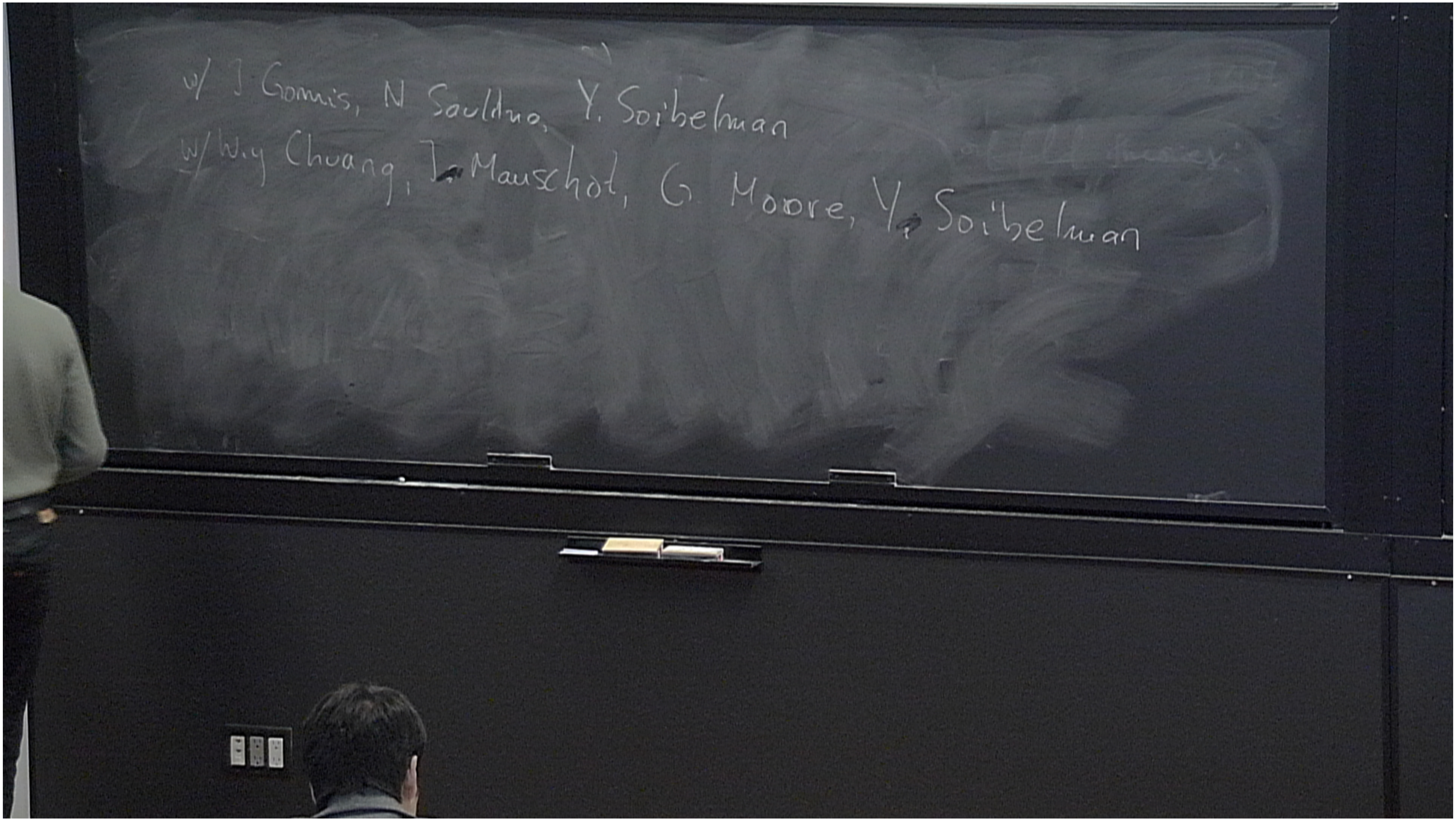
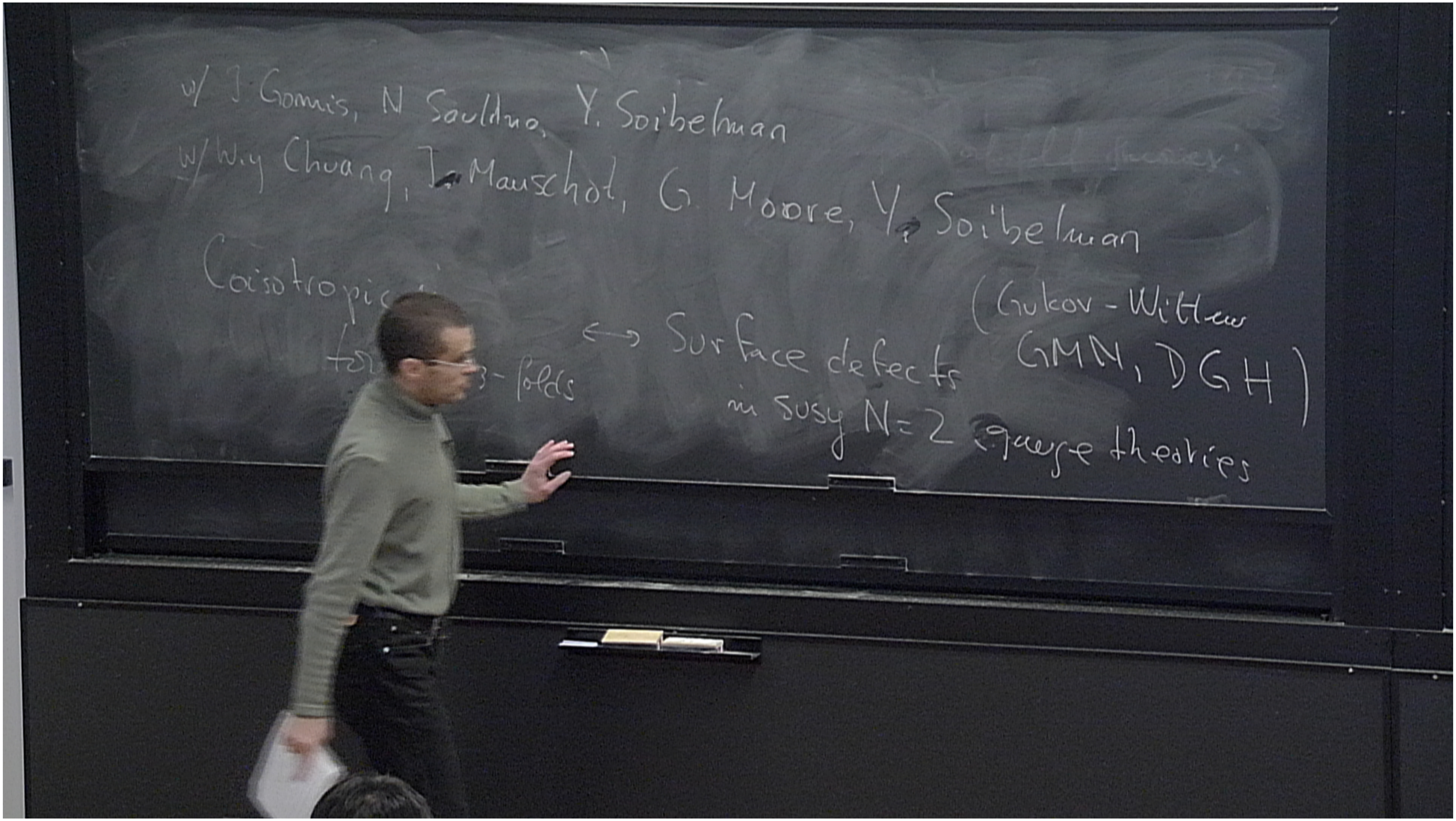
Title: Coisotropic branes, surface defects and mirror symmetry

Date: Oct 21, 2013 02:00 PM

URL: http://pirsa.org/13100113

Abstract:


w/ J Gomis, N Saulina, Y. Soibelman
w/ W.y Chuang, J. Manschot, G. Moore, Y. Soibelman


w/ J. Gomis, N Saulina, Y. Soibelman
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⟷ Surface defects
in susy N=2 gauge theories
(Gukov - Witten
GMN, DGH)

Soibelman

Coisotropic in toric CY_3-folds $\longleftrightarrow$ Surface defects in susy $N=2$ gauge theories (Gukov-Witten, GMN, DGH)

① Coisotropic A-branes [Kapustin-Orlov]

(X, ω) symp manifold $\dim_{\mathbb{R}} X = 6$

Soibelman

Coisotropic in toric CY_3-folds $\longleftrightarrow$ Surface defects in susy $N=2$ gauge theories (Gukov-Witten GMN, DGH)

① Coisotropic A-branes [Kapstin-Orlov]

- (X, ω) symp manifold $\dim_{\mathbb{R}} X = 6$
- (M, L, A) coisotropic A-brane

a) $M \subset X$ coisotropic 5-cycle

$$T^{\perp} M \subset TM$$

a) $M \subset X$ coisotropic S-cycle

$$T^{\perp}M \subset TM$$

$$\wr\Vert \leftarrow \text{induced by } \omega$$

$$N^{*}_{M/X}$$

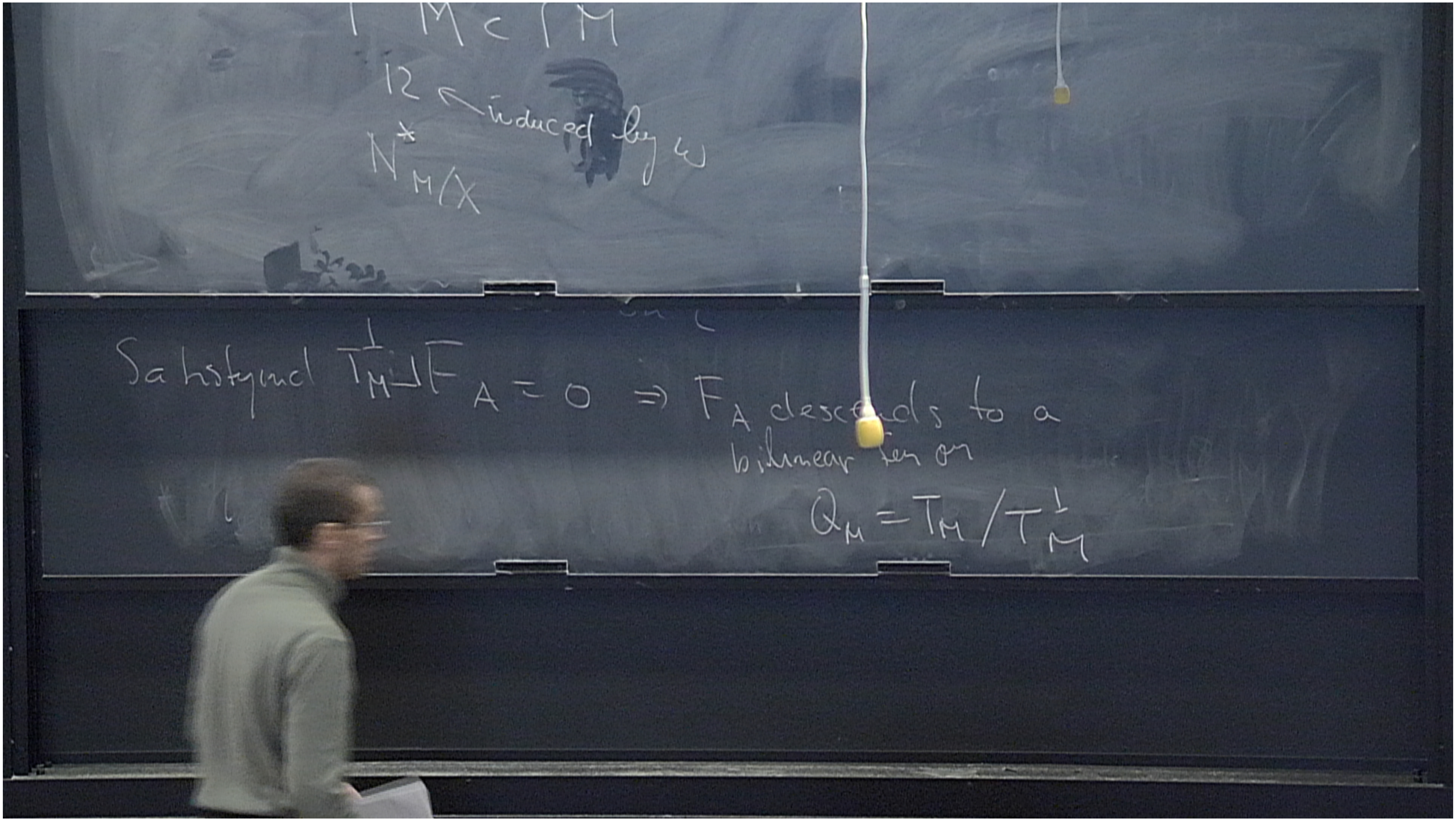
$TM \subset TM$

$\Omega \leftarrow$ induced by ω

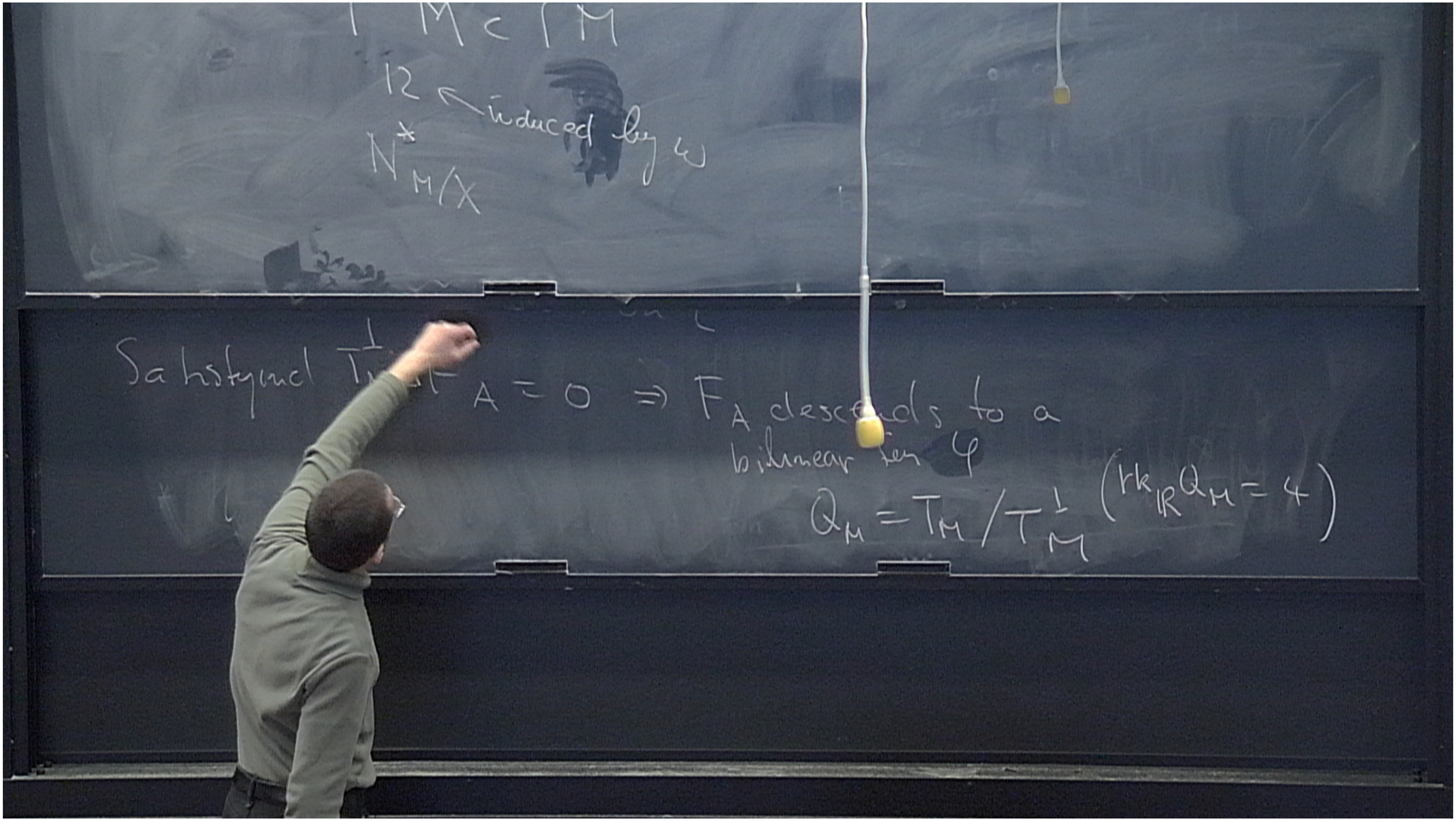
$N^*_{M/X}$

(b) L cpx rk 1 bundle on M

(c) A $U(1)$ connect on L



$T^\perp M \subset TM$
$\Omega \leftarrow$ induced by ω
$N^*_{M/X}$
Satisfying $T^\perp_M \lrcorner F_A = 0 \Rightarrow F_A$ descends to a bilinear fcn on
$Q_M = T_M / T^\perp_M$



induced by ω
Satisfying
A = 0 ⇒ F_A descends to a
bilinear fcn φ
Q_M = T_M / T^⊥_M (rk_ℝ Q_M = 4)

, ... descends to a bil. form $\tilde{\omega}$ on Q_M

$\mathbb{R} \leftarrow$ induced by ω

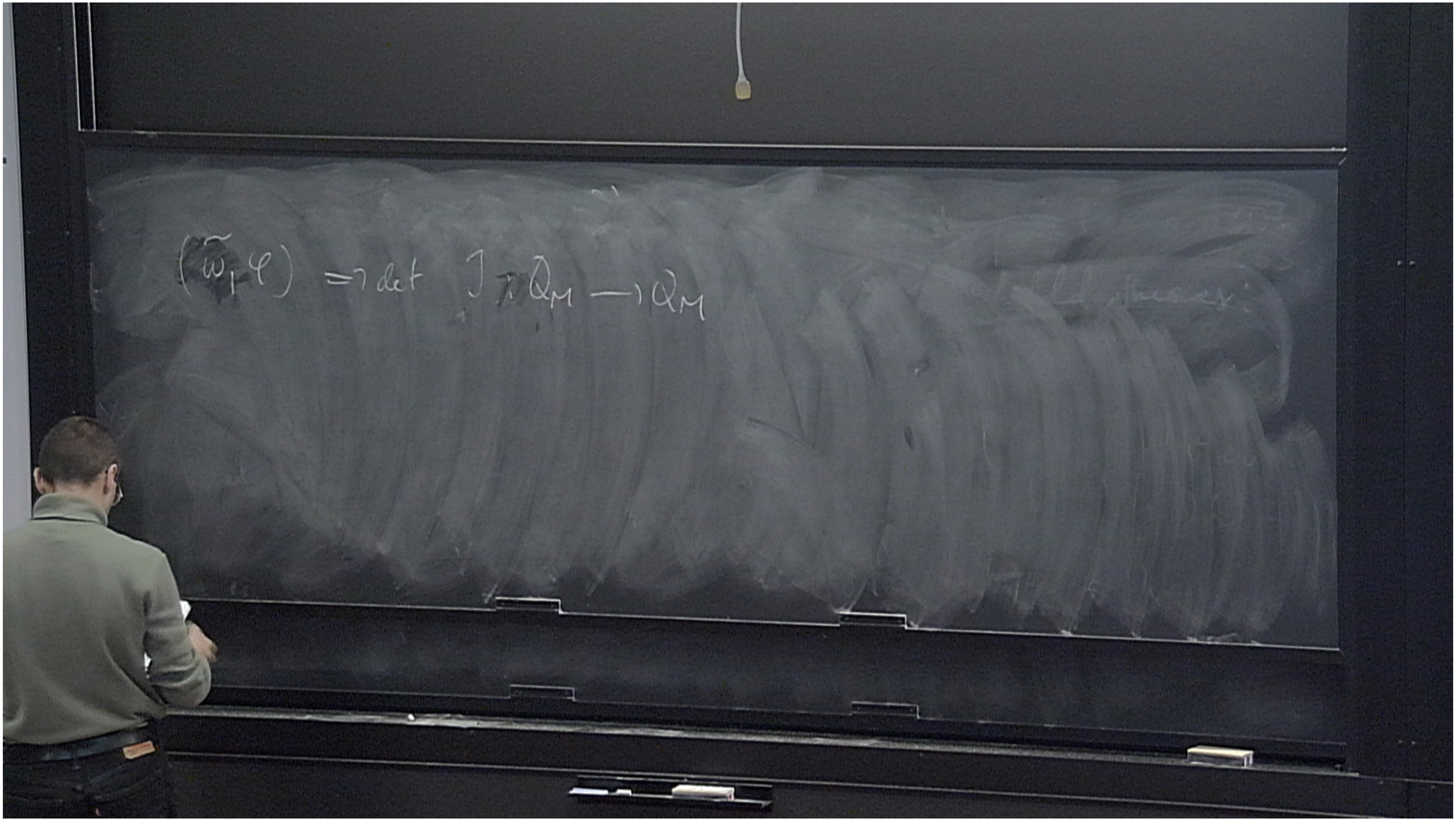
$N^*_{M/X}$

(b) L cpx rk 1 bundle on M

(c) A $U(1)$ connect. on L

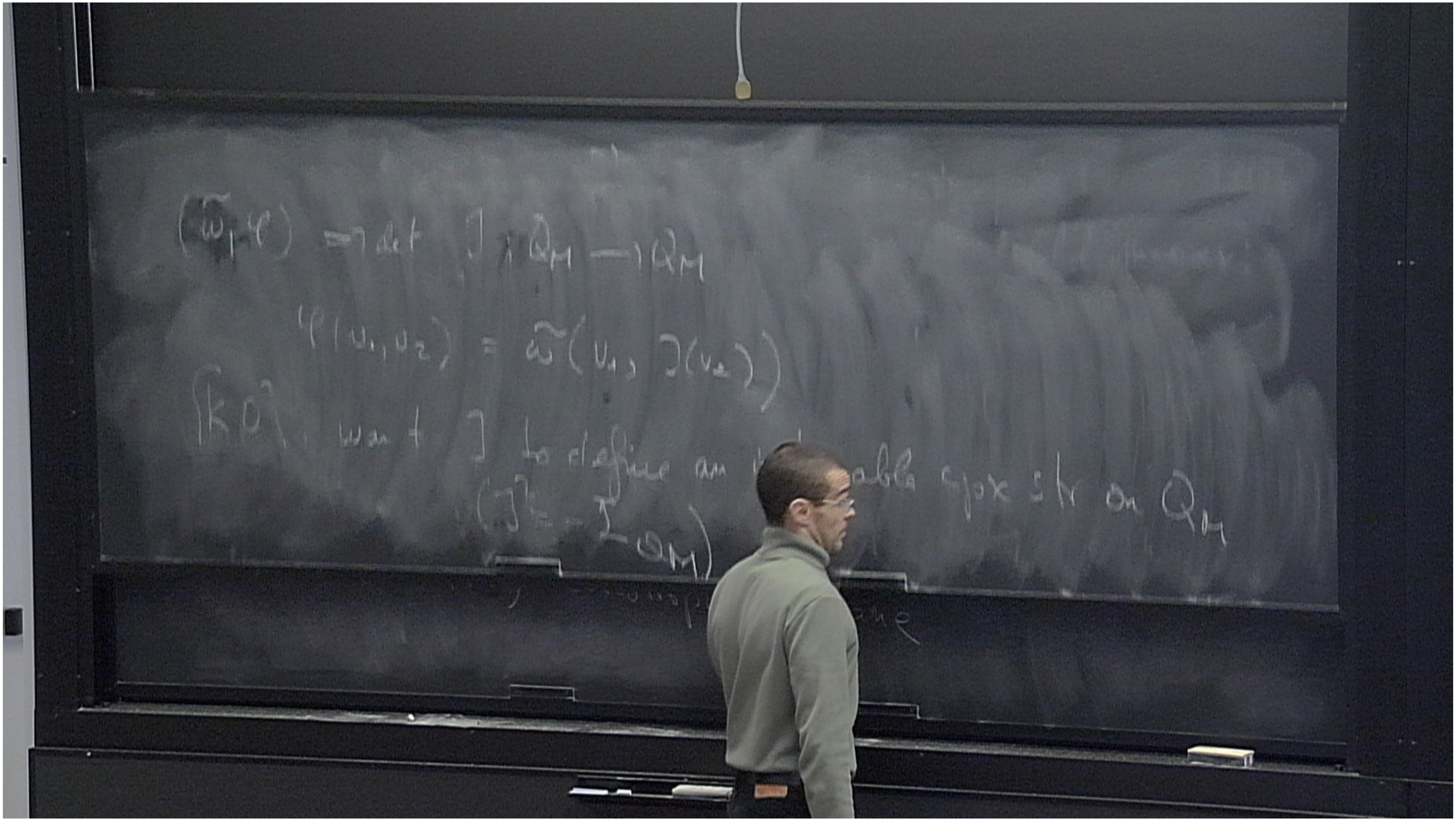
Suppose $T_M^{\perp} \lrcorner F_A = 0 \Rightarrow F_A$ descends to a bilinear form φ

$$Q_M = T_M / T_M^{\perp} \quad (\mathrm{rk}_{\mathbb{R}} Q_M = 4)$$


$(\tilde{\omega}, \varphi) \Rightarrow \det J \Rightarrow Q_M \longrightarrow Q_M$

$$(\tilde{\omega}, \varphi) \Rightarrow \det \quad J : Q_M \longrightarrow Q_M$$

$$\varphi(v_1, v_2) = \tilde{\omega}(v_1, J(v_2))$$



$(\tilde{\omega}, \varphi) \Rightarrow \det J : Q_M \longrightarrow Q_M$

$$\varphi(v_1, v_2) = \tilde{\omega}(v_1, J(v_2))$$

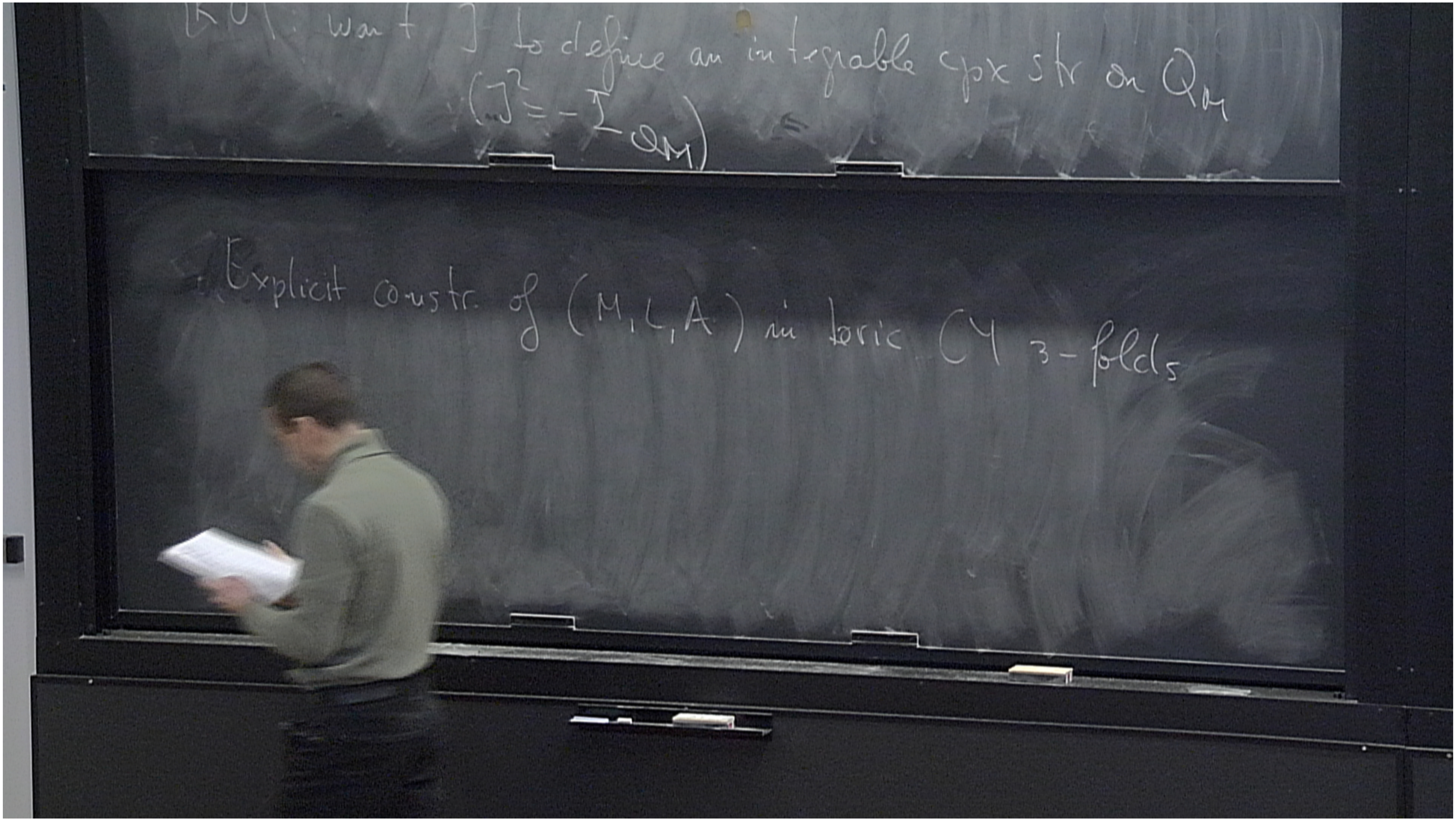
[KO]: want J to define an integrable cpx str on Q_M

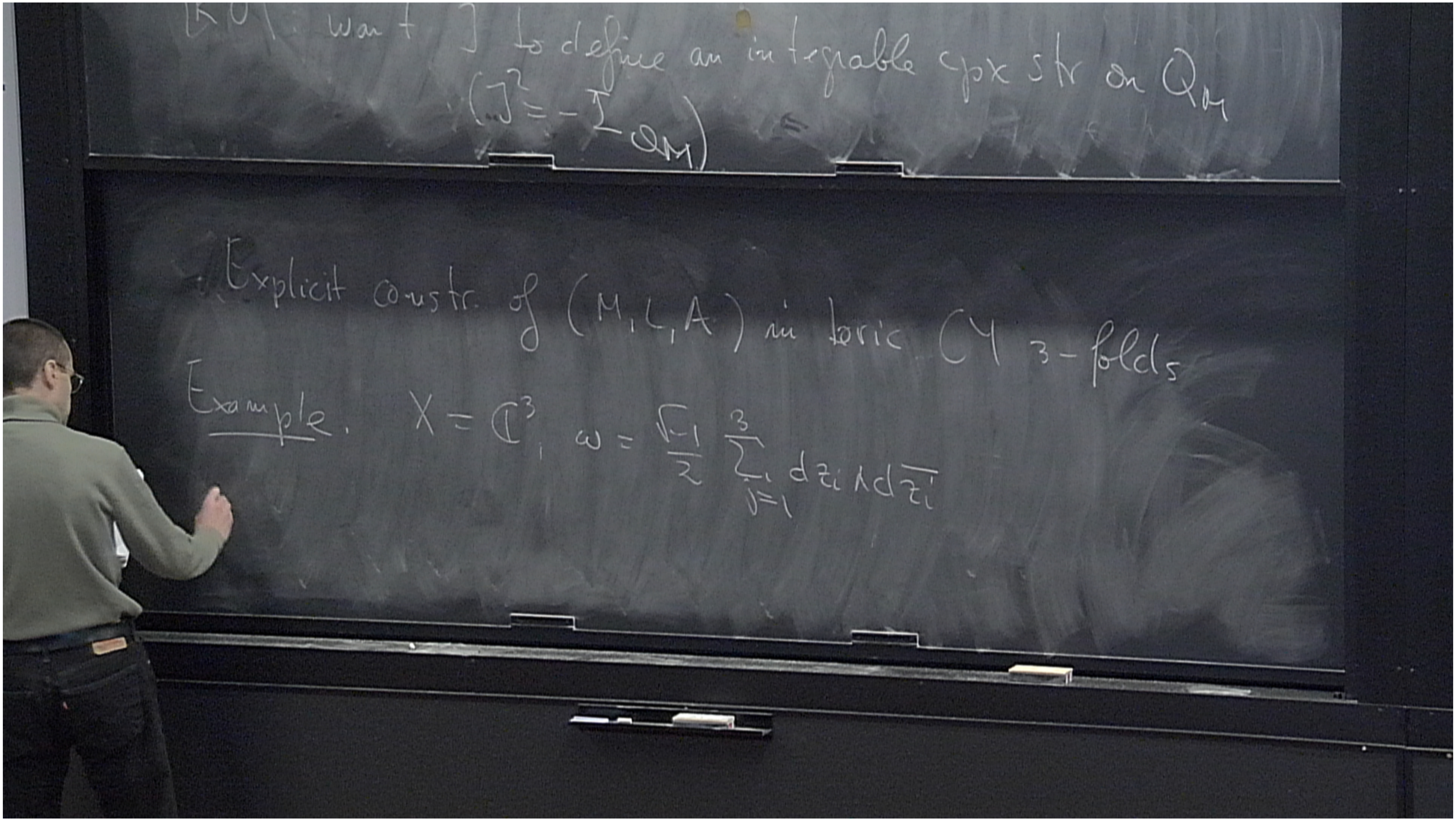
$(J^2 = -I_{Q_M})$

...c A-branes [Kapustin - Orlov]

) symp manifold $\dim_{\mathbb{R}} X = 6$

(L, A) coisotropic ...


want J to define an integrable cpx str on Q_M
(J² = −1_{Q_M})
Explicit constr. of (M, L, A) in toric CY 3-folds



[KO]: want J to define an integrable cpx str on Q_M
$(J^2 = -I_{Q_M})$
Explicit constr. of (M, L, A) in toric CY 3-folds
Example. $X = \mathbb{C}^3$, $\omega = \frac{\sqrt{-1}}{2} \sum_{i=1}^{3} dz_i \wedge d\bar{z}_i$

RK: want J to define an integrable cpx str on Q_M
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Explicit constr. of (M, L, A) in toric CY 3-folds

Example. $X = \mathbb{C}^3$, $\omega = \frac{\sqrt{-1}}{2} \sum_{j=1}^{3} dz_i \wedge d\bar{z}_i$

$M \subset X$. $\{ |z_1| = 1 \} \cong S^1 \times \mathbb{C}^2$

[illegible]: want J to define an integrable cpx str on Q_M

$(J^2 = -I_{Q_M})$

Explicit constr. of (M, L, A) in toric CY 3-folds

<u>Example</u>. $X = \mathbb{C}^3$, $\omega = \frac{\sqrt{-1}}{2} \sum_{j=1}^{3} dz_i \wedge d\overline{z_i}$

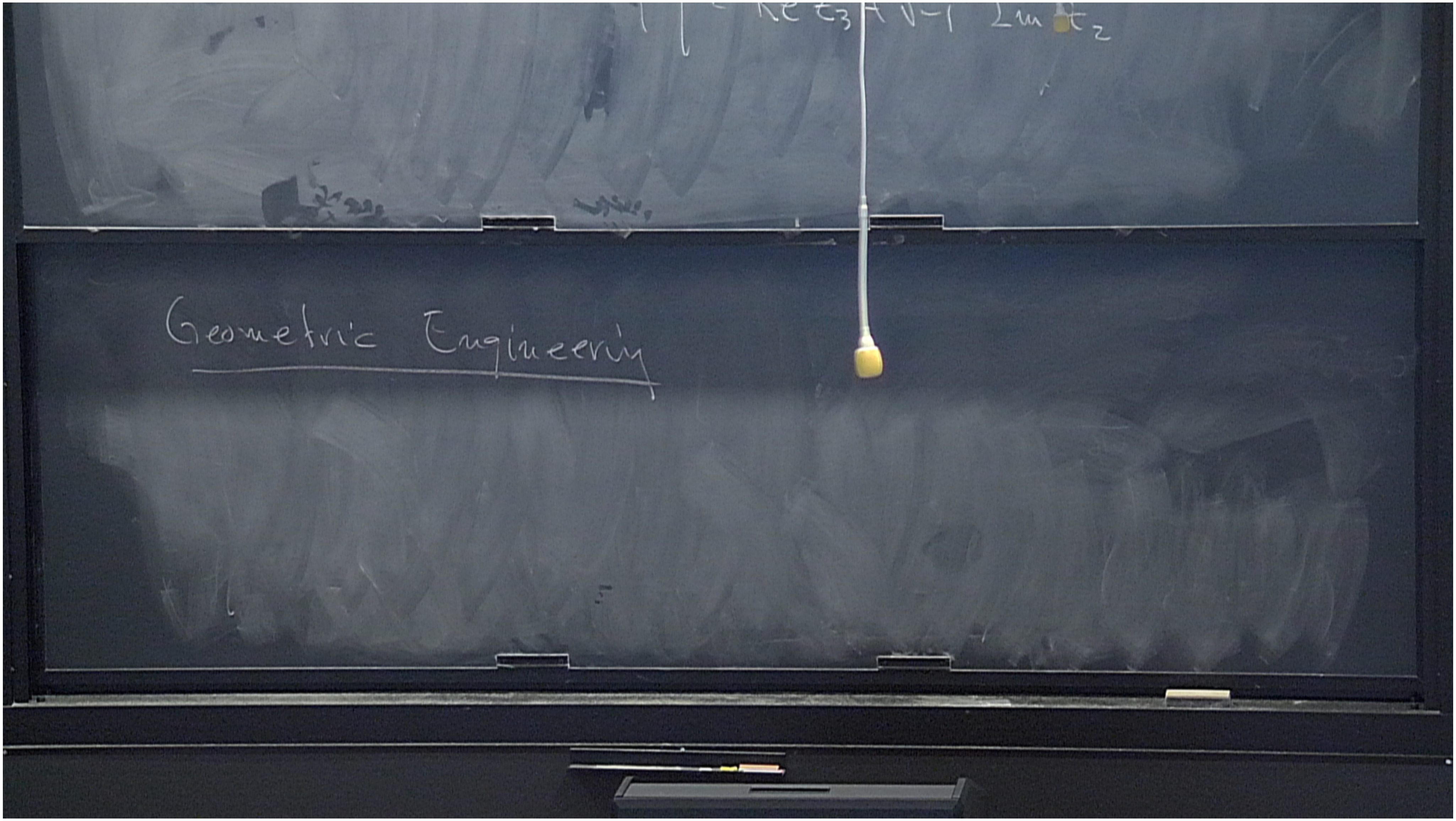
$M \subset X$ $\{ |z_1| = 1 \} \cong S^1 \times \mathbb{C}^2$, $L \cong$ trivial line bdle

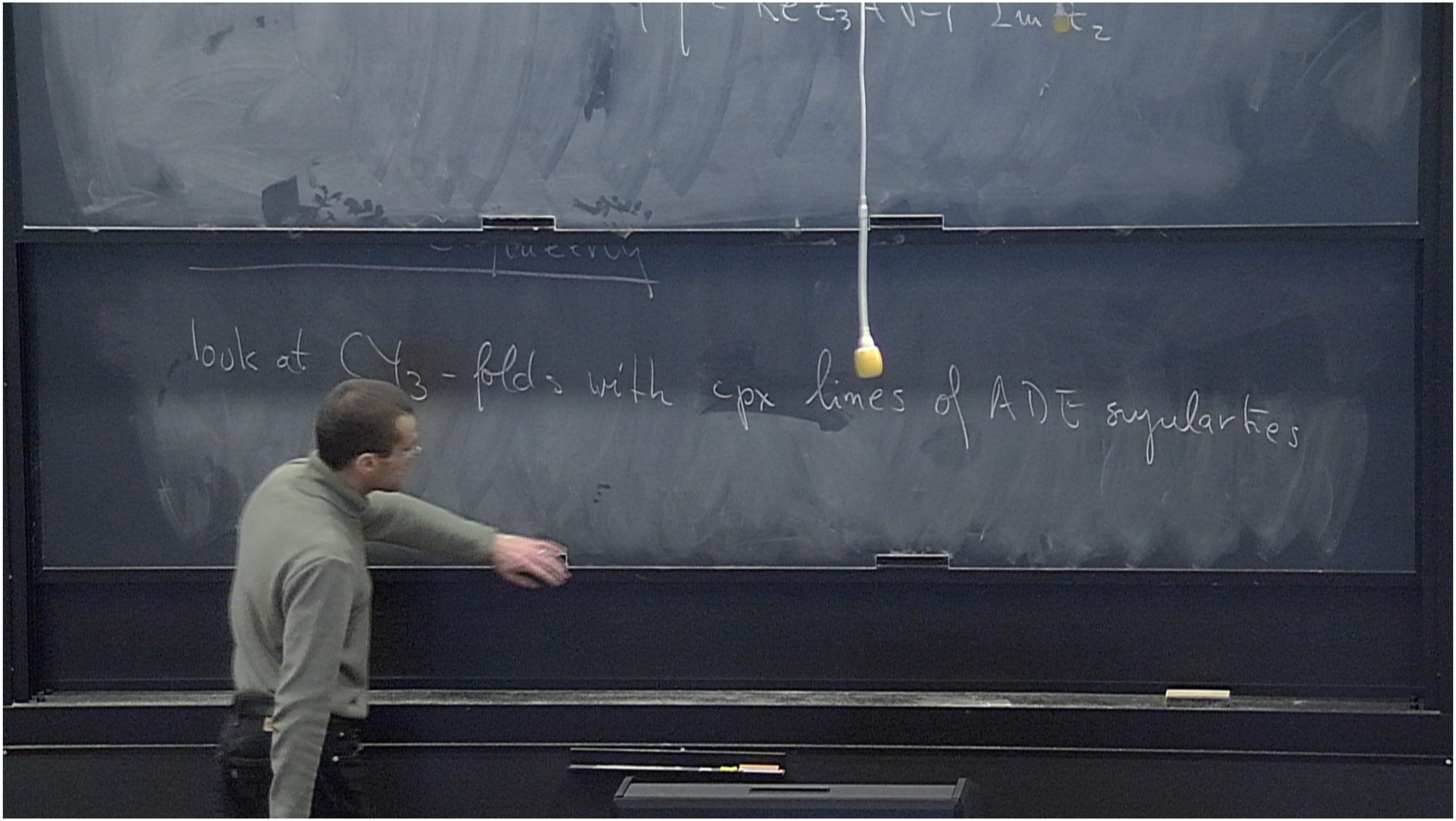
$$A = \frac{1}{2}\left(z_2\, dz_3 + \bar{z}_2\, d\bar{z}_3\right)$$

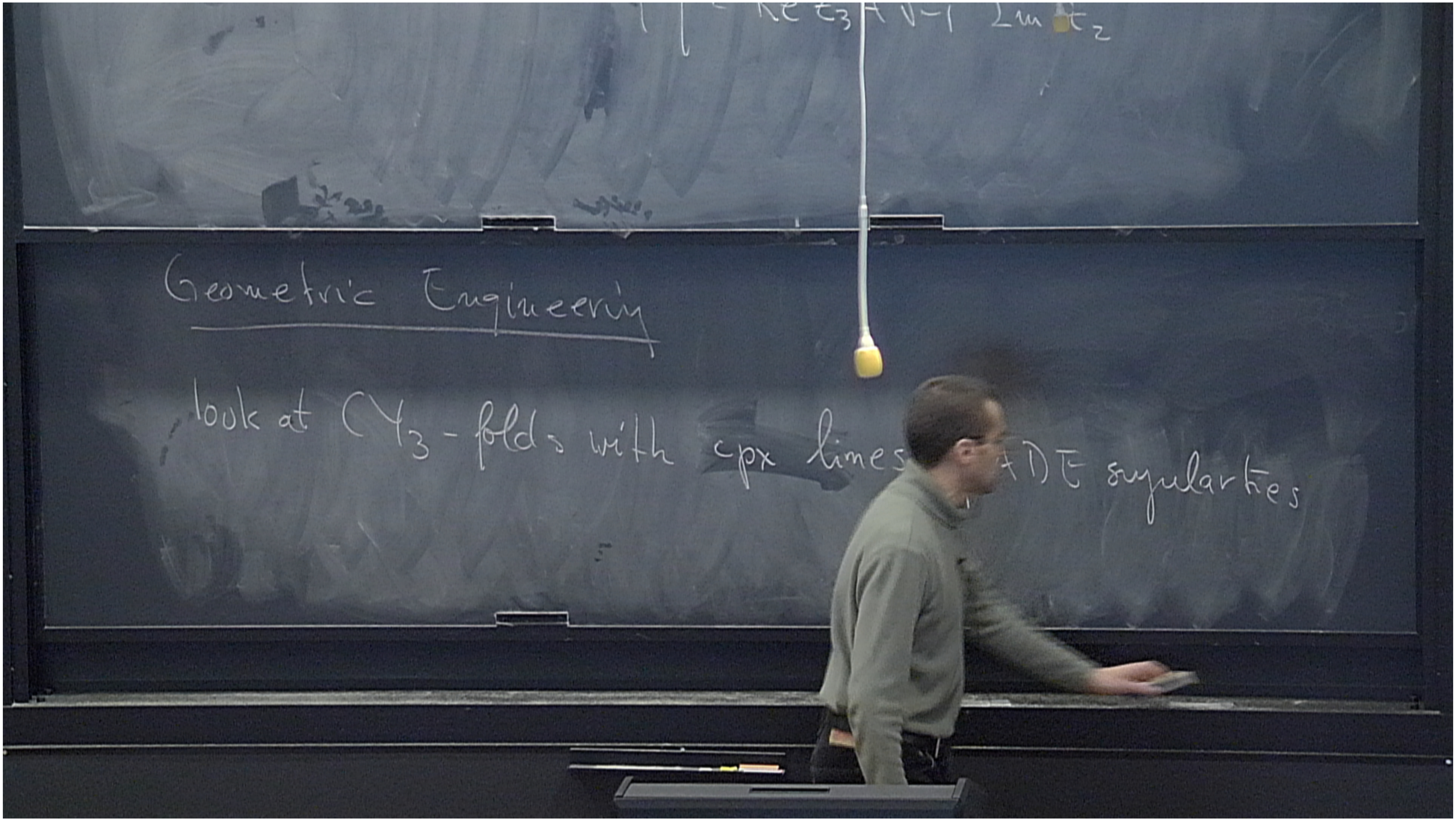
$$A = \frac{1}{2}\left(z_2\, dz_3 + \bar{z}_2\, d\bar{z}_3 \right)$$

--- $\Longrightarrow$ cpx str J

$$\begin{cases} \xi = \mathrm{Re}\, z_2 + \sqrt{-1}\, \mathrm{Im}\, z_3 \\ \eta = \mathrm{Re}\, z_3 + \sqrt{-1}\, \mathrm{Im}\, z_2 \end{cases}$$


Geometric Engineering


look at CY3-folds with cpx lines of ADE singularities

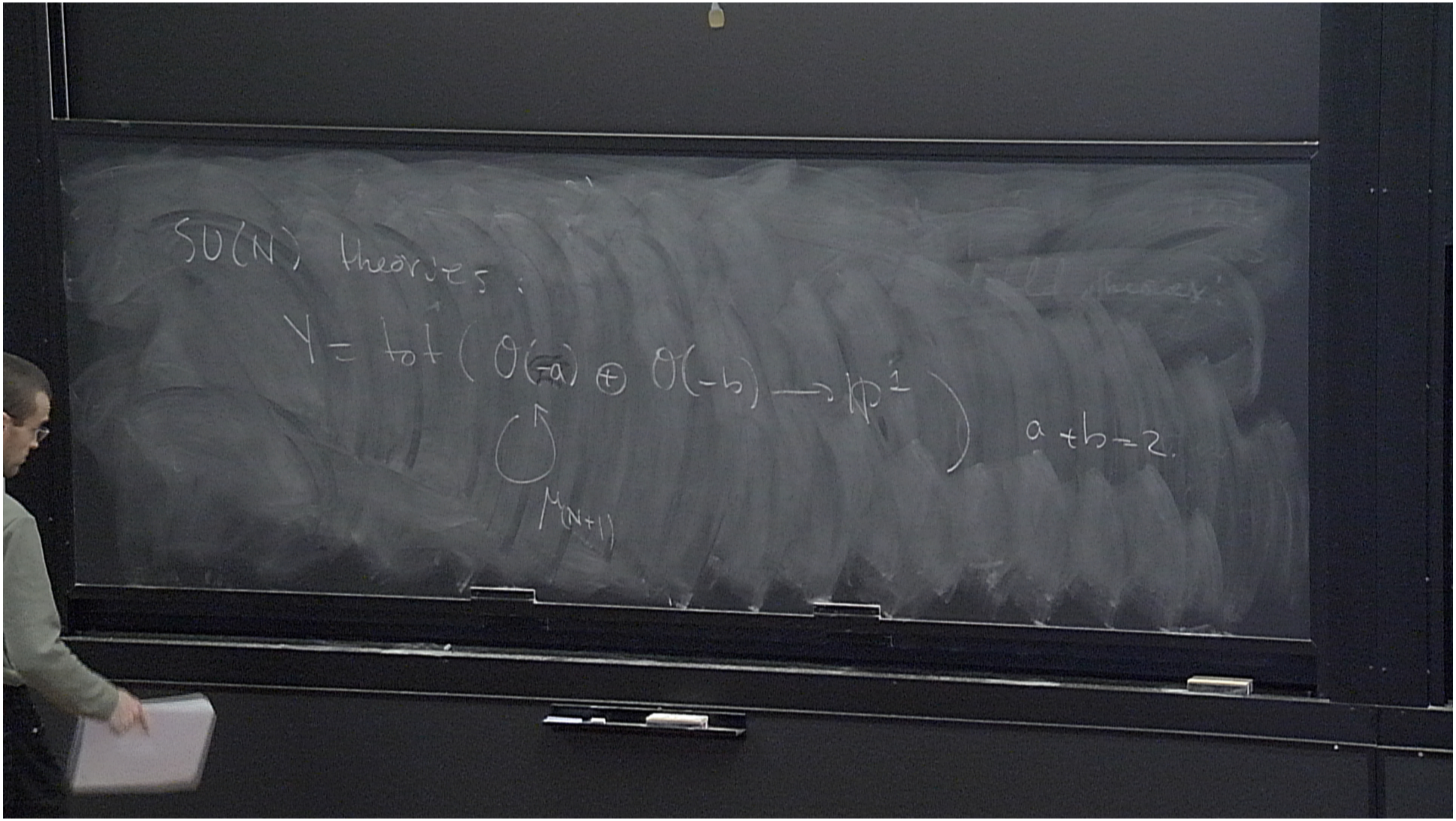

Geometric Engineering
look at CY3-folds with cpx lines
DE singularties

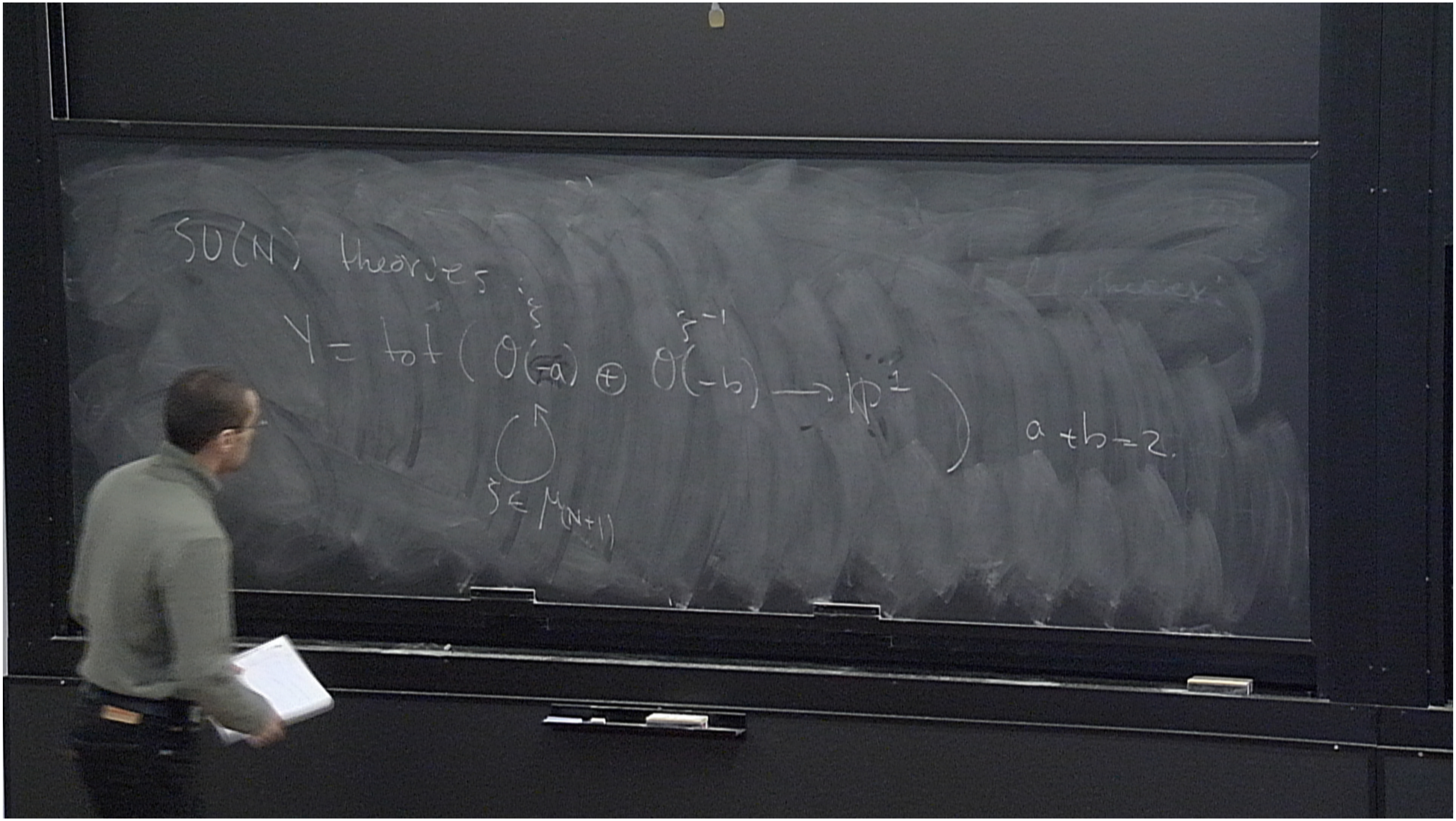
$= \mathrm{Re}\, t_3 + \sqrt{-1}\, \mathrm{Im}\, t_2$

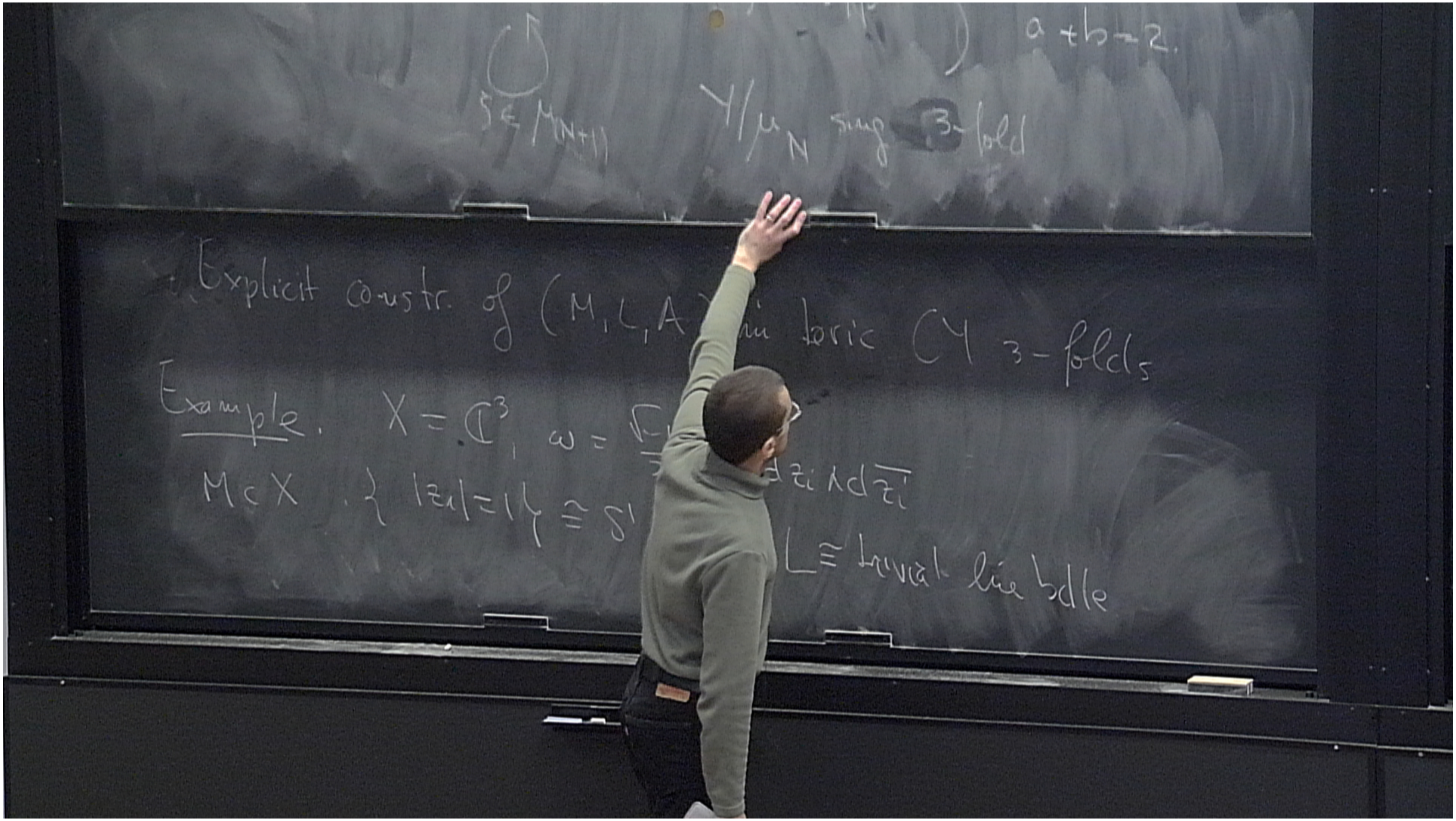
Geometric Engineering

look at CY_3-folds with cpx lines of ADE singularities

$\Rightarrow$ local toric models


SU(N) theories:
$Y = \mathrm{tot}\left(\mathcal{O}(-a) \oplus \mathcal{O}(-b) \longrightarrow \mathbb{P}^1\right)$
$a+b=2.$
$\mu_{(N+1)}$


SU(N) theories:
Y = tot (O(-a) ⊕ O(-b) → P^1)
ξ
ξ^-1
ξ ∈ μ(N+1)
a + b = 2.


a + b = 2.
$\xi \in \mu_{(N+1)}$
Y/μ_N sing 3-fold
Explicit constr. of (M, L, A) in toric CY 3-folds
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$dz_i \wedge d\bar{z}_i$
$M \subset X$: $\{|z_1| = 1\} \cong S^1$
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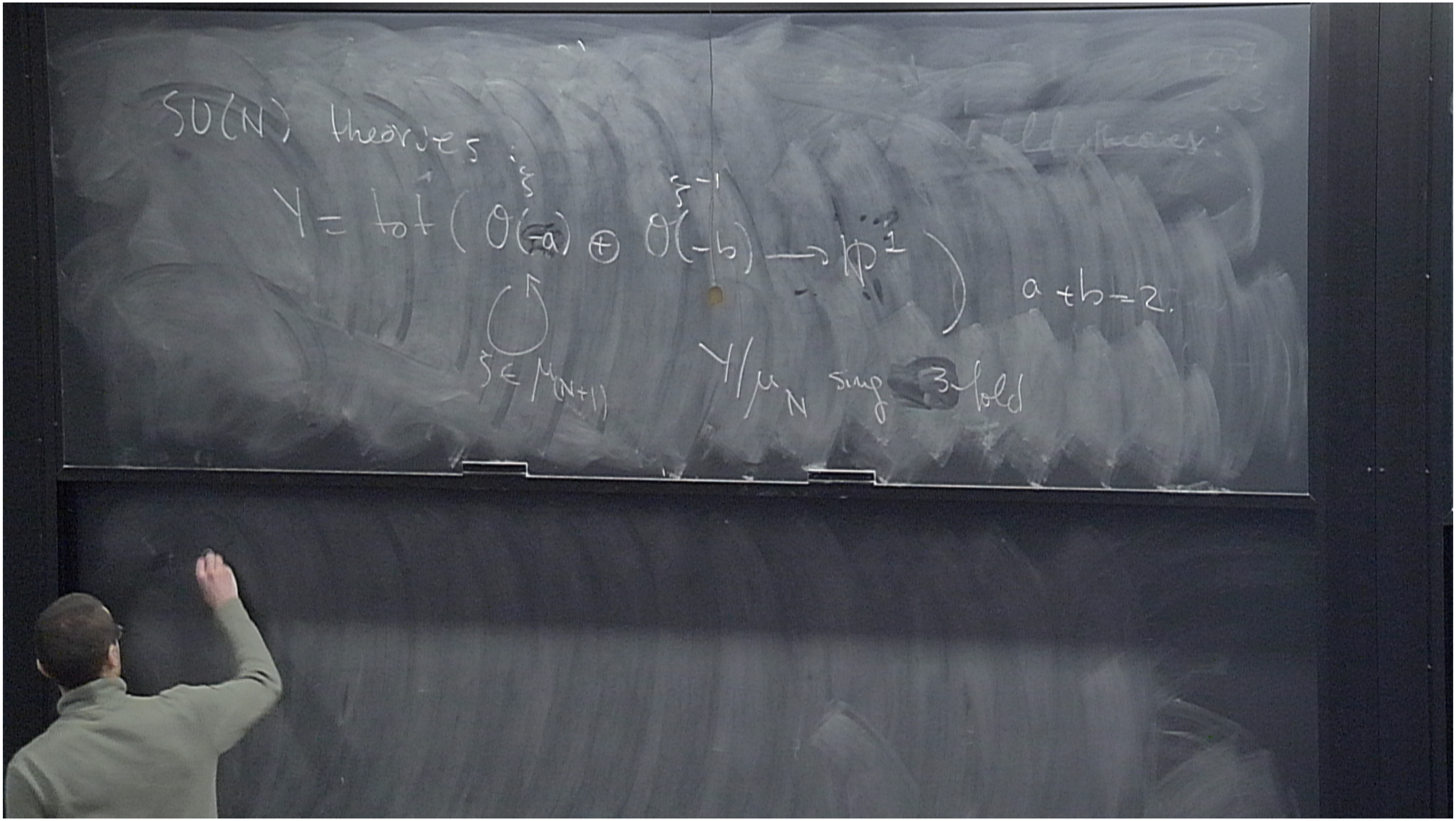
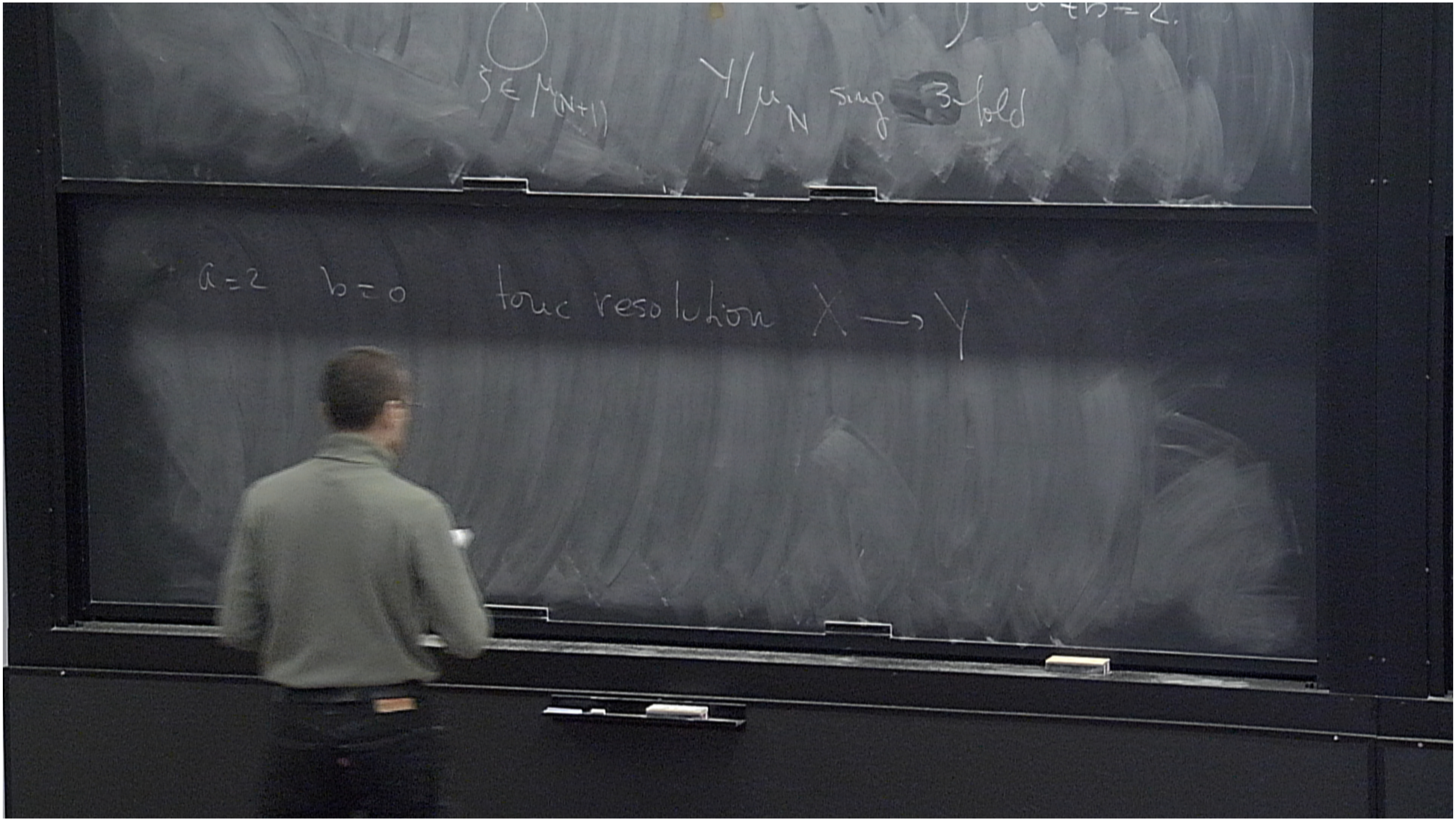
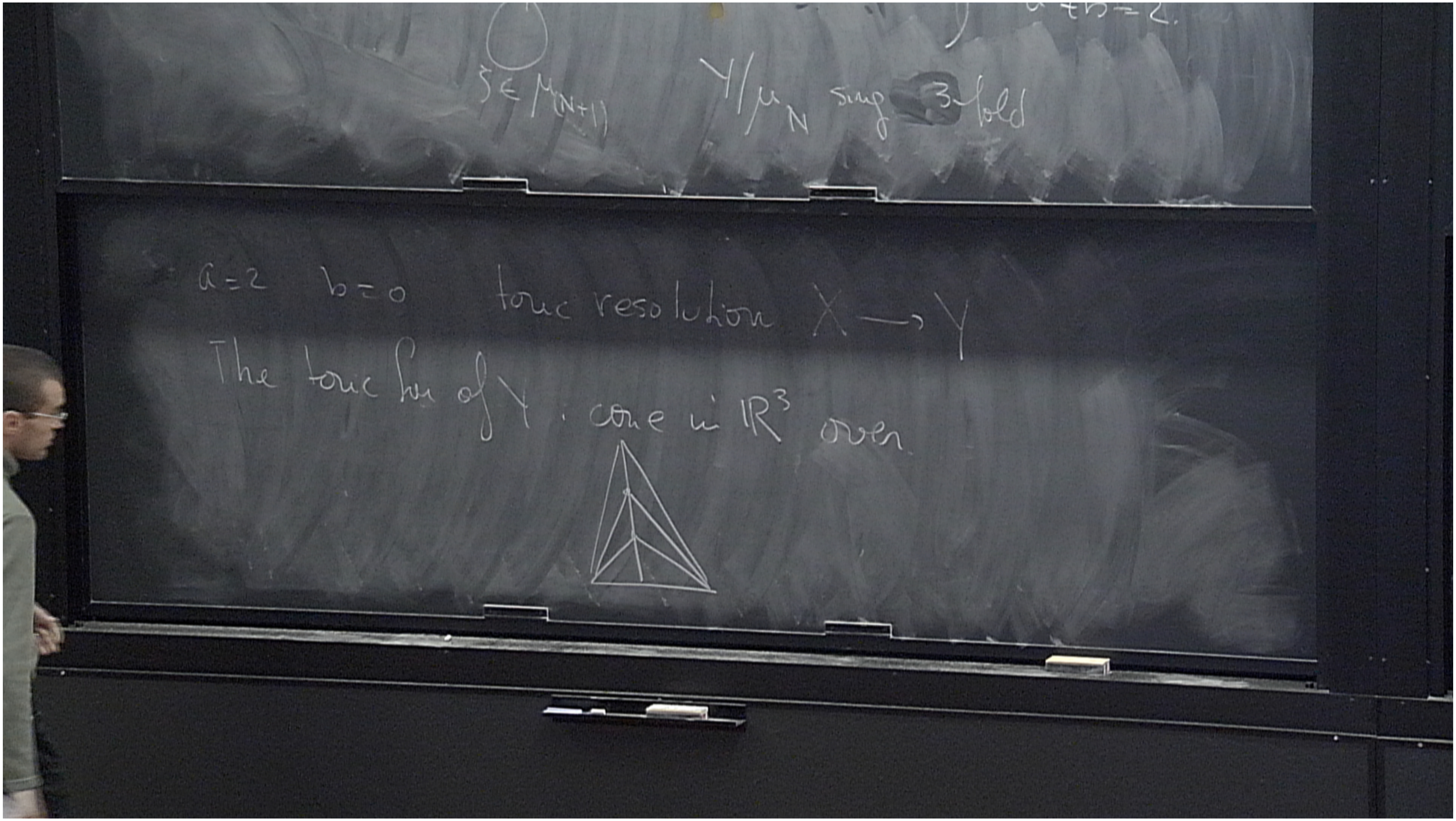
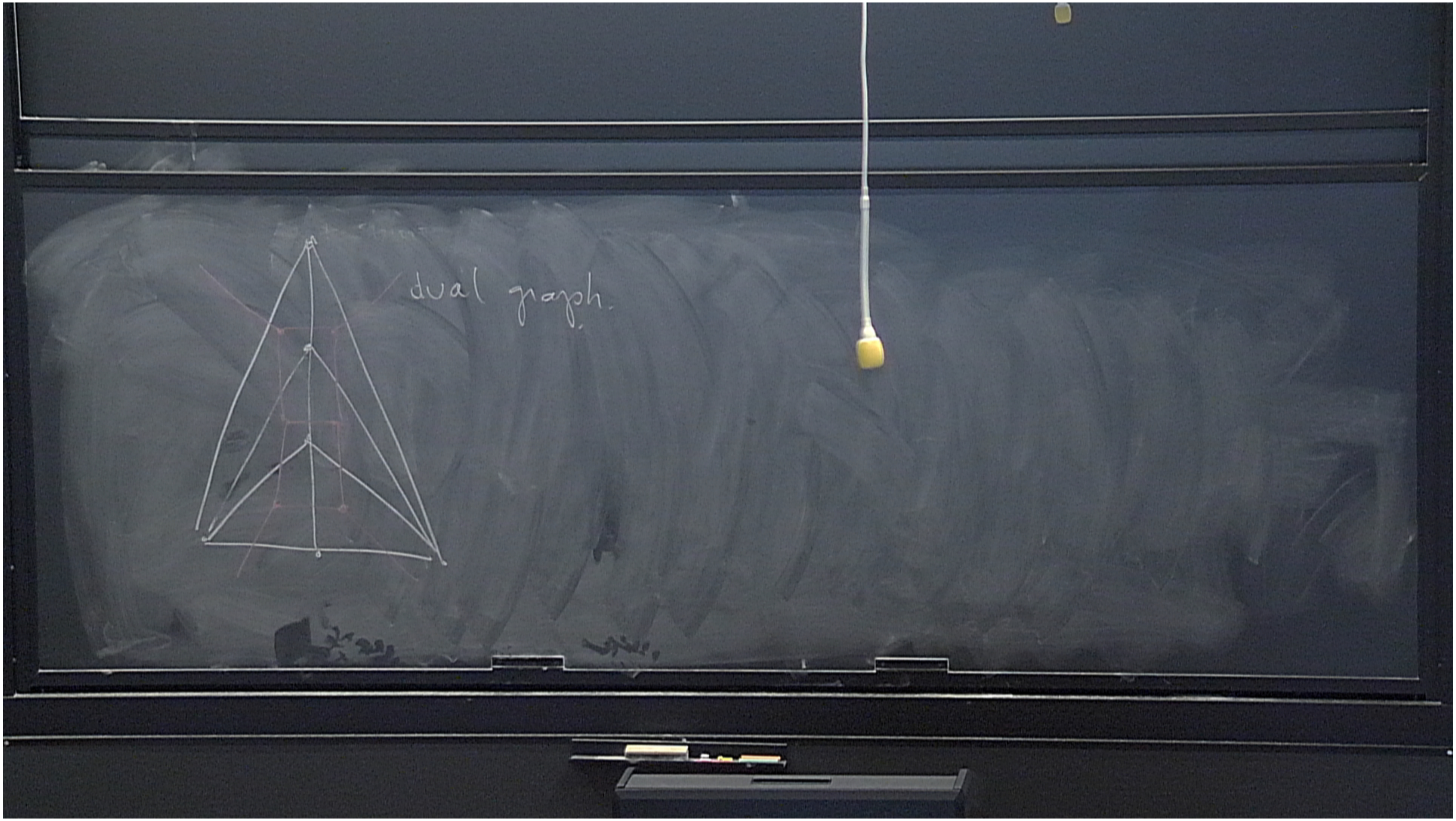
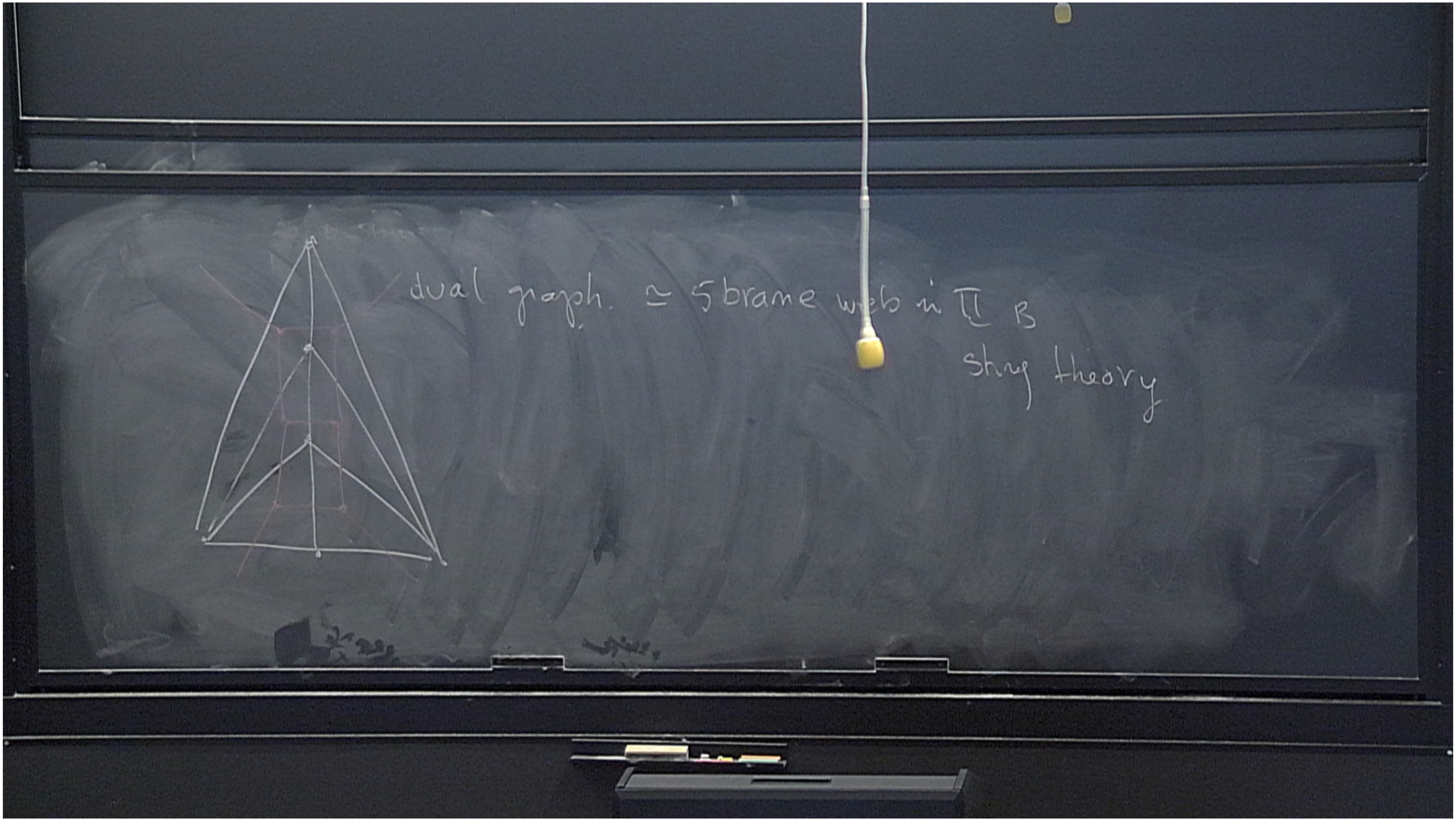
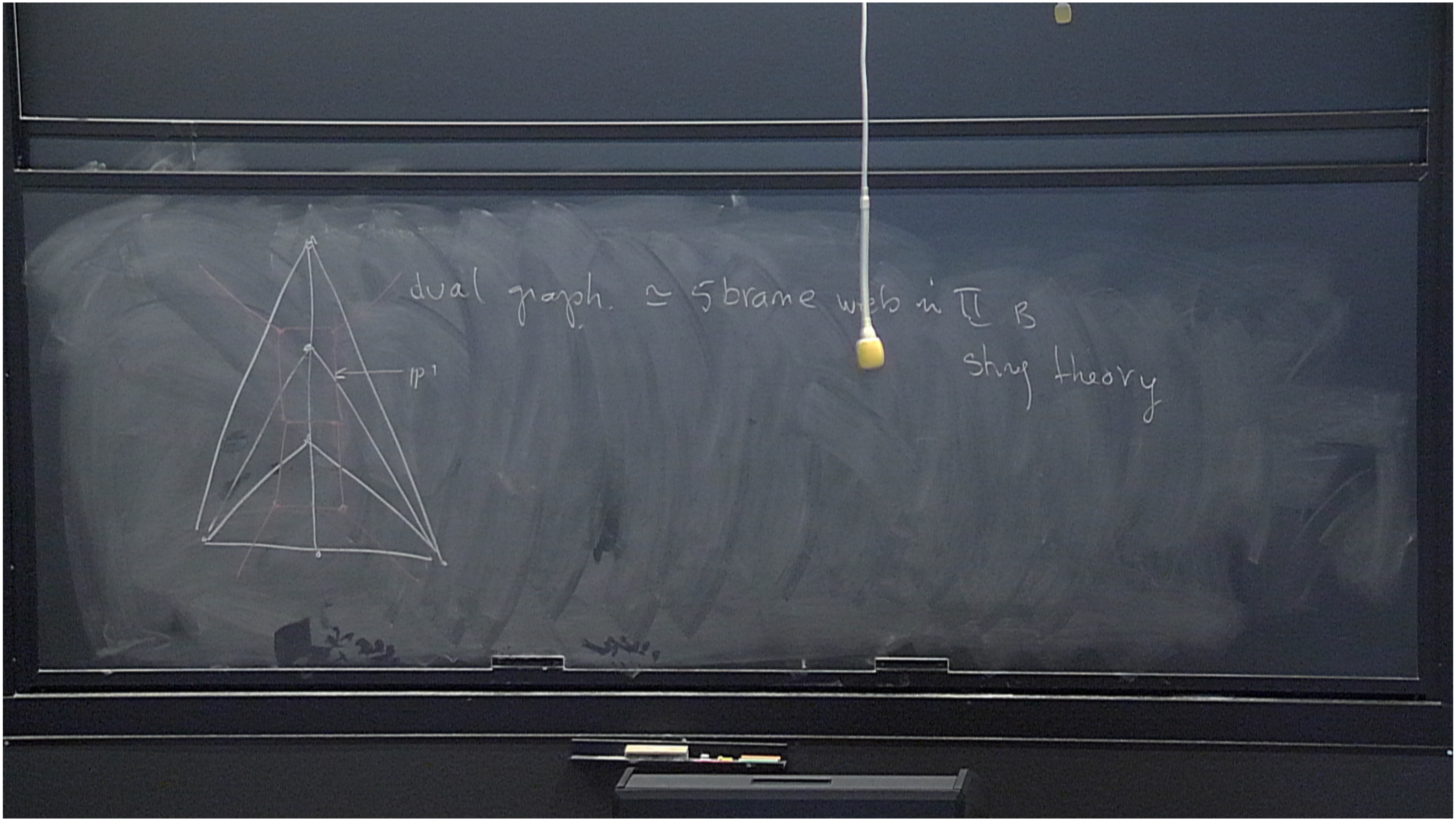

SU(N) theories:
ξ ξ⁻¹
Y = tot (O(-a) ⊕ O(-b) → P¹)
a + b = 2.
ξ ∈ μ(N+1)
Y/μ_N sing 3-fold


ξ ∈ μ(N+1)
Y/μN sing 3-fold
a=2 b=0 toric resolution X → Y

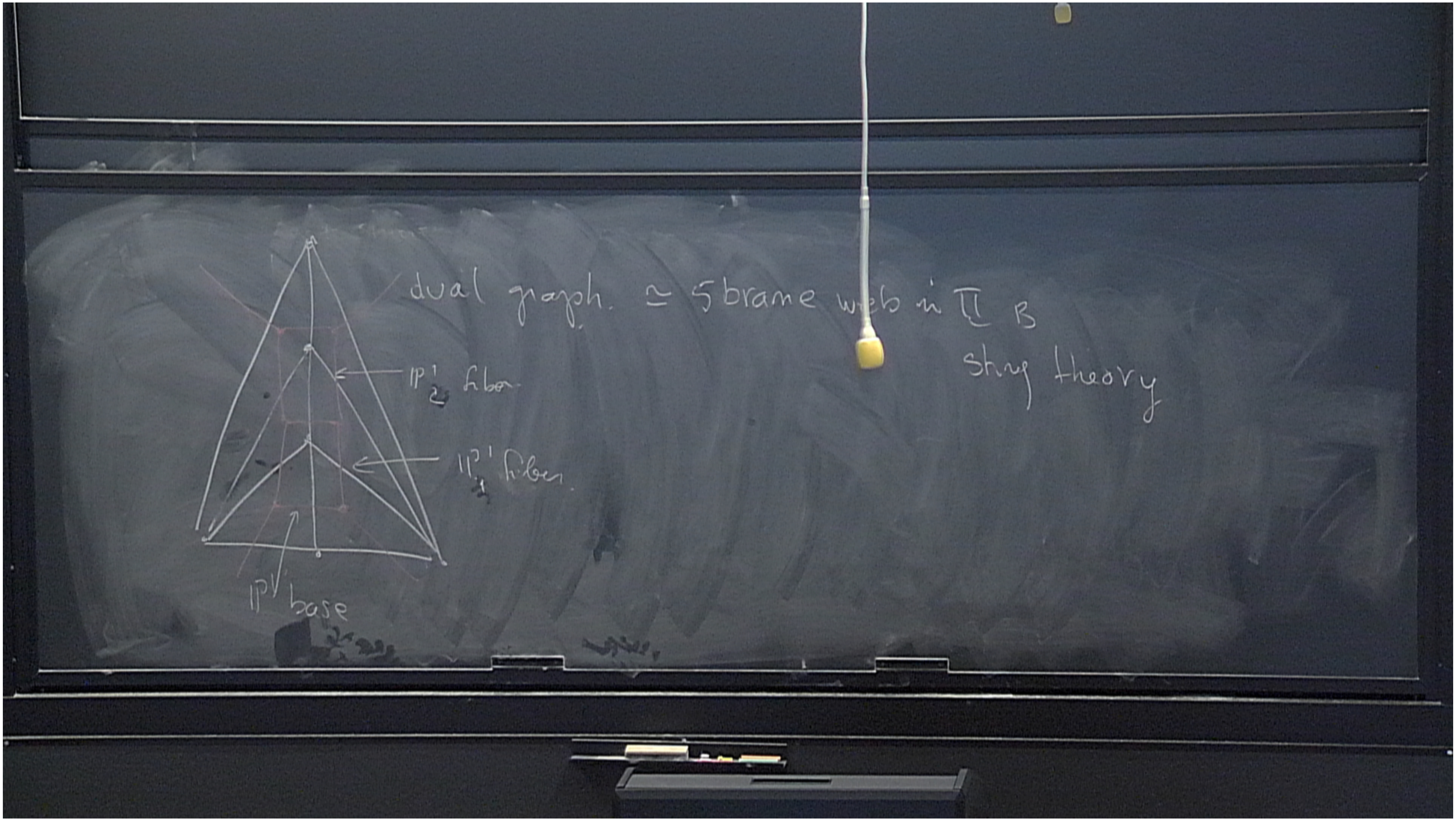

ξ ∈ μ(N+1)
Y/μN sing 3-fold
a=2 b=0 toric resolution X → Y
The toric fan of Y: cone in R3 over

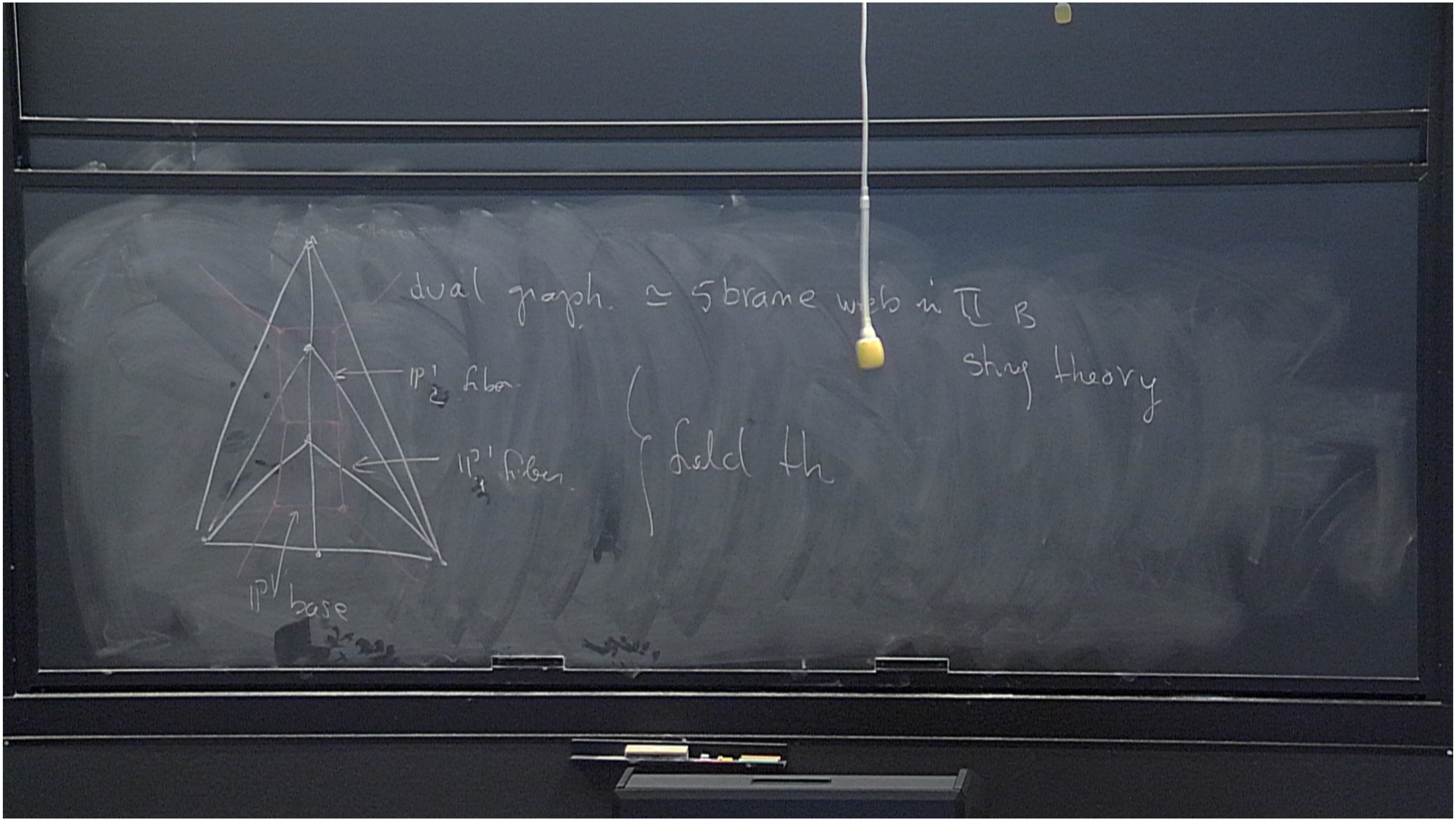

dual graph.

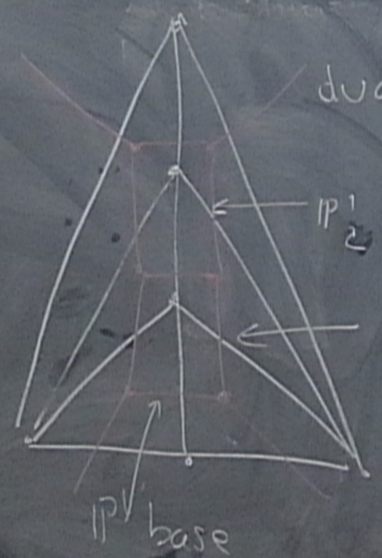

dual graph. ≃ 5brane web in II B
string theory



dual graph. ≃ 5brane web in II B
string theory
$\mathbb{P}^1$

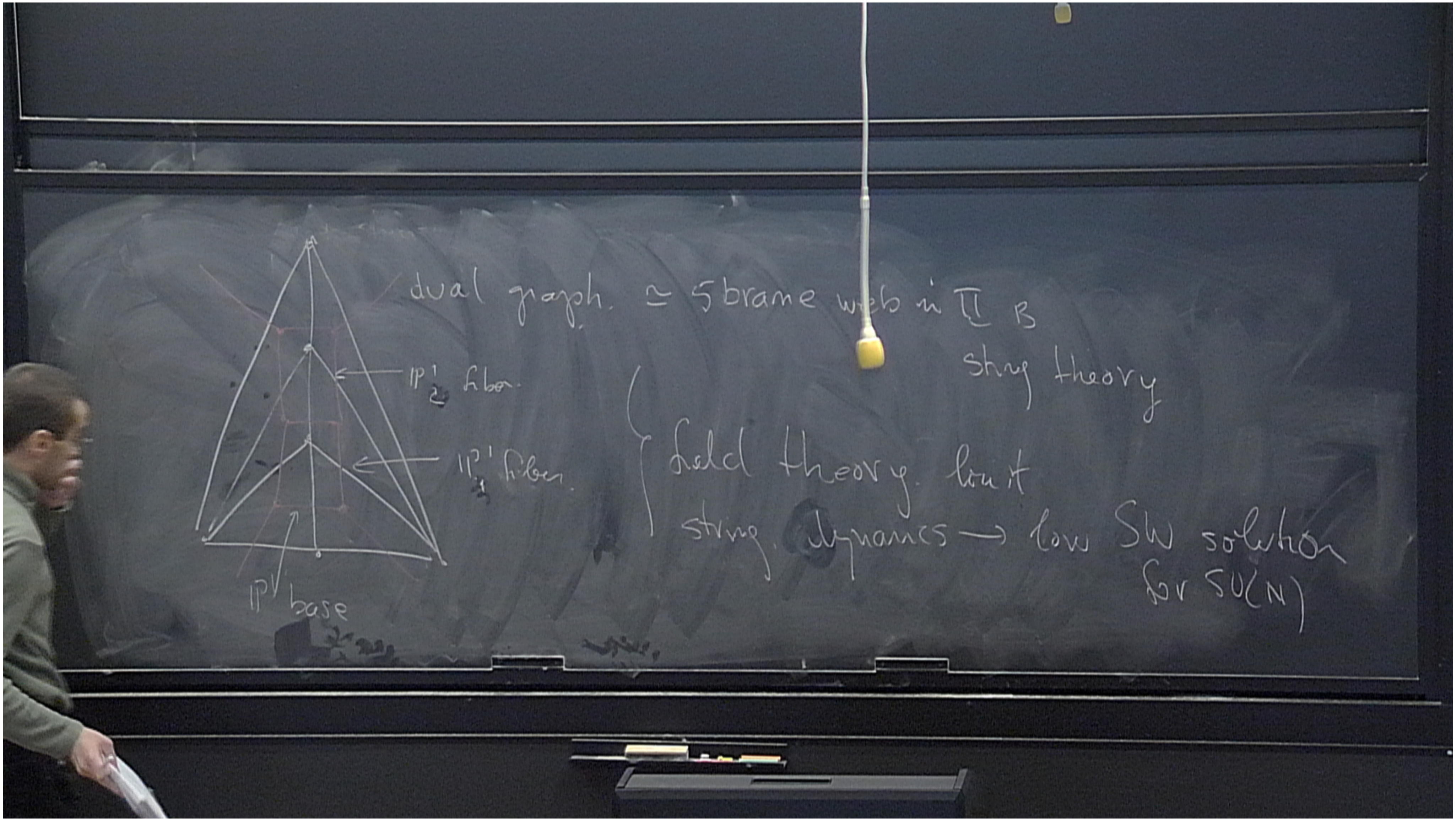

dual graph ≃ 5brane web in IIB
string theory
$\mathbb{P}^1_2$ fiber
$\mathbb{P}^1_1$ fiber
$\mathbb{P}^1$ base


dual graph ≃ 5brane web in IIB
string theory
P¹₂ fiber
P¹₁ fiber
field th
P¹ base

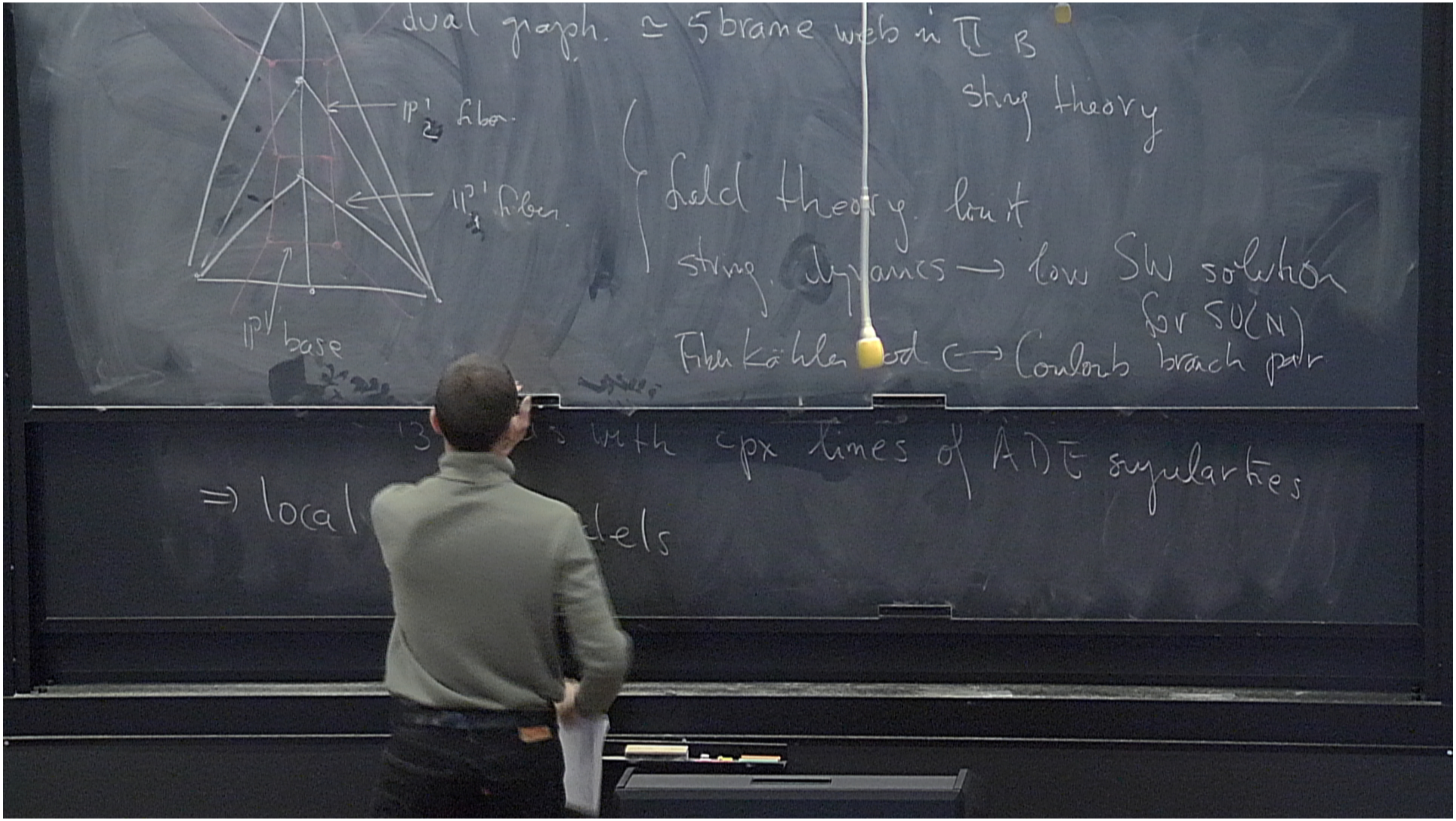


dual graph $\simeq$ 5brane web in II B string theory

field theory limit



dual graph ≃ 5brane web in IIB
string theory
IP¹ fiber
IP¹ fiber
IP¹ base
field theory limit
string dynamics → low SW solution
for SU(N)


dual graph. ≃ 5brane web in IIB
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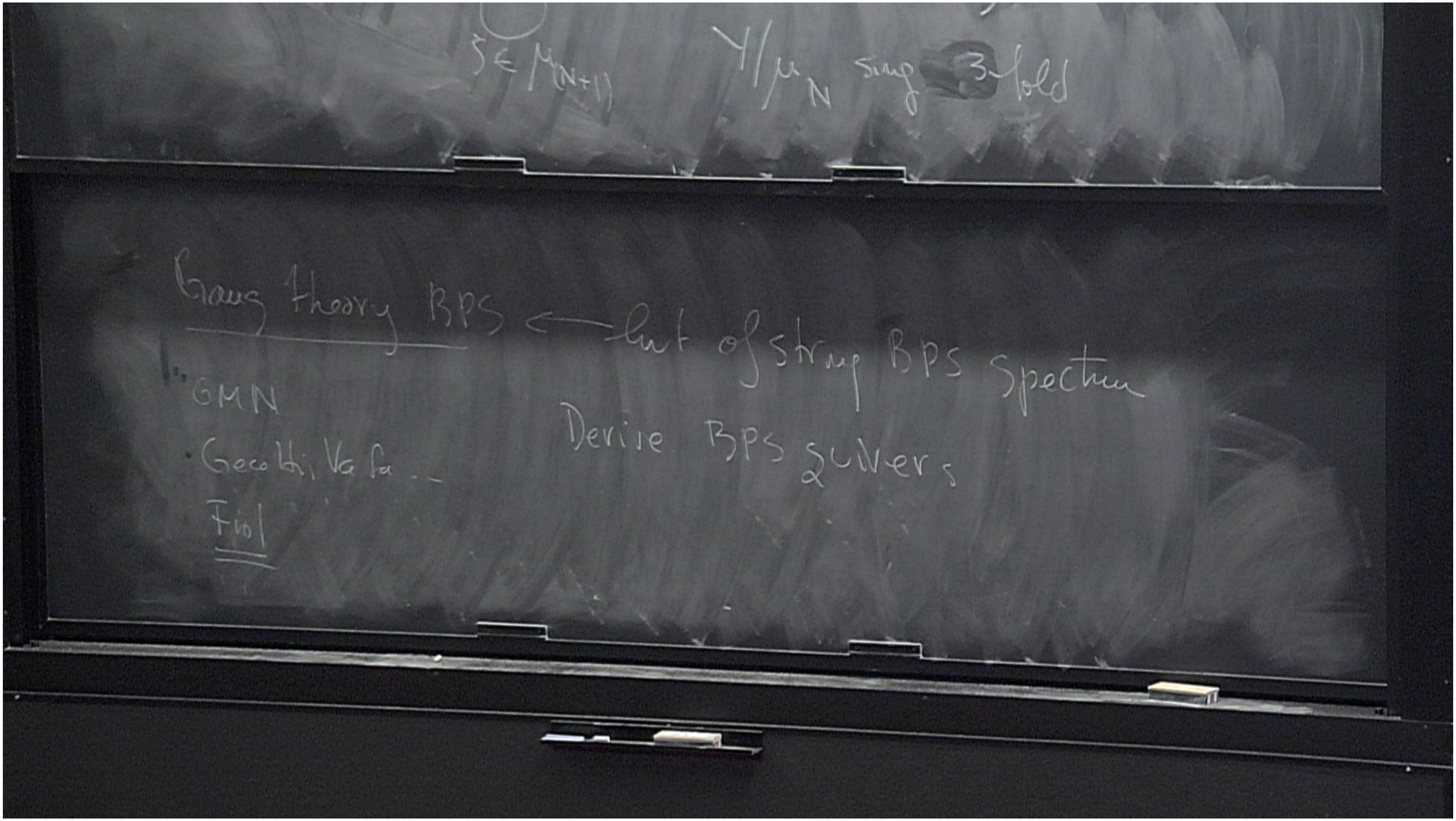
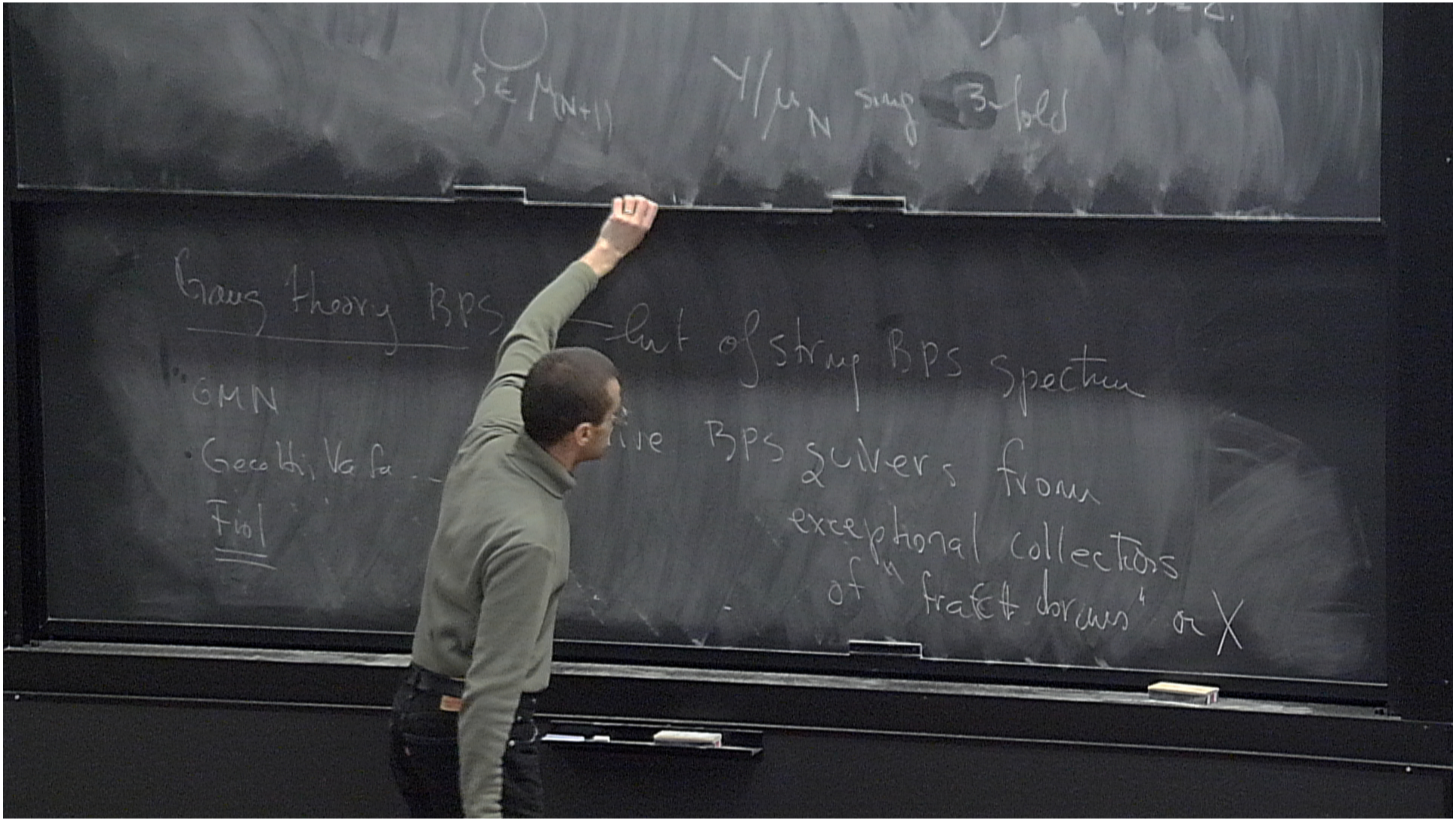
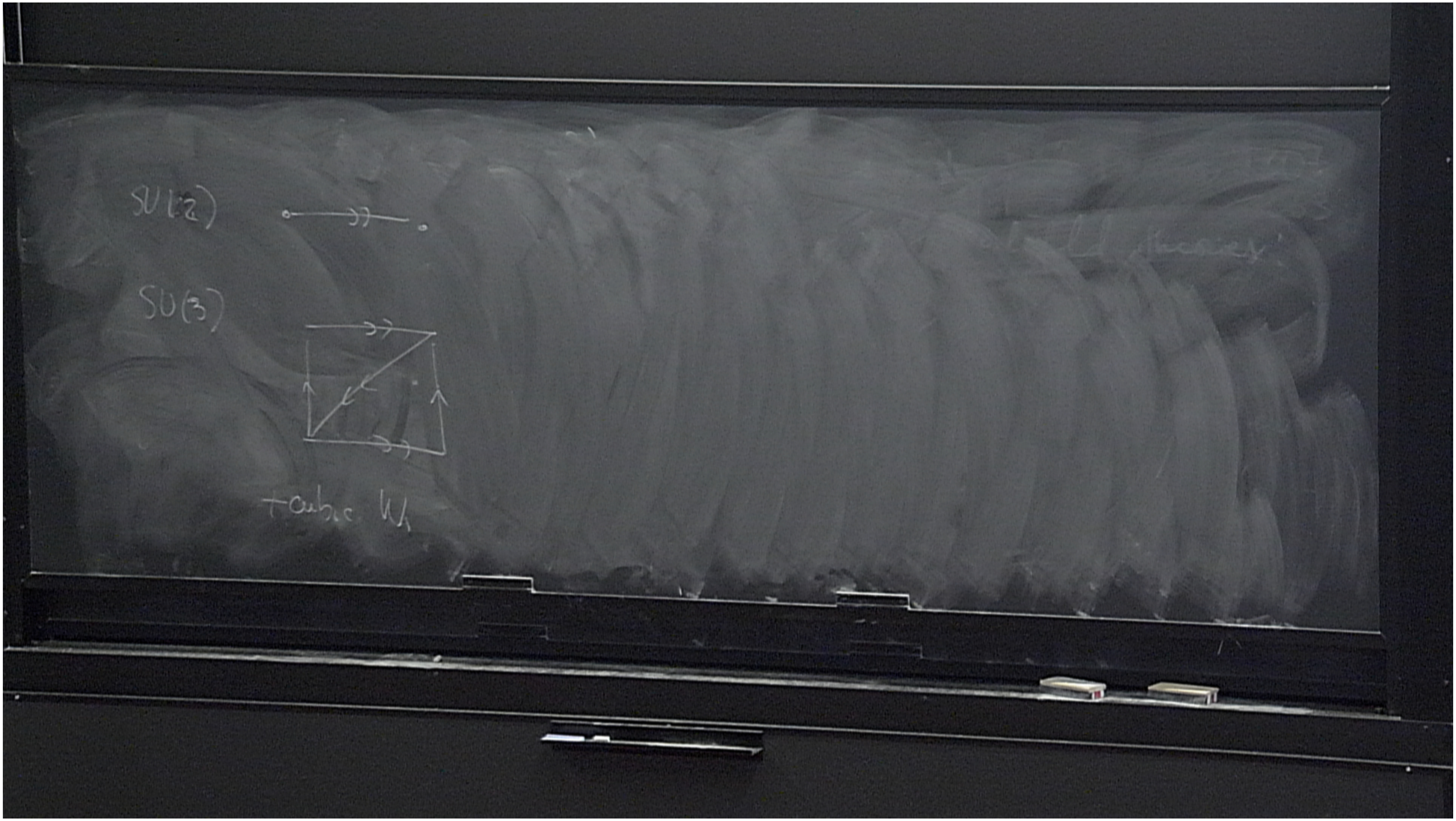
Coulomb branch par
$\mathbb{P}^1_2$ fiber
$\mathbb{P}^1_1$ fiber
$\mathbb{P}^1$ base
with cpx lines of ADE singularities
⇒ local

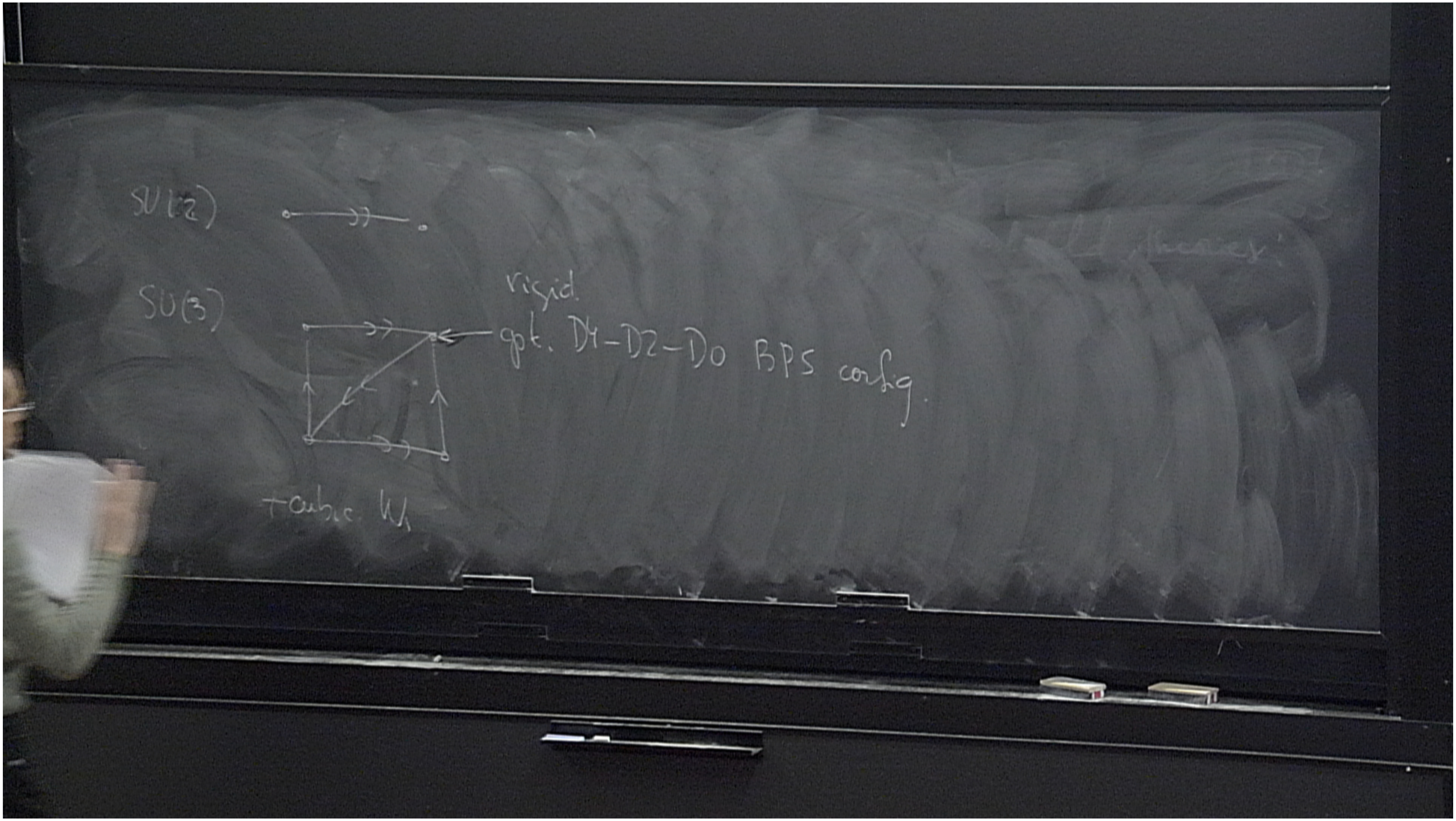
$\zeta \in \mu_{(N+1)}$ $\quad Y/\mu_N$ sing 3-fold

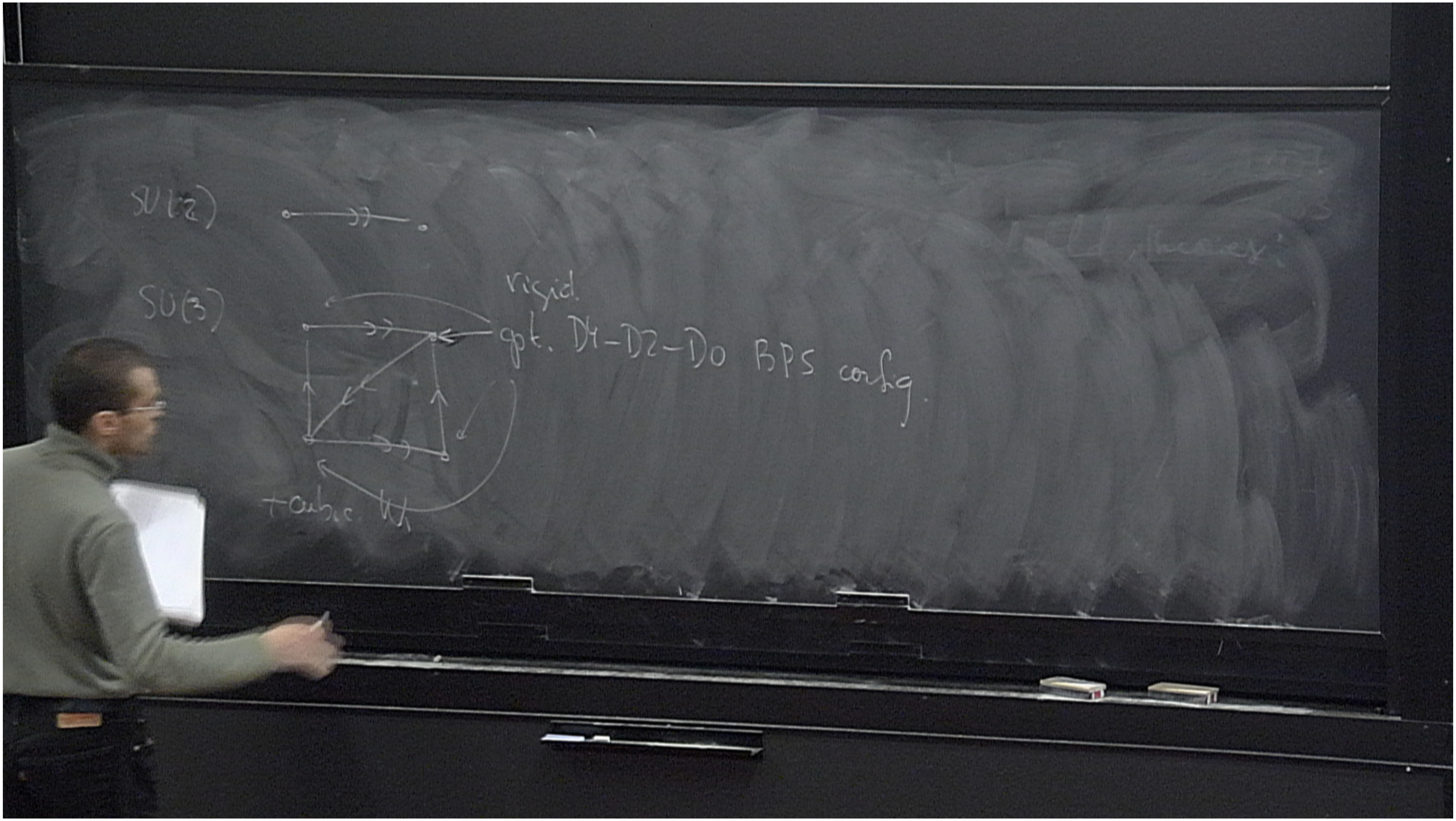
Gauge theory BPS $\longleftarrow$ limit of string BPS spectrum


ξ ∈ μ(N+1)
Y/μN sing 3-fold
Gauge theory BPS ← limit of string BPS spectrum
GMN
Cecotti, Vafa ..
Fiol
Derive BPS quivers


Y/μ_N sing 3-fold
Gauge theory BPS
of string BPS spectrum
GMN
ive BPS quivers from
exceptional collections
Fiol
of "fract. branes" on X


SU(2)
SU(3)


SU(2)
SU(3)
rigid
qpt. D4–D2–D0 BPS config.
+cubic W



SU(2)
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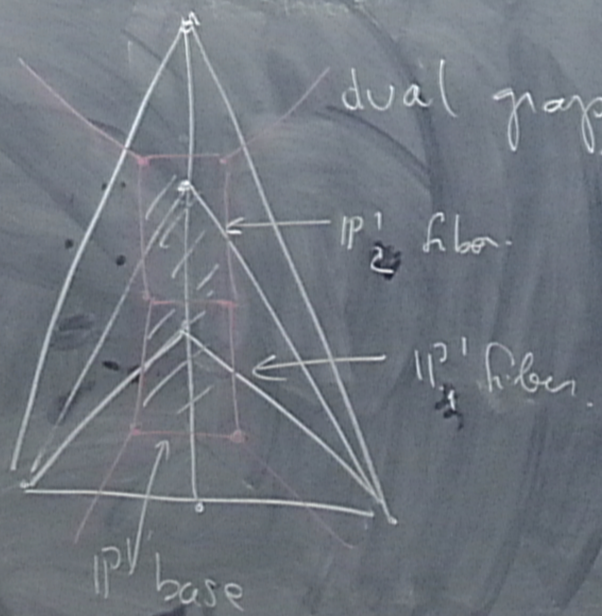
dual graph. $\simeq$ 5brane web in IIB string theory

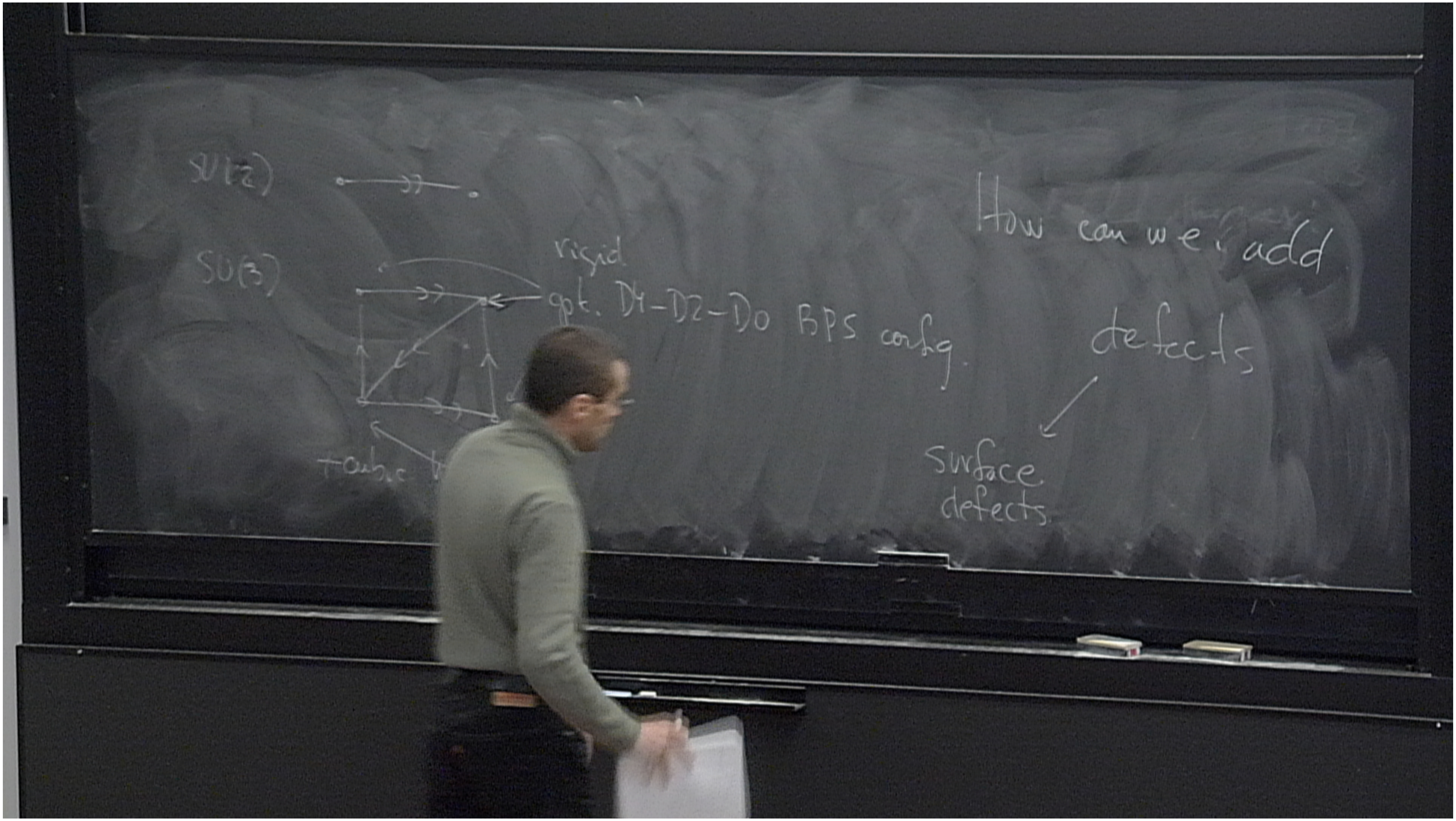


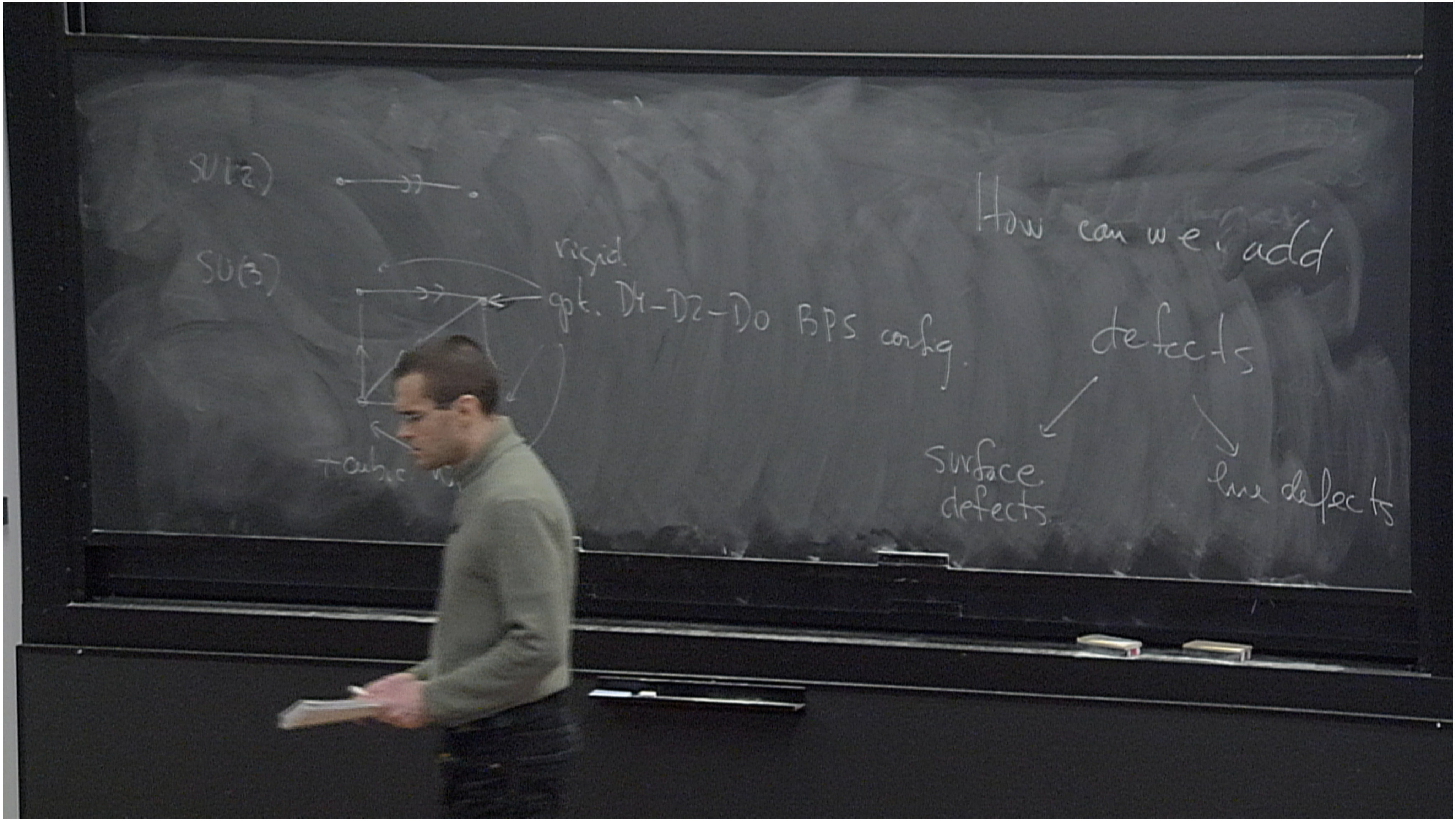
field theory limit

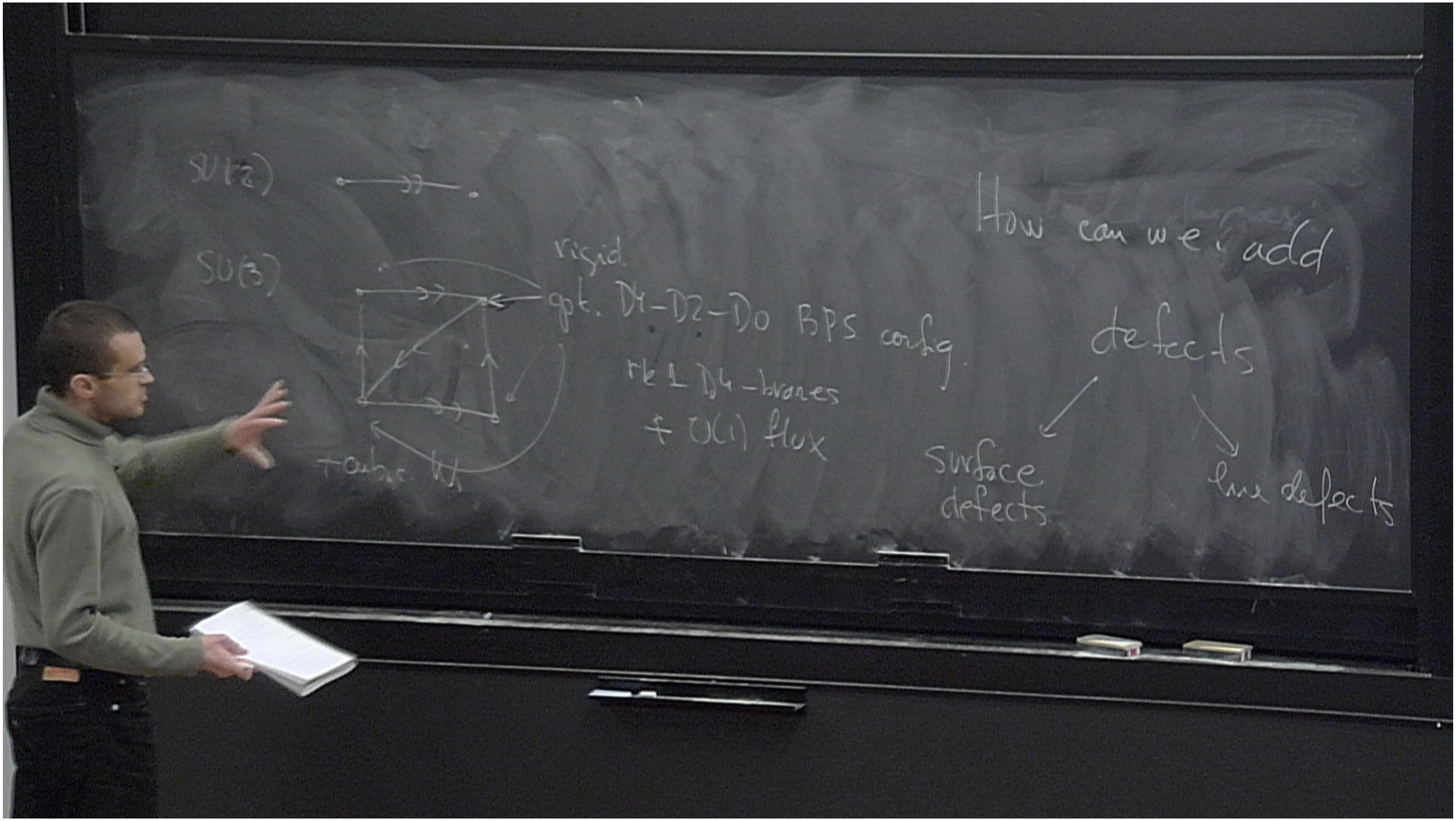
string dynamics $\longrightarrow$ low SW soluti[...] for SU(N)

Fiber Kähler mod $\longleftrightarrow$ Coulomb branch p[...]

look at CY_3-folds with [...]


SU(2)
SU(3)
rigid
D4–D2–D0 BPS config.
How can we add
defects
surface
defects


SU(2)
SU(3)
rigid
qpt. D4–D2–D0 BPS config.
How can we add
defects
surface defects
line defects


SU(2)
SU(3)
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rk 1 D4-branes
+ U(1) flux
How can we add
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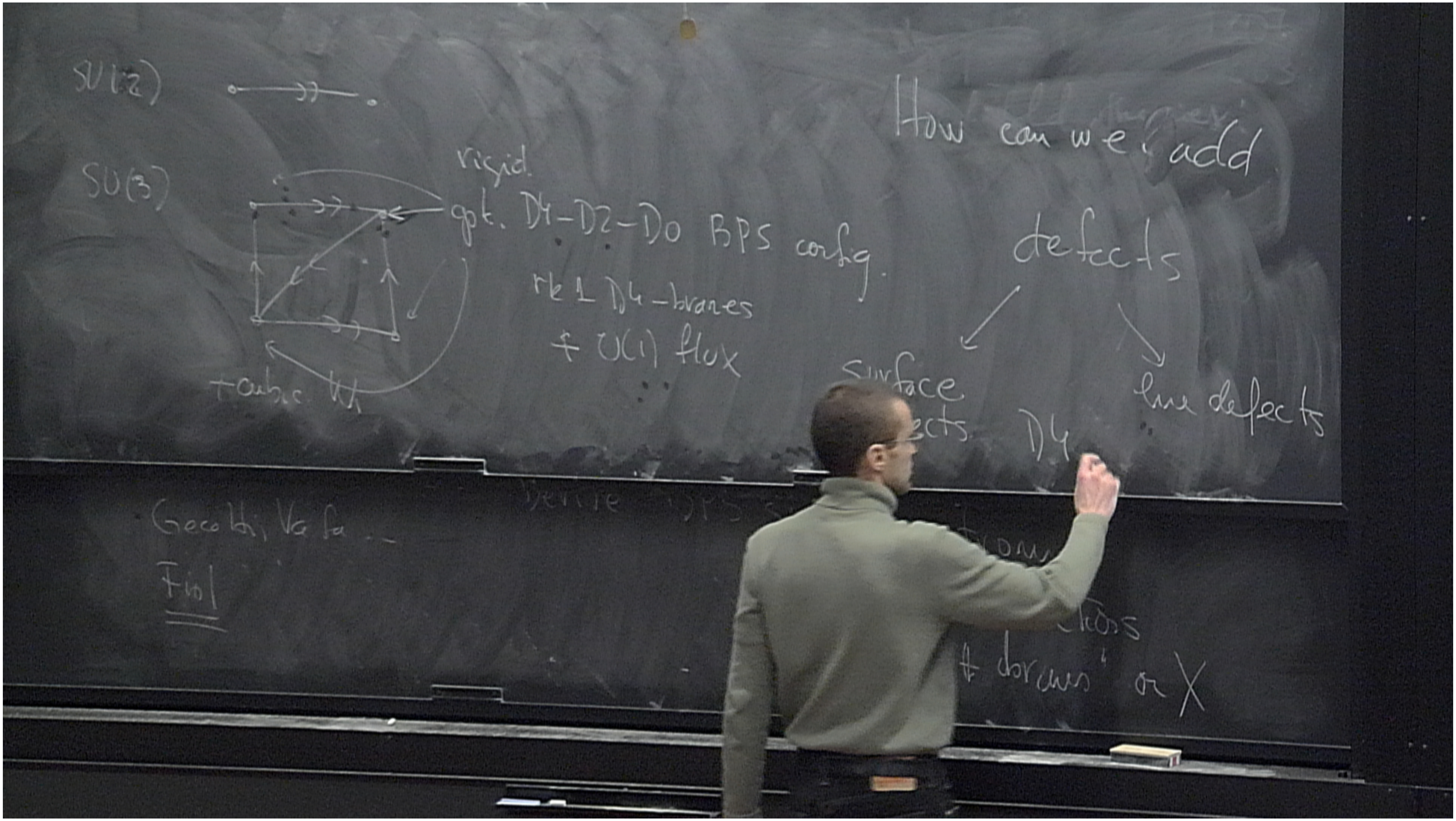
SU(2)
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surface
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D4
Gecotti, Vafa ...
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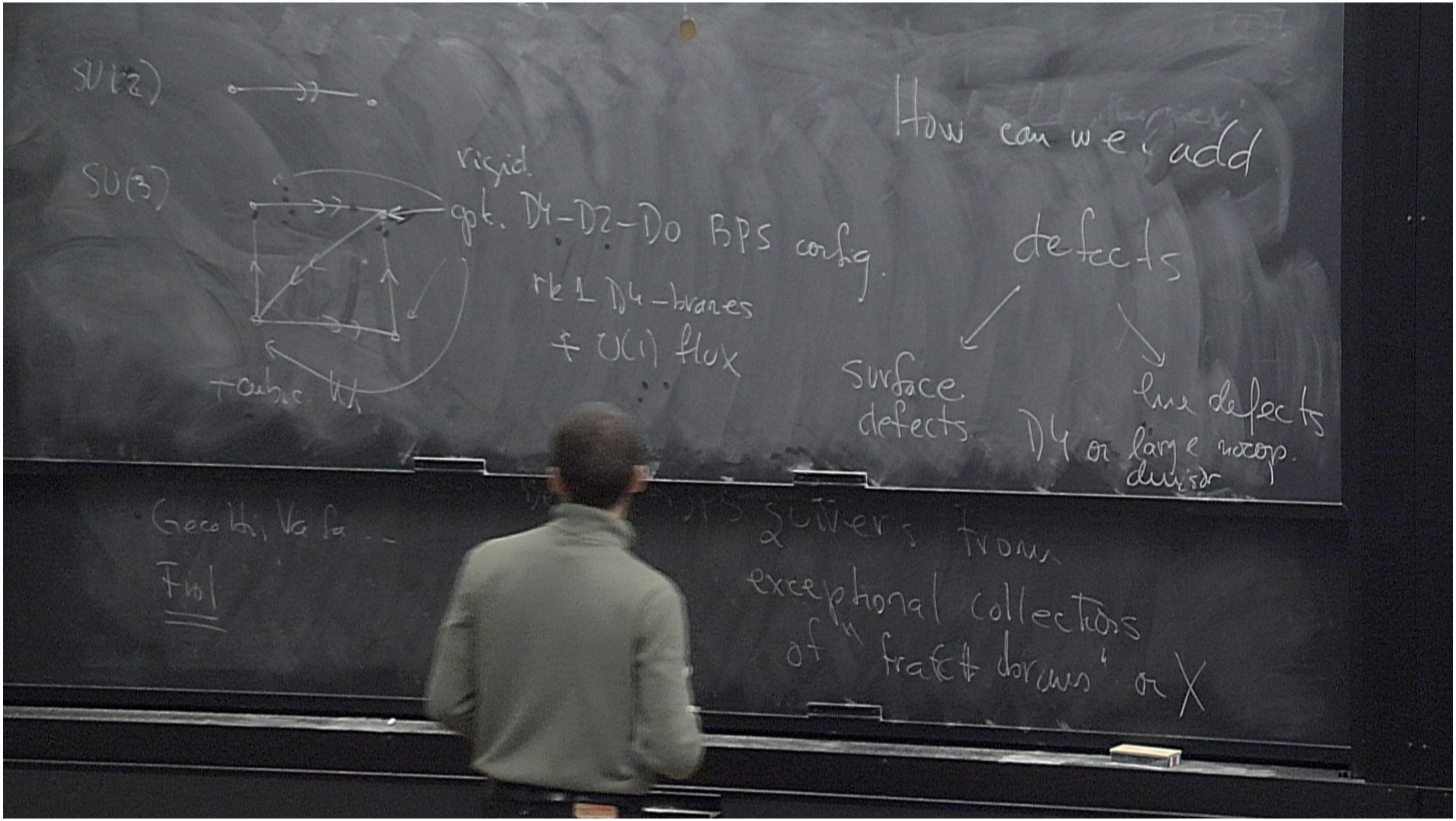
SU(2)
SU(3)
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qpt. D4-D2-D0 BPS config.
rk 1 D4-branes
+ U(1) flux
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How can we add
defects
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D4 or large noncomp. divisor
quivers from
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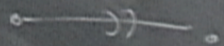
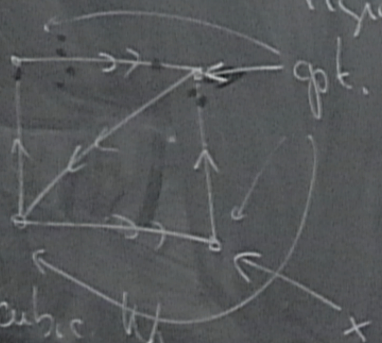
of "fract branes" on X

SU(2) 

SU(3) 

+ cubic W

rigid
gpt. D4–D2–D0 BPS config.

rk 1 D4-branes
+ U(1) flux

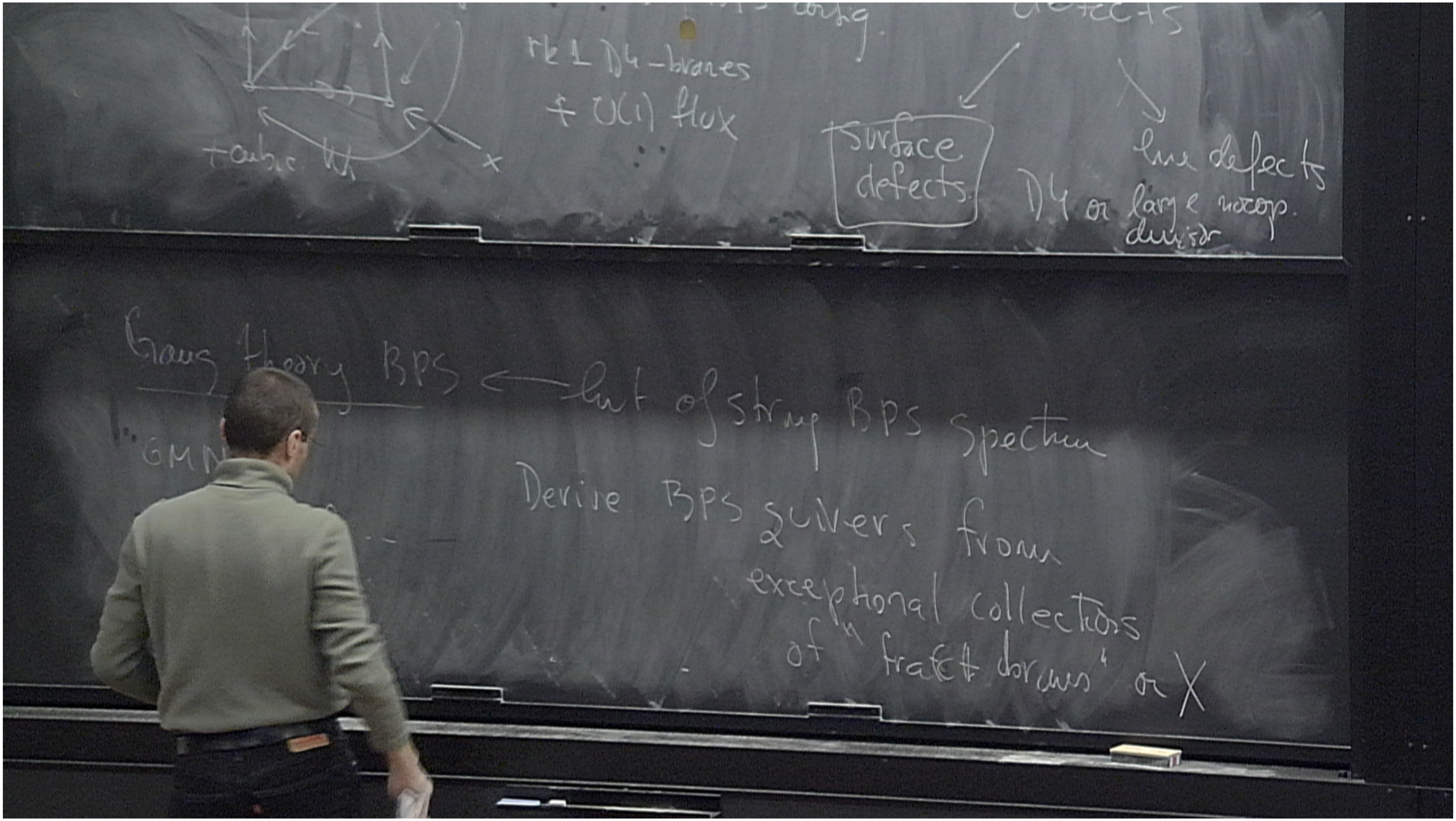
How can we add
defects

surface defects

line defects

D4 or large noncp. divisor

of "fract branes" or X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surface defects
line defects
D4 or large noncp. divisor
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Derive BPS quivers from
exceptional collections
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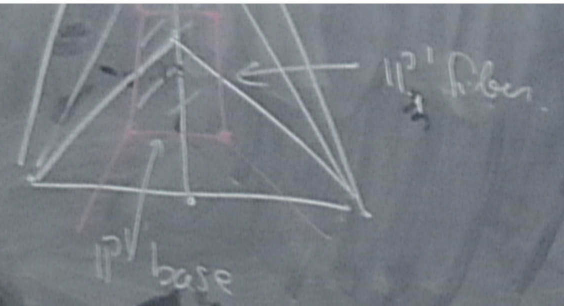
field theory limit

string dynamics $\longrightarrow$ low SW solution for $SU(N)$

Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par

DGH

surface defects $\cong$ D4 on a toric slag cycle in X



field theory limit

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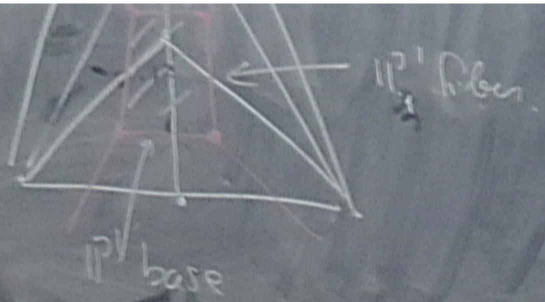
Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par

DGH

Surface defects $\cong$ D4 on a toric slag cycle in X $(S^1 \times \mathbb{R}^2)$



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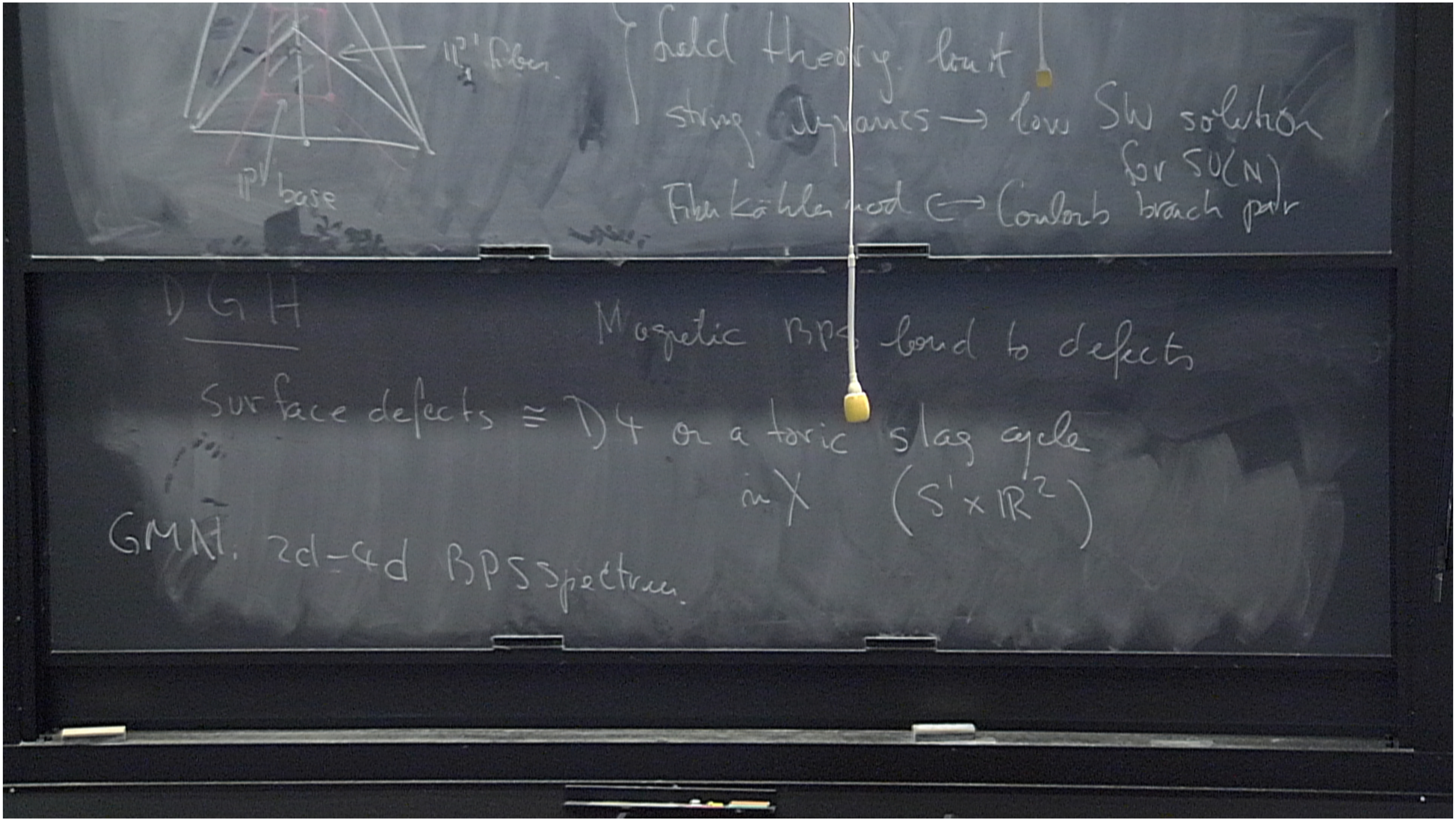
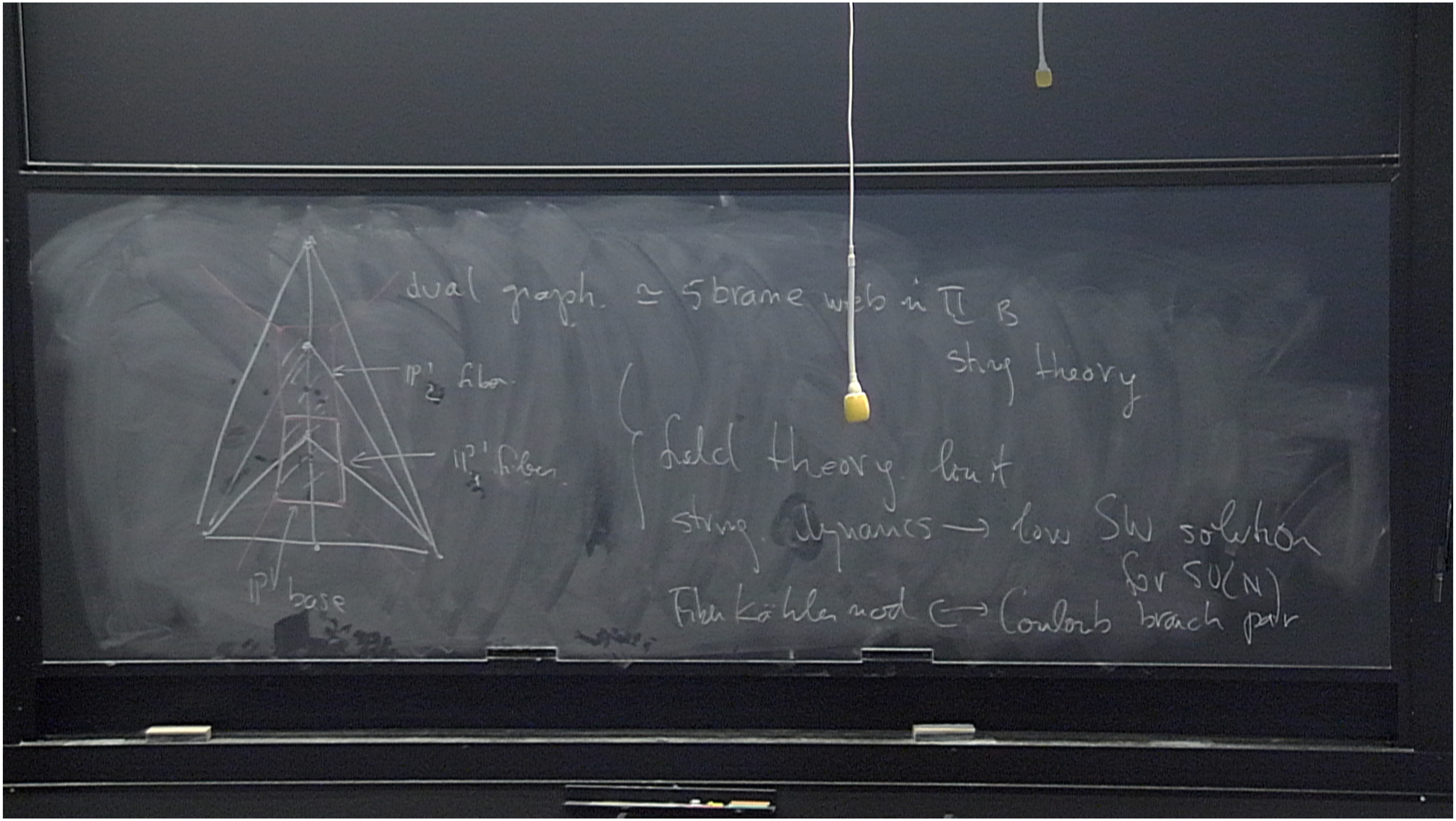
DGH

Surface defects $\cong$ D4 on a toric slag cycle in X $(S^1 \times \mathbb{R}^2)$

GMN: 2d-4d BPS spectrum.


$\mathbb{P}^1$ fiber
$\mathbb{P}^1$ base
field theory limit
string dynamics → low SW solution
for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
DGH
Magnetic BPS bound to defects
Surface defects ≅ D4 on a toric slag cycle
in X ($S^1 \times \mathbb{R}^2$)
GMN: 2d-4d BPS spectrum


dual graph ≃ 5brane web in IIB
string theory
IP¹ fiber
IP¹ fiber
field theory limit
string dynamics ⟶ low SW solution
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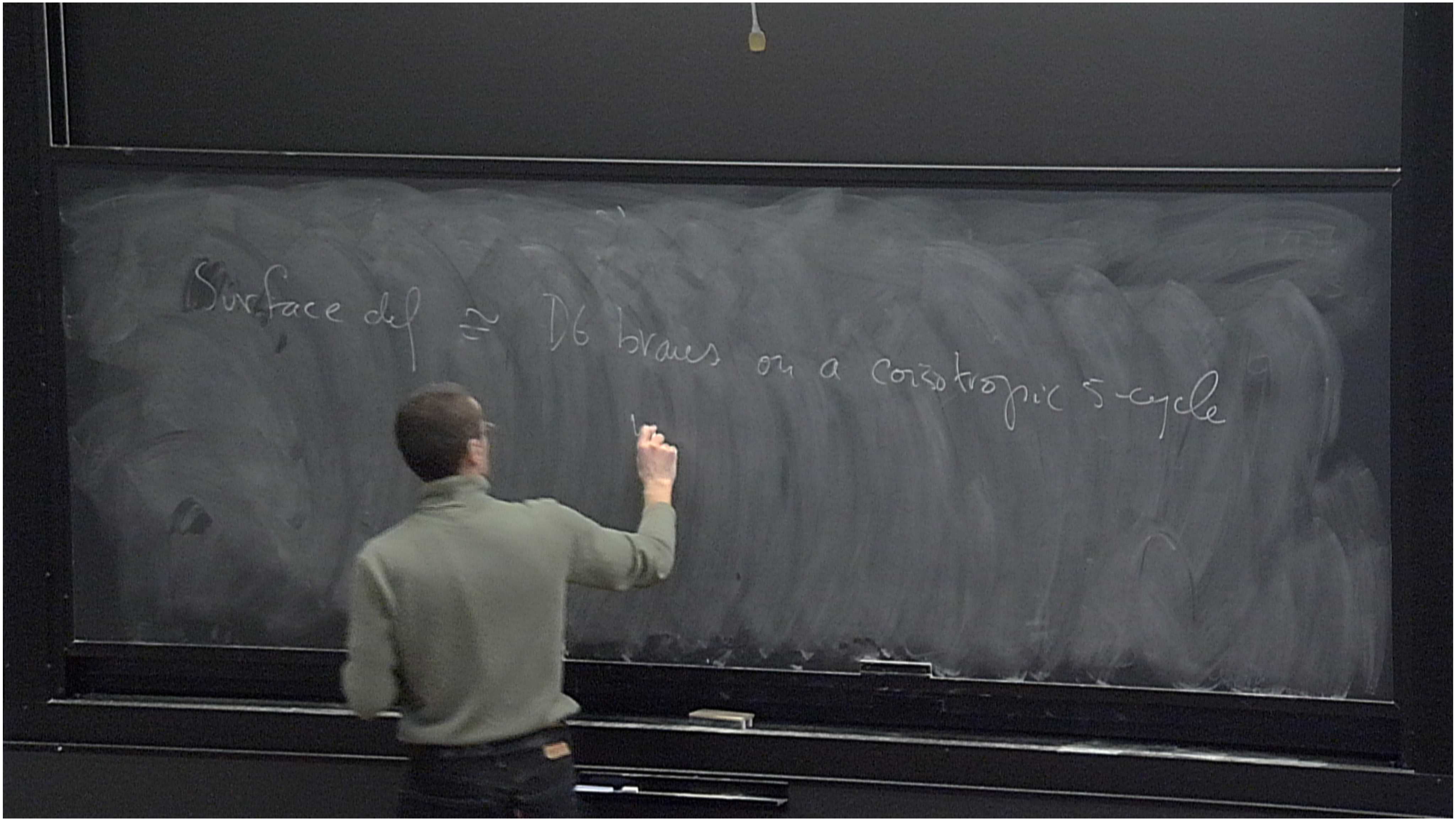
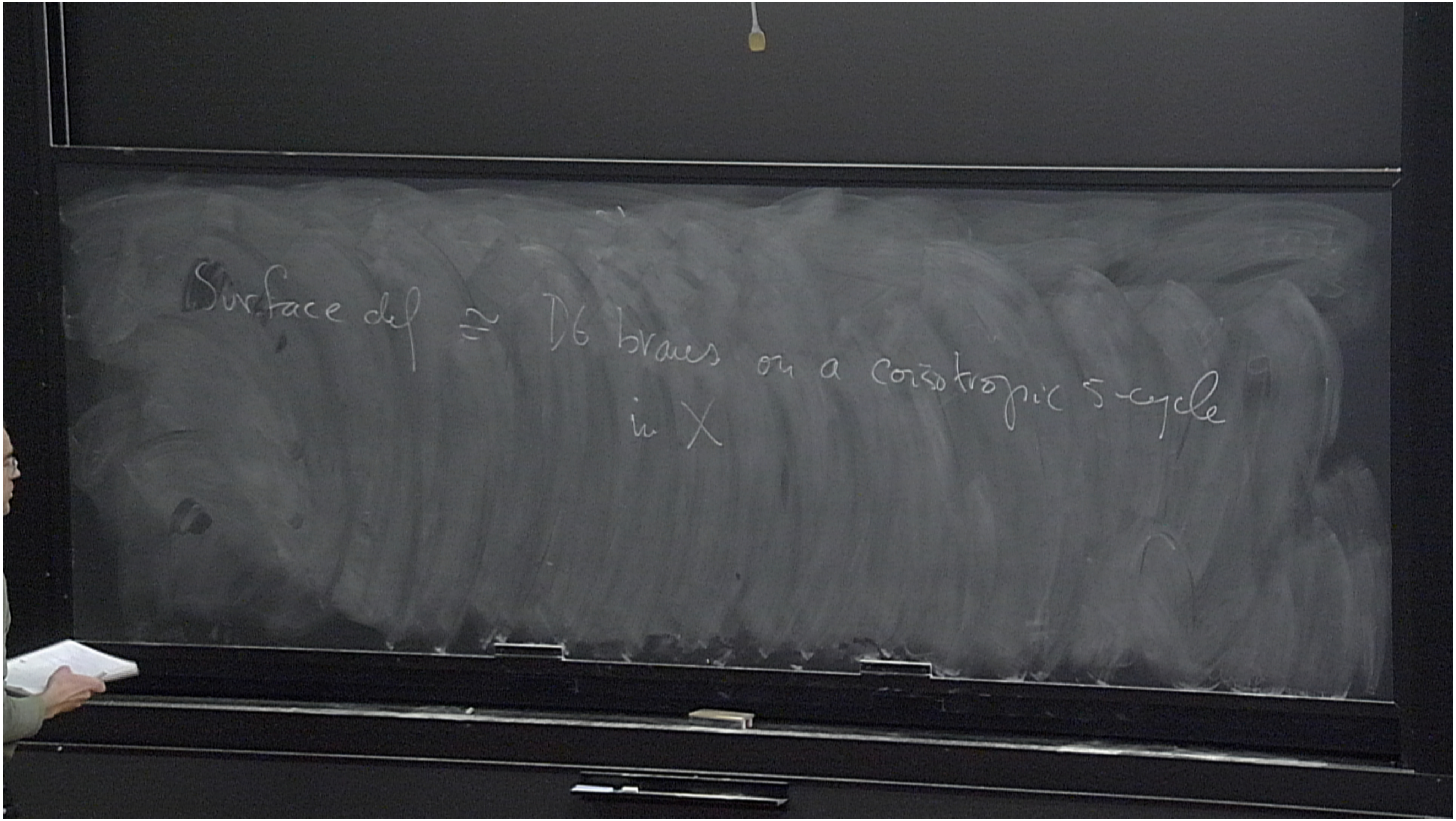
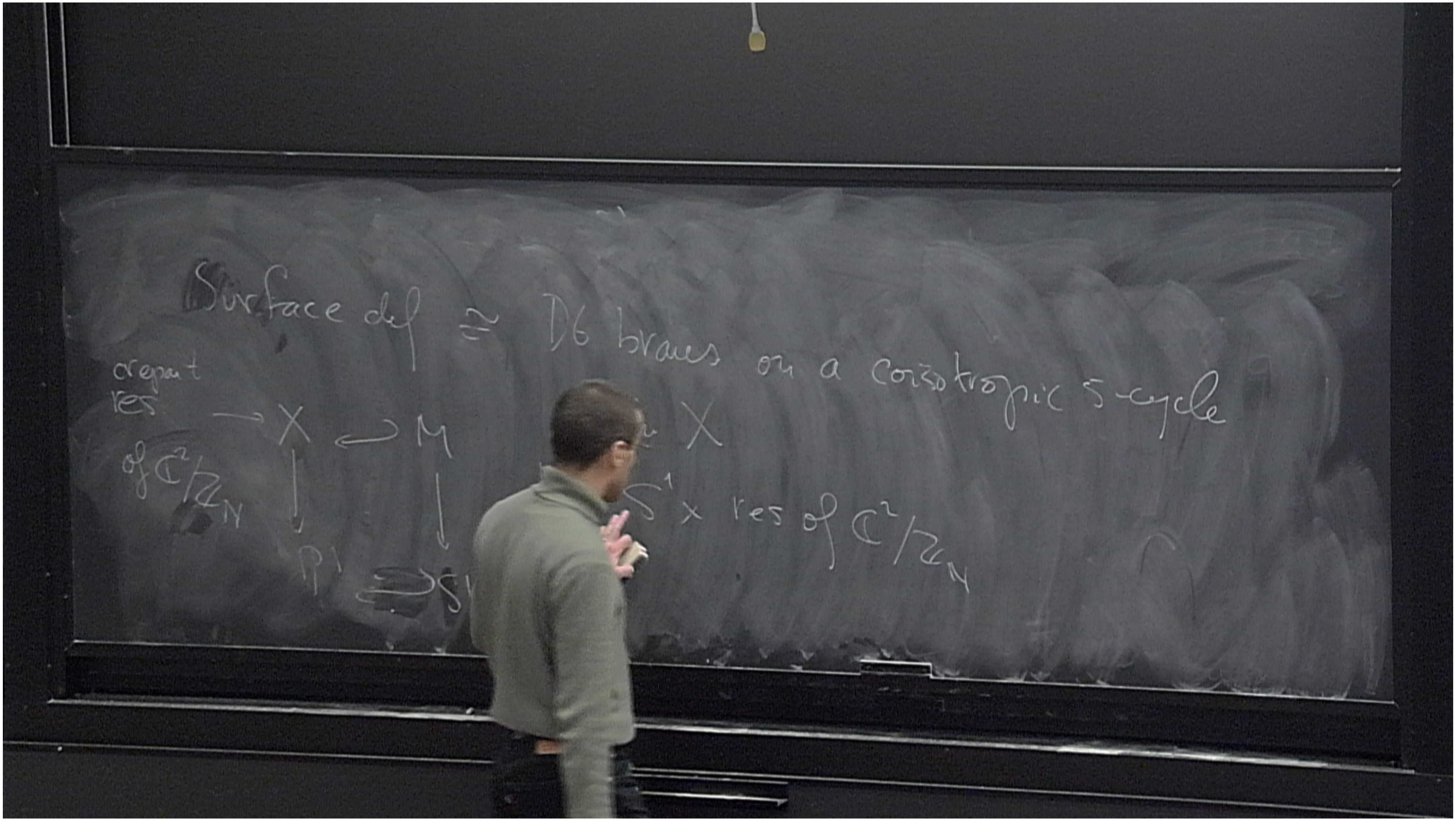
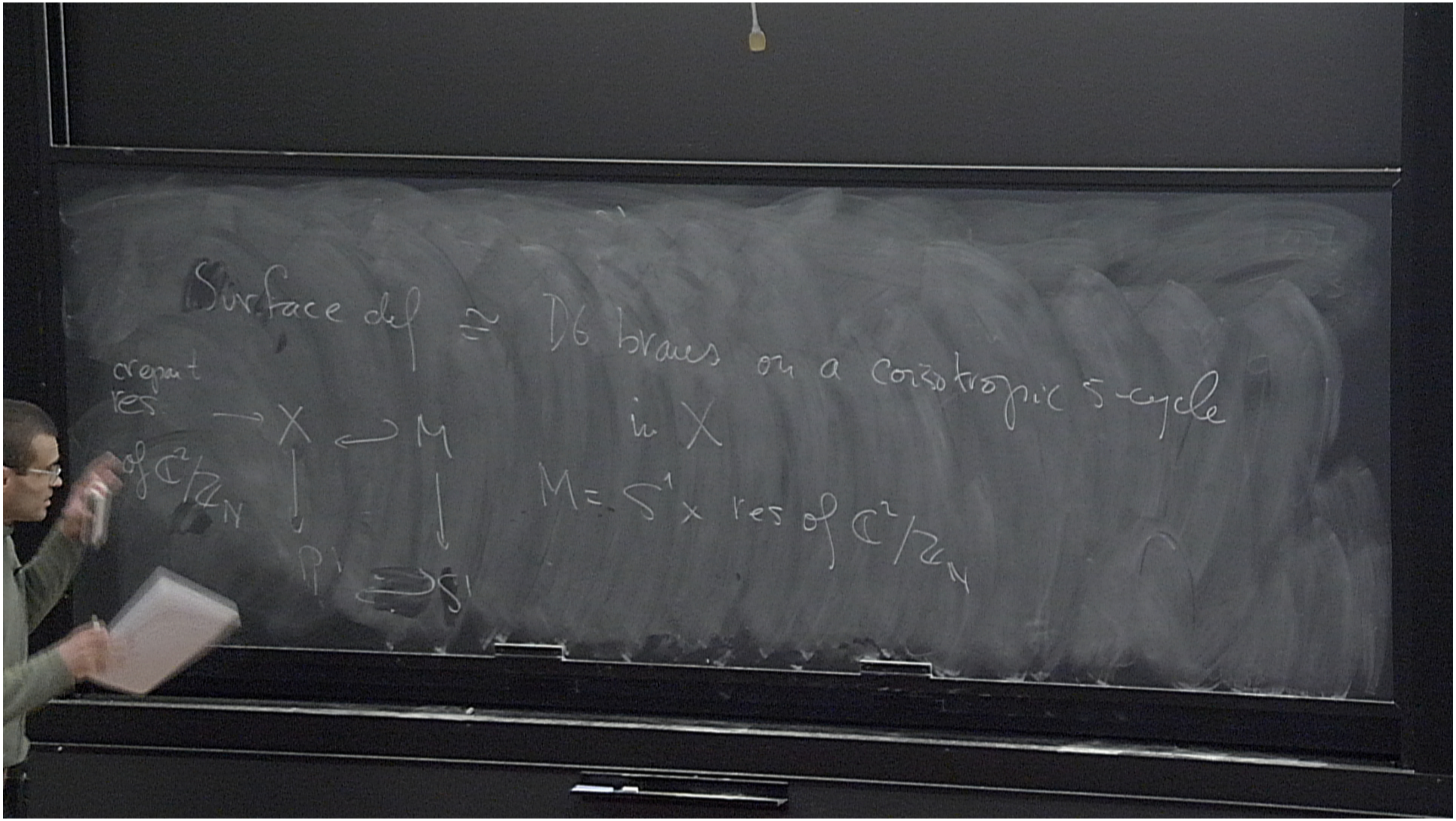
IP¹ base
Fiber Kähler mod ⟷ Coulomb branch par


Surface def ≃ D6 branes on a coisotropic 5-cycle


Surface def ≅ D6 branes on a coisotropic 5-cycle
in X

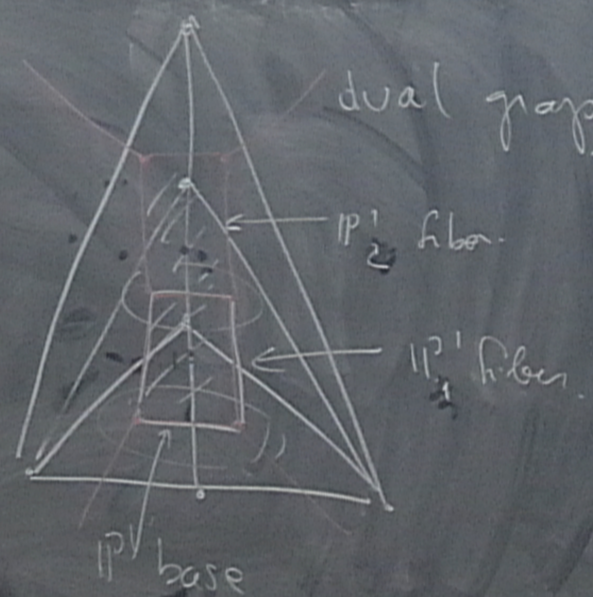

Surface def ≅ D6 branes on a coisotropic 5-cycle
crepant res
of $\mathbb{C}^2/\mathbb{Z}_N$
$X \hookrightarrow M$
X
$S^1 \times$ res of $\mathbb{C}^2/\mathbb{Z}_N$


Surface def ≅ D6 branes on a coisotropic 5-cycle
crepant res of C²/Z_N → X ↩ M
in X
M = S¹ × res of C²/Z_N
P¹
S¹

Surface def $\cong$ D6 branes on a coisotropic 5-cycle in X

$$
\begin{tikzcd}
\text{crepant res. of } \mathbb{C}^2/\mathbb{Z}_N \arrow[r] & X \arrow[r, hook] \arrow[d] & M \arrow[d] \\
& \mathbb{P}^1 & S^1 \arrow[l, hook']
\end{tikzcd}
$$

$M = S^1 \times \text{res of } \mathbb{C}^2/\mathbb{Z}_N$

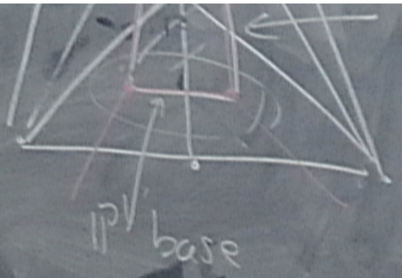


dual graph $\simeq$ 5brane web in IIB string theory

field theory limit

string dynamics $\longrightarrow$ low SW solution for $SU(N)$

Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par

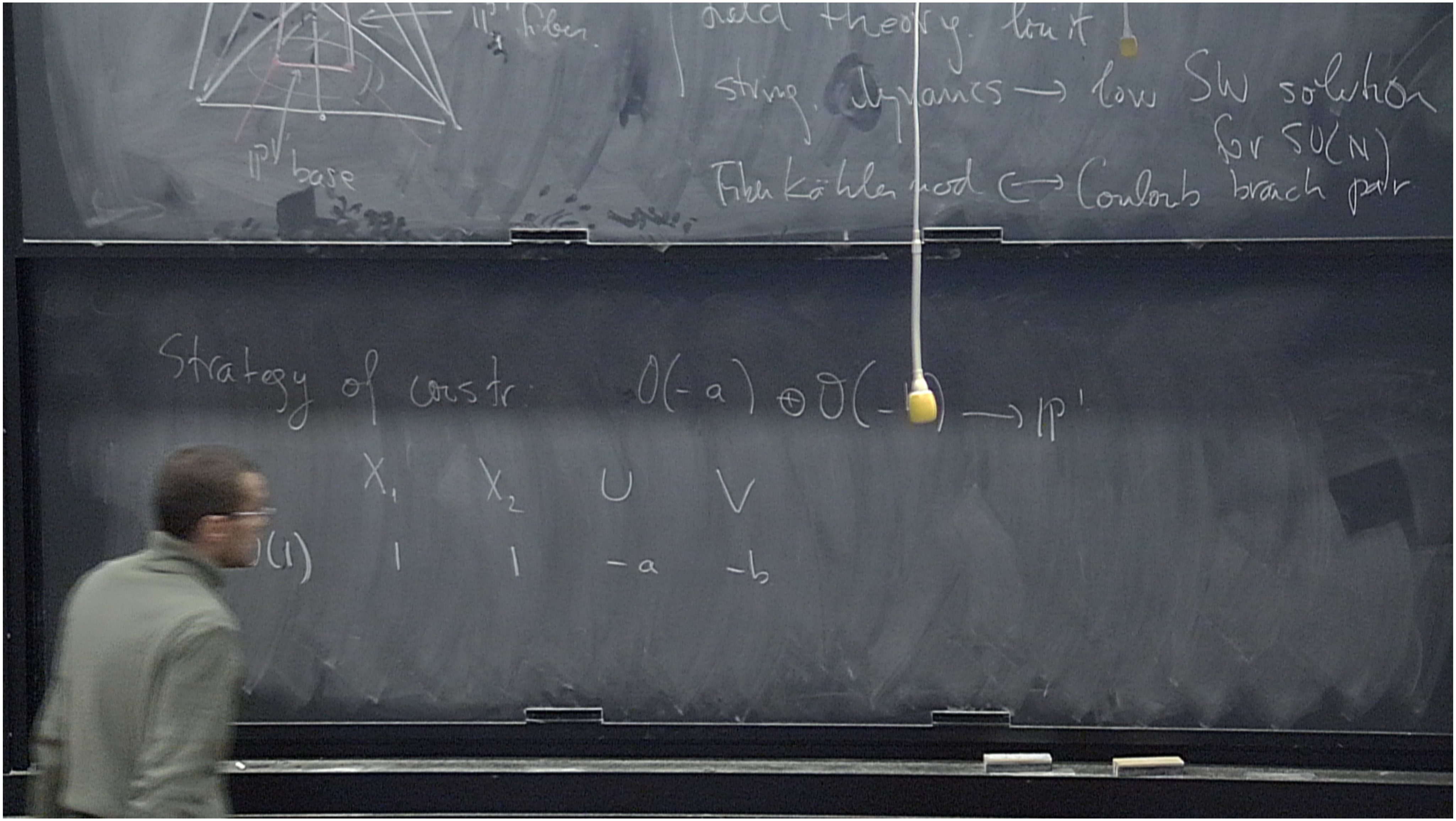


field theory limit

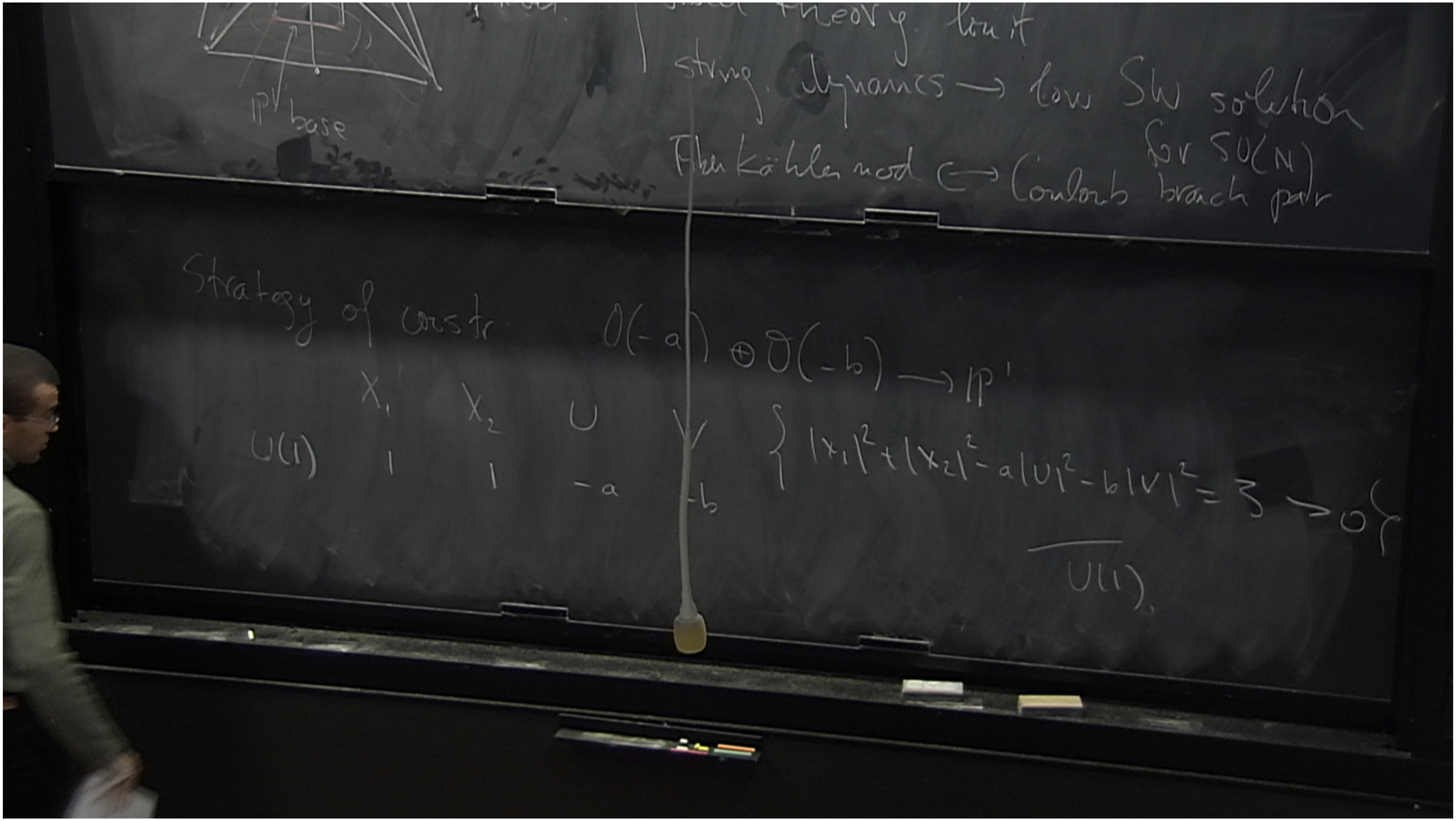
string dynamics $\longrightarrow$ low SW solution for $SU(N)$

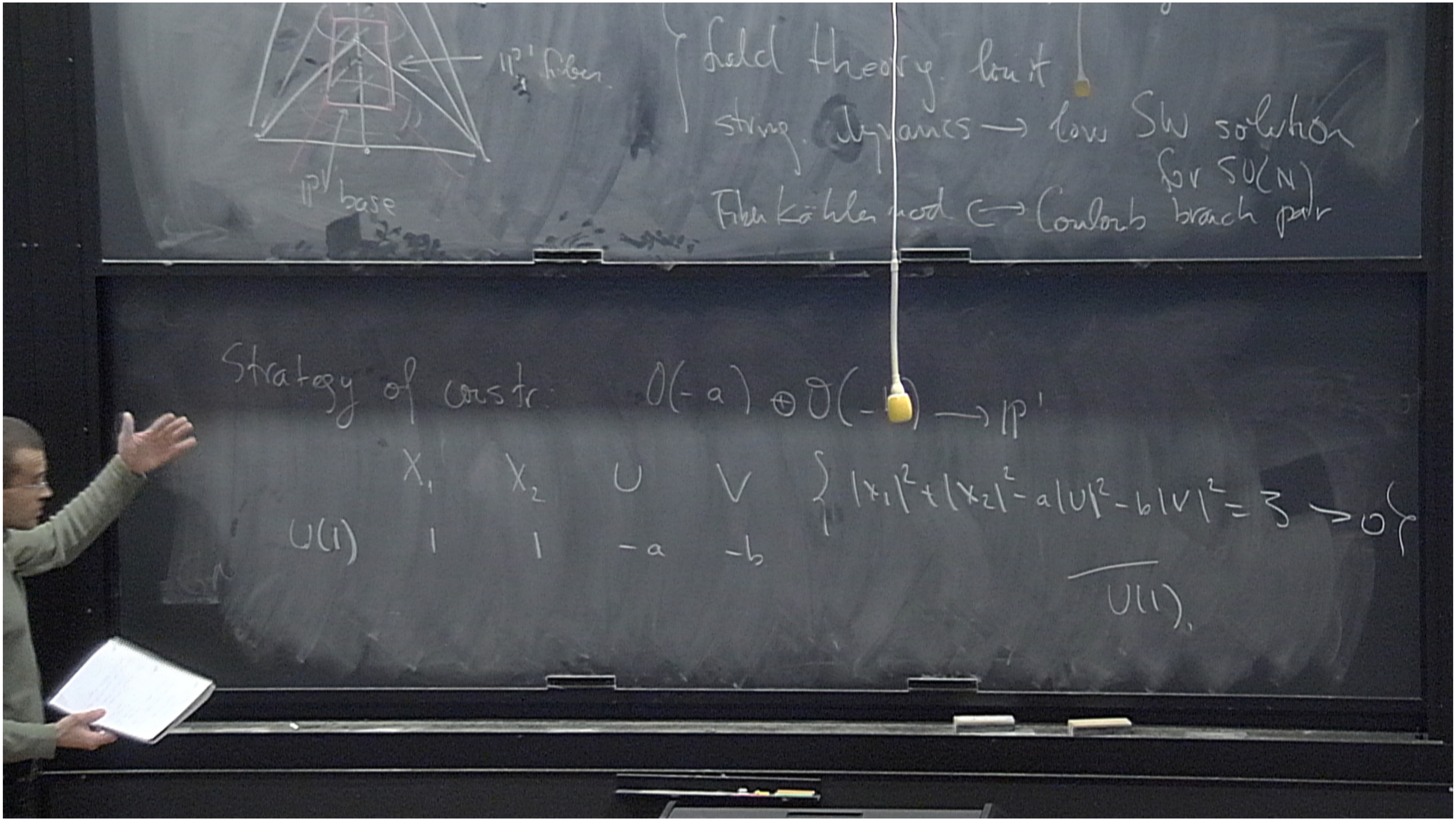
Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par.

Strategy of constr.

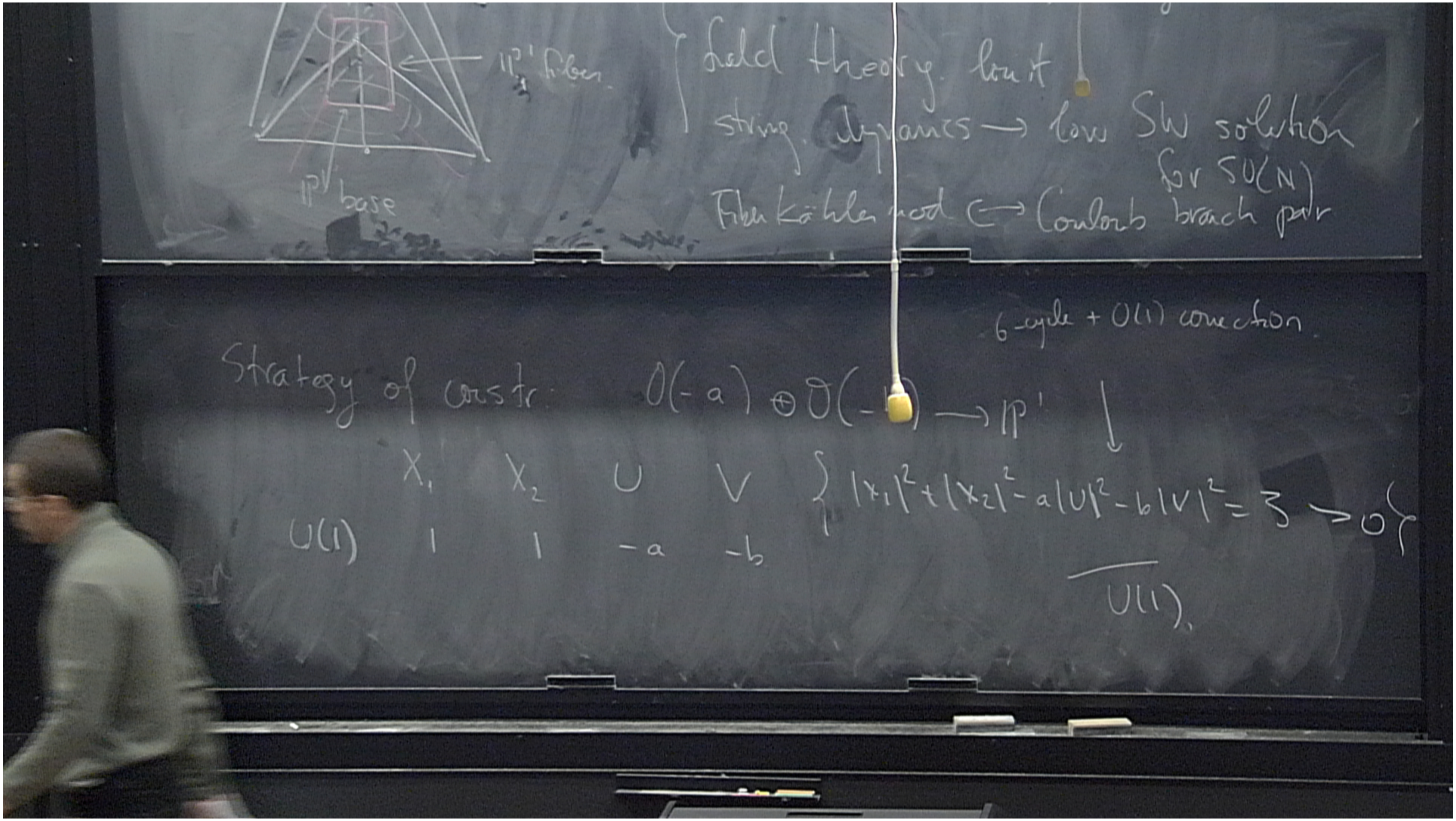


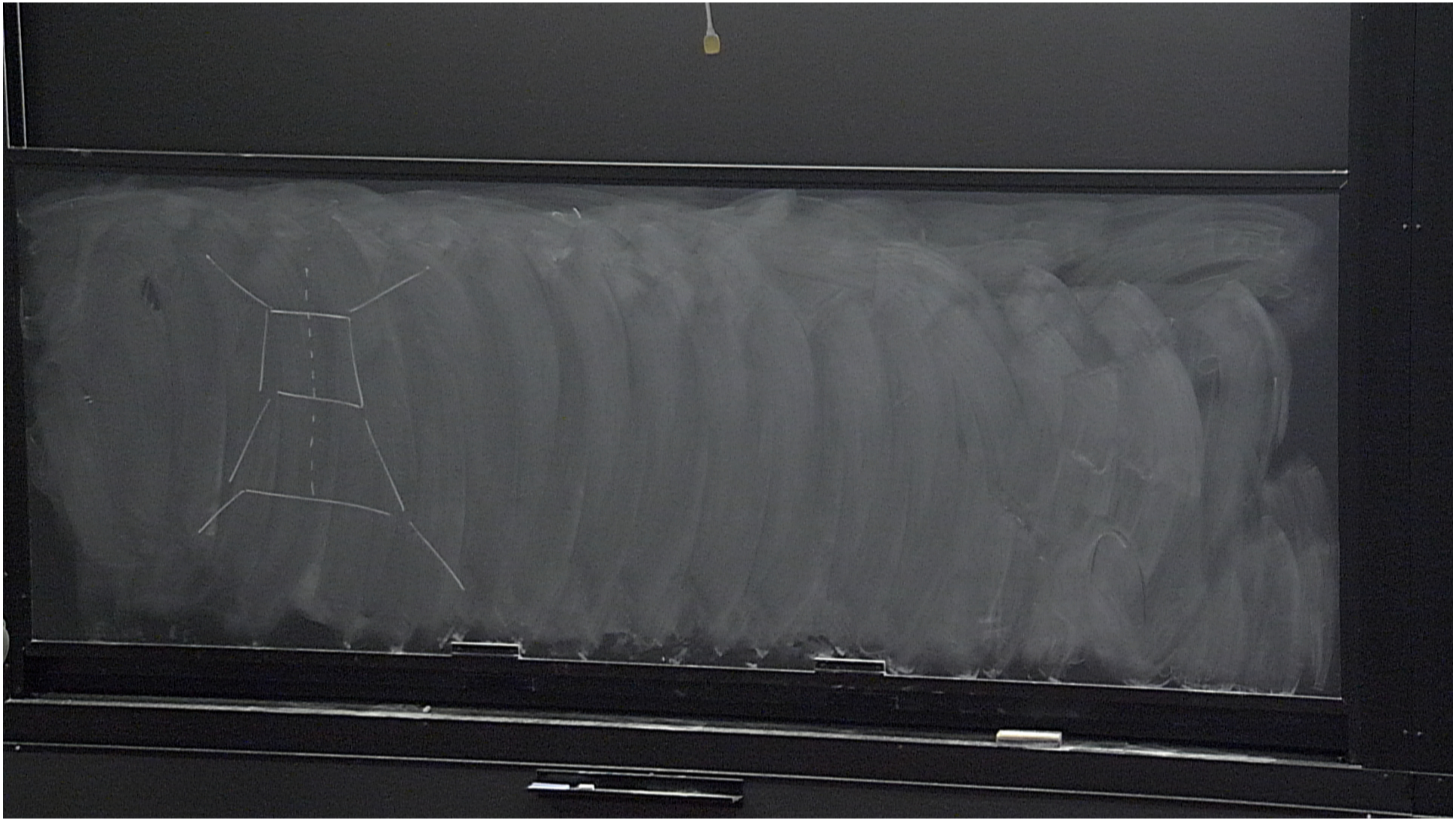
fiber
base
theory limit
string dynamics → low SW solution for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
Strategy of constr.
X1 X2 U V
1 1 -a -b

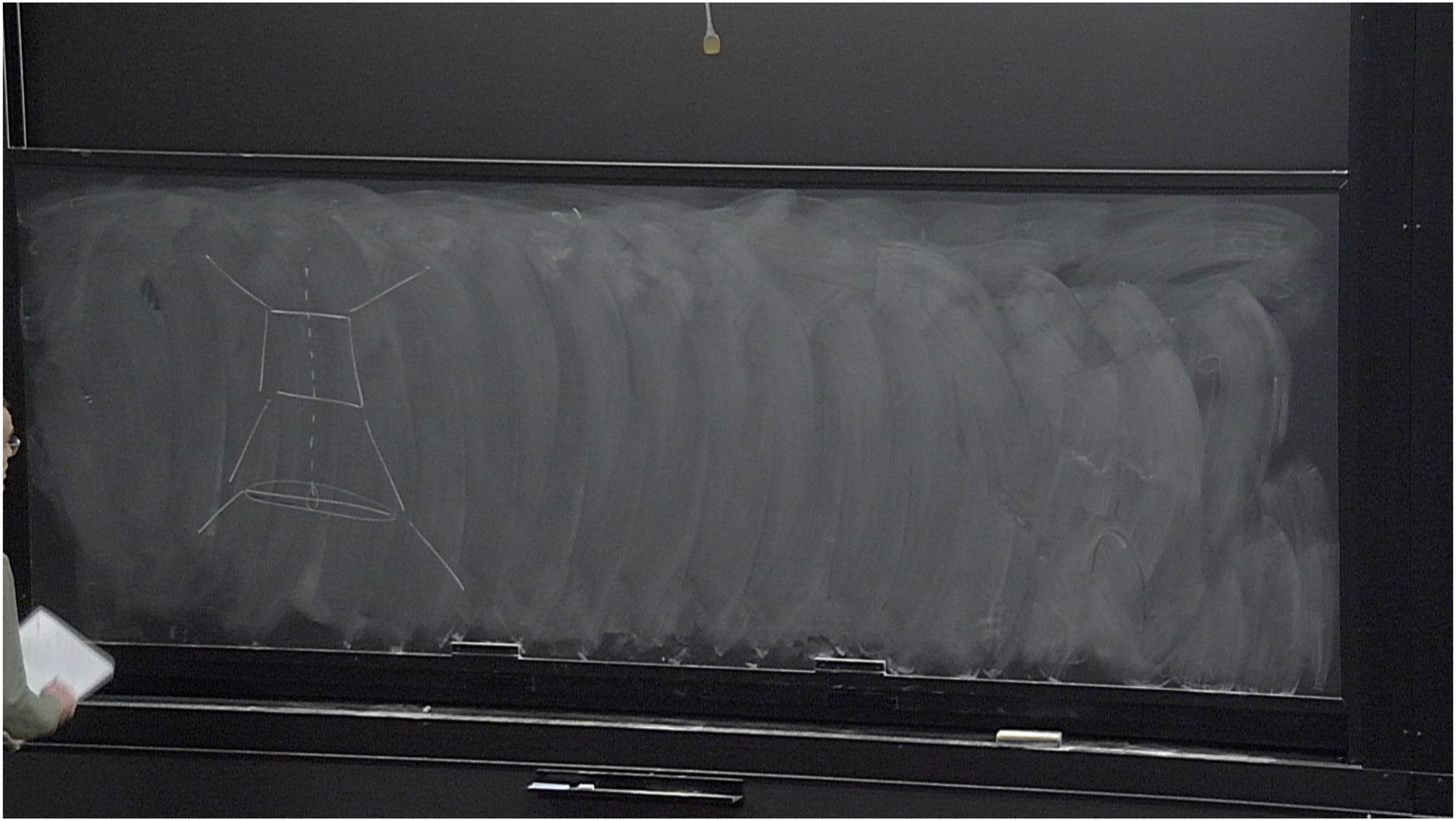

theory limit
string dynamics → low SW solution for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
$\mathbb{P}^1$ base
Strategy of constr $\mathcal{O}(-a)\oplus\mathcal{O}(-b)\to\mathbb{P}^1$
X_1 X_2 U V
$U(1)$ 1 1 $-a$ $-b$
$\{|x_1|^2+|x_2|^2-a|U|^2-b|V|^2=\zeta>0\}/U(1)$

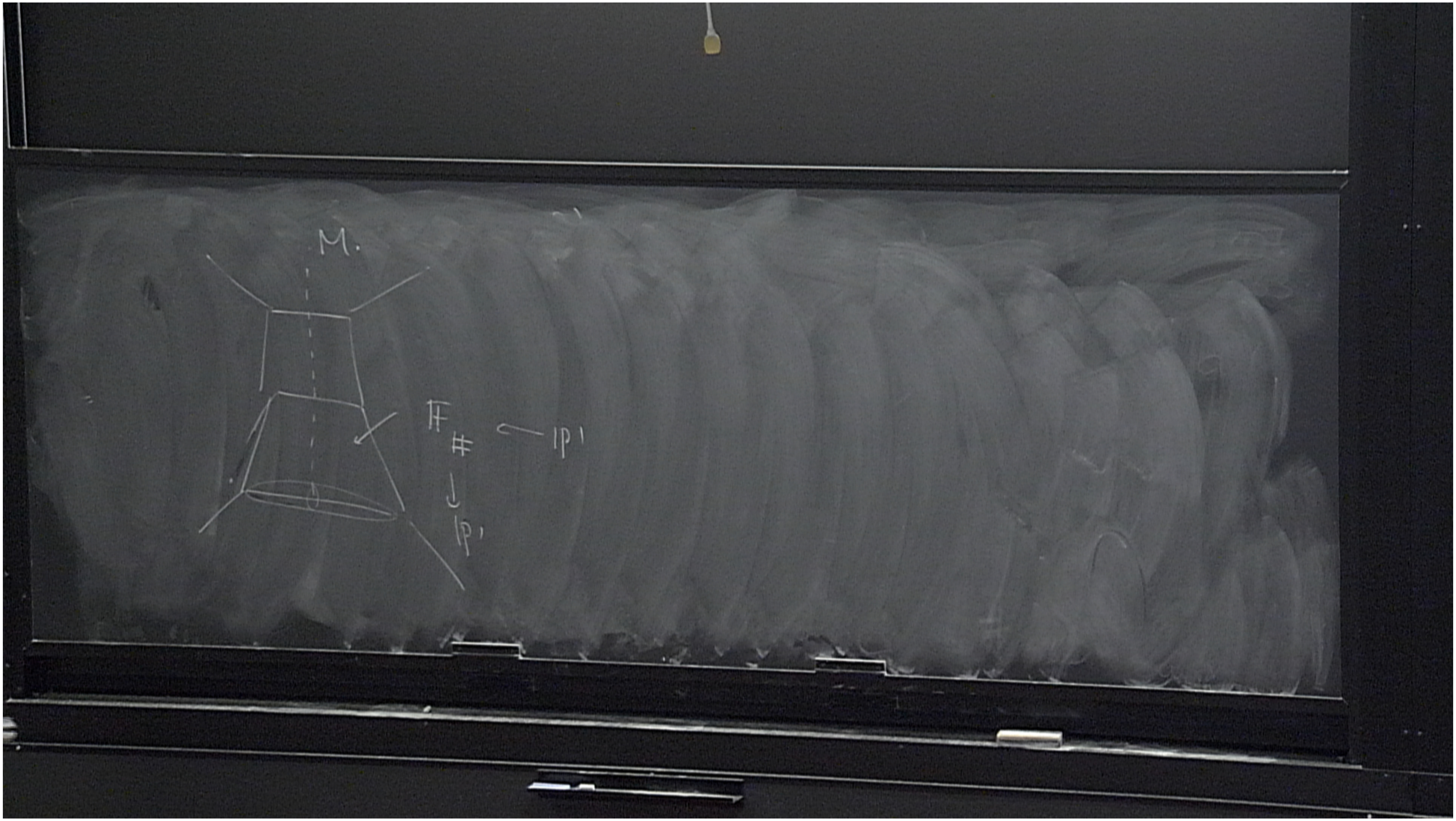


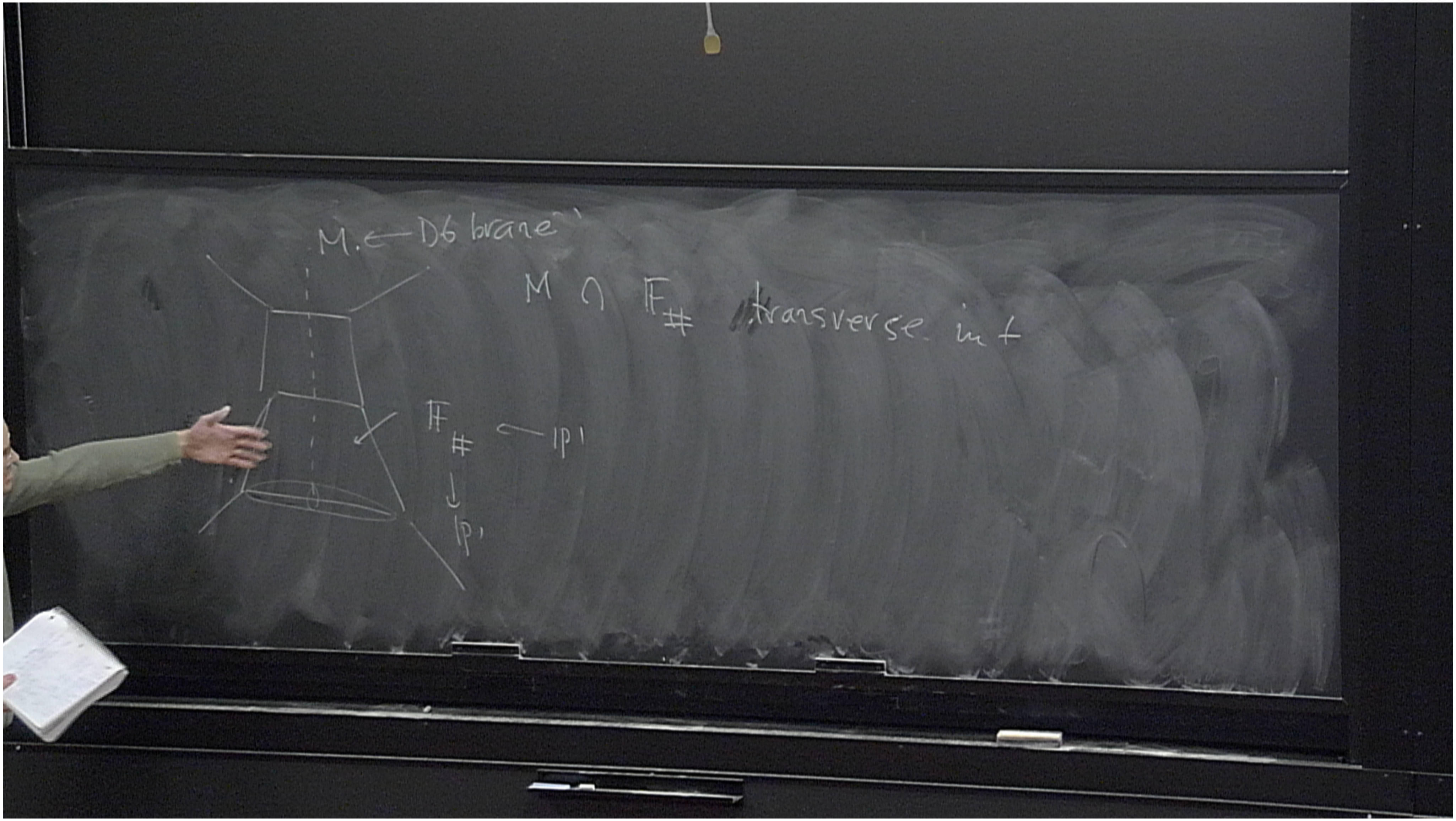
P' fiber
P' base
field theory limit
string dynamics → low SW solution
for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
Strategy of constr: O(-a) ⊕ O(-
→ P'
X1 X2 U V
U(1) 1 1 -a -b
|x1|²+|x2|²-a|U|²-b|V|²=ξ>0
U(1)


P¹ fiber
P¹ base
field theory limit
string dynamics → low SW solution
for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
6-cycle + U(1) connection
Strategy of constr: O(-a) ⊕ O(-b) → P¹
X1 X2 U V
U(1) 1 1 -a -b
{ |x1|² + |x2|² - a|U|² - b|V|² = ζ > 0 } / U(1)

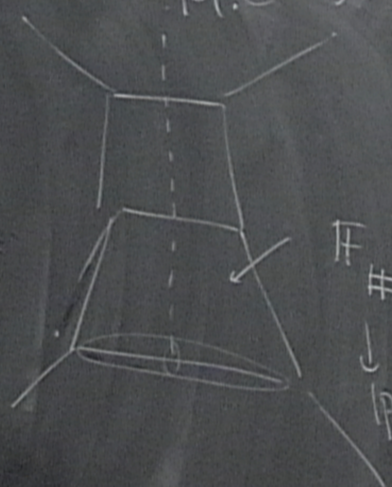





M.


M. ← D6 brane
M ∩ F# transverse in t
F# ← P1
P1

M. ← D6 brane



$\mathbb{F}_\# \leftarrow \mathbb{P}^1$

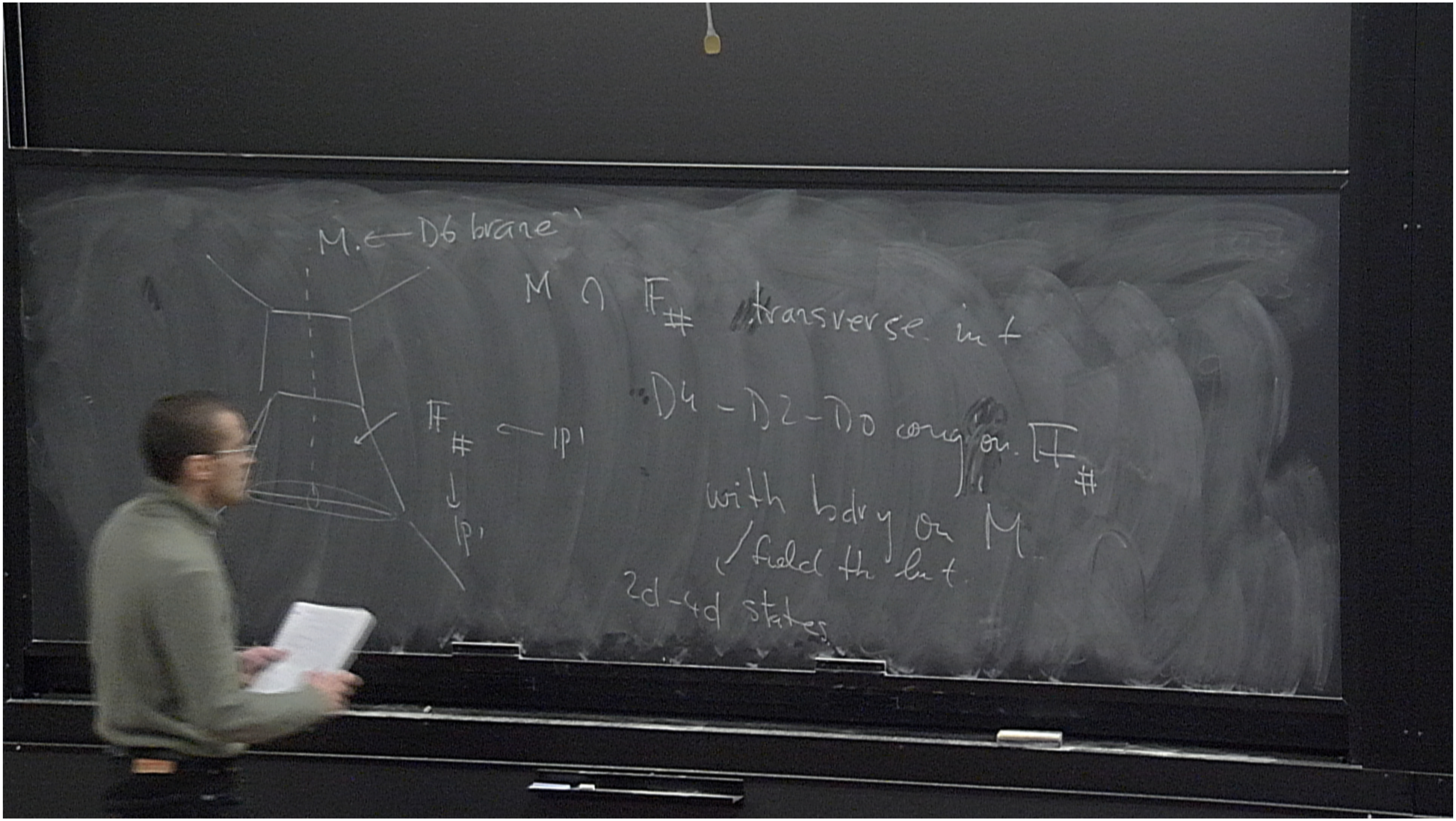
$\downarrow$

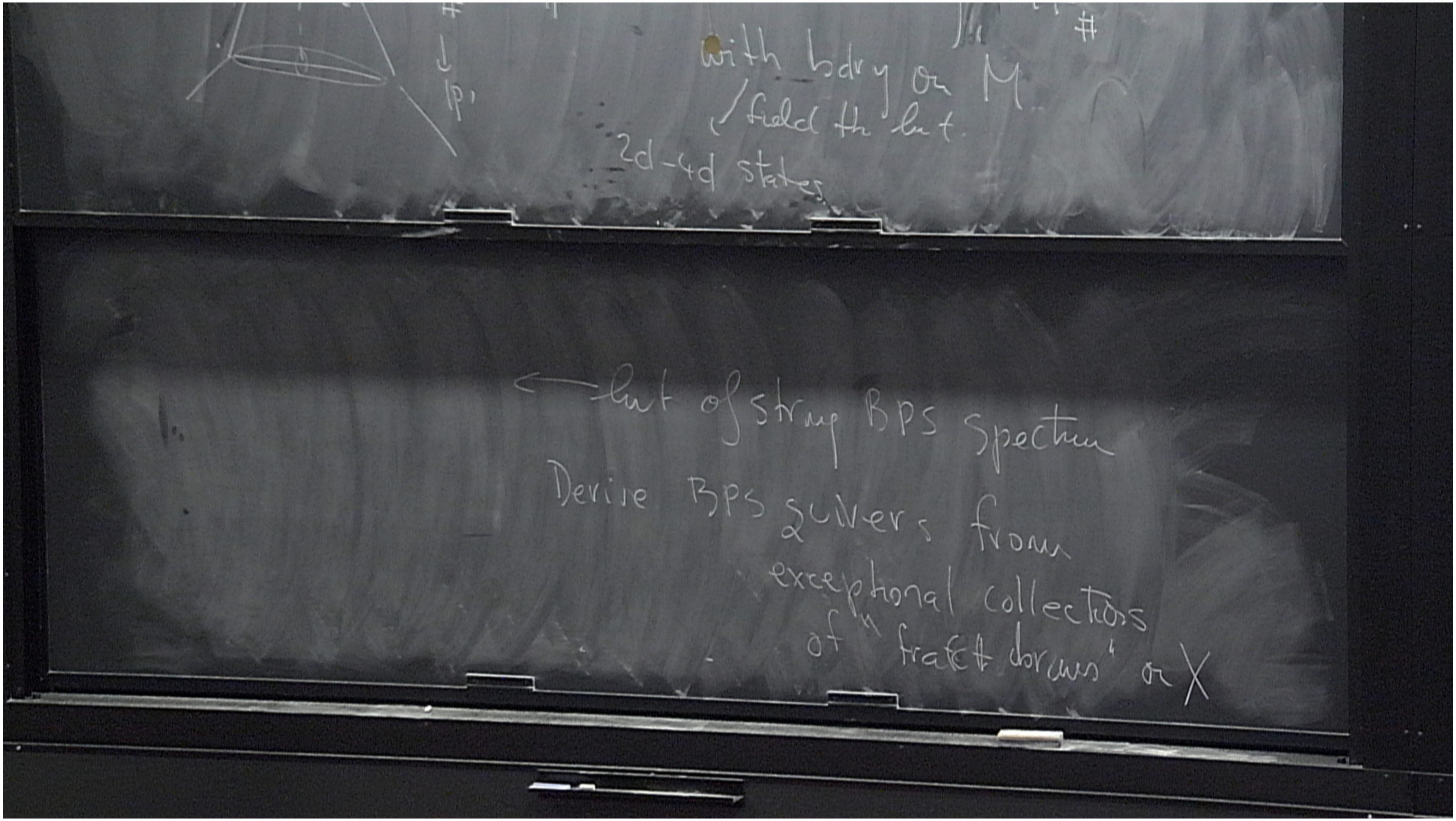
$\mathbb{P}^1$

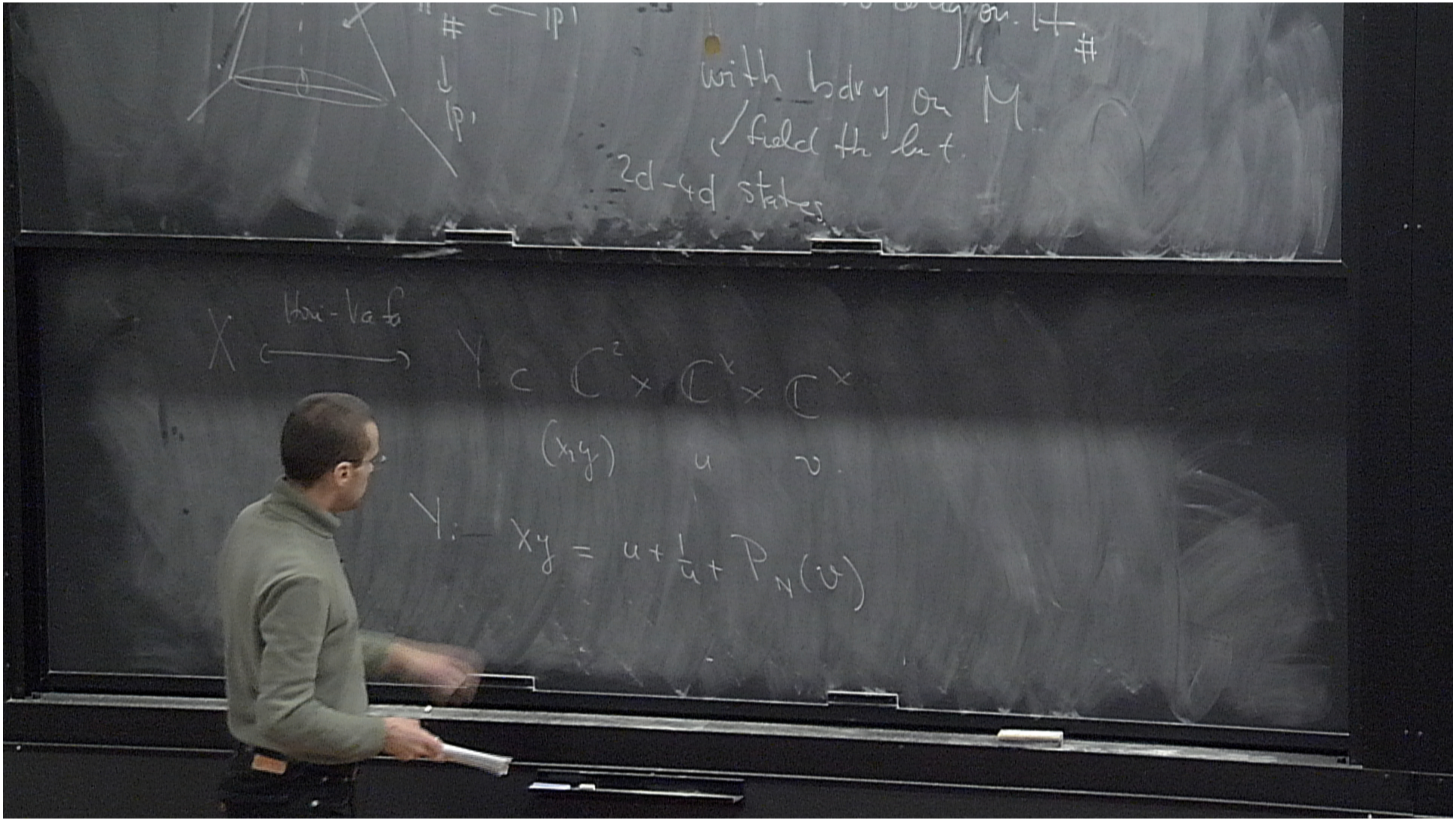
$M \cap \mathbb{F}_\#$ transverse in t

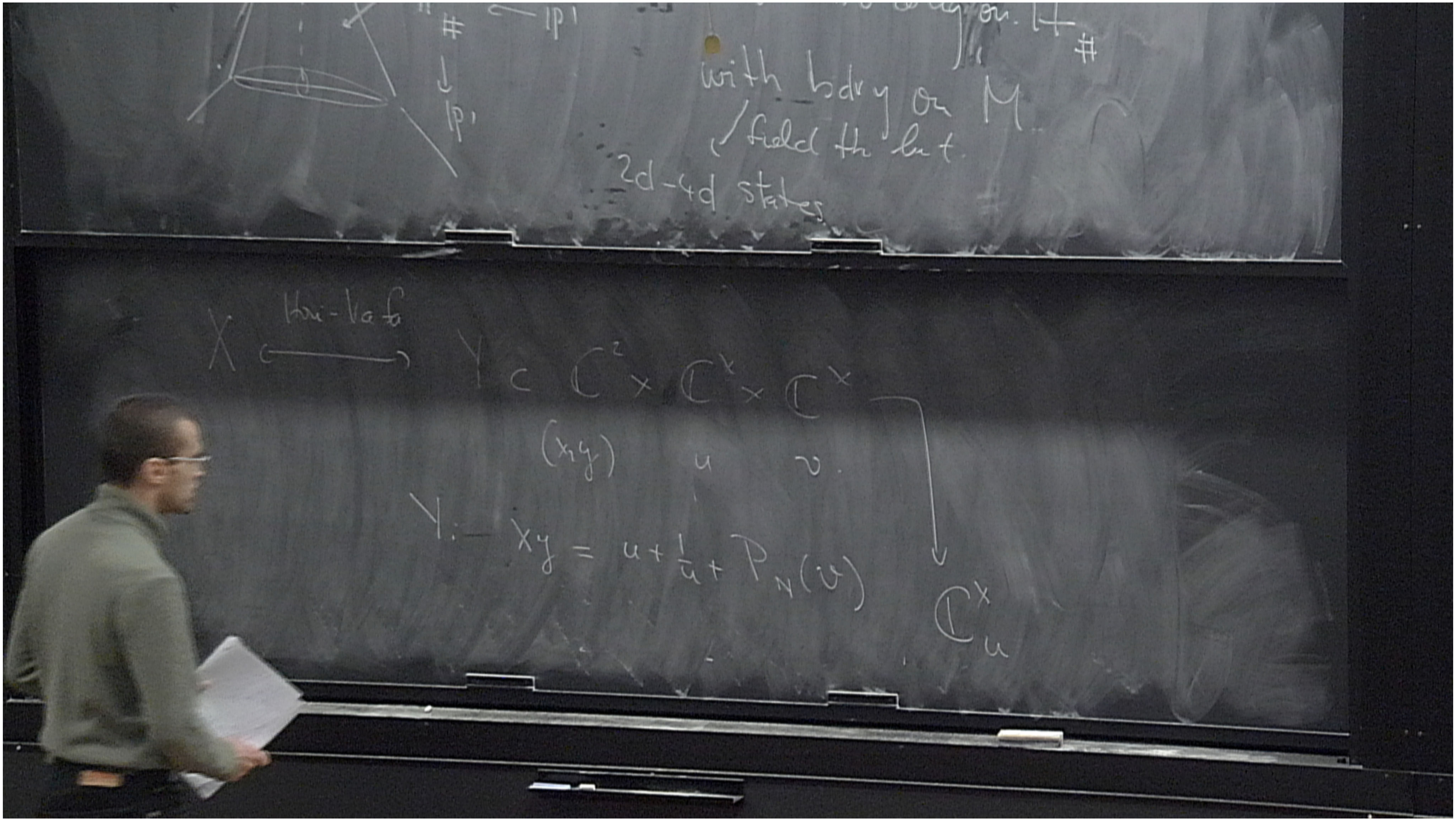
D4 – D2 – D0 config. on $\mathbb{F}_\#$

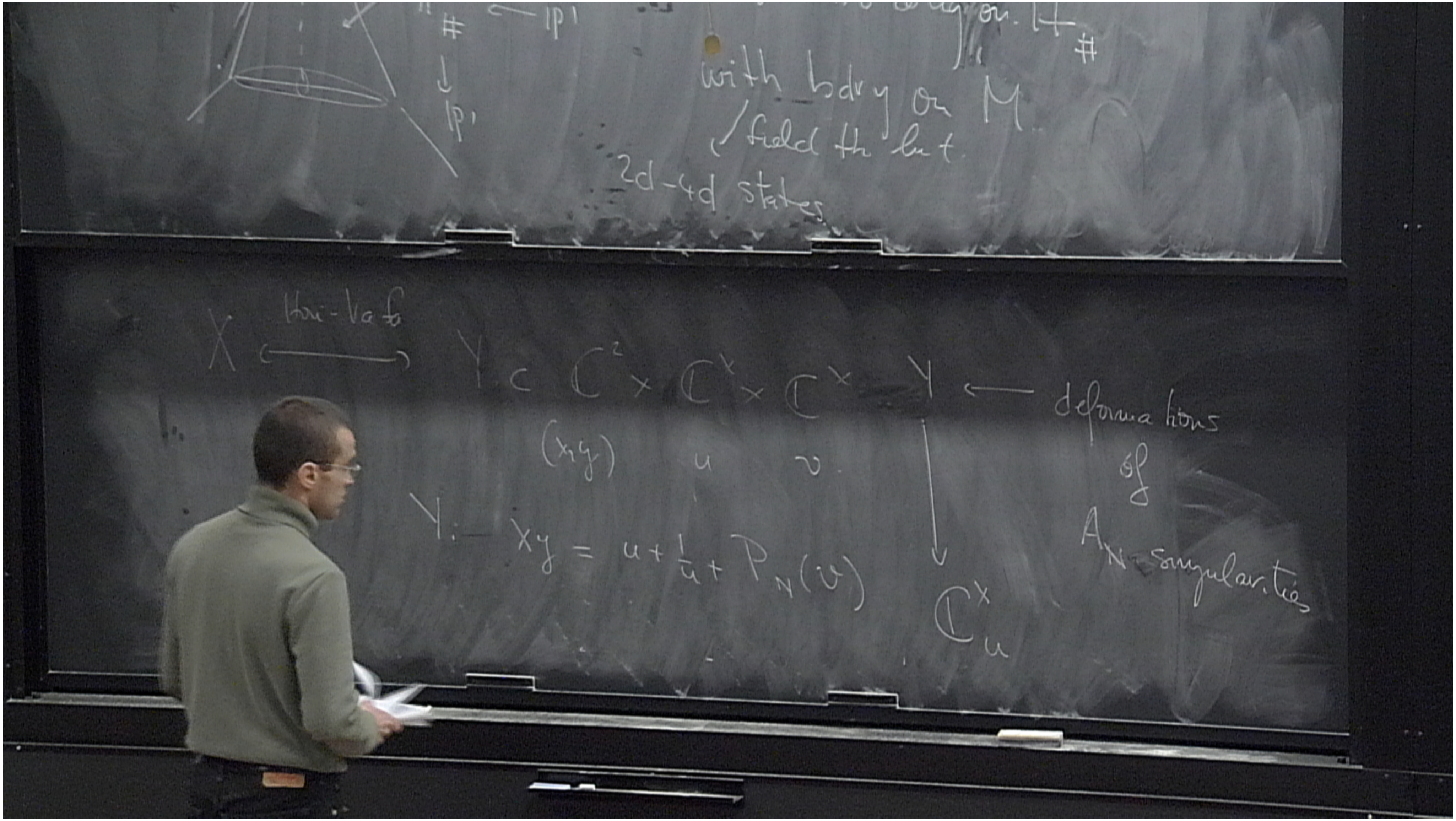
with bdry on M

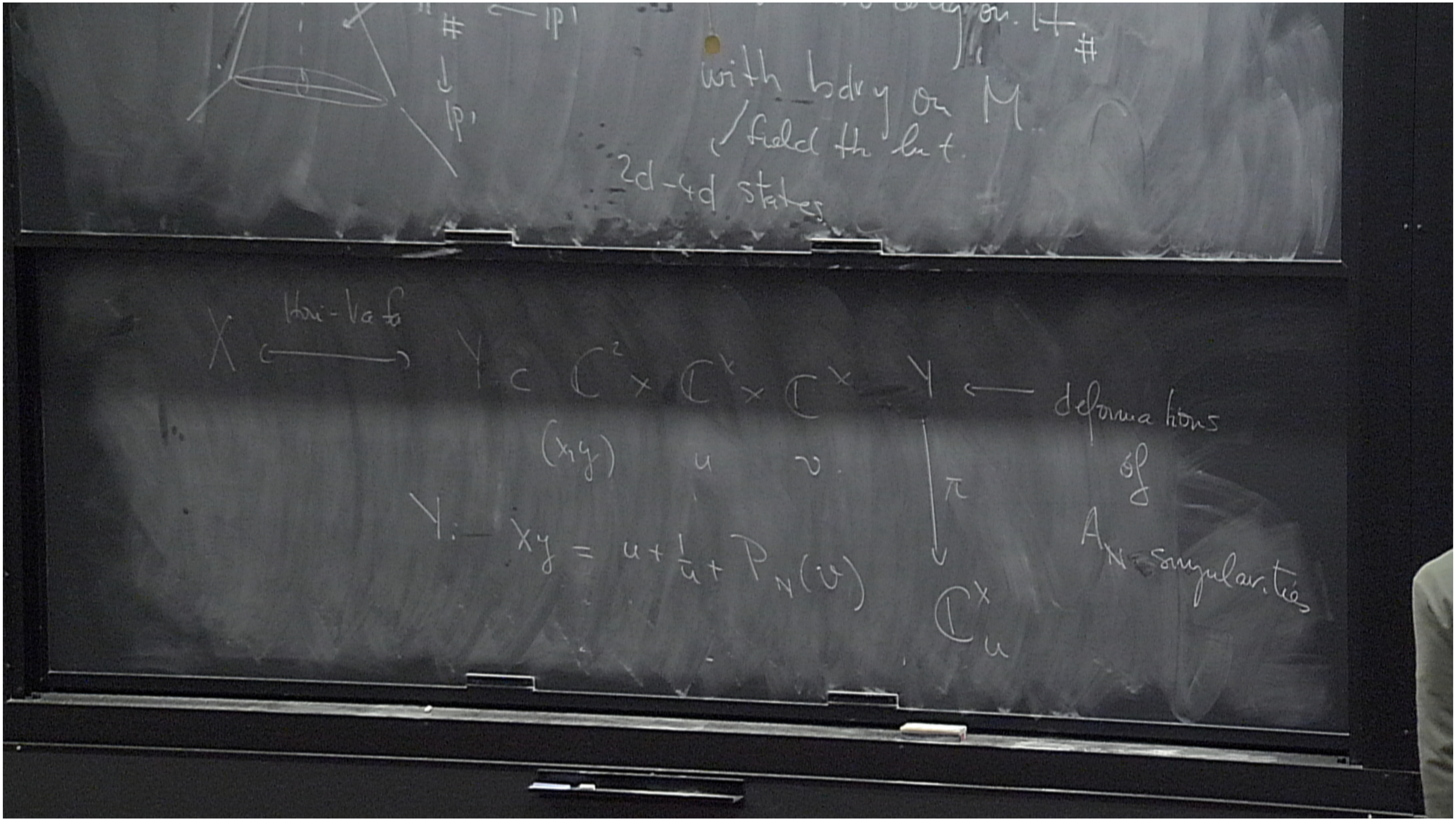

M. ← D6 brane
M ∩ F# transverse int
F# ← IP'
IP'
D4 - D2 - D0 config on F#
with bdry on M
/ field th int.
2d-4d states

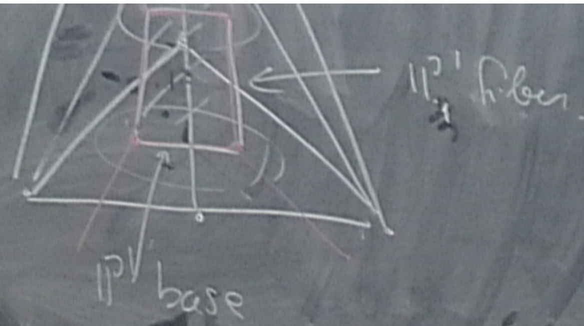

with bdry on M
2d-4d states
lut of strng BPS spectrum
Derive BPS quivers from
exceptional collections
of "fract branes" or X


with bdry on M
2d-4d states
Hori-Vafa
X ⟶ Y ⊂ C² × C^× × C^×
(x,y) u v
Y: xy = u + 1/u + P_N(v)


with bdry on M
2d-4d states
Hori-Vafa
X
Y ⊂ C^2 × C^× × C^×
(x,y) u v
Y: xy = u + 1/u + P_N(v)
C^×_u


with bdry on M
2d-4d states
Hori-Vafa
X Y ⊂ C^2 × C^× × C^×
(x,y) u v
Y: xy = u + 1/u + P_N(v)
C^×_u
deformations of A_N singularities


with bdry on M
2d-4d states
Hori-Vafa
X ⟷ Y ⊂ C^2 × C^× × C^×
(x,y) u v
Y: xy = u + 1/u + P_N(v)
deformations of A_N singularities
π
C^×_u

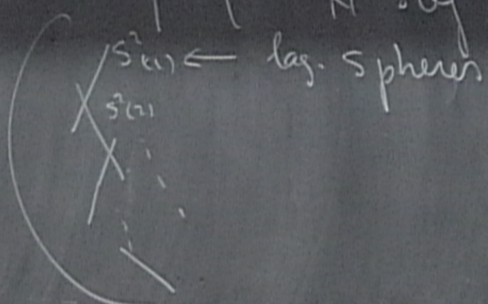


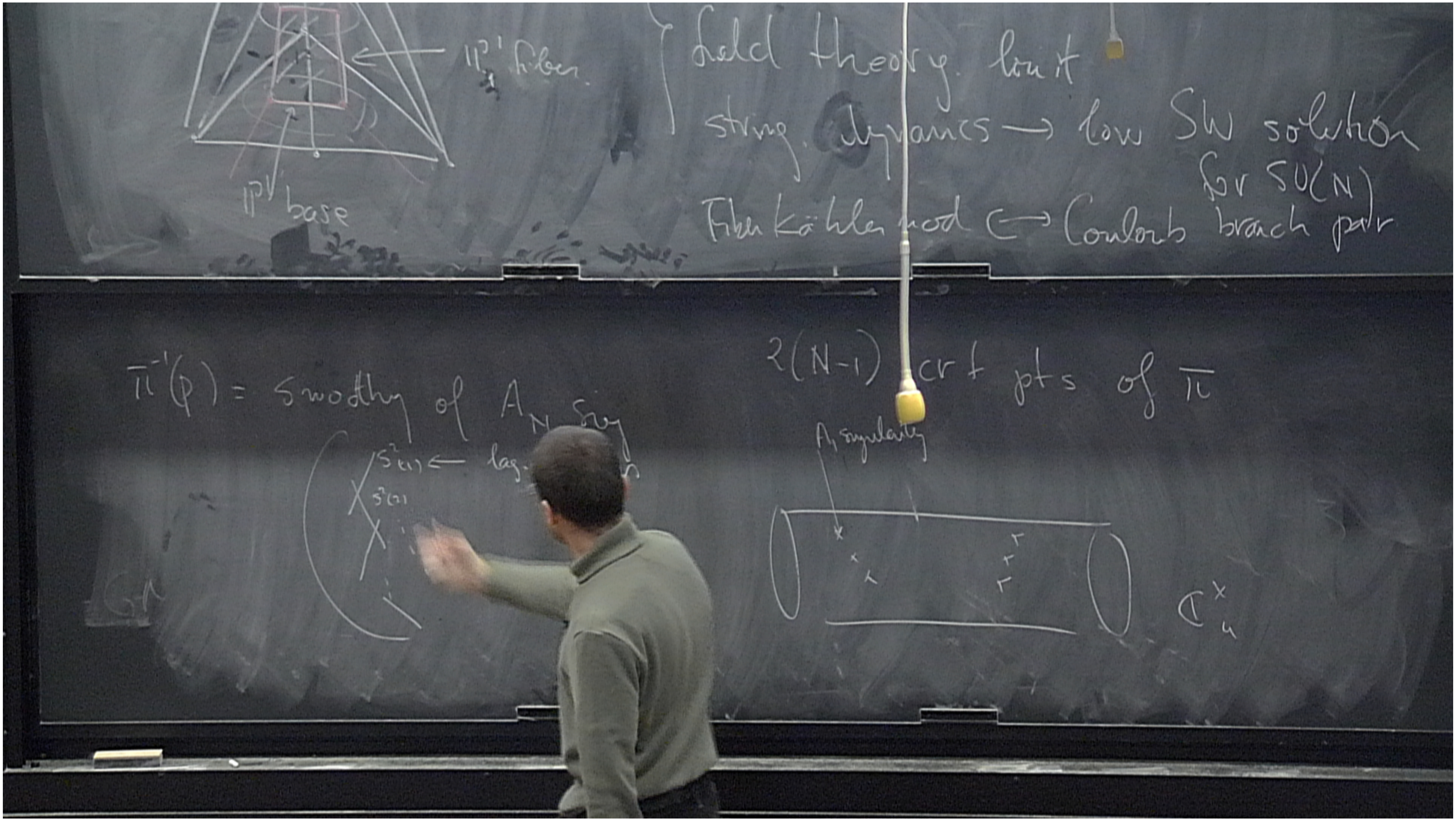
field theory limit

string dynamics $\longrightarrow$ low SW solution for $SU(N)$

Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par

$\pi^{-1}(p)$ = smoothing of A_N sing




field theory limit
string dynamics → low SW solution
for SU(N)
Fiber Kähler mod ⟷ Coulomb branch par
$\mathbb{P}^1$ fiber
$\mathbb{P}^1$ base
$\pi^{-1}(p)$ = smoothing of A_N sing
$2(N-1)$ crit pts of π
A_1 singularity
$\mathbb{C}^\times_u$

Fiber Kähler mod $\longleftrightarrow$ Coulomb

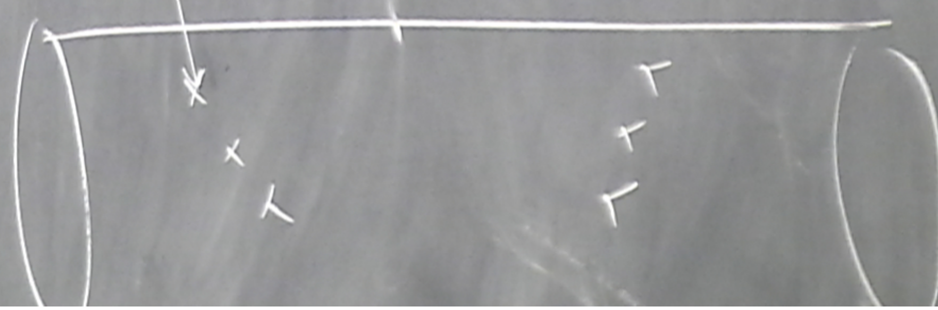
$2(N-1)$ crt pts of $\overline{\pi}$

isolated
A_1 singularity

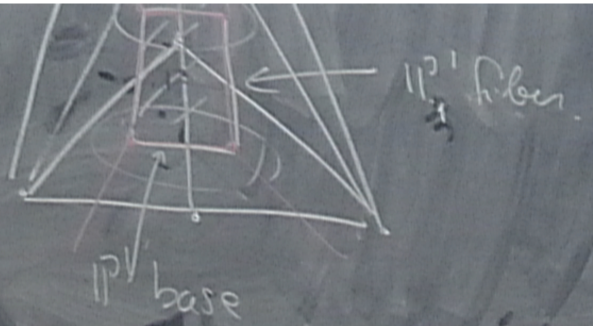
1 crt / cr

A_N Sing

lag. spheres



$\mathbb{C}^\times_u$

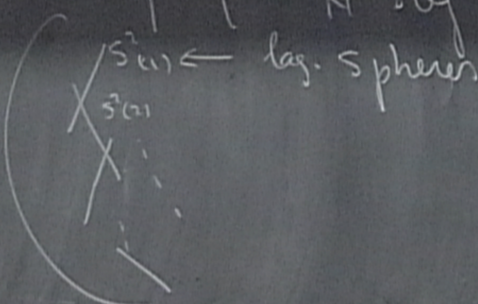


field theory limit

string dynamics $\longrightarrow$ low SW solution for $SU(N)$

Fiber Kähler mod $\longleftrightarrow$ Coulomb branch par

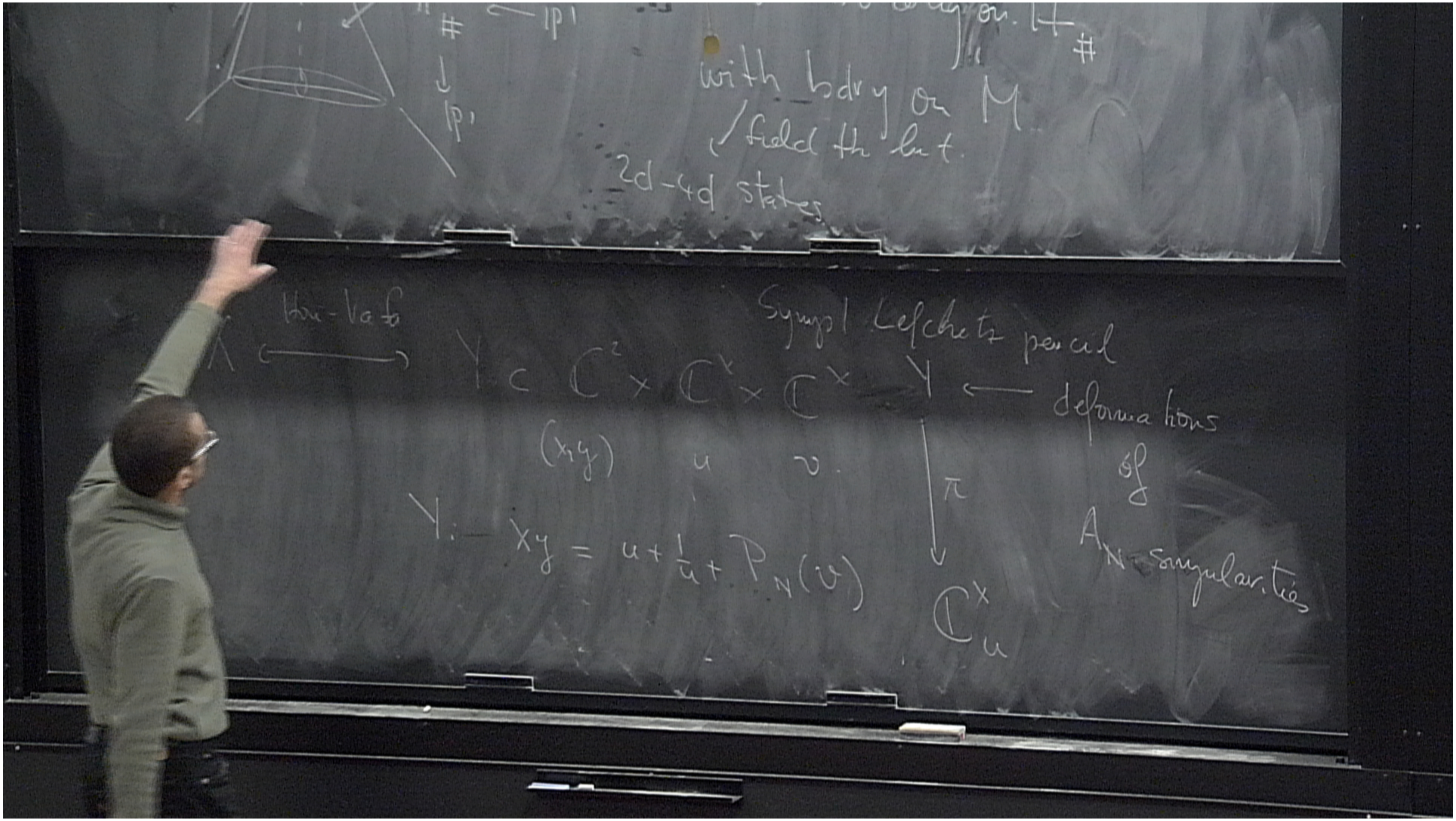
$\pi^{-1}(p)$ = smoothing of A_N sing

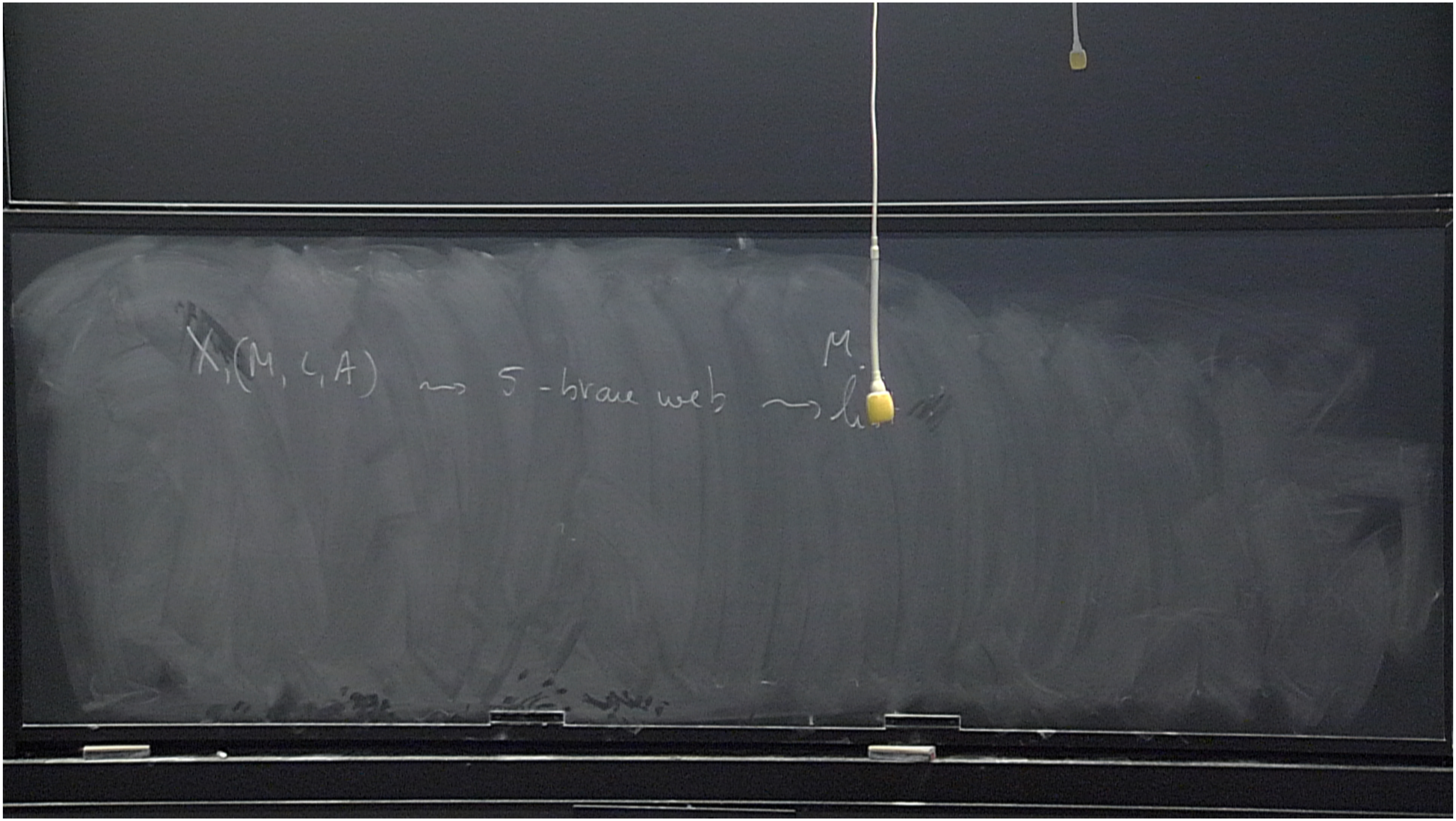


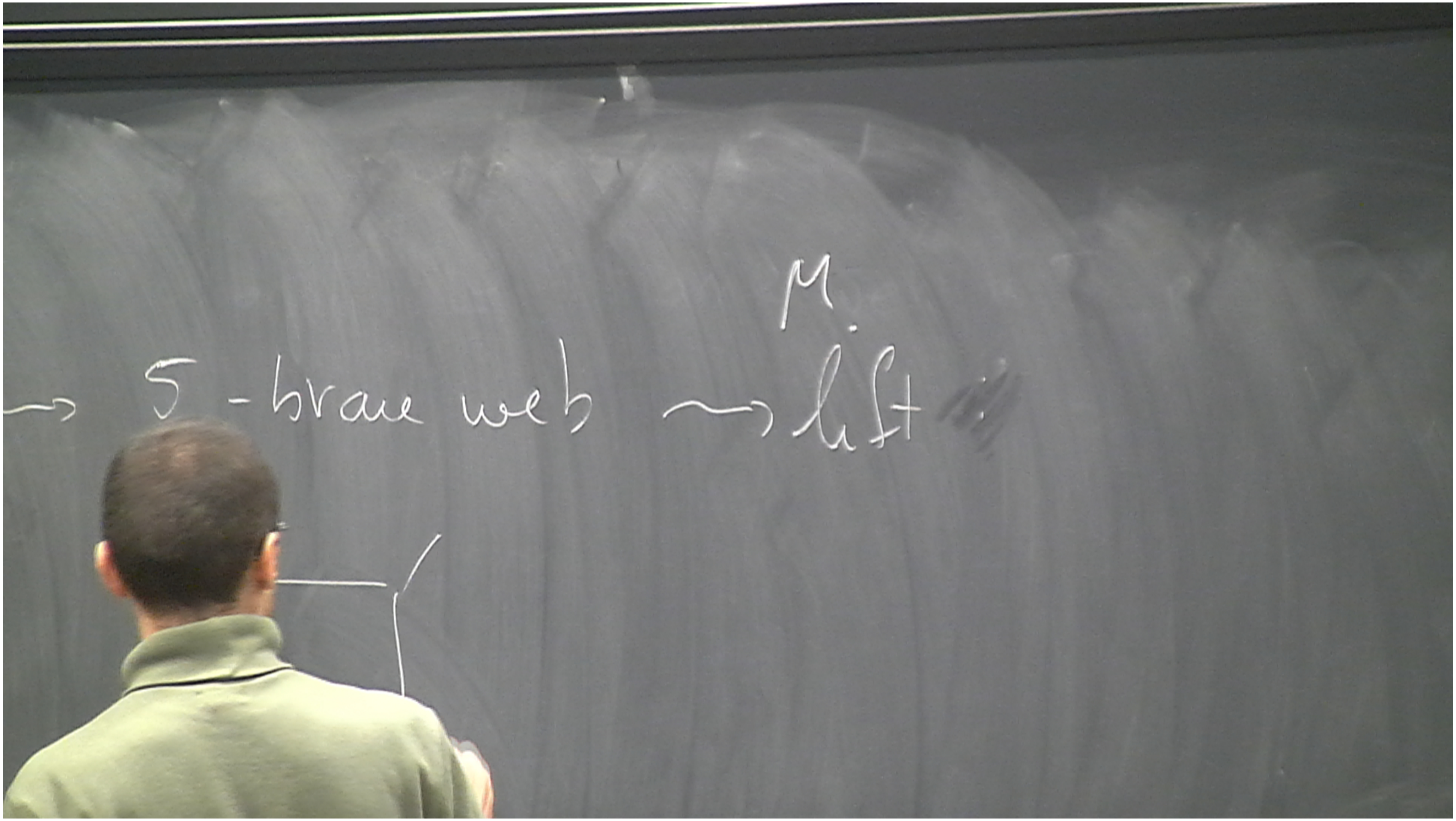
$2(N-1)$ crt pts of π

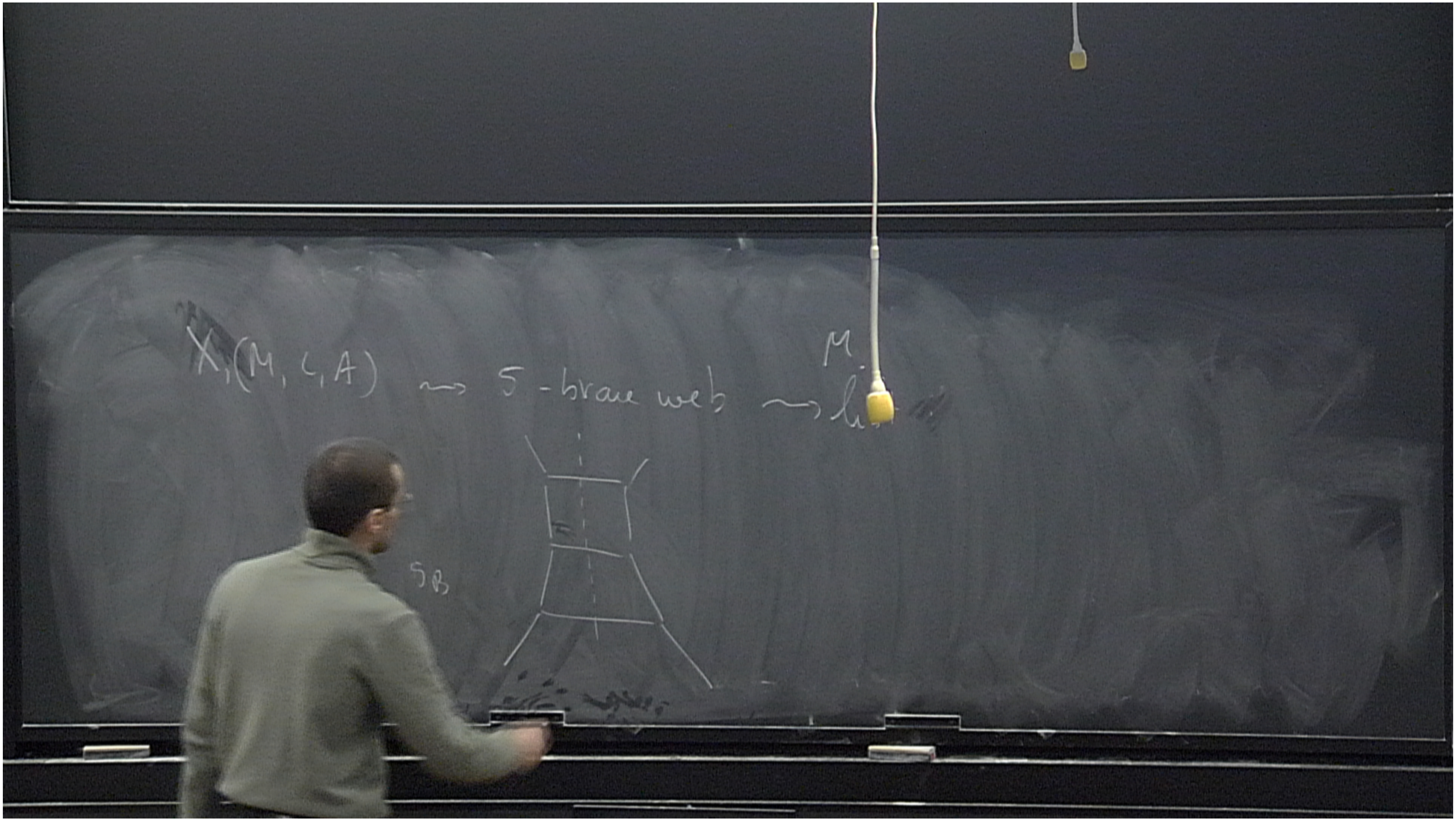
isolated A_1 singularity

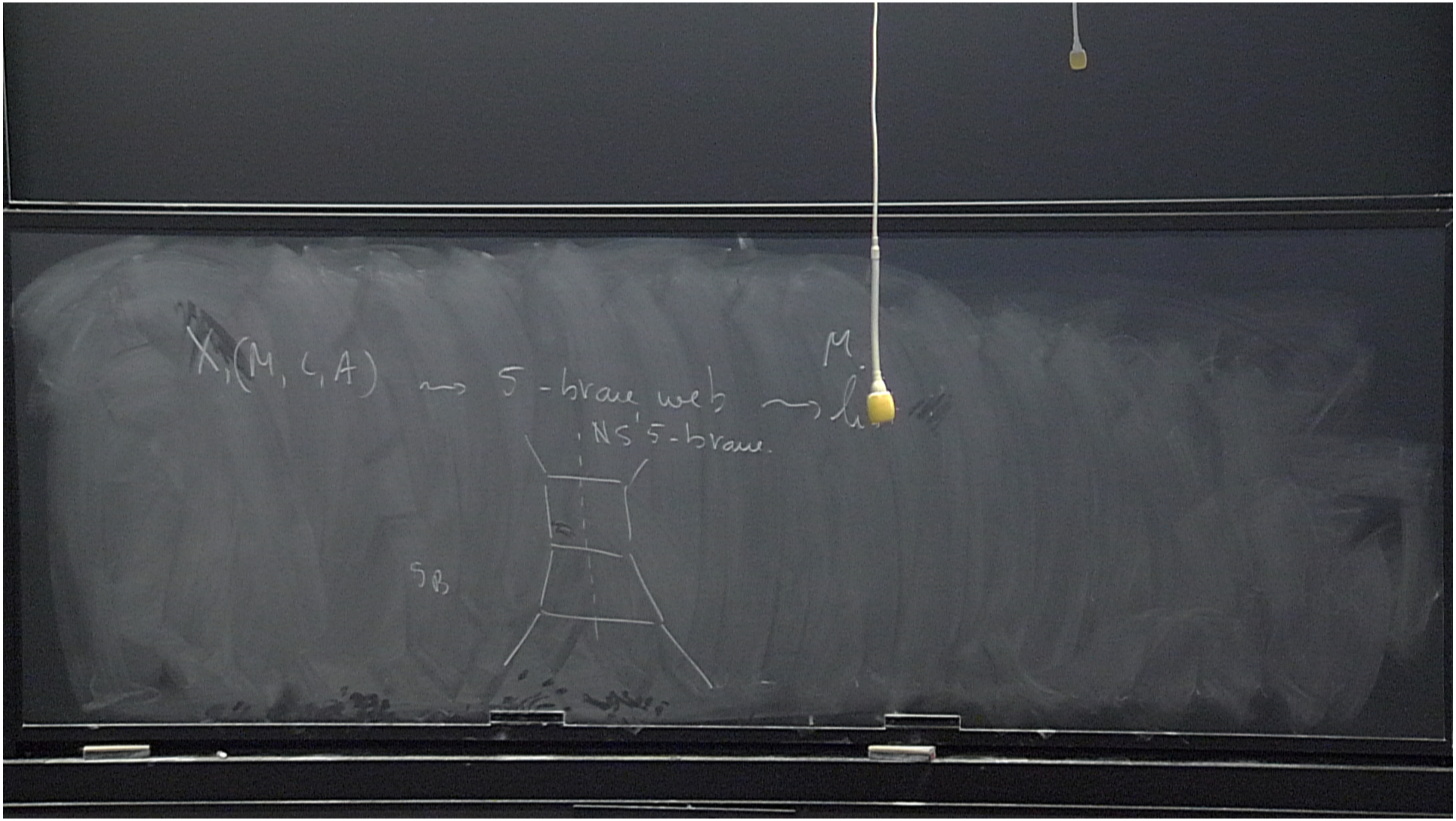
1 crt / crt value


with bdry on M
2d-4d states
Hori-Vafa
Sympl Lefchetz pencil
deformations
of
A_N singularities
Y: xy = u + 1/u + P_N(v)


$X,(M, L, A) \rightsquigarrow$ 5-brane web $\rightsquigarrow$
M


→ 5-brane web → M lift


5 - brane web
5B

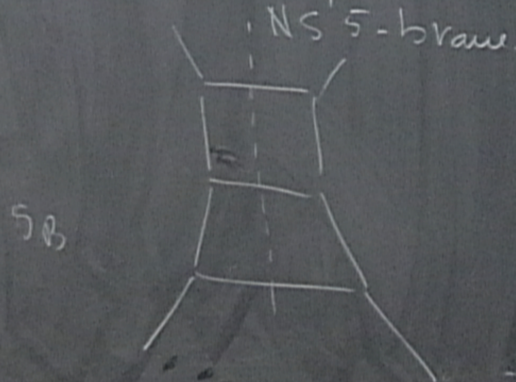

X, (M, C, A) ⇝ 5-brane web ⇝
M
NS' 5-brane
5B

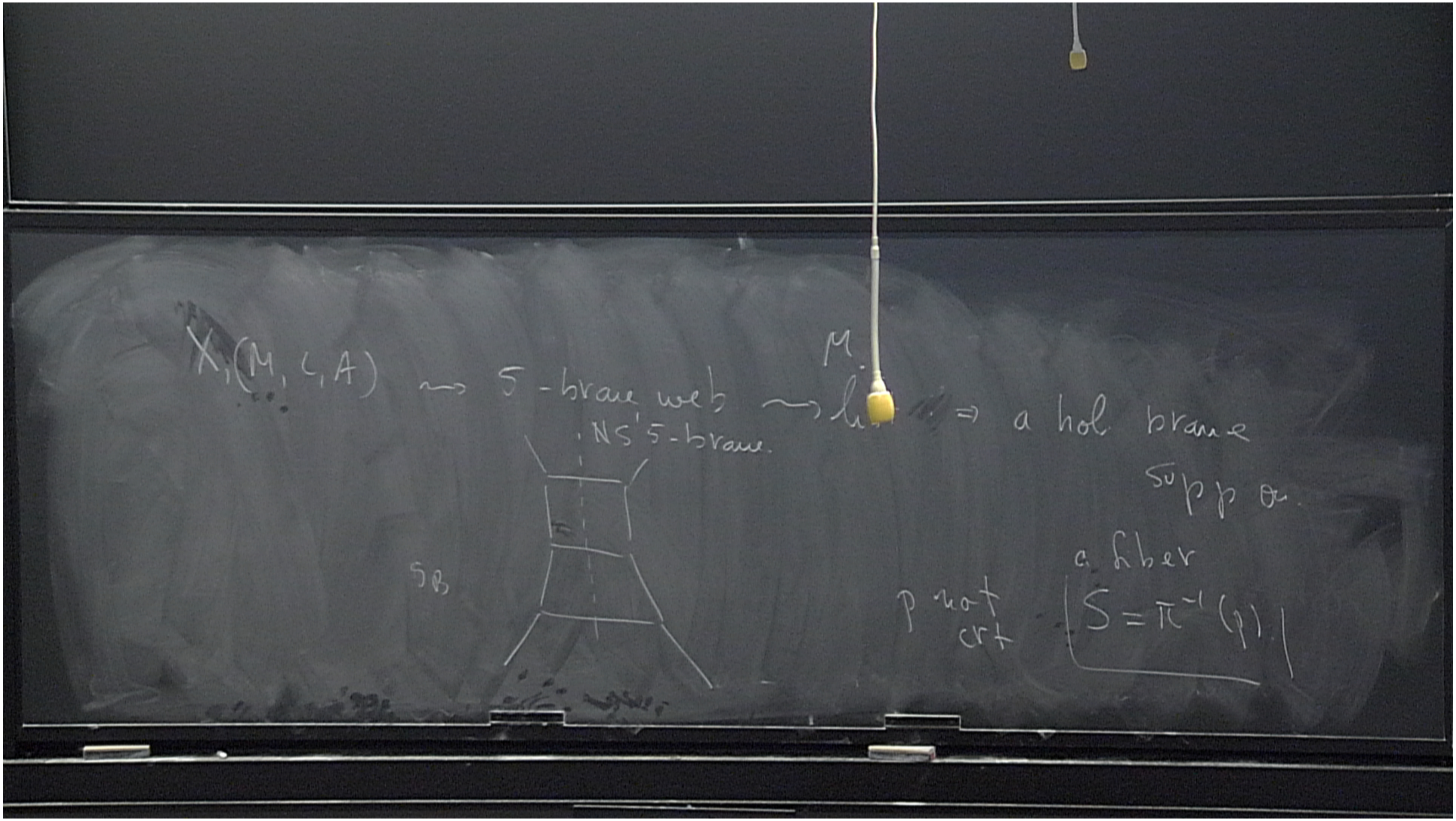
$X, (M, \zeta, A) \rightsquigarrow$ 5-brane web $\rightsquigarrow \overset{M.}{li}$ $\Rightarrow$ a hol. brane

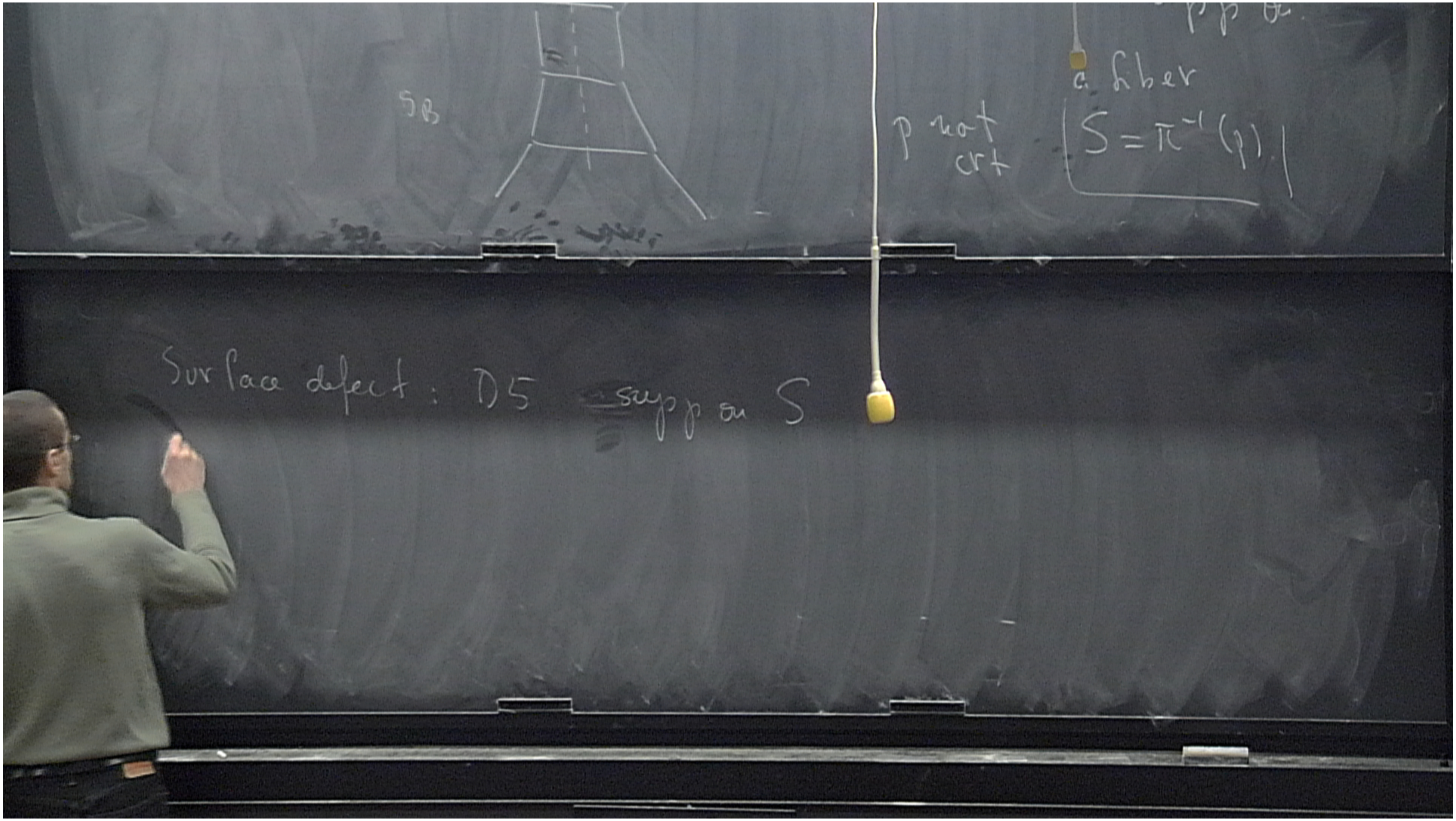
supp on

a fiber

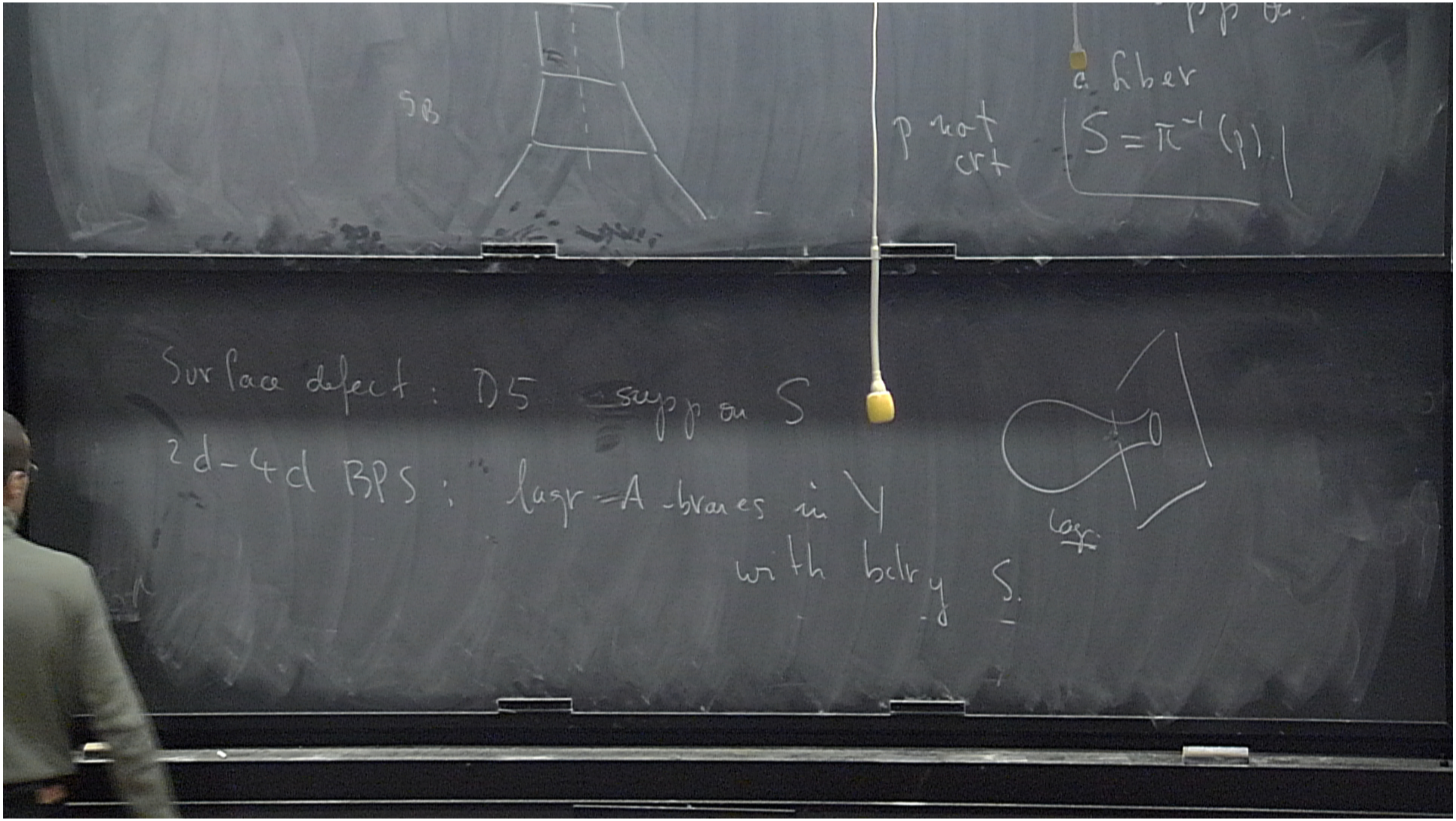
$S = \pi^{-1}(q)$.

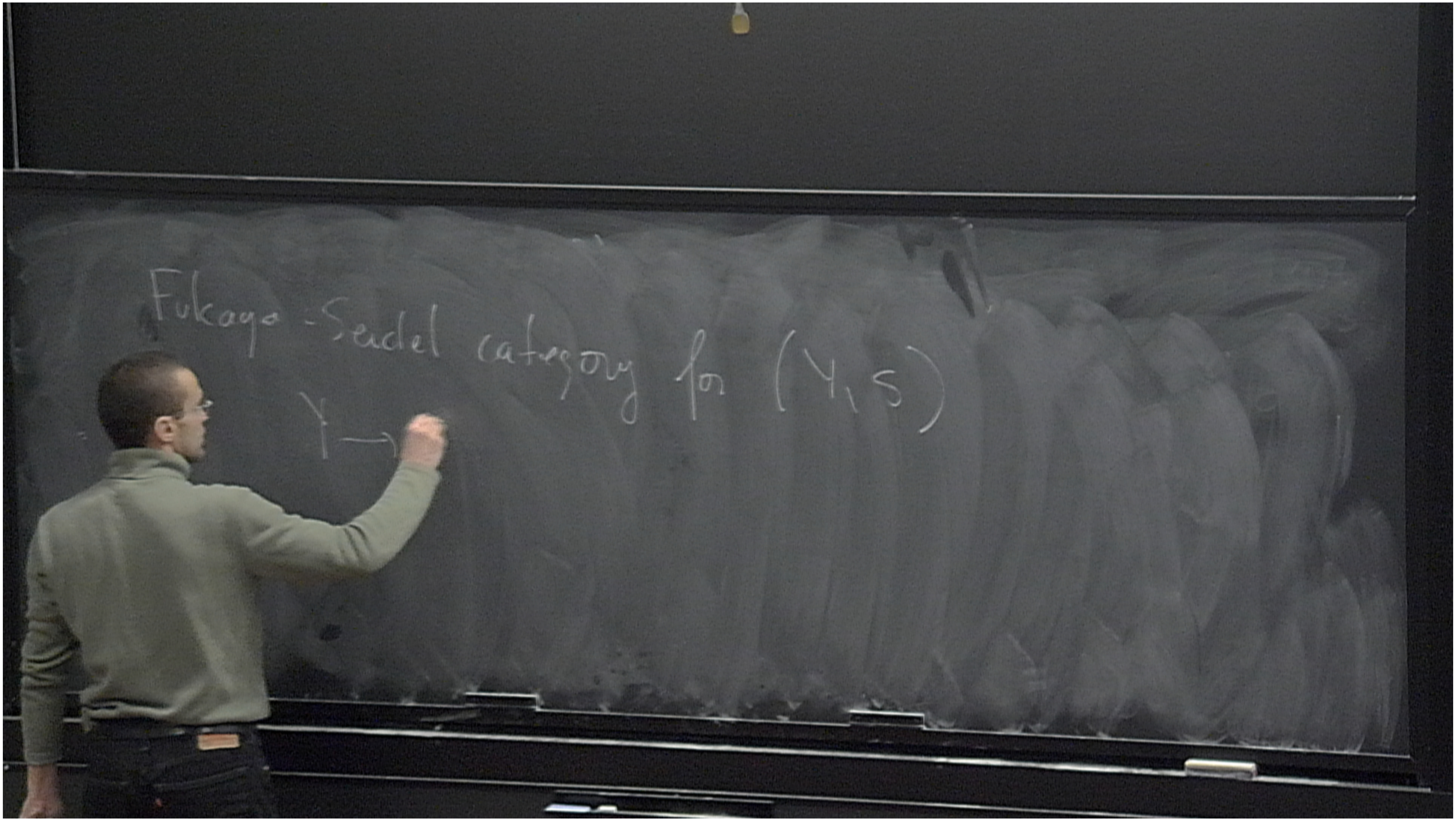



X, (M, L, A) ⇝ 5-brane web ⇝
⇒ a hol. brane
NS'5-brane
supp on
a fiber
p not
crt
S = π⁻¹(p)



SB
fiber
p not crt
S = π⁻¹(p)
Surface defect : D5 supp on S


SB
p not crt
c fiber
S = π^{-1}(p)
Surface defect: D5 supp on S
2d-4d BPS: lagr A-branes in Y
with bdry S.
lagr


Fukaya-Seidel category for (Y, S)
Y ⟶

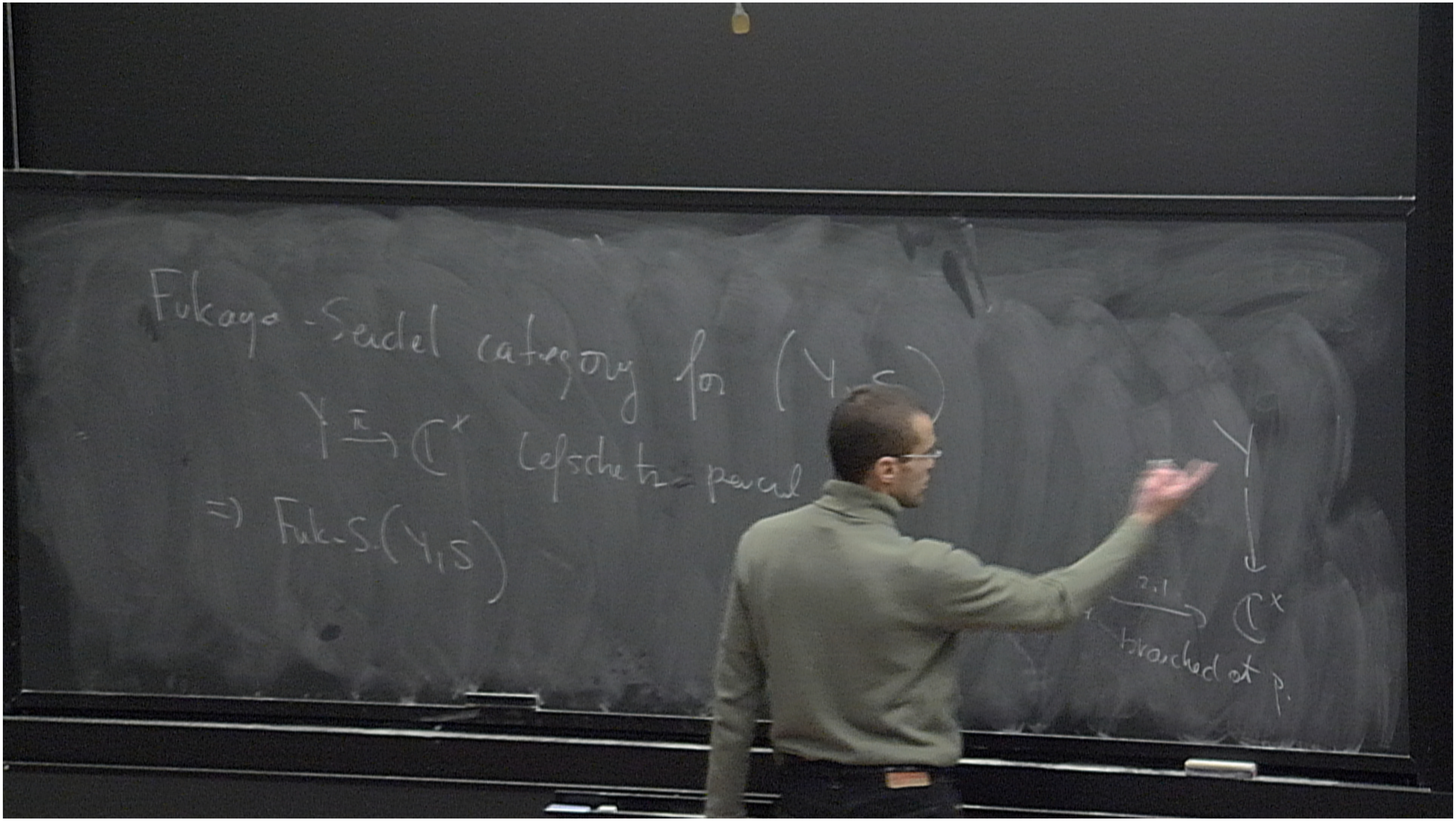
Fukaya-Seidel category for (Y, S)

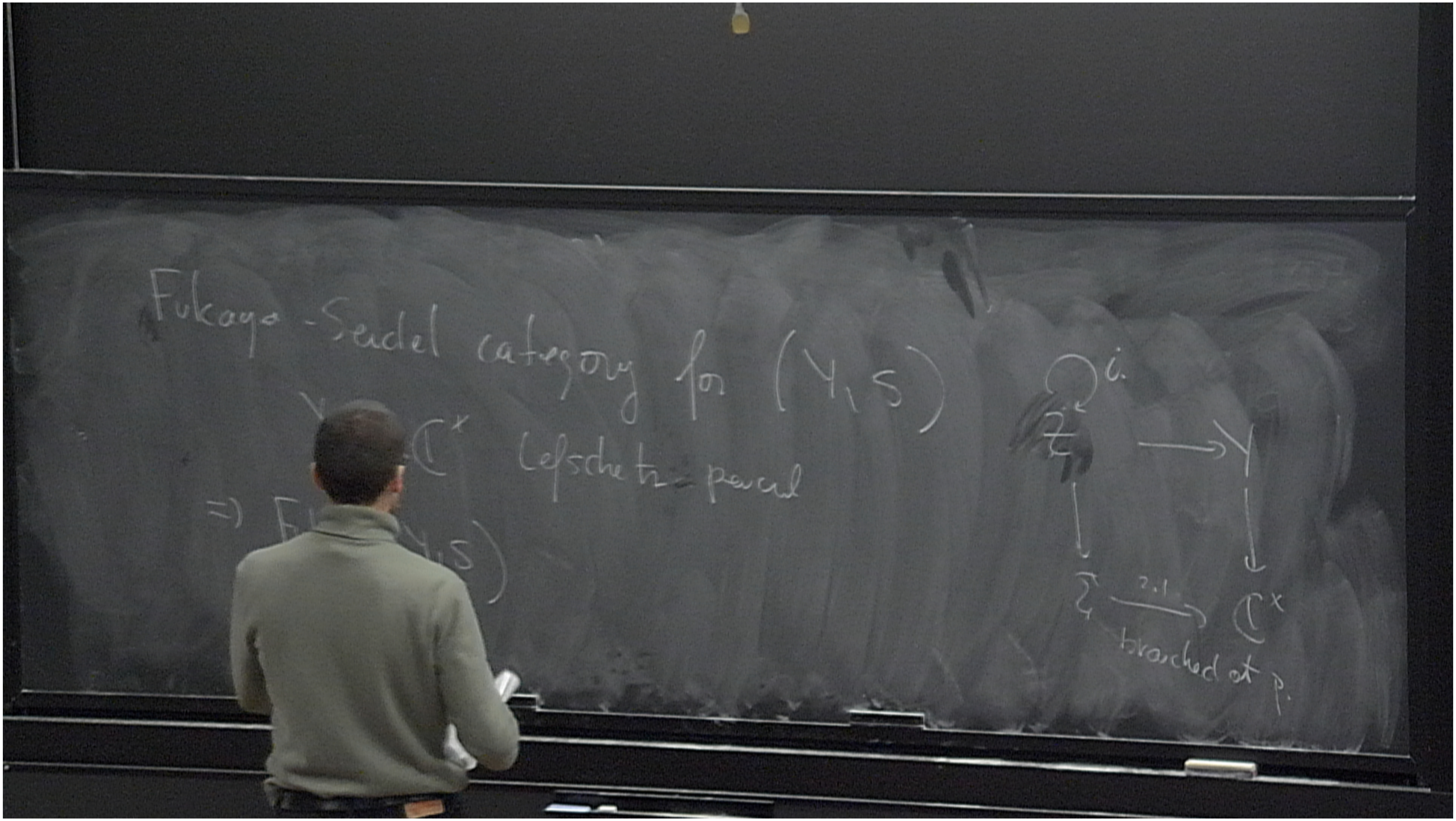
$Y \xrightarrow{\pi} \mathbb{C}^*$ Lefschetz pencil

Fukaya-Seidel category for (Y, S)

$Y \xrightarrow{\pi} \mathbb{C}^*$ Lefschetz pencil

$\Rightarrow$ Fuk-S. (Y, S)


Fukaya-Seidel category for (Y,S)
Y →π ℂ^× Lefschetz pencil
⇒ Fuk-S.(Y,S)
2:1
ℂ^×
branched at p.


Fukaya-Seidel category for (Y, S)
Lefschetz pencil
branched at p.
2:1

Fukaya-Seidel category for (Y, S)

$Y \xrightarrow{\pi} \mathbb{C}^\times$ Lefschetz pencil

$\Rightarrow$ Fuk-S.$(Y, S) \cong \mathrm{Fuk}^i(Z)$ invariant Fukaya of Z

$$\begin{tikzcd} \tilde{Z} \arrow[loop above, "i"] \arrow[r, "2:1"] \arrow[d] & Y \arrow[d] \\ Z \arrow[r, "2:1"] & \mathbb{C}^\times \end{tikzcd}$$

branched along S.

branched at p.

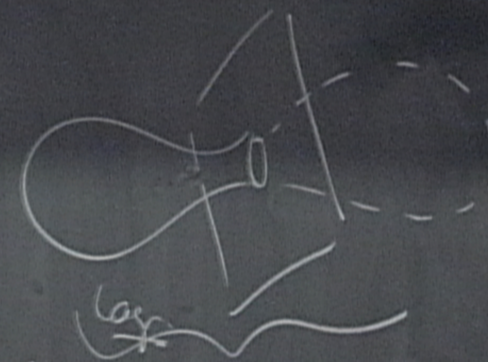
p not crt

c. fiber

$\boxed{S = \pi^{-1}(p)}$

Surface defect: D5 supp on S

2d-4d BPS: lagr. A-branes in Y with bdry S.



lag. on Σ with [illegible]

$\Rightarrow$ Fuk-S.$(Y, S) \cong$ Fuk(Σ) invariant Fukaya of Σ $\qquad \tilde{\Sigma} \xrightarrow{2:1} \mathbb{C}^\times$ branched at p.

Open problem

- compute some BPS deg for categ (GMN)
- derive the 2d-4d WCF from KS for the cat.
- COHA of 2d-4d.

$\Rightarrow \text{Fuk-S.}(Y, S) \cong \text{Fuk}(\tilde{Z})$ invariant Fukaya of $\tilde{Z}$

$\tilde{Z} \xrightarrow{2:1} \mathbb{C}^\times$ branched at p.

Open problem

Φ

- compute some BPS deg for categ (GMN)
- derive the 2d-4d WCF from KS for the cat.
- COHA of 2d-4d.

$\Phi + \frac{1}{\Phi} + \ldots$