Title: Conformal symmetry and cosmology

Date: Oct 22, 2013 11:00 AM

URL: http://pirsa.org/13100070

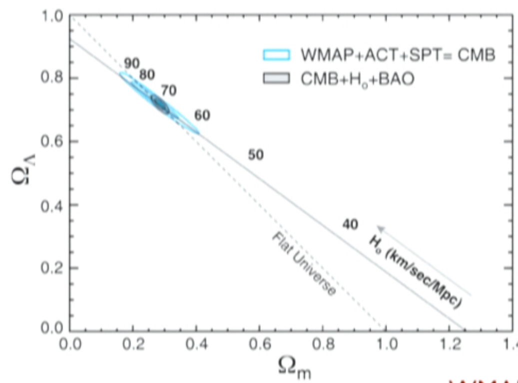
Abstract: We
will explore the role that conformal symmetries may play in cosmology. First,
we will discuss the symmetries underlying the statistics of the
primordial perturbations which seeded the temperature anisotropies of the
Cosmic Microwave Background. I will show how symmetry considerations lead us to
three broad classes of theories to explain these perturbations: single-field
inflation, multi-field inflation, and the conformal mechanism. We will discuss
the symmetries in each case and derive their model-independent consequences.
Finally, we will examine the possibility of violating the null energy condition
with a well-behaved quantum field theory.

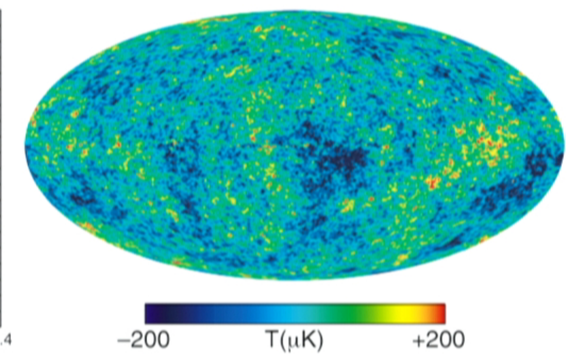
Cosmological puzzles

On the largest scales, we observe the early universe to be

- Flat
- Homogeneous
- Isotropic

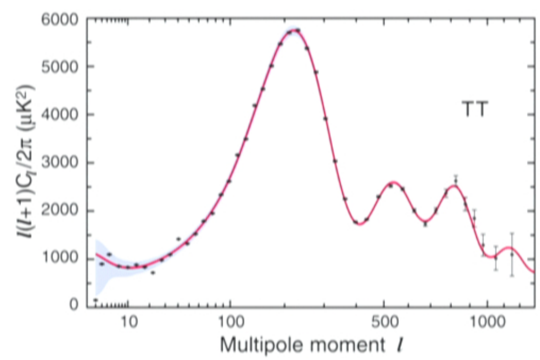
even in apparently causally disconnected regions!





WMAP9 1212.5225

Further, we observe a spectrum of perturbations which imply nearly scale invariant and Gaussian initial conditions



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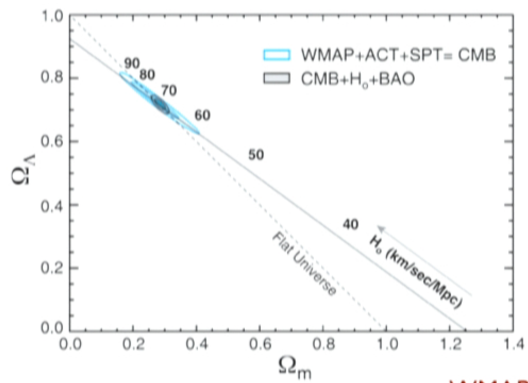

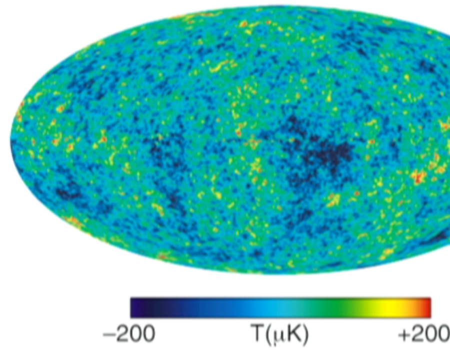

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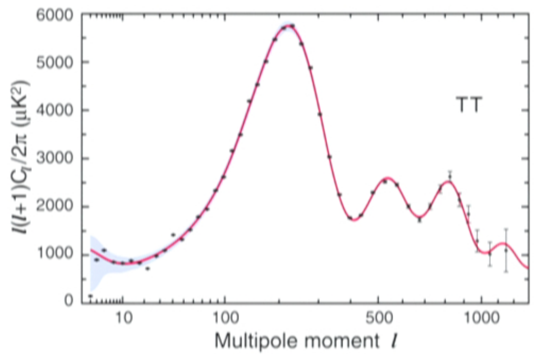

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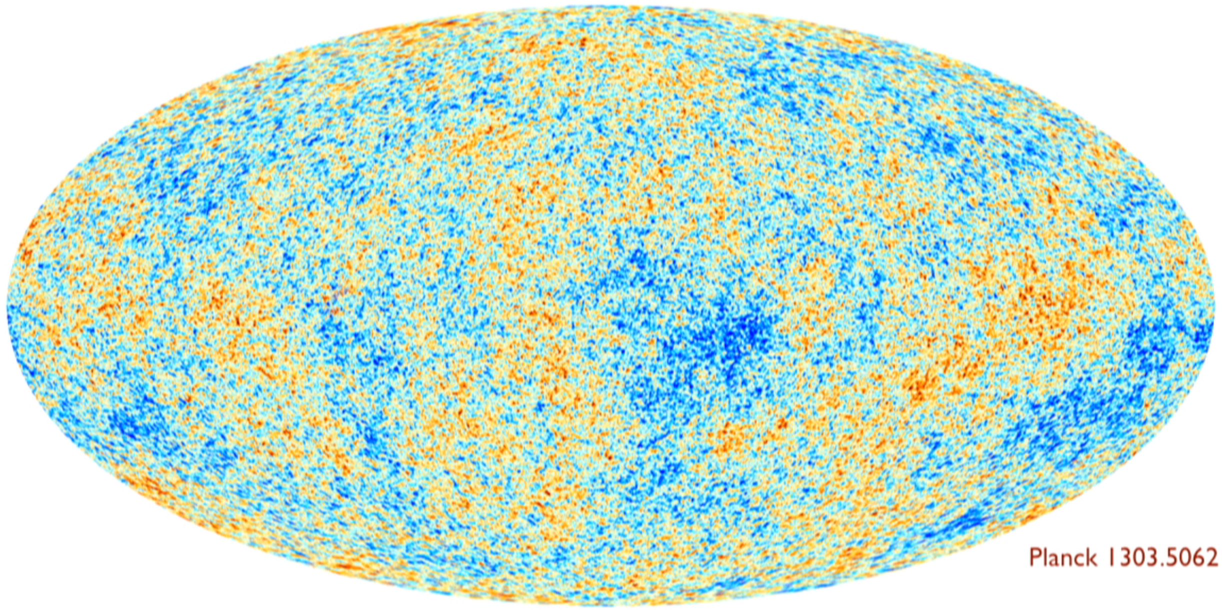

Planck 1303.5062

We decompose the temperature fluctuations in spherical harmonics

$$\langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle = \sum_\ell C_\ell \left(\frac{2\pi + 1}{4\pi} \right) P_\ell(\hat{n} \cdot \hat{n}')$$

Measurement of the C_ℓ tells us about the primordial perturbations which seeded the CMB

Primordial inhomogeneities

Temperature fluctuations are related to primordial fluctuations through

$$C_\ell \sim \int \frac{\mathrm{d}k}{k} k^3 P_\zeta(k) \Delta_\ell^2(k)$$

CMB measurements indicate that the primordial perturbations are

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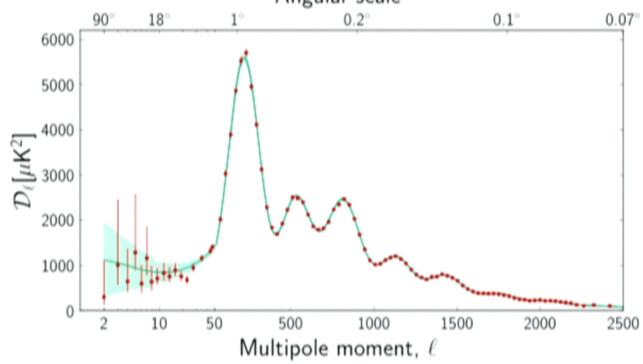

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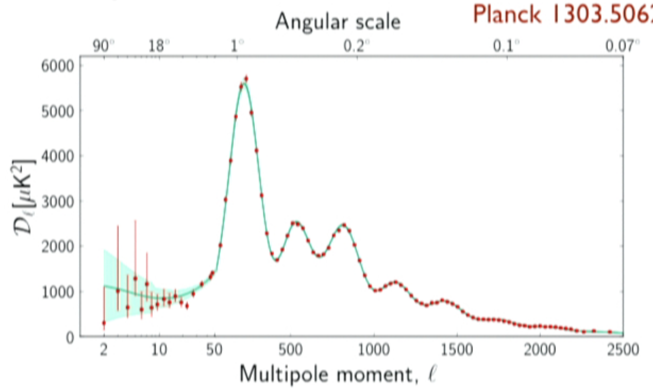

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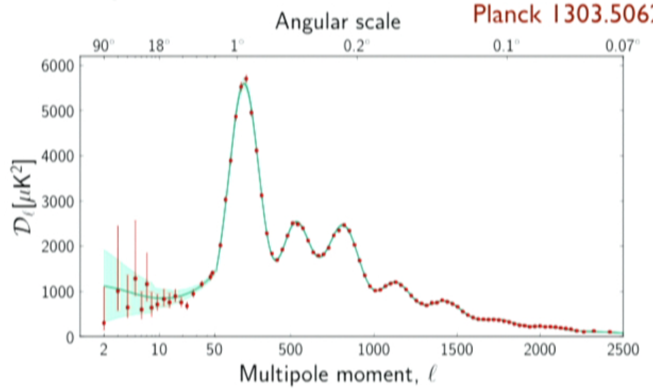

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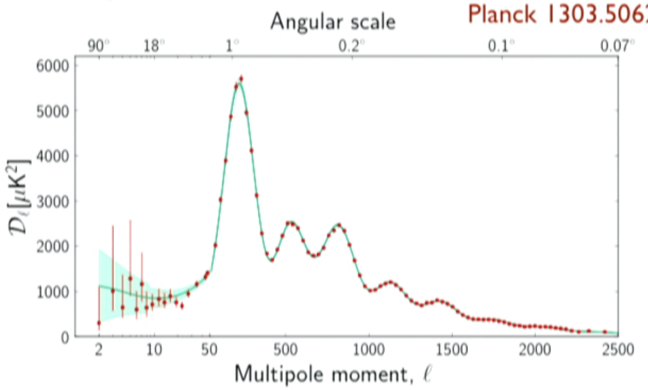

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$$k^3 P_\zeta(k) \sim k^{2(n_s-1)}$$

$$n_s = 0.960 \pm 0.0073$$

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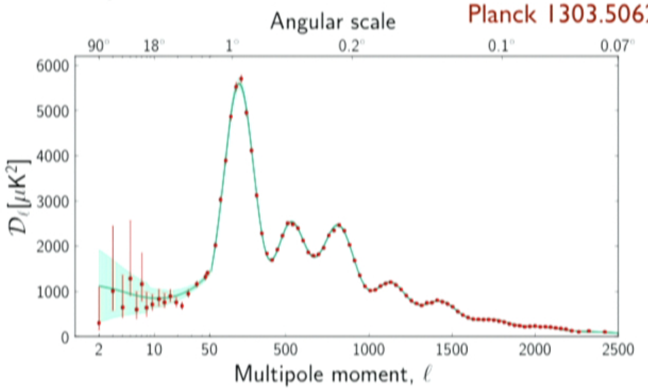

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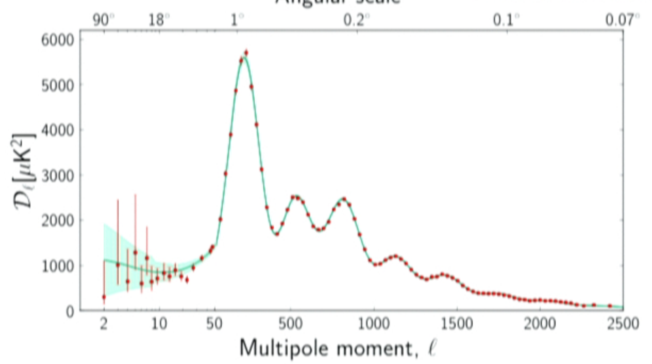

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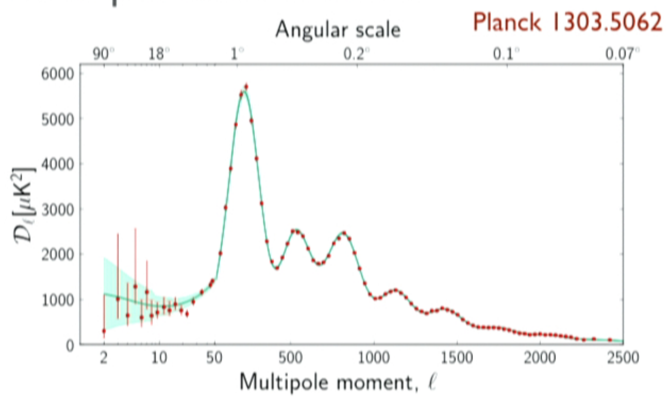

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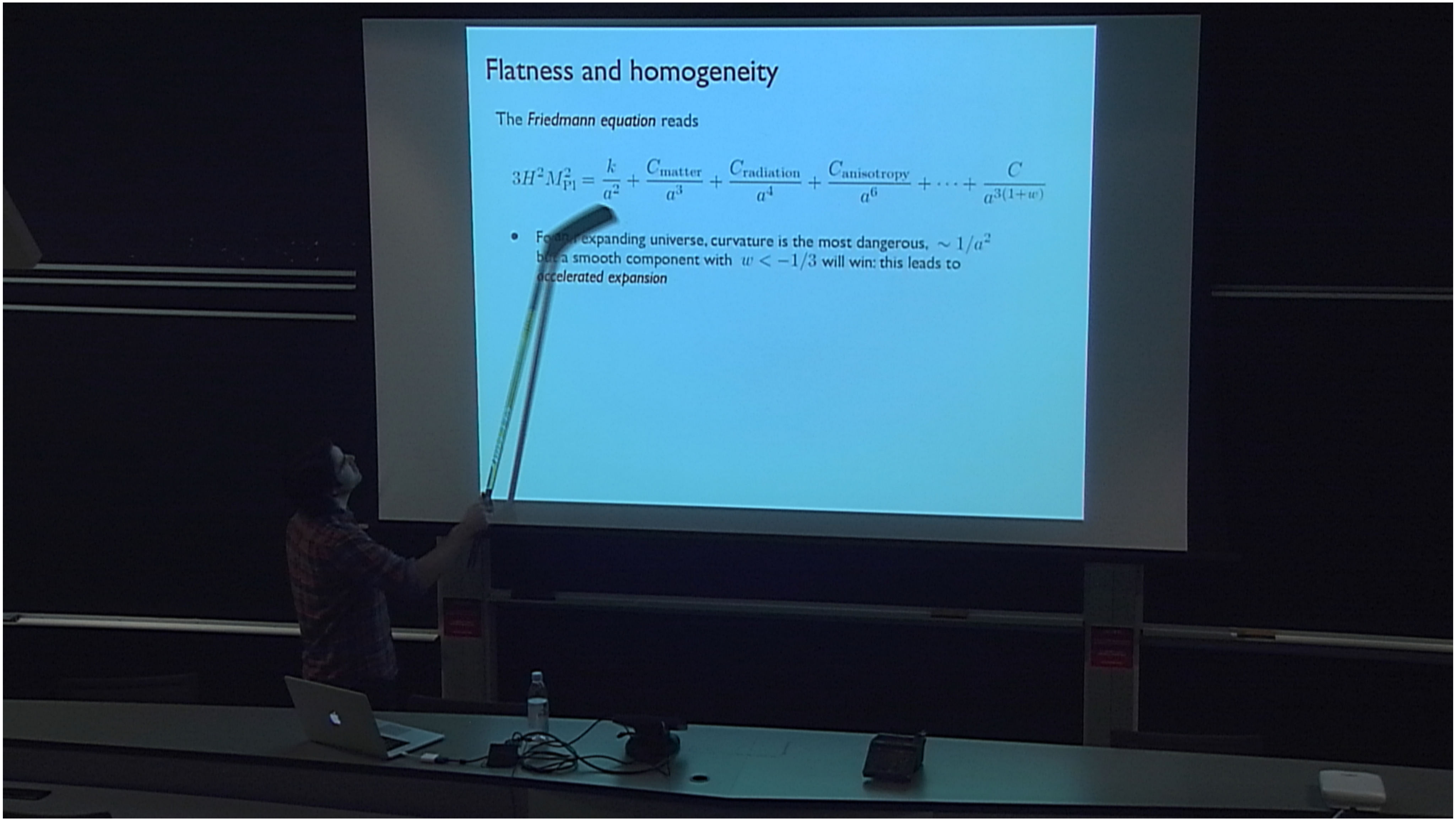
Flatness and homogeneity

The *Friedmann equation* reads

$$3H^2 M_{\rm Pl}^2 = \frac{k}{a^2} + \frac{C_{\rm matter}}{a^3} + \frac{C_{\rm radiation}}{a^4} + \frac{C_{\rm anisotropy}}{a^6} + \cdots + \frac{C}{a^{3(1+w)}}$$

- For an expanding universe, curvature is the most dangerous, $\sim 1/a^2$ but a smooth component with $w < -1/3$ will win: this leads to *accelerated expansion*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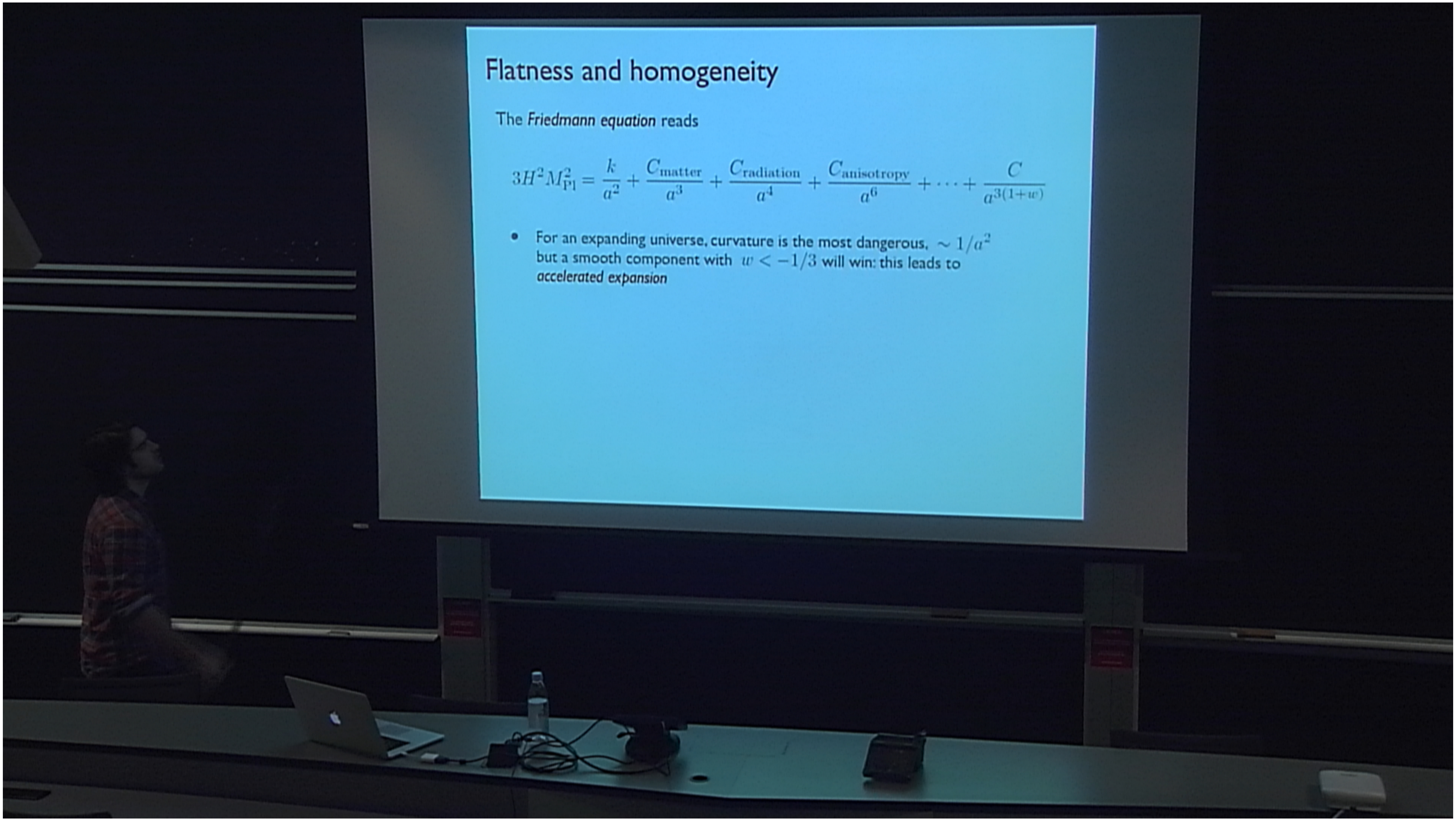
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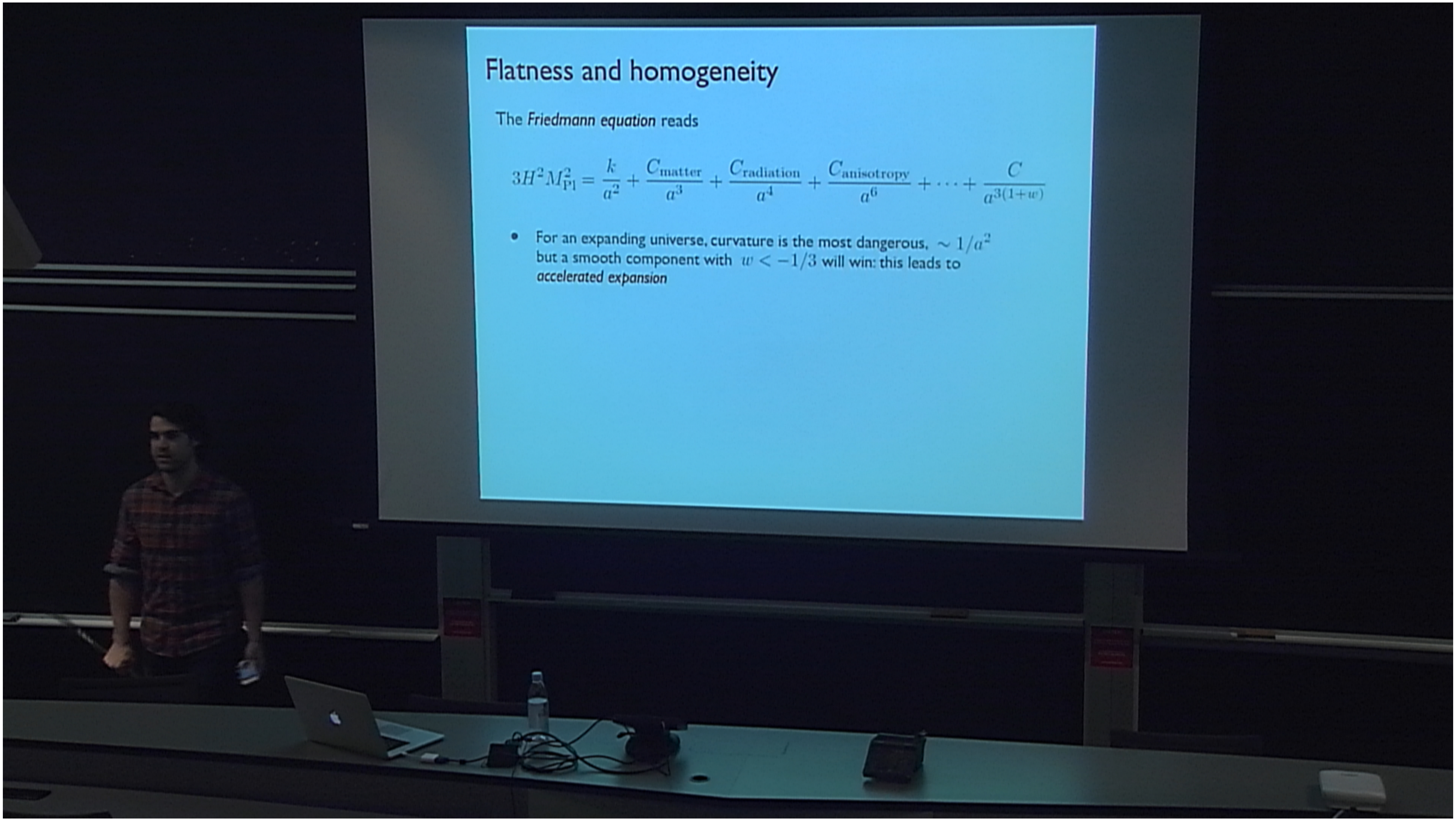
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  Gratton, Khoury, Steinhardt, Turok astro-ph/0301395

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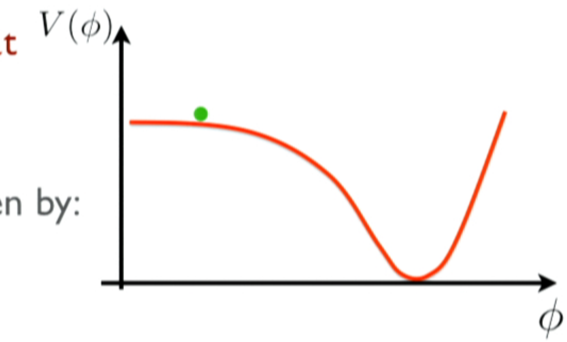
Inflation

Inflation explains the large-scale universe by postulating a phase of de Sitter expansion:

$$ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2$$

- This is driven by a component with nearly constant energy density

Concrete example: a scalar field rolling down a nearly flat potential



- The *equation of state parameter* $P = w\rho$ is given by:

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \sim -1$$

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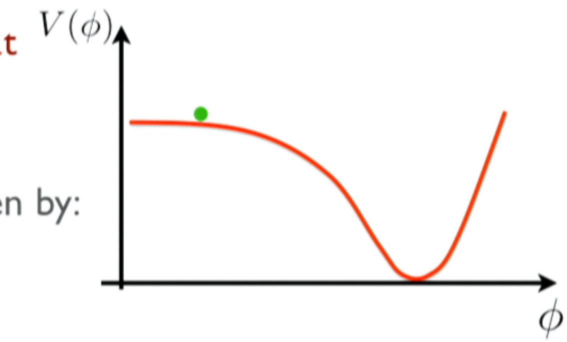

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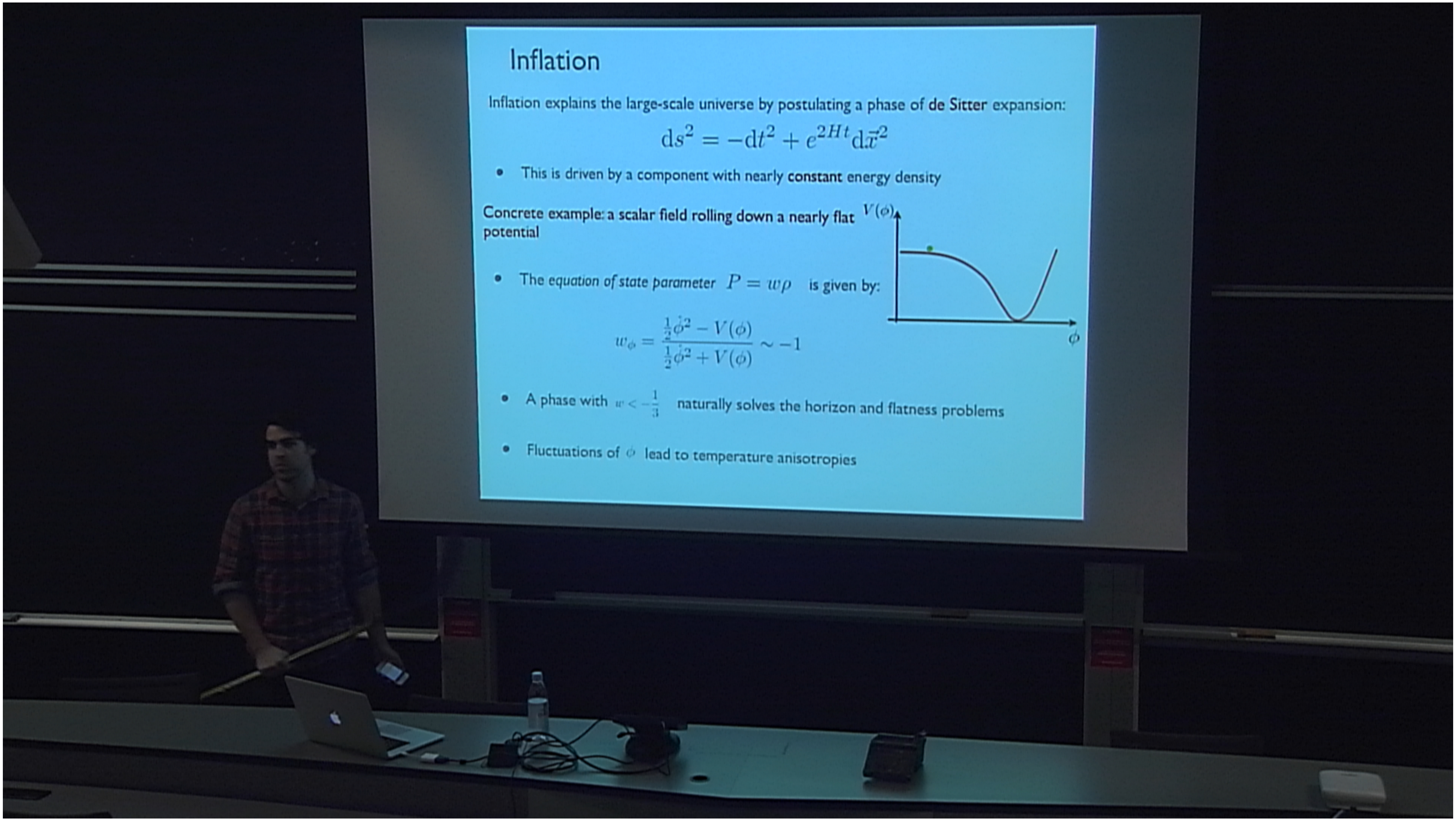
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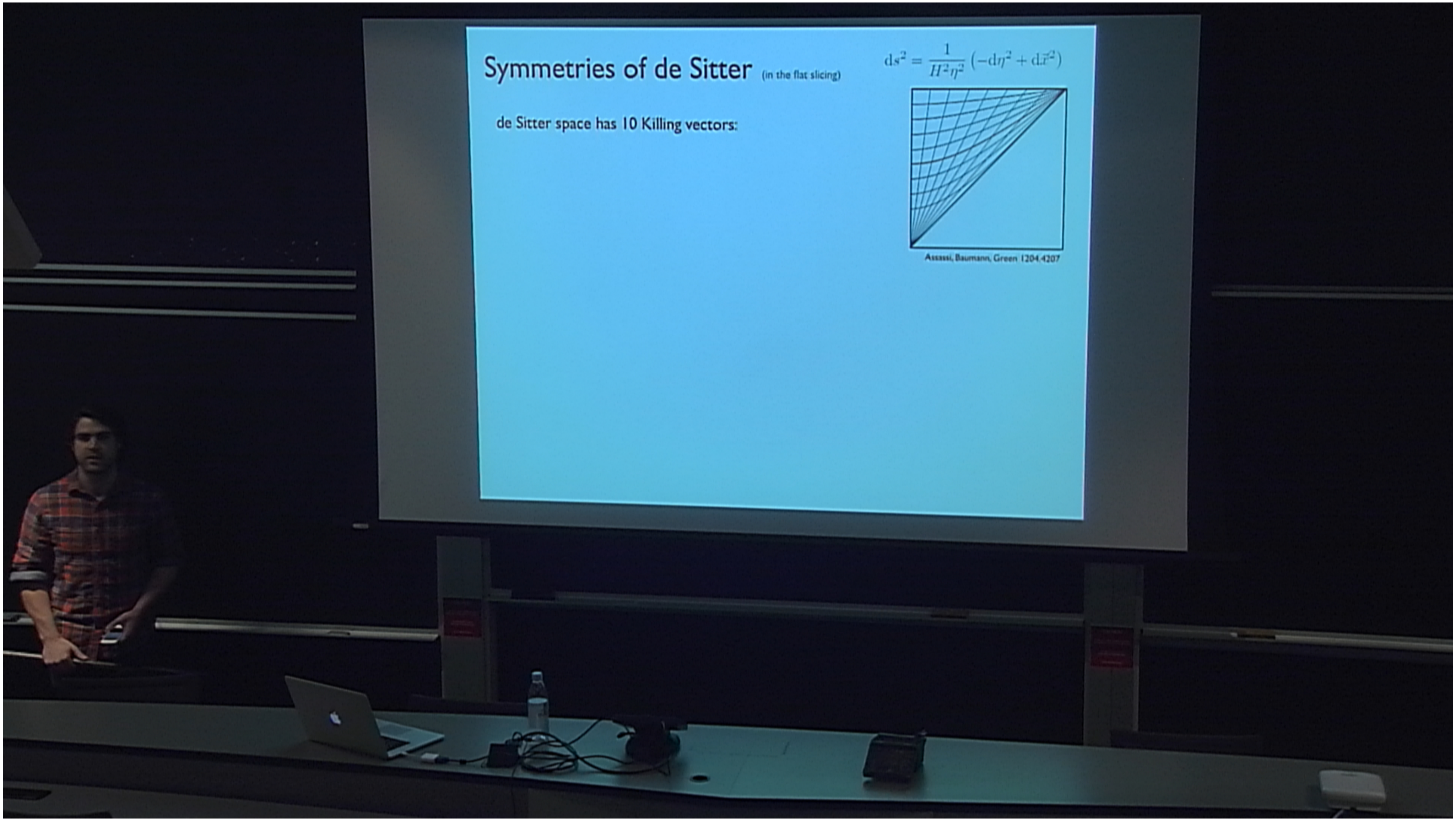
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Assassi, Baumann, Green 1204.4207

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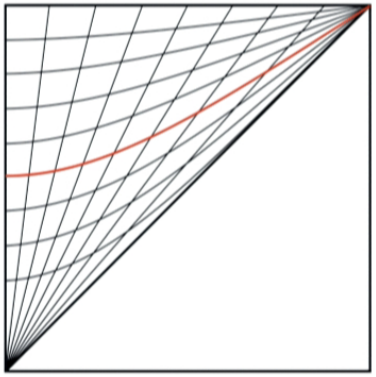

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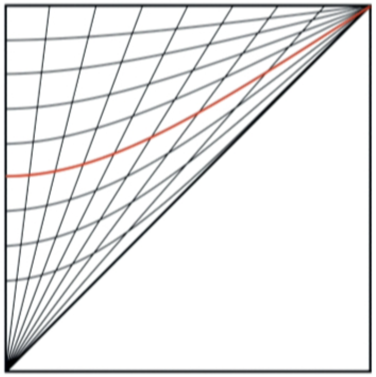

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de Sitter space has 10 Killing vectors:

- 3 translations and 3 rotations of a spatial slice $\mathbb{R}^3$

$$P_i = \nabla_i \qquad\qquad J_{ij} = x_i\nabla_j - x_j\nabla_i$$

- a dilation and 3 SCT-like transformations:

$$D = x^\mu\partial_\mu = \eta\partial_\eta + \vec{x}\cdot\vec{\nabla} \qquad K_i = 2x_i\eta\partial_\eta - (-\eta^2 + \vec{x}^2)\nabla_i + 2x_ix^j\nabla_j$$

Together, these form the isometry algebra of de Sitter space $so(4,1)$

- How do these symmetries act on fields?

- We will see that de Sitter symmetry *naturally* leads to the observed statistics of the CMB

Scalar fields living on de Sitter

Antoniadis, Mazur, Mottola astro-ph/9611208
Maldacena, Pimentel 1104.2846
Creminelli 1108.0874
Bzowski, McFadden, Skenderis 1112.1967

Consider a massive scalar

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{m_\phi^2}{2}\phi^2\right) \implies \phi_k'' - \frac{2}{\eta}\phi_k' + \left(k^2 + \frac{m_\phi^2}{H^2\eta^2}\right)\phi_k = 0$$

In the late time limit, the time dependence of the field is given by its mass for $0 \leq m_\phi^2 \leq 9H^2/4$

$$\phi_k \sim \eta^{\Delta_\pm} \quad \text{with} \quad \Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m_\phi^2}{H^2}}$$

Then, we can trade $\eta\partial_\eta \longrightarrow \Delta_- \equiv \Delta$ and the isometries act on the future boundary as

$$\delta_D = -\left(\Delta + \vec{x}\cdot\vec{\nabla}\right)\phi \qquad \delta_{J_{ij}} = (x_i\nabla_j - x_j\nabla_i)\,\phi$$

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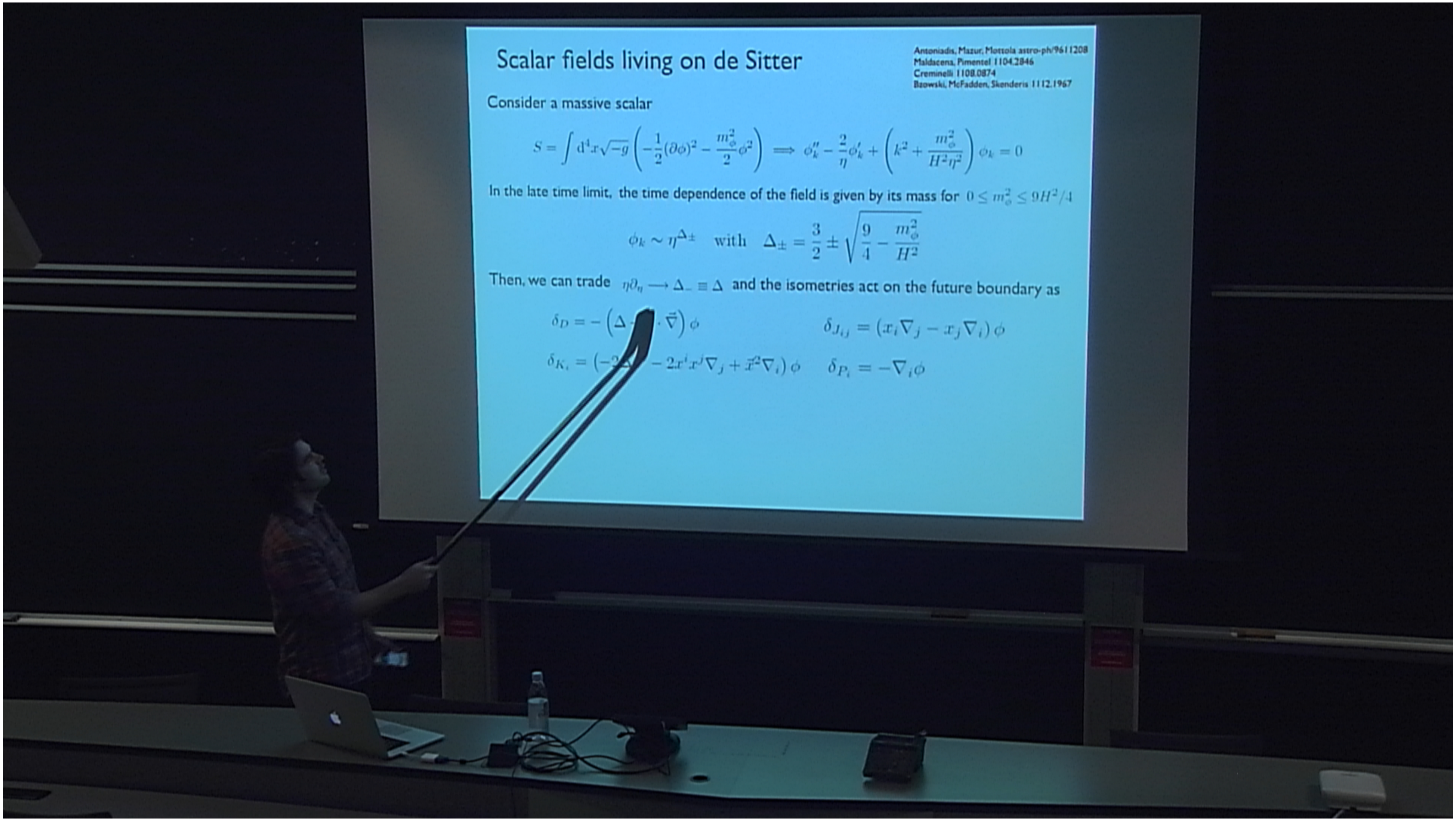
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The de Sitter isometries act as conformal transformations on the fields at late times!

- The isometry algebra of de Sitter space, $so(4,1)$, is *identical* to the conformal algebra on Euclidean 3-space

Spectator correlation functions

In cosmology, we are interested in late time correlation functions; these are constrained by the conformal symmetries: fields transform as primary operators of weight Δ

- 2-point function of a spectator in real space

$$\langle \phi(\vec{x},t)\phi(\vec{x}',\eta)\rangle \sim |\vec{x}-\vec{x}'|^{-2\Delta} \qquad \Delta = \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{m_\phi^2}{H^2}}$$

- Or, in Fourier space

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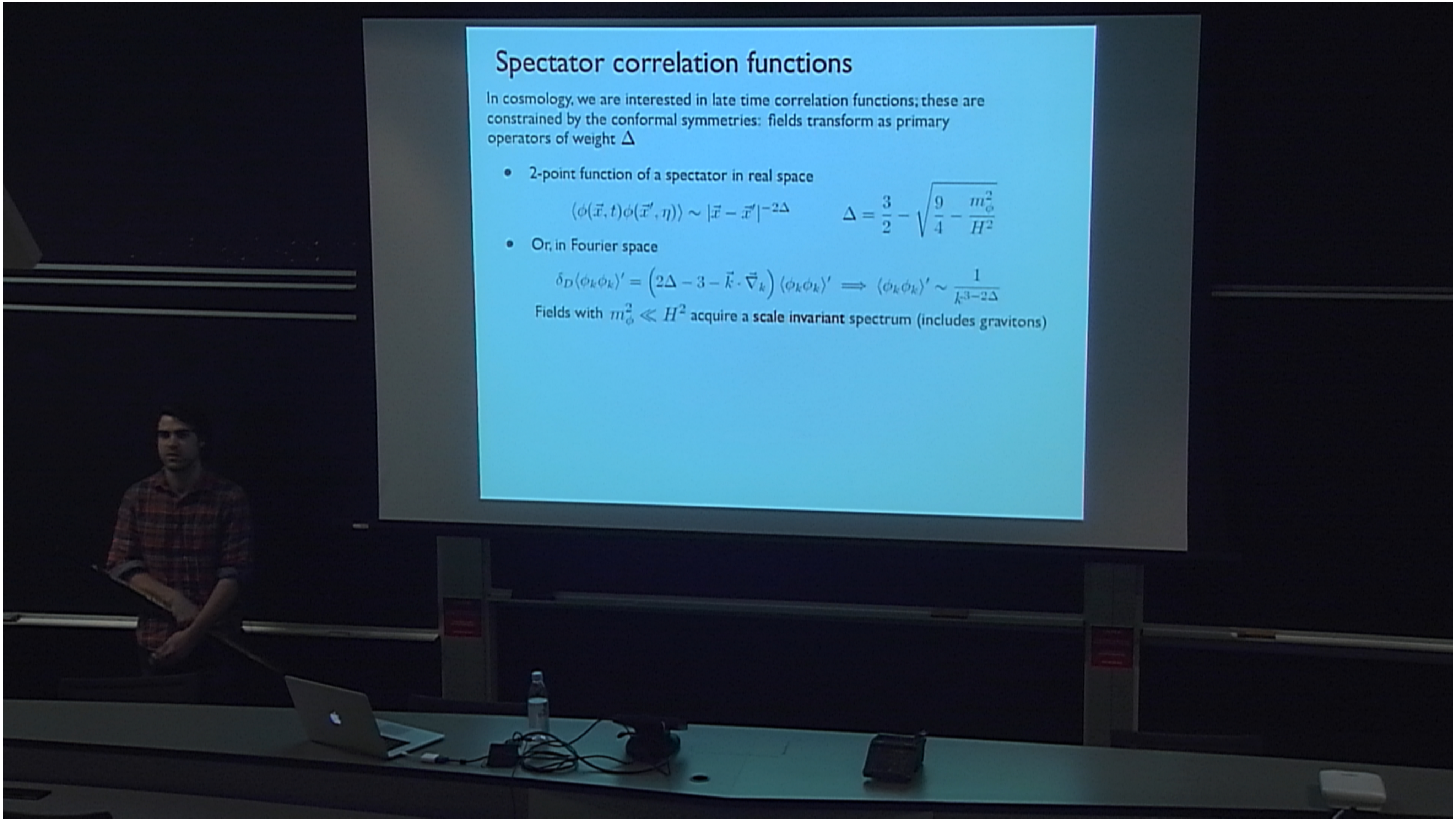
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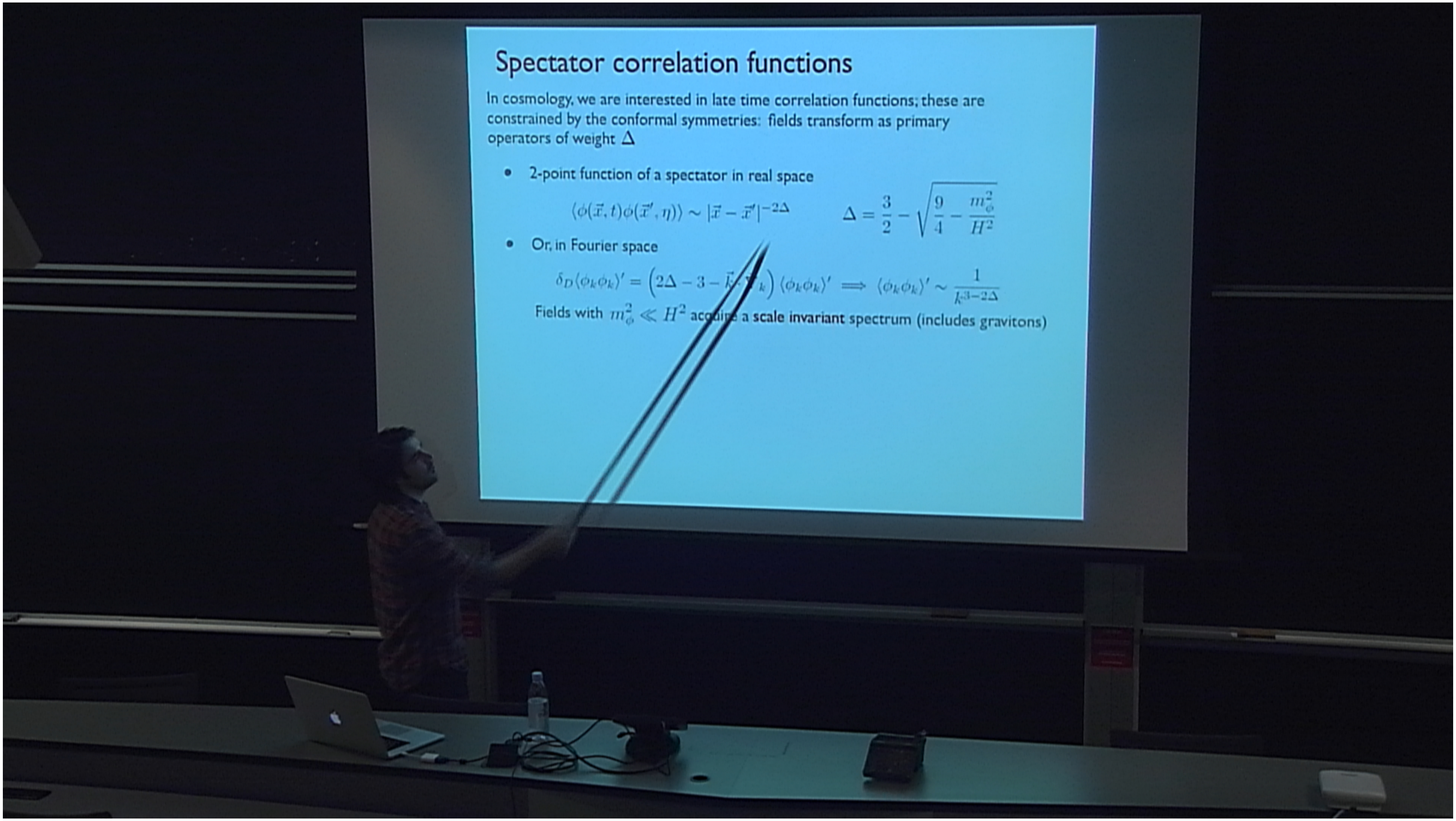
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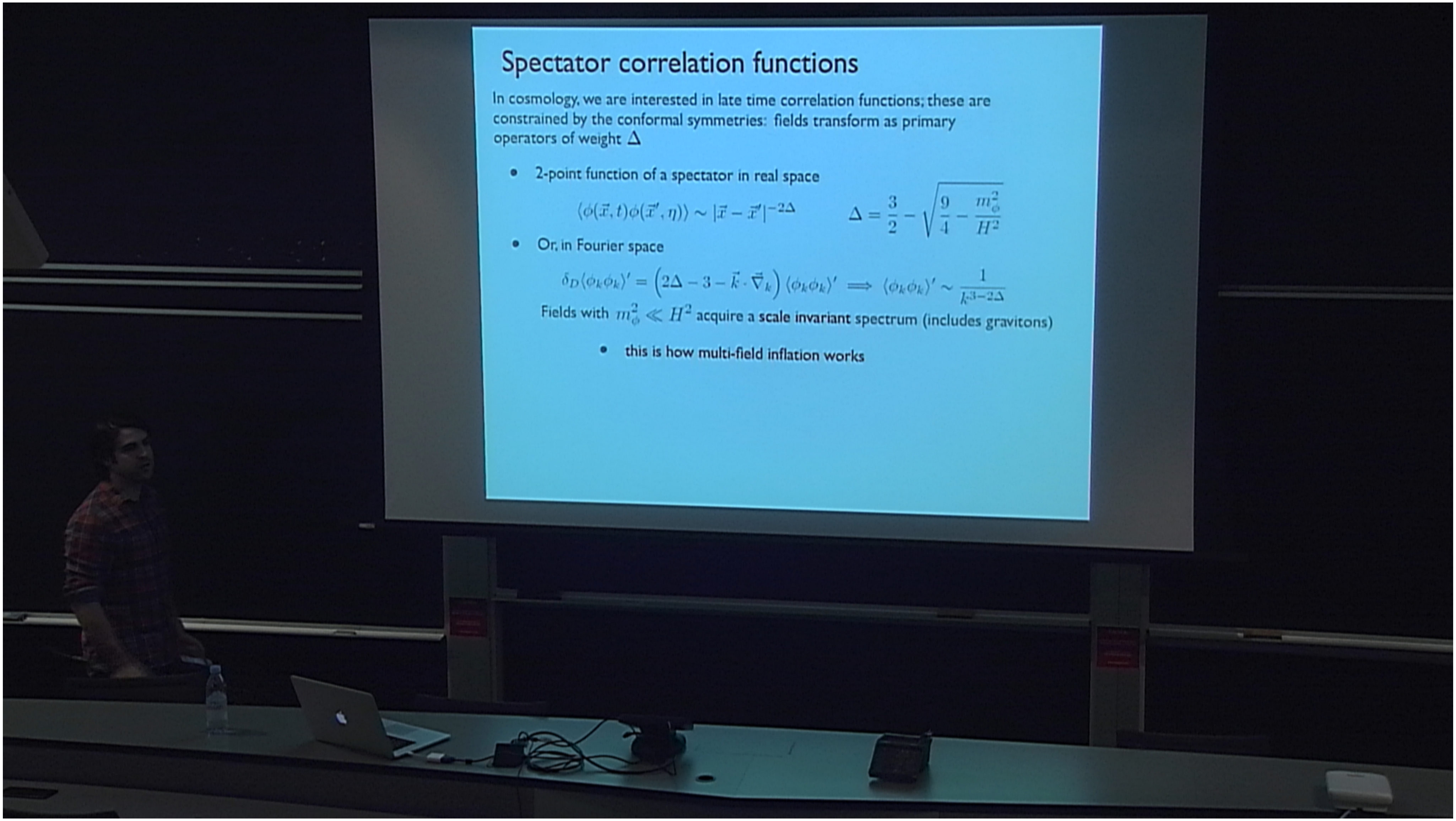
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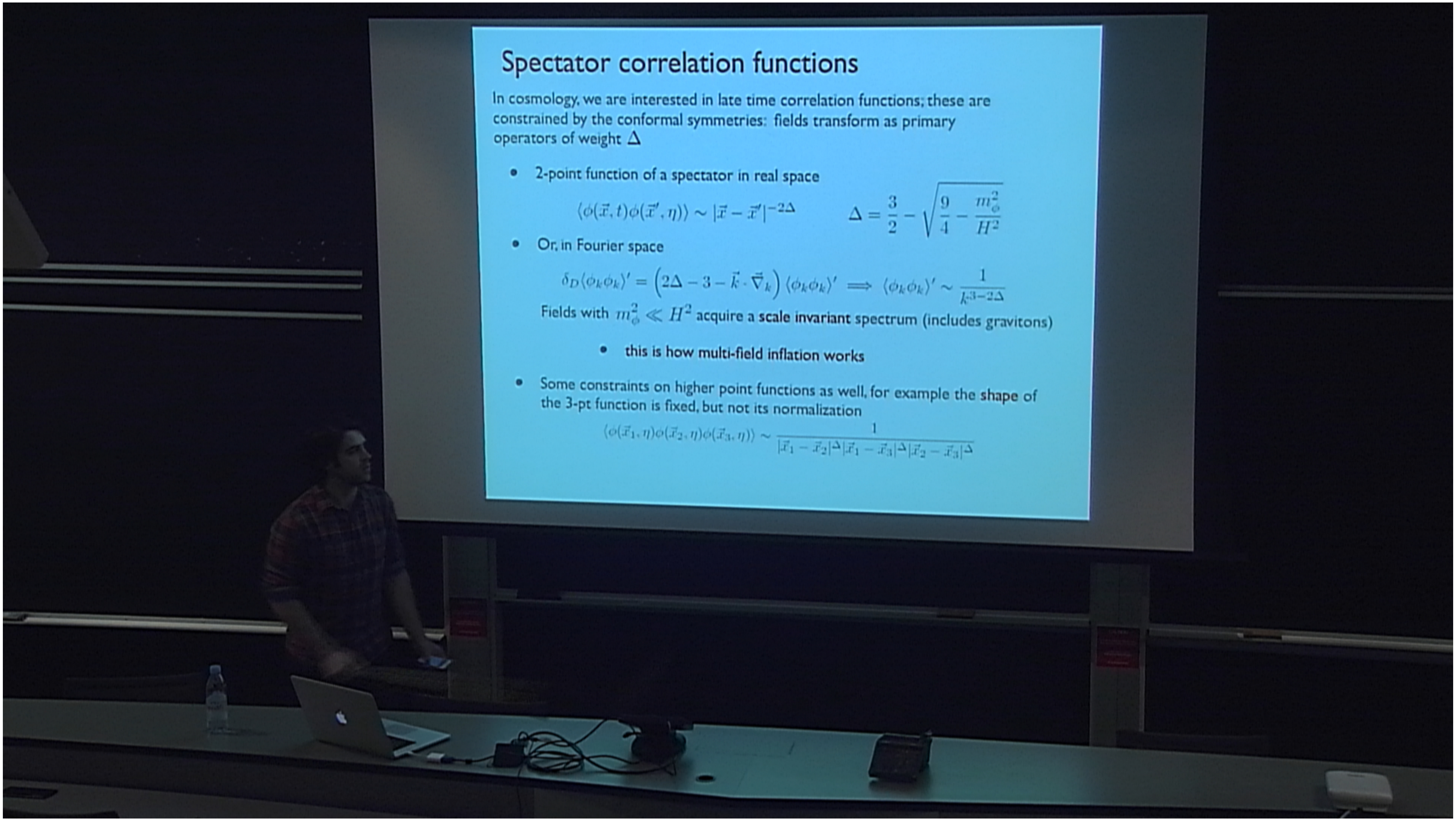
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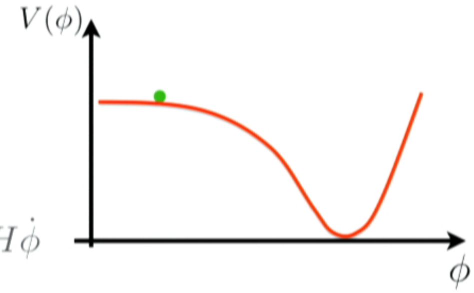
Slow roll inflation

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

We have a scalar field rolling down a nearly flat potential

$$3M_{\rm Pl}^2H^2 = \frac{1}{2}\dot\phi^2 + V(\phi)$$
$$\ddot\phi + 3H\dot\phi + V_{,\phi} = 0$$



- Slow-roll conditions: $\epsilon = \dot\phi^2/2M_{\rm Pl}^2H^2 \ll 1$; $\ddot\phi \ll H\dot\phi$
- The rolling field is the clock, inflation ends when it starts rolling too fast
- Fluctuations of this field are responsible for anisotropies:

  Work in ζ-gauge: $\delta\phi = 0$; $h_{ij} = a^2(t)e^{2\zeta(\vec{x},t)}\delta_{ij}$

- The quadratic action is then $S = M_{\rm Pl}^2\int \mathrm{d}^3x\mathrm{d}\eta\; a^2\epsilon\left(\zeta'^2 - (\vec\nabla\zeta)^2\right)$
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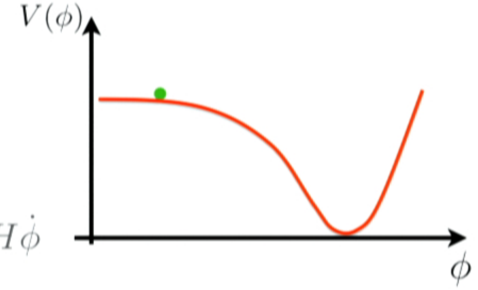

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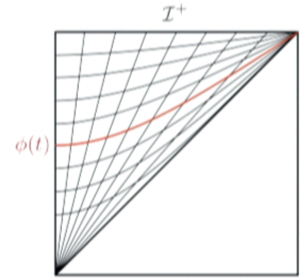
The inflaton as a Goldstone



Assassi, Baumann, Green 1204.4207

Inflation ends so the spacetime cannot be **exact** de Sitter

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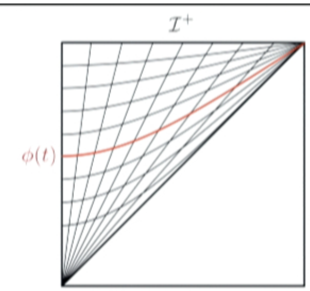

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Creminelli, Norena, Simonovic 1203.4595
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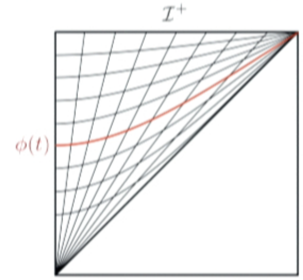

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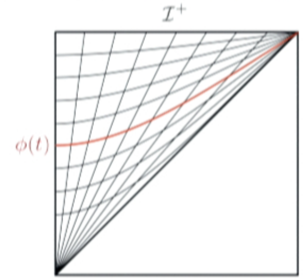

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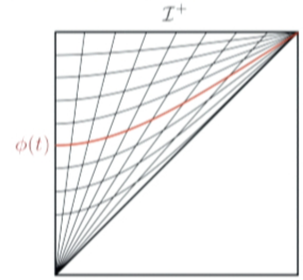

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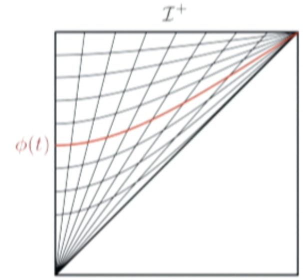

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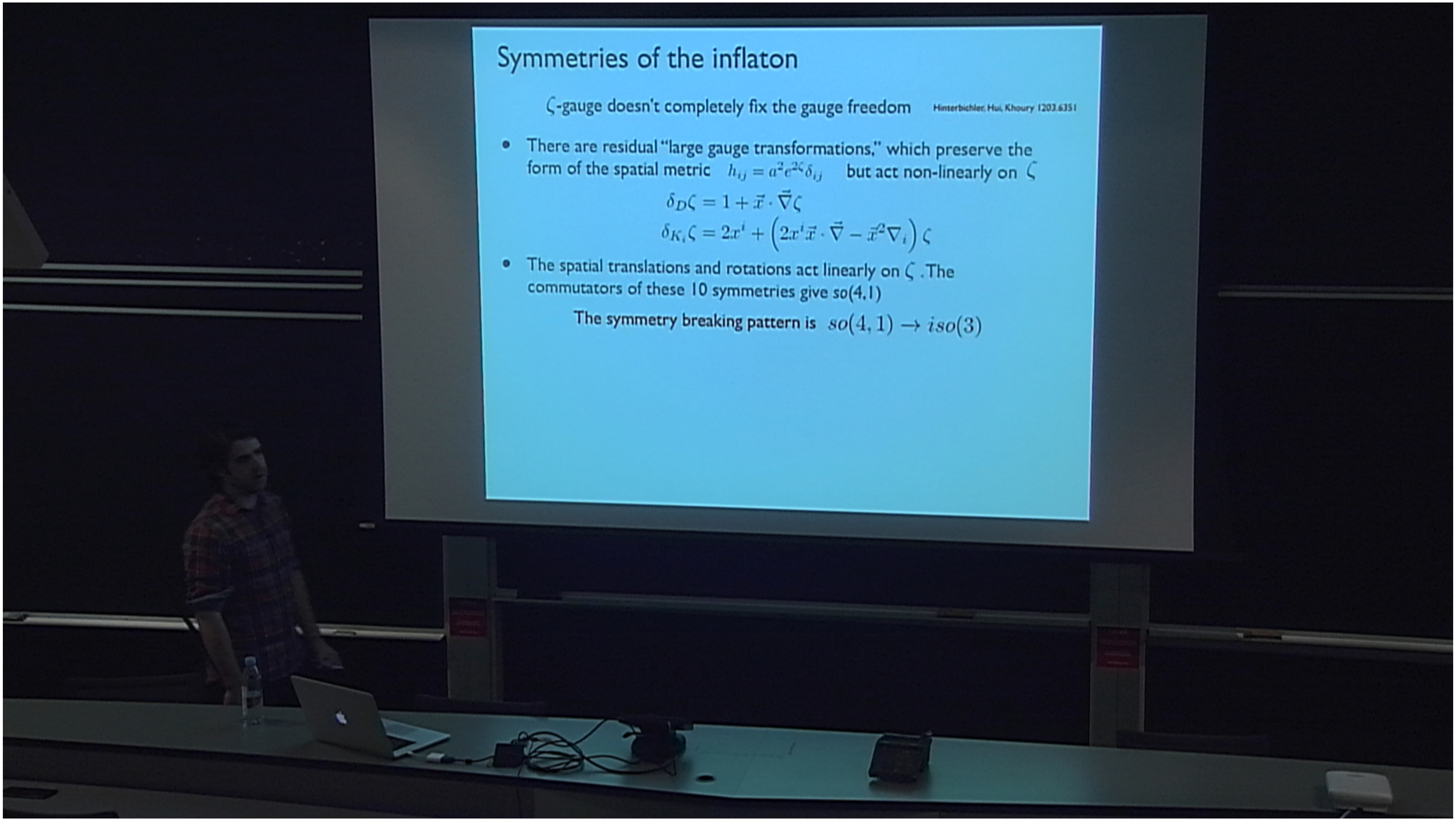
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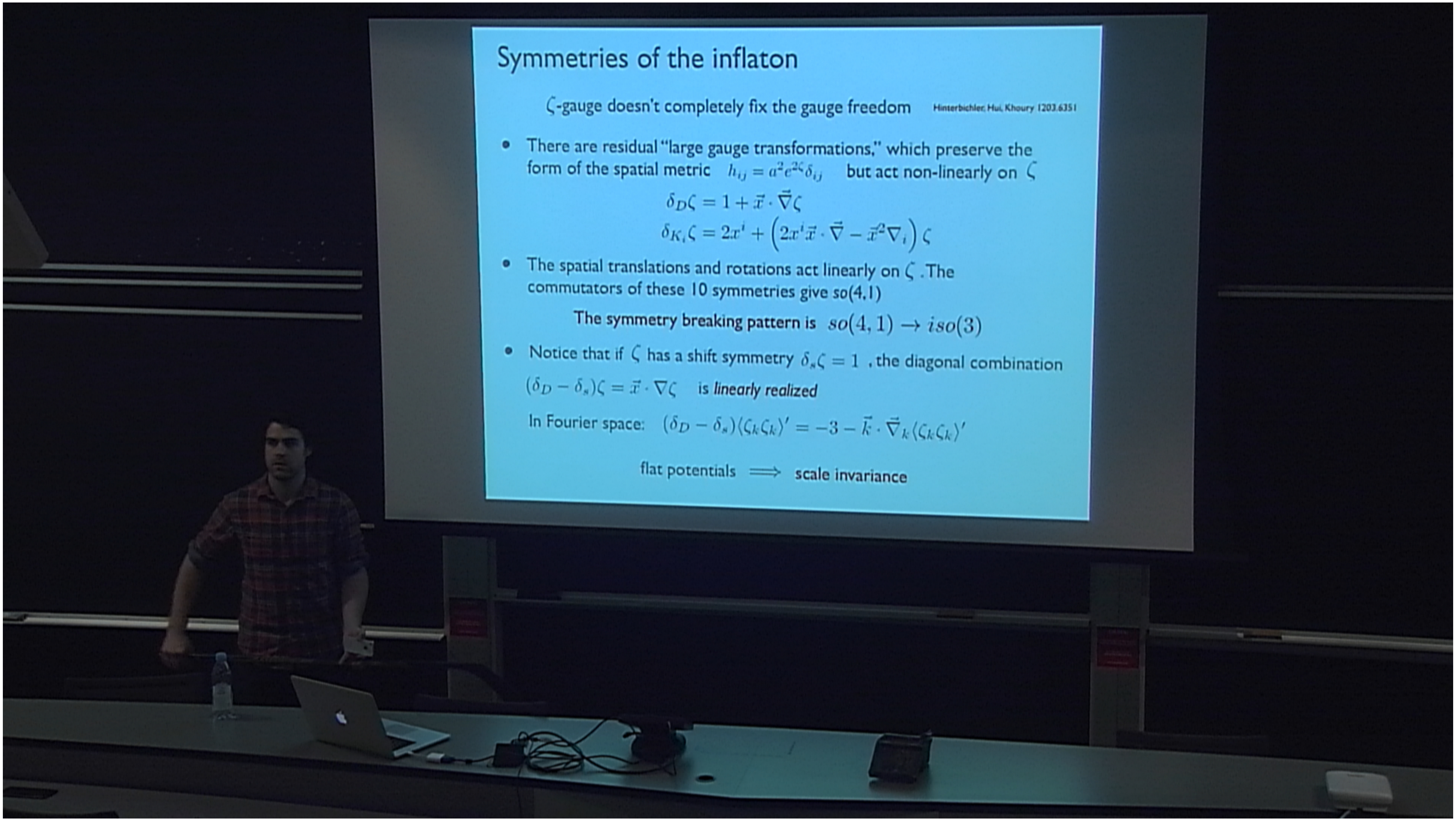
$(\delta_D - \delta_s)\zeta = \vec{x} \cdot \nabla\zeta$ is *linearly realized*

In Fourier space: $(\delta_D - \delta_s)\langle\zeta_k \zeta_k\rangle' = -3 - \vec{k} \cdot \vec{\nabla}_k \langle\zeta_k \zeta_k\rangle'$

flat potentials $\Longrightarrow$ scale invariance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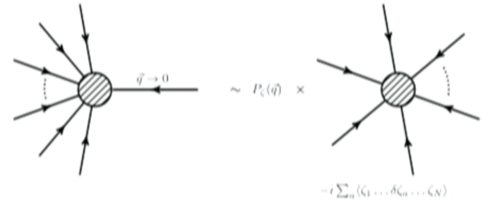
Consistency relations

Maldacena astro-ph/0210603
Creminelli, Zaldarriaga astro-ph/0407059
Cheung, Fitzpatrick, Kaplan, Senatore 0709.0295

The non-linearly realized symmetries can also tell us something about correlation functions



- They constrain correlators in the limit where one of the external momenta is soft
- Originally derived using background wave arguments, but can also be seen as Ward identities (operator formalism, effective action, wavefunctionals)

Creminelli, Norena, Simonovic 1203.4595
Hinterbichler, Hui, Khoury 1203.6351
Assassi, Baumann, Green 1204.4207
Goldberger, Hui, Nicolis 1303.1193
Pimentel 1309.1793
Berezhiani, Khoury 1309.4461

More soft legs?

Senatore, Zaldarriaga 1203.6884
AJ, Khoury, Simonovic, to appear

A reasonable question to ask is: *what happens if we take more legs soft?*

- Inspired by Pion physics, where the single-soft relation is trivial, but taking multiple soft legs is very interesting

$$\lim_{q_a, q_b \to 0} \langle \pi^a(q_a)\pi^b(q_b)\pi^{i_1}(k_1)\cdots\pi^{i_n}(k_n)\rangle = \frac{1}{2}\sum_j \frac{(q_a - q_b)\cdot k_j}{(q_a + q_b)\cdot k_j}\epsilon^{abc}\langle\pi^{i_1}(k_1)\cdots T_c\pi^{i_j}(k_j)\cdots\pi^{i_n}(k_n)\rangle$$

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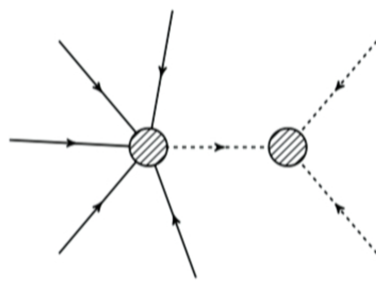
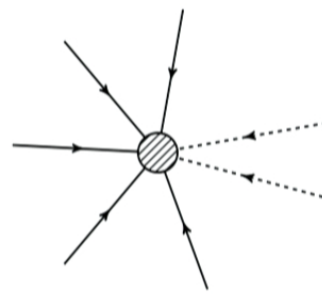
- Diagrammatically, we expect two types of contribution





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- Indeed, this ends up being the case

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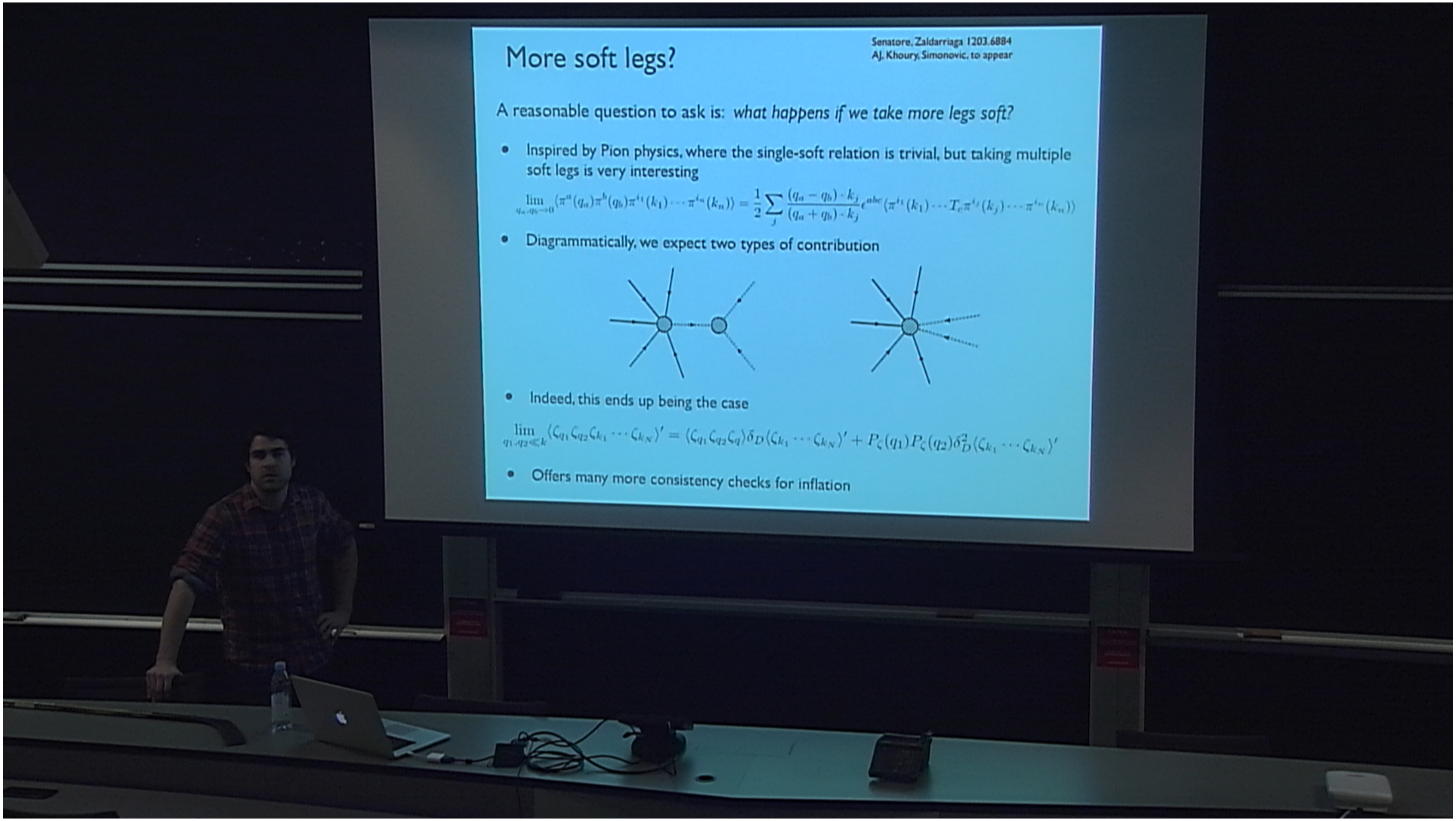

More soft legs?
Senatore, Zaldarriaga 1203.6884
AJ, Khoury, Simonovic, to appear
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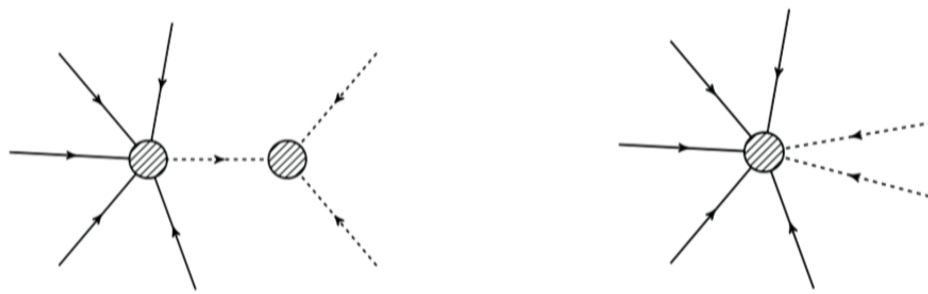

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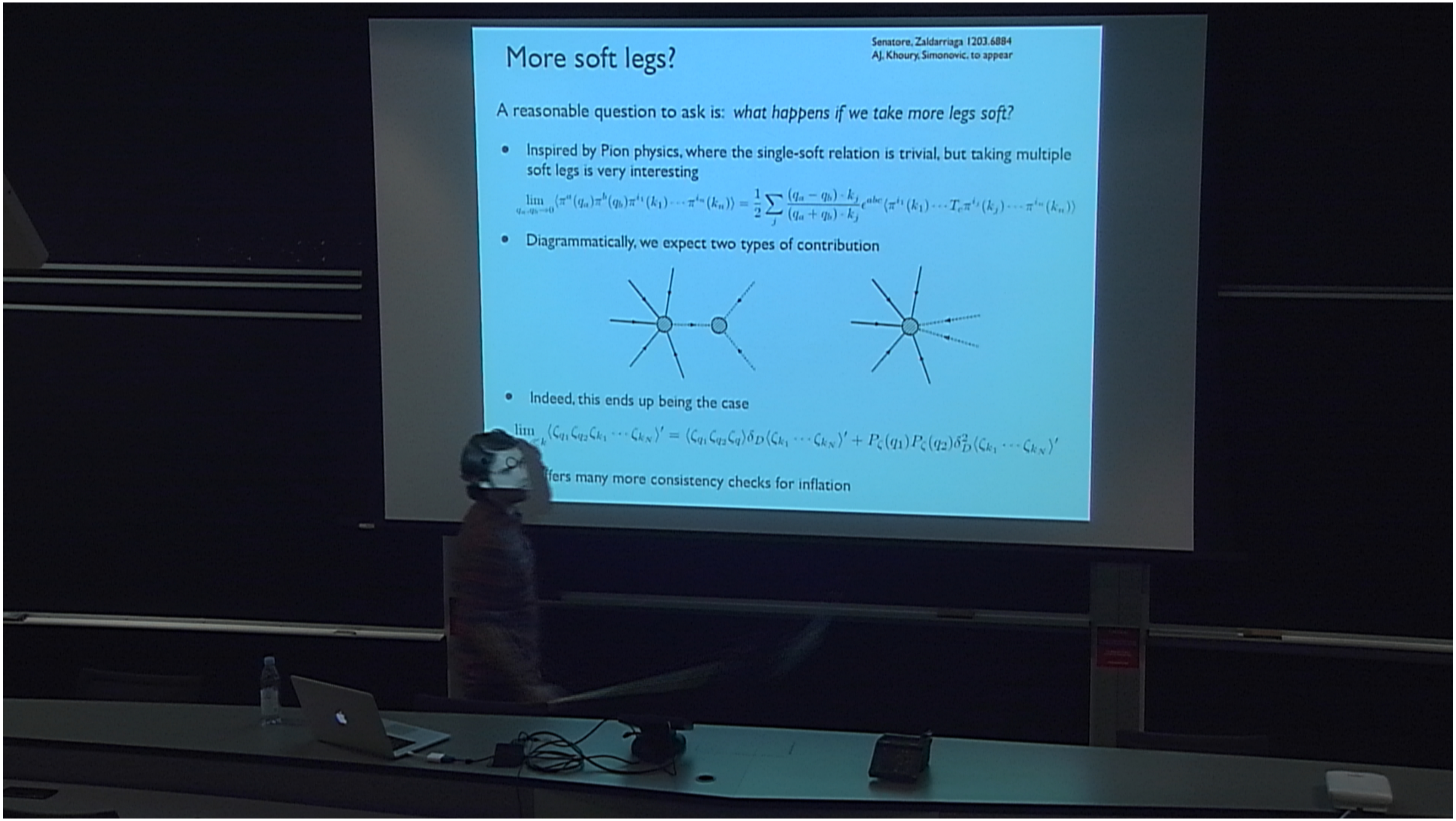
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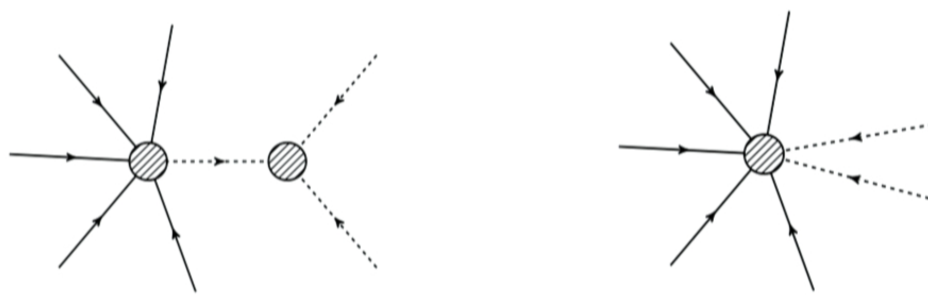

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Khoury, Miller 1012.0846
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Background Cosmology

Hinterbichler, AJ, Khoury 1202.6056
Hinterbichler, AJ, Khoury, Miller 1209.5742

- Imagine coupling the CFT minimally in Einstein frame $S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_{\rm cft}[g]\right)$
- Dilation symmetry tells us that we must have

$$\rho_{\rm cft} = 0\ ;\quad P_{\rm cft} = \frac{\beta}{t^4}$$

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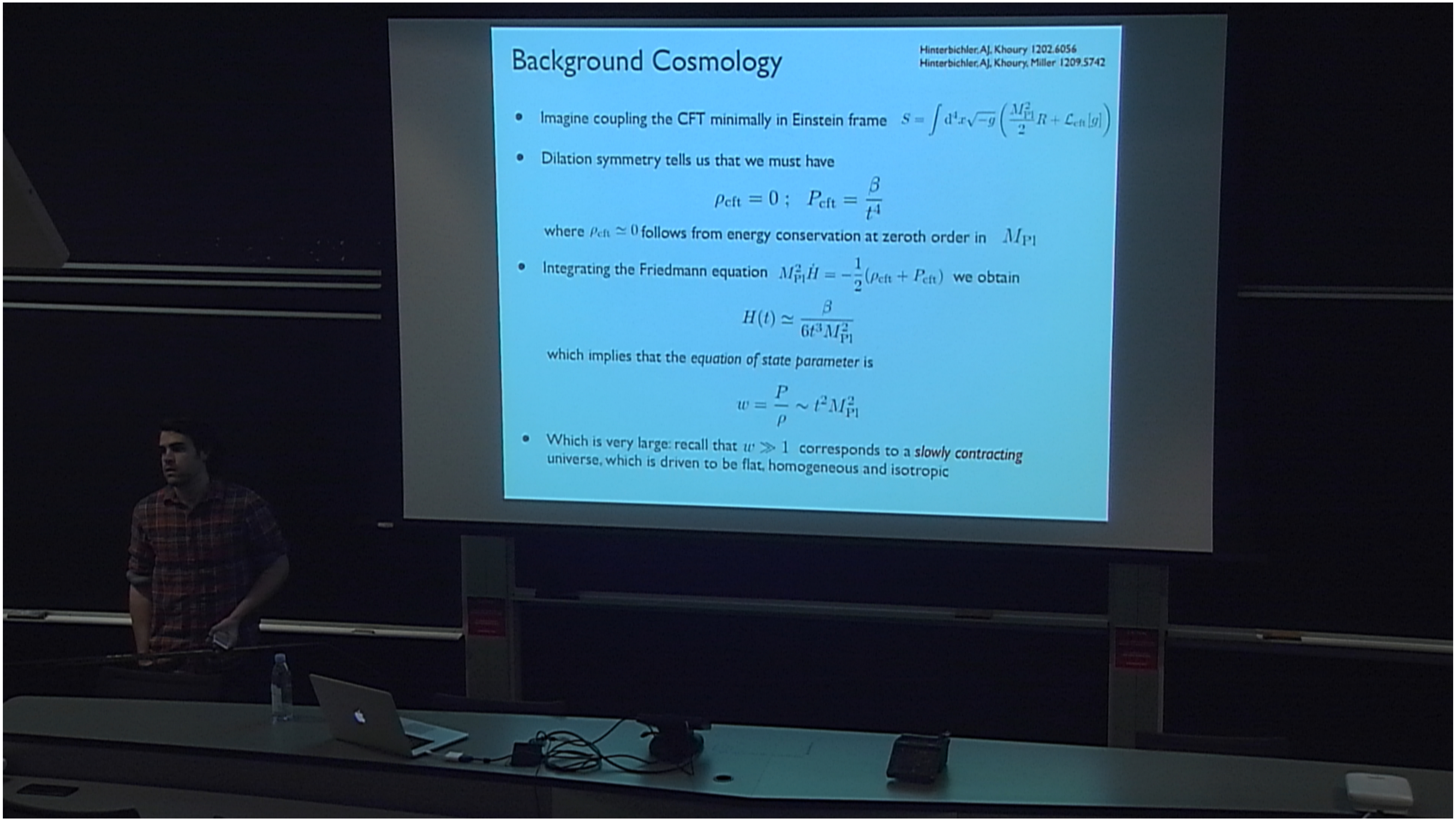
$$H(t) \simeq \frac{\beta}{6t^3M_{\rm Pl}^2}$$

which implies that the *equation of state parameter* is

$$w = \frac{P}{\rho} \sim t^2M_{\rm Pl}^2$$

- Which is very large: recall that $w \gg 1$ corresponds to a *slowly contracting* universe, which is driven to be flat, homogeneous and isotropic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$$\bar{\phi}(t) = \sqrt{\frac{2}{\lambda}} \frac{1}{(-t)}$$

- The field getting a profile breaks some of the symmetries
  - Time-translations
  - Boosts
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- The remaining symmetries can be repackaged into the algebra $so(4,1)$
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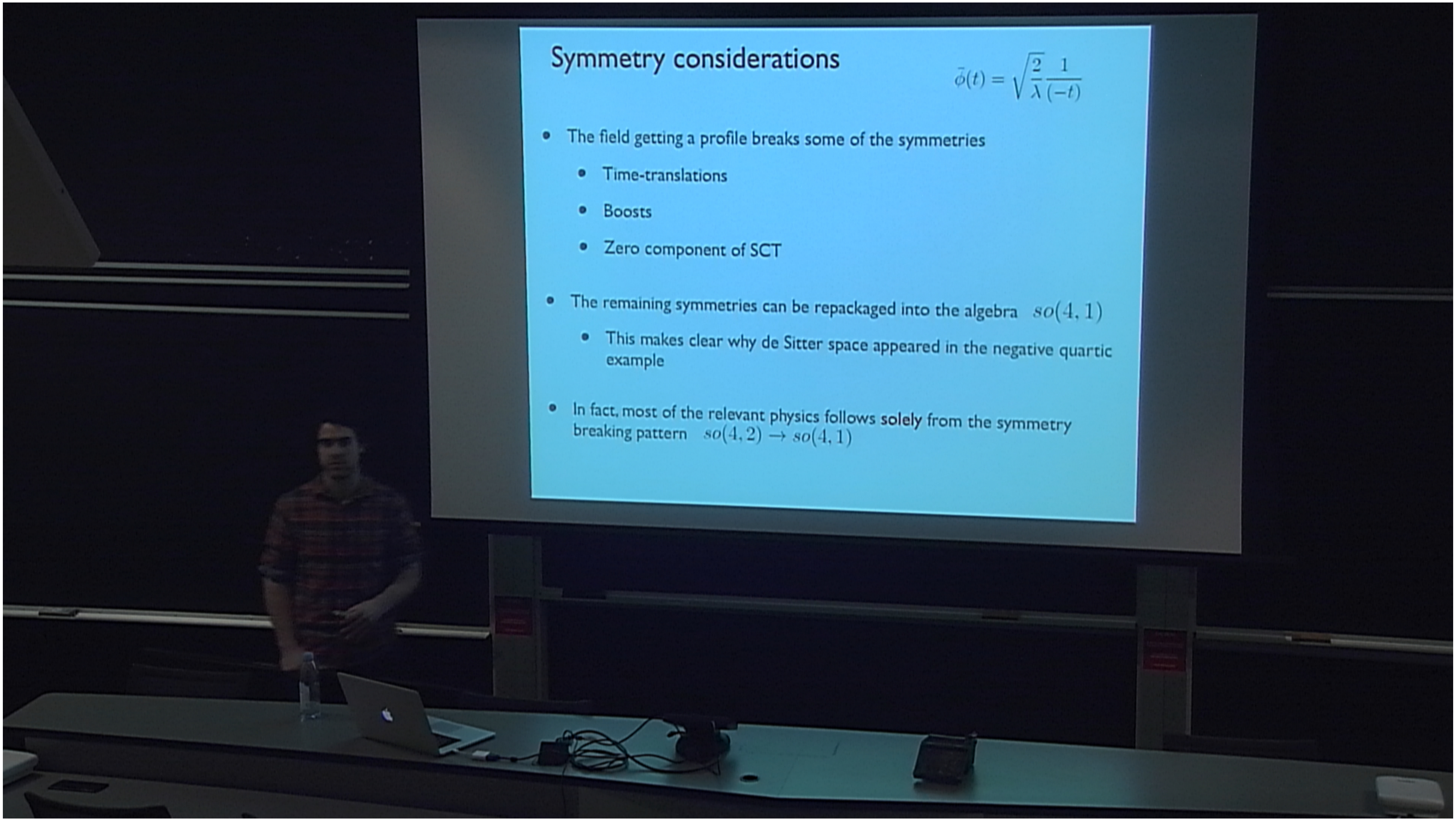
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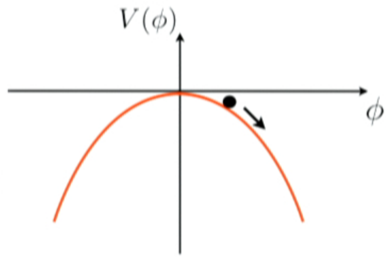
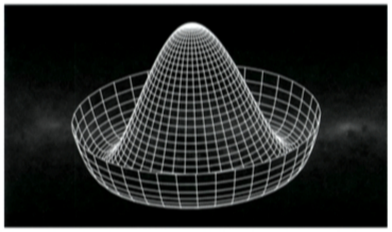
In fact, most of the relevant physics follows **solely** from the symmetry breaking pattern $so(4,2) \to so(4,1)$

Other examples

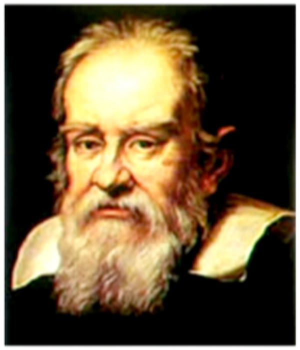




- Negative quartic; negative U(1)

  Craps, Hertog, Turok 0712.4180
  Rubakov 0906.3693
  Hinterbichler, Khoury 1106.1428

- Galilean genesis



utilizes a higher-derivative scalar field theory - conformal galileon

the symmetry-breaking solution violates the null energy condition

Creminelli, Nicolis, Trincherini 1007.0027
Creminelli, Hinterbichler, Khoury, Nicolis, Trincherini 1209.3768

- DBI - a brane probing AdS

  exploits the isomorphism between the conformal group and the group of isometries of AdS in one higher dimension

Hinterbichler, Khoury 1106.1428
Hinterbichler, AJ, Khoury, Miller 1209.5742

Generalities

Hinterbichler, Khoury 1106.1428
Hinterbichler, AJ, Khoury 1202.6056

- We can use non-linear realization techniques to construct the most general low-energy effective action for the symmetry breaking pattern
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$$\omega_{\hat{P}}^m = \mathrm{d}e^{\pi}\bar{e}_\mu^m \mathrm{d}y^\mu \,,$$

$$\omega_D = \mathrm{d}\pi + 2e^{\pi}\xi_m \bar{e}_\mu^m \mathrm{d}y^\mu \,,$$

Inverse Higgs: $\xi_\mu = -\frac{1}{2}e^{-\pi}\partial_\mu \pi$

$$\omega_K^m = \mathrm{d}\xi^m - \omega_{\rm spin}^{mn}\xi_n + 2e^{\pi}\xi_n\xi^m\bar{e}_\mu^n\mathrm{d}y^\mu - e^{\pi}\xi^2\bar{e}_\mu^m\mathrm{d}y^\mu - \frac{H^2}{2}\sinh\pi\bar{e}_\mu^m\mathrm{d}y^\mu + \xi^m\mathrm{d}\pi \,,$$

$$\frac{1}{2}\omega_J^{mn} = e^{\pi}\mathrm{d}y^\mu\left(\xi^n\bar{e}_\mu^m - \xi^m\bar{e}_\mu^n\right) + \omega_{\rm spin}^{mn} \,.$$

The two derivative action for breaking $so(4,2) \to so(4,1)$ is then given by

$$S_\pi = M_\pi^2 \int \mathrm{d}^4x\sqrt{-g_{\rm dS}}\left(-\frac{1}{2}e^{2\pi}(\partial\pi)^2 - H^2e^{2\pi} + \frac{H^2}{2}e^{4\pi}\right) \qquad g_{\mu\nu}^{\rm eff} = \frac{1}{H^2t^2}\eta_{\mu\nu}$$

Non-linearly realized symmetries

Creminelli, AJ, Khoury, Simonovic 1212.3329

- The spectator fields enjoy linearly realized *so*(4,1) symmetry by construction, this constrains their correlation functions - same amount as spectator in inflation
- However, here there are *additional* non-linearly realized symmetries due to the broken conformal symmetry. This leads to relations between correlation functions with soft Goldstone fields
- The broken symmetries are non-linearly realized on π as

$$\delta_{P_0}\pi = \frac{1}{t} - \partial_t \pi \; ;$$

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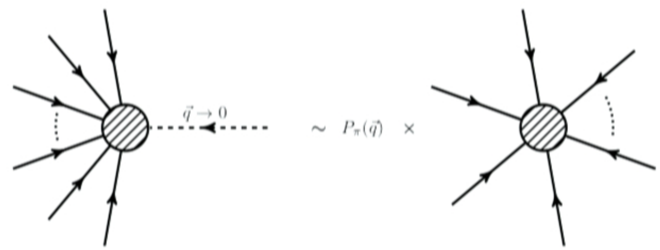
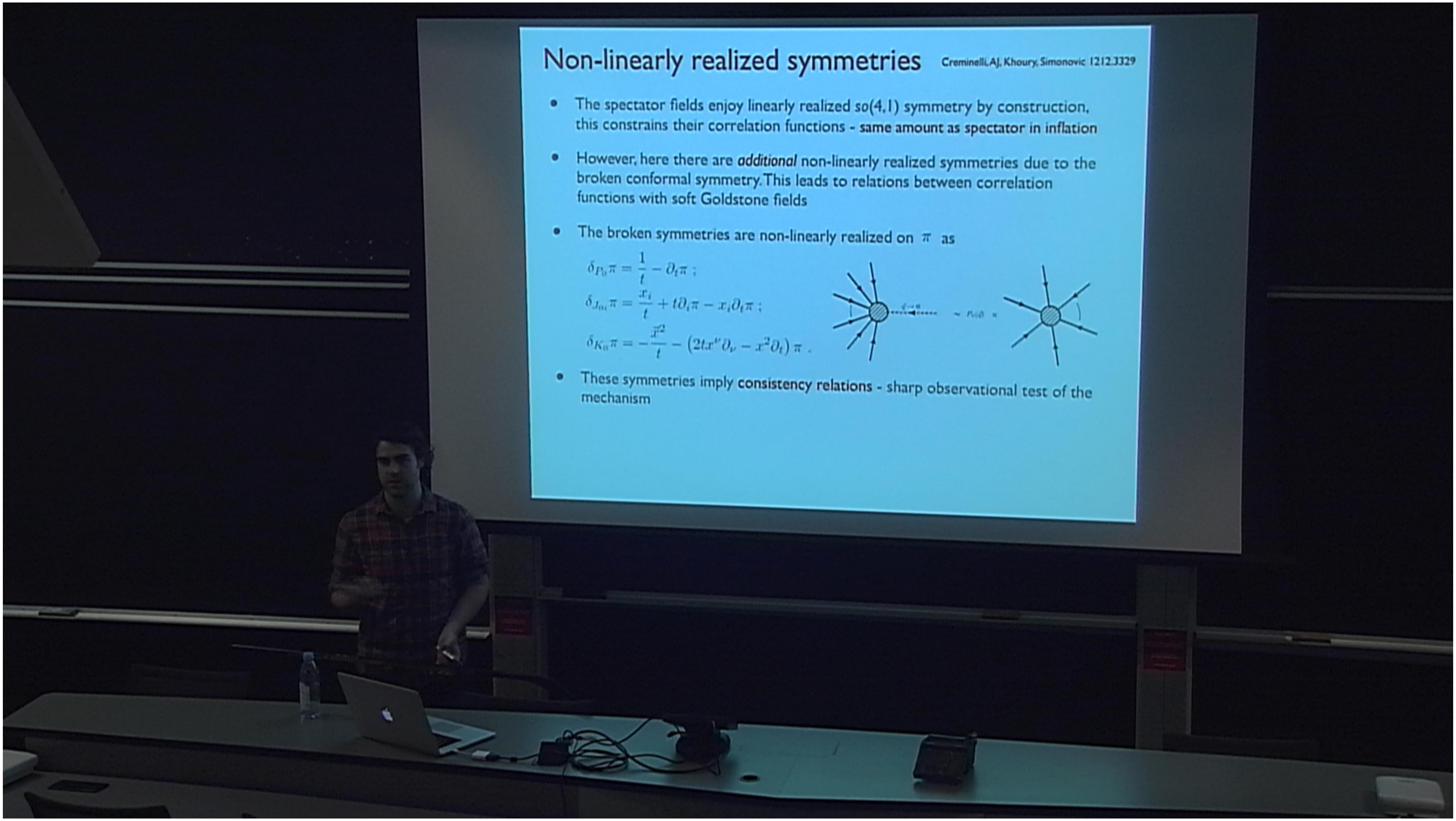


- These symmetries imply consistency relations - sharp observational test of the mechanism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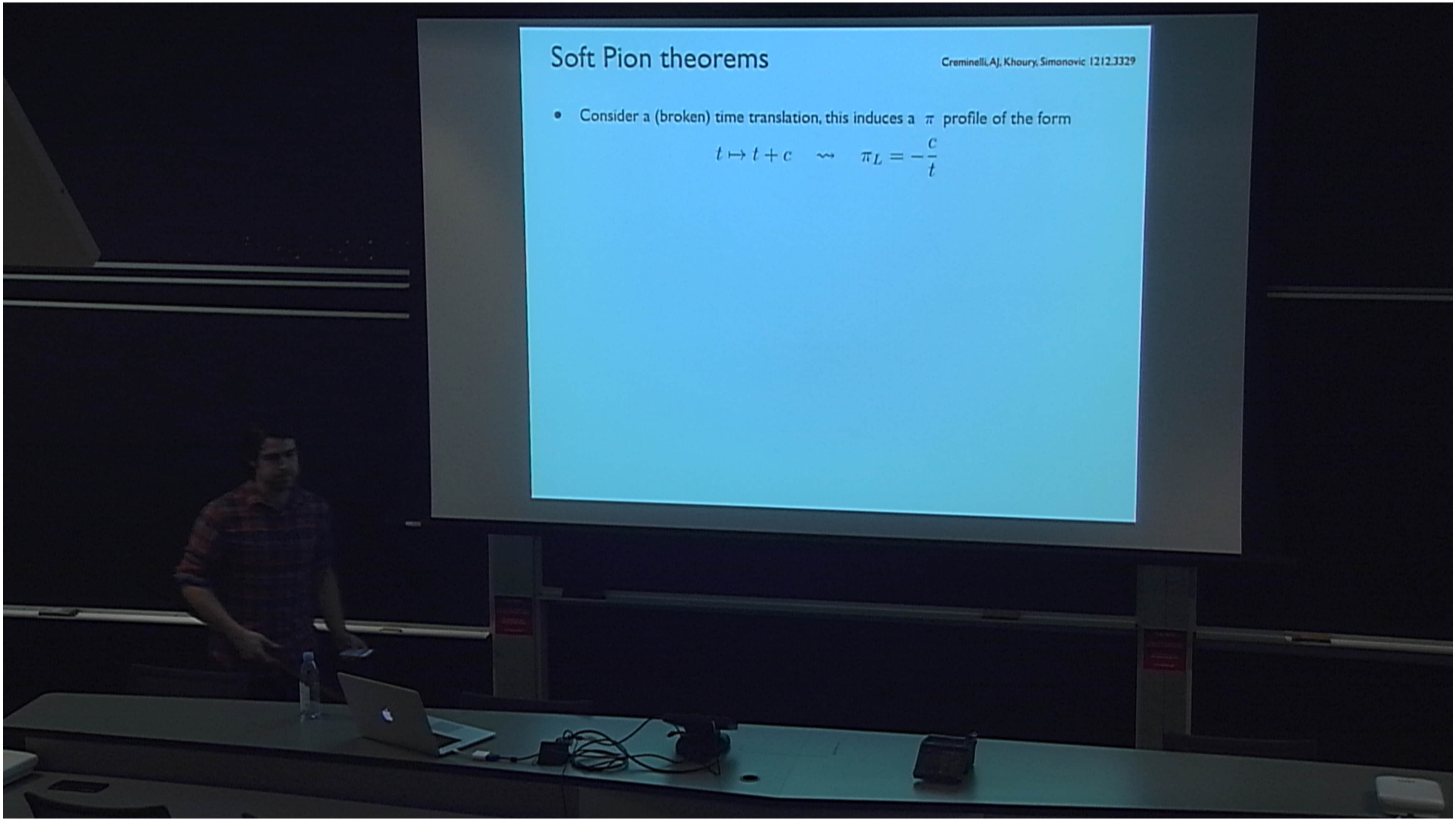
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- There are *also* identities corresponding to boosts and SCT

$$\langle \pi_{\vec{q}} \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_N} \rangle'_{q \to 0} \overset{\text{avg}}{=} -P_\pi(q) \left(1 + \frac{1}{N} q_i \sum_a \partial_{k_{ai}} + \frac{1}{6N} \vec{q}^{\,2} \sum_a \partial^2_{k_a} \right) t \frac{\mathrm{d}}{\mathrm{d}t} \langle \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_N} \rangle'$$

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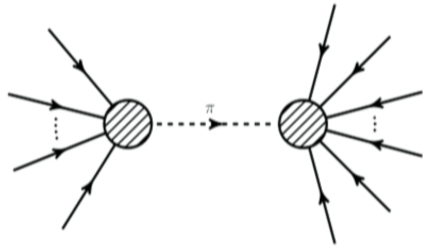
$$\langle \pi_{\vec{q}} \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_N} \rangle'_{q \to 0} \overset{\text{avg}}{=} -P_\pi(q) \left(1 + \frac{1}{N} q_i \sum_a \partial_{k_{ai}} + \frac{1}{6N} \vec{q}^{\,2} \sum_a \partial^2_{k_a} \right) t \frac{\mathrm{d}}{\mathrm{d}t} \langle \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_N} \rangle'$$

Soft internal lines



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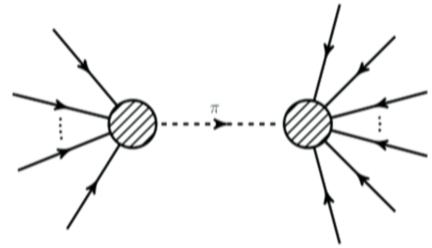

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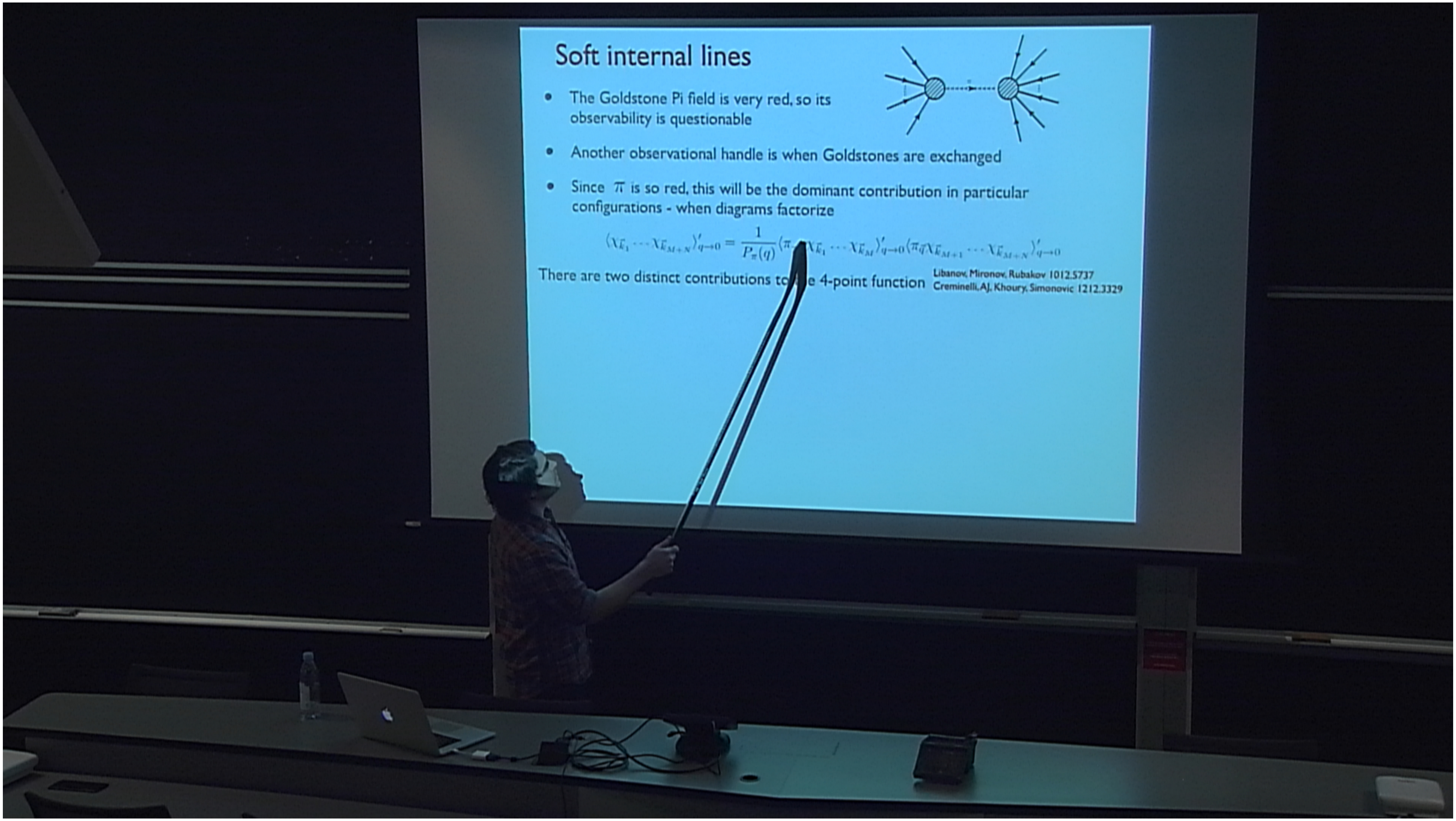
Libanov, Mironov, Rubakov 1012.5737
Creminelli, AJ, Khoury, Simonovic 1212.3329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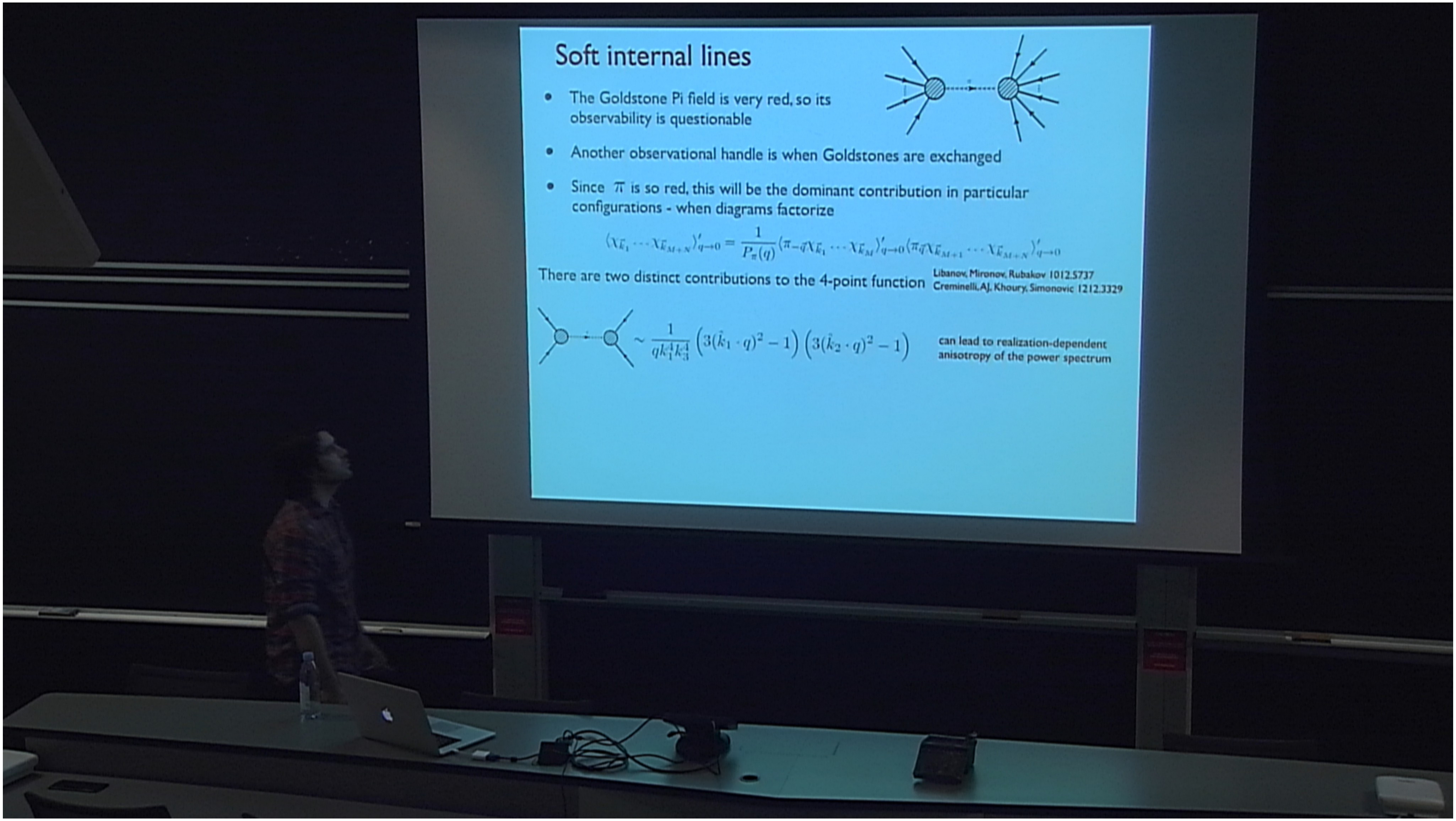
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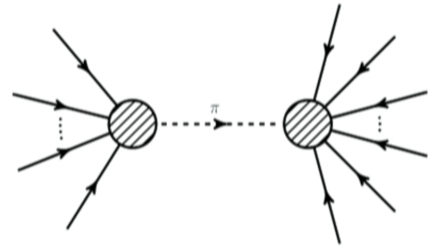

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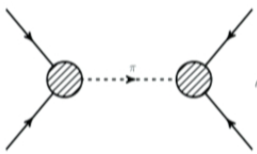

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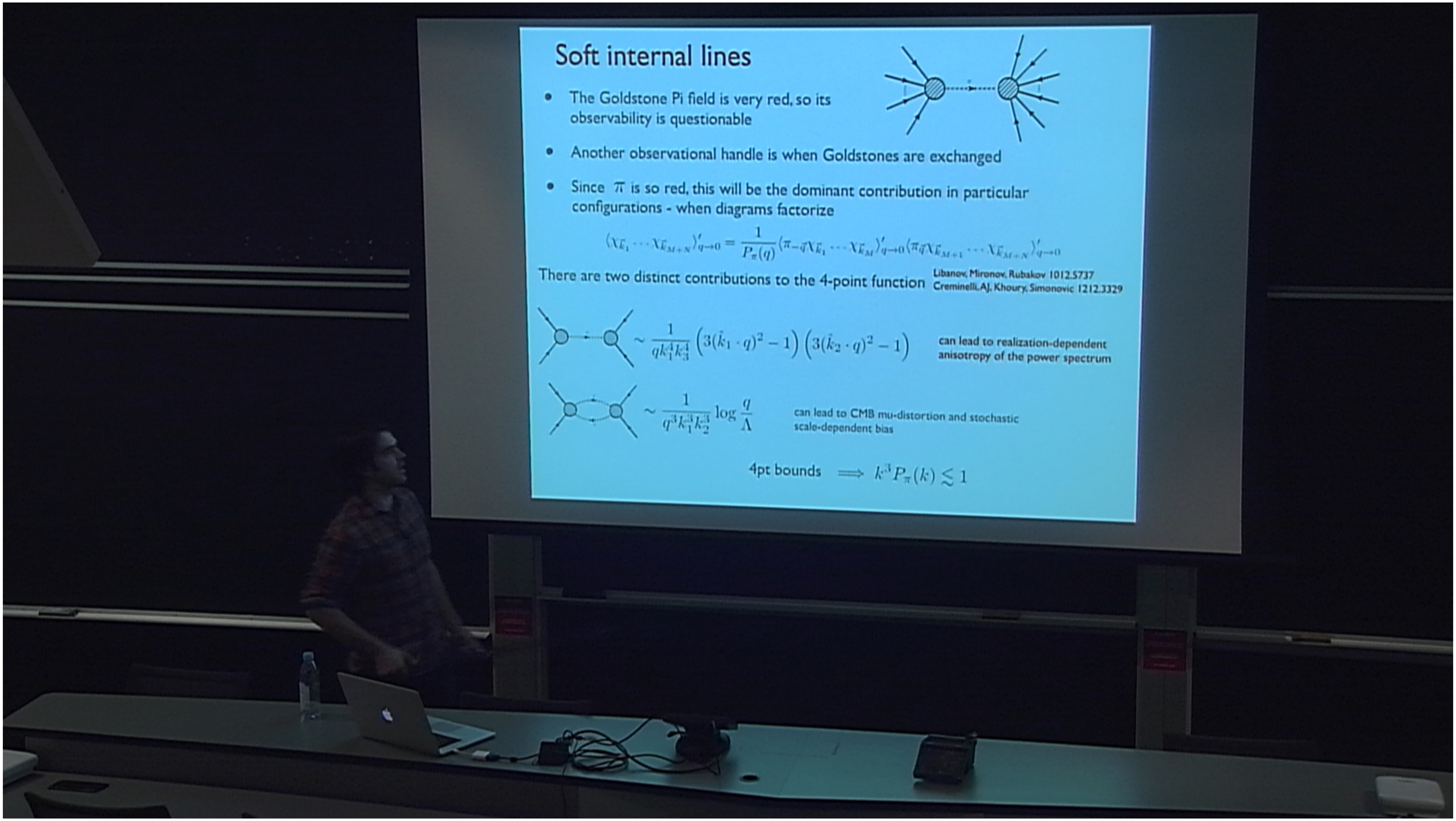
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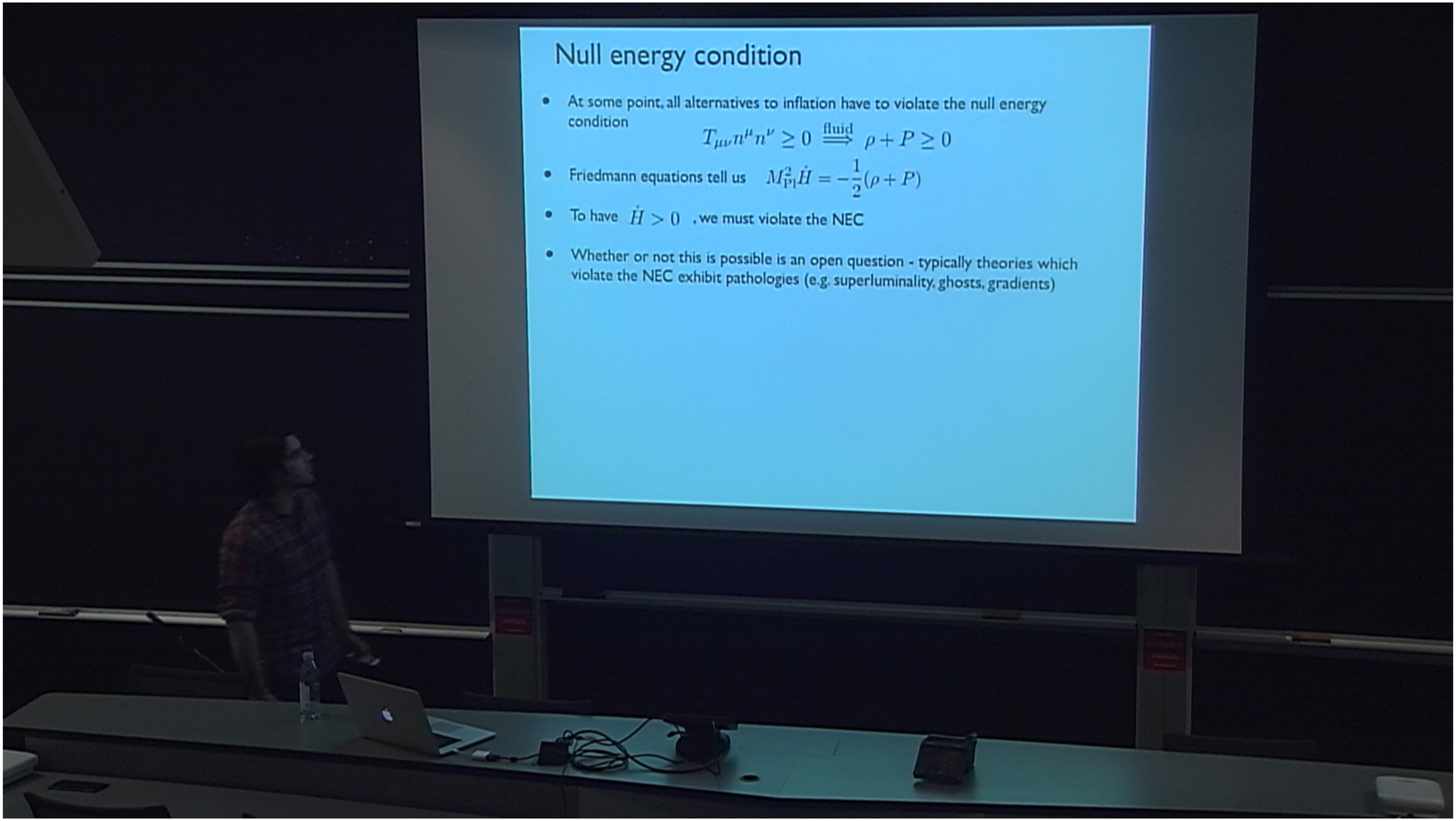
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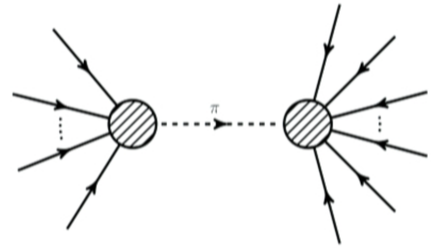

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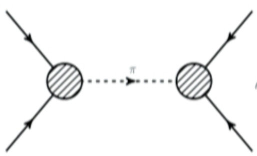

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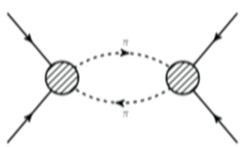

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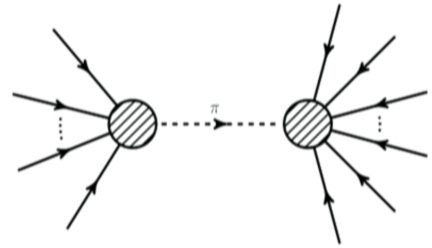

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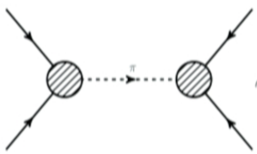

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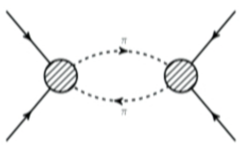

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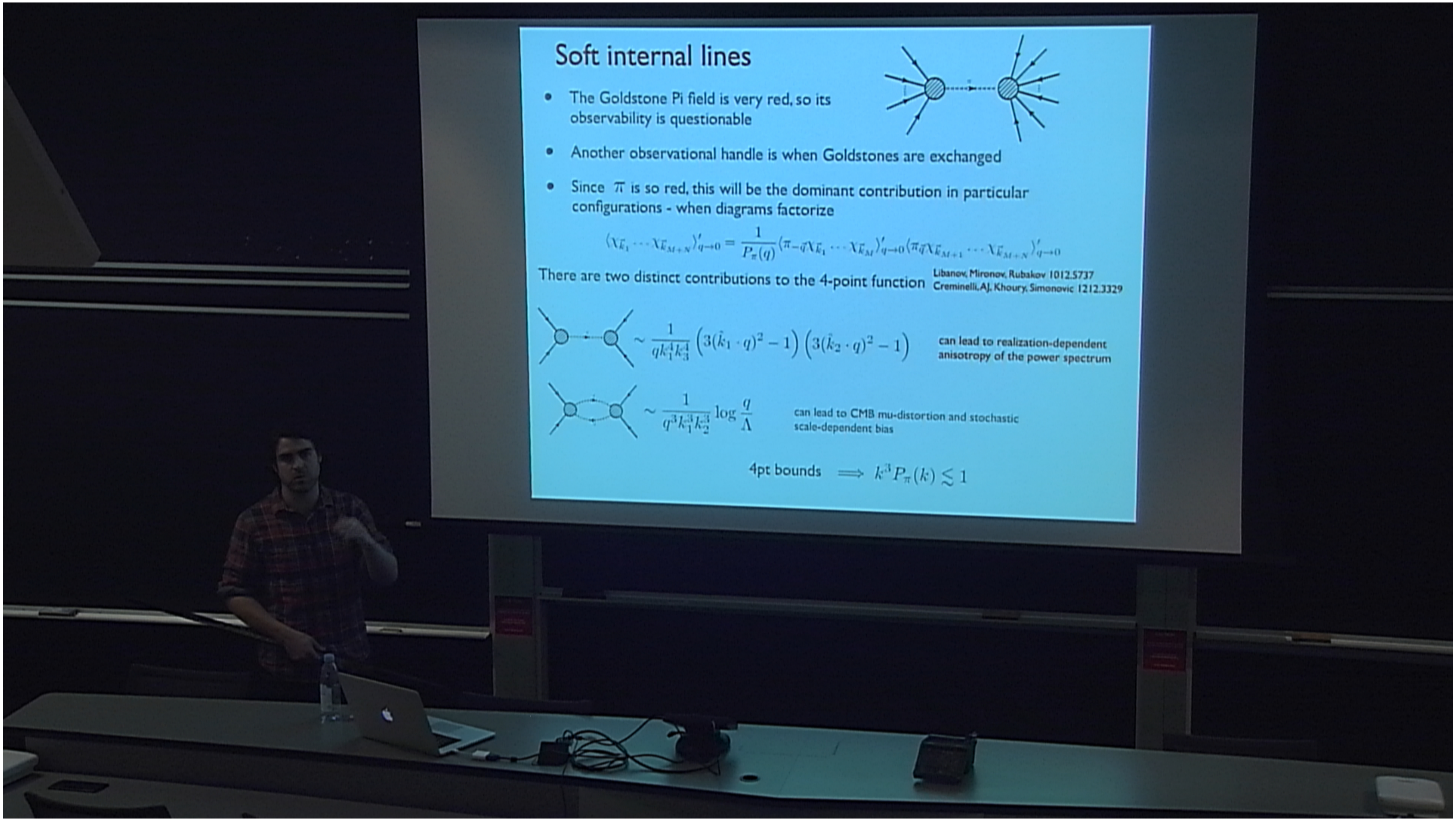
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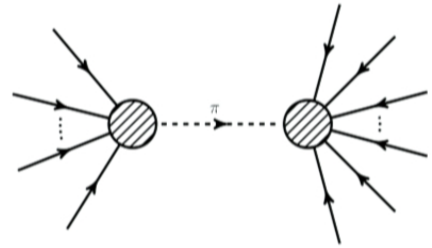

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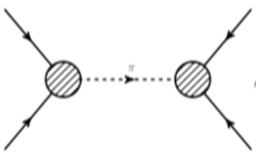

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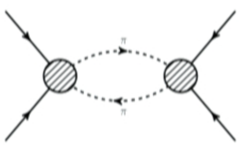

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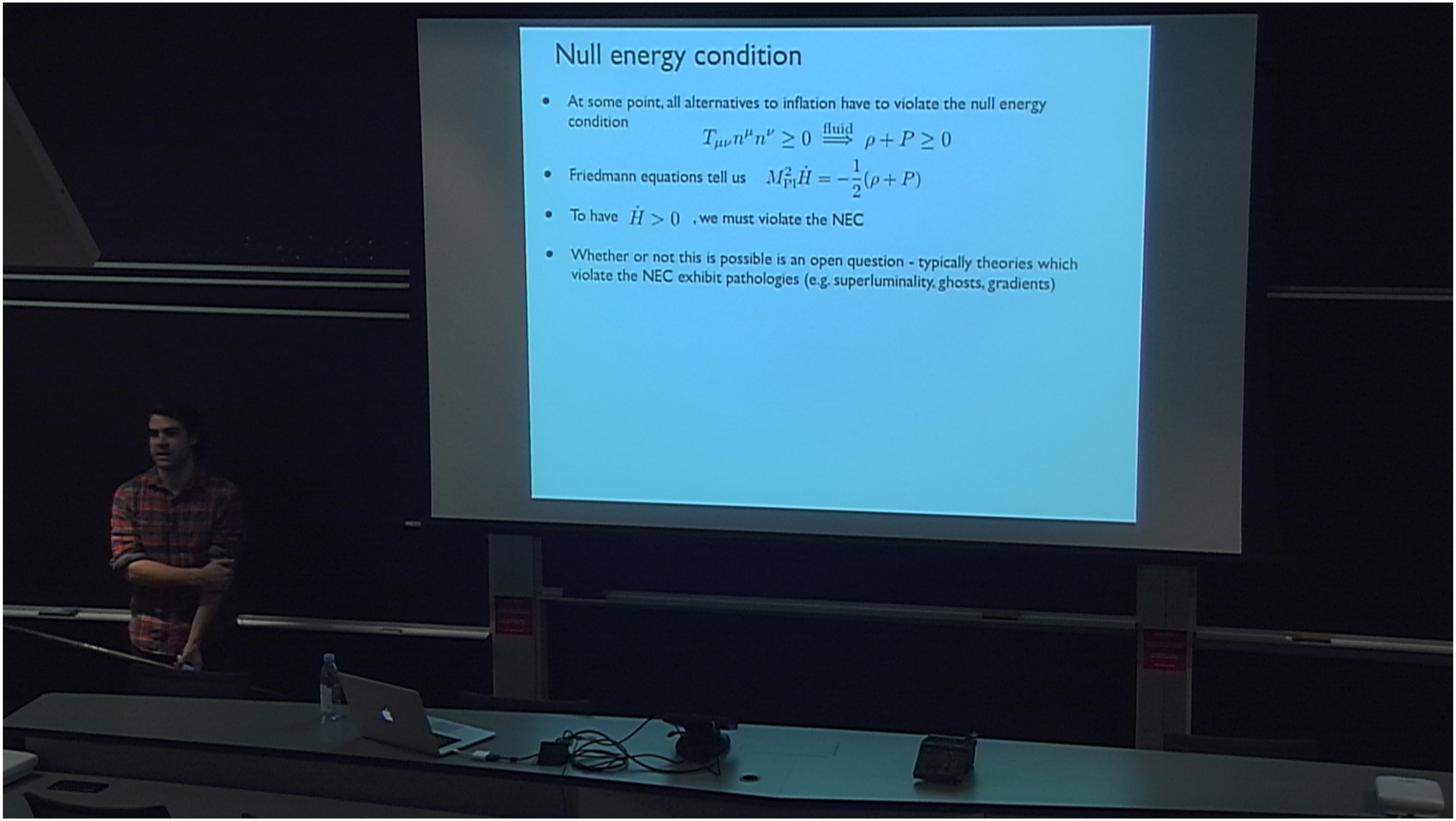
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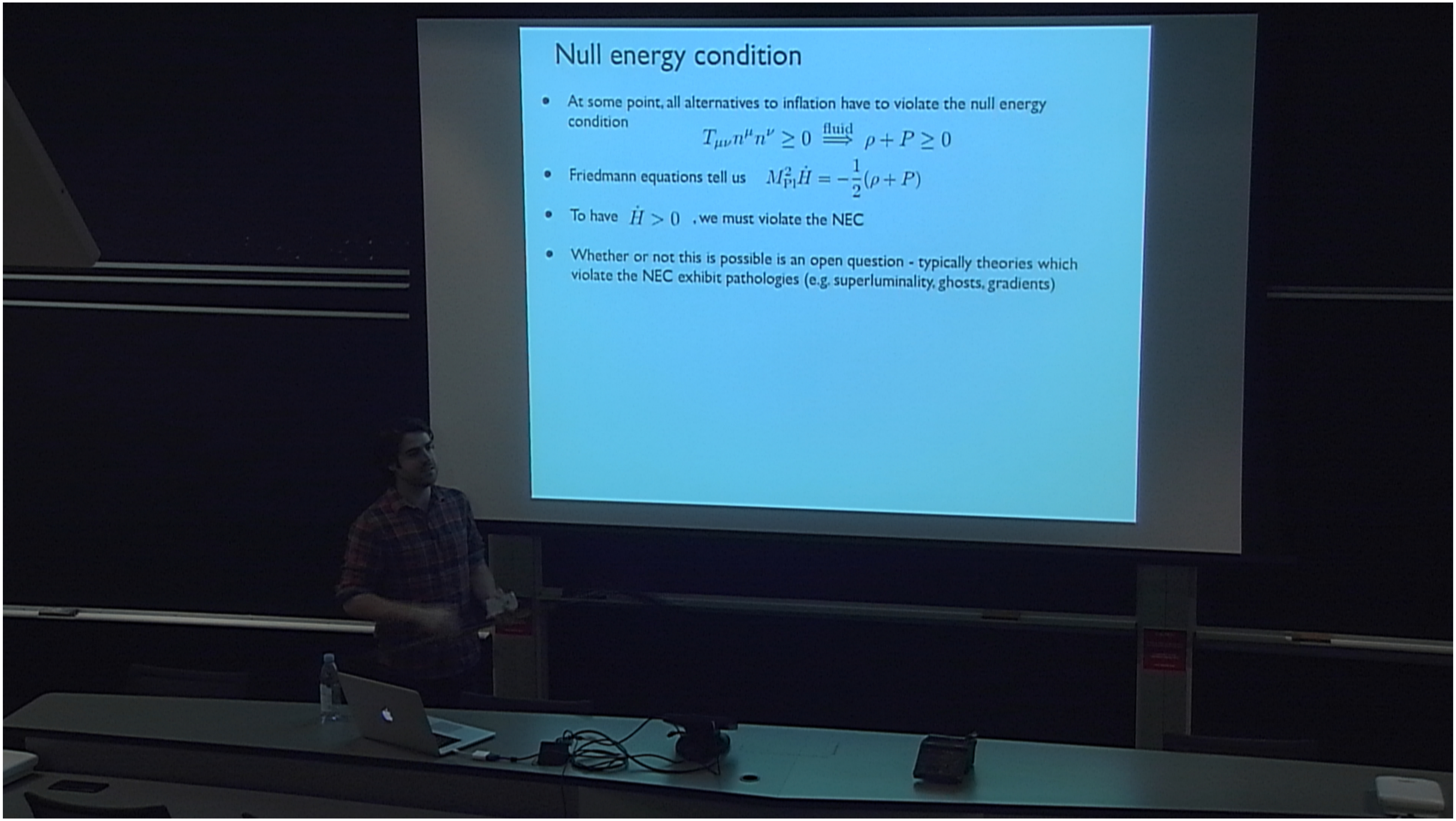
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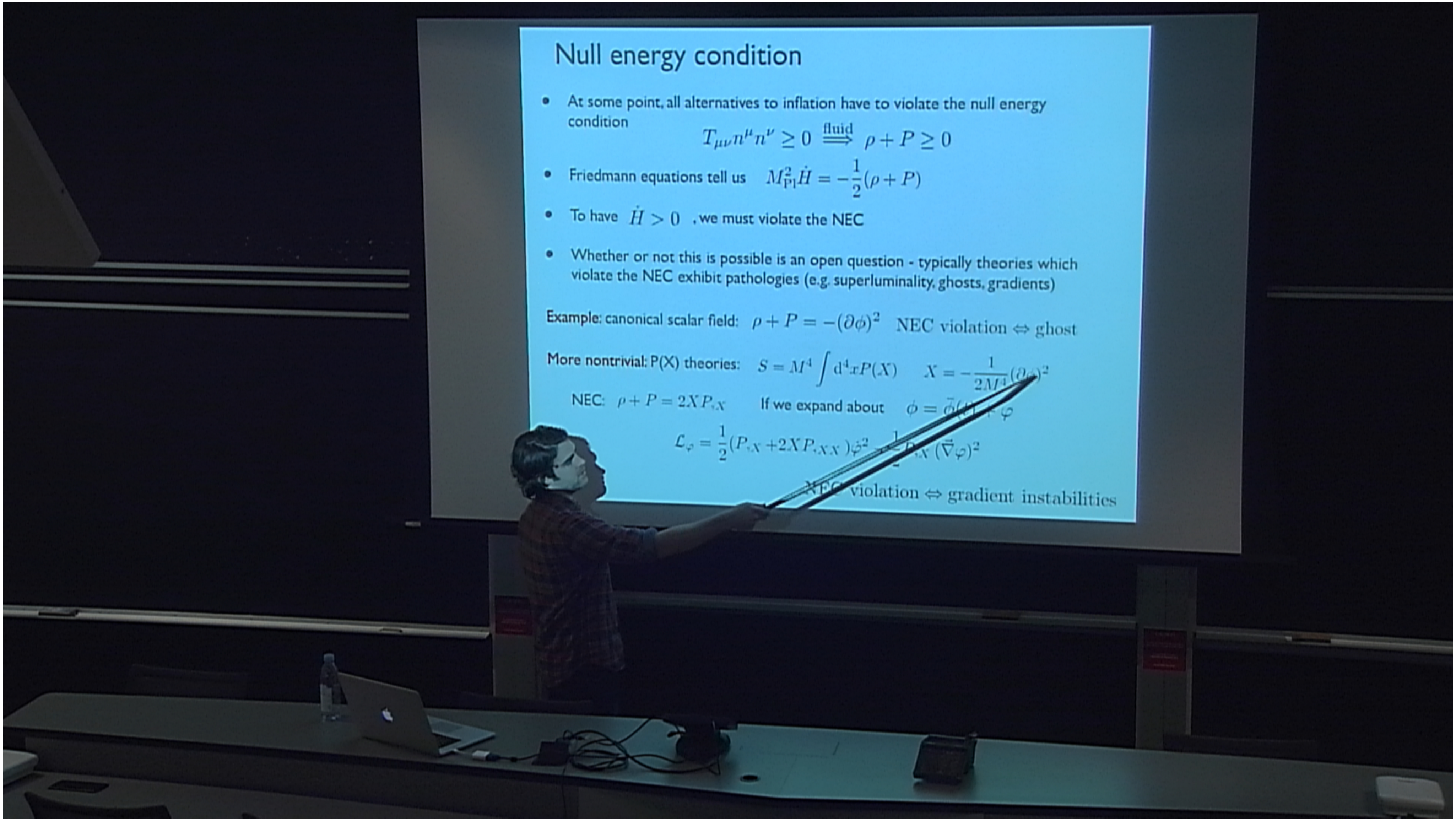
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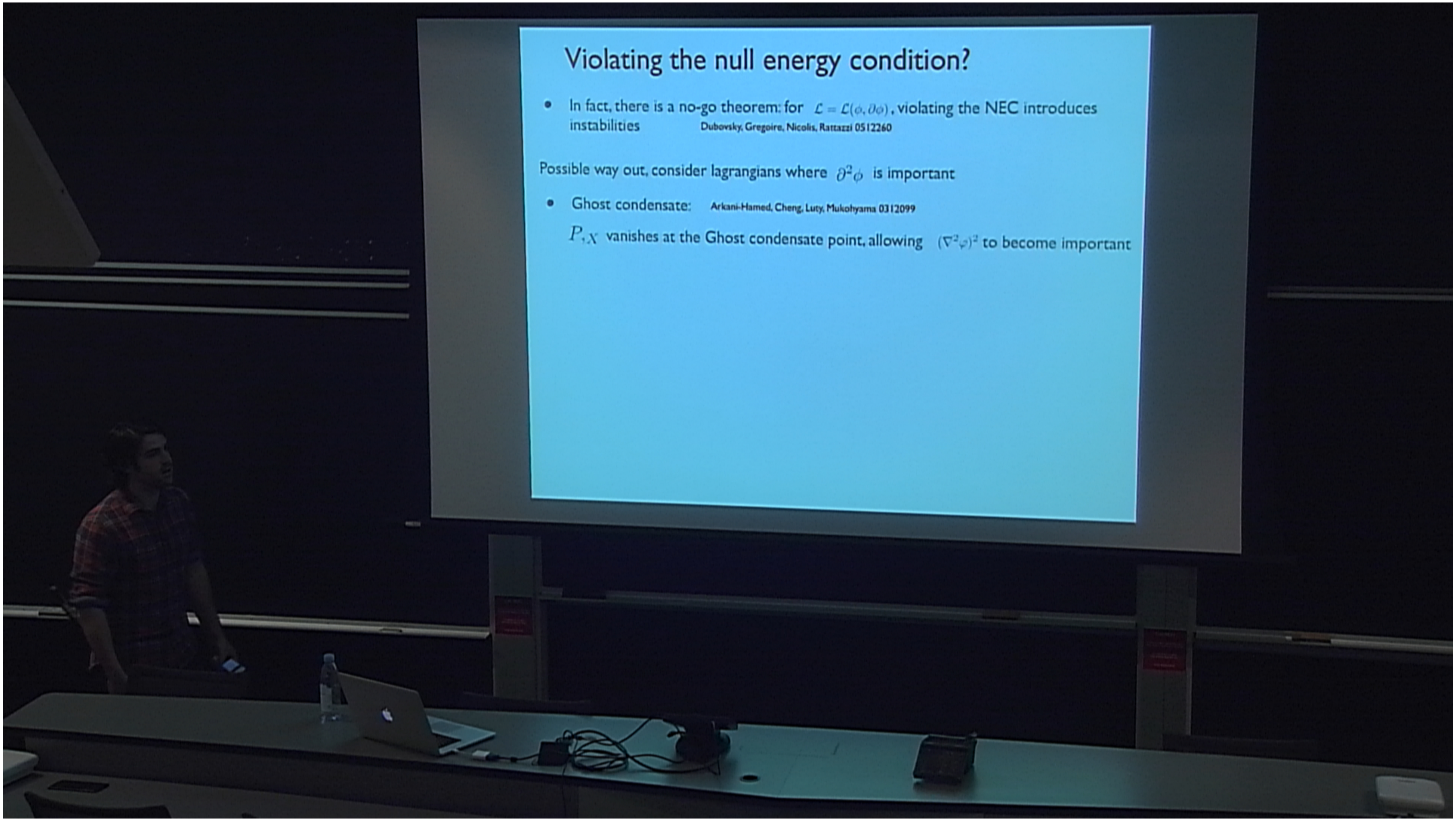
Violating the null energy condition?

- In fact, there is a no-go theorem: for $\mathcal{L} = \mathcal{L}(\phi, \partial\phi)$, violating the NEC introduces instabilities Dubovsky, Gregoire, Nicolis, Rattazzi 0512260

Possible way out, consider lagrangians where $\partial^2\phi$ is important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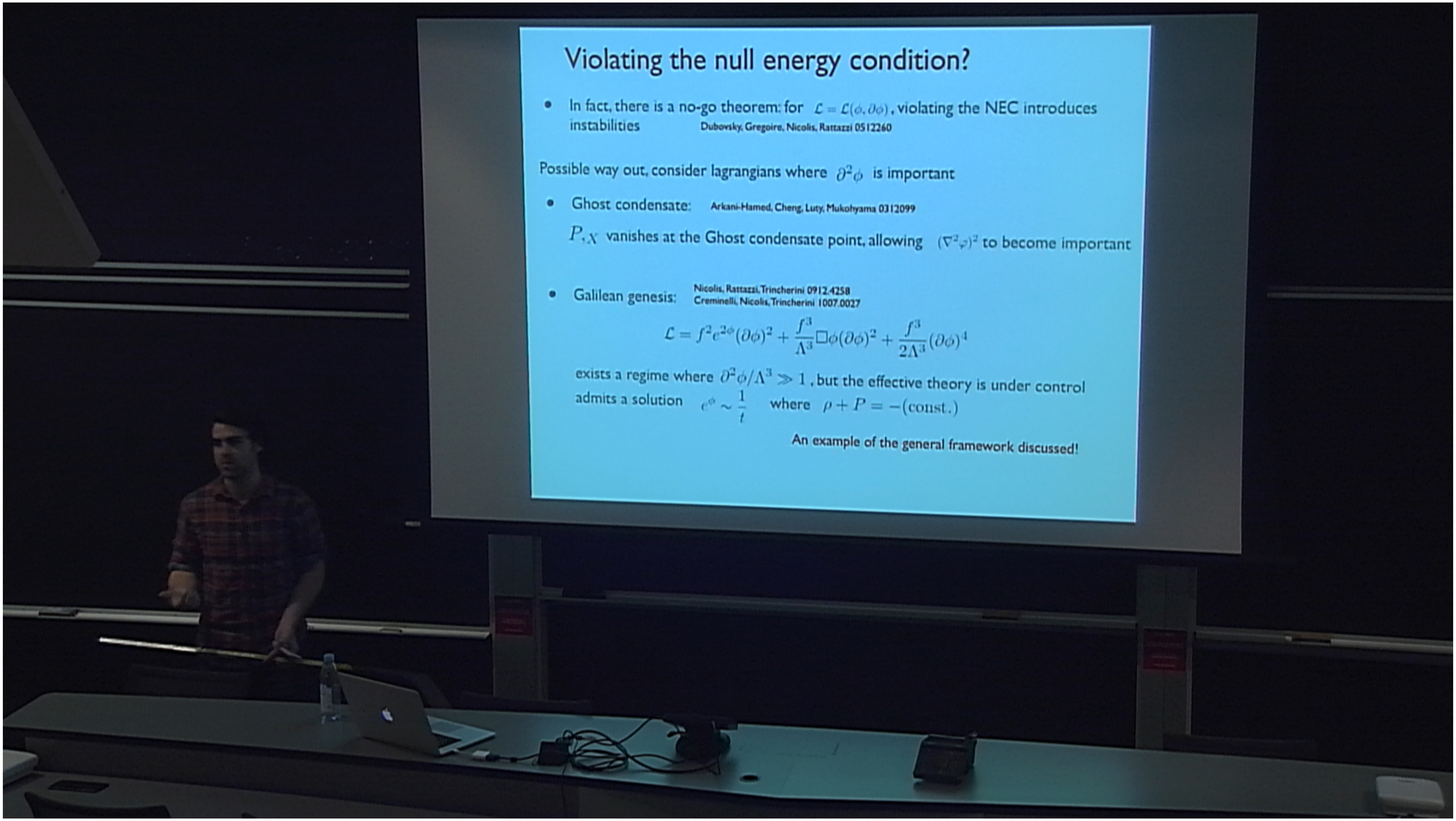
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Major drawback: neither of these theories have a Poincare vacuum

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$$\mathcal{L} = f^2 e^{2\phi}(\partial\phi)^2 + \frac{f^3}{\Lambda^3}\Box\phi(\partial\phi)^2 + \frac{f^3}{2\Lambda^3}(\partial\phi)^4$$

exists a regime where $\partial^2\phi/\Lambda^3 \gg 1$, but the effective theory is under control
admits a solution $e^\phi \sim \frac{1}{t}$ where $\rho + P = -(\text{const.})$

An example of the general framework discussed!

Major drawback: neither of these theories have a Poincare vacuum

Violating the null energy condition?

- In fact, there is a no-go theorem: for $\mathcal{L} = \mathcal{L}(\phi, \partial\phi)$, violating the NEC introduces instabilities Dubovsky, Gregoire, Nicolis, Rattazzi 0512260

Possible way out, consider lagrangians where $\partial^2\phi$ is important

- Ghost condensate: Arkani-Hamed, Cheng, Luty, Mukohyama 0312099

  $P_{,X}$ vanishes at the Ghost condensate point, allowing $(\nabla^2\varphi)^2$ to become important

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Toward a stable violation - DBI genesis

Hinterbichler, AJ, Khoury, Miller 1212.3607, PRL (2013)

This can be fixed!

- Consider the theory of a brane probing AdS

$$\mathrm{d}s^2 = Z^{-2}\mathrm{d}Z^2 + Z^2\eta_{\mu\nu}\mathrm{d}X^\mu\mathrm{d}X^\nu$$

- The induced metric on the brane is

$$\bar{g}_{\mu\nu} = \phi^2\eta_{\mu\nu} + \phi^{-2}\partial_\mu\phi\partial_\nu\phi$$

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- We consider the world-volume theory of Lovelock invariants and their boundary terms

$$\mathcal{L} = c_1\mathcal{L}_1 + c_2\mathcal{L}_2 + c_3\mathcal{L}_3 + c_4\mathcal{L}_4 + c_5\mathcal{L}_5$$

$$\begin{aligned}
\mathcal{L}_1 &= -\frac{1}{4}\phi^4 \\
\mathcal{L}_2 &= -\sqrt{-\bar{g}} \\
\mathcal{L}_3 &= \sqrt{-\bar{g}}K \\
\mathcal{L}_4 &= -\sqrt{-\bar{g}}\bar{R} \\
\mathcal{L}_5 &= \frac{3}{2}\sqrt{-\bar{g}}\left(-\frac{K^3}{3} + K^2_{\mu\nu}K - \frac{2}{3}K^3_{\mu\nu} - 2\bar{G}_{\mu\nu}K^{\mu\nu}\right)
\end{aligned}$$

- Then the goal is to choose the coefficients so that the theory has nice properties

How do we do?

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- Coefficients can be chosen to violate the NEC; it represents an improvement over previous attempts

	Ghost condensate	**Galilean genesis**	**DBI genesis**
~~NEC~~ vacuum	✓	✓	✓
No ghosts	✓	✓	✓
Sub-luminality	✓	✗	✓
Poincaré vacuum	✗	✗	✓
No ghosts			✓
S-Matrix analyticity			✓
Sub-luminality			✗
Radiative stability	✓	✓	✓
BH Thermodynamics	✗	?	?

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Conclusions

- Three different sets of symmetries to describe the near scale-invariance of primordial perturbations
  - multi-field inflation - $so(4,1)$
  - single-field inflation - $so(4,1) \rightarrow \text{translations \& rotations}$
  - conformal mechanism - $so(4,2) \rightarrow so(4,1)$
- Large scale homogeneity and isotropy:
  - inflation - accelerated expansion
  - conformal mechanism - very stiff equation of state; no gravity waves

The different symmetries show up in correlation functions (particularly through soft limits), and are observationally distinguishable

- Is it possible to violate the NEC in a completely healthy way?

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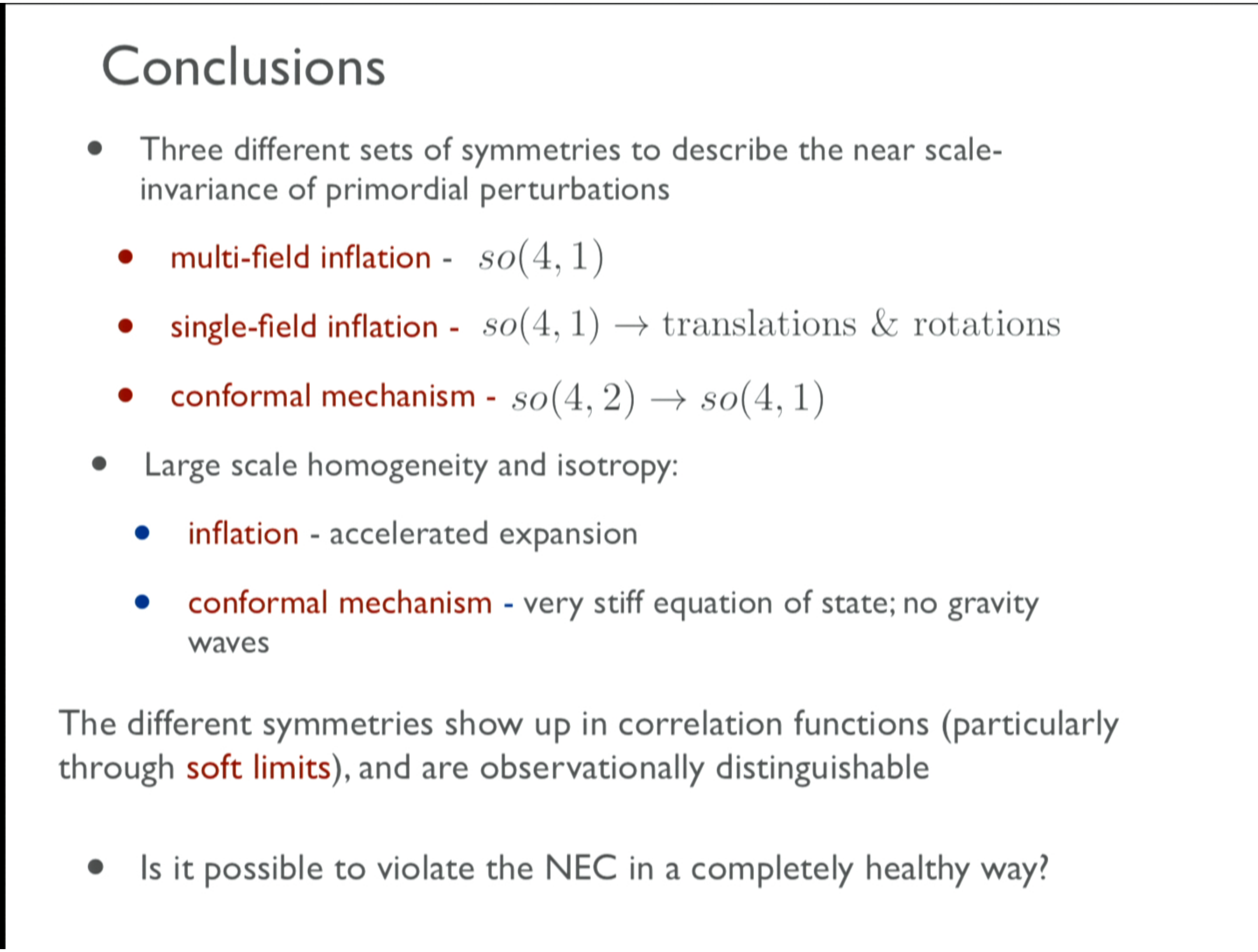
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