Title: Geometry and Topology in the Fractional Quantum Hall Effect

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URL: http://pirsa.org/13100086

Abstract: The FQHE is exhibited by electrons moving on a 2D surface
through which a magnetic flux passes, giving rise to

flat bands with extensive degeneracy (Landau
levels).   The degeneracy

of a partially-filled Landau level is lifted by Coulomb
repulsion between the electrons, which at certain rational fillings, leads
to gapped incompressible
topologically-ordered fluid states exhibiting the FQHE. Successful model wavefunctions for FQHE states, such as the Laughlin and Moore-Read states, are surprisingly related to Euclidean conformal field
theory, even though they are gapped incompressiible quantum fluids with a
fundamental unit of area set by the area per magnetic flux quantum h/e.

The model wavefunctions are parametrized by a
continuously-variable Euclidean metric,
just like the Euclidean conformal group of the cft to which they are related.

This metric is fixed locally both by the form of the
projected Coulomb interaction within the partially-filled Landau level, and by
local gradients of the tangential electric field on the 2D surface, promoting
it from a static flat metric fixed globally by the cft, to a dynamic local
physical degree of freedom of the FQHE fluid with area-preserving zero-point fluctuations
that leave an imprint in the ground-state structure function.

The curious connection to cft appears to be that the Virasoro algebra plays a fundament role in
both cft and FQHE, for apparently-unrelated reasons. In the FQHE it
derives from a chiral
"gravitional"

(geometric) topologically-protected anomaly at the edge of the fluid that is also
revealed in the entanglement spectrum of a cut through the bulk fluid.

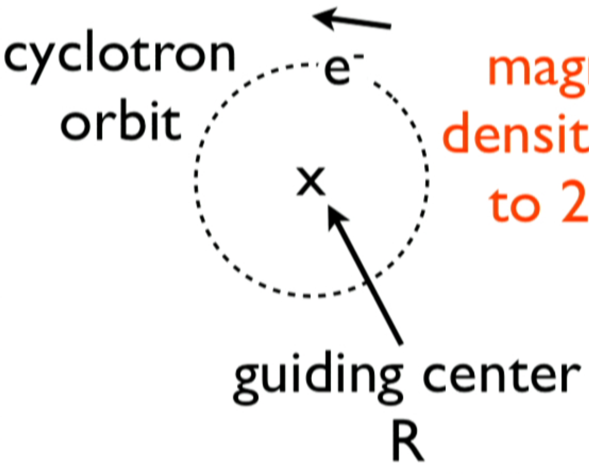
Perimeter Institute, October 7, 2013

Geometry and topology in the fractional quantum Hall effect

F. Duncan M. Haldane
Princeton University

- Non-commutative geometry
- Origin of FQHE incompressibility
- geometry of flux attachment
- Topological (chiral) Virasoro anomaly,

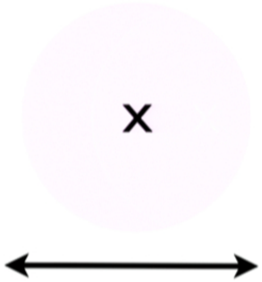
- electron in 2D Landau orbit (bound to 2D surface)



magnetic flux density B normal to 2D surface

=

Becomes a "fuzzy object" after kinetic energy is quantized

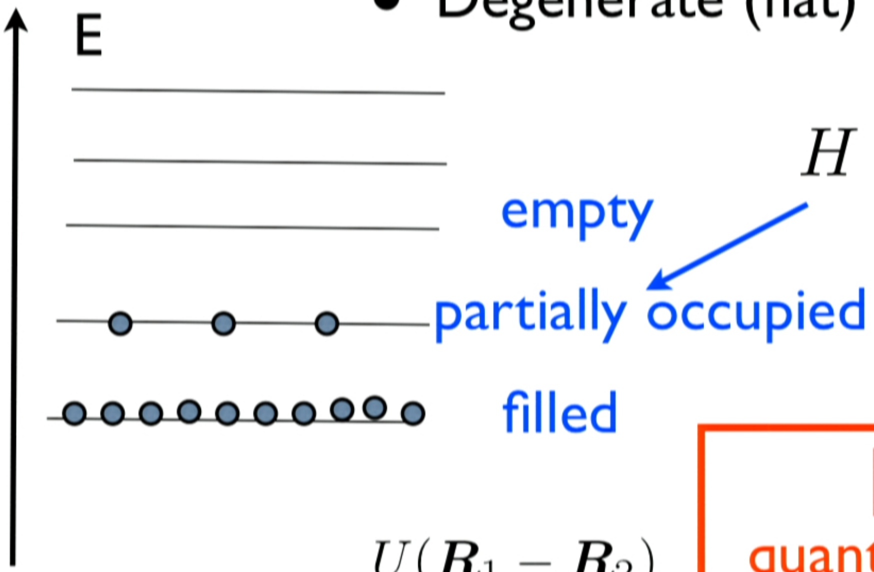


$$\ell_B = \left(\frac{\hbar}{|eB|}\right)^{\frac{1}{2}}$$

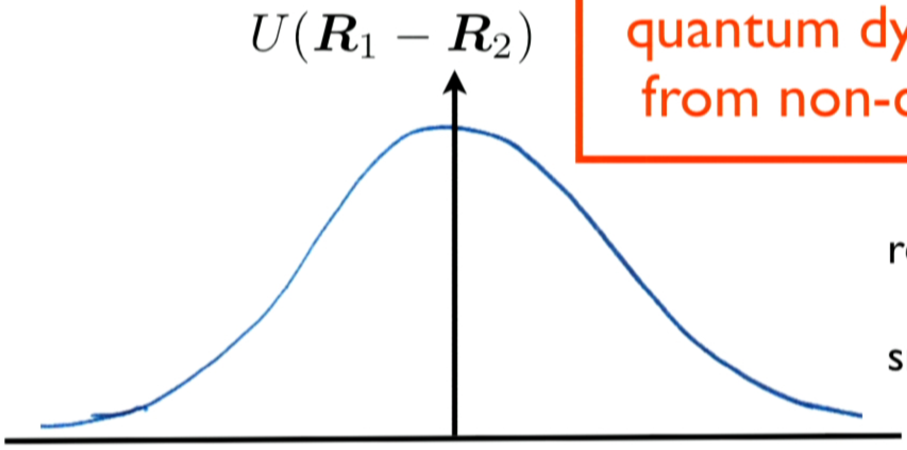
$$\boxed{[R^x, R^y] = -i\ell_B^2}$$

non-commutative geometry

- Degenerate (flat) Landau levels



$$H = \sum_{i<j} U(\boldsymbol{R}_i - \boldsymbol{R}_j)$$



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quantum dynamics comes from non-commutativity

effective Coulomb repulsion is analytic at origin because of smoothing by Landau-orbit form factor

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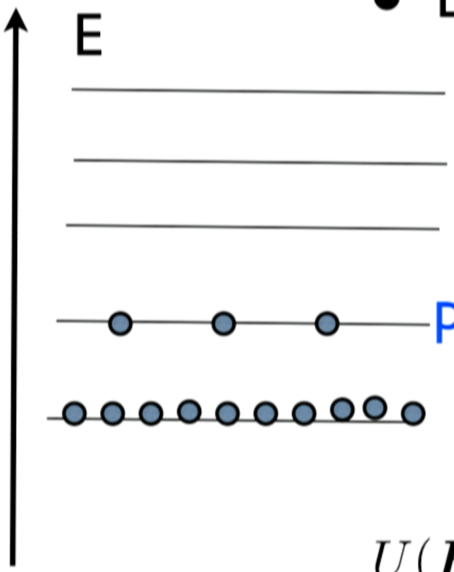

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partially occupied

filled

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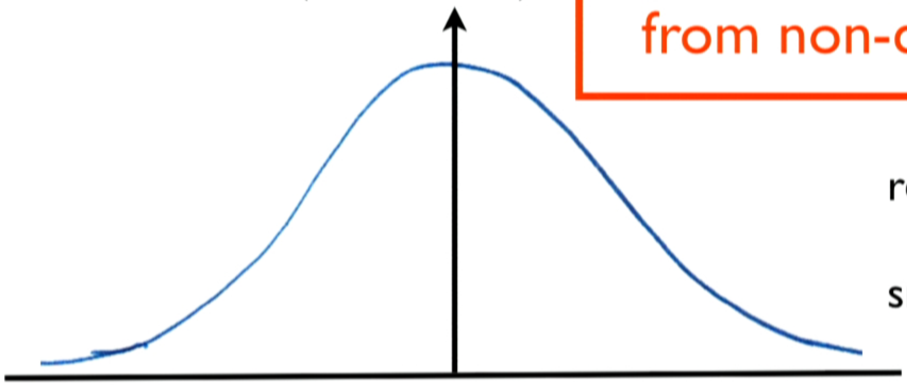

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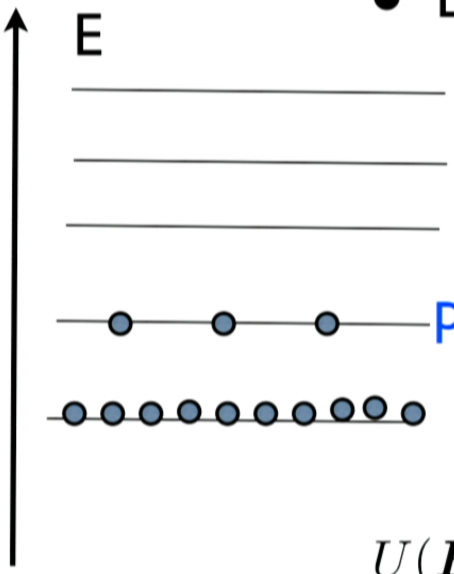

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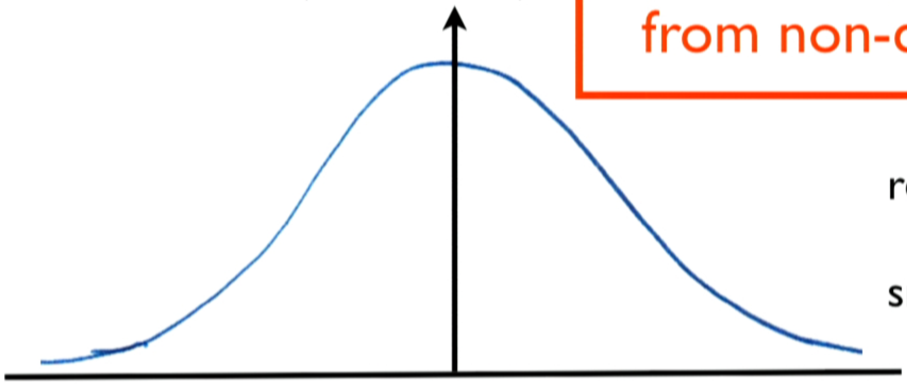

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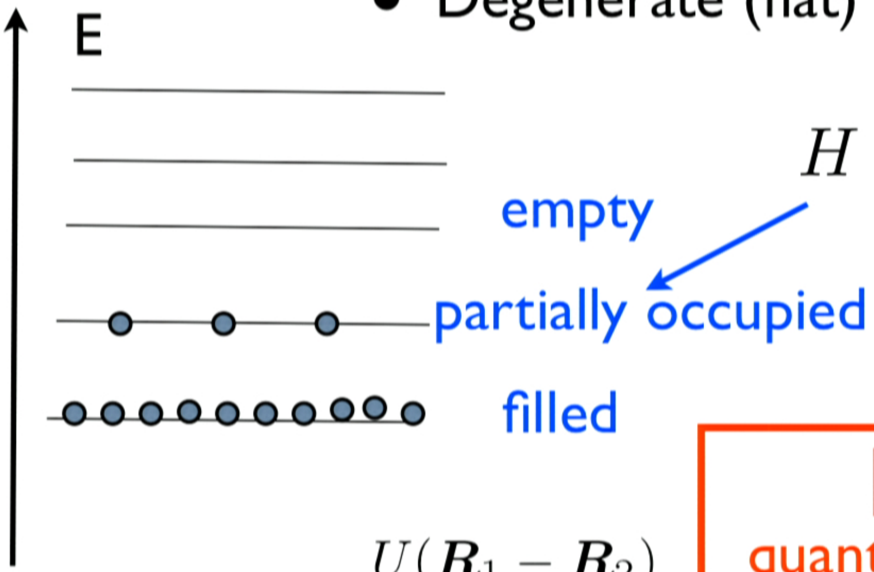

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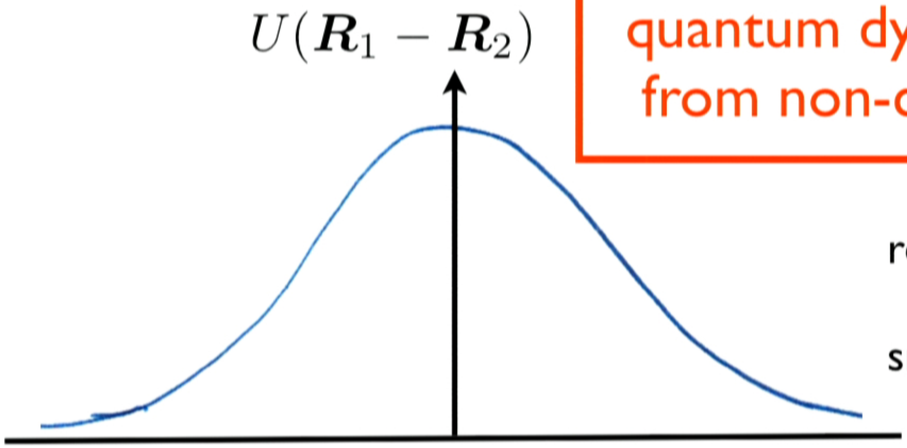

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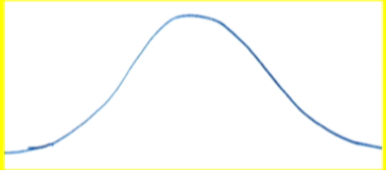

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like phase-space, has Heisenberg uncertainty principle

- H has translation and inversion symmetry

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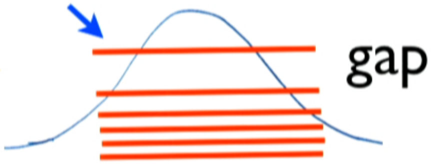
$$[H, \textstyle\sum_i \boldsymbol{R}_i] = 0$$

- generator of translations and electric dipole moment!

$$[(R_1^x - R_2^x), (R_1^y - R_2^y)] = -2i\ell_B^2$$

- relative coordinate of a pair of particles behaves like a single particle

want to avoid this state



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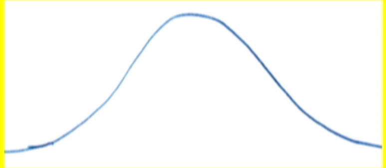

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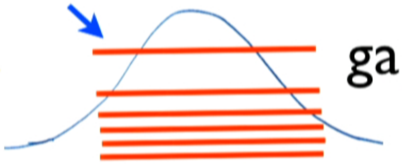

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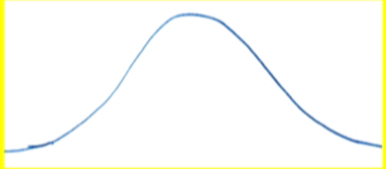

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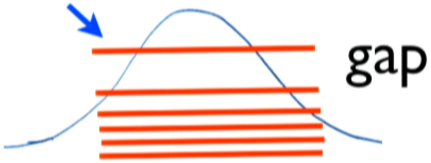

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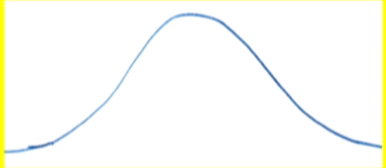

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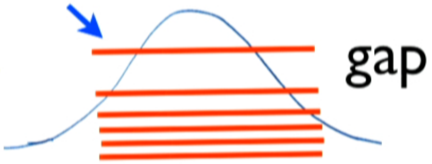

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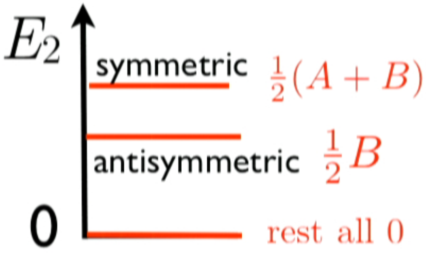
- Solvable model! ("short-range pseudopotential")

$$\boxed{U(r_{12}) = \left(A + B\left(\frac{(r_{12})^2}{\ell_B^2}\right)\right) e^{-\frac{(r_{12})^2}{2\ell_B^2}}}$$



- Laughlin state

$$|\Psi_L^m\rangle = \prod_{i<j}\left(a_i^\dagger - a_j^\dagger\right)^m |0\rangle$$

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$$E_L = 0 \qquad [a_i, a_j^\dagger] = \delta_{ij}$$

maximum density null state

- m=2: (bosons): all pairs avoid the symmetric state E_2 = ½(A+B)
- m=3: (fermions): all pairs avoid the antisymmetric state E_2 = ½B

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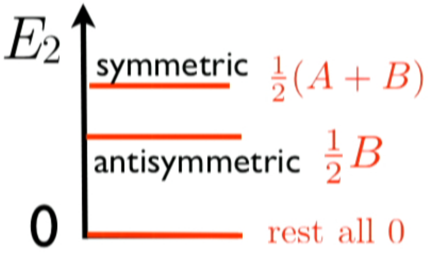

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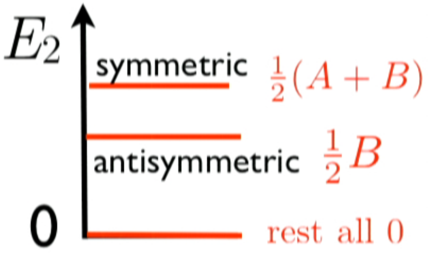

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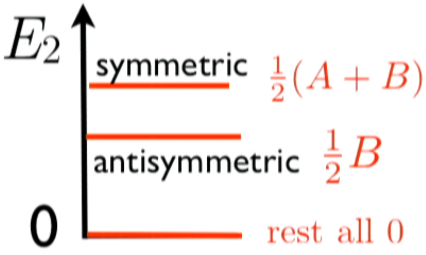

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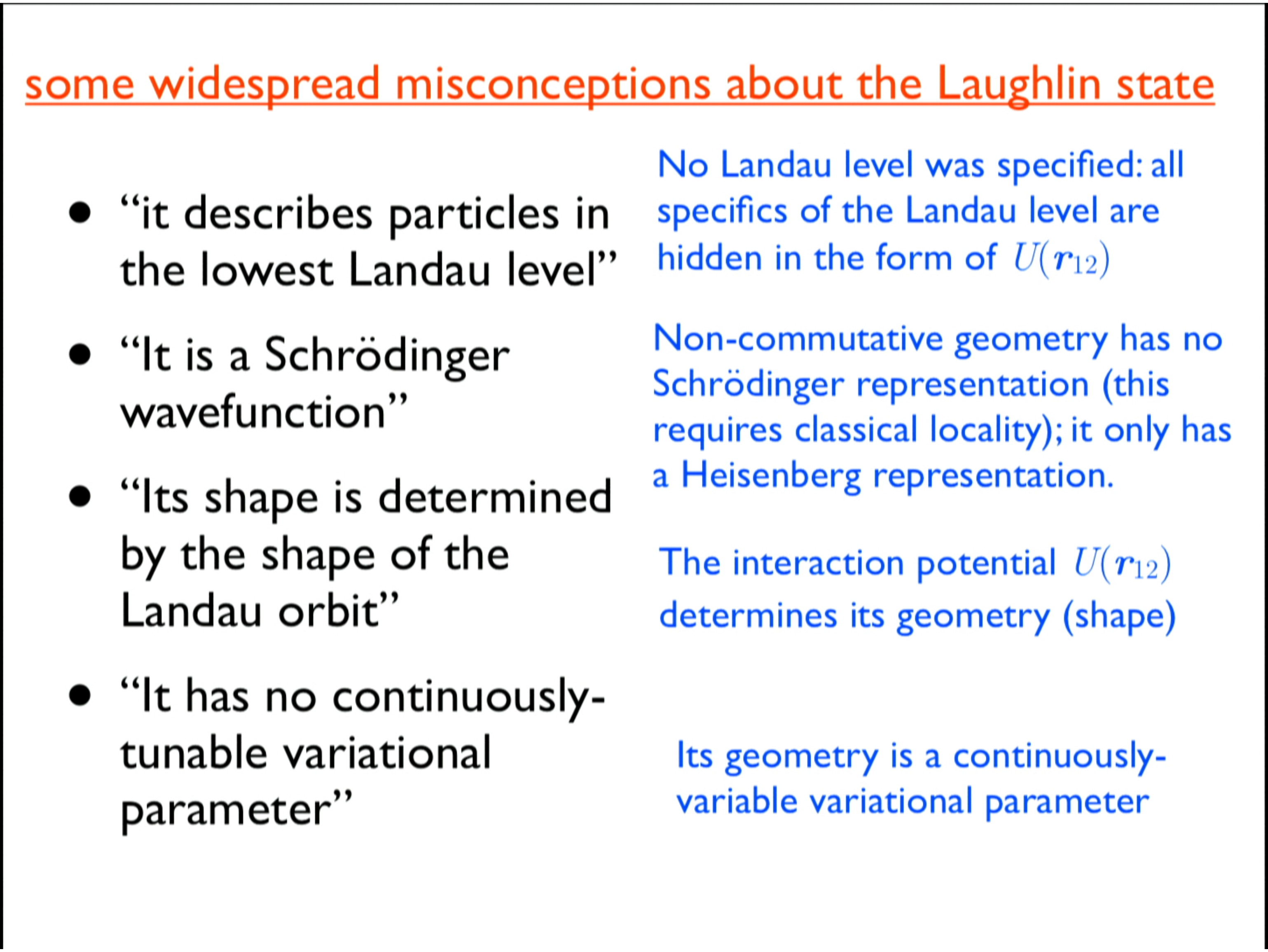

some widespread misconceptions about the Laughlin state

• "it describes particles in the lowest Landau level"	No Landau level was specified: all specifics of the Landau level are hidden in the form of $U(\boldsymbol{r}_{12})$
• "It is a Schrödinger wavefunction"	Non-commutative geometry has no Schrödinger representation (this requires classical locality); it only has a Heisenberg representation.
• "Its shape is determined by the shape of the Landau orbit"	The interaction potential $U(\boldsymbol{r}_{12})$ determines its geometry (shape)
• "It has no continuously-tunable variational parameter"	Its geometry is a continuously-variable variational parameter

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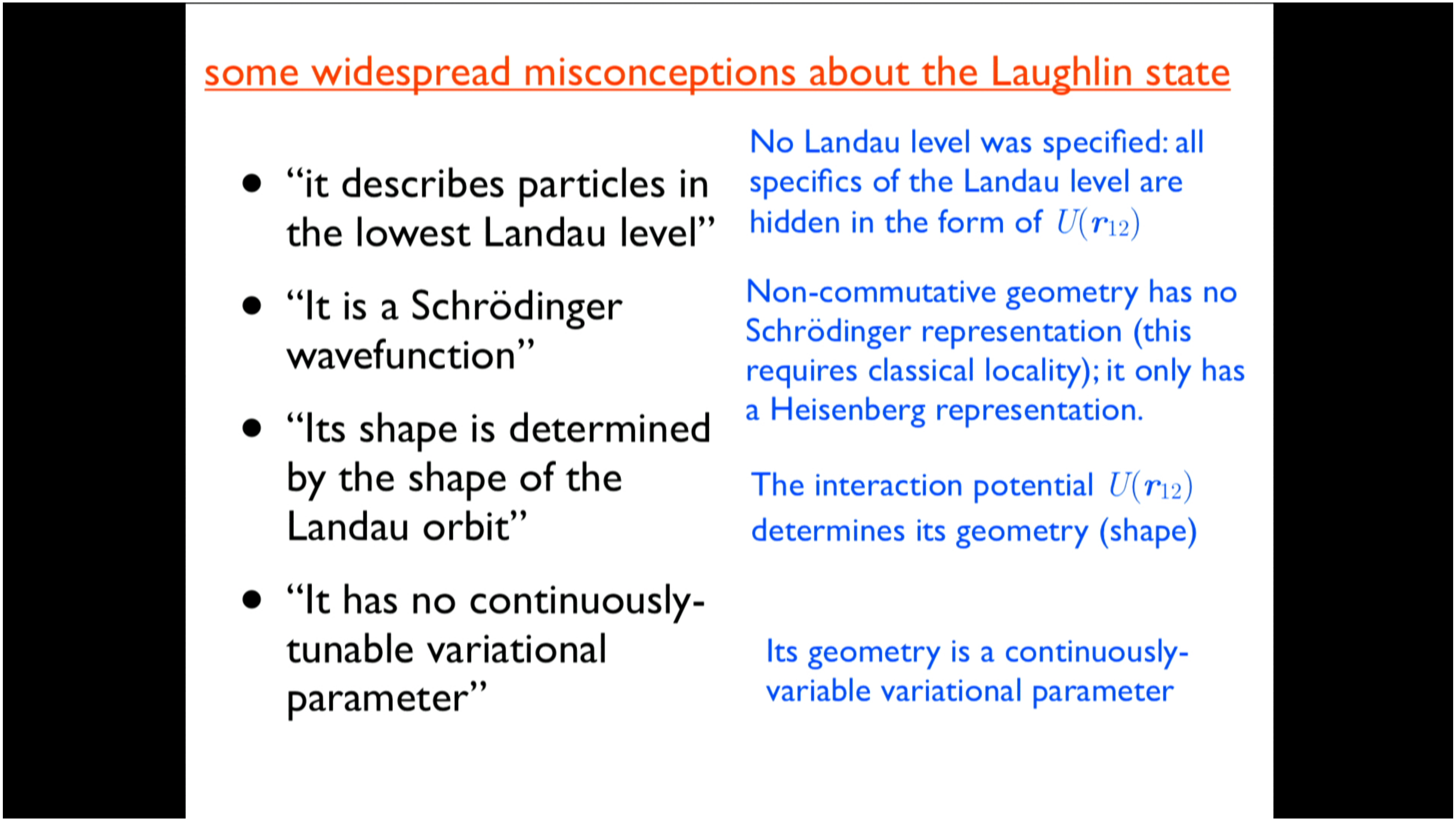

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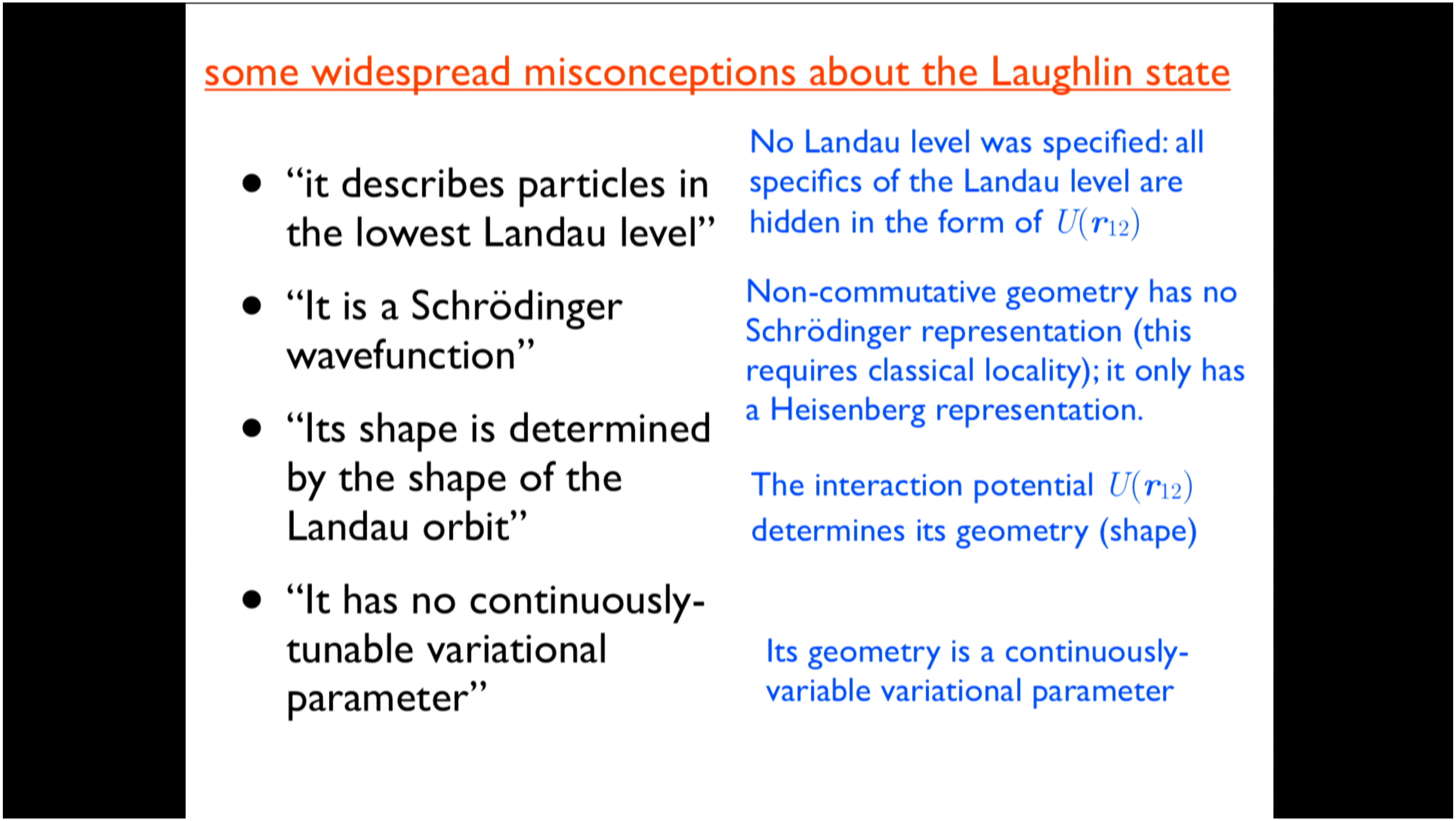

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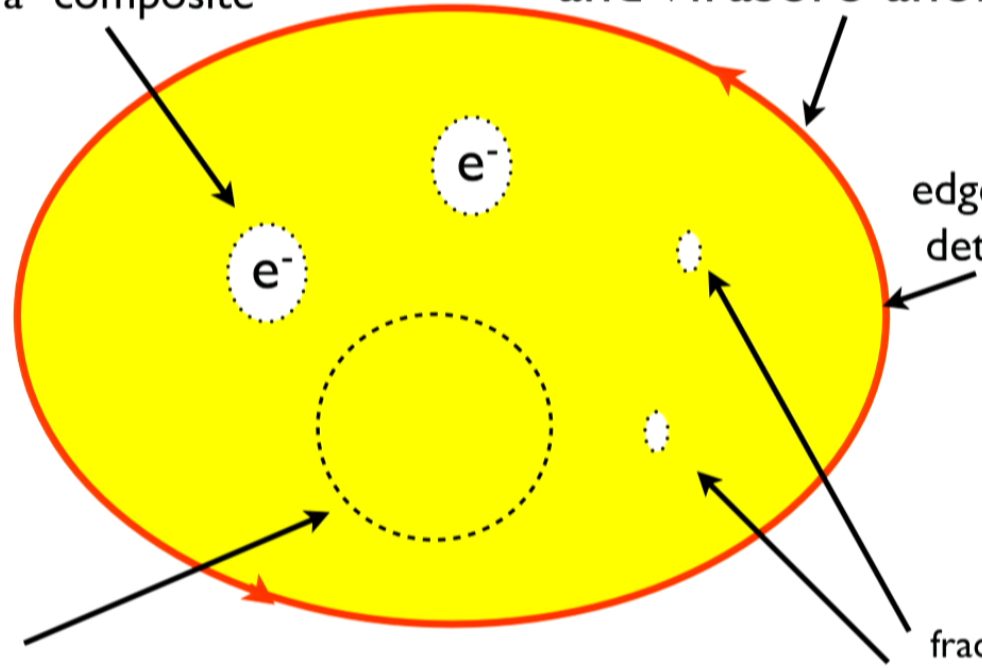
• Anatomy of Laughlin state



electron with "flux attachment" to form a "composite boson"

Chiral edge mode with chiral anomaly and Virasoro anomaly

geometric edge dipole moment determined by Hall viscosity

Topological and geometric bulk properties revealed by entanglement spectrum of cut

fractionally-charged e/3 quasiholes obeying (Abelian) fractional statistics

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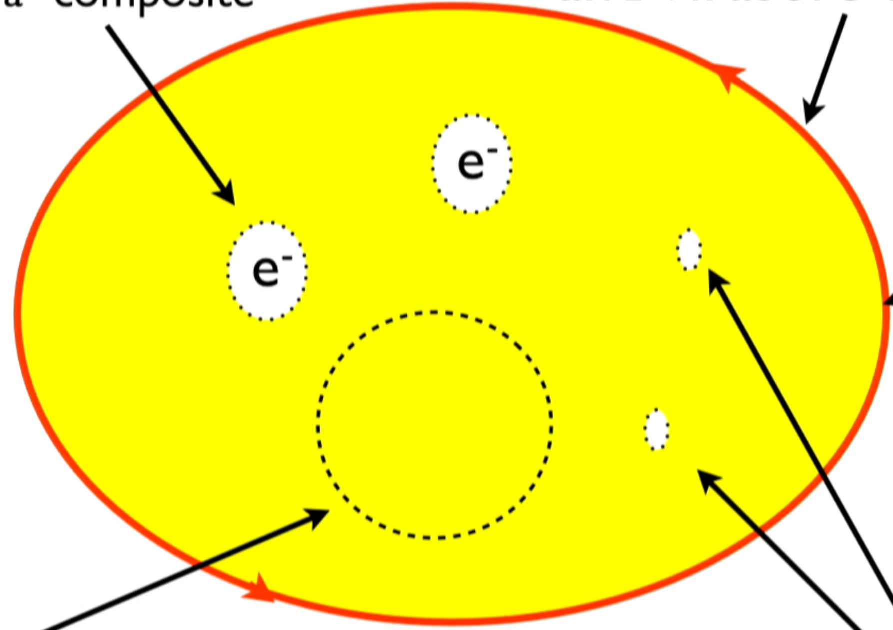

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where is the geometry hiding?

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- in the definition of a_i

$$L(g) = \frac{g_{ab}R^aR^b}{2\ell_B^2} = \frac{1}{2}\left(a^\dagger a + aa^\dagger\right)$$

“guiding-center spin”

“unimodular” Euclidean metric ($\det\ g = 1$)

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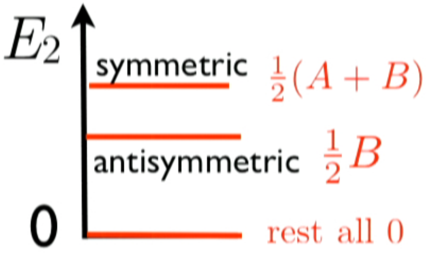

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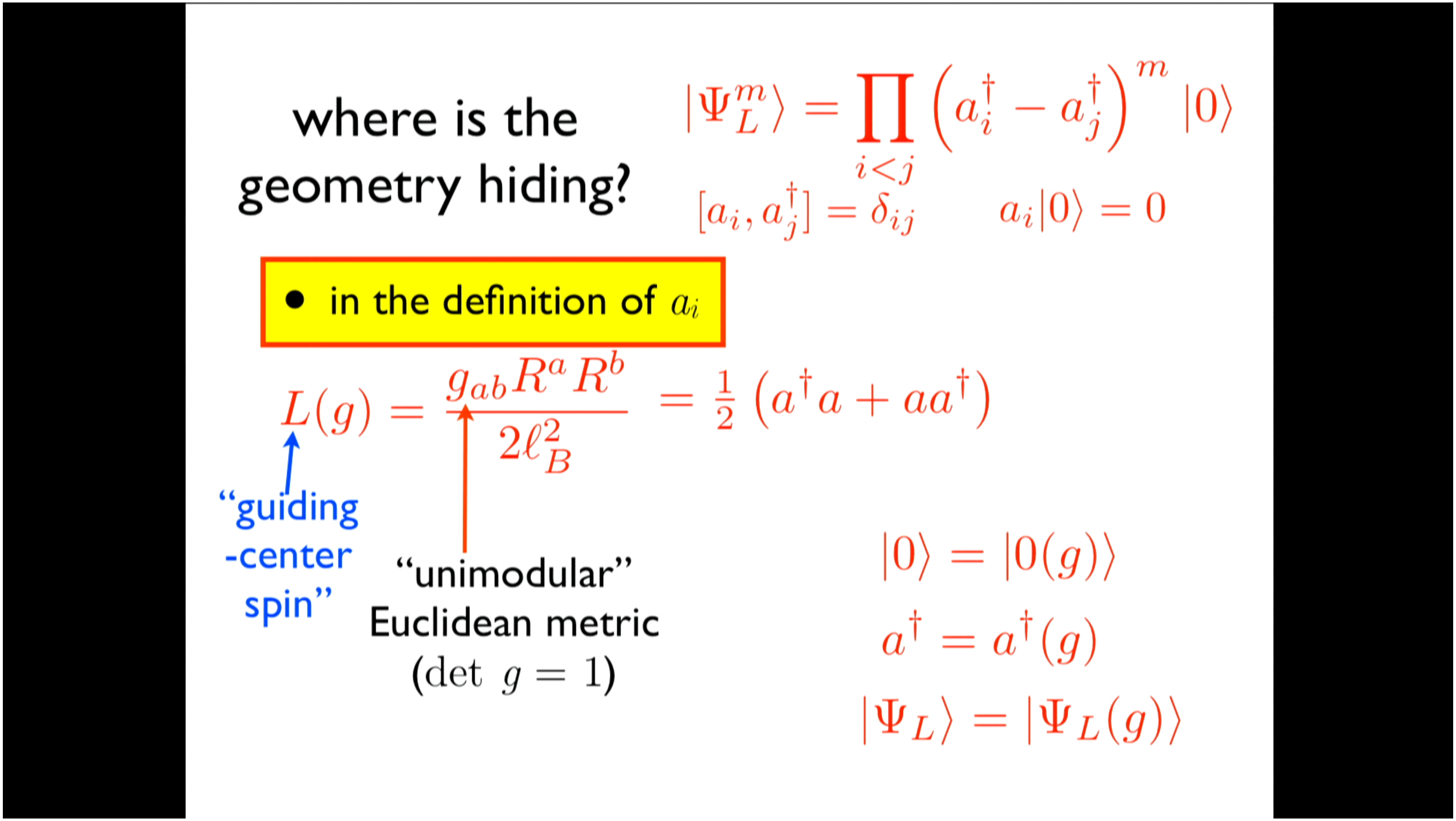

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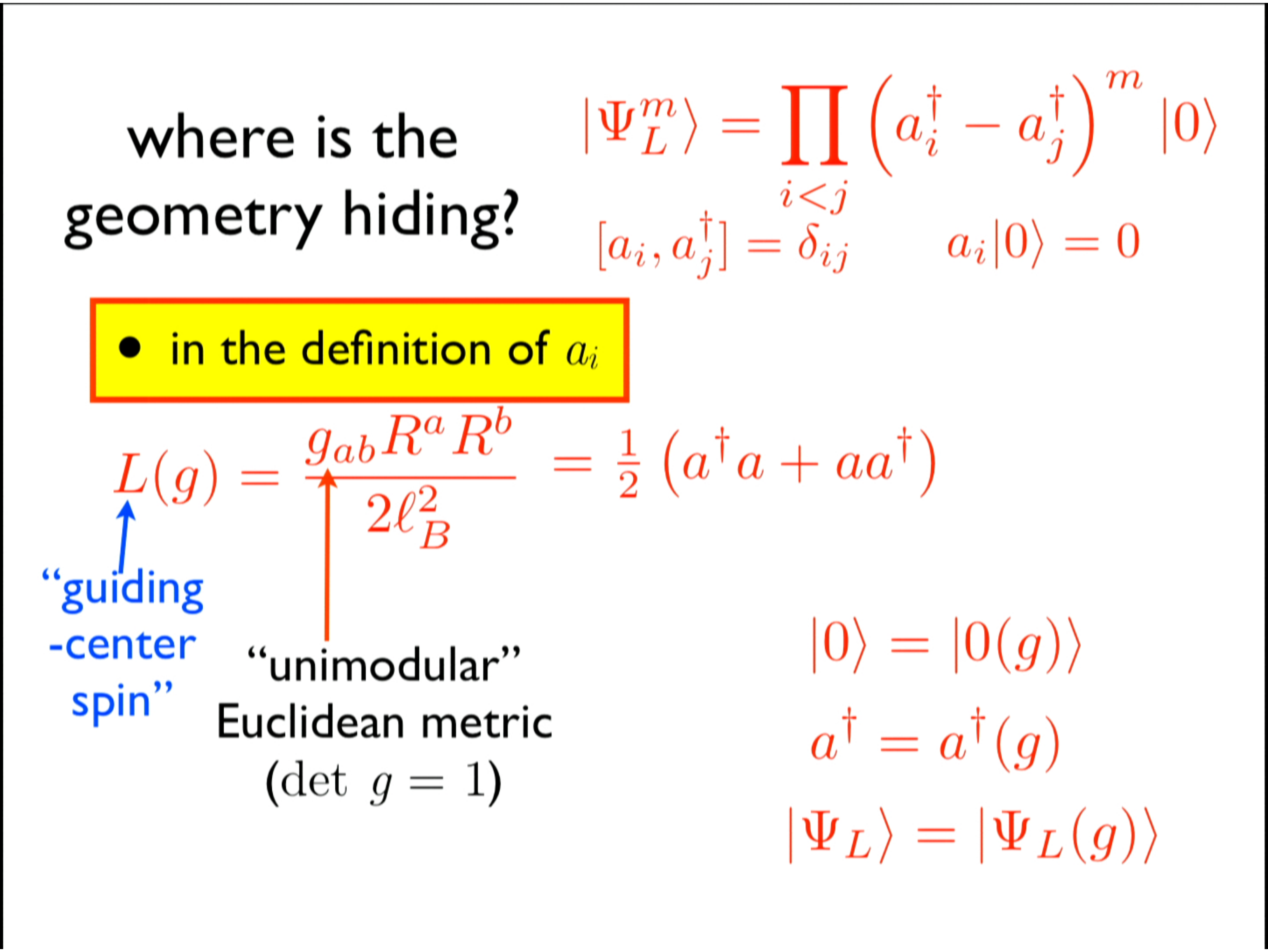

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$$L(g) = \frac{g_{ab}R^aR^b}{2\ell_B^2} = \tfrac{1}{2}\left(a^\dagger a + aa^\dagger\right)$$

"guiding-center spin"

"unimodular" Euclidean metric $(\det\ g = 1)$

$$|0\rangle = |0(g)\rangle$$

$$a^\dagger = a^\dagger(g)$$

$$|\Psi_L\rangle = |\Psi_L(g)\rangle$$

where is the geometry hiding?

$$|\Psi_L^m\rangle = \prod_{i<j} \left(a_i^\dagger - a_j^\dagger\right)^m |0\rangle$$

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$$g_{ab} = \omega_a^* \omega_b + \omega_b^* \omega_a$$

$$\frac{1}{2}\left(\underline{g_{ab}} + i\epsilon_{ab}\right) = \omega_a \omega_b$$

$$a^\dagger = \frac{\omega_a R^a}{\ell_B}$$

$$R^x + iR^y$$

$$\omega = \frac{1}{\sqrt{2}\ell_B}(1, i)$$

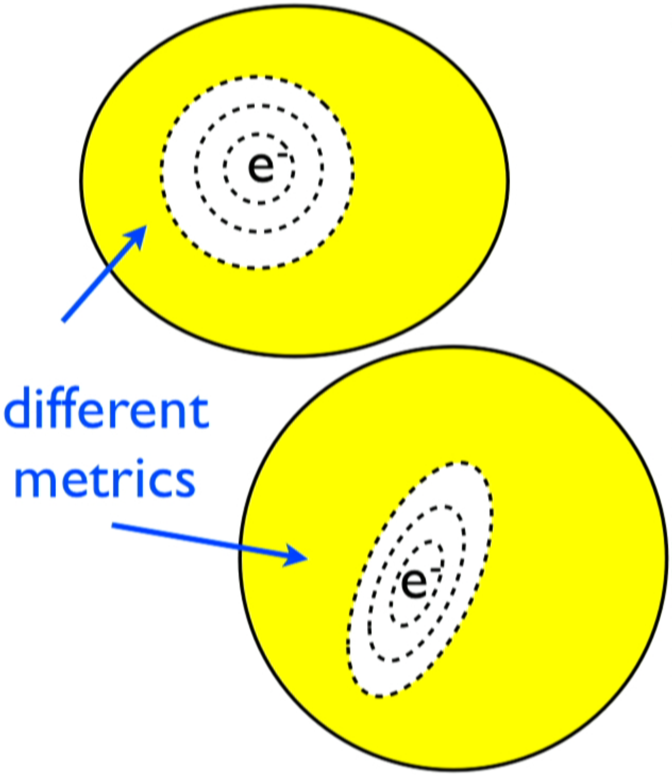
- composite boson: if the central orbital of a basis of eigenstates of L(g) is filled, the next two are empty

- this correlation hole is equivalent to "attachment of three flux quanta" or vortices that travel with the particle, generating a Berry phase that cancels the Bohm-Aharonov phase and transmutes Fermi to Bose exchange statistics.

- this shape of the corelation hole - and hence its correlation energy - varies with the metric g_{ab}

$$|\Psi^3_L\rangle = \prod_{i<j}\left(a^\dagger_i - a^\dagger_j\right)^3 |0\rangle$$

$$L(g)|\psi_m\rangle = (m + \tfrac{1}{2})|\psi_m\rangle$$



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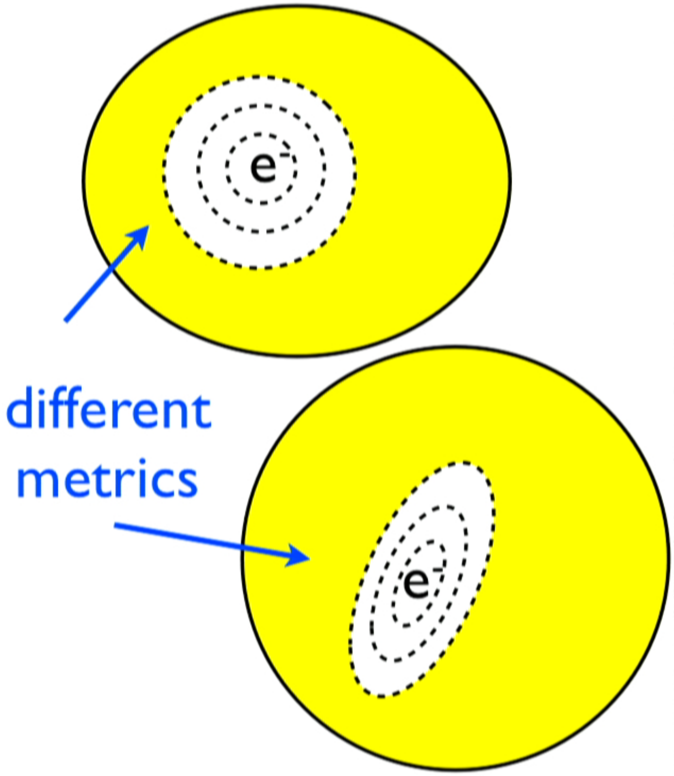

different metrics

where is the geometry hiding?

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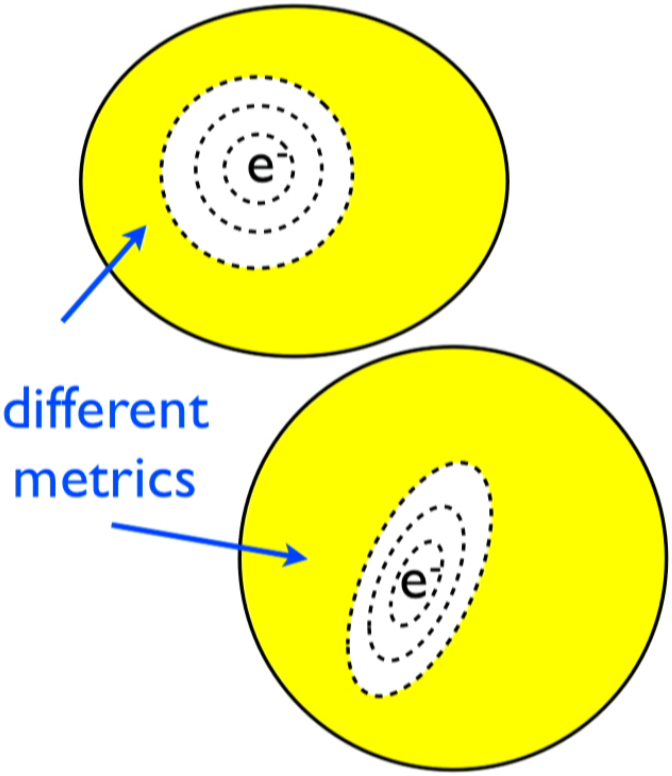

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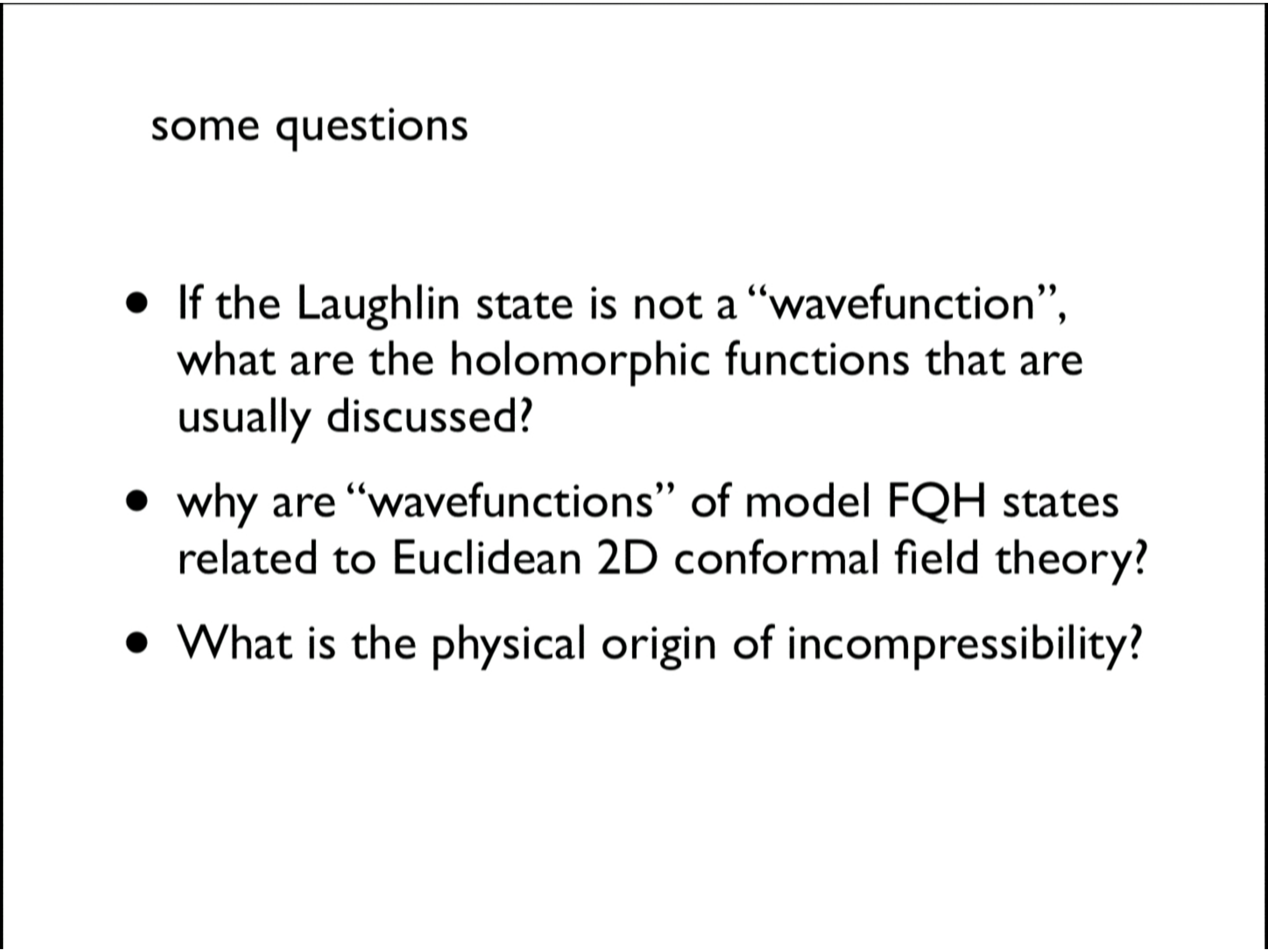
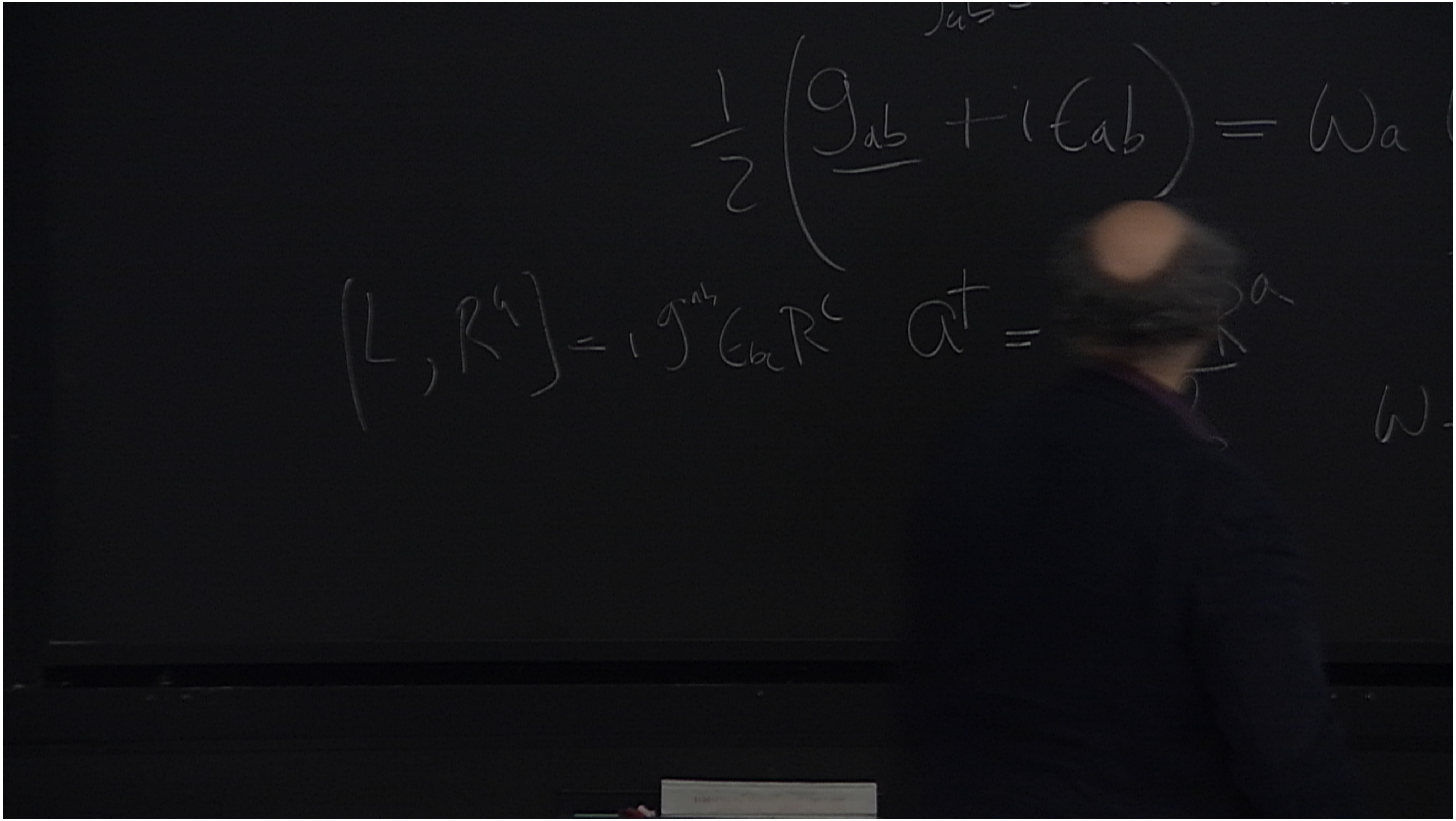


some questions

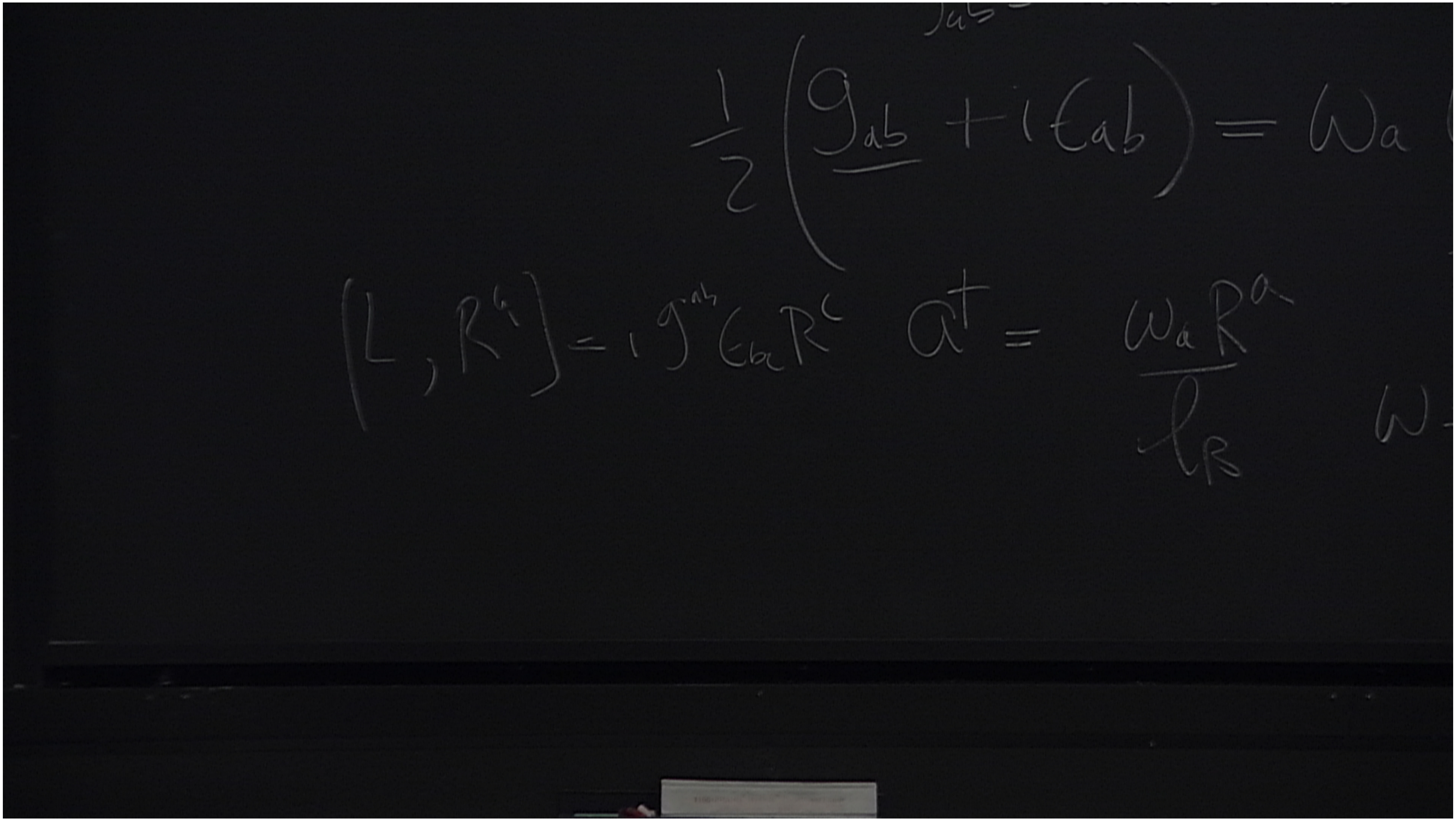
- If the Laughlin state is not a “wavefunction”, what are the holomorphic functions that are usually discussed?
- why are “wavefunctions” of model FQH states related to Euclidean 2D conformal field theory?
- What is the physical origin of incompressibility?



$$\frac{1}{2}\left(\underline{g_{ab}} + i\epsilon_{ab}\right) = \omega_a$$

$$[L, R^a] = i g^{ab}\epsilon_{bc}R^c \qquad a^\dagger =$$

$$\omega$$



g_{ab}

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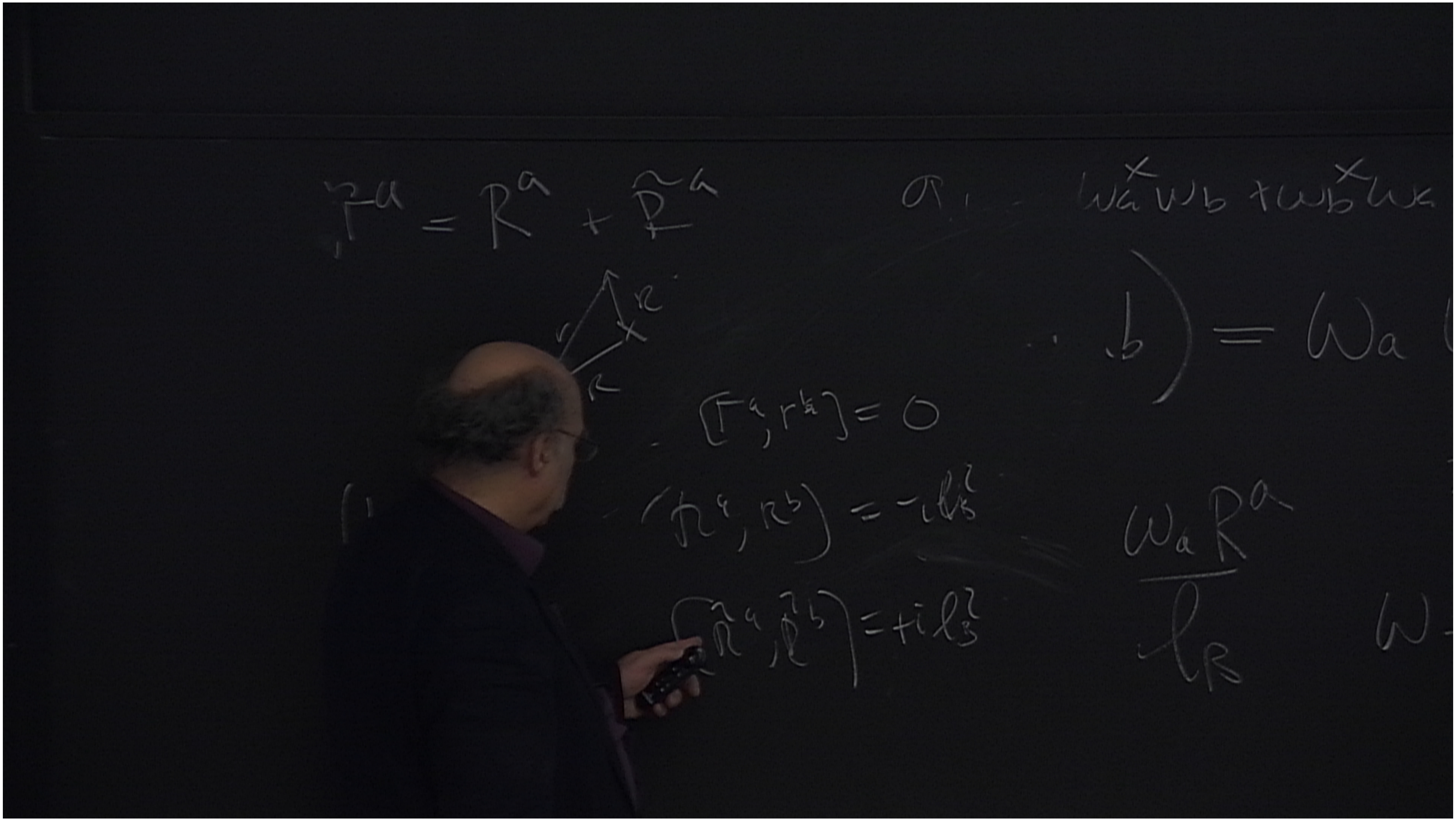


$\tilde{r}^a = R^a + \tilde{R}^a$

$\omega_a^\times \omega_b + \omega_b^\times \omega_a$

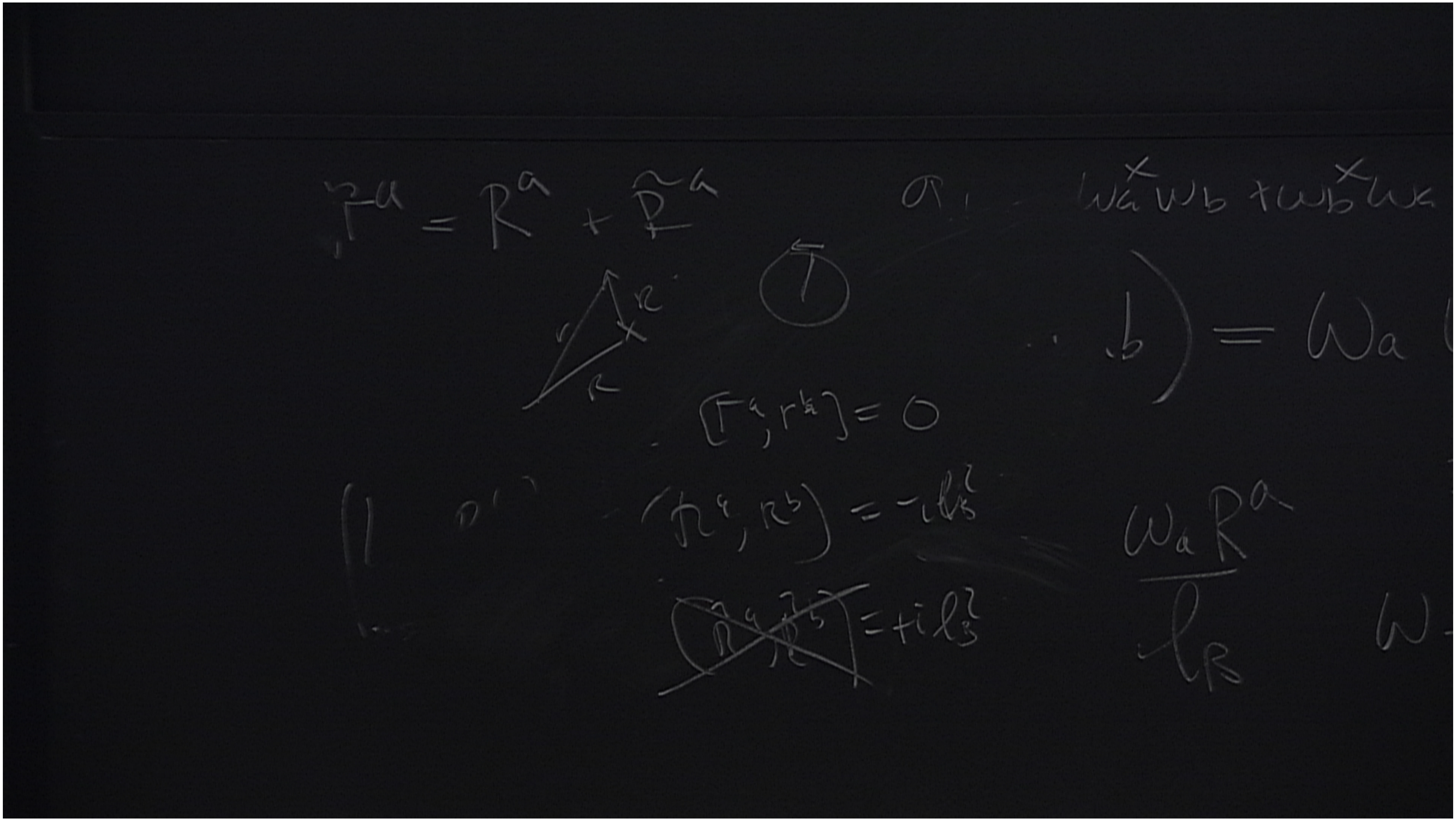
$b) = \omega_a$

$[r^a, r^b] = 0$

$(R^a, R^b) = -i l_B^2$

$(\tilde{R}^a, \tilde{R}^b) = +i l_B^2$

$\frac{\omega_a R^a}{l_B}$



$r^a = R^a + \tilde{R}^a$

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~~“wavefunction”~~ → coherent state amplitude

- coherent states are parameterized by shape (g_{ab}) and location $z(\boldsymbol{r})$

$$a(g)|\boldsymbol{r},g\rangle = z^*(\boldsymbol{r})|\boldsymbol{r},g\rangle$$

- these are non-orthogonal and overcomplete

$$S(\boldsymbol{r},\boldsymbol{r}';g) = \langle \boldsymbol{r},g|\boldsymbol{r}',g\rangle = e^{z^* z'} e^{-\frac{1}{2}z^* z} e^{-\frac{1}{2}z'^* z'}$$

- eigenfunctions: $\displaystyle\int \frac{d^2\boldsymbol{r}'}{2\pi\ell_B^2} S(\boldsymbol{r},\boldsymbol{r}';g)\Psi(\boldsymbol{r}') = s\Psi(\boldsymbol{r})$

solution:
$s = 1$
(degenerate)

$$\Psi(\boldsymbol{r}) = F(z)e^{-\frac{1}{2}z^* z}$$

holomorphic!

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$$|\Psi(F,g)\rangle = F(a^\dagger(g))|0(g)\rangle$$

- coherent state representation

$$|\Psi(F,g)\rangle = \int \frac{d^2\boldsymbol{r}}{2\pi\ell_B^2} e^{-\frac{1}{2}z^*z} F(z)|\boldsymbol{r}(z),g\rangle$$

- Schrodinger wavefunction in some Landau level

$$|\psi_n(\boldsymbol{r},g)\rangle = |\boldsymbol{r}(z),g\rangle \otimes |\psi_n\rangle$$

Landau-level coherent state with metric g	guiding-center coherent state with metric g	Landau orbit state

$$\Psi_{nF}(\boldsymbol{r}') = \int \frac{d^2\boldsymbol{r}}{2\pi\ell_B^2} e^{-\frac{1}{2}z^*z} F(z)\langle r'|\psi_n(\boldsymbol{r},g)\rangle$$

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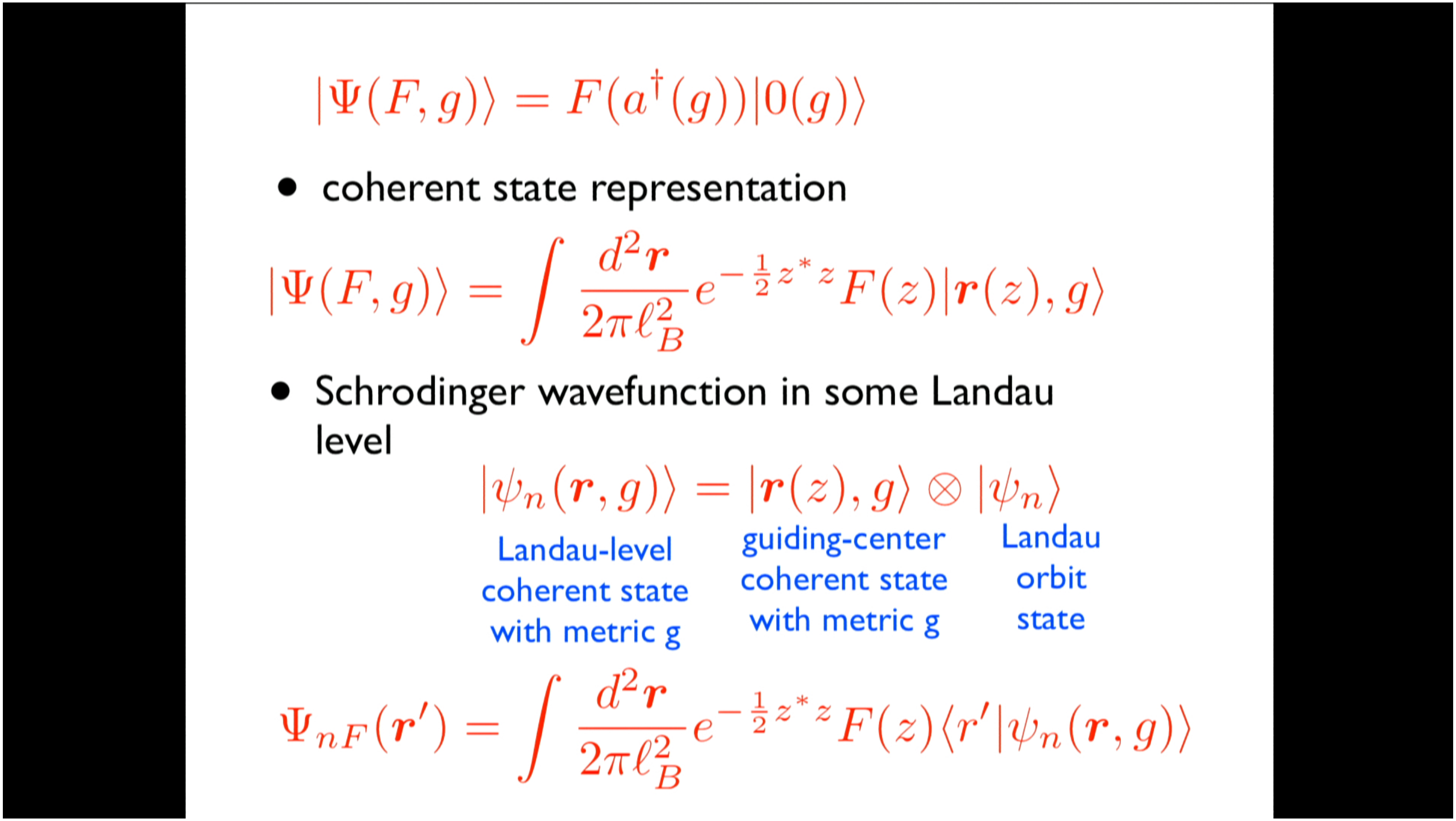
- if **"accidentally"**, the Landau level is the n = 0 level of Galilean-invariant Landau levels with cyclotron effective mass tensor mg_{ab}

$$\Psi_{0F}(\boldsymbol{r}) = \ e^{-\frac{1}{2}z^* z}F(z) \quad ! \qquad z = \frac{x+iy}{\sqrt{2}\ell_B}$$

- People who though they were working with lowest-Landau-level FQHE wavefunctions were in fact working with Landau-level-agnostic coherent state amplitudes!



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- Laughlin (model state) coherent state amplitude

$$\Psi(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) = \boxed{\prod_{i<j}(z_i - z_j)^m} \prod_i e^{-\frac{1}{2}z_i^* z_i}$$

This was recognized as a conformal block of a (free boson) 2D Euclidean conformal field theory

- Same thing for Moore-Read, Read-Rezayi states, etc.

- WHY? (there is no scale invariance in the gapped, incompressible FQH states!)

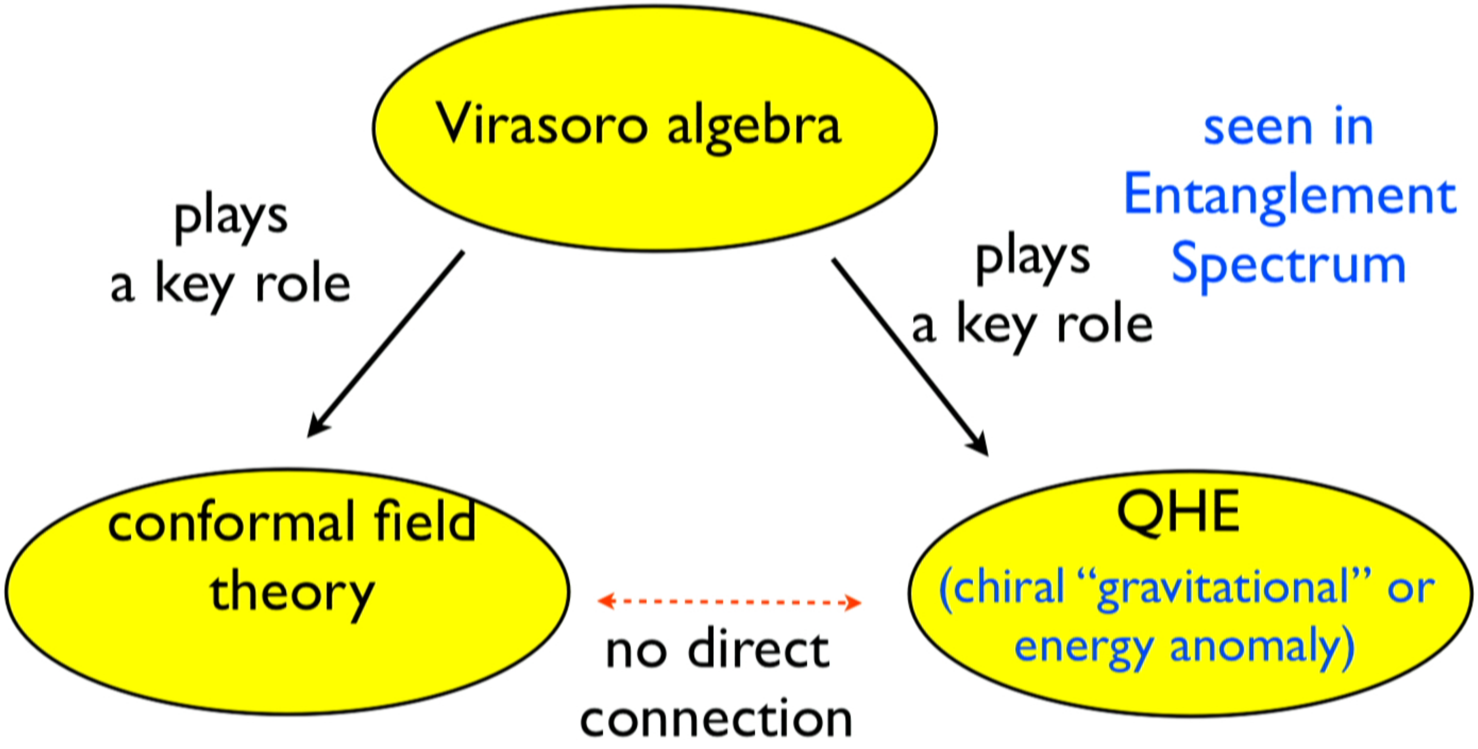
- usual suggestion: edge states of FQH states are described by 1+1d cft.
- But NO Lorentz invariance! so not cft.
- 1+1 cft requires a space-time metric

$$\xcancel{ds^2 = dx^2 - v^2 dt^2}$$

universal speed of massless excitations

There is **no** universal speed of FQH edge modes!
They propagate with different speeds!

- suggested explanation (later)



Virasoro algebra

plays
a key role

plays
a key role

seen in
Entanglement
Spectrum

conformal field
theory

no direct
connection

QHE
(chiral "gravitational" or
energy anomaly)

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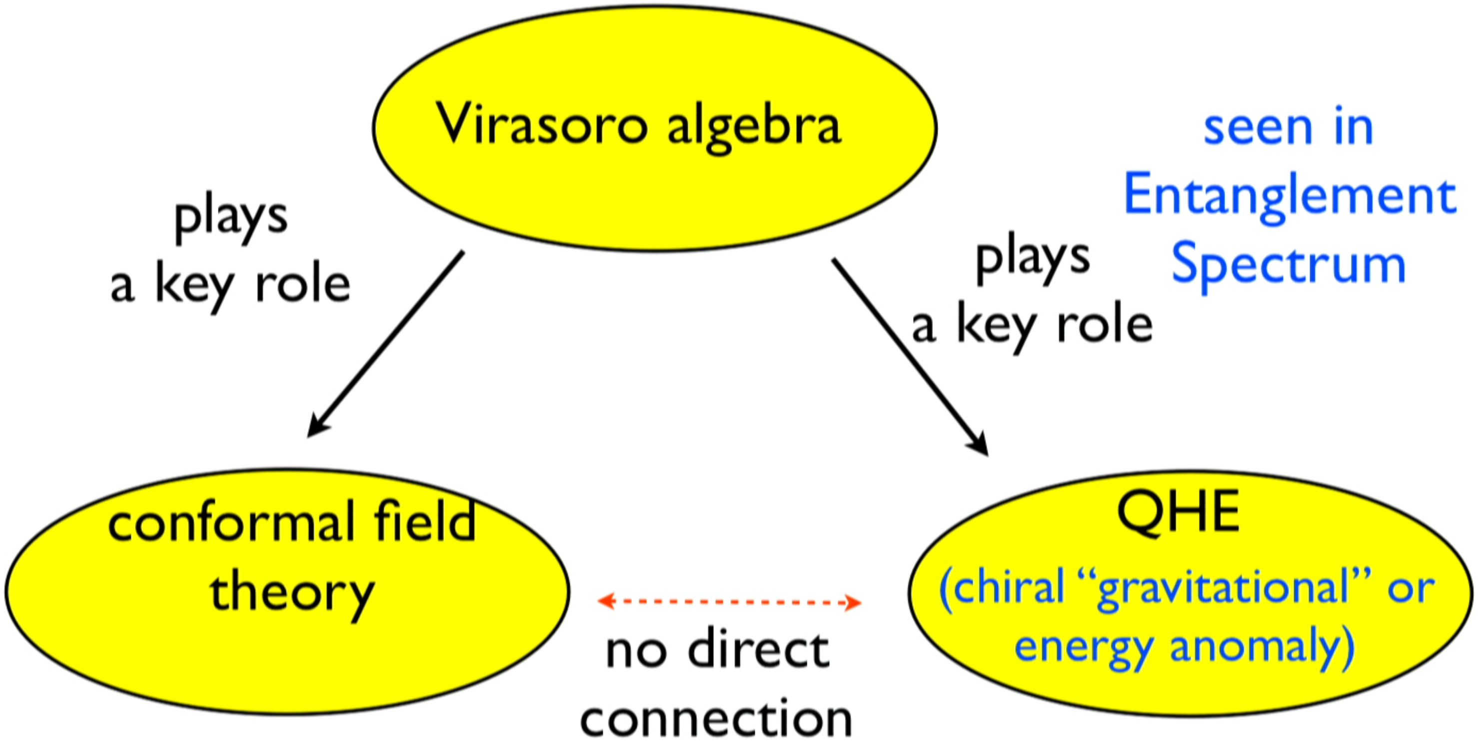

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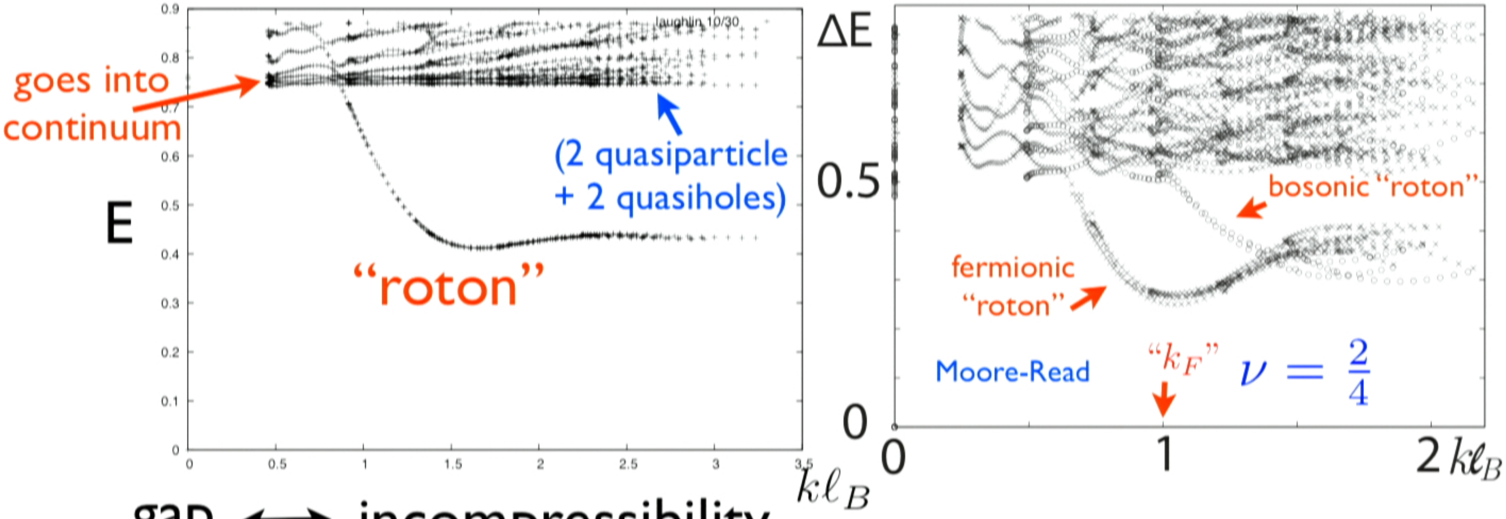


gap ⟷ incompressibility

Collective mode with short-range V$_1$ pseudopotential, 1/3 filling (Laughlin state is exact ground state in that case)

Collective mode with short-range three-body pseudopotential, 1/2 filling (Moore-Read state is exact ground state in that case)

- momentum $\hbar$k of a quasiparticle-quasihole pair is proportional to its **electric dipole moment p$_e$** $\hbar k_a = \epsilon_{ab} B p_e^b$

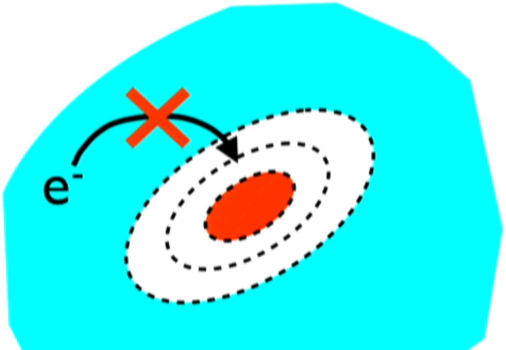
gap for electric dipole excitations is a MUCH stronger condition than just gap for charged excitations

- Model cft-based states such as the Laughlin state have a **constant** (flat, rigidly-fixed) metric
- In real FQH states of electrons contained in a non-uniform background potential, the metric **varies locally and dynamically** to allow the incompressible fluid to adjust to non-uniform flow induced by the background.
- The metric $g_{ab}(\boldsymbol{r}, t)$ then becomes an emergent dynamical collective degree of freedom of the FQH state.

FDMH, Phys. Rev. Lett. **107**, 116801 (2011)

- Origin of FQHE incompressibility is analogous to origin of **Mott-Hubbard gap** in lattice systems.
- There is an energy gap for putting an **extra particle** in a quantized region that is **already occupied**

- **On the lattice** the "quantized region" is an atomic orbital with a fixed shape
- **In the FQHE** only the **area** of the "quantized region" is fixed. The **shape** must adjust to minimize the correlation energy.



energy gap prevents additional electrons from entering the region covered by the composite boson

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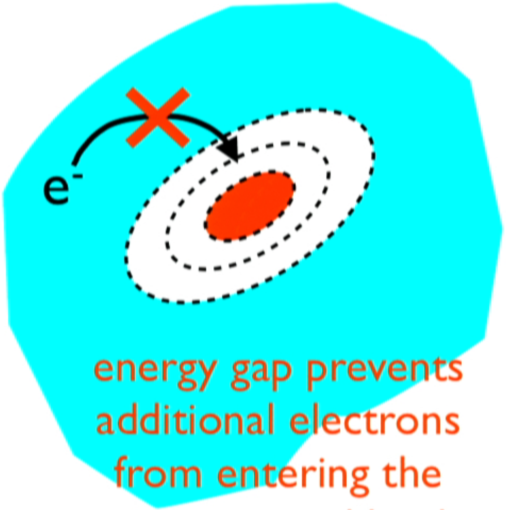


energy gap prevents additional electrons from entering the region covered by the composite boson

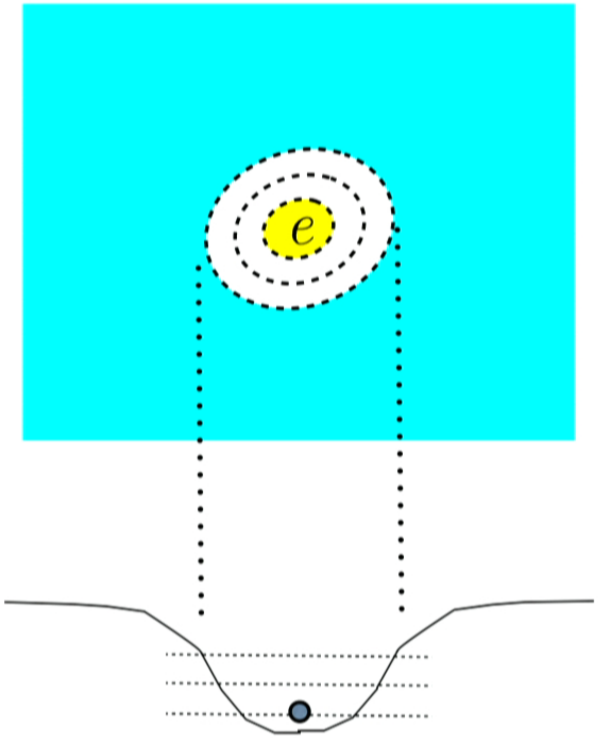
$1/3$ Laughlin state

If the central orbital is filled, the next two are empty

The composite boson has inversion symmetry about its center

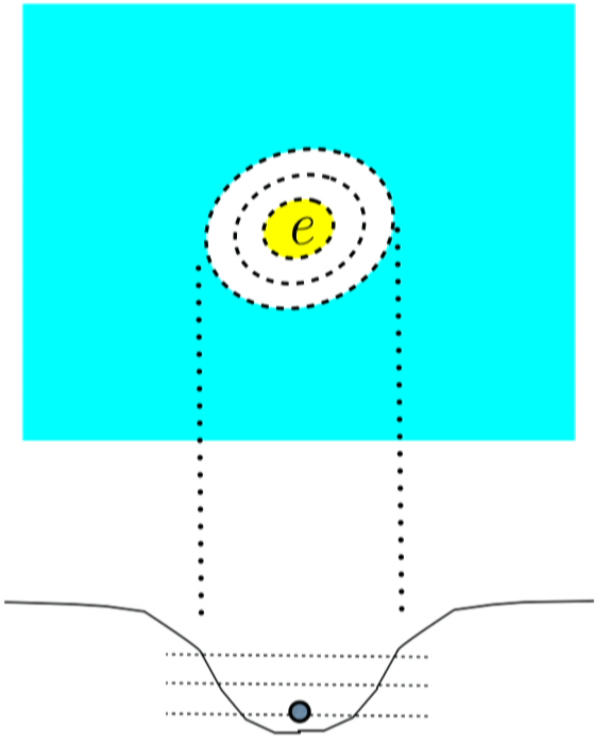
It has a "spin"

$$\begin{array}{rccccr} & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & & \\ & \boxed{1} & \boxed{0} & \boxed{0} & \cdots\cdots & L = \frac{1}{2} \\ - & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \cdots\cdots & -\ L = \frac{3}{2} \\ \cline{6-6} & & & & & s = -1 \end{array}$$



the electron excludes other particles from a region containing 3 flux quanta, creating a potential well in which it is bound

$1/3$ Laughlin state



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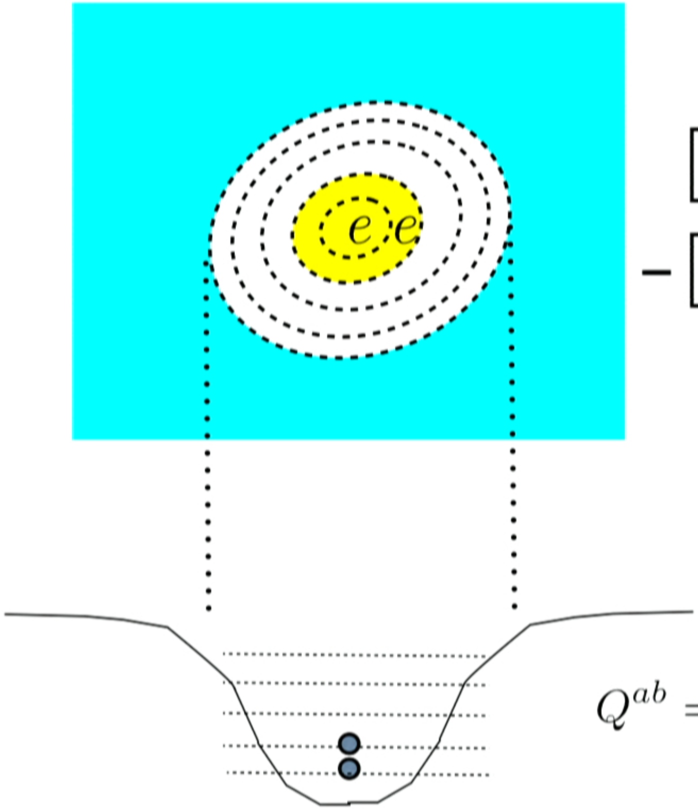
$2/5$ state



$$\overset{\frac{1}{2}}{\boxed{1}}\overset{\frac{3}{2}}{\boxed{1}}\overset{\frac{5}{2}}{\boxed{0}}\boxed{0}\boxed{0}\cdots\cdots \qquad L = 2$$

$$-\boxed{\tfrac{2}{5}}\boxed{\tfrac{2}{5}}\boxed{\tfrac{2}{5}}\boxed{\tfrac{2}{5}}\boxed{\tfrac{2}{5}}\cdots\cdots \qquad -L = 5$$

$$s = -3$$

$$L = \frac{g_{ab}}{2\ell_B^2}\sum_i R_i^a R_i^b$$

$$Q^{ab} = \int d^2r\, r^a r^b \delta\rho(r) = s\ell_B^2 g^{ab}$$

second moment of neutral
composite boson
charge distribution

- The composite boson behaves as a neutral particle because the Berry phase (from the disturbance of the the other particles as its “exclusion zone” moves with it) cancels the Bohm-Aharonov phase

- It behaves as a boson provided its statistical spin cancels the particle exchange factor when two composite bosons are exchanged

p particles $(-1)^{pq} = (-1)^p$ fermions
q orbitals $(-1)^{pq} = 1$ bosons

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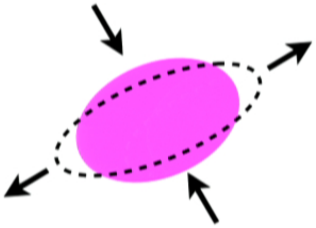
p particles	$(-1)^{pq} = (-1)^p$	fermions
q orbitals	$(-1)^{pq} = 1$	bosons

- The metric (shape of the composite boson) has a preferred shape that minimizes the correlation energy, but fluctuates around that shape



$\delta E \propto (\text{distortion})^2$

- The zero-point fluctuations of the metric are seen as the O(q^4) behavior of the "guiding-center structure factor" (Girvin et al, (GMP), 1985)
- long-wavelength limit of GMP collective mode is fluctuations of (spatial) metric (analog of "graviton")

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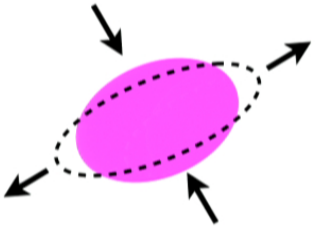

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FDMH, Phys. Rev. Lett. **107**, 116801 (2011)

- generators of area preserving diffeomorphisms

$$\rho(\boldsymbol{q}) = \sum_j e^{i\boldsymbol{q}\cdot\boldsymbol{R}_j}$$

Girvin-MacDonald-Platzman

$$[\rho(\boldsymbol{q}), \rho(\boldsymbol{q}')] = 2i\sin(\tfrac{1}{2}\boldsymbol{q}\times\boldsymbol{q}'\ell_B^2)\rho(\boldsymbol{q}+\boldsymbol{q}')$$

- static guiding-center structure factor

$$s(\boldsymbol{q}) = \frac{\nu}{N}\left(\langle\rho(\boldsymbol{q})\rho(-\boldsymbol{q})\rangle - \langle\rho(\boldsymbol{q})\rangle\langle\rho(-\boldsymbol{q})\rangle\right)$$

characterizes zero-point quantum fluctuations
vanishes as $|q|^4$ at long wavelengths,
this is quantum fluctuations of the metric

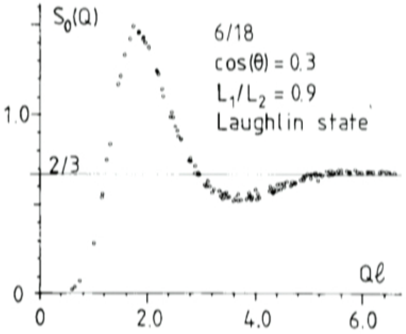

Figure 8.7 The Laughlin state data of Fig. 8.6, converted to the Landau-level-independent form $S_0(Q)$ (see Eq. 8.3.9). The asymptotic large-Q value $1+\nu\langle P\rangle = 2/3$, where P is the pair-exchange operator, is attained within the 'window' of reciprocal space that can be studied in a six-particle system.

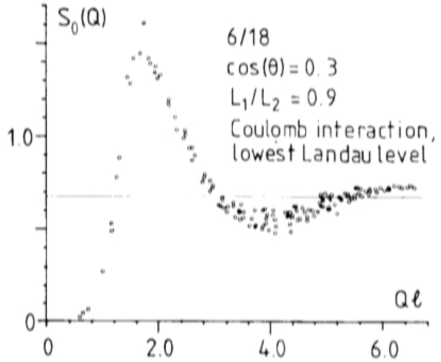

Figure 8.8 Same quantities as Fig. 8.7, but calculated for the true ground state of the lowest-Landau-level Coulomb interaction rather than the Laughlin state.

Text

$$S_0(\boldsymbol{q}) = \frac{1}{N}\sum_{ij}\langle e^{i\boldsymbol{q}\cdot\boldsymbol{R}_i} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}_j}\rangle - \langle e^{i\boldsymbol{q}\cdot\boldsymbol{R}_i}\rangle\langle e^{-i\boldsymbol{q}\cdot\boldsymbol{R}_j}\rangle$$

$$\nu = \frac{N}{N_{\mathrm{orb}}}$$

number of orbitals = number of flux quanta through 2D surface

$$s(\boldsymbol{q}) = \nu S_0(\boldsymbol{q})$$

(per flux , instead of per particle)

variational upper bound to collective excitation energy

$$0 < \Delta E(k) < \frac{O(k^4)}{s(k)} \longleftarrow$$

must be O(k^4) if gapped

- crucial new physics:

composite bosons couple to the combination

$$\boxed{peB(\boldsymbol{r}) - \hbar s K(r)}$$

Gaussian curvature of metric

charge of composite boson

guiding-center "spin" of boson

* gauge field is

$$\boxed{peA_\mu(\boldsymbol{r}) - \hbar s \Omega_\mu(r)}$$

analog of spin-connection

+ Chern-Simons

related to Wen and Zee 1992

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+ Chern-Simons

- The shape of the composite boson is determined by minimizing the sum of the correlation energy and the background potential energy.
- If there is no background potential, the metric is flat and the charge density is uniform
- If there is a background potential $g_{ab}(\boldsymbol{r})$ varies with position to give a charge density fluctuation

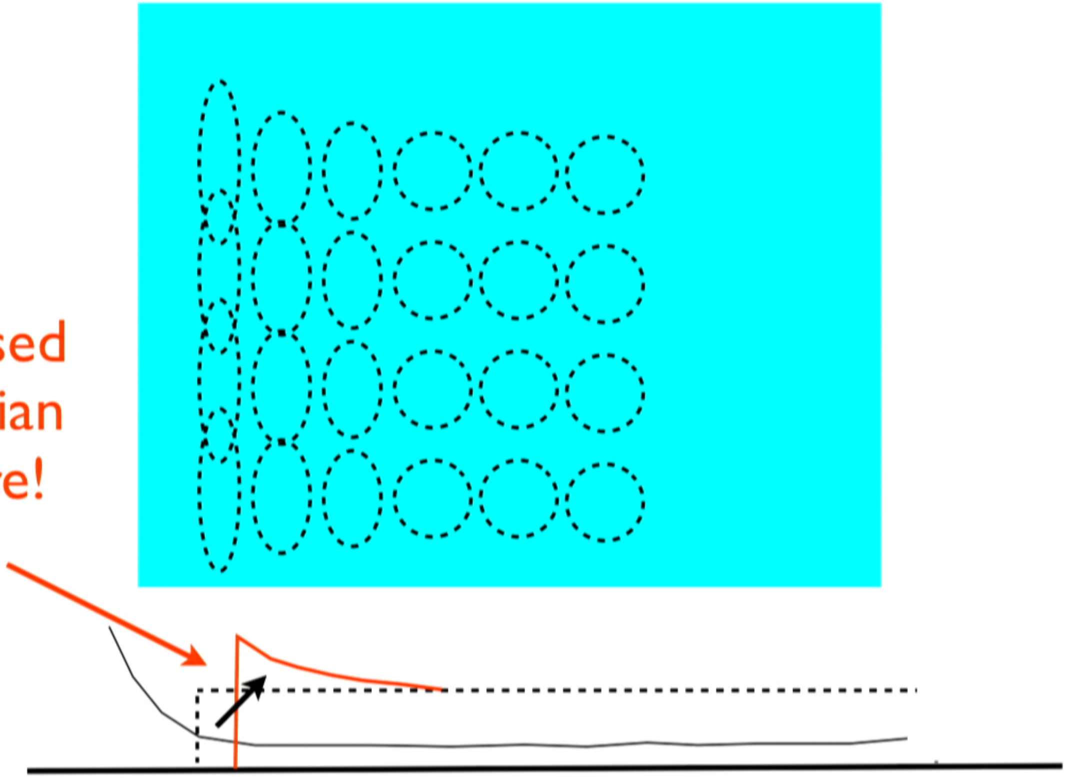
Gaussian curvature of metric

$$\delta\rho(\boldsymbol{r}) = esK(\boldsymbol{r})$$

“spin”

$$K(r) = \underbrace{\tfrac{1}{2}\partial_a\partial_b g^{ab}}_{\text{from variation of second moment of charge distribution}} + \underbrace{\tfrac{1}{8}g_{ab}\epsilon_{cd}\epsilon^{ef}\partial_e g^{ac}\partial_f g^{bd}}_{\text{from Berry phase associated with shape change}}$$

- metric deforms (preserving $\det\ g = 1$)in presence of non-uniform electric field



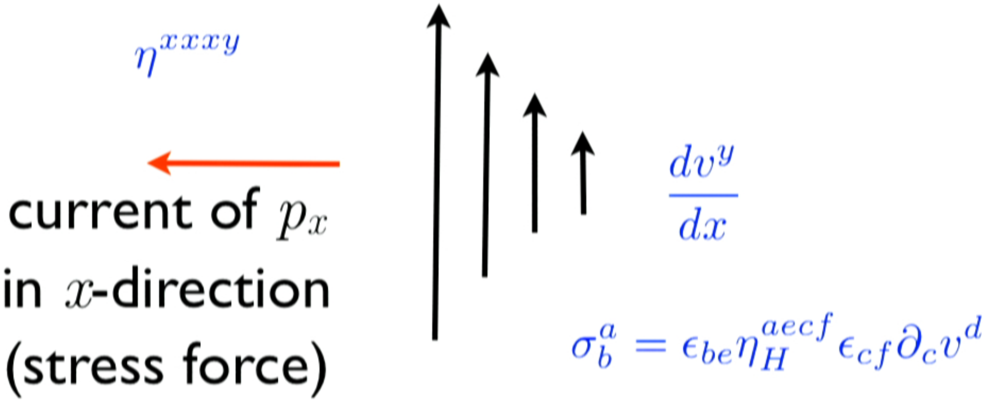
fluid compressed by Gaussian curvature!

potential near edge

produces a dipole momemt

- Hall viscosity $\eta^{abcd} = \frac{eBs}{4\pi q}\frac{1}{2}\left(g^{ac}\epsilon^{bd} + g^{bd}\epsilon^{ac} + a \leftrightarrow b\right)$

(plus a similar term from the Landau orbit degrees of freedom (Avron et al))

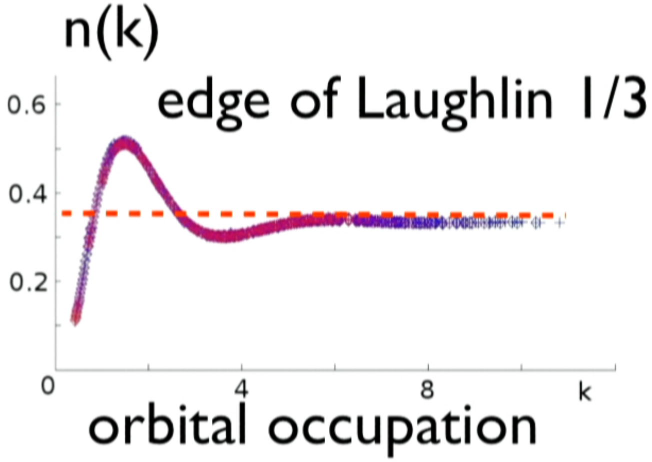


η^{xxxy}

current of p_x
in x-direction
(stress force)

$\frac{dv^y}{dx}$

$\sigma_b^a = \epsilon_{be}\eta_H^{aecf}\epsilon_{cf}\partial_c v^d$



The dipole at a segment of the edge has a momentum

$$dP_a = \frac{\hbar}{e\ell_B^2}\epsilon_{ab}dp^b$$

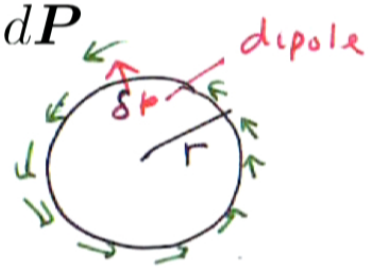
momentum

dipole

doesn't contribute to total momentum:

$$\oint dP_a = 0$$

$\text{momentum } d\boldsymbol{P}$



circular droplet

it <u>does</u> contribute an extra term to total angular momentum:

$$\Delta L^z(\boldsymbol{g}) = \hbar \oint \epsilon^{ab} g_{bc} r^c dP_a \neq 0$$

- multicomponent quantum Hall edge states do not have a universal speed, so are **not** Lorentz and conformally invariant.
- components of the cft energy momentum tensor:

Momentum density is independent of v:

$$T_x^0 = T - \bar{T}$$

Energy density and stress are proportional to v

$$T_0^0 = -T_x^x = v(T + \bar{T})$$

Tracelessness (in flat space-time) is independent of v

$$T_0^0 + T_x^x = 0$$

Energy current density is proportional to v^2

$$T_0^x = v^2(T - \bar{T})$$

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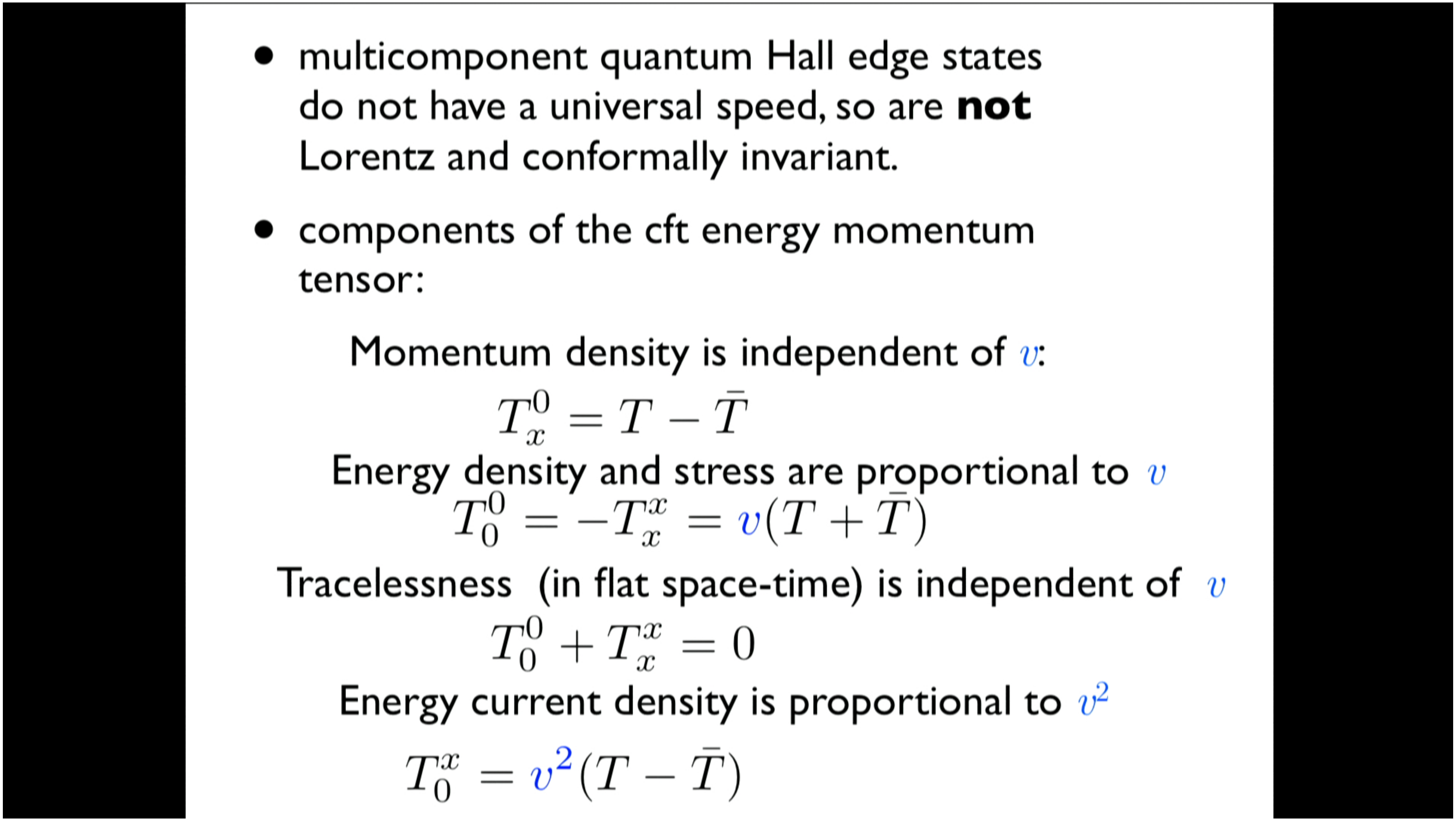

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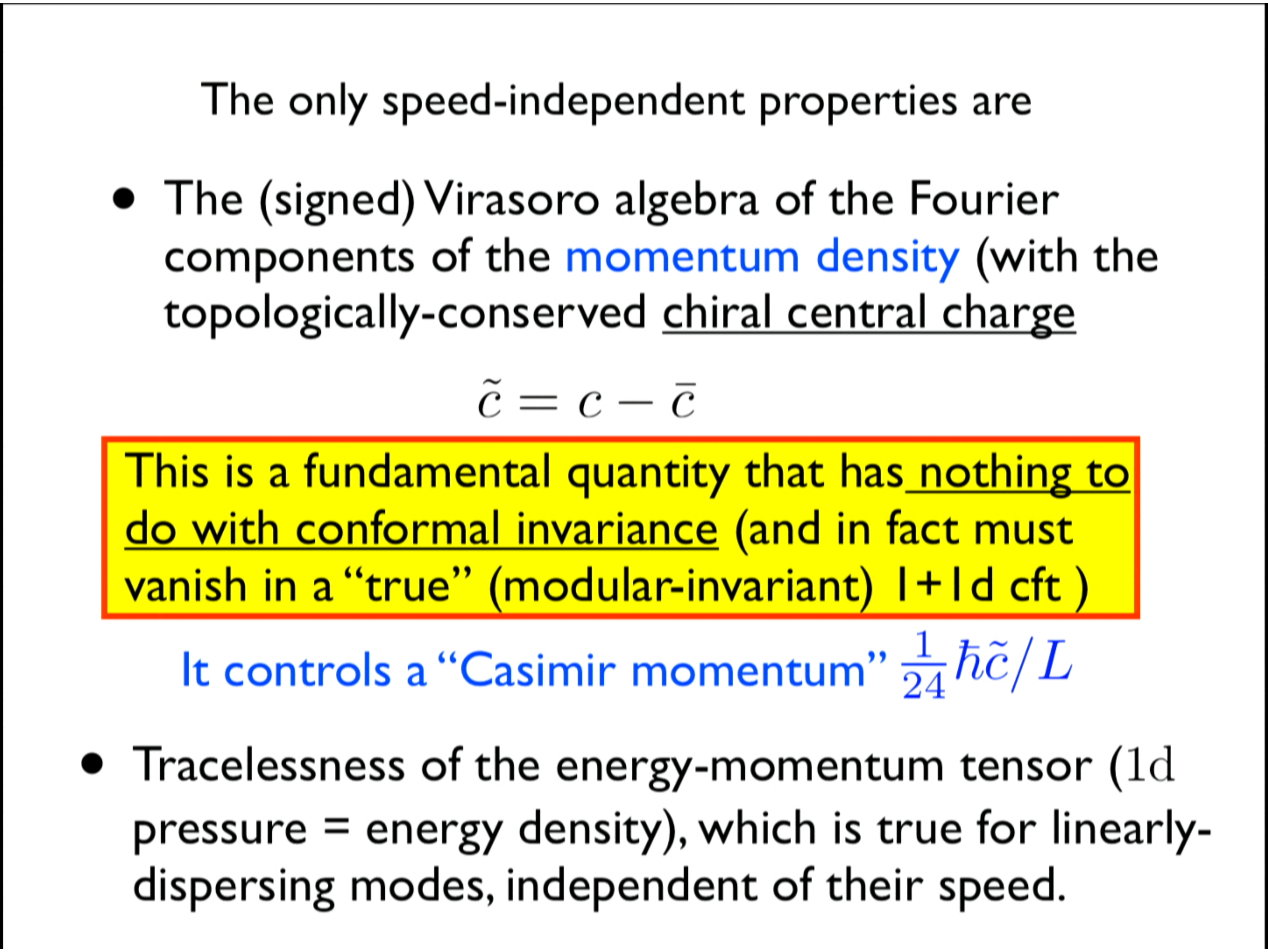

The only speed-independent properties are

- The (signed) Virasoro algebra of the Fourier components of the momentum density (with the topologically-conserved <u>chiral central charge</u>

$$\tilde{c} = c - \bar{c}$$

This is a fundamental quantity that has <u>nothing to do with conformal invariance</u> (and in fact must vanish in a "true" (modular-invariant) 1+1d cft)

It controls a "Casimir momentum" $\frac{1}{24}\hbar\tilde{c}/L$

- Tracelessness of the energy-momentum tensor ($1\mathrm{d}$ pressure = energy density), which is true for linearly-dispersing modes, independent of their speed.

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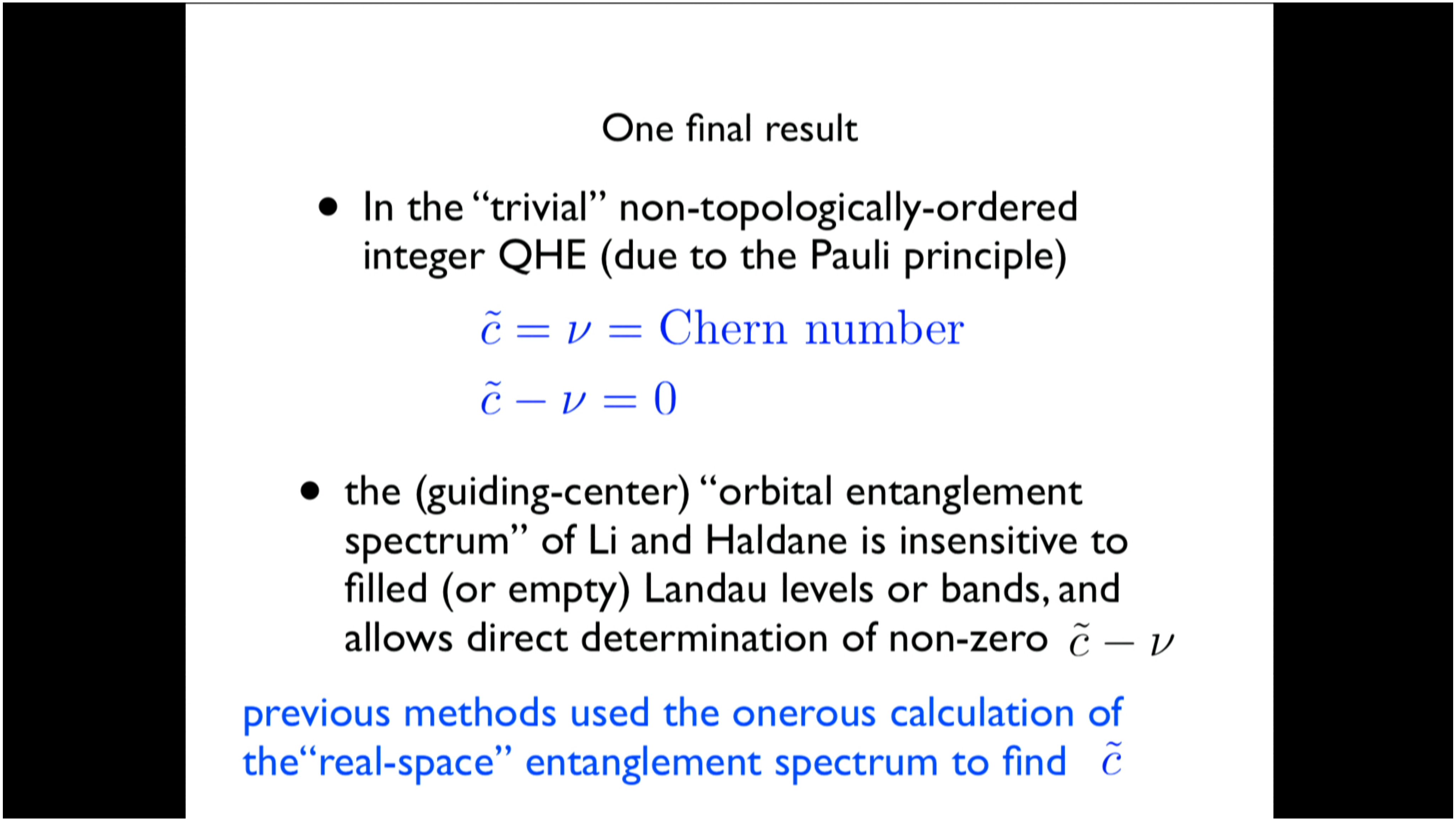


One final result

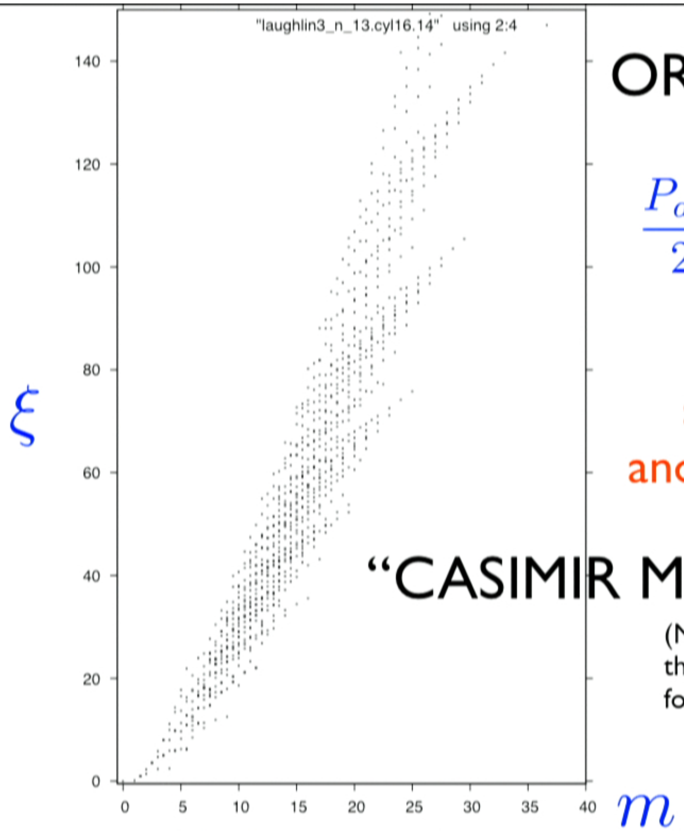
- In the "trivial" non-topologically-ordered integer QHE (due to the Pauli principle)

$$\tilde{c} = \nu = \text{Chern number}$$

$$\tilde{c} - \nu = 0$$

- the (guiding-center) "orbital entanglement spectrum" of Li and Haldane is insensitive to filled (or empty) Landau levels or bands, and allows direct determination of non-zero $\tilde{c} - \nu$

previous methods used the onerous calculation of the"real-space" entanglement spectrum to find $\tilde{c}$



ORBITAL CUT

$$\frac{P_a L^a}{2\pi} = \frac{\sum_\alpha m_\alpha e^{-\xi_\alpha}}{\sum_\alpha e^{-\xi_\alpha}} = \eta_H^{cd}\epsilon_{ac}\epsilon_{bd}\frac{L^a L^b}{2\pi\ell_B^2}$$

$$+\frac{1}{24}(\tilde{c}-\nu) - h$$

signed conformal anomaly (chiral stress-energy anomaly)

chiral anomaly

virasoro level of sector

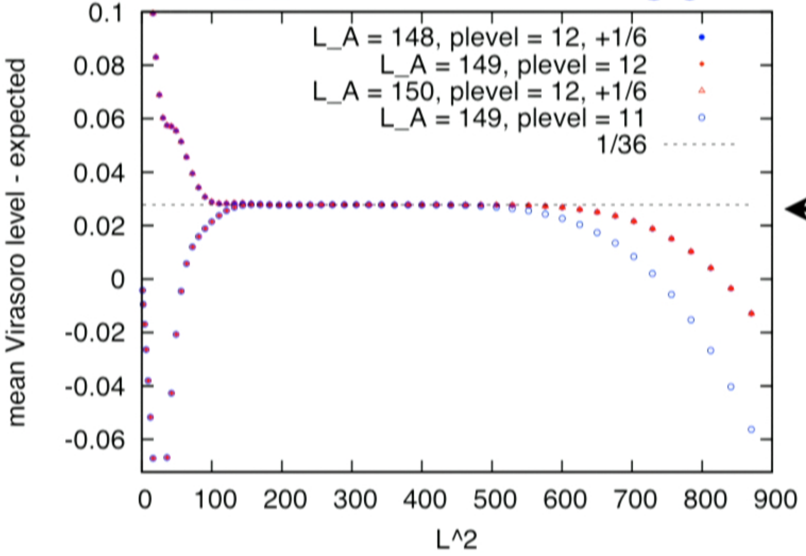
“CASIMIR MOMENTUM” term

(NOT “real-space cut” which requires the Landau orbit degrees of freedom and their form factor to be included

- Hall viscosity gives “thermally excited” momentum density on entanglement cut, relative to “vacuum”, at von Neumann temperature T = 1

Yeje Park, Z Papic, N Regnault

$$\frac{1}{24}\left(\tilde{c}-\nu\right)=\frac{1}{24}\left(1-\tfrac{1}{3}\right)=\frac{1}{36}$$



$\longleftarrow \frac{1}{36}$

Matrix-product state calculation on cylinder with circumference L
("plevel" is Virasoro level at which the auxilliary space is truncated)

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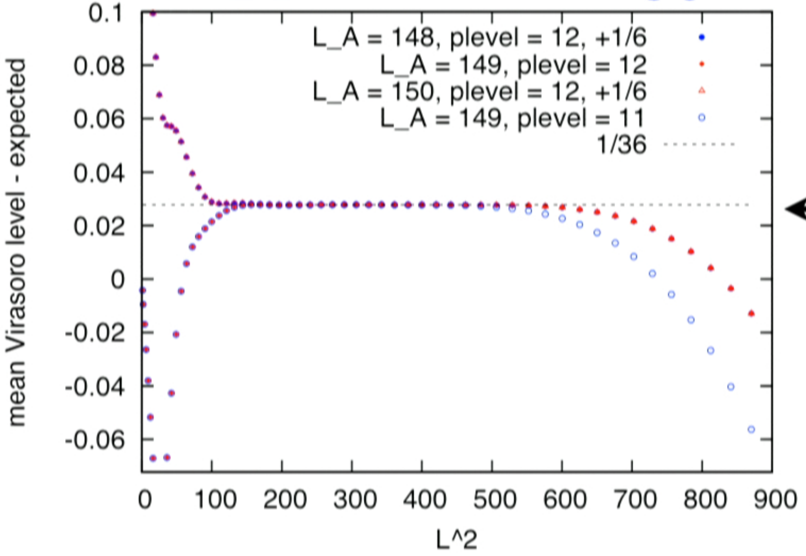


$$\longleftarrow \frac{1}{36}$$

Matrix-product state calculation on cylinder with circumference L
(“plevel” is Virasoro level at which the auxialliary space is truncated)

- How universal is the Thermal Hall effect formula? It also depends on
- If Lorentz invariance is present, its $\tilde{c}$ essentially the same calculation as Casimir momentum
- When Lorentz invariance is broken by different speeds for different modes, but the remain independent, the result still stands
- How much information about the Hamiltonian ($T^0{}_0$) is needed? Is there a clean "gravitational" derivation just based on the momentum $T^0{}_x$ Virasoro anomaly?

$$J_E^a = \frac{\tilde{c}}{12}\frac{(2\pi k_B T)^2}{2\pi\hbar}\epsilon^{ab}\frac{g_b}{c^2} \qquad \frac{\vec{g}}{c^2} = -\frac{\vec{\nabla} T}{T}$$

Momentum density is universal:

$$T^0_x = \frac{1}{L} \sum_m T_m \exp(2\pi i x/L)$$

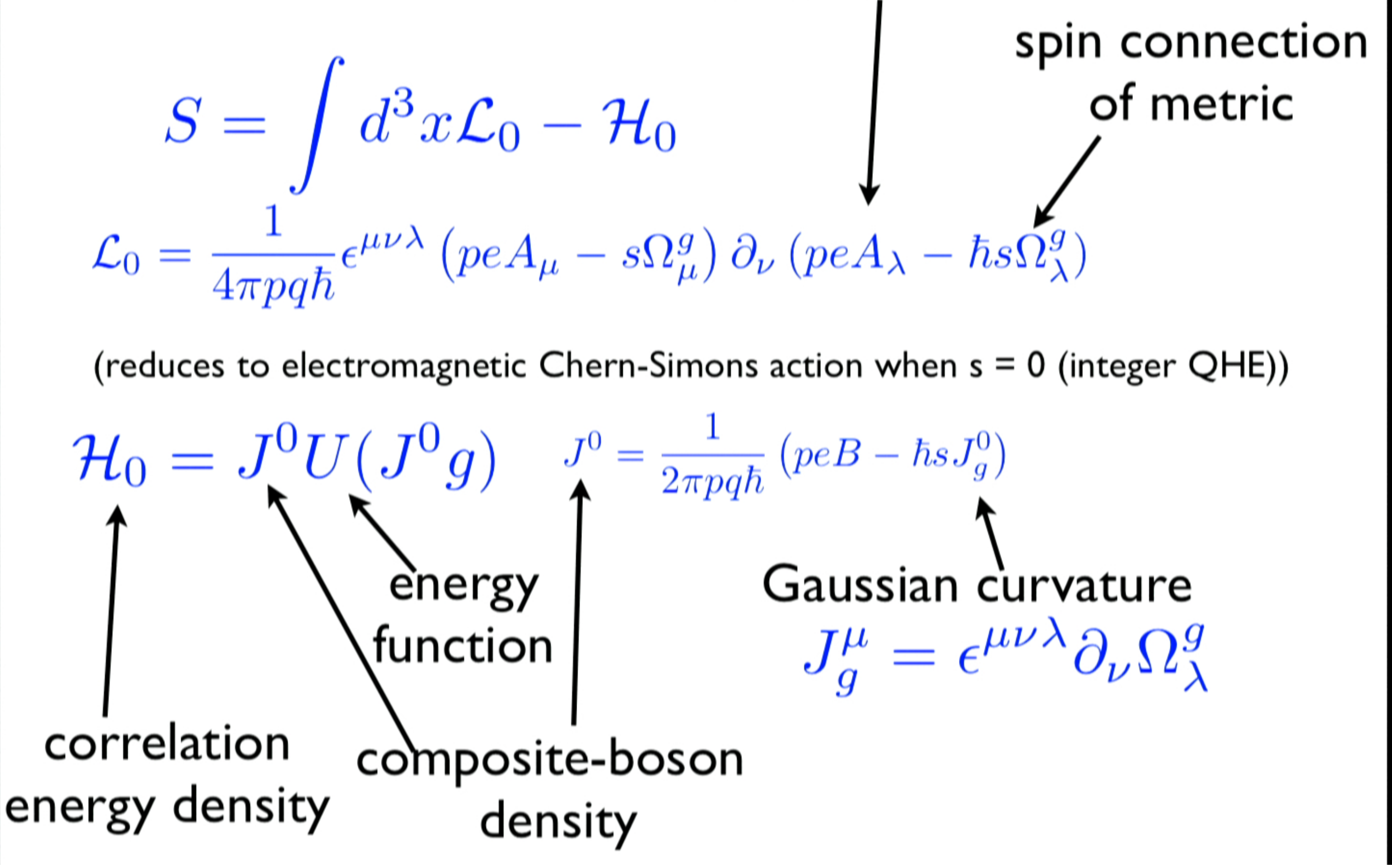
$$[T_m, T_n] = (m-n)T_{m+n} + \tfrac{1}{12}\tilde{c}m(m^2-1)\delta_{m+n,0}$$

chiral central charge

Can we obtain the Thermal Hall effect just from this plus "gravity"?

- Geometric action

(after Chern-Simons fields are integrated over)



electromagnetic gauge potentials

spin connection of metric

$$S = \int d^3x \mathcal{L}_0 - \mathcal{H}_0$$

$$\mathcal{L}_0 = \frac{1}{4\pi pq\hbar}\epsilon^{\mu\nu\lambda}\left(peA_\mu - s\Omega^g_\mu\right)\partial_\nu\left(peA_\lambda - \hbar s\Omega^g_\lambda\right)$$

(reduces to electromagnetic Chern-Simons action when s = 0 (integer QHE))

$$\mathcal{H}_0 = J^0 U(J^0 g) \qquad J^0 = \frac{1}{2\pi pq\hbar}\left(peB - \hbar s J^0_g\right)$$

energy function

Gaussian curvature

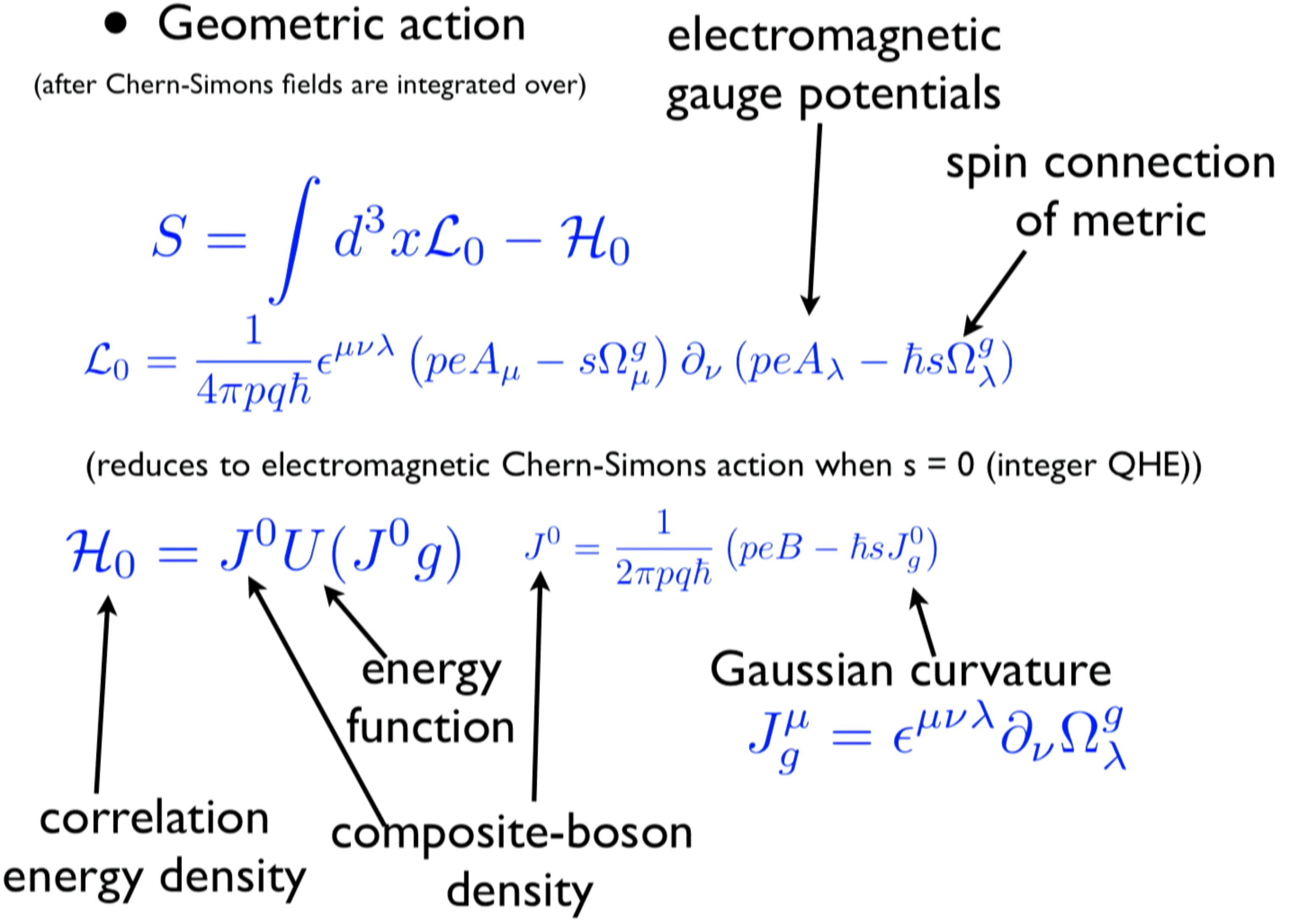
$$J^\mu_g = \epsilon^{\mu\nu\lambda}\partial_\nu\Omega^g_\lambda$$

correlation energy density

composite-boson density

- Geometric action

(after Chern-Simons fields are integrated over)



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correlation energy density

composite-boson density

Geometric distortion energy

correlation
energy density

$$\mathcal{H}_0 = (\det G)^{1/2} U(G) = J^0 U(J^0 g)$$

geometric chemical potential
(of composite bosons)

$$\boxed{\mu_g = U(G) + G_{ab} \frac{\partial U}{\partial G_{ab}}}$$

shear-stress tensor
(traceless)

$$\boxed{\sigma^a_b = 2G_{bc} \frac{\partial U}{\partial G_{ac}} - \delta^a_b G_{cd} \frac{\partial U}{\partial G_{cd}}}$$

$$\boxed{\sigma^a_a = 0}$$

$$\sigma^a_c(x)\epsilon^{bc} = \sigma^{bc}(x)\epsilon^{ac}$$

$$\sigma^a_c(x)g^{bc}(x) = \sigma^{bc}(x)g^{ac}(x)$$

both expressions are symmetric in $a \leftrightarrow b$

Stress tensor is traceless because the gapped quantum incompressible fluid does not transmit pressure

(unlike incompressible limit of classical incompressible fluid, which has speed of sound $v_s \to \infty$

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Euler equation

- action is minimized by Hall viscosity condition

$$\boxed{J^0\sigma^a_b(G) = \eta^{ac}_{bd}(G)\nabla^g_c J^d}$$

composite boson current

Traceless stress-tensor

covariant spatial gradient of $J^a = J^0 v^a$

fluid flow-velocity

Hall viscosity

$$\eta^{ac}_{bd}(G) = \tfrac{1}{2}\hbar s \epsilon_{be}\epsilon_{df} J^0 \Gamma^{aecf}_H(g)$$

$$\Gamma^{abcd}_H(g) = \tfrac{1}{2}\left(\epsilon^{ac}g^{bd} + \epsilon^{ad}g^{bc} + \epsilon^{bc}g^{ad} + \epsilon^{bd}g^{ac}\right)$$

$\eta^{ac}_{bd} = -\eta^{ca}_{db}$ dissipationless

$\eta^{ab}_{ac} = \eta^{ba}_{ca} = 0$ incompressible

Geometric distortion energy

correlation
energy density

$$\mathcal{H}_0 = (\det G)^{1/2} U(G) = J^0 U(J^0 g)$$

geometric chemical potential
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- composite boson current

Gaussian curvature density and current

$$J^0 = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab} peB - \hbar s J_g^0\right)$$

$$J^a = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab}(peE_b - \partial_b \mu_g) - \hbar s J_g^a\right)$$

responds to gradient of geometric chemical potential as well as electric field

$$peE_a J^a \neq 0$$

Energy flow from electromagnetic field to FQH fluid

$$pe(J^0 E_a + \epsilon_{ab} J^a B) \neq 0$$

tangential momentum flow from electromagnetic field to FQH fluid

- Action gives gapped spin-2 (graviton-like) collective mode that coincides at long wavelengths with the "single-mode approximation" of Girvin-MacDonald and Platzman.

- charge fluctuations relative to the back-ground charge density fixed by the magnetic flux are given by the Gaussian curvature

$$J^0_g = -\tfrac{1}{2}\partial_a\partial_b g^{ab} + \tfrac{1}{8} g_{ac}\epsilon_{bd}\epsilon^{ef}\left(\partial_e g^{ab}\right)\left(\partial_f g^{cd}\right)$$

$$\delta J^0_e = \frac{e^* s}{2\pi} J^0_g$$

second derivative of metric

zero-point fluctuations of gaussian curvature give quantitatively correct $O(q^4)$ structure factor

- composite boson current

Gaussian curvature density and current

$$J^0 = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab} peB - \hbar s J_g^0\right)$$

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