

Title: Algebraic structures in massive (2,2) theories

Date: Oct 22, 2013 02:30 PM

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Abstract: I will review some ongoing work on the low energy properties of D-branes/boundary conditions in massive two-dimensional field theories with (2,2) supersymmetry.

$(2,2)$

$$\{Q_L^+, Q_L^-\} = E + P$$

$$\{Q_R^+, Q_R^-\} = E - P$$

$$\{Q_L^+, Q_R^-\} = Z$$

$$\{Q_R^+, Q_L^-\} = \bar{Z}$$

(2,2)

$$\{Q_L^+, Q_L^-\} = E + P$$

$$\{Q_R^+, Q_R^-\} = E - P$$

$$\{Q_R^+, Q_R^-\} = Z$$

$$\{Q_R^+, Q_L^-\} = \bar{E}$$

$$[J, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

$$(2,2) \quad \{Q_L^+, Q_L^-\} = E + P \quad [J, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

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$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$[J, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

PRESERVED BY
BRANES

LG ON $\mathbb{C}^N$ (ON CY)

A-BRANES

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

LG ON $\mathbb{C}^N$ (ON CY)
A-BRANES

GLSM $\mathbb{C}P^N$
B-BRANES



$$\{Q_R^+, Q_L^-\} = \overline{E}$$

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LG ON $\mathbb{C}^N$ (ON CY)
A-BRANES

GLSM $\mathbb{C}P^N$
B-BRANES

(2,2)

$$\{Q_L^+, Q_L^-\} = E + P$$

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$$\{Q_L^+, Q_R^+\} = 0$$

$$\{Q_L^-, Q_R^-\} = 0$$

$$[T, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

PRESERVED BY
BRANES

MASSIVE VACUA

Z

(2,2)

$$\{Q_L^+, Q_L^-\} = E + P$$

$$\{Q_R^+, Q_R^-\} = E - P$$

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$$[T, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

PRESERVED BY
BIANES

MASSIVE VACUA

$$Z_{ij} = Z_i - Z_j + m \cdot q$$

(2,2)

$$\{Q_L^+, Q_L^-\} = E + P$$

$$\{Q_R^+, Q_R^-\} = E - P$$

$$\{Q_L^+, Q_R^-\} = Z$$

$$\{Q_L^-, Q_R^+\} = \bar{Z}$$

$$\{Q_L^+, Q_L^-\} = 0 \quad \{Q_R^+, Q_R^-\} = 0$$

$$[T, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

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BRANES

MASSIVE VACUA

$$Z_{ij} = Z_i - Z_j + \cancel{m \cdot g}$$

(2,2)

$$\{Q_L^+, Q_L^-\} = E + P$$

$$\{Q_R^+, Q_R^-\} = E - P$$

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$$[T, Q_{L,R}^\pm] = \pm Q_{L,R}^\pm$$

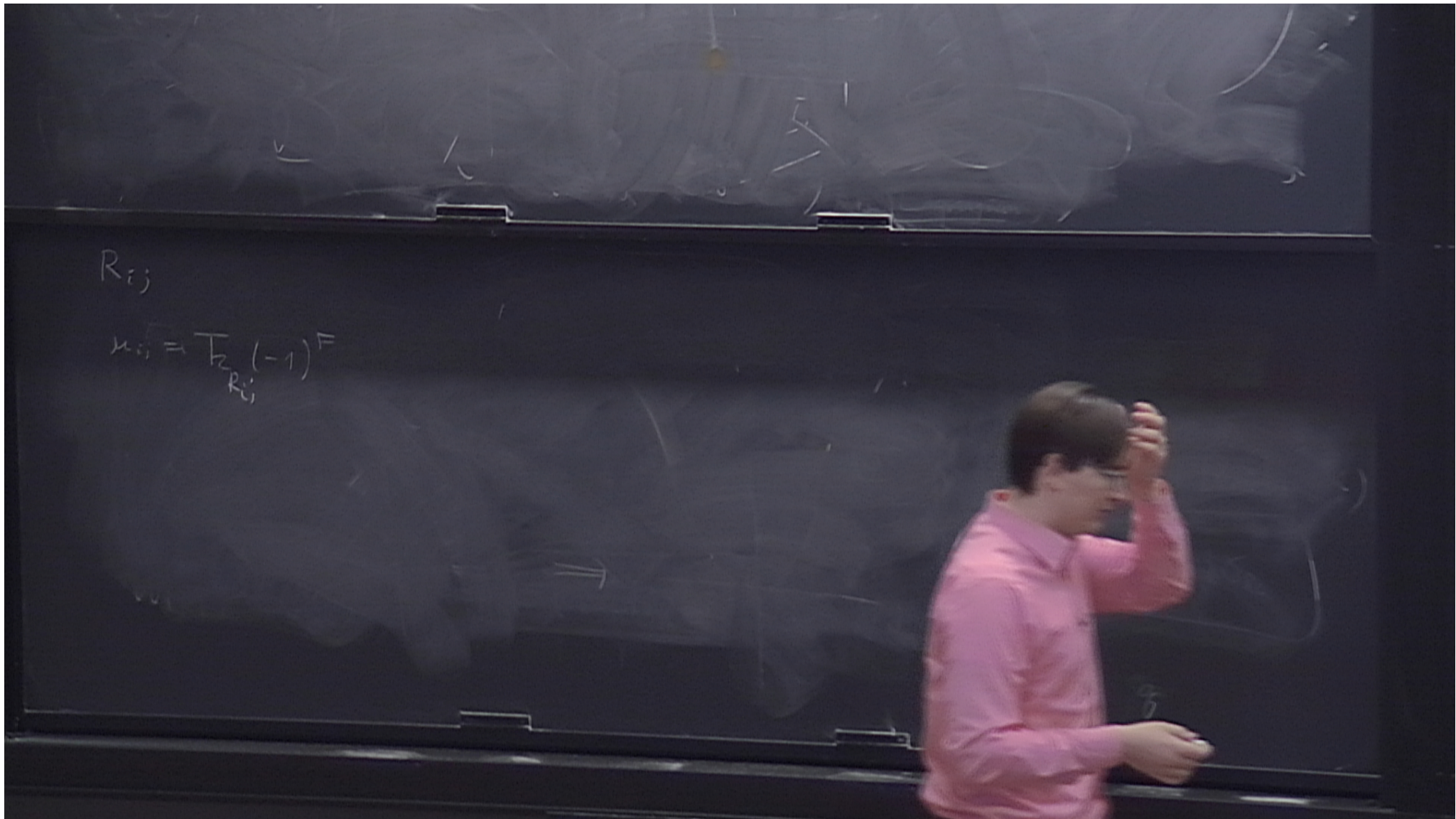
PRESERVED BY
BRANES

MASSIVE VACUA

$$Z_{ij} = Z_L - Z_j + \cancel{m} \cdot q$$

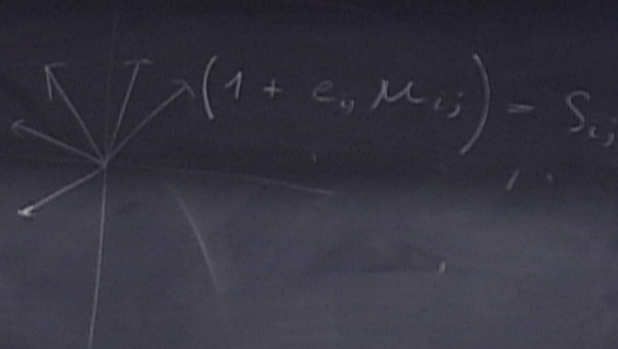
$$M_{ij} \geq |Z_{ij}|$$





R_{ij}

$$\mu_{ij} = T_{R_{ij}} (-1)^F$$



any δ

$R_{i,j}$

$$\mu_{i,j} = T_{R_{i,j}}(-1)^F$$

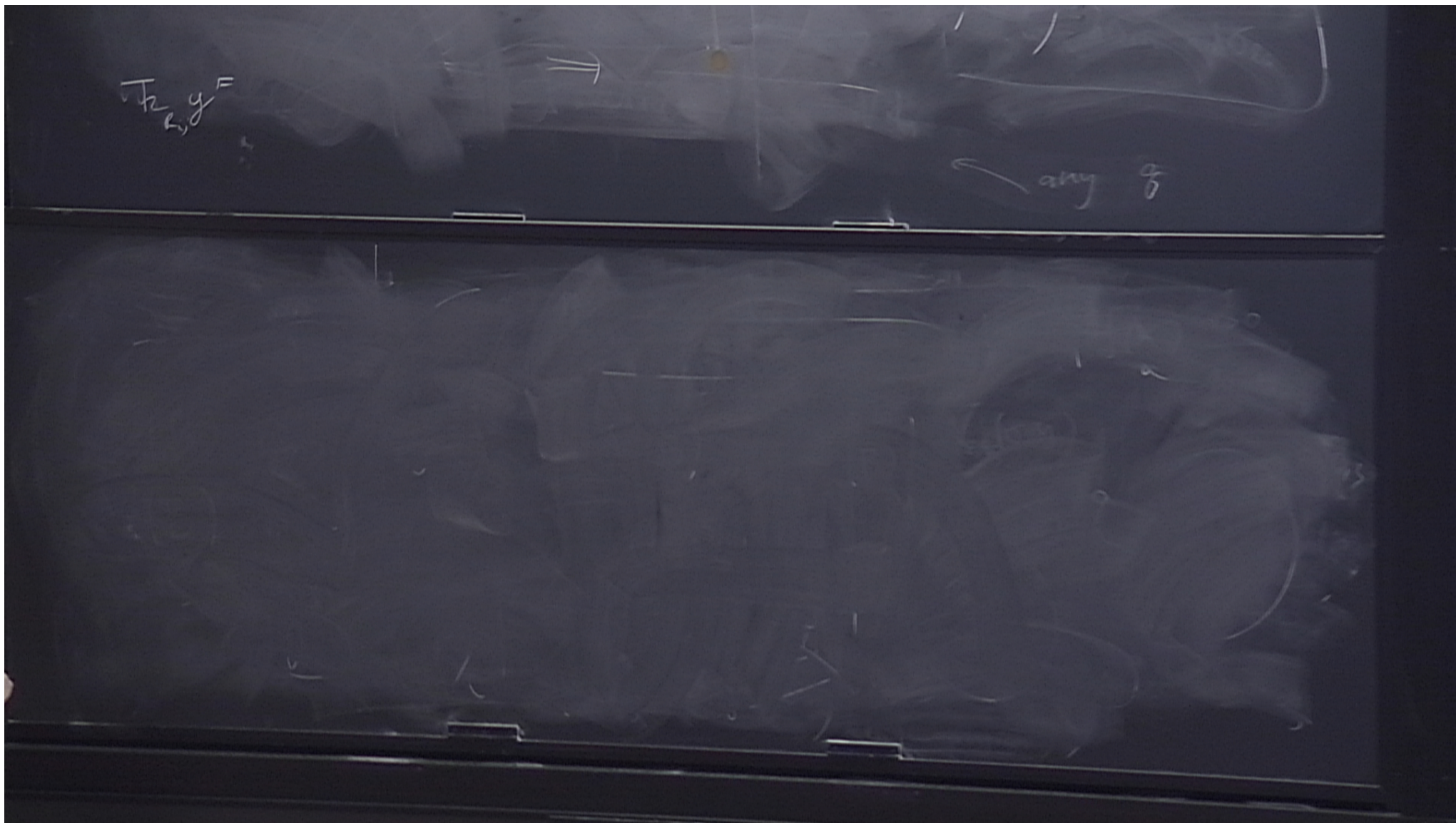
H_0

$$(1 + e_{i,j} \mu_{i,j}) = S_{i,j}$$

$$\prod_{\sigma_j Z_{i,j} \in H_0} S_{i,j} = S_{H_0}$$

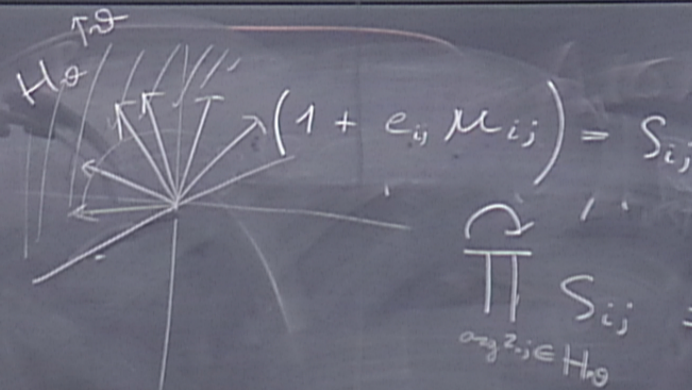
S_{H_0}

any g



R_{ij}

$$\mu_{ij} = T_{R_{ij}}(-1)^F$$



$$\prod_{\substack{\text{any } z_{ij} \in H_0}} S_{ij} = S_0$$

$$T_{R_{ij}} y^F$$

$\Rightarrow$

any $\frac{a}{b}$

$$\prod (\mathbb{Z} S_{ij} + R_{ij} e_{ij}) =$$

$$\mathbb{H}_{x,y} =$$

any g

$$\prod (Z_{i,j} + R_{i,j} e_{i,j}) = \left(\hat{R}_{i,j} \right)$$

$$\mathbb{H}_{\mathbb{R}, y}^F$$

any g

$$\prod (Z s_{ij} + R_{ij} e_{ij}) = \begin{pmatrix} \hat{R}_{ij} \\ \hat{R}_{ij}' \end{pmatrix}$$

$$(\hat{R}_{ij}, d_{ij})$$

$$(\hat{R}_{ij}', d_{ij}')$$

$$T_{x,y}^F$$

any g

$$1) \bigotimes_{\text{any } Z_{ij} \in H_0} \left(\sum S_{ij} + R_{ij} e_{ij} \right) = \left(\hat{R}_{ij} \right)$$

$$\left(\hat{R}_{ij}, d_{ij} \right) \xrightarrow{f_{ij}} \left(\hat{R}'_{ij}, d'_{ij} \right)$$

$$d_{ij} f_{ij} = f_{ij} d'_{ij}$$

$$\mathcal{H}_{x,y}^F$$

any g

$$1) \bigotimes_{\text{arg } z_{ij} \in H_0} (R_{ij} s_{ij} + R_{ij} e_{ij}) = (\hat{R}_{ij})$$

$$(R_{ij}, d_{ij}) \xrightarrow{f_{ij}} (\hat{R}_{ij}, d'_{ij})$$

$$d_{ij} f_{ij} = f_{ij} d'_{ij}$$

$$\mathcal{H}_{x,y}^F$$

any g

$$1) \quad \bigotimes_{\text{any } Z_{ij} \in H_0} (R_{ij} s_{ij} \oplus R_{ij} e_{ij}) = (\hat{R}_{ij})$$

$$(R_{ij}, d_{ij}) \xrightarrow{f_{ij}} (\hat{R}_{ij}', d_{ij}')$$

$$d_{ij} f_{ij} = f_{ij} d_{ij}'$$

$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

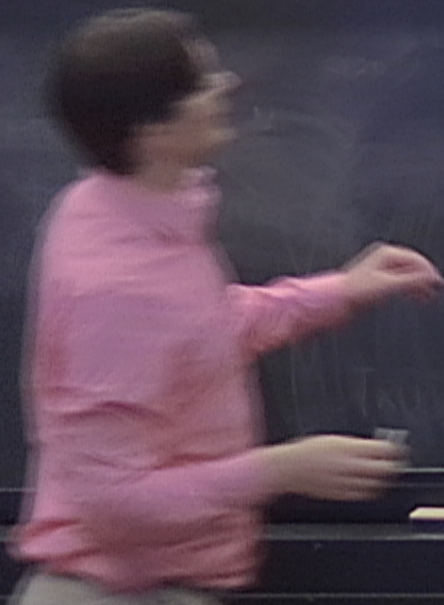
$$Z_{ij} = Z_i - Z_j + m - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = X^3 - aX$$

$$X^2 = a$$

$$X = \pm \sqrt{a}$$



$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m_{ij} - q$$

$$M_{ij} \geq |Z_{ij}|$$

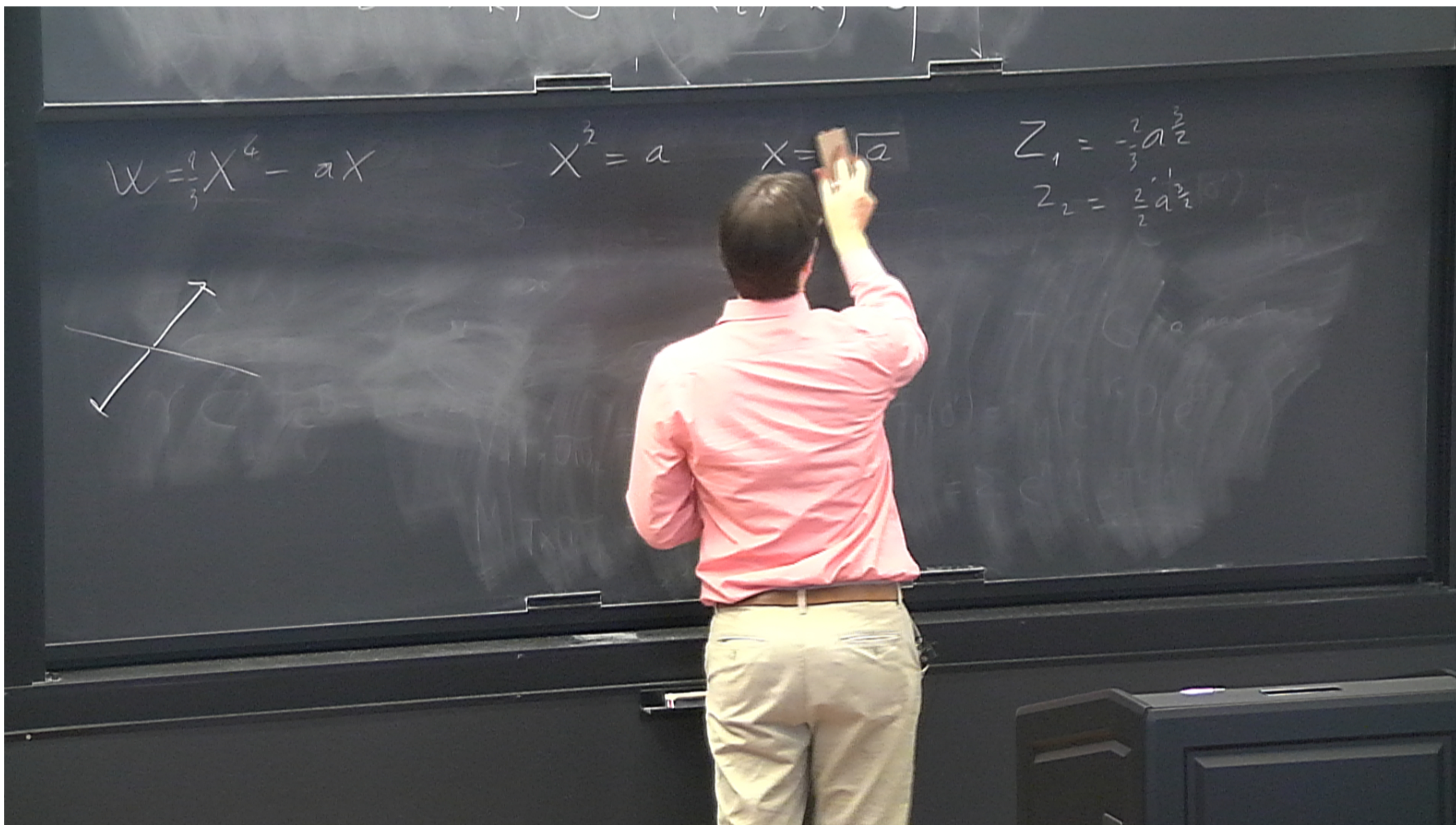
$$W = \frac{1}{3}X^3 - aX$$

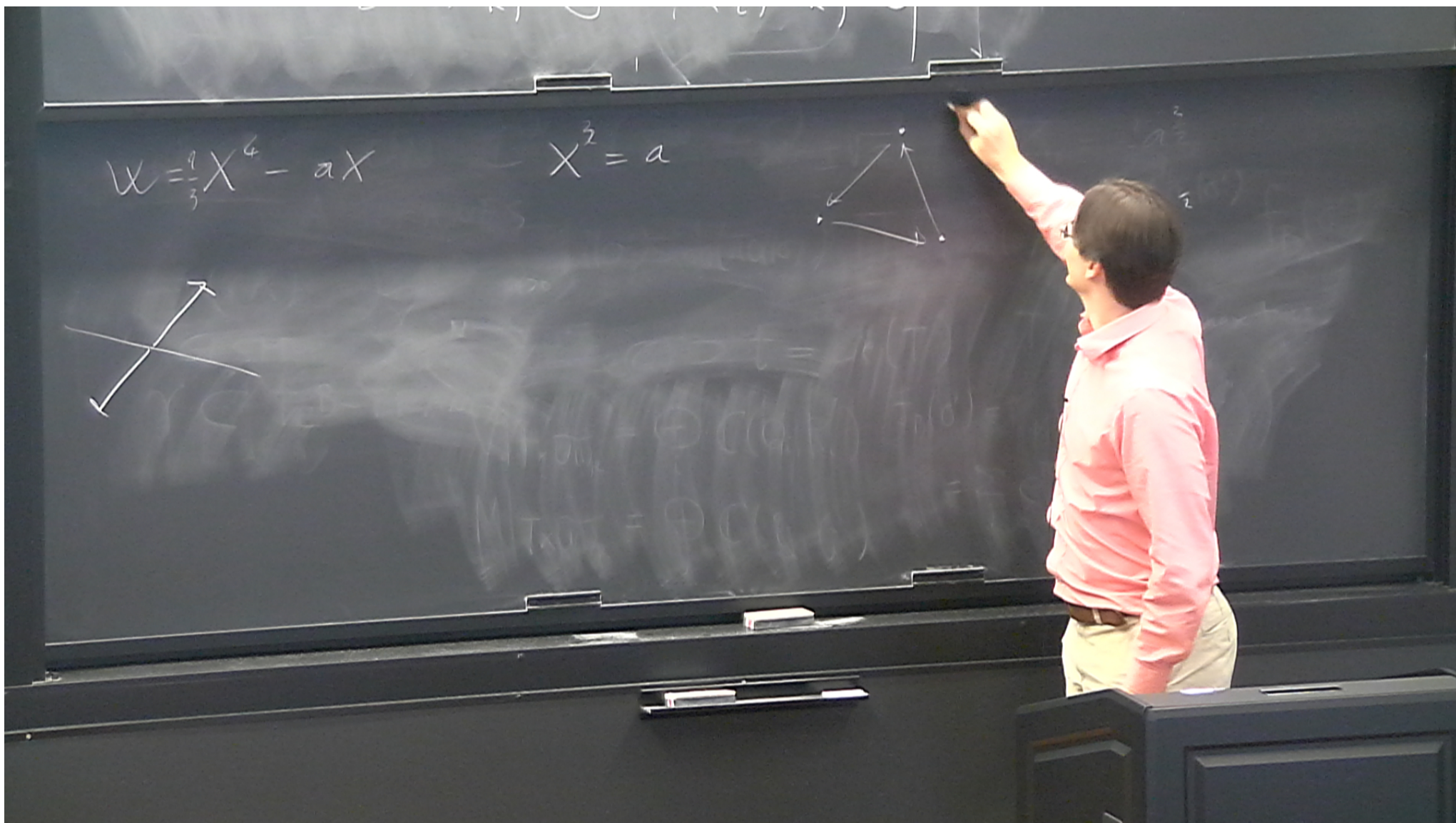
$$X^2 = a$$

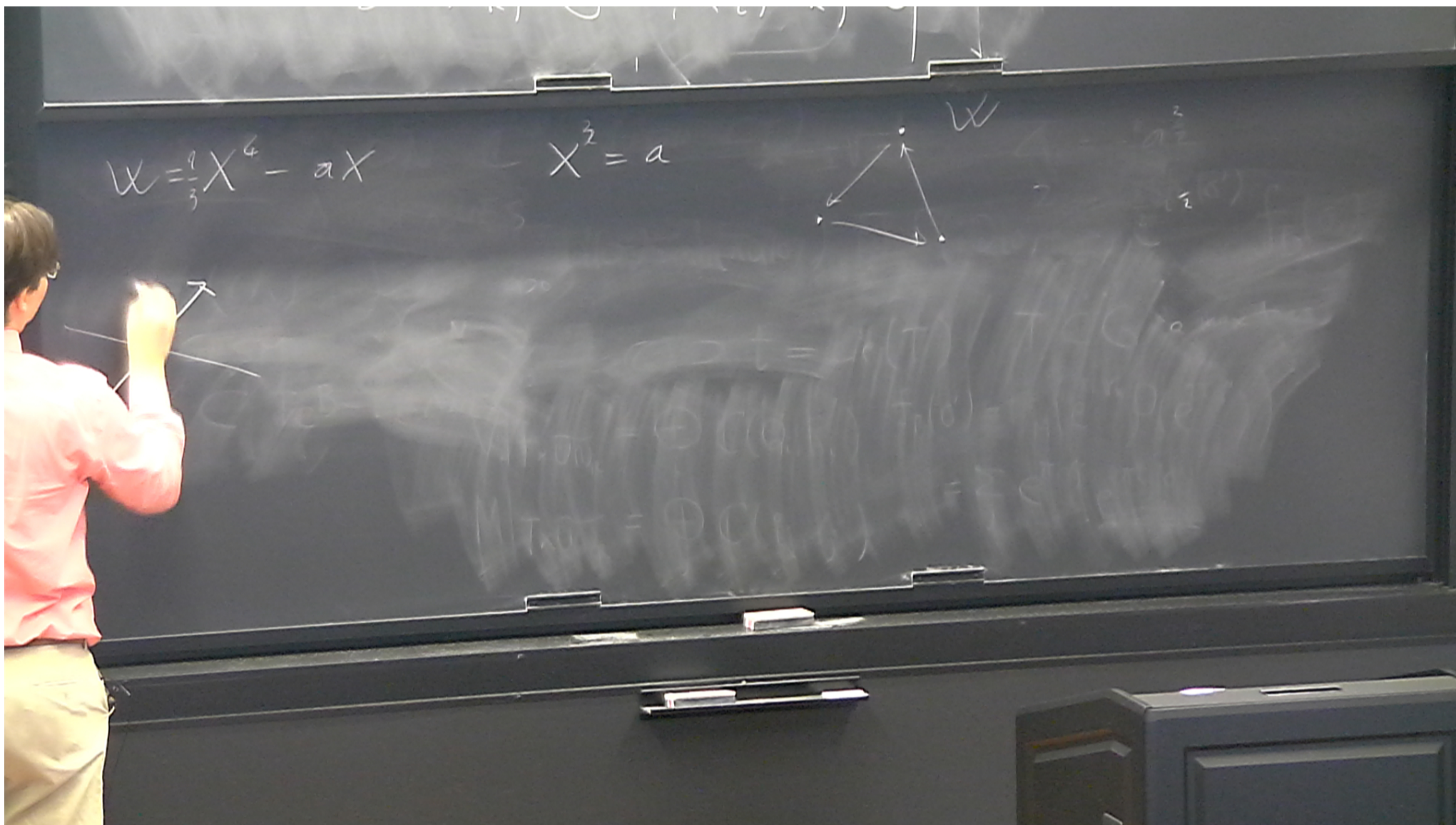
$$X = \pm \sqrt{a}$$

$$Z_1 = -\frac{2}{3}a^{\frac{3}{2}}$$

$$Z_2 = \frac{2}{3}a^{\frac{3}{2}}$$

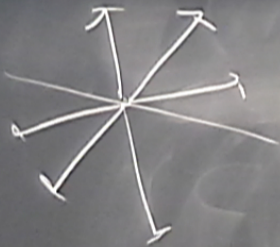
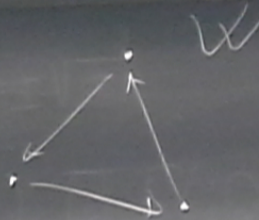






$$W = \frac{1}{3}X^4 - aX$$

$$X^2 = a$$



$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

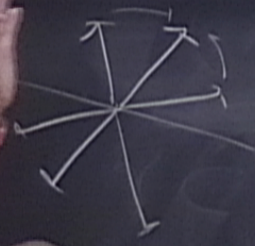
$$Z_{ij} = Z_i - Z_j + m - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_f(x)$$

$$x^2 = a$$

W



$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

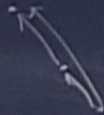
$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m - q$$

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(L, R)

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

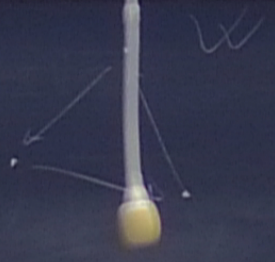
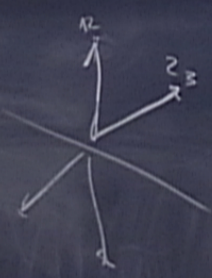
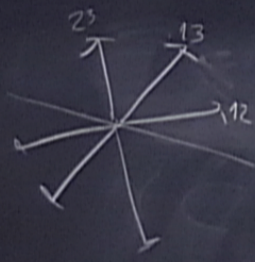
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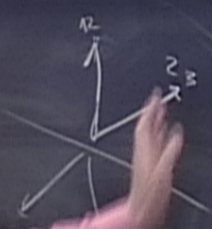
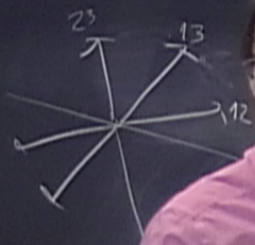
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$$(1 + e_{12})(1 + e_{23})$$

$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

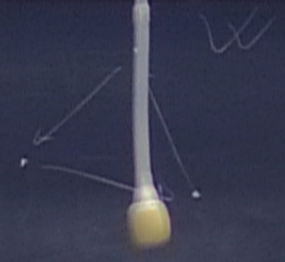
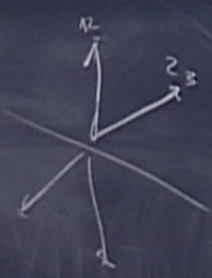
$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m_{ij} - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_f(x)$$

$$x^2 = a$$



$$(1 + e_{12})(1 + e_{23}) = 1 + e_{12} + e_{23} + e_{13}$$

(L, R)

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

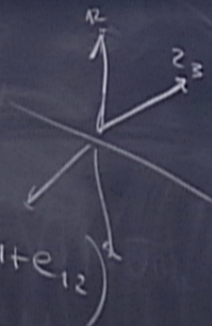
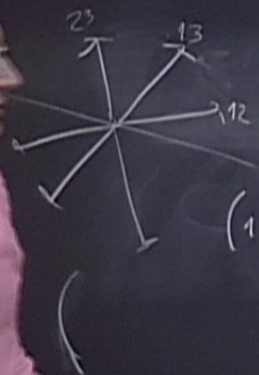
$$Z_{ij} = Z_i - Z_j + m - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_f(X)$$

$$X^2 = a$$

W



$$(1+e_{23})(1+e_{13})(1+e_{12})$$

$$(1+e_{12})(1+e_{23}) = 1 + e_{12} + e_{23} + e_{13}$$

(Q_L, Q_R)

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

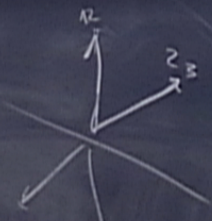
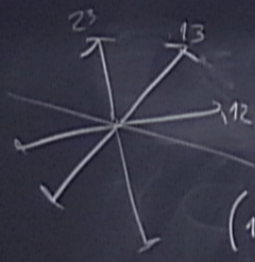
$$Z_{ij} = Z_i - Z_j + m - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_4(X)$$

$$X^2 = a$$

W



$$(1+e_{12})(1+e_{23}) = 1 + e_{12} + e_{23} + e_{13}$$

$$(1+e_{23})(1+e_{13})(1+e_{12}) = 1 + e_{12} + e_{13} + e_{23}$$

$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

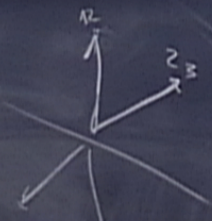
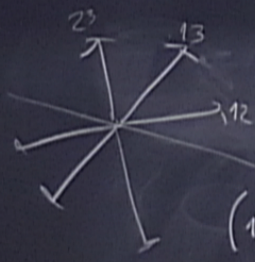
$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m_{ij} - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_4(X)$$

$$X^2 = a$$



$$(1+e_{23})(1+e_{13})(1+e_{12}) = 1+e_{12}+e_{13}+e_{23}$$

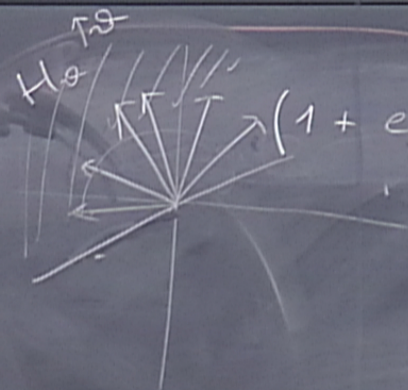
$$\begin{pmatrix} R & R & R \\ & R & R \\ & & R \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{pmatrix}$$



R_{ij}

$$\mu_{ij} = T_{R_{ij}}(-1)^F$$



$$(1 + e_{ij} \mu_{ij}) = S_{ij}$$

$$\prod_{\text{any } z_{ij} \in H_0} S_{ij} = S_{\mathcal{Q}}$$

$$S_{\mathcal{Q}}$$

$$T_{R_{ij}} y^F$$

$$\mu_{ij} = -\mu_{ji}$$

any g

$$1) \bigotimes_{\text{any } z_{ij} \in H_0} (R_{ij} S_{ij} \oplus R_{ij} e_{ij}) = \left(\hat{R}_{ij} \right)$$

$$\{Q_R^+, Q_L^-\} = \overline{P}$$

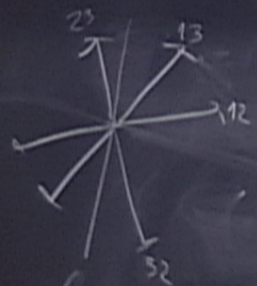
$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$W = P_4(X)$$

$$X^2 = a$$

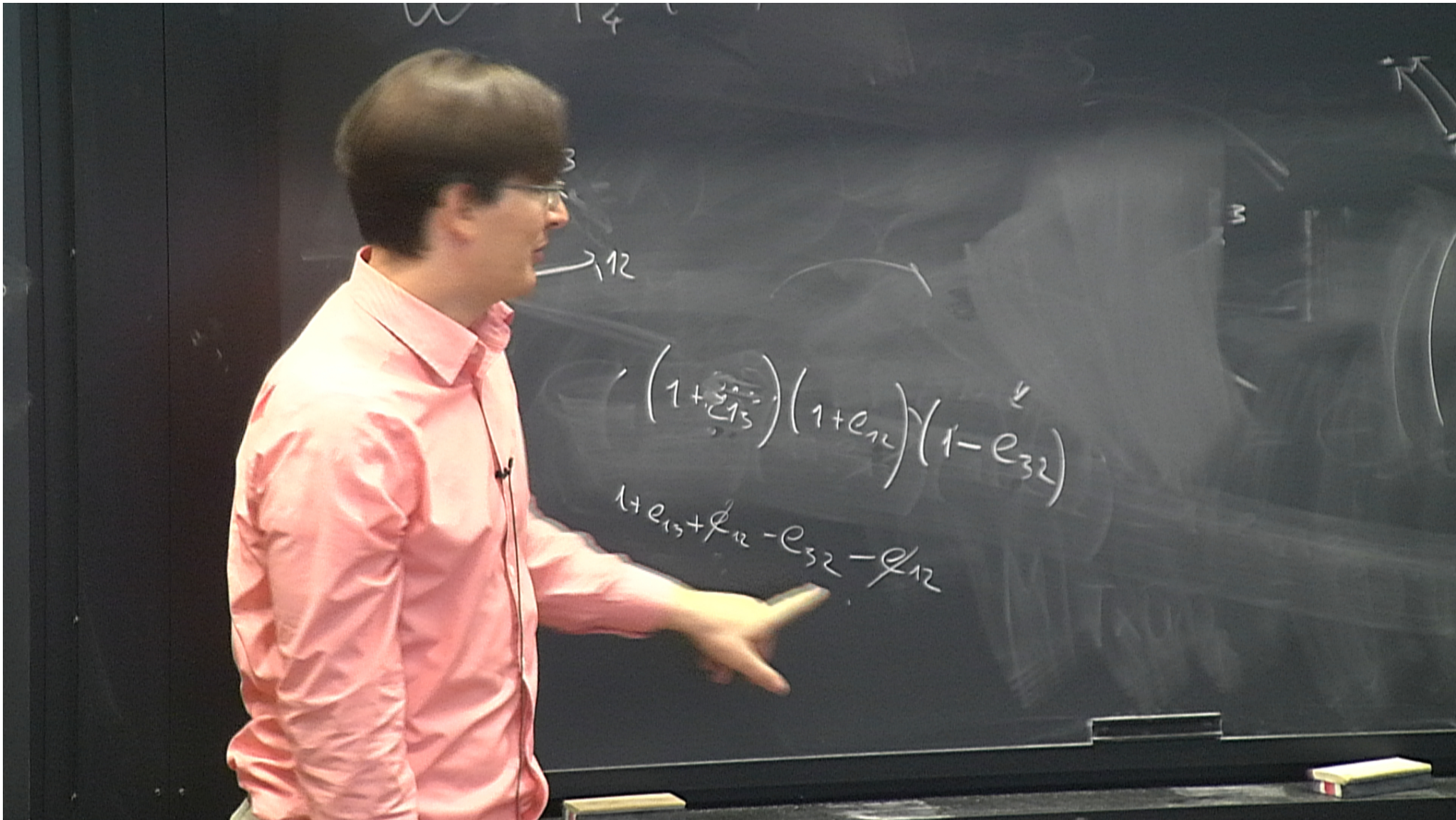


$$(1+e_{12})(1+e_{13})(1-e_{32})$$



$$\begin{pmatrix} R & R & R \\ R & R & R \\ R & R & R \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + \cancel{m} - q$$

$$M_{ij} \geq |Z_{ij}|$$

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m \cdot g$$

$$M_{ij} \geq |Z_{ij}|$$

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{ijk} : \hat{R}_{ij} \otimes \hat{R}_{jk} \rightarrow \hat{R}_{ik}$$

$$d_{ijkl} : \hat{R}_{ij} \otimes \hat{R}_{jk} \otimes \hat{R}_{kl} \rightarrow \hat{R}_{il}$$

...

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{i,jn} : \hat{R}_{ij} \otimes \hat{R}_{jn} \rightarrow \hat{R}_{in}$$

$$d_{i,jnt} : \hat{R}_{ij} \otimes \hat{R}_{jn} \otimes \hat{R}_{nt} \rightarrow \hat{R}_{it}$$

$$\sum_{\pm} d_{\pm}(\dots, d_{\pm}(\dots), \dots) = 0$$

$$(Q_L, Q_R)$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + m \cdot g$$

$$M_{ij} \geq |Z_{ij}|$$

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{ijn} : \hat{R}_{ij} \otimes \hat{R}_{jn} \rightarrow \hat{R}_{in}$$

$$d_{ijnt} : \hat{R}_{ij} \otimes \hat{R}_{jn} \otimes \hat{R}_{nt} \rightarrow \hat{R}_{it}$$

$$\begin{aligned} & \dots \\ & f_{ij} \\ & f_{ijn} \\ & \dots \end{aligned}$$

$$\sum d_*(\dots, d_*(\dots), \dots) = 0$$

$$\{Q_R^+, Q_L^-\} = \overline{E}$$

$$\{Q_L^+, Q_R^+\} = 0 \quad \{Q_L^-, Q_R^-\} = 0$$

$$Z_{ij} = Z_i - Z_j + \cancel{m \cdot g}$$

$$M_{ij} \geq |Z_{ij}|$$

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{ijn} : \hat{R}_{ij} \otimes \hat{R}_{jn} \rightarrow \hat{R}_{in}$$

$$d_{ijnt} : \hat{R}_{ij} \otimes \hat{R}_{jn} \otimes \hat{R}_{nt} \rightarrow \hat{R}_{it}$$

$$f_{ij} : \hat{R} \rightarrow \hat{R}'$$

$$f_{ijn} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}'$$

$$\sum_{\pm} d_{\pm}(\dots, d_{\pm}(\dots), \dots) = 0$$

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{ijk} : \hat{R}_{ij} \otimes \hat{R}_{jk} \rightarrow \hat{R}_{ik}$$

$$d_{ijkt} : \hat{R}_{ij} \otimes \hat{R}_{jk} \otimes \hat{R}_{kt} \rightarrow \hat{R}_{it}$$

$$\begin{aligned} \hat{R} &\rightarrow \hat{R}' \\ \hat{R} \otimes \hat{R} &\rightarrow \hat{R}' \end{aligned}$$

$$\sum_{\pm} d_{\pm}(\dots, d_{\pm}(\dots), \dots) = 0$$

$$\sum d(\dots, f(\dots)) = \sum f(\dots) d(\dots)$$

LEVEL 2

$$d_{ij} : \hat{R}_{ij} \rightarrow \hat{R}_{ij}$$

$$d_{ijn} : \hat{R}_{ij} \otimes \hat{R}_{jn} \rightarrow \hat{R}_{in}$$

$$d_{ijnt} : \hat{R}_{ij} \otimes \hat{R}_{jn} \otimes \hat{R}_{nt} \rightarrow \hat{R}_{it}$$

$$f_{ij} : \hat{R} \rightarrow \hat{R}'$$

$$f_{ijn} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}'$$

$$\sum_i d_i(\dots, d_i(\dots), \dots) = 0$$

$$\sum d(\dots, f(\dots)) = \sum f(\dots) d(\dots)$$

LEVEL 3

BRANE $\Rightarrow \{ \mathcal{E}_i \}$

$$B_{ij} \cdot \hat{R}_{ij} \otimes \mathcal{E}_j \rightarrow \mathcal{E}_i$$

LEVEL 3

BRANE $\Rightarrow \{ \mathcal{E}_i \}$

d_{ij}

$$B_{ij} \in \mathcal{E}_i \otimes \hat{R}_{ij} \otimes \mathcal{E}_j^*$$

LEVEL 3

BRANE $\Rightarrow \{E_i\}$

$$B_{ij} \in E_i \otimes \hat{R}_{ij} \otimes E_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk})$$

LEVEL 3

BRANE $\Rightarrow \{ \mathcal{E}_i \}$

$$B_{ij} \in \mathcal{E}_i \otimes \hat{R}_{ij} \otimes \mathcal{E}_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots$$

$$K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

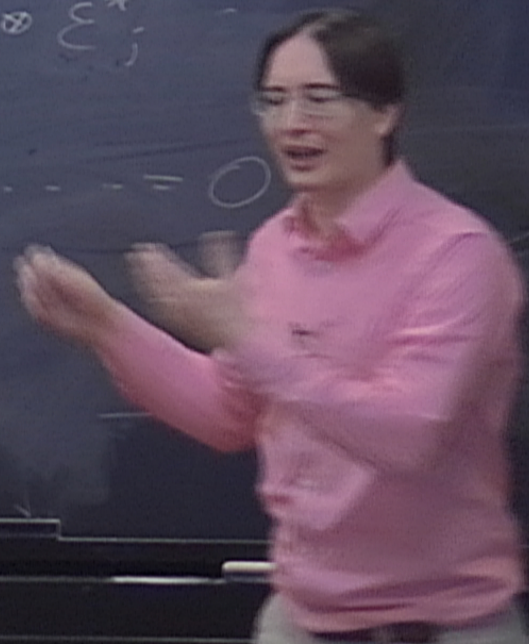
$$\begin{matrix} f_{ij} & \hat{R} \rightarrow \hat{R}' \\ f_{ijh} & \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots & \dots \end{matrix}$$

LEVEL 3

$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \varepsilon_i \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$



$$\text{int} : K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{array}{l} f_{ij} : \hat{R} \rightarrow \hat{R}' \\ f_{ijk} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots \end{array}$$

LEVEL 3

$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \varepsilon_i \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_i(B_i) + d_{ijn}(B_{ij}, B_{jk}) + \dots = 0$$

$$\text{unit} : K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{array}{l} f_{ij} : \hat{R} \rightarrow \hat{R}' \\ f_{ijk} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots \end{array}$$

LEVEL 3

$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \varepsilon_i \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$

LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

BRANE $\Rightarrow$

$$\{ \varepsilon_i \}$$

$$B_{ij} \in \varepsilon_i \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij} + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$

$$K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{aligned} f_{ij} &: \hat{R} \rightarrow \hat{R}' \\ f_{ijk} &: \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ &\vdots \end{aligned}$$

LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

BRANE

$$\Rightarrow$$

$$\{\varepsilon_i\}$$

$$B_{ij} \in \Sigma \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$

$$B_i \rightarrow$$

$$K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{aligned} f_{ij} &: \hat{R} \rightarrow \hat{R}' \\ f_{ijk} &: \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ &\vdots \end{aligned}$$

LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

BRANE

$$\Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \Sigma \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$



$$K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{matrix} f_{ij} & \hat{R} \rightarrow \hat{R}' \\ f_{ijk} & \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots & \dots \end{matrix}$$

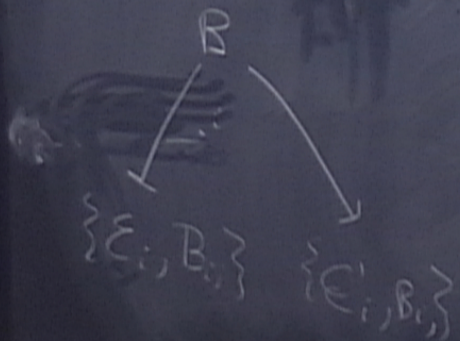
LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \Sigma \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$



$$\text{int} : K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{array}{l} f_{ij} : \hat{R} \rightarrow \hat{R}' \\ f_{ijk} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots \end{array}$$

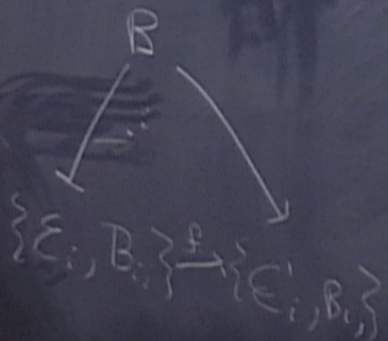
LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

$$B_{ij} \in \Sigma \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$



$$\text{int} : K_{ij} \otimes K_{jk} \otimes R_{kt} \rightarrow R_{it}$$

$$\begin{array}{l} f_{ij} : \hat{R} \rightarrow \hat{R}' \\ f_{ijk} : \hat{R} \otimes \hat{R} \rightarrow \hat{R}' \\ \dots \end{array}$$

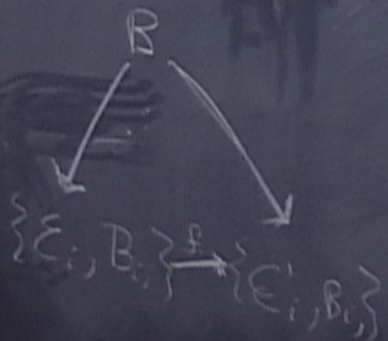
LEVEL 3

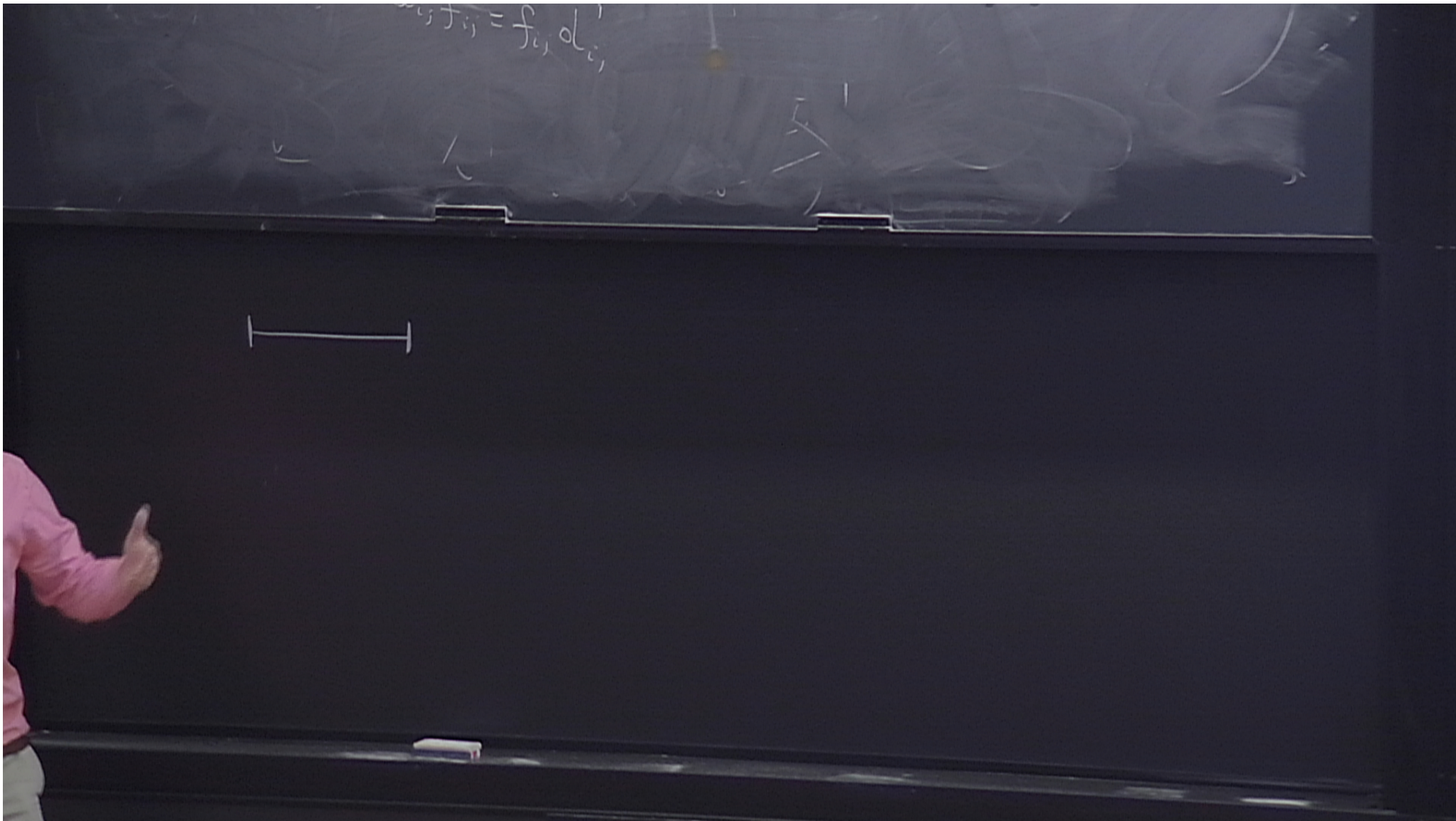
$$\sum \varepsilon_i \otimes T_i$$

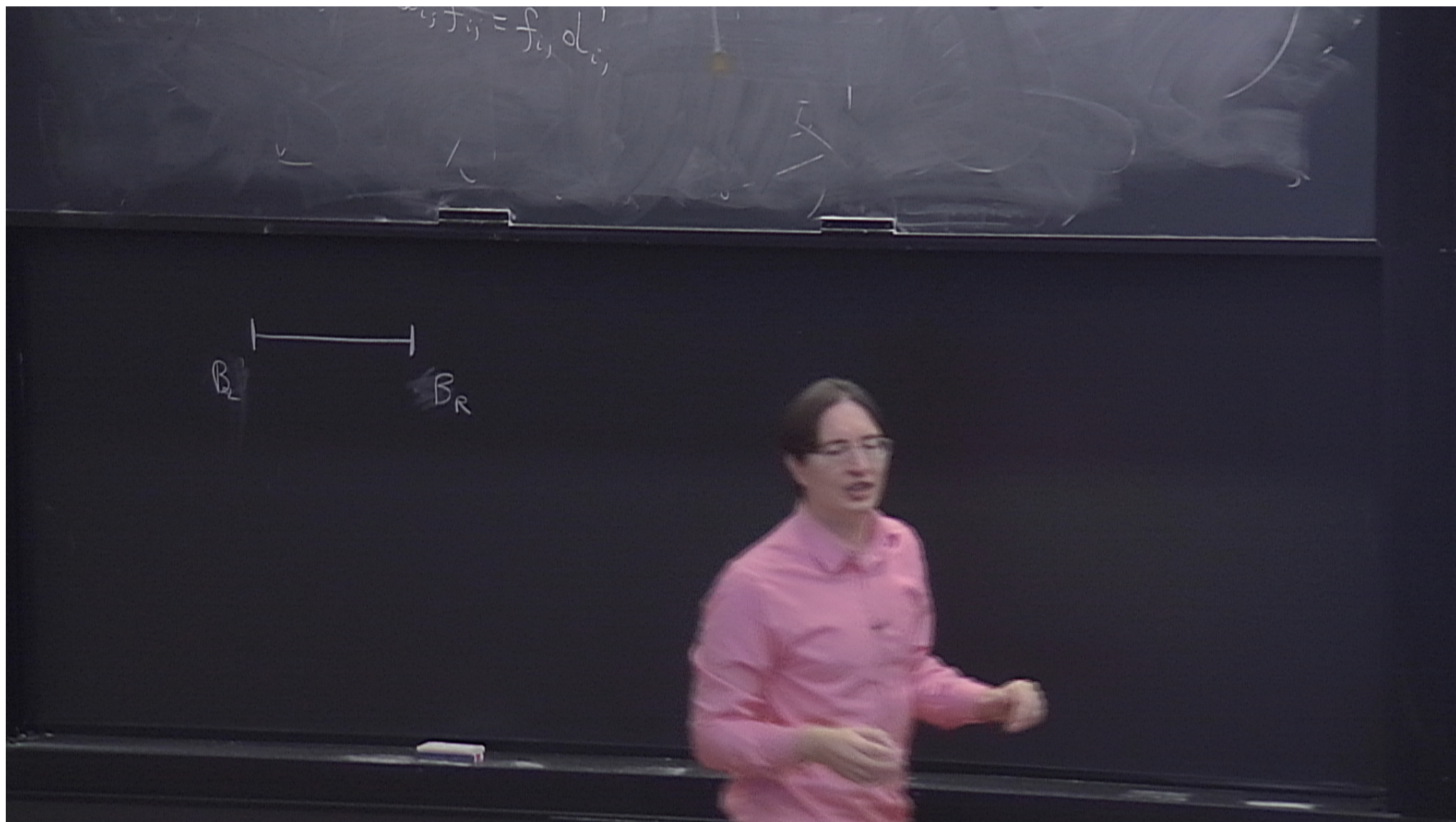
$$\text{BRANE} \Rightarrow \{\varepsilon_i\}$$

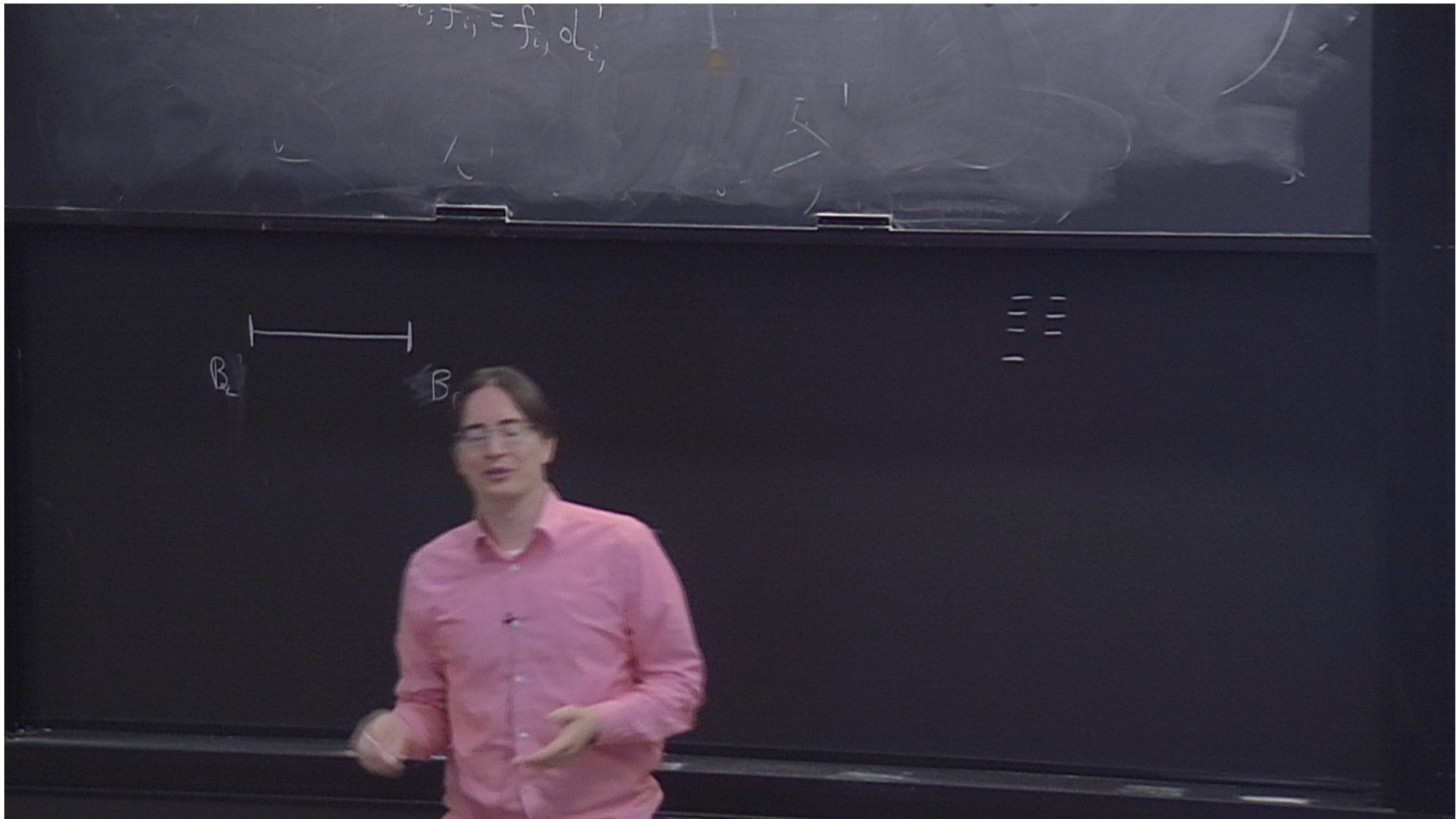
$$B_{ij} \in \Sigma \otimes \hat{R}_{ij} \otimes \varepsilon_j^*$$

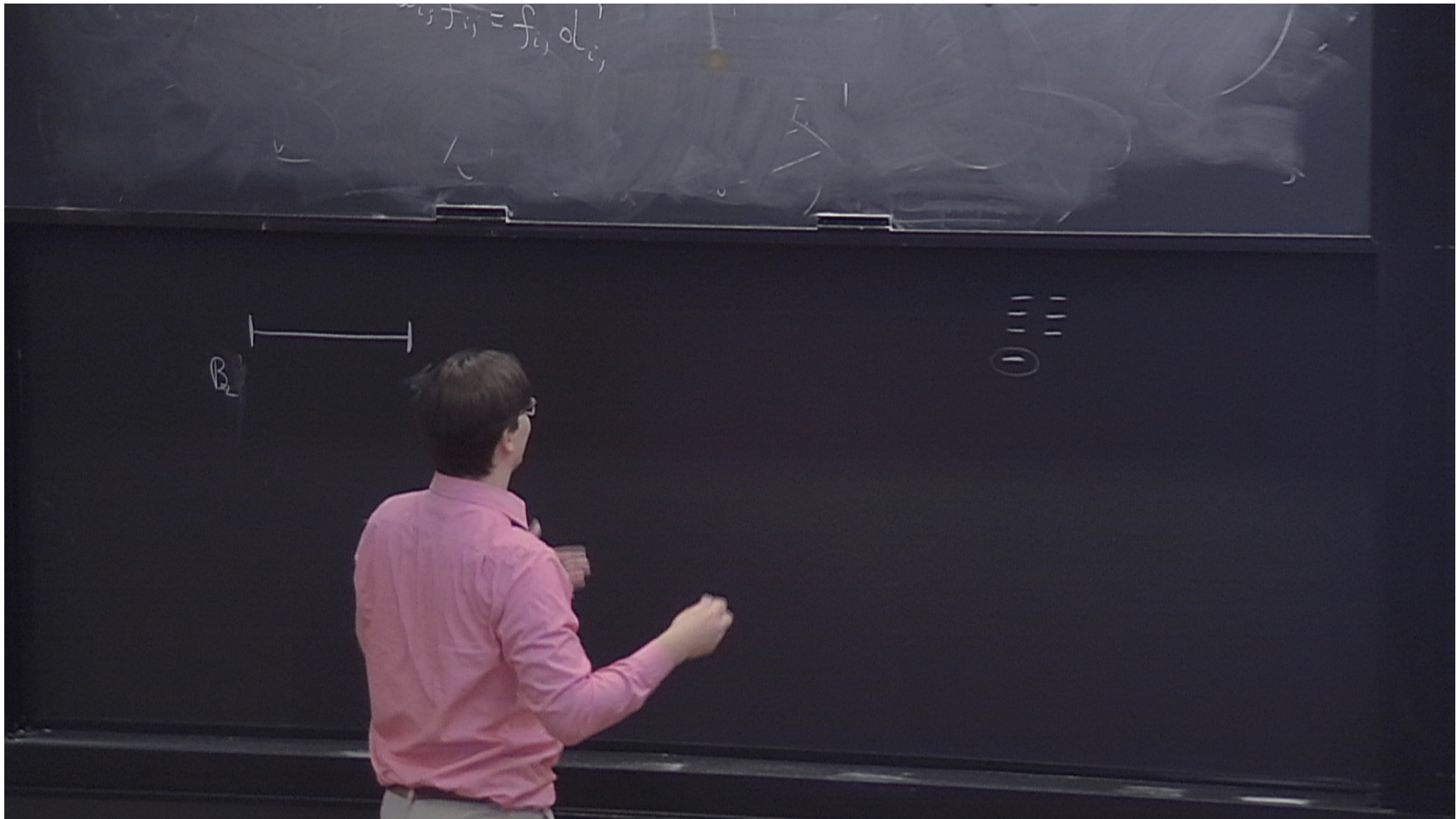
$$d_i(B_i) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$

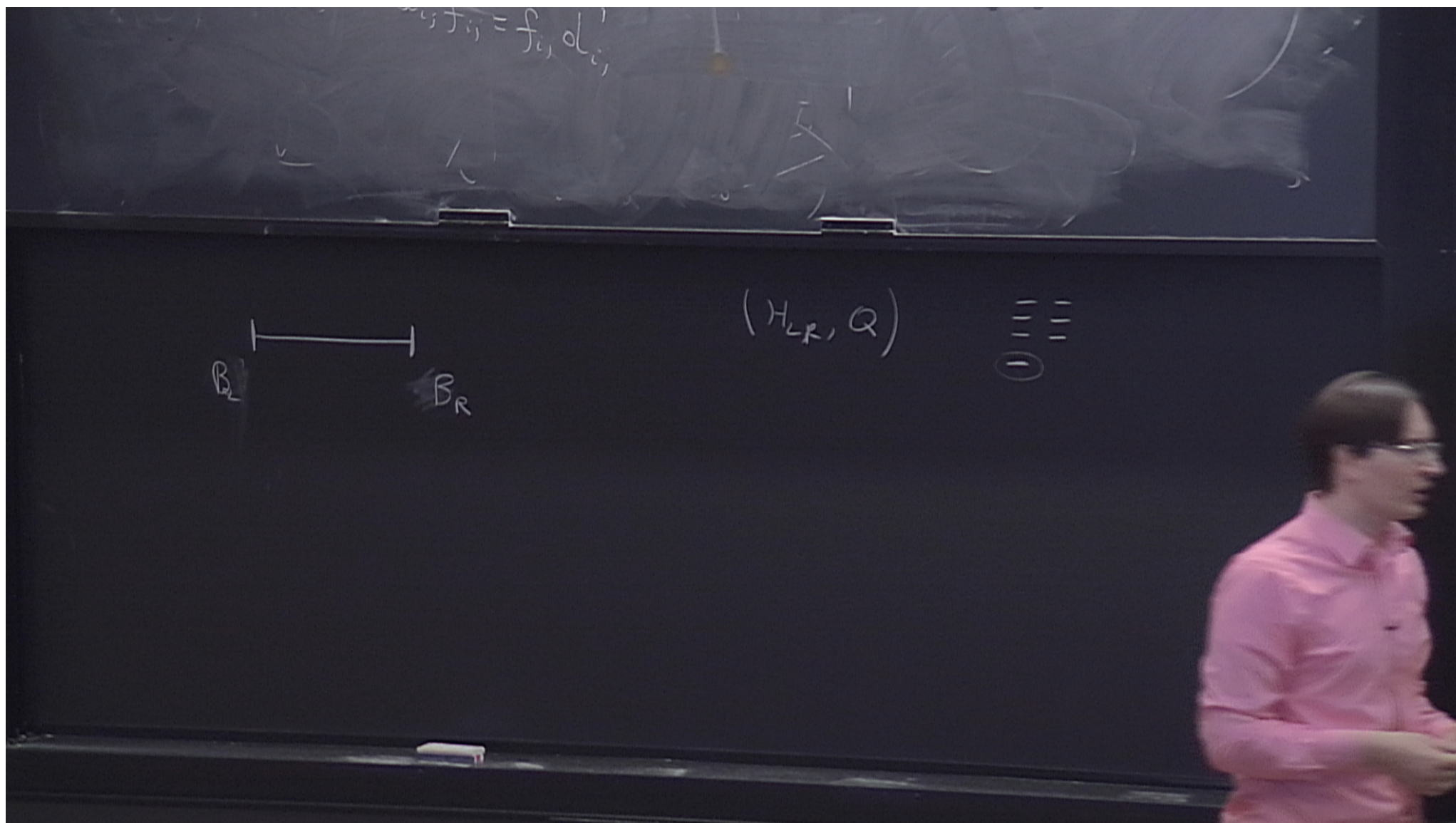


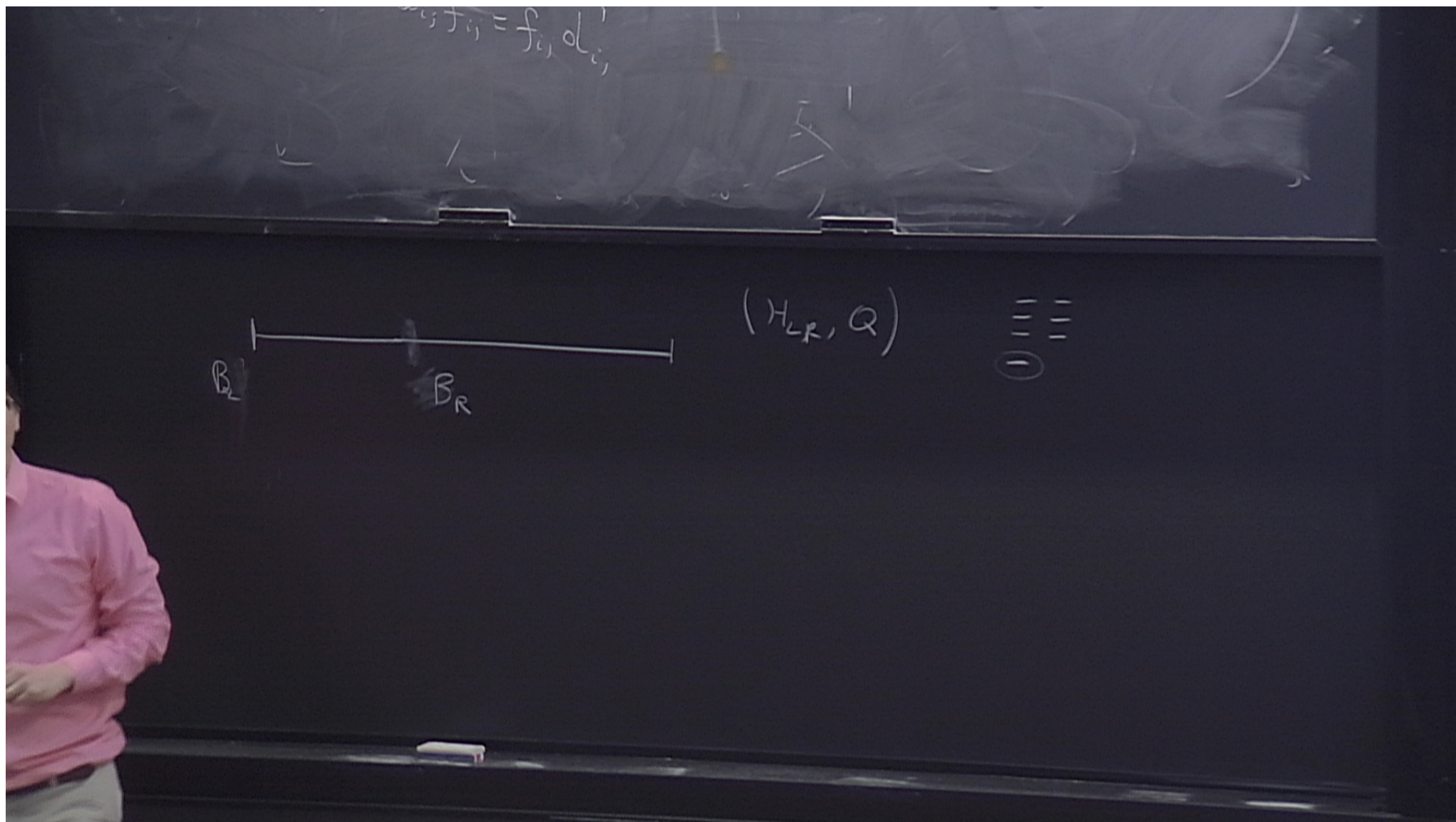


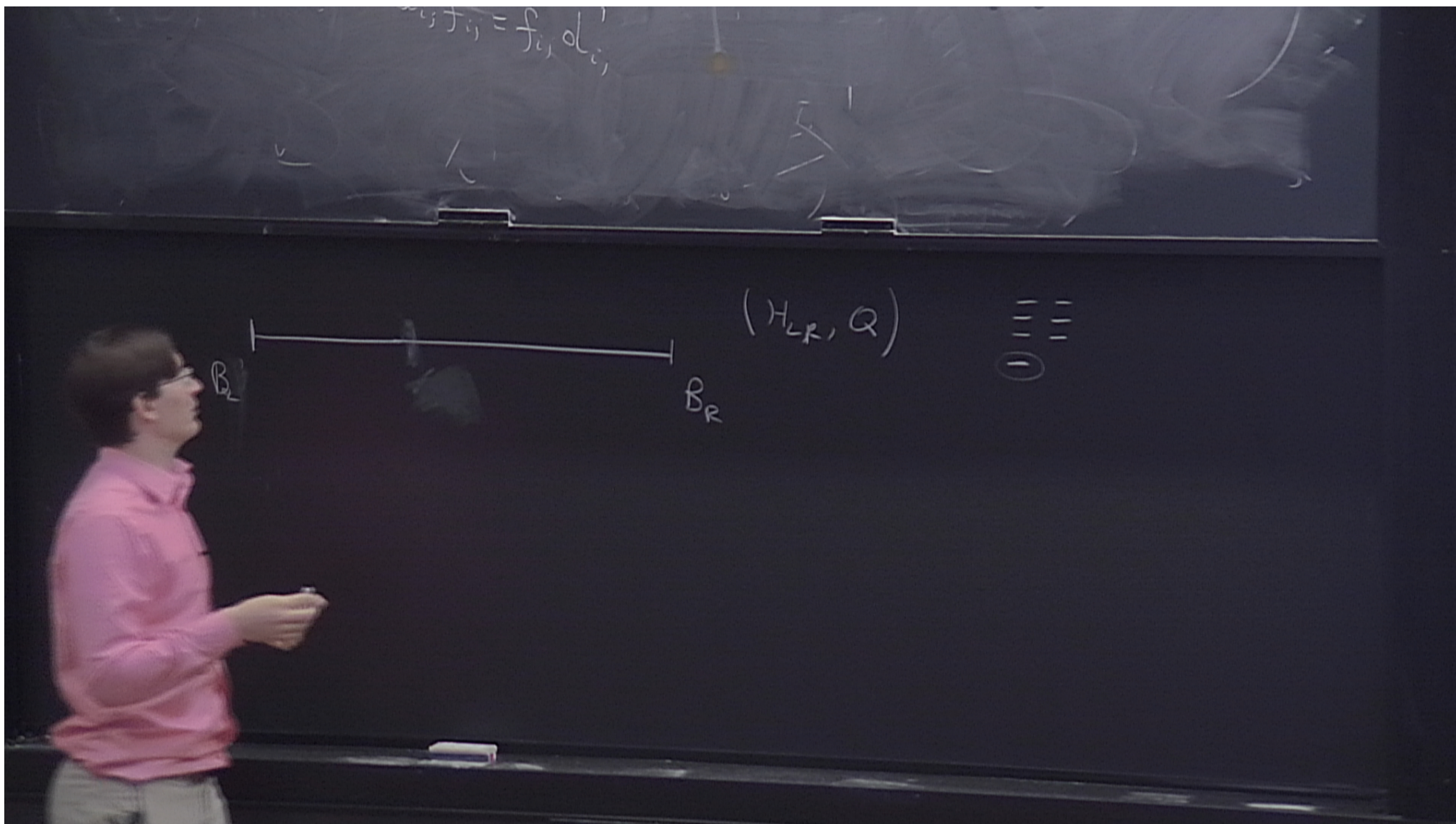


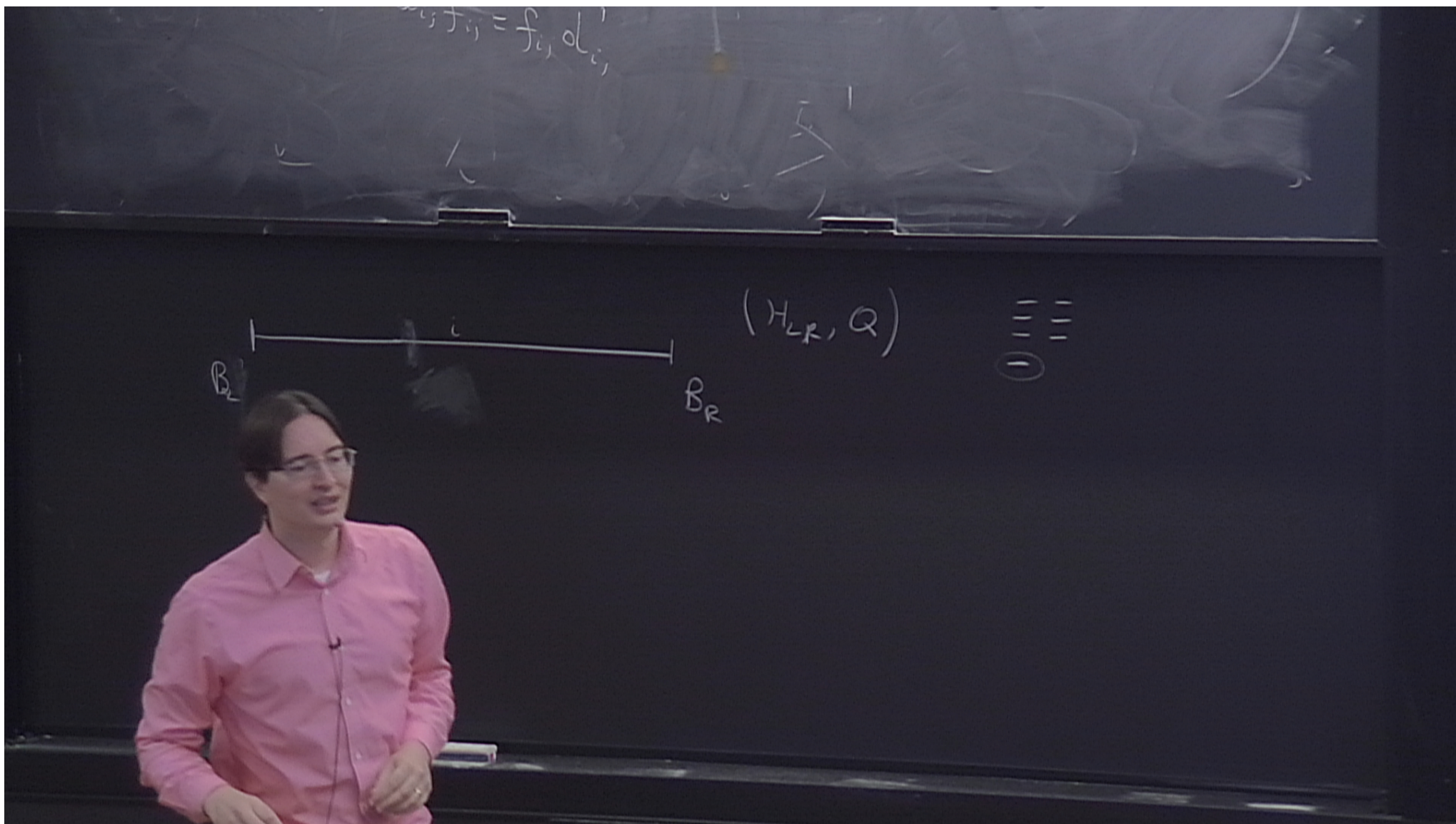


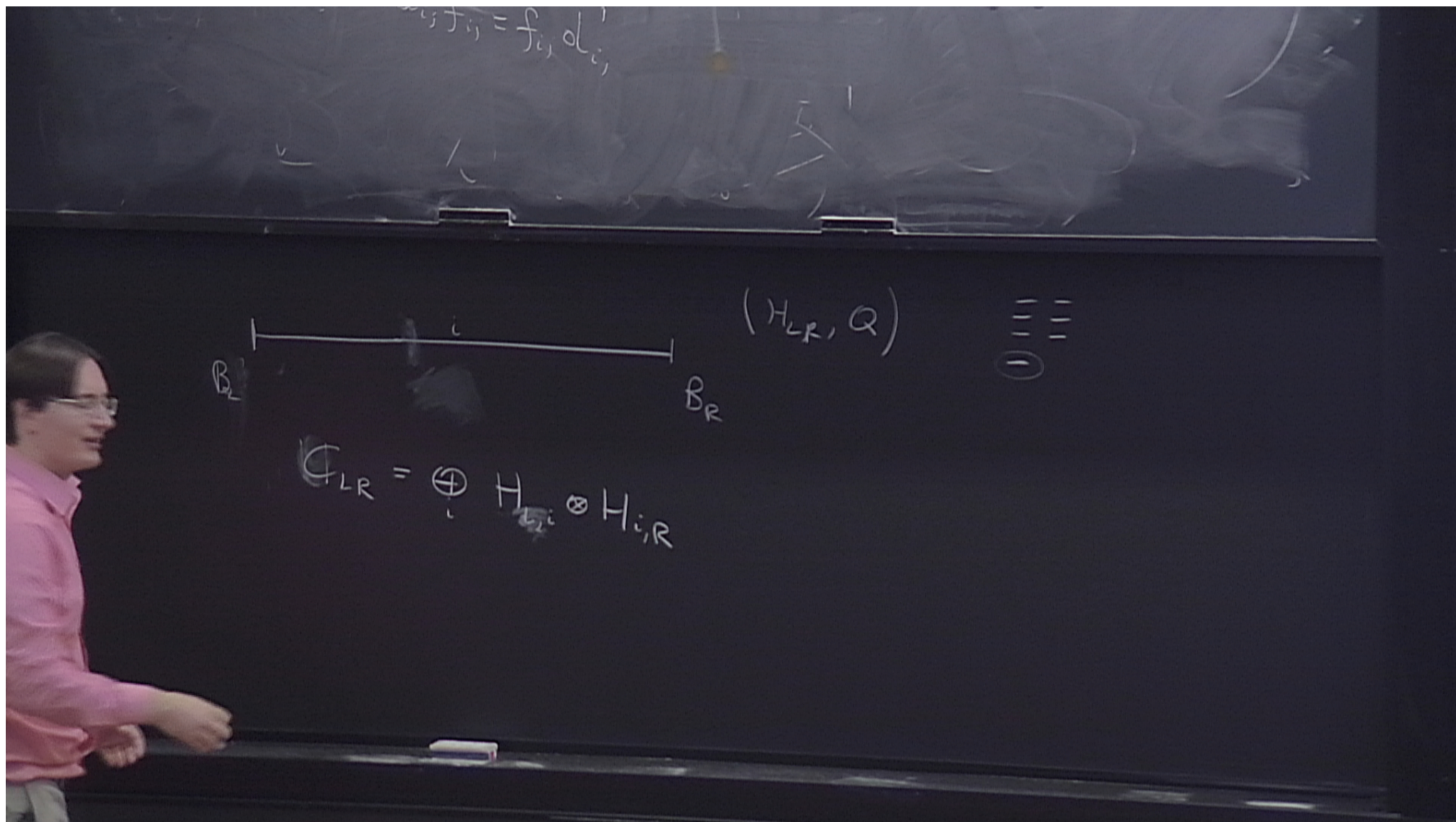




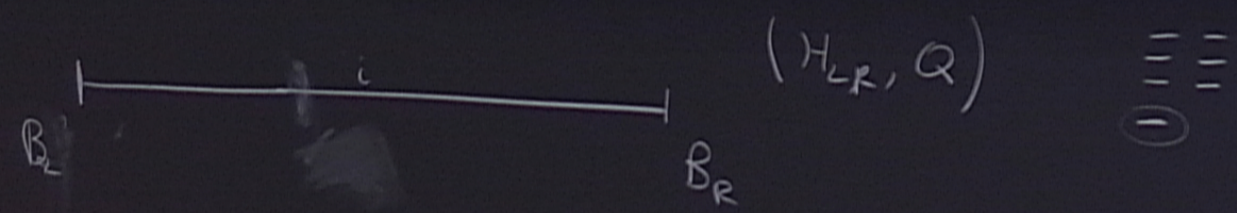








$$f_{ij} = f_{ij}, d_{ij}$$



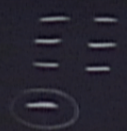
$$C_{LR} = \bigoplus_i H_{L,i} \otimes H_{i,R}$$



$$f_{i,j} = f_{i,j}, d_{i,j}$$



$$(H_{LR}, Q)$$

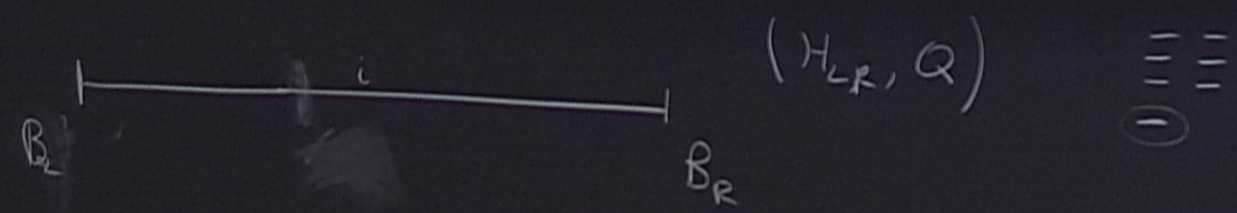


$$C_{LR} = \bigoplus_i H_{L,i} \otimes H_{i,R}$$

$$(C_{LR}, d_{LR})$$

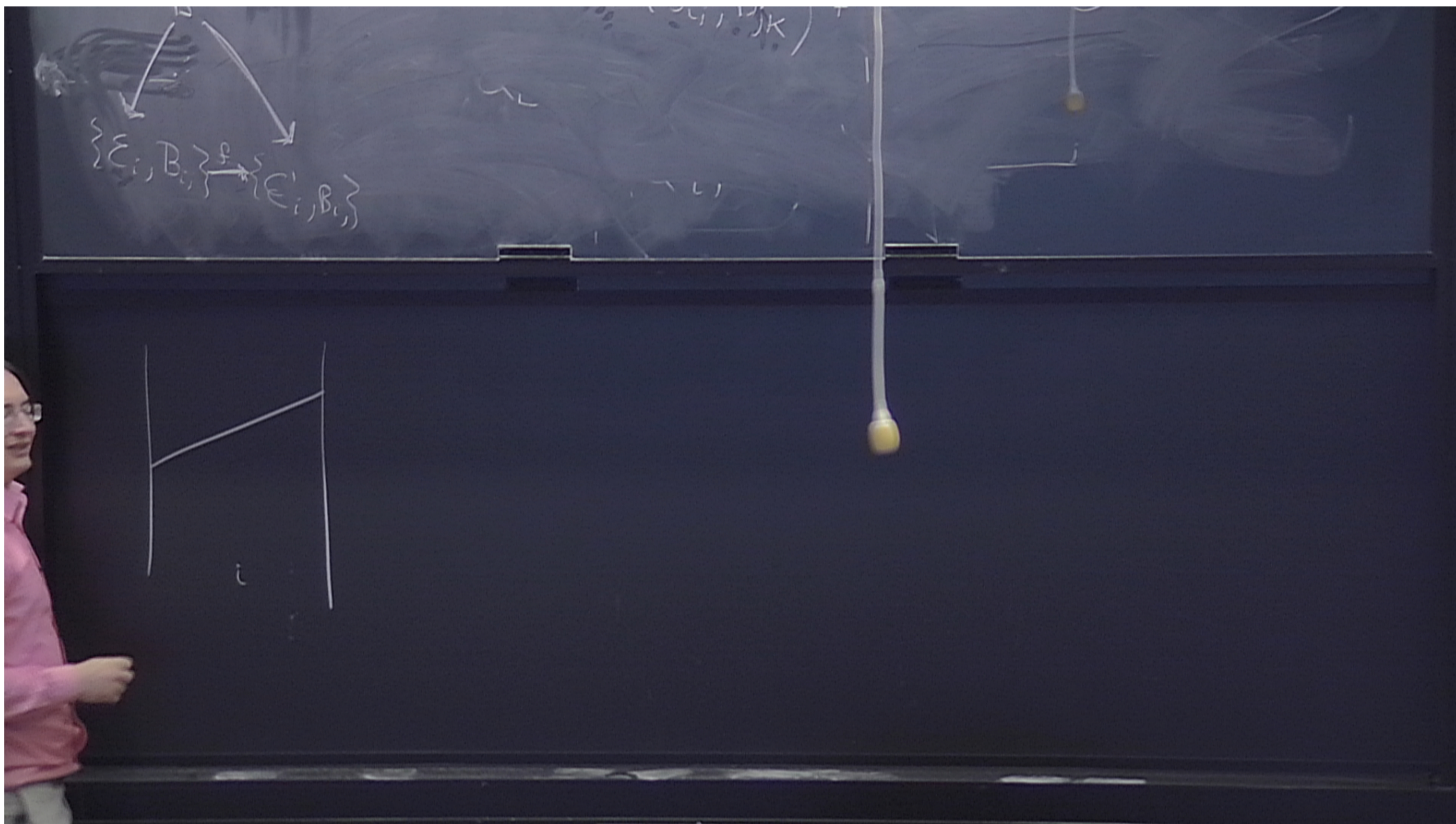


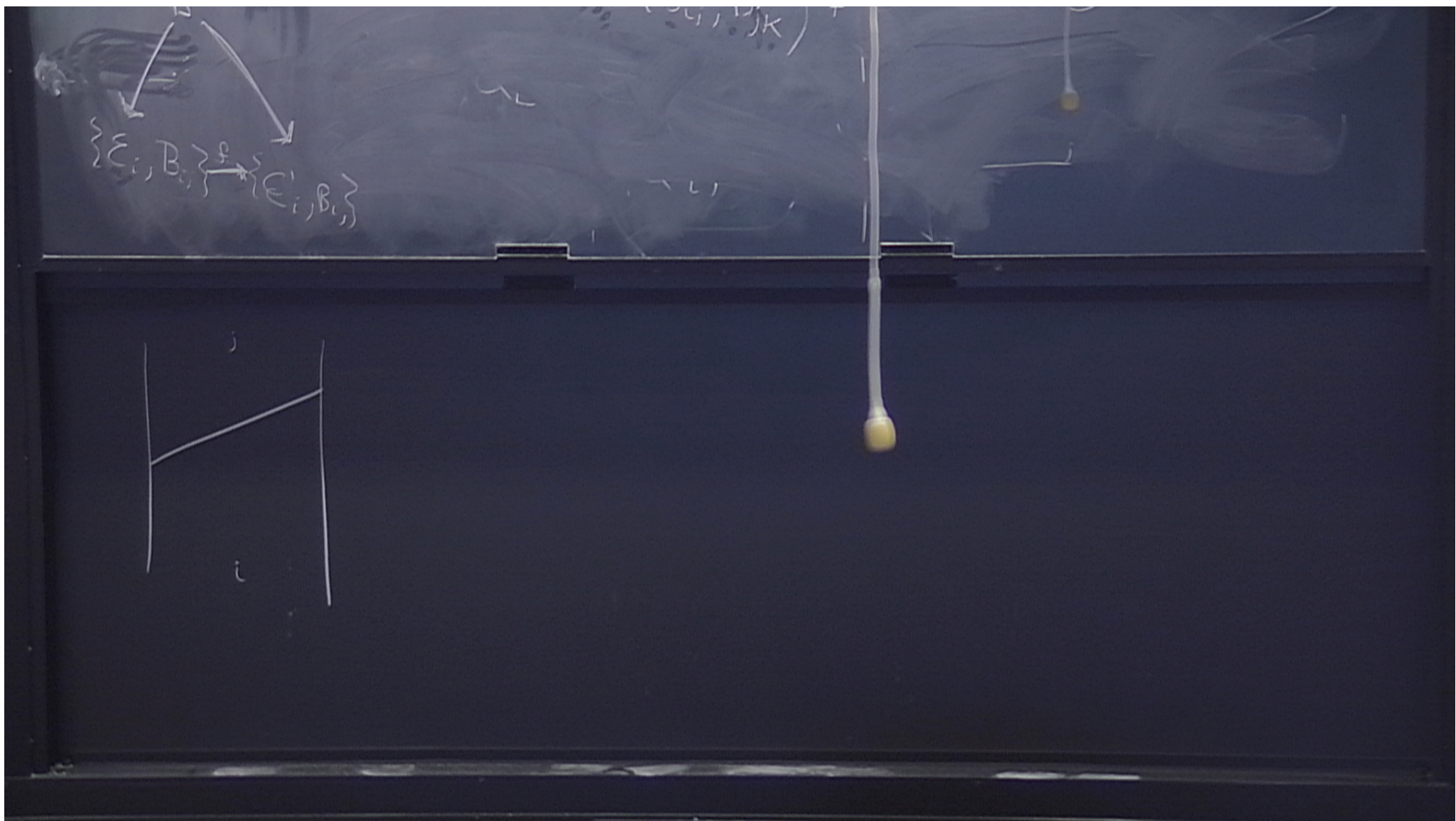
$$f_{i,j} = f_{i,j}, d_{i,j}$$

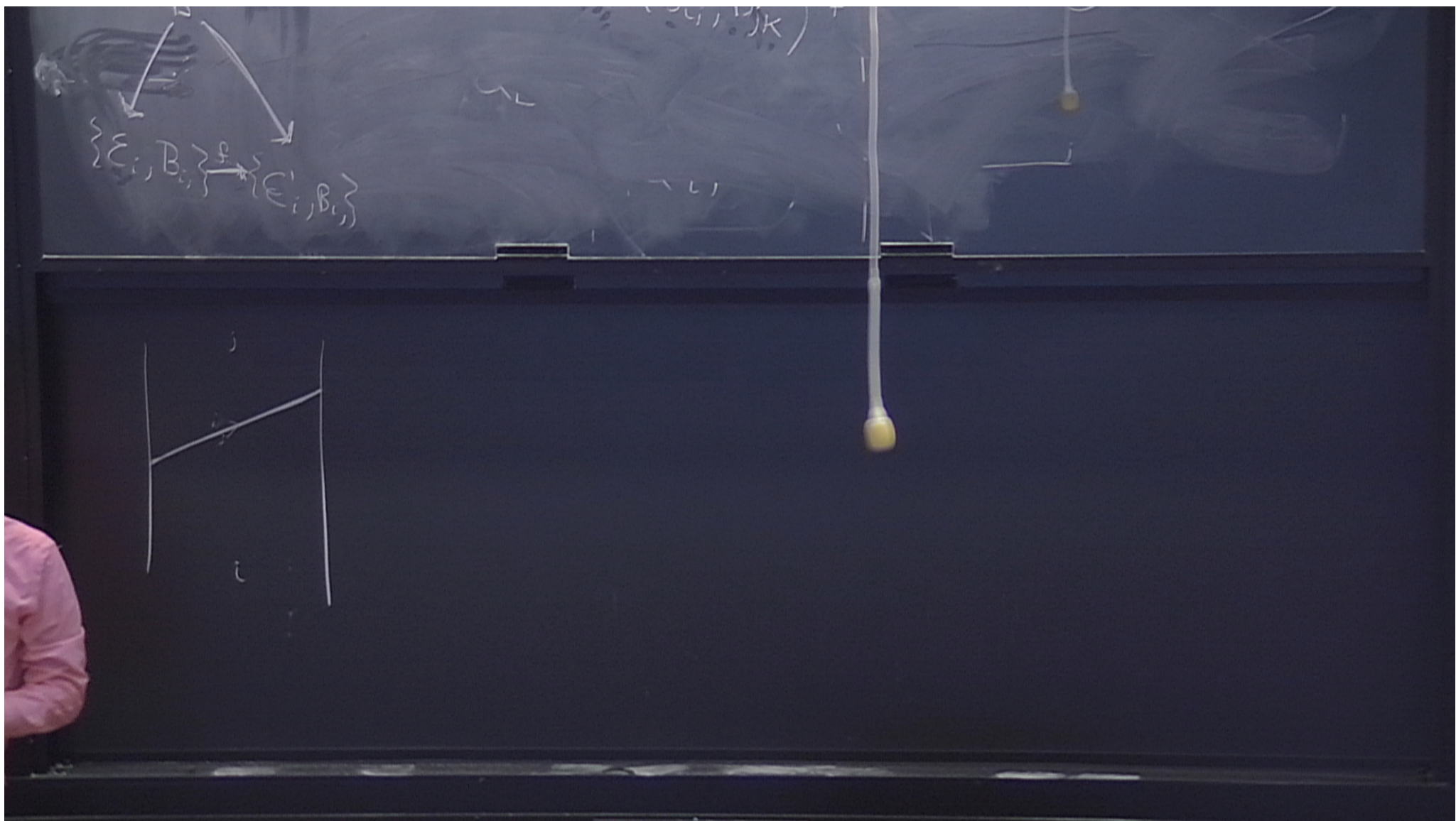


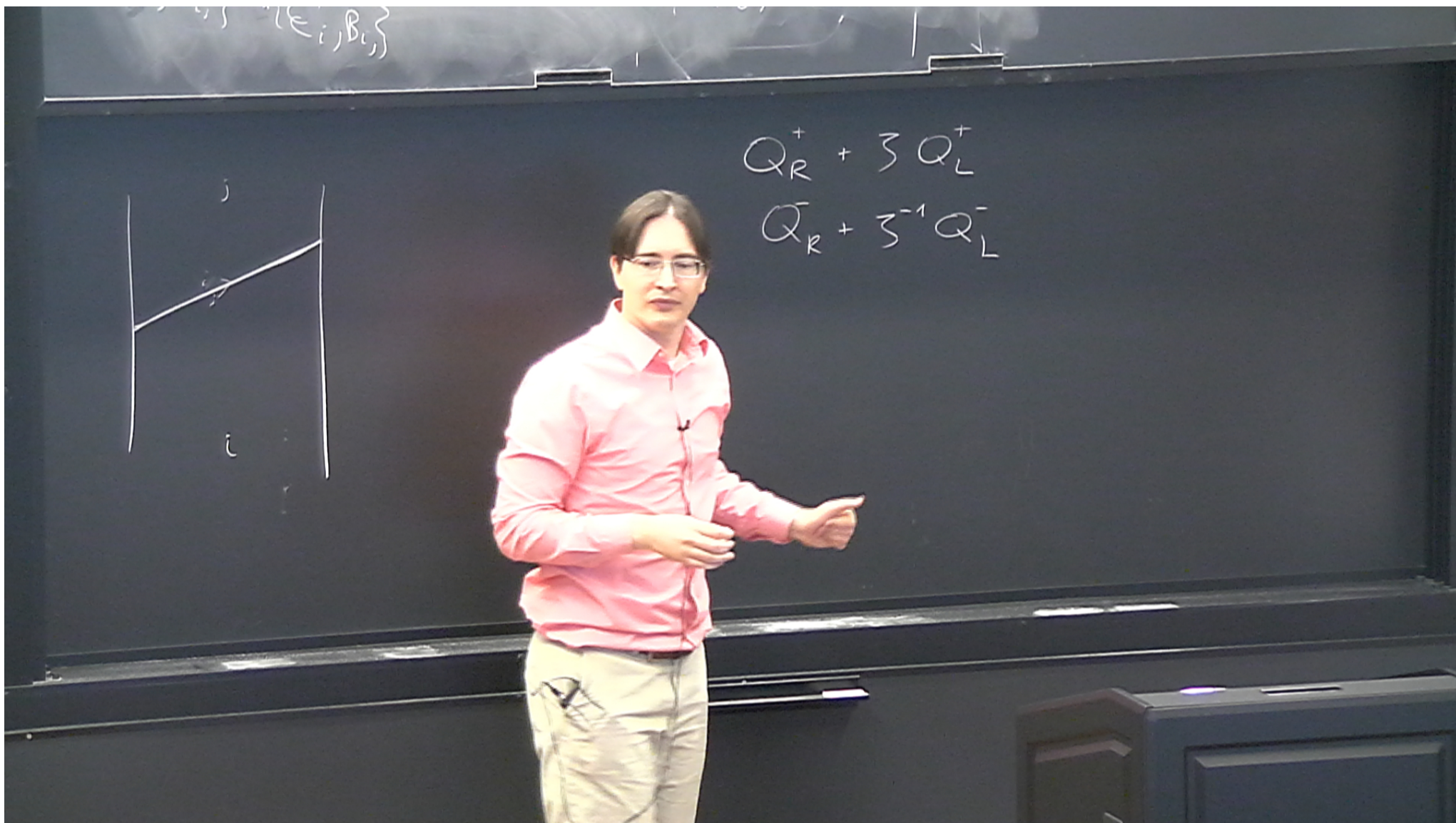
$$C_{LR} = \bigoplus_i H_{L,i} \otimes H_{i,R}$$

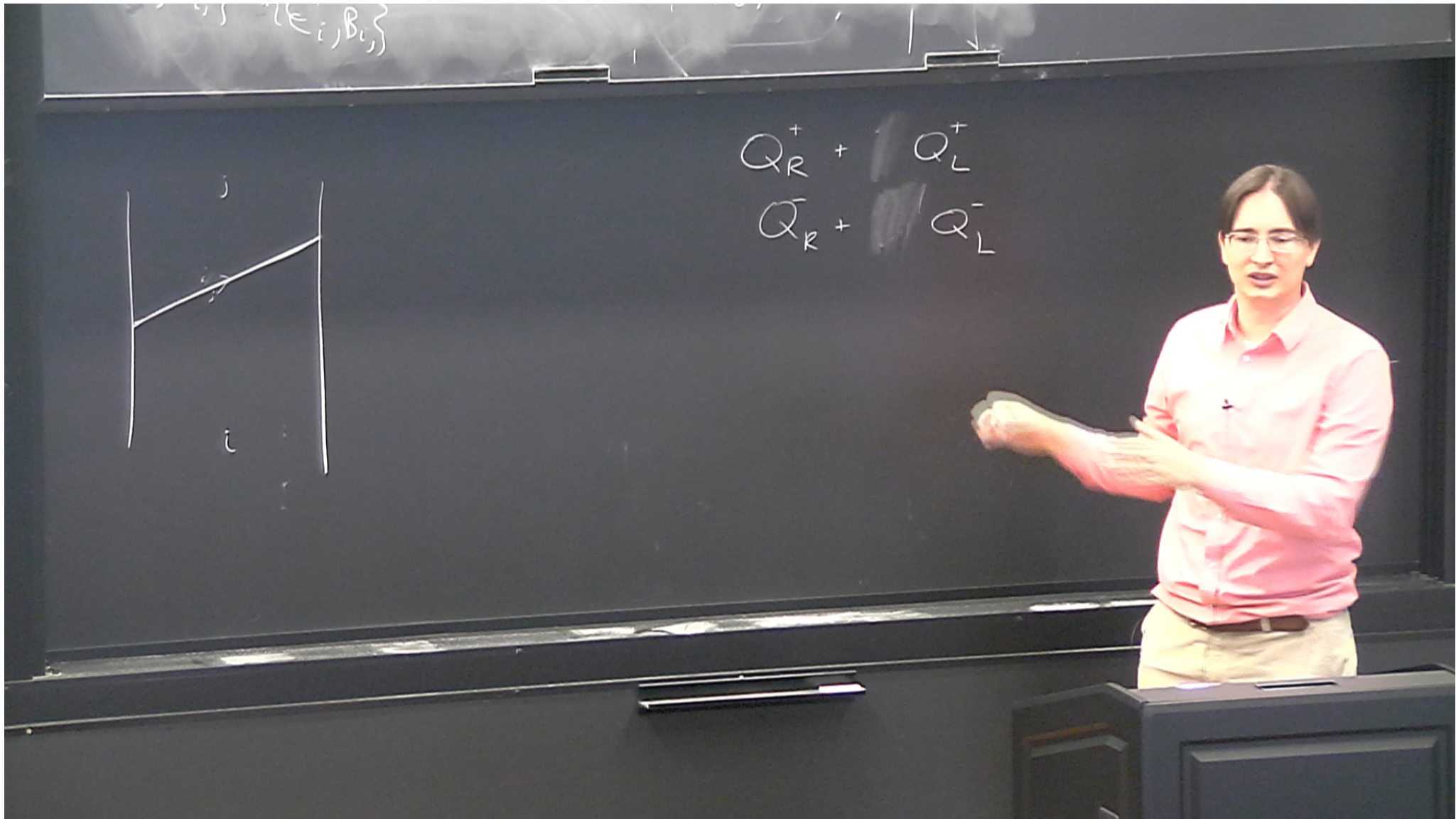
(C_{LR}, d_{LR})

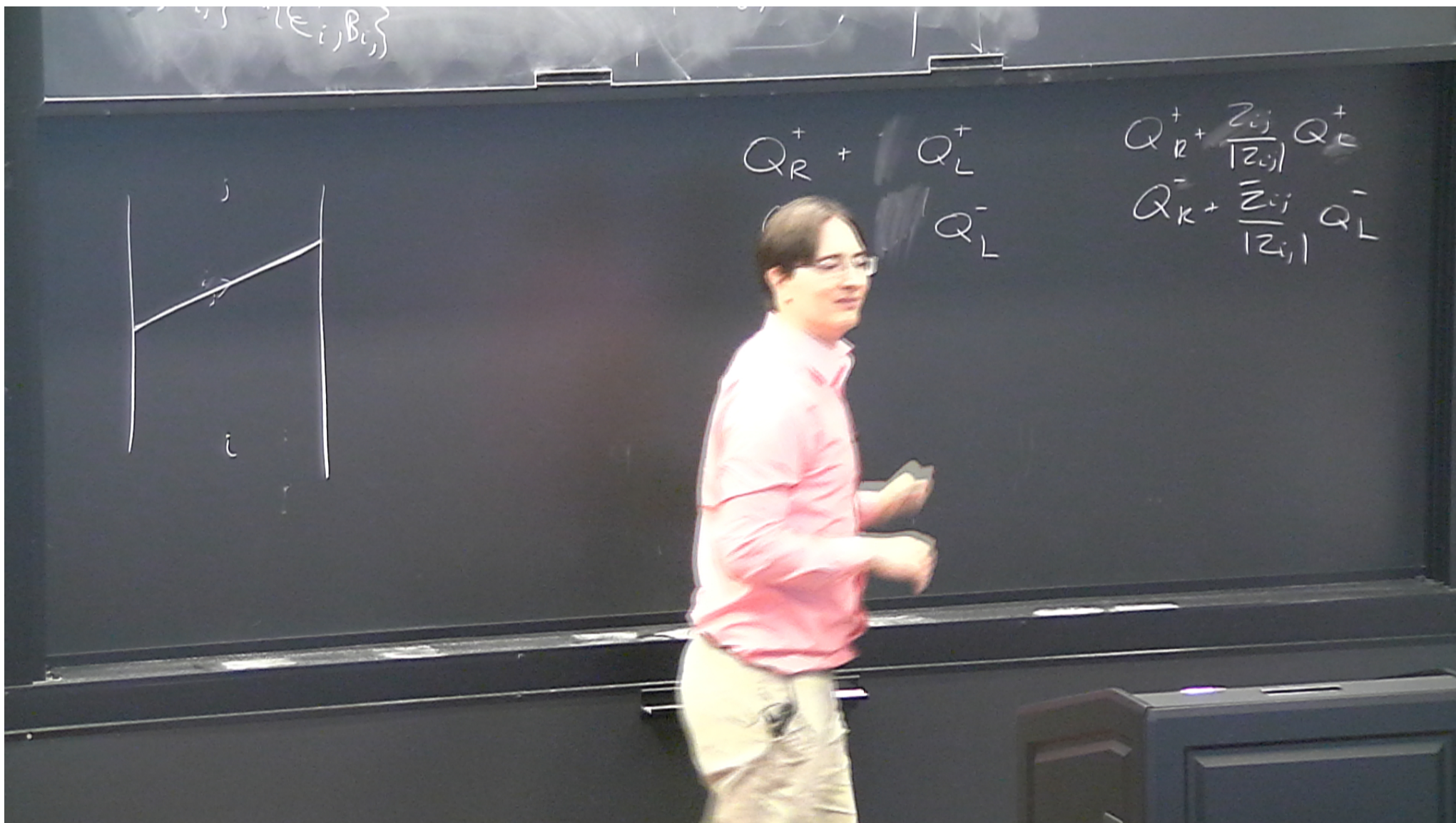


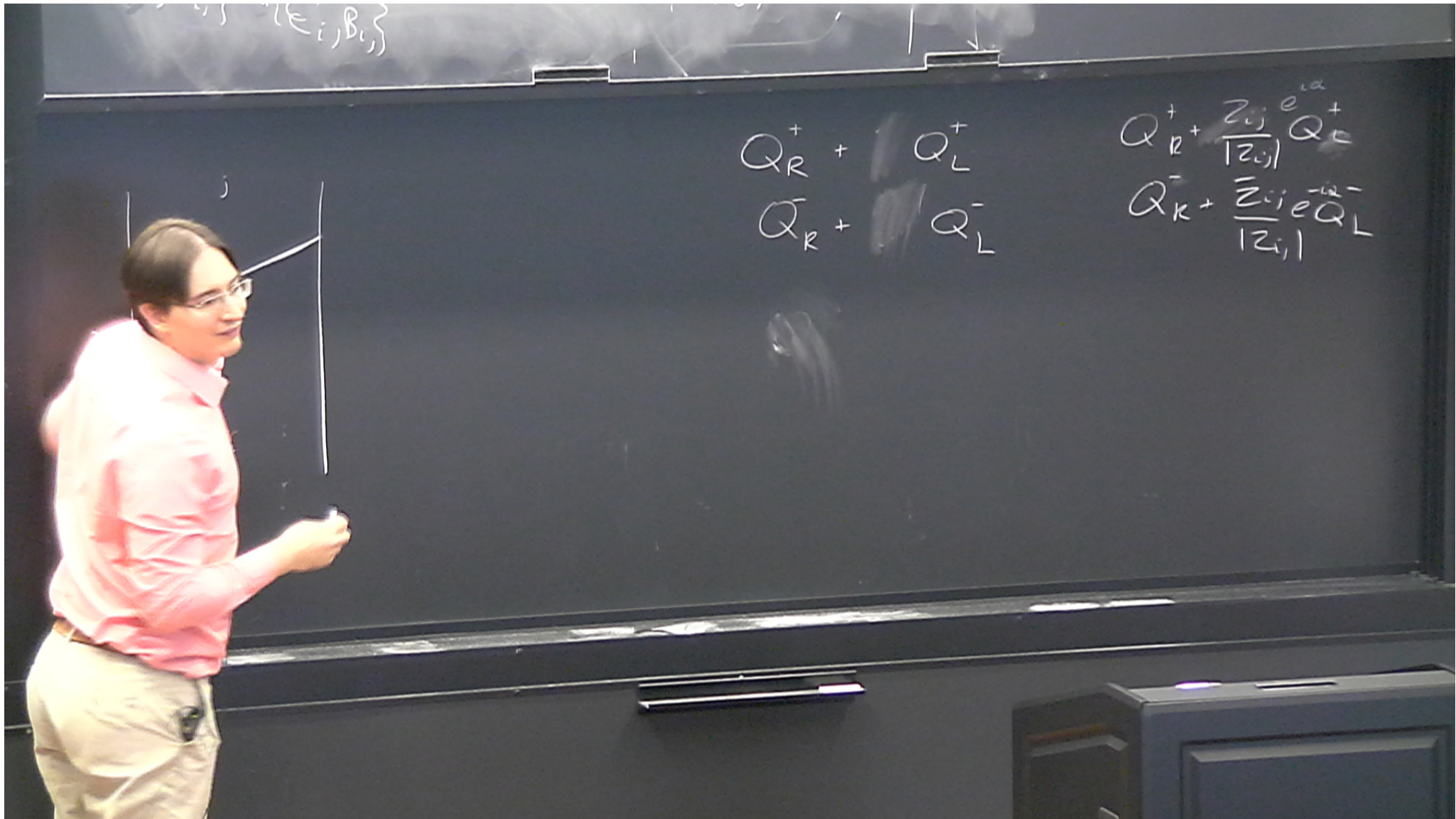


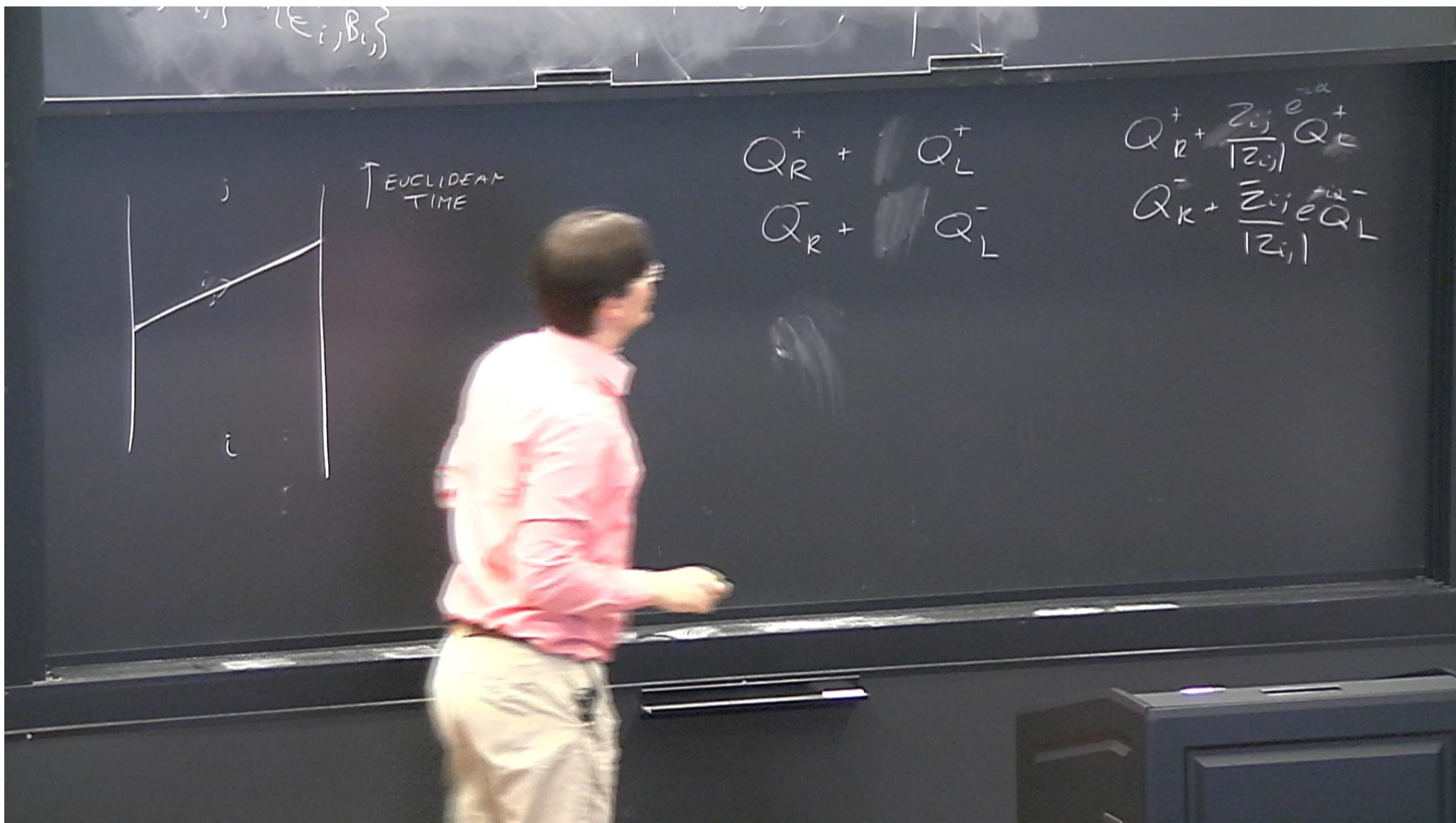


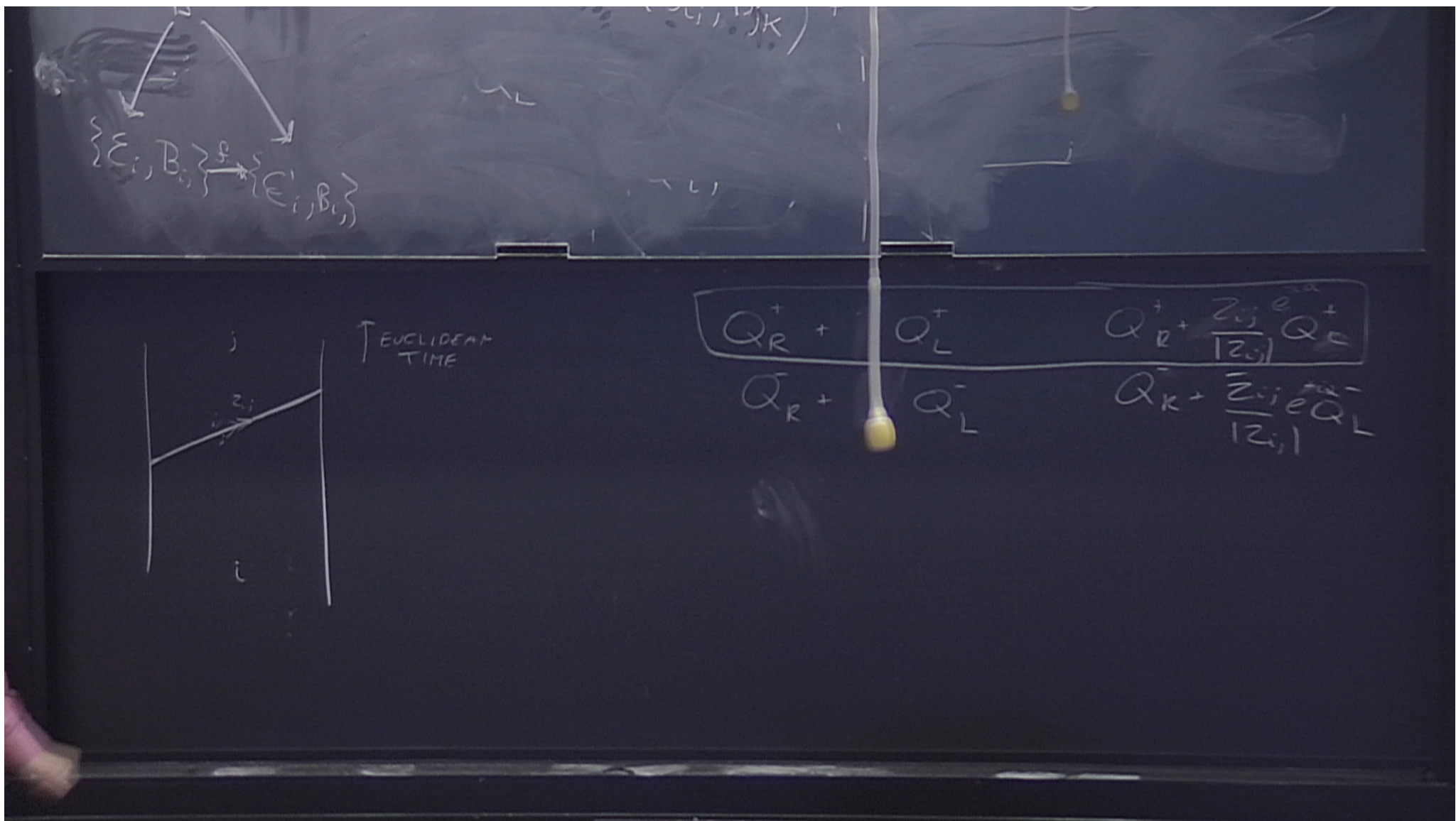


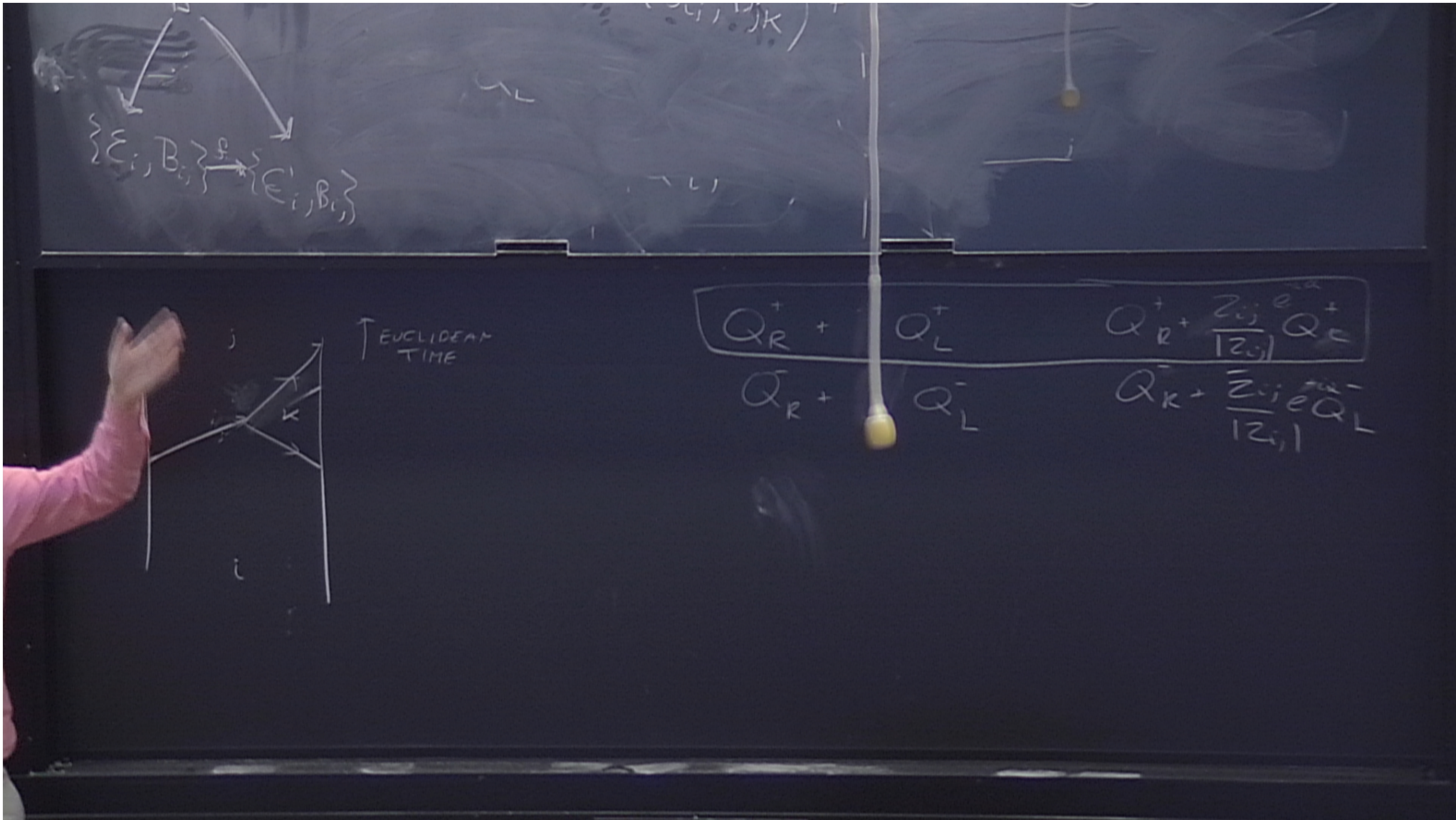












$$\{E_i, B_i\} \xrightarrow{F} \{E'_i, B'_i\}$$



↑ EUCLIDEAN
TIME

$$\begin{array}{cc} Q_R^+ + Q_L^+ & Q_R^+ + \frac{z_{ij} e^{+i\alpha}}{|z_{ij}|} Q_L^+ \\ \hline Q_R^- + Q_L^- & Q_R^- + \frac{\bar{z}_{ij} e^{-i\alpha}}{|z_{ij}|} Q_L^- \end{array}$$

$$\{E_i, B_i\} \xrightarrow{F} \{E'_i, B'_i\}$$



↑ EUCLIDEAN TIME

$$d_{LR} = \sum_{\text{RIGID WEBS ON STRIP}}$$

$$\begin{array}{cc} Q_R^+ & Q_L^+ \\ Q_R^- & Q_L^- \end{array} \quad Q_R^+ + \frac{Z_{ij} e^{i\alpha}}{|Z_{ij}|} Q_L^+ \quad Q_R^- + \frac{\bar{Z}_{ij} e^{i\alpha}}{|Z_{ij}|} Q_L^-$$

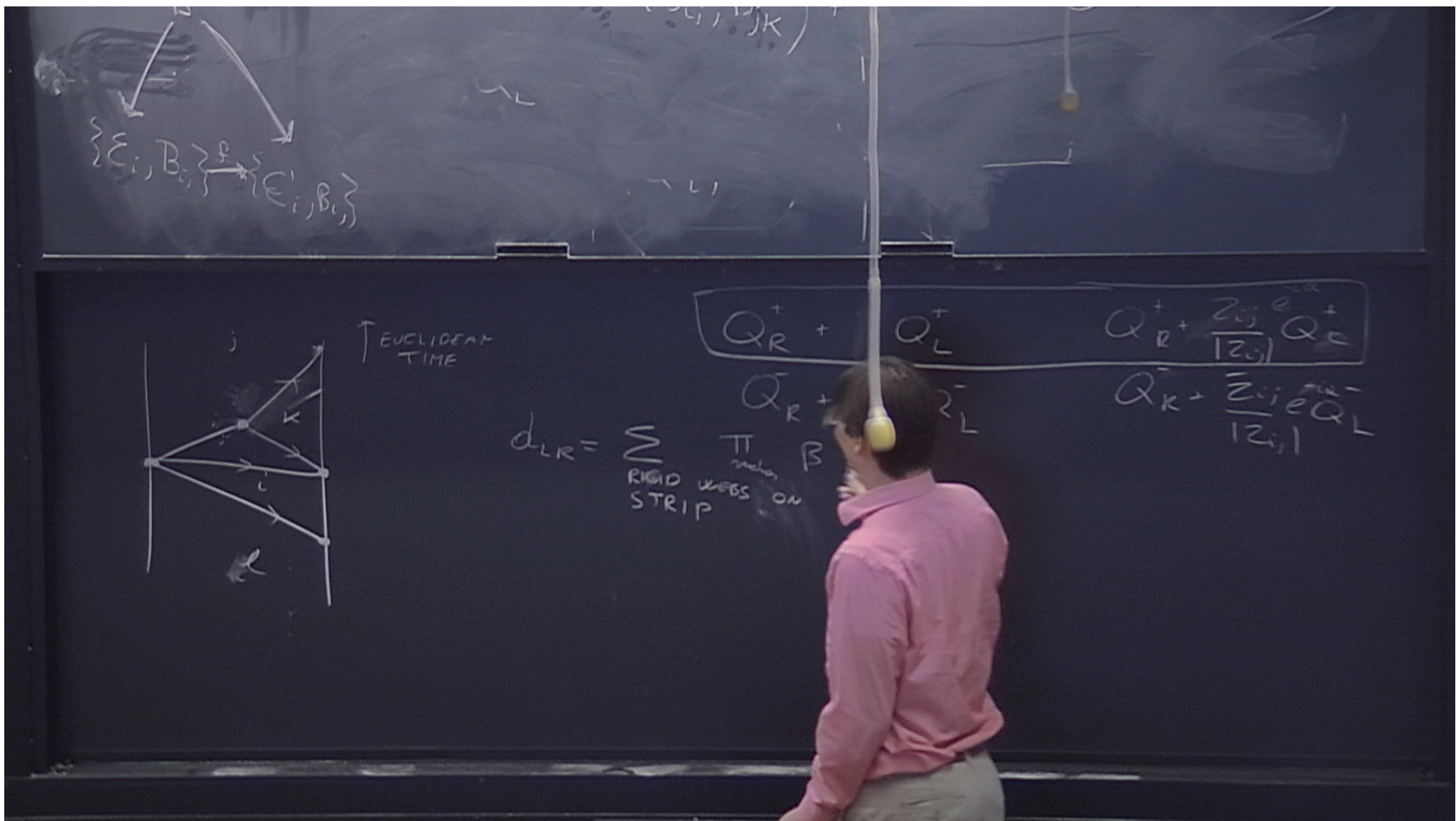
$$\{E_i, B_i\} \xrightarrow{F} \{E'_i, B'_i\}$$

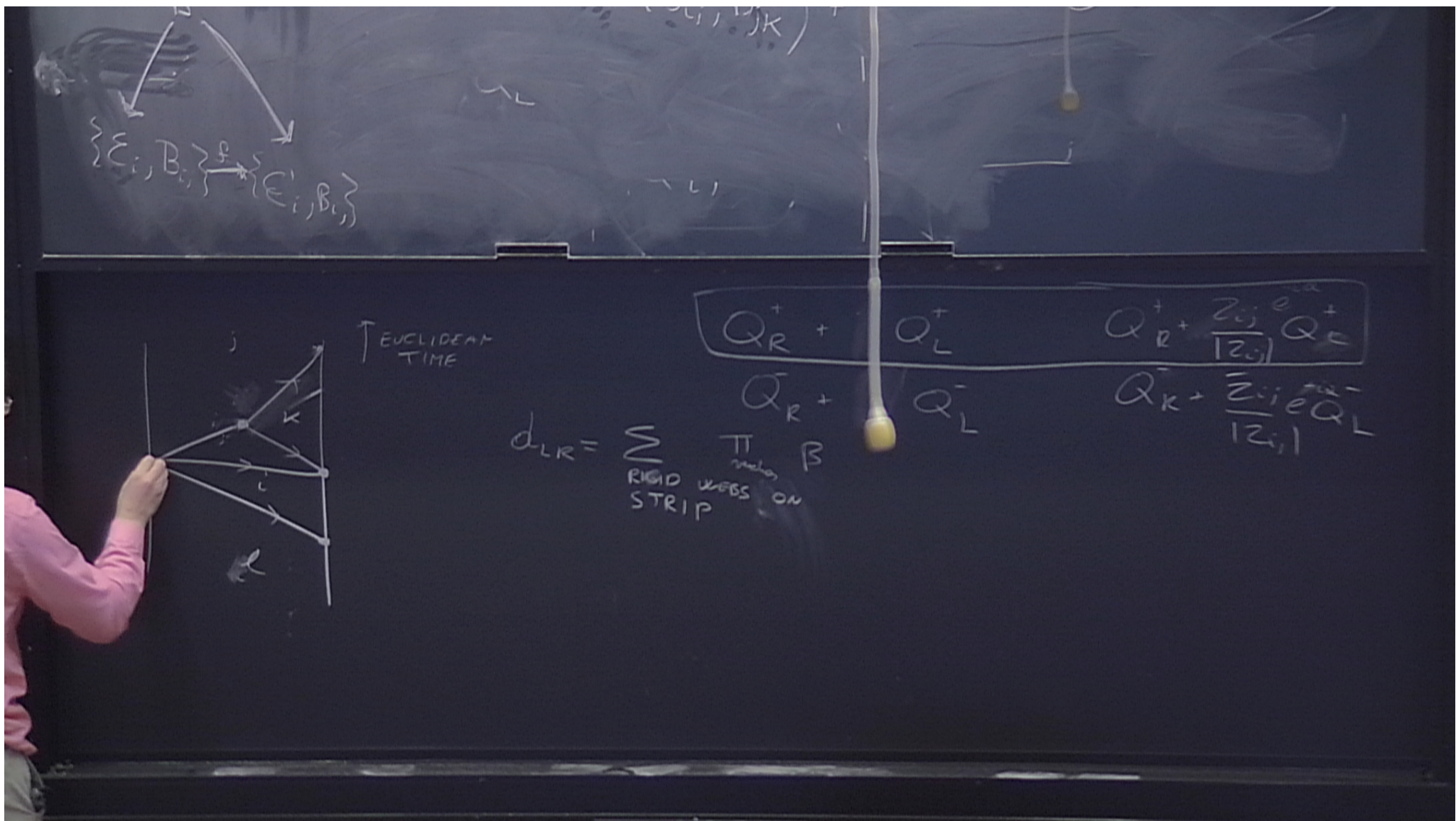


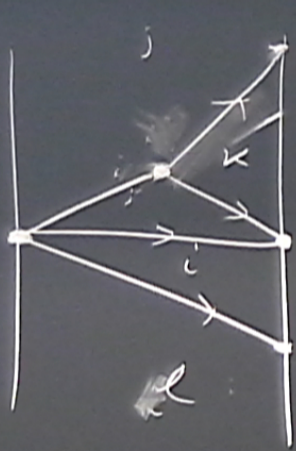
↑ EUCLIDEAN TIME

$$d_{LR} = \sum_{\text{RIGID WEBS ON STRIP}}$$

$$\begin{array}{cc} Q_R^+ & Q_L^+ \\ Q_R^- & Q_L^- \end{array} \quad \begin{array}{c} Q_R^+ + \frac{Z_{ij} e^{+i\alpha}}{|Z_{ij}|} Q_L^+ \\ Q_R^- + \frac{\bar{Z}_{ij} e^{-i\alpha}}{|Z_{ij}|} Q_L^- \end{array}$$







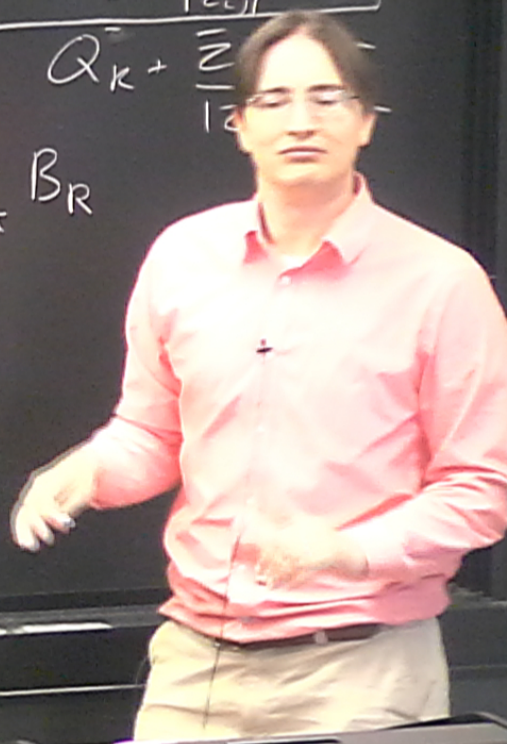
↑ EUCLIDEAN TIME

$$Q_R^+ + Q_L^+ \quad Q_R^+ + \frac{z_{ij} e^{-\alpha}}{|z_{ij}|} Q_L^+$$

$$Q_R^+ \quad Q_L^+$$

$$Q_R^+ + \frac{\bar{z}_{ij}}{|\bar{z}_{ij}|} Q_L^+$$

$$d_{LR} = \sum_{\text{RIGID WEBS ON STRIP}} \pi_{\text{web}} \beta \quad \pi_{\text{LEFT}} \beta_L \quad \pi_{\text{RIGHT}} \beta_R$$



$$\{E_i, B_i\} \xrightarrow{P} \{E'_i, B'_i\}$$



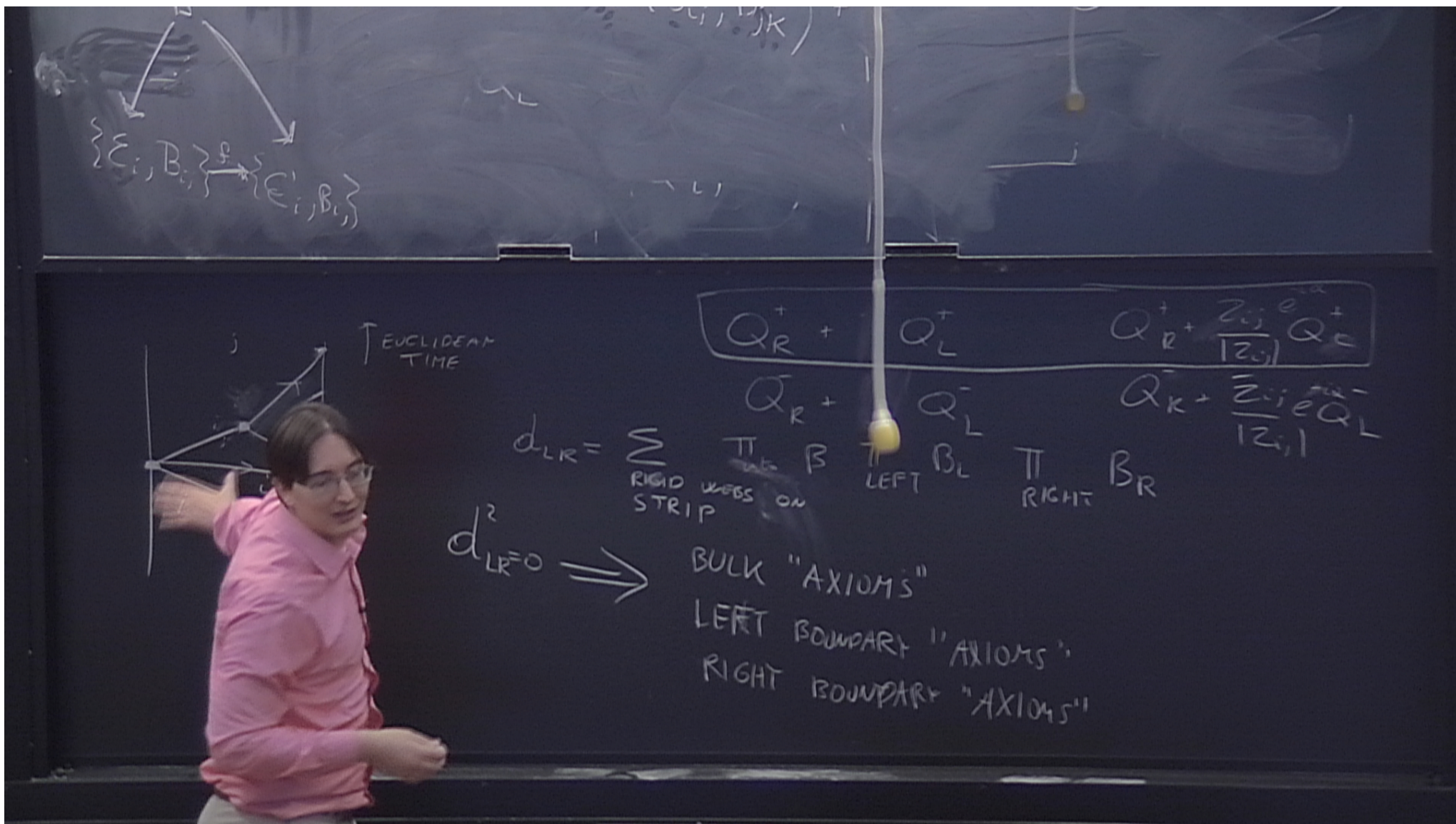
↑ EUCLIDEAN TIME

$$d_{LR}^2 \Rightarrow$$

$$d_{LR} = \sum_{\text{RIGID WEBS ON STRIP}} \pi_{\text{left}} \beta$$

Q_R^+	Q_L^+	$Q_R^+ + \frac{z_{ij} e^{+i\alpha}}{ z_{ij} } Q_L^+$
Q_R^-	Q_L^-	$Q_R^- + \frac{\bar{z}_{ij} e^{-i\alpha}}{ \bar{z}_{ij} } Q_L^-$

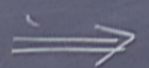
$\pi_{\text{LEFT}} \beta_L$ $\pi_{\text{RIGHT}} \beta_R$



LEVEL 3

$$\sum \varepsilon_i \otimes T_i$$

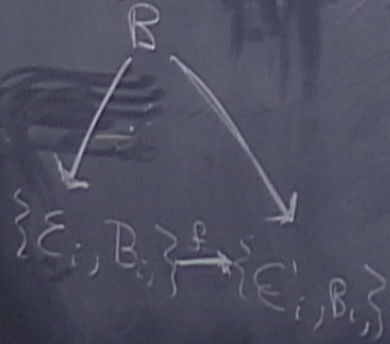
BRANE



$$\{\varepsilon_i\}$$

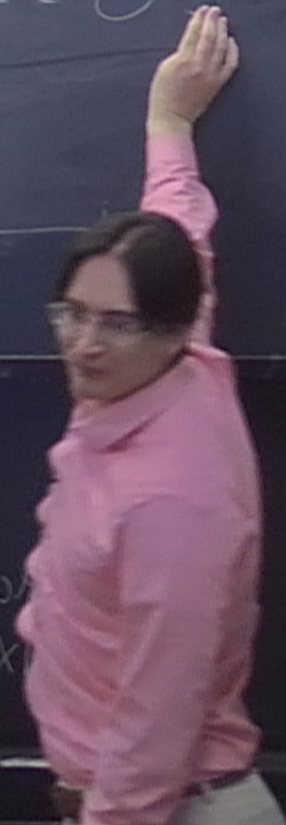
$$B_{ij} \in \varepsilon_i \otimes R_{ij} \otimes \varepsilon_j^*$$

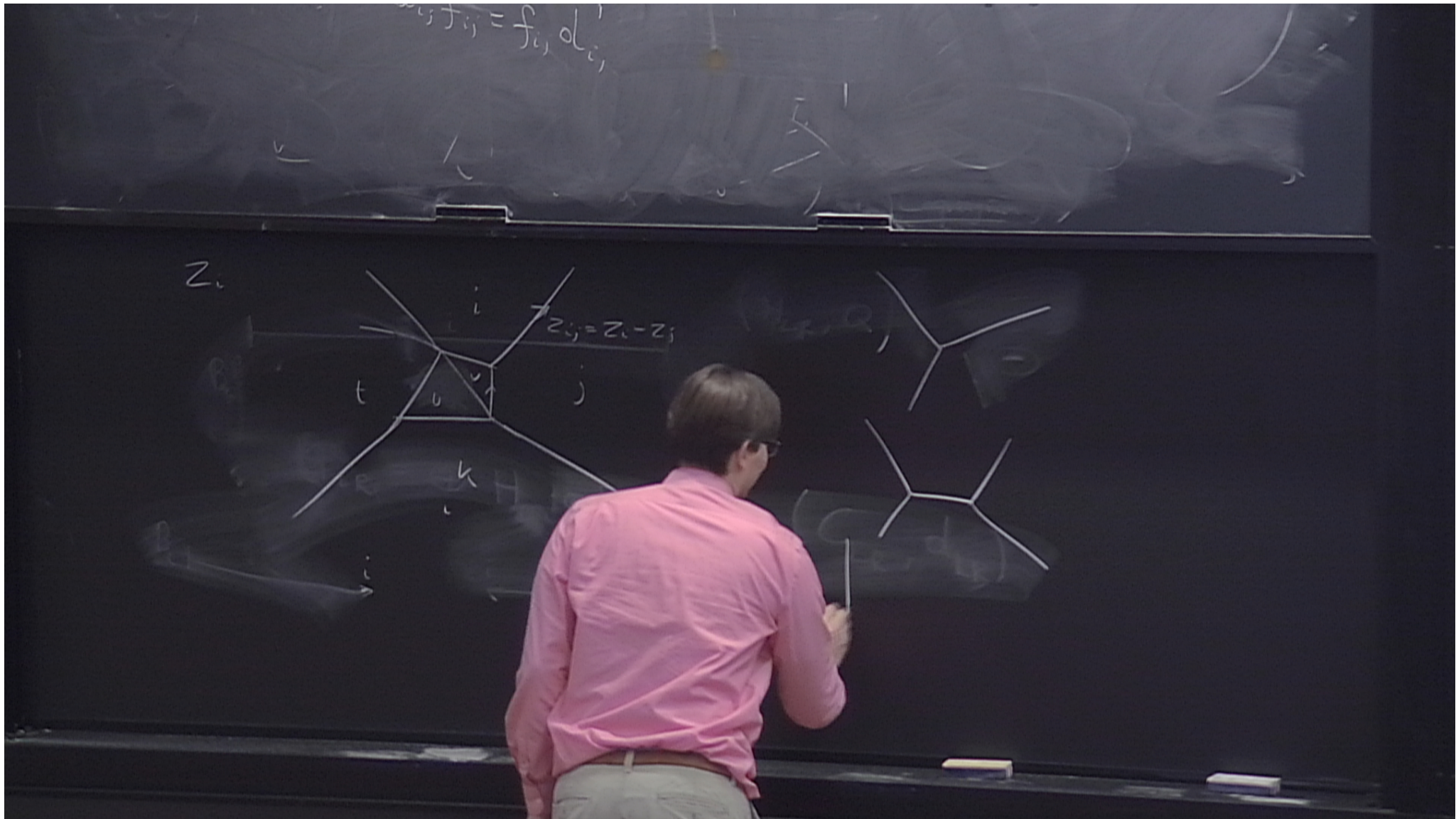
$$d_{ij}(B_{ij}) + d_{ijk}(B_{ij}, B_{jk}) + \dots = 0$$

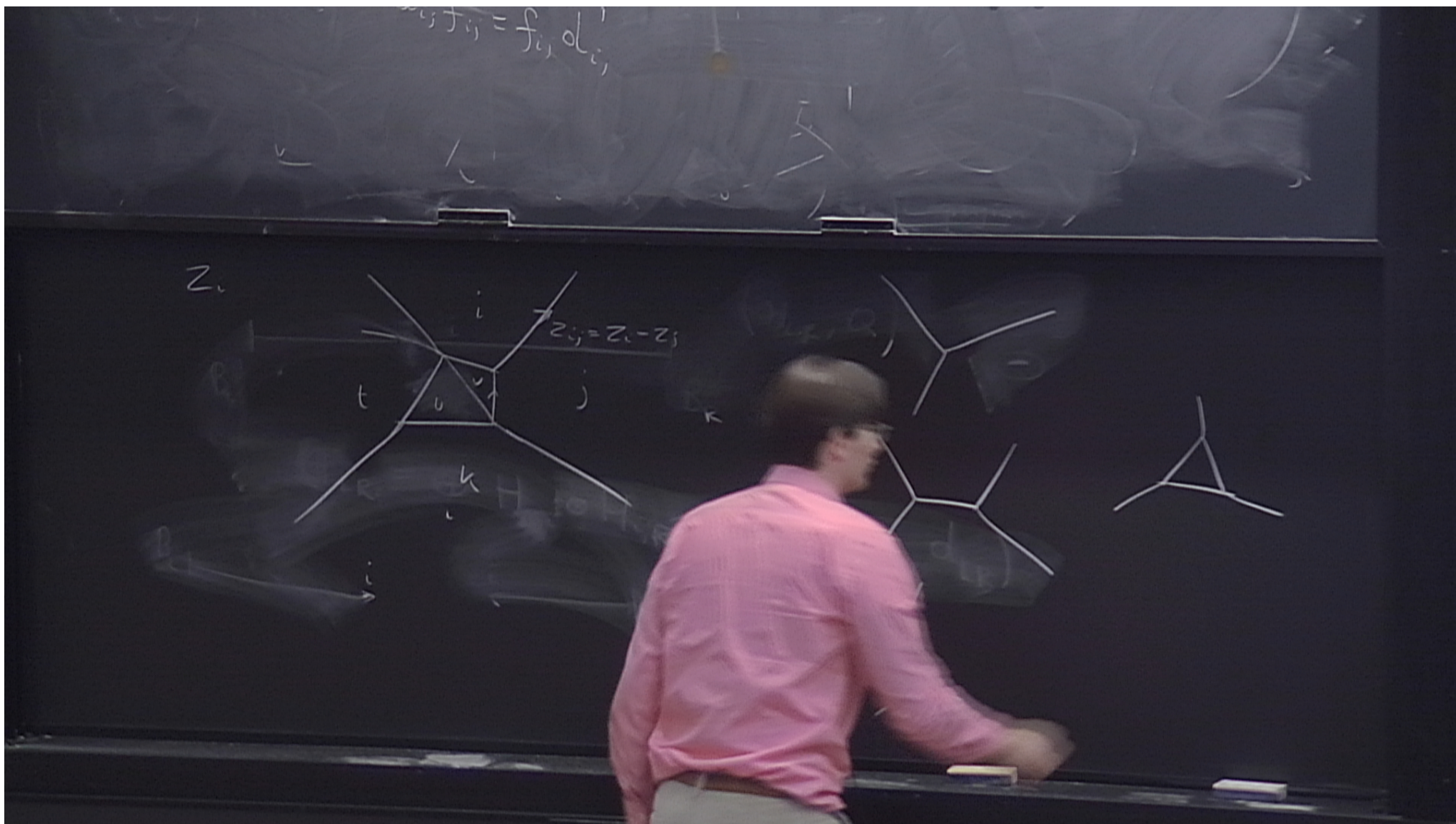


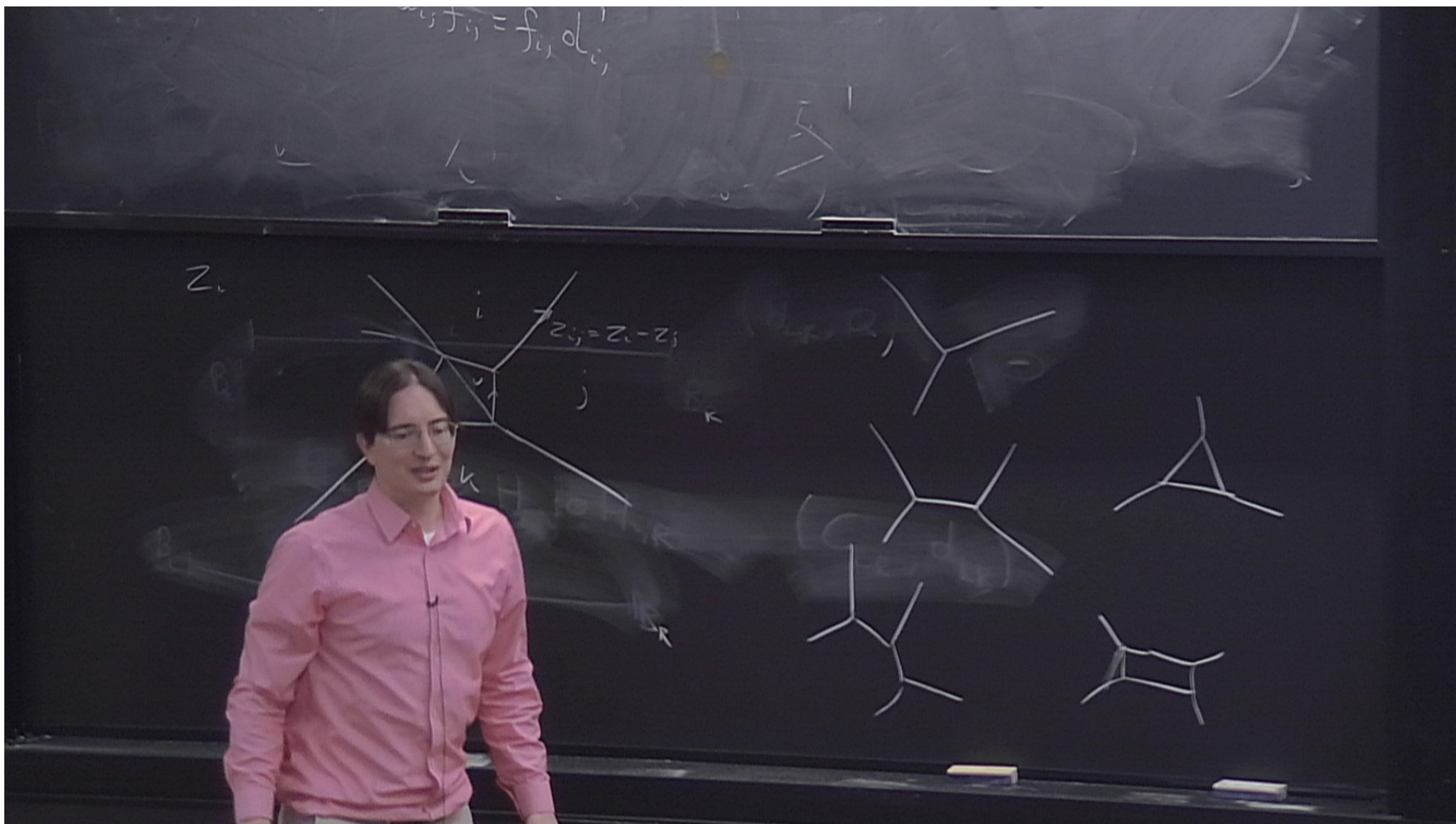
$$d_{LR=0} \Rightarrow$$

BULK "AXIOMS"
LEFT BOUNDARY "AXIOM"
RIGHT BOUNDARY "AXIOM"

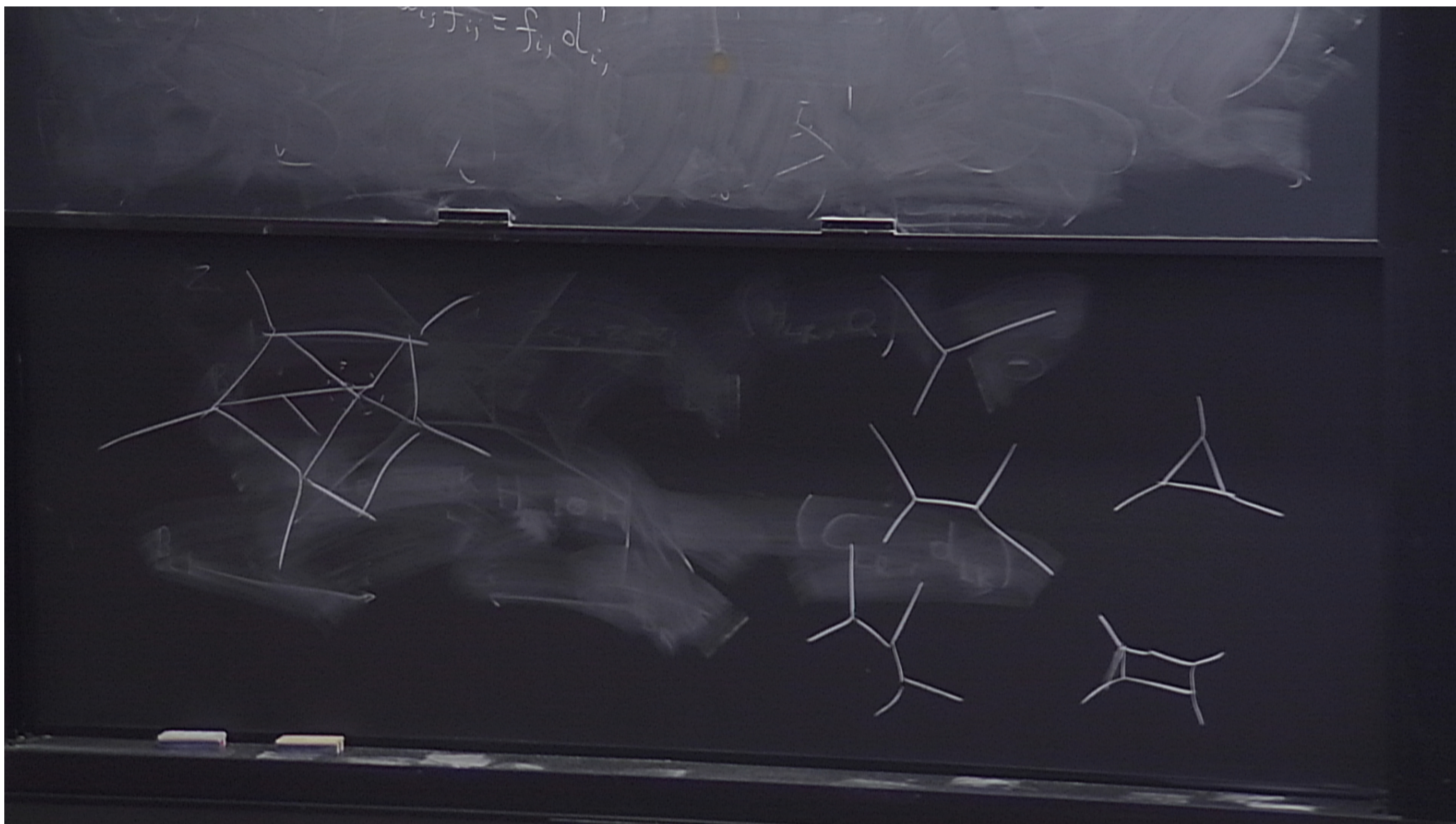


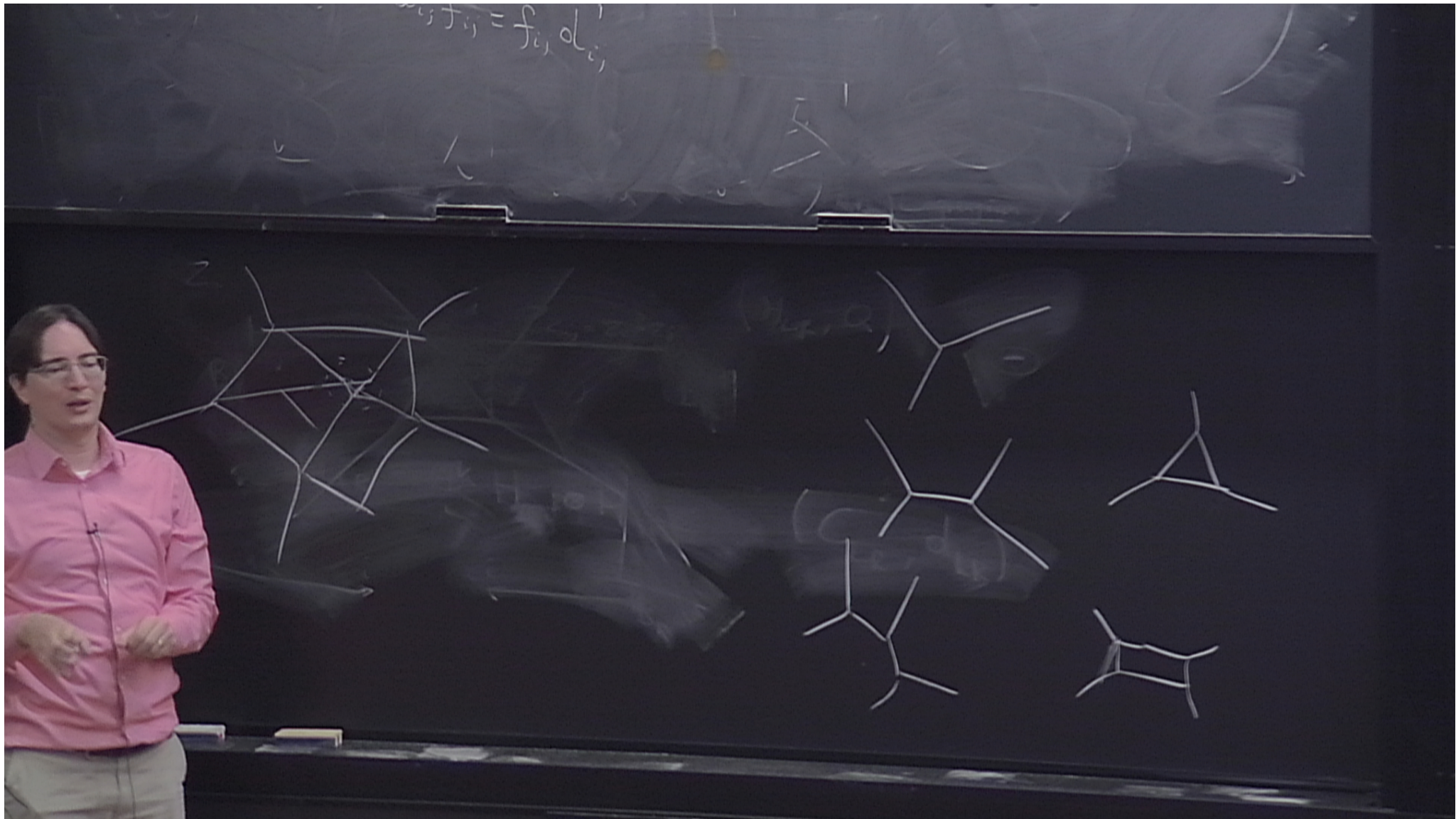


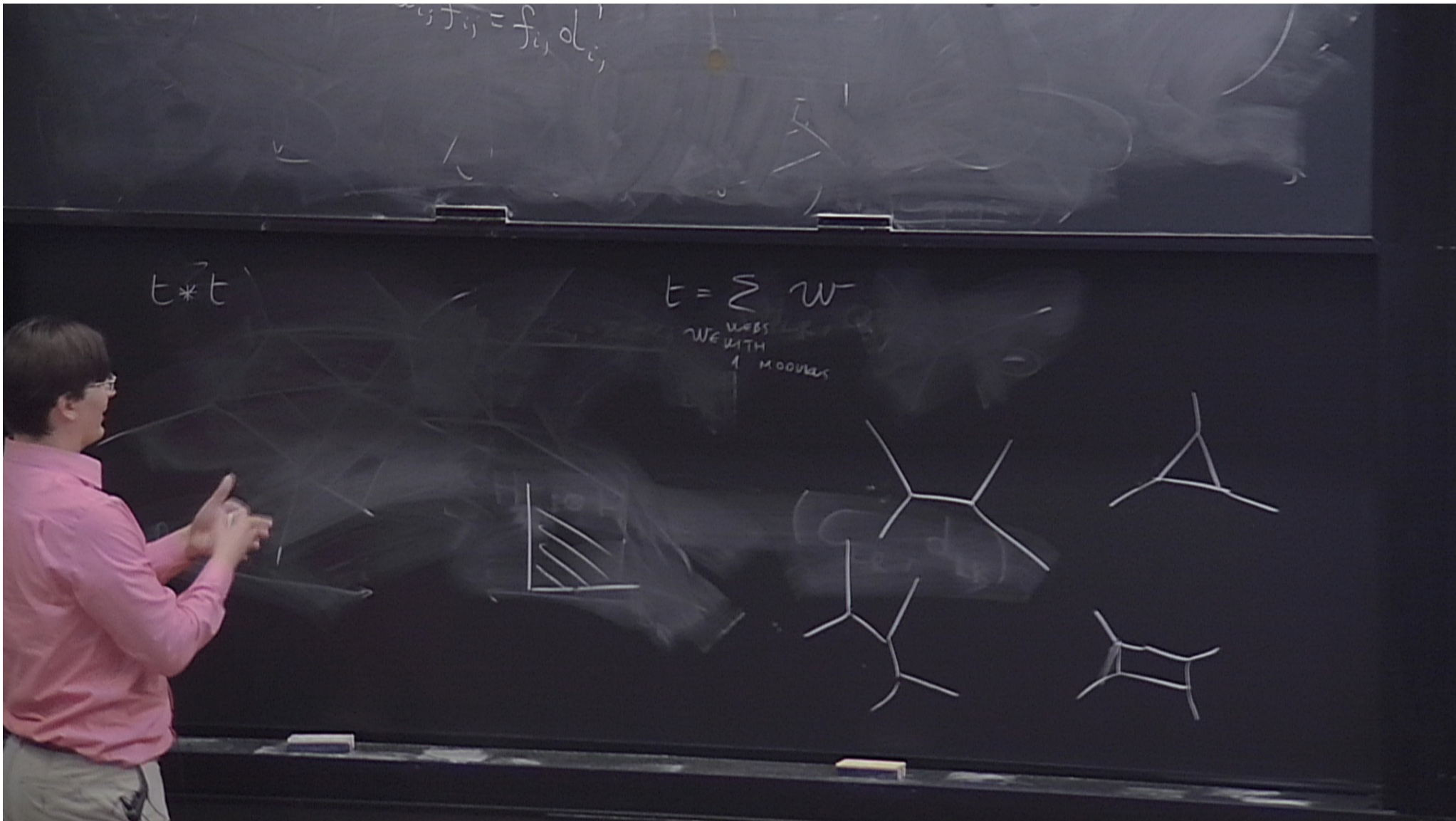


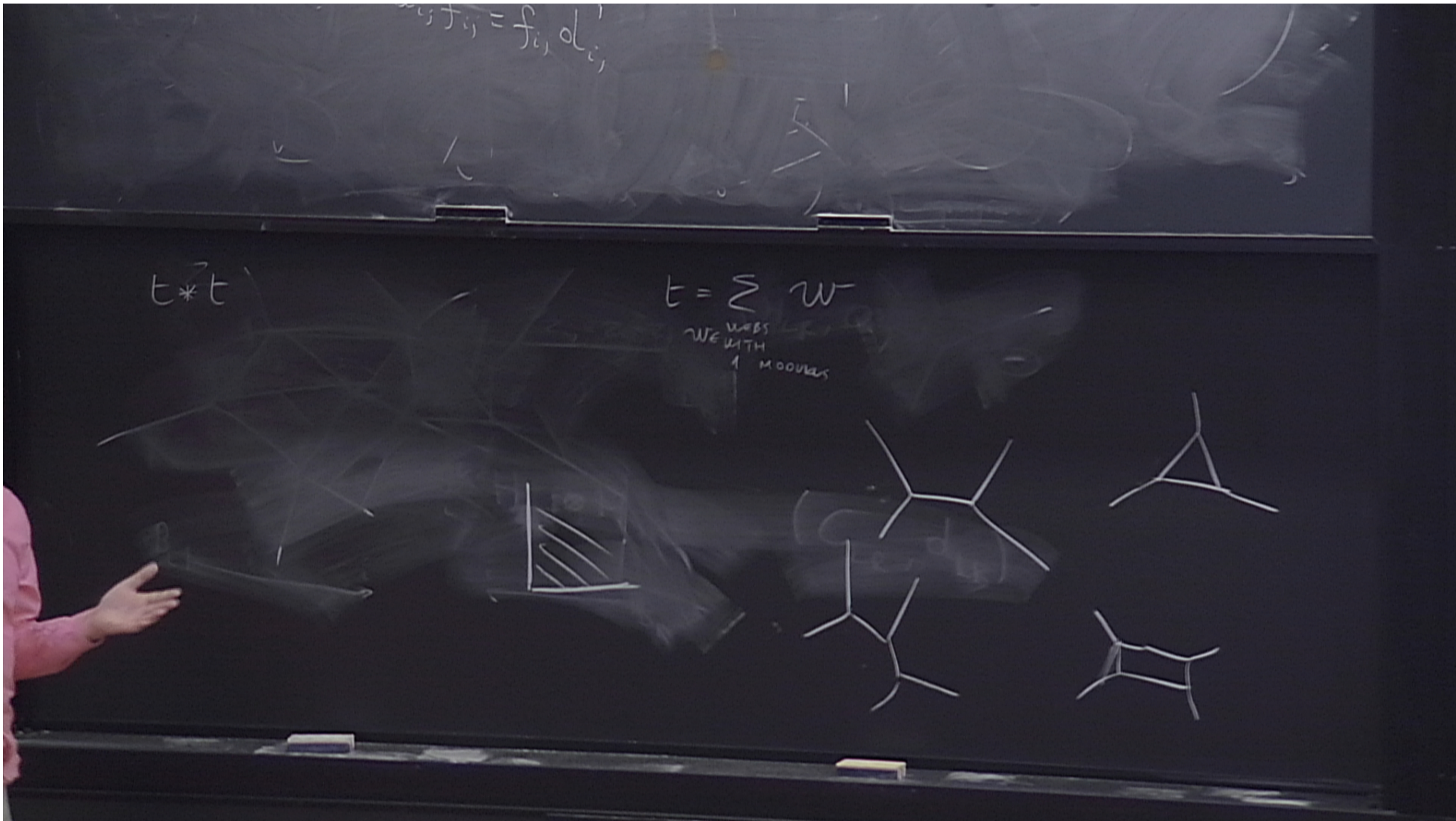












$$f_{ij} = f_{ji}, d_{ij}$$

$$t * t = 0$$

$$\sum T(\dots T(\dots), \dots)$$

$$t_H^* t + t_H * t_H = 0$$

$$\sum T_{T_H}$$

$$t = \sum w$$

WEBS WITH 1 MOOVES

