Title: A causal set action

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URL: http://pirsa.org/13100063

Abstract: Causal set theory is discrete, fully covariant theory of quantum gravity. The discrete framework makes it necessary to reformulate continuum concepts. One of these concepts is that of a derivative operator. It is possible to define a derivative operator in causal sets that in the continuum limit agrees with the d'Alembertian for a scalar field. This operator can be used to define a causal set action, which enables Monte-Carlo simulations. In this seminar I will present this operator and action and then show some results of Monte-Carlo simulations in 2 dimensions.

A causal set action

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Niels Bohr Institute, Copenhagen

October 31, 2013

Why does causality encode space-time?

Theorem

A bijective map between two past and future distinguishing spacetimes that preserves their causal structure is a conformal isomorphism.

(Hawking, Malament)

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A bijective map between two past and future distinguishing spacetimes that preserves their causal structure is a conformal isomorphism.

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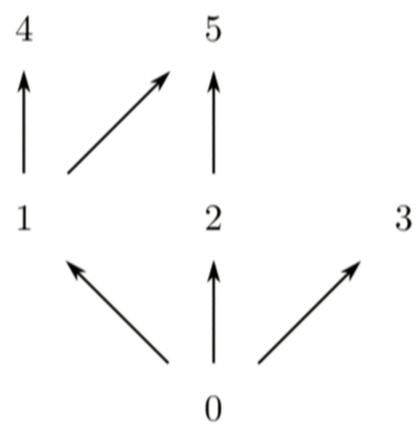
- If we know the causal structure all we need is the volume element
- Discreteness and a fundamental volume scale

Definition of a causal set

Mathematically a causal set is a set of elements $\mathcal{C}$ with a partial order relation $\preceq$, which denotes causal relations, which is



- **reflexive** for all $x \in \mathcal{C}$ $x \preceq x$
- **transitive** for all $x, y, z \in \mathcal{C}$ and $x \preceq y$ and $y \preceq z$ then $x \preceq z$
- **antisymmetric** if $x, y \in \mathcal{C}$ and $x \preceq y \preceq x$ then $x = y$
- **locally finite** for all $x, y \in \mathcal{C}$ $|I(x, y)| < \infty$

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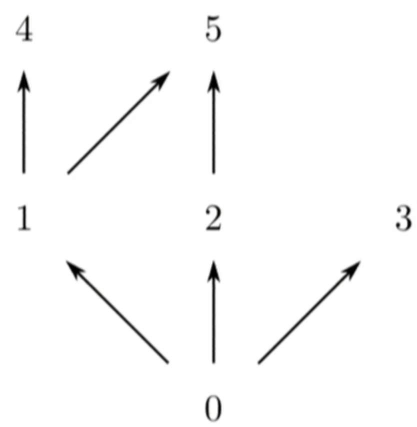

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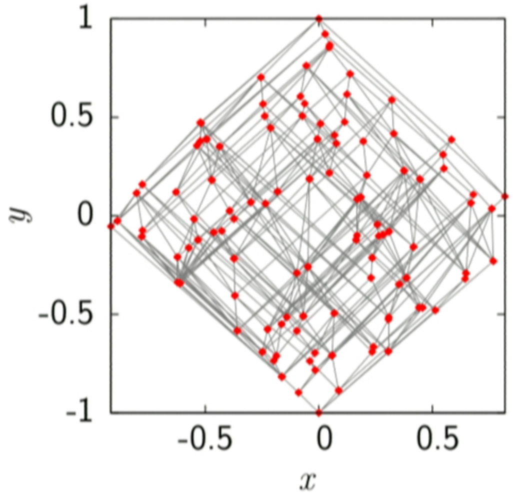
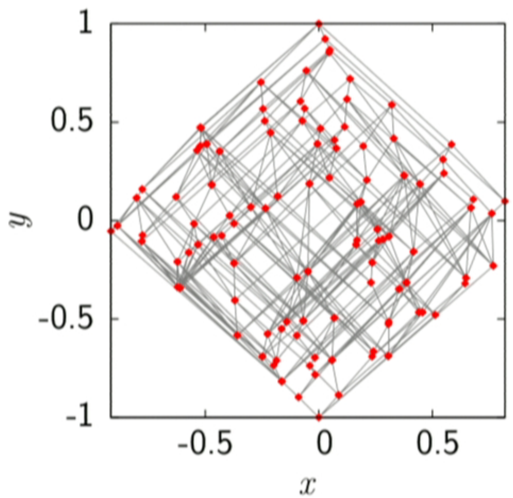
How does a manifold-like causal set look?



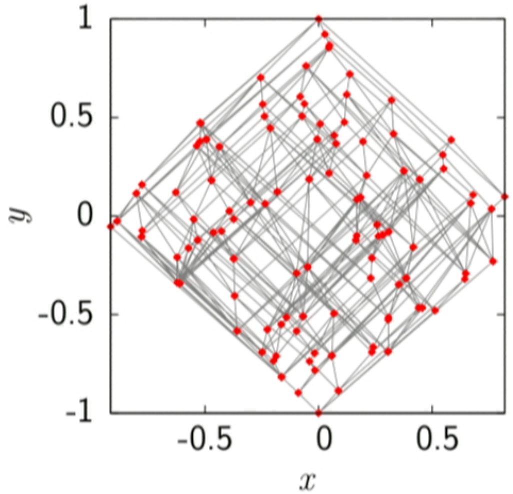
- A causal set $\mathcal{C}$ is approximated by a manifold $\mathcal{M}$ if it allows for a faithful embedding into this manifold.

How does a manifold-like causal set look?



- A causal set $\mathcal{C}$ is approximated by a manifold $\mathcal{M}$ if it allows for a faithful embedding into this manifold.
- An embedding is said to be faithful if it would arise with a high likelihood trough Poisson "sprinkling" into that manifold.

How does a manifold-like causal set look?



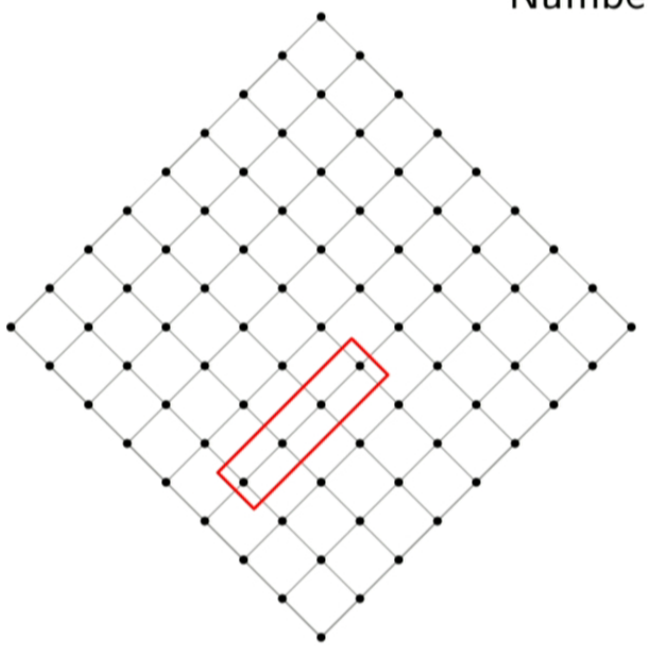
- pick N points from $\mathcal{M}$ according to a Poisson distribution

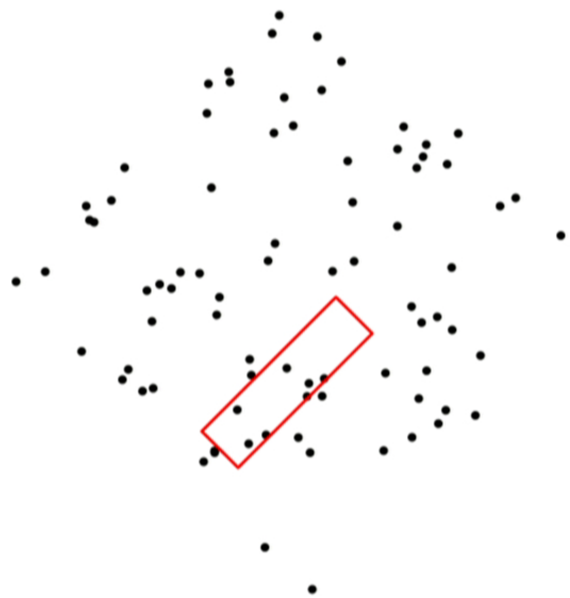
$$P(m, V, \rho) = \frac{(\rho V)^m}{m!} e^{-\rho V}$$

- partial order is induced through the causal structure of the manifold

Why not a regular lattice?

Number ↔ Volume?





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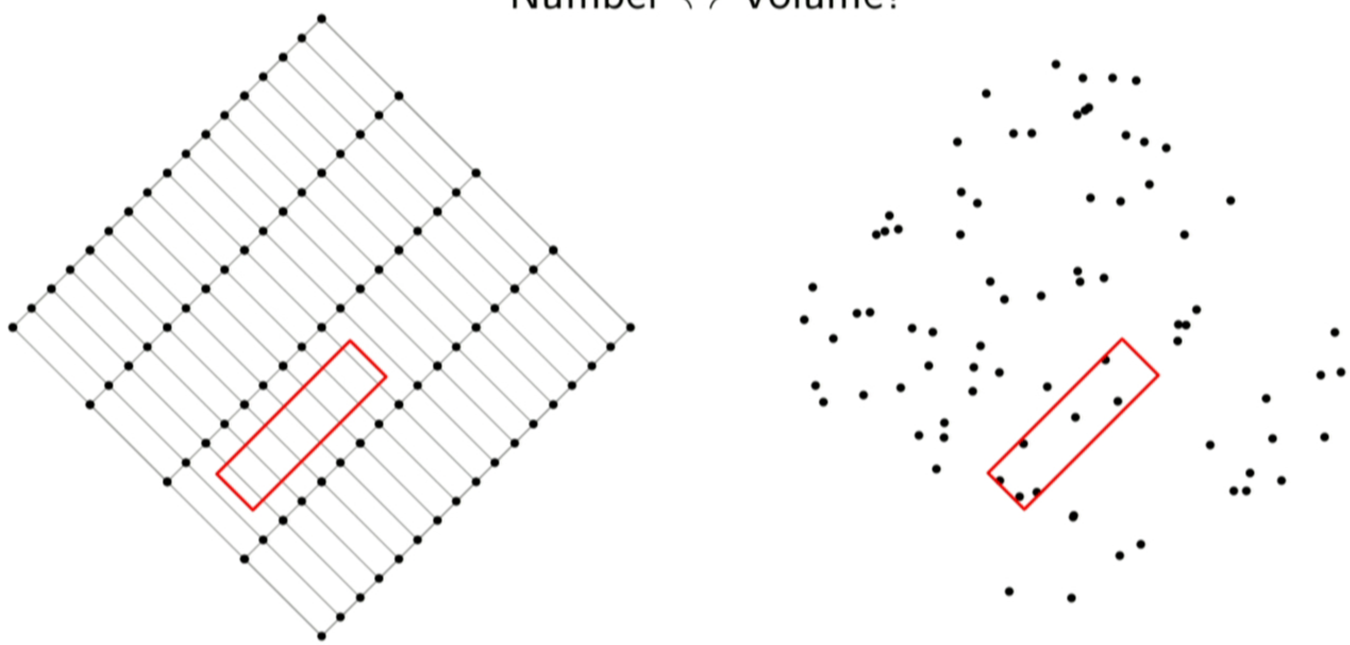
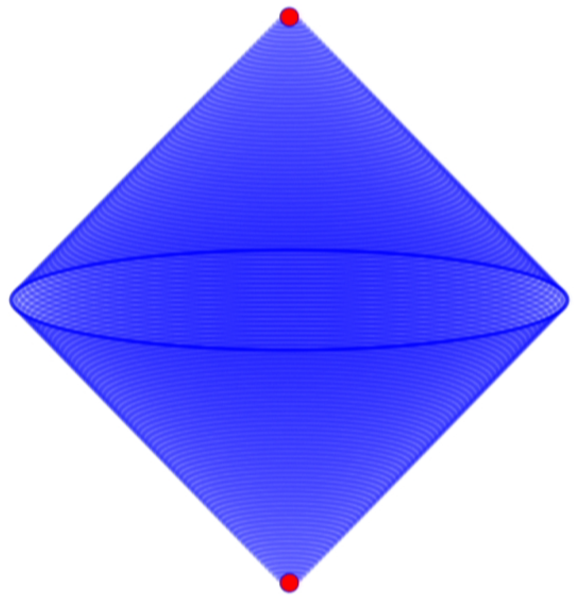
Number $\leftrightarrow$ Volume?

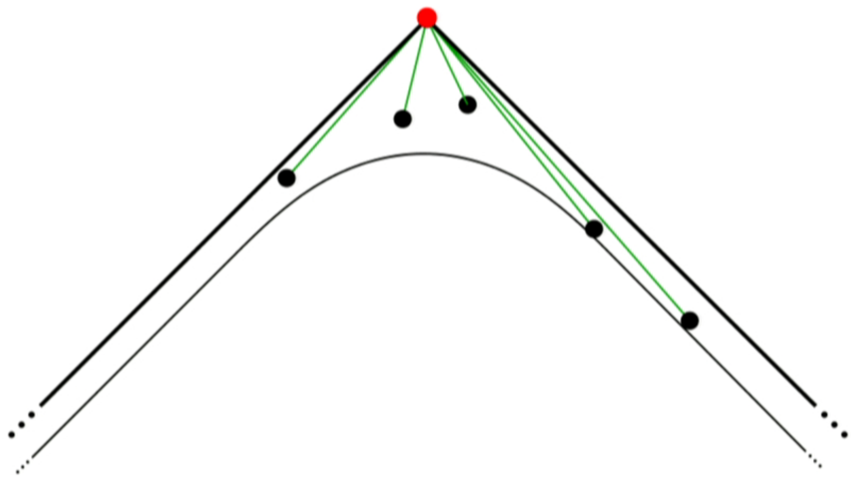


Alexandrov intervals



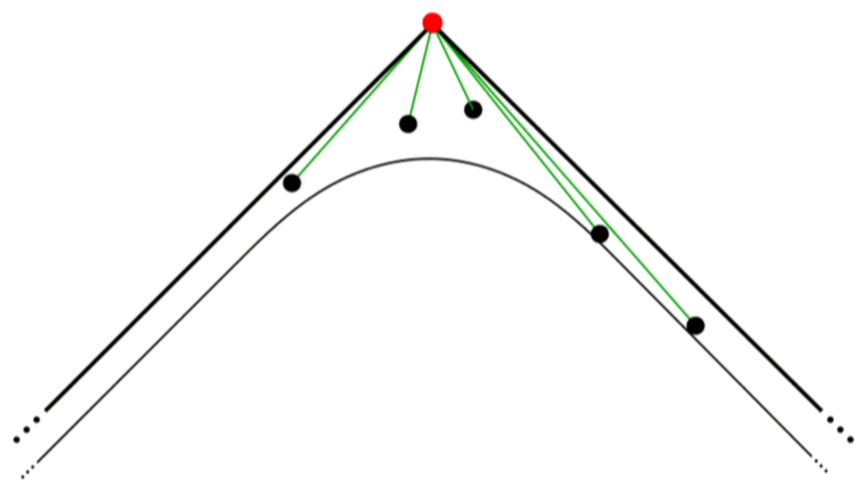
$$V_{0\,d}(x,y) = S_{d-2}\frac{1}{d(d-1)2^{d-1}}\tau_{x-y}^{d}$$

Infinite valency graph



Every point has infinitely many nearest neighbours!

Infinite valency graph



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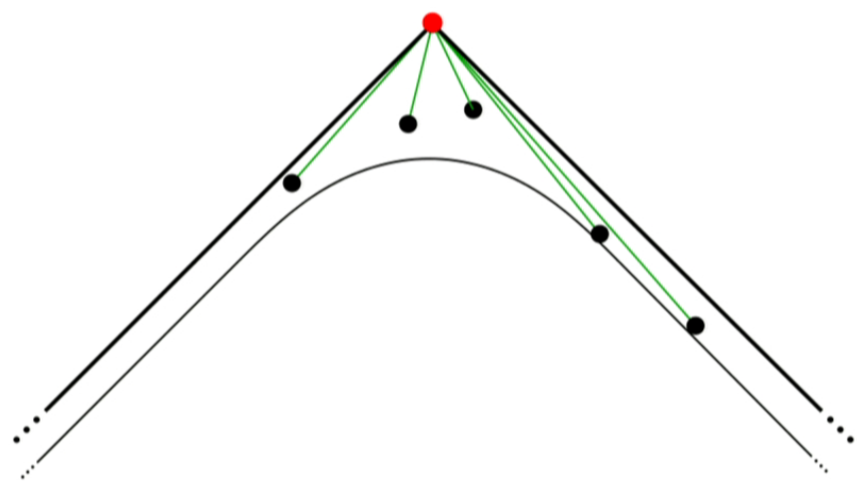


Every point has infinitely many nearest neighbours!

What is a causal set?

The causal set d'Alembertian

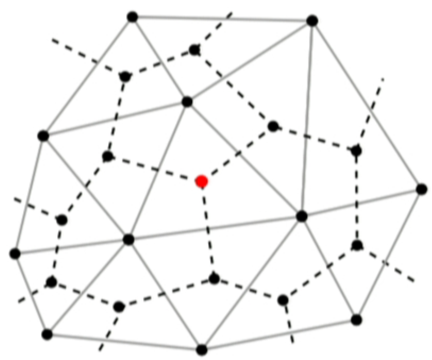
The Causal Set action

A 'local' region?

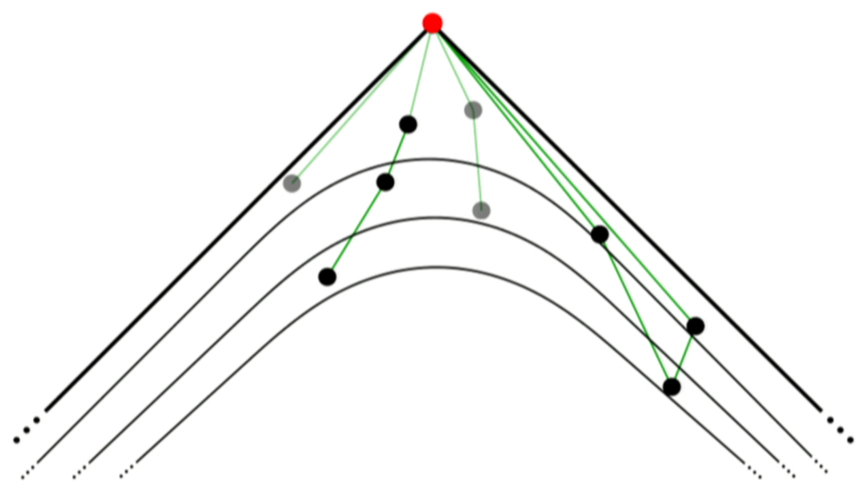
The d'Alembertian operator

Task

Discretize $\Box\phi(x, y)$
on a simplical complex



The 2-d causal set d'Alembertian operator



$$B^{(2)}\phi(x) := \frac{1}{l^2}\Big[-2\phi(x) + 4\Big(\sum_{y\in L_1(x)}\phi(y) - 2\sum_{y\in L_2(x)}\phi(y) + \sum_{y\in L_3(x)}\phi(y)\Big)\Big]$$

(Sorkin arXiv:gr-qc/0703099;Benincasa,Dowker arXiv:1001.2725)

The d-d causal set d'Alembertian operator

The idea is a sum over 'layers' of the causal set.

$$B^{(d)}\phi(x) = \frac{1}{l^2}\left(\alpha_d \phi(x) + \beta_d \sum_{i=1}^{n_d} C_i^{(d)} \sum_{y \in L_i} \phi(y)\right)$$

α_d, β_d, $C_i^{(d)}$ and n_d are dimension dependent constants.

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Non-locality scale

This operator fluctuates strongly.
In fact, in the infinite density limit it will fluctuate infinitely much.

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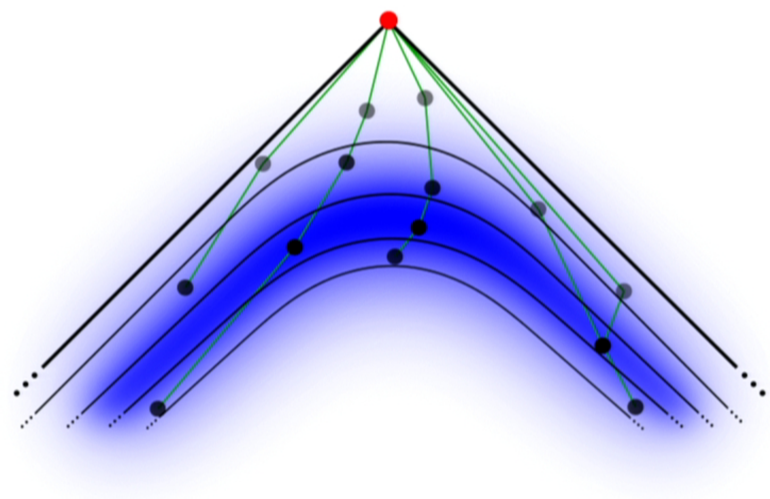
Sorkins solution

Introduce a second, intermediate non-locality scale l .

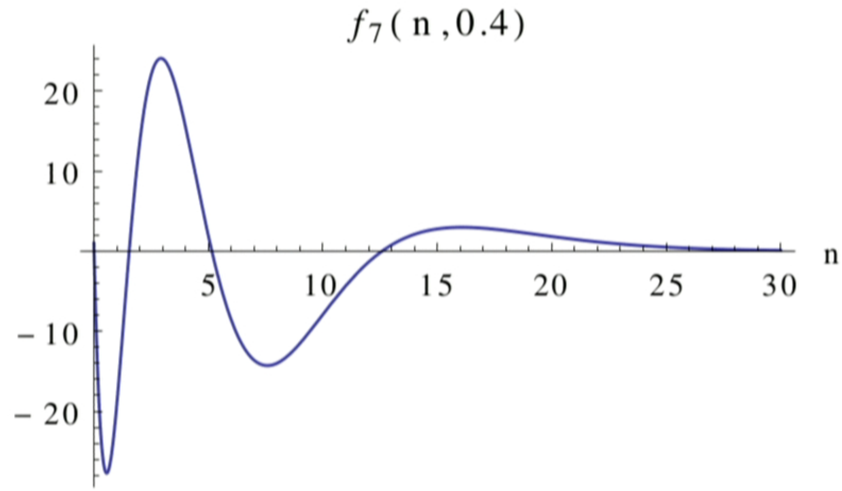
(Sorkin arXiv:gr-qc/0703099)



How do we introduce it?

smearing function $\epsilon = \left(\frac{l_p}{l}\right)^d$

$$f_d(n,\epsilon) := (1-\epsilon)^n \sum_{i=1}^{n_d} C_i^{(d)} \binom{n}{i-1} \left(\frac{\epsilon}{1-\epsilon}\right)^{i-1} . \qquad (1)$$



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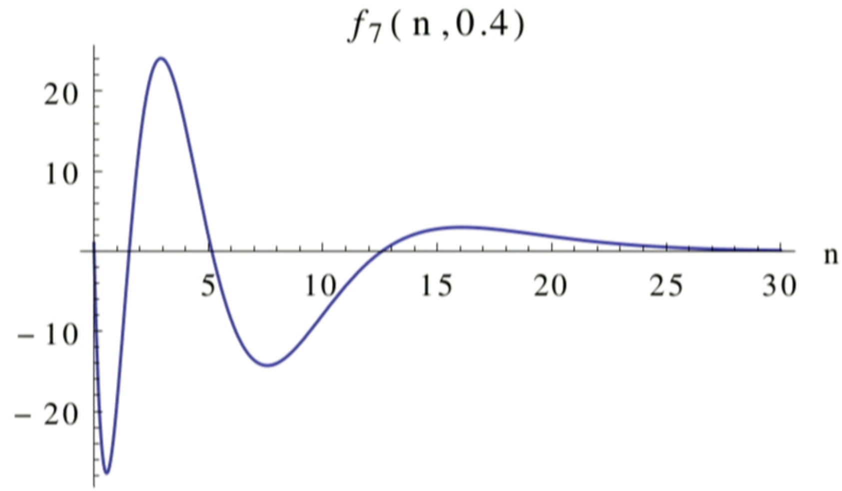

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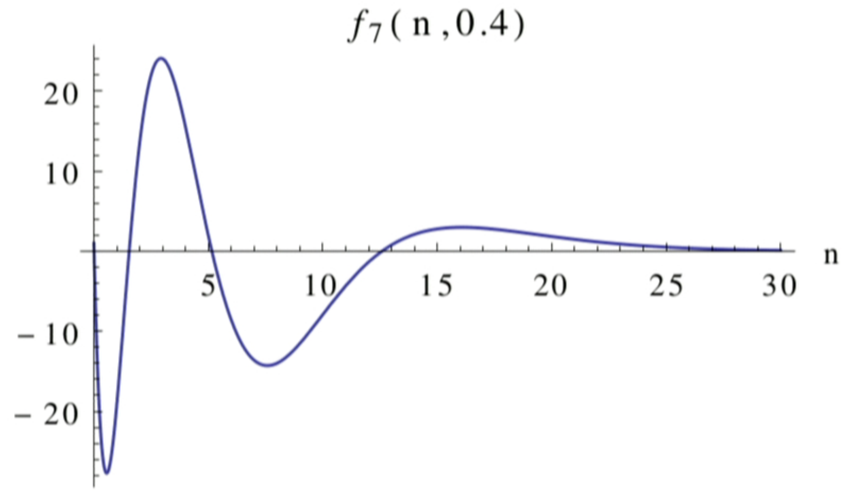
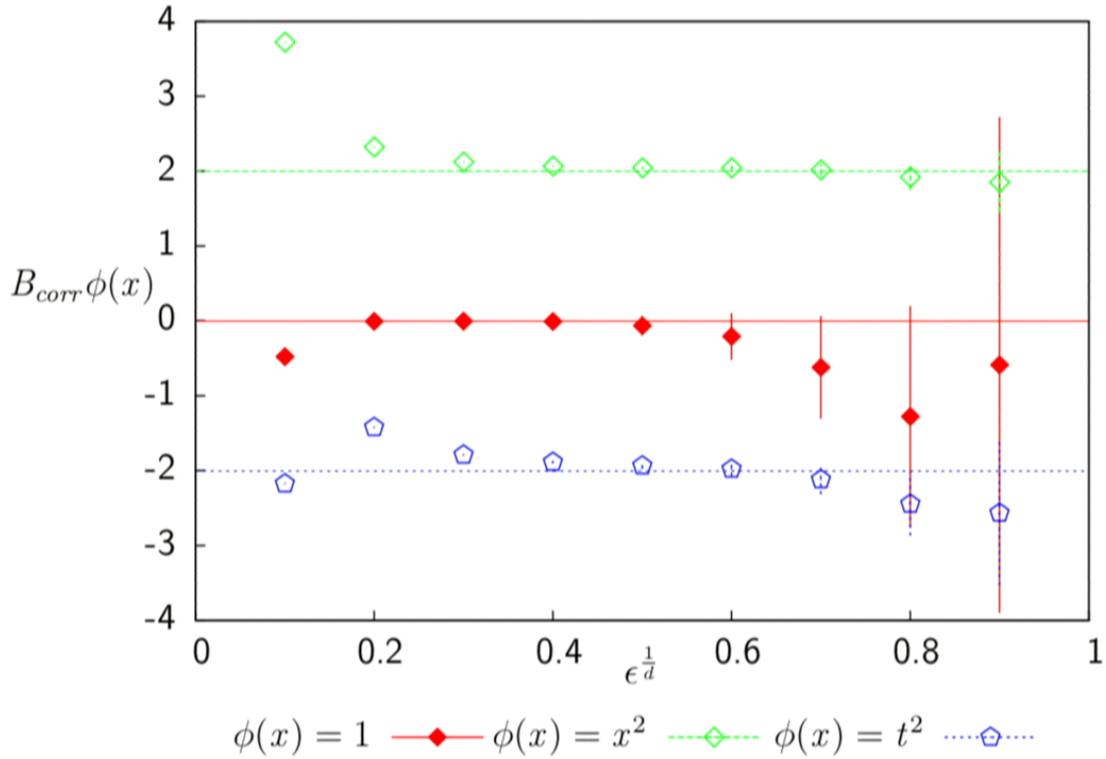


Simulation results



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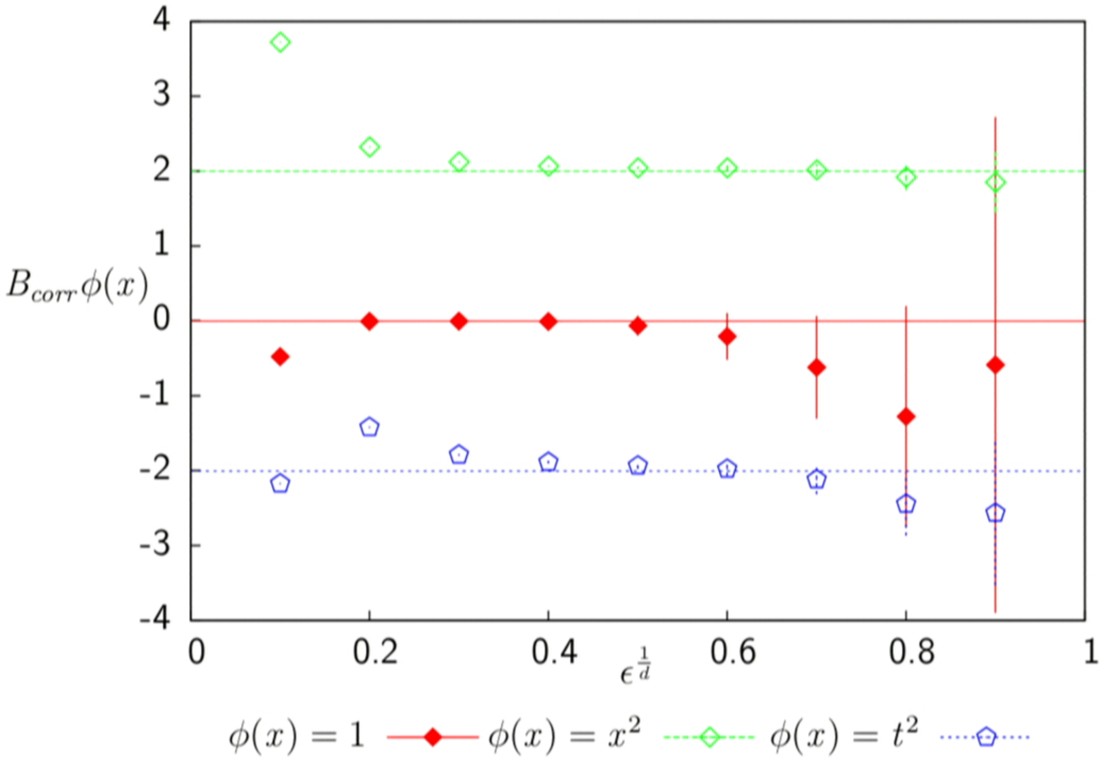

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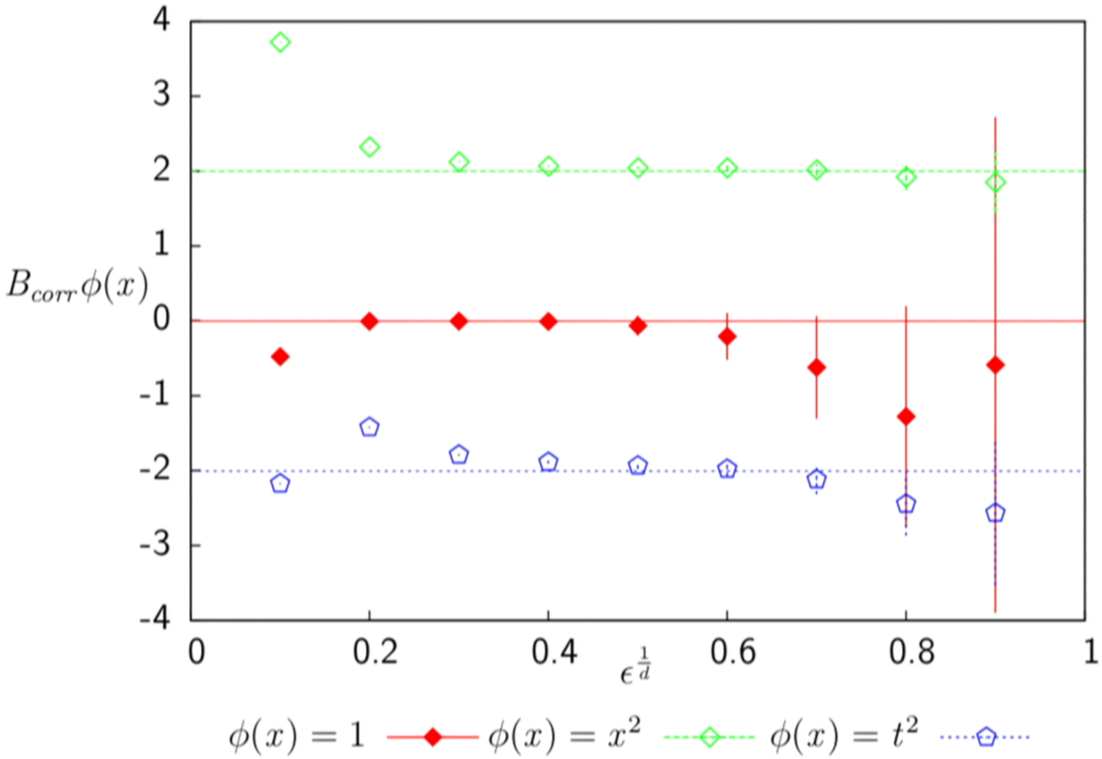

The action

On a curved spacetime

$$\lim_{l \to 0} \bar{B}^{(d)} \phi(x) = \Box^{(d)} \phi(x) + \frac{1}{2} R(x) \phi(x) \, ,$$

Then we can sum over all points in a region to obtain an action

$$\int R(x) \simeq \sum_x B^{(2)}(2)$$

$$\frac{1}{\hbar} S_{2D} = N - 2N_0 + 4N_1 - 2N_2$$

Assuming the discreteness is at the Planck scale $l = l_p$

(Benincasa,Dowker,Schmitzer arXiv:1011.5191)

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Monte-Carlo Simulations

The action can be for Monte Carlo Simulations

$$Z_N = \sum_{C \in \Omega_{2d}} e^{-\frac{\beta}{\hbar} S_{2D}(C,\epsilon)} \tag{3}$$

β is a Wick rotated inverse temperature and Ω_{2d} is a class of 2d orders

(Surya arXiv:1110.6244)

▸ Extra

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▸ Extra

What is Ω_{2d} exactly?

Ω_{2d} is the set of N -element "2D orders"

Definition

Let $S = (1, ..., N)$ and $U = (u_1, u_2, ..., u_N)$, $V = (v_1, v_2, ..., v_N)$, with $u_i, v_i \in S$. U and V are then total orders with $\prec$ given by the natural ordering $<$ in S.
An N -element 2D order is the intersection $C = U \cap V$ of two total N-element orders U and V , i.e., $e_i \prec e_j$ in C iff $u_i < u_j$ and $v_i < v_j$.

This corresponds to lightcone coordinates.

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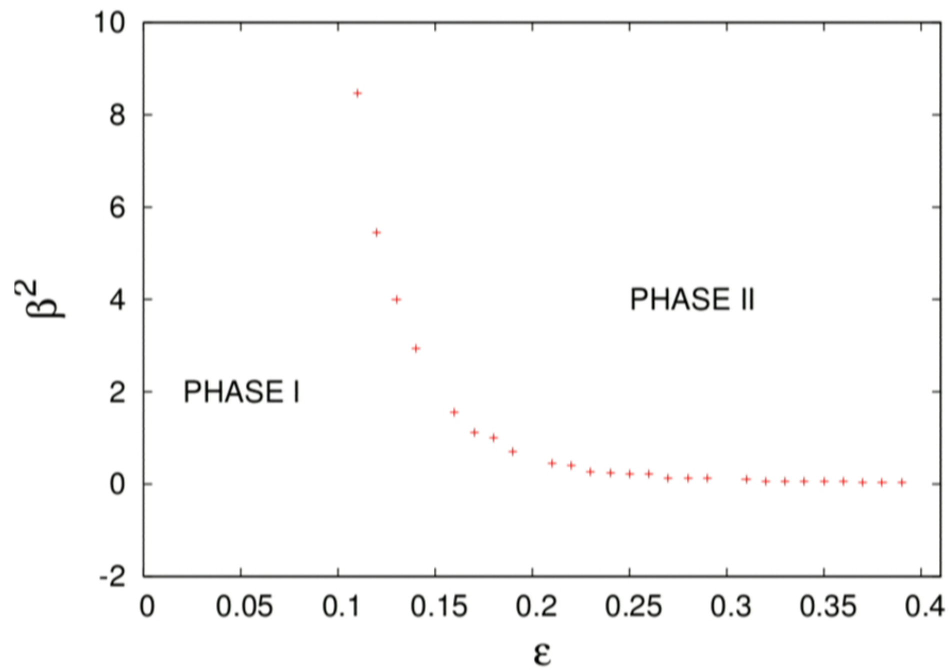
Phase transition



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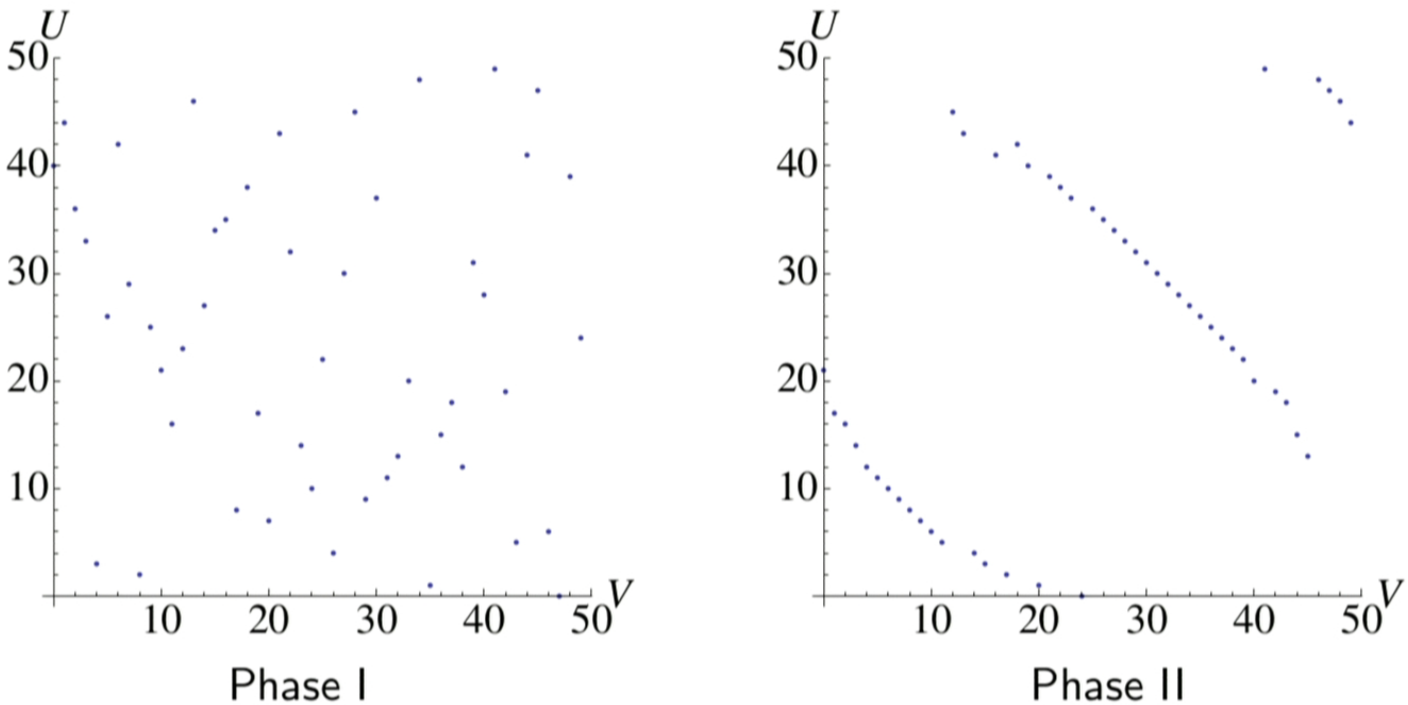

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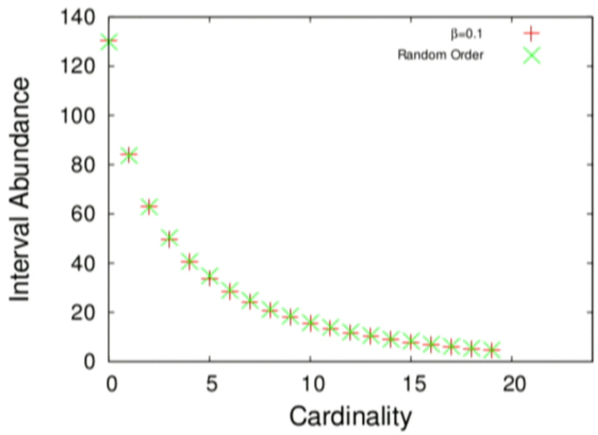
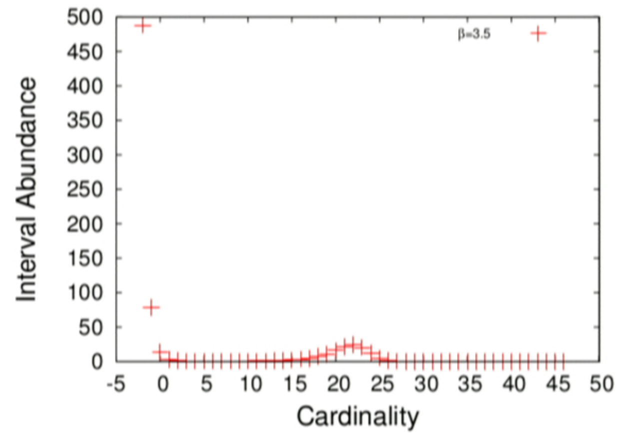
A sign of manifold-likeness?

Counting the number of sub intervals of a given size



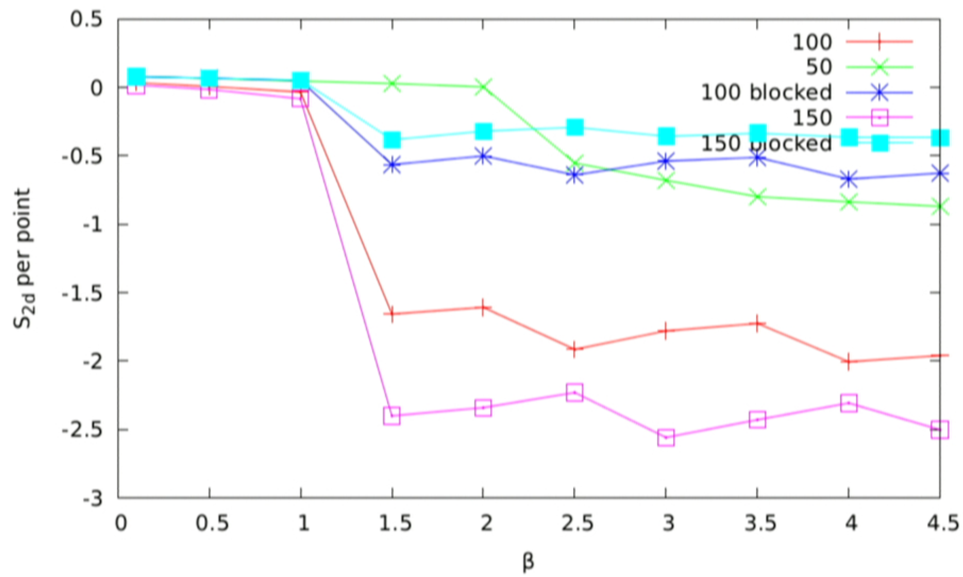
Phase I



Phase II

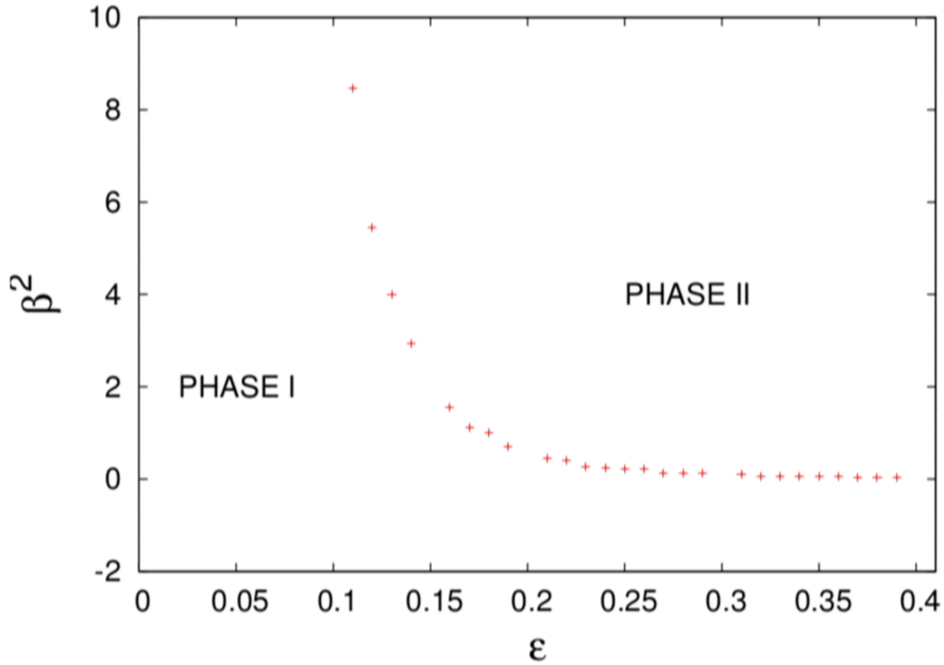
Open questions in 2d MC

- How does the phase transition behave for other volumes ?
- Does the random phase carry over to negative β^2, the quantum theory?



(Glaser,Surya, ongoing work)

Phase transition



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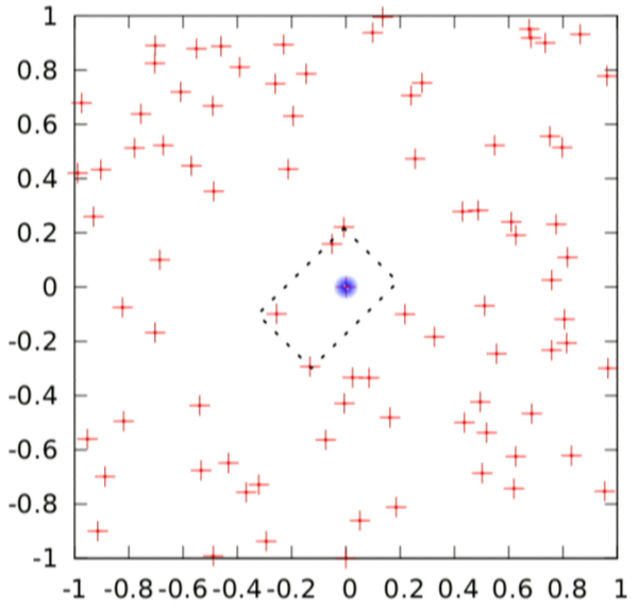
Assuming the discreteness is at the Planck scale $l = l_p$

(Dowker,Glaser arXiv:1305.2588)

What is a local neighbourhood?



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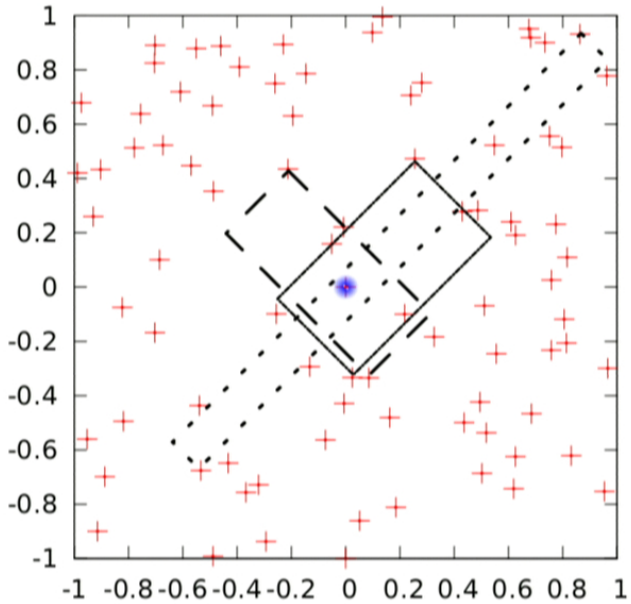

Which of these is local?

In a causal set they are all equal!

Calculating the interval abundance

$$\langle N_m^d \rangle(\rho, V) = \rho^2 \int_{\diamond} \mathrm{d}\, V_y \int_{\diamond_y} \mathrm{d}\, V_x \frac{(\rho V)^m}{m!} e^{-\rho V}$$

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$$= \frac{(\rho V)^{m+2}}{(m+2)!} \frac{\Gamma(d)^2}{\left(\frac{d}{2}(m+1)+1\right)_{d-1}} \frac{1}{\left(\frac{d}{2}m+1\right)_{d-1}}$$

$${}_dF_d \left(\begin{matrix} 1+m, \frac{2k}{d}+m \\ 3+m, \frac{2k}{d}+m+2 \end{matrix} \middle| -\rho V \right)$$

$$k = 0 \ldots d-2$$

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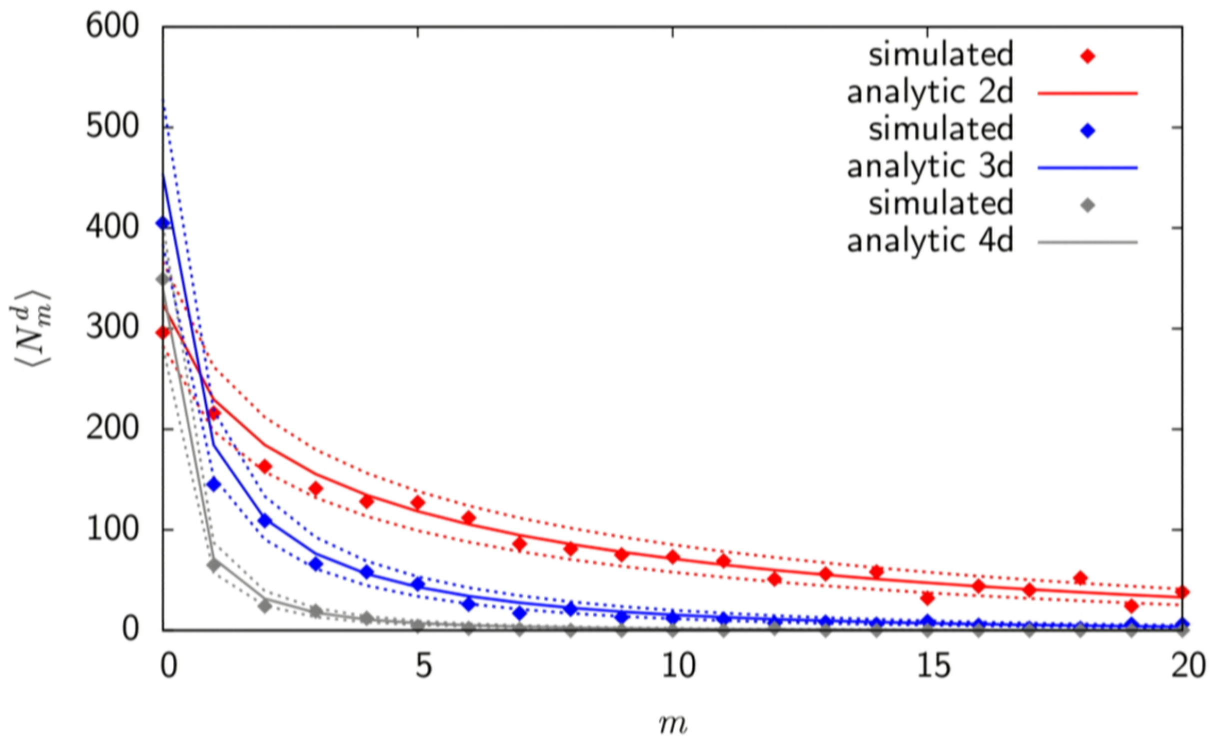
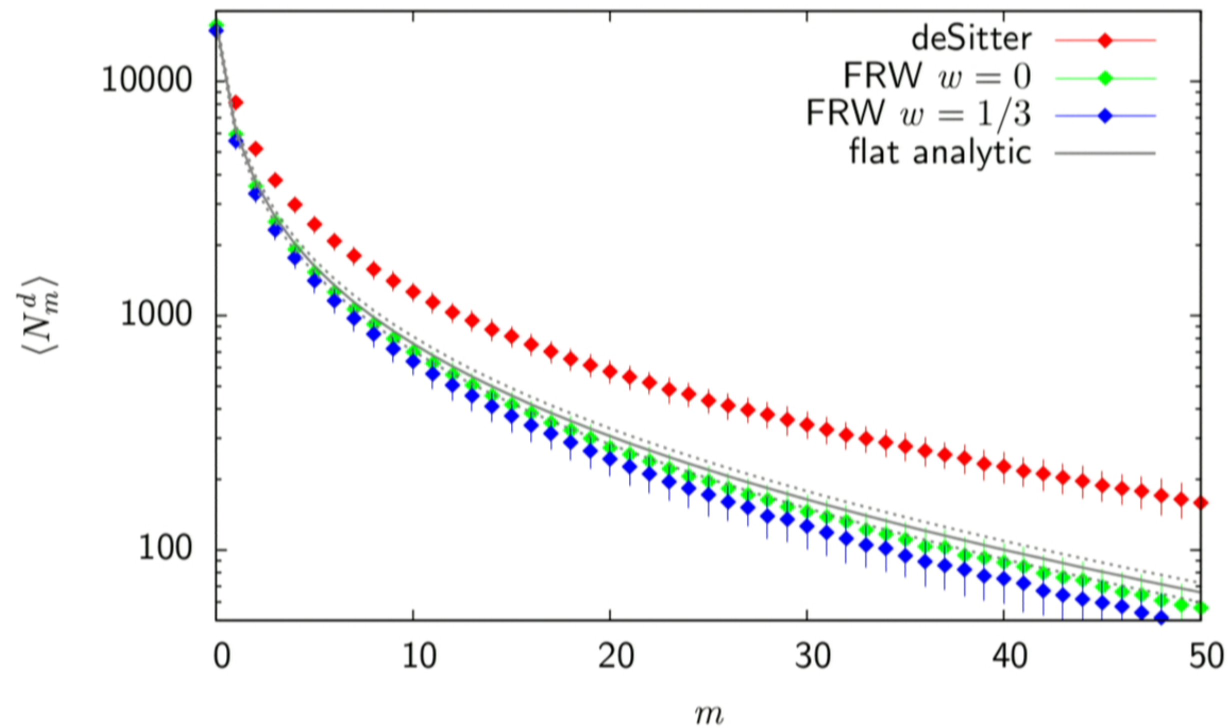
Single sprinkled Causet 100 elements

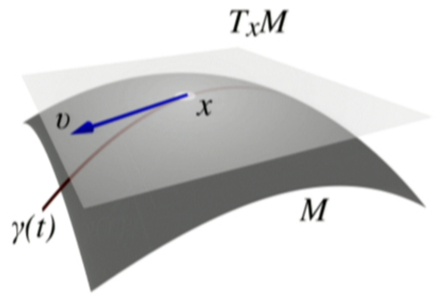


FRW 1000 point Intervals

averaged over 100 realisations

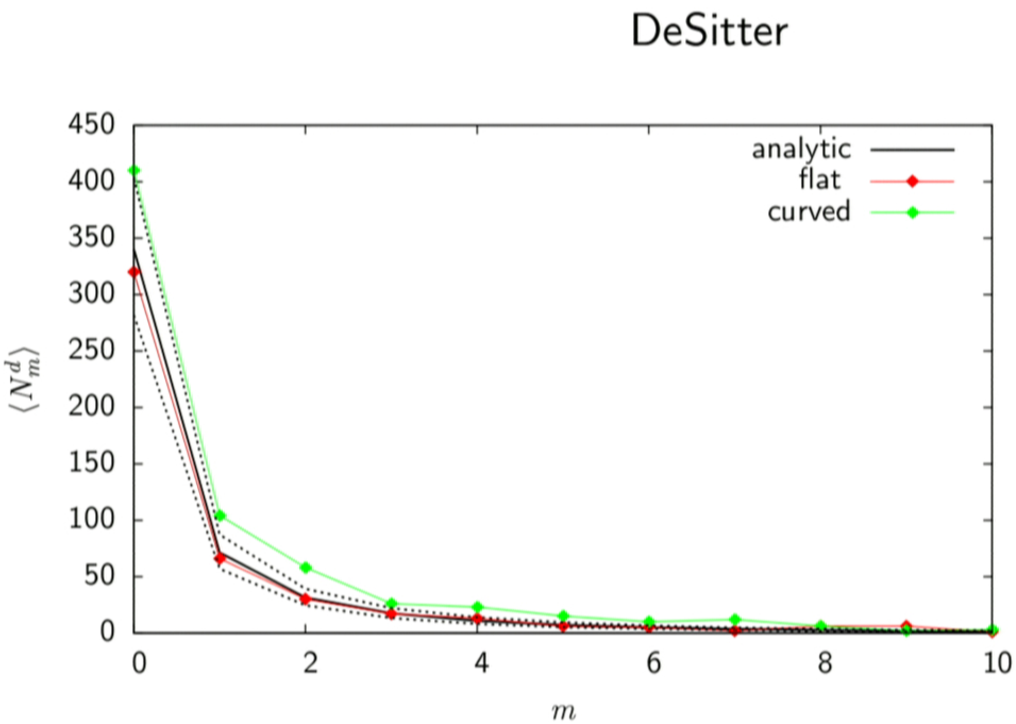


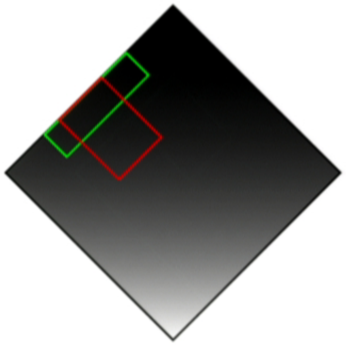
But what does that have to do with locality?



- a smooth manifold will be locally flat
- any local region smaller than the curvature scale will thus be indistinguishable from flat space

FRW Small Intervals





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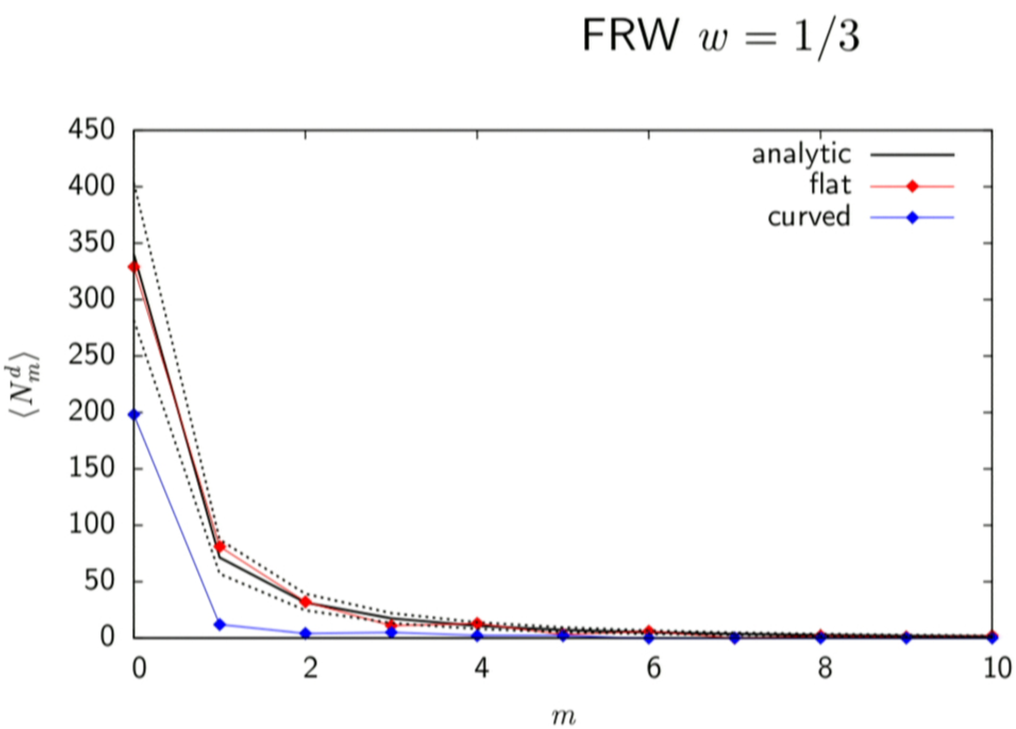

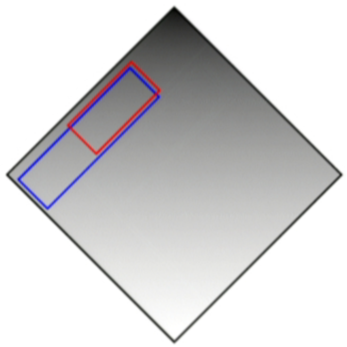

Summary & Conclusion

Summary:

- d'Alembertian in d-dimensions $\rightarrow$ Action!
- intermediate non-locality scale
- 2d Monte Carlo simulations
- a tenative definition of locality

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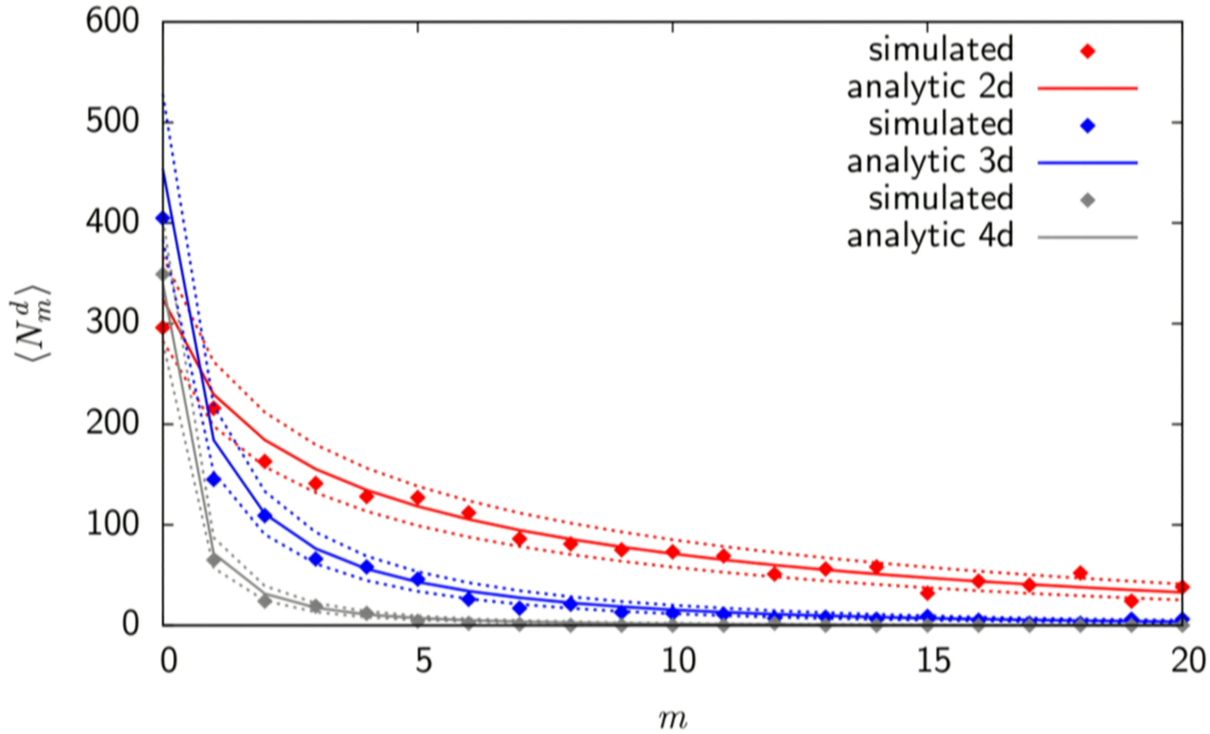

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Open problems:

- Monte Carlo in higher dimensions
- Does the non-locality scale have any phenomenological effects
- Compare to other estimators for R (Roy, Sinha, Surya, arXiv:1212.0631)
- ... what all can we do with locality...?

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