Title: Heavy particle effective field theory: formalism, dark matter and the proton size puzzle

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Abstract: Heavy particle expansions, familiar from heavy quark physics, have found important applications in the analysis of dark matter candidates and their interactions with the Standard Model. From a different direction, precision spectroscopy of muonic hydrogen has challenged QED and required more precise knowledge of proton structure. These problems have forced a closer examination of the construction of general heavy particle lagrangians at high orders in the 1/M expansion, and in the absence of known ultraviolet completions. Key aspects of this formalism, including the emergence of Lorentz invariance from "nonrelativistic" lagrangians, are reviewed, and several applications are presented.   A status report on the proton radius puzzle is given.

Heavy particle effective field theory: formalism and new applications to dark matter and atoms

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18 October 2013

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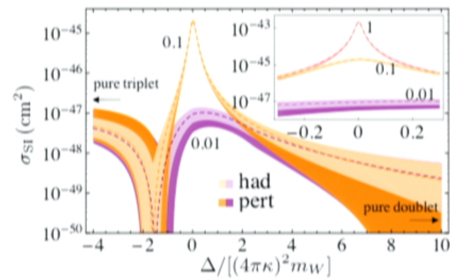
Outline

1) Lorentz invariance in heavy particle effective theories

Johannes Heinonen, RJH and Mikhail Solon, PRD 86, (2012) 094020

2) Universal behavior in the scattering of heavy, weakly interacting dark matter

RJH and Mikhail Solon, PLB 707 (2012) 539, 1309.4092



3) Proton structure in hydrogenic bound states

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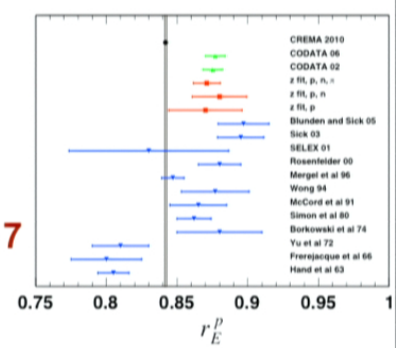

1) Lorentz invariance in heavy particle effective theories

"what is a chiral lagrangian ?" implementation of chiral symmetries on pions (nonlinear realization).

"what is a heavy particle lagrangian ?"
"what is SCET ?"

chiral symmetry	$\leftrightarrow$	Lorentz symmetry
chiral lagrangian	$\leftrightarrow$	heavy particle effective theory
nonlinear realization	$\leftrightarrow$	induced representation

- develop theory of induced representations for EFT
- find interesting disagreement with standard reparameterization analysis starting at $1/M^4$

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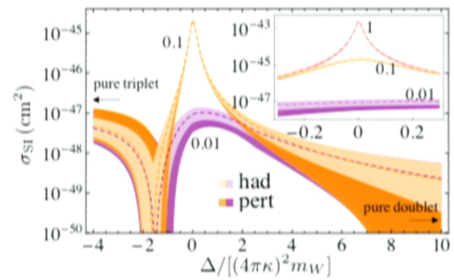

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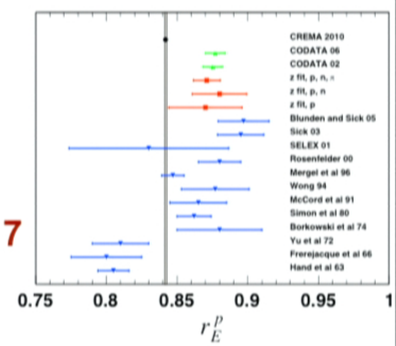

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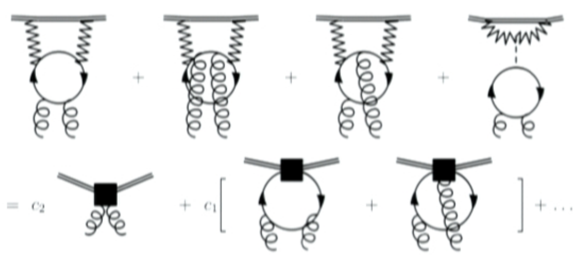
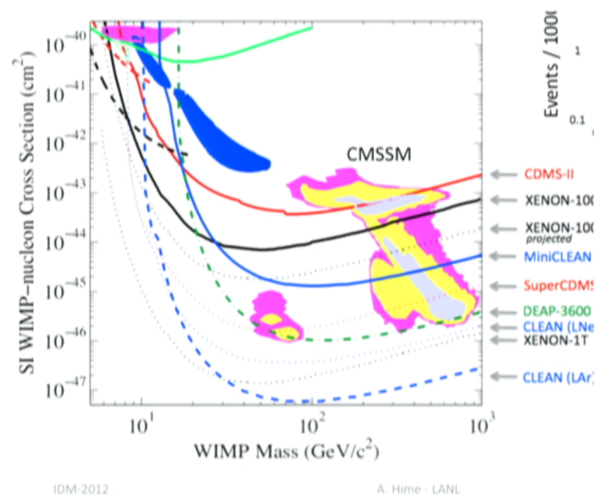
2) Standard Model anatomy of dark matter detection

- simple WIMP candidate: Lorentz scalar, electroweak triplet

- M ~ TeV from thermal relic abundance. $M \gg m_W$: model-independent analysis, predictive scattering cross section. But estimates range over several orders of magnitude (!)

- large gluon matrix element:
2 loop required for leading analysis





⇒ prototype for QCD effects in general models

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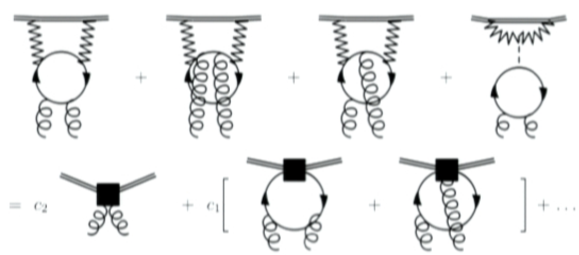

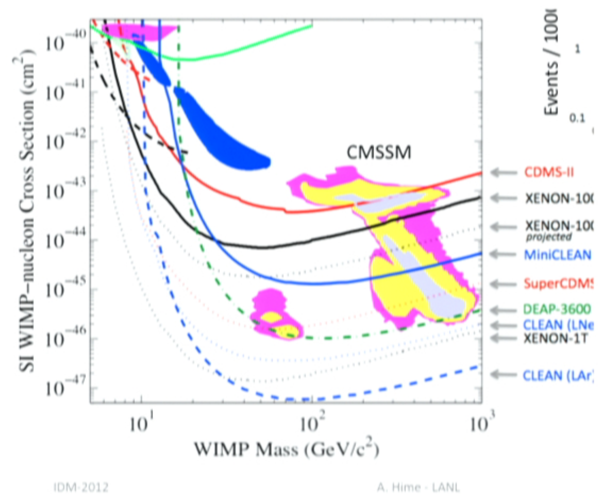

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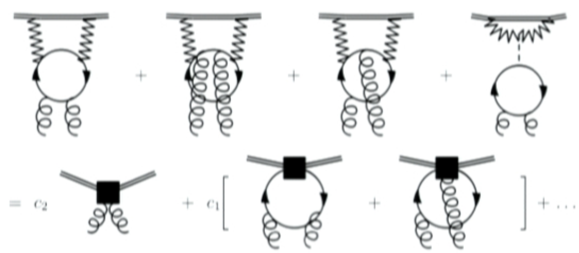

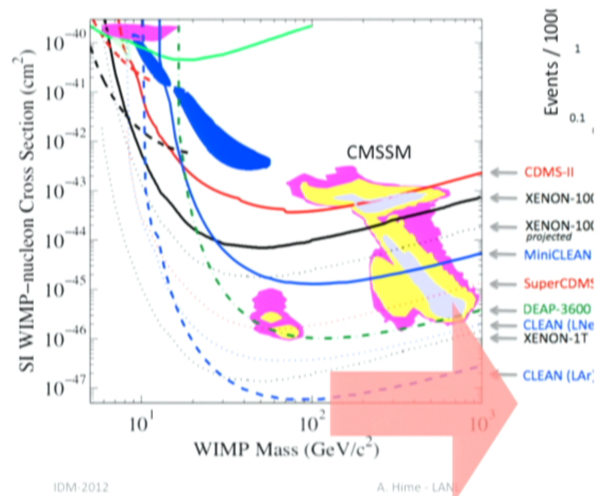

⇒ prototype for QCD effects in general models

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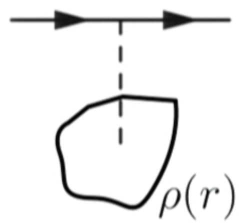
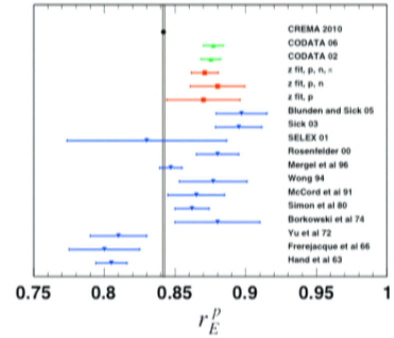
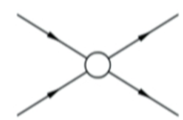




Modern analysis of proton structure in NRQED

- identify conflicting definitions of charge radius, (small) double counting in muonic hydrogen analysis

- isolate undetermined contact interaction



EFT and the little group

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recall the "usual" construction of Lorentz invariant field theory

$$\Psi(x)_a \to M(\Lambda)_{ab}\Psi(\Lambda^{-1}x)_b$$

where M is a finite-dimensional (non-unitary or trivial) representation of Lorentz

Straightforward to build general Lorentz invariant lagrangian,

$$\mathcal{L} \sim \bar{\Psi} i\gamma^\mu \partial_\mu \Psi + \ldots$$

Can prove pedantically that the S matrix is Lorentz invariant:

Translation invariance:

$$\Psi(x) \to \Psi(x+a)$$

⇒ conserved current

$$T^{\mu\nu} = g^{\mu\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\nu \Psi_a)}\partial^\mu \Psi_a$$

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Lorentz invariance of the lagrangian:

$$\Psi_a(x) \to \left(\delta_{ab} + \frac{i}{2}\omega^{\mu\nu}[\mathcal{J}_{\mu\nu}]_{ab}\right)\Psi_b(\Lambda^{-1}x)$$

⇒ conserved *symmetric* current (Belinfante)

$$\Theta^{\mu\nu} = T^{\mu\nu} - \frac{i}{2}\partial_\rho \frac{\partial \mathcal{L}}{\partial(\partial_\sigma \Psi_a)}[\mathcal{J}^{\alpha\beta}]_{ab}\Psi_b \left[\delta^\rho_\sigma \delta^\mu_\alpha \delta^\nu_\beta - \delta^\rho_\alpha \delta^\mu_\sigma \delta^\nu_\beta - \delta^\rho_\alpha \delta^\mu_\beta \delta^\nu_\sigma\right]$$

Define:

$$\begin{aligned} H &= \int d^3x\, \Theta^{00} \\ P^i &= \int d^3x\, \Theta^{0i} \\ J^{ij} &= \int d^3x\, (x^i \Theta^{0j} - x^j \Theta^{0i}) \\ K^i &= \int d^3x\, (\dot{x}^i \Theta^{00} - x^0 \Theta^{0k}) \end{aligned}$$

These charges obey the commutation relations of the Poincare algebra, as can be shown e.g. by canonical quantization.:

$$
\begin{aligned}
[P^i, P^j] &= 0 \\
[P^i, H] &= 0 \\
[J^i, H] &= 0 \\
[J^i, J^j] &= i\epsilon^{ijk} J^k \\
[J^i, K^j] &= i\epsilon^{ijk} K^k \\
[K^i, K^j] &= -i\epsilon^{ijk} J^k \\
[J^i, P^j] &= i\epsilon^{ijk} P^k \\
[K^i, P^j] &= iH\delta^{ij} \\
[K^i, H] &= iP^i
\end{aligned}
$$

Finally, consider the scattering operator,

$$S = \Omega(\infty)^\dagger \Omega(-\infty)$$

$$\Omega(t) = e^{iHt} e^{-iH_0 t}$$

If we can show that for free particle operators,

$$[H_0, S] = [\boldsymbol{P}_0, S] = [\boldsymbol{J}_0, S] = [\boldsymbol{K}_0, S] = 0$$

then we are done.

Assume (reasonably) $\boldsymbol{P} = \boldsymbol{P}_0, \quad \boldsymbol{J} = \boldsymbol{J}_0$

and that we have shown $[\boldsymbol{K}, H] = i\boldsymbol{P}$

Then

$$[K_0, e^{iH_0 t}] = -tP_0 e^{iH_0 t}$$

$$[K, e^{iHt}] = -tPe^{iHt} = -tP_0 e^{iHt}$$

$$[\boldsymbol{K}_0, \Omega(t)^\dagger \Omega(t_0)] = -\left[e^{iH_0 t}(\boldsymbol{K} - \boldsymbol{K}_0)e^{-iH_0 t}\right]\Omega(t)^\dagger \Omega(t_0) + \Omega(t)^\dagger \Omega(t_0)\left[e^{iH_0 t_0}(\boldsymbol{K} - \boldsymbol{K}_0)e^{-iH_0 t_0}\right]$$

[e.g. Weinberg, QFT vol 1]

$\to 0$ as $t \to \infty$, $t_0 \to -\infty$, by smoothness condition (~ existence of S matrix)

So $[\boldsymbol{K}_0, S] = 0$ QED.

Is this all we need ?

$$\Psi(x)_a \to M(\Lambda)_{ab}\Psi(\Lambda^{-1}x)_b \qquad (3.8)$$

"... there is no advantage in considering transformation laws more general than (3.8)"

[from a very good textbook]

But many particles have *only* a heavy-particle effective description, e.g. proton, composite dark matter, ...

And many other particles are efficiently described by effective theories, e.g. heavy quark, atomic electron, neutralino DM, ...

How does relativistic invariance work for these, ?

The alternative: induced representations

Three classes of infinite-dimensional, unitary representations of Lorentz, based on the "little group" for invariant vectors: (Wigner 1939, ...)

$$k^\mu = M v^\mu \,, \quad k^2 > 0 \qquad G = \{\Lambda \,|\, \Lambda v = v\} \cong SO(3)$$

(HQET/NRQCD)

$$k^\mu = E n^\mu \,, \quad k^2 = 0 \qquad G = \{\Lambda \,|\, \Lambda n = n\} \cong E(2)$$

(SCET)

$$k^\mu = Q s^\mu \qquad k^2 < 0 \qquad G = \{\Lambda \,|\, \Lambda k = k\} \cong SO(2,1)$$

(tachyon effective field theory?)

These representations are known to implement Lorentz on physical states - we want to apply them to *fields*

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nuts and bolts for induced reps:

For arbitrary Λ, define a matrix function of vectors p^μ, $p^2=M^2$ such that

$$L(p)^\mu{}_\nu v^\nu = \frac{1}{M} p^\mu$$

Then the matrix

$$W(\Lambda, p) = L^{-1}(\Lambda p)\Lambda L(p)$$

belongs to the little group ($p \rightarrow p$), and any representation of the little group induces a representation of the full Lorentz group

$$\phi(p)_i \to D[W(\Lambda, p)]_{ji}\phi(\Lambda p)_j$$

where $D: \Lambda \rightarrow D(\Lambda)$ is a representation of the little group

continuous variable p ⇒ infinite dimensional

2) Lorentz invariance for interacting theory

$$\Psi_a(x) \to D[W(\Lambda, iD)]_{ab}\Psi_b(\Lambda^{-1}x)$$

[Choose a definite ordering for covariant derivates.]

Claim: if we make the lagrangian invariant under this modified transformation law, then the resulting S matrix will be Lorentz invariant

Proof: First notice that

$$0 = \frac{d}{dt}K^i = \frac{\partial}{\partial t}K^i + i[H, K^i] = P^i + i[H, K^i]$$

$$\Rightarrow \qquad [K^i, H] = iP^i$$

Since we have already demonstrated Lorentz invariance for the free theory (g=0), Lorentz invariance of the S matrix follows as before.

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Return to enforcing invariance under specified generators

For a general v^μ, rotations and boost look complicated. Instead, consider

a) 3-parameter group of Lorentz transformations Λ that keep v^μ fixed

Can choose L(p) such that $W(\Lambda, p) = \Lambda$

All other fields in the lagrangian transform "as usual" and since we also have

$$v^\mu \to \Lambda^\mu{}_\nu v^\nu$$

we find invariance by treating all fields in the usual covariant way, e.g.

$$\bar{\Psi}\left[iv\cdot D + c\frac{D^2}{2M} + \dots\right]\Psi$$

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b) 3-parameter set of Lorentz transformations Λ that shift v^μ

$$\Lambda^\mu{}_\nu v^\nu \equiv v^\mu + q^\mu/M \qquad \Psi_a(x) \to D[W(\Lambda, iD)]_{ab}\Psi_b(\Lambda^{-1}x)$$

These transformations take the place of "reparameterization invariance constraints" [Luke, Manohar 1992]

- note that we are not "reparameterizing" anything, i.e., we are not shifting v^μ
- immediate extension to arbitrary spin
- inconsistencies arise at $1/M^4$ order in Luke-Manohar ansatz

$$\Psi_v = \Gamma(v, iD)\psi_v \qquad \Psi_v \to e^{iq\cdot x}\Psi_v$$

$$\Gamma(v + q/M, iD - q)\Lambda^{-1}W(\Lambda, iD + Mv) = \Gamma(v, iD)$$

interplay of Lorentz and gauge symmetry: solution to "invariance equation" is nontrivial starting at $1/M^4$

In detail, must solve the invariance equation,

$$\Gamma(v + q/M, iD - q)\mathcal{B}^{-1}\tilde{W}(\mathcal{B}, iD + Mv) = \Gamma(v, iD)$$

which can be done order by order,

$$X \equiv \mathcal{B}^{-1}W = 1 + q^\mu X_\mu = 1 + q^\mu \left[\frac{1}{M}X_\mu^{(1)} + \frac{1}{M^2}X_\mu^{(2)} + \dots\right],$$

$$\Gamma = 1 + \frac{1}{M}\Gamma^{(1)} + \frac{1}{M^2}\Gamma^{(2)} + \dots .$$

if we can solve:

$$\delta\Gamma = \Gamma(v + q/M, iD - q) - \Gamma(v, iD) = q^\mu\left(-\frac{\partial}{\partial iD^\mu}\Gamma + \frac{1}{M}\frac{\partial}{\partial v^\mu}\Gamma\right)$$

$$\frac{\partial}{\partial iD^\mu}\Gamma^{(n)} = \frac{\partial}{\partial v^\mu}\Gamma^{(n-1)} + \Gamma^{(n-1)}X_\mu^{(1)} + \Gamma^{(n-2)}X_\mu^{(2)} + \dots + \Gamma^{(0)}X_\mu^{(n)} \equiv Y_\mu^{(n)}$$

(like $\nabla\varphi = E$ for noncommuting coordinates)

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solution:

$$\Gamma^{(n)} = \sum_{m=1}^{n} \frac{(-1)^{m-1}}{m!} iD_\perp^{\mu_1} iD_\perp^{\mu_2} \dots iD_\perp^{\mu_m} \frac{\partial}{\partial iD^{\mu_1}} \frac{\partial}{\partial iD^{\mu_2}} \cdots \frac{\partial}{\partial iD^{\mu_{m-1}}} Y_{\mu_m}^{(n)}$$

$$= iD_\perp^{\mu} Y_\mu^{(n)} - \frac{1}{2!} iD_\perp^{\mu} iD_\perp^{\nu} \frac{\partial}{\partial iD^{\mu}} Y_\nu^{(n)} + \dots ,$$

provided,

$$\frac{\partial}{\partial iD^{[\nu}} Y_{\mu]}^{(n)} = 0 \, ,$$

From the definition of Y, this is equivalent to:

$$\frac{\partial}{\partial iD^{[\nu}} X_{\mu]}^{(n)} = -\frac{\partial}{\partial v^{[\mu}} X_{\nu]}^{(n-1)} + X_{[\mu}^{(n-1)} X_{\nu]}^{(1)} + X_{[\mu}^{(n-2)} X_{\nu]}^{(2)} + \dots + X_{[\mu}^{(1)} X_{\nu]}^{(n-1)} \equiv Z_{\mu\nu}^{(n)}$$

and a solution requires (like ∇.B=0 for ∇xA=B, with noncommuting coordinates)

$$0 = v_\sigma \epsilon^{\mu\nu\rho\sigma} \frac{\partial}{\partial iD^{\rho}} Z_{\mu\nu}^{(n)}$$

In detail, must solve the invariance equation,

$$\Gamma(v + q/M, iD - q)\mathcal{B}^{-1}\tilde{W}(\mathcal{B}, iD + Mv) = \Gamma(v, iD)$$

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Can show by induction that a solution exists

In particular, can find an X that reduces to a given free solution,

$$X_\mu^{(n)} = \hat{X}_\mu^{(n)} + 2\sum_{m=1}^{n-1} \frac{(-1)^m}{(m+1)!} iD_\perp^{\nu_1} \cdots iD_\perp^{\nu_m} \frac{\partial}{\partial iD^{\nu_1}} \cdots \frac{\partial}{\partial iD^{\nu_{m-1}}} \left(Z_{\nu_m \mu}^{(n)} - \hat{Z}_{\nu_m \mu}^{(n)} \right)$$

$$\hat{Z}_{\mu\nu}^{(n)} \equiv \frac{\partial}{\partial iD^{[\nu}} \hat{X}_{\mu]}^{(n)}$$

naive covariantization (∂→D with arbitrary ordering)

Finally, turn the crank to obtain solution to invariance equation

$$\Gamma^{(1)} = \frac{1}{2} i\not{D}_\perp$$

$$\Gamma^{(2)} = -\frac{1}{8}(iD_\perp)^2 - \frac{1}{2} i\not{D}_\perp iv\cdot D$$

$$\Gamma^{(3)} = \frac{1}{4}(iD_\perp)^2 iv\cdot D + \frac{i\not{D}_\perp}{2}\left[-\frac{3}{8} i\not{D}_\perp (iD_\perp)^2 + (iv\cdot D)^2\right] \boxed{- \frac{g}{8} v^\alpha G_{\alpha\beta} D_\perp^\beta - \frac{g}{16}\sigma^\perp_{\alpha\beta} G^{\alpha\beta} i\not{D}_\perp}$$
$$+ \frac{g}{8}\left[i\gamma_\perp^\beta \sigma_\perp^{\mu\alpha}[D_\mu, G_{\beta\alpha}] - v^\alpha [D_\perp^\mu, G_{\alpha\mu}] - [D_\perp^\mu, G^\perp_{\mu\beta}]\gamma_\perp^\beta \right].$$

Extra terms missed by "covariantization" of free result,

$$\Gamma^{\rm naive}(v, iD) = 1 + \frac{i\not{D}_\perp}{2M} + \frac{1}{M^2}\left[-\frac{1}{8}(iD_\perp)^2 - \frac{1}{2} i\not{D}_\perp iv\cdot D\right]$$
$$+ \frac{1}{M^3}\left[\frac{1}{4}(iD_\perp)^2 iv\cdot D + \frac{i\not{D}_\perp}{2}\left(-\frac{3}{8}(iD_\perp)^2 + (iv\cdot D)^2\right)\right] + \mathcal{O}(1/M^4)$$

Not related by field redefinition

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$$+ \frac{1}{M^3}\left[\frac{1}{4}(iD_\perp)^2 iv\cdot D + \frac{i\not{D}_\perp}{2}\left(-\frac{3}{8}(iD_\perp)^2 + (iv\cdot D)^2\right)\right] + \mathcal{O}(1/M^4)$$

Not related by field redefinition

Explicit example: heavy fermion

Irreducible representation of little group on Dirac spinors with

$$\not{v}\,\psi_v = \psi_v$$

(arbitrary spin: convenient to embed in products of Dirac spin-1/2 and vector spin-1)

Extract rest mass via

$$\psi_v(x) = e^{-iMv\cdot x}\psi'_v(x)$$

a) Λ doesn't change v (rotation) Λv=v

$$\psi_v(x) \to \Lambda_{\frac{1}{2}}\psi_v(\Lambda^{-1}x)$$

b) Λ changes v (boost) Λv=v+q/M

$$\psi_v(x) \to e^{iq\cdot x}\left(1 + \frac{i}{2}\sigma^{\mu\nu}\omega(\Lambda, iD)_{\mu\nu}\right)\psi_v(\Lambda^{-1}x)$$

1/M^4 NRQED:

$$
\begin{aligned}
\mathcal{L} = \psi^\dagger \Big\{ & iD_t + c_2 \frac{\boldsymbol{D}^2}{2M} + c_4 \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + i c_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} \\
& + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma}\cdot\boldsymbol{B} D^i}{4M^3} + c_{p'p} g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D} + \boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}}{8M^3} \\
& + i c_M g \frac{\{D^i, [\boldsymbol{\partial}\times\boldsymbol{B}]^i\}}{8M^3} + c_{A1} g^2 \frac{\boldsymbol{B}^2 - \boldsymbol{E}^2}{8M^3} - c_{A2} g^2 \frac{\boldsymbol{E}^2}{16M^3} + c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{D}]}{M^4} \\
& + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{M^4} + c_{X3} g \frac{[\boldsymbol{\partial}^2\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + i c_{X4} g^2 \frac{\{D^i, [\boldsymbol{E}\times\boldsymbol{B}]^i\}}{M^4} \\
& + i c_{X6} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) D^i}{M^4} + i c_{X8} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{M^4} + c_{X9} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} \\
& + c_{X10} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{M^4} + c_{X11} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{M^4} + c_{X12} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X13} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} \\
& + c_{X14} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t\boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{M^4} \Big\} \psi \,. \qquad (
\end{aligned}
$$

Invariance under:

$$
\psi \to e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left\{ 1 + \frac{i\boldsymbol{q}\cdot\boldsymbol{D}}{2M^2} + \frac{i\boldsymbol{q}\cdot\boldsymbol{D}\boldsymbol{D}^2}{4M^4} - \frac{\boldsymbol{\sigma}\times\boldsymbol{q}\cdot\boldsymbol{D}}{4M^2}\left[1 + \frac{\boldsymbol{D}^2}{4M^2}\right] + \dots \right\} \psi \,,
$$

$$
\boldsymbol{B} \to \boldsymbol{B} - \frac{1}{M}\boldsymbol{q}\times\boldsymbol{E} \,, \quad \boldsymbol{E} \to \boldsymbol{E} + \frac{1}{M}\boldsymbol{q}\times\boldsymbol{B} \,, \quad \boldsymbol{D} \to \boldsymbol{D} + \frac{1}{M}\boldsymbol{q} D_t \,, \quad D_t \to D_t + \frac{1}{M}\boldsymbol{q}\cdot\boldsymbol{D}
$$

With formalism in place, simple matter to enforce Lorentz invariance:

$$\delta\mathcal{L}_4 = \psi^\dagger \left[\frac{ig}{8}[\boldsymbol{D}^2, \boldsymbol{q}\cdot\boldsymbol{E}] \left(\frac{5Q}{4} - c_F + c_D - 32c_{X1} \right) + \frac{ig}{8}\{\boldsymbol{q}\cdot\boldsymbol{D}, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\} \left(-\frac{Q}{4} + c_F - 16c_{X2} \right) \right.$$
$$+ \frac{g^2}{8}\boldsymbol{q}\cdot\boldsymbol{E}\times\boldsymbol{B} \left(\frac{Q^2}{2} + 2c_F(Q - c_F) - 2Qc_D + c_{A2} + 16c_{X4} \right)$$
$$+ \frac{g}{8}[\boldsymbol{q}\cdot\boldsymbol{\sigma}\times\boldsymbol{\partial}\boldsymbol{\partial}\cdot\boldsymbol{E}] \left(-Q + c_F - \frac{1}{4}c_D - c_{W1} + 8c_{X8} \right)$$
$$\left. + \frac{g}{8}D^i \left(q^i(\boldsymbol{E}\times\boldsymbol{\sigma})^j + (\boldsymbol{E}\times\boldsymbol{\sigma})^i q^j + \boldsymbol{\sigma}\times\boldsymbol{q}\cdot\boldsymbol{E}\delta^{ij} \right) D^j \left(\frac{Q}{2} - 2c_F + 16c_{X6} \right) \right] \psi \,,$$

New non-renormalization theorems:

$$32c_{X1} = \frac{5Q}{4} - c_F + c_D \,,$$
$$32c_{X2} = -\frac{Q}{2} + 2c_F \,,$$
$$32c_{X4} = -Q^2 - 4c_F(Q - c_F) + 4Qc_D - 2c_{A2} \,,$$
$$32c_{X6} = -Q + 4c_F \,,$$
$$32c_{X8} = 4(Q - c_F) + c_D - 4c_{W1} \,,$$

A subset of these required in muonic hydrogen analysis

1/M⁴ NRQED:

$$
\begin{aligned}
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$$

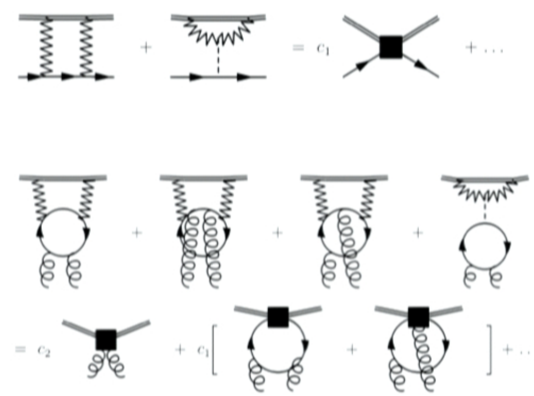
$$
\boldsymbol{B} \to \boldsymbol{B} - \frac{1}{M}\boldsymbol{q}\times\boldsymbol{E}\,, \quad \boldsymbol{E} \to \boldsymbol{E} + \frac{1}{M}\boldsymbol{q}\times\boldsymbol{B}\,, \quad \boldsymbol{D} \to \boldsymbol{D} + \frac{1}{M}\boldsymbol{q} D_t\,, \quad D_t \to D_t + \frac{1}{M}\boldsymbol{q}\cdot\boldsymbol{D}
$$

Dark matter

Heavy WIMP effective theory

Consider e.g. the neutral component of SU(2) electroweak triplet (Weakly interacting stable pion, wino LSP, or other)

Universal properties emerge in the limit M >> m_W, described by the relevant heavy particle effective theory (+ SM)



E.g., Scattering on nucleon is completely determined, up to controlled corrections

$$m_W/M, \quad \Lambda^2_{\rm QCD}/m_c^2, \quad m_b/m_W \ldots$$

27

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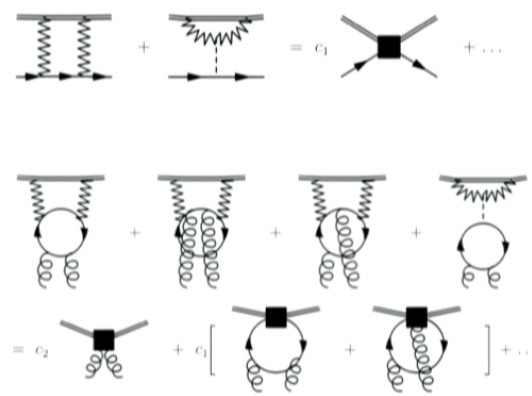

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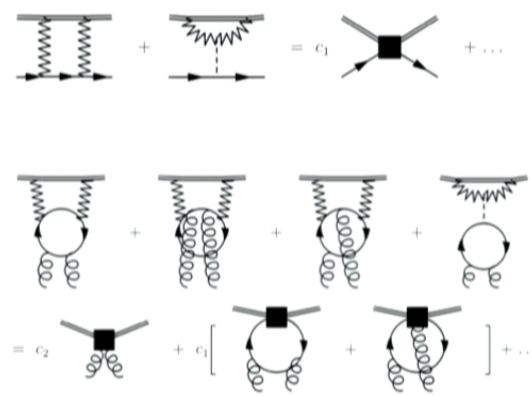

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$$m_W/M, \quad \Lambda^2_{\rm QCD}/m_c^2, \quad m_b/m_W \ldots$$

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Multiple scales:

Renormalization analysis required to sum large logarithms

$$\alpha_s(\mu)\log\frac{m_t}{\mu} \sim \alpha_s(1\,\mathrm{GeV})\log\frac{170\,\mathrm{GeV}}{1\,\mathrm{GeV}}$$

Consider effective theory at each scale:

$$?\,,\mathrm{SM}$$

M ....

$$\phi_v\,,\mathrm{SM}$$

$m_W \sim m_h \sim m_t$ ....

$$\phi_v^{(Q=0)}\,,u,d,s,c,b,g$$

$m_b\,,\quad m_c$ ....

$$\phi_v^{(Q=0)}\,,u,d,s,g$$

Λ_{QCD} ....

(EW symmetric) heavy scalar effective theory:

Operator basis

Building blocks: $\phi_v(x)\,, \quad v^\mu\,, \quad D_{\perp\mu} = D_\mu - v^\mu v\cdot D$

Everything not forbidden is allowed:

$$\begin{aligned}\mathcal{L}_\phi = \phi_v^* \Big\{ & iv\cdot D - c_1 \frac{D_\perp^2}{2M} + c_2 \frac{D_\perp^4}{8M^3} + g_2 c_D \frac{v^\alpha [D_\perp^\beta, W_{\alpha\beta}]}{8M^2} + i g_2 c_M \frac{\{D_\perp^\alpha, [D_\perp^\beta, W_{\alpha\beta}]\}}{16M^3} \\ & + g_2^2 c_{A1} \frac{W^{\alpha\beta}W_{\alpha\beta}}{16M^3} + g_2^2 c_{A2} \frac{v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta}}{16M^3} + g_2^2 c_{A3} \frac{\mathrm{Tr}(W^{\alpha\beta}W_{\alpha\beta})}{16M^3} + g_2^2 c_{A4} \frac{\mathrm{Tr}(v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta})}{16M^3} \\ & + g_2^2 c'_{A1} \frac{\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A2} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu W_{\nu\alpha} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A3} \frac{\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(W_{\mu\nu} W_{\rho\sigma})}{16M^3} \\ & + g_2^2 c'_{A4} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu \mathrm{Tr}(W_{\nu\alpha} W_{\rho\sigma})}{16M^3} + \dots \Big\} \phi_v\,,\end{aligned}$$

Lorentz invariance: $c_1 = c_2 = 1\,, \quad c_M = c_D$

$\Rightarrow$Through $O(1/M^3)$, heavy gauged scalar determined by 2 numbers (mass and "charge radius"), plus polarizabilities

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Standard model interactions

$$\mathcal{L}_{\phi,\mathrm{SM}} = \phi_v^* \Big\{ c_H \frac{H^\dagger H}{M} + \cdots + c_Q \frac{t_J^a \bar{Q}_L \tau^a \not{v} Q_L}{M^2} + c_X \frac{i\bar{Q}_L \tau^a \gamma^\mu Q_L \{t_J^a, D_\mu\}}{2M^3} + c_{DQ} \frac{\bar{Q}_L \not{v}\, iv\cdot D Q_L}{M^3}$$
$$+ c_{Du} \frac{\bar{u}_R \not{v}\, iv\cdot D u_R}{M^3} + c_{Dd} \frac{\bar{d}_R \not{v}\, iv\cdot D d_R}{M^3} + c_{Hd} \frac{\bar{Q}_L H d_R + h.c.}{M^3} + c_{Hu} \frac{\bar{Q}_L \tilde{H} u_R + h.c.}{M^3}$$
$$+ g_3^2 c_{A1}^{(G)} \frac{G^{A\,\alpha\beta} G^A_{\alpha\beta}}{16M^3} + g_3^2 c_{A2}^{(G)} \frac{v_\alpha v^\beta G^{A\,\mu\alpha} G^A_{\mu\beta}}{16M^3} + g_3^2 c_{A1}^{(G)\,\prime} \frac{\epsilon^{\mu\nu\rho\sigma} G^A_{\mu\nu} G^A_{\rho\sigma}}{16M^3} + g_3^2 c_{A2}^{(G)\,\prime} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu G^A_{\nu\alpha} G^A_{\rho\sigma}}{16M^3}$$
$$+ \ldots \Big\} \phi_v \,.$$

Lorentz invariance:

$$c_Q = c_X$$

All of these are suppressed by 1/M

Low energy theory

Operator basis

$$\mathcal{L} = \mathcal{L}_{\phi_0} + \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi_0,{\rm SM}} + \dots \,,$$

Heavy neutral scalar:

$$\mathcal{L}_{\phi_0} = \phi^*_{v,Q=0}\left\{ iv\cdot\partial - \frac{\partial_\perp^2}{2M_{(Q=0)}} + \mathcal{O}(1/m_W^3)\right\}\phi_{v,Q=0}$$

c_D=0 (reality constraint)

SM interactions:

$$\mathcal{L}_{\phi_0,{\rm SM}} = \frac{1}{m_W^3}\phi_v^*\phi_v\left\{\sum_q\left[c_{1q}^{(0)}O_{1q}^{(0)} + c_{1q}^{(2)}v_\mu v_\nu O_{1q}^{(2)\mu\nu}\right] + c_2^{(0)}O_2^{(0)} + c_2^{(2)}v_\mu v_\nu O_2^{(2)\mu\nu}\right\} + \dots$$

Convenient to choose basis of definite spin

$$O_{1q}^{(0)} = m_q\bar{q}q\,, \qquad O_2^{(0)} = (G^A_{\mu\nu})^2\,,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q}\left(\gamma^{\{\mu}iD^{\nu\}} - \frac{1}{d}g^{\mu\nu}i\not{D}\right)q\,, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_\lambda + \frac{1}{d}g^{\mu\nu}(G^A_{\alpha\beta})^2\,.$$

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Universal mass shift induced by EWSB

$-i\Sigma(p) =$

$$-i\Sigma_2(v\cdot p) = -g_2^2 \int \frac{d^dL}{(2\pi)^L}\frac{1}{v\cdot(L+p)}\left[J^2\frac{1}{L^2-m_W^2} + J_3^2\left(\frac{c_W^2}{L^2-m_Z^2} - \frac{1}{L^2-m_W^2} + \frac{s_W^2}{L^2}\right)\right] + \mathcal{O}(1/M)$$

heavy particle Feynman rules

$$\delta M = \Sigma(v\cdot p = 0) = \alpha_2 m_W\left[-\frac{1}{2}J^2 + \sin^2\frac{\theta_W}{2}J_3^2\right]$$

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2\frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170\,\mathrm{MeV})Q^2$$

Different pole masses for each charge eigenstate in low-energy theory (or residual mass terms)

Universal mass shift induced by EWSB

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$+ \ldots .$

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Different pole masses for each charge eigenstate in low-energy theory (or residual mass terms)

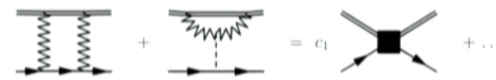
Matching (μ≈m$_W$)

Heavy particle Feynman rules simplify matching calculations

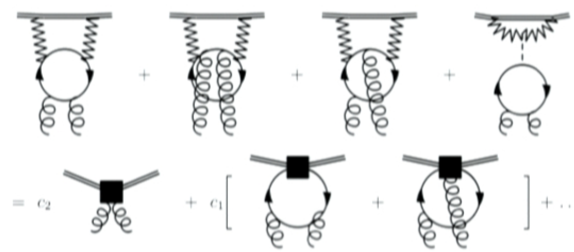
quark operators

$$c_{1U}^{(0)}(\mu_t) = \mathcal{C}\left[-\frac{1}{x_h^2}\right], \qquad c_{1D}^{(0)}(\mu_t) = \mathcal{C}\left[-\frac{1}{x_h^2} - |V_{tD}|^2\frac{x_t}{4(1+x_t)^3}\right],$$

$$c_{1U}^{(2)}(\mu_t) = \mathcal{C}\left[\frac{2}{3}\right], \qquad c_{1D}^{(2)}(\mu_t) = \mathcal{C}\left[\frac{2}{3} - |V_{tD}|^2\frac{x_t(3+6x_t+2x_t^2)}{3(1+x_t)^3}\right],$$



gluon operators



$$c_2^{(0)}(\mu_t) = \mathcal{C}\frac{\alpha_s(\mu_t)}{4\pi}\left[\frac{1}{3x_h^2} + \frac{3+4x_t+2x_t^2}{6(1+x_t)^2}\right],$$

$$c_2^{(2)}(\mu_t) = \mathcal{C}\frac{\alpha_s(\mu_t)}{4\pi}\left[-\frac{32}{9}\log\frac{\mu_t}{m_W} - 4 - \frac{4(2+3x_t)}{9(1+x_t)^3}\log\frac{\mu_t}{m_W(1+x_t)}\right.$$
$$- \frac{4(12x_t^5 - 36x_t^4 + 36x_t^3 - 12x_t^2 + 3x_t - 2)}{9(x_t-1)^3}\log\frac{x_t}{1+x_t} - \frac{8x_t(-3+7x_t^2)}{9(x_t^2-1)^3}\log 2$$
$$\left. - \frac{48x_t^6 + 24x_t^5 - 104x_t^4 - 35x_t^3 + 20x_t^2 + 13x_t + 18}{9(x_t^2-1)^2(1+x_t)}\right].$$

(

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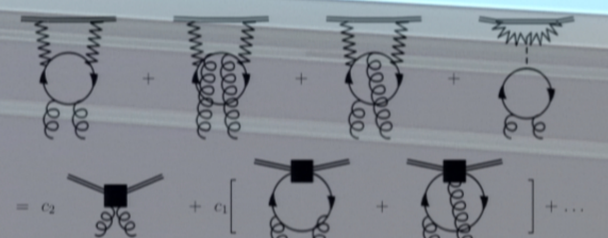

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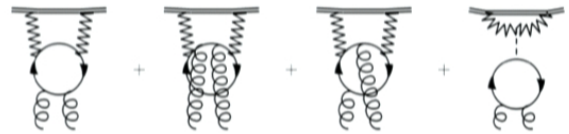
E.g. full theory:



$$i\mathcal{M} = -g_2^2 \int (dL) \left[\frac{1}{-v\cdot L + i0} + \frac{1}{v\cdot L + i0} \right] \frac{1}{(L^2 - m_W^2 + i0)^2} v_\mu v_\nu \Pi^{\mu\nu}(L)$$

electroweak polarizability tensor
in background gluon field

Electroweak gauge invariance is immediate:

$$v^\mu \left[g_{\mu\mu'} - (1-\xi)\frac{L_\mu L_{\mu'}}{L^2 - \xi m_W^2} \right] = v_{\mu'} + \mathcal{O}(v\cdot L)$$

crossed and uncrossed diagrams cancel

Fock-Schwinger gauge (x.A=0) in dim.reg. :

$$iS(p) = \frac{i}{\not{p} - m} + g\int (dq) \frac{i}{\not{p}-m} i\not{A}(q) \frac{i}{\not{p} - \not{q} - m}$$
$$+ g^2 \int (dq_1)(dq_2) \frac{i}{\not{p}-m} i\not{A}(q_1) \frac{i}{\not{p}-\not{q}_1 - m} i\not{A}(q_2) \frac{i}{\not{p} - \not{q}_1 - \not{q}_2 - m} + \ldots$$

Solution to RG equations

$$O_{1q}^{(0)} = m_q \bar{q} q \,, \qquad O_2^{(0)} = (G^A_{\mu\nu})^2 \,,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q}\left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i \not{D}\right) q \,, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}{}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G^A_{\alpha\beta})^2 \,.$$

$$\frac{d}{d\log\mu} O_i^{(S)} = -\sum_j \gamma_{ij}^{(S)} O_j \qquad \frac{d}{d\log\mu} c_i^{(S)} = \sum_j \gamma_{ji}^{(S)} c_j^{(S)}$$

Spin 0:

$$c_2^{(0)}(\mu) = c_2^{(0)}(\mu_t) \frac{\frac{\beta}{g}[\alpha_s(\mu)]}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

$$\hat{\gamma}^{(0)} = \left(\begin{array}{ccc|c} 0 & & & 0 \\ & \ddots & & \vdots \\ & & 0 & 0 \\ \hline -2\gamma_m' & \cdots & -2\gamma_m' & (\beta/g)' \end{array}\right)$$

$$c_1^{(0)}(\mu) = c_1^{(0)}(\mu_t) - 2[\gamma_m(\mu) - \gamma_m(\mu_t)] \frac{c_2^{(0)}(\mu_t)}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

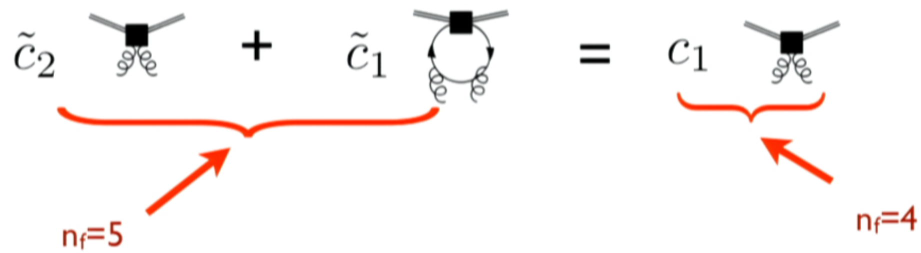
Spin 2:

Diagonalize anomalous dimension matrix
(familiar from PDF analysis)

$$\hat{\gamma}^{(2)} = \frac{\alpha_s}{4\pi} \left(\begin{array}{ccc|c} \frac{64}{9} & & & -\frac{4}{3} \\ & \ddots & & \vdots \\ & & \frac{64}{9} & -\frac{4}{3} \\ \hline -\frac{64}{9} & \cdots & -\frac{64}{9} & \frac{4n_f}{3} \end{array}\right) + \dots$$

As check, can evaluated spin-2 matrix elements at high scale (spin-0 and spin-2 decoupled)

Integrate out heavy quarks



$$c_2^{(0)}(\mu_b) = \tilde{c}_2^{(0)}(\mu_b)\left(1+\frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\right) - \frac{\tilde{a}}{3}\tilde{c}_{1b}^{(0)}(\mu_b)\left[1+\tilde{a}\left(11+\frac{4}{3}\log\frac{m_b}{\mu_b}\right)\right] + \mathcal{O}(\tilde{a}^3)$$

$$c_{1q}^{(0)}(\mu_b) = \tilde{c}_{1q}^{(0)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

$$c_2^{(2)}(\mu_b) = \tilde{c}_2^{(2)}(\mu_b) - \frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\tilde{c}_{1b}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

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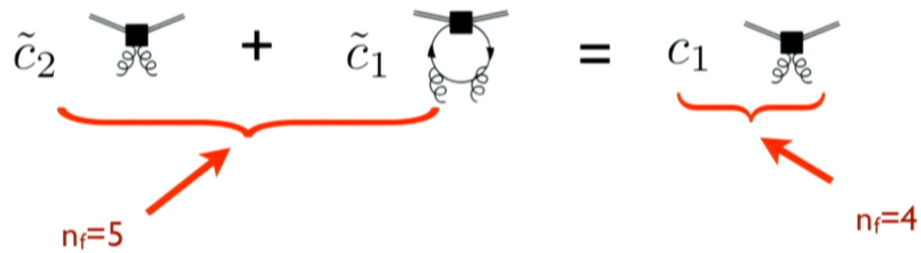
[Ovrut, Schnitzer, 1982]
[Inami, Kubota, Okada, 1983]

Contribution to gluon operators familiar from h→gg

Heavy quark mass scheme enters at higher order

Charm quark treated similarly (after running to m_c)

Integrate out heavy quarks



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Contribution to gluon operators familiar from h→gg

Heavy quark mass scheme enters at higher order

Charm quark treated similarly (after running to m_c)

Spin - 0

$$\langle N(k)|T^{\mu\nu}|N(k)\rangle = \frac{k^\mu k^\nu}{m_N} = \frac{1}{m_N}\left(k^\mu k^\nu - \frac{1}{4}g^{\mu\nu}m_N^2\right) + m_N\frac{1}{4}g^{\mu\nu}$$

Spin-0 operators determine contributions to nucleon mass

$$m_N = (1-\gamma_m)\sum_q \langle N|m_q\bar{q}q|N\rangle + \frac{\beta}{2g}\langle N|(G^a_{\mu\nu})^2|N\rangle$$

$$\langle N|O^{(0)}_{1q}|N\rangle \equiv m_N f^{(0)}_{q,N}\,, \qquad \frac{-9\alpha_s(\mu)}{8\pi}\langle N|O^{(0)}_2(\mu)|N\rangle \equiv m_N f^{(0)}_{G,N}(\mu)$$

significant uncertainty in this quantity

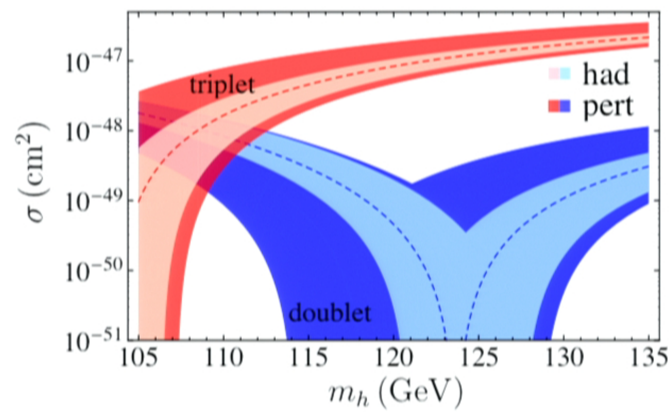
$$m_N(f^{(0)}_{u,N} + f^{(0)}_{d,N}) \approx \Sigma_{\pi N}\,, \quad m_N f^{(0)}_{s,N} = \frac{m_s}{m_u+m_d}(\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$

$$f^{(0)}_{G,N}(\mu) \approx 1 - \sum_{q=u,d,s} f^{(0)}_{q,N}$$

but NLO, NNLO corrections significant

Parameter-free WIMP-nucleon cross section for pure states ("wino", "higgsino")

Parameter	Value	Reference
$\lvert V_{td}\rvert$	~ 0	-
$\lvert V_{ts}\rvert$	~ 0	-
$\lvert V_{tb}\rvert$	~ 1	-
m_u/m_d	0.49(13)	[20]
m_s/m_d	19.5(2.5)	[20]
$\Sigma_{\pi N}^{\rm lat}$	0.047(9) GeV	[21]
$\Sigma_s^{\rm lat}$	0.050(8) GeV	[22]
$\Sigma_{\pi N}$	0.064(7) GeV	[23]
Σ_0	0.036(7) GeV	[24]
m_W	80.4 GeV	[20]
m_t	172 GeV	[15]
m_b	4.75 GeV	[15]
m_c	1.4 GeV	[15]
m_N	0.94 GeV	-
$\alpha_s(m_Z)$	0.118	[20]
$\alpha_2(m_Z)$	0.0338	[20]



(Numerical benchmark: low velocity, spin independent cross section on nucleon)

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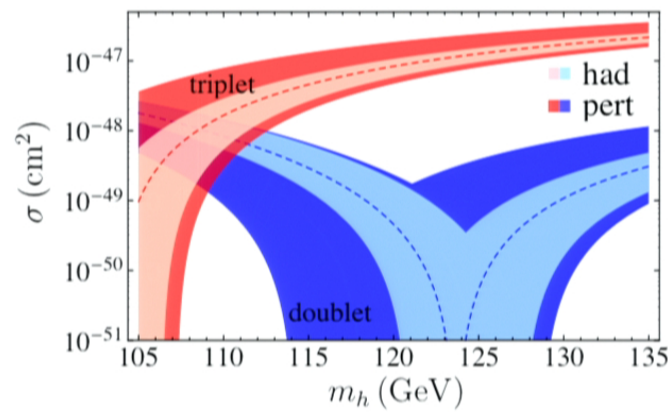

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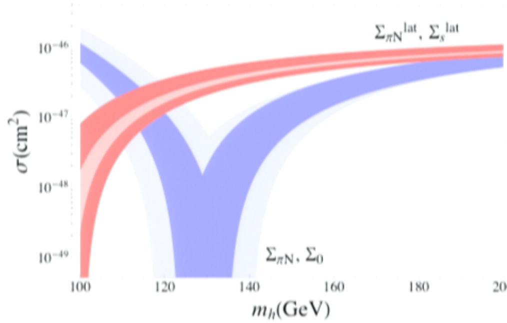
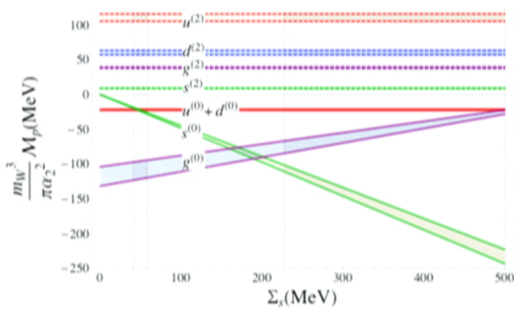
Dependence on strange scalar matrix element



Robust cancellation between spin-0 and spin-2 amplitudes, the former affected by higgs mass, and quark scalar matrix elements



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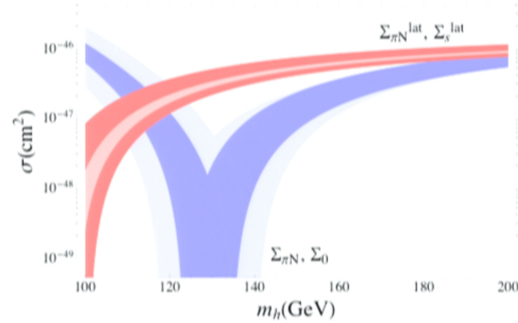

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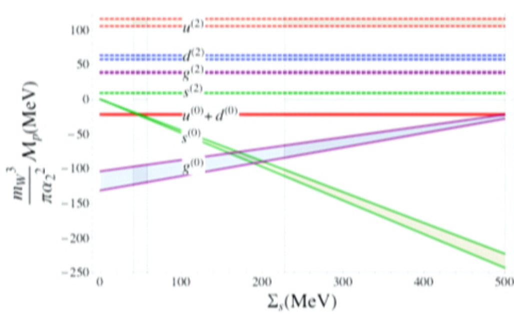
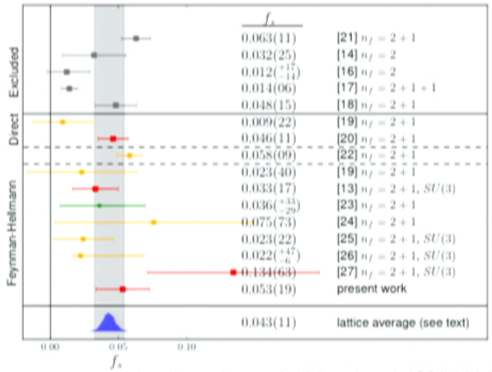
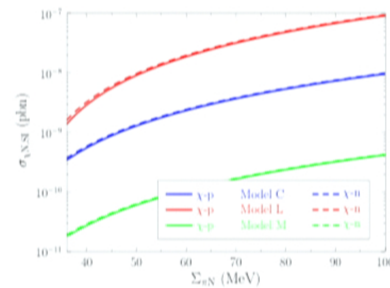


- strange quark scalar matrix element the subject of recent controversy



summary plot: Junnarkar and Walker-Loud, 1301.1114

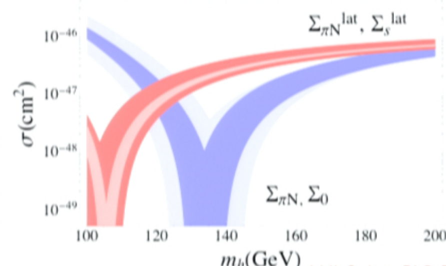


Ellis, Olive, Savage, PRD 77 065026 (2008)

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d) | p \rangle \qquad \Sigma_0 = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d - 2\bar{s}s) | p \rangle$$

$$m_N (f^{(0)}_{u,N} + f^{(0)}_{d,N}) \approx \Sigma_{\pi N}$$

$$m_N f^{(0)}_{s,N} = \frac{m_s}{m_u + m_d} (\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$



Hill, Solon, PLB 707 539 (2012)

- lattice results still noisy but converging on small value compared traditional SU(3) Ch.P.T. (cf. Alarcon et al., 1209.2870)

- beneficial to also have lattice constraints on charm scalar matrix element

Mixed cases

With the scale separation $M \gg m_W$, can argue that "pure states" are generic: effects of higher states suppressed by $m_W/(M'-M)$

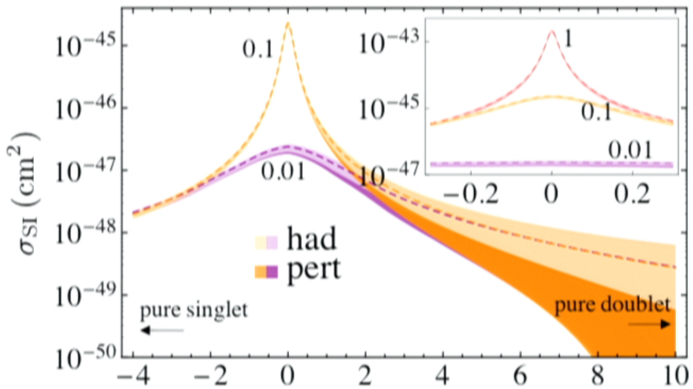
Intricate interplay of $m_W/(M'-M)$ suppressed higgs exchange versus pure-state loop corrections

Again, analyze in the $M, M' \gg m_W$ limit.

Inclusion of additional multiplets introduces residual mass and direct Higgs coupling

$$\mathcal{L} = \bar{h}_v \left[iv \cdot D - \delta m - f(H) \right] h_v + \mathcal{O}(1/M)$$

bino-higgsino

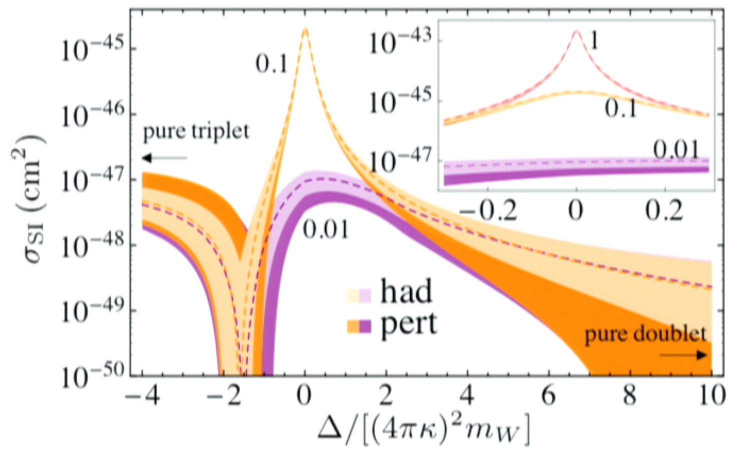


Pure states recovered in limit of large Δ / small κ

10-parameter space of coefficients reduced to 2-parameter space

Direct detection just beginning to probe largest of these cross sections

wino-higgsino



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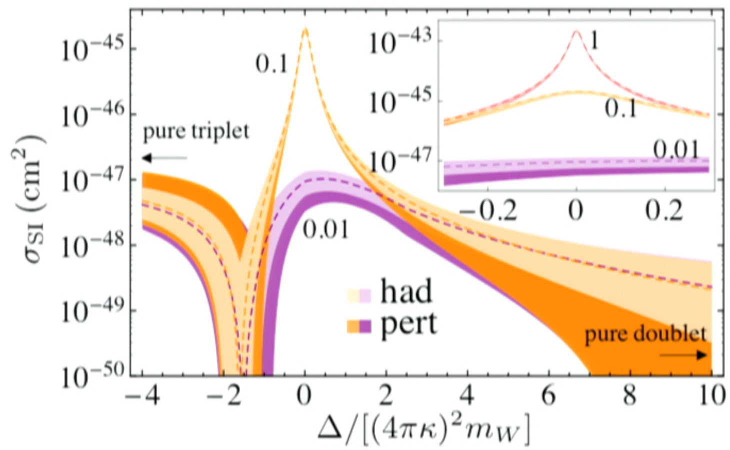

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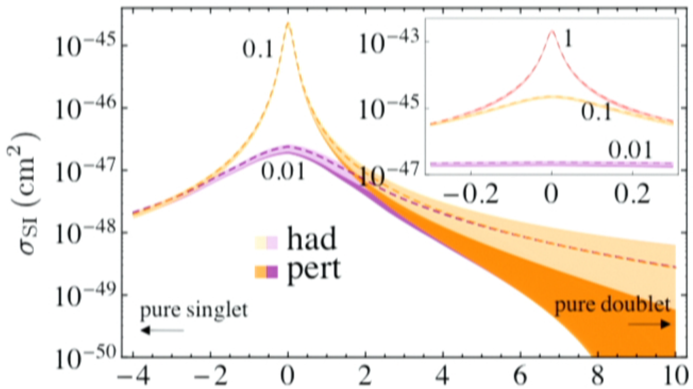

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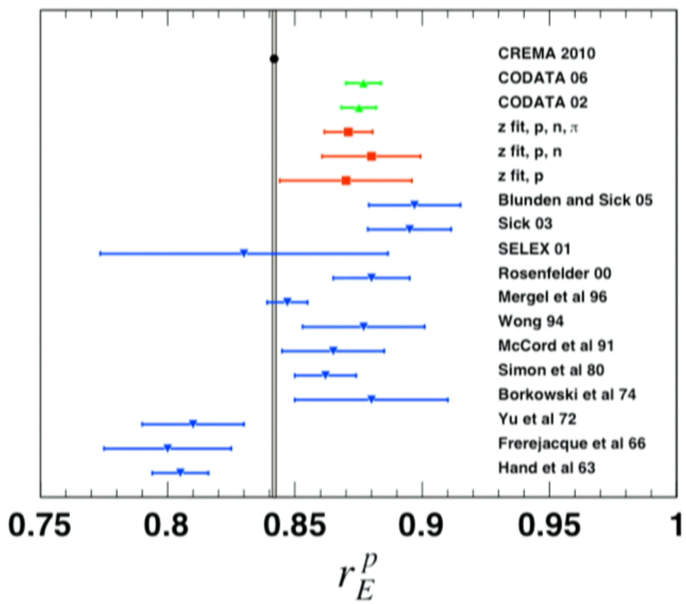
proton radius puzzle



- inferred from muonic H (for a review, R. Pohl Moriond 2011)

- inferred from electronic H

- extraction from e p, e n scattering, ππNN data

- previous extractions from e p scattering (as tabulated in PDG)

muon g-2 hadronic uncertainties

Theory Error Budget

$a_\mu(EW)$ = 154 (2) x 10^{-11}

$a_\mu(HVP\text{-}LO)$ = 6894 (40) x 10^{-11}

$a_\mu(HVP\text{-}NLO)$ = -98 (1) x 10^{-11}

$a_\mu(HLBL)$ = 116 (39) x 10^{-11}
105 (26) x 10^{-11} — Most challenging

δa_μ^{TH} = $\pm$ 48 x 10^{-11}

δa_μ^{EXP} = $\pm$ 63 x 10^{-11} — BNL E821
$\pm$ 15 x 10^{-11} — FNAL New g-2

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{TH}$ = 316 (79) x 10^{-11}

W. Marciano, arXiv: 1001.4528/hep-ph

[Slides stolen from M. Ramsey Musolf, Confinement 2012]

HLBL: Compilation

Nyffeler 1001.3970

Contribution	BPP [8]	HKS, HK [9]	KN [10]	MV [11]	BP [5], MdRR [1]	PdRV [6]	N [13], JN [3]
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15±10	22±5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

KTE, MRM	-16 $\pm$ 20 ?	-67 $\pm$ 20 ?
New Total ?	+78 $\pm$ 24 ?	+55 $\pm$ 27 ?
Asymptopia	✔	✖
DSE — Goeke, Fischer, Williams (B1, E2)		+188 $\pm$ 4 (stat)

possible resolutions and side effects:

- option 1: experimental issue in muonic hydrogen ?
  - central value would imply dramatic shift

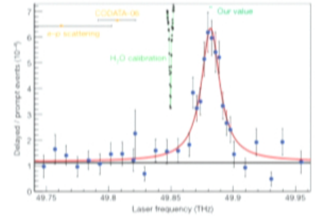
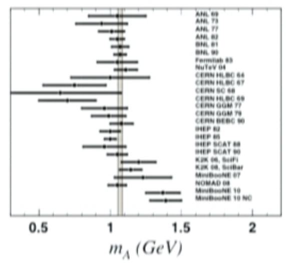
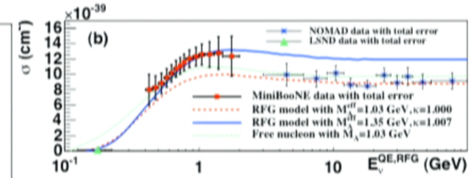


Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[1] or electron scattering[2,3] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the ±1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

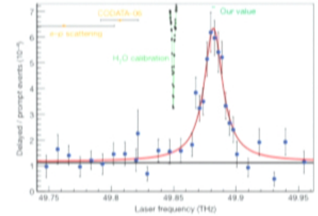
- option 2: systematic shifts in *both* electronic hydrogen (correlated errors?) *and* electron scattering (rad. cor., Q^2 extrapolations?) ?
  - would imply ~5 σ shift in Rydberg
  - related basic issues in lepton-nucleon scattering, e.g. signal process for $\nu_\mu \rightarrow \nu_e$ oscillations searches





- option 3: new structure effects in muonic hydrogen ?
  - related effects in electromagnetic contribution to nucleon masses
  - potential impact on muon-proton scattering
- option 4: something else ?

possible resolutions and side effects:



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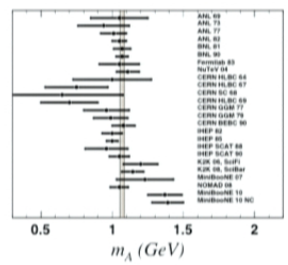

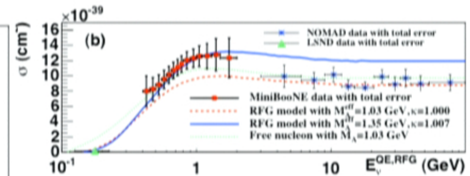

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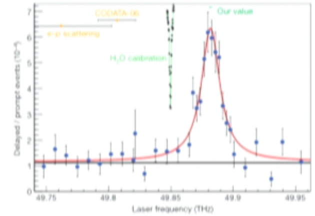
possible resolutions and side effects:



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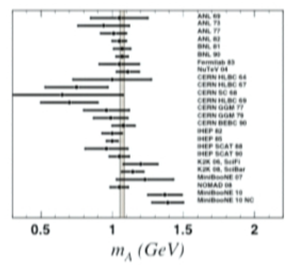

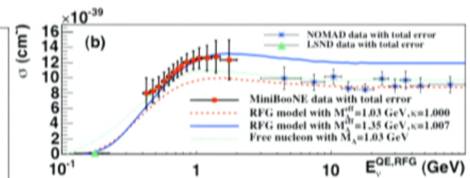

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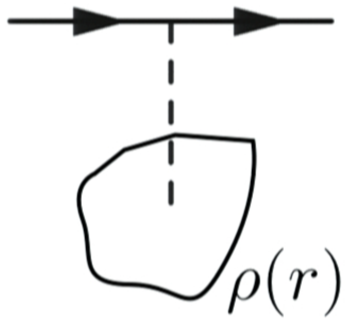
historical interlude



$$H = H_0 + \Delta V \,,$$

$$\Delta V \sim -\int d^3r\, \rho(r) \left(\frac{Z\alpha}{|r - r'|} - \frac{Z\alpha}{r'} \right)$$

[Friar, 1979]

Semiclassical picture: multipole expand and carry out perturbation theory:

$$\Delta E_n = \frac{2\pi}{3} \,|\, \phi_n(0)|^2\, Z\alpha \left(\langle r^2 \rangle - \frac{Z\alpha\mu}{2} \langle r^3 \rangle_{(2)} + (Z\alpha)^2 F_{\mathrm{REL}} + (Z\alpha\mu)^2 F_{\mathrm{NR}} \right)$$

$$\langle r^3 \rangle_{(2)} = \int d^3r\, r^3 \rho^{(2)}(r)$$

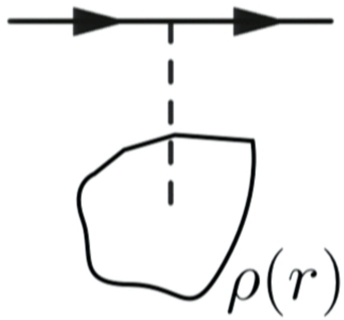
$$\rho^{(2)}(r) = \int d^3r'\, \rho(|\, \mathbf{r}' - \mathbf{r} \,|)\, \rho(r') \qquad \text{etc.}$$

Demands assumption for charge distribution, nonrecoil expansion (~ $m_\rho/m_{\mathrm{proton}} \ll 1$)

Not clear how to merge in model-independent way with "polarizability"

$\Longrightarrow$ *Enter effective field theory* (= QM + relativity + calculus)

historical interlude



$$H = H_0 + \Delta V \,,$$

$$\Delta V \sim -\int d^3r\,\rho(r)\left(\frac{Z\alpha}{|r-r'|} - \frac{Z\alpha}{r'}\right)$$

[Friar, 1979]

Semiclassical picture: multipole expand and carry out perturbation theory:

$$\Delta E_n = \frac{2\pi}{3}\,|\,\phi_n(0)|^2\, Z\alpha\left(\langle r^2\rangle - \frac{Z\alpha\mu}{2}\langle r^3\rangle_{(2)} + (Z\alpha)^2 F_{\mathrm{REL}} + (Z\alpha\mu)^2 F_{\mathrm{NR}}\right)$$

$$\langle r^3\rangle_{(2)} = \int d^3r\; r^3 \rho^{(2)}(r)$$

$$\rho^{(2)}(r) = \int d^3r'\,\rho(|\,\mathbf{r}' - \mathbf{r}\,|)\,\rho(r') \qquad \text{etc.}$$

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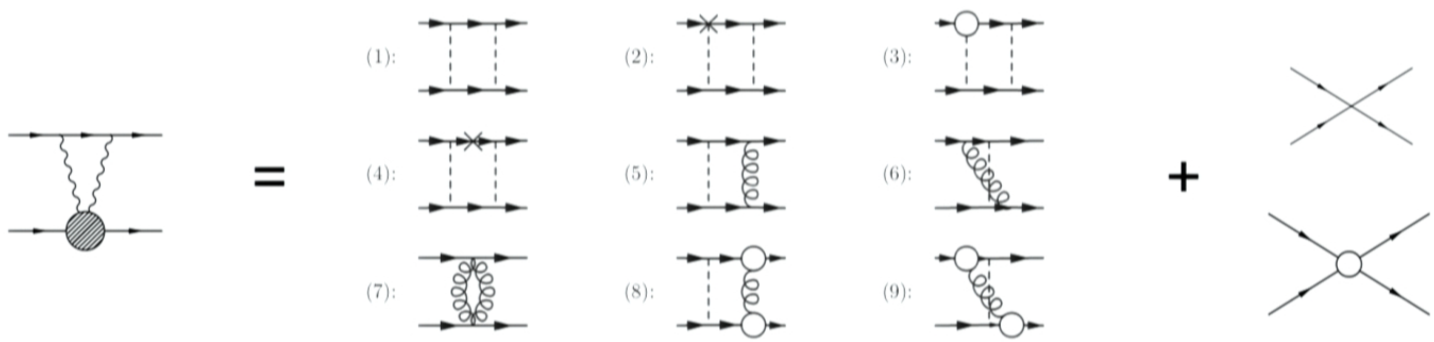
NRQED d_2: two-photon exchange



- to reproduce large momentum regions, include four-fermion counterterms

$$\mathcal{L}_{\rm ct} = \frac{d_1}{M^2}\psi_p^\dagger\psi_p\psi_e^\dagger\psi_e + \frac{d_2}{M^2}\psi_p^\dagger\vec{\sigma}\psi_p\cdot\psi_e^\dagger\vec{\sigma}\psi_e + \ldots$$

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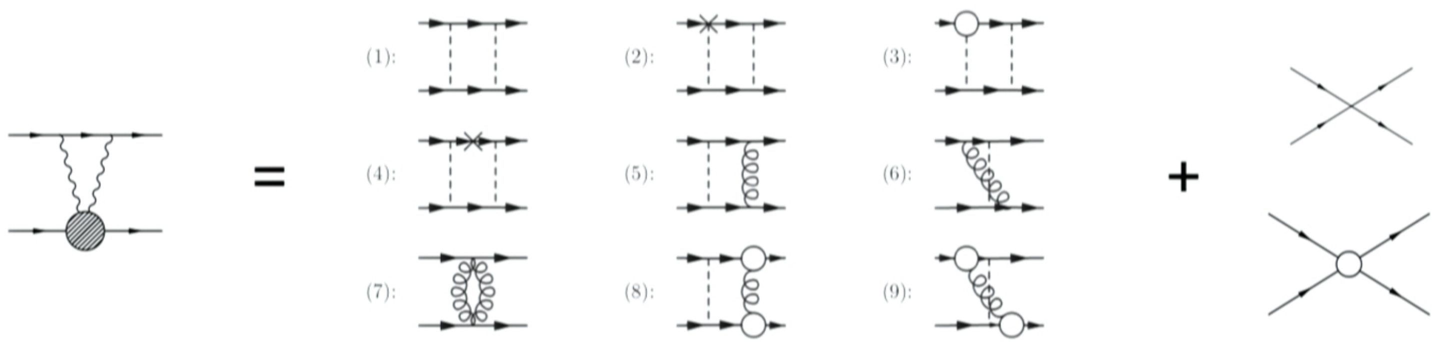

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- model independently, in small lepton mass limit, 2-photon contribution to energy is

universal hadronic parameters

$$\Delta E(nS) = \frac{m_r^3 (Z\alpha)^3}{\pi n^3} \frac{\delta d_2}{m_e m_p}$$

$$= (Z\alpha)^5 \frac{m_r^3}{\pi n^3} \frac{m_\ell}{m_p^3} \left\{ \left[m_p^3 \frac{4\pi}{e^2} (5\bar{\alpha} - \bar{\beta}) - 3a_p^2 \right] \log m_\ell + c_0 + c_1 m_\ell + \dots \right\}$$

$$\approx (Z\alpha)^5 \frac{m_r^3}{\pi n^3} \frac{m_\ell}{m_p^3} \left\{ 100 \log m_\ell + c_0 + c_1 m_\ell + \dots \right\}$$

- for electronic hydrogen, contribution is numerically large compared to experimental error, but not compared to present theory error

  - loose constraint $c_0 \lesssim 10^4$ from 1S-2S hydrogen-deuterium isotope shift

- unfortunately, for muonic hydrogen m_μ/m_π , $m_\mu/(m_\Delta - m_p)$ not small. What reasonable values can δd_2 take ?

$$c_0 + \cdots \sim 1000 \implies \delta E(2S) \sim 0.1\,\mathrm{meV}$$

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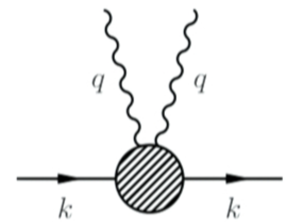
forward Compton amplitude

- QCD input summarized by amplitudes of forward scattering



$$W^{\mu\nu}(q,k) = i\int d^4x\, e^{iq\cdot x}\langle \text{proton}(k,s)|T\{J^\mu(x)J^\nu(0)\}|\text{proton}(k,s)\rangle$$

$$= \bar{u}_s(k)\Big\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1(\nu,Q^2) + \left(k^\mu - \frac{k\cdot q}{q^2}q^\mu\right)\left(k^\nu - \frac{k\cdot q}{q^2}q^\nu\right) W_2(\nu,Q^2)$$

$$+ H_1(\nu,Q^2)\{[\gamma_\nu, \not{q}]k_\mu - [\gamma_\mu,\not{q}]k_\nu + [\gamma_\mu,\gamma_\nu]k\cdot q\}$$

$$+ H_2(\nu,Q^2)\{[\gamma_\nu,\not{q}]q_\mu - [\gamma_\mu,\not{q}]q_\nu + [\gamma_\mu,\gamma_\nu]q^2\}\Big\}u_s(k)$$

- four invariant functions of photon energy ν and invariant Q^2: two spin-independent, two spin-dependent

- in DIS, just interested in imaginary part, but here need the whole thing

$$\frac{4\pi m_r}{\lambda^3} - \frac{\pi m_r}{2m_e m_p \lambda} - \frac{2\pi m_r}{m_p^2\lambda}\left[F_2(0) + 4m_p^2 F_1'(0)\right] - \frac{2}{m_e m_p}\left[\frac{2}{3} + \frac{1}{m_p^2 - m_e^2}\left(m_e^2 \log\frac{m_p}{\lambda} - m_p^2\log\frac{m_e}{\lambda}\right)\right] + \frac{\delta d_2 (Z\alpha)^{-2}}{m_e m_p}$$

$$= -\frac{m_e}{m_p}\int_{-1}^{1} dx\sqrt{1-x^2}\int_0^\infty dQ\, \frac{Q^3\left[(1+2x^2)W_1(2im_pQx,Q^2) - (1-x^2)m_p^2 W_2(2im_pQx,Q^2)\right]}{(Q^2+\lambda^2)^2(Q^2+4m_e^2x^2)},$$

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dispersion relation with subtraction:

- $W_1(\nu,Q^2)$ *not* determined by imaginary part

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\rm cut}^2}^{\infty} d\nu'^2 \frac{{\rm Im} W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

- no intrinsic meaning to "proton contribution" and "non-proton contribution" to $W_1(0,Q^2)$

- can analyze at low momentum using NRQED: double scattering of proton off external static electromagnetic field

Previous analyses employ a "sticking in form factors" (SIFF) ansatz

$$W_1(0,Q^2) = 2(-1+{c_F}^2) + \frac{Q^2}{2M^2}\left({c_F}^2 - 2c_F c_{W1} + 2c_M + c_{A1}\right) + \ldots$$

$$= \underbrace{2a_p(2+a_p) + \frac{Q^2}{2M^2}\left(-8a_pF_1'(0) - 8(1+a_p)F_2'(0)\right.}_{2F_2(-Q^2)[2F_1(-Q^2)+F_2(-Q^2)]} \left. + \underbrace{M^3\frac{4\pi}{e^2}\bar{\beta}}_{\frac{4\pi}{e^2}\bar{\beta}\frac{MQ^2}{2}\frac{1}{(1+Q^2/0.71\,\mathrm{GeV}^2)^4}}\right) + \ldots$$

e.g. Pachuckii 1999

- assumes ad hoc separation into "proton" versus "non-proton" states and assigns form factors to the former

- model dependent extrapolation, wrong behavior at large Q^2

caution is warranted..

This ansatz fails dramatically for experimentally accessible spin-dependent structure function

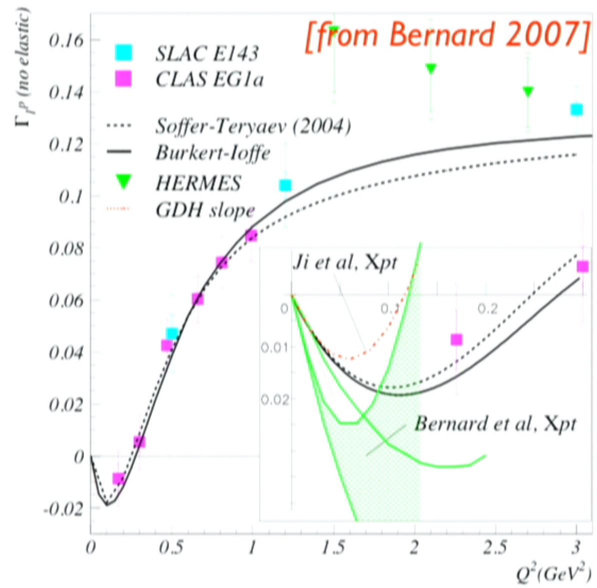
$$\Gamma_1 = \frac{Q^2}{8}\bar{S}_1 = \frac{MQ^2}{4}\bar{H}_1$$

Drell Hearn

~ 0.8 GeV^{-2}

$$\begin{aligned} S_1(0,Q^2) - S_1^{\text{proton}}(0,Q^2) &= -\frac{1}{M}a_p^2 + cQ^2 + \ldots \\ &= \frac{1}{\pi}\int d\nu^2 \frac{\text{Im}S_1(\nu,Q^2)}{\nu^2} \end{aligned}$$

data (unsubtracted dispersion relation)



From the bully pulpit:

"we believe that the uncertainty assigned to this contribution (<0.004 meV) is underestimated by at least an order of magnitude."

[RJH, G. Paz 2011]

Not easy to find significantly more with "reasonable" functions. But an unsatisfactory state of affairs (cf hadronic LBL in muon g-2)

[Pineda, 2005, Nevado, Pineda 2008]
[Carlsen, Vanderhaegen 2011]
[Walker-Loud et al 2012] ...

For example, Q^4 term studied in a single-parameter ansatz for $W_1(0,Q^2)$

[Birrse, McGoverne, 2012]

$$\mathrm{W}_1(0,Q^2) = W_1^{\mathrm{SIFF}}(0,Q^2) + \frac{4\pi}{e^2}\bar{\beta}\,\frac{MQ^2}{2}\,\frac{1}{\left(1+\frac{Q^2}{2M_\beta^2}\right)^2}$$

claimed uncertainty of 0.001 meV on LS difficult to reconcile with our current state of knowledge

$$W_i^{c,1}(\nu,Q^2) = \frac{\nu^2}{\pi}\int_{\nu^2_{\rm cut}(Q^2)}^{\infty} \frac{d\nu'^2}{\nu'^2}\frac{{\rm Im}W_i(\nu',Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

real photon terms expanded at small ν:

$$\lim_{Q^2=0} W_2(\nu,Q^2) = 0$$

$$W_1(\nu,0) = -2 + \frac{\nu^2}{8M^4}\left(-c_F^2 + c_F c_S + 2c_M - \frac{1}{2}c_{A2}\right) + \mathcal{O}(\nu^4)$$

Combined with pole and subtraction terms these determine continuum contributions:

$$W_1^{c,1}(\nu,0) = \frac{\nu^2}{2M^4}(\bar\alpha + \bar\beta) + \mathcal{O}(\nu^4)\,,$$

$$W_2^{c,1}(\nu,0) = 0\,.$$

Finally, insert into matching formula for d_2, and determine:

unambiguously determined by NRQED

$$\delta E(nS) = \frac{m_r^3(Z\alpha)^5}{\pi n^3}\frac{m_\ell}{M^3}\{\log m_\ell\left[M^3\left(5\bar\alpha - \bar\beta\right)/\alpha - 3a_p(1+a_p) + 2M^2(r_E^p)^2\right] + \dots\}$$

[Khriplovich, Sen'kov 1998]

Summary of leading radiative corrections to nuclear structure

Contribution		Ref. [20] *[Pachucki]*	Ref. [23] *[Borie]*	This work *[Hill, Paz]*
$\delta E^{\rm vertex}$		-0.0099	-0.0096	-0.0108
$\delta E^{\rm two-\gamma}$	$\delta E^{\rm proton}_{\mu H}$	0.035	0.051	-0.016
	$\delta E^{W_1(0,Q^2)}_{\mu H}$			Model Dependent
	$\delta E^{\rm continuum}_{\mu H}$			0.013 [19] *[Carlson, Vanderhaegen]*
Total		0.025	0.042	

TABLE I: Comparison between this and previous works for $\mathcal{O}(\alpha^5)$ proton structure corrections to the $2P-2S$ Lamb shift in muonic hydrogen, in meV.

Model-dependent part is largest (this statement is convention dependent)

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Investigate the impact of modified structure corrections and larger uncertainty on the two-photon exchange contribution

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vertex correction	-0.0096 meV	-0.0108	mismatch in r definitions
two photon (d_2)	0.051	? ~ 0.05 +/- 0.05	model dependent
"recoil finite size"	0.013	0	double counting
total	210.0011(45) $-5.2262r^2$	209.987(50) $-5.2262\ r^2$	
extracted radius	0.8421(6)	0.841(6)	

H	CODATA06	0.876(8)	4.2σ	3.5σ
e-p	Sick 2005	0.895(18)	2.9σ	2.8σ
	JLab 2011	0.875(10)	3.3σ	2.9σ
	Mainz	0.879(8)	4.6σ	3.8σ
H and e-p	CODATA10	0.8775(51)	6.9σ	4.6σ
	ep	0.870(26)	1.1σ	1.1σ
	ep, en	0.880(20)	1.9σ	1.9σ
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EFT and the little group:

- consistent implementation of Lorentz invariance in heavy particle EFT
- extends to arbitary order, arbitrary spin, and light-like ($n^2=0$, SCET) expansions

Heavy particle EFT and weakly interacting DM

- universal cross sections: (small) targets for future experiments
- Simplified computations, e.g. 2-loop matching for gluon operators

NRQED and proton structure

- definition of charge radius in presence of radiative corrections, and connection to scattering observables
- model-dependent assumptions in previous analyses
- discrepancies still present, further analysis of e-p scattering underway, await more data