Title: Hybrid quantization of the Gowdy model within loop quantum cosmology

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Abstract: Loop quantum cosmology (LQC) proposes a quantization for homogeneous cosmologies which success in solving the classical singularity problem. Realistic scenarios call for the consideration of inhomogeneities. Focusing on the simplest inhomogenous cosmological model, the Gowdy model with three-torus spatial topology and linearly polarized gravitational waves, I'll describe an approach to treat inhomogeneities in the framework of loop quantum cosmology. This is a hybrid approach that combines LQC methods with Fock quantization. Furthermore, I'll discuss justified approximations that allow us to find approximate solutions to the (very complicated) Hamiltonian constraint of the model.

Hybrid quantization of the Gowdy model within loop quantum cosmology

Mercedes Martín-Benito

Perimeter Institute

In collaboration with:
D. Martín-de Blas and G.A. Mena Marugán
arXiv:1307.1420

October 15th @ Perimeter Institute


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Introduction

- LQC is a quantum approach for cosmological modes inspired by LQG that provides a satisfactory quantization leading to the resolution of singularities in terms of a *quantum bounce*
- Our aim: to study the effects of LQC phenomena in **inhomogeneous cosmological models**
- Our proposal: **Hybrid quantization** combining LQC quantization of the homogeneous d.o.f. with a Fock quantization for the inhomogeneities
- We need to develop approximation methods to solve the (very involved) Hamiltonian constraint

Introduction

- The Gowdy model with 3-torus topology and linear polarization is a most suitable arena to start with:
  - Classical solutions well known. The subfamily of homogeneous solutions represent Bianchi I spacetimes
  - A Fock quantization of the deparametrized system has been achieved and shown to be essentially unique

$\rightarrow$ gravitational waves over a Bianchi I background

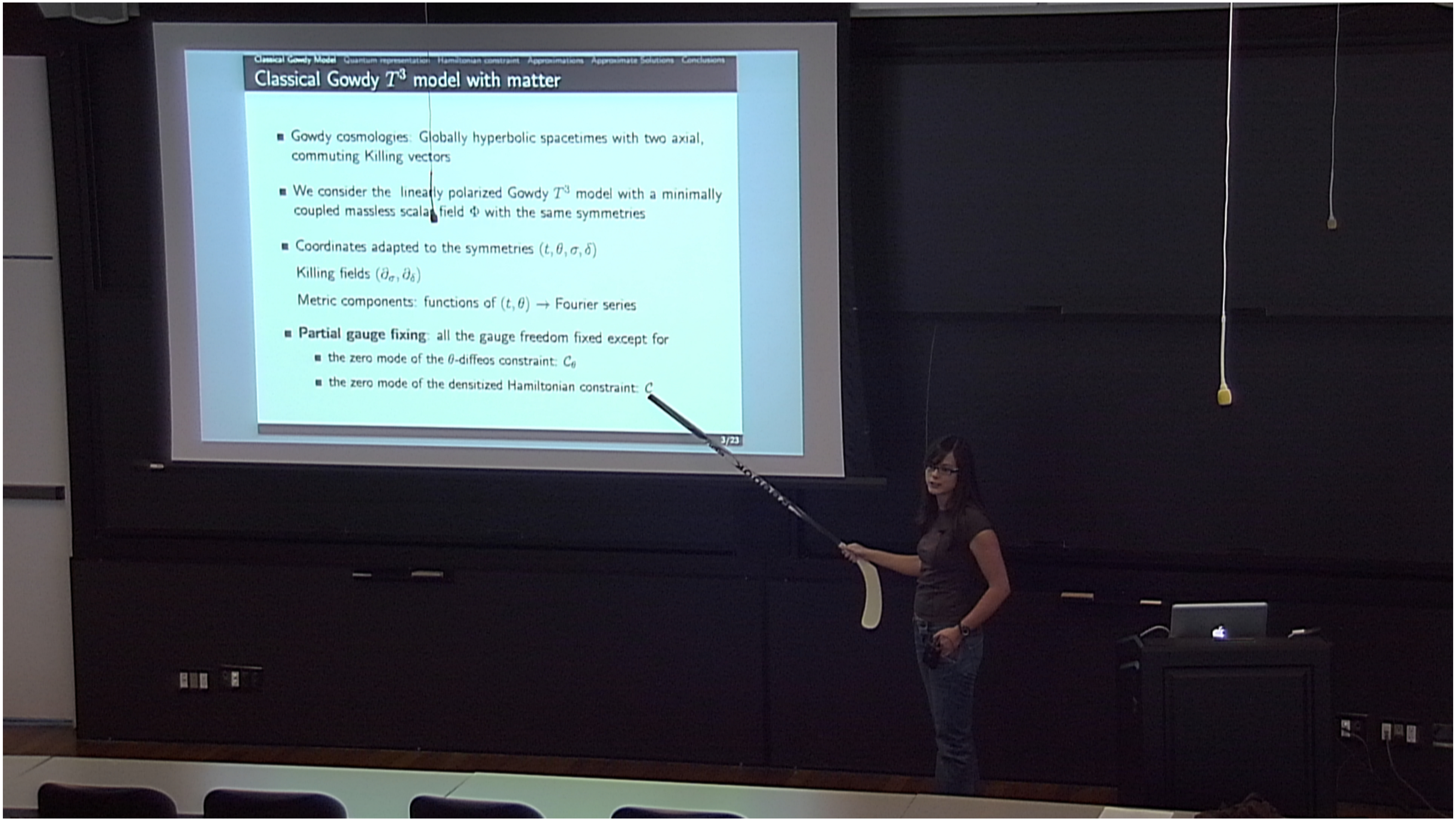
- Inclusion of a massless scalar field $\Phi \rightarrow$ the homogeneous sector contains **flat FRW solutions**

Classical Gowdy T^3 model with matter

- Gowdy cosmologies: Globally hyperbolic spacetimes with two axial, commuting Killing vectors
- We consider the linearly polarized Gowdy T^3 model with a minimally coupled massless scalar field Φ with the same symmetries
- Coordinates adapted to the symmetries $(t, \theta, \sigma, \delta)$

  Killing fields $(\partial_\sigma, \partial_\delta)$

  Metric components: functions of $(t, \theta) \to$ Fourier series
- **Partial gauge fixing**: all the gauge freedom fixed except for
  - the zero mode of the θ-diffeos constraint: $\mathcal{C}_\theta$
  - the zero mode of the densitized Hamiltonian constraint: $\mathcal{C}$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Reduced phase space: Homogeneous sector

- **Gravitational sector**: phase space of the Bianchi I model with 3-torus topology
  - Ashtekar variables: three connection coefficients c_i and three densitized-triad coefficients p_i $\quad (|p_i| = a_j a_k) \quad (i,j,k = \theta,\sigma,\delta)$
  - For simplicity: local rotational symmetry (LRS) $p_\delta = p_\sigma \equiv p_\perp$

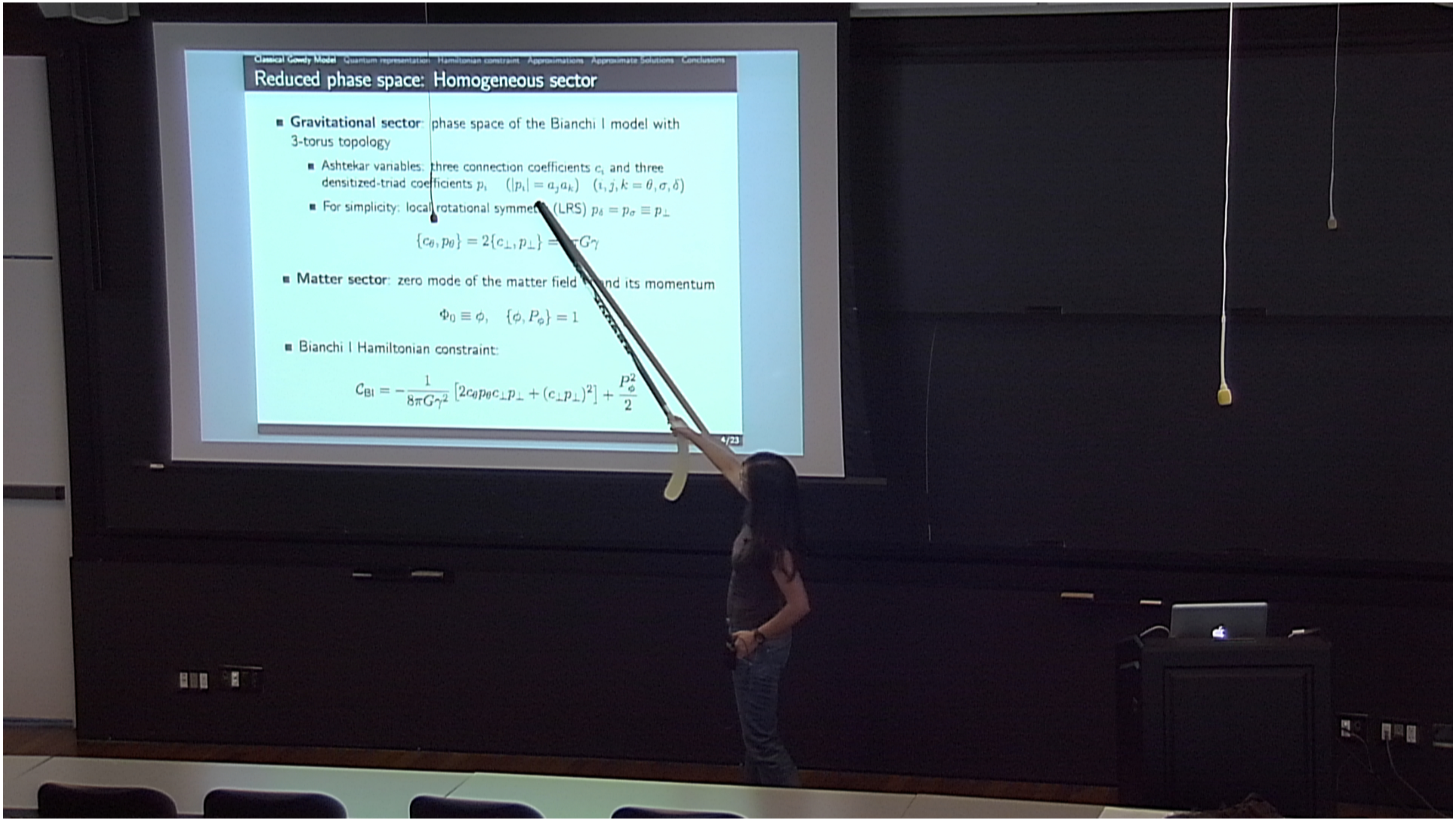
$$\{c_\theta, p_\theta\} = 2\{c_\perp, p_\perp\} = 8\pi G\gamma$$

- **Matter sector**: zero mode of the matter field Φ and its momentum

$$\Phi_0 \equiv \phi, \quad \{\phi, P_\phi\} = 1$$

- Bianchi I Hamiltonian constraint:

$$\mathcal{C}_{\mathrm{BI}} = -\frac{1}{8\pi G\gamma^2}\left[2c_\theta p_\theta c_\perp p_\perp + (c_\perp p_\perp)^2\right] + \frac{P_\phi^2}{2}$$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Reduced phase space: Inhomogeneous sector

Non-zero Fourier modes of a gravitational field $\xi(\theta)$ and those of the matter field $\Phi(\theta)$, together with their conjugate momenta.

- In the deparametrized model (only $\mathcal{C}_\theta$ remains) there exists a privileged description:
  - $\xi(\theta)$ and $\varphi(\theta) \equiv \frac{\Phi(\theta)}{|p_\theta|}$ such that they verify the same e.o.m, that of a scalar field with a time dependent mass in a statistic spacetime of 1+1 dimensions
  - Annihilation and creation like variables associated to a free massless scalar field

$$\{a_m^\alpha, a_{\tilde{m}}^{\alpha *}\} = -i\delta_{m\tilde{m}}, \quad \alpha = \xi, \varphi$$

→ This description leads to a Fock quantization with unitary dynamics and vacuum state invariant under S^1. It is the unique one with these properties (up to unitary equivalence)

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We choose the same variables to describe our inhomogeneous sector

$$\{a_m^\alpha, a_{\tilde{m}}^{\alpha *}\} = -i\delta_{m\tilde{m}}, \quad m \in \mathbb{Z} - \{0\}, \quad \alpha = \xi, \varphi$$

Remaining global constraints

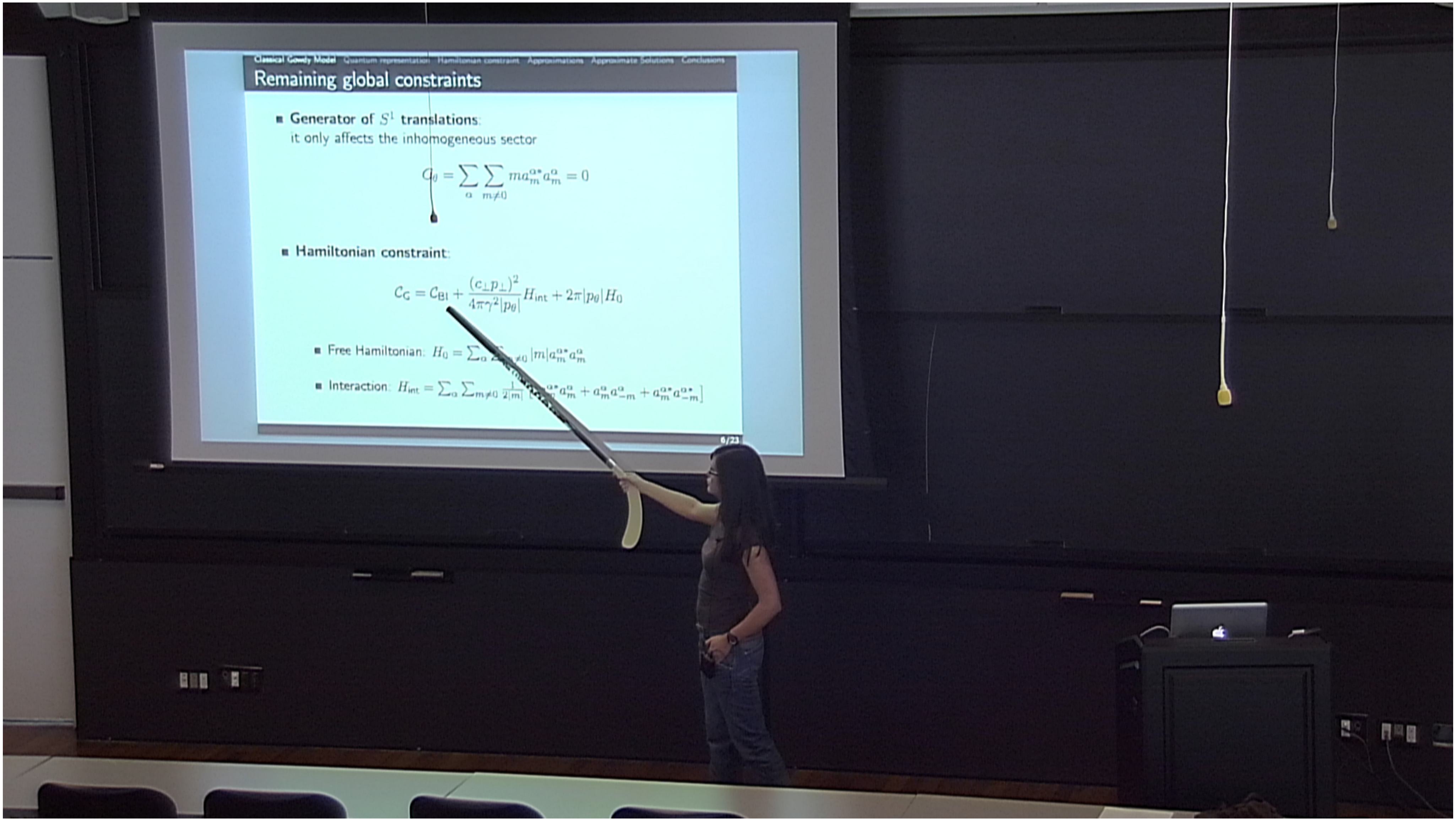
- **Generator of S^1 translations**:
  it only affects the inhomogeneous sector

$$C_\theta = \sum_{\alpha} \sum_{m \neq 0} m a_m^{\alpha *} a_m^{\alpha} = 0$$

- **Hamiltonian constraint**:

$$\mathcal{C}_{\mathsf{G}} = \mathcal{C}_{\mathsf{BI}} + \frac{(c_\perp p_\perp)^2}{4\pi\gamma^2 |p_\theta|} H_{\mathsf{int}} + 2\pi |p_\theta| H_0$$

  - Free Hamiltonian: $H_0 = \sum_\alpha \sum_{m \neq 0} |m| a_m^{\alpha *} a_m^{\alpha}$
  - Interaction: $H_{\mathsf{int}} = \sum_\alpha \sum_{m \neq 0} \frac{1}{2|m|} \left[2a_m^{\alpha *} a_m^{\alpha} + a_m^{\alpha} a_{-m}^{\alpha} + a_m^{\alpha *} a_{-m}^{\alpha *} \right]$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Quantum representation of the inhomogeneous sector

Fock quantization:

- The variables a_m^α and $a_m^{\alpha *}$ are promoted to annihilation and creation operators, $\hat{a}_m^\alpha$ and $\hat{a}_m^{\alpha\dagger}$, over $\mathcal{F}^\alpha$

$$[\hat{a}_m^\alpha, \hat{a}_{\tilde{m}}^{\alpha\dagger}] = \delta_{m\tilde{m}}$$

- $\mathcal{F}^\alpha \supset \mathcal{S}^\alpha = \text{span}(|\mathfrak{n}^\alpha\rangle)$, $|\mathfrak{n}^\alpha\rangle := |..., n_{-2}^\alpha, n_{-1}^\alpha, n_1^\alpha, n_2^\alpha, ...\rangle$

Generator of S^1-translations:

$$\widehat{C}_\theta = \widehat{C}_\theta^\xi + \widehat{C}_\theta^\varphi, \quad \widehat{C}_\theta^\alpha = \sum_{m\neq 0} m \hat{a}_m^{\alpha\dagger} \hat{a}_m^\alpha$$

The n-particle states $|\mathfrak{n}^\xi\rangle \otimes |\mathfrak{n}^\varphi\rangle$ annihilated by $\widehat{C}_\theta$ form a proper subspace of $\mathcal{F}^\xi \otimes \mathcal{F}^\varphi$: $\mathcal{F}_p$

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Quantum representation of the homogeneous sector

LQC representation of the Bianchi I model:

- **Matter sector** (ϕ): Standard quantization

$$L^2(\mathbb{R}, d\phi), \quad \hat{P}_\phi = -i\hbar\partial_\phi$$

- **Gravitational sector**: Loop quantization
  - Kinematical Hilbert space:

    $\mathcal{H}_{\text{kin}}^{\text{BI}} = \overline{\text{span}\{|p_\theta, p_\perp\rangle\}}$, $\langle p_i|p_i'\rangle = \delta_{p_i,p_i'}$, discrete inner product

    $\hat{p}_i|p_i\rangle = p_i|p_i\rangle \rightarrow$ discrete spectrum equal to $\mathbb{R}$
  - There is no well-defined operator representing the connection coefficients c_i, but rather its holonomies $e^{i\mu_i c_i}$, that we consider along straight edges of length μ_i in the fiducial directions

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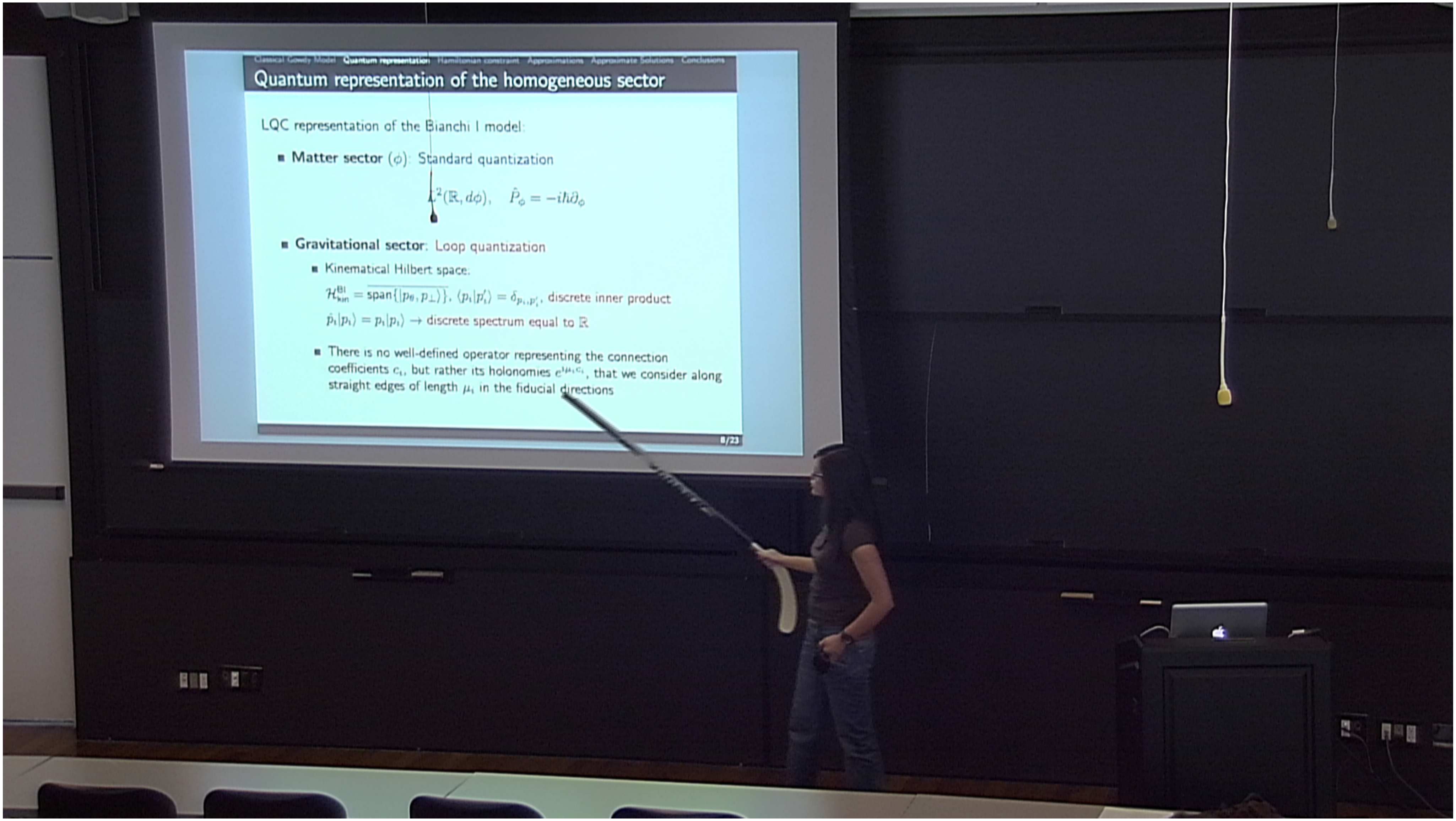
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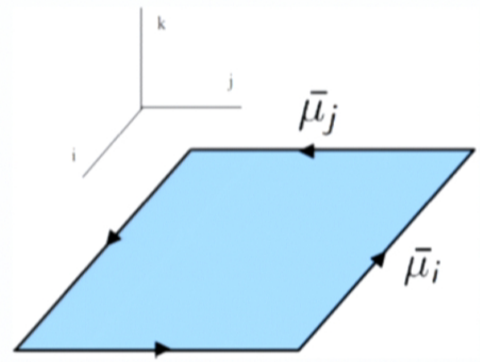
Improved dynamics prescription

- In order to define the curvature tensor of the connection, we take a rectangular loop of holonomies.
- The limit when the enclosed area tends to zero is not well-defined, but there exists a minimum nonzero fiducial length for the holonomies, measured by $\bar{\mu}_j$, $j \in \{\theta, \sigma, \delta\}$.



- The kinematical area $\bar{\mu}_i \bar{\mu}_j p_k$ of that loop equals the minimum nonzero eigenvalue Δ of the area operator in LQG

$$\bar{\mu}_i = \sqrt{\frac{|p_i|\Delta}{|p_j p_k|}}$$

Holonomy operators

- The holonomies $\widehat{e^{i\frac{\bar{\mu}_i c_i}{2}}}$ generate a complicated **state-dependent** transformation
- It is convenient to define $\lambda_i(p_i) \propto \text{sgn}(p_i)\sqrt{|p_i|}$, $v = 2\lambda_\theta \lambda_\perp^2$ and relabel the basis states $|p_\theta, p_\perp\rangle \longrightarrow |v\ , \lambda_\theta\rangle$
- $\bar{\mu}_\theta c_\theta = \frac{\sqrt{\Delta |p_\theta|}}{|p_\perp|} c_\theta \equiv b_\theta, \qquad \bar{\mu}_\perp c_\perp = \sqrt{\frac{\Delta}{|p_\theta|}} c_\perp \equiv b$
- Polymeric representation:

$$\hat{v}|v, \lambda_\theta\rangle = v|v, \lambda_\theta\rangle; \qquad \hat{\lambda}_\theta |v, \lambda_\theta\rangle = \lambda_\theta |v, \lambda_\theta\rangle$$

$$[\hat{v}, \widehat{e^{\pm ib}}] = i\hbar \widehat{\{v, e^{\pm ib}\}}; \qquad [\hat{\lambda}_\theta, \widehat{e^{\pm ib_\theta}}] = i\hbar \widehat{\{\lambda_\theta, e^{\pm ib_\theta}\}}$$

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Hamiltonian constraint: Bianchi I term

- "Polimerization": $c_i \to \frac{\sin(\bar{\mu}_i c_i)}{\bar{\mu}_i}$

- $c_\theta p_\theta \to 2\kappa\gamma :\widehat{v\sin(b_\theta)}: , \qquad c_\perp p_\perp \to 2\kappa\gamma :\widehat{v\sin(b)}: \qquad (\kappa = \pi G\hbar)$

$$2:\widehat{v\sin(b)}: \equiv \hat{\Omega} = \sqrt{|\hat{v}|}\left[\widehat{\text{sign}(v)}\widehat{\sin(b)} + \widehat{\sin(b)}\widehat{\text{sign}(v)}\right]\sqrt{|\hat{v}|}$$

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- One can restrict to $v, \lambda_\theta \in \mathbb{R}^+$. **Zero-volume states decoupled**

- Bianchi I Hamiltonian constraint:

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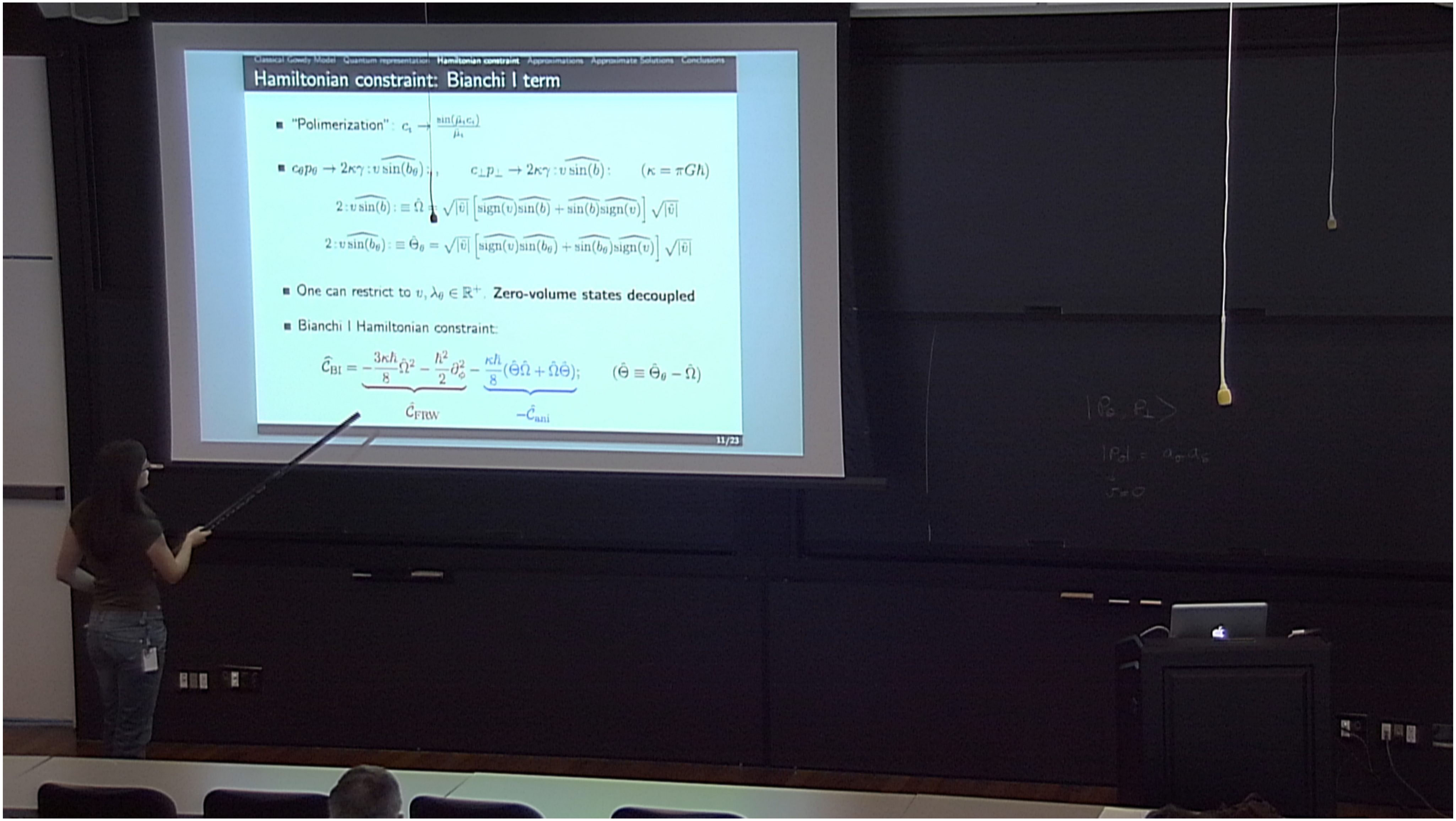

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$\hat{C}_{\mathrm{FRW}}$
$-\hat{C}_{\mathrm{ani}}$
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Superselection sectors

- Kinematical Hilbert space non-separable since $v, \lambda_\theta \in \mathbb{R}^+$.
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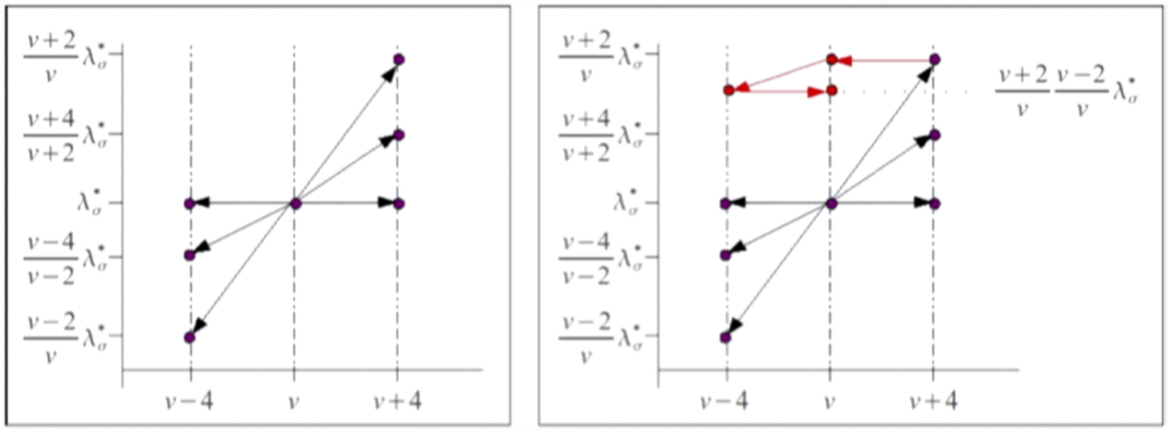

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- $\widehat{\mathcal{C}}_{\rm BI}$ acting on $|v, \lambda_\theta^*\rangle \otimes |\phi\rangle$
  - $v = \varepsilon + 4n$, $\varepsilon \in (0, 4]$, $n \in \mathbb{N}$ semilattice of step 4
  - $\lambda_\theta = w_\varepsilon \lambda_\theta^*$, $w_\varepsilon \in \mathcal{W}_\varepsilon$: countable set, dense in $\mathbb{R}^+$

    (e.g: $\varepsilon = \lambda_a^* = 1 \rightarrow \lambda_a = 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ..., 3, \frac{3}{5}, \frac{3}{7}, ..., \left(\frac{11}{7}\right)^2 \frac{5}{3}, ...$)
  - $|v, \lambda_\theta\rangle \rightarrow |v, \Lambda\rangle$, $\Lambda = \log(\lambda_\theta)$, $\Lambda - \Lambda^\star = w_\varepsilon \in \mathcal{W}_\varepsilon$
- The subspaces spanned by states $|\varepsilon + 4n, \Lambda^\star + w_\varepsilon\rangle$, with $n \in \mathbb{N}$ and $w_\varepsilon \in \mathcal{W}_\varepsilon$, provide separable superselection sectors

$$\mathcal{H}_{(\varepsilon,\Lambda^\star)} = \mathcal{H}_\varepsilon \otimes \mathcal{H}^\varepsilon_{\Lambda^\star}$$

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Gowdy Hamiltonian constraint

$$\hat{\mathcal{C}}_{\mathrm{G}} = \underbrace{-\frac{3\kappa\hbar}{8}\hat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2}_{\hat{\mathcal{C}}_{\mathrm{FRW}}} - \underbrace{\frac{\kappa\hbar}{8}(\hat{\Theta}\hat{\Omega} + \hat{\Omega}\hat{\Theta})}_{-\hat{\mathcal{C}}_{\mathrm{ani}}} + \underbrace{\frac{2\kappa\hbar}{\beta}\widehat{e^{2\Lambda}}\hat{H}_0}_{\hat{\mathcal{C}}_0} + \underbrace{\frac{\kappa\hbar\beta}{4}\widehat{e^{-2\Lambda}}\hat{\Omega}^2\hat{H}_{\mathrm{I}}}_{\hat{\mathcal{C}}_{\mathrm{I}}}$$

- $\beta \equiv [G\hbar/(16\pi^2\gamma^2\Delta)]^{1/3}$
- $\hat{H}_0$: free field contribution of both inhomogeneities
- $\hat{H}_{\mathrm{I}}$: self-interaction contribution
- Constraint defined on: $\mathcal{H}_\varepsilon \otimes \mathcal{H}^\varepsilon_{\Lambda^\star} \otimes \mathcal{H}_\phi \otimes \mathcal{F}^\xi \otimes \mathcal{F}^\varphi$

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- In order to construct solutions the problematic terms are:

1. Self-interaction term ($\hat{\mathcal{C}}_{\rm I}$): $\hat{H}_{\rm I}$ creates and annihilates a pair of particles in each mode
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   - It does not commute with $\hat{\Omega}^2$
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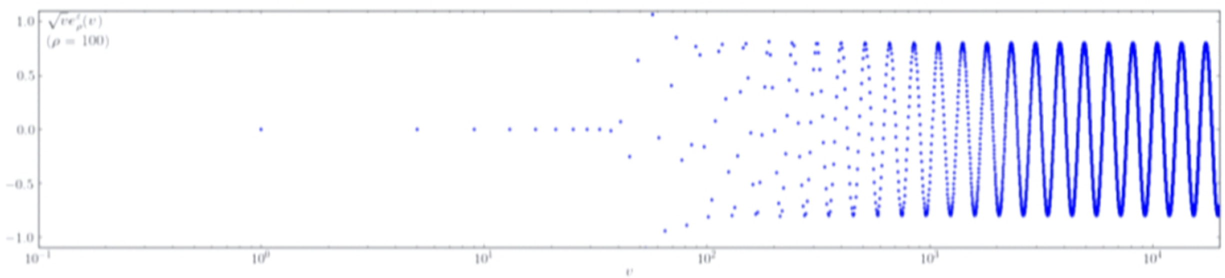
Approximations

- **Approximation strategy:**

  With the aim of obtaining approximations for the problematic terms we consider the **eigenstates** $|e^{\varepsilon}_{\rho}\rangle$ of the **FRW operator** $\hat{\Omega}^2$:

  - They provide a resolution of the identity in $\mathcal{H}_\varepsilon$: $\mathbb{I}_{\mathcal{H}_\varepsilon} = \int_0^\infty d\rho |e^{\varepsilon}_{\rho}\rangle\langle e^{\varepsilon}_{\rho}|$
  - Given an eigenvalue $\rho^2 \in \mathbb{R}^+$, $e^{\varepsilon}_{\rho}(v) \equiv \langle v | e^{\varepsilon}_{\rho}\rangle$ is a real function



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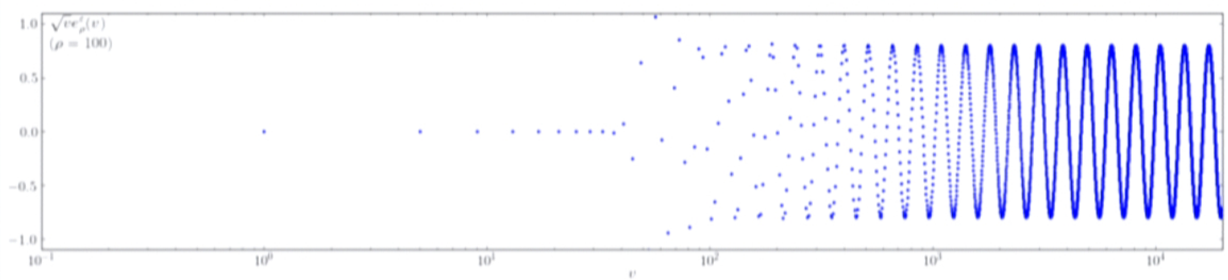

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Approximations: Anisotropy term - I

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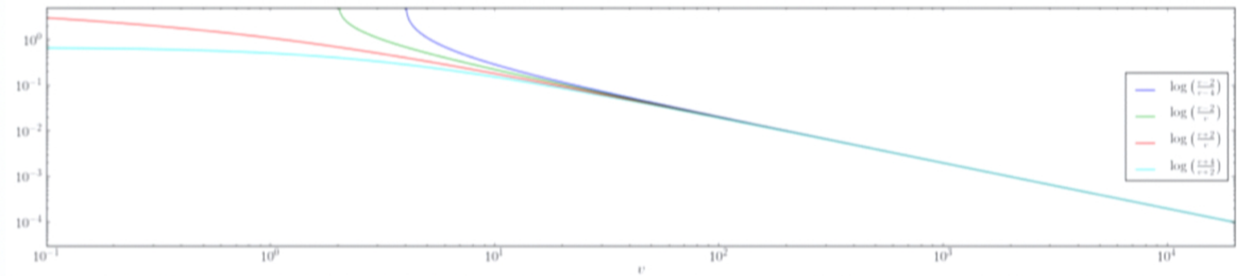

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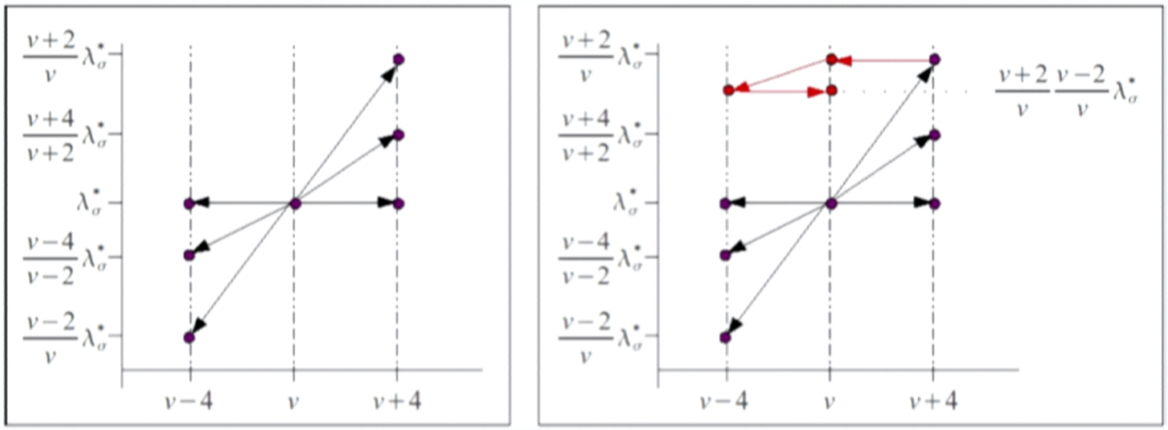

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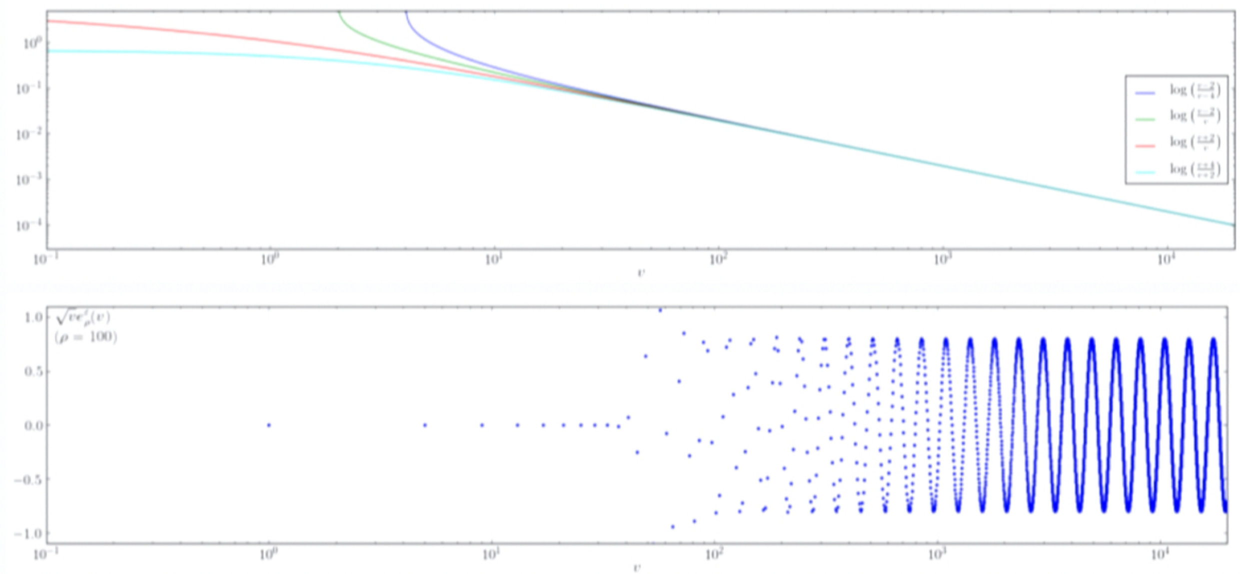
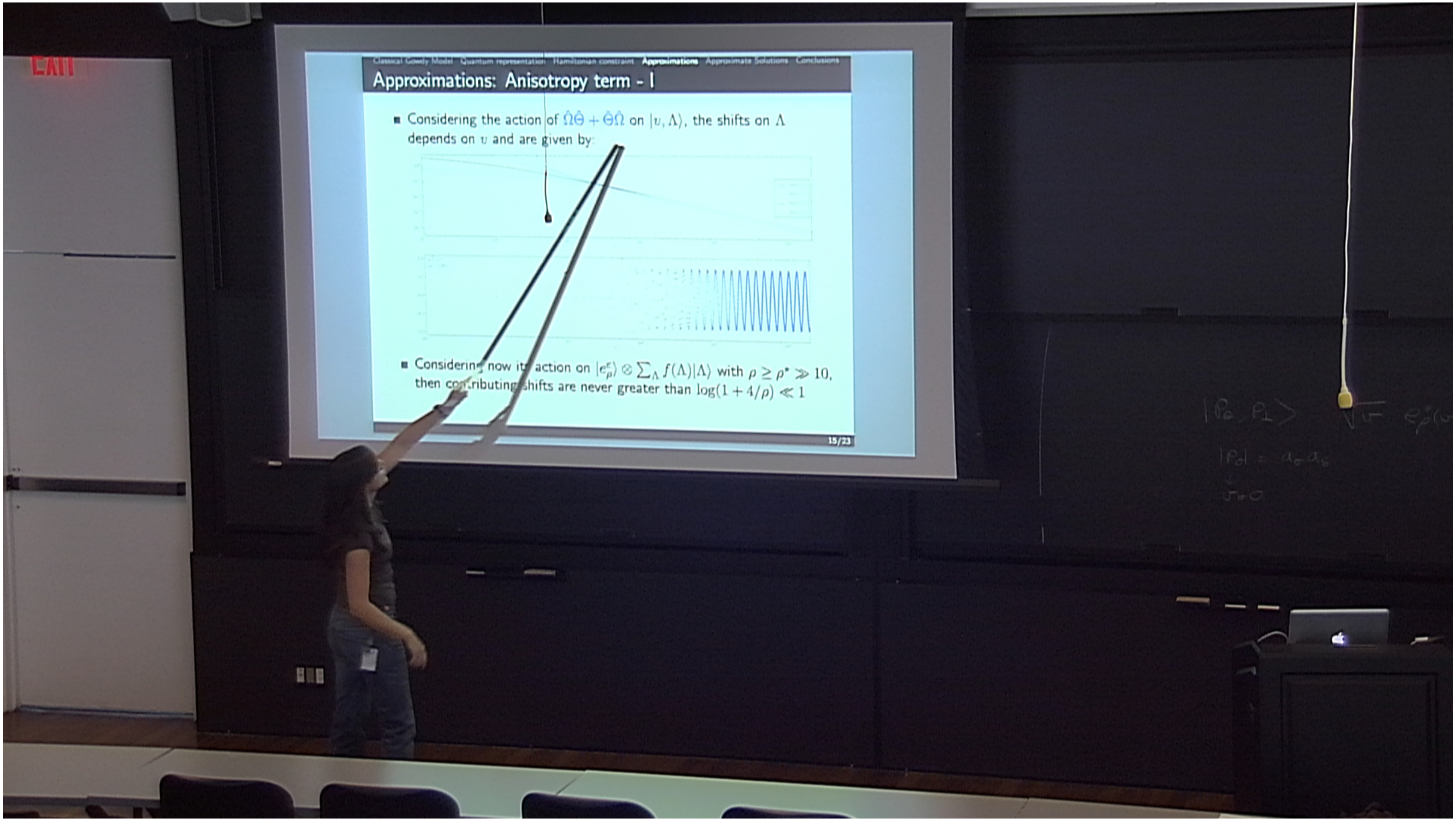


- Considering now its action on $|e^{\varepsilon}_{\rho}\rangle \otimes \sum_{\Lambda} f(\Lambda)|\Lambda\rangle$ with $\rho \geq \rho^{\star} \gg 10$, then contributing shifts are never greater than $\log(1 + 4/\rho) \ll 1$


EXIT
Classical Gowdy Model Quantum representation Hamiltonian constraint Approximations Approximate Solutions Conclusions
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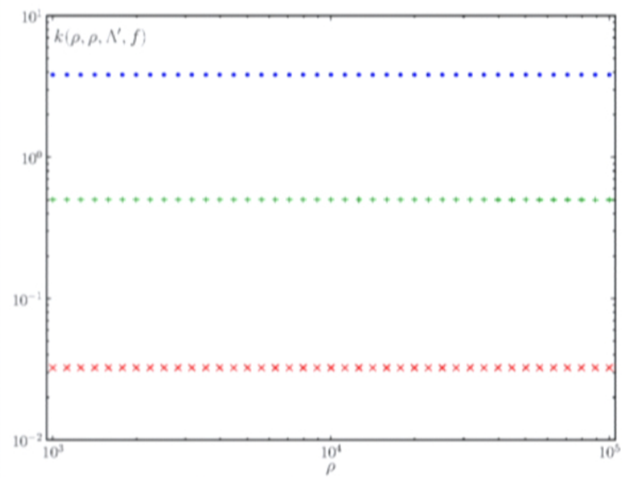
Approximations: Anisotropy term - III

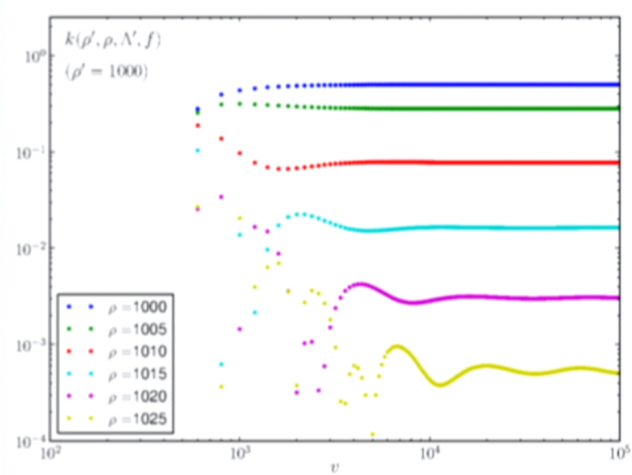
- The approximation has been checked numerically by computing

$$\langle\Lambda'|\otimes\langle e^{\epsilon}_{\rho'}|\left[2\hat{\Omega}'\hat{\Theta}'-(\hat{\Omega}\hat{\Theta}+\hat{\Theta}\hat{\Omega})\right]\sum_{\Lambda}f(\Lambda)|e^{\varepsilon}_{\rho}\rangle\otimes|\Lambda\rangle$$

with $f(\Lambda)=\frac{1}{\sqrt{2\pi\sigma_{\Lambda}^2}}e^{-\frac{1}{2\sigma_{\Lambda}^2}(\Lambda-\bar{\Lambda})^2}$ and different values of the step q_{ε}



Diagonal matrix elem. $(\sigma_{\Lambda}=0.5,1.0,2.5)$



Non-diagonal matrix elem. $(\sigma_{\Lambda}=1.0)$

$$\Lambda'=0.1,\ \bar{\Lambda}=0.0,\quad \mathcal{W}_{\varepsilon}\ni q_{\epsilon}\gtrsim\log(1+4/\rho^{\star})\ (\rho^{\star}=1000)$$

Approximations: Interaction and anisotropy terms

- $\hat{\mathcal{C}}'_{\mathrm{G}} = -\frac{3\kappa\hbar}{8}\hat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2 - \frac{\kappa\hbar}{4}\hat{\Omega}'\hat{\Theta}' + \frac{2\kappa\hbar}{\beta}\widehat{e^{2\Lambda}}\hat{H}_0 + \frac{\kappa\hbar\beta}{4}\widehat{e^{-2\Lambda}}\hat{\Omega}^2\hat{H}_{\mathrm{I}}$
  - $\hat{\Omega}'$ does not commute $\hat{\Omega}^2$
  - $\hat{\Theta}'$ does not commute with $\widehat{e^{2\Lambda}}$ and $\widehat{e^{-2\Lambda}}$
  - Presence of the self-interaction contribution $\hat{H}_{\mathrm{I}}$
- Considering anisotropy Gaussian-like profiles $|\psi_{\bar{\Lambda}}\rangle$ peaked at $s=0$

$$\begin{aligned} |\psi_{\bar{\Lambda}}\rangle &= \int_{-4/q_\varepsilon}^{4/q_\varepsilon} ds \sqrt{\frac{q_\varepsilon}{4\sigma_s\sqrt{\pi}\cos[x(s)]}}\, e^{-\frac{x(s)^2}{2\sigma_s^2} - ix(s)\frac{\bar{\Lambda}}{q_\varepsilon}} |e_s^{(1)}\rangle \\ &\simeq \sum_{\Lambda\in\mathcal{L}_{\Lambda^\star}^{q_\varepsilon}} \frac{\sqrt{\sigma_s}}{\sqrt[4]{\pi}} e^{-\frac{\sigma_s^2}{2q_\varepsilon^2}(\Lambda-\bar{\Lambda})^2} |\Lambda\rangle, \qquad \text{when } \sigma_s \ll \pi/2 \end{aligned}$$

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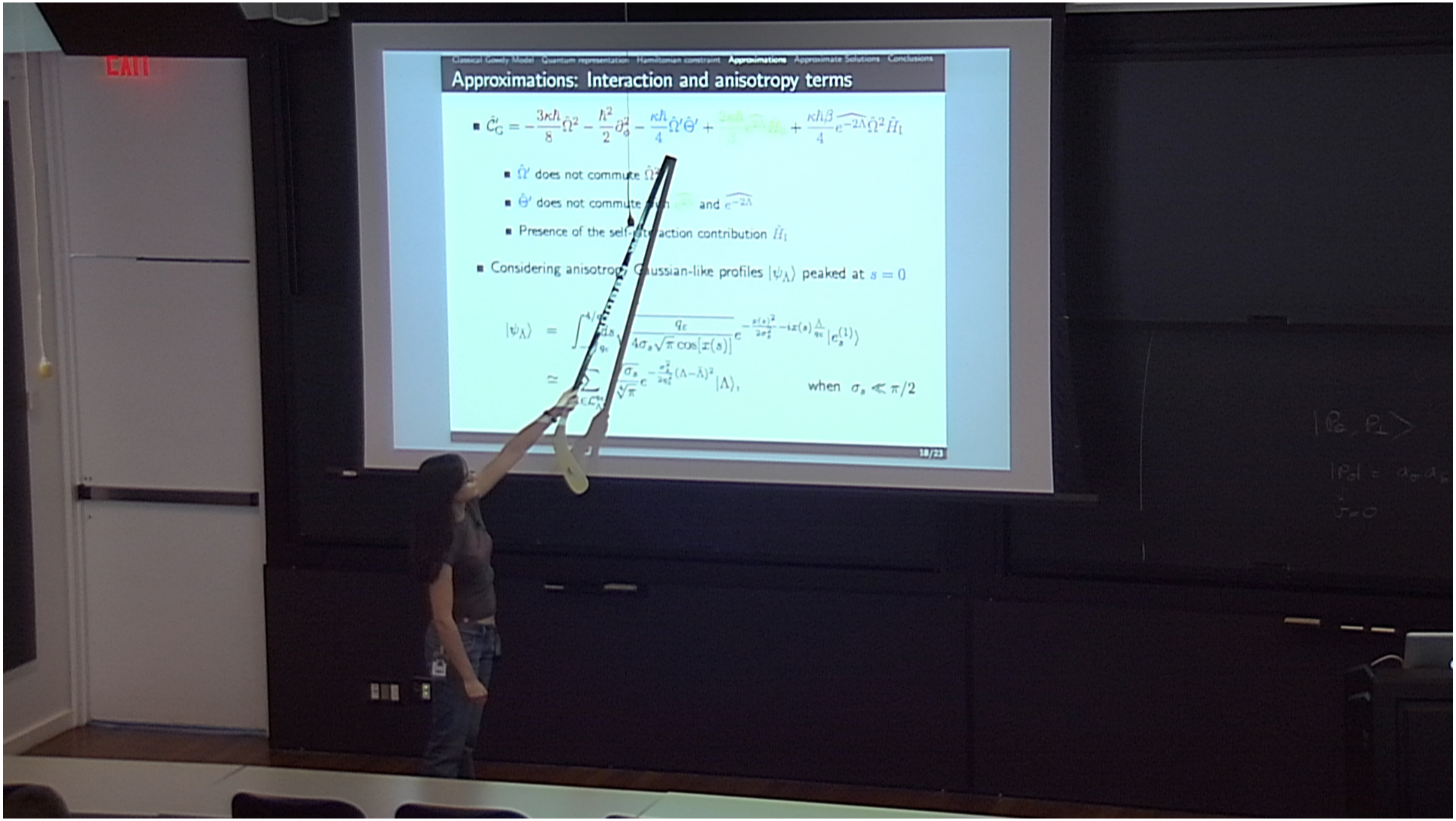
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Approximate Solutions - I

- Solvable Hamiltonian constraint:

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- $\hat{\mathcal{C}}_{\text{app}}$ exact solutions:

$$(\Psi| = \int_{-\infty}^{\infty} d\phi \sum_{v\in\mathcal{L}_\varepsilon^+} \sum_{\Lambda\in\mathcal{L}_{\Lambda^\star}^{q\varepsilon}} \sum_{\mathfrak{n}} \Psi(\phi, v, \Lambda, \mathfrak{n})\langle\phi, v, \Lambda, \mathfrak{n}|$$

with profiles given by

$$\Psi(\phi, v, \Lambda, \mathfrak{n}) = \int_{-\infty}^{\infty} dp_\phi \Psi(p_\phi, \Lambda, \mathfrak{n}) e^{\varepsilon}_{\rho(p_\phi\Lambda,\mathfrak{n})}(v) e_{p_\phi}(\phi)$$

where

$$\rho(p_\phi, \Lambda, \mathfrak{n}) = \sqrt{\frac{4}{3\kappa\hbar}p_\phi^2 + \frac{16}{3\beta}e^{2\Lambda}H_0(\mathfrak{n})}$$

  - Note that $H_0(\mathfrak{n}) = \langle\mathfrak{n}|\hat{H}_0|\mathfrak{n}\rangle > 0$

Approximate Solutions - II

- Approximate solutions for Gowdy: $\Psi(p_\phi, \Lambda, \mathfrak{n}) = \Psi(p_\phi, \mathfrak{n})\psi(\Lambda)$

$$\psi(\Lambda) = \frac{\sqrt{\sigma_s}}{\sqrt[4]{\pi}} e^{-\frac{\sigma_s^2}{2q_\varepsilon^2}(\Lambda-\bar{\Lambda})^2}, \quad \text{with} \quad \sigma_s^2 \ll \frac{\pi}{2}, \quad \bar{\Lambda} \gg \frac{q_\varepsilon^2}{\sigma_s^2}$$

  Additionally it is necessary to demand:

  - $\rho \gg 10 \quad \Rightarrow \quad p_\phi^2 \gg 75\kappa\hbar \approx 200G\hbar^2$ (large enough field momentum)
  - small content of inhomogeneities and the n-particle states $|\mathfrak{n}\rangle$ must satisfy the momentum constraint ($\hat{C}_\theta|\mathfrak{n}\rangle = 0$)

- Convenient choice for q_ε:

  - p_ϕ is a constant of motion and provides a natural scale in the system
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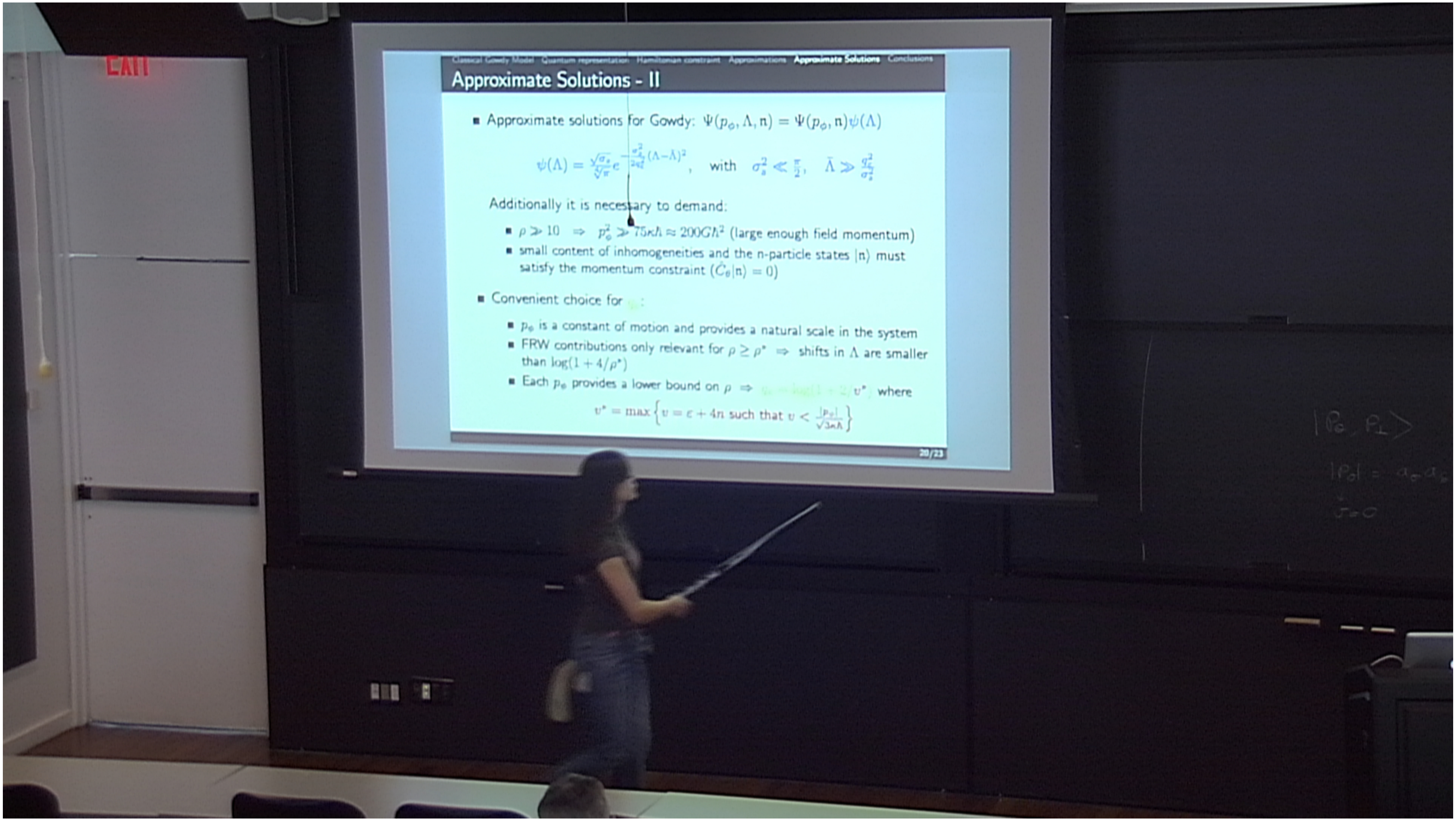
$$v^\star = \max\left\{v = \varepsilon + 4n \text{ such that } v < \frac{|p_\phi|}{\sqrt{3\kappa\hbar}}\right\}$$


Classical Gowdy Model Quantum representation Hamiltonian constraint Approximations Approximate Solutions Conclusions
Approximate Solutions - II
Approximate solutions for Gowdy: $\Psi(p_\phi, \Lambda, \mathfrak{n}) = \Psi(p_\phi, \mathfrak{n})\psi(\Lambda)$
with $\sigma_s^2 \ll \frac{\pi}{2}$,
Additionally it is necessary to demand:
$\rho \gg 10 \Rightarrow p_\phi^2 \gg 75\kappa\hbar \approx 200G\hbar^2$ (large enough field momentum)
small content of inhomogeneities and the n-particle states $|\mathfrak{n}\rangle$ must satisfy the momentum constraint $(\hat{C}_\theta|\mathfrak{n}\rangle = 0)$
Convenient choice for
p_ϕ is a constant of motion and provides a natural scale in the system
FRW contributions only relevant for $\rho \geq \rho^*$ $\Rightarrow$ shifts in Λ are smaller than $\log(1 + 4/\rho^*)$
Each p_ϕ provides a lower bound on ρ $\Rightarrow$ where
$v^* = \max\{v = \varepsilon + 4n$ such that $v <$
20/23

Conclusions

- We have **completed the quantization** of the linearly polarized Gowdy T3 model with an inhomogeneous scalar field using hybrid techniques in LQC
- The analogs of the cosmological singularities are eliminated quantum mechanically
- We have studied **approximation methods** in the context of LQC to construct **quantum solutions** of inhomogeneous and anisotropic cosmological models
- Using the behavior of the FRW eigenfunctions we have approximated the anisotropy term by other simpler operator that factorizes
- We have constructed states with peaked anisotropy profiles, such that both anisotropy and self-interaction terms can be disregarded, and thus provided **approximate quantum solution** for the Gowdy model

Outlook

- Analysis of the quantum evolution of these solutions to check the robustness of the bounce scenario in presence of inhomogeneities
- Apply the same analysis for more realistic scenario: FRW with cosmological perturbations

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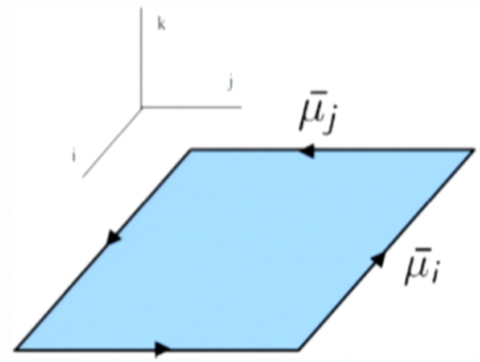
Thanks you for your attention

Improved dynamics prescription

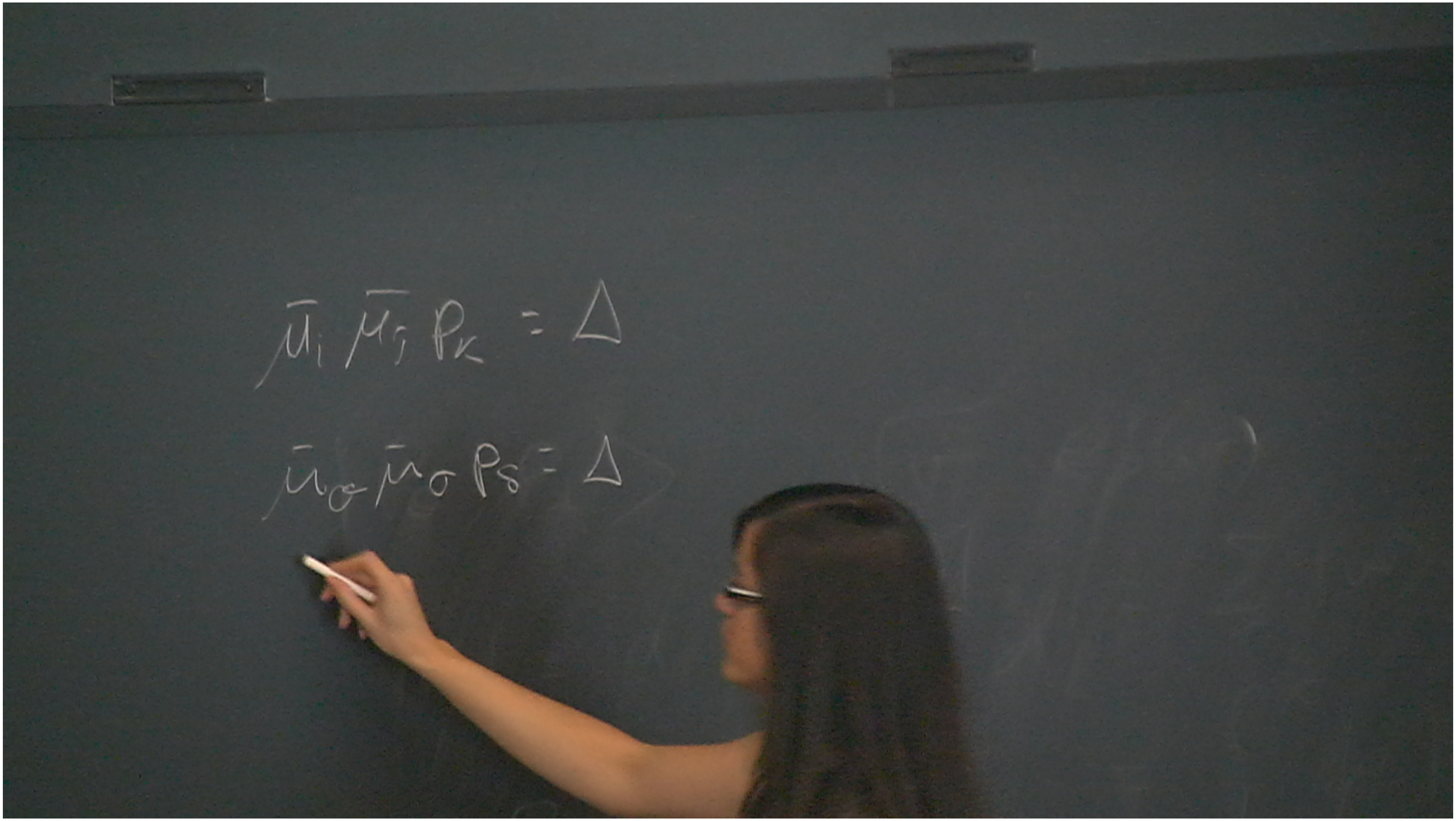
- In order to define the curvature tensor of the connection, we take a rectangular loop of holonomies.

- The limit when the enclosed area tends to zero is not well-defined, but there exists a minimum nonzero fiducial length for the holonomies, measured by $\bar{\mu}_j$, $j \in \{\theta, \sigma, \delta\}$.



- The kinematical area $\bar{\mu}_i \bar{\mu}_j p_k$ of that loop equals the minimum nonzero eigenvalue Δ of the area operator in LQG

$$\bar{\mu}_i = \sqrt{\frac{|p_i|\Delta}{|p_j p_k|}}$$


μ̄i μ̄j Pk = Δ
μ̄σ μ̄σ Ps = Δ