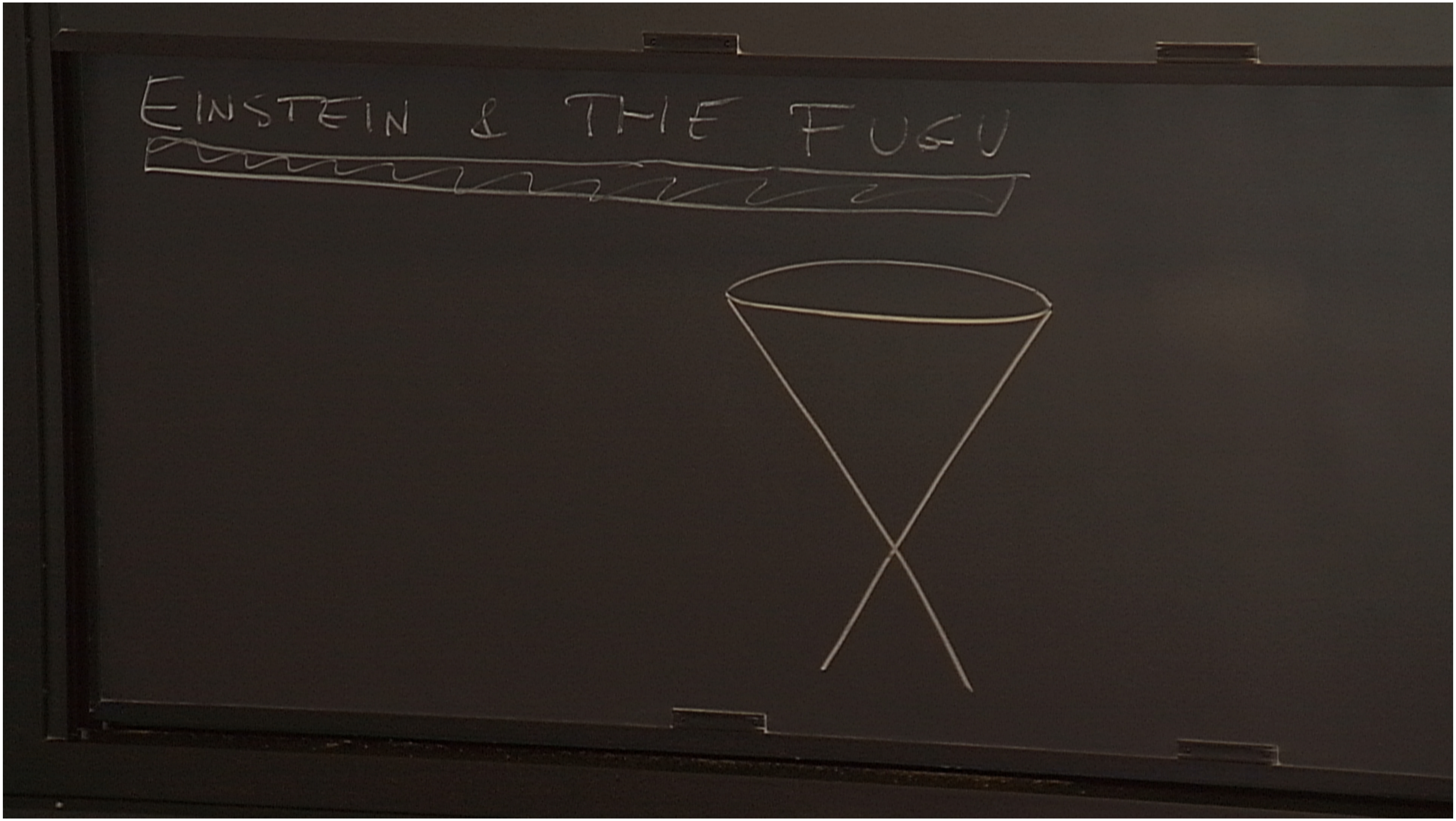
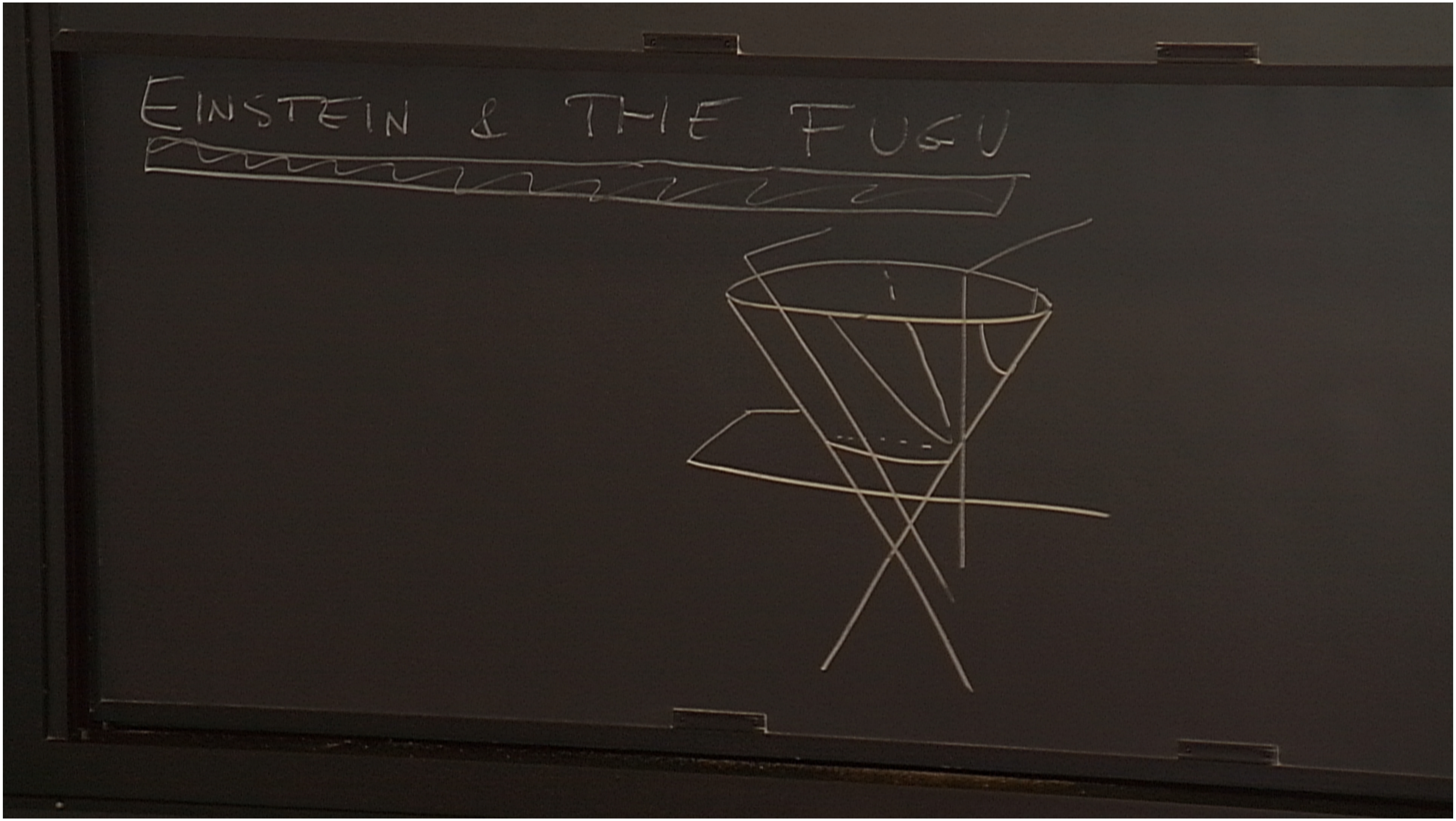
Title: Einstein and the Fugu

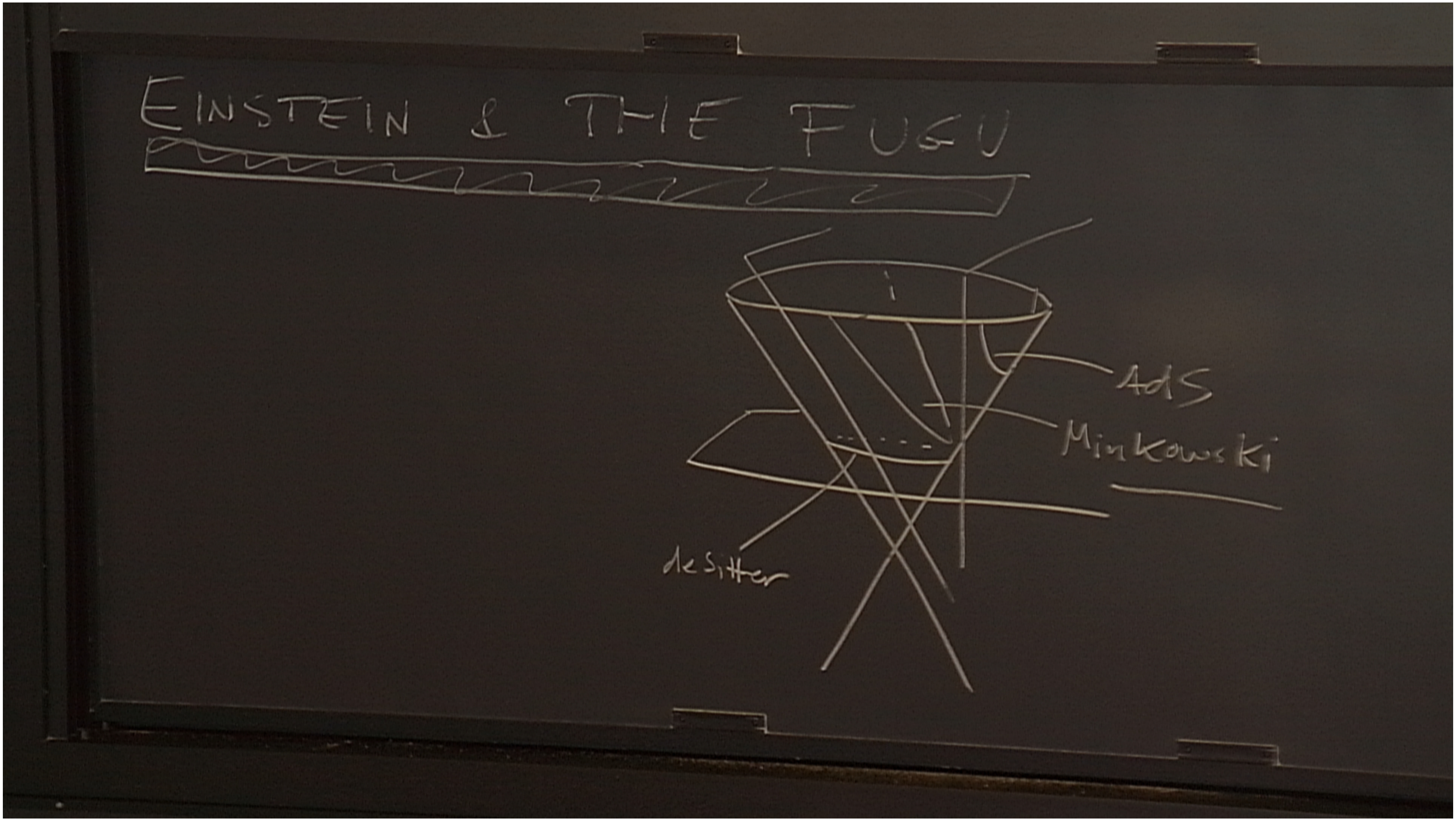
Date: Oct 10, 2013 10:00 AM

URL: http://pirsa.org/13100101

Abstract:


EINSTEIN & THE FUGU


EINSTEIN & THE FUGU

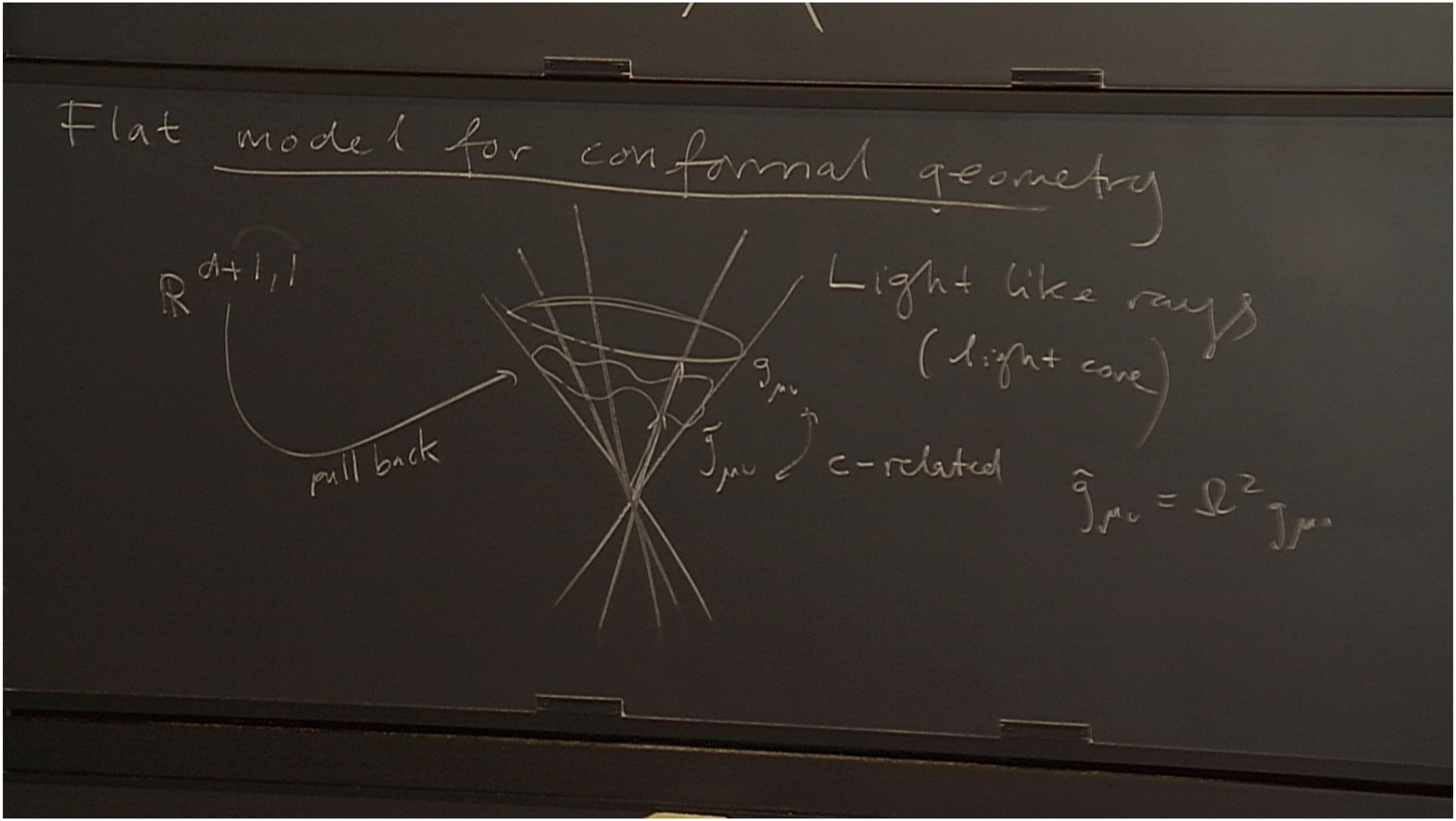

EINSTEIN & THE FUGU
AdS
Minkowski
de Sitter

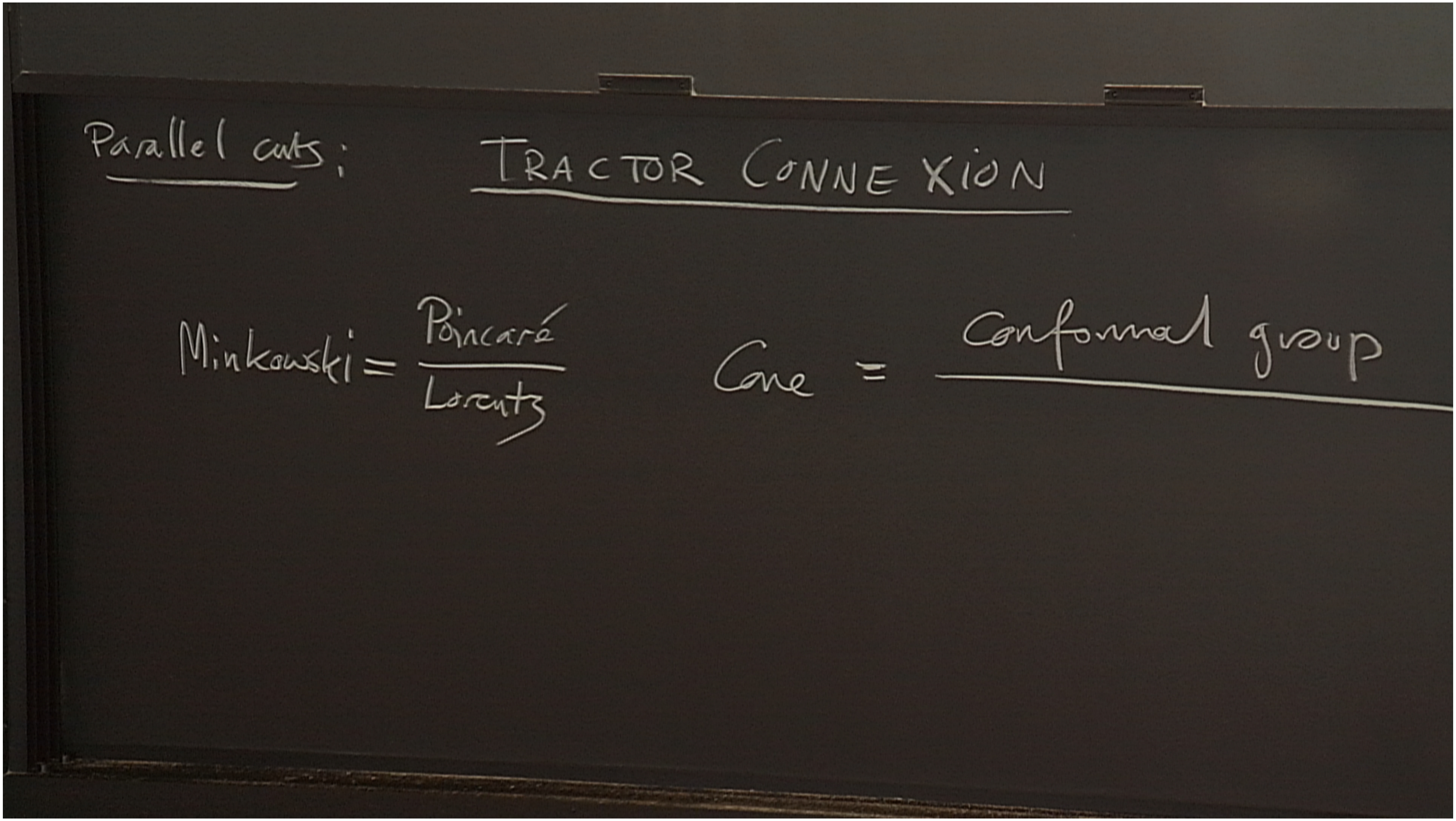
Flat model for conformal geometry

$\mathbb{R}^{d+1,1}$

Light like rays

(light cone)


Flat model for conformal geometry
$\mathbb{R}^{d+1,1}$
pull back
Light like rays
(light cone)
$g_{\mu\nu}$
$\tilde{g}_{\mu\nu}$
c-related
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$


Parallel cuts:
TRACTOR CONNEXION
Minkowski = Poincaré / Lorentz
Cone = Conformal group /

...nts:

TRACTOR CONNEXION

$$\text{...owski} = \frac{\text{Poincaré}}{\text{Lorentz}}$$

$$\text{Cone} = \frac{\overbrace{\text{Conformal group}}^{SO(4,2)}}{\text{Stab (lightlike ray)}}$$

$\to$ ($\underset{\mathbb{R}^4}{\text{C-boost}}$, $\underset{\mathbb{R}_+}{\text{dilations}}$, $\underset{SO(3,1)}{\text{Lorentz}}$)

Parallel cuts:

TRACTOR CONNEXION

$$\text{Minkowski} = \frac{\text{Poincaré}}{\text{Lorentz}}$$

$$\text{Cone} = \frac{\text{Conformal group}}{\text{Stab (lightlike}}$$

$SO(4,$

c-boost, $\mathbb{R}^4$ /

$SO(4,2)$

$g^{-1} d g$

Parallel cuts:

TRACTOR CONNEXION

$$\text{Minkowski} = \frac{\text{Poincaré}}{\text{Lorentz}} \qquad \text{Cone} = \frac{\overset{so(4,}{\text{Conformal group}}}{\text{Stab (lightlike}}$$

$\searrow$ c-boost, $\mathbb{R}^4$ /

$$\theta = g^{-1}d\overset{so(4,2)}{\overset{\downarrow}{g}} \quad \text{Cartan Maurer} \qquad d\theta + \theta\wedge\theta = 0$$

$so(4,2)$ inv^{t}

Parallel cuts:

TRACTOR CONNEXION

$$\text{Minkowski} = \frac{\text{Poincaré}}{\text{Lorentz}} \qquad \text{Cone} = \frac{\text{Conformal group}}{\text{Stab (lightlike}}$$

so(4,

c-boost, $\mathbb{R}^4$

so(4,2)

$$\theta = g^{-1}dg \quad \text{Cartan Maurer} \quad F_\theta = d\theta + \theta\wedge\theta = 0$$

so(4,2) inv^t

θ - flat connexion

CONNEXION

$$\text{Cone} = \frac{\overbrace{\text{Conformal}}^{SO(4,2)}\ \text{group}}{\text{Stab (lightlike ray)}}$$

$\searrow$ C-boost, dilations, Lorentz

$\mathbb{R}^4$ / $\mathbb{R}_+$, $SO(3,1)$

$$F_\theta = d\theta + \theta \wedge \theta = 0$$

θ-flat connexion

$$\nabla^T = d + \theta$$

C-flat

tractor connexion

Pla

CONNEXION

$$\text{Cone} = \frac{\overset{SO(4,2)}{\text{Conformal group}}}{\text{Stab (lightlike ray)}}$$

c-boost $\mathbb{R}^4$, dilations $\mathbb{R}_+$, Lorentz $SO(3,1)$

$$F_\theta = d\theta + \theta \wedge \theta = 0$$

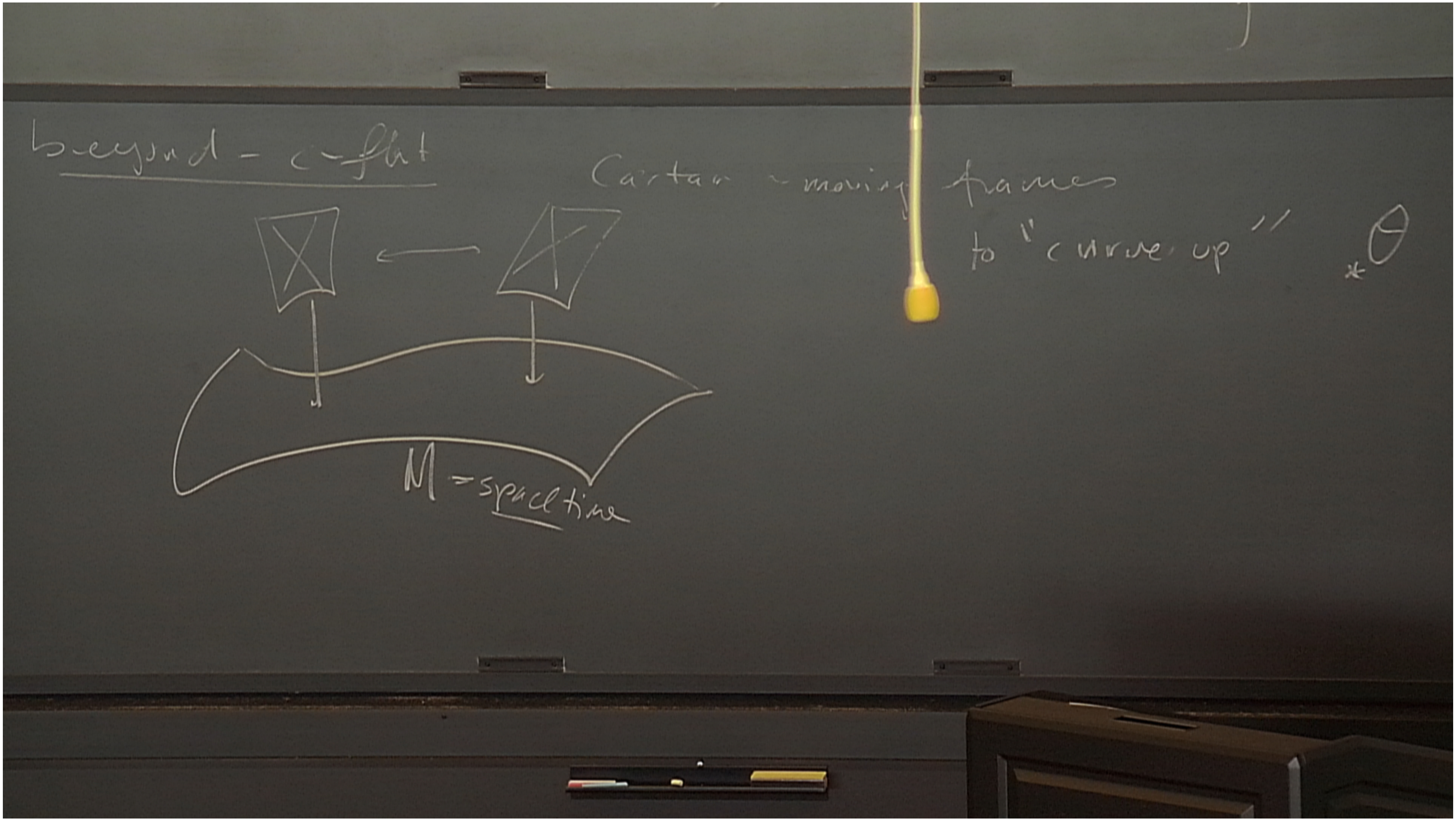
θ-flat connexion

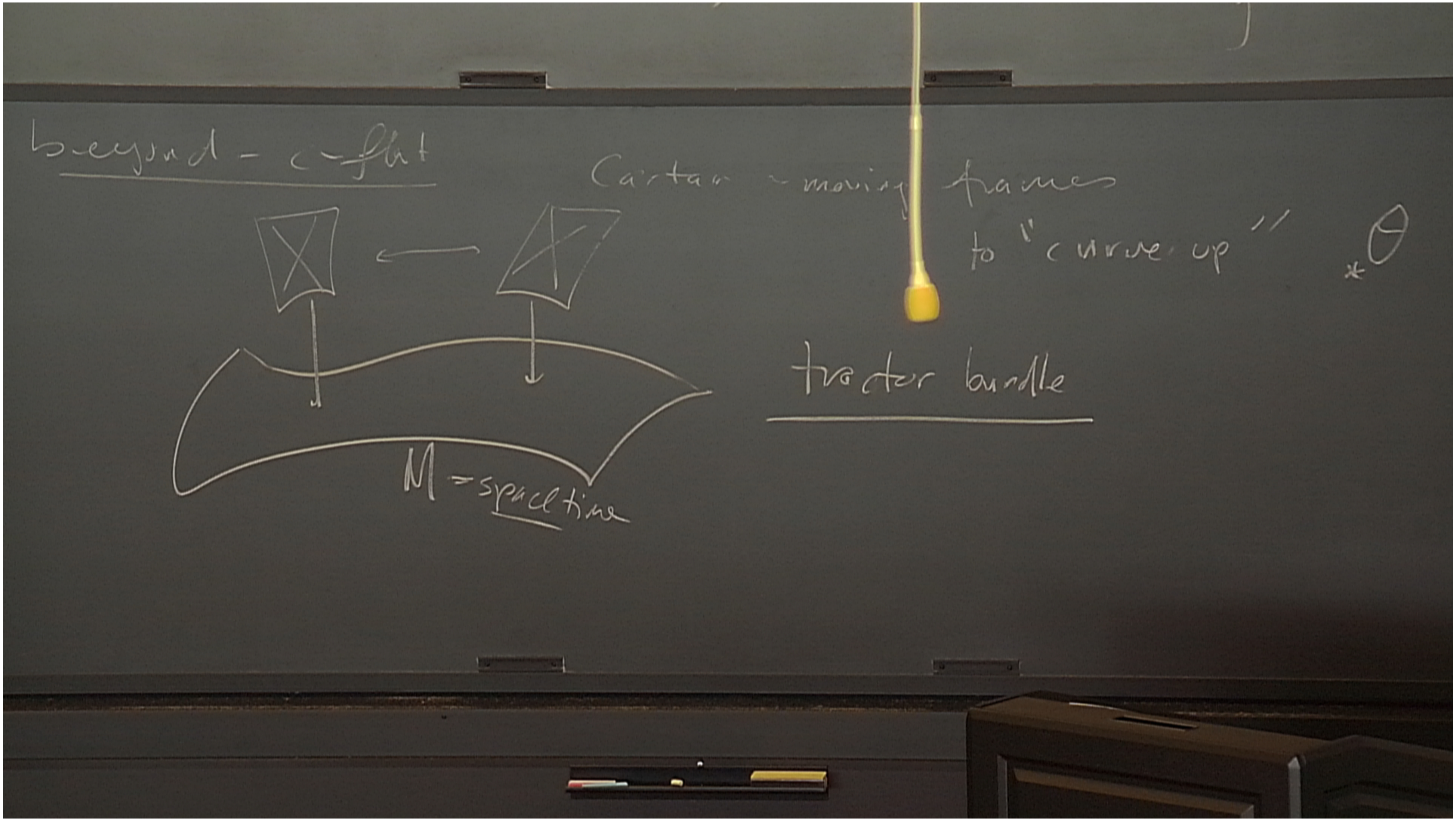
$$\nabla^T = d + \theta^*$$

pullback

c-flat

tractor connexion


Cartan ~ moving frames
to "curve up"
M = spacetime


beyond - c-flat
Cartan - moving frames
to "curve up"
tractor bundle
M = spacetime

Tractor connexion & physics

∇^T flat

(zero curvature)

$\nabla^T I = 0$

parallel sections

Tractor connexion & physics

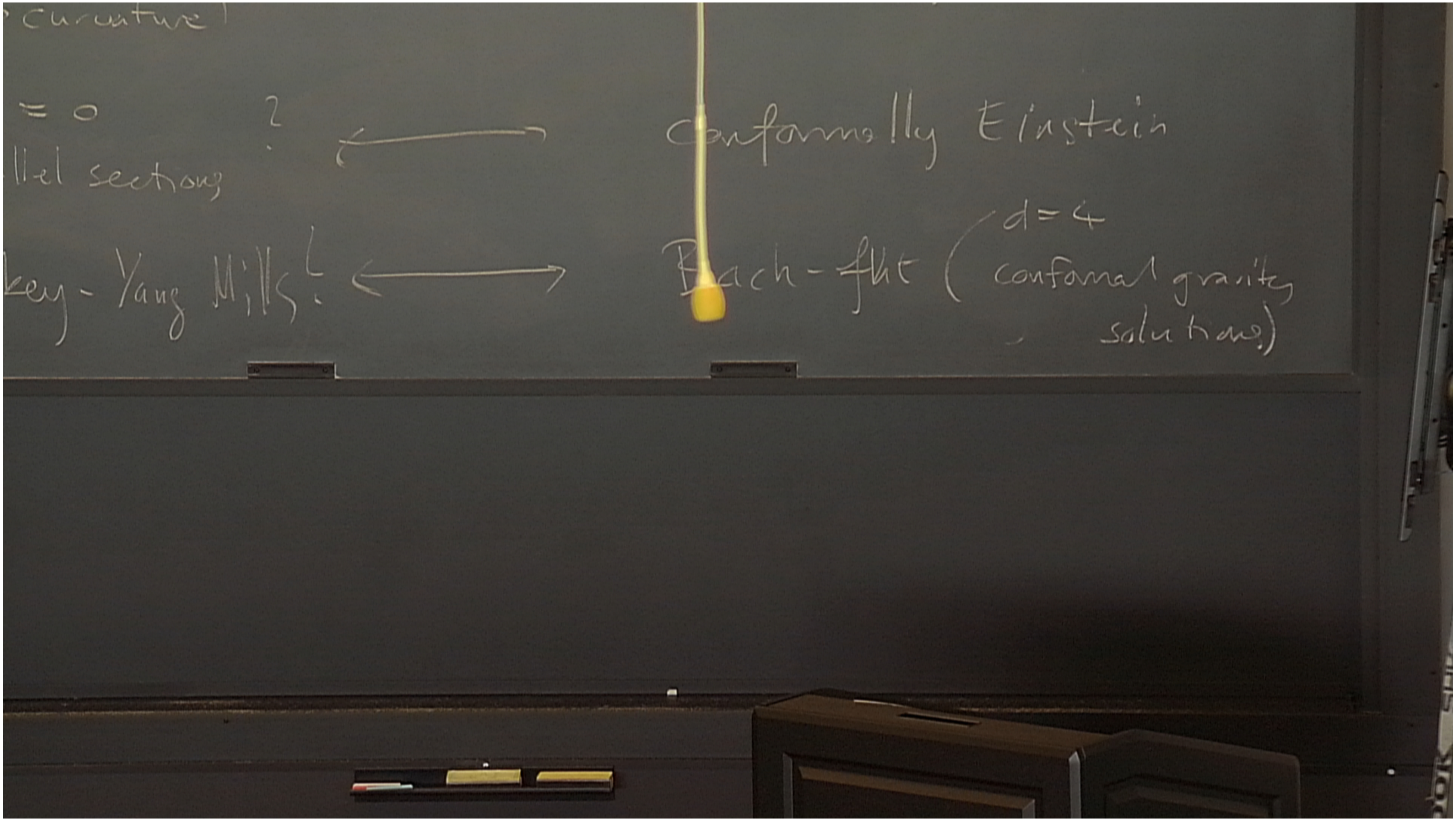
∇^T flat ? $\longrightarrow$

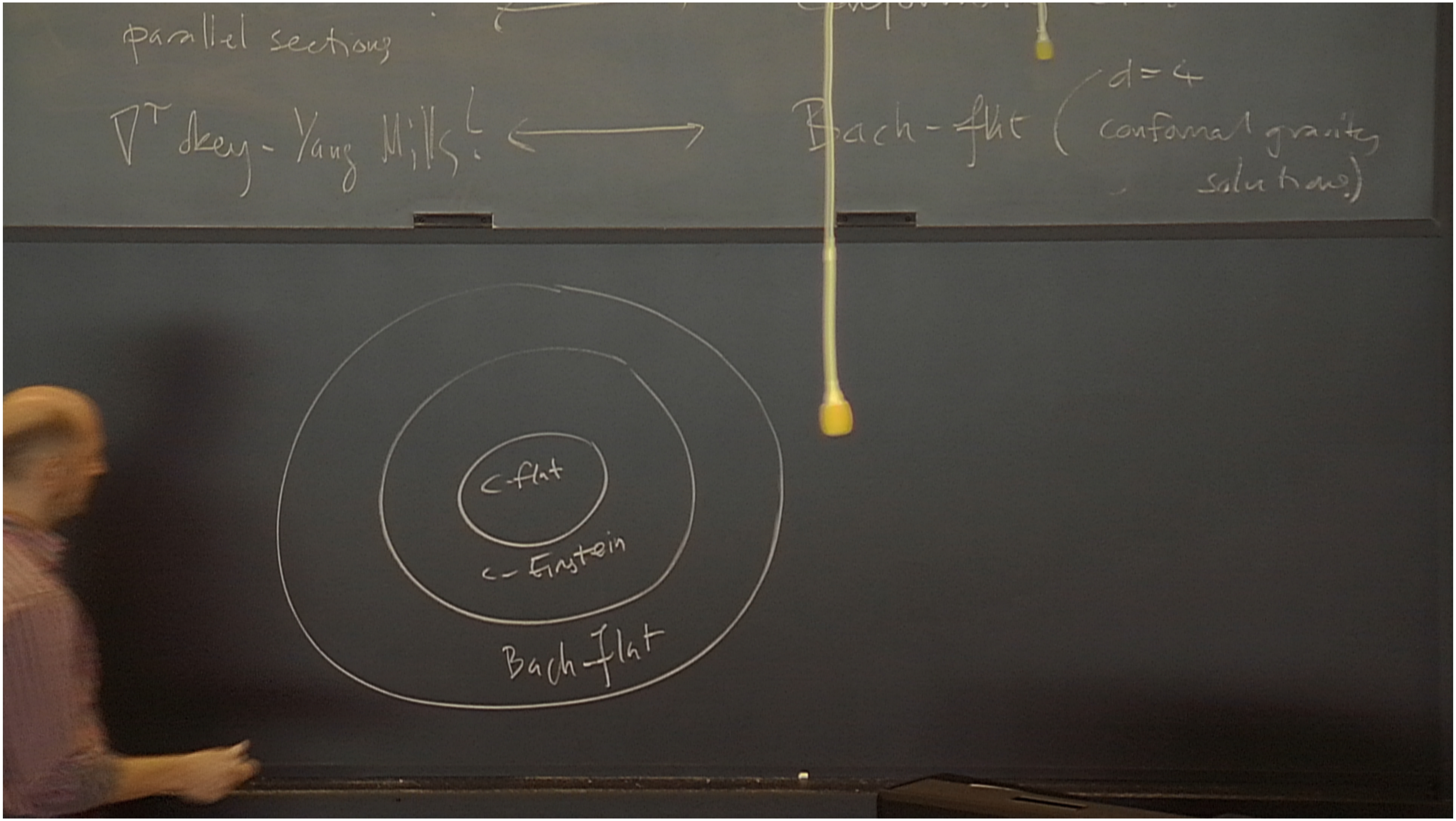
(zero curvature)

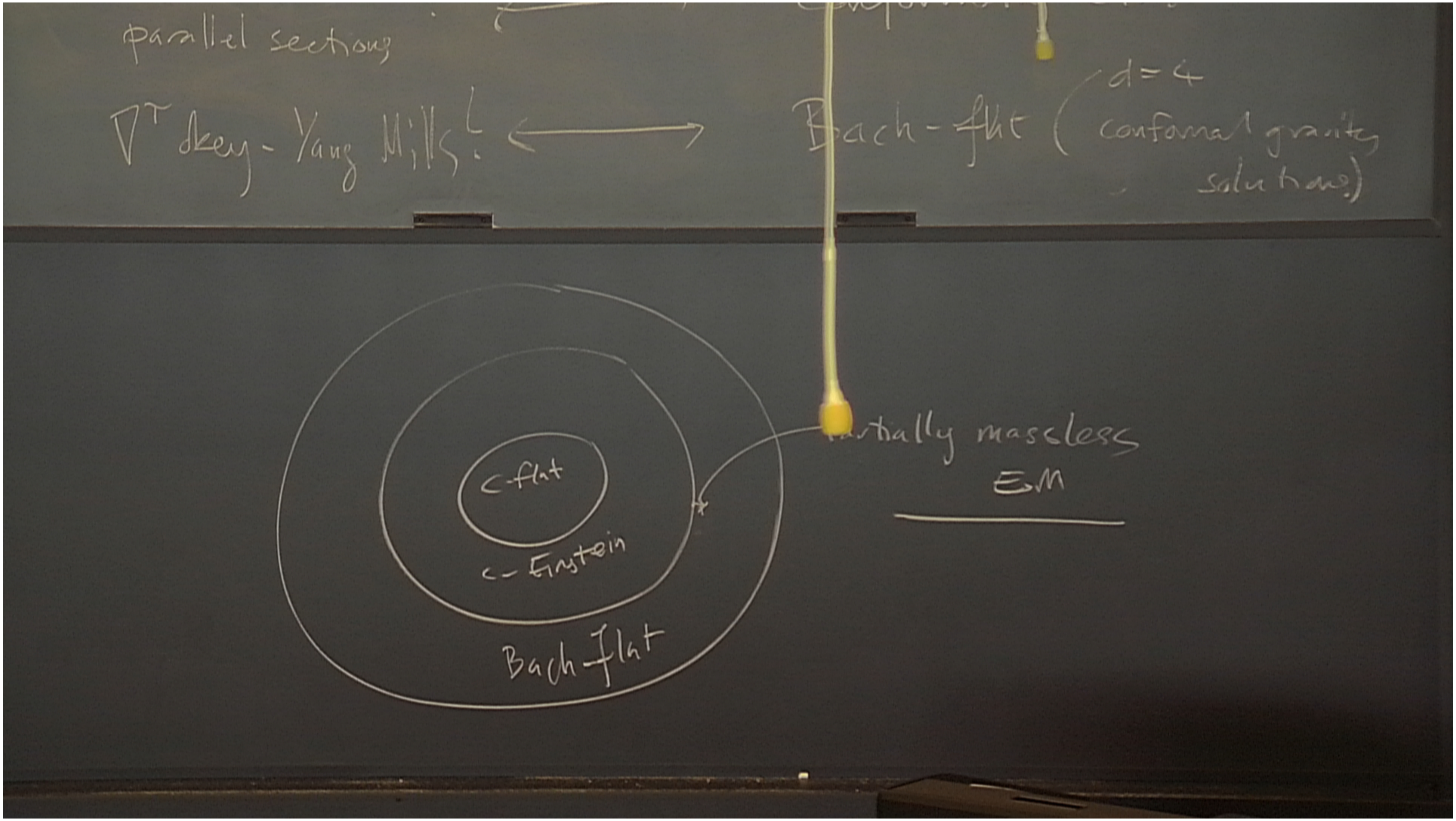
$\nabla^T I = 0$? $\longrightarrow$

parallel sections

∇^T obey Yang Mills! $\longrightarrow$


curvature)
= 0
?
llel sections
Conformally Einstein
key-Yang Mills!
d=4
Bach-flat (conformal gravity
solutions)


parallel sections
∇^T skey-Yang Mills!
Bach-flat
d=4
conformal gravity
solutions)
c-flat
c-Einstein
Bach-flat

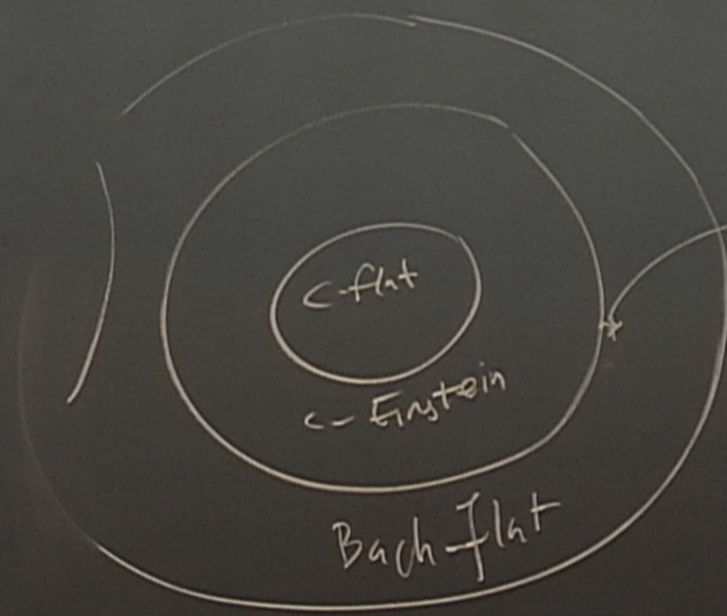

parallel sections
Yang Mills
Bach-flat (
d=4
conformal gravity
solutions)
partially massless
EM
c-flat
c-Einstein
Bach-flat

parallel sections

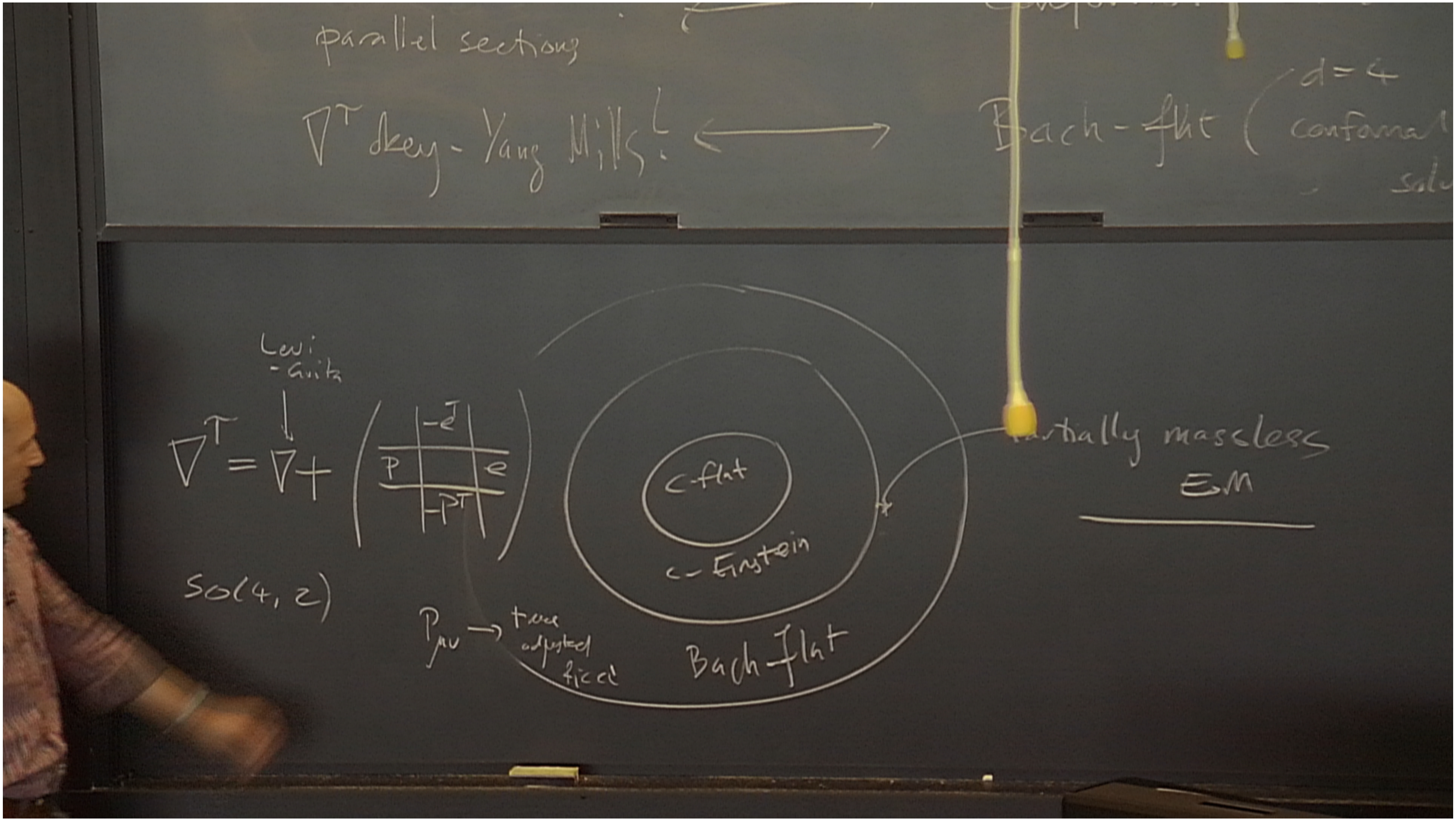
∇^T obey Yang-Mills! $\longleftrightarrow$ Bach-flat (d=4 conformal ... sol

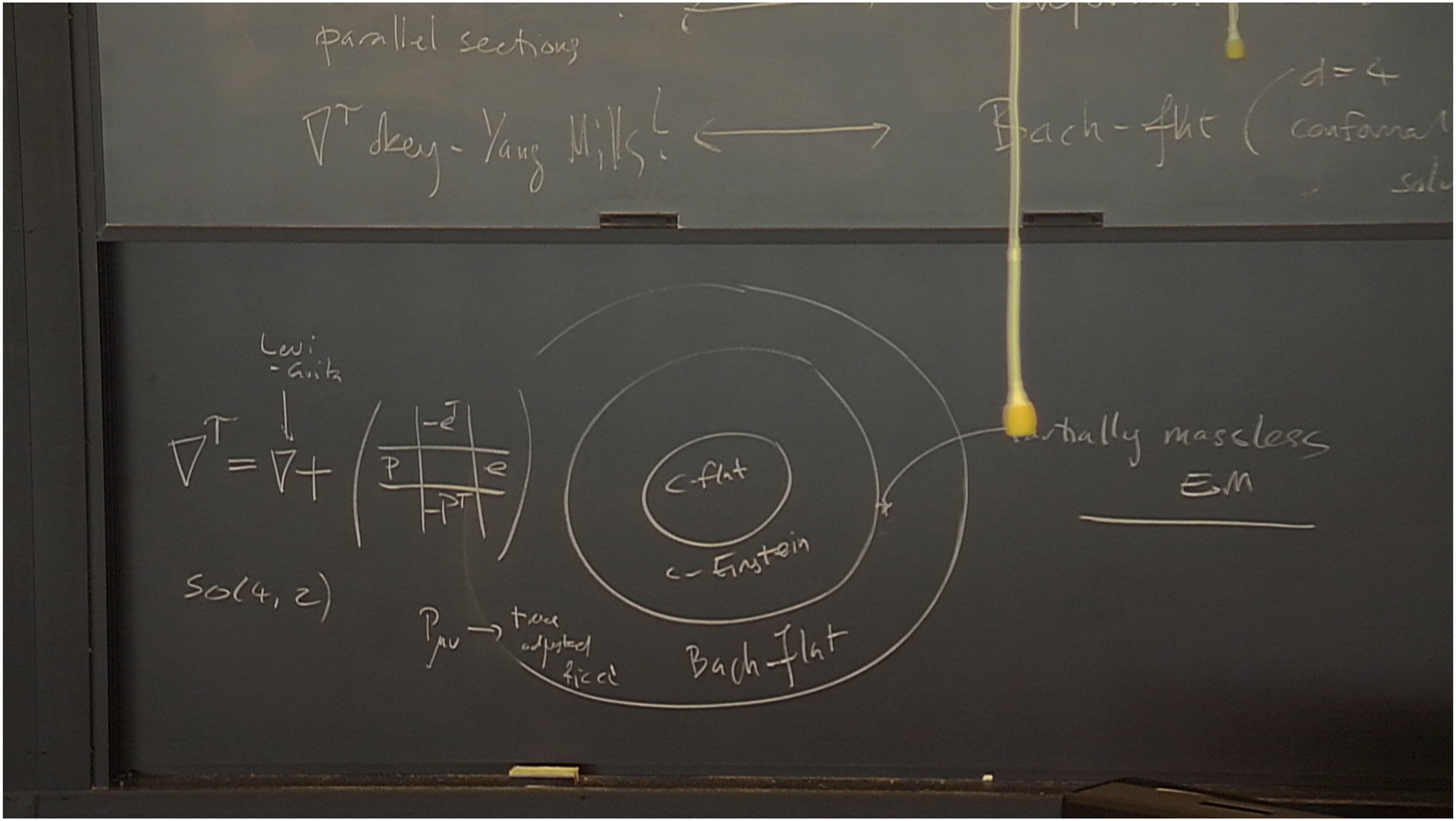
Levi-Civita

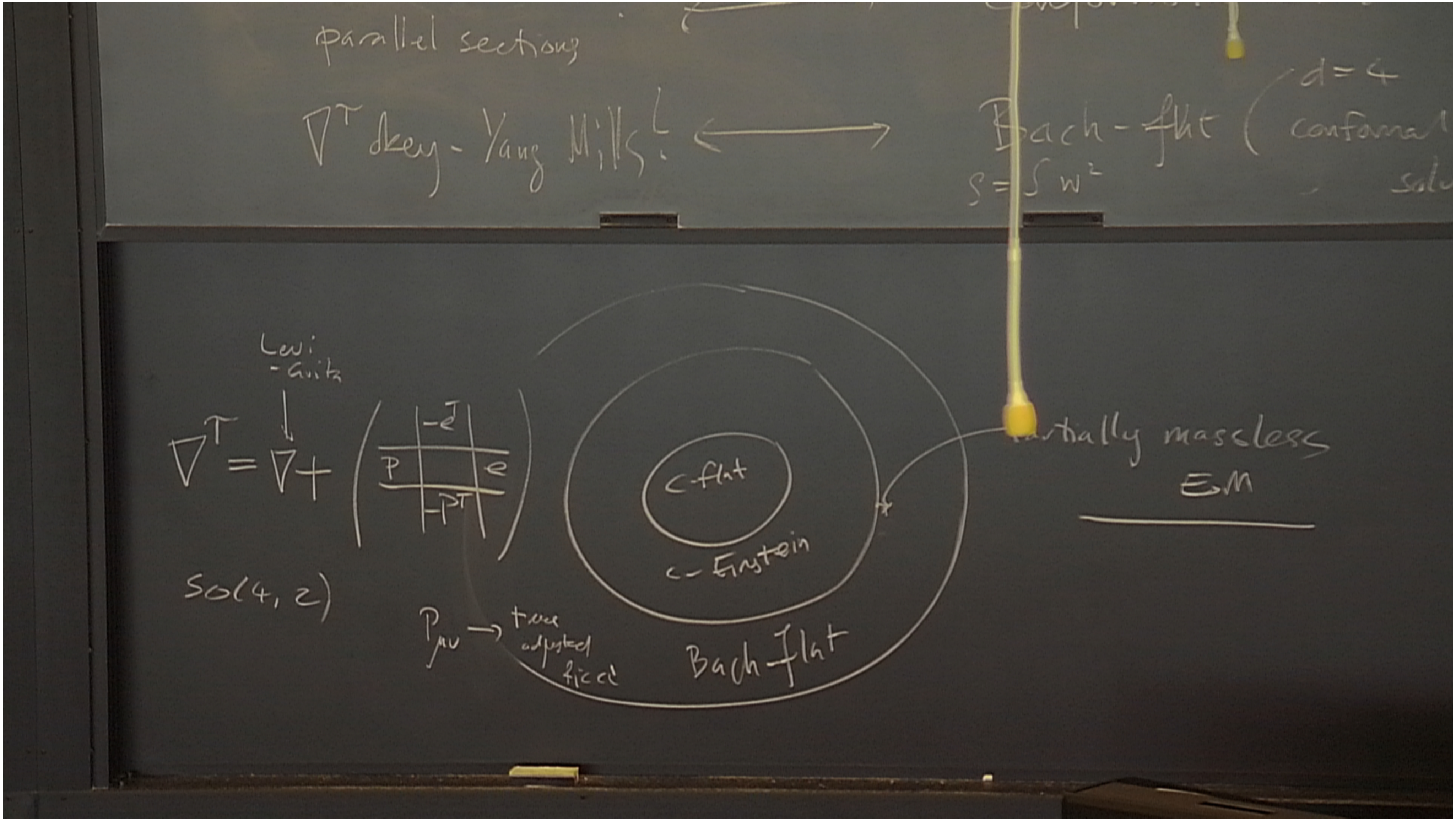
$\nabla^T = \nabla + \Big(\quad \Big)$

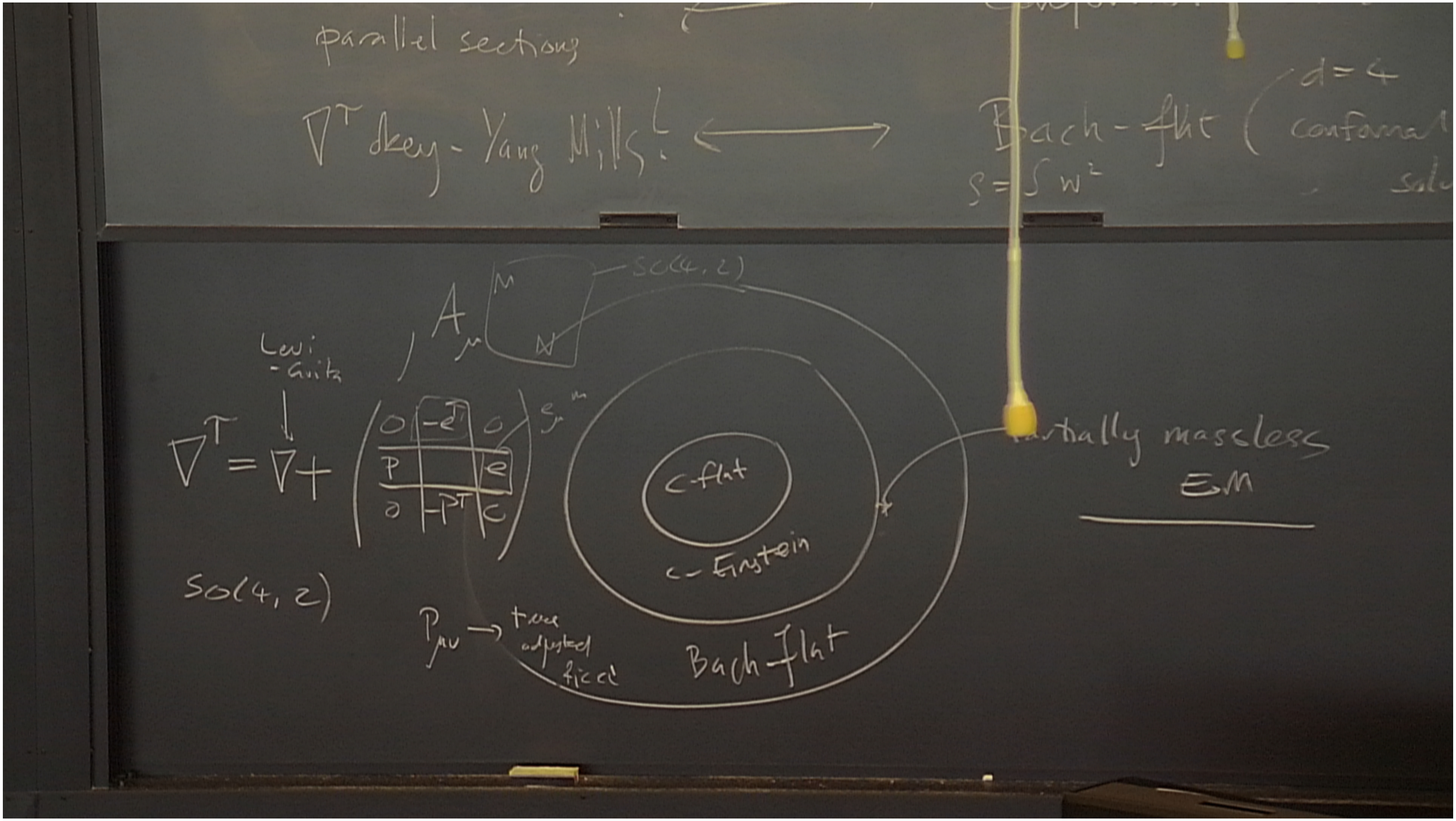


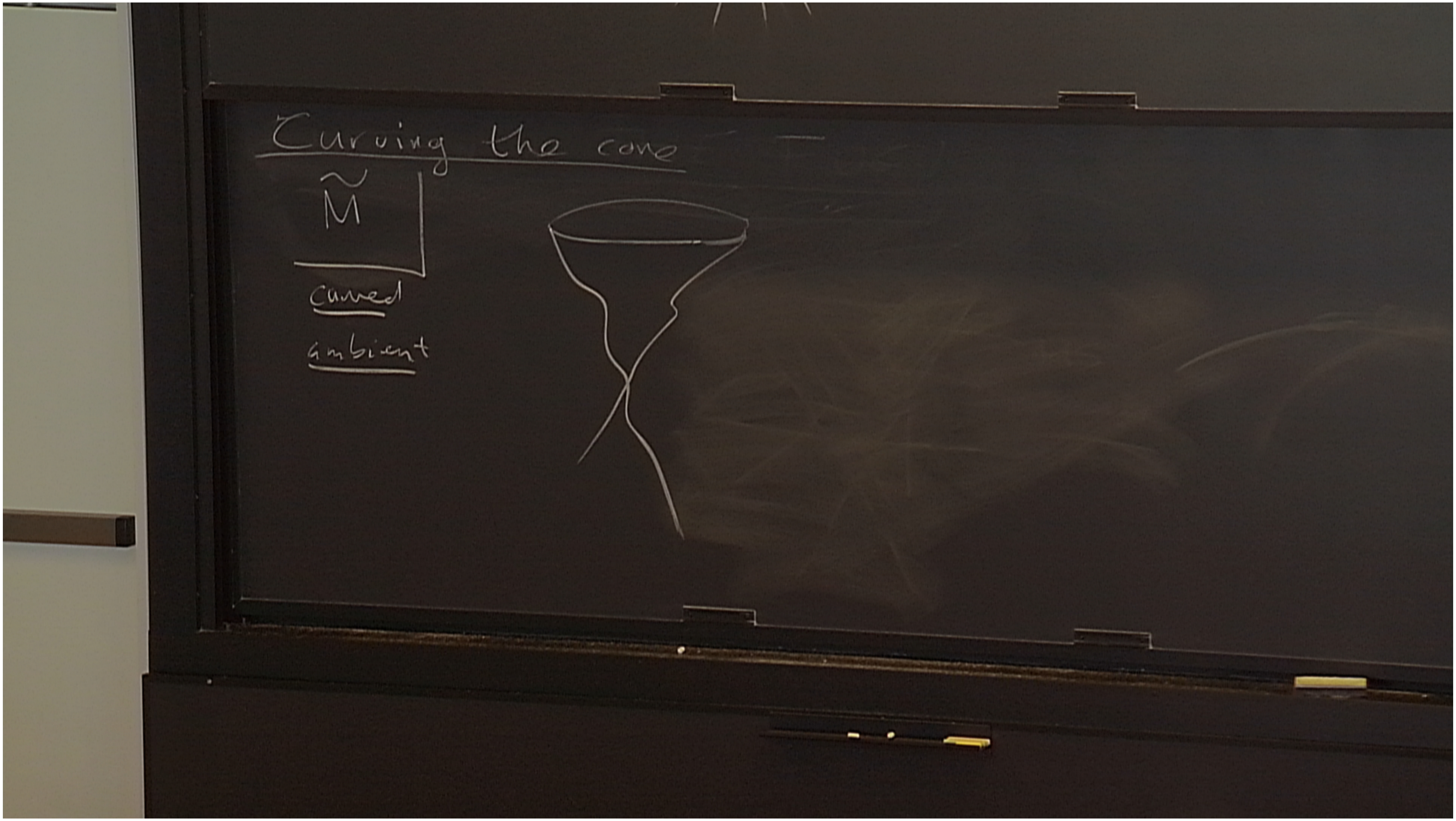
partially massless EM

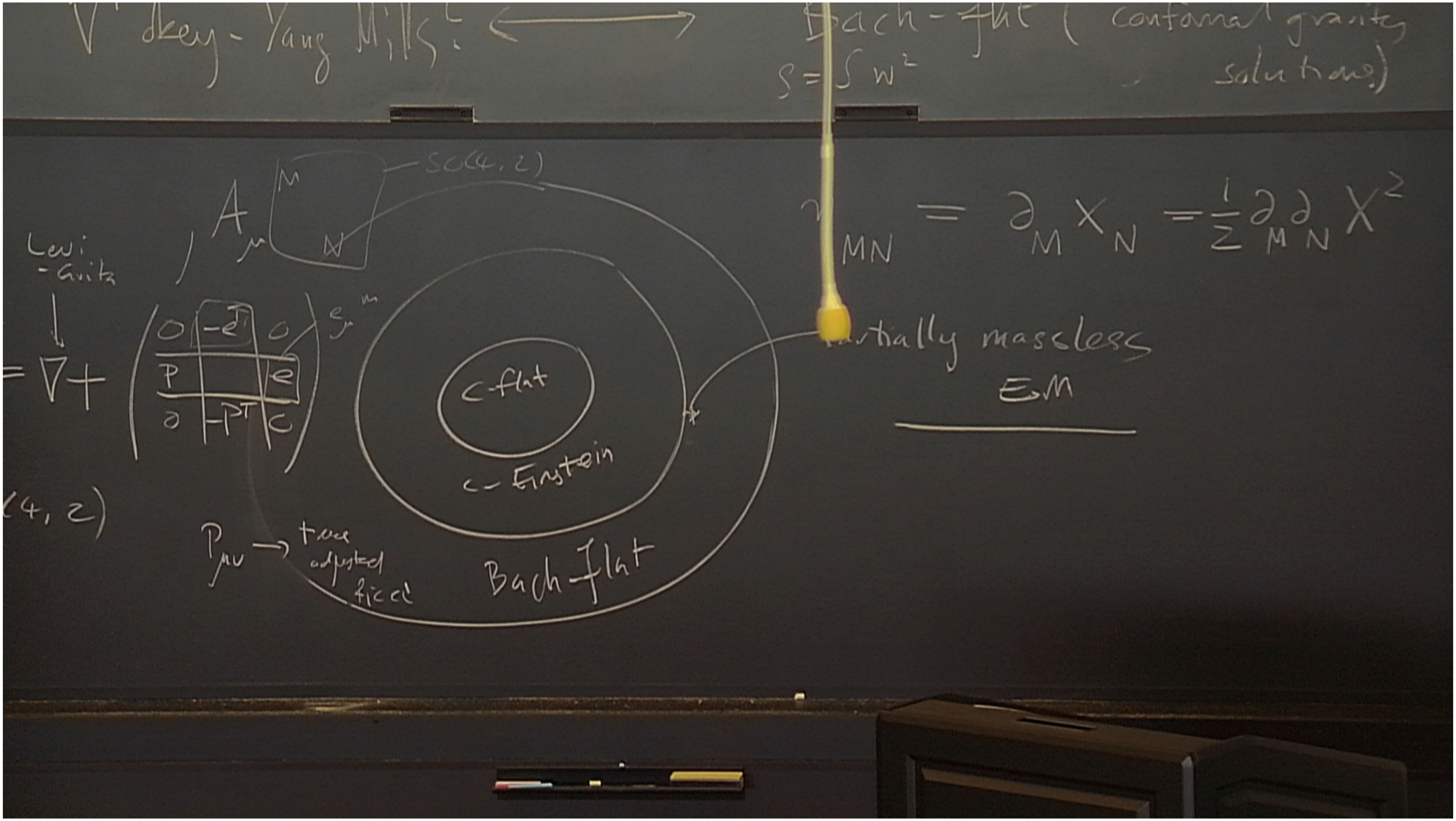

parallel sections
Yang Mills
Bach-flat
d=4
Levi-Civita
partially massless
EM
c-flat
c-Einstein
Bach-flat
so(4,2)
trace adjusted Ricci

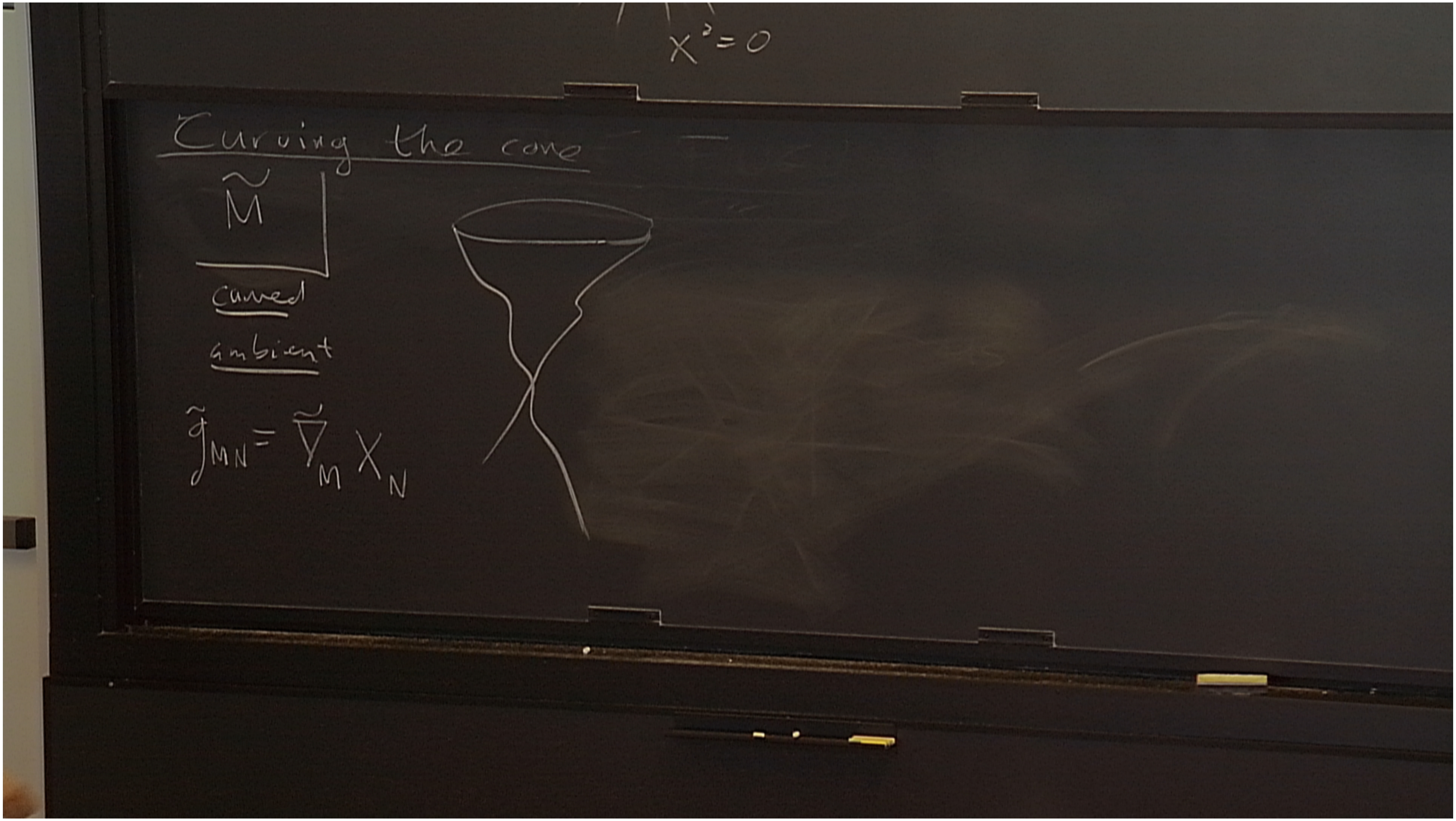

parallel sections
Yang Mills
Bach-flat
d=4
conformal
Levi-Civita
so(4,2)
$P_{\mu\nu} \to$ trace adjusted Ricci
c-flat
c-Einstein
Bach-flat
partially massless
E.M

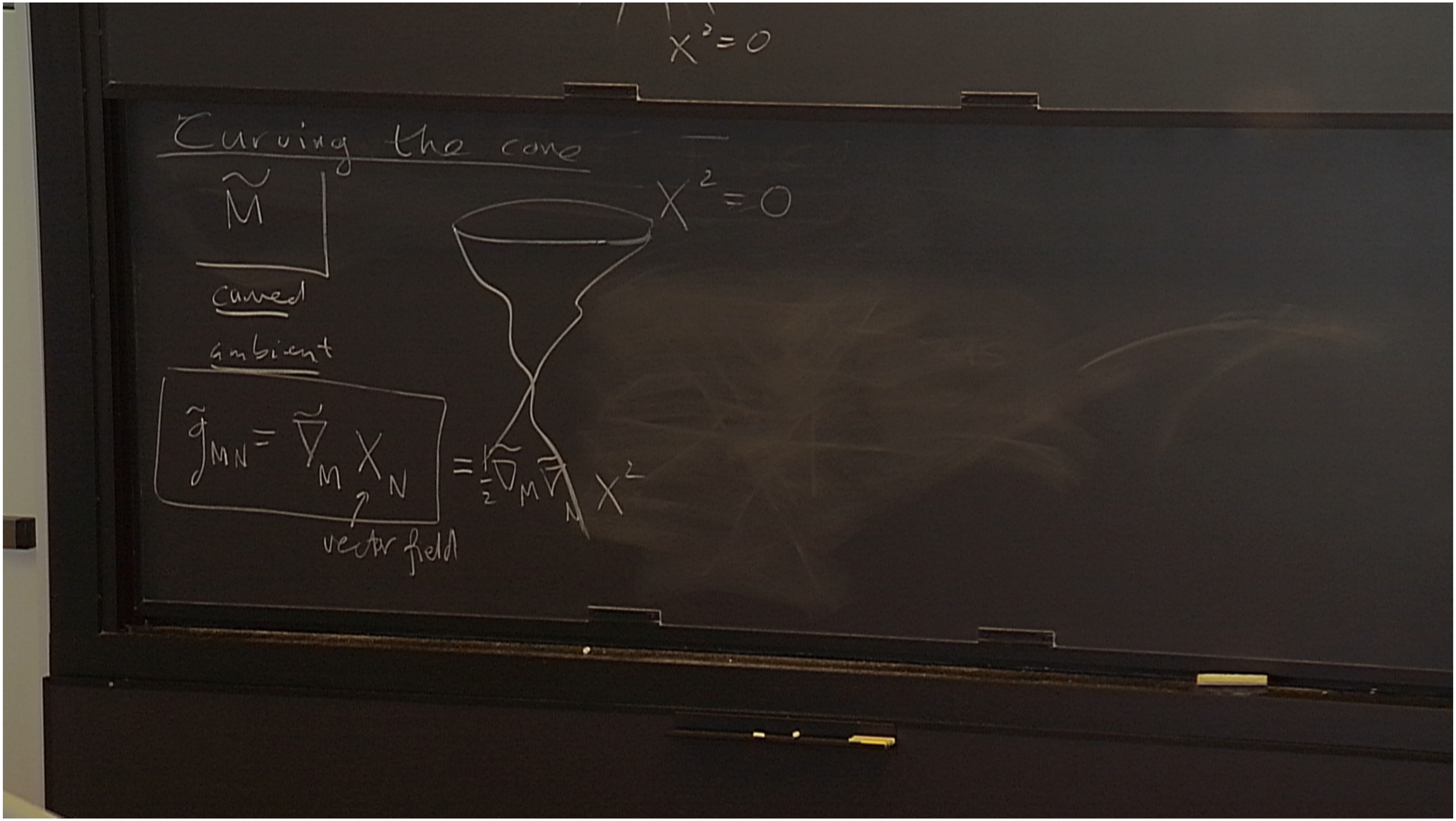

parallel sections
Bach-flat
d=4
S = ∫W²
Levi-Civita
so(4,2)
c-flat
c-Einstein
Bach-flat
P_μν → trace adjusted Ricci
partially massless
EM

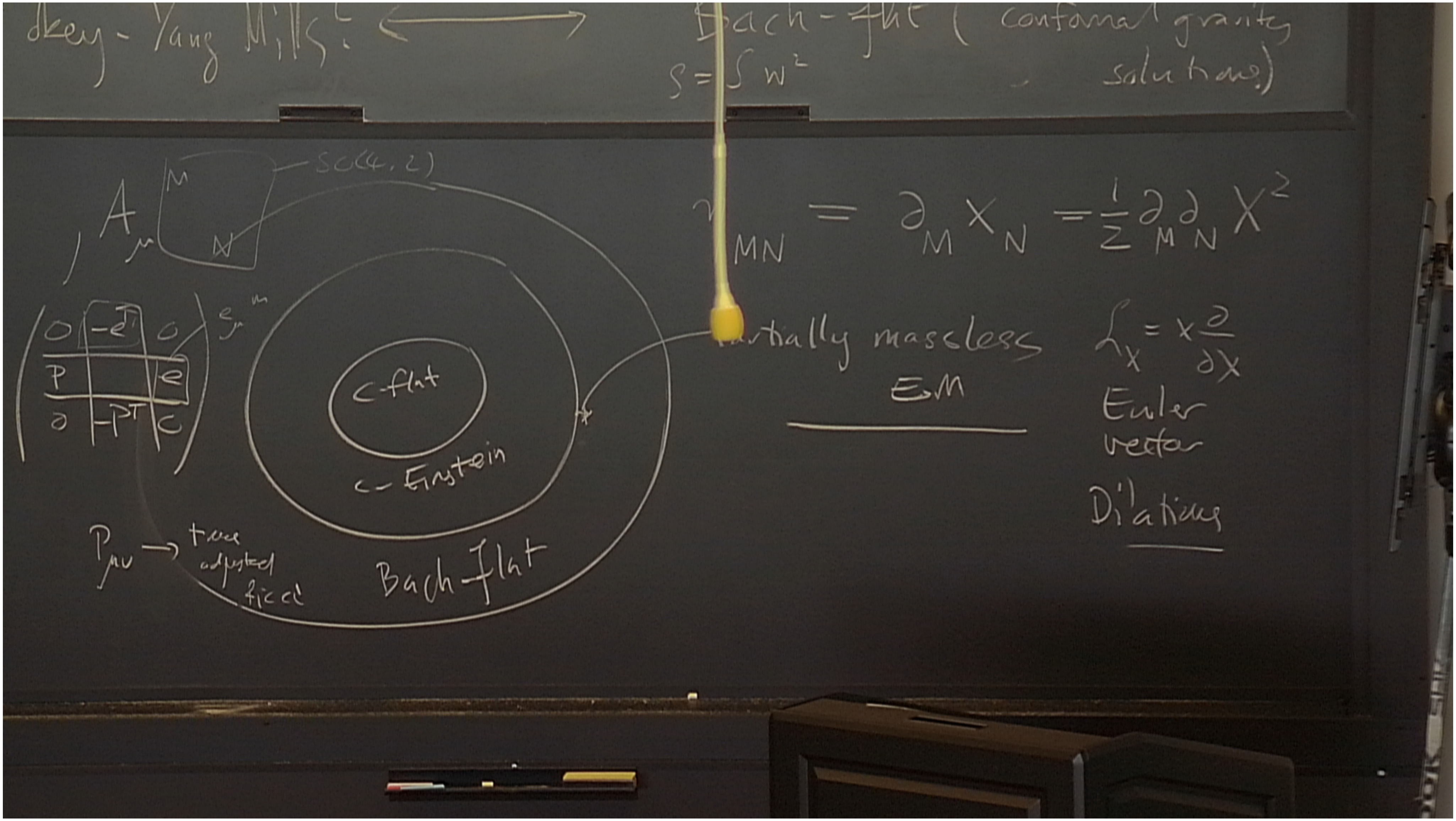

parallel sections
Yang Mills
Bach-flat
d=4
conformal
SO(4, 2)
Levi-Civita
partially massless
EM
c-flat
c-Einstein
trace adjusted Ricci
SO(4, 2)
Bach-flat

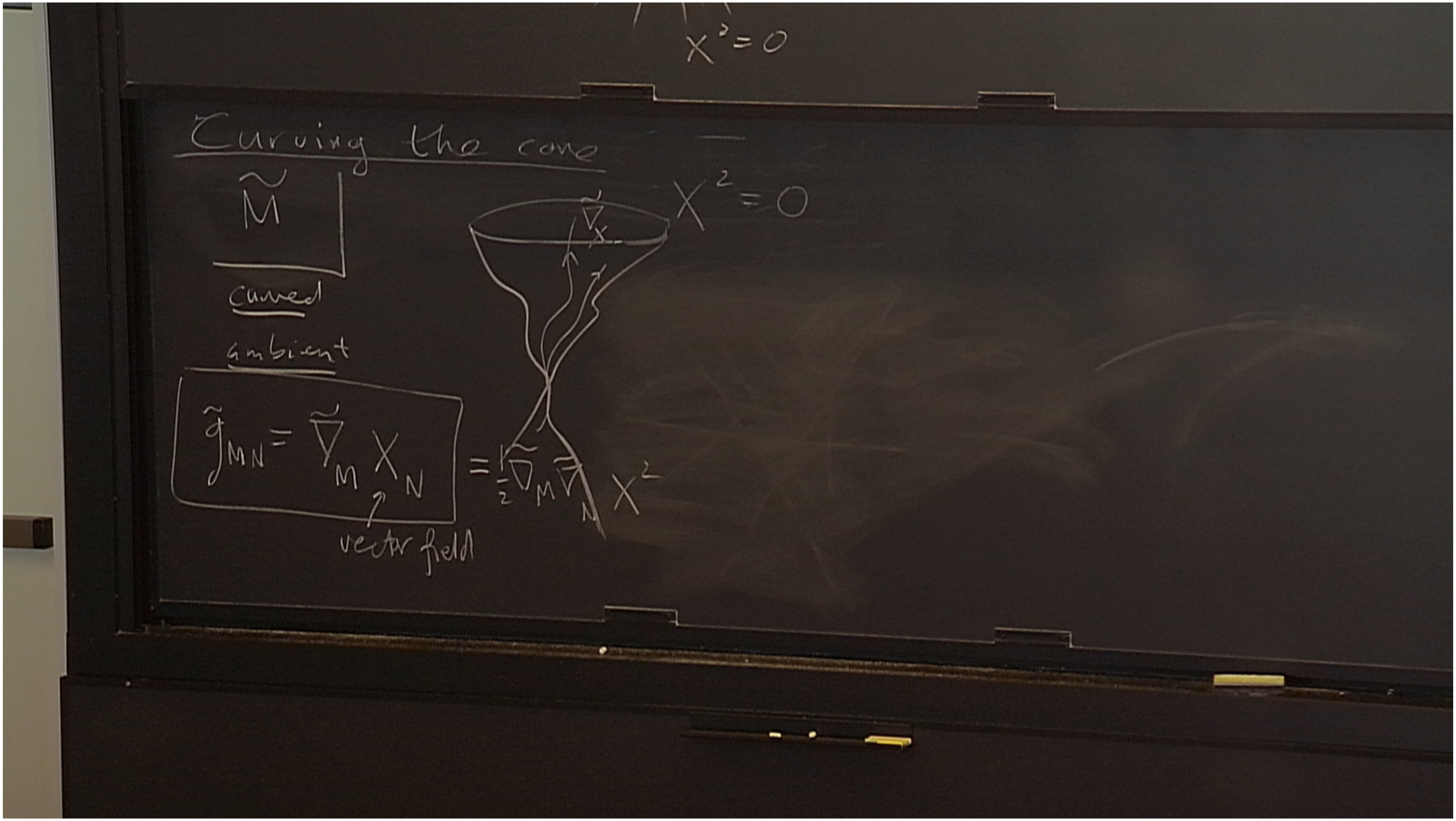

Curving the cone
M̃
curved
ambient


Yang Mills
Bach-flat (conformal gravity solutions)
so(4,2)
Levi-Civita
c-flat
c-Einstein
Bach-flat
partially massless
EM
trace adjusted ricci


$X^2=0$
Curving the cone
$\tilde{M}$
curved
ambient
$\tilde{g}_{MN} = \tilde{\nabla}_M X_N$


$X^2 = 0$
Curving the cone
$\tilde{M}$
curved
ambient
$X^2 = 0$
$\boxed{\tilde{g}_{MN} = \tilde{\nabla}_M X_N} = \frac{1}{2}\tilde{\nabla}_M \tilde{\nabla}_N X^2$
vector field


Yang Mills:
Bach-flat (conformal gravity solutions)
S = ∫ W²
SO(4,2)
c-flat
c-Einstein
Bach-flat
P_μν → trace adjusted Ricci
partially massless EM
Euler vector
Dilations


$X^2 = 0$
Curving the cone
$\tilde{M}$
$X^2 = 0$
curved
ambient
$\tilde{g}_{MN} = \tilde{\nabla}_M X_N = \frac{1}{2}\tilde{\nabla}_M \tilde{\nabla}_N X^2$
vector field

$X^2=0$

Curving the cone

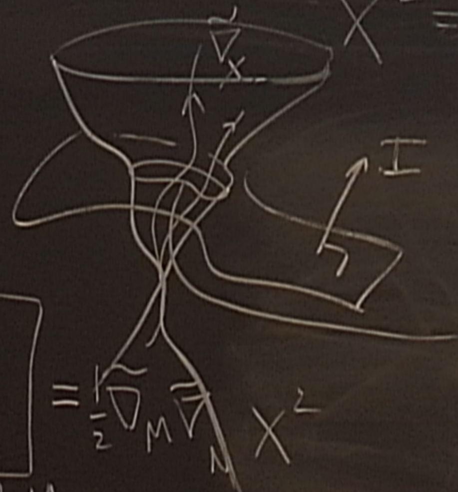
$\tilde{M}$

curved

ambient

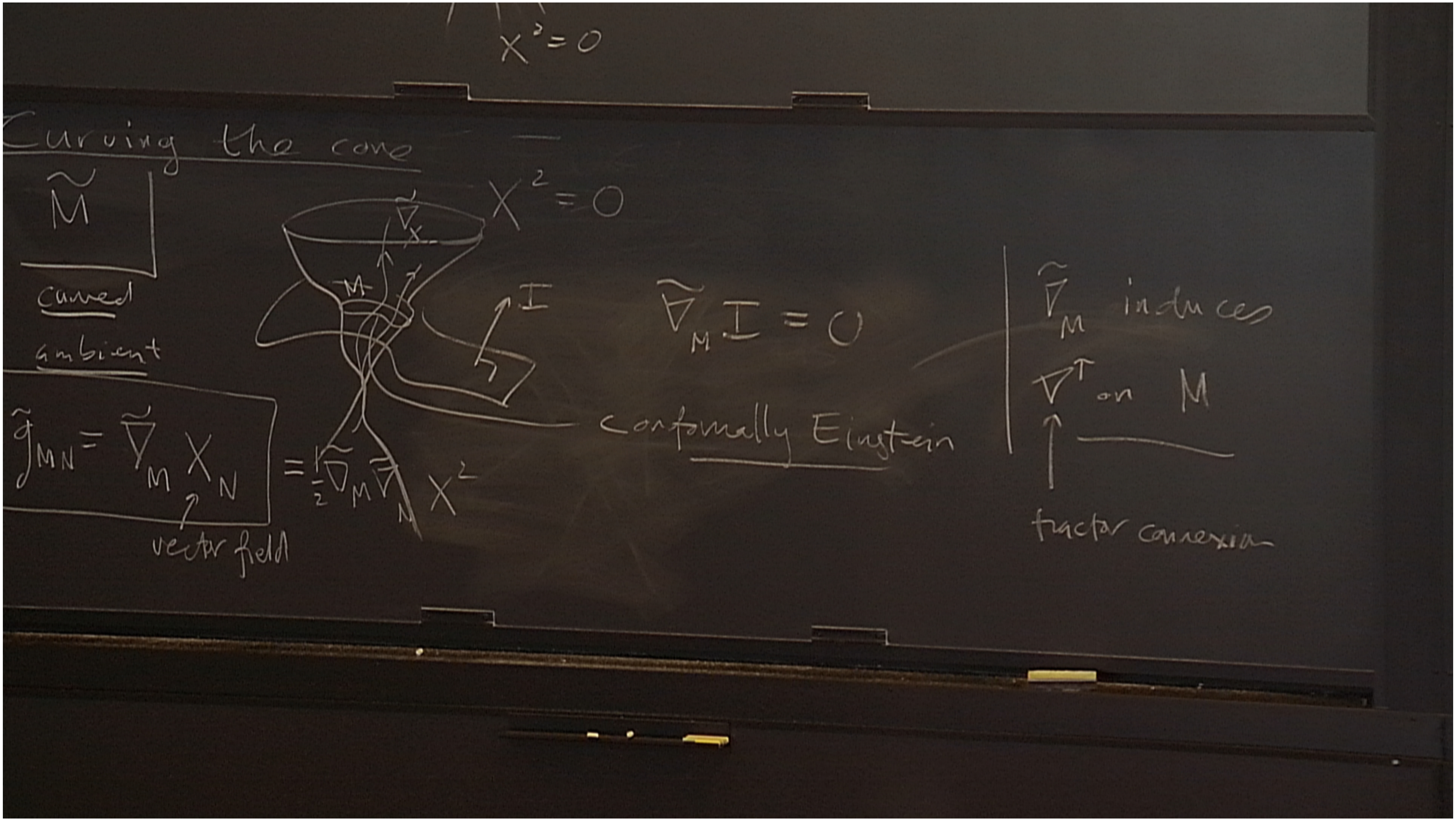
$$\boxed{\tilde{g}_{MN} = \tilde{\nabla}_M X_N} = \tfrac{1}{2}\tilde{\nabla}_M \tilde{\nabla}_N X^2$$

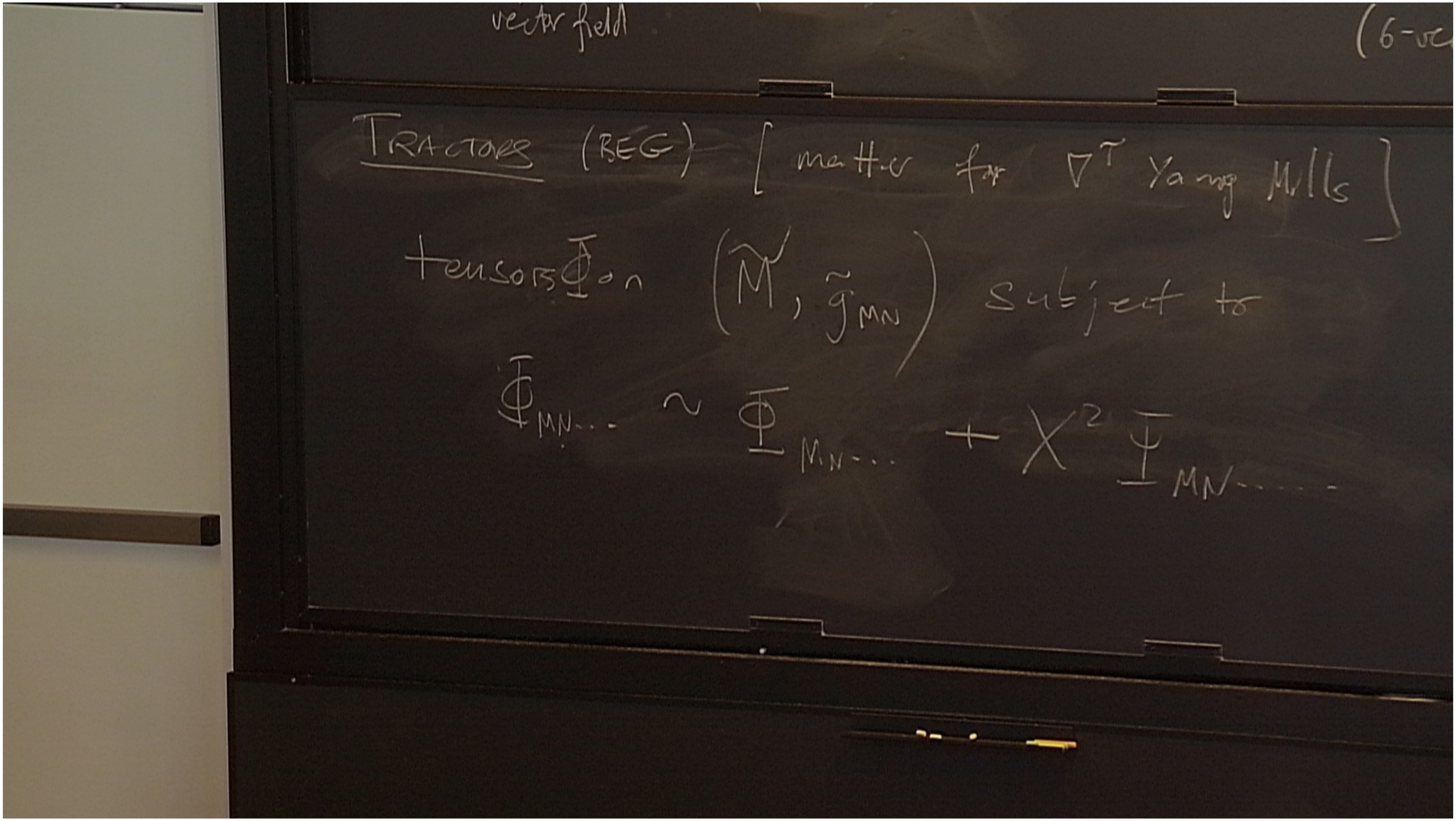
vector field



$\tilde{\nabla}_M I = 0$

conformally Einstein


$X^2 = 0$
Curving the cone
$\tilde{M}$
curved
ambient
$X^2 = 0$
$\tilde{\nabla} X$
M
I
$\tilde{\nabla}_M I = 0$
conformally Einstein
$\tilde{g}_{MN} = \tilde{\nabla}_M X_N = \frac{1}{2} \tilde{\nabla}_M \tilde{\nabla}_N X^2$
vector field
$\tilde{\nabla}_M$ induces
∇^T on M
tractor connexion


vector field
(6-ve
TRACTORS (BEG) [matter for ∇^T Yang Mills]
tensors Φ on $(\tilde{M}, \tilde{g}_{MN})$ subject to
$\Phi_{MN\ldots} \sim \Phi_{MN\ldots} + X^2 \Phi_{MN\ldots}$

$$\Phi_{MN\cdots} \sim \Phi_{MN\cdots} + X^2\, \Phi_{MN\cdots} \cdots$$

classified by weight

∇

$$\theta = g^{-1} d\overset{\overset{SO(4,2)}{\downarrow}}{g}$$ Cartan Maurer $$F_\theta = d\theta + \theta \wedge \theta = 0$$

SO(4,2) inv^t

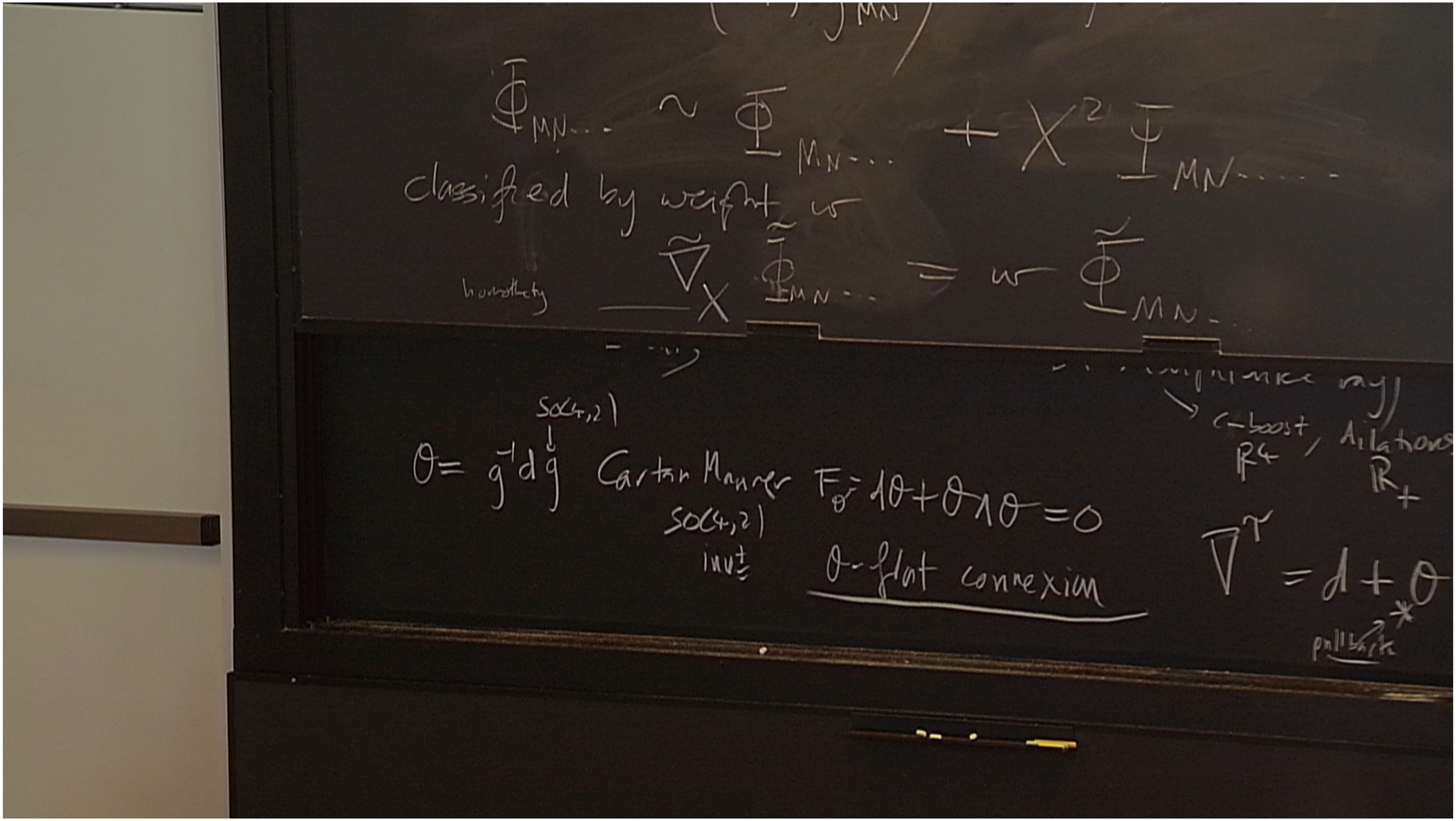
θ-flat connexion

→ C-boost, dilations

$\mathbb{R}^4$, $\mathbb{R}_+$

$$\nabla^\tau = d + \theta^*$$

pullback


classified by weight w
homothety
SO(4,2)
θ = g⁻¹dg Cartan Maurer
F = dθ + θ∧θ = 0
SO(4,2) inv't
θ-flat connexion
c-boost / dilations
pullback

TRACTORS (BEG) [matter for ∇^T Yang Mills]

tensors Φ on $(\tilde{M}, \tilde{g}_{MN})$ subject to

$$\tilde{\Phi}_{MN\ldots} \sim \Phi_{MN\ldots} + X^2 \Psi_{MN\ldots}$$

classified by weight w

4 dims

$$\underbrace{\tilde{\nabla}_X}_{\text{homothety}} \tilde{\Phi}_{MN\ldots} = w\, \tilde{\Phi}_{MN\ldots} \Rightarrow \tilde{\Phi} = \tilde{\Phi}(x)$$

SO(4,2) inv't

θ-flat connexion $\nabla^T = d + \theta^*$ (pullback) c-rant

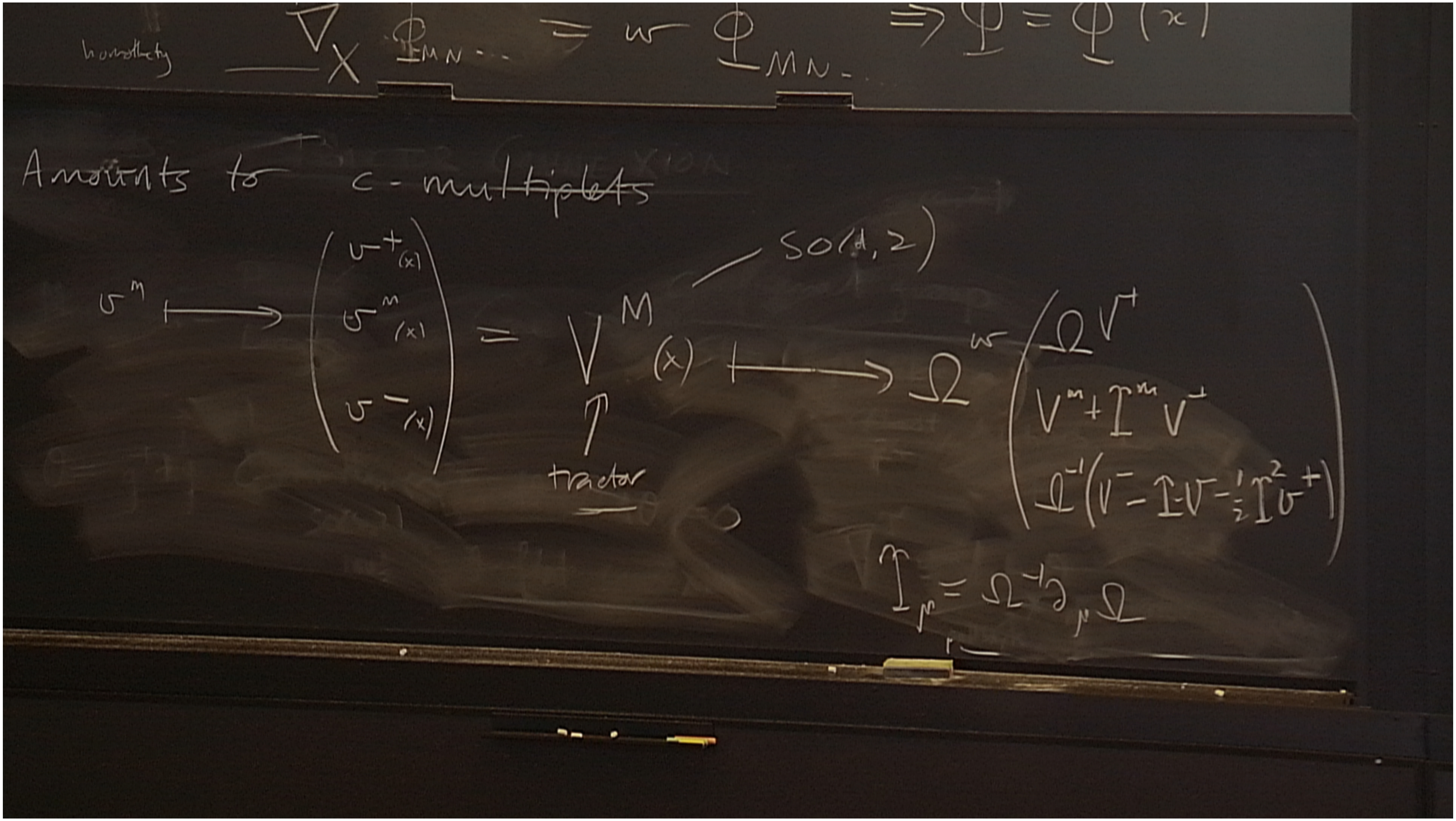
tractor connexion

homothety $\nabla_X \Phi_{MN\cdots} = w\, \Phi_{MN\cdots} \Rightarrow \Phi = \Phi(x)$

Amounts to c-multiplets

$$v^m \longmapsto \begin{pmatrix} v^+(x) \\ v^m(x) \\ v^-(x) \end{pmatrix} = V^M(x)$$

so(d,2)


homothety
$\nabla_X \Phi_{MN\cdots} = w\, \Phi_{MN\cdots} \Rightarrow \Phi = \Phi(x)$
Amounts to c-multiplets
$SO(d,2)$
$v^m \longmapsto \begin{pmatrix} v^+(x) \\ v^m(x) \\ v^-(x) \end{pmatrix} = V^M(x) \longmapsto \Omega^w \begin{pmatrix} \Omega V^+ \\ V^m + \Upsilon^m V^+ \\ \Omega^{-1}\left(V^- - \Upsilon \cdot V - \frac{1}{2}\Upsilon^2 V^+\right) \end{pmatrix}$
tractor
$\Upsilon_\mu = \Omega^{-1}\partial_\mu \Omega$

classified by weight w

$$\underset{\text{homothety}}{\tilde{\nabla}_X} \tilde{\Phi}_{MN\cdots} = w\, \tilde{\Phi}_{MN\cdots} \Rightarrow \tilde{\Phi} = \tilde{\Phi}(x)$$

Amounts to c - mul~~tiplets~~

SO(d, 2)

$$v^m \longmapsto \begin{pmatrix} v^+(x) \\ v^m(x) \\ v^-(x) \end{pmatrix} = V^M(x) \longmapsto \Omega^w \begin{pmatrix} \Omega V^+ \\ V^m + \Upsilon^m V^+ \\ \Omega^{-1}\left(V^- - \Upsilon \cdot V - \frac{1}{2}\Upsilon^2 V^+\right) \end{pmatrix}$$

tractor

$$\tilde{\nabla}_\mu V^M$$

$$\Upsilon_\mu = \Omega^{-1}\partial_\mu \Omega$$

∇^T obey Yang Mills! $\longleftrightarrow$ Bach-flat ($d=4$ conformal gravity solutions)

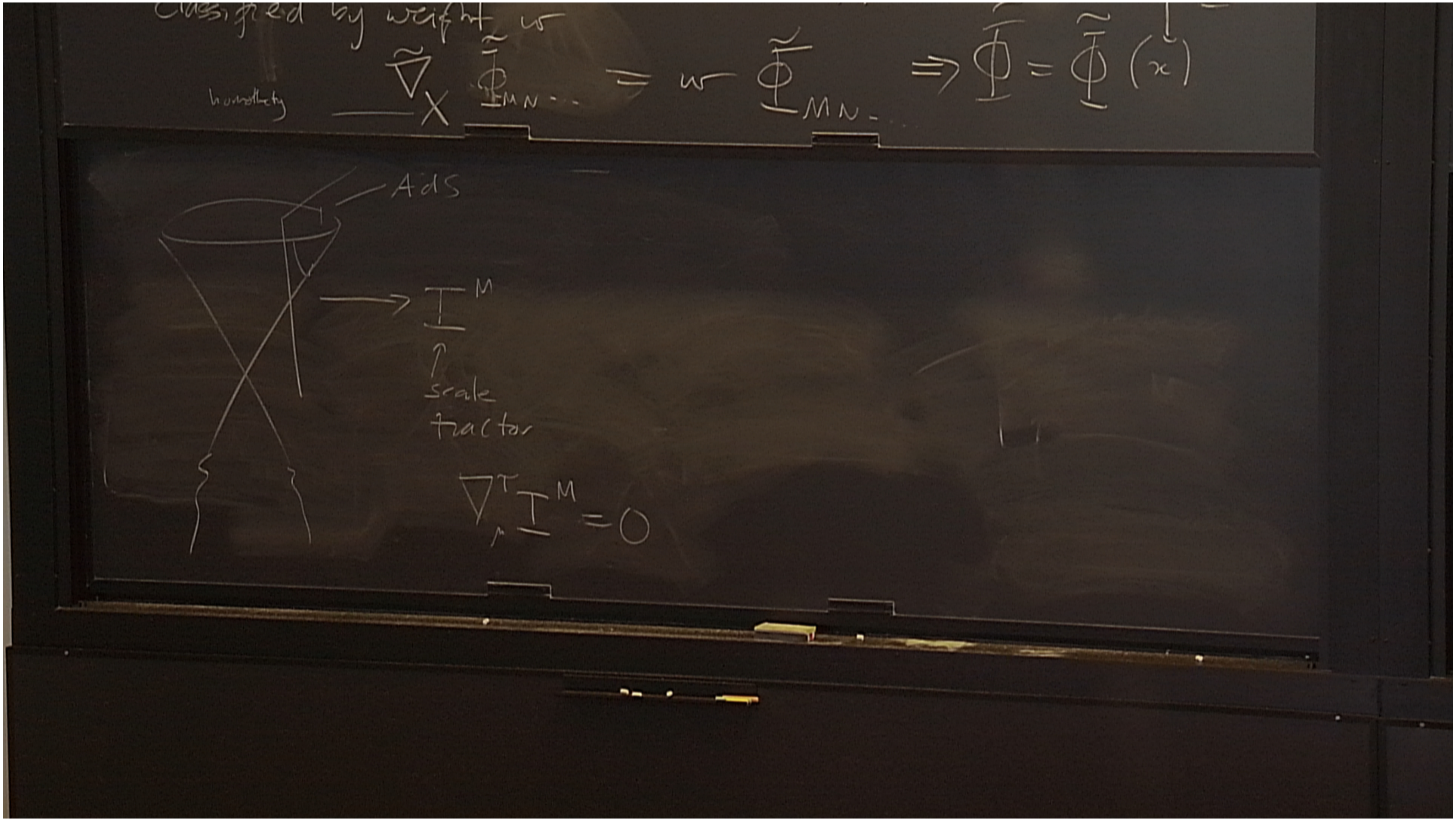
$S = \int W^2$

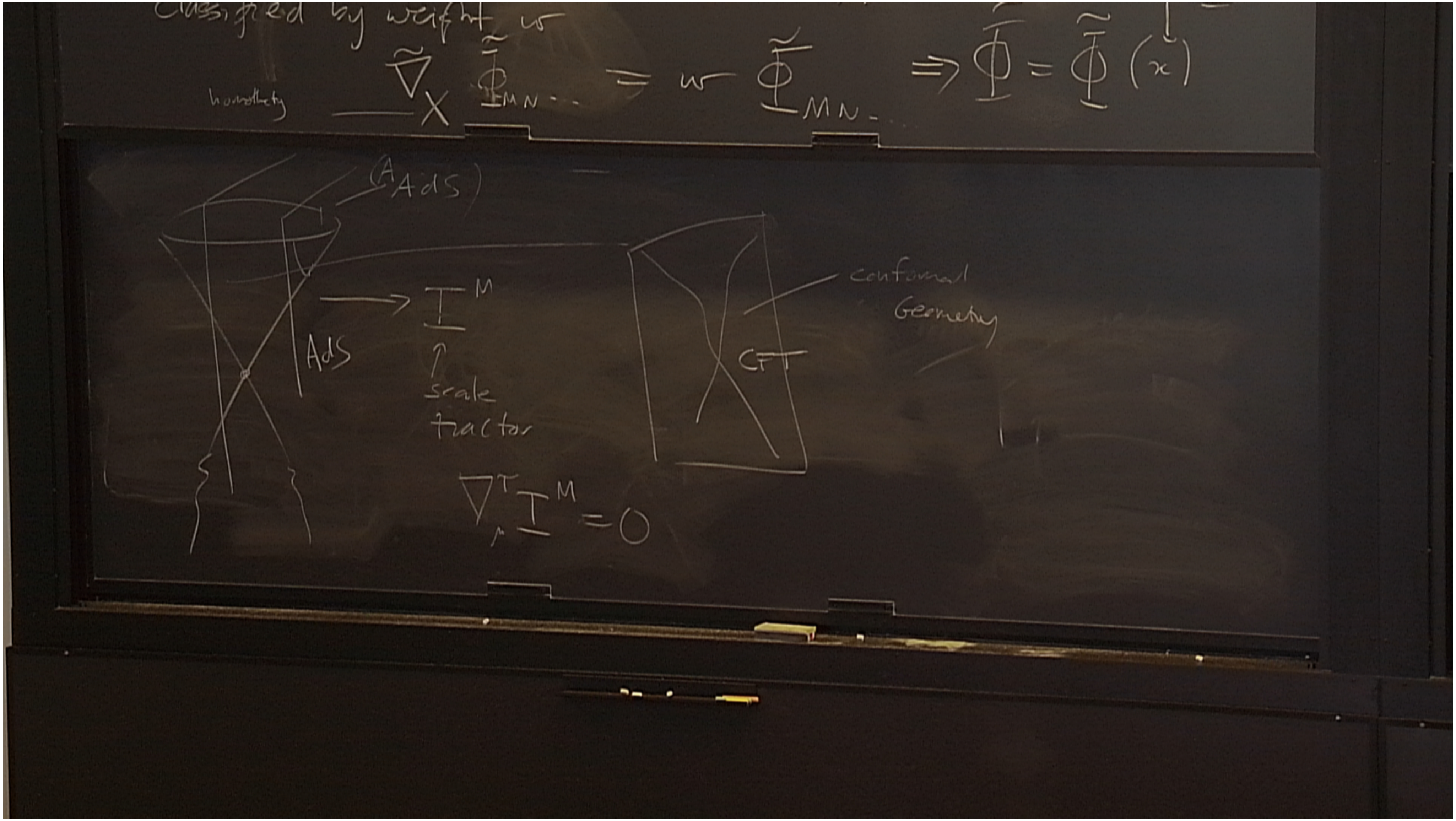
$\exists$ tractor calculus

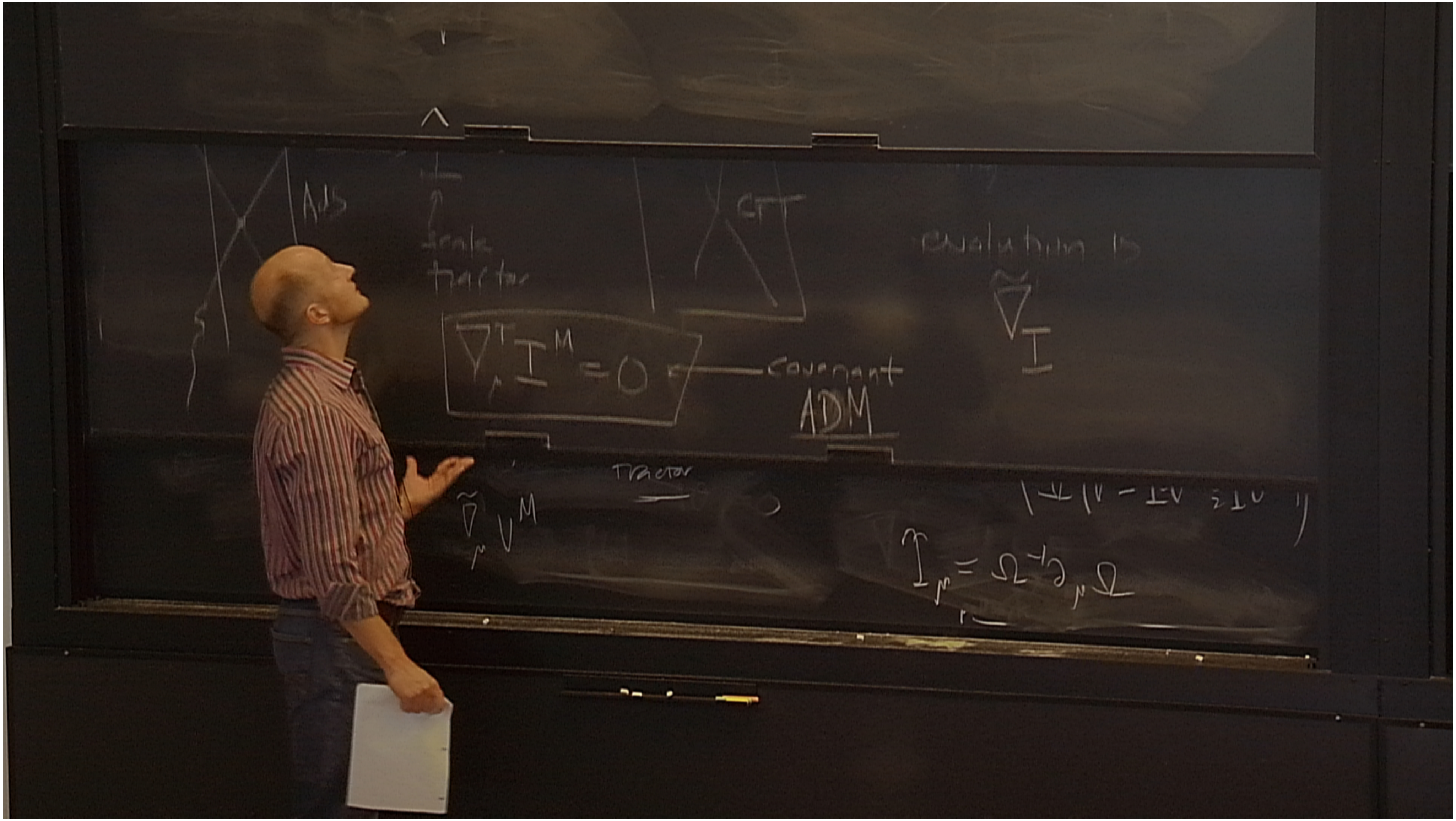
$$D^M = \begin{pmatrix} w\,(d+2w-2) \\ (d+2w-2)\,\nabla^T_m \\ -\left(\Delta_T + w\underset{\uparrow \atop Sc}{J}\right) \end{pmatrix}$$

$\exists$ tractor calculus

$$\text{``tractor } D\text{''} \nearrow D^M = \begin{pmatrix} w\,(d+2w-2) \\ (d+2w-2)\,\nabla^{\tau}_{m} \\ -\left(\Delta_\tau + w \underset{\substack{\uparrow \\ Sc}}{J}\right) \end{pmatrix}$$


classified by weight w
homothety
AdS
scale
tractor


classified by weight w
homothety
(AAdS)
AdS
scale
tractor
conformal
Geometry
CFT


AdS
scale
tractor
CFT
evolution is
covariant
ADM
Tractor

$$\boxed{\nabla^T_\mu I^M = 0}$$ ← Covariant

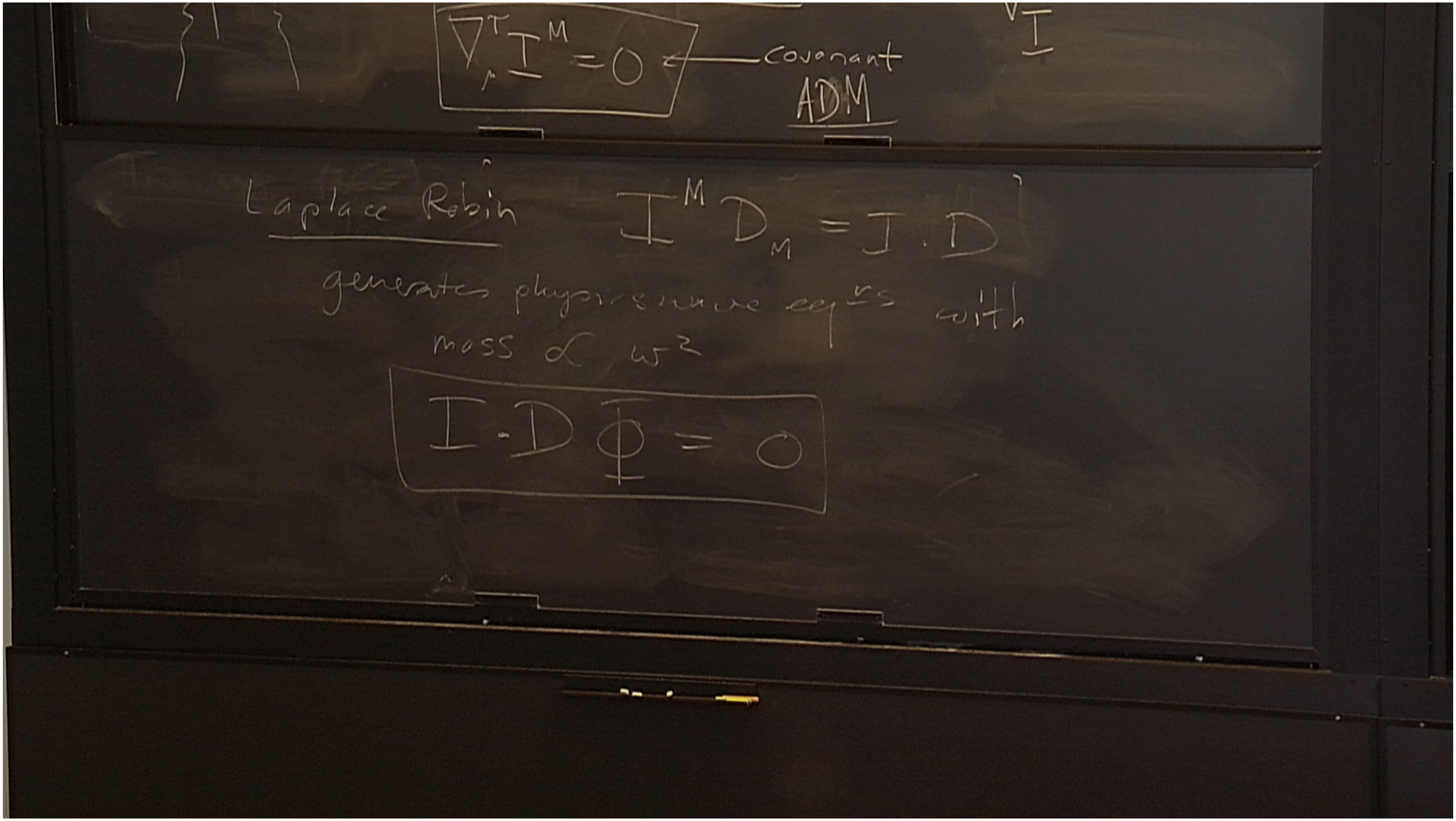
ADM

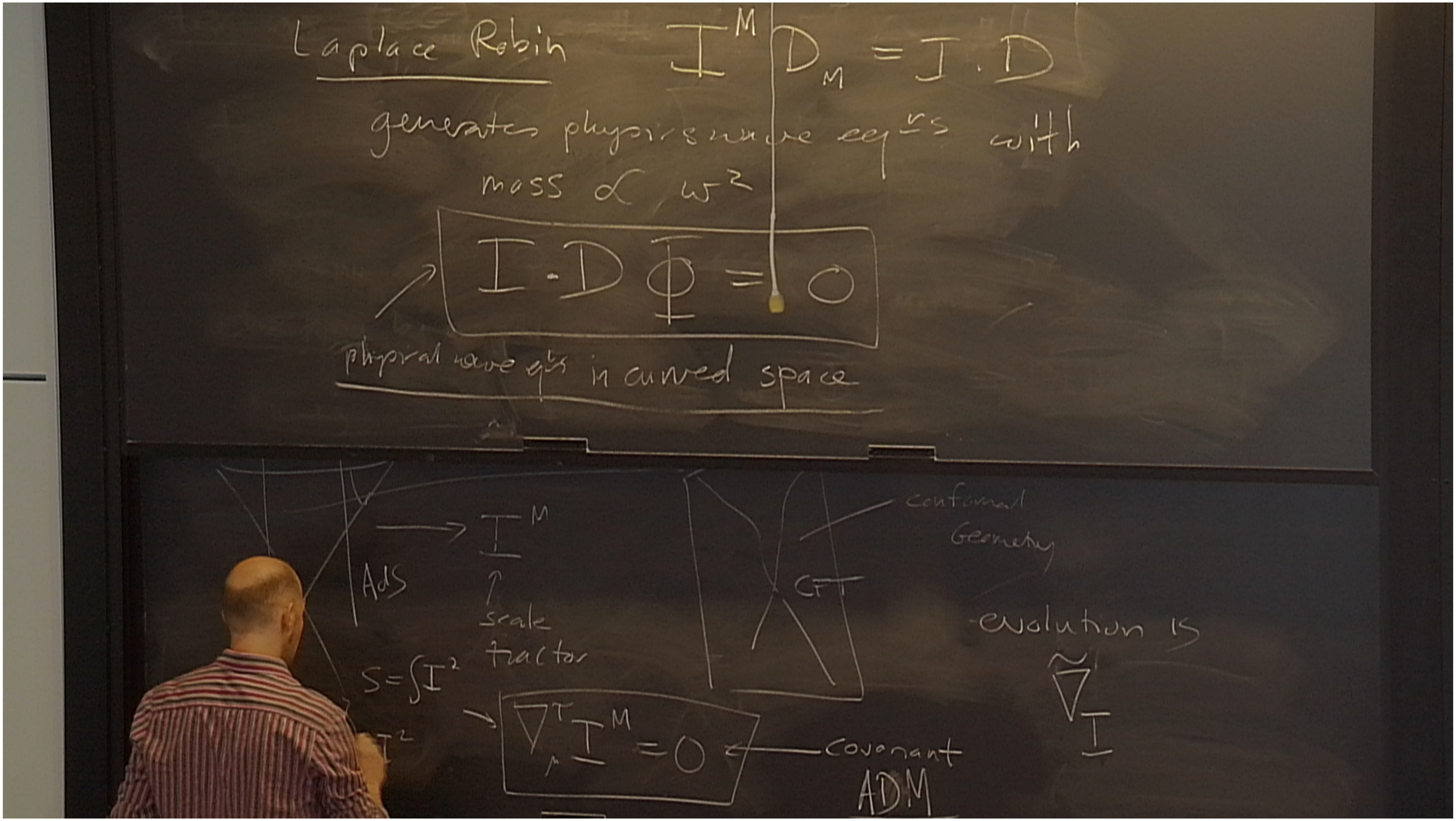
∇I

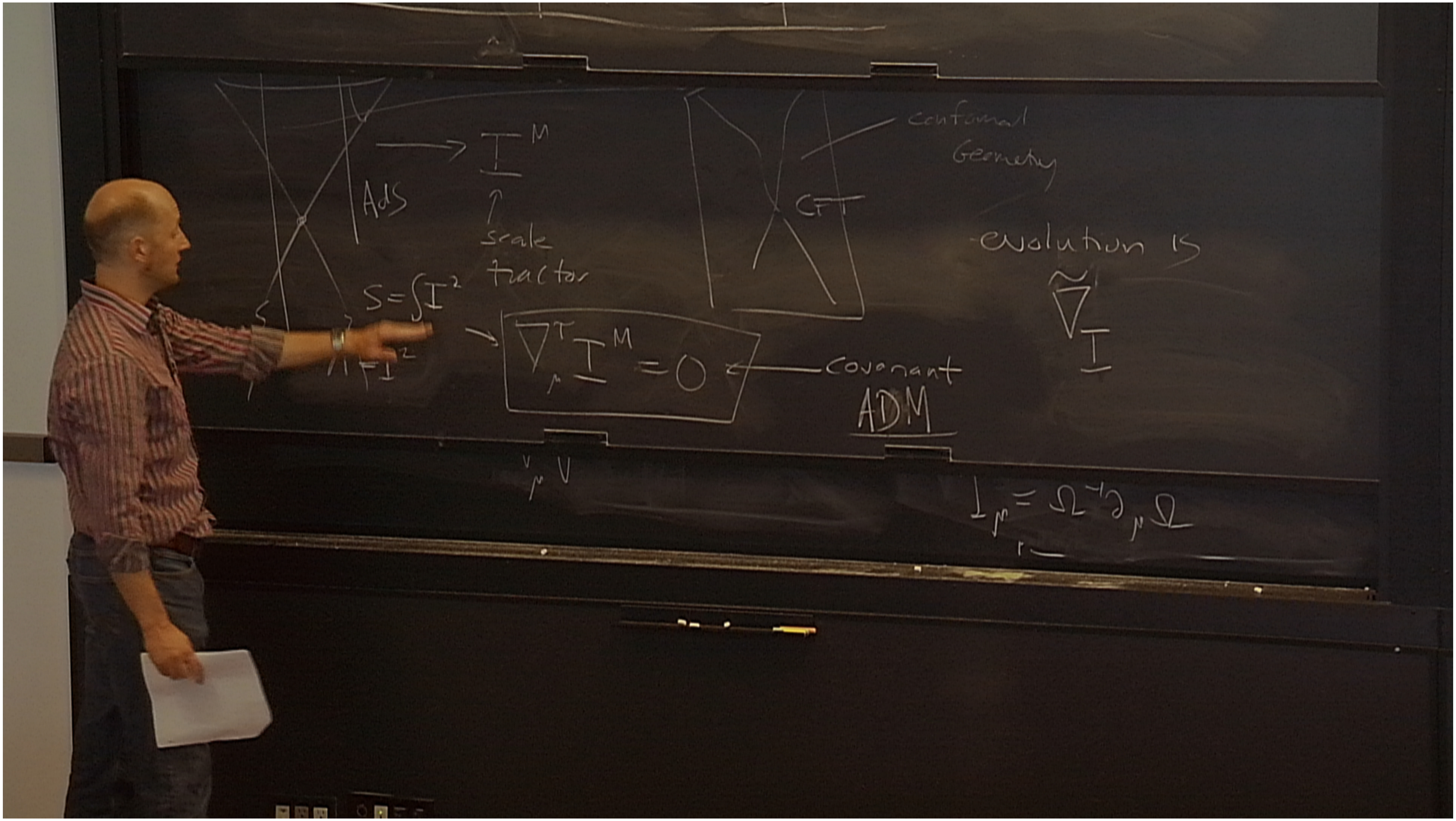
Laplace Robin $\quad I^M D_M = I \cdot D$

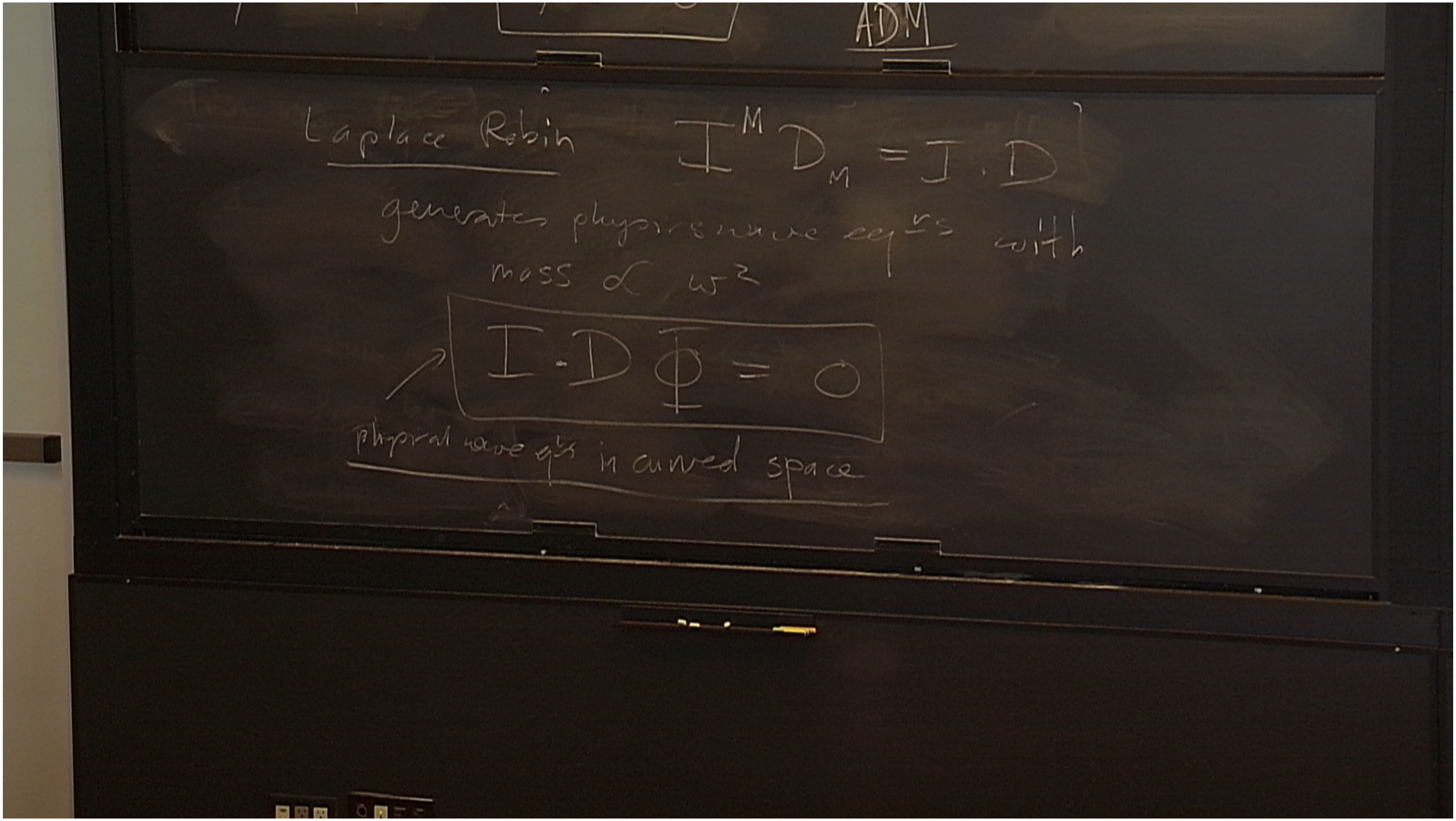
generates physics wave eq^{ns} with mass $\propto w^2$

$I \cdot D \, \Phi$


$\nabla^T_\mu I^M = 0$
covariant
ADM
Laplace Robin
$I^M D_M = I \cdot D$
generates physics wave eq^ns with
mass $\propto \omega^2$
$I \cdot D \Phi = 0$


Laplace Robin
generates physics wave eq^ns with
mass ∝ w²
physical wave eq^ns in curved space
AdS
scale
tractor
S = ∫I²
covariant
ADM
conformal
Geometry
CFT
evolution is


AdS
scale
tractor
conformal
Geometry
CFT
evolution is
Covariant
ADM


ADM
Laplace Robin
I^M D_M = I · D
generates physics wave eq^ns with
mass ∝ ω²
I · D Φ = 0
physical wave eq^ns in curved space

$\Lambda = I^2$

$$\boxed{\nabla^T_\mu I^M = 0} \longleftarrow \text{covariant ADM}$$

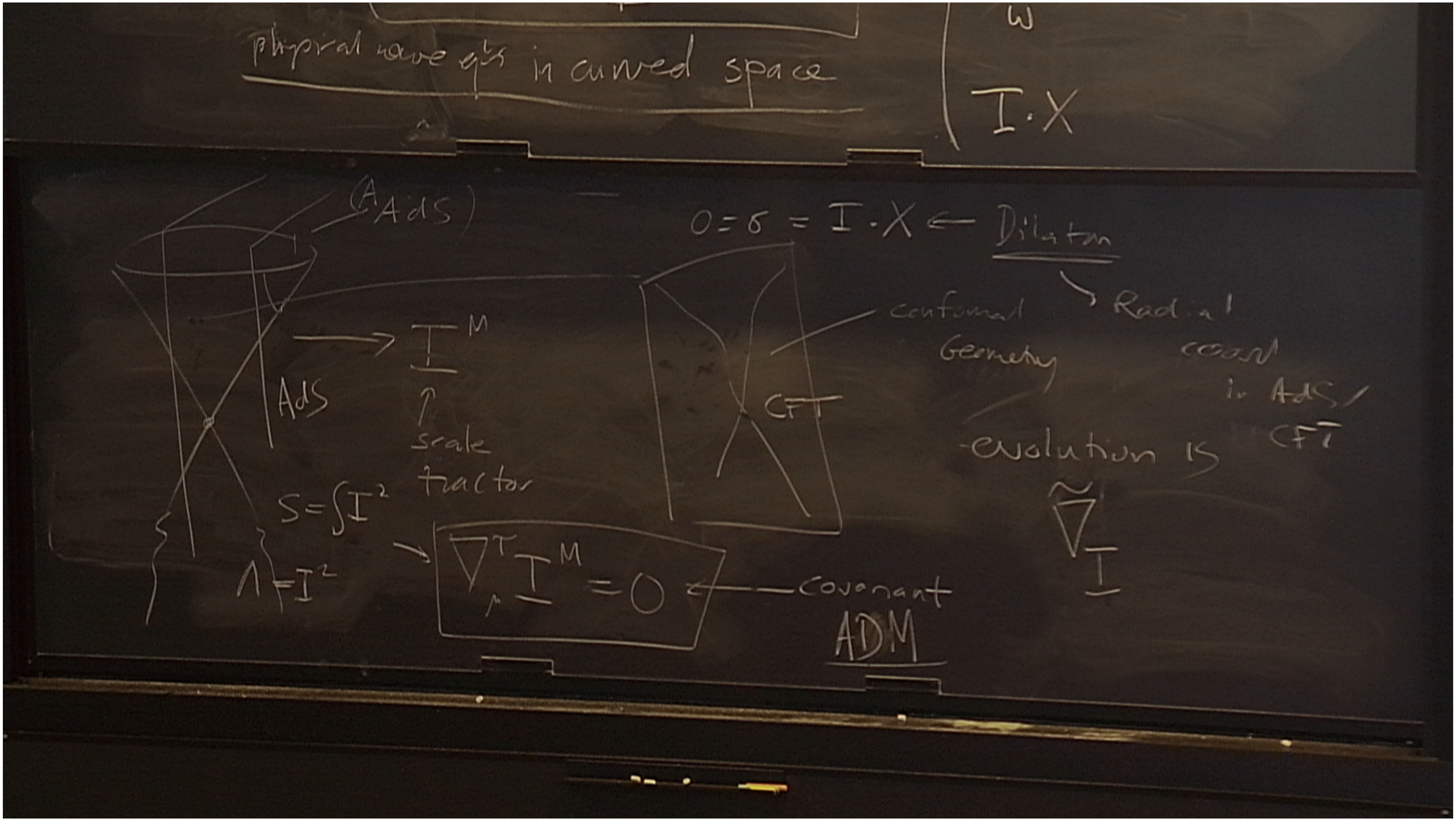
Laplace Robin $\quad I^M D_M = I \cdot D$

generates physics wave eq^{ns} with mass $\propto \omega^2$

$$\boxed{I \cdot D\, \Phi = 0}$$

hidden sl_2 $\left\{ \begin{array}{l} I \cdot D \\ \omega \\ I \cdot X \end{array} \right.$

physical wave eq^{ns} in curved space


in curved space
I · X
(AAdS)
O = σ = I · X ← Dilaton
Radial
coord
in AdS/CFT
conformal
Geometry
CFT
AdS
scale
factor
S = ∫I²
Λ = I²
covariant
ADM
evolution is

tractor calculus

$$D^M = \begin{pmatrix} w\,(d+2w-2) \\ (d+2w-2)\,\nabla^{\tau}_m \\ -\left(\Delta_\gamma + w\,J\right) \end{pmatrix}$$

"tractor D"

Sc

Weyl anomaly

$$\left(I\cdot D\right)^{d-1} \log\tau \Big|_{Bdy}$$