Title: Asymptotically safe matrix models

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Abstract: The functional renormalization group is a tool in the systematic search for Euclidean QFTs that works with very little input: All one needs to specify is a field content, symmetries and a notion of locality. The functional renormalization group then allows one to scan this theory space for bare actions for which the path integral can be performed nonperturbatively. These actions appear as fixed points (and relevant deformations) of the renormalization group flow (so-called asymptotic safety). Such a systematic search has so far not been performed for the tensor model approach to quantum gravity. We investigate matrix models for 2-dimensional gravity and the Grosse-Wulkenhaar model, which is a simple tensor model for a 4D noncommutative scalar field theory, as a first step. I will report on work, done in collaboration with Astrid Eichhorn and Alessandro Sfondrini, where we confirmed asymptotic safety purely through functional RG methods. Based on the lessons learned from these models I will summarize how the usual continuum RG approach has to be generalized for the investigation of tensor models and conclude with a recipe for the investigation of general tensor models.

Asymptotically Safe Matrix Models

work with A. Eichhorn, A. Stondrini, I. Vilensky

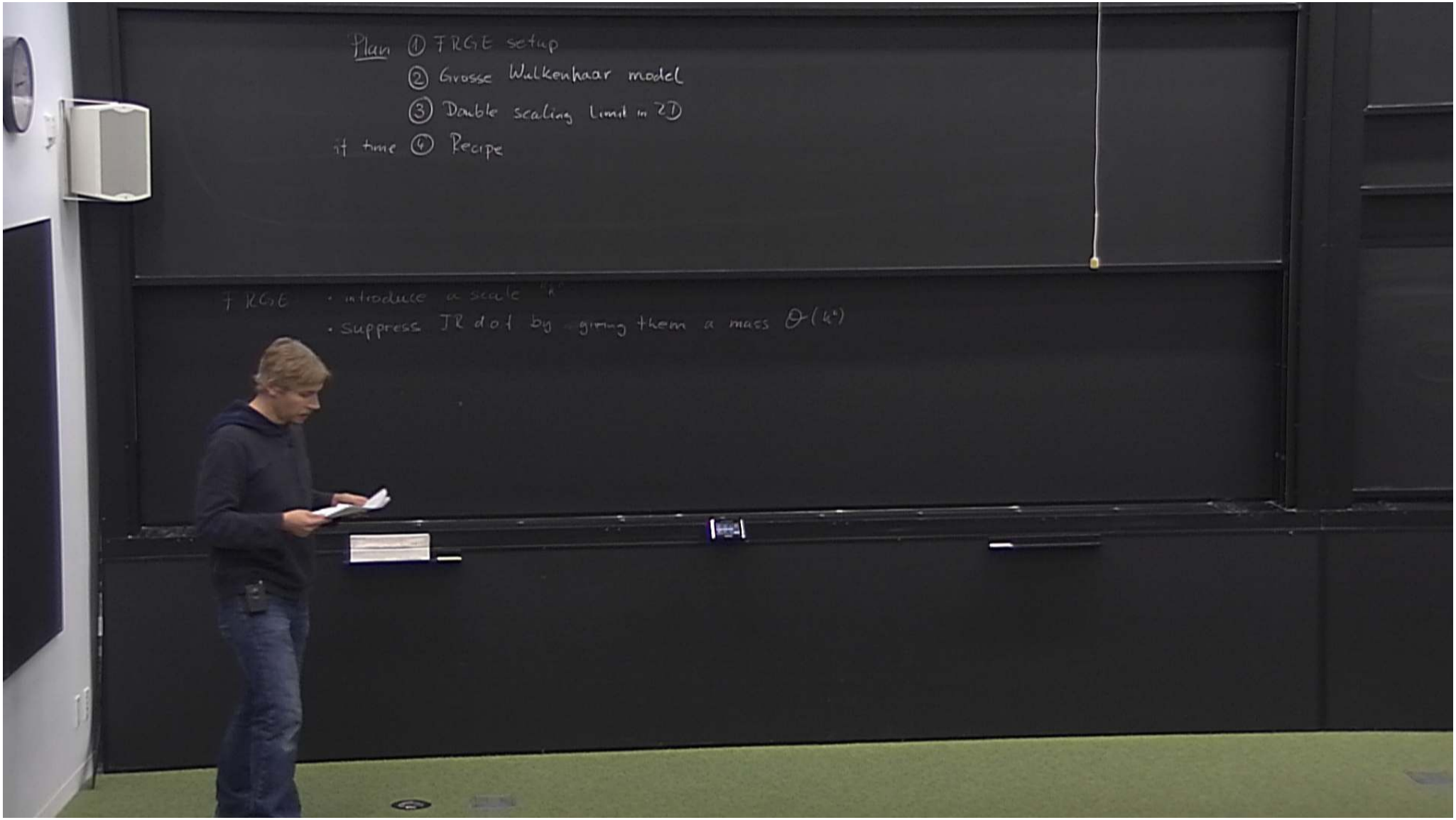
→ T.K., A Stondrini: IJMP A26 (2011) 4009

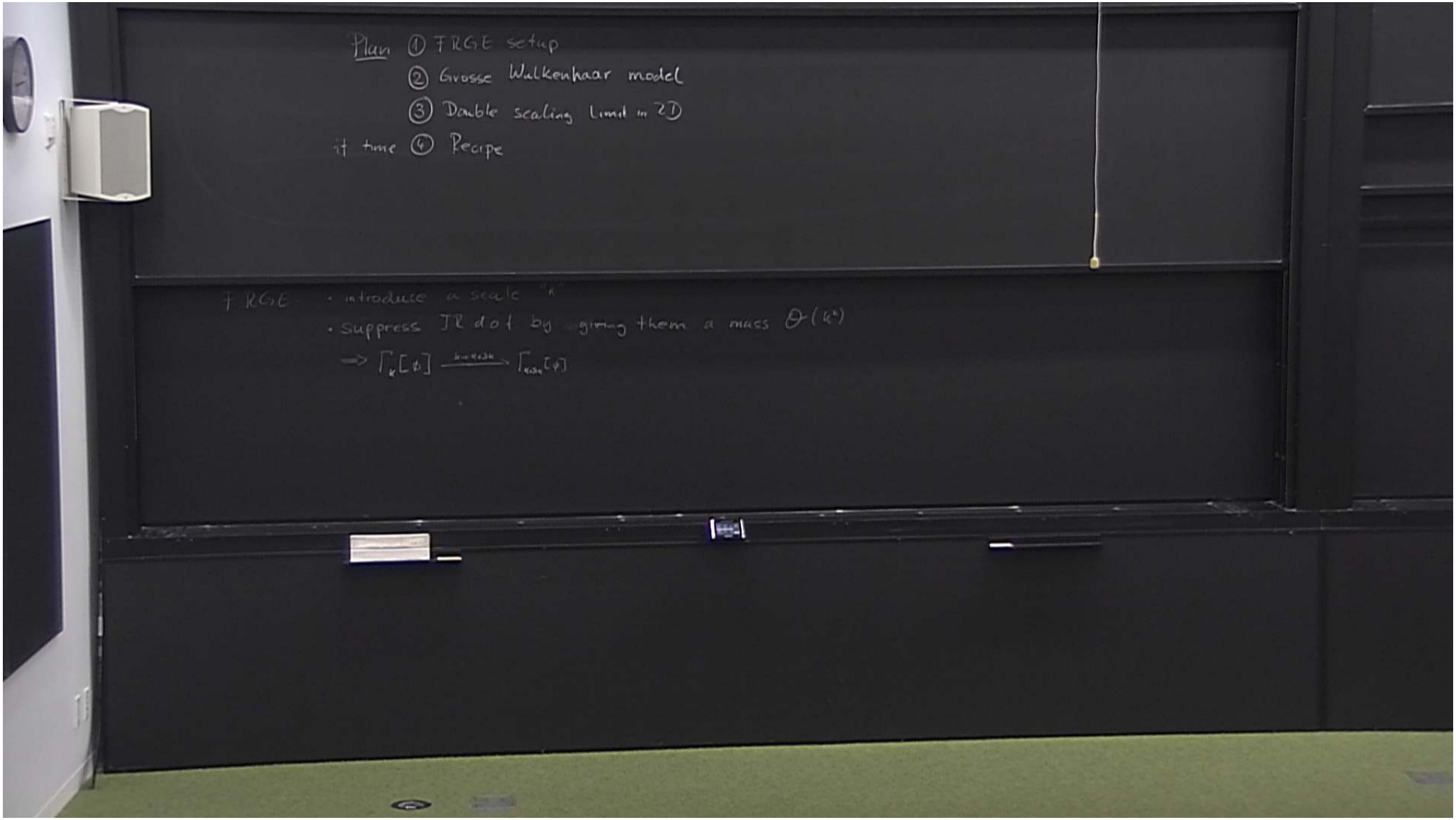
→ A Eichhorn, T.K.: PRD 88 (2013) iss 8, 084016

→ unpublished w/h I. Vilensky

Main message: ① establish FRGE (main tool in AS program) as a tool in matrix and tensor models

② confirm known AS (from constructive prog) w/h FRGE


Plan ① FRGE setup
② Grosse Wulkenhaar model
③ Double scaling limit in 2D
if time ④ Recipe
f KGE • introduce a scale 'k'
• suppress IR d.o.f by giving them a mass $\Theta(k^2)$


Plan ① FRGE setup
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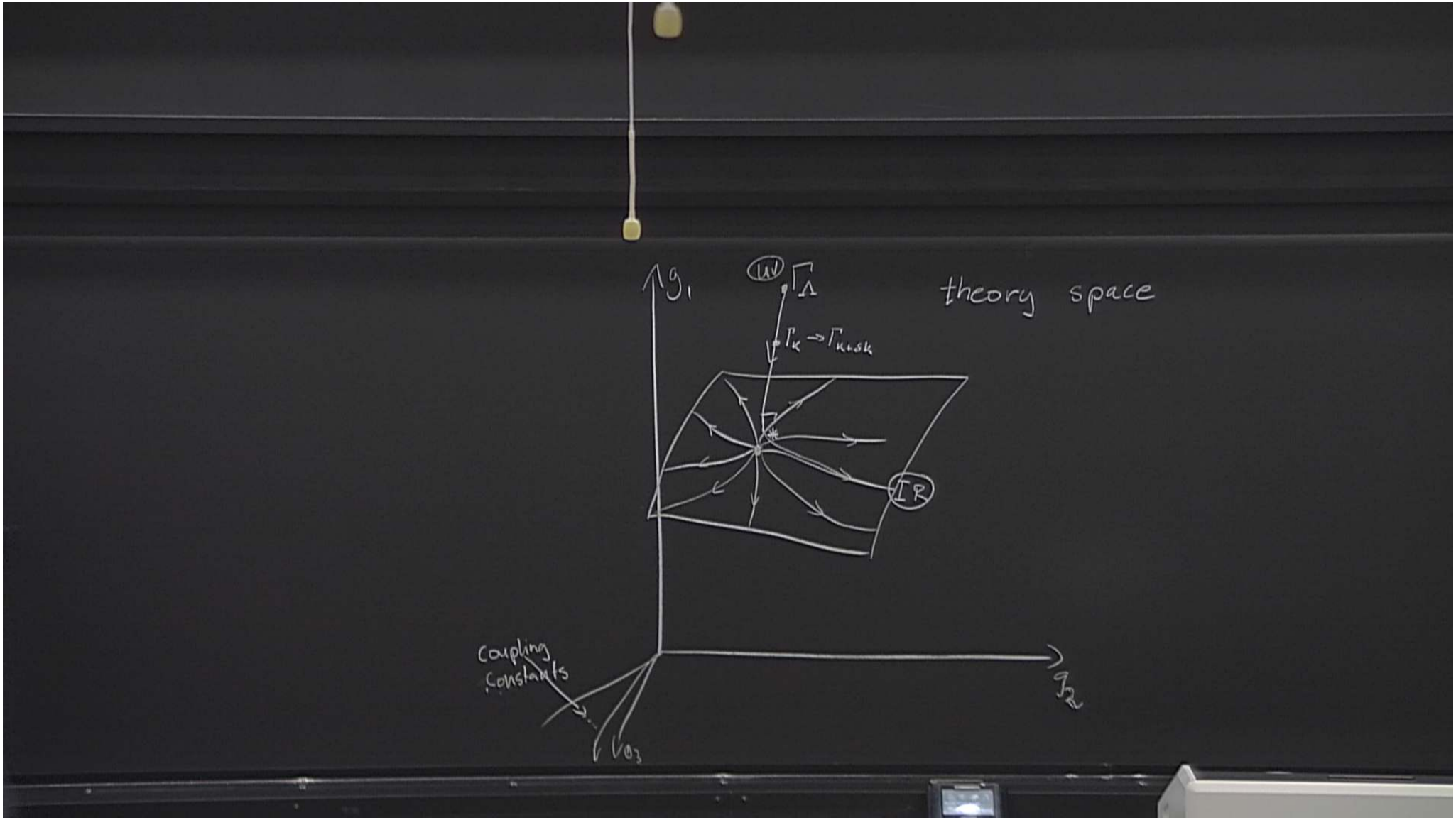

theory space
UV
IR
Coupling Constants

FRGE

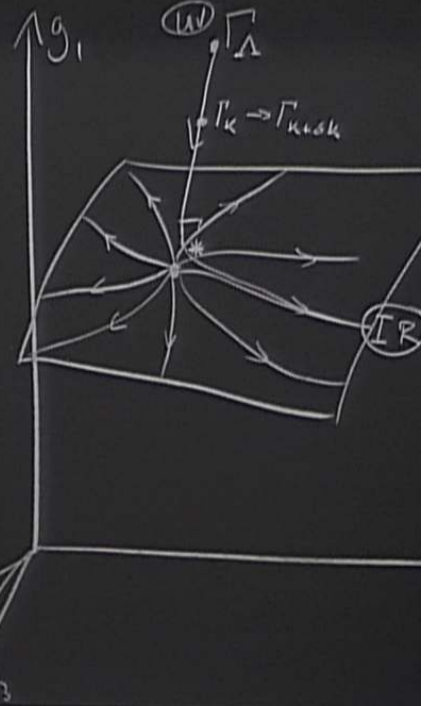
- introduce a scale "k"
- suppress IR dof by giving them a mass $\Theta(k^2)$

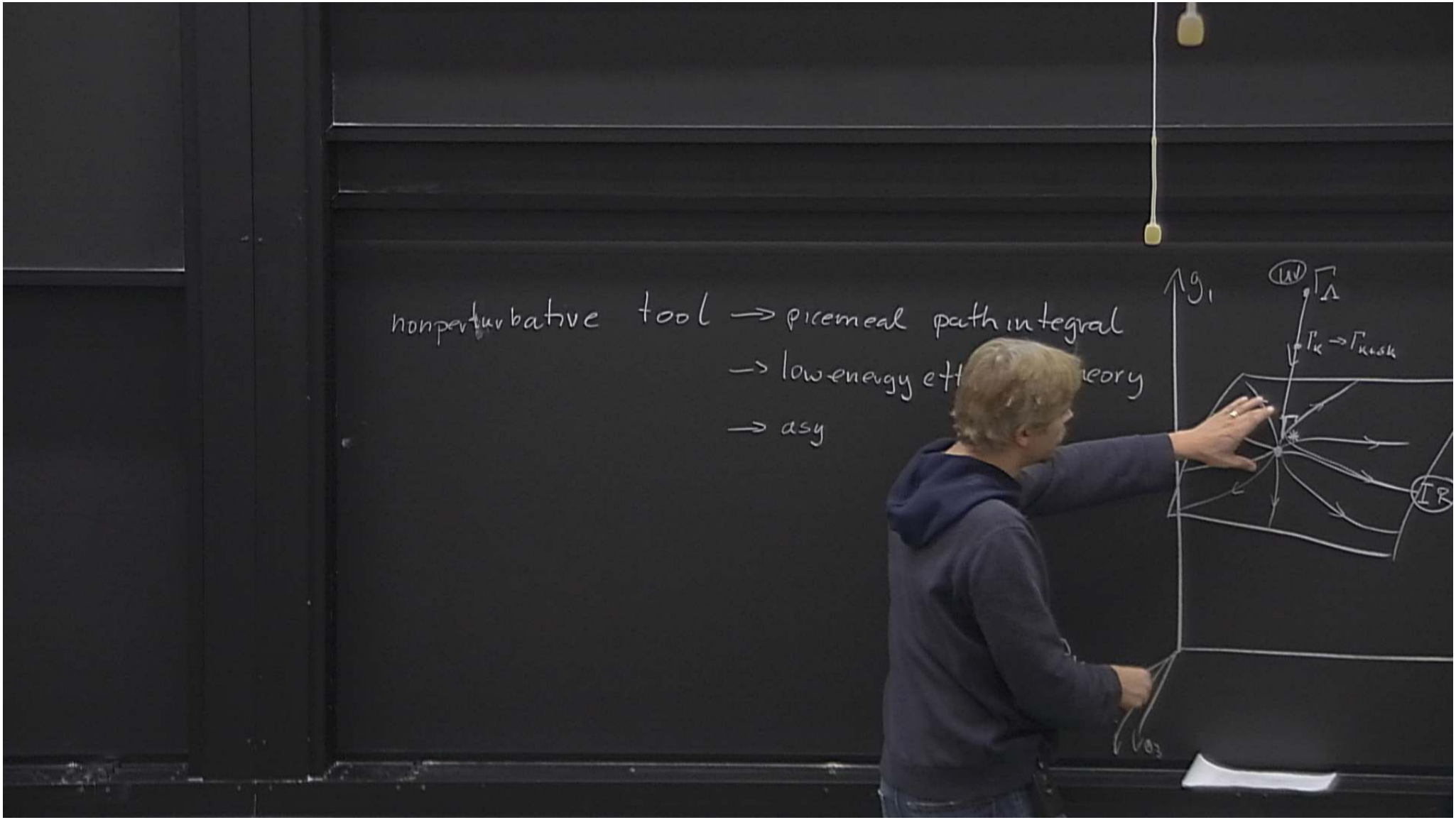
$$\Rightarrow \Gamma_k[\phi] \xrightarrow{k \to k+\delta k} \Gamma_{k+\delta k}[\phi]$$

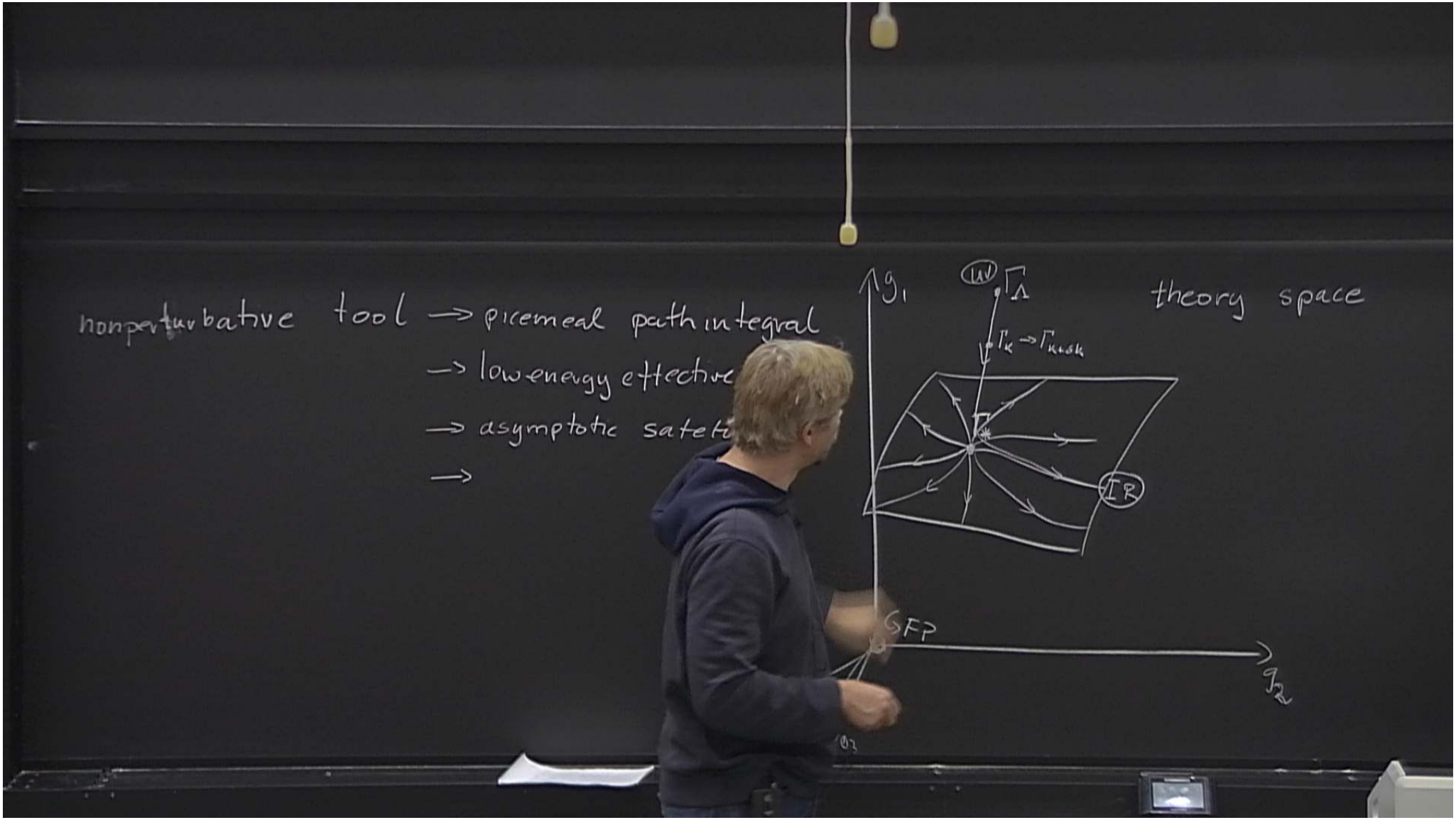
$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left(\frac{k\partial_k R_k(q)}{\Gamma_k^{(2)}[\phi] + R_k} \right) - k\partial_k \mathrm{Tr} \ln\left(\Gamma_S^{(2)} + R_k \right)\Big|_{S=k}$$

nonperturbative tool → piecemeal path integral

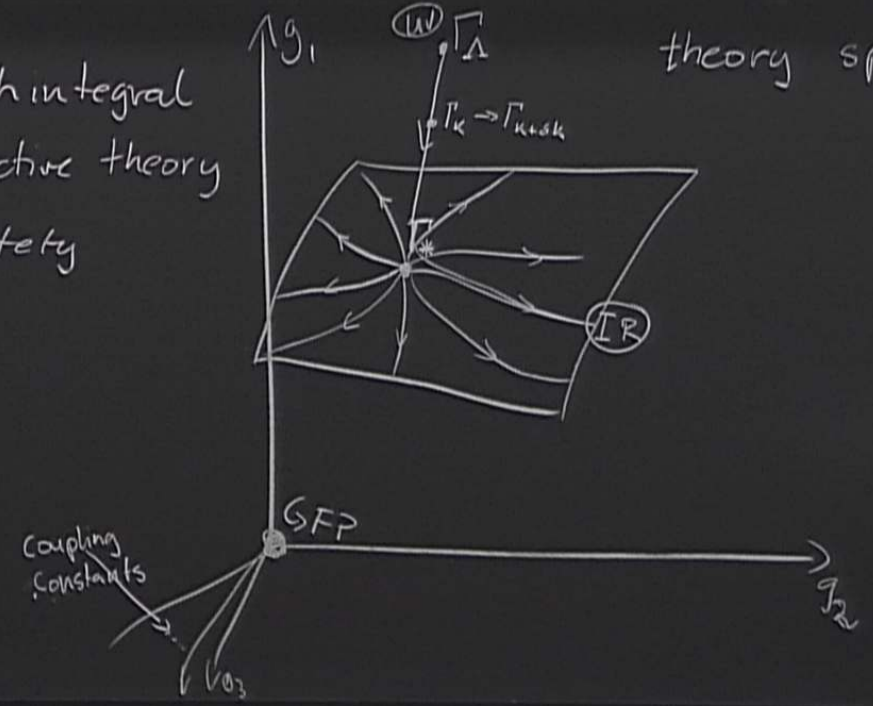
→

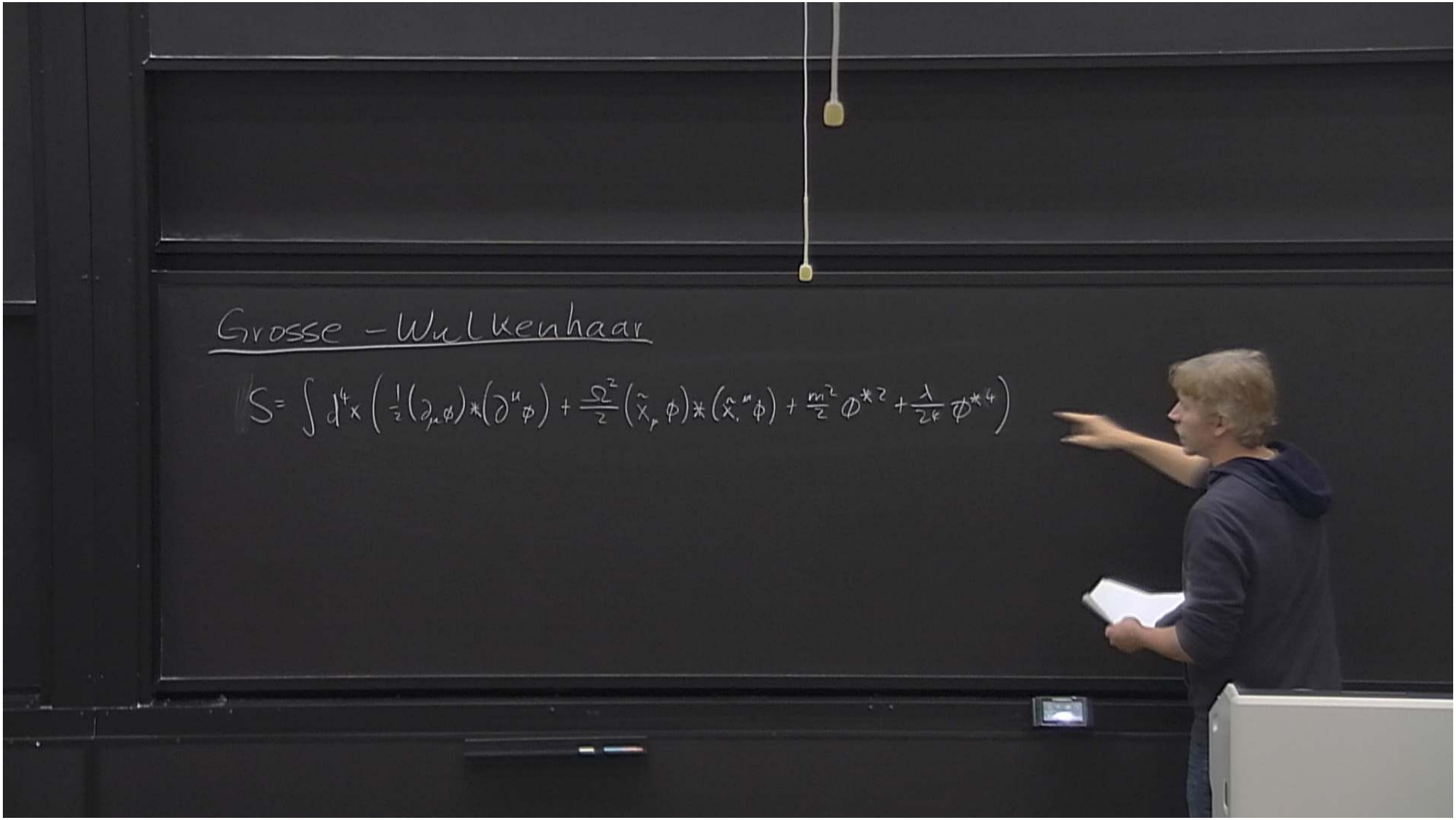



nonperturbative tool → piecemeal path integral
→ low-energy eff eory
→ asy
g_1
UV
Γ_Λ
$\Gamma_k \to \Gamma_{k=0}$
IR


nonperturbative tool → piecemeal path integral
→ low-energy effective
→ asymptotic safety
→
theory space
UV
IR
FP

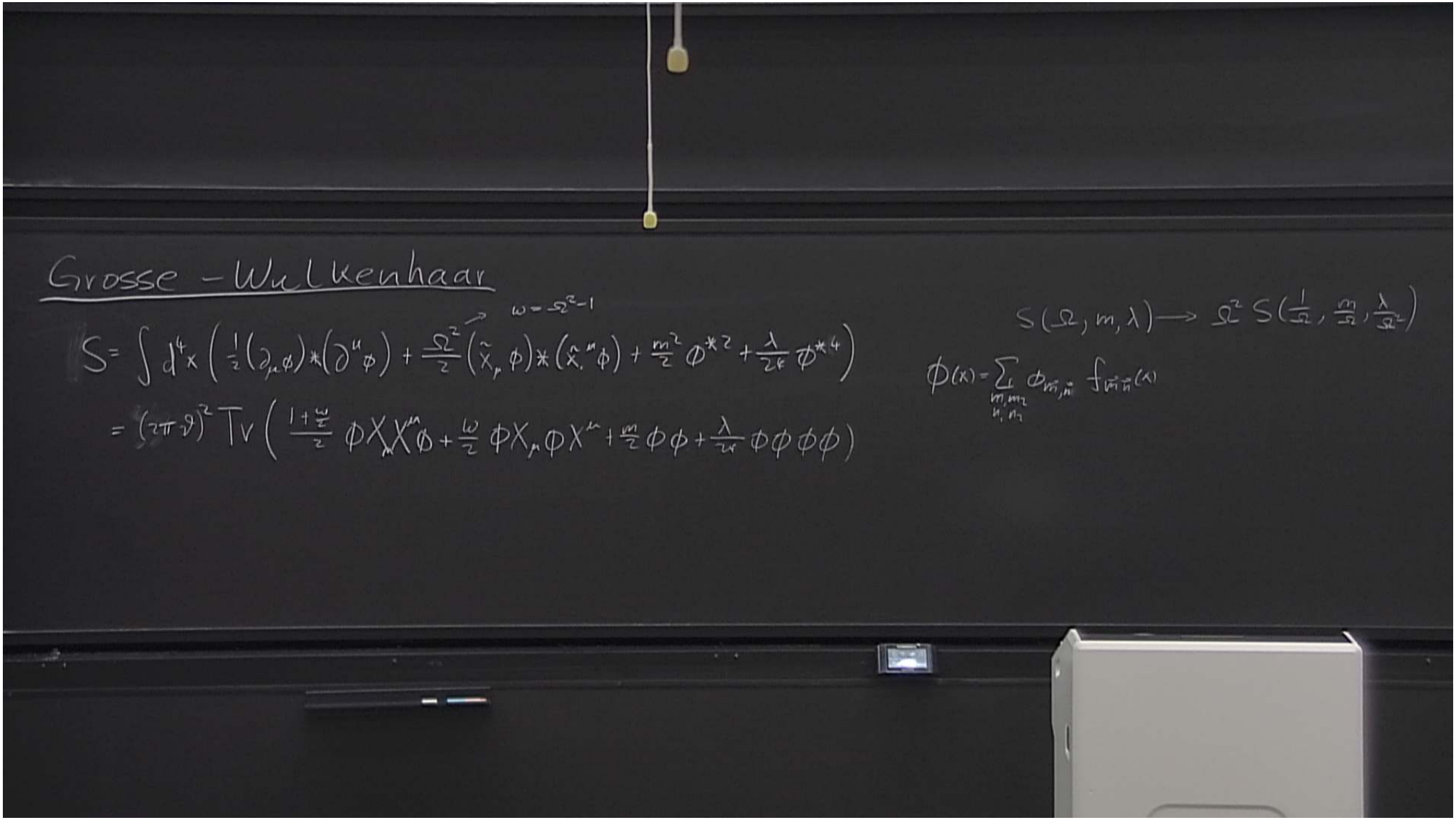
nonperturbative tool → piecemeal path integral
→ low-energy effective theory
→ asymptotic safety
→ phase diagram




Grosse - Wulkenhaar
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{24}\phi^{*4} \right)$

Grosse - Wulkenhaar

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{2\kappa}\phi^{*4} \right)$$


Grosse - Wulkenhaar
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{24}\phi^{*4} \right)$
$\omega = \Omega^2 - 1$
$= (2\pi\vartheta)^2 \mathrm{Tr}\left(\frac{1+\omega}{2}\phi X_m X^m \phi + \frac{\omega}{2}\phi X_\mu \phi X^\mu + \frac{m}{2}\phi\phi + \frac{\lambda}{24}\phi\phi\phi\phi \right)$
$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S\left(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2}\right)$
$\phi(x) = \sum_{\substack{m_1, m_2 \\ n_1, n_2}} \phi_{\vec{m},\vec{n}} f_{\vec{m},\vec{n}}(x)$

Grosse - Wulkenhaar

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{24}\phi^{*4} \right) \qquad \omega = \Omega^2 - 1$$

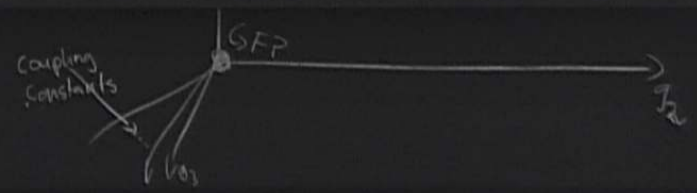
$$= (2\pi\vartheta)^2 \mathrm{Tr}\left(\frac{1+\omega}{2}\phi X_\mu X^\mu \phi + \frac{\omega}{2}\phi X_\mu \phi X^\mu + \frac{m^2}{2}\phi\phi + \frac{\lambda}{24}\phi\phi\phi\phi \right)$$

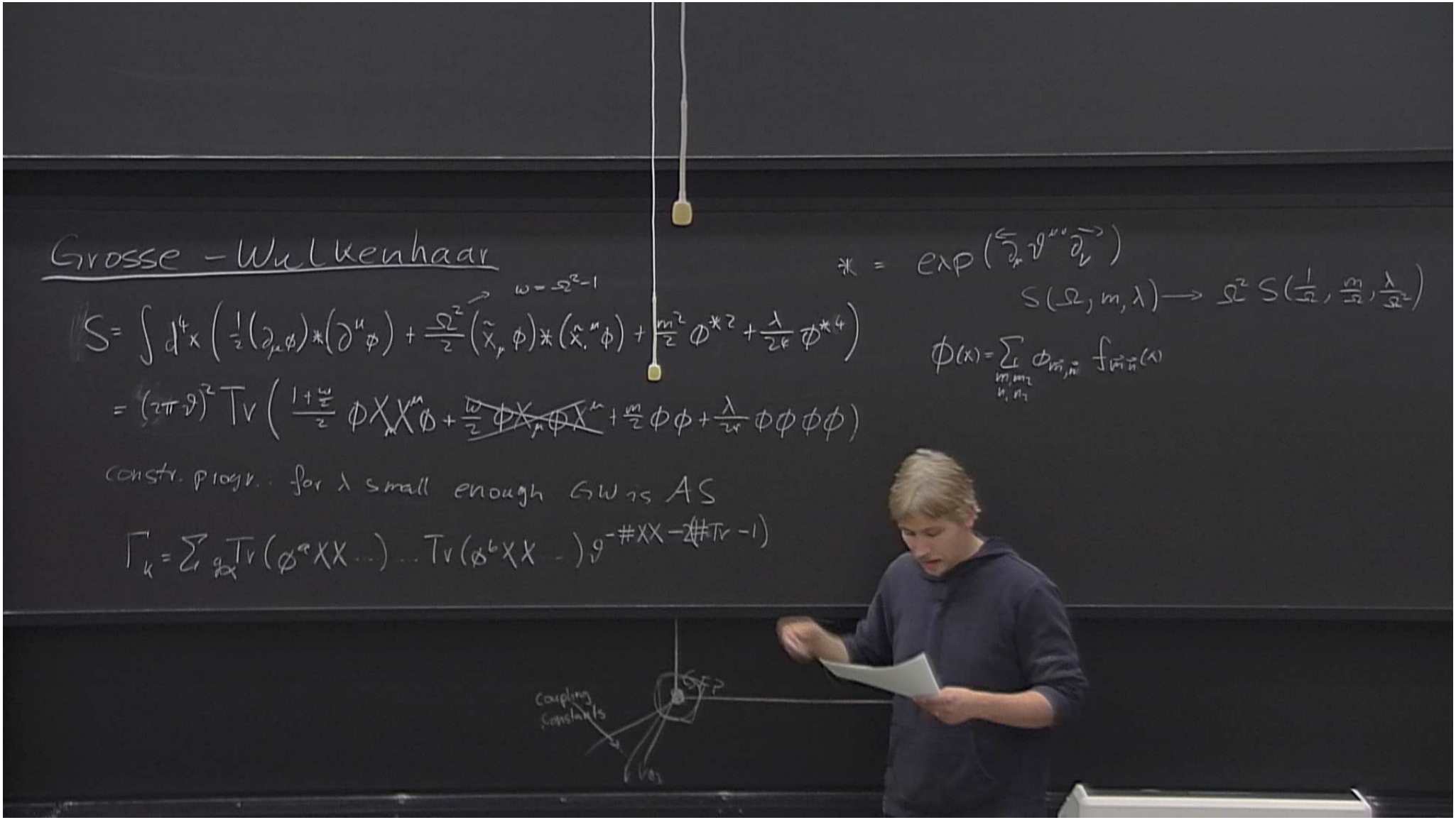
$$* = \exp\left(\overleftarrow{\partial_\mu} \vartheta^{\mu\nu} \overrightarrow{\partial_\nu} \right)$$

$$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S\left(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2}\right)$$

$$\phi(x) = \sum_{\substack{m,n \\ m_2,n_2}} \phi_{\vec{m},\vec{n}}\, f_{\vec{m},\vec{n}}(x)$$

cons




Grosse - Wulkenhaar
$\omega = \Omega^2 - 1$
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{2^4}\phi^{*4} \right)$
$= (2\pi\vartheta)^2 \mathrm{Tr}\left(\frac{1+\omega}{2}\phi X X^\mu \phi + \frac{\omega}{2}\phi X_\mu \phi X^\mu + \frac{m^2}{2}\phi\phi + \frac{\lambda}{2^4}\phi\phi\phi\phi \right)$
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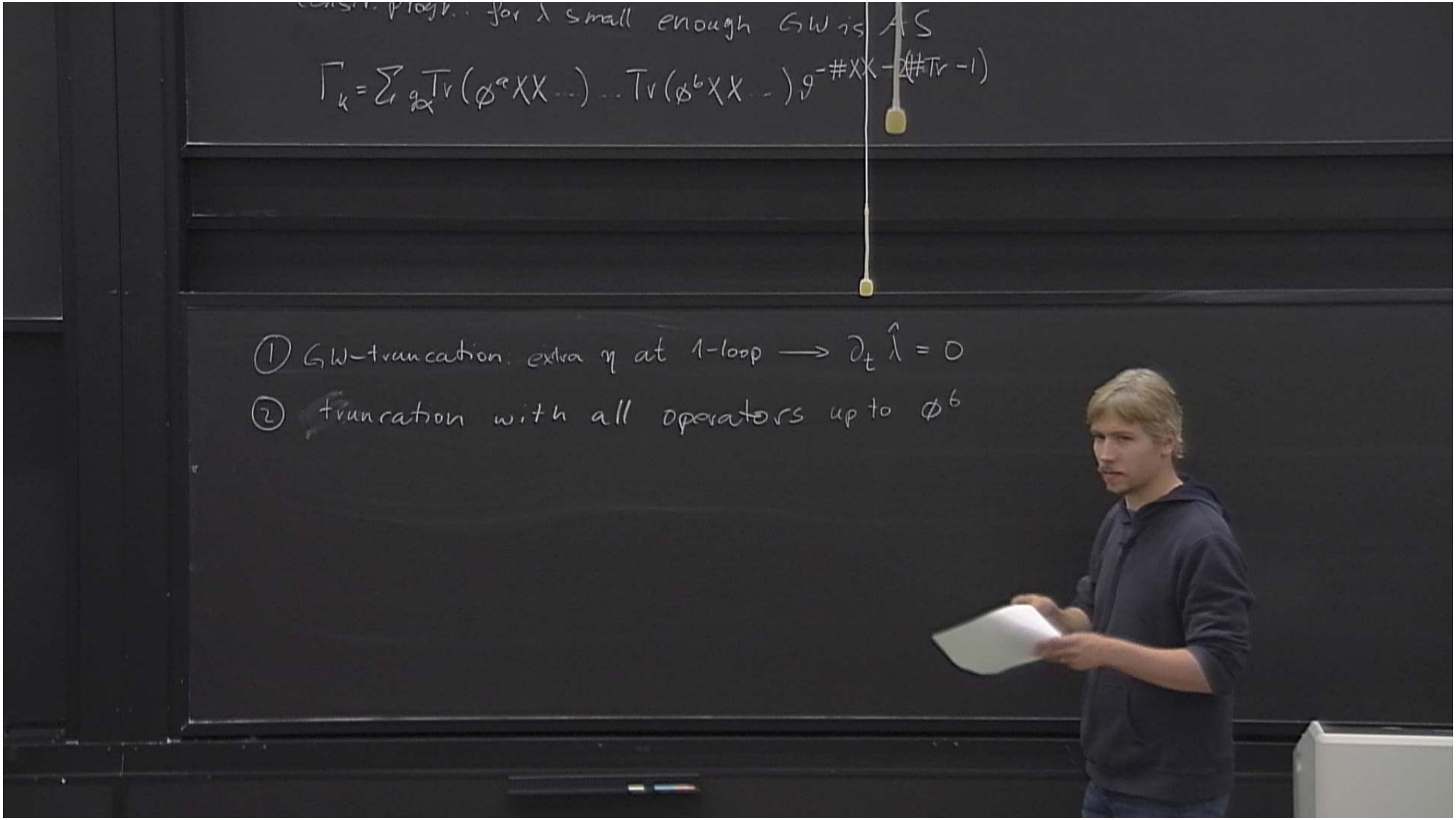
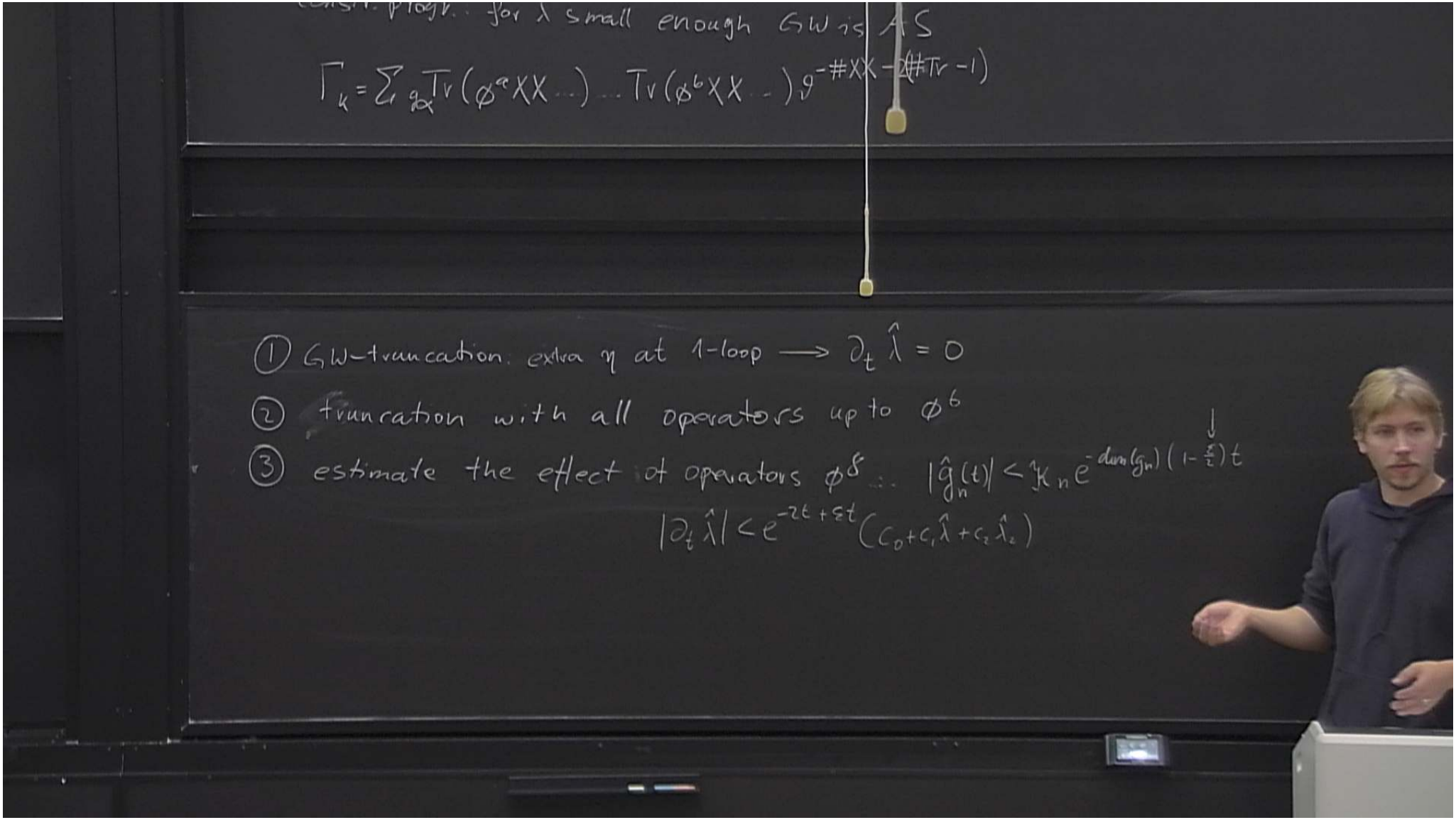
$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S\left(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2}\right)$
$\phi(x) = \sum_{\substack{m_1, m_2 \\ n_1, n_2}} \phi_{m_1 m_2, n_1 n_2} f_{m_1 n_1}(x)$
constr. progr.: for λ small enough GW is AS
$\Gamma_k = \sum_i g_i \mathrm{Tr}(\phi^a X X \ldots) \ldots \mathrm{Tr}(\phi^b X X \ldots) \vartheta^{-\#XX - 2(\#\mathrm{Tr} - 1)}$
coupling constants
RG?


for λ small enough GW is AS
$\Gamma_k = \sum_i g_{\alpha_i} \mathrm{Tr}(\phi^a XX\ldots)\ldots\mathrm{Tr}(\phi^b XX\ldots)\, g^{-\#XX-(\#\mathrm{Tr}-1)}$
① GW-truncation: extra η at 1-loop $\longrightarrow \partial_t \hat{\lambda} = 0$
② truncation with all operators up to ϕ^6


for λ small enough GW is AS
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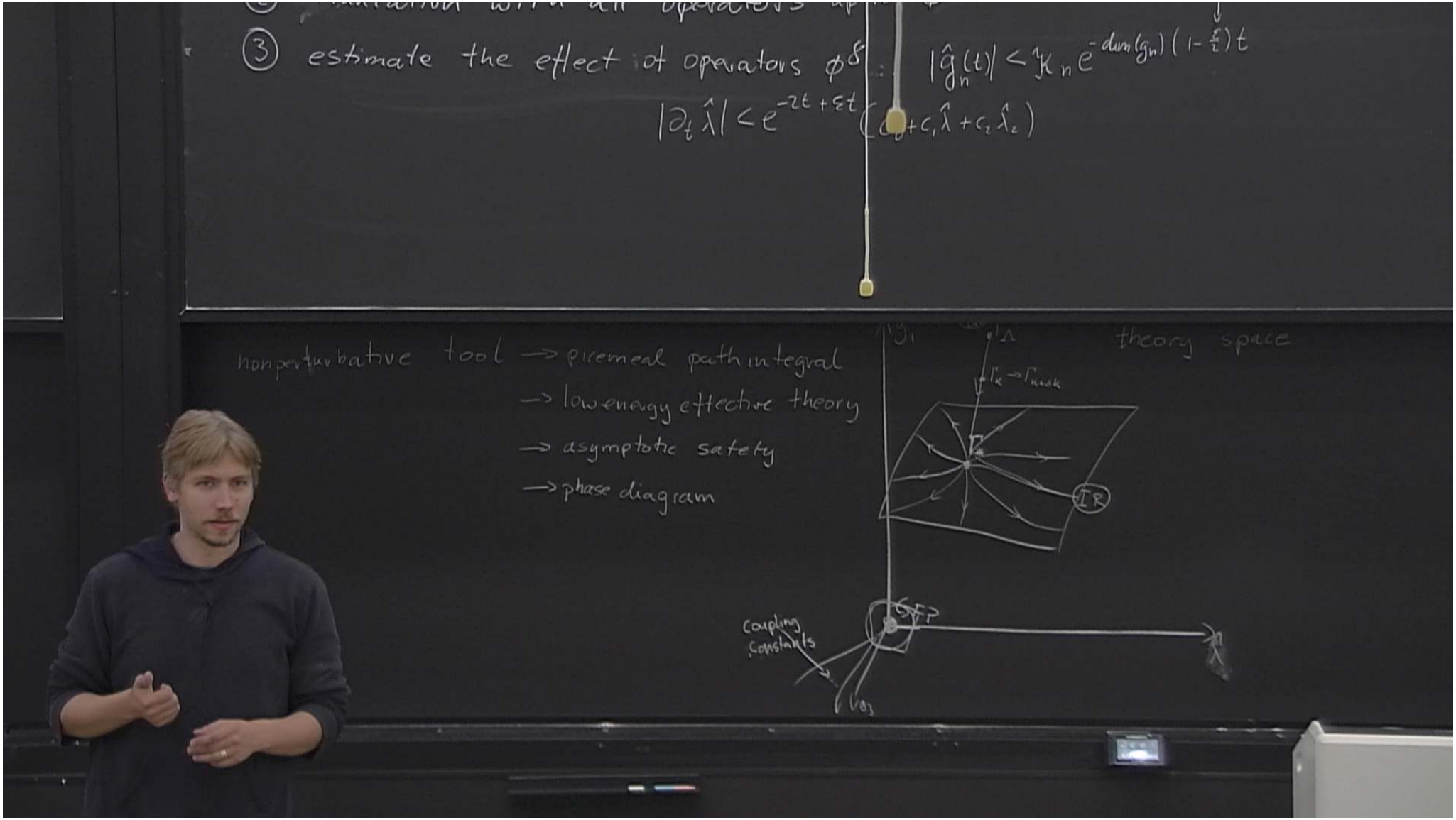
① GW-truncation: extra η at 1-loop $\longrightarrow \partial_t \hat{\lambda} = 0$
② truncation with all operators up to ϕ^6
③ estimate the effect of operators ϕ^8 ... $|\hat{g}_n(t)| < \gamma_{k_n} e^{-\dim(g_n)\left(1-\frac{\varepsilon}{2}\right)t}$
$|\partial_t \hat{\lambda}| < e^{-2t+\varepsilon t}(c_0 + c_1\hat{\lambda} + c_2\hat{\lambda}^2)$

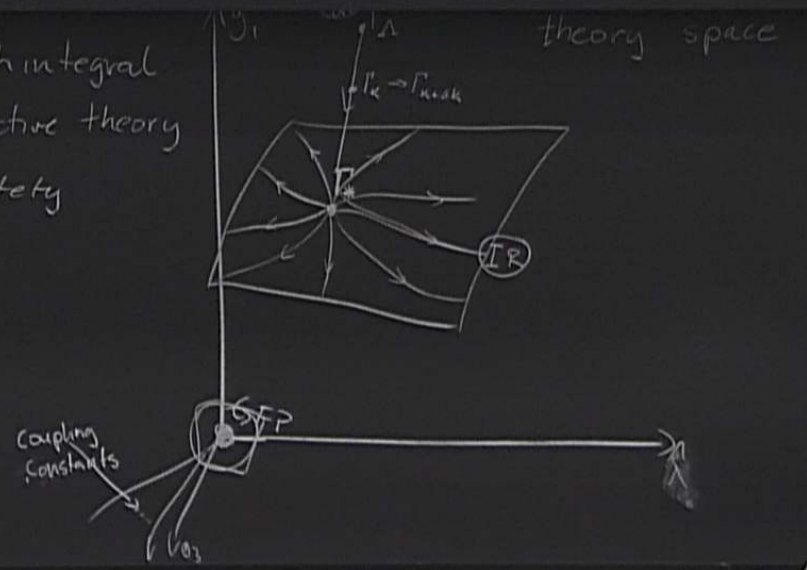

③ estimate the effect of operators ϕ^δ : $|\hat{g}_n(t)| < \mathcal{K}_n e^{-\dim(g_n)(1-\frac{\delta}{2})t}$
$|\partial_t \hat{\lambda}| < e^{-2t+\varepsilon t}(c_0 + c_1\hat{\lambda} + c_2\hat{\lambda}_2)$
nonperturbative tool → piecemeal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram
theory space
IR
GFP
coupling constants

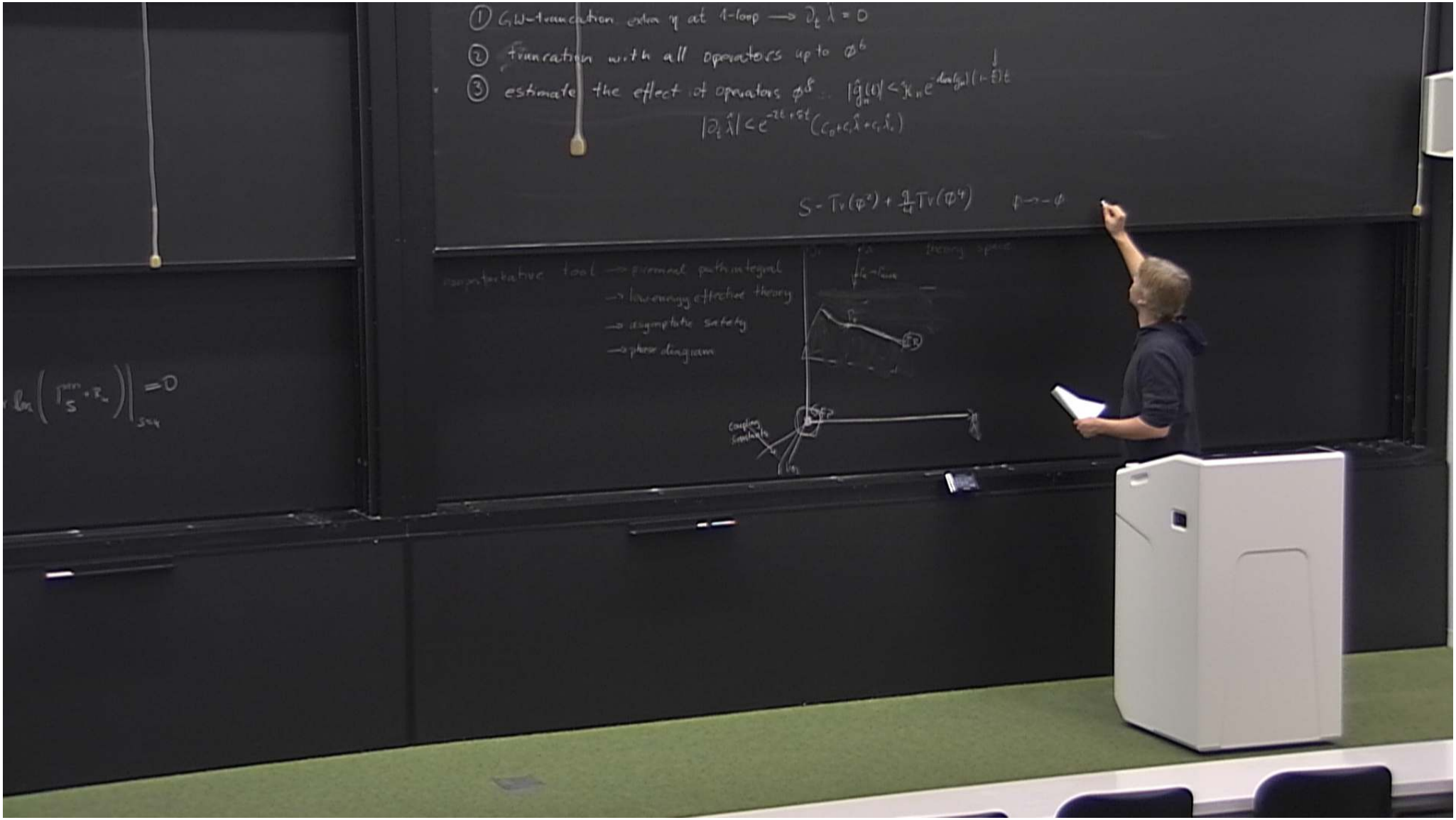
③ estimate the effect of operators ϕ^δ : $|\hat{g}_n(t)| < \kappa_n e^{-\dim(g_n)\left(1-\frac{\delta}{2}\right)t}$

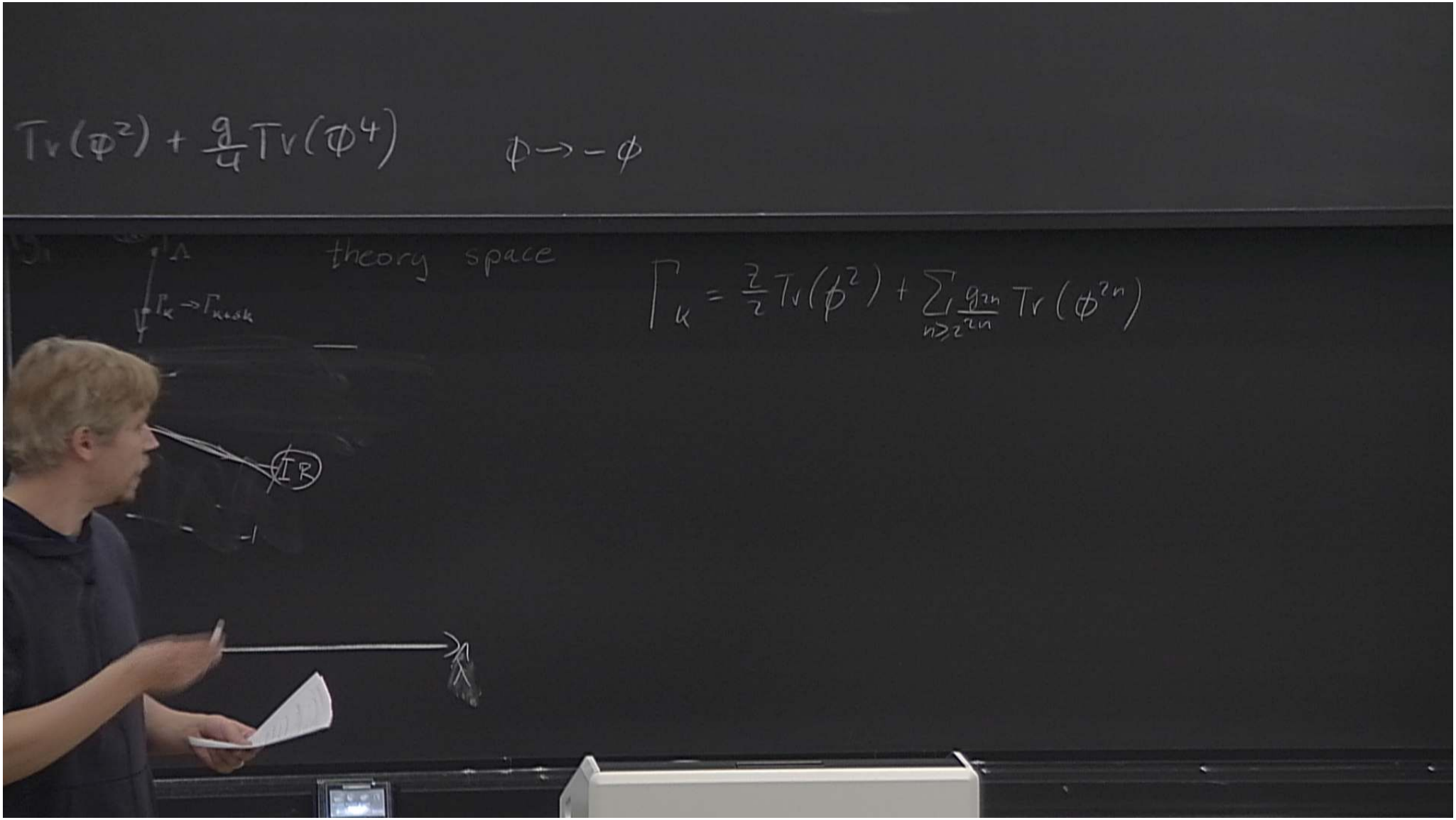
$|\partial_t \hat{\lambda}| < e^{-2t+\varepsilon t}\left(c_0 + c_1\hat{\lambda} + c_2\hat{\lambda}_2\right)$

$S = \mathrm{Tr}(\phi^2) + \frac{g}{4}\mathrm{Tr}(\phi^4)$

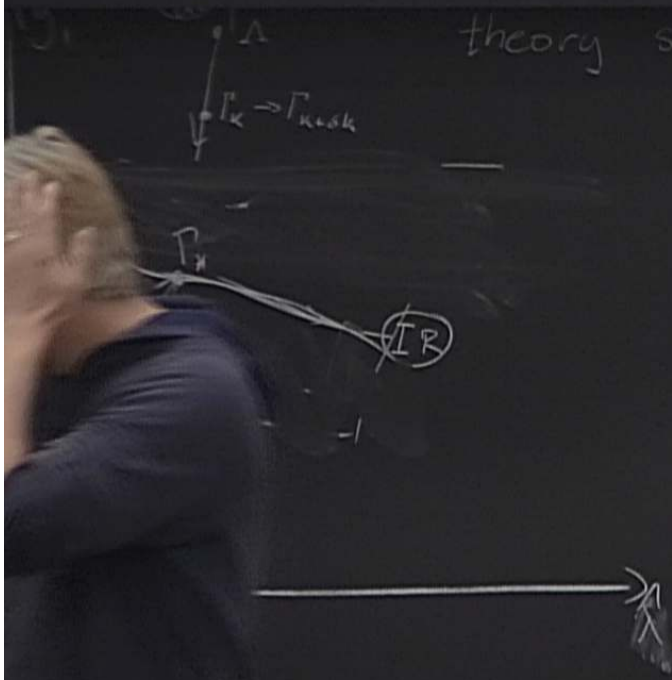
nonperturbative tool → piecemeal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram




① GW-truncation extra η at 1-loop → ∂_t λ = 0
② truncation with all operators up to φ^6
③ estimate the effect of operators φ^8
S = Tr(φ^2) + 1/4 Tr(φ^4)
φ → −φ
comparative tool → pivotal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram
theory space
coupling constants


$\mathrm{Tr}(\Phi^2) + \frac{g}{4}\mathrm{Tr}(\Phi^4)$
$\Phi \to -\Phi$
theory space
Λ
$\Gamma_k \to \Gamma_{k=0}$
IR
$\Gamma_k = \frac{Z}{2}\mathrm{Tr}(\phi^2) + \sum_{n \geq 2} \frac{g_{2n}}{2^{2n}}\mathrm{Tr}(\phi^{2n})$

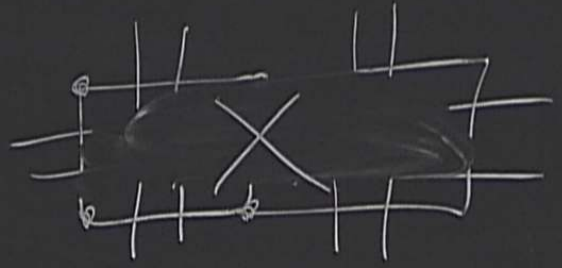
$Tr(\Phi^2) + \frac{g}{4} Tr(\Phi^4)$ $\quad \Phi \to -\Phi$



theory space

$$\Gamma_k = \frac{z}{2} Tr(\phi^2) + \sum_{n \geq 2} \frac{g_{2n}}{z^{2n}} Tr(\phi^{2n})$$

$(\dot{R} P^n)$

$Tr(\phi^4)$ = 

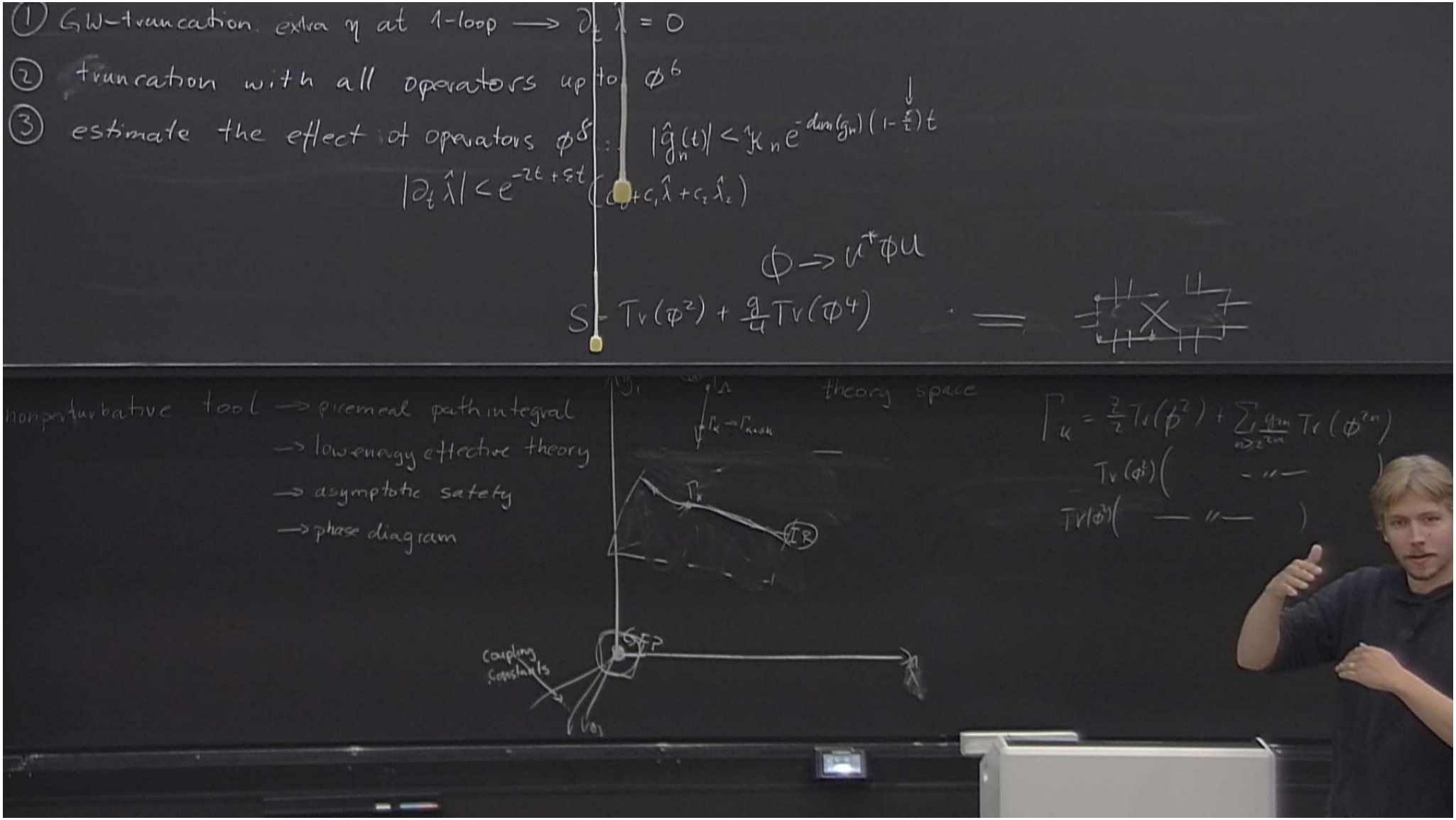
theory space

$$\Gamma_k = \frac{z}{2} Tr(\phi^2) + \sum_{n \geq 2} \frac{g_{2n}}{z^{2n}} Tr(\phi^{2n})$$

$Tr(\phi^2)$(— // —)

$Tr(\phi^2)$(— // —)

IR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② truncation with all operators up to φ^6
③ estimate the effect of operators φ^8
|∂_t λ̂| < e^{-2t} + ε_3 t (c_0 + c_1 λ̂ + c_2 λ̂_2)
|ĝ_n(t)| < κ_n e^{-dim(g_n)(1 - ξ/2)t}
φ → U* φ U
S = Tr(φ^2) + g/4! Tr(φ^4) = X
nonperturbative tool → piecemeal path integral
→ low-energy effective theory
→ asymptotic safety
→ phase diagram
theory space
Γ_k → Γ_{k=0}
Γ_k
IR
FP
coupling constants
Γ_k = Z/2 Tr(φ^2) + Σ g_{2n}/(n! 2^n) Tr(φ^{2n})
Tr(φ^3)(— " —)
Tr(φ^4)(— " —)