

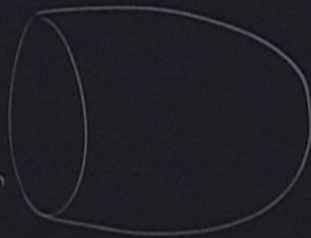
Title: Exact Results In Two-Dimensional (2,2) Supersymmetric Gauge Theories With Boundary

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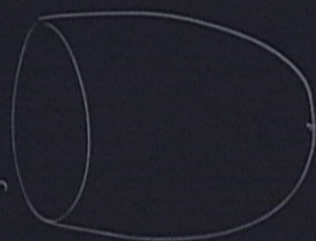
Abstract: We compute the partition function on the hemisphere of a class of two-dimensional (2,2) supersymmetric field theories including gauged linear sigma models. The result provides a general exact formula for the central charge of the D-brane placed at the boundary. It takes the form of Mellin-Barnes integral and the question of its convergence leads to the grade restriction rule concerning branes near the phase boundaries. We find expressions in various phases including the large volume formula in which a characteristic class called the Gamma class shows up. The two sphere partition function factorizes into two hemispheres glued by inverse to the annulus. The result can also be written in a form familiar in mirror symmetry, and suggests a way to find explicit mirror correspondence between branes.

With Mauricio Romo,



boundary condition
= D-brane.

With Mauricio Romo,

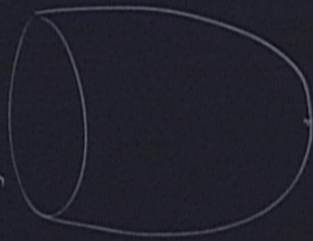


boundary condition
~ D-brane.

A-branes $\leftrightarrow$ B-type SUSY

B-branes $\leftrightarrow$ A-type SUSY

With Mauricio Romo,



boundary condition
= D-brane.

A-brane

B-branes

$= Z(\text{brane})$

USY

USY

Romo,

$= Z(\text{brane})$ central charge

A-branes $\leftrightarrow$ SUSY

B-branes

10,

$$= Z(\text{brane}) \quad \text{central charge}$$

-branes $\leftrightarrow$ B-type SUSY

-branes $\leftrightarrow$ A-type SUSY

$$= \left(\begin{array}{c} P_x \\ P_x \end{array} \right) \frac{\mathcal{F}(B)}{1+r^4} \quad \text{"Macroscopic loop"}$$

$\sim$ "J-function"

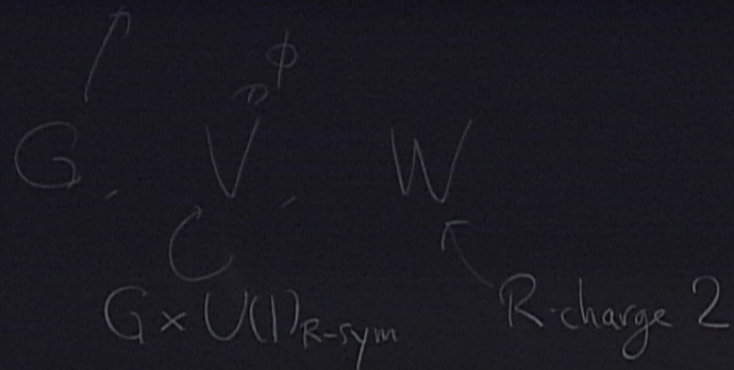
$$\begin{aligned}
 &= Z(\text{brane}) \quad \text{central charge} \\
 &= \left(\begin{array}{c} P_x \\ \circ \\ P_x \end{array} \right) \frac{\mathcal{F}(B)}{1+r^4} \quad \text{"Macroscopic loop"} \\
 &\sim \text{"J-function"}
 \end{aligned}$$

branes $\leftrightarrow$ B-type SUSY
 branes $\leftrightarrow$ A-type SUSY

gauged LSM $\rightarrow$ (D-branes $\leftrightarrow$ A-type SUSY)

G V W
 $\quad \quad \quad \swarrow$
 $G \times U(1)_{R\text{-sym}}$ R-charge 2

gauged LSM $\rightarrow$ (D-branes $\leftrightarrow$ A-type SUSY)



Charge Integrality
(A-twistable)

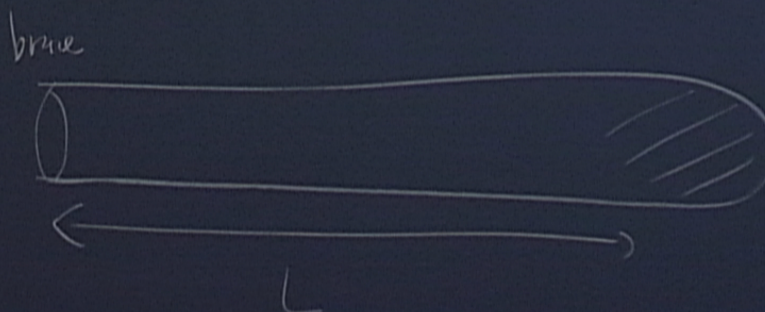
$$(g, \lambda) : \phi \rightarrow \lambda^R g \phi$$

$$W(\lambda^R \phi) = \lambda^2 W(\phi)$$

$$W(g\phi) = W(\phi)$$

$$Z(\text{brane}) =$$

$$\lim_{L \rightarrow \infty}$$



B-twist
A-twist

$$\begin{array}{ccc}
 & \phi & \\
 G & V & W \\
 & \downarrow & \uparrow \\
 & G \times U(1)_{R\text{-sym}} & R\text{-charge } 2
 \end{array}$$

$$(g, \lambda) : \phi \rightarrow \lambda^R g \phi$$

$$W(\lambda^R \phi) = \lambda^2 W(\phi)$$

$$W(g\phi) = W(\phi)$$

Charge Integrality
(A-twistable)

$$J \in G \text{ s.t. } e^{\pi i R} = J \text{ on } V$$

$$R \rightarrow R + \Delta$$

ϕ
 V W
 $\times U(1)_{R\text{-sym}}$ charge 2
 $\lambda) : \phi \rightarrow$
 $(\lambda^R \phi) =$
 $V(g\phi)$

Charge Integrality

(A-twisted)

$$J \in G \quad \text{s.t.} \quad e^{\pi i R} = \int_{\text{on } V} e^{-\pi i \Delta} \rightarrow J \in (\mathbb{Z}_G)^{\circ}$$

$$R \rightarrow R + \Delta = R'$$

$$e^{\pi i R'} = 1$$

$$\begin{array}{c} \phi \\ \downarrow \\ V, W \\ \uparrow \\ \times U(1)_{R\text{-sym}} \quad R\text{-charge } 2 \end{array}$$

$$\lambda : \phi \rightarrow \lambda^R g \phi$$

$$(\lambda^R \phi) = \lambda^2 W(\phi)$$

$$W(g\phi) = W(\phi)$$

Charge Integrality

(A-twisted)

$$J \in G \quad \text{s.t.} \quad e^{\pi i R} = J \text{ on } V$$

$$R \rightarrow R + \Delta = R'$$

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$$e^{-\pi i \Delta} \rightarrow J \in (\mathbb{Z}_G)^\vee$$

Brane data . gauge inv. homog. matrix factorization of W .

Brane data : gauge inv. homog. matrix factorization of W .

(M, Q, p, r)

$\nwarrow G \times U(1)_R$

Brane data : gauge inv. homog. matrix factorization of W .

$$(M, Q, P, r_*)$$

$$\curvearrowright G \times \widetilde{U(1)}_R \ni (g, \lambda) \mapsto P(g) \lambda^{r_*}$$

Brane data : gauge inv. homog. matrix factorization of W .

$$(M, Q, P, \mathbb{R}_*)$$

Chan-Paton $\curvearrowright$ $G \times \widetilde{U(1)}_R \ni (g, \lambda) \mapsto P(g) \lambda^{\mathbb{R}_*}$

$$Q(\phi)^2 = W(\phi) \text{id}_M$$

$$P(g)^{-1} Q(g\phi) P(g) = Q(\phi)$$

$$\lambda^{\mathbb{R}_*} Q(\lambda^R \phi) \lambda^{-\mathbb{R}_*} = \lambda Q(\phi)$$

Brane data : gauge inv. homog. matrix factorization of W .

$$(M, Q, \rho, \mathbb{V}_*)$$

Chan-Paton

$$G \times \widetilde{U(1)}_R \ni (g, \lambda) \mapsto \rho(g) \lambda^{\mathbb{V}_*}$$

$\mathbb{Z}_2$ -graded

$$Q(\phi)^2 = W(\phi) \text{id}_M$$

$$\rho(g)^{-1} Q(g\phi) \rho(g) = Q(\phi)$$

$$\lambda^{\mathbb{V}_*} Q(\lambda^R \phi) \lambda^{-\mathbb{V}_*} = \lambda Q(\phi)$$

$$e^{\pi i \mathbb{V}_*} \rho(\mathcal{J}) =$$

ata : gauge inv. homog. matrix factorization of W .

$$Q, \rho, \mathbb{R}_*$$

$$G \times \widetilde{U(1)}_R \ni (g, \lambda) \mapsto \rho(g) \lambda^{\mathbb{R}_*}$$

$$Q(\phi)^2 = W(\phi) \text{id}_M$$

$$Q(g\phi)\rho(g) = Q(\phi)$$

$$Q(\lambda^R \phi) \lambda^{-\mathbb{R}_*} = \lambda Q(\phi)$$

$$e^{\pi i \mathbb{R}_*} \rho(J) = \begin{cases} +1 & M^{\text{ev}} \\ -1 & M^{\text{od}} \end{cases}$$

ata : gauge inv. homog. matrix factorization of W .

Q, p, r_*

$G \times \widetilde{U(1)}_R$

$(\phi)^2 =$

$Q(g\phi)$

$Q(x)$

$$\mapsto P(g) \lambda^{r_*}$$

$$\pi^* r_* P(J) = \begin{cases} +1 & M^{ev} \\ -1 & M^{od} \end{cases}$$

expression $B = (M, \Omega, p, r_0)$

$$= (r\lambda)^{\hat{c}/2} \int_{\gamma} d^{\text{tg}} \sigma' \prod_{\alpha > 0} d(\sigma')$$

expression $B = (M, Q, P, V_0)$

$$= (r\lambda)^{\hat{c}/2} \int_{\gamma} d^{\text{to}} \sigma' \prod_{\alpha > 0} d(\sigma') \sinh(\pi v(\sigma')) \prod_i$$

$$\prod_0 \alpha(\sigma') \sinh(\pi \alpha(\sigma')) \prod_i \Gamma\left(i Q_i(\sigma') + \frac{R_i}{2}\right) e^{i t_R(\sigma')} f_B(\sigma')$$

Brane data : gauge inv. homog. matrix factorization of W .

$$(M, Q, P, r_*)$$

Chan-Paton

$\mathbb{Z}_2$ -graded

$$G \times \widetilde{U(1)}_R$$

$$Q(\phi)^2 = W$$

$$P(g)Q(g\phi)P(g)^{-1}$$

$$\lambda^{r_*} Q(\lambda^R \phi)$$

$$\mapsto P(g)\lambda^{r_*}$$

charge integrality

$$\Rightarrow e^{\pi i r_*} \rho(J) = \begin{cases} +1 & M \\ -1 & M^* \end{cases}$$

$$\int_{\sigma'} \prod_{\alpha > 0} d(\sigma') \sinh(\pi \alpha(\sigma')) \prod_i \Gamma(i Q_i(\sigma') + \frac{R_i}{2}) e^{i t_R(\sigma')} f_B(\sigma')$$

$$\mathfrak{g} = \text{Lie}(G) \supset \mathfrak{t} = \text{Lie}(T) \quad T \subset G \text{ a max torus}$$

$$V|_{T \times \widetilde{U(1)}_R} = \bigoplus_i \mathbb{C}(Q_i, R_i) \quad f_B(\sigma') = \text{tr}_M \left(e^{\pi i V_*} \rho(e^{2\pi \sigma'}) \right)$$

$$M|_{T \times \widetilde{U(1)}_R} = \bigoplus_j \mathbb{C}(Q_j, R_j) \quad = \sum_j e^{\pi i V_j} e^{2\pi i Q_j(\sigma')}$$

$$\sigma' = \underbrace{r\sigma}_{\text{lowest component of vect}} \quad (\text{TCG})$$

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$$t_R(\sigma) = t(\sigma) - \sum_i Q_i(\sigma) \log(r\Lambda)$$

Λ - some renormalization scale

$$\hat{c} = \sum_i (1 - R_i) - d_G$$

$$d_G = d$$

of vect ($T \subset G$)

$\log(r\Lambda)$

$$d_G = \dim_{\mathbb{R}} G$$

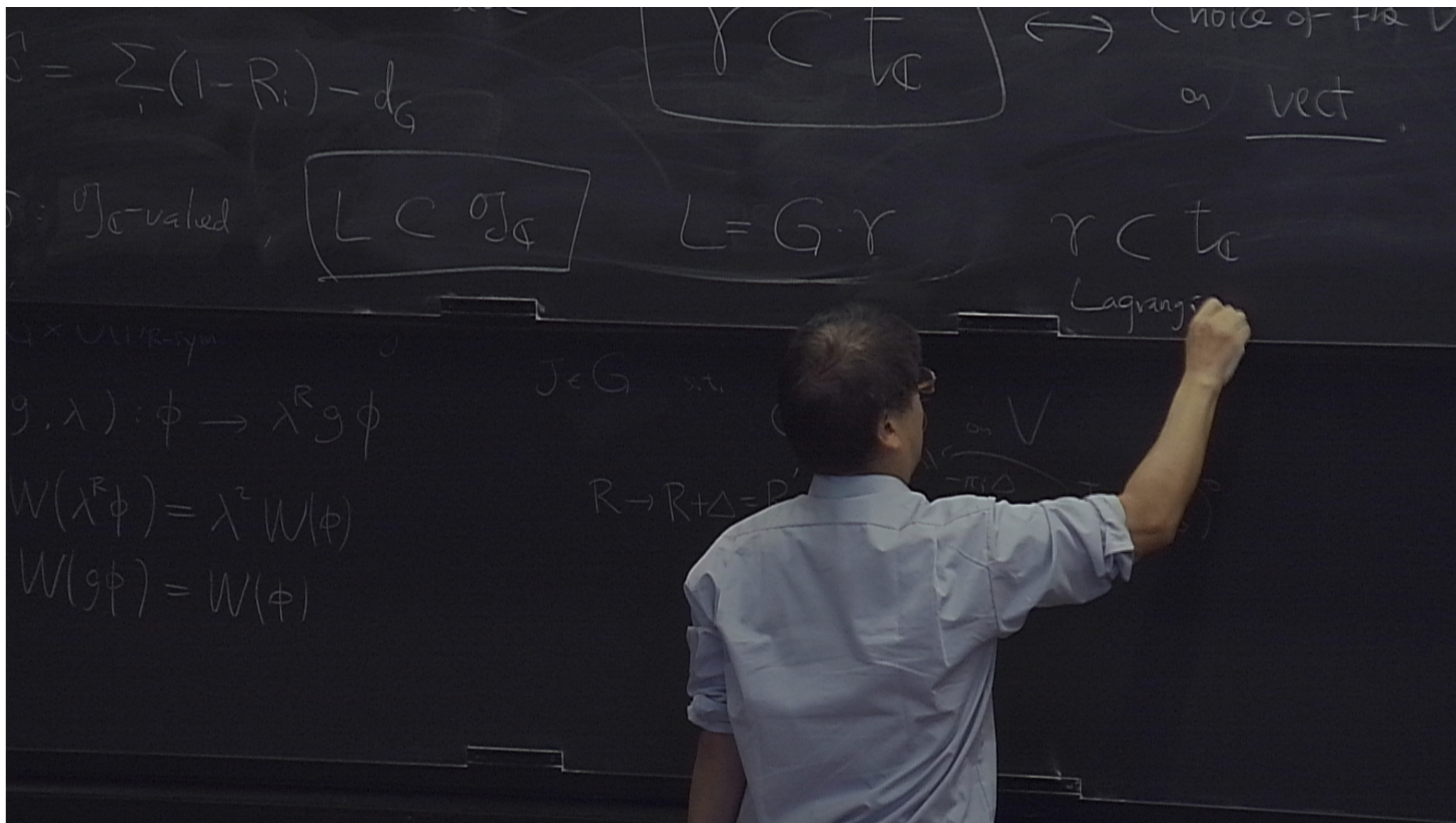
$$l_G = \dim_{\mathbb{R}} T (= \text{rank } G)$$

$$\sum_i Q_i(\sigma) \log(r\Lambda)$$

$$l_G = \dim_{\mathbb{R}} T (= \text{rank } G)$$

$$\gamma \subset t_G$$

$\longleftrightarrow$ Choice of the Boundary Condition
on vect.



Lagrangian

ϕ
 W
 $\mathcal{N}_{R\text{-sym}}$ $\nwarrow$ $R\text{-charge } 2$

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$$) = \lambda^2 W(\phi)$$

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charge integrality
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$$J \in G \text{ s.t. } e^{\pi i R} = J \text{ on } V$$

$$R \rightarrow R + \Delta = R'$$

$$e^{\pi i \Delta} \rightarrow J \in (\mathbb{Z}_G)^\circ$$

$$e^{\pi i R'} = 1$$

The simplest $\gamma = \underbrace{it}_{\text{hermitian}} \subset t_c$ "real"

$\mathbb{D}$ is-holomorphic in twisted chiral parameters
 - indep of chiral parameters

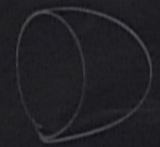
$$(W, \mathbb{Q}) \quad \mathbb{Q}^2 = W \text{id}_M$$

Lagrangian

The simplest

$$\gamma = i \underbrace{t_c}_{\text{hermitian}} \underbrace{t_c}_{\text{anti hermitian}} \text{ "real"}$$

$f(z)$



is - holomorphic in twisted chiral po
- indep of chiral parameters

(W, Q)

Lagrangian

$\gamma = \underbrace{it}_{\text{hermitian}} \subset t_0$ "real"
anti hermitian

$f(z)$

is - holomorphic in twisted chiral parameters
- indep of chiral parameters

(W, Q) $Q^2 = W \text{id}_M$

Lagrangian

$\gamma = \underbrace{it}_{\text{hermitian}} \subset t_0$ "real"
 $\uparrow$
 antihermitian

is - holomorphic in twisted chiral parameters
 - indep of chiral parameters

$$(W, Q) \quad Q^2 = W \text{id}_M$$

$f(z)$

$$d^{lc} \sigma = d\sigma_{lc} \wedge \dots \wedge d\sigma_{lc}$$

Lagrangian

$\gamma = \underbrace{it}_{\text{hermitian}} \subset \text{to "real"}$
 $\uparrow$
 antihermitian

is - holomorphic in twisted chiral parameters
 - indep of chiral parameters

$$(W, Q) \quad Q^2 = W \text{id}_M$$

$f(z)$

$$d^{\text{lc}} \sigma = d\sigma_1 \wedge \dots \wedge d\sigma_{\text{lc}}$$

Properties of Γ -function

$$\textcircled{1} \quad \Gamma(z) \sim \frac{(-1)^n}{n!} \frac{1}{z+n}$$

$$z \sim -n \quad (n=0, 1, 2, \dots)$$

Properties of Γ -function

$$\textcircled{1} \quad \Gamma(z) \sim \frac{(-1)^n}{n!} \frac{1}{z+n} \quad z \sim -n \quad (n=0, 1, 2, \dots)$$

$$\textcircled{2} \quad \Gamma(z) \sim \sqrt{2\pi} e^{-z} z^{z-\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right)$$


function

$$\frac{(-1)^n}{n!} \frac{1}{z+n}$$

$$z \sim -n \quad (n=0, 1, \dots)$$

$$\sqrt{2\pi} e^{-z} z^{z-\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right)$$


Stirling's formula

Properties of Γ -function

① $\Gamma(z) \sim \frac{(-1)^n}{n!} \frac{1}{z+n} \quad z \sim -n \quad (n=0, 1, 2, \dots)$

② $\Gamma(z) \sim \sqrt{\pi} e^{-z} z^{z-\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right)$  Stirling

③ $\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$

Properties of Γ -function

$$(1) \quad \Gamma(z) \sim \frac{(-1)^n}{n!} \frac{1}{z+n} \quad z \sim -n \quad (n=0, 1, 2, \dots)$$

$$(2) \quad \Gamma(z) \sim \sqrt{2\pi} e^{-z} z^{z-\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right)$$
A diagram illustrating Stirling's approximation. It shows a rectangular area with diagonal lines, representing the asymptotic behavior of the Gamma function for large z. The word "Stir" is written to the right of the diagram.

Stir

$$(3) \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$$

$$(4) \quad \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad \operatorname{Re}(z) > 0$$

Properties of Γ -function

$$(1) \quad \Gamma(z) \sim \frac{(-1)^n}{n!} \frac{1}{z+n} \quad z \sim -n \quad (n=0, 1, 2, \dots)$$

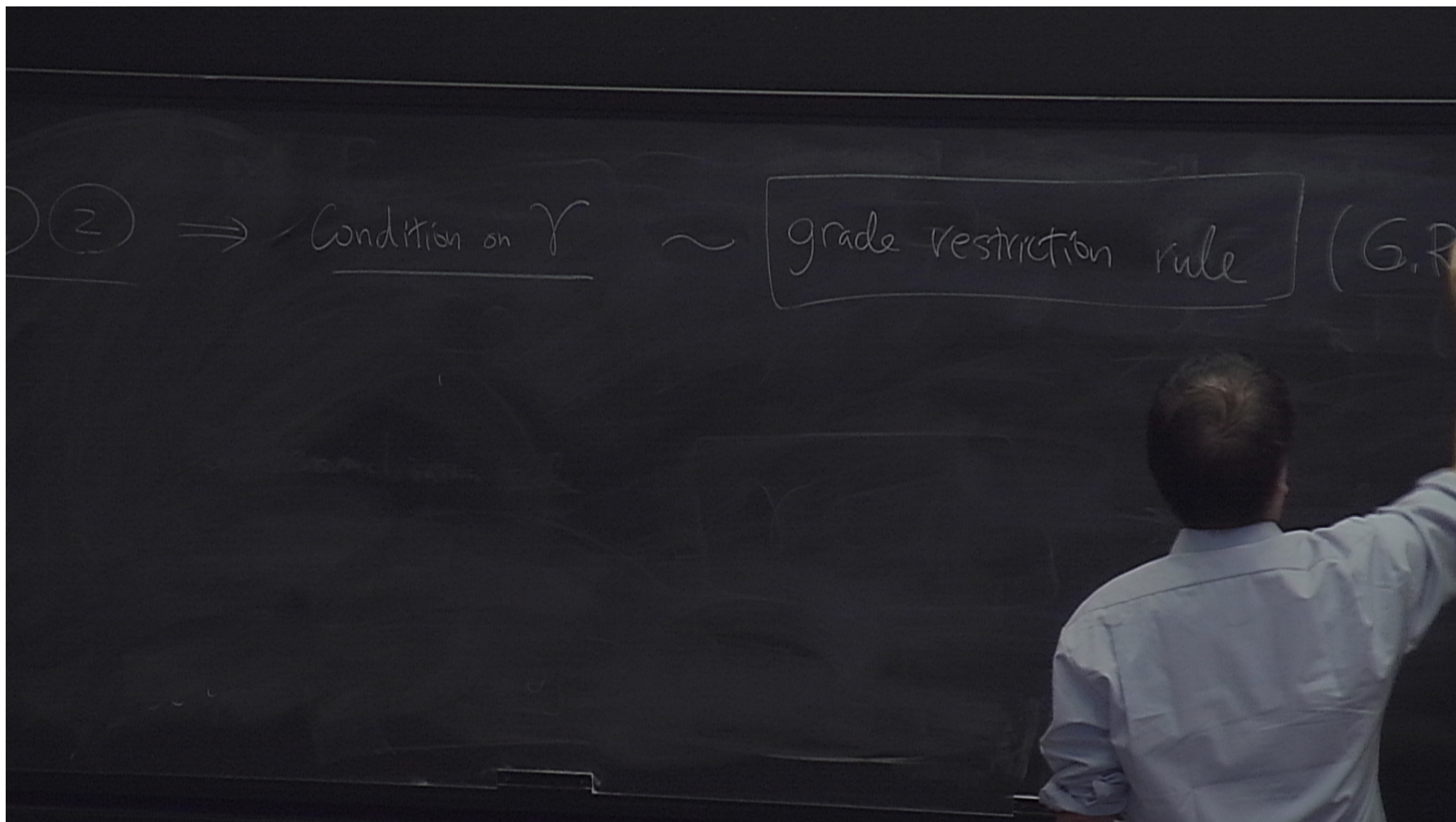
$$(2) \quad \Gamma(z) \sim \sqrt{2\pi} e^{-z} z^{z-\frac{1}{2}} \left(1 + O\left(\frac{1}{z}\right)\right)$$
A diagram illustrating Stirling's approximation. It shows a rectangular area filled with diagonal lines, with a vertical line intersecting it. The word "Stir" is written to the right of the diagram.

Stir

$$(3) \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$$

$$(4) \quad \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad \operatorname{Re}(z) > 0$$

①② $\Rightarrow$ Condition on γ $\sim$ grade



② $\Rightarrow$ Condition on γ $\sim$ grade restriction rule (G.R)

① $\Rightarrow$ RG flow of the boundary condition (decoupling of "empty br" by tachyon cond)

③ $\Rightarrow$ expression in geometric phase.

② - Condition on γ $\sim$ grade restriction rule (G.R.R.)

R.G. flow of the boundary condition (decoupling of "empty brane"
by tachyon condensation)

Expression in geometric phase.

①② $\Rightarrow$ Condition on γ $\sim$ grade restriction rule (G.)

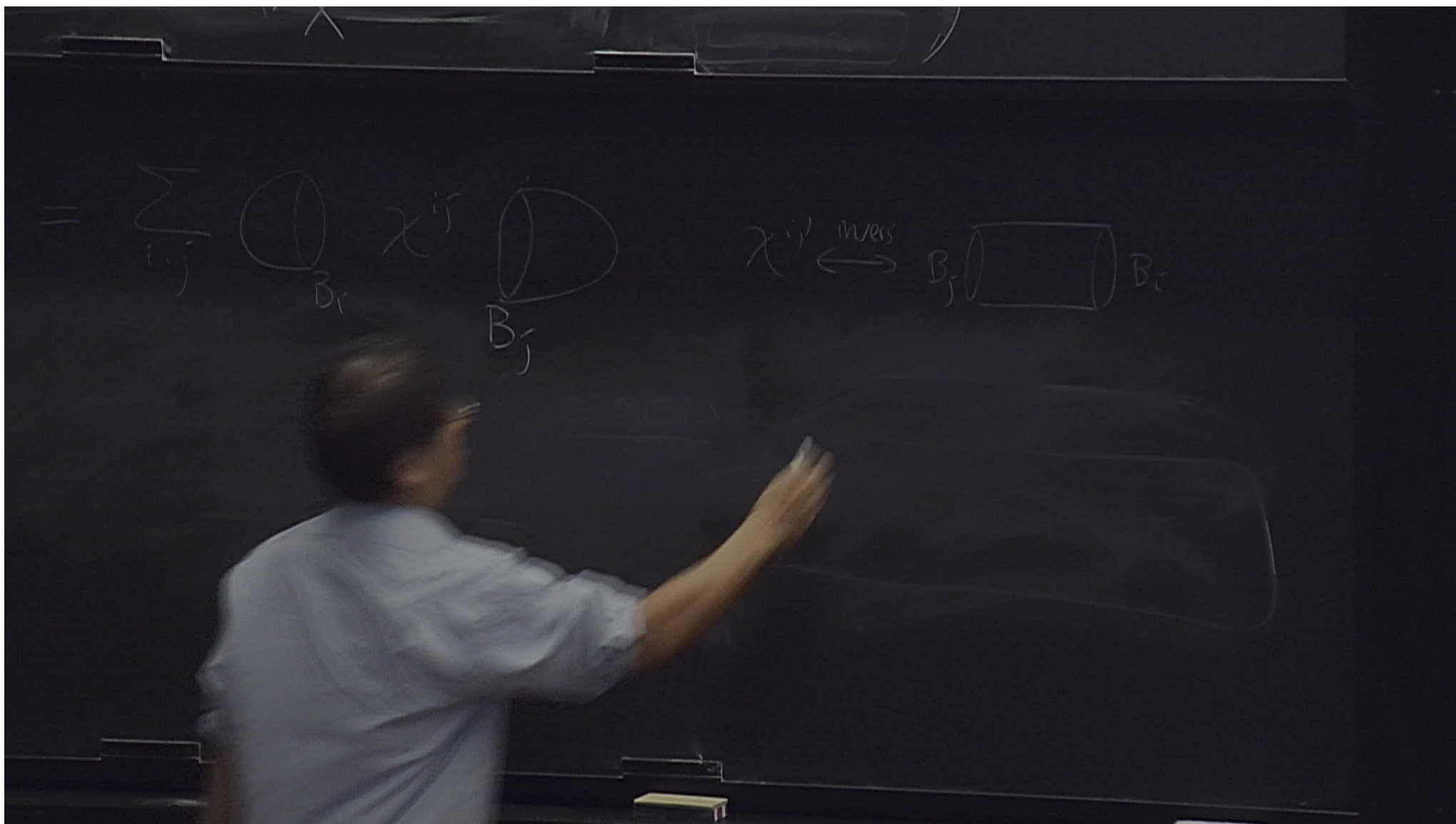
① $\Rightarrow$ R.G. flow of the boundary condition (decoupling of "empty" by tachyon)

③ $\Rightarrow$ Expression in geometric phase. B -field

$$\int_B \hat{\Gamma}_X e^{B+i\omega} = (rA)^{\dim X/c} \left(\int_X \hat{\Gamma}_X e^{B+i\omega} \right) + \dots$$

brane $\downarrow$ $ch(B)$

$$\textcircled{3} \Rightarrow \bigcirc = \sum_{i,j} \bigcirc_{B_i} \chi^{ij} \bigcirc_{B_j} \chi^{ji}$$



$$= \sum_{i,j} \bigcirc_{B_i} \chi_{ij} \bigcirc_{B_j}$$

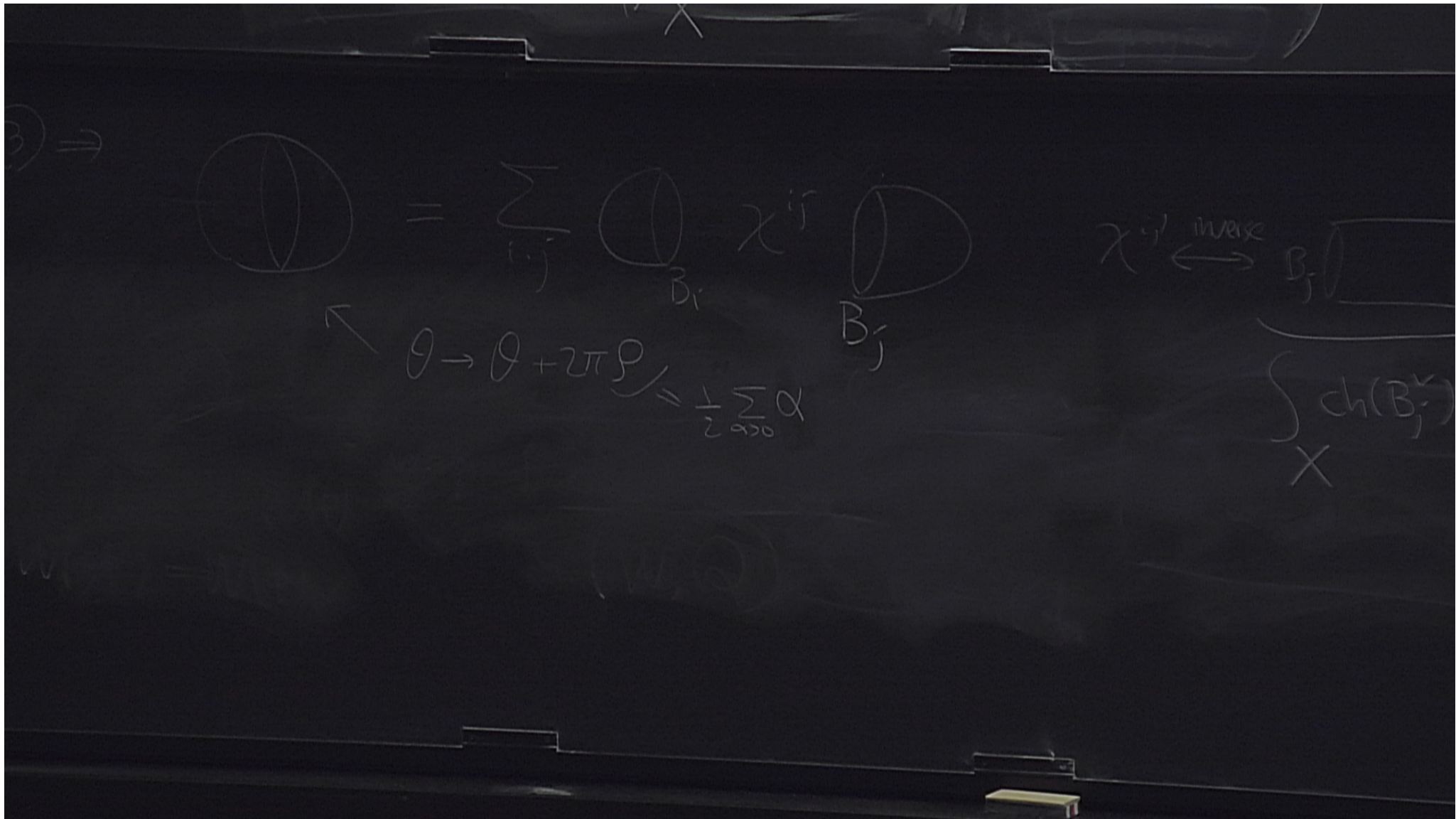
$$\chi_{ij} \xleftrightarrow{\text{inverse}} \underbrace{B_j \square B_i}$$

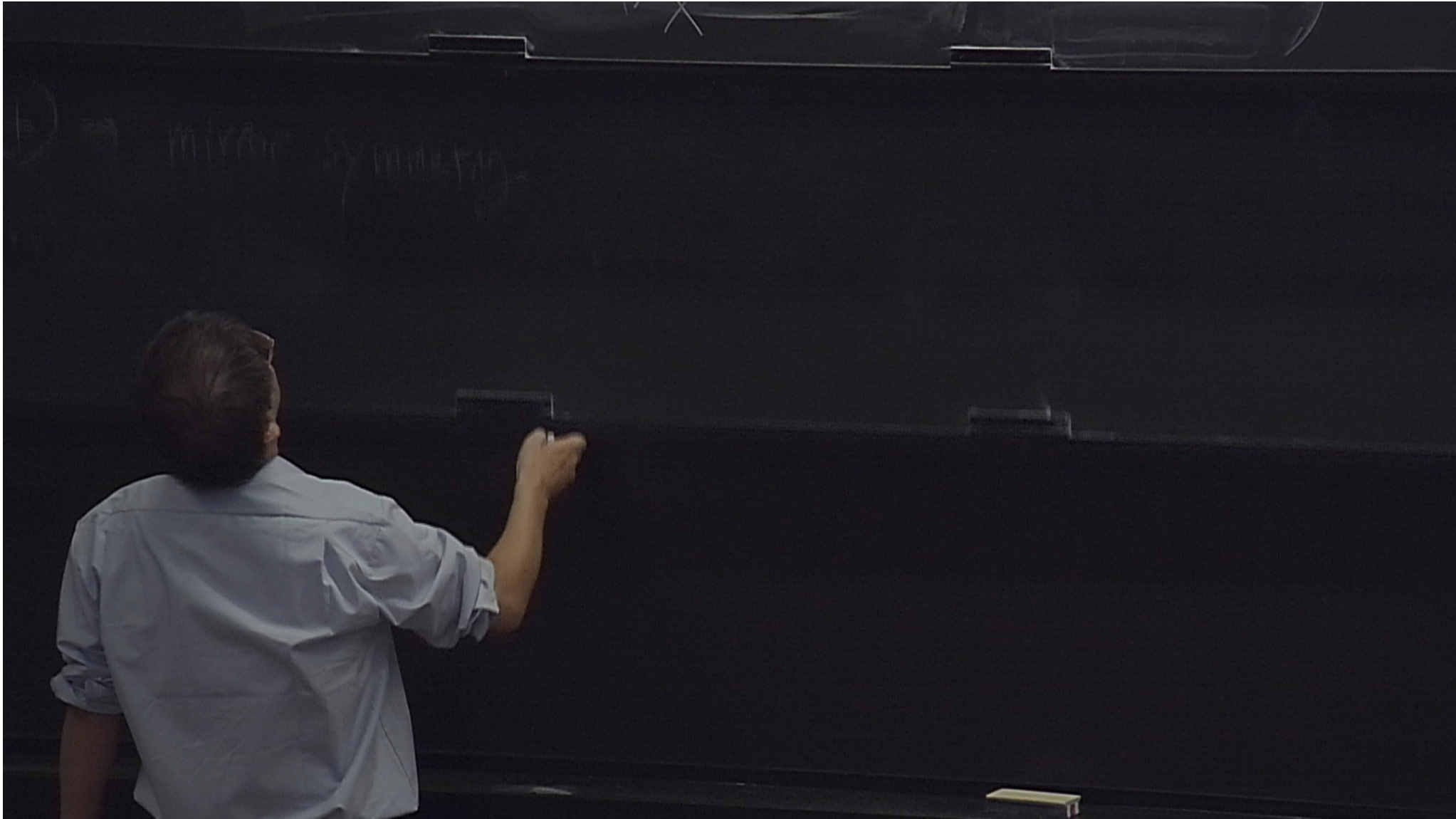
$$\int_X \text{ch}(B_j^\vee) \text{ch}(B_i) \hat{A}(x)$$

$$= \sum_{i,j} \bigcirc_{B_i} \chi^{ij} \bigcirc_{B_j}$$

$$\chi^{ij} \xleftrightarrow{\text{inverse}} \underbrace{B_j \square B_i}$$

$$\int_X \text{ch}(B_j^\vee) \text{ch}(B_i) \hat{A}(x)$$





$$B = \frac{d\mathbf{r} + d\mathbf{a}}{2} \sum_{\mathbf{e}_i, j} +$$

$$\mathbb{B} = \frac{d_V + d_G}{2} \sum_{\epsilon, j} \left(\prod_{\alpha > 0} \frac{\epsilon_\alpha}{2} \right) e^{\pi i r_j}$$

$$\begin{aligned}
 \mathbb{D} &= \star (r\Lambda)^{\frac{d_v+d_g}{2}} \sum_{\varepsilon, j} \left(\prod_{\alpha>0} \frac{\varepsilon_\alpha}{2} \right) e^{\pi i r_j} + \\
 &\int_{Y \times \mathbb{R}^{d_v}} d^{d_g} \sigma \, d^{d_v} y \prod_{\alpha>0} d(\sigma) \cdot \prod_i e^{-\frac{R_i}{2} y_i} e^{-i r \widetilde{W}_{\varepsilon, g}(\sigma, y)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{D} &= \star (r\Lambda)^{\frac{d_v+d_g}{2}} \sum_{\varepsilon, j} \left(\prod_{\alpha>0} \frac{\varepsilon_\alpha}{2} \right) e^{\pi i \varphi_j} \\
 &\times \int_{Y \times \mathbb{R}^{d_v}} d^{d_g} \sigma \, d^{d_v} y \, \prod_{\alpha>0} d(\sigma) \cdot \prod_i e^{-\frac{R_i}{2} y_i} e^{-i r \widetilde{W}_{\varepsilon, q_j}(\sigma, y)} \\
 \widetilde{W}_{\varepsilon, q_j} &= \left(\sum_i Q_i y_i - t_{\varepsilon, q_j} \right) (\sigma) + (-i\Lambda) \sum_i e^{-y_i}
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{D} &= \star(r\Lambda)^{\frac{d_v+d_a}{2}} \sum_{\varepsilon, j} \left(\prod_{\alpha>0} \frac{\varepsilon_\alpha}{2} \right) e^{\pi i r_j} \\
 &\times \int_{Y \times \mathbb{R}^{d_v}} d^{d_a} \sigma \, d^{d_v} y \, \prod_{\alpha>0} d(\sigma) \cdot \prod_i e^{-\frac{R_i}{2} y_i} e^{-i r \widetilde{W}_{\varepsilon, j}(\sigma, y)}
 \end{aligned}$$

where

$$\begin{aligned}
 \widetilde{W}_{\varepsilon, j} &= \left(\sum_i \partial_i y_i - t_{\varepsilon, j} \right) (\sigma) + (-i\Lambda) \sum_i e^{-y_i} \\
 t_{\varepsilon, j} &= t - i \left(2\pi e_j + \sum_{\alpha>0} \varepsilon_\alpha \pi \alpha \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{d\nu + d\alpha}{2} \sum_{\varepsilon, j} \left(\prod_{\alpha > 0} \frac{\varepsilon_\alpha}{2} \right) e^{\pi i \nu_j} + \\
 & \int_{Y \times \mathbb{R}^+} d^{\text{da}} \sigma \, d^{\text{dv}} y \prod_{\alpha > 0} d(\sigma) \cdot \prod_i e^{-\frac{R_i}{2} y_i} e^{-i r \widetilde{W}_{\varepsilon, y}(\sigma, y)} \\
 & \left(\partial_i y_i - t_{\varepsilon, y_i} \right)(\sigma) + (-i\Lambda) \sum_i e^{-y_i} \quad (\text{HV 2000}) \\
 & = t - i \left(2\pi e_{j_i} + \sum_{\alpha > 0} \varepsilon_\alpha \pi \alpha \right)
 \end{aligned}$$

$$C_{E,q} = (-1)^{q+1} \left(C_{E,q-1} + \sum_{\alpha>0} \Sigma_{\alpha} \cdot \pi(\alpha) \right)$$

$$\text{Integrand}_q \sim P(\sigma) e^{-ir \widetilde{W}_{\text{eff},q}(\sigma)}$$

$\nwarrow$
 Power in σ

$$\widetilde{W}_{\text{eff},q}(\sigma) = \sum_{\alpha>0} \pm \pi i d(\sigma) - \sum_i Q_i(\sigma) \left(\log \left(\frac{Q_i(\sigma)}{-i\Lambda} \right) - 1 \right) - t(\sigma) + 2\pi i q(\sigma)$$

$$C_{E, \epsilon} = \left(-1 \right) \left(C_{E, \epsilon} + \sum_{\alpha > 0} \epsilon_{\alpha} \cdot \pi \alpha \right)$$

Integrand_q $\sim$ $P(\sigma) \cdot e^{-ir \widetilde{W}_{\text{eff}, q}(\sigma)}$

$\nwarrow$
 Power in σ

$$\widetilde{W}_{\text{eff}, q}(\sigma) = \sum_{\alpha > 0} \pm \pi i d(\sigma) - \sum_i Q_i(\sigma) \left(\log \left(\frac{Q_i(\sigma)}{-i\Lambda} \right) - 1 \right) - t(\sigma) + 2\pi i q(\sigma)$$

$$C_{E, \epsilon} = (-1) \left(C_{H, \epsilon} + \sum_{\alpha > 0} \epsilon_{\alpha} \pi \alpha \right)$$

Integrand_q $\sim$ $P(\sigma) \cdot e^{-ir \widetilde{W}_{\text{eff}, q}(\sigma)}$

$\nwarrow$
 Power in σ

$$\begin{aligned} \widetilde{W}_{\text{eff}, q}(\sigma) = & \sum_{\alpha > 0} (\pm \pi i d(\sigma)) - \sum_i Q_i(\sigma) \left(\log \left(\frac{Q_i(\sigma)}{-i\Lambda} \right) - 1 \right) \\ & - t(\sigma) + 2\pi i q(\sigma) \end{aligned}$$

$$C_{E, \epsilon} = (-1) \left(C_{E, \epsilon} + \sum_{\alpha > 0} \epsilon_{\alpha} \pi(\alpha) \right)$$

integrand_q $\sim$ $P(\sigma) e^{-ir \widetilde{W}_{\text{eff}, q}(\sigma)}$

↑
Power in σ

$$h_{\epsilon}(\sigma) = \sum_{\alpha > 0} (\pm \pi i d(\sigma)) - \sum_i Q_i(\sigma) \left(\log \left(\frac{Q_i(\sigma)}{-i\Lambda} \right) - 1 \right) - t(\sigma) + 2\pi i q(\sigma)$$

W-boson

$$C_{\mathcal{E}, \mathcal{F}} = \left(-1 \right)^{\sum_{\alpha} C_{\mathcal{E}, \mathcal{F}}^{\alpha}} + \sum_{\alpha > 0} \mathcal{E}_{\alpha} \cdot \pi(\alpha)$$

integrand_q $\sim$ $P(\sigma) \cdot e^{-ir \underbrace{\widetilde{W}_{\text{eff}, q}(\sigma)}_{\text{Power in } \sigma}} = \text{effective twisted superpotential}$

$$\widetilde{W}_{\text{eff}, q}(\sigma) = \sum_{\alpha > 0} \left(\pm \pi i d(\sigma) \right) - \sum_i Q_i(\sigma) \left(\log \left(\frac{Q_i(\sigma)}{-i\Lambda} \right) - 1 \right) \quad \text{Coulomb}$$

$\boxed{W\text{-boson}}$ $-t(\sigma) + 2\pi i q(\sigma)$

Proposal for γ . γ is a

Proposal for γ . γ is a deformation of $it \subset t_c$

s.t. ① avoid the poles $Q_i(\sigma') = i \left(n_i + \frac{R}{2} \right)$

for γ . γ is a deformation of $it \in t_{\mathbb{C}}$

s.t. ① avoid the poles $Q_i(\sigma') = i \left(n_i + \frac{R_i}{2} \right)$ $n_i = 0, 1, 2, \dots$

②

or γ . γ is a deformation of $\text{it} \subset t_{\mathbb{C}}$

s.t. ① avoid the poles $Q_i(\sigma') = i \left(n_i + \frac{R_i}{2} \right)$ $n_i = 0, 1, 2, \dots$

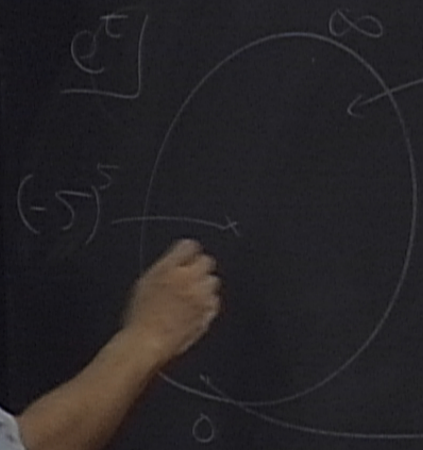
② $A_q(\sigma) \rightarrow \infty$ for every σ of γ
for $\forall q$ of the brane.

e.g. $G = U(1), \quad V = \mathbb{C}(-5) \oplus \mathbb{C}(1)^{\oplus 5} \Rightarrow (p, x_1, \dots, x_5)$

$$V = \mathbb{C}(-5) \oplus \mathbb{C}(1)^{\oplus 5} \ni (p, x_1, \dots, x_5), \quad W = pf(x) \quad \leftarrow \text{quintic}$$

e.g. $G = U(1)$, $V = \mathbb{C}(-5) \oplus \mathbb{C}(1)^{\oplus 5} \Rightarrow (p, x_1, \dots, x_5)$

NLO-M on $X_f = (f=0) \subset \mathbb{CP}^4$

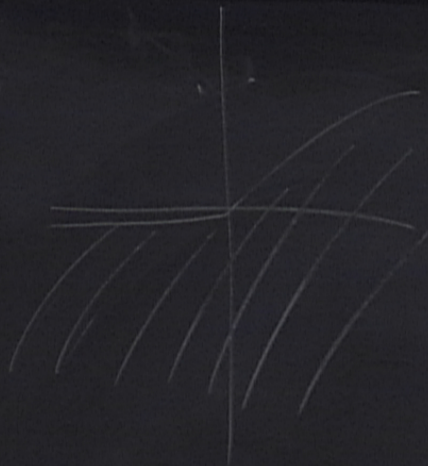


LGO $W(x) = f(x) / Z_5: (x_1, \dots, x_5) \mapsto (\omega x_1, \dots, \omega x_5)$

$\omega^5 = 1$

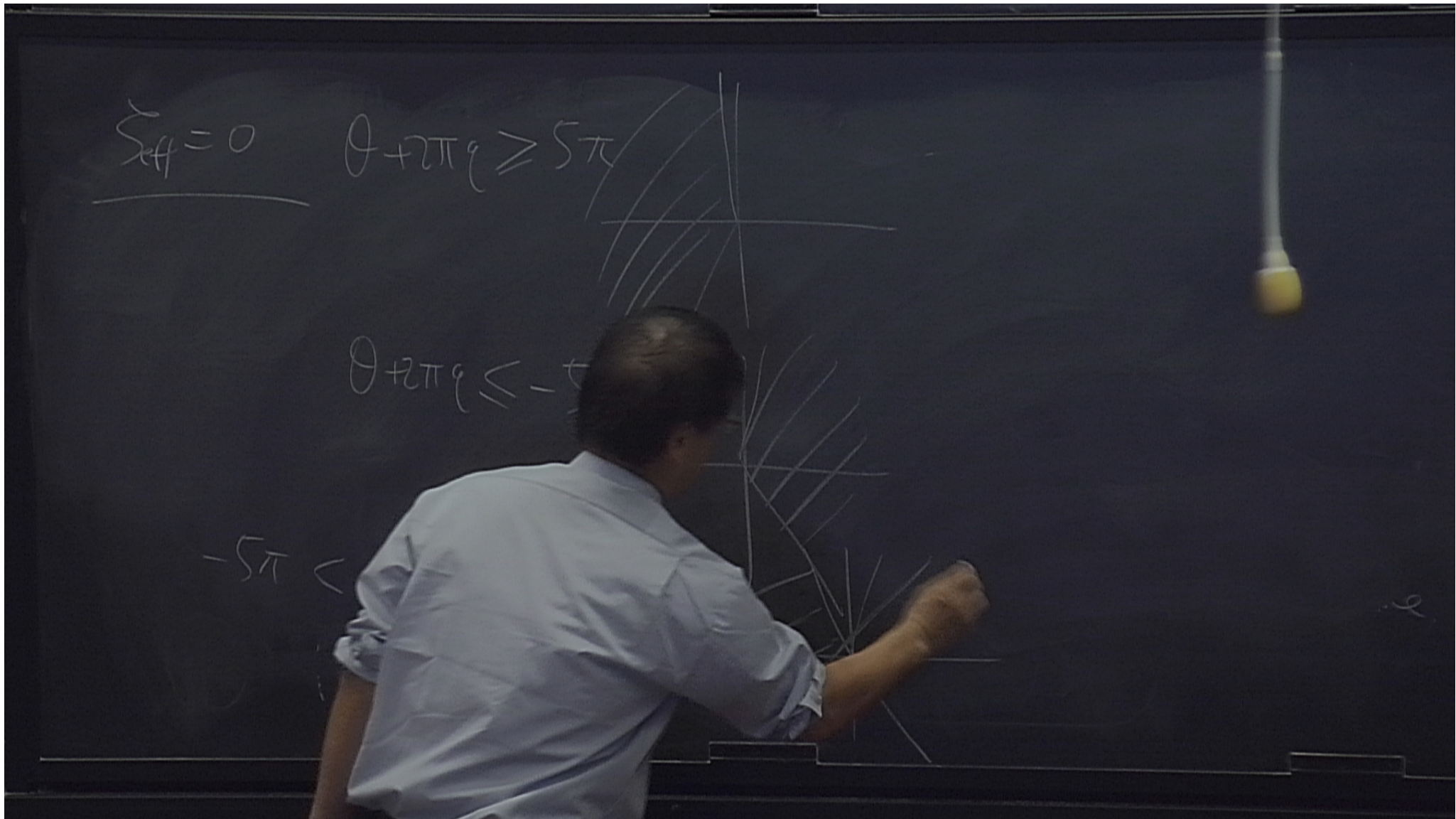
$$A_q(\sigma) = \sum_{\text{eff}} \left[\text{Im} \sigma + \left(5\pi - \text{sgn}(\text{Re} \sigma) (\beta + 2\pi q) \right) |\text{Re} \sigma| \right]$$

$$\sum \epsilon_{eff} < 0$$


 $\Rightarrow$


$$W = p f$$

$$V_{eff}(E)$$

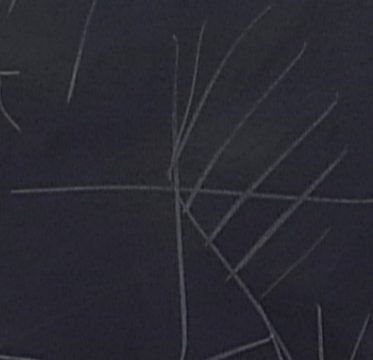


$$\underline{\vec{S}_H = 0} \quad \theta + 2\pi q \geq 5\pi$$


 $\Rightarrow$

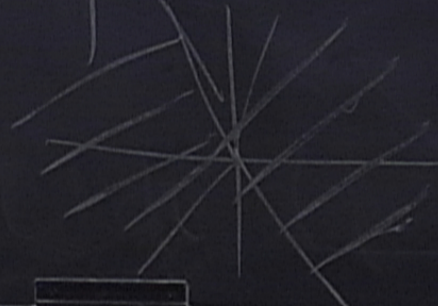
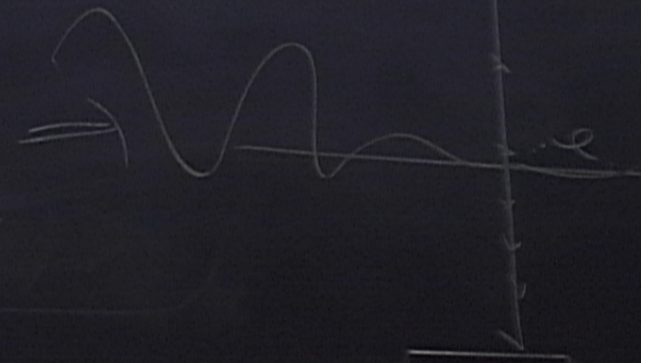
Not possible

$$\theta + 2\pi q \leq -5\pi$$

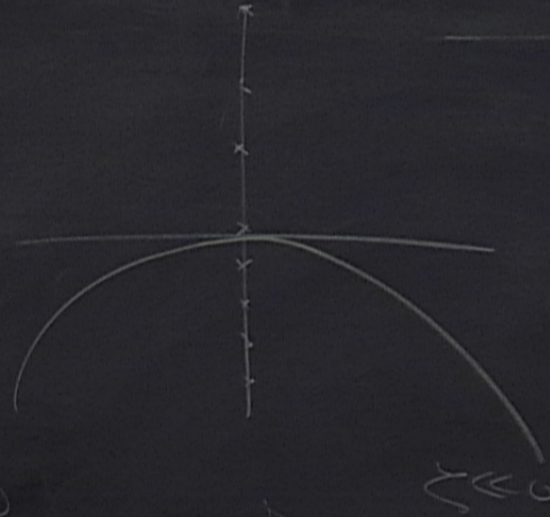
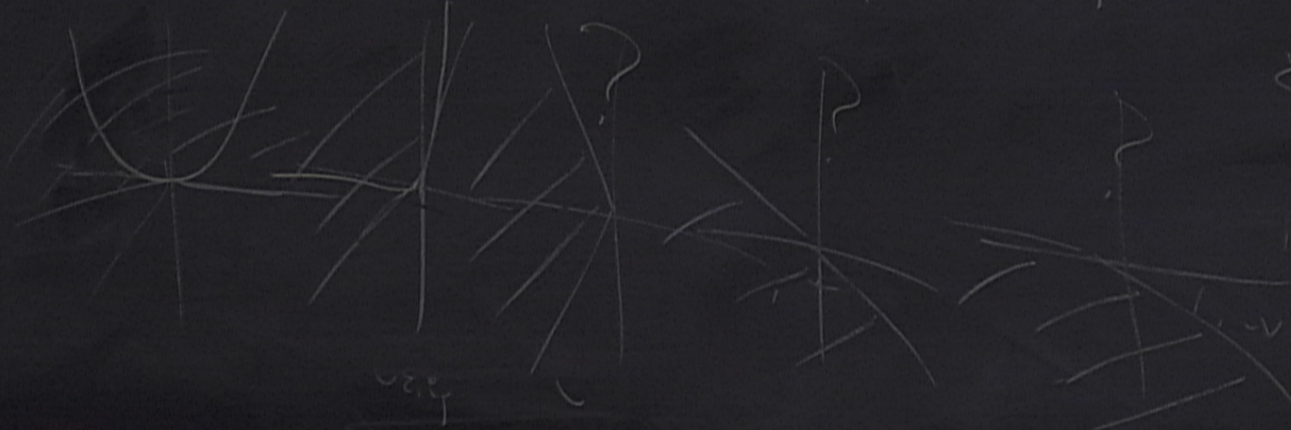

 $\Rightarrow$

Not possible

$$-5\pi < \theta + 2\pi q < 5\pi$$


 $\Rightarrow$


$$\sum_{i=1}^n x_i \leq 0$$


 $\Rightarrow$

 W
 $\Rightarrow 0$

 u_1, u_2

