Title: Fractional solitons on the edge of the Fractional Quantum Hall states.

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Abstract: We argue that dynamics of gapless Fractional Quantum Hall Edge states is essentially non-linear and that it features fractionally quantized solitons propagating along the edge. Observation of solitons would be a direct evidence of fractional charges. We show that the non-linear dynamics of the Laughlin's FQH state is governed by the quantum Benjamin-Ono equation.

Non-linear Theory of FQH Edge:

Fractionally Charged Solitons, Emergent Topology in Non-linear Waves, Quantum Hydrodynamics of FQH liquid

P. Wiegmann

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Perimeter

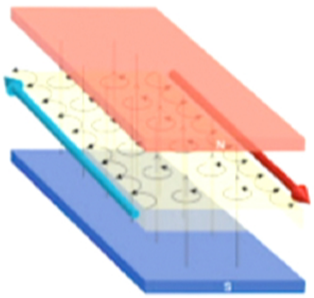
October 10, 2013

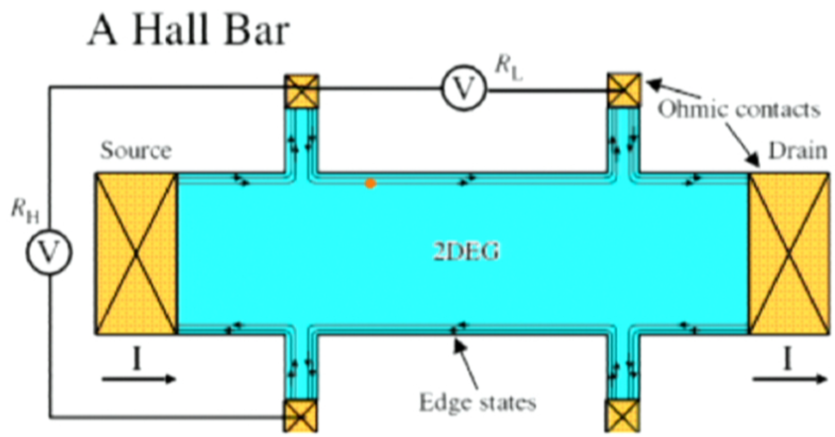
Messages

* Waves on the Edge of FQH are essentially non-linear;
* Emergence of quantization in non-linear dynamics;
* A universal corrections to Chern-Simon "theory"
* Relation between FQHE and CFT - revised

Only Laughlin's states (for now).

Fractional Quantum Hall Effect





2D electron gases placed in a transverse magnetic field: the longitudinal resistance vanishes, while the Hall conductance $\sigma_{xy} = \nu(e^2/h)$ is quantized to a rational multiple of e^2/h

SCALES

* Energies
  * Cyclotron energy - distance between Landau levels;

$$\hbar\omega_c = \frac{e\hbar B}{mc} \sim 30K$$

  * Coulomb interaction → a gap at fractional filling: number of electrons per flux quantum $\nu = 1/3$,

$$\Delta \sim \frac{e^2}{\ell} \sim 10K$$

* Length scale: $\ell = \sqrt{\frac{\hbar c}{eB}} \sim 10nm$
* Size of the device $\sim 10 - 100\mu m$
* Number of electrons $N \sim 10^6$

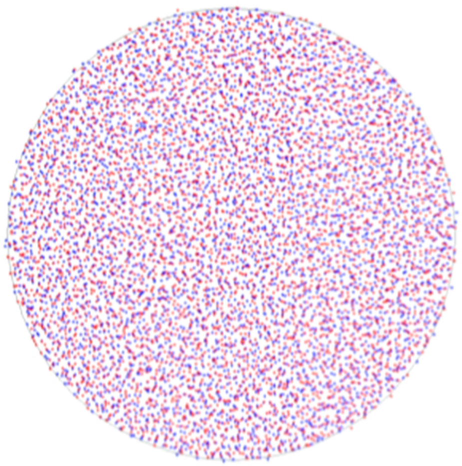
Fractional Quantum Hall States Exist only because

$$\hbar\omega_c \gg \Delta$$

Laughlin state(s)

All these features are encompassed by the Laughlin w.f.

$$\Psi_0(z_1,\ldots,z_N) = \left[\prod_{i\neq j}^{N}(z_i - z_j)\right]^{\beta} e^{-\sum_i |z_i|^2/4\ell_B^2}$$



- ℓ_B -magnetic length;
- $\nu = 1/\beta$ - is a filling fraction;
- $\beta = 1$ - IQHE; $\beta = 3$- FQHE.

Important features:

- Wave-function is holomorphic;
- Degree of zero at $z_i \to z_j$ is larger than 1;

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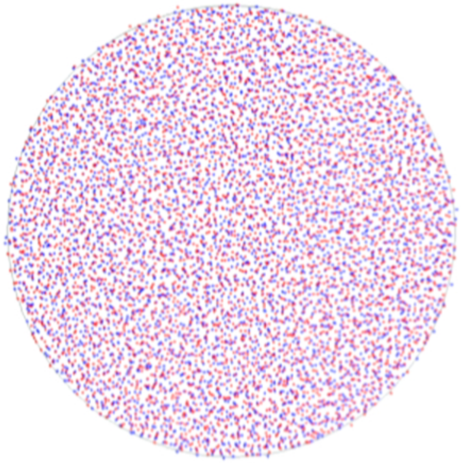

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Edge states: FQHE in a confining potential

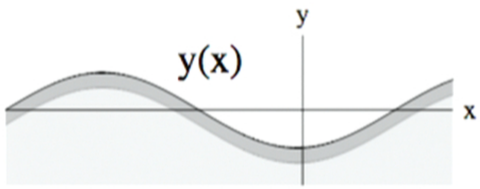
* Potential well lifts a degeneracy:

$$H_0 \to H = H_0 + \sum_i U(|r_i|)$$

* Low energy states emerge. They are localized on an edge → Edge States;

* Smooth potential: Curvature of the potential is small compared to the gap but a slope is larger than electric field

$$\ell_B^2 \nabla_y^2 U \ll \Delta_\nu, \quad \ell_B^2 \nabla_y U \gg e^2$$



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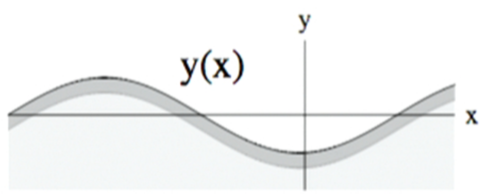

Linear Edge States theory (Wen, 1991)

* Density is a chiral field:

$$\rho(x) = \sum_k e^{ikx}\rho_k, \quad \rho_k = \rho^\dagger_{-k},$$

$$[\rho_k, \rho_l] = \nu k \delta_{k+l,0},$$

$$(\partial_t - c_0 \nabla_x)\rho = 0,$$

* $c_0 = \hbar^{-1}\ell_B^2|\nabla_y U|$ is a slope of the potential well (non-universal);

* factor ν proliferates to the exponent in edge tunneling

* Common believe (I disagree with):

  $c = 1$ - CFT of free bosons with a compactification radius $\nu = \beta^{-1}$.

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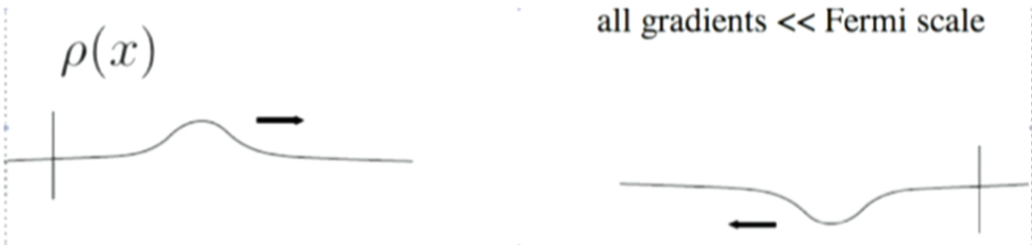
Propagation of a wave-packet



The linear theory does not answer a question:
how does a smooth non-equilibrium state (a wave packet) propagate?

$$\dot{\rho} - c_0 \nabla \rho = 0, \quad \text{wave equation,}$$
$$\rho(x,t) = \rho(x - c_0 t, t = 0)$$

The shape does not change !?

A new scale must be included in the theory;

New scale: $\Delta_{1/3} \ll \hbar\omega_c$ - energy of a hole or a vortex (non-universal scale);

Most phenomena do not depend on the scale and are universal;

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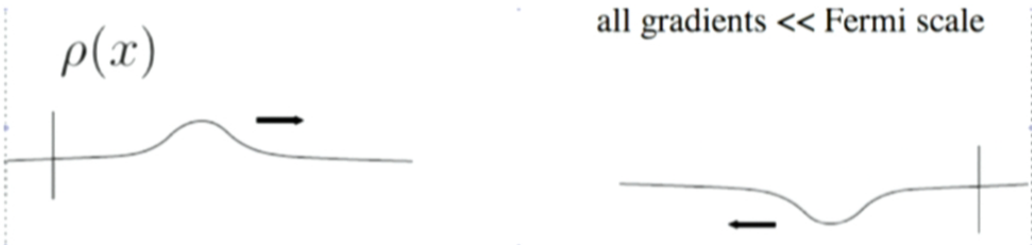

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* Riemann equation:

$$\dot{\rho} + \rho \nabla_x \rho = 0;$$

* Burgers equation:

$$\dot{\rho} + \rho \nabla_x \rho + \eta \nabla_x^2 \rho = 0;$$

* KdV equation:

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* Benjamin-Ono equation:

$$\dot{\rho} + \rho \nabla_x \rho + \eta \nabla_x^2 \rho_H = 0; \quad f_H(x) = \frac{1}{\pi} P.V. \int \frac{f(x')}{x - x'} dx'$$

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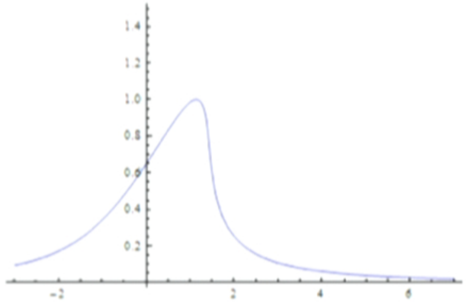
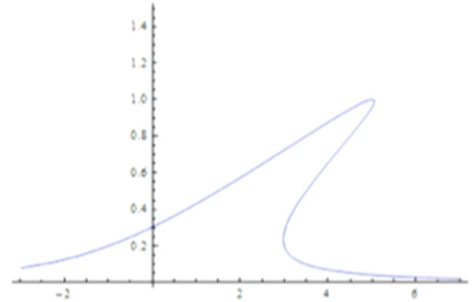
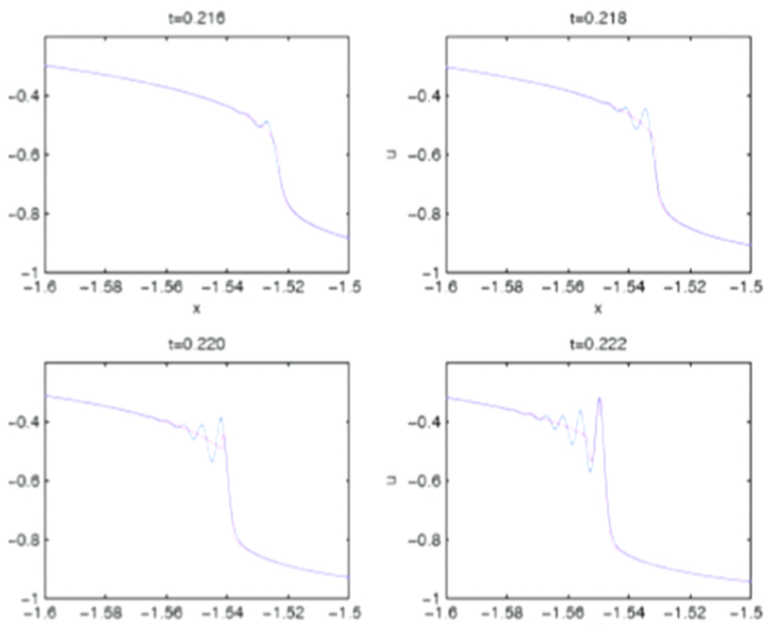


Small gradients (initially)



Infinitely Large gradients



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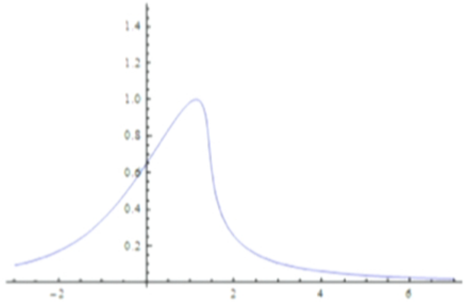

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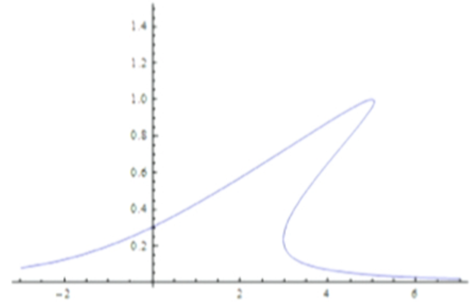

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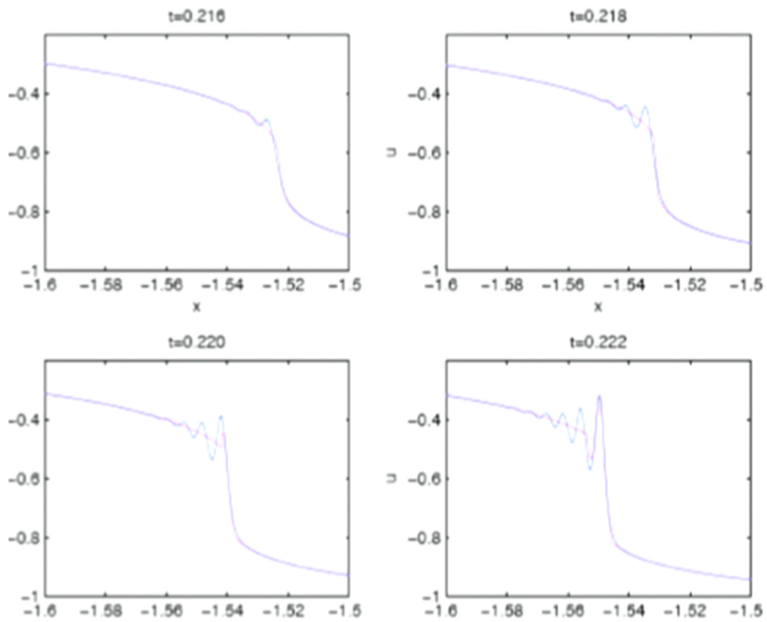

Non-linear theory of Edge States

* Linearized version (fails in 10ps):

$$(\partial_t - c_0\nabla_x)\rho = 0$$

* Universal description of non-linear chiral boson at FQHE edge

$$\boxed{(\partial_t - c_0\nabla_x)\rho - \kappa\nabla\left(\tfrac{1}{2}\rho^2 - \tfrac{1-\nu}{4\pi}\nabla\rho_H\right) = 0}$$

$$[\rho(x),\, \rho_H(x')] = \nu\nabla\delta(x-x'),$$

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* New scale: $\kappa \sim \Delta_\nu \ell_B^2/\hbar$ - energy of a quasi-hole less cyclotron energy (non-universal scale) but the form of equation is universal;
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* Classical Benjamin-Ono equation describes surface waves of interface of stratified fluids;
* Integrable (despite being non-local);

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* Two branches of solitons:
  * **subsonic**: *holes* propagating to the left;

    Charge $\nu = 1/\beta$: $\int \rho_h dx = \text{integer} \times \nu$

  * **ultrasonic**: *particles* propagating to the right:

    Charge 1: $\int \rho_p dx = \text{integer}$

$$\rho = \frac{q}{\pi}\frac{A}{(x - V_q t)^2 + A^2} \qquad q = 1,\ -\nu, \quad V_q = q\kappa A$$

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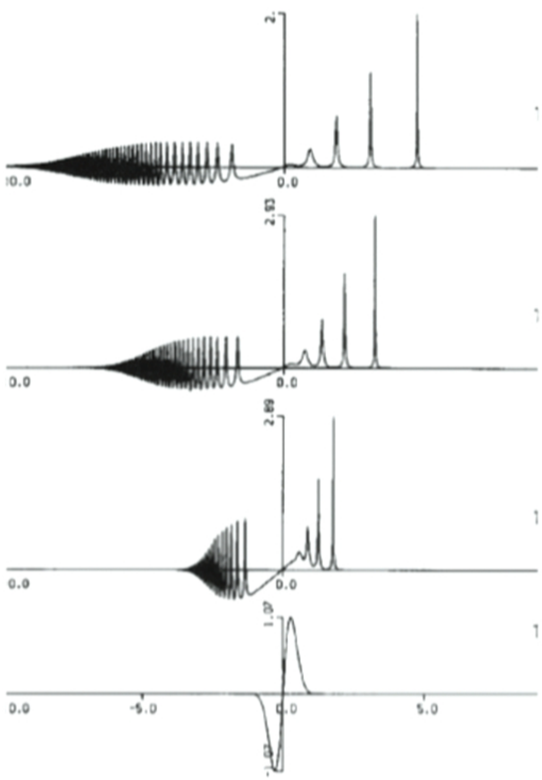
MORNING GLORY: QUANTIZED SOLITONS

Quantization through evolution

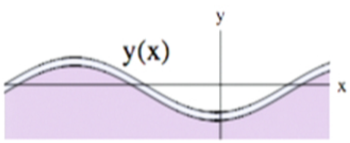


Kinematic Boundary Condition and the Chiral constraint

* *Kinematic Boundary Condition:*
  Fluid moves together with the boundary (no slipping)

$$\dot{y} + \mathrm{v}_x \nabla_x y + \mathrm{v}_y = 0$$

$$y(x) = \int \rho(x,y)dx, \quad \text{"height" of the boundary}$$



* *Chiral Constraint:*
  On the first Landau level position of particles determine velocity of particles.
  Density of the fluid ρ determines velocity v of the fluid:

$$\rho \Rightarrow \mathrm{v}$$

Hydrodynamic description of FQHE

* Liquid is incompressible

$$\nabla \cdot \mathbf{v} = 0$$

* Chiral constraint: Relation between velocity and density: Simplified version (adopted for presentations)

$$\boxed{\text{vorticity} \times \text{fraction} = \text{electronic density}}$$

* Less simplified (Zabrodin and P.W., 2006)

$$\boxed{\nu(\nabla \times \mathbf{v}) = \rho - \rho_I + \frac{1-\nu}{4\pi} \Delta \log \rho}$$

* Sum rules:

$$\boxed{\eta = \int y(\rho - \rho_I) dy = \frac{1-\nu}{4\pi}}$$

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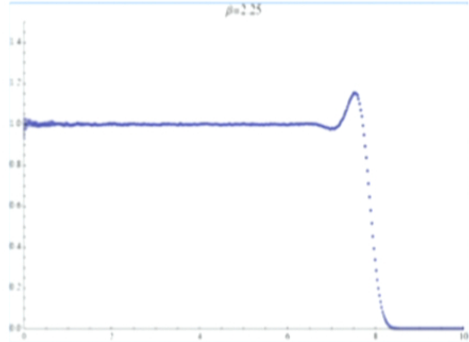
Overshoot: dipole moment of FQHE droplet



$$\Psi_0 = \left[\prod_{i>j}(z_i - z_j)\right]^{\beta} e^{-\frac{1}{2}\sum_i |z_i|^2/4\ell_B^2},$$

$$\mathbf{d} = \int_0^R (r-R)\rho(r)dr = \frac{1-2\nu}{8\pi}$$

Dipole moment of a spherical droplet

$$\langle\rho\rangle = \lim_{N\to\infty}\int |z - z_i|^{2\beta}|\Delta|^{2\beta} e^{-\sum_i |z_i|^2/2\ell_B^2}\prod_i d^2 z_i$$

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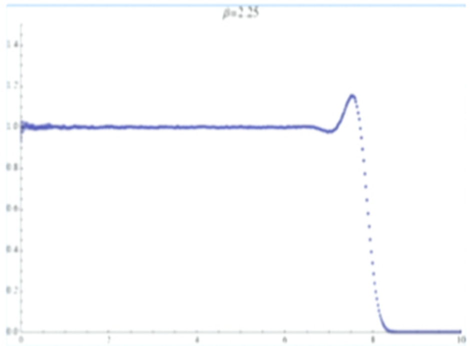

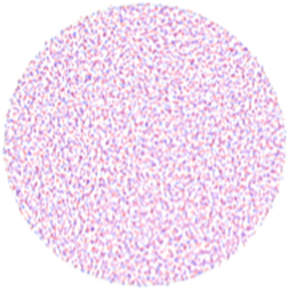

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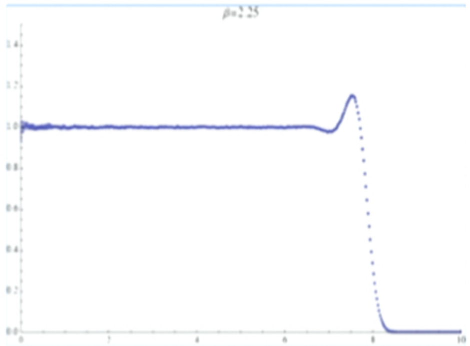

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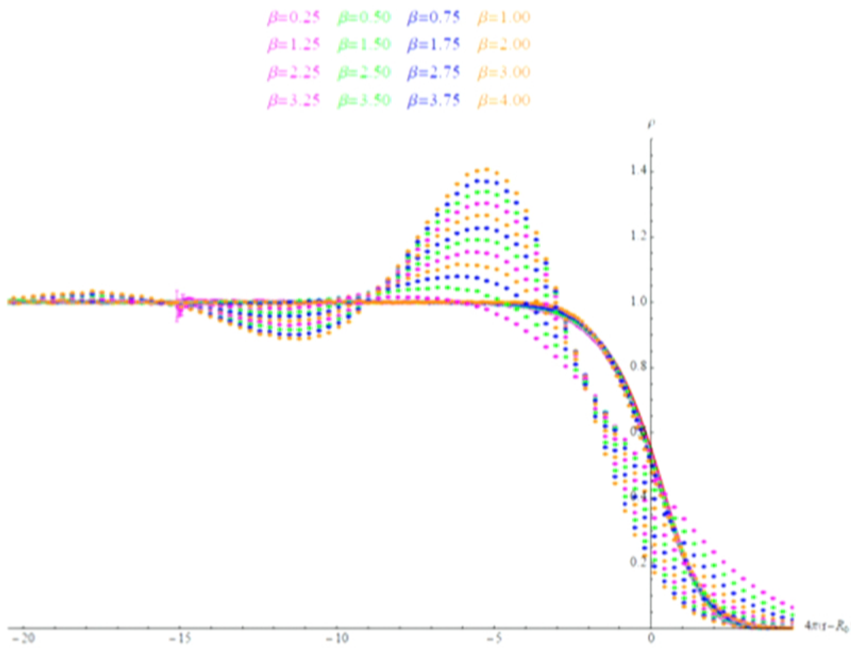
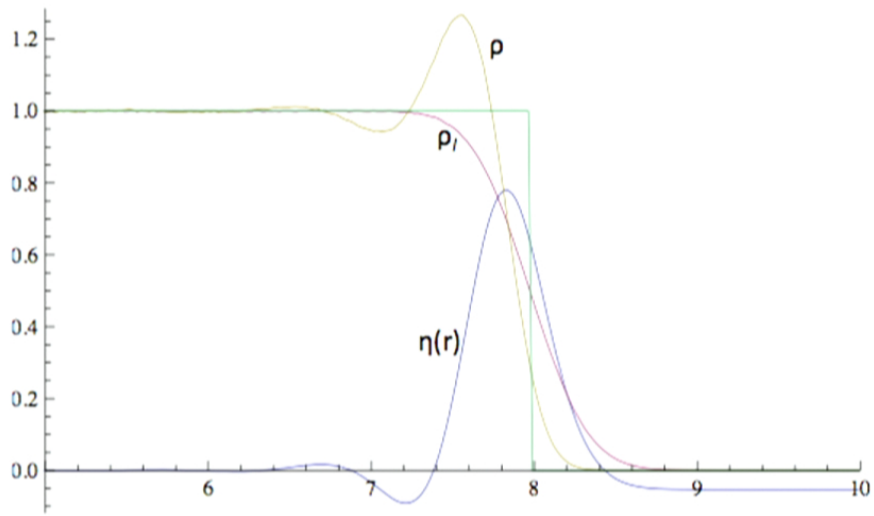
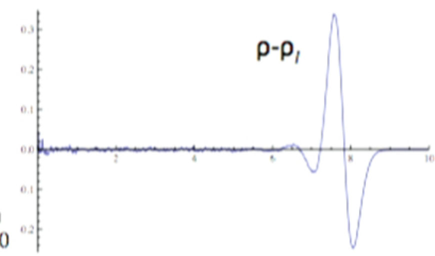


No overshoot at $\beta = \frac{1}{\nu} \leq 1$.





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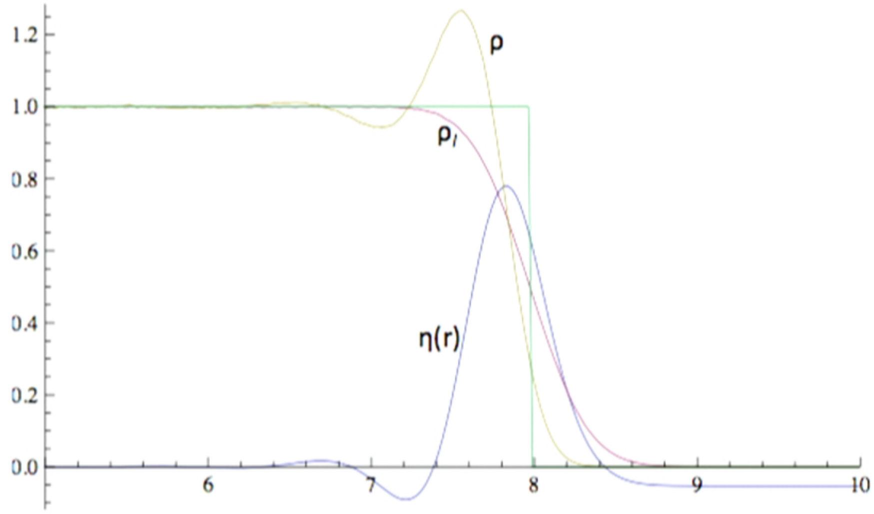

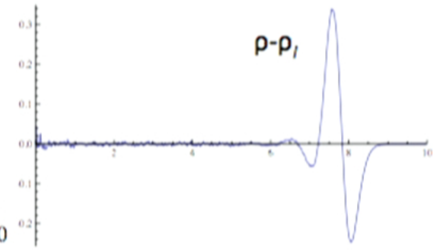

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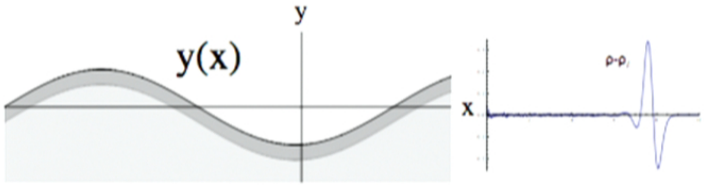
Boundary Waves

FIGURE: Boundary waves: the boundary layer is highlighted



$$\nabla \times \mathbf{v}) = \rho - \rho_I + \frac{1-\nu}{4\pi}\Delta \log \rho$$

Boundary value of velocities

$$(\mathbf{v}_x - c_0) = -\nu^{-1}\bar{\rho}y(x), \quad \mathbf{v}_y = \frac{1-\nu}{4\pi\nu}y^H_{xx}$$

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leads to Quantum Benjamin Ono Equation.

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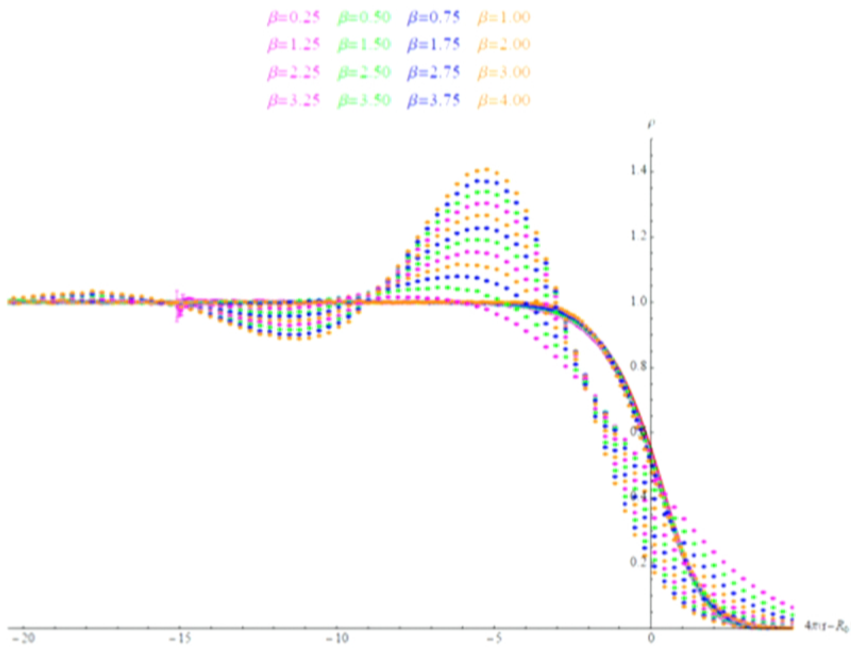
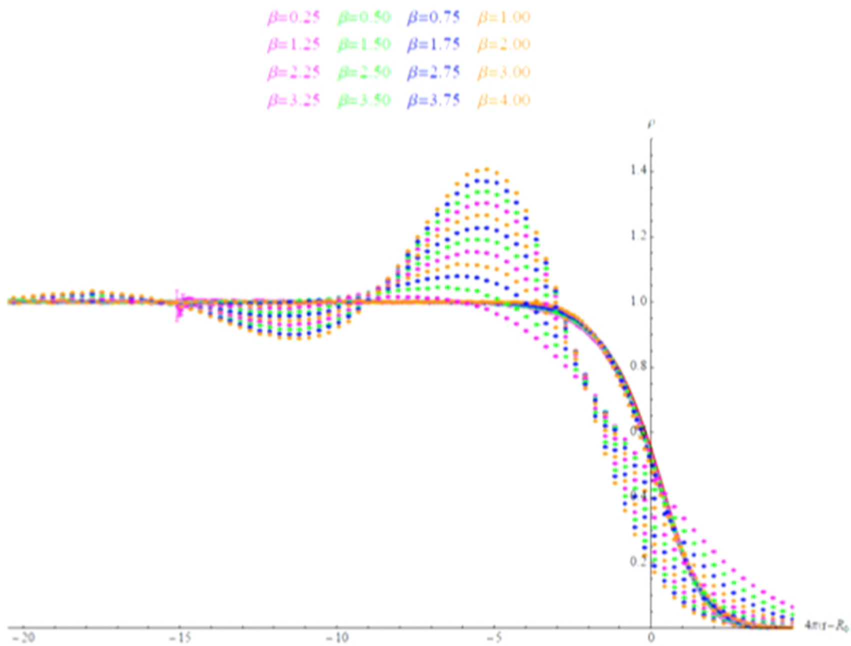
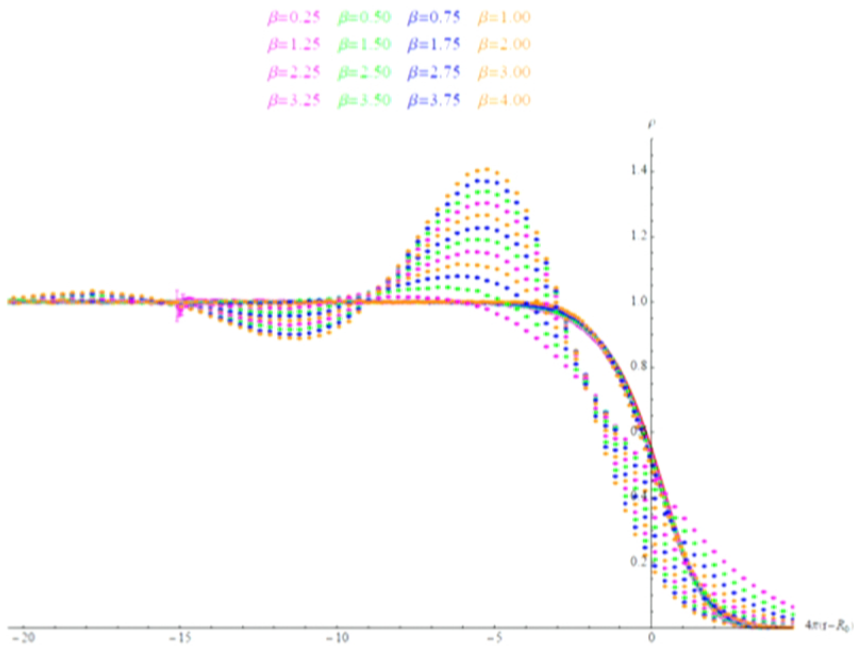


No overshoot at $\beta = \frac{1}{\nu} \leq 1$.



No overshoot at $\beta = \frac{1}{\nu} \leq 1$.



No overshoot at $\beta = \frac{1}{\nu} \leq 1$.