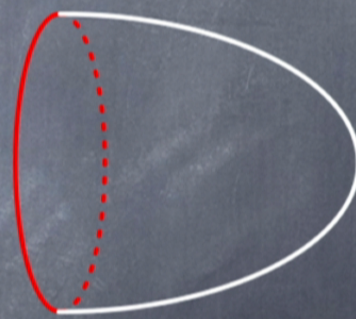
Title: Exact results for boundaries and domain walls in 2d supersymmetric theories

Date: Oct 22, 2013 10:00 AM

URL: http://pirsa.org/13100115

Abstract: We apply supersymmetric localization to N=(2,2) gauged linear sigma models on a hemisphere, with boundary conditions, i.e., D-branes, preserving B-type supersymmetries. We explain how to compute the hemisphere partition function for each object in the derived category of equivariant coherent sheaves, and argue that it depends only on its K theory class. The hemisphere partition function computes exactly the central charge of the D-brane, completing the well-known formula obtained by an anomaly inflow argument. We also formulate supersymmetric domain walls as D-branes in the product of two theories. We exhibit domain walls that realize the sl(2) affine Hecke algebra. Based on arXiv:1308.2217.

There are string theory and gauge theory motivations to study the hemisphere partition function



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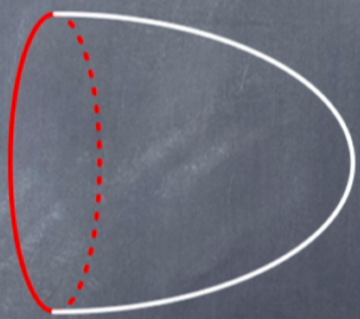
- String theory motivations
  - Z_{hem} is the central charge of a D-brane
  - Z_{hem} is related to Gromov-Witten theory



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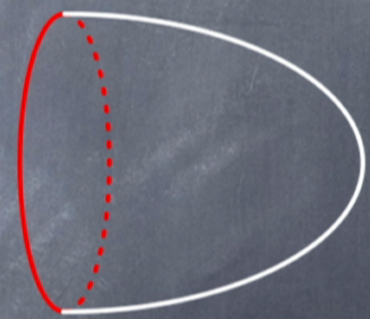

- Gauge theory motivations
  - Yet another example of SUSY localization
  - Connects the A-model (tip) with the B-model (boundary)
  - Domain wall expectation values

Bulk data characterize an N=(2,2) SUSY gauge theory

G	gauge group (compact Lie group)
V_{mat} ($\ni\phi$)	matter representation (G-rep)
$W=W(\phi)$	superpotential (G-inv polynomial)
$t=r-i\theta$	FI parameter and theta angle
$m=(m_a)$	twisted masses in Cartan of $(G_F)_c$

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We will focus on the geometric phase

- Assume $\phi=(P_\alpha,x^i)$, $W=P_\alpha G^\alpha(x)$, $G^\alpha(x)$: polynomials. $R(x^i)=0$, $R(P_\alpha)=-2$.
- Gauge theory flows to a non-linear sigma model in IR
- Low-energy target space (assumed smooth):

$$M= (V_{mat} \backslash \text{deleted set}) \cap G^{-1}(0)/G_C$$

- Flavor symmetry G_F acts on M as isometries

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Boundary data include boundary interactions

- V: Chan-Paton vector space. Representation of $G \times G_F \times U(1)_R$. **Z**- and $\mathbf{Z}_2$-graded by R-charges.
- $Q(\phi)$: odd linear operator on V, called the tachyon profile.
- B:=(V,Q)

[Herbst,Hori,Page]

The boundary interaction is constructed from the boundary data B=(V,Q)

$$\mathcal{A}_\varphi \sim A_\varphi + i\sigma_2 + m + \{\mathcal{Q}, \bar{\mathcal{Q}}\} + \psi^i \partial_i \mathcal{Q} + \bar{\psi}_i \partial^i \bar{\mathcal{Q}}$$

- In the path integral, insert

$$\mathrm{Str}_{\mathcal{V}}\left[P \exp\left(i \oint d\varphi \mathcal{A}_\varphi\right)\right]$$

- Warner term canceled in SUSY variation if $Q^2 = W \cdot 1_V$.
- Non-abelian + equivariant (straightforward) generalization of the abelian result in [Herbst,Hori,Page].

SUSY localization gives exact answers for some quantities

- If S and Q·V are Q-invariant,

$$0 = \frac{\partial}{\partial T} \int DA...e^{-S-TQ\cdot V}$$

- Take T to +∞. Do Gaussian integrals. Sum(integrate) over saddle points.

- Use the SUSY Lagrangian and transformations used for $\mathbf{S}^2$ localization.
[Benini,Cremonesi][Doroud,Gomis,Lee,Le Floch][Gomis,Lee]

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For given bulk and boundary data, the hemisphere partition function can be computed

$$Z_{\text{hem}}(\mathcal{B};t;m)$$
$$= \frac{1}{|W(G)|}\int_{\sigma\in it}\frac{d^{\text{rk}(G)}\sigma}{(2\pi i)^{\text{rk}(G)}}\text{Str}_{\mathcal{V}}[e^{-2\pi i(\sigma+m)}]e^{t\cdot\sigma}Z_{\text{1-loop}}(\sigma;m)$$

The one-loop determinant is given as

$$Z_{\text{1-loop}}(\sigma;m)$$
$$= \left(\prod_{\alpha>0}\alpha\cdot\sigma\sin(\pi\alpha\cdot\sigma)\right)\prod_{a}\prod_{w\in R_a}\Gamma(w\cdot\sigma+m_a)$$

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D-branes preserving B-type SUSY are objects in the derived category of coherent sheaves

- Any object (B-brane) in the derived category can be represented as a complex of holomorphic vector bundles (space-filling branes).
- It is enough to consider Neumann boundary conditions.

Given an object in the derived category, the boundary data B=(V,Q) can be constructed

- $\exists$Algorithm: object E in derived cateogory -> boundary data B=(V,Q). [Herbst,Hori,Page]
- Example: structure sheaf of the quintic.

$$\phi=(P,x^1,\dots,x^5),\ W=P\cdot G(x)$$

$$\{\eta,\bar{\eta}\}=1,\ \eta|0\rangle=0,\ V=\mathbb{C}|0\rangle+\mathbb{C}\bar{\eta}|0\rangle,\ Q=G(x)\eta+P\bar{\eta}$$

$$Z_{\text{hem}} = \int_{i\mathbb{R}} \frac{d\sigma}{2\pi i}(e^{-5\pi i\sigma} - e^{5\pi i\sigma})e^{t\sigma}\Gamma(\sigma)^5\Gamma(1-5\sigma)$$

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We argue that Z_{hem} computes the central charge of the D-brane

- Z_{hem} expected to be invariant under a certain metric deformation.
- The theory is in the Ramond-Ramond sector in the large deformation limit.

- Overlap <B|1>.
- In the mirror case and in a similar set-up, this is $\int_L \Omega$, the exact central charge of an A-brane. [[Ooguri-Oz-Yin]

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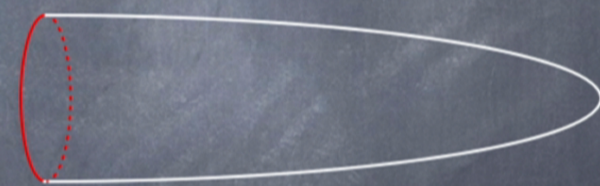

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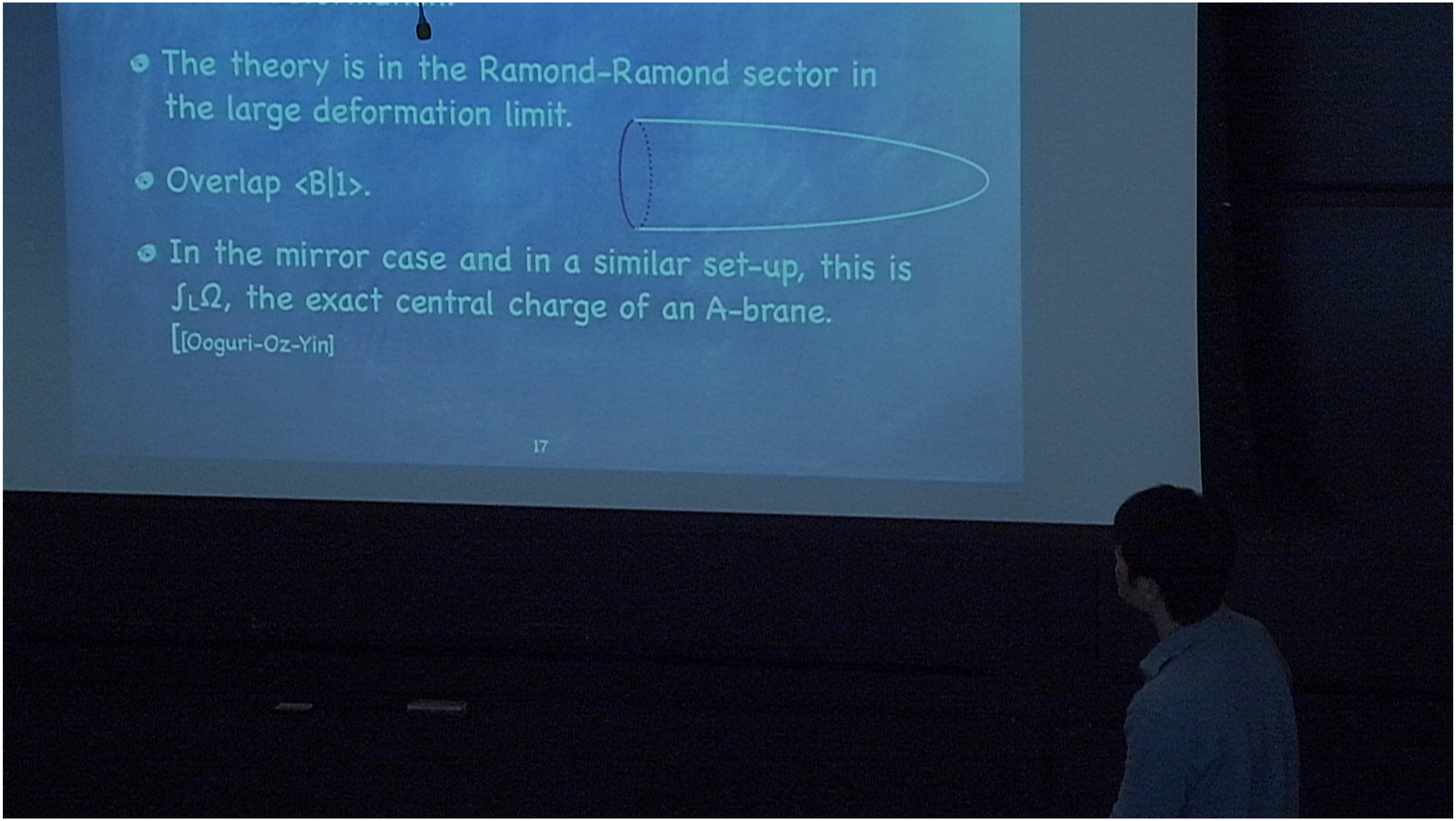
The sphere partition function can be factorized using Z_{hem}

- For models with target $T^*Gr(N,N_F)$, we get $Z_{hem}(B)=\Sigma_v <B|v><v|1>$, $<v|1>\sim Z^v{}_{vortex}(t,m)$, and $Z_{sphere}=\Sigma_v <1|v><v|1>$, using the same $<v|1>$.
- True if we include flux-dependent weights in Z_{sphere}.
- We can also write this as $Z_{sphere}=\Sigma_{i,j}<1|B_i>\chi^{ij}<B_j|1>$, $\chi^{ij}<B_j|B_k>=\delta^i{}_k$.

2nd test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Domain walls

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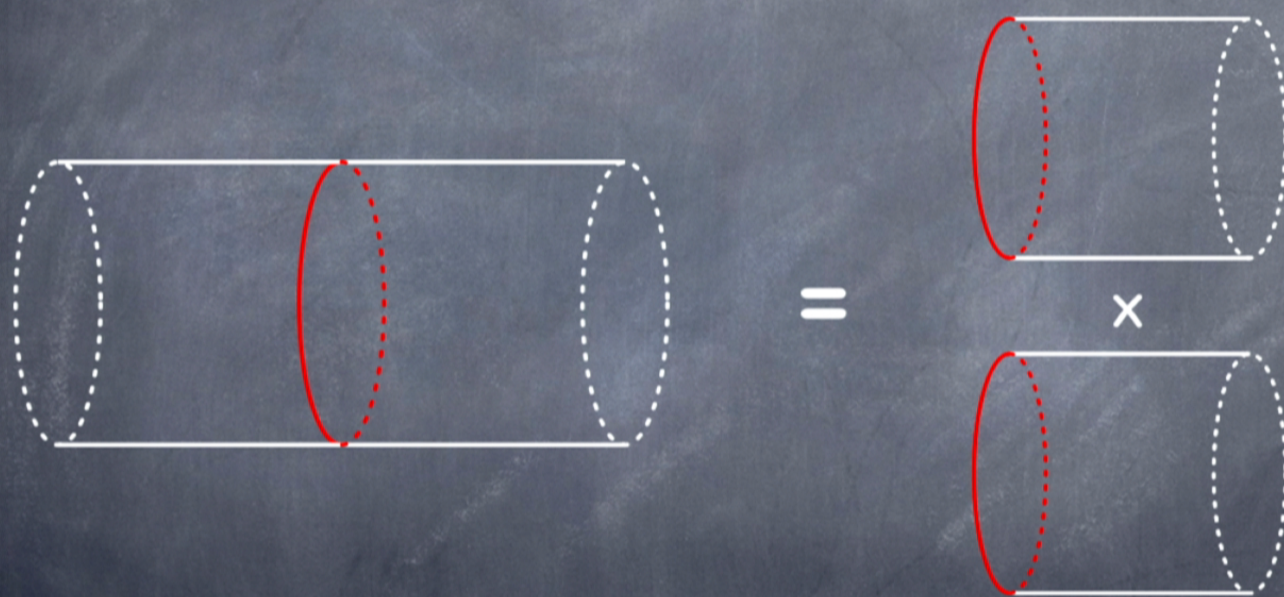
A doman wall is a boundary in the folded theory



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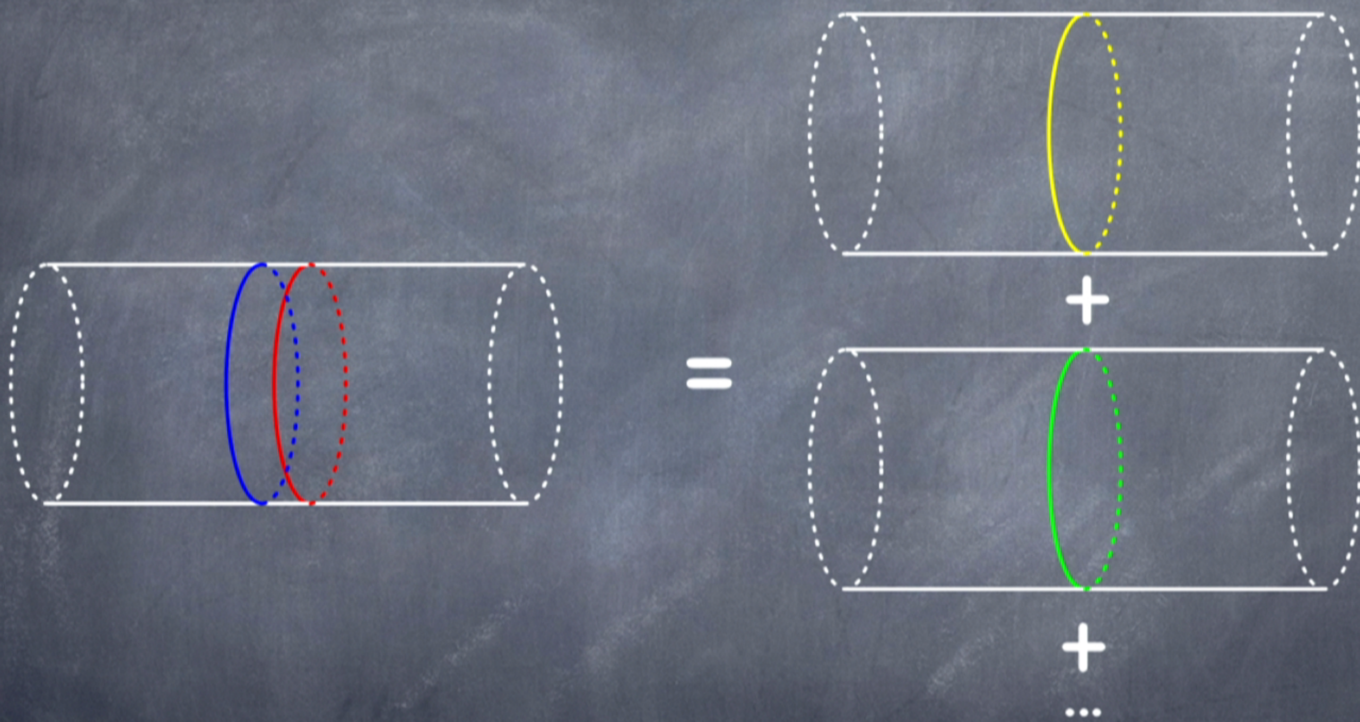
Domain walls form algebras



Proposal by Nekrasov and Shatashvili: there should be connections to geometric representation theory.

Gauge theories are related to quantum integrable systems
[Nekrasov-Shatashvili]

- Collection of $T^*Gr(N,N_F)$ for $0 \leq N \leq N_F$ gives the XXX spin chain model.
- The chiral ring relations are the Bethe ansatz equations.
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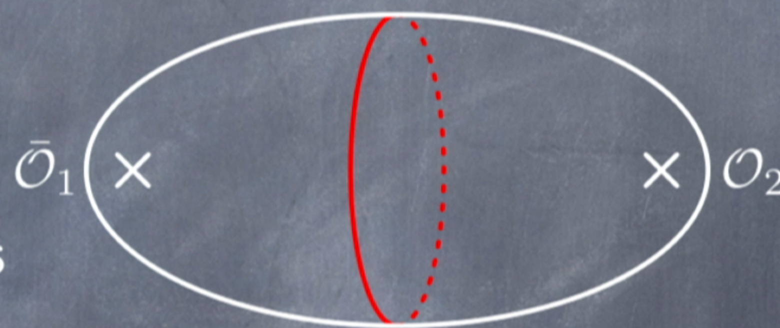
One can use the (extended) hemisphere partition function to compute domain wall matrix elements

We can insert chiral and anti-chiral operators at the two tips.

Read off matrix elements <v|W|w> from the correlation function.



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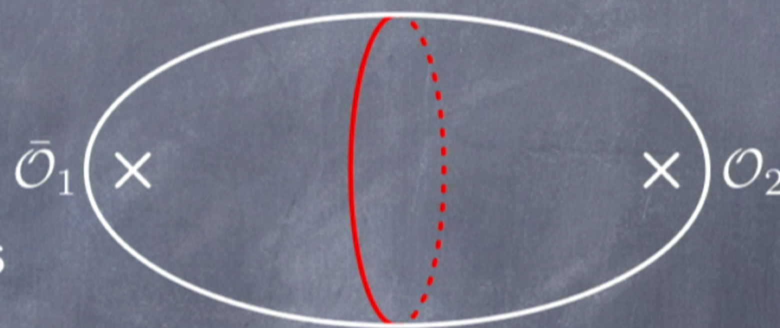

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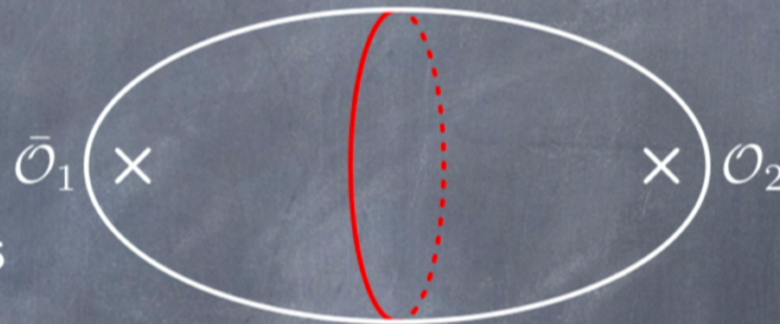

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As an example we obtain the sl(2) affine Hecke algebra

- sl(2) affine Hecke algebra is generated by T and X satisfying

  $(T+1)(T-q)=0,\ TX^{-1}-XT=(1-q)X.$

  q: parameter

- Domain walls in $T^*\mathbf{P}^1$ model realize this algebra.

As an example we obtain the sl(2) affine Hecke algebra

1: diagonal of $T^*\mathbf{P}^1 \mathrm{x} T^*\mathbf{P}^1$

X: charge -1 Wilson loop

-1-T: push-forward of $q^{-1/2}O(-1,-1)$ by $\mathbf{P}^1\mathrm{x}\mathbf{P}^1\text{->}T^*(\mathbf{P}^1\mathrm{x}\mathbf{P}^1)$

Agrees with geometric representation (Kazhdan-Lusztig) theory up to convention change.

In progress: Yangian/quantum affine algebra

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Conclusion

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