Title: A Nearly Gaussian Hubble-patch in a non-Gaussian Universe

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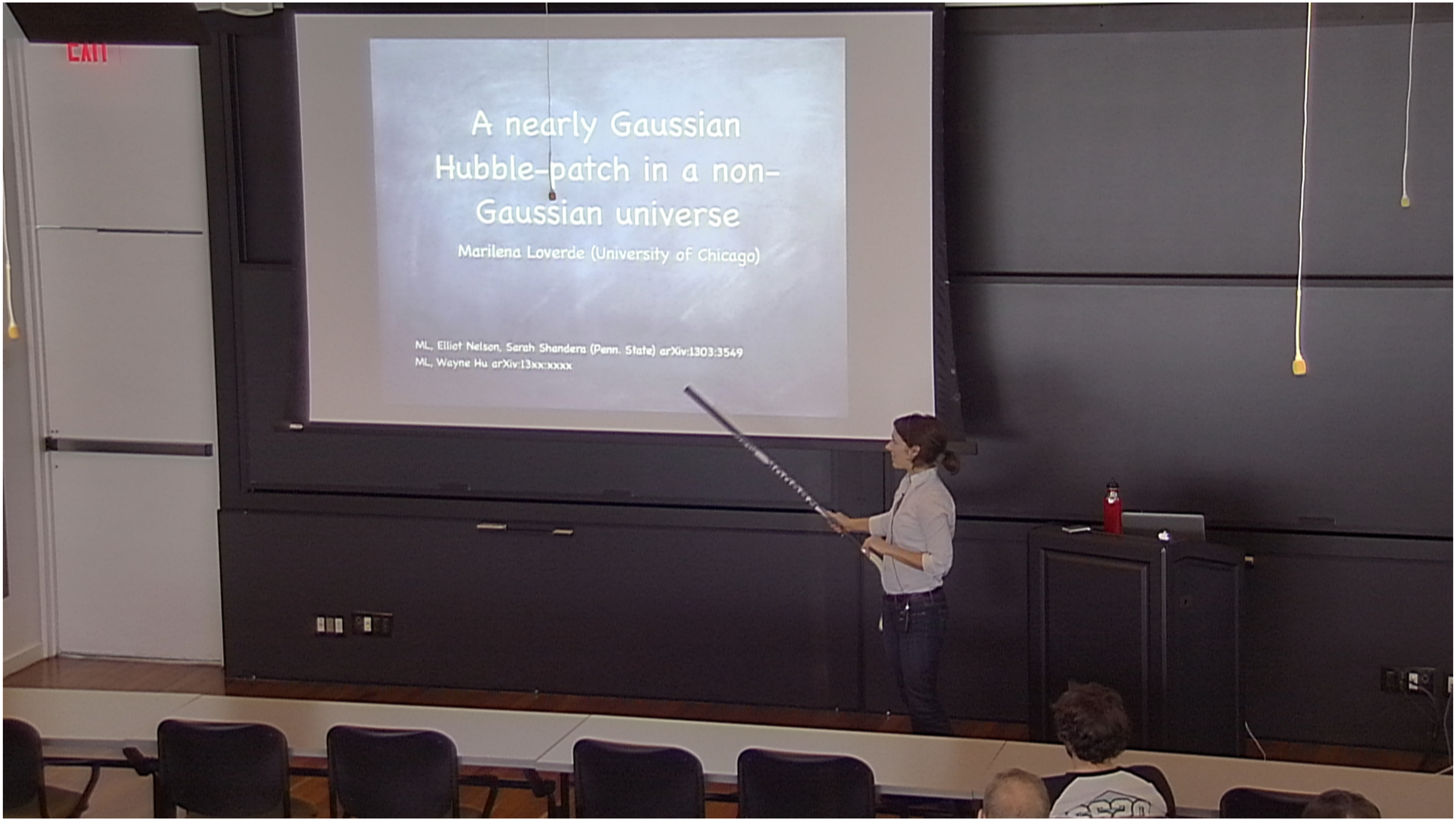
Abstract: Local-type primordial non-Gaussianity couples statistics of the curvature perturbation \zeta on vastly different physical scales. Because of this coupling, statistics (i.e. the polyspectra) of \zeta in our Hubble volume may not be representative of those in the larger universe -- that is, they may be biased. The bias depends on the local background value of \zeta, which includes contributions from all modes with wavelength k ~< H_0 and is therefore enhanced if the entire post-inflationary patch is large compared with our Hubble volume. I will discuss the bias to locally-measured statistics for general local-type non-Gaussianity. I will discuss three examples in detail: (i) the usual fNL, gNL model, (ii) a strongly non-Gaussian model with \zeta ~ \zeta_G^p, and (iii) two-field non-Gaussian initial conditions. In each scenario one may generate statistics in a Hubble-size patch that are weakly Gaussian and consistent with observations despite the fact that the statistics in the larger, post-inflationary patch look very different. Finally, I will present a worked example of how the variation in local statistics arises in the curvaton scenario.

A nearly Gaussian Hubble-patch in a non-Gaussian universe

Marilena Loverde (University of Chicago)

ML, Elliot Nelson, Sarah Shandera (Penn. State) arXiv:1303:3549
ML, Wayne Hu arXiv:13xx:xxxx


EXIT
A nearly Gaussian Hubble-patch in a non-Gaussian universe
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Idea:

Statistics of curvature perturbation ζ (i.e. inhomogeneities) are our primary means to learn about inflation

But, we only observe ζ in our Hubble patch and *local statistics may not be representative*

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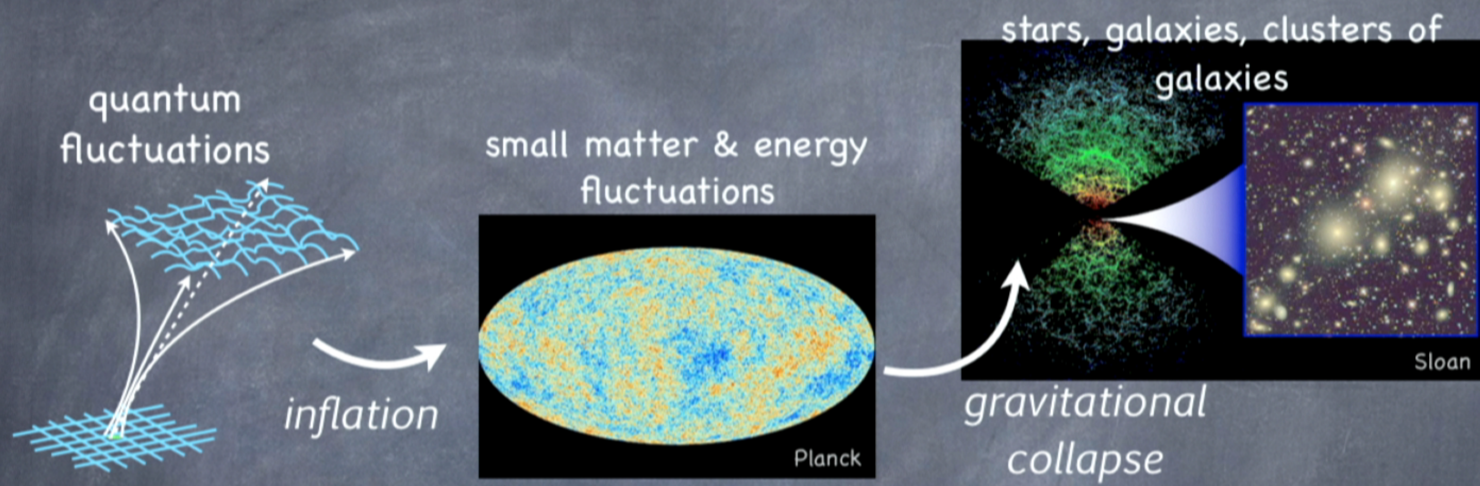
But, we only observe ζ in our Hubble patch and *local statistics may not be representative*

Outline

- How are local and global statistics related?
- Three examples of statistics for ζ :
  - Single-source weakly non-Gaussian IC's
  - Single-source strongly non-Gaussian IC's
  - Multi-source initial conditions
- A worked example from the curvaton
- Conclusions

Inflation as the origin of structure



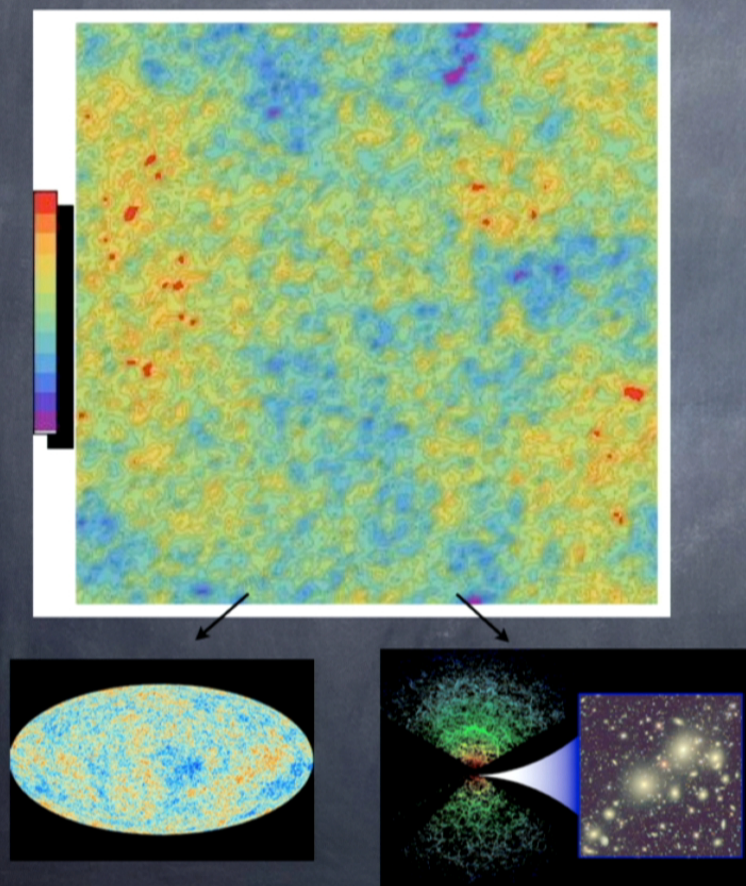
$\delta\ \phi_{\text{inflaton}} \longrightarrow \zeta_{\text{curvature}} \rightsquigarrow$

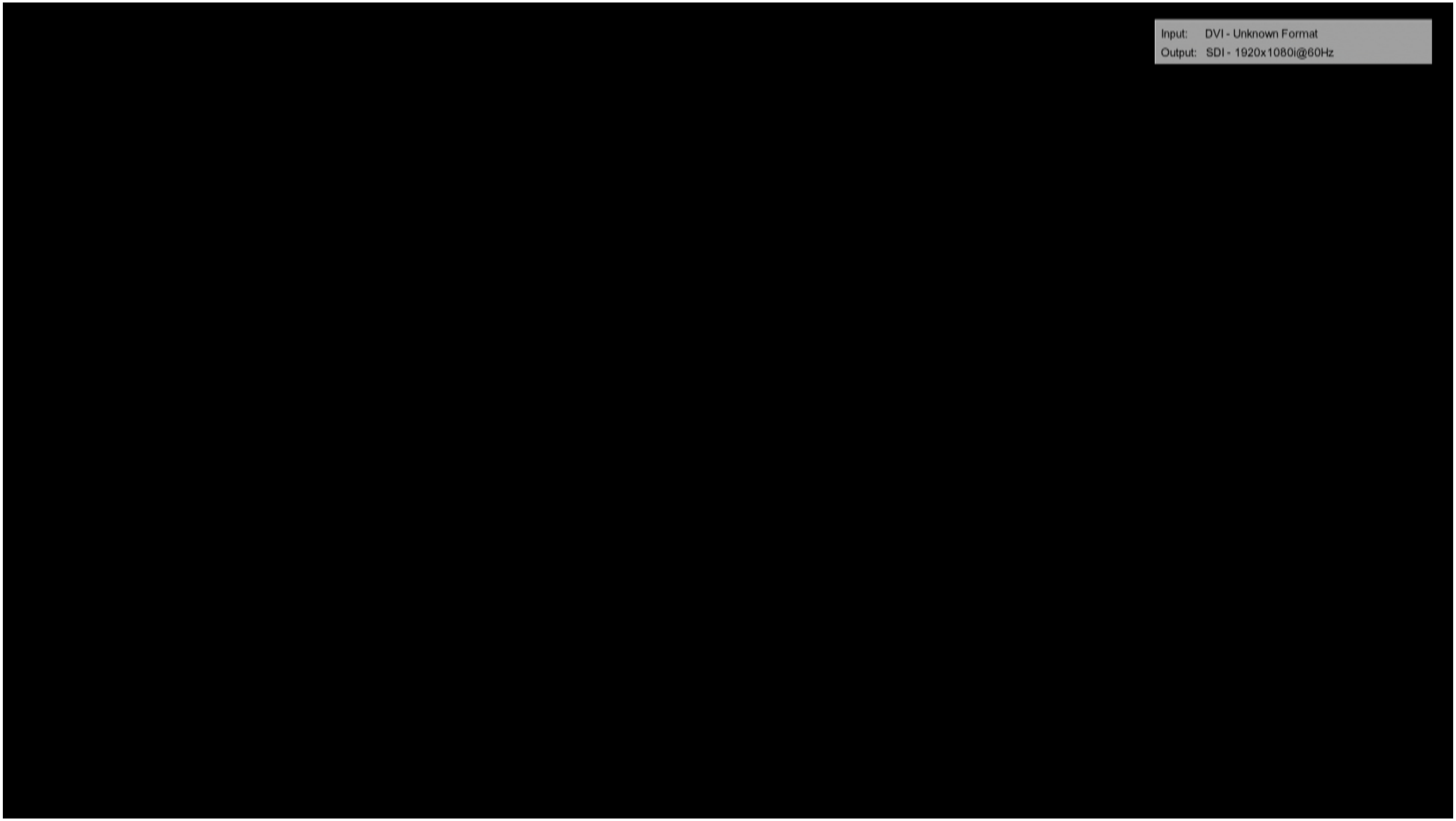
$\delta\ T_{\text{CMB}}$

$\delta\ \rho_{\text{matter}}$

$\delta\ n_{\text{galaxies}}$

For example, we can measure the power spectrum

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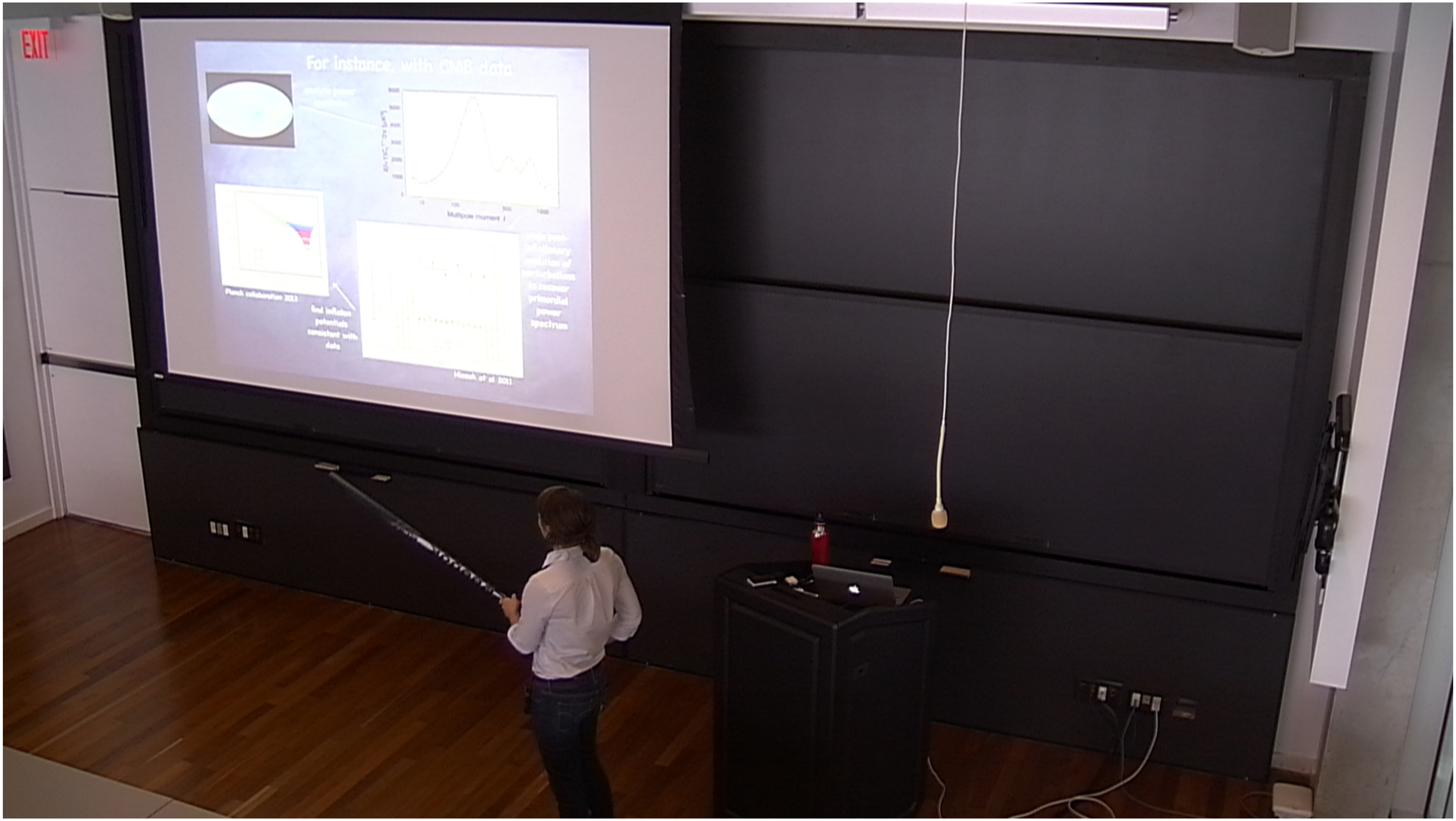
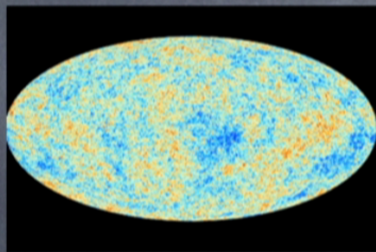
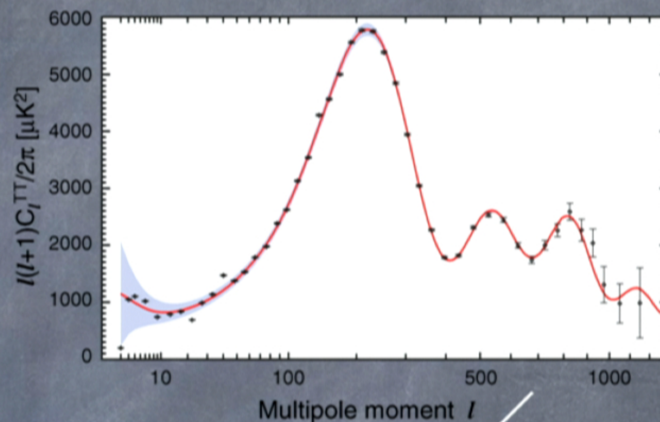
Output: SDI - 1920x1080i@60Hz


EXIT
For instance, with CMB data
Multipole moment l
Planck collaboration 2013
find inflation potentials consistent with data
primordial power spectrum
Hlozek et al 2011

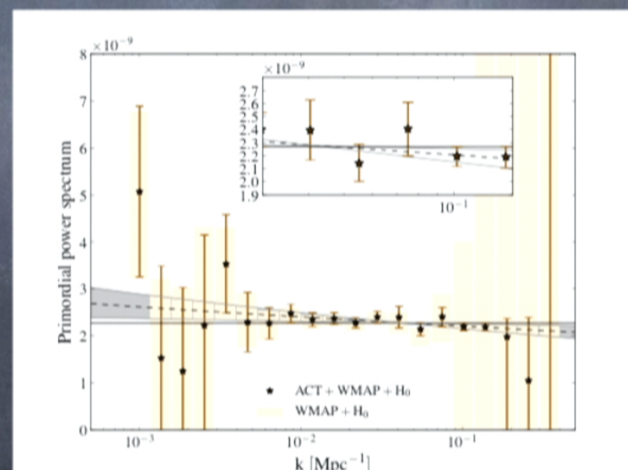
For instance, with CMB data



analyze power spectrum

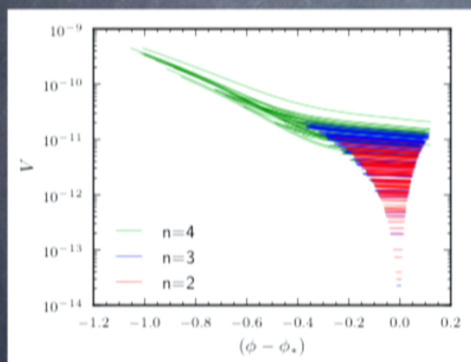


undo post-inflationary evolution of perturbations to recover primordial power spectrum



Hlozek et al 2011

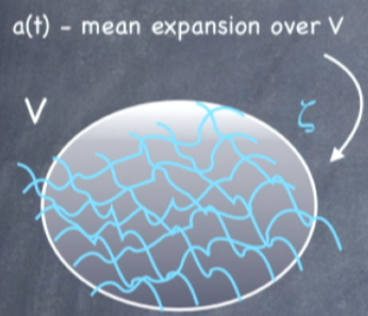
find inflaton potentials consistent with data



Planck collaboration 2013

How do the statistics we observe in our Hubble volume relate to what's predicted from inflation?

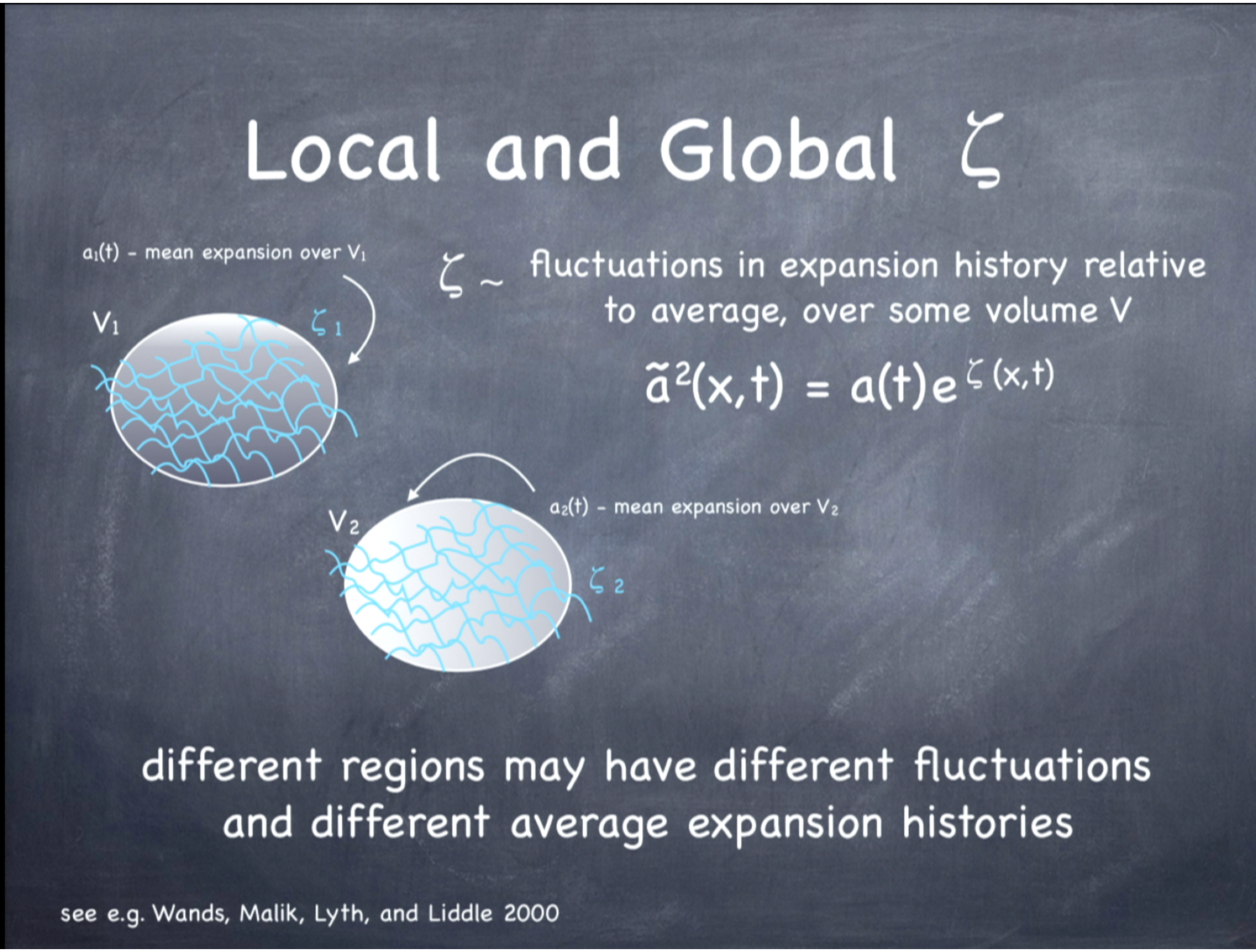
What's ζ ?



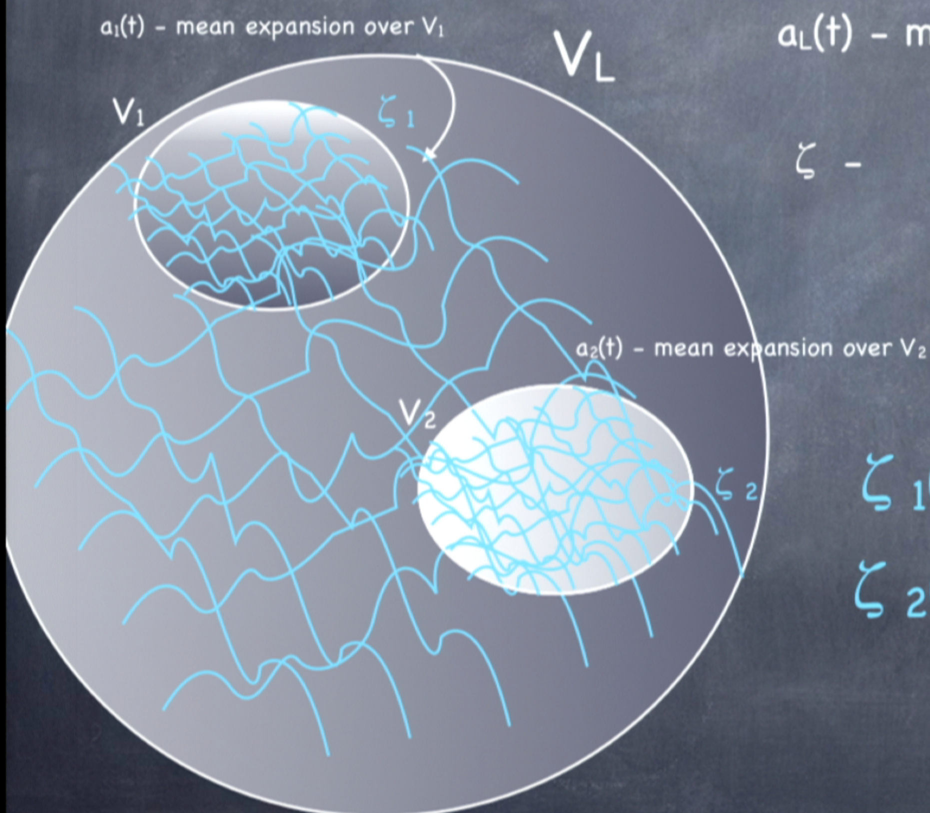
$\zeta \sim$ fluctuations in expansion history relative to average, over some volume V

$$\tilde{a}^2(x,t) = a(t)e^{\zeta(x,t)}$$

see e.g. Wands, Malik, Lyth, and Liddle 2000


Local and Global ζ
$a_1(t)$ - mean expansion over V_1
V_1
ζ_1
ζ ~ fluctuations in expansion history relative to average, over some volume V
$\tilde{a}^2(x,t) = a(t)e^{\zeta(x,t)}$
$a_2(t)$ - mean expansion over V_2
V_2
ζ_2
different regions may have different fluctuations and different average expansion histories
see e.g. Wands, Malik, Lyth, and Liddle 2000

Local and Global ζ



$a_L(t)$ - mean expansion over V_L

ζ - perturbation with respect to average expansion in V_L

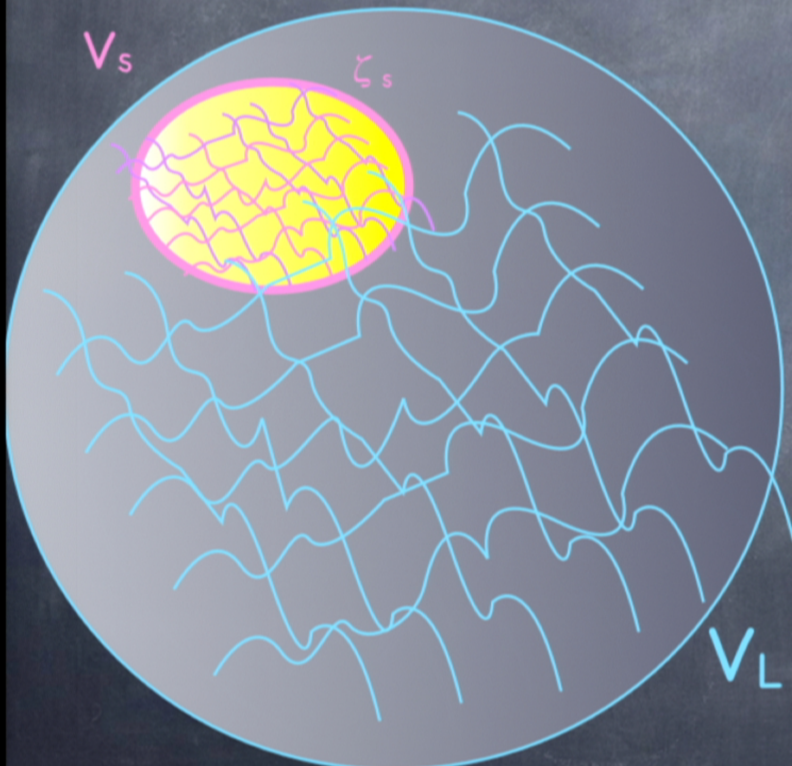
↓

$$\zeta_1(x) = \zeta(x) - \langle \zeta \rangle_1$$

$$\zeta_2(x) = \zeta(x) - \langle \zeta \rangle_2$$

see e.g. Wands, Malik, Lyth, and Liddle 2000

Local and Global ζ



DEFINITIONS

local curvature perturbation

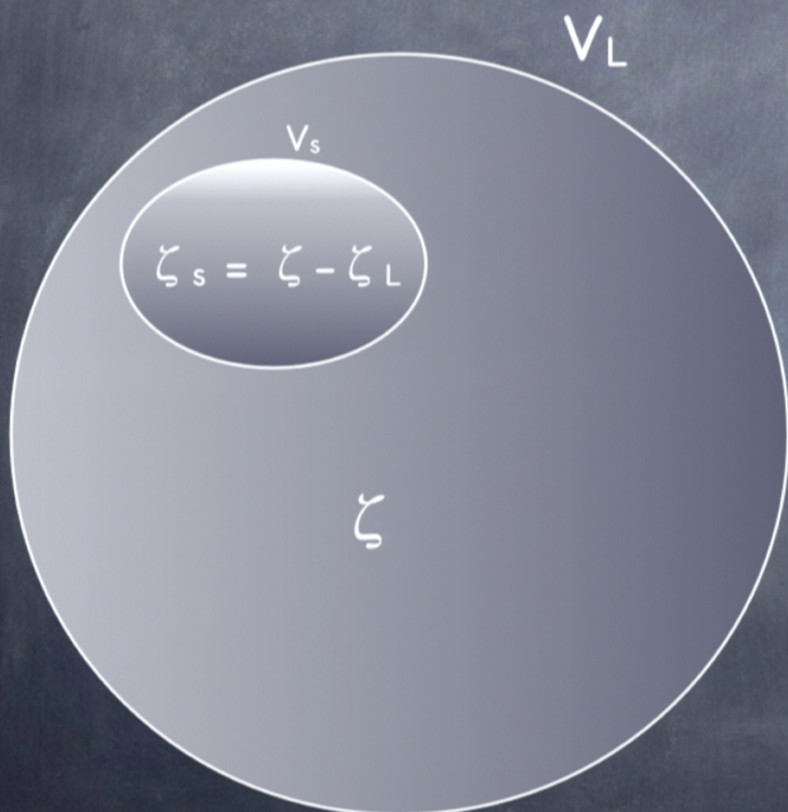
$$\zeta_s(x) \equiv \zeta(x) - \zeta_L$$

$$\zeta_L \equiv \langle \zeta \rangle_{V_s}$$

↑

average over volume V_s

Local and Global ζ



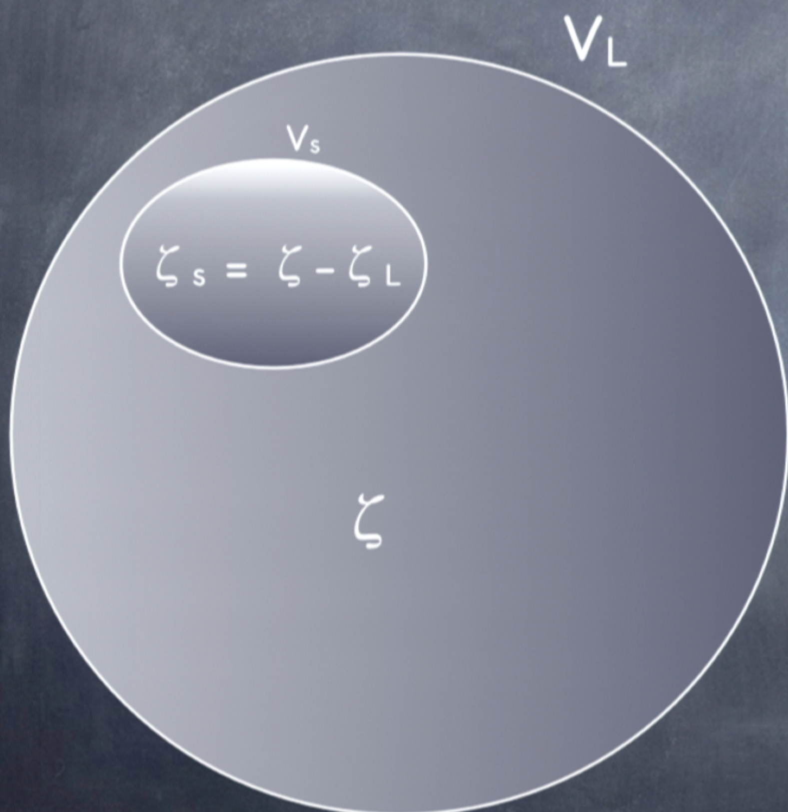
if ζ is Gaussian,

ζ_s and ζ_L

are uncorrelated*

* ok, strictly speaking this is only true in Fourier space. corrections depend on how we define volumes, but we can calculate them

Local and Global ζ



if ζ is Gaussian,

ζ_s and ζ_L

are uncorrelated*

BUT if ζ is non-Gaussian ζ_s *can depend on the value of* ζ_L

* ok, strictly speaking this is only true in Fourier space. corrections depend on how we define volumes, but we can calculate them

possibly familiar example:

Non-linear couplings

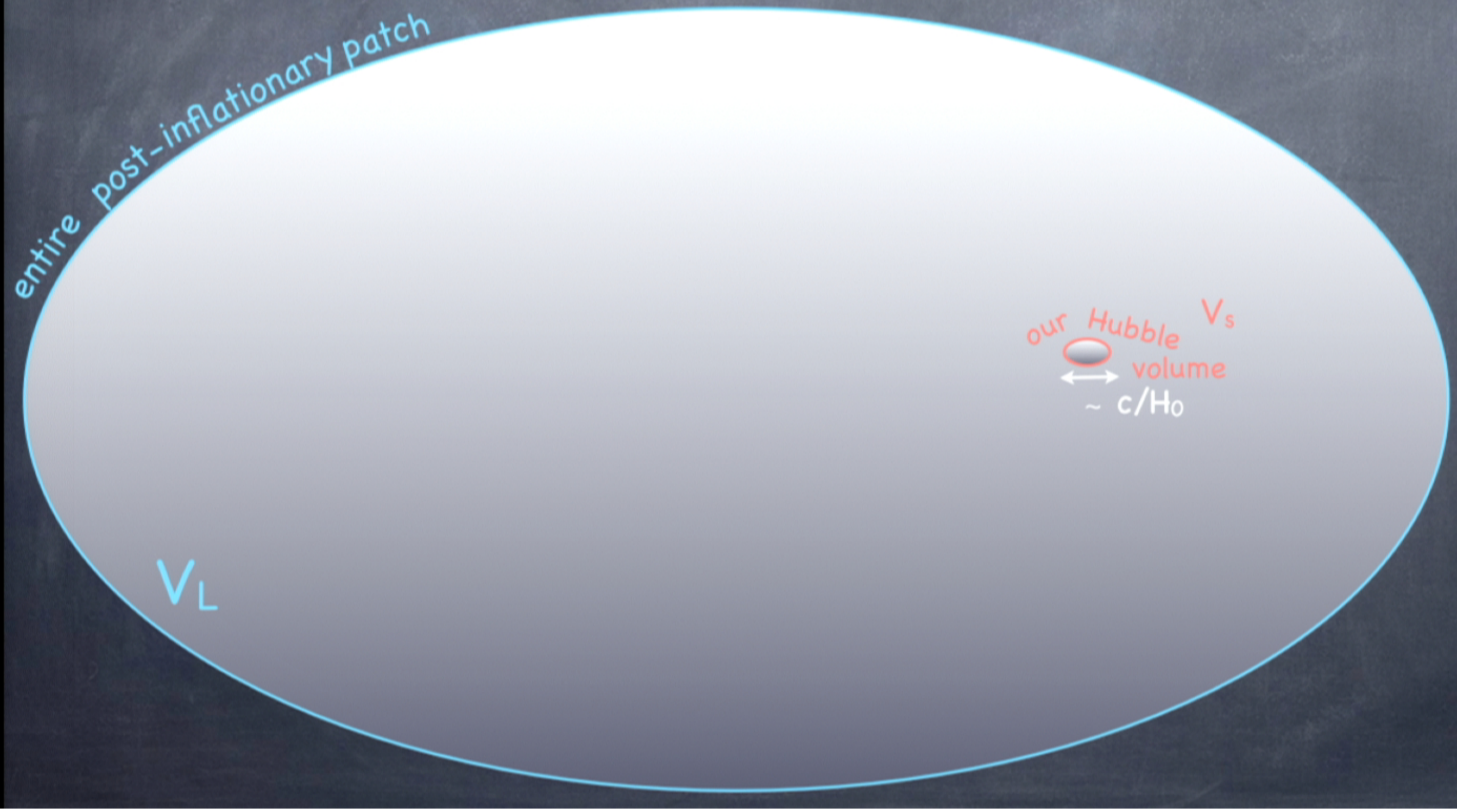
For instance, the quadratic local ansatz: $\zeta = \zeta_G + f_{NL}\, \zeta_G^2$

$$\langle \zeta_s^2 \rangle = \langle \zeta_{G,s}^2 \rangle \left(1 + 4\, f_{NL}\, \zeta_{G,L}(x)\right)$$

small-scale power depends on large-scale fluctuations

Dalal, Doré, Huterer, Shirokov 2007

Suppose our Hubble volume is small compared with the entire post-inflationary patch



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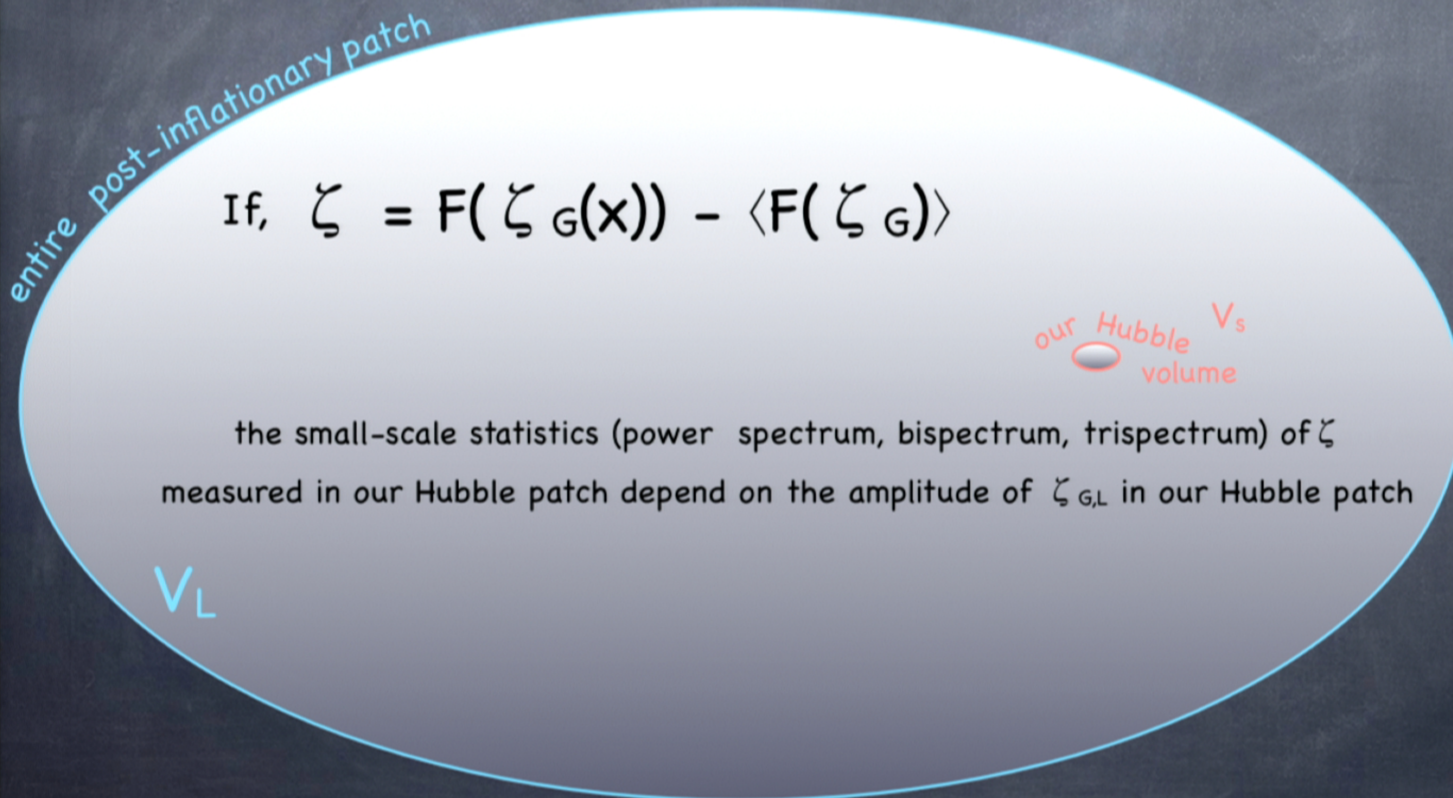
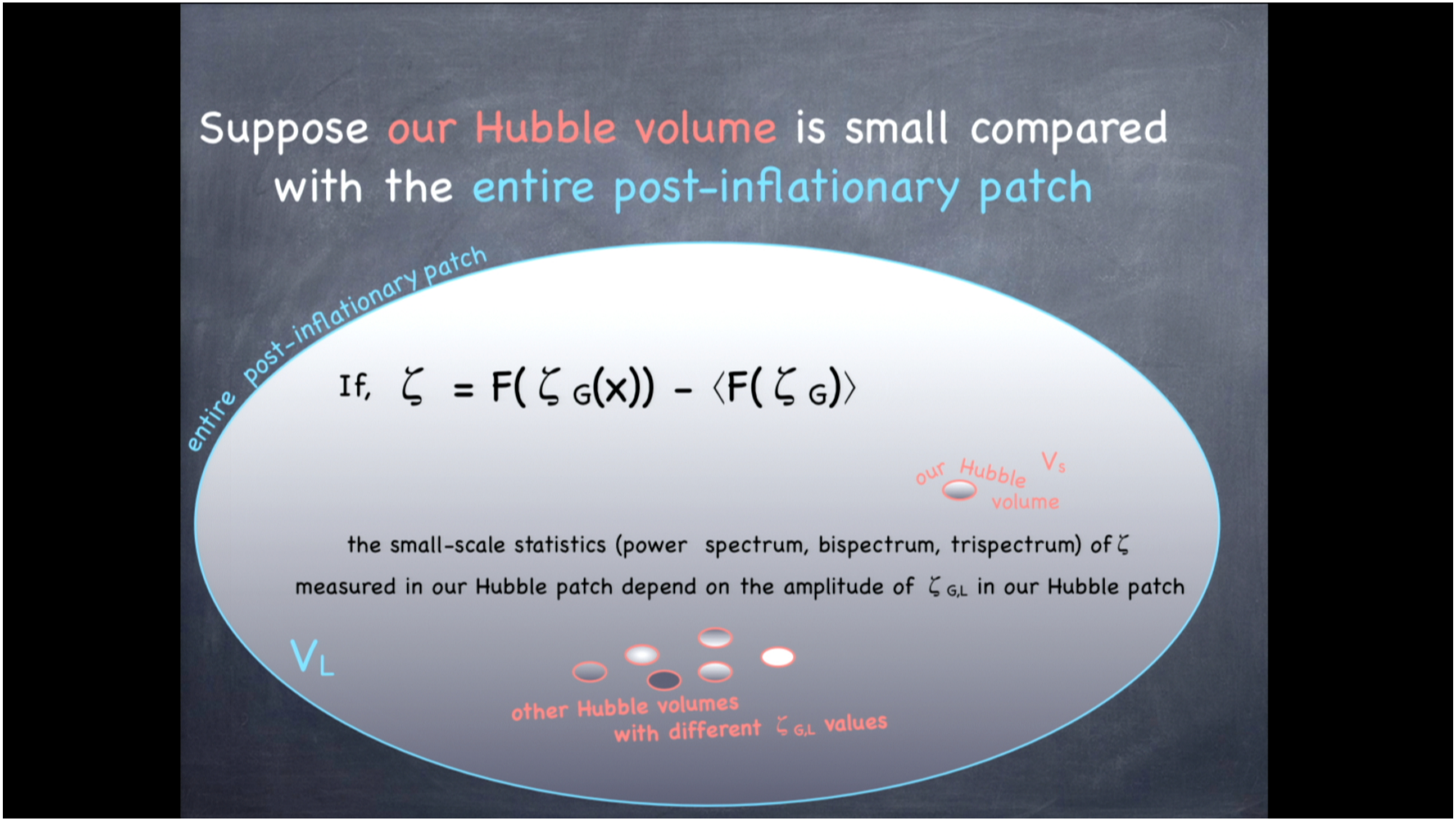
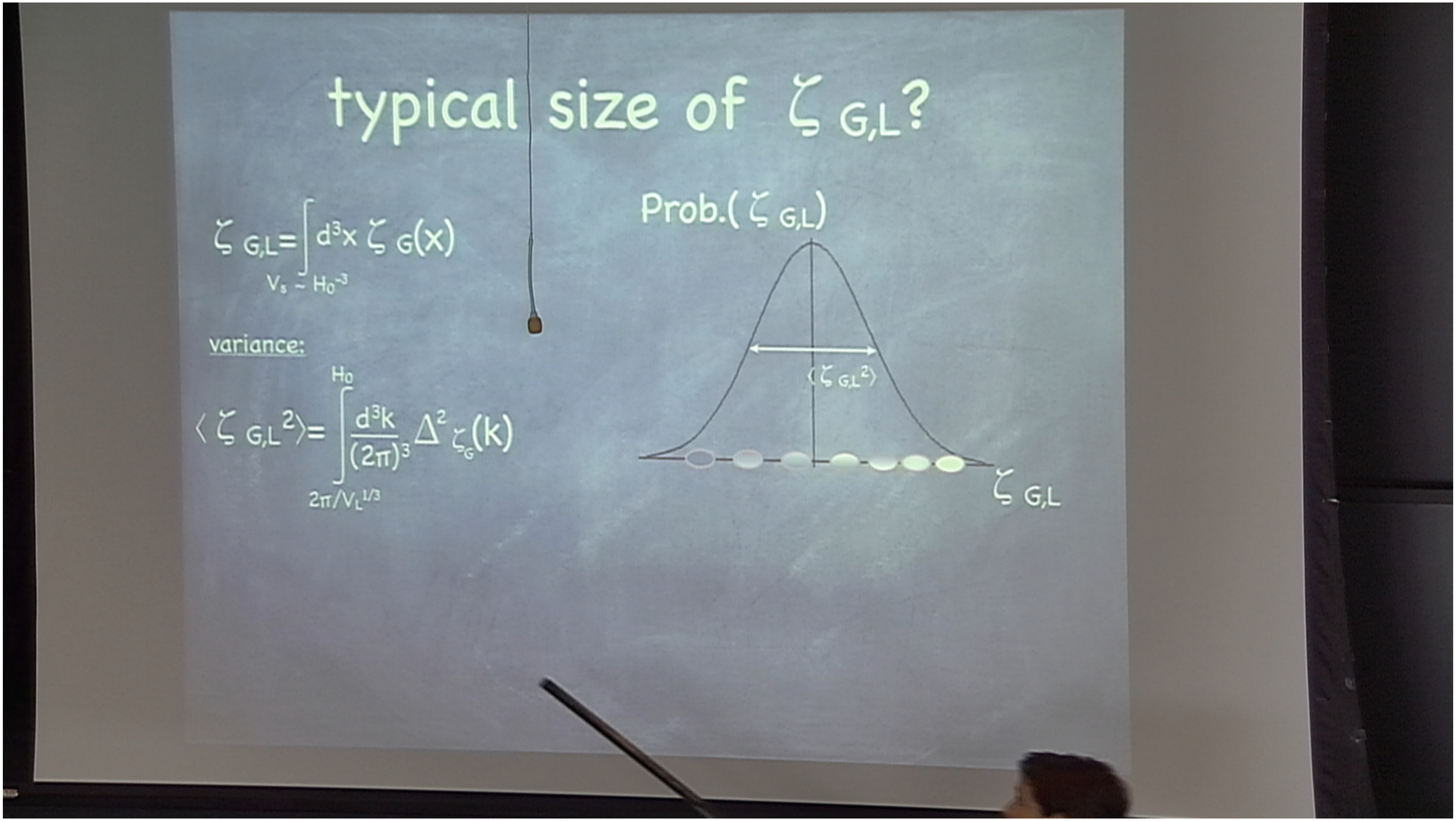


If, $\zeta = F(\zeta_G(x)) - \langle F(\zeta_G) \rangle$

the small-scale statistics (power spectrum, bispectrum, trispectrum) of ζ measured in our Hubble patch depend on the amplitude of $\zeta_{G,L}$ in our Hubble patch


Suppose our Hubble volume is small compared with the entire post-inflationary patch
entire post-inflationary patch
If, $\zeta = F(\zeta_G(x)) - \langle F(\zeta_G) \rangle$
our Hubble volume V_s
the small-scale statistics (power spectrum, bispectrum, trispectrum) of ζ
measured in our Hubble patch depend on the amplitude of $\zeta_{G,L}$ in our Hubble patch
V_L
other Hubble volumes with different $\zeta_{G,L}$ values


typical size of ζ G,L?
ζ G,L = ∫ d³x ζ G(x)
Vs ~ H0^-3
variance:
⟨ζ G,L²⟩ = ∫ d³k/(2π)³ Δ² ζG(k)
H0
2π/VL^1/3
Prob.(ζ G,L)
⟨ζ G,L²⟩
ζ G,L

typical size of $\zeta_{G,L}$?

$$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x \, \zeta_G(x)$$

sum over all modes with $k \lesssim H_0$

<u>variance:</u>

$$\langle \zeta_{G,L}{}^2 \rangle = \int_{2\pi/V_L^{1/3}}^{H_0} \frac{d^3k}{(2\pi)^3} \Delta^2_{\zeta_G}(k)$$

depends on power spectrum outside horizon! which we don't know

$$\langle \zeta_{G,L}{}^2 \rangle \sim \Delta^2_{\zeta_G} N$$ for scale invariant ($n_s=1$)

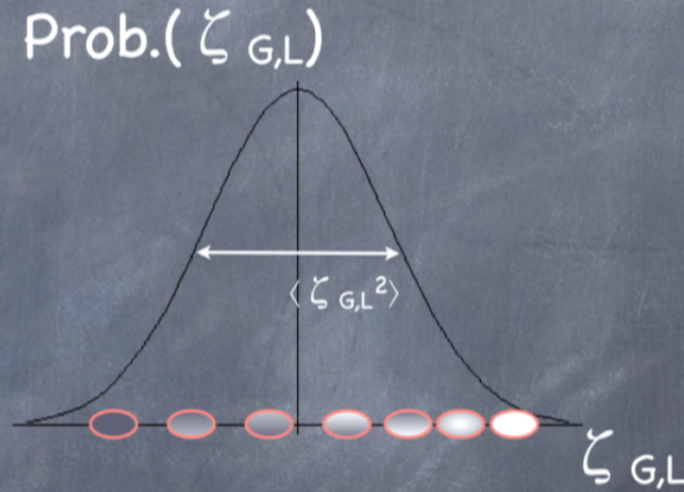
$$N \equiv \frac{1}{3} \ln \frac{V_L}{V_s}$$

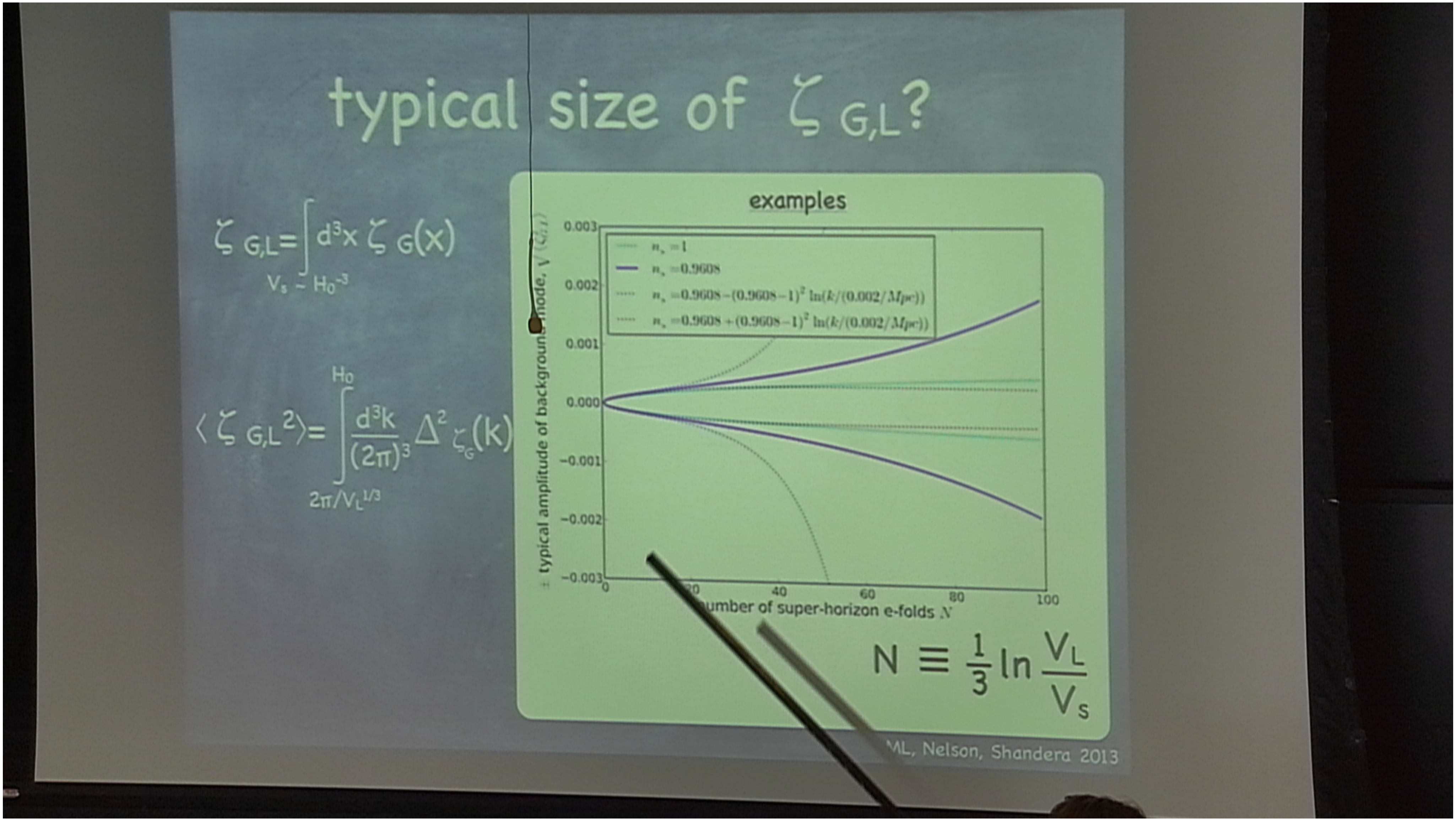
typical size of $\zeta_{G,L}$?

$$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x\, \zeta_G(x)$$

variance:

$$\langle \zeta_{G,L}^2 \rangle = \int_{2\pi/V_L^{1/3}}^{H_0} \frac{d^3k}{(2\pi)^3} \Delta^2_{\zeta_G}(k)$$

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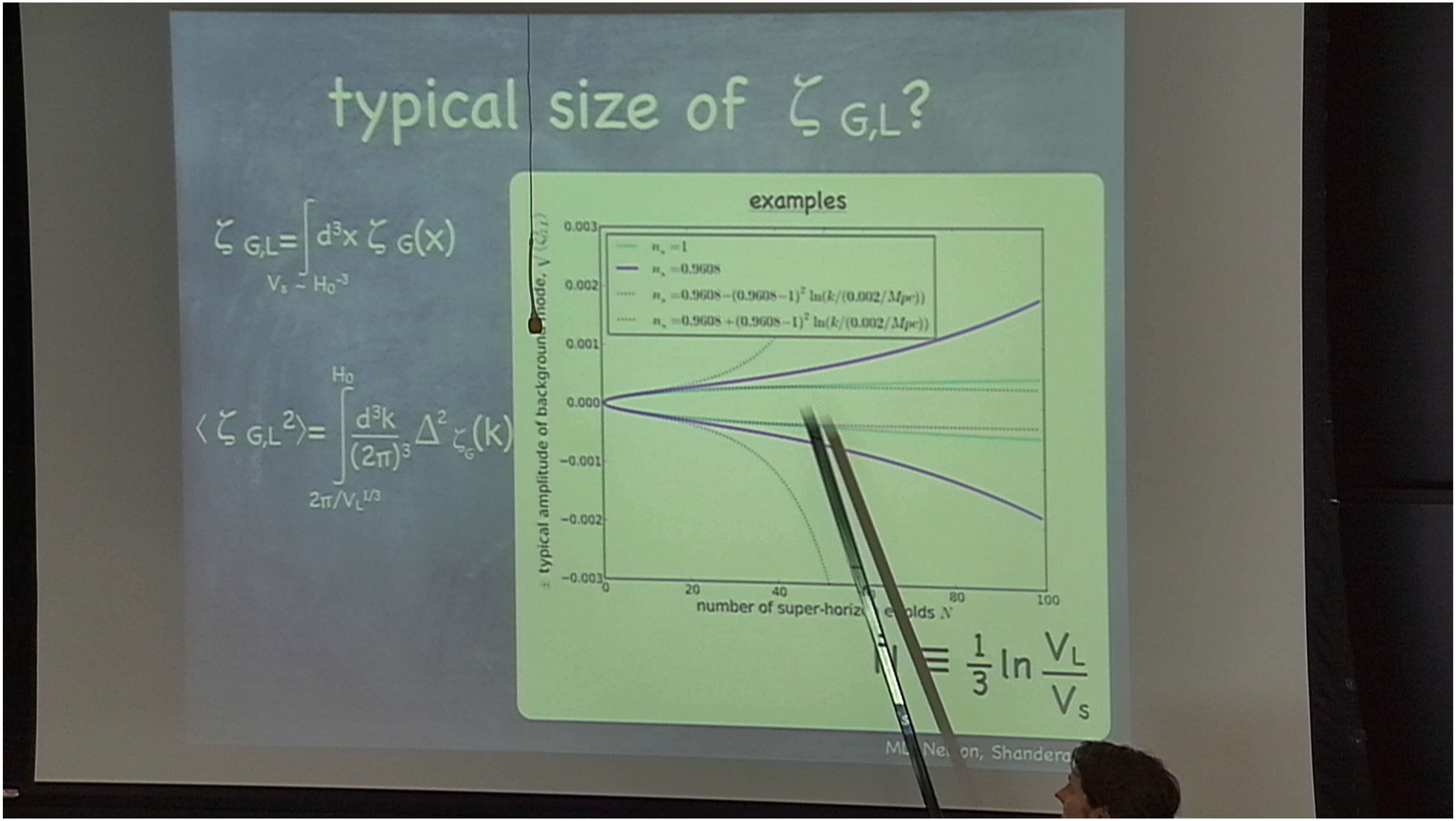
Vs ~ H0^-3
⟨ζ G,L²⟩ = ∫ d³k/(2π)³ Δ² ζG(k)
H0
2π/VL^1/3
examples
n_s = 1
n_s = 0.9608
n_s = 0.9608 − (0.9608 − 1)² ln(k/(0.002/Mpc))
n_s = 0.9608 + (0.9608 − 1)² ln(k/(0.002/Mpc))
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
0
20
40
60
80
100
number of super-horizon e-folds N
N ≡ 1/3 ln VL/Vs
ML, Nelson, Shandera 2013



typical size of $\zeta_{G,L}$?
$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x \, \zeta_G(x)$
$\langle \zeta_{G,L}^2 \rangle = \int_{2\pi/V_L^{1/3}}^{H_0} \frac{d^3k}{(2\pi)^3} \Delta^2_{\zeta_G}(k)$
examples
$n_s = 1$
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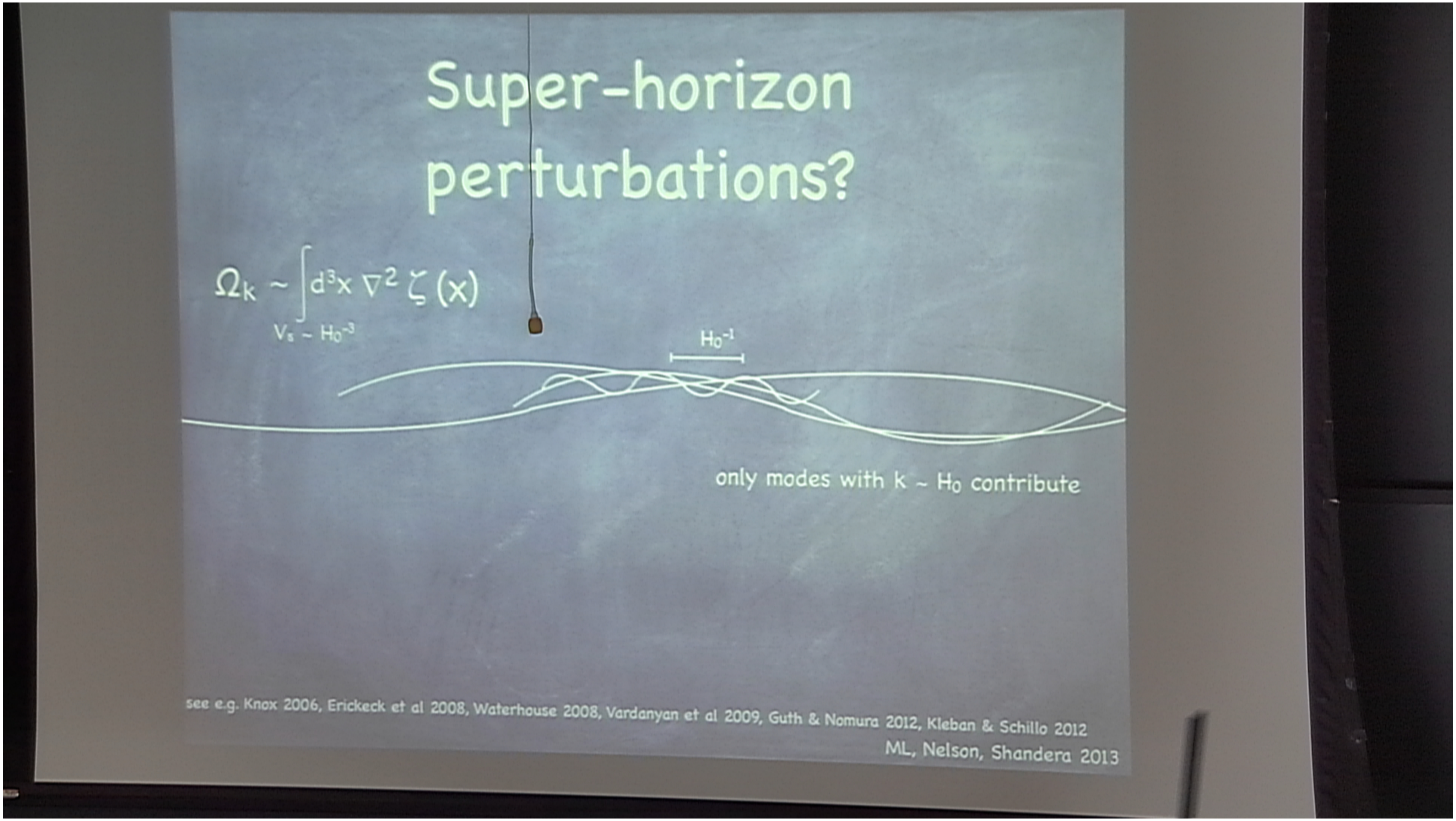
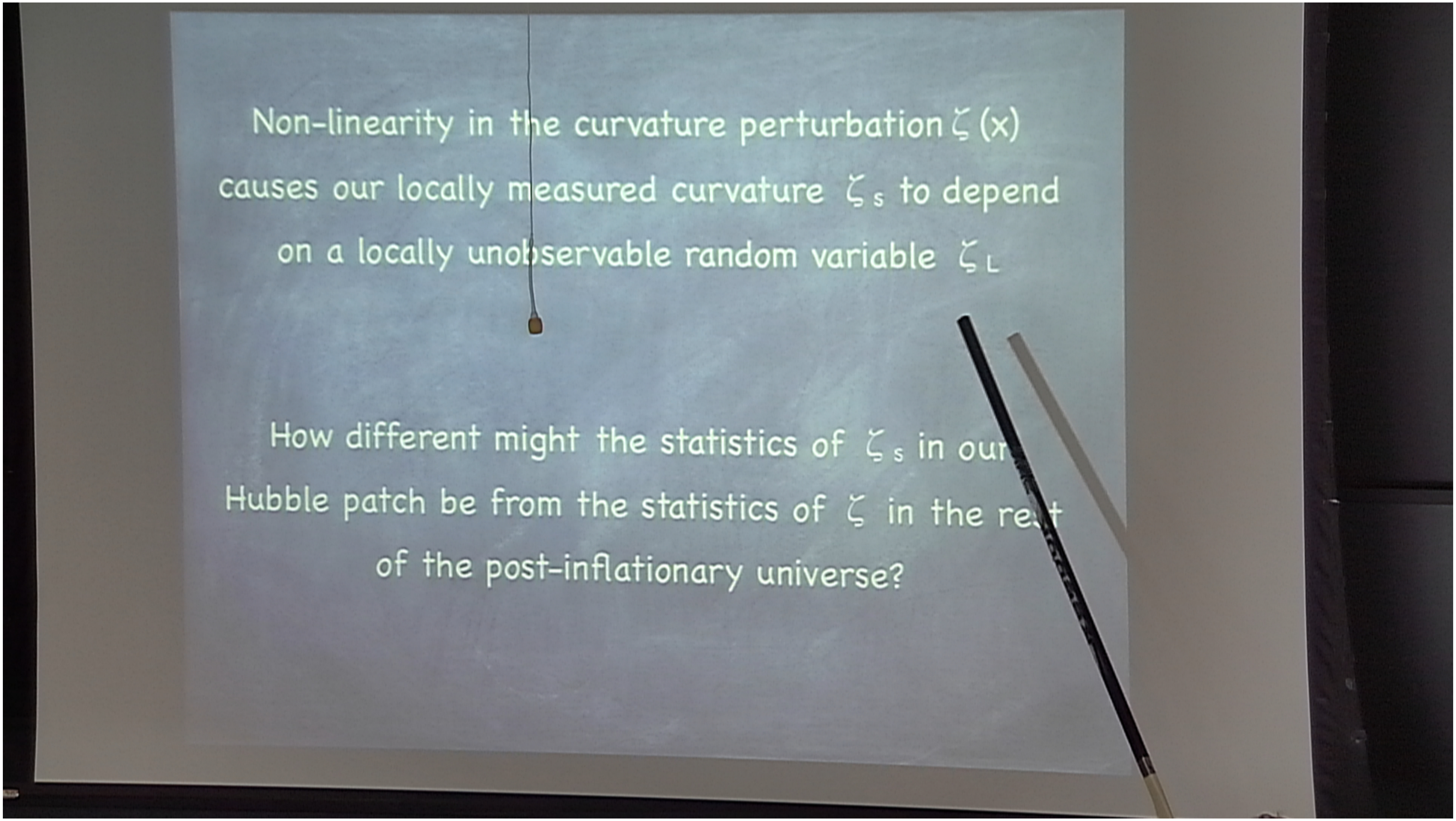
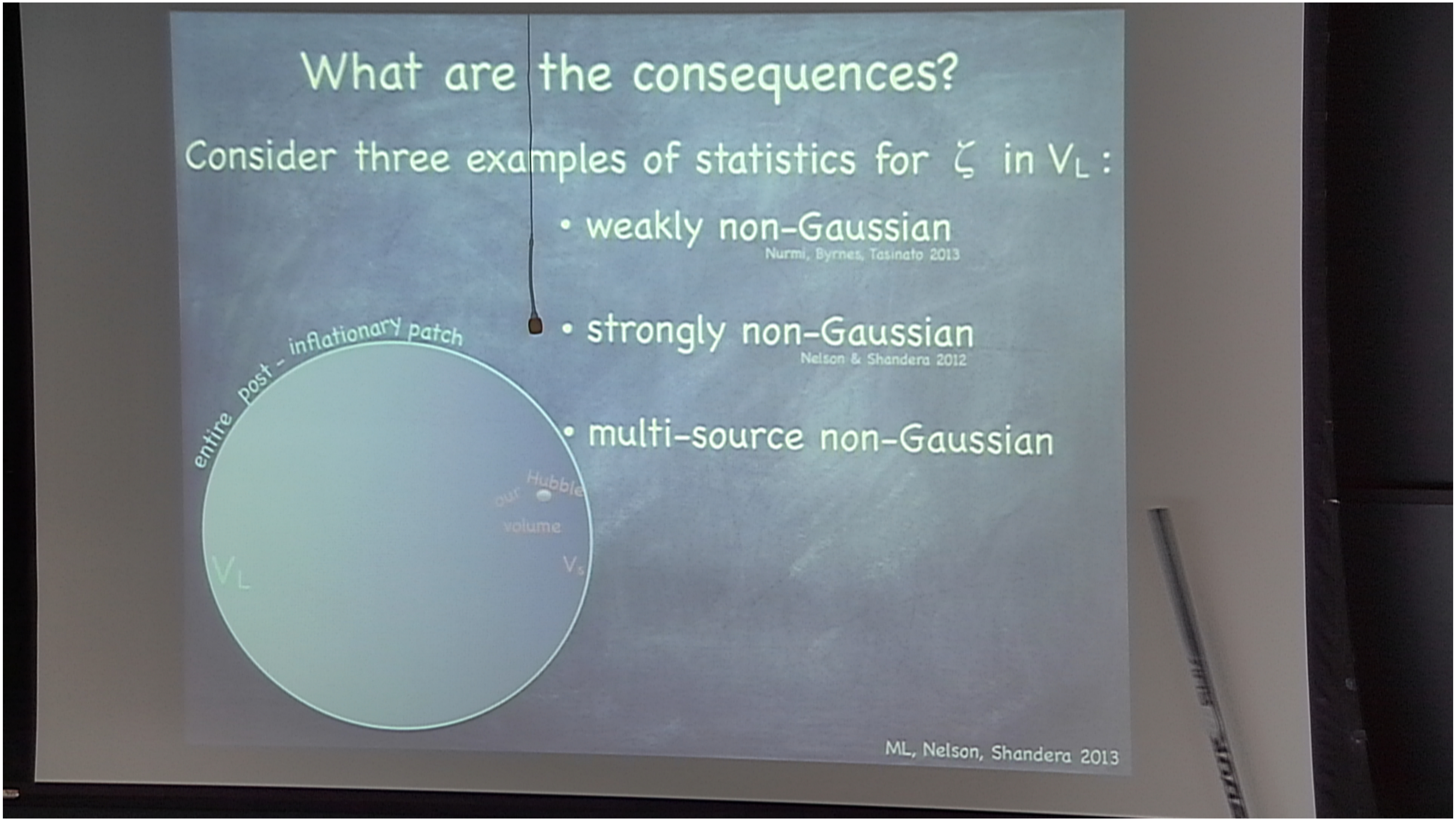
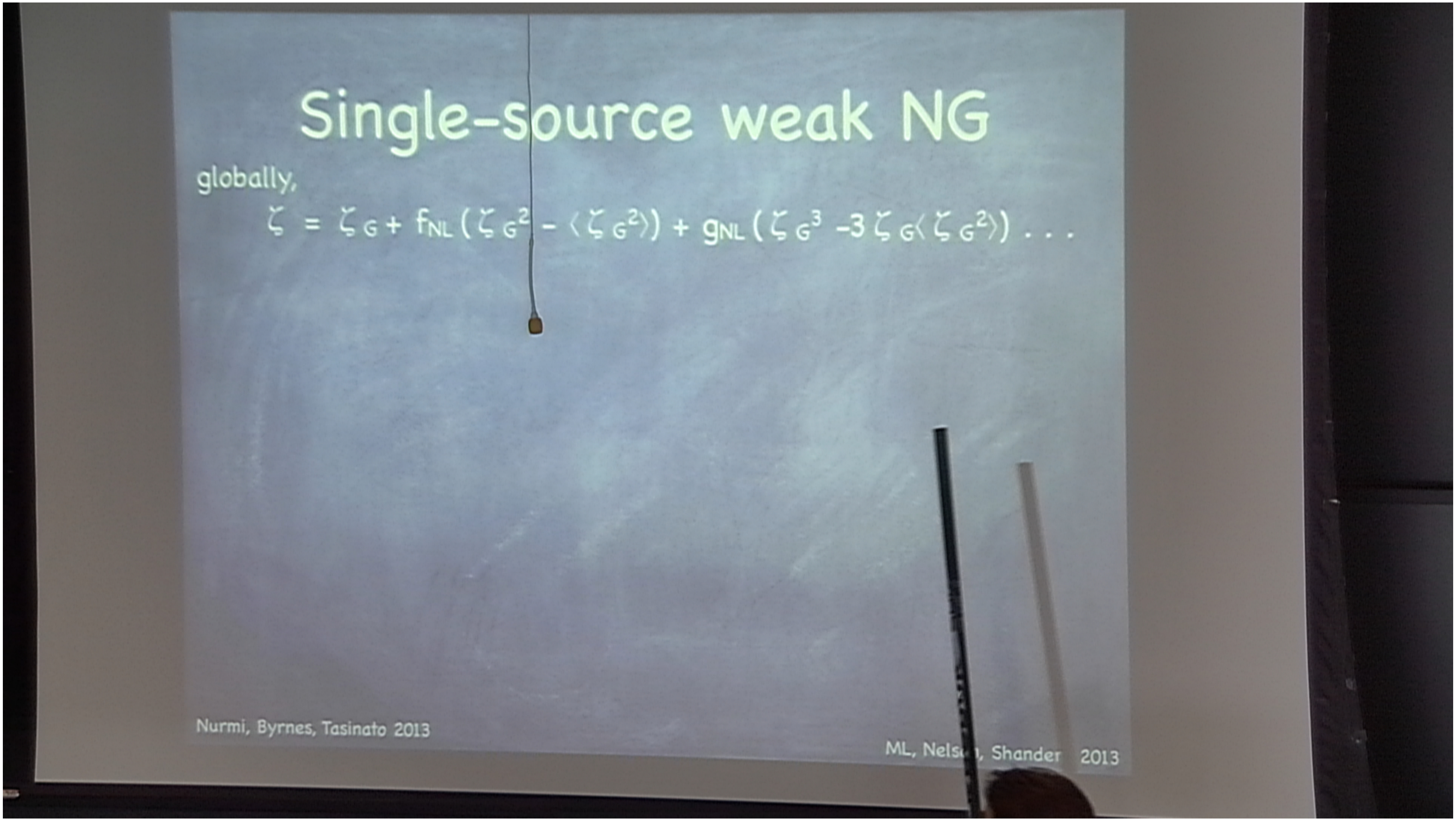
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
0
20
40
80
100
$N \equiv \frac{1}{3} \ln \frac{V_L}{V_s}$
Shandera

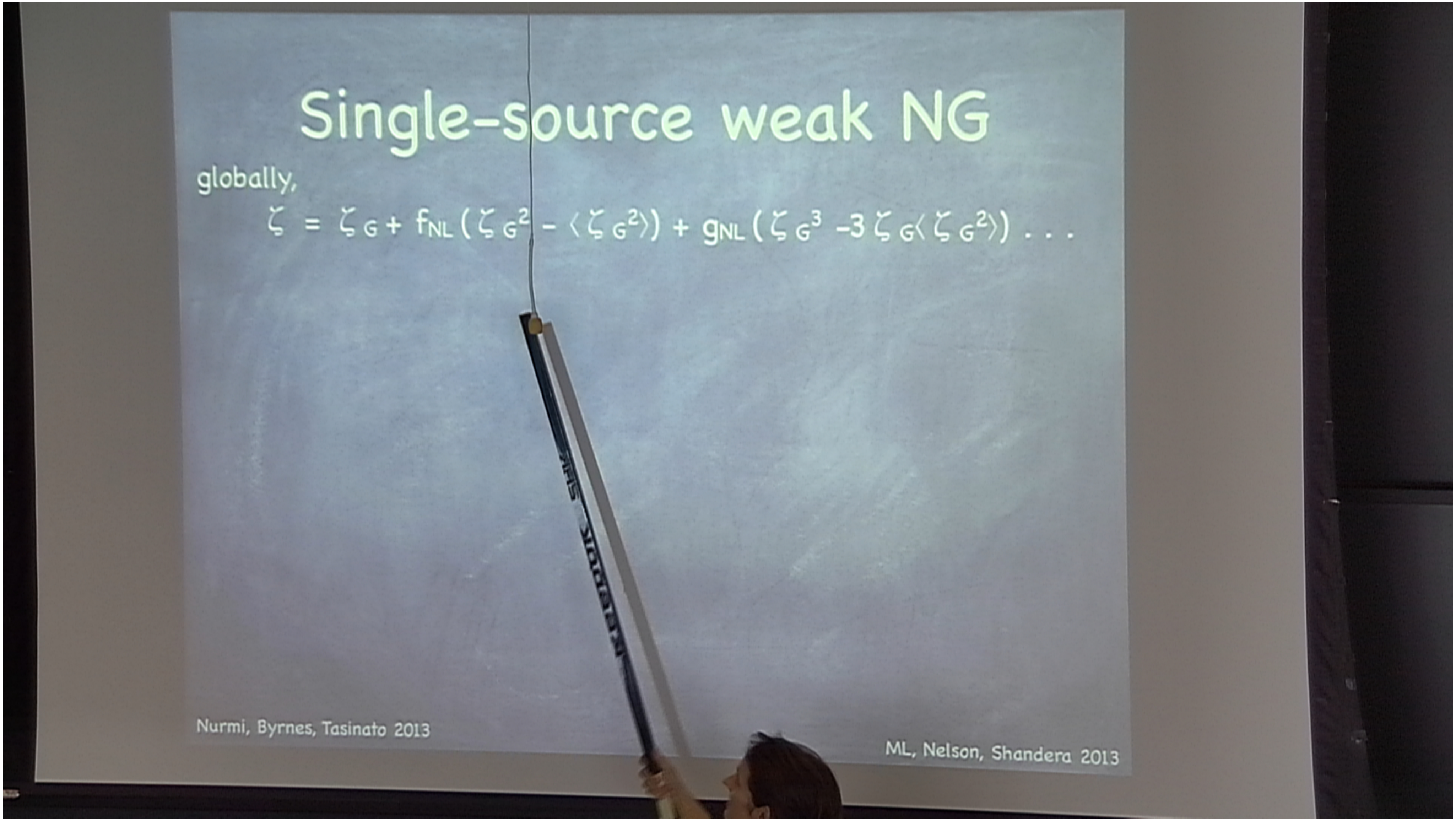

Super-horizon perturbations?
$\Omega_k \sim \int_{V_s \sim H_0^{-3}} d^3x \, \nabla^2 \zeta(x)$
H_0^{-1}
only modes with k ~ H_0 contribute
see e.g. Knox 2006, Erickeck et al 2008, Waterhouse 2008, Vardanyan et al 2009, Guth & Nomura 2012, Kleban & Schillo 2012
ML, Nelson, Shandera 2013



Non-linearity in the curvature perturbation $\zeta(x)$
causes our locally measured curvature ζ_s to depend
on a locally unobservable random variable ζ_L
How different might the statistics of ζ_s in our
Hubble patch be from the statistics of ζ in the rest
of the post-inflationary universe?


What are the consequences?
Consider three examples of statistics for ζ in V_L :
• weakly non-Gaussian
Nurmi, Byrnes, Tasinato 2013
• strongly non-Gaussian
Nelson & Shandera 2012
• multi-source non-Gaussian
entire post - inflationary patch
our Hubble volume
V_L
V_s
ML, Nelson, Shandera 2013


Single-source weak NG
globally,
$\zeta = \zeta_G + f_{NL}(\zeta_G^2 - \langle \zeta_G^2 \rangle) + g_{NL}(\zeta_G^3 - 3\zeta_G \langle \zeta_G^2 \rangle) \ldots$
Nurmi, Byrnes, Tasinato 2013
ML, Nels, Shander 2013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Single-source weak NG

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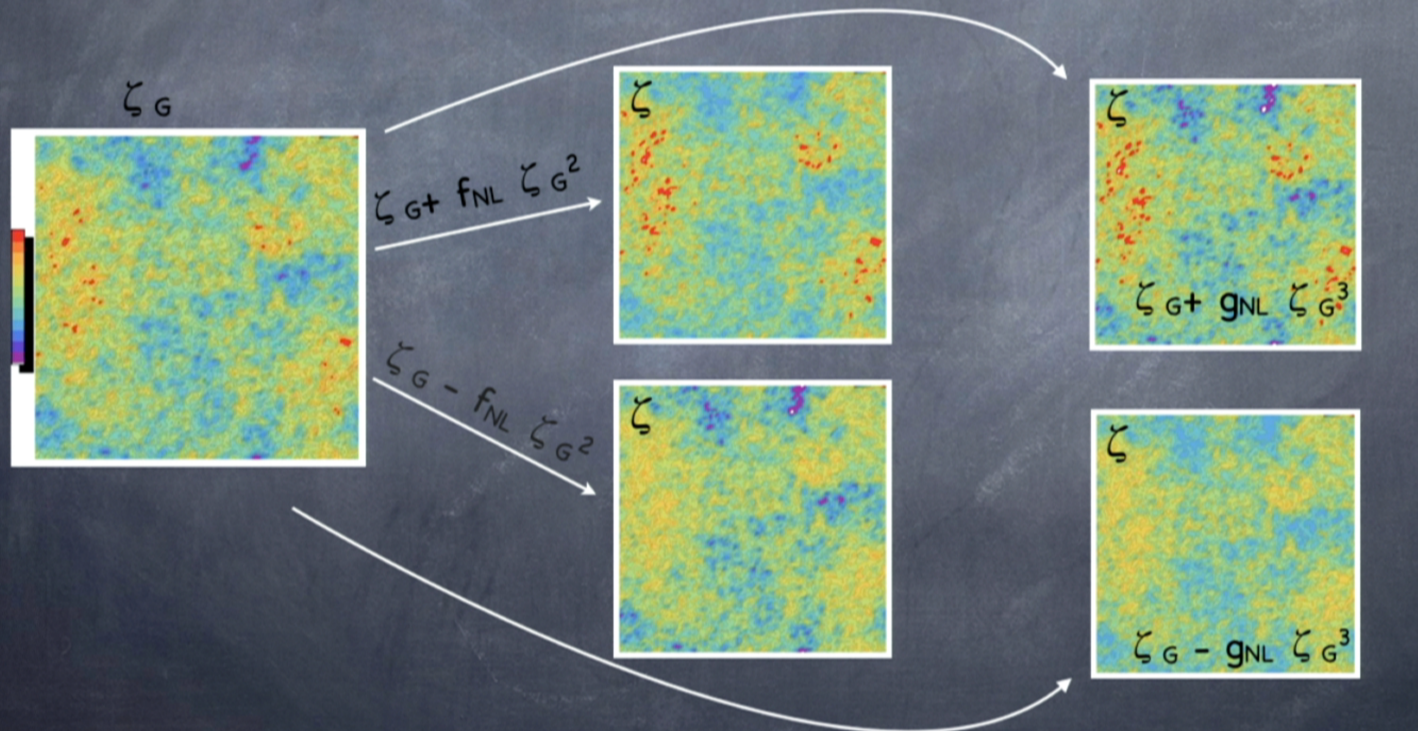
$$\zeta = \zeta_G + f_{NL}\left(\zeta_G^2 - \langle \zeta_G^2 \rangle\right) + g_{NL}\left(\zeta_G^3 - 3\,\zeta_G \langle \zeta_G^2 \rangle\right) \ldots$$



Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

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$$\zeta = \zeta_G + f_{NL}(\zeta_G{}^2 - \langle \zeta_G{}^2 \rangle) + g_{NL}(\zeta_G{}^3 - 3\zeta_G \langle \zeta_G{}^2 \rangle) \ldots$$

power spectrum

$$\langle \zeta\, \zeta \rangle \sim \langle \zeta_G \zeta_G \rangle$$

bispectrum

$$\langle \zeta\, \zeta\, \zeta \rangle \sim f_{NL} \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle$$

trispectrum

$$\langle \zeta\, \zeta\, \zeta\, \zeta \rangle - 3 \langle \zeta\, \zeta \rangle^2 \sim f_{NL}{}^2 \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle + g_{NL} \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

globally,

$$\zeta = \zeta_G + f_{NL}(\zeta_G^2 - \langle \zeta_G^2 \rangle) + g_{NL}(\zeta_G^3 - 3\zeta_G \langle \zeta_G^2 \rangle) \ldots$$

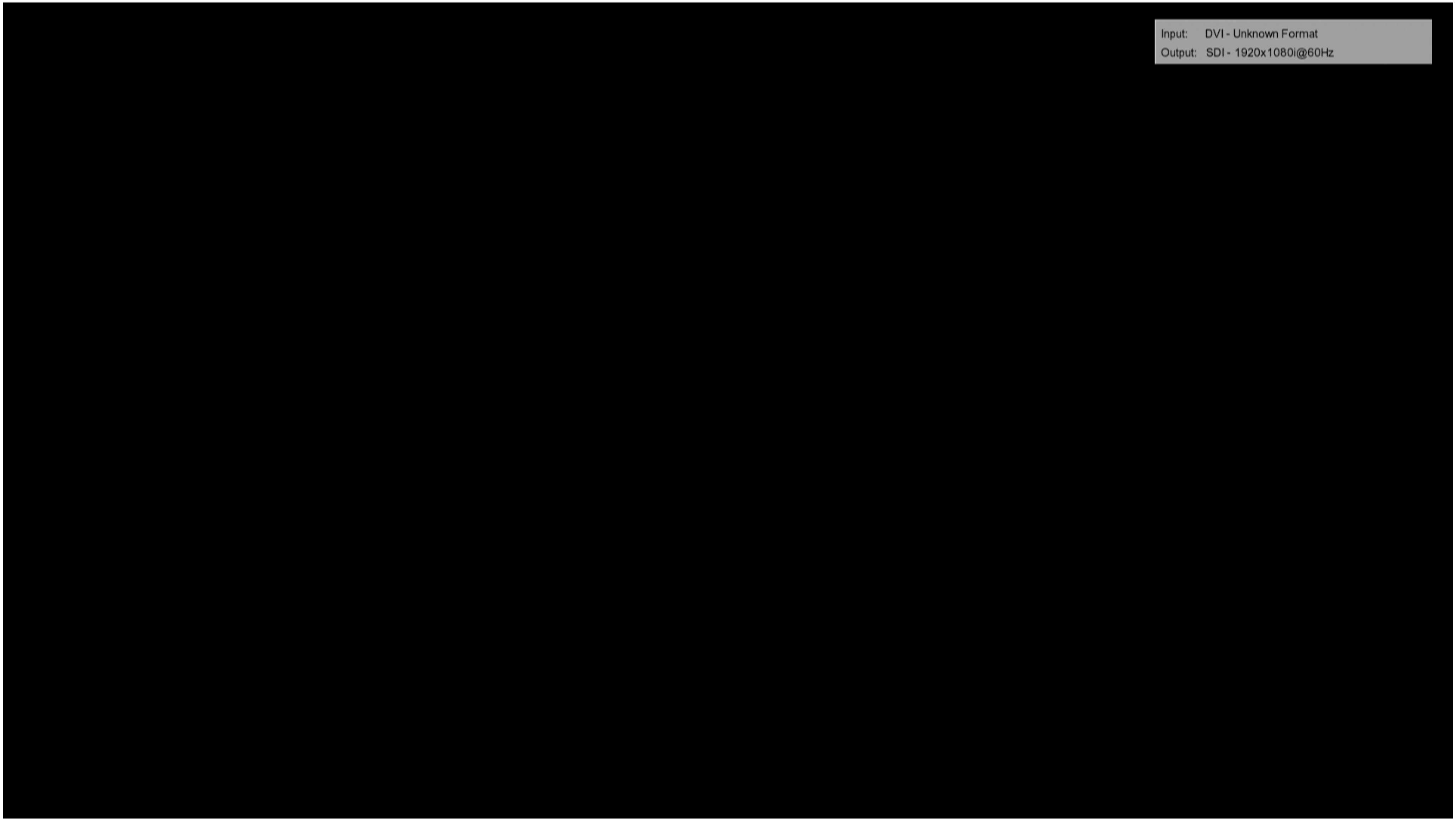
$$\Downarrow$$

locally,

$$\zeta_s = \zeta_{G,s}(1 + 2f_{NL}\zeta_{G,L}) + (f_{NL} + \tfrac{9}{5} g_{NL}\zeta_{G,L})(\zeta_G^2 - \langle \zeta_G^2 \rangle) + \ldots$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013


Input: DVI - Unknown Format
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Single-source weak NG

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$$\downarrow$$

locally,

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$$\downarrow$$

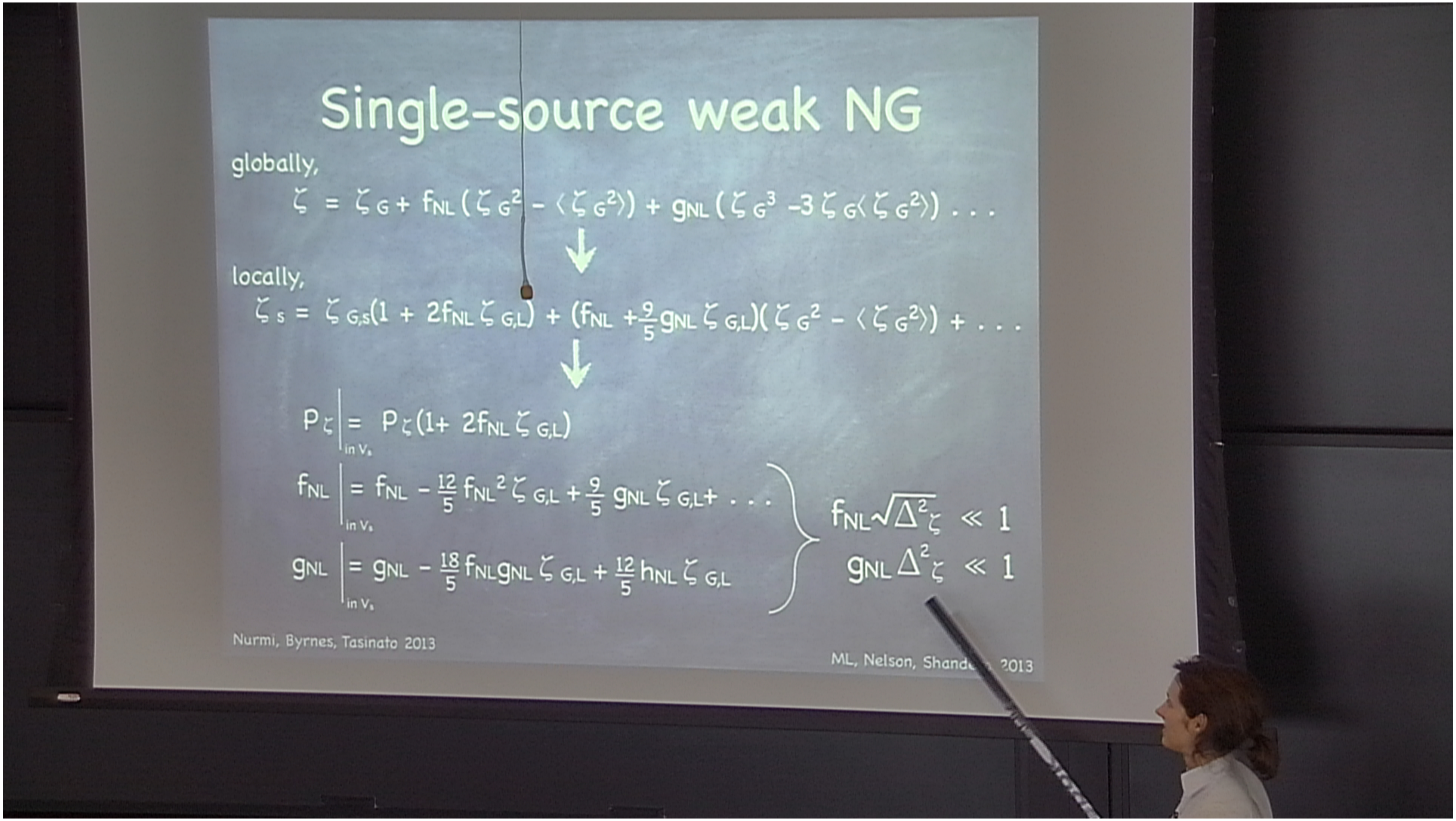
$$P_\zeta \Big|_{\text{in } V_s} = P_\zeta (1 + 2f_{NL}\,\zeta_{G,L})$$

$$f_{NL} \Big|_{\text{in } V_s} = f_{NL} - \frac{12}{5} f_{NL}^2\,\zeta_{G,L} + \frac{9}{5} g_{NL}\,\zeta_{G,L} + \ldots$$

$$g_{NL} \Big|_{\text{in } V_s} = g_{NL} - \frac{18}{5} f_{NL} g_{NL}\,\zeta_{G,L} + \frac{12}{5} h_{NL}\,\zeta_{G,L}$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013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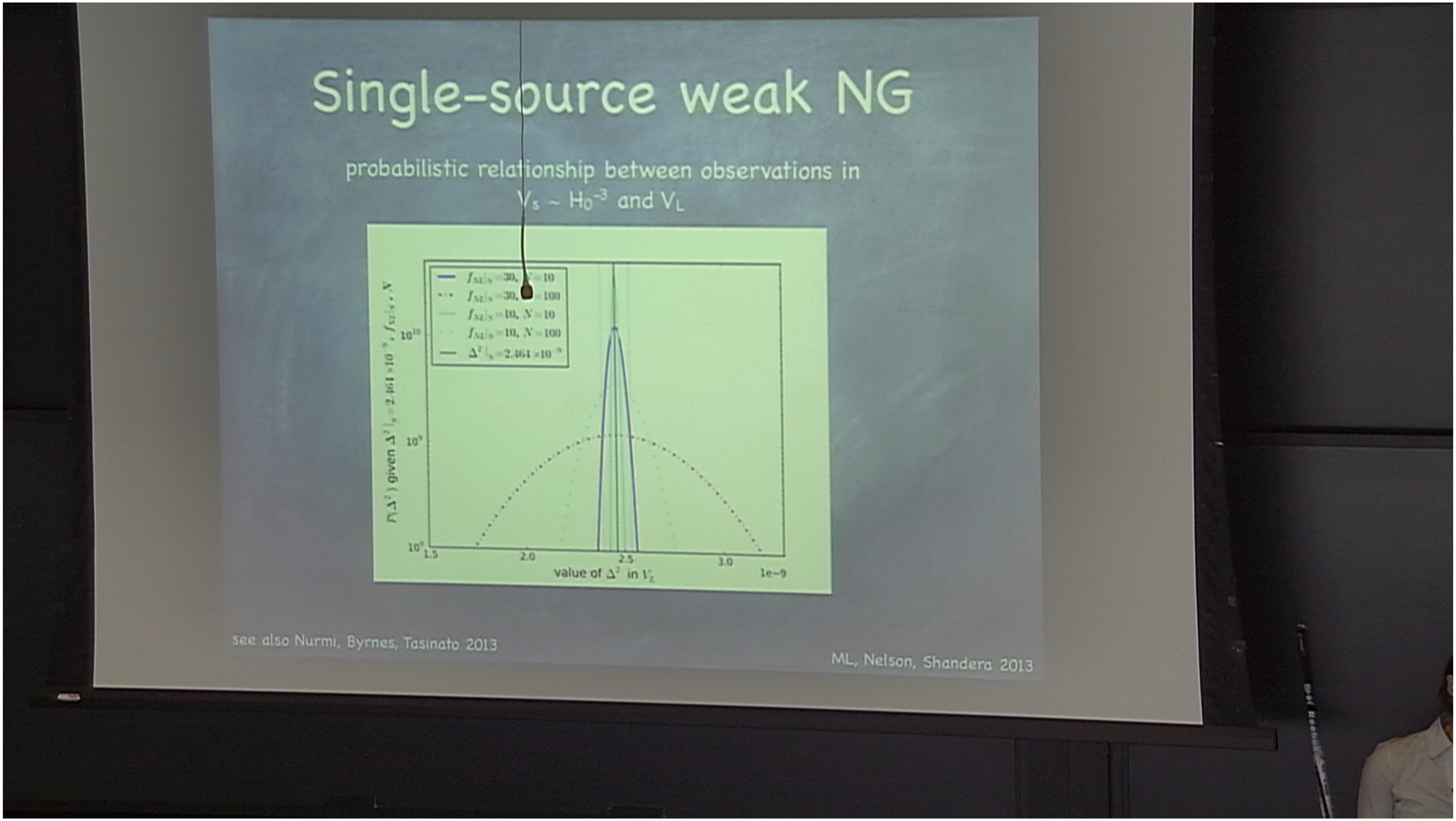
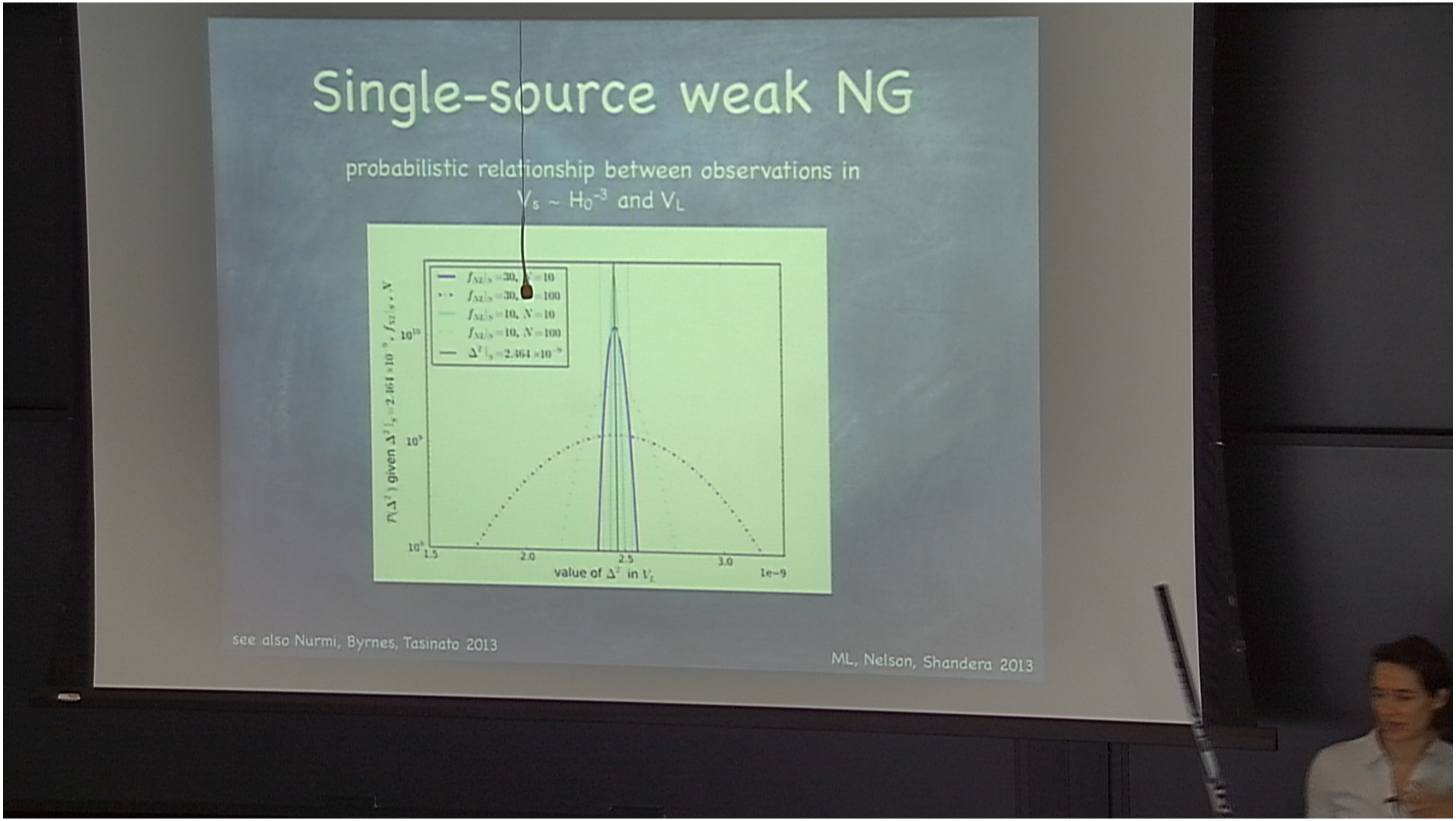
locally,
$\zeta_s = \zeta_{G,s}(1 + 2f_{NL}\zeta_{G,L}) + (f_{NL} + \frac{9}{5}g_{NL}\zeta_{G,L})(\zeta_G^2 - \langle\zeta_G^2\rangle) + \ldots$
$P_\zeta\Big|_{\text{in } V_s} = P_\zeta(1 + 2f_{NL}\zeta_{G,L})$
$f_{NL}\Big|_{\text{in } V_s} = f_{NL} - \frac{12}{5}f_{NL}^2\zeta_{G,L} + \frac{9}{5}g_{NL}\zeta_{G,L} + \ldots$
$g_{NL}\Big|_{\text{in } V_s} = g_{NL} - \frac{18}{5}f_{NL}g_{NL}\zeta_{G,L} + \frac{12}{5}h_{NL}\zeta_{G,L}$
$f_{NL}\sqrt{\Delta^2_\zeta} \ll 1$
$g_{NL}\Delta^2_\zeta \ll 1$
Nurmi, Byrnes, Tasinato 2013
ML, Nelson, Shande... 2013


Single-source weak NG
probabilistic relationship between observations in
Vs ~ H0-3 and VL
1.5
2.0
2.5
3.0
1e−9
value of Δ² in VL
see also Nurmi, Byrnes, Tasinato 2013
ML, Nelson, Shandera 2013



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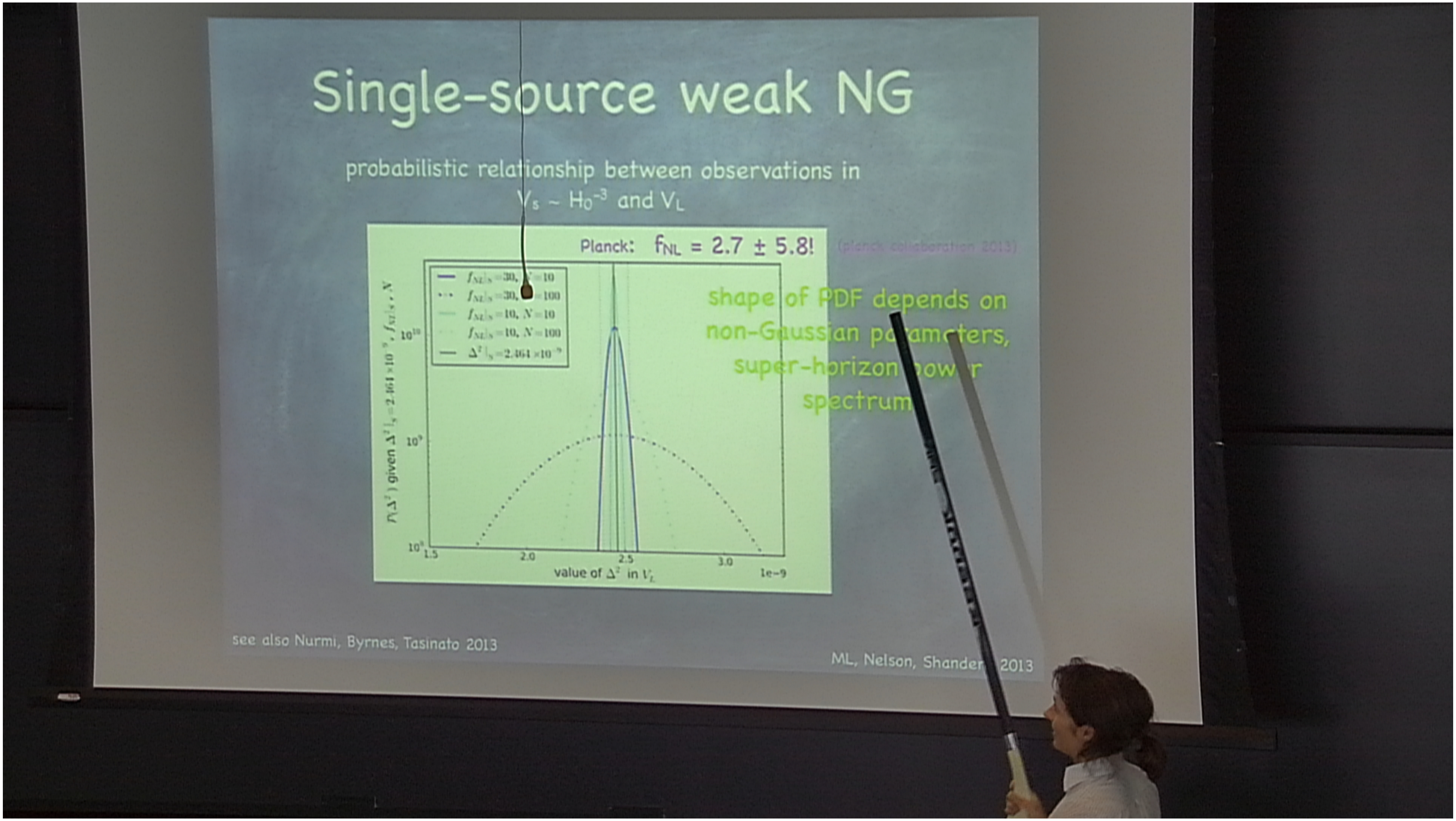
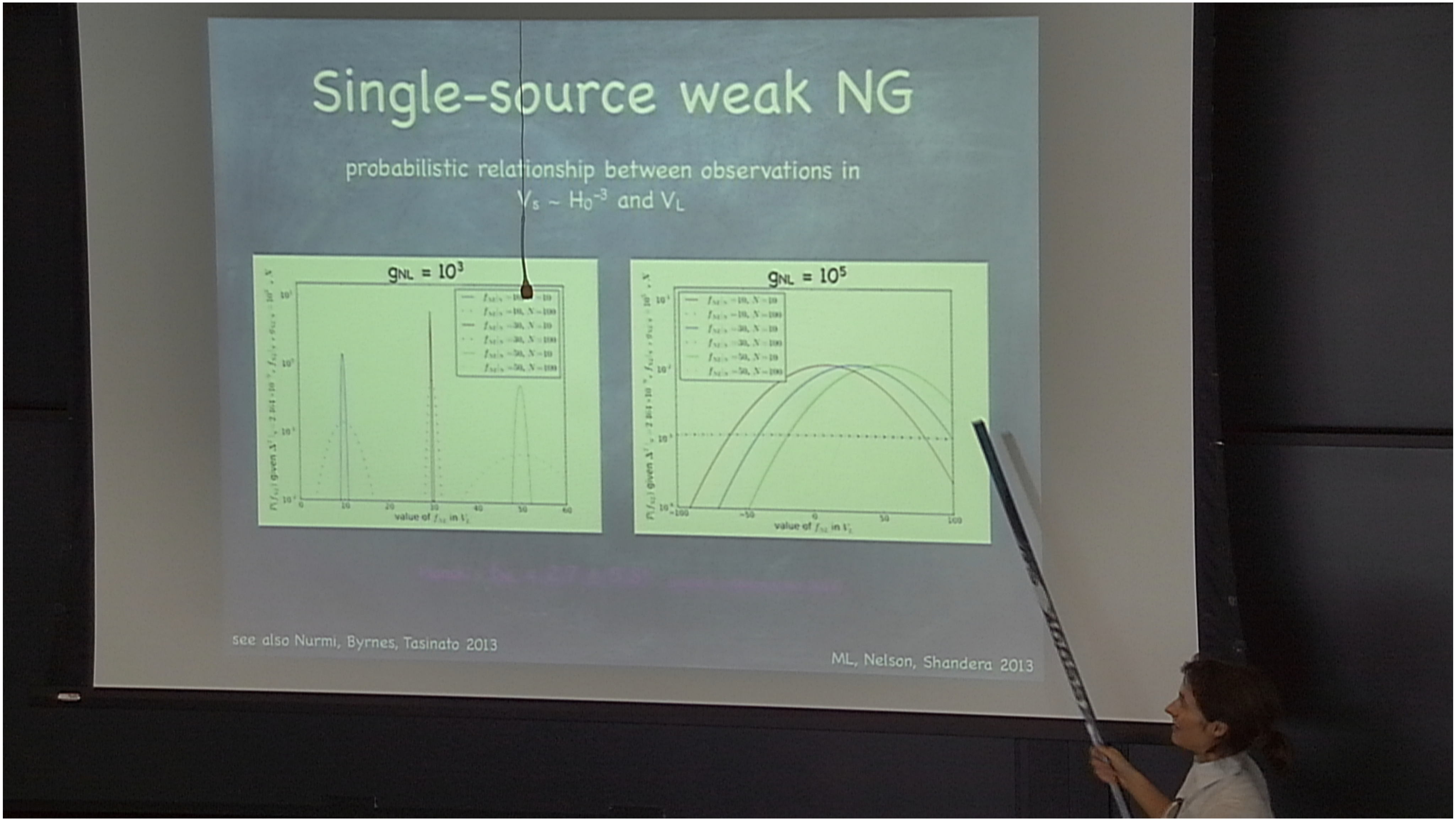
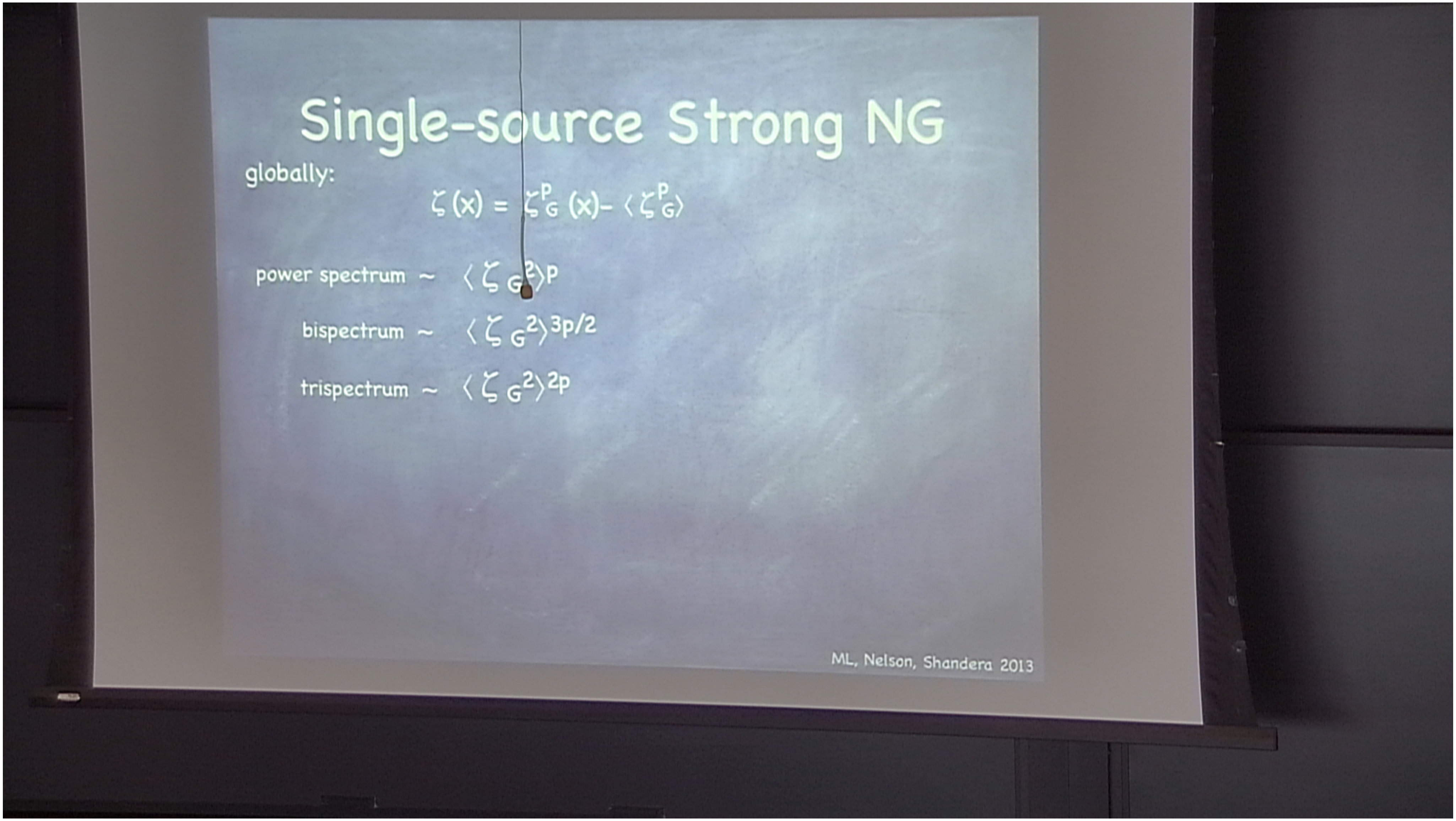
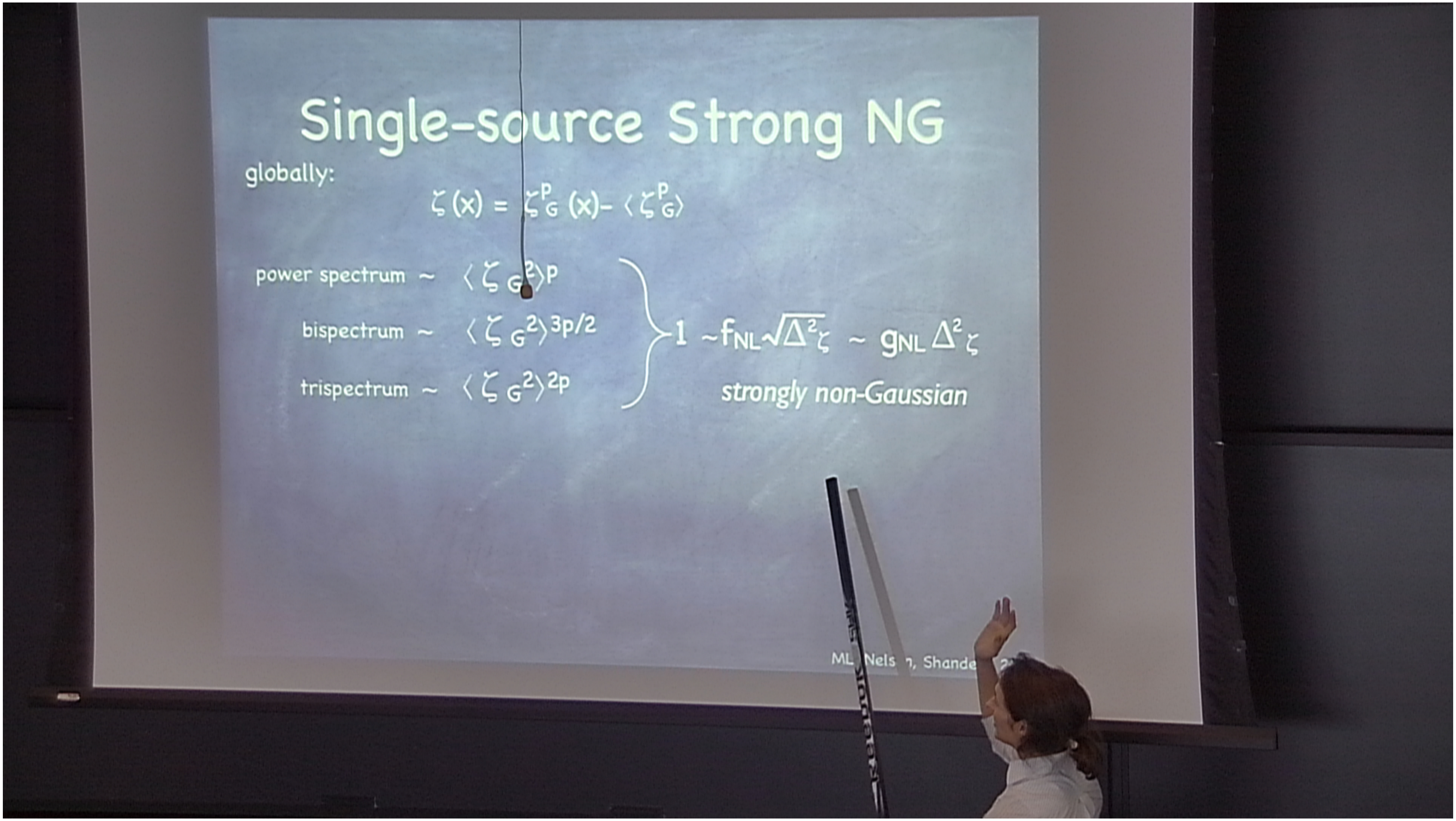

Single-source weak NG
probabilistic relationship between observations in
$V_s \sim H_0^{-3}$ and V_L
Planck: $f_{NL} = 2.7 \pm 5.8$!
shape of PDF depends on
value of Δ^2 in V_L
1e-9
see also Nurmi, Byrnes, Tasinato 2013


Single-source weak NG
probabilistic relationship between observations in
$V_s \sim H_0^{-3}$ and V_L
$g_{NL} = 10^3$
$g_{NL} = 10^5$
value of f_{NL} in V_L
value of f_{NL} in V_L
see also Nurmi, Byrnes, Tasinato 2013
ML, Nelson, Shandera 2013


Single-source Strong NG
globally:
$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$
power spectrum ~ $\langle \zeta_G^2 \rangle^p$
bispectrum ~ $\langle \zeta_G^2 \rangle^{3p/2}$
trispectrum ~ $\langle \zeta_G^2 \rangle^{2p}$
ML, Nelson, Shandera 2013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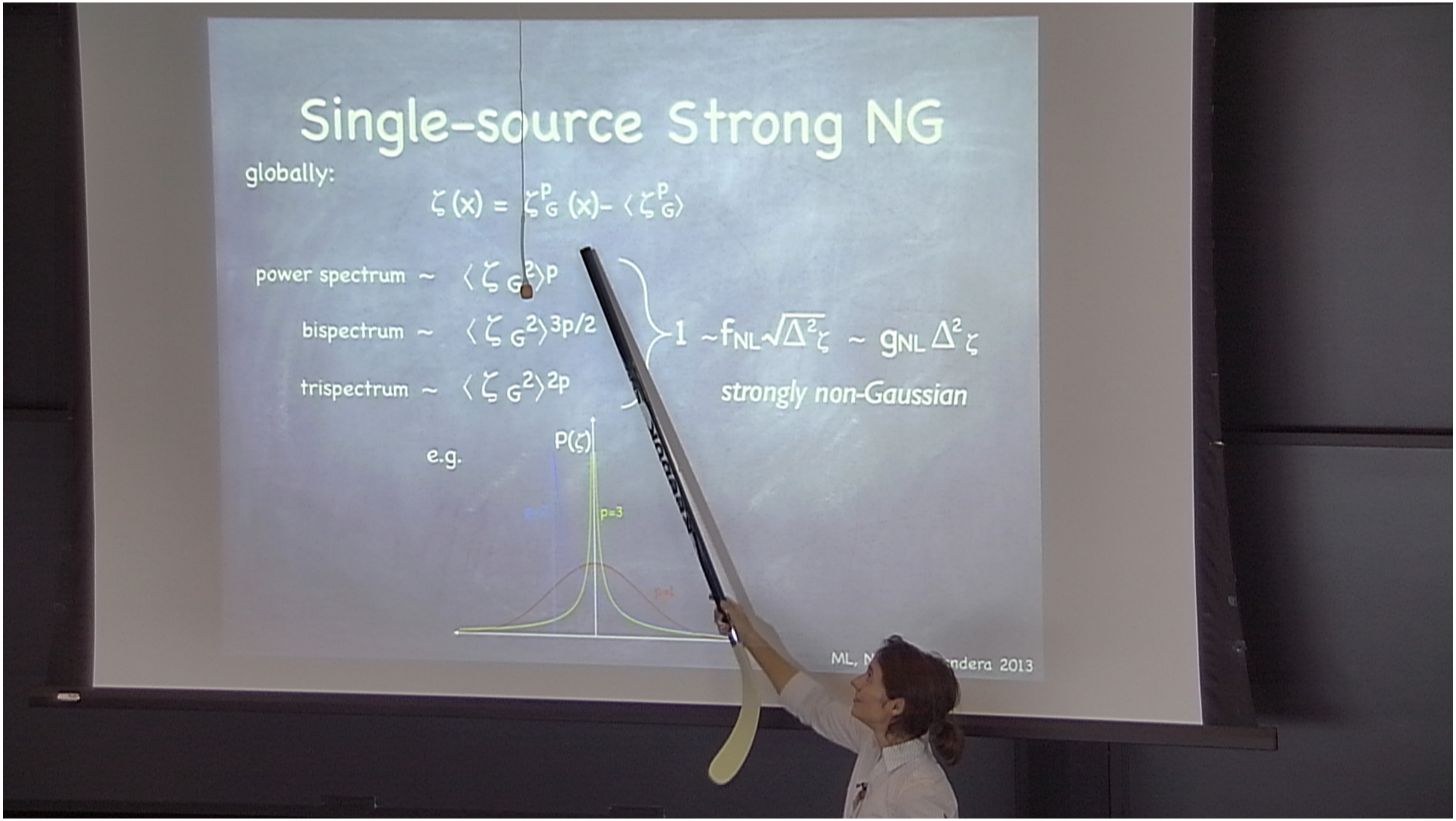
$1 \sim f_{NL}\sqrt{\Delta^2_\zeta} \sim g_{NL}\,\Delta^2_\zeta$
strongly non-Gaussian



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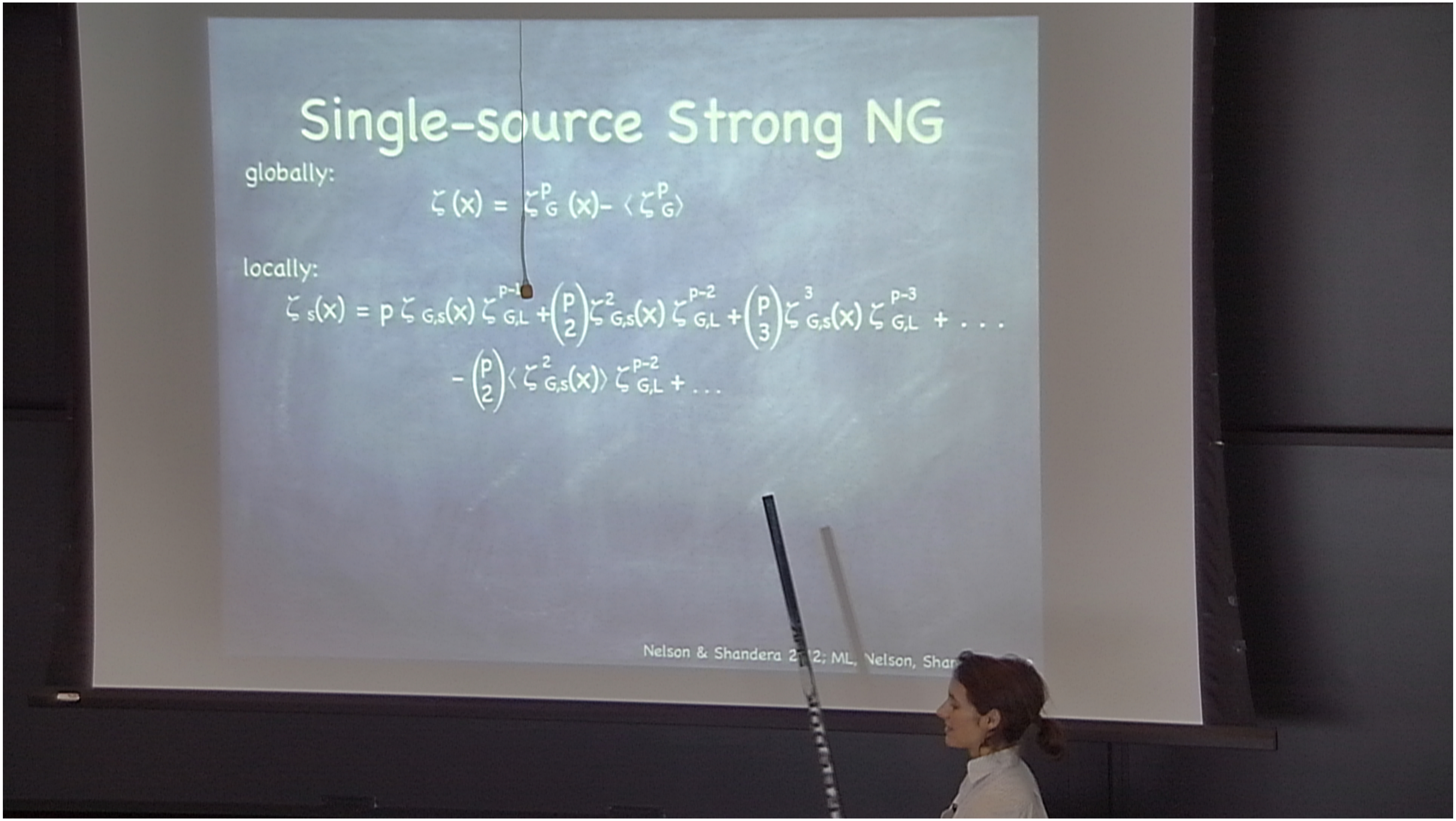
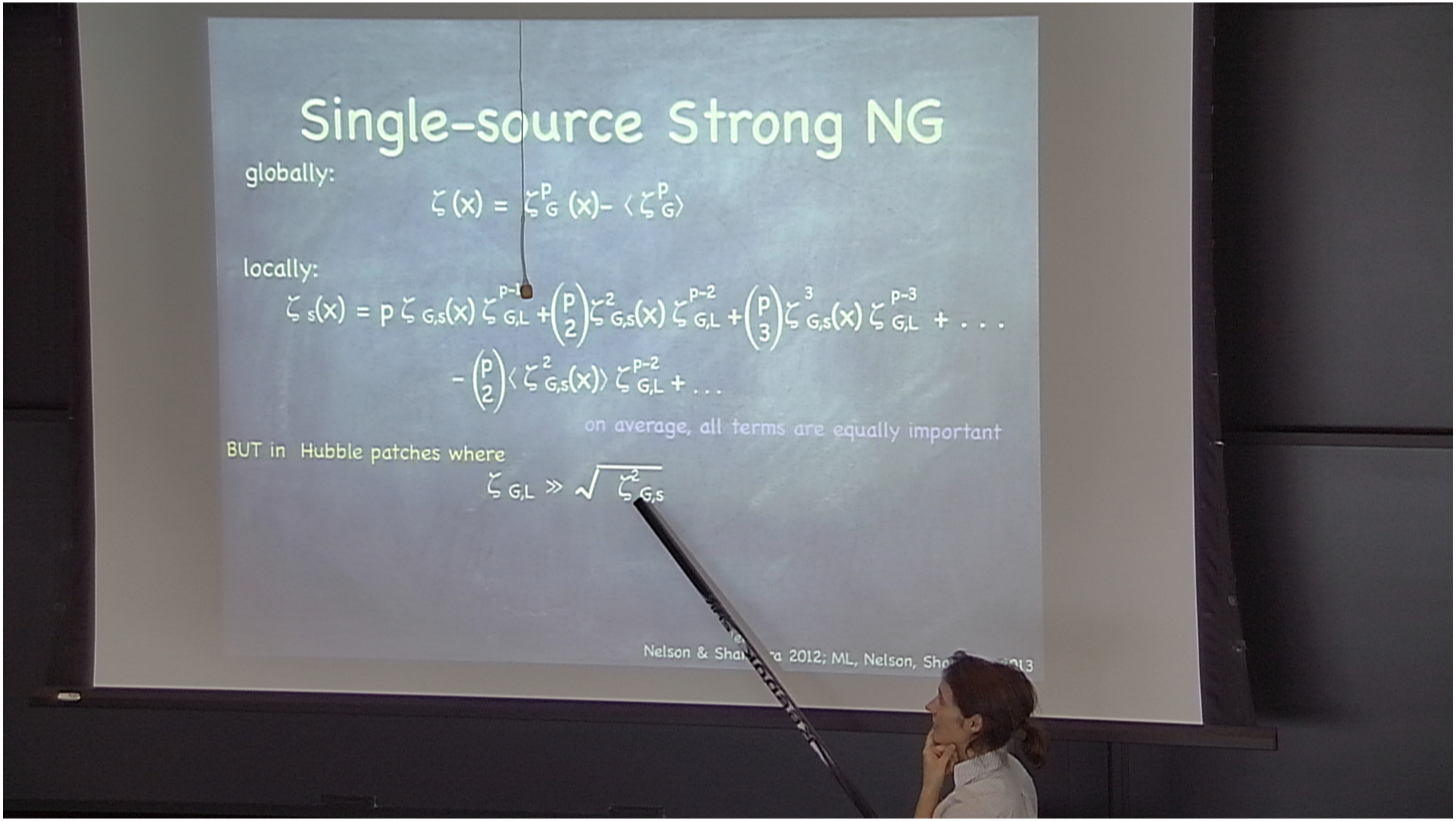
$1 \sim f_{NL}\sqrt{\Delta^2_\zeta} \sim g_{NL}\Delta^2_\zeta$
strongly non-Gaussian
e.g.
$P(\zeta)$
p=3


Single-source Strong NG
globally:
$\zeta(x) = \zeta^{p}_{G}(x) - \langle \zeta^{p}_{G} \rangle$
locally:
$\zeta_{s}(x) = p\, \zeta_{G,s}(x)\, \zeta^{p-1}_{G,L} + \binom{p}{2} \zeta^{2}_{G,s}(x)\, \zeta^{p-2}_{G,L} + \binom{p}{3} \zeta^{3}_{G,s}(x)\, \zeta^{p-3}_{G,L} + \ldots$
$- \binom{p}{2} \langle \zeta^{2}_{G,s}(x) \rangle\, \zeta^{p-2}_{G,L} + \ldots$
Nelson & Shandera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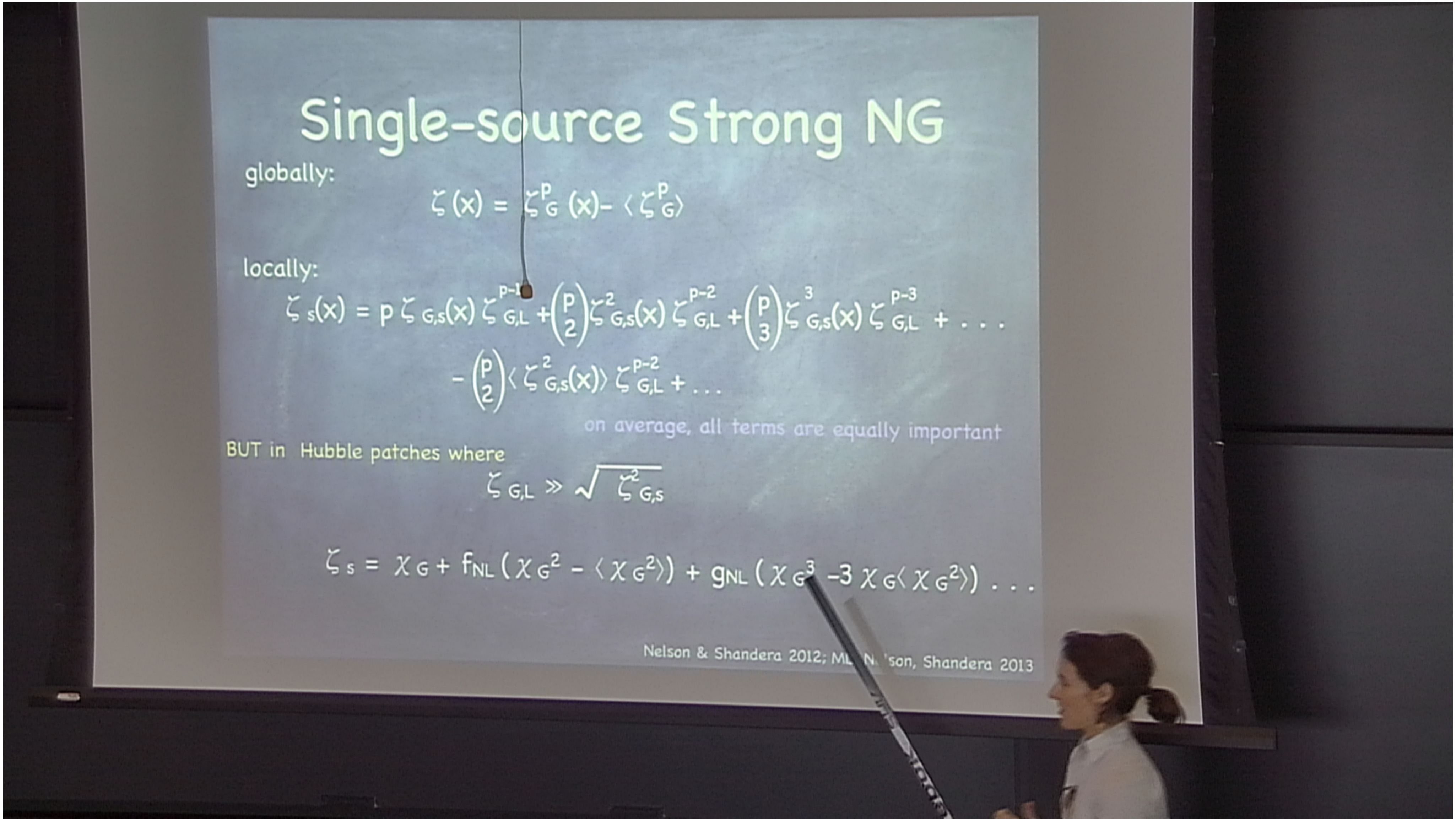
$- \binom{p}{2}\langle \zeta^{2}_{G,s}(x)\rangle\,\zeta^{p-2}_{G,L} + \dots$
on average, all terms are equally important
BUT in Hubble patches where
$\zeta_{G,L} \gg \sqrt{\zeta^{2}_{G,s}}$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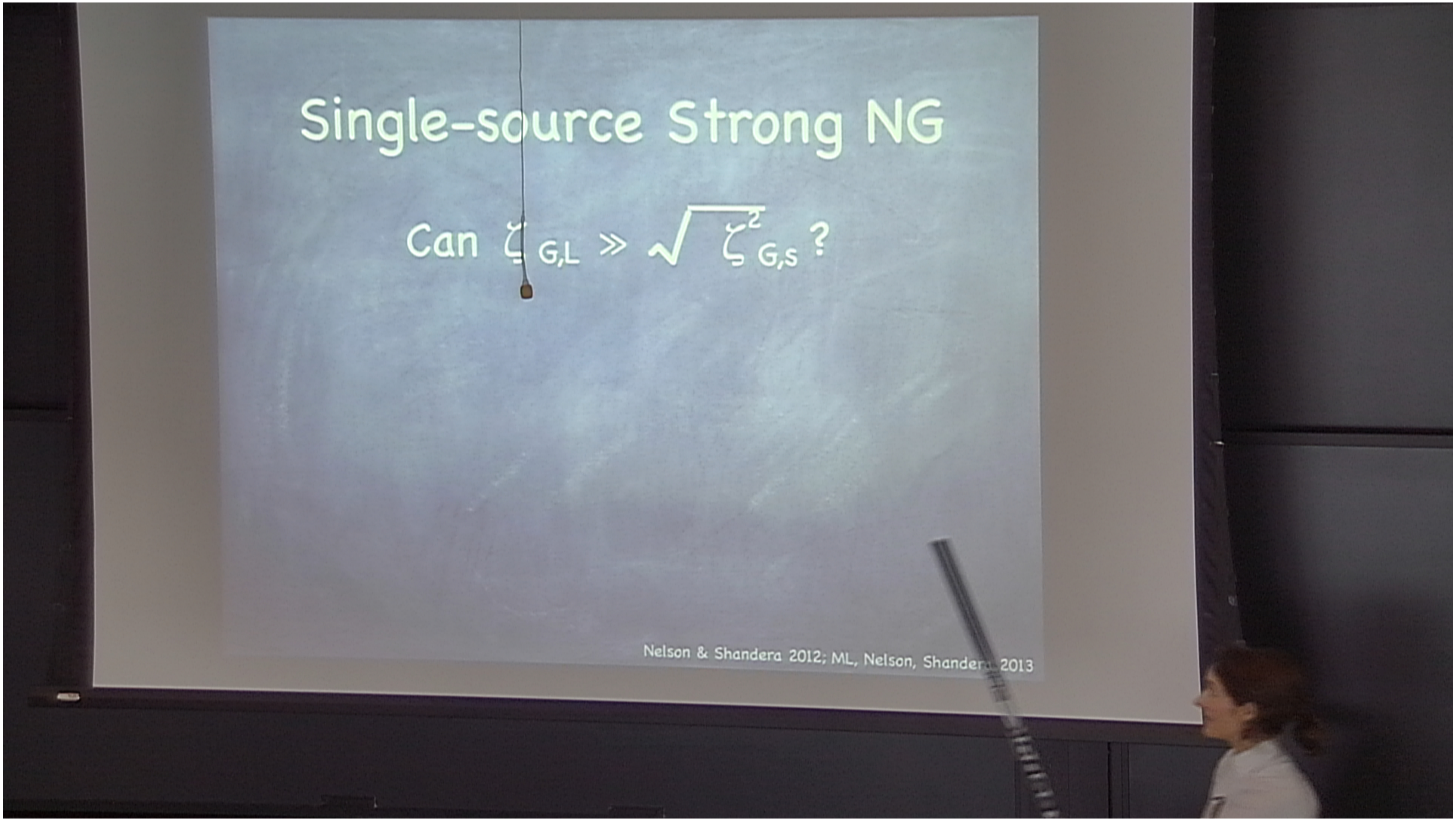
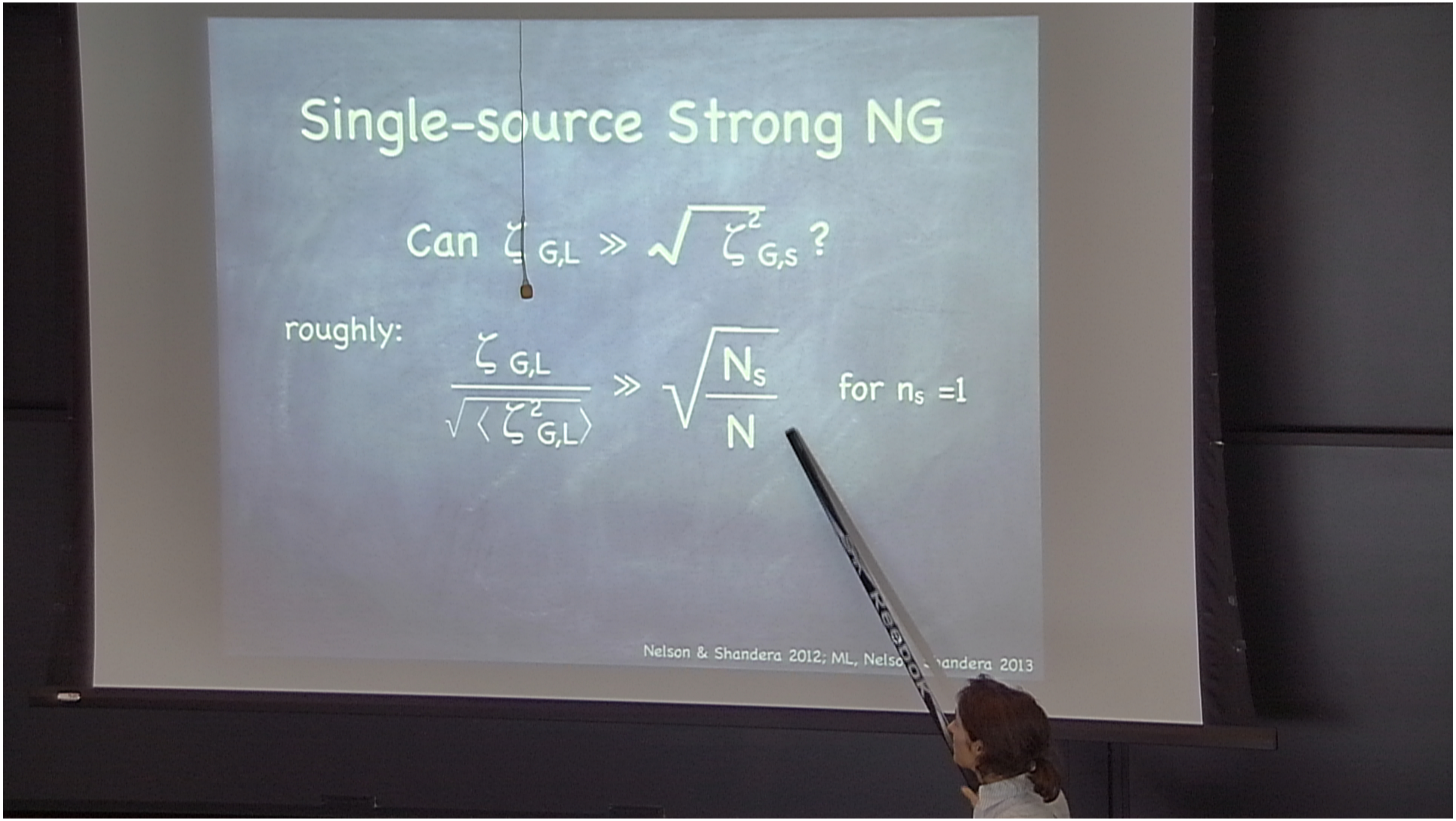
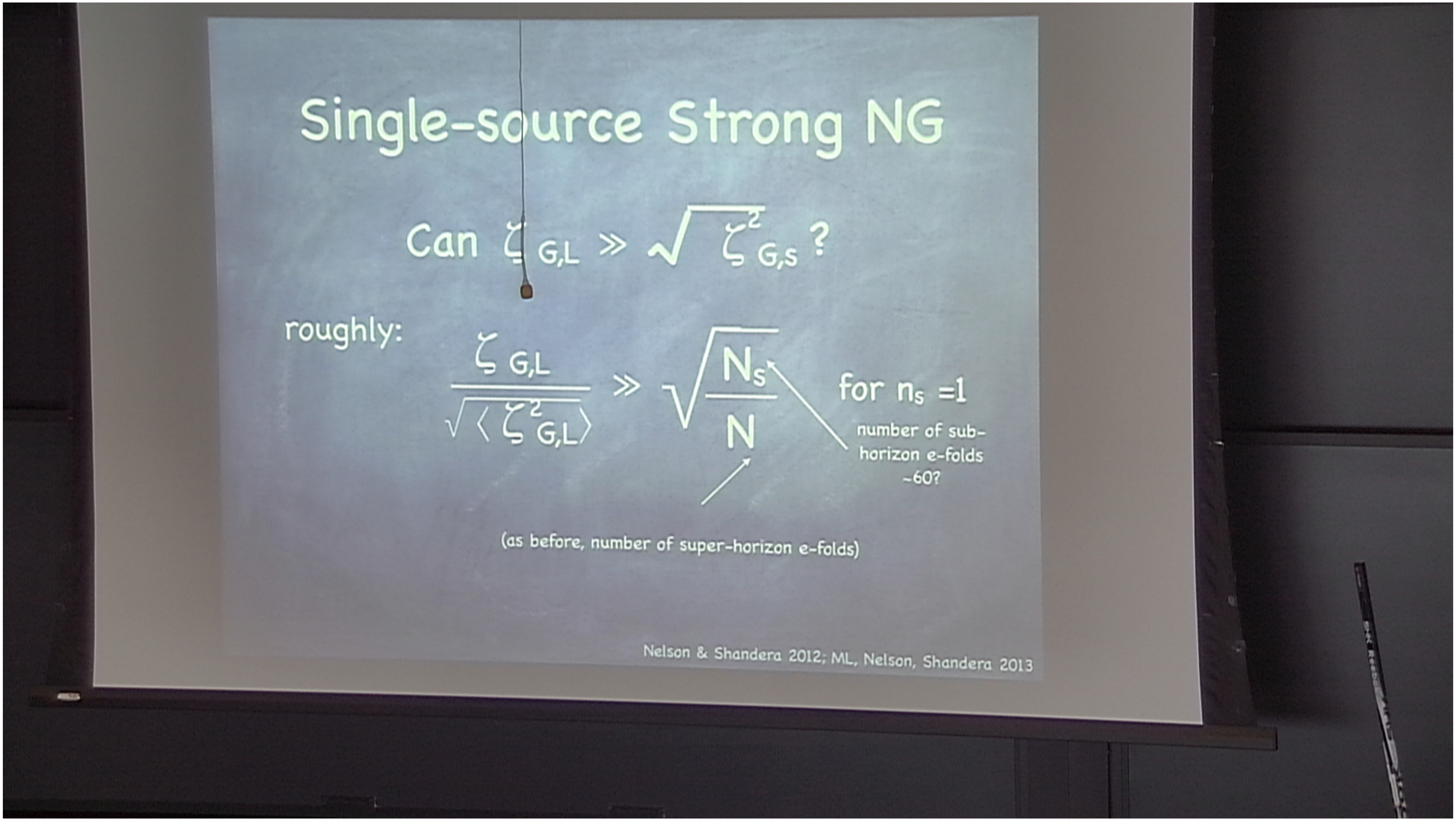
BUT in Hubble patches where
$\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$
$\zeta_s = \chi_G + f_{NL}(\chi_G^2 - \langle \chi_G^2 \rangle) + g_{NL}(\chi_G^3 - 3\chi_G\langle \chi_G^2 \rangle) \ldots$
Nelson & Shandera 2012; ML


Single-source Strong NG
Can $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$?
Nelson & Shandera 2012; ML, Nelson, Shandera 2013


Single-source Strong NG
Can $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$?
roughly:
$\frac{\zeta_{G,L}}{\sqrt{\langle \zeta^2_{G,L} \rangle}} \gg \sqrt{\frac{N_s}{N}}$ for $n_s = 1$
Nelson & Shandera 2012; ML, Nelso
andera 2013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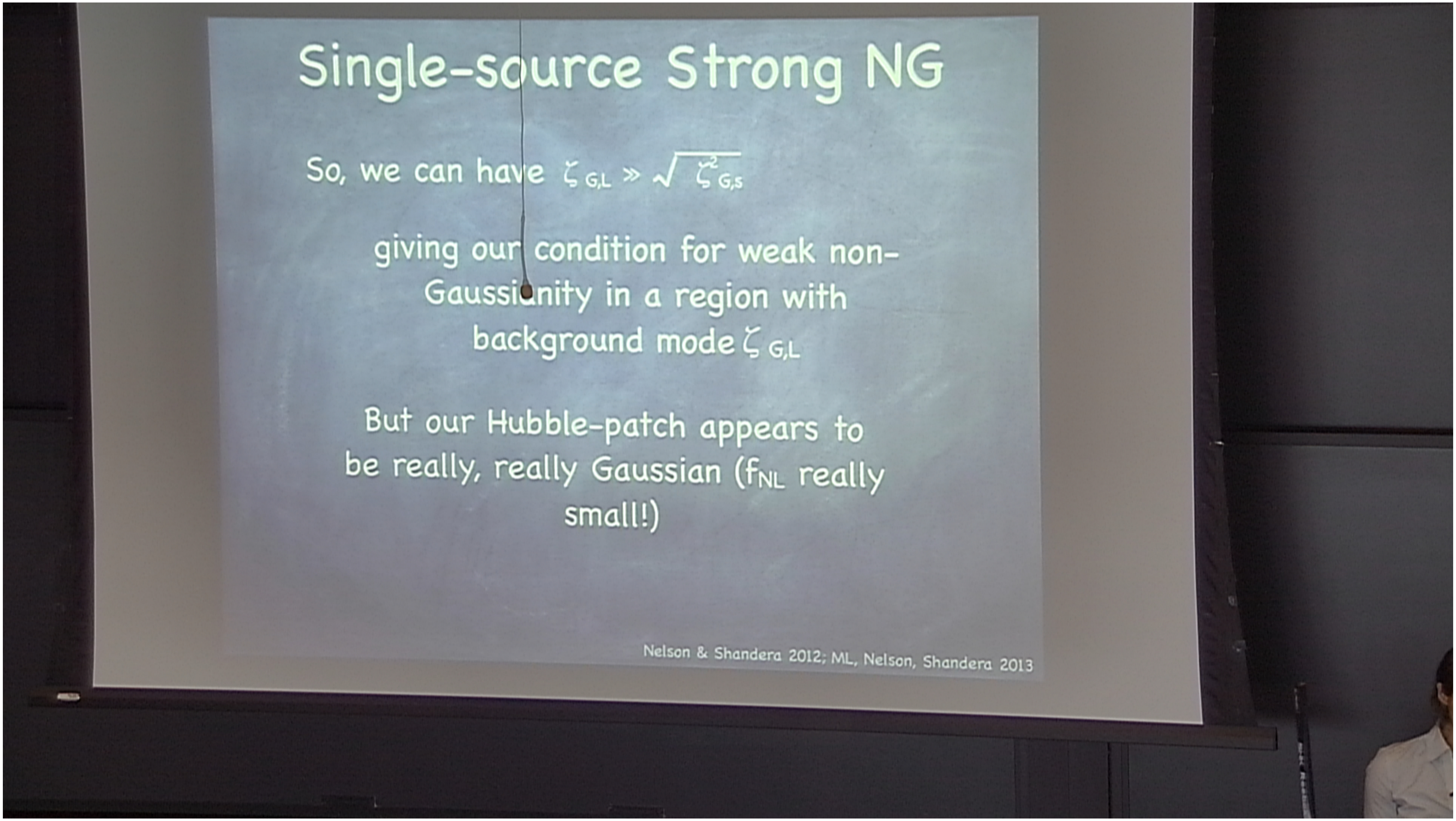
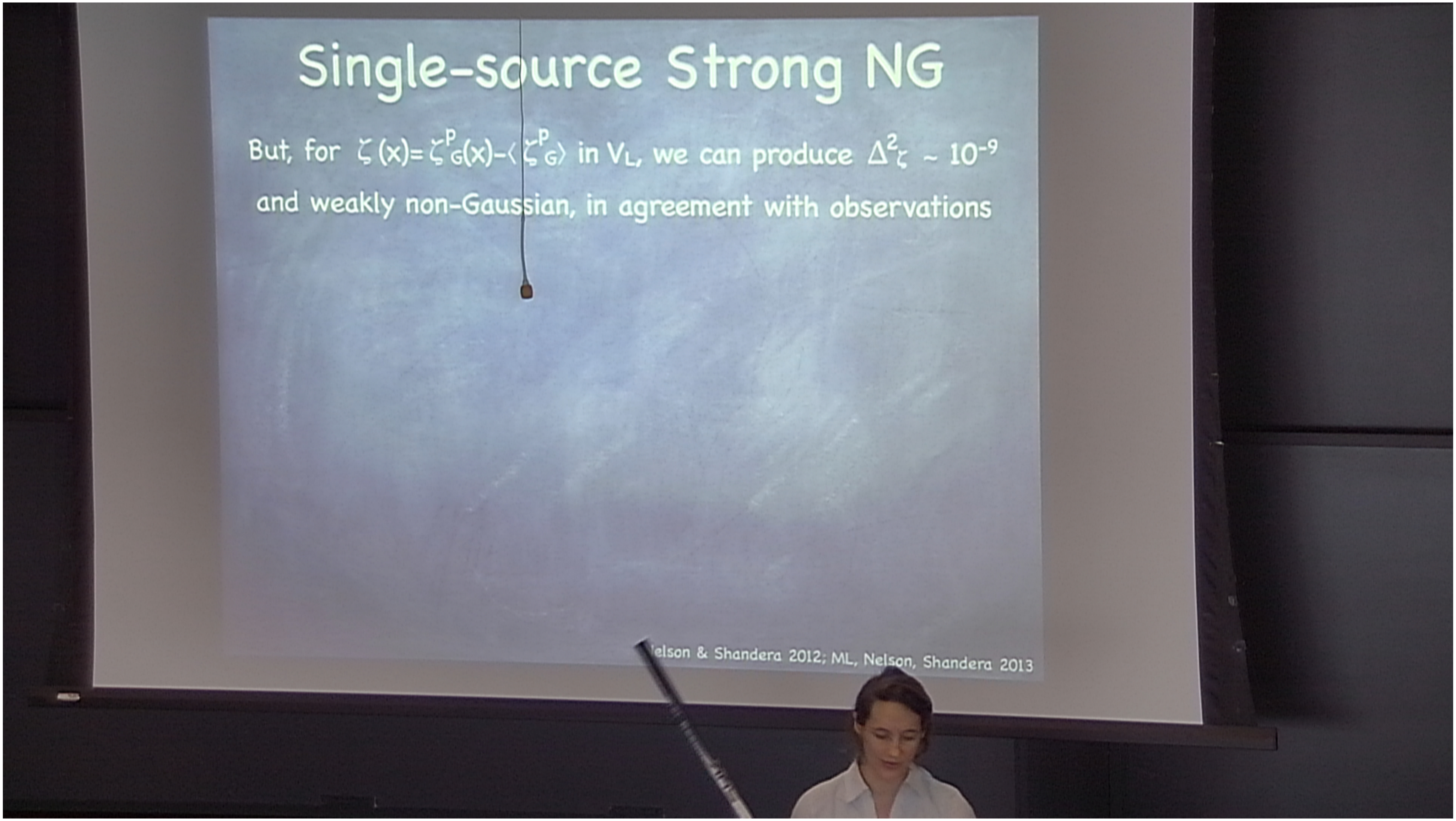
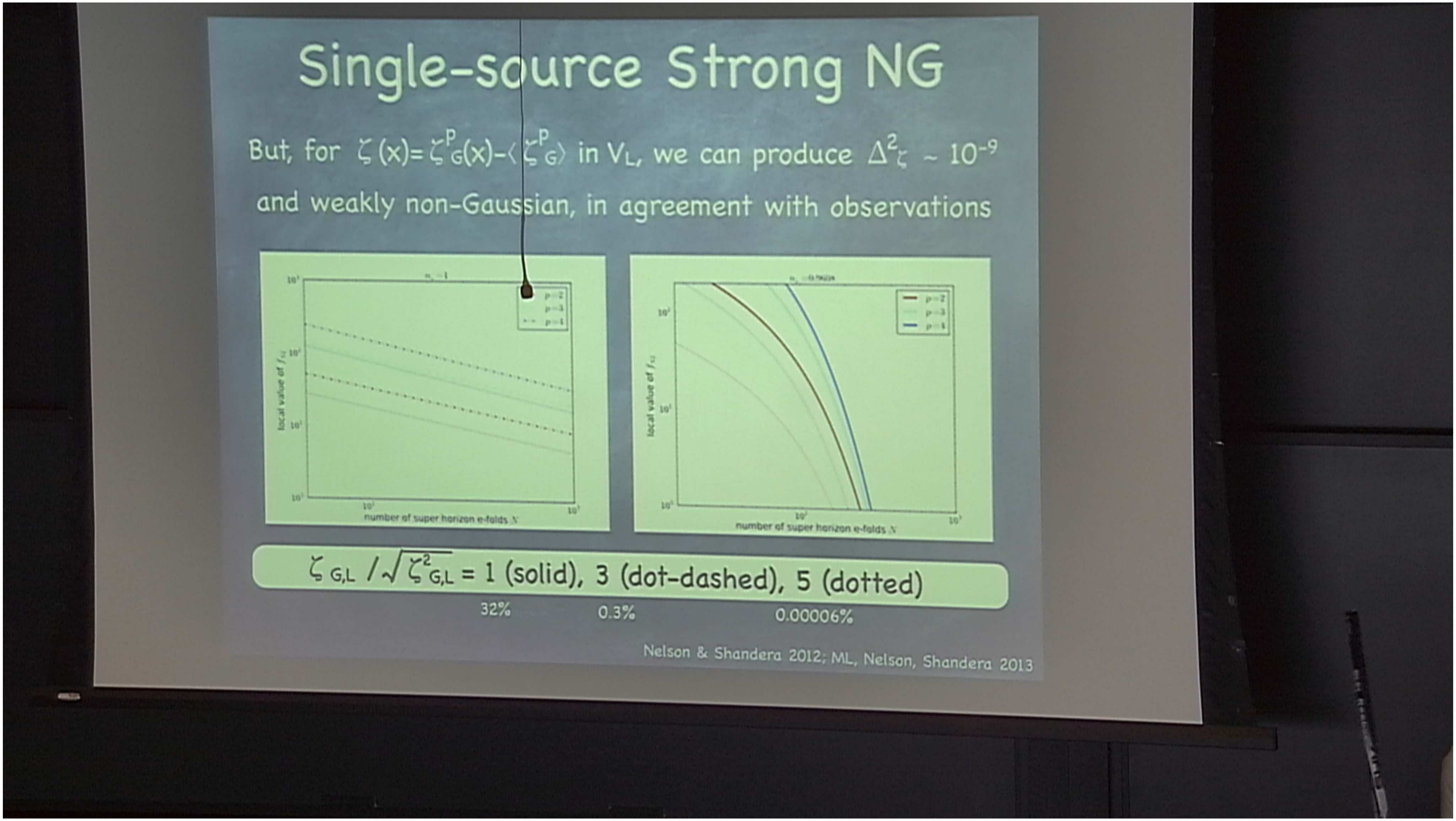
roughly:
$\frac{\zeta_{G,L}}{\sqrt{\langle \zeta^2_{G,L} \rangle}} \gg \sqrt{\frac{N_s}{N}}$
for $n_s = 1$
number of sub-horizon e-folds ~60?
(as before, number of super-horizon e-folds)
Nelson & Shandera 2012; ML, Nelson, Shandera 2013


Single-source Strong NG
So, we can have $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$
giving our condition for weak non-Gaussianity in a region with background mode $\zeta_{G,L}$
But our Hubble-patch appears to be really, really Gaussian (f_{NL} really small!)
Nelson & Shandera 2012; ML, Nelson, Shandera 2013


Single-source Strong NG
But, for $\zeta(x)=\zeta^{p}_{G}(x)-\langle\zeta^{p}_{G}\rangle$ in V_L, we can produce $\Delta^2_\zeta \sim 10^{-9}$
and weakly non-Gaussian, in agreement with observations
elson & Shandera 2012; ML, Nelson, Shandera 2013

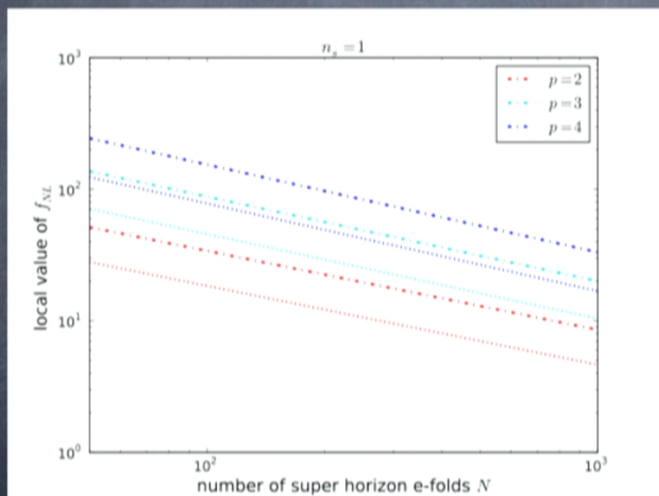


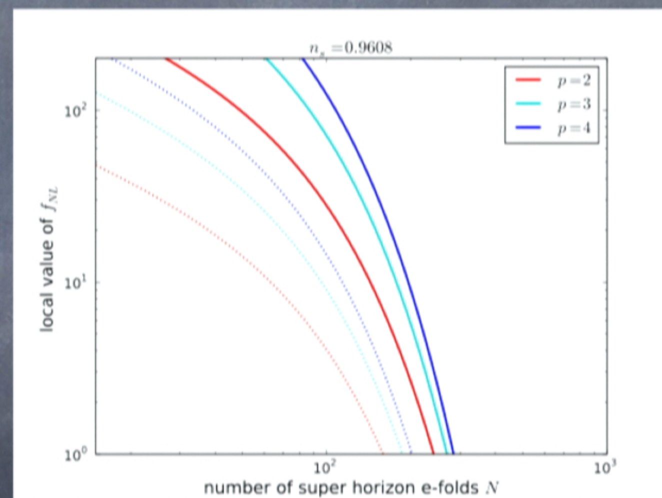
Single-source Strong NG
But, for $\zeta(x)=\zeta^p_G(x)-\langle\zeta^p_G\rangle$ in V_L, we can produce $\Delta^2_\zeta \sim 10^{-9}$
and weakly non-Gaussian, in agreement with observations
local value of f_{NL}
number of super horizon e-folds N
$p=2$
$p=3$
$p=4$
local value of f_{NL}
number of super horizon e-folds N
$p=2$
$p=3$
$p=4$
$\zeta_{G,L} / \sqrt{\zeta^2_{G,L}}$ = 1 (solid), 3 (dot-dashed), 5 (dotted)
32%
0.3%
0.00006%
Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

But, for $\zeta(x)=\zeta^{p}_{G}(x)-\langle\zeta^{p}_{G}\rangle$ in V_L, we can produce $\Delta^2_\zeta \sim 10^{-9}$

and weakly non-Gaussian, in agreement with observations

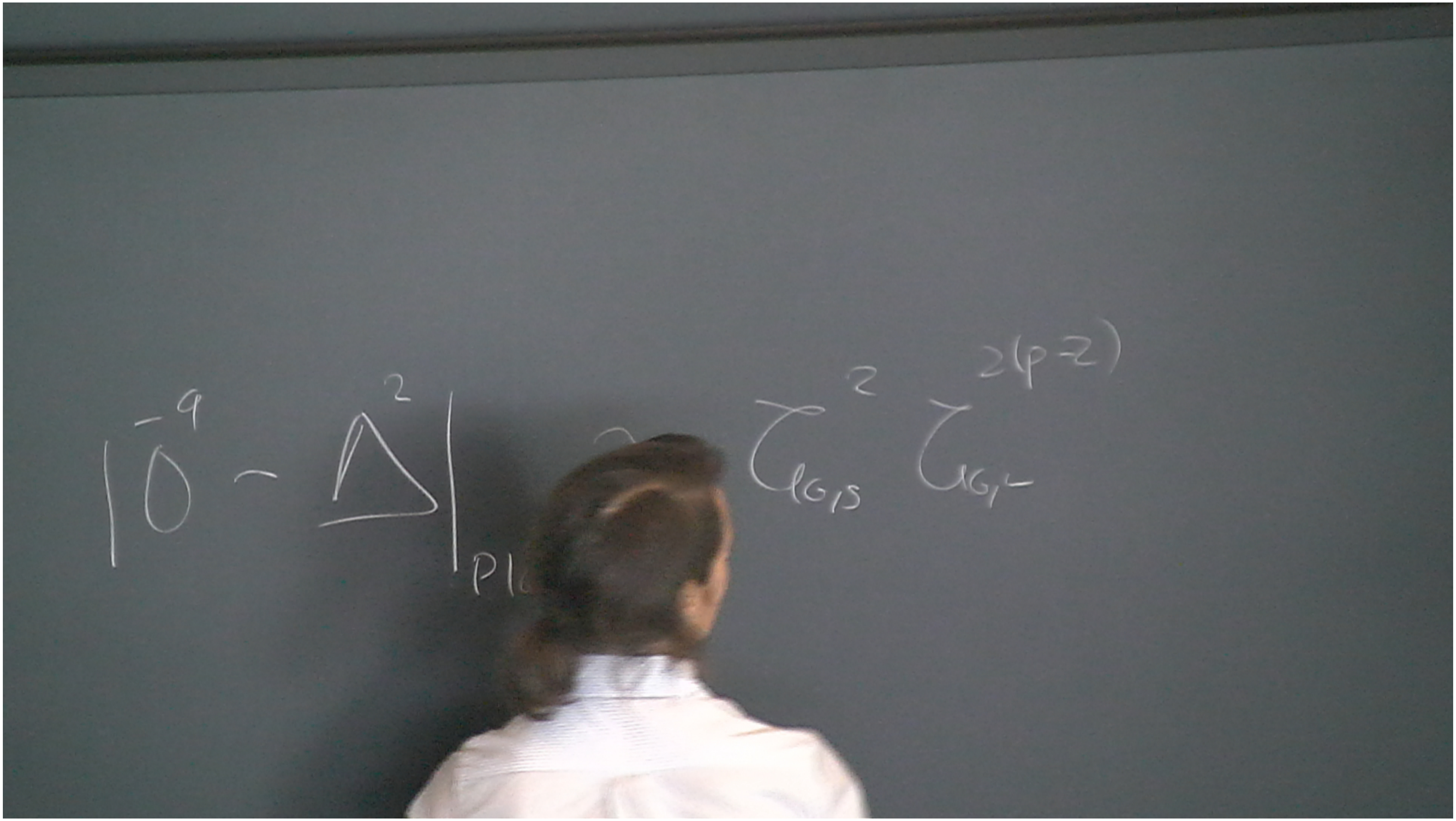


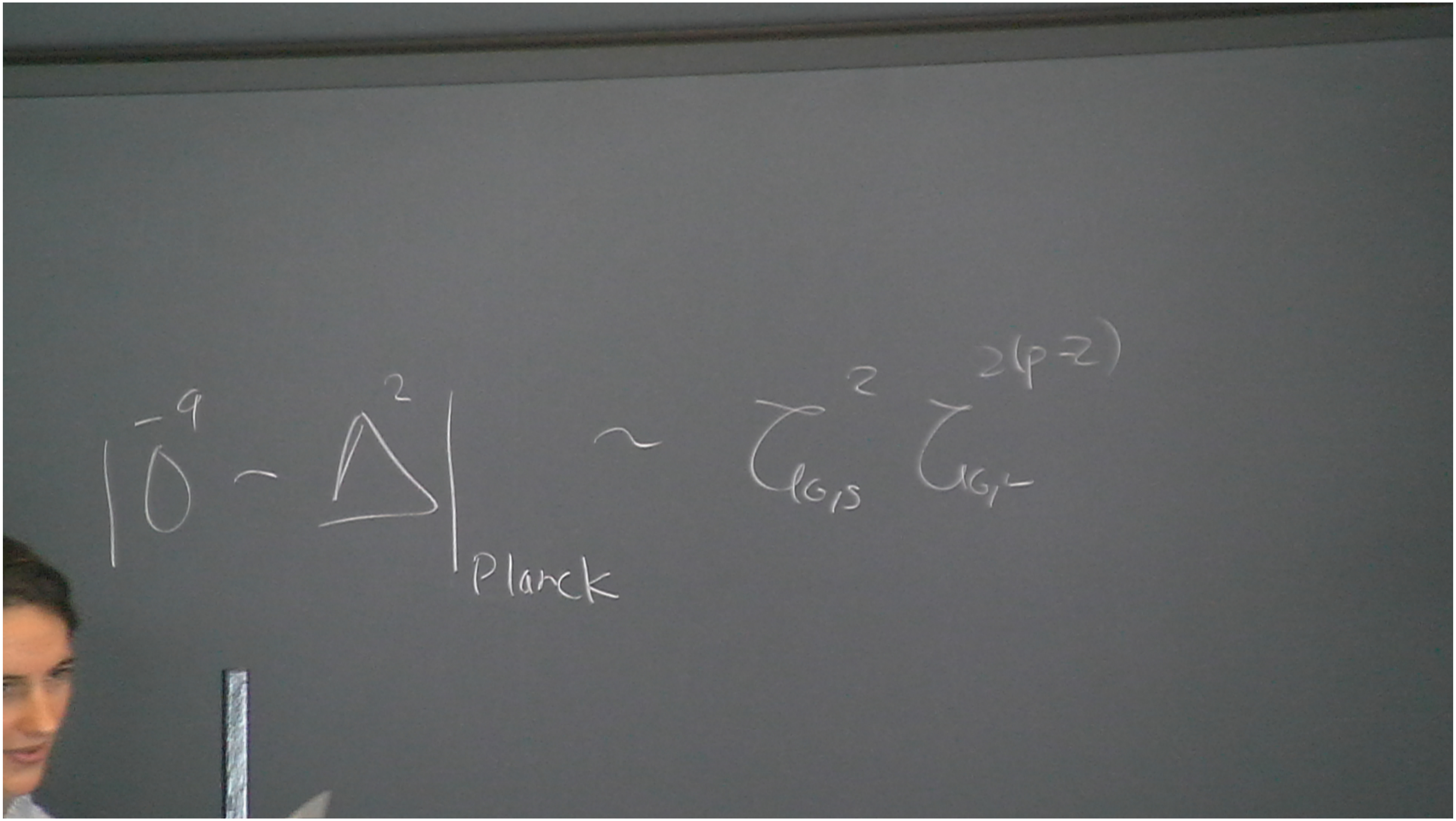


$\zeta_{G,L} / \sqrt{\zeta^2_{G,L}}$ = 1 (solid), 3 (dot-dashed), 5 (dotted)

32% 0.3% 0.00006%

Nelson & Shandera 2012; ML, Nelson, Shandera 2013




$10^{-9} \sim \Delta^2 \big|_{\text{Planck}} \sim \zeta_{G,s}^{2} \, \zeta_{G,-}^{2(p-2)}$

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

locally:

$$\zeta_s(x) = p\,\zeta_{G,s}(x)\,\zeta_{G,L}^{p-1} + \binom{p}{2}\zeta^2_{G,s}(x)\,\zeta_{G,L}^{p-2} + \binom{p}{3}\zeta^3_{G,s}(x)\,\zeta_{G,L}^{p-3} + \ldots$$

$$- \binom{p}{2}\langle \zeta^2_{G,s}(x)\rangle\,\zeta_{G,L}^{p-2} + \ldots$$

on average, all terms are equally important

BUT in Hubble patches where

$$\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$$

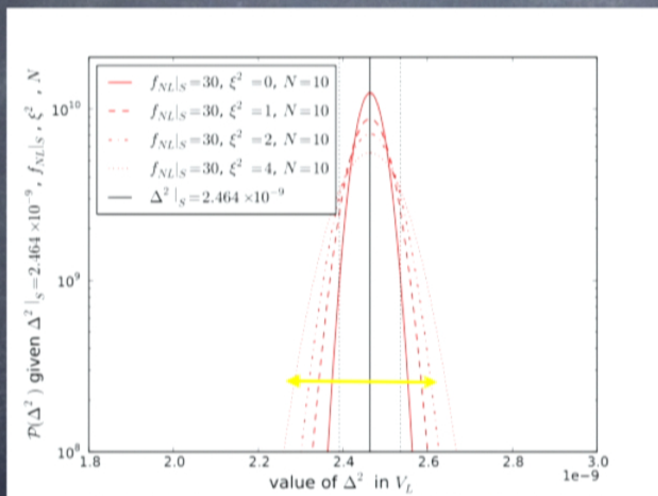
$$\zeta_s = \chi_G + f_{NL}\left(\chi_G^2 - \langle \chi_G^2 \rangle\right) + g_{NL}\left(\chi_G^3 - 3\chi_G \langle \chi_G^2 \rangle\right) \ldots$$

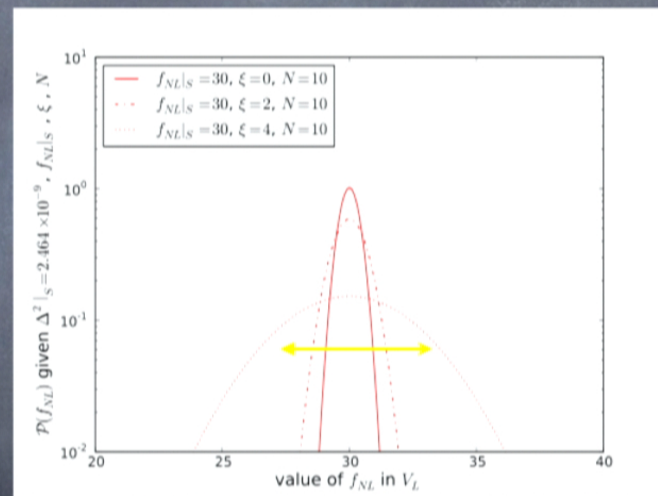
the statistics look only weakly non-Gaussian

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Multi-source weak NG I

for fixed f_{NL}, P_ζ typical modulation is larger



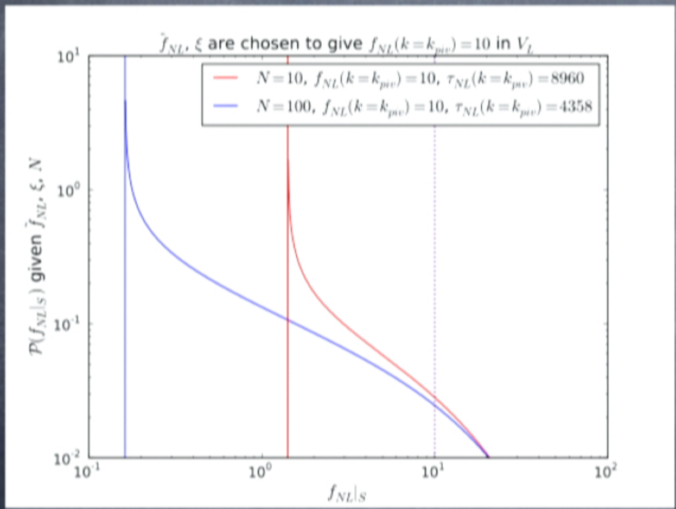


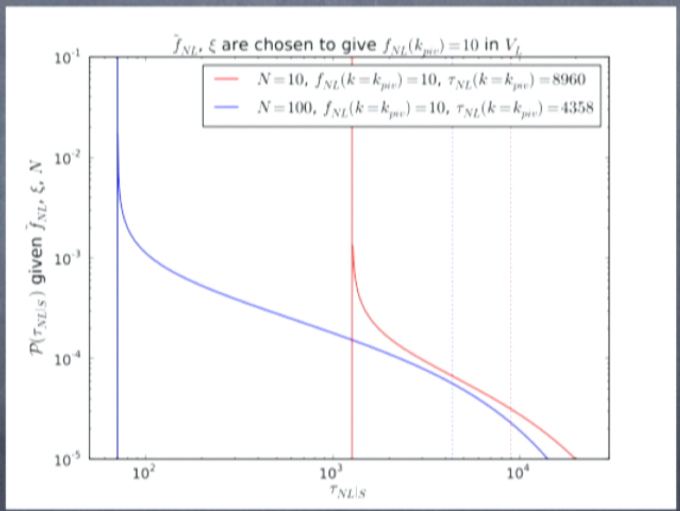
increasing the power coming from
inflaton (keeping f_{NL} fixed) broadens
distribution of observed f_{NL} values
in Hubble-size volumes

ML, Nelson, Shandera 2013

Multi-source weak NG II

Local parameters are modulated by $\sigma^2_{G,L}$, instead of $\sigma_{G,L}$ so probability distributions are highly skewed!



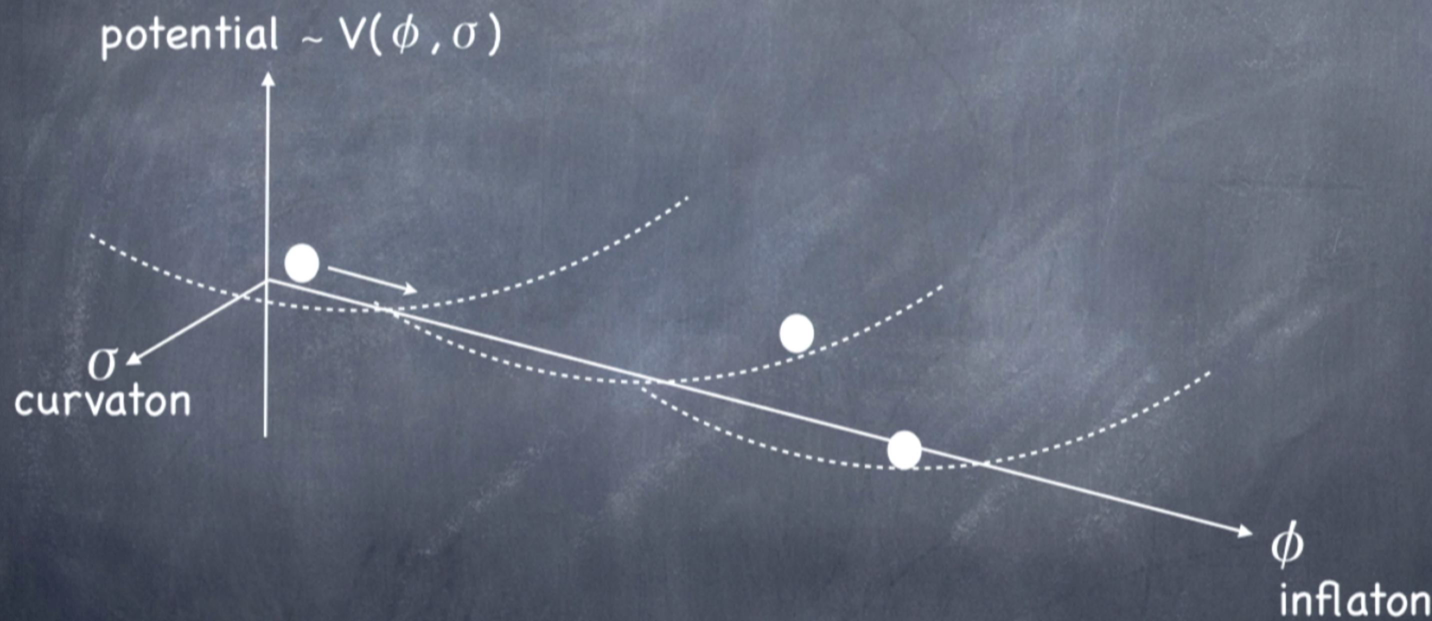


ML, Nelson, Shandera 2013

worked example: curvaton, no perturbations from inflaton

total energy dominated by inflaton: $H^2 = 8\pi G/3\ V(\phi)$

perturbations dominated by curvaton: $P_\Phi(k) \approx P_\sigma(k)$



Linde & Mukhanov 1997; Lyth and Wands 2002

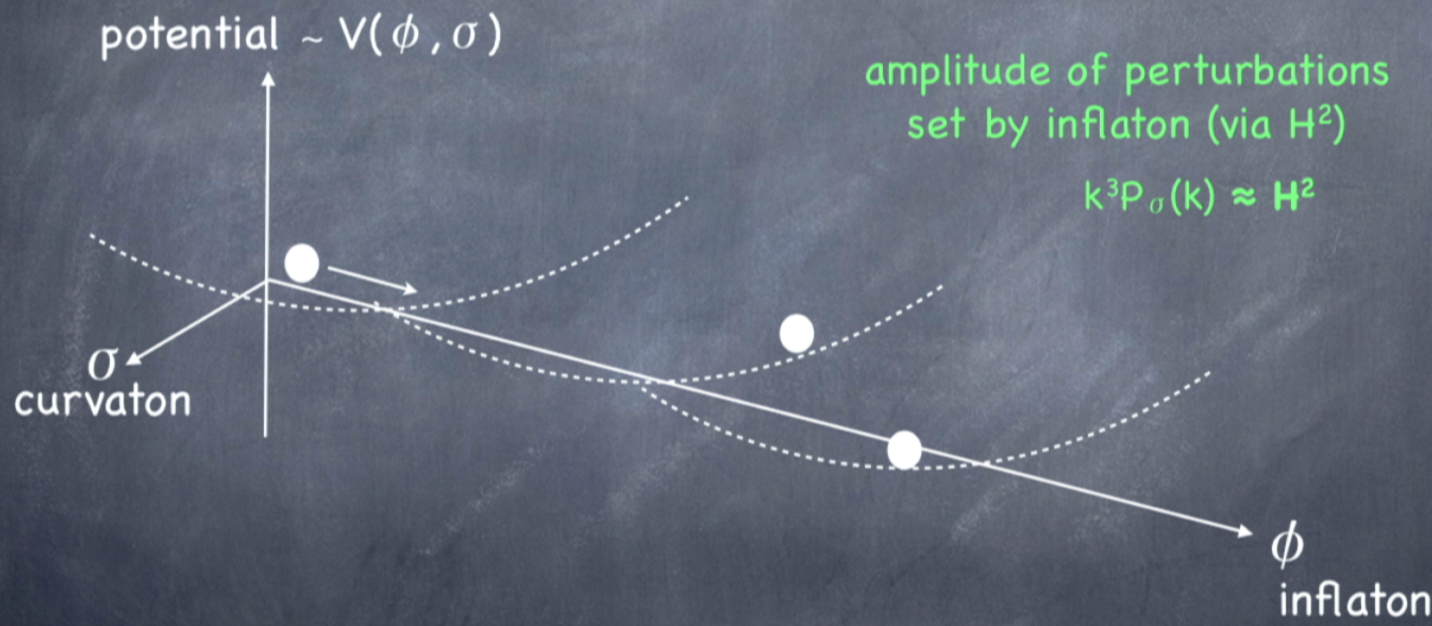
worked example: curvaton, no perturbations from inflaton

total energy dominated by inflaton: $H^2 = 8\pi G/3\ V(\phi)$

perturbations dominated by curvaton: $P_\Phi(k) \approx P_\sigma(k)$

amplitude of perturbations
set by inflaton (via H^2)

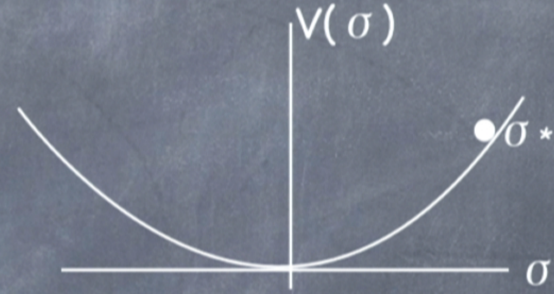
$k^3 P_\sigma(k) \approx \mathbf{H^2}$



Linde & Mukhanov 1997; Lyth and Wands 2002

worked example: curvaton, no perturbations from inflaton

inflation ends, curvaton stuck here at σ_*



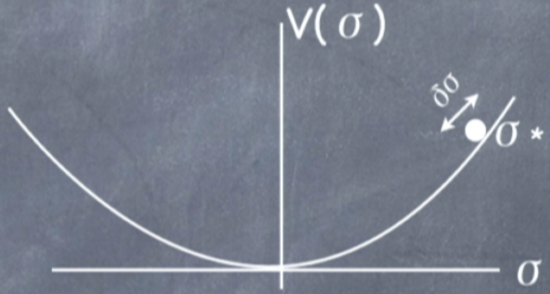
curvature perturbations generated when curvaton decays

Linde & Mukhanov 1997; Lyth and Wands 2002

worked example: curvaton, no perturbations from inflaton

curvaton only at curvaton decay, $V(\sigma) = m^2 \sigma^2$

curvature perturbations generated when curvaton decays



curvaton decays when the local Hubble value is equal to the decay rate: $H^2 = \Gamma^2$. Regions with different $\delta\sigma$ will have perturbations in expansion

$$\rho_\sigma(a)\left(1 + \frac{\delta\sigma_1}{\sigma_*}\right) e^{-3\zeta_1} = \rho_\sigma(a)\left(1 + \frac{\delta\sigma_2}{\sigma_*}\right) e^{-3\zeta_2}$$

Linde & Mukhanov 1997; Lyth and Wands 2002

worked example: curvaton, no perturbations from inflaton

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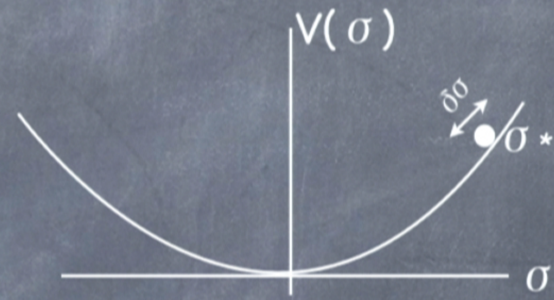

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Linde & Mukhanov 1997; Lyth and Wands 2002

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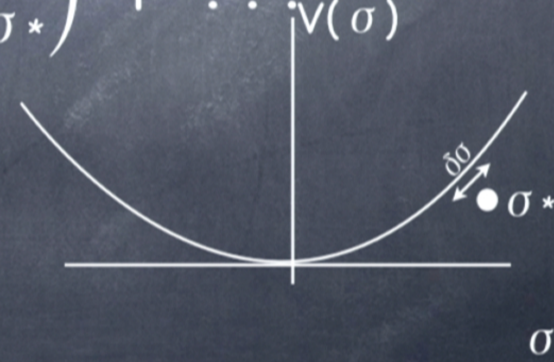
gives non-linear solution for ζ in terms of $\delta\sigma$

$$\zeta = \frac{2\delta\sigma}{3\sigma_*} - \frac{5}{4}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^2 + \frac{25}{12}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^3 + \cdots$$



Linde & Mukhanov 1997; Lyth and Wands 2002

ML

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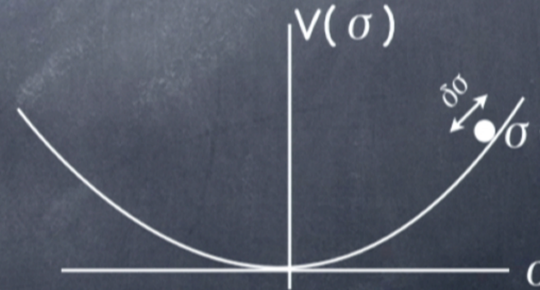
Local power spectrum in a patch with $\delta\sigma_L$?

just shifts σ_*

$$\Delta_\zeta^2 \rightarrow \Delta_\zeta^2 (1 - 3\zeta_L)$$

$$f_{NL} \rightarrow f_{NL}$$



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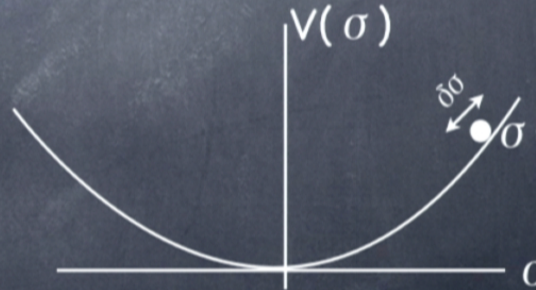
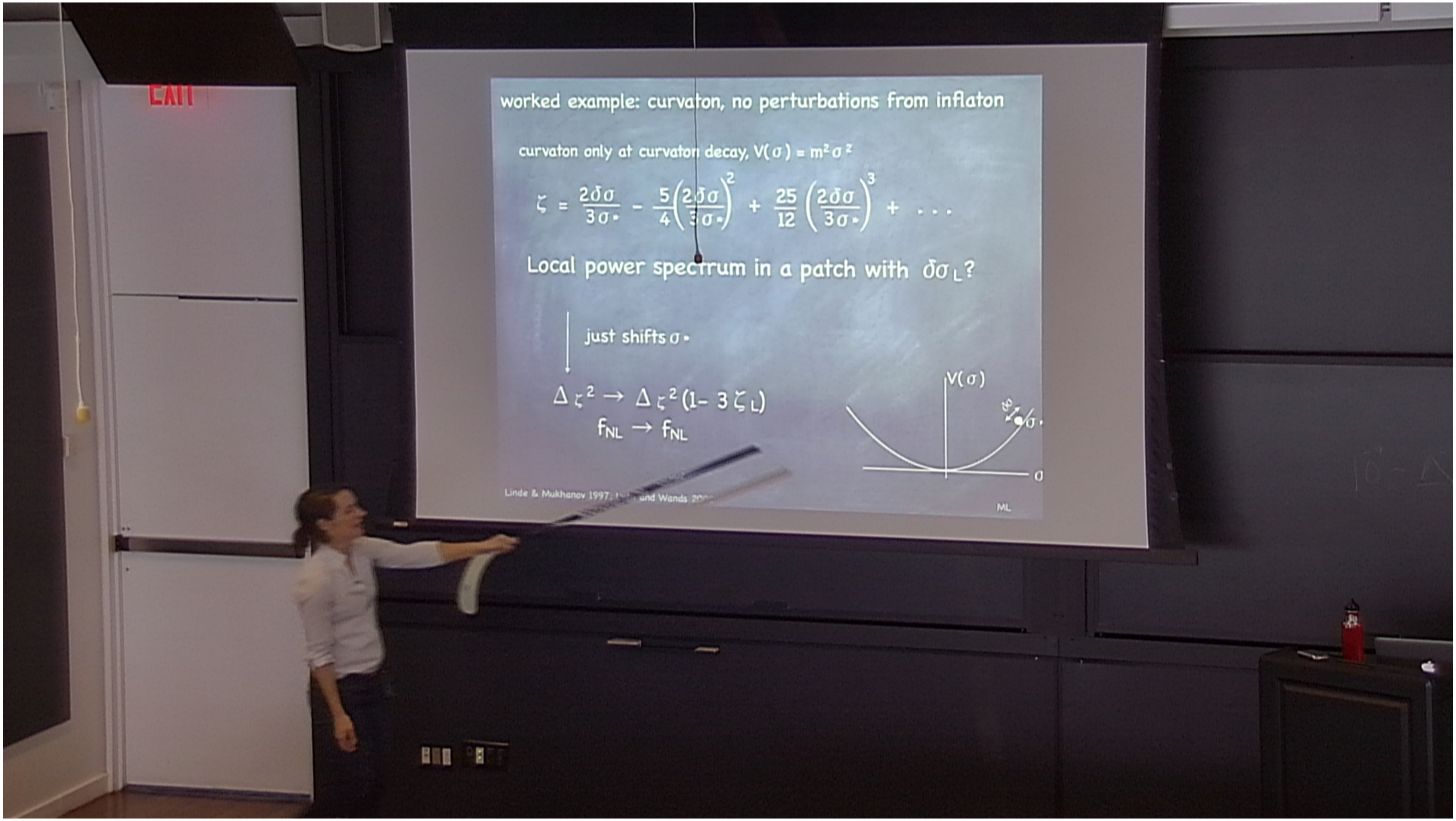


Linde & Mukhanov 1997; Lyth and Wands 2002

ML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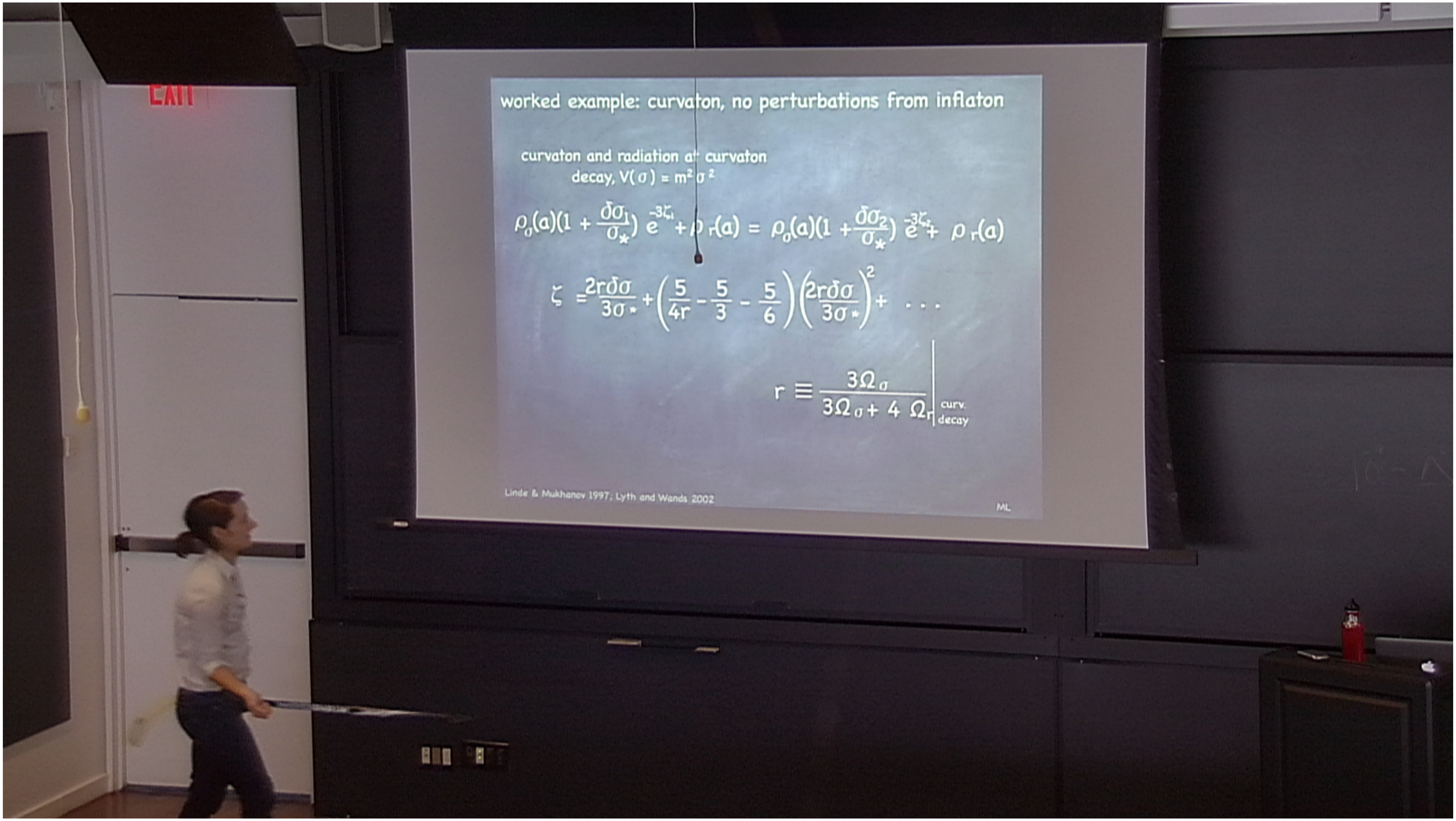
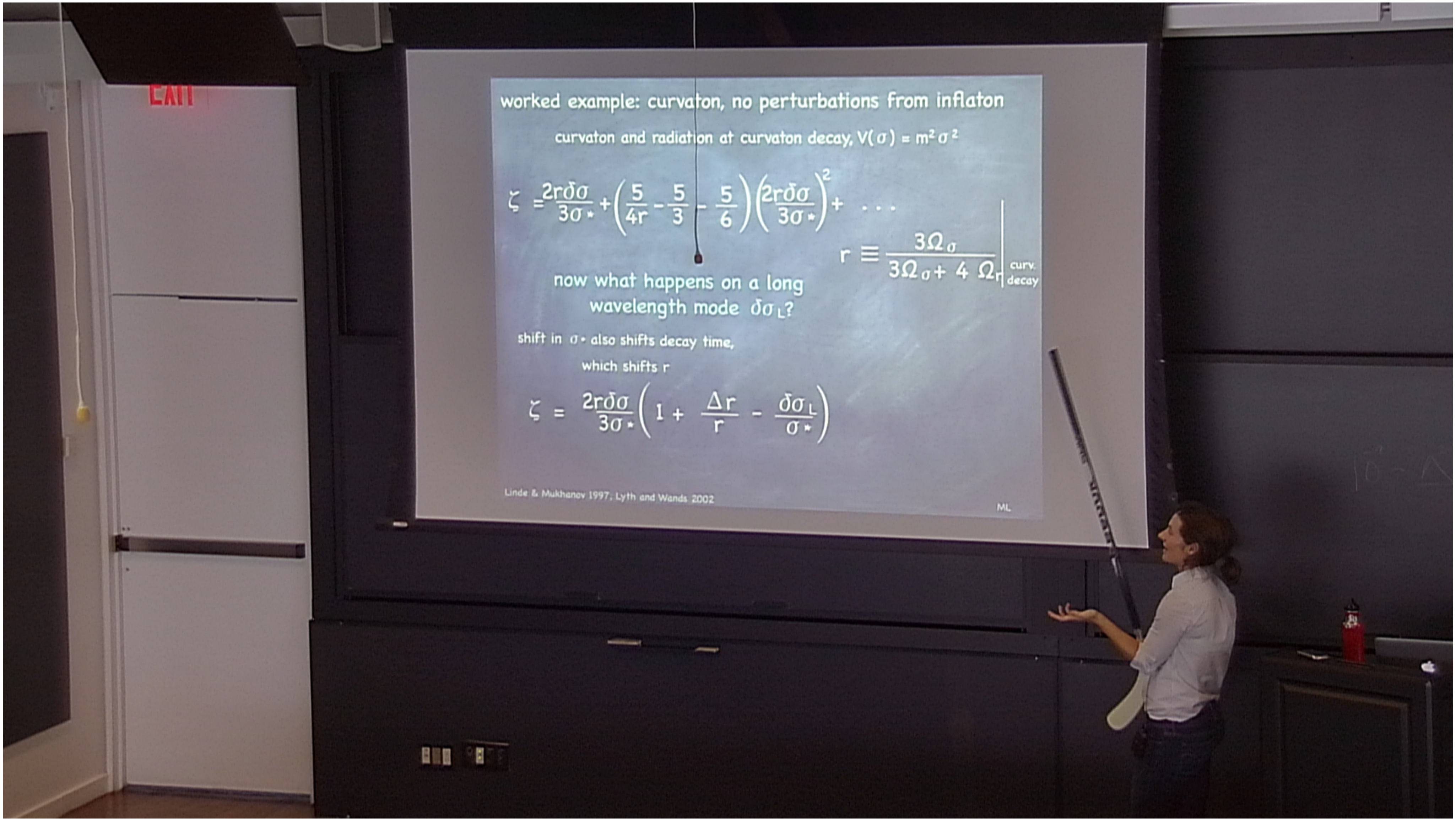
$f_{NL} \rightarrow f_{NL}$
$V(\sigma)$
σ_*
σ
ML


worked example: curvaton, no perturbations from inflaton
curvaton and radiation at curvaton decay, $V(\sigma) = m^2 \sigma^2$
$\rho_\sigma(a)(1 + \frac{\delta\sigma_1}{\sigma_*}) e^{-3\zeta_1} + \rho_r(a) = \rho_\sigma(a)(1 + \frac{\delta\sigma_2}{\sigma_*}) e^{-3\zeta_2} + \rho_r(a)$
$\zeta = \frac{2r\delta\sigma}{3\sigma_*} + \left(\frac{5}{4r} - \frac{5}{3} - \frac{5}{6}\right)\left(\frac{2r\delta\sigma}{3\sigma_*}\right)^2 + \ldots$
$r \equiv \left.\frac{3\Omega_\sigma}{3\Omega_\sigma + 4\,\Omega_r}\right|_{\text{curv. decay}}$
Linde & Mukhanov 1997; Lyth and Wands 2002
ML



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$r \equiv \frac{3\Omega_\sigma}{3\Omega_\sigma + 4\Omega_r}\Big|_{\text{curv. decay}}$
now what happens on a long wavelength mode $\delta\sigma_L$?
shift in σ_* also shifts decay time,
which shifts r
$\zeta = \frac{2r\delta\sigma}{3\sigma_*}\left(1 + \frac{\Delta r}{r} - \frac{\delta\sigma_L}{\sigma_*}\right)$
Linde & Mukhanov 1997; Lyth and Wands 2002
ML

worked example: curvaton, no perturbations from inflaton

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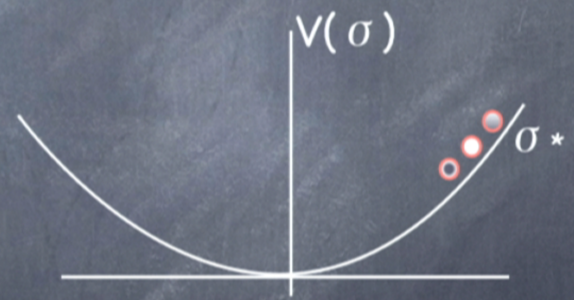
Linde & Mukhanov 1997; Lyth and Wands 2002

ML

worked example: curvaton

Interpretation: mode coupling calculation captures dependence of ζ on local background

in quadratic curvaton, "super cosmic variance" just reflecting range of σ_* values



ML

Summary

- If the curvature perturbation ζ has local non-Gaussianity (even at a relatively small level) the statistics observed in our Hubble volume may be a biased sample
- We explicitly computed local/global relationship in three simple examples that each give local statistics consistent with observations, even if globally, the statistics are very different and inconsistent with observations
- The local/global (mode-coupling) mappings agree with calculations by local observers with different backgrounds
- More examples:

Linde & Mukhanov 2006
Demozzi, Linde, Mukhanov 2011
Nelson & Shandera 2012
Nurmi, Byrnes, Tasinato 2013
Bramante, Kumar, Nelson, Shandera 2013


Reebok