Title: Asymptotically safe matrix models

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Abstract: The functional renormalization group is a tool in the systematic search for Euclidean QFTs that works with very little input: All one needs to specify is a field content, symmetries and a notion of locality. The functional renormalization group then allows one to scan this theory space for bare actions for which the path integral can be performed nonperturbatively. These actions appear as fixed points (and relevant deformations) of the renormalization group flow (so-called asymptotic safety). Such a systematic search has so far not been performed for the tensor model approach to quantum gravity. We investigate matrix models for 2-dimensional gravity and the Grosse-Wulkenhaar model, which is a simple tensor model for a 4D noncommutative scalar field theory, as a first step. I will report on work, done in collaboration with Astrid Eichhorn and Alessandro Sfondrini, where we confirmed asymptotic safety purely through functional RG methods. Based on the lessons learned from these models I will summarize how the usual continuum RG approach has to be generalized for the investigation of tensor models and conclude with a recipe for the investigation of general tensor models.

Asymptotically Safe Matrix Models

work with A. Eichhorn, A. Stondrini, I. Vilensky

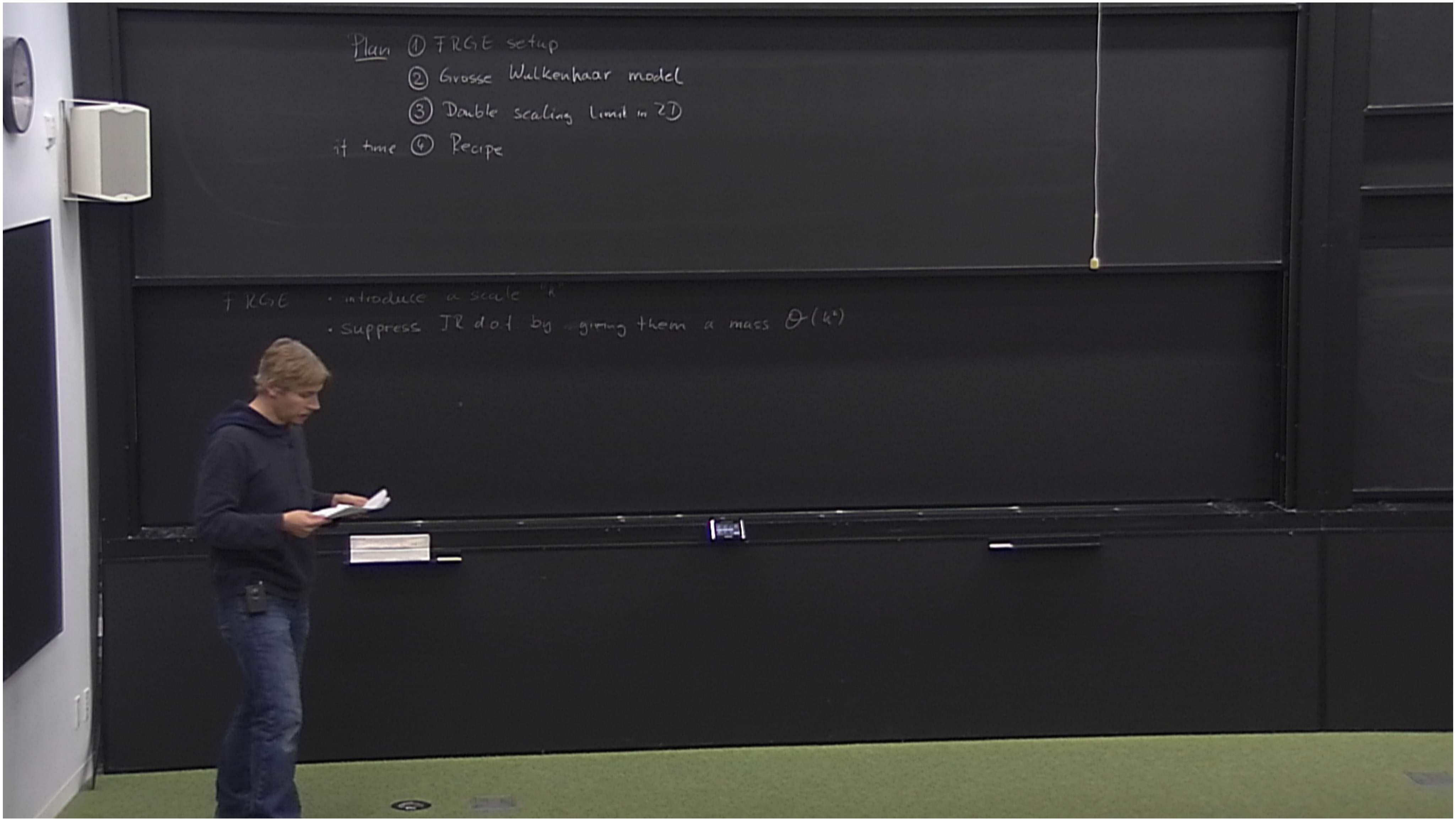
→ T.K., A. Stondrini: IJMP A26 (2011) 4009

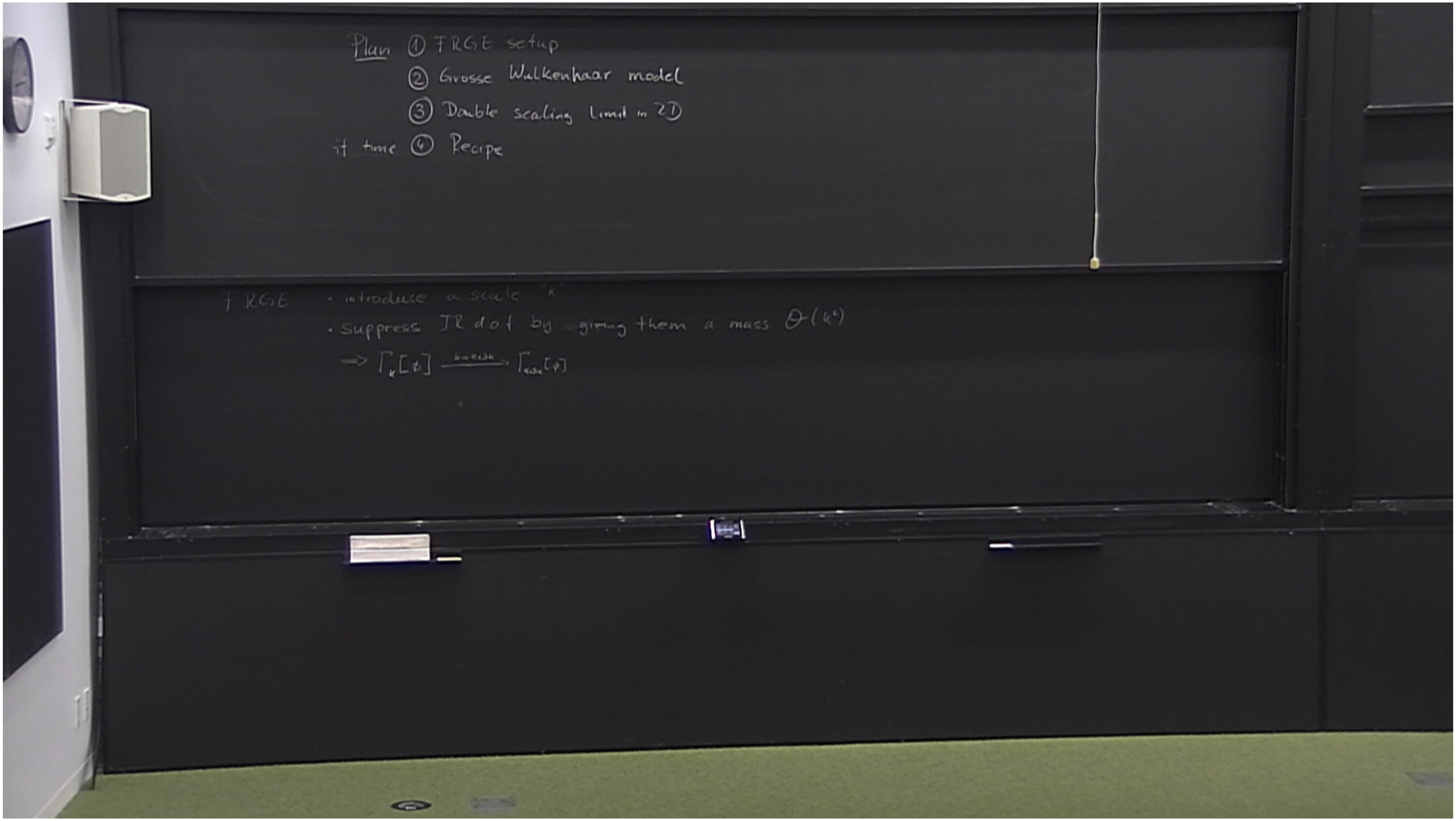
→ A. Eichhorn, T.K.: PRD 88 (2013) iss. 8, 084016

→ unpublished w/h I. Vilensky

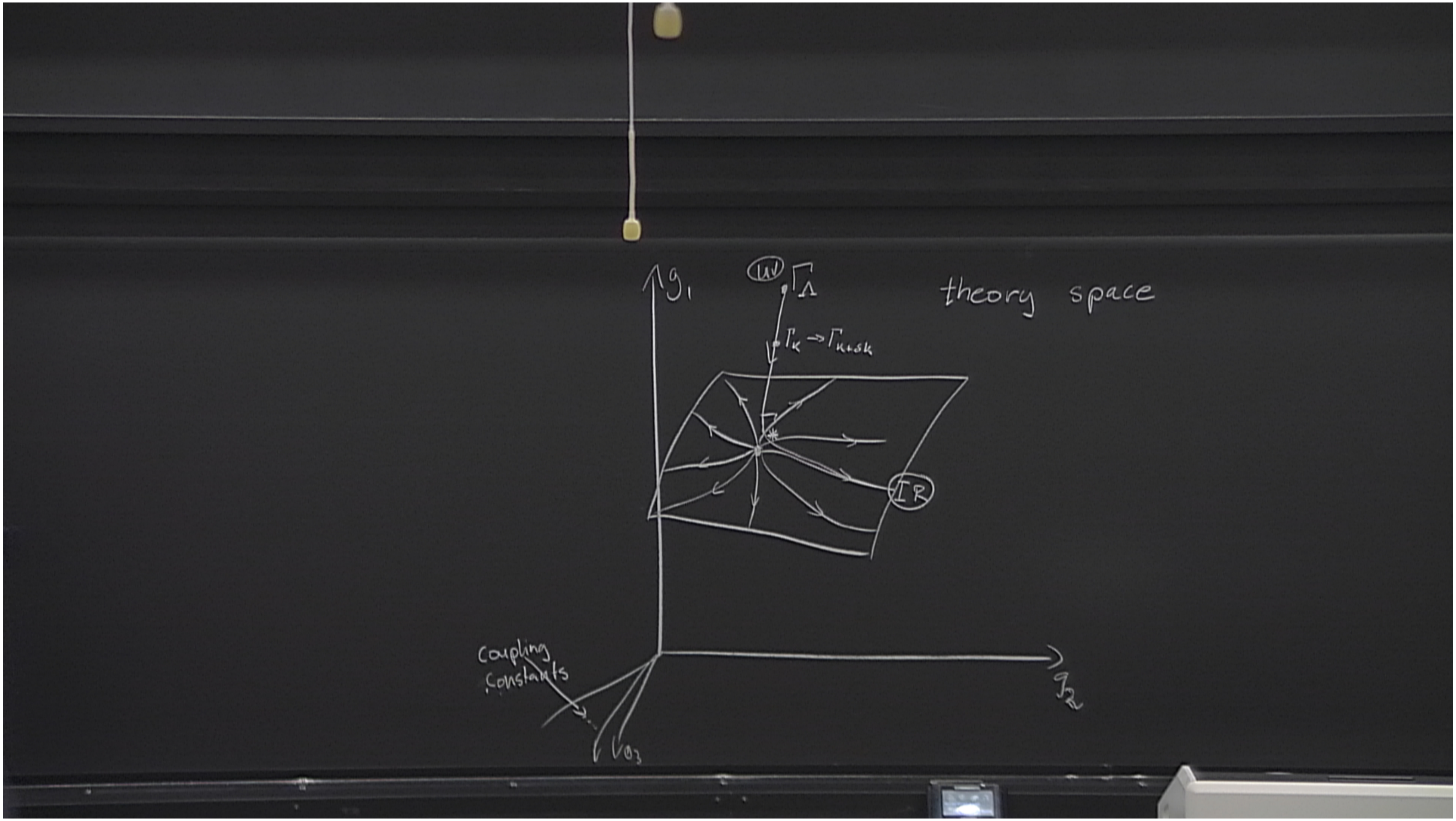
Main message: 1 establish FRGE (main tool in AS program) as a tool in matrix and tensor models

2 confirm known AS (from constructive prog) w/h FRGE


Plan ① FRGE setup
② Grosse Wulkenhaar model
③ Double scaling limit in 2D
if time ④ Recipe
FRGE · introduce a scale "k"
· suppress IR dof by giving them a mass O(k²)



Plan ① FRGE setup
② Grosse Wulkenhaar model
③ Double scaling limit in 2D
if time ④ Recipe
FRGE · introduce a scale "k"
· suppress IR dof by giving them a mass $\mathcal{O}(k^2)$
$\Rightarrow \Gamma_k[\phi] \xrightarrow{k \to k+\delta k} \Gamma_{k+\delta k}[\phi]$



theory space
UV
Γ_Λ
$\Gamma_k \to \Gamma_{k+\delta k}$
IR
g_1
g_2
g_3
coupling constants

FRGE

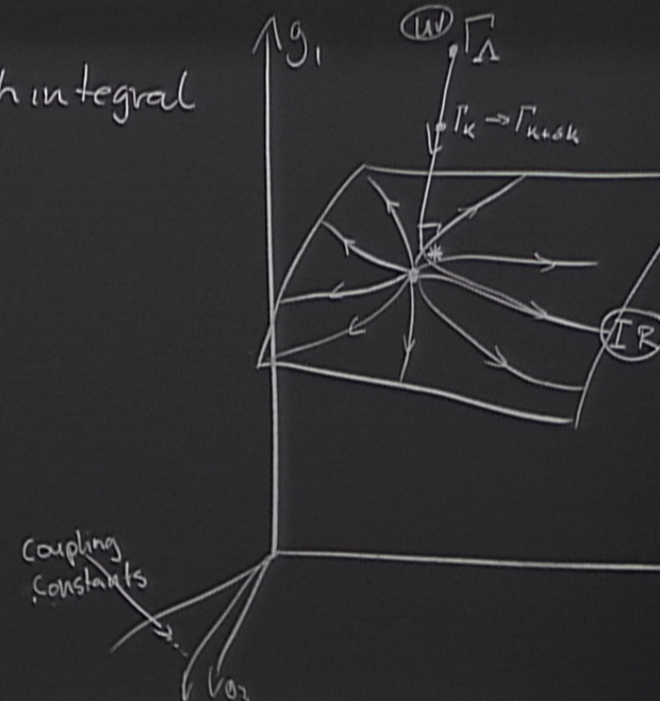
- introduce a scale "k"
- suppress IR d.o.f. by giving them a mass $\mathcal{O}(k^2)$

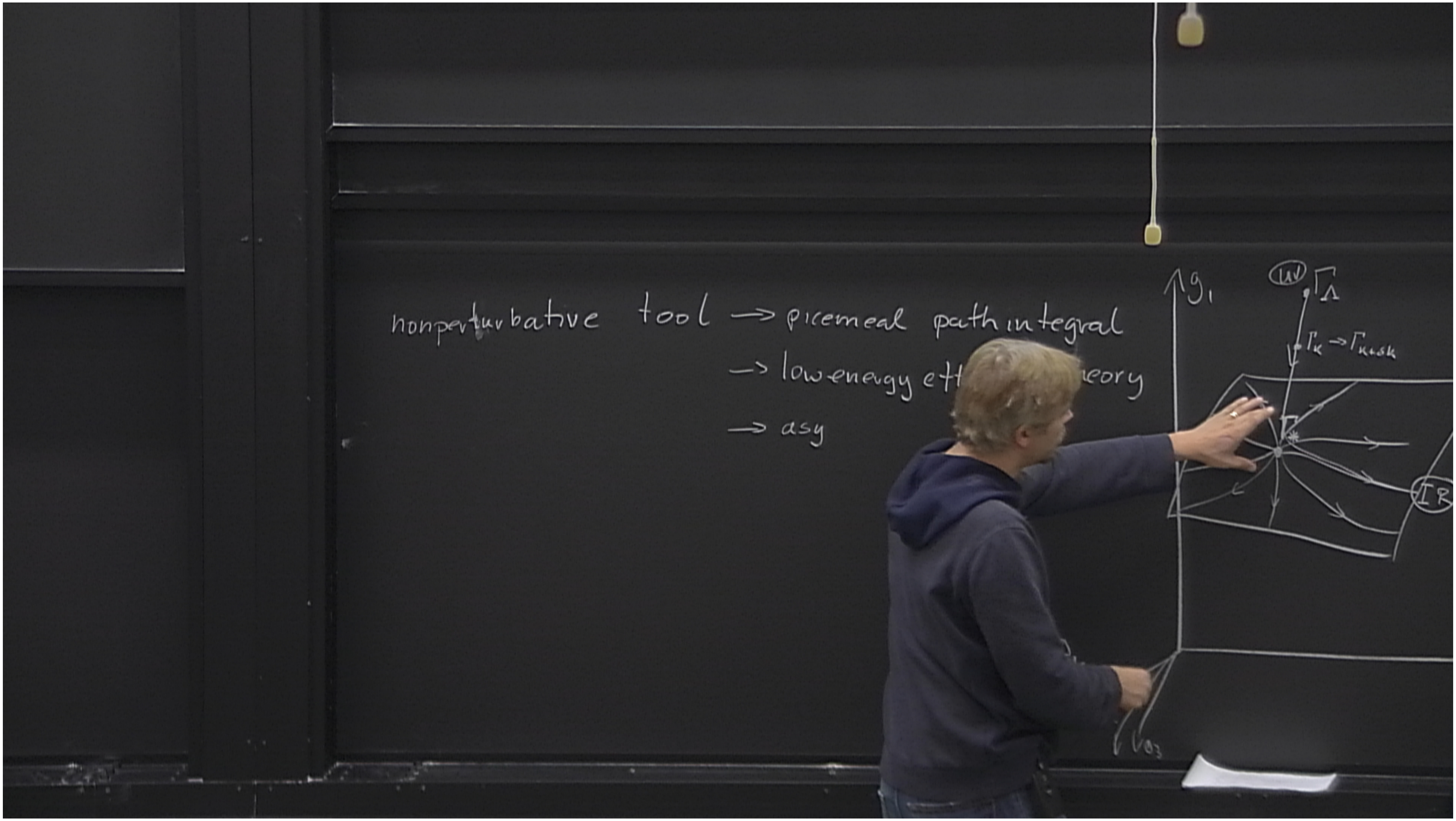
$\Longrightarrow \Gamma_k[\phi] \xrightarrow{k \to k+\delta k} \Gamma_{k+\delta k}[\phi]$

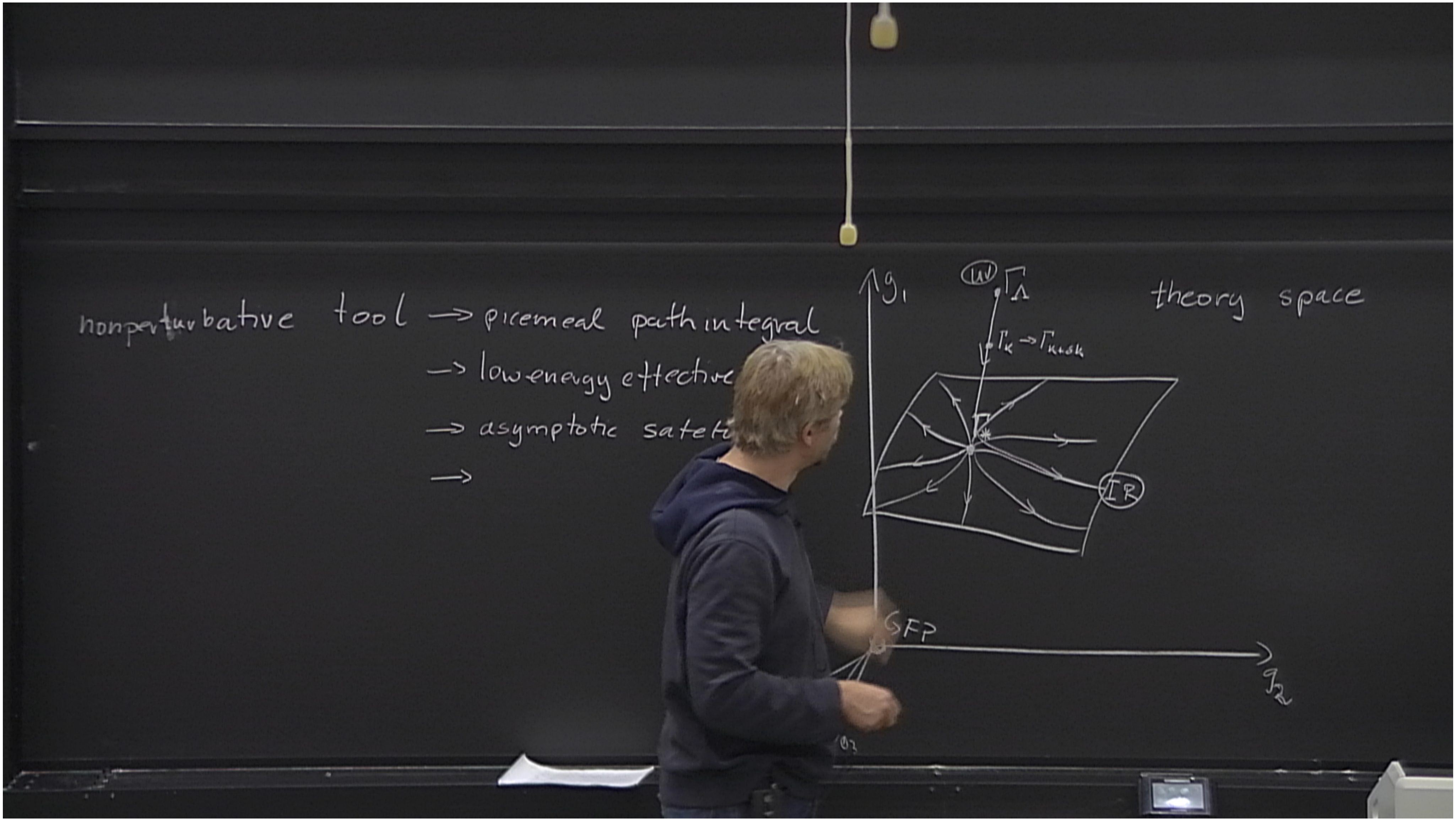
$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left(\frac{k\partial_k R_k(\Delta)}{\Gamma_k^{(2)}[\phi] + R_k} \right) - k\partial_k \mathrm{Tr} \ln\left(\Gamma_s^{(2)} + R_k \right)\Big|_{s=k}$$

nonperturbative tool → piecemeal path integral

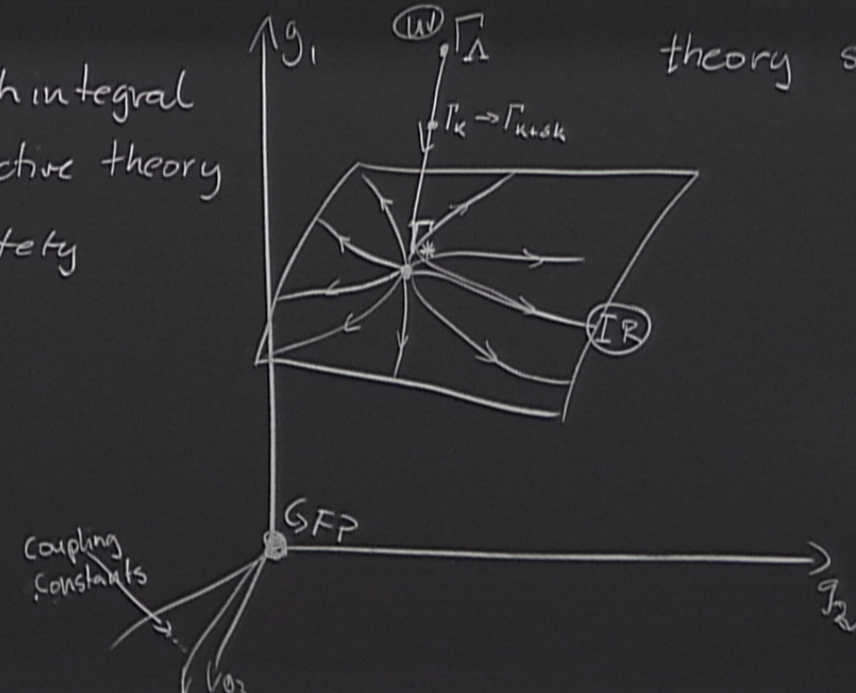
→

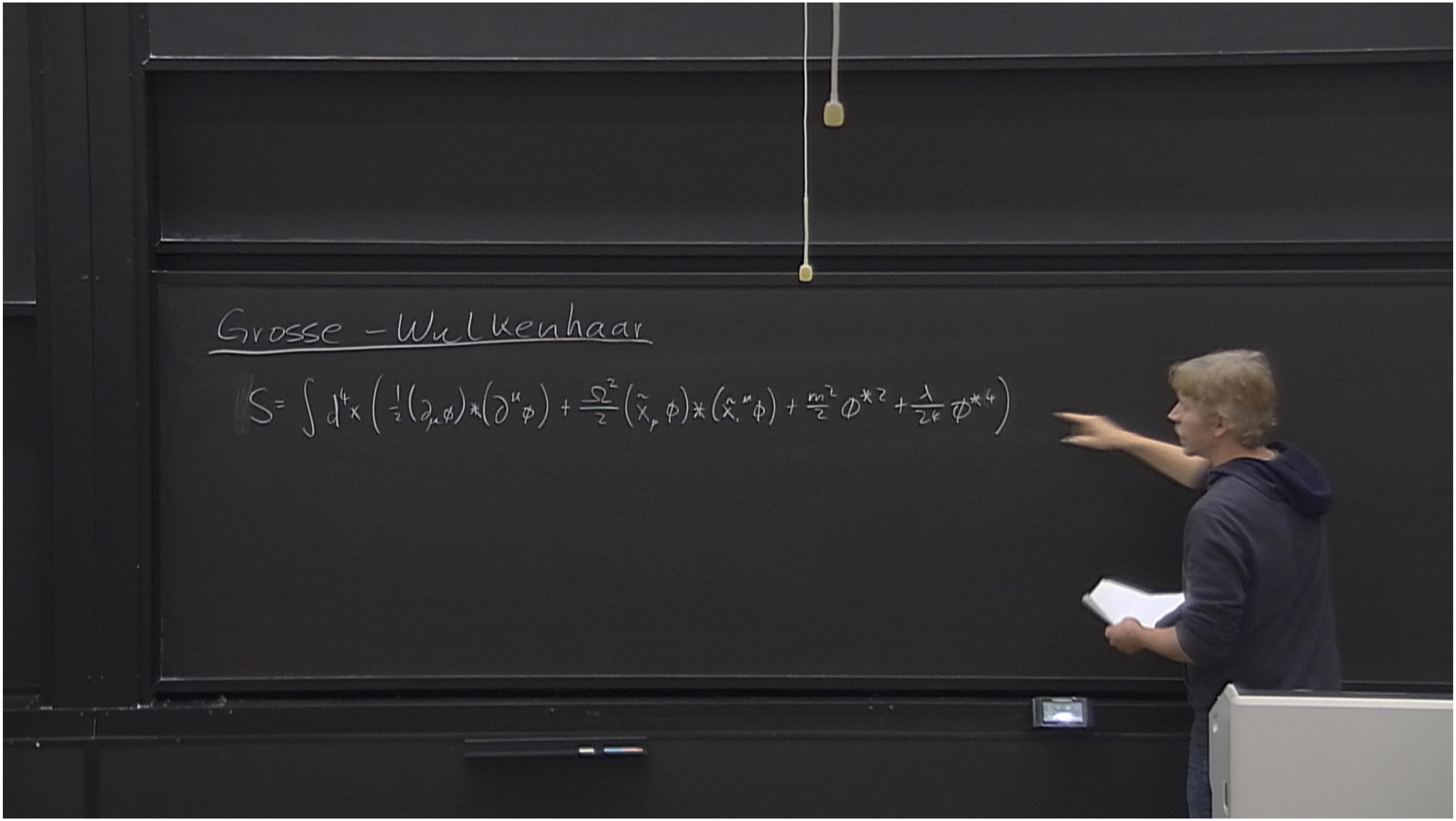



nonperturbative tool → piecemeal path integral
→ low energy eff
eory
→ asy
g1
UV
Γ_Λ
Γ_k → Γ_k+δk
IR


nonperturbative tool
piecemeal path integral
low energy effective
asymptotic safet
theory space
g1
UV
Γ_Λ
Γ_k → Γ_k+δk
IR
GFP
g2

nonperturbative tool → piecemeal path integral
→ low-energy effective theory
→ asymptotic safety
→ phase diagram




Grosse - Wulkenhaar
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{4!}\phi^{*4} \right)$

Grosse - Wulkenhaar

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{24}\phi^{*4} \right)$$

Grosse - Wulkenhaar

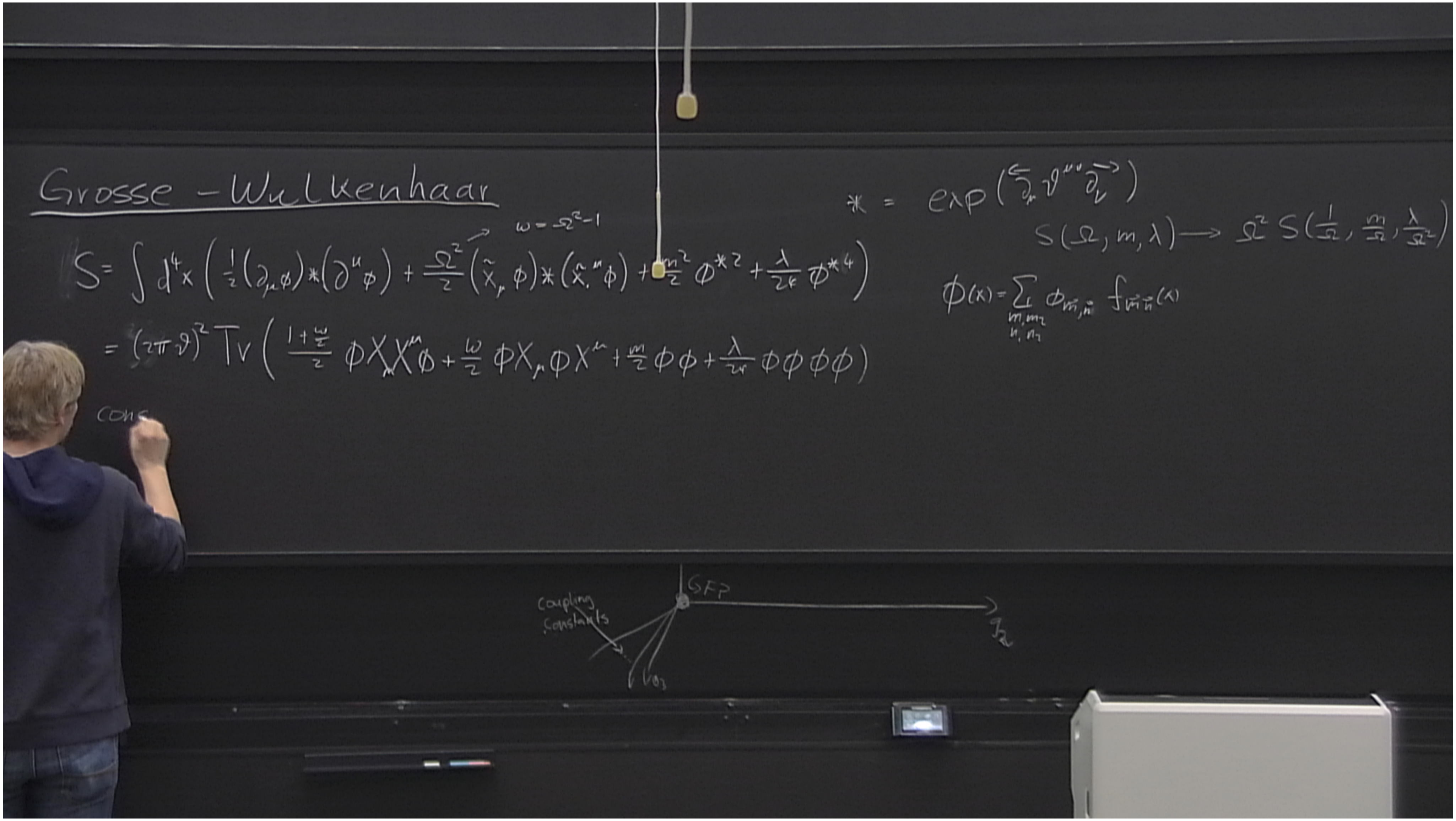
$$\omega = \Omega^2 - 1$$

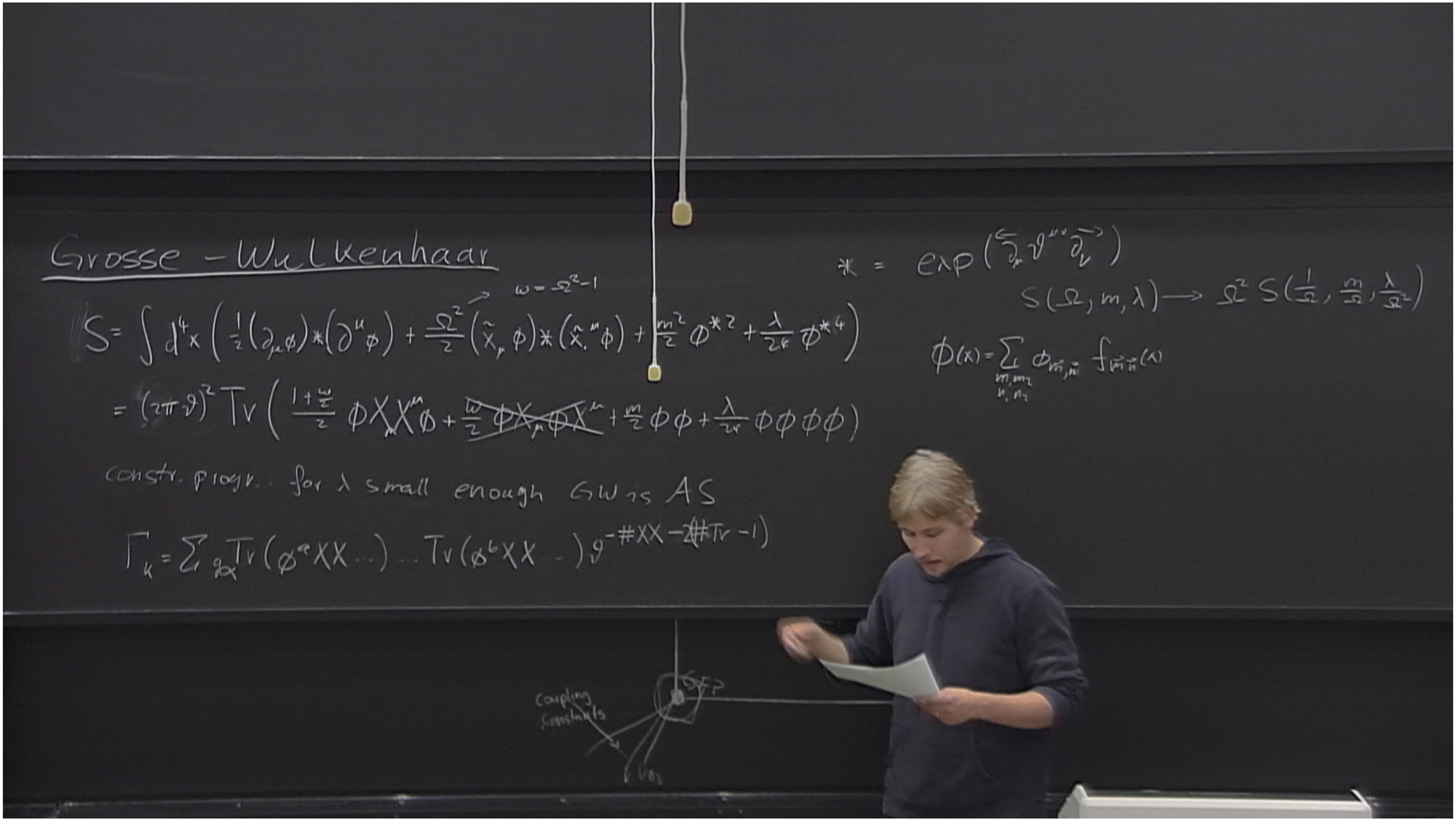
$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{24}\phi^{*4} \right)$$

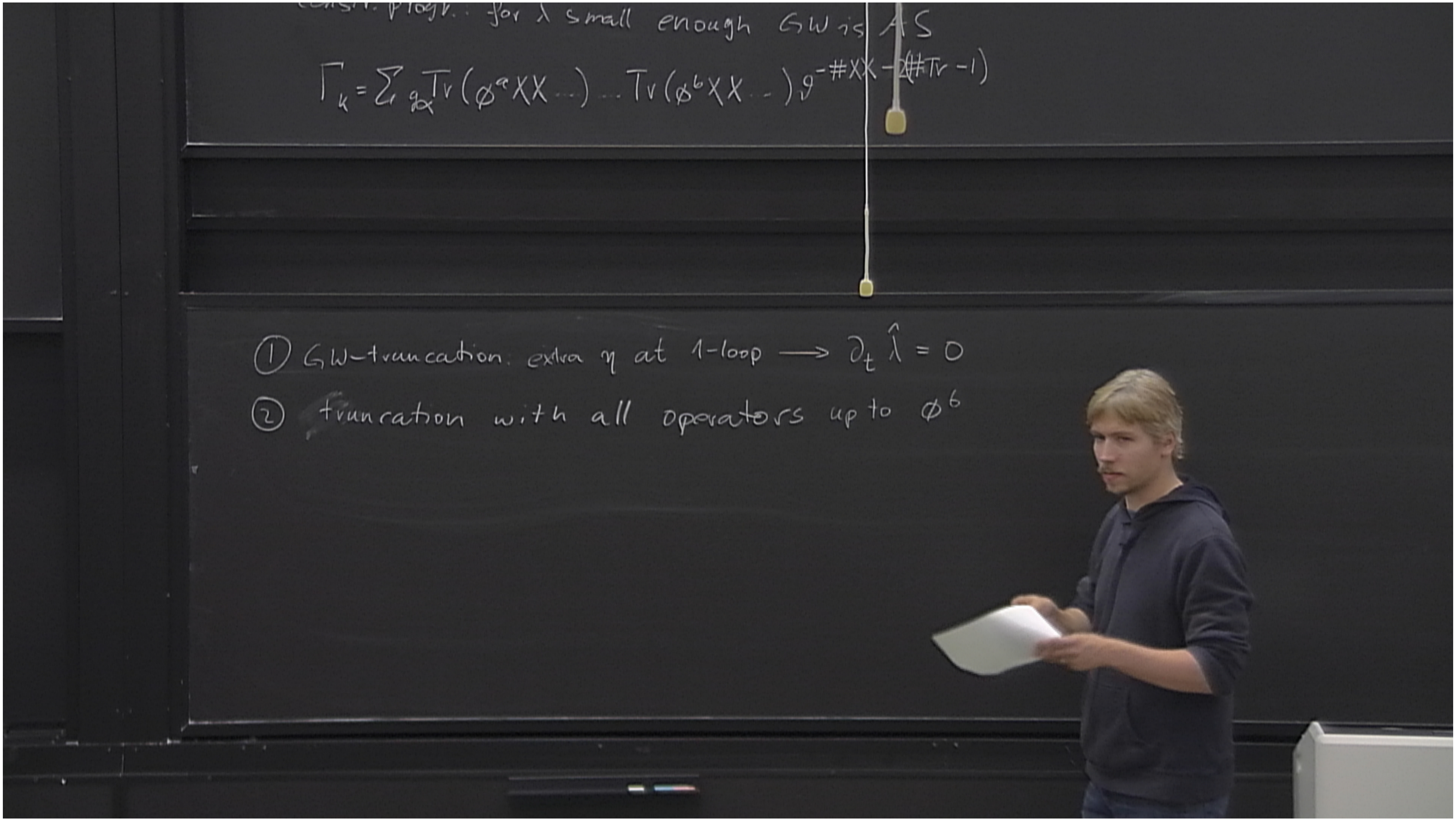
$$= (2\pi\vartheta)^2 \operatorname{Tr}\left(\frac{1+\frac{\omega}{2}}{2} \phi X_\mu X^\mu \phi + \frac{\omega}{2} \phi X_\mu \phi X^\mu + \frac{m}{2}\phi\phi + \frac{\lambda}{24}\phi\phi\phi\phi \right)$$

$$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S\left(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2}\right)$$

$$\phi(x) = \sum_{\substack{m_1, m_2 \\ n_1, n_2}} \phi_{\vec{m},\vec{n}} \, f_{\vec{m}\vec{n}}(x)$$


Grosse - Wulkenhaar
$\omega = \Omega^2 - 1$
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2} (\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2} \phi^{*2} + \frac{\lambda}{24} \phi^{*4} \right)$
$= (2\pi\vartheta)^2 \mathrm{Tr} \left(\frac{1+\frac{\omega}{2}}{2} \phi X_\mu X^\mu \phi + \frac{\omega}{2} \phi X_\mu \phi X^\mu + \frac{m}{2} \phi\phi + \frac{\lambda}{24} \phi\phi\phi\phi \right)$
cons
$* = \exp(\overleftarrow{\partial_\mu} \vartheta^{\mu\nu} \overrightarrow{\partial_\nu})$
$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2})$
$\phi(x) = \sum_{m_1, m_2, n_1, n_2} \phi_{\vec{m}, \vec{n}} f_{\vec{m}\vec{n}}(x)$
GFP
coupling constants
g_2


Grosse - Wulkenhaar
$\ast = \exp(\overleftarrow{\partial_\mu}\vartheta^{\mu\nu}\overrightarrow{\partial_\nu})$
$\omega = \Omega^2 - 1$
$S(\Omega, m, \lambda) \longrightarrow \Omega^2 S(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2})$
$S = \int d^4x \left(\frac{1}{2}(\partial_\mu\phi)\ast(\partial^\mu\phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu\phi)\ast(\tilde{x}^\mu\phi) + \frac{m^2}{2}\phi^{\ast 2} + \frac{\lambda}{24}\phi^{\ast 4}\right)$
$\phi(x) = \sum_{m_1,m_2,n_1,n_2} \phi_{\vec{m},\vec{n}} f_{\vec{m}\vec{n}}(x)$
$= (2\pi\vartheta)^2 \mathrm{Tr}\left(\frac{1+\frac{\omega}{2}}{2}\phi X_\mu X^\mu\phi + \cancel{\frac{\omega}{2}\phi X_\mu \phi X^\mu} + \frac{m}{2}\phi\phi + \frac{\lambda}{24}\phi\phi\phi\phi\right)$
constr. progr.. for λ small enough GW is AS
$\Gamma_k = \sum_i g_\alpha \mathrm{Tr}(\phi^a XX\ldots)\ldots\mathrm{Tr}(\phi^b XX\ldots)\vartheta^{-\#XX-2\#\mathrm{Tr}-1)}$
coupling constants
NGFP


for λ small enough GW is AS
$\Gamma_k = \sum g_\alpha \mathrm{Tr}(\phi^a XX\dots)\dots\mathrm{Tr}(\phi^b XX\dots)\,\vartheta^{-\#XX-2(\#\mathrm{Tr}-1)}$
① GW-truncation: extra η at 1-loop ⟶ $\partial_t \hat{\lambda} = 0$
② truncation with all operators up to ϕ^6

consv. progr.: for λ small enough GW is AS

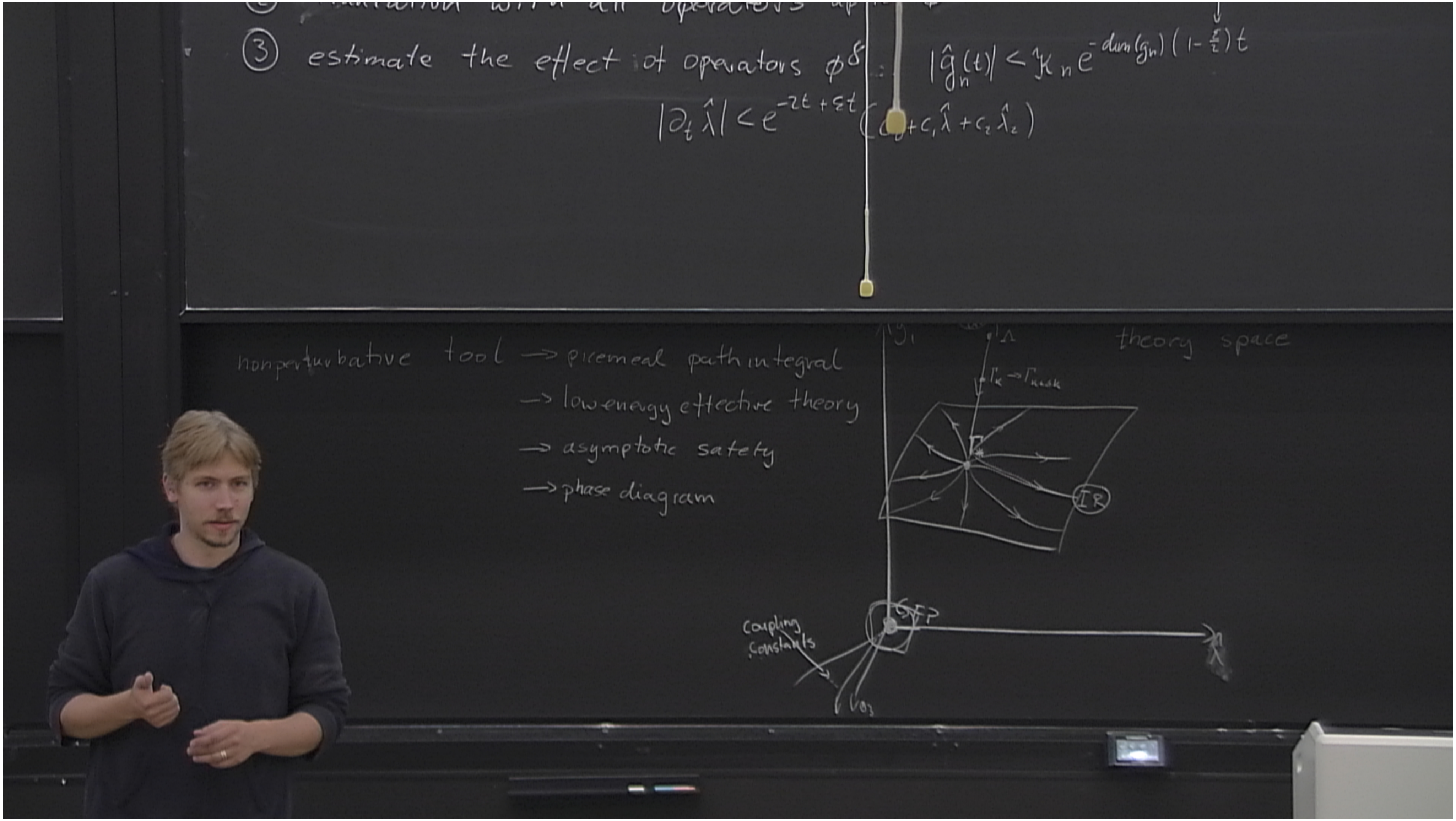
$$\Gamma_k = \sum_i g_\alpha \mathrm{Tr}(\phi^a XX \ldots) \ldots \mathrm{Tr}(\phi^b XX \ldots)\, \vartheta^{-\#XX - 2(\#\mathrm{Tr} - 1)}$$

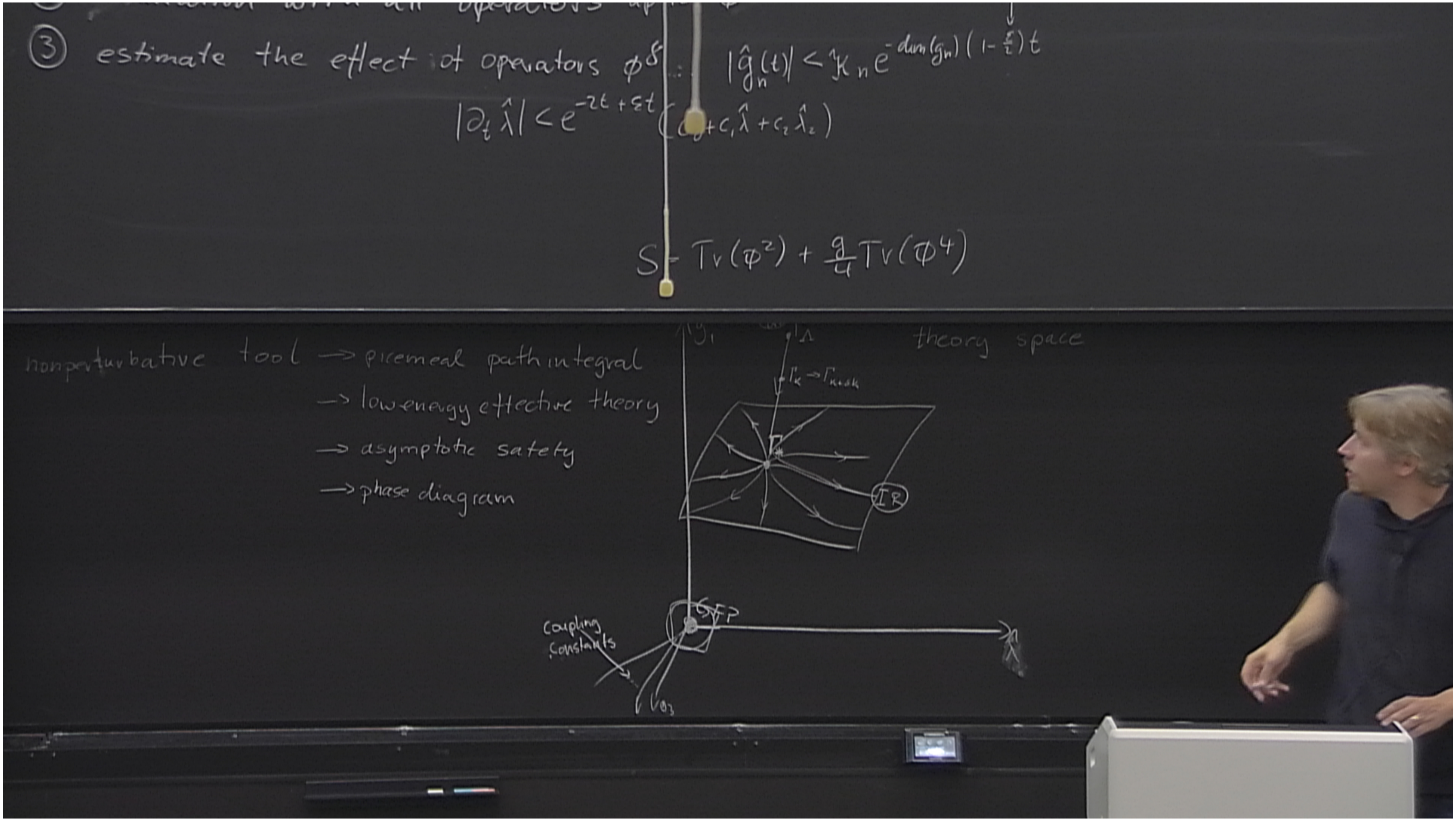
① GW-truncation: extra η at 1-loop $\longrightarrow \partial_t \hat{\lambda} = 0$

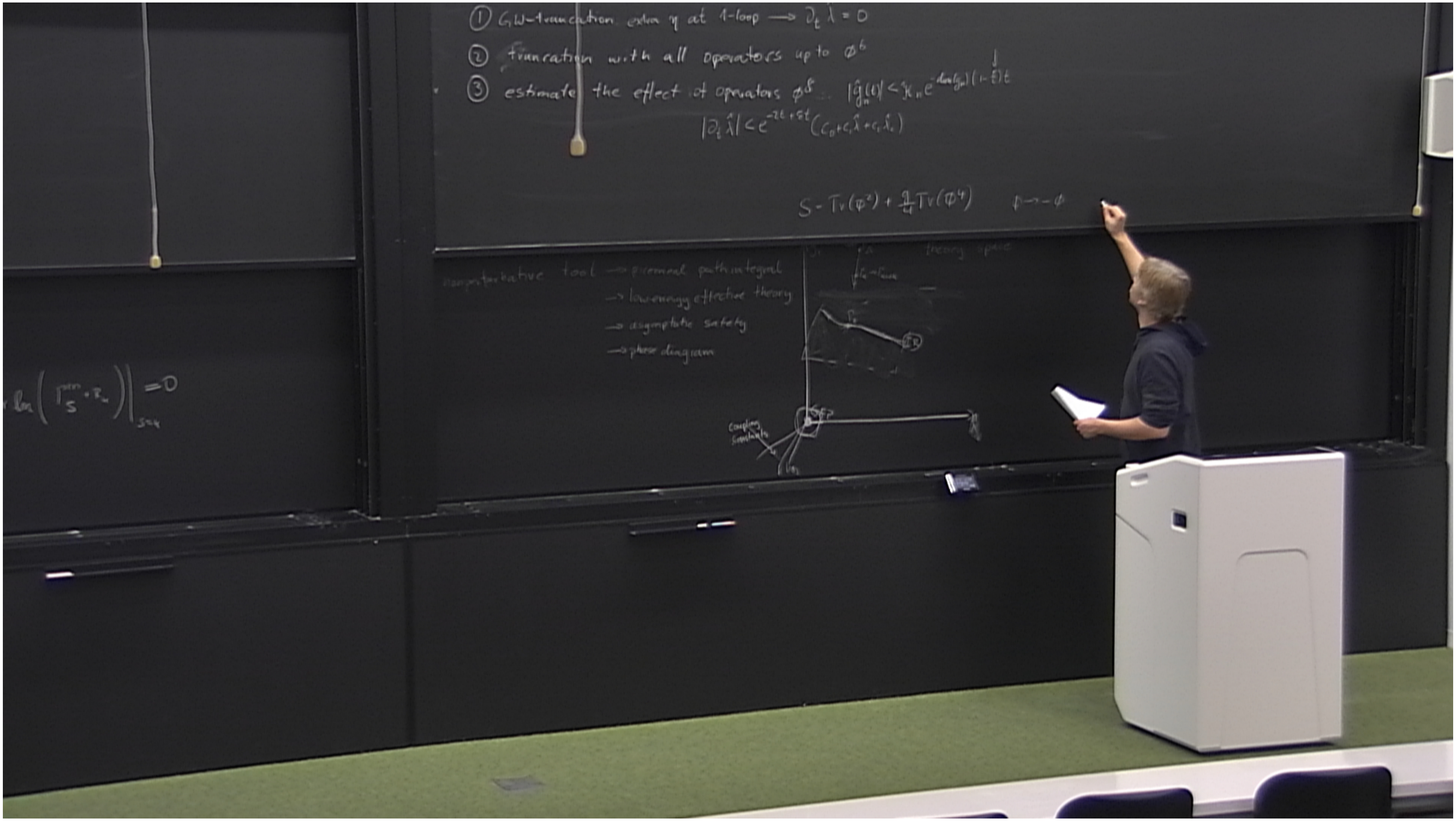
② truncation with all operators up to ϕ^6

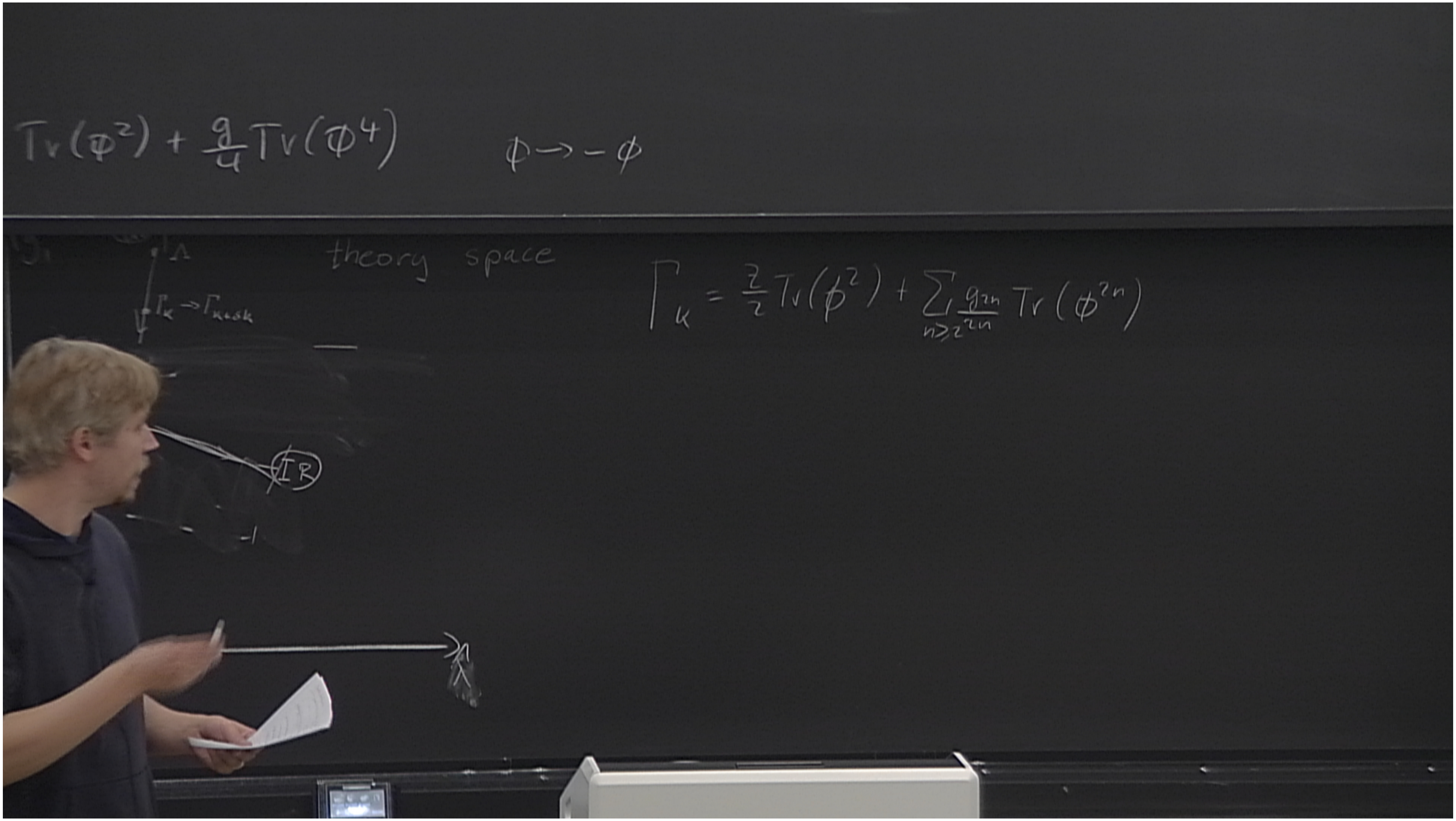
③ estimate the effect of operators ϕ^8 … $|\hat{g}_n(t)| < K_n e^{-\dim(g_n)(1-\frac{\varepsilon}{2})t}$

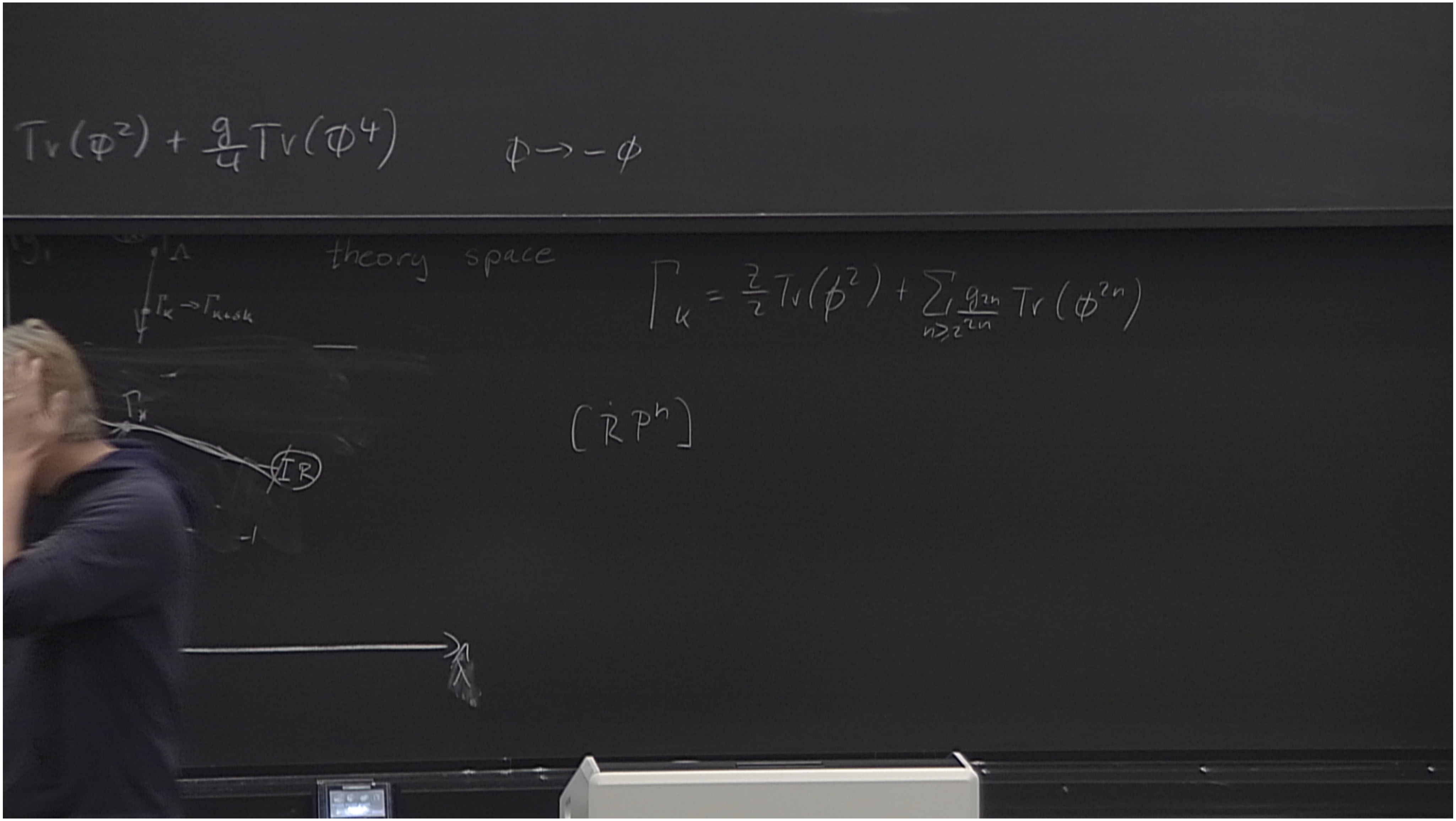
$$|\partial_t \hat{\lambda}| < e^{-2t+\varepsilon t}(c_0 + c_1\hat{\lambda} + c_2\hat{\lambda}_2)$$

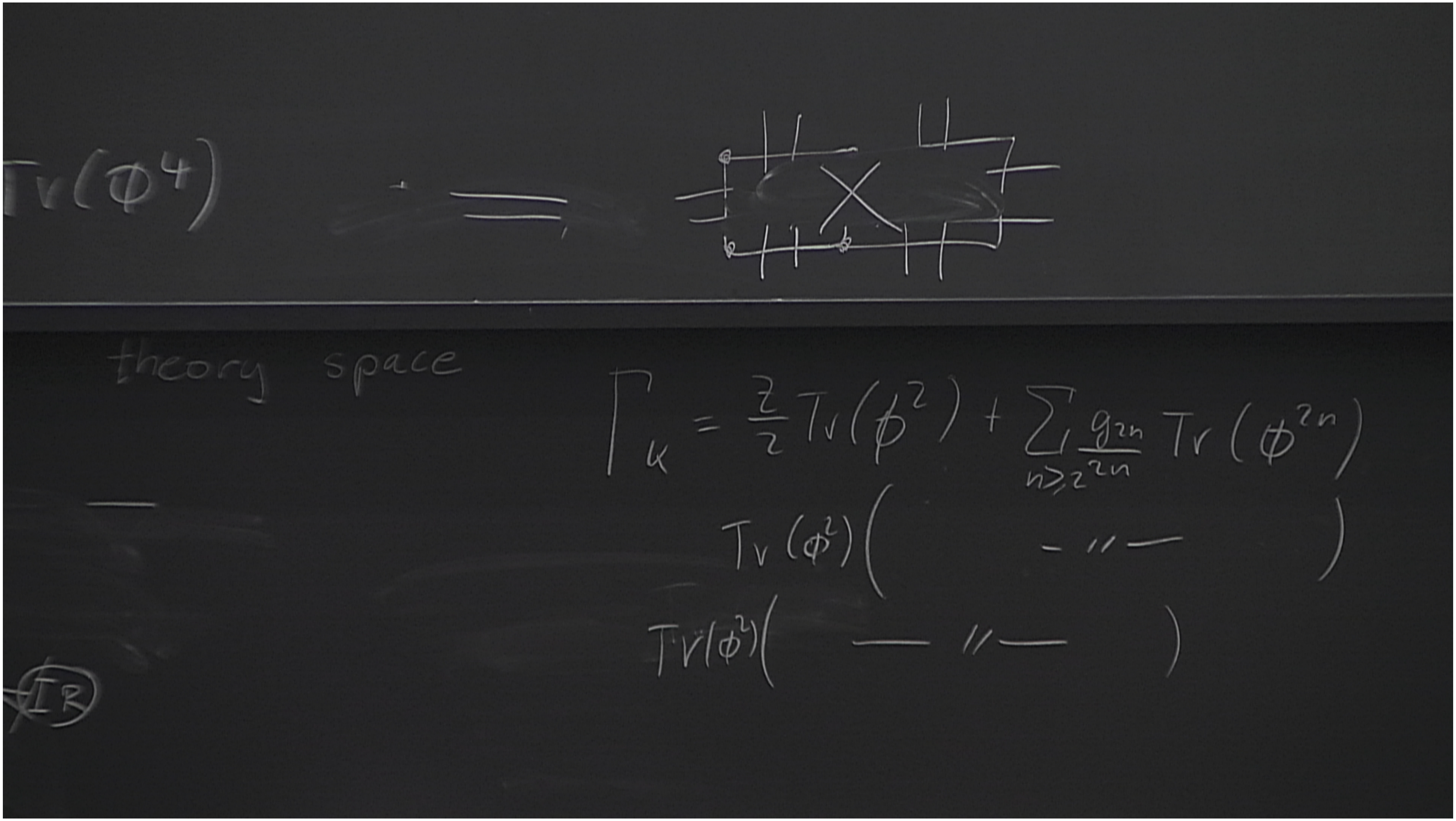

③ estimate the effect of operators φ⁸
nonperturbative tool → piecemeal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram
theory space
IR
GFP
coupling constants

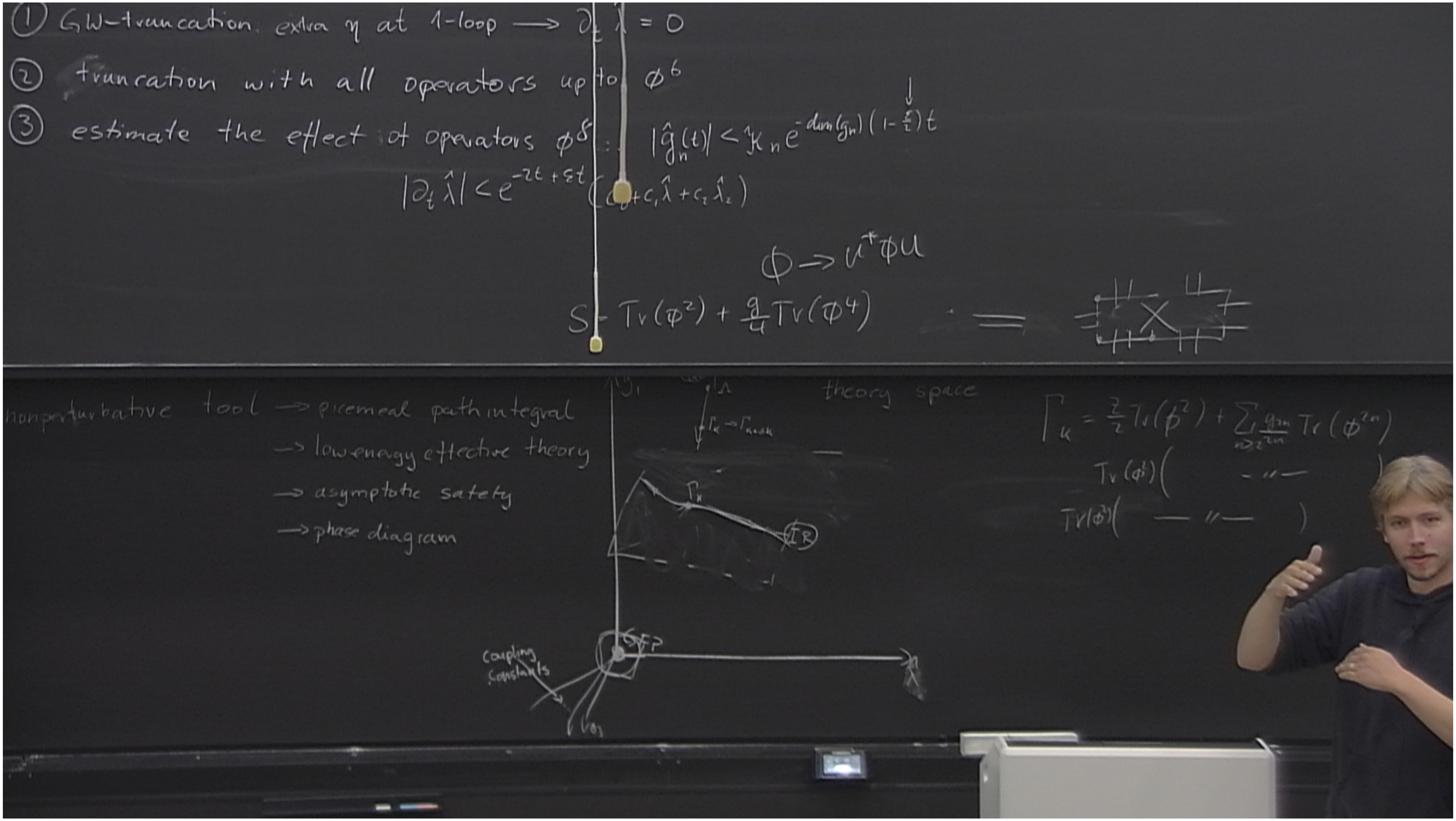

③ estimate the effect of operators ϕ^8 : $|\hat{g}_n(t)| < \kappa_n e^{-\dim(g_n)(1-\frac{\varepsilon}{2})t}$
$|\partial_t \hat{\lambda}| < e^{-2t+\varepsilon t}(c_0 + c_1\hat{\lambda} + c_2\hat{\lambda}_2)$
$S = \mathrm{Tr}(\Phi^2) + \frac{g}{4}\mathrm{Tr}(\Phi^4)$
nonperturbative tool → piecemeal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram
theory space
$\Gamma_k \to \Gamma_{k+dk}$
IR
GFP
coupling constants


① GW-truncation extra η at 1-loop → ∂_t λ = 0
② truncation with all operators up to φ^6
③ estimate the effect of operators φ^8
S = Tr(φ^2) + g/4 Tr(φ^4)
φ → −φ
→ low-energy effective theory
→ asymptotic safety
→ phase diagram
theory space
coupling constants


$\mathrm{Tr}(\Phi^2) + \frac{g}{4}\mathrm{Tr}(\Phi^4)$
$\phi \to -\phi$
Λ
theory space
$\Gamma_k \to \Gamma_{k+\delta k}$
$\Gamma_k = \frac{Z}{2}\mathrm{Tr}(\phi^2) + \sum_{n\geq 2} \frac{g_{2n}}{2^{2n}} \mathrm{Tr}(\phi^{2n})$
IR


$\mathrm{Tr}(\Phi^2) + \frac{g}{4}\mathrm{Tr}(\Phi^4)$
$\phi \to -\phi$
theory space
Λ
$\Gamma_k \to \Gamma_{k+\delta k}$
$\Gamma_k = \frac{Z}{2}\mathrm{Tr}(\phi^2) + \sum_{n\geq 2} \frac{g_{2n}}{2^{2n}} \mathrm{Tr}(\phi^{2n})$
$[\dot{R} P^n]$
IR


Tr(Φ⁴)
theory space
Γk = Z/2 Tr(φ²) + Σ n≥2 g2n/2²ⁿ Tr(φ²ⁿ)
Tr(φ²)(— // —)
Tr(φ²)(— // —)
IR


① GW-truncation: extra η at 1-loop → ∂t λ = 0
② truncation with all operators up to φ^6
③ estimate the effect of operators φ^8
φ → u^+ φ u
S = Tr(φ^2) + g/4 Tr(φ^4)
nonperturbative tool → piecemeal path integral
→ low energy effective theory
→ asymptotic safety
→ phase diagram
theory space
IR
coupling constants
Γ_k = Z/2 Tr(φ^2) + Σ g_2n/2^2n Tr(φ^2n)