Title: Hybrid conformal field theories

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Abstract: I will discuss a class of limiting points in the moduli space of d=2 (2,2) superconformal field theories. These SCFTs arise as IR limits of "hybrid" UV theories constructed as a fibration of a Landau-Ginzburg theory over a base Kaehler geometry. A significant generalization of Landau-Ginzburg and large radius geometric limit points, the hybrid theories can be used to probe general features of (2,2) and (0,2) SCFT moduli spaces.

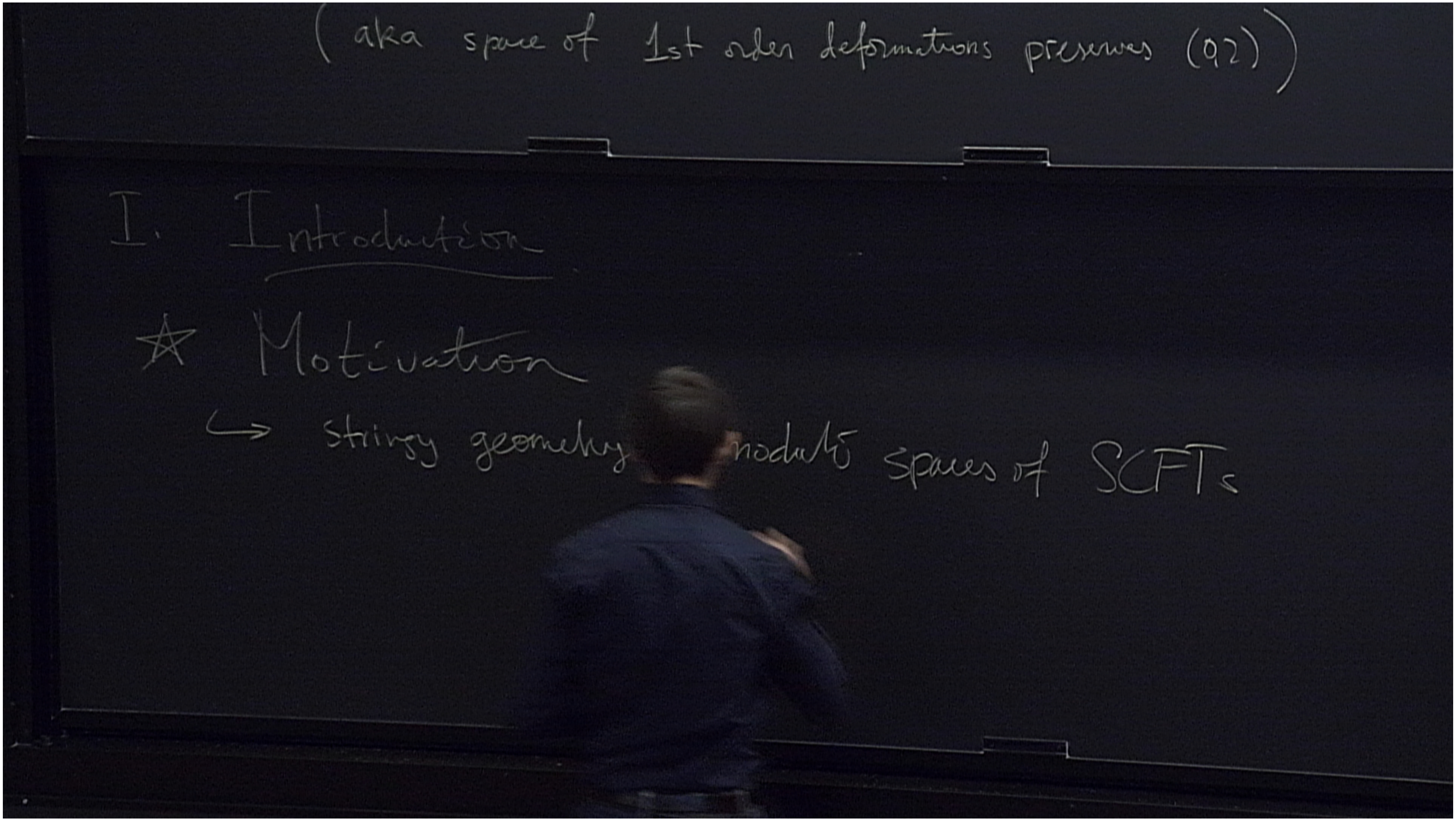
Hybrid Conformal Theories with M. Bertolini & R. Plesser 1307.7063

Hybrid Conformal Theories with M. Bertolini & R. Plesser

focus: $d=2$ $(2,2)$ SCFTs with $c_L = c_R = 9$ & $q_L, q_R \in \mathbb{Z}$

results:

1) intrinsic defn of hybrids
2) analysis of massless spacetime spectrum
   (aka space of 1st order deformations preserving $(0,2)$)


(aka space of 1st order deformations preserves (0,2))
I. Introduction
★ Motivation
↪ stringy geometry
moduli spaces of SCFTs

(aka space of 1st order deformations preserves (0,2))

I. Introduction

☆ Motivation

↪ stringy geometry; moduli spaces of SCFTs
mirror symmetry.

↪ many questions!

* is there a finite # of families of (2,2) SCFTs?
* does every SCFT $\longrightarrow CY_3$?
* # of marginal (0,2) defs?
* $\mathcal{M}_{(2,2)} \subseteq$

(2,2) SCFTs?

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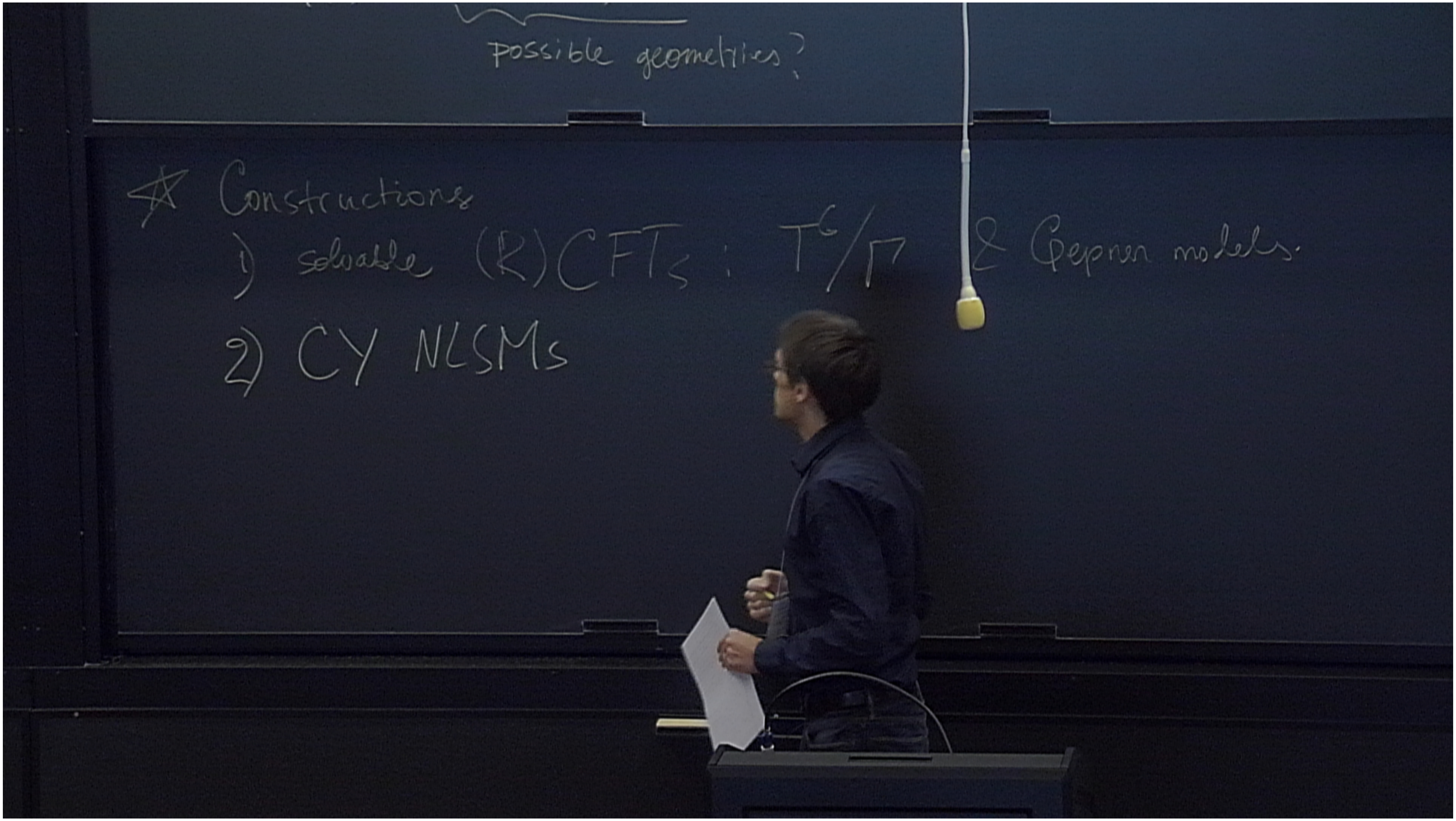
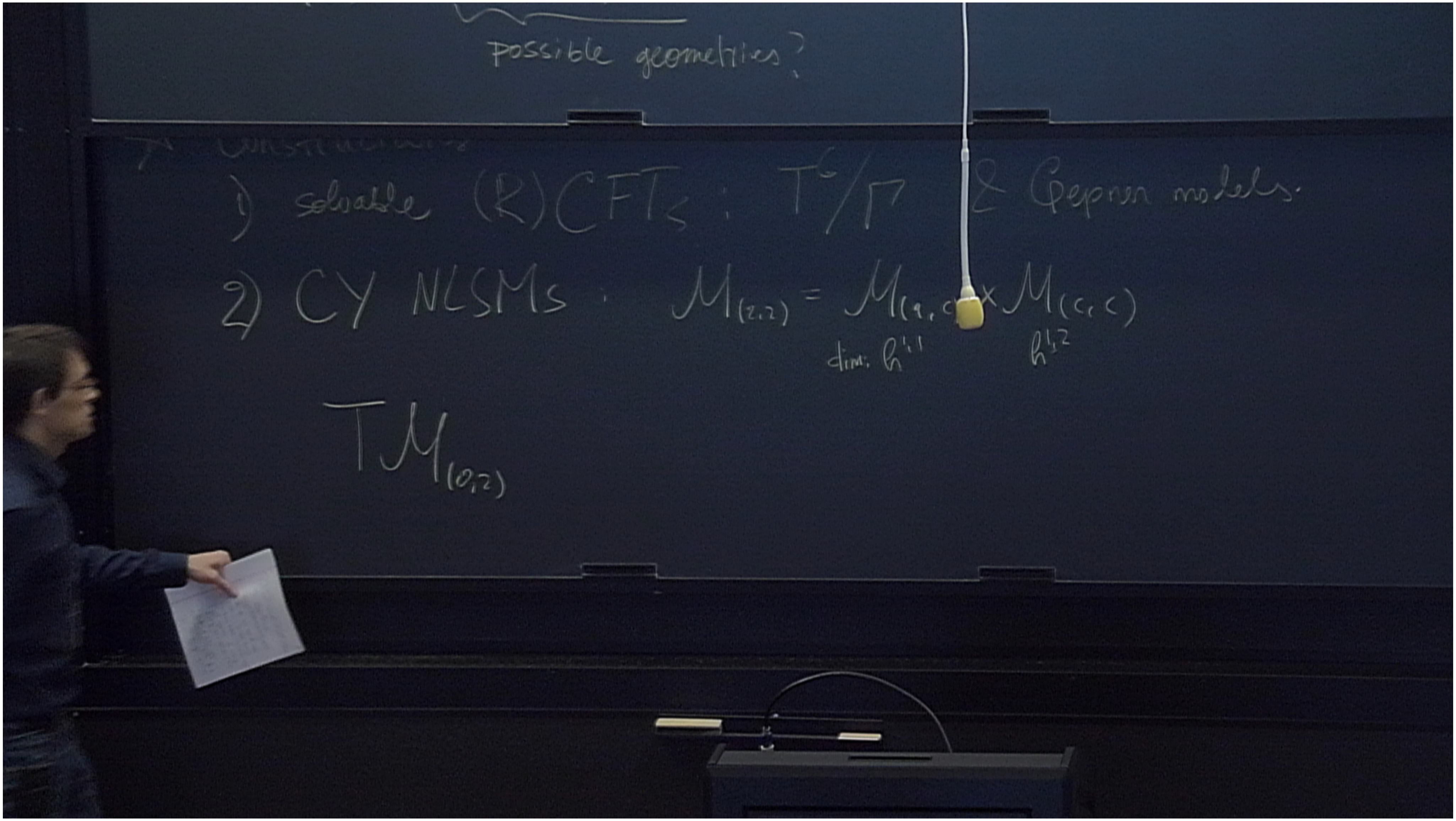
* # of marginal (0,2) defs?

* $\mathcal{M}_{(2,2)} \subseteq \underbrace{\mathcal{M}_{(0,2)}}_{\text{possible geometries?}}$


possible geometries?
Constructions
1) solvable (R)CFTs : T^6/Γ & Gepner models.
2) CY NLSMs


possible geometries?
1) solvable (R)CFTs : T^6/Γ & Gepner models.
2) CY NLSMs : M(2,2) = M(a,c) × M(c,c)
dim: h^1,1
h^1,2
TM(0,2)

possible geometries?

1) solvable (R)CFTs : T^4/Γ, Gepner models

2) CY NLSMs : $\mathcal{M}_{(2,2)} = \mathcal{M}_{(a,c)} \times \mathcal{M}_{(c,c)}$

dim: $h^{1,1}$ $h^{1,2}$

$$T\mathcal{M}_{(0,2)}\Big|_{\mathcal{M}(2,2)} = H^1(M, T_M^*) \oplus H^1(M, T_M) \oplus H^1(M, T_M \otimes T_M^*)$$

3) RG flow from (2,2) SUSY UV theory.

↳ GLSMs

3) RG flow from (2,2) SUSY UV theory.

$\hookrightarrow$ GLSMs ($d=2$ gauge theories)

$\cup$

$\hookrightarrow$ Landau-Ginzburg Orbifolds ($\supset$ Gepner model)

"NLSM with "superpotential""

* target : $\mathbb{C}^n$
* $W \subset \mathbb{C}[\phi_1, \dots, \phi_n]$ $\quad W(t^{m_i}\phi_i) = t^N W(\phi)$ $\quad m_i, N \in \mathbb{Z}_{>0}$
* RG flows to SCFT with:

$$c_L = c_R = 3 \sum_i \left(1 - 2\frac{m_i}{N}\right)$$

$$U(1)_L \times U(1)_R, \quad q_{L,R}(\phi_i) = \frac{m_i}{N} \equiv q_i$$

(c,c) ring $R_{cc} = \mathbb{C}[\phi_1 \dots \phi_n] / \langle dW \rangle$

(a,c) ring $R_{a,c} = 1$

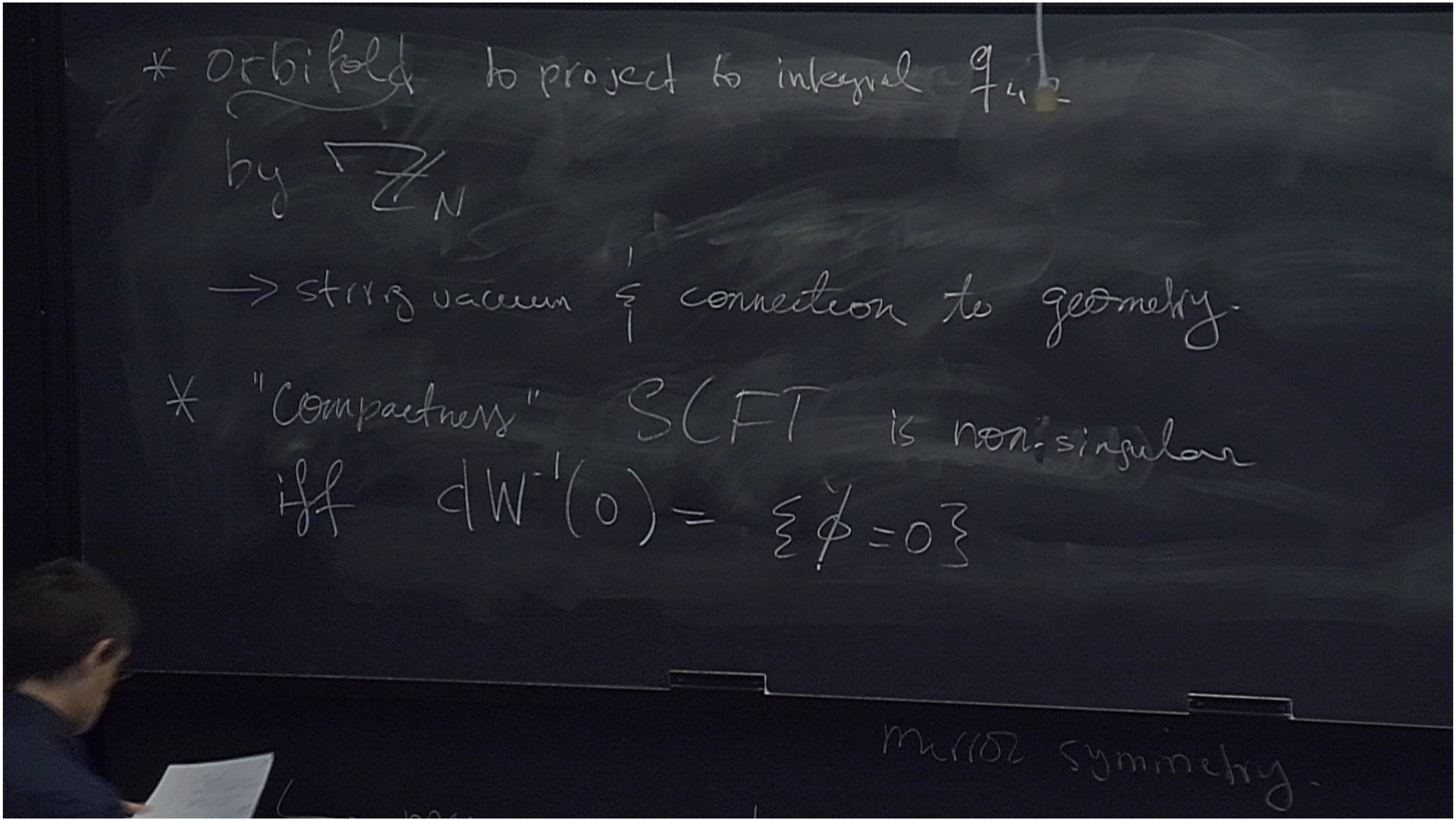
2) CY NLSMs : $\mathcal{M}_{(2,2)} = \mathcal{M}_{(a,c)} \times \mathcal{M}_{(c,c)}$

dim: $h^{1,1}$ $\quad h^{1,2}$

* Orbifold to project to integral q_{4R}
  by $\mathbb{Z}_N$

  $\rightarrow$ string vacuum & connection to geometry.

* "Compactness" SCFT is non-singular
  iff $dW^{-1}(0) = \{\phi = 0\}$


* Orbifold to project to integral q_4
by $\mathbb{Z}_N$
→ string vacuum & connection to geometry.
* "Compactness" SCFT is non-singular
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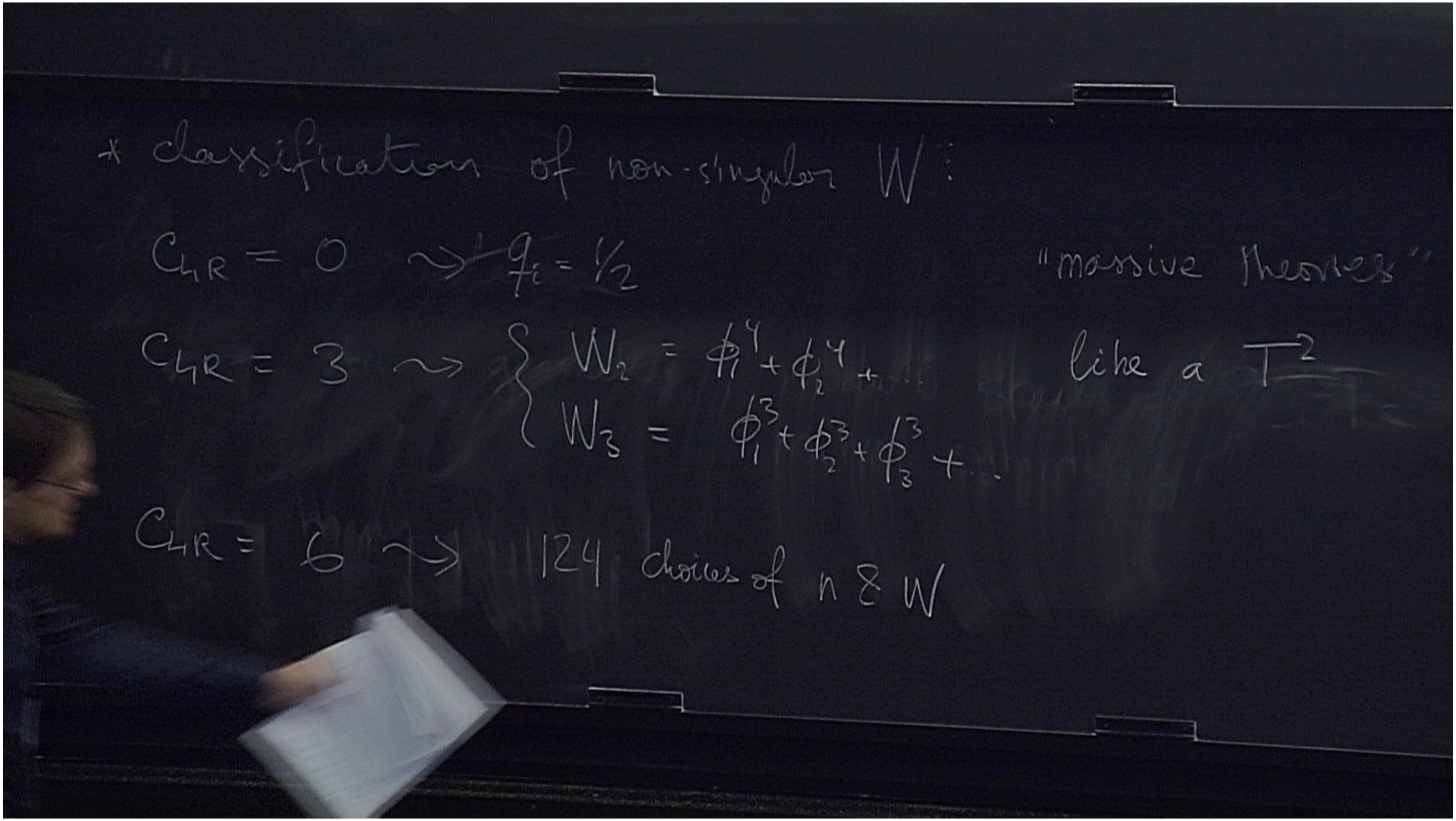
mirror symmetry.

* classification of non-singular W :

$$c_{L,R} = 0 \rightsquigarrow q_i = 1/2$$


* classification of non-singular W:
$c_{4R} = 0 \rightsquigarrow q_i = 1/2$
"massive theories"
$c_{4R} = 3 \rightsquigarrow \begin{cases} W_2 = \phi_1^4 + \phi_2^4 + \dots \\ W_3 = \phi_1^3 + \phi_2^3 + \phi_3^3 + \dots \end{cases}$
like a T^2
$c_{4R} = 6 \rightsquigarrow$ 124 choices of n & W

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$c_{4R} = 6 \rightsquigarrow$ 124 choices of n & W like a K3

$c_{4R} = 9 \rightsquigarrow$

* classification of non-singular W:

$c_{LR} = 0 \rightsquigarrow q_i = 1/2$ "massive theories"

$c_{LR} = 3 \rightsquigarrow \begin{cases} W_2 = \phi_1^4 + \phi_2^4 + \dots \\ W_3 = \phi_1^3 + \phi_2^3 + \phi_3^3 + \dots \\ \tilde{W}_2 \quad \phi_1^6 + \phi_2^3 \end{cases}$ like a T^2

$c_{LR} = 6 \rightsquigarrow$ 124 choices of n & W like a K3

$c_{LR} = 9 \rightsquigarrow$ 10,839 choices " " CY_3

★ Hybrid inevitability!

↪ CY/LGO correspondence

$$E = \{ W = \phi_1^3 + \phi_2^3 + \phi_3^3 = 0 \} \subset \mathbb{CP}^2[\phi_1, \phi_2, \phi_3]$$

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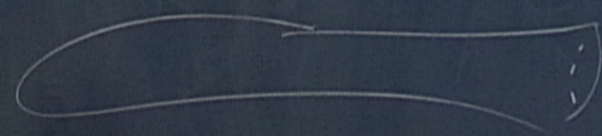

$\mathcal{M}_{(a,c)}$

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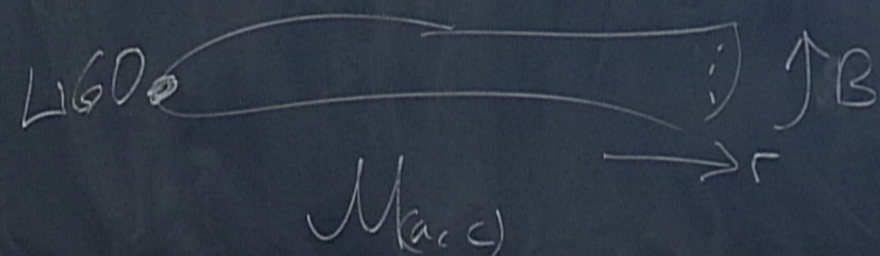
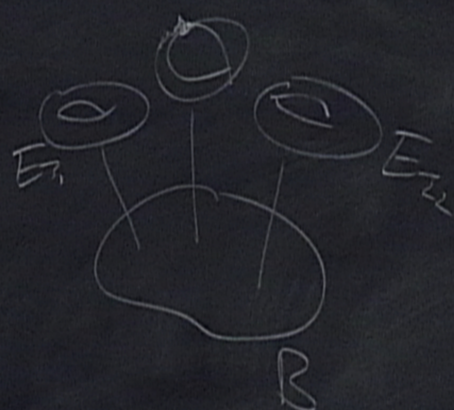
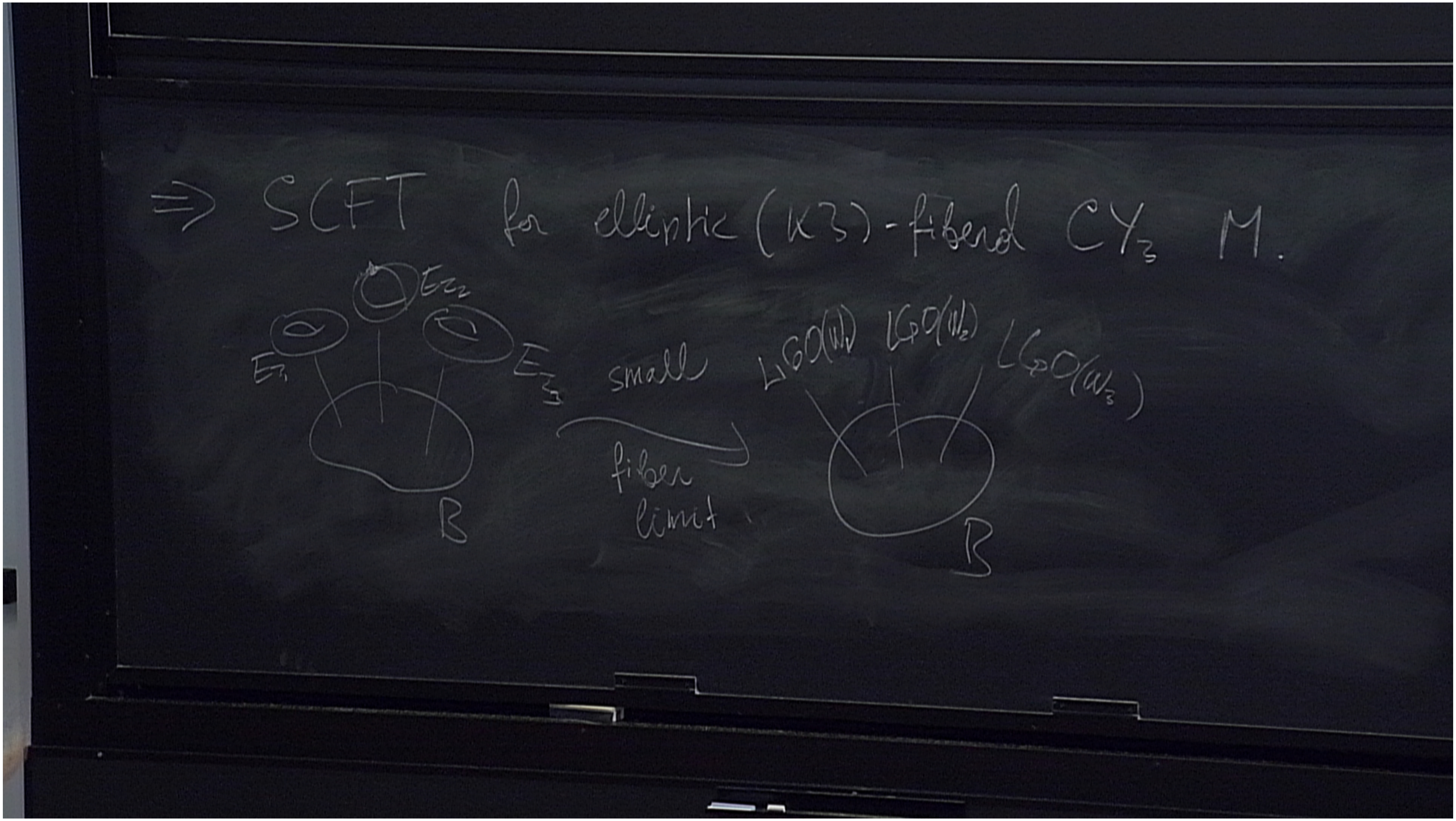
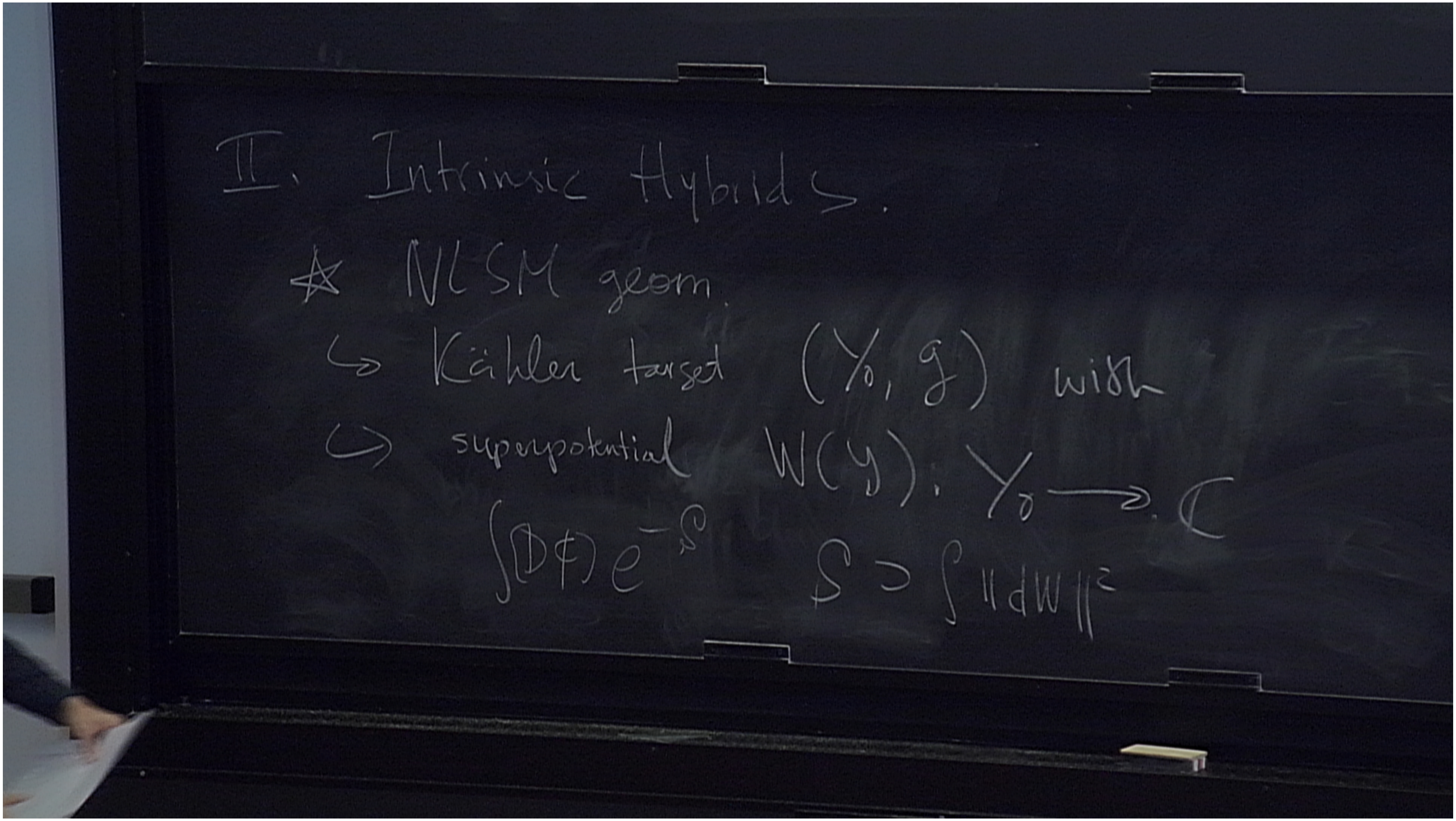


$\Rightarrow$ SCFT for elliptic (K3)-fibered CY_3 M.




⇒ SCFT for elliptic (K3)-fibered CY_3 M.
E_{z_1}
E_{z_2}
E_{z_3}
B
small
fiber limit
$LG\,O(W_1)$
$LG\,O(W_2)$
$LG\,O(W_3)$
B


II. Intrinsic Hybrids.
NLSM geom.
Kähler target (Y_0, g) with
superpotential $W(y): Y_0 \to \mathbb{C}$
$\int (D\phi)\, e^{-S}$
$S \supset \int \|dW\|^2$

$\hookrightarrow$ potential condition :

$dW^{-1}(0) = B$, smooth, compact mfld $\dim_{\mathbb{C}} B = d$

$\Longrightarrow$

$\hookrightarrow$ potential condition :

$dW^{-1}(0) = B$, smooth, compact mfld $\dim_{\mathbb{C}} B = d$

$\Rightarrow$ model for IR physics by

$* \; Y = \mathrm{tot}(X \to B)$

$y^\alpha = (u, \phi)$

$* \; W = \sum'_{\vec{m}} f_{\vec{m}}(u)$

$$\mathcal{M}_{(0,2)}\Big|_{\mathcal{M}(2,2)} = H^1(M, T_M^*) \oplus H^1(M, T_M) \oplus H^1(M, T_M \otimes T_M$$

$\hookrightarrow$ potential condition :

$dW^{-1}(0) = B$, smooth, compact mfld $\dim_{\mathbb{C}} B = d$

$\Rightarrow$ model for IR physics by

* $Y \equiv \mathrm{tot}(X \to B)$

$y^\alpha = (u, \phi)$

* $W = \sum_{\vec{m}} f_{\vec{m}}(u) \prod_{i=1}^{n} (\phi_i)^{m_i} \equiv$ section of $\mathcal{O}_Y$

Simplest case

$X = \bigoplus_i L_i$

2) CY NLSMs : $\mathcal{M}_{(2,2)} = \mathcal{M}_{(a,c)} \times \mathcal{M}_{(c,c)}$

$$* \; W = \sum_{\vec{m}} f_{\vec{m}}(u) \prod_{i=1} (\phi_i)^{m_i} \equiv \text{section of } \mathcal{O}_Y$$

Symmetries :

$\hookrightarrow \; U(1)_L \times U(1)_R \Longleftrightarrow$

$* \; c_1(T_Y) = 0 \Longleftrightarrow c_1(T_B) + c_1(X) = 0$

$* \; \exists \, (!) \; V \in H^0(Y, T_Y) \text{ s.t. } \mathcal{L}_V W = W$

↳ "good" vs "pseudo" hybrids..

* <u>good hybrid</u> iff V is vertical v.f. $\Rightarrow$ $V = \sum_{i=1}^{p} q_i \phi^i \frac{\partial}{\partial \phi^i}$

charges associated to LG fiber

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$\rightarrow$ SCFT $\quad c_{L,R} = \underline{3d} + 3 \sum_i (1 - 2q_i)$

* if V is not vertical then $U(1)_L \times U(1)_R$ action is base-dependent!

$\Leftrightarrow$ "good" vs "pseudo" hybrids..

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$\rightsquigarrow$ singular ?? (Aspinwall & Plesser '09) associated ... fiber

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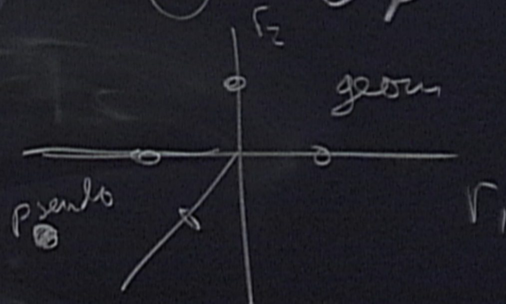

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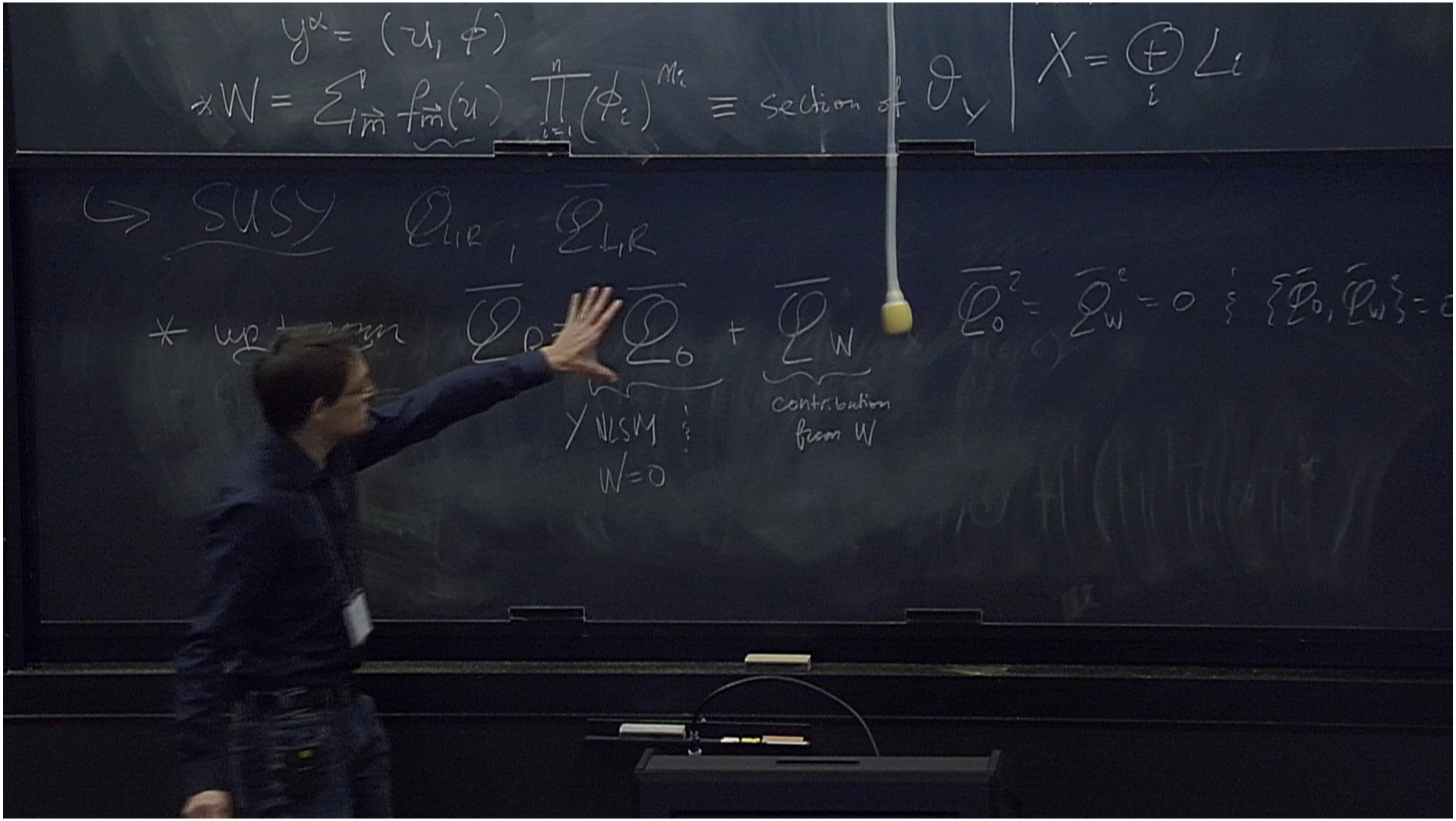
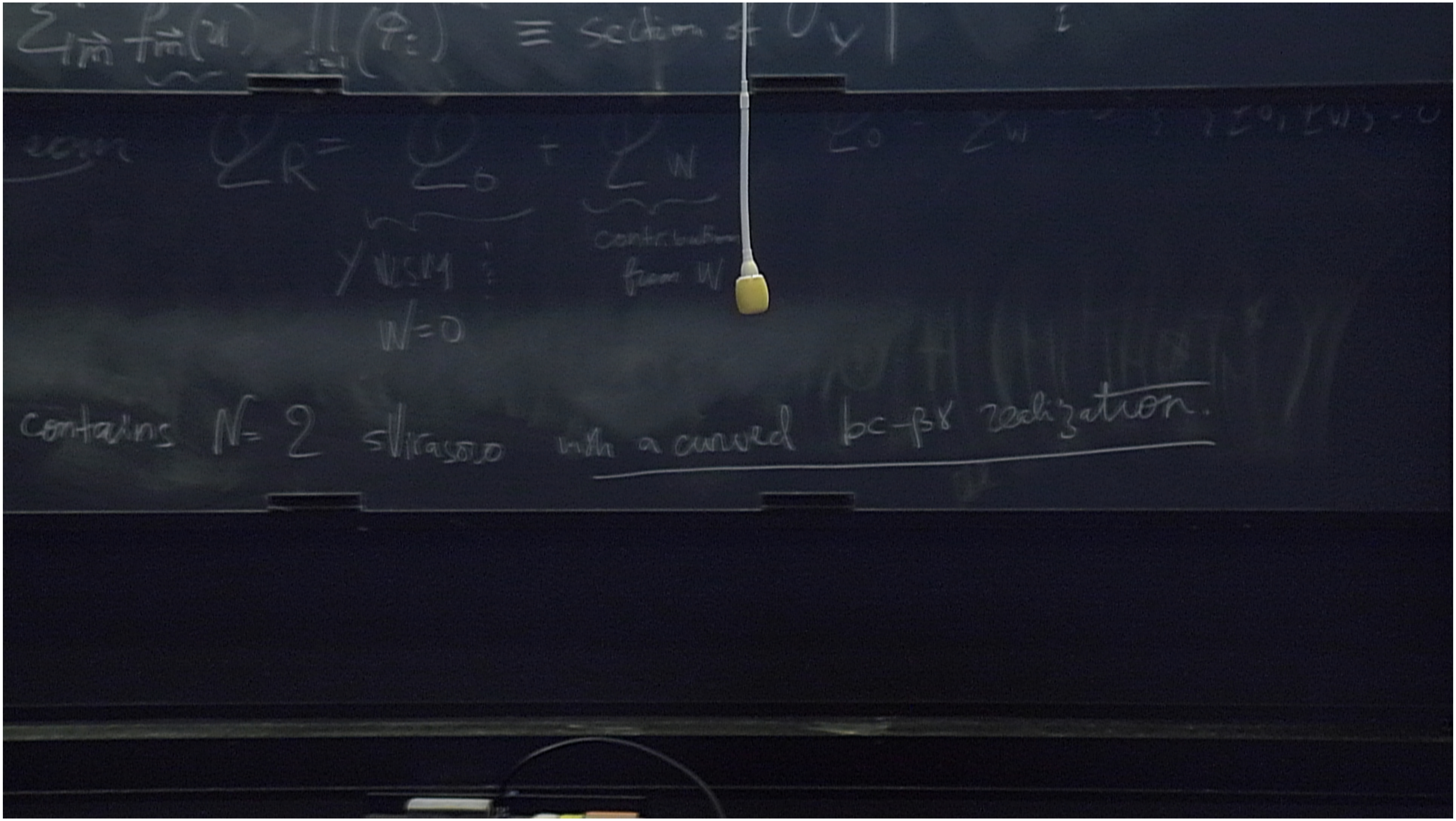
(Aspinwall & Plesser '09)


$y^\alpha = (u, \phi)$
$* W = \sum_{\vec{m}} f_{\vec{m}}(u) \prod_{i=1}^{n} (\phi_i)^{m_i} \equiv$ section of $\mathcal{O}_X$
$X = \bigoplus_i L_i$
$\hookrightarrow$ SUSY $Q_{L,R}$, $\bar{Q}_{L,R}$
$\bar{Q}_D = \bar{Q}_0 + \bar{Q}_W$
$\bar{Q}_0^2 = \bar{Q}_W^2 = 0$; $\{\bar{Q}_0, \bar{Q}_W\} = 0$
Y NLSM
W=0
contribution from W


≡ section of
contribution from W
W=0
contains N=2 sVirasoro with a curved bc-βγ realization.

III. Hybrid limit & (0,2) defs.

↪ orbifold by $\mathbb{Z}_N$ as in LG ⇝ hybrid orbifold theories:

* string vacuum & possible connection to geom.

$Y = \text{tot}(X \to B)$ * "Orbi-bundles" e.g. $X = \underline{\mathcal{O}(-5/2) \oplus \mathcal{O}(-3/2)} \longrightarrow \mathbb{P}^3$

↪ take <u>hybrid limit</u>

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III. Hybrid limit & (0,2) defs

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↪ take <u>hybrid limit</u> : large volume of base B.

↪ "good" vs "pseudo" hybrids..

☆ 1st order marginal defs : (0,2)!

↪ CY large radius limit : $\underbrace{h^1(T) + h^1(T^*)}_{T\mathcal{M}_{(2,2)}} + \underbrace{h^1(T\otimes T^*)}$

not topological!
depend on c-x structure of M.
& there can be quantum obstructions!

$$2)!$$

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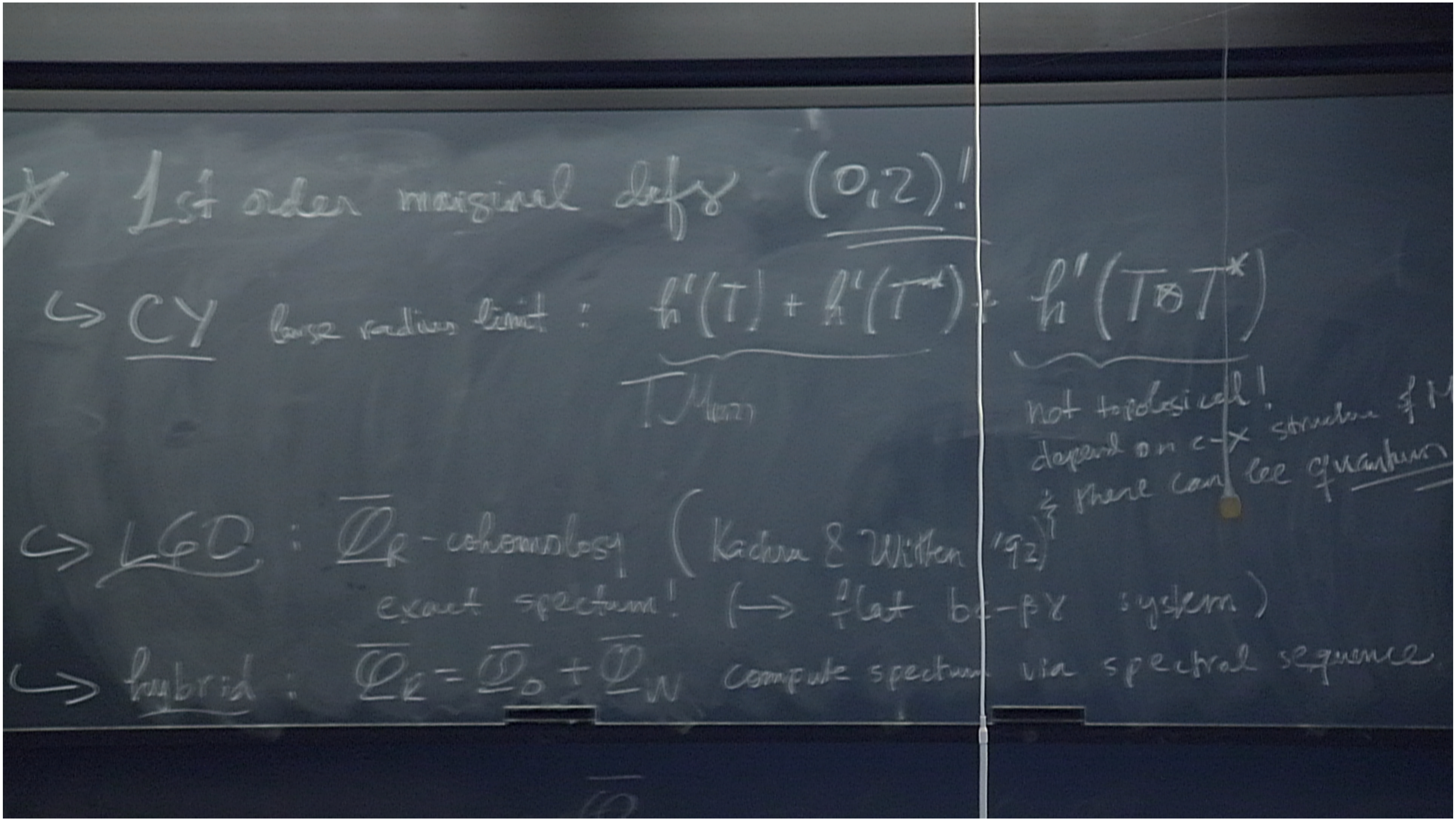
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↪ LGO: $\bar{Q}_R$-cohomology (Kachru & Witten '93)
exact spectrum! (→ flat bc-βγ system)
↪ hybrid: $\bar{Q}_R = \bar{Q}_0 + \bar{Q}_W$ compute spectrum via spectral sequence

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exact spectrum! ($\longrightarrow$ flat bc-$\beta\gamma$ system)

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$\ast$ exact results up to worldsheet instantons in B.

$\ast$ different geometries in different twisted sectors eO : $\overline{Q}_R$-cohomology (Kachru & Witten '92)

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* exact results up to worldsheet instantons in B.

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* dependence on c-x of W & B.

* suggestions of new non-renormalization theorems.

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not topological! depend on c-x structure & [illegible] & there can be quantum [illegible]

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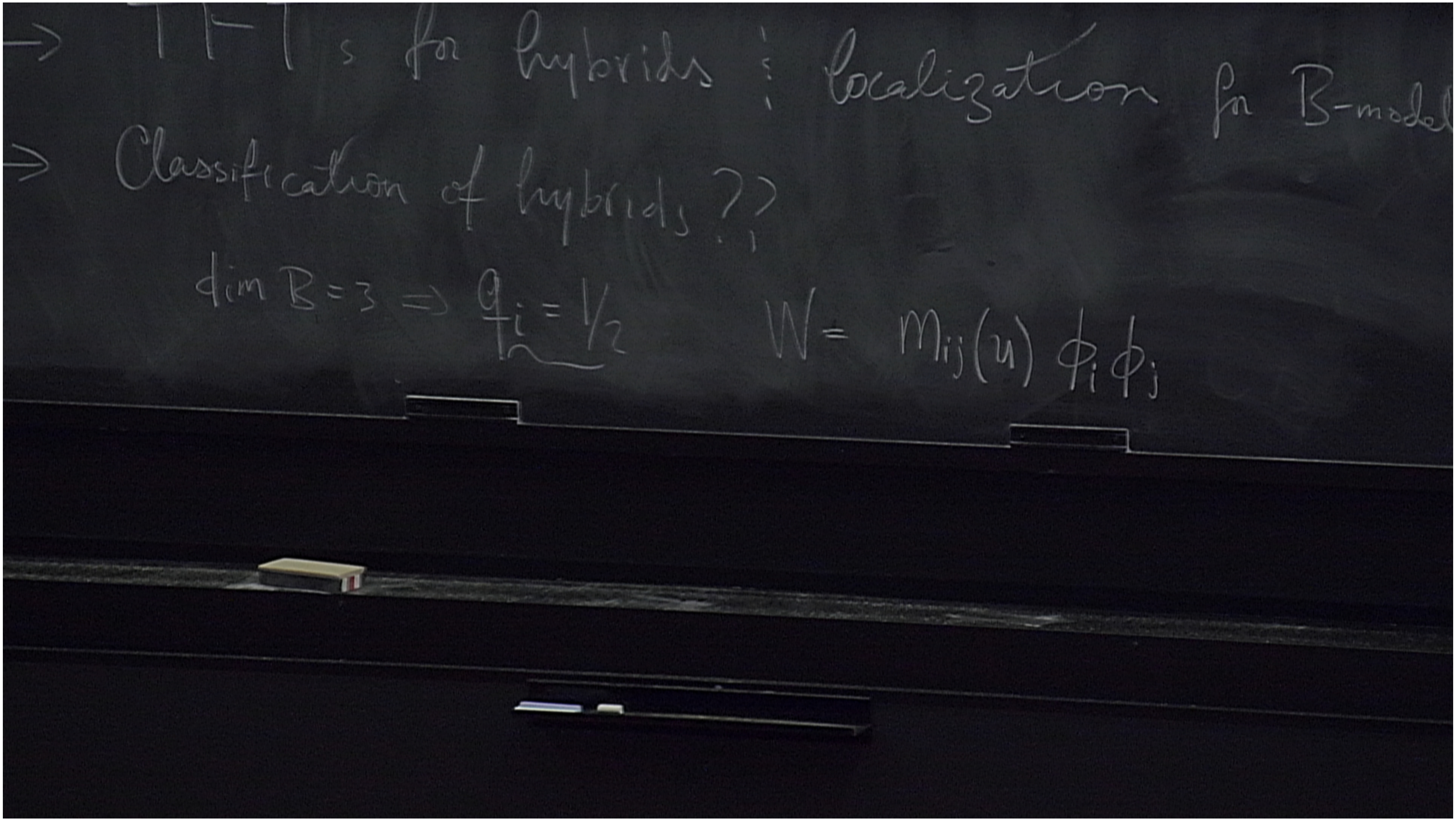
IV Hybrid prospects.

☆ (2,2)

↪ TFTs for hybrids ; localization for B-model?

↪ Classification of hybrids??


⇒ TFTs for hybrids & localization for B-model
⇒ Classification of hybrids??
dim B = 3 ⇒ $q_i = 1/2$
$W = m_{ij}(u)\,\phi_i \phi_j$

hybrids : localization for B-model ?

hybrids ??

$= 1/2$ $\qquad W = \underline{m_{ij}(u)}\, \phi_i \phi_j \longrightarrow$ branched covers of B_3

hybrids : localization for B-model ?

hybrids ??

$= 1/2$ $\quad W = \underline{M_{ij}(u)}\,\phi_i\phi_j \longrightarrow$ branched covers of $B_3 \hookrightarrow$ singular CYs defs ?

☆ (0,2) Thoughts! $\rightsquigarrow$ hybrid orbifold th

$\hookrightarrow$ new classes of SCFTs ??

$\hookrightarrow$ interplay between heterotic anomalies associated to base & fiber

$\hookrightarrow$ non-Kähler B ? fiber as (0,2) LG

$\dim B = 3 \Rightarrow q_i = 1/2$ $\quad W = M_{ij}(u)\,\phi_i \phi_j \longrightarrow$ branched B