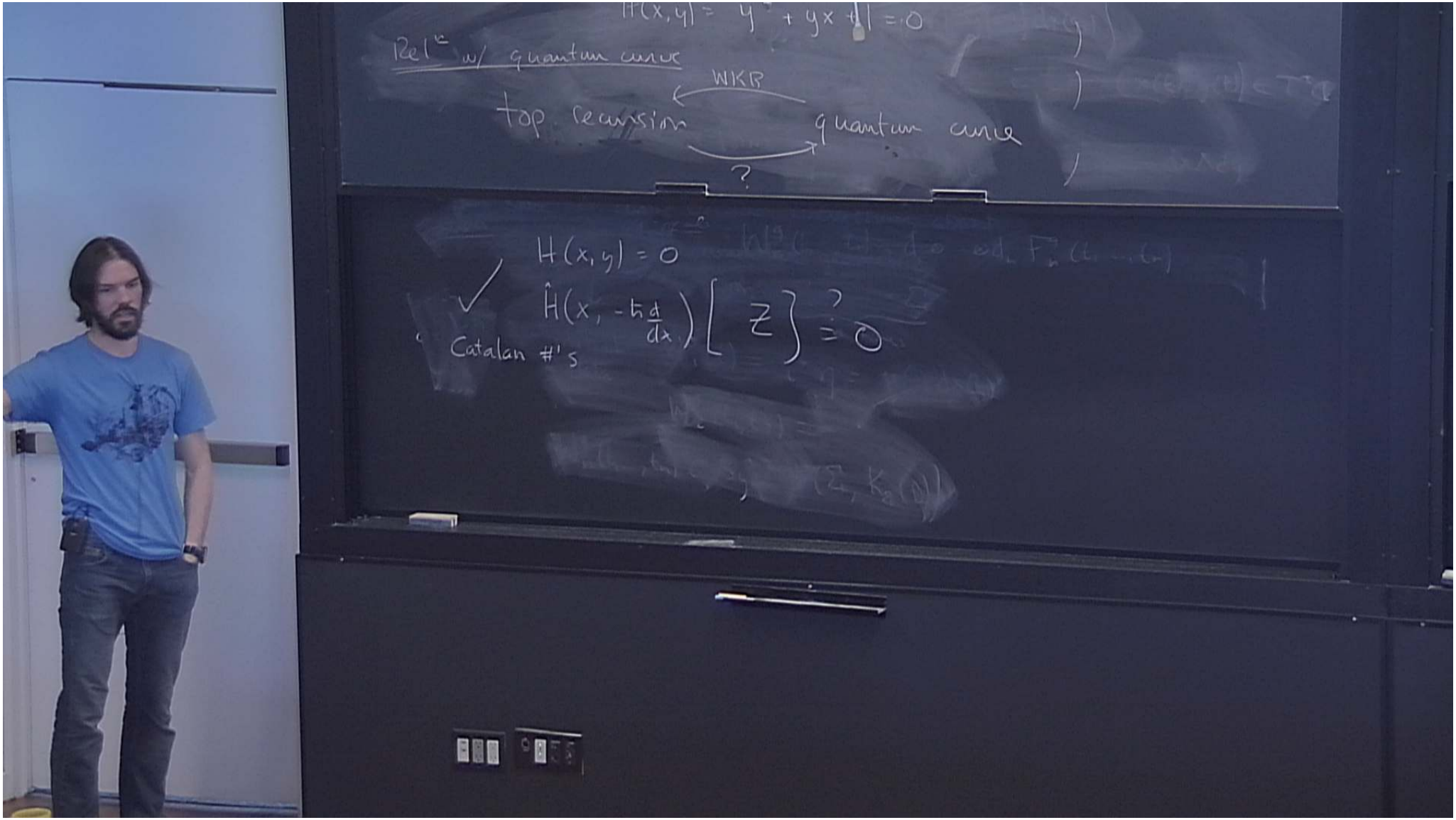

Thm: W_n^g are obtained from top recursion
$t \mapsto (x(t), y(t)) = (t + \frac{1}{t}, -t)$
$H(x,y) = y^2 + yx + 1 = 0$
Rel
$W_n^g(t_1, \ldots, t_n) \in Sym^2 H^0(\Sigma, K_\Sigma(0))$

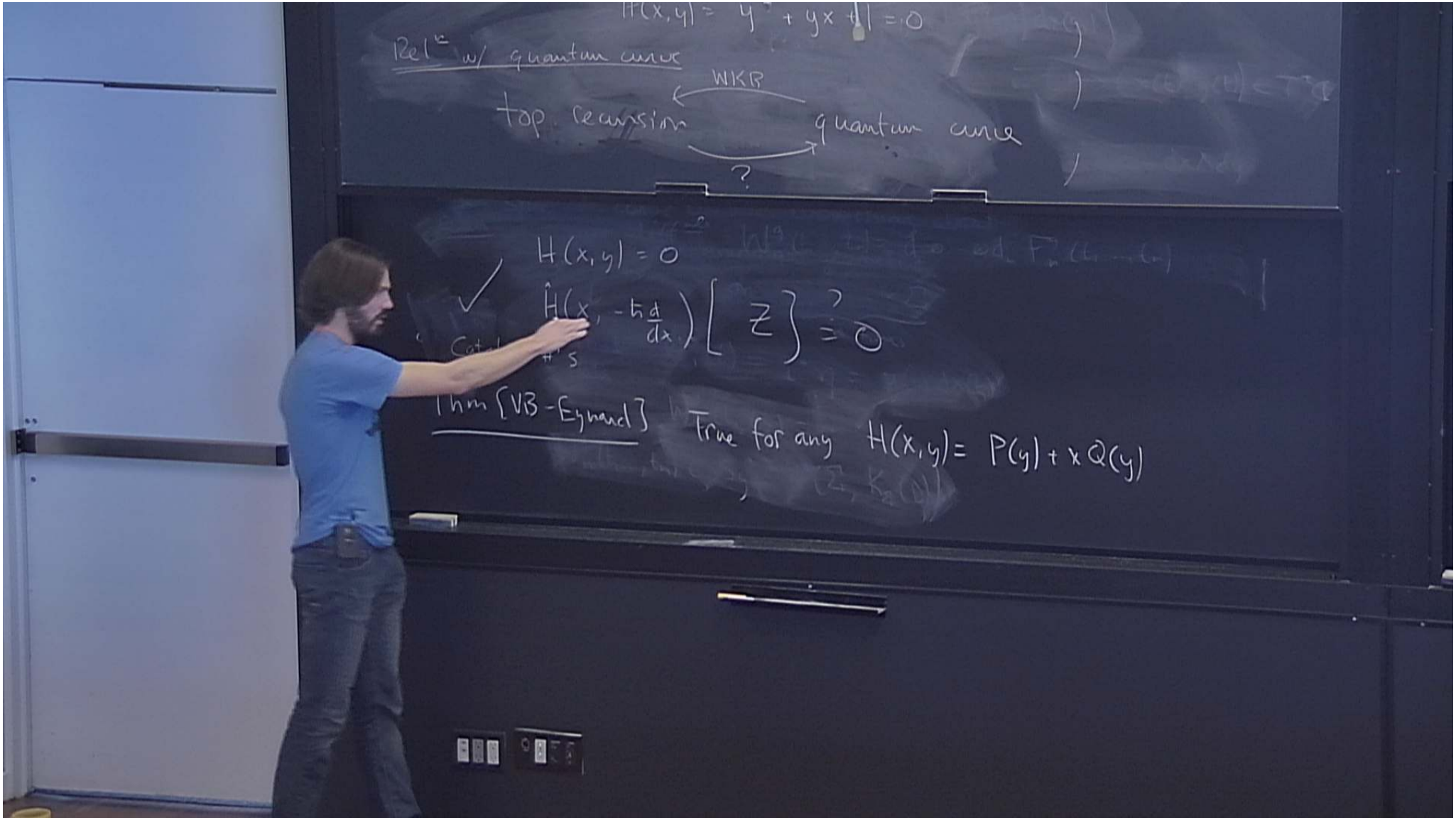

Thm: W_n^g are obtained from top. recursion
i. $t \mapsto (x(t), y(t)) = (t + \frac{1}{t}, -t)$
$H(x,y) = y^2 + yx + 1 = 0$
WKB
quantum curve
$\in Sym^2 H^0(\Sigma, K_\Sigma(0))$

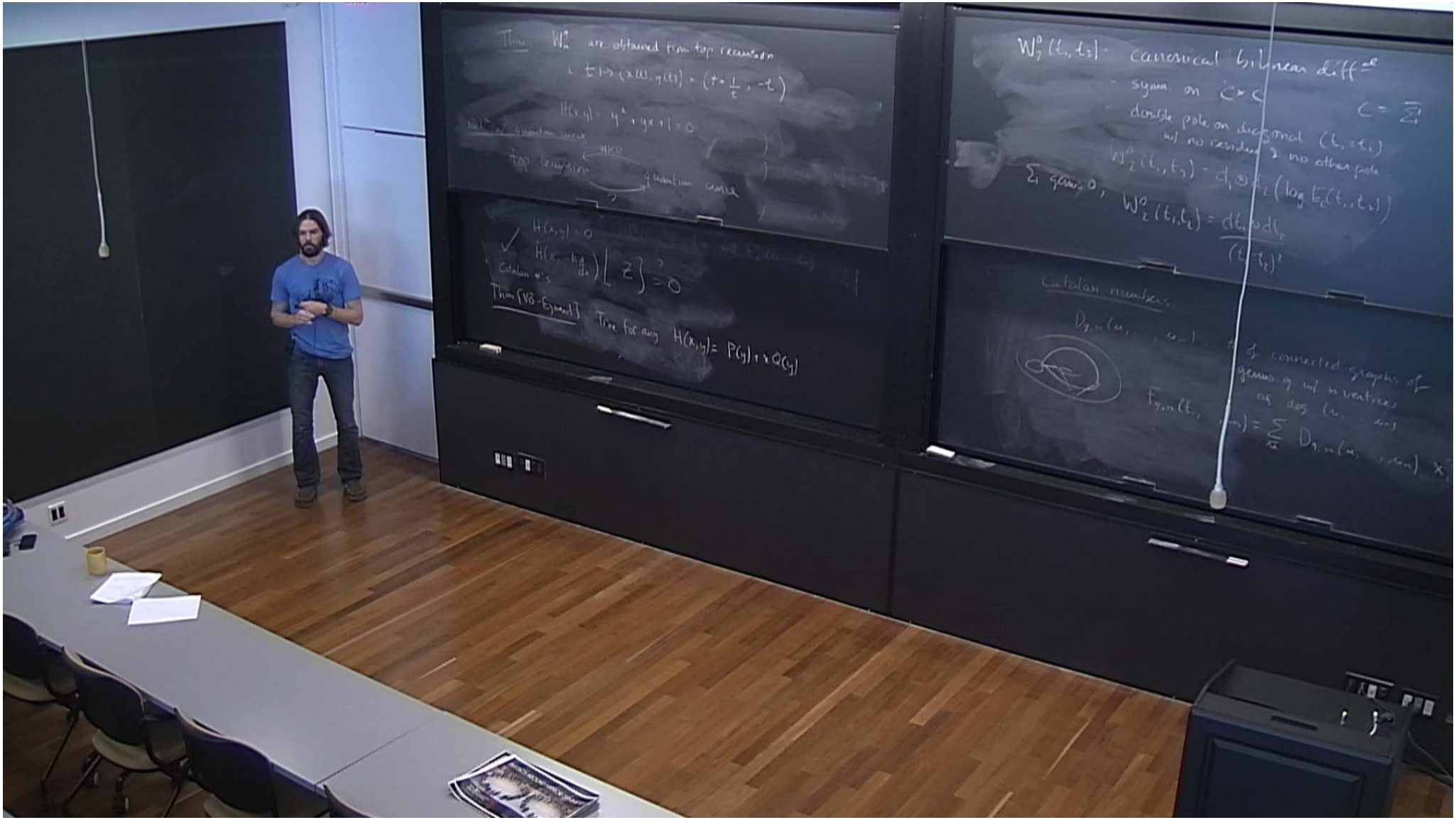

H(x,y) = y² + yx + 1 = 0
Rel^n w/ quantum curve
WKB
top. recursion
quantum curve
?
H(x,y) = 0

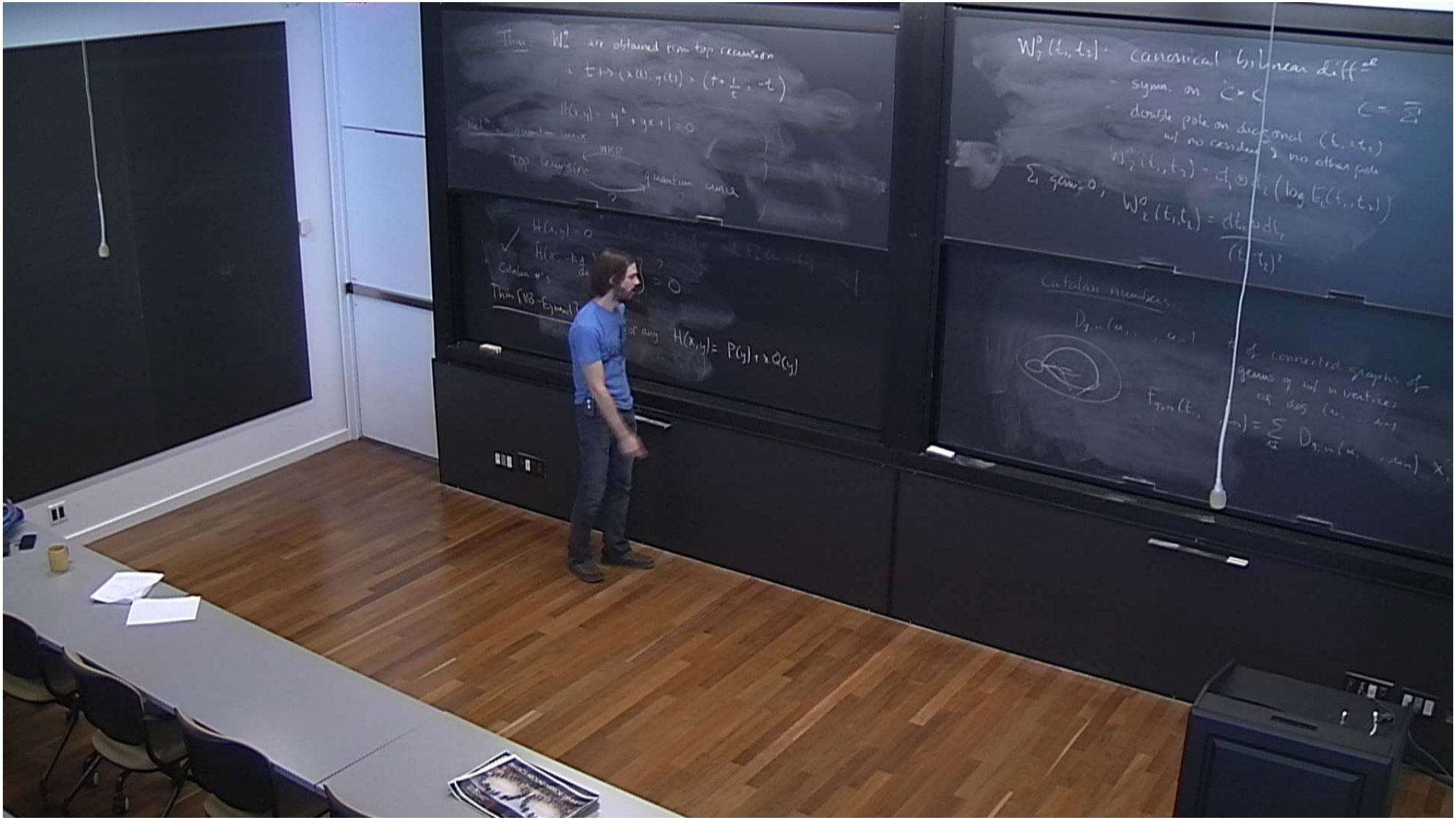

H(x,y) = y² + yx + 1 = 0
Rel^n w/ quantum curve
WKB
top. recursion
quantum curve
?
H(x,y) = 0
Ĥ(x, -ħ d/dx) [Z] = ?
0

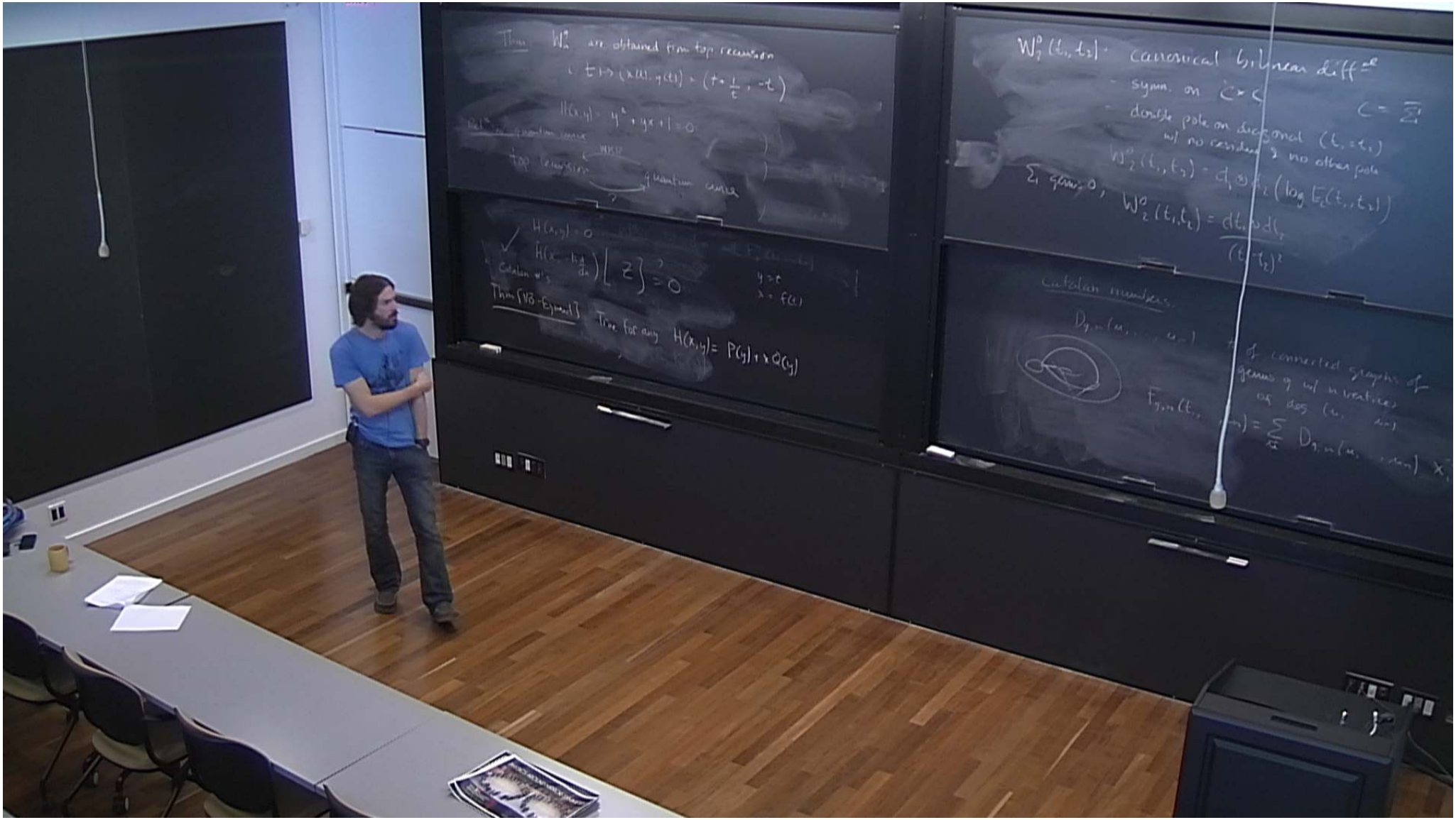

H(x,y) = y² + yx + 1 = 0
Rel^n w/ quantum curve
WKB
top. recursion
quantum curve
?
H(x,y) = 0
Ĥ(x, -ħ d/dx) [Z] = ?
0


H(x,y) = y² + yx + 1 = 0
Rel^n w/ quantum curve
WKR
top recursion
quantum curve
?
H(x,y) = 0
Ĥ(x, -ħ d/dx) [Z] = 0
Catalan #'s


H(x,y) = y² + yx + 1 = 0
Rel^n w/ quantum curve
WKB
top. recursion
quantum curve
?
H(x,y) = 0
Ĥ(x, -ħ d/dx) [Z] = 0
Thm [VB-Eynard] True for any H(x,y) = P(y) + x Q(y)


Thm
are obtained from top recursion
top recursion
quantum curve
WKB
quantum curve
Catalan #'s
Thm [VB-Eynard]
Catalan numbers
connected graphs of
genus g w/ n vertices
of deg


Thm
are obtained from top recursion
quantum curve
top recursion
MKP
Catalan #'s
Catalan numbers
connected graphs of
genus
of degs


Thm
are obtained from top recursion
top recursion
quantum curve
Catalan numbers
Thm [VB-Eynard]
Tree for any
connected graphs of
genus
Pirsa: 13100122
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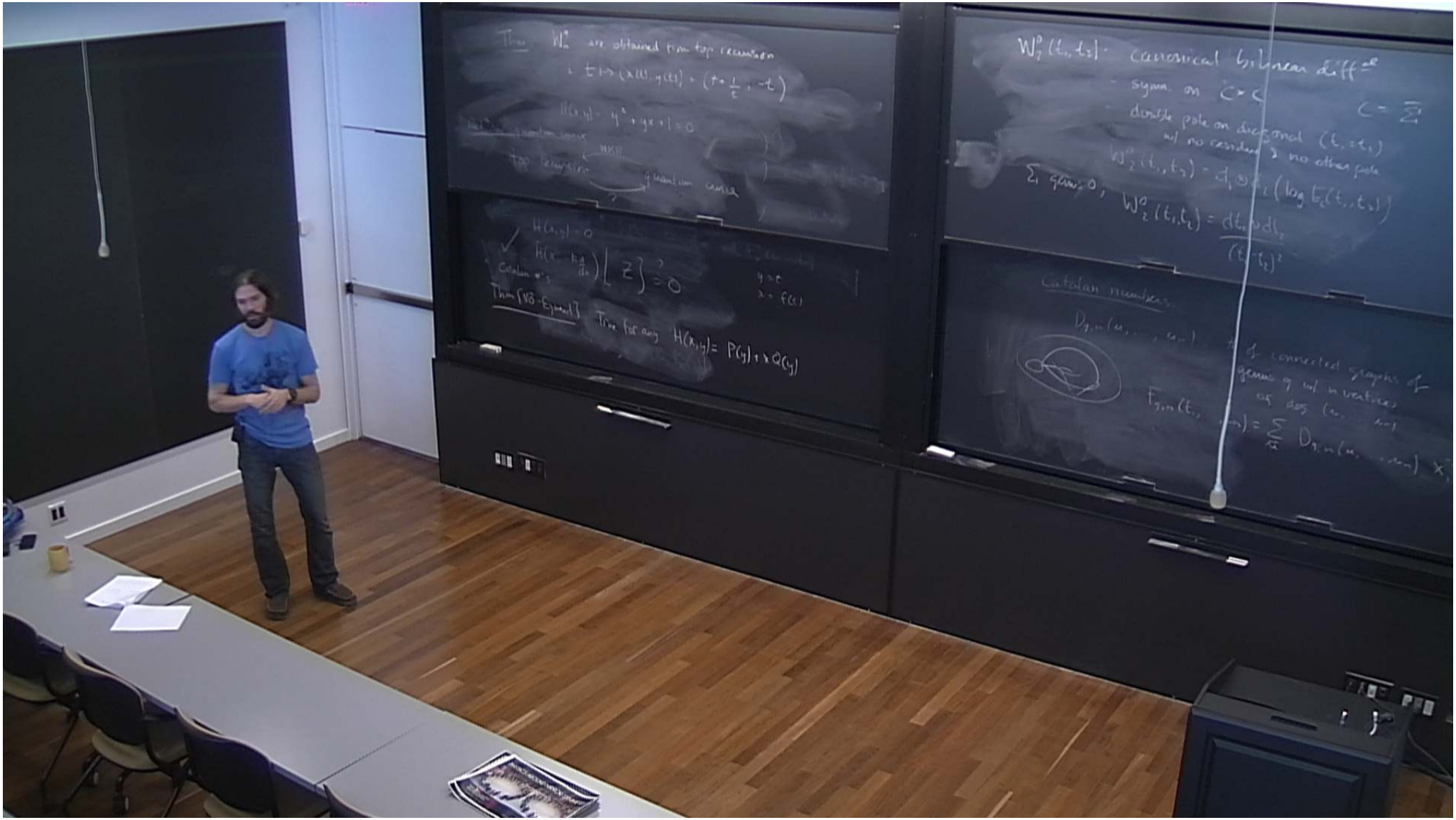
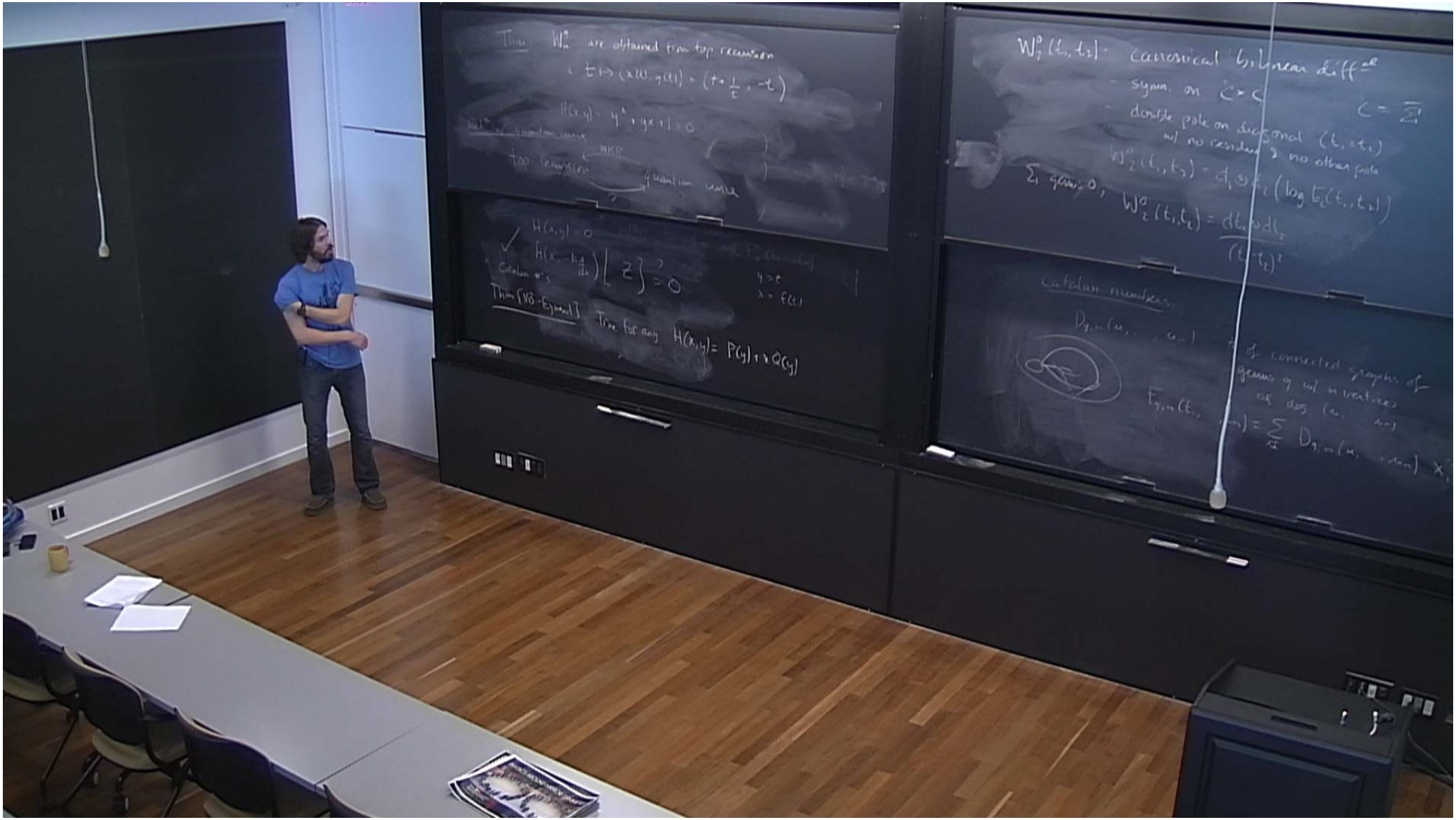

Catalan numbers
Thm [V8-Eynard] True for any H(x,y) = P(y) + xQ(y)


Thm
are obtained from top recursion
top recursion
quantum curve
WKB
quantum curve
Catalan #'s
Thm [VB-Eynard]
Tree for any
catalan numbers
connected graphs of
genus
of degs