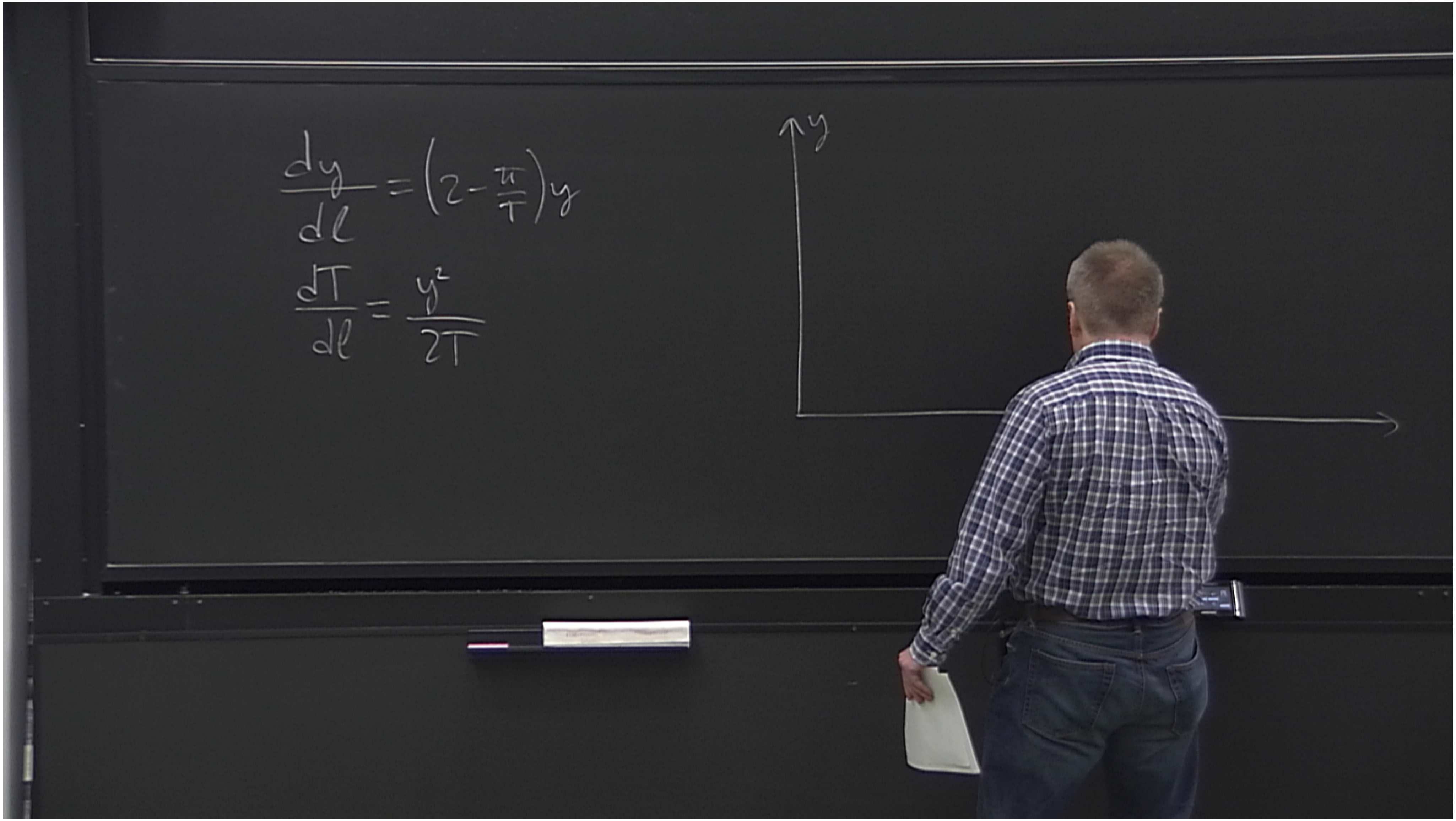
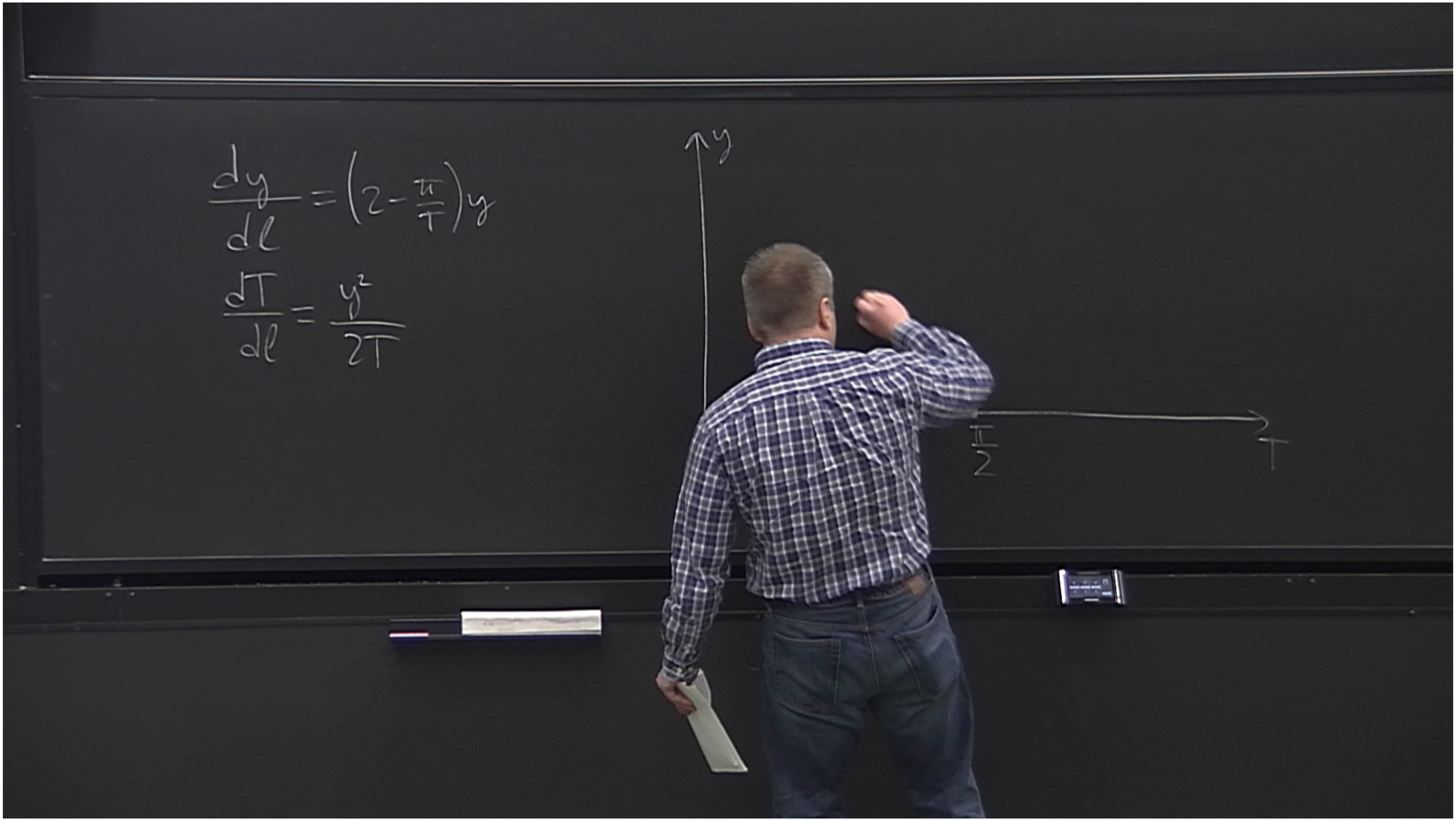
Title: 13/14 PSI - Statistical Mechanics - Lecture 14

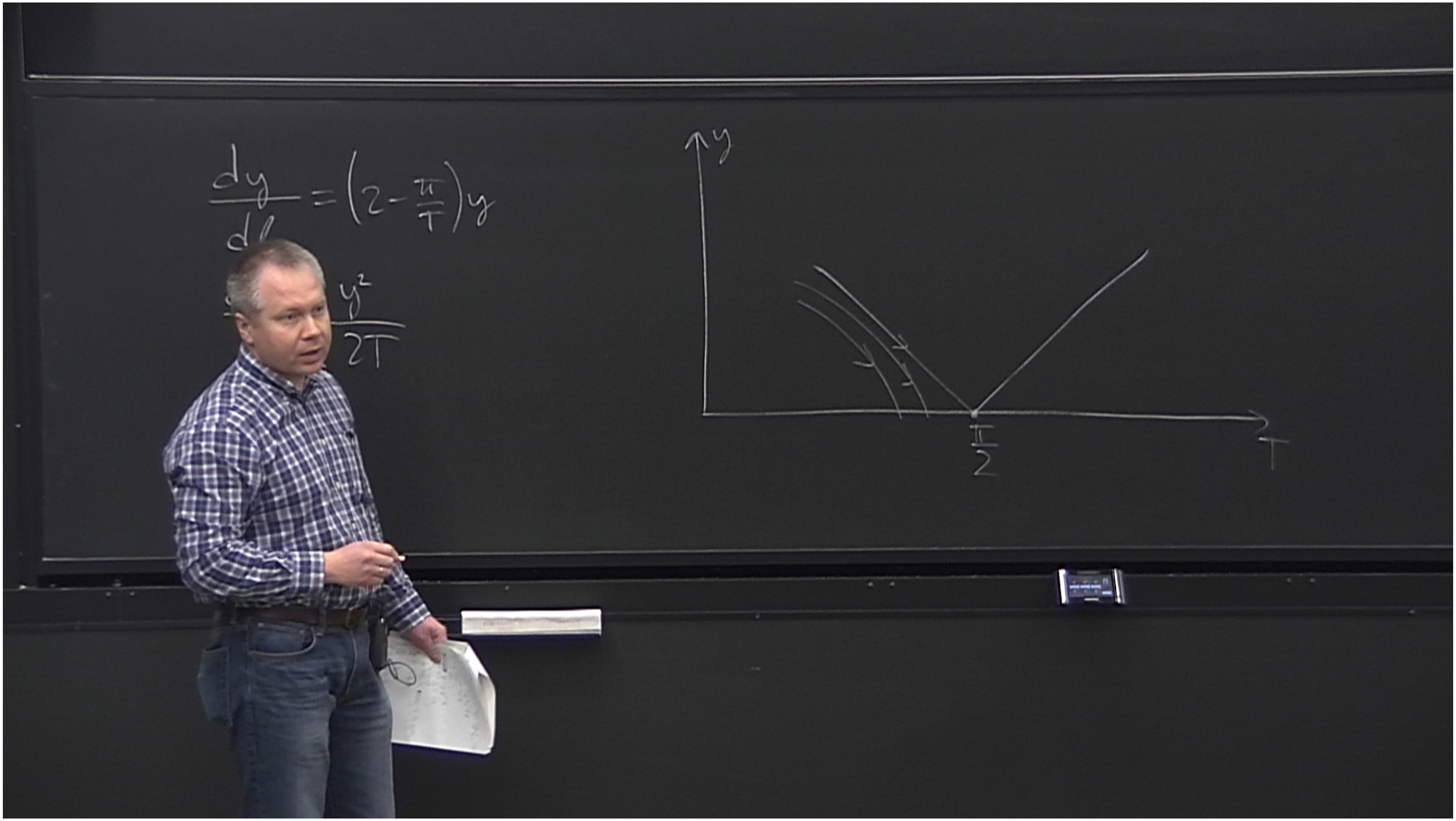
Date: Oct 25, 2013 10:45 AM

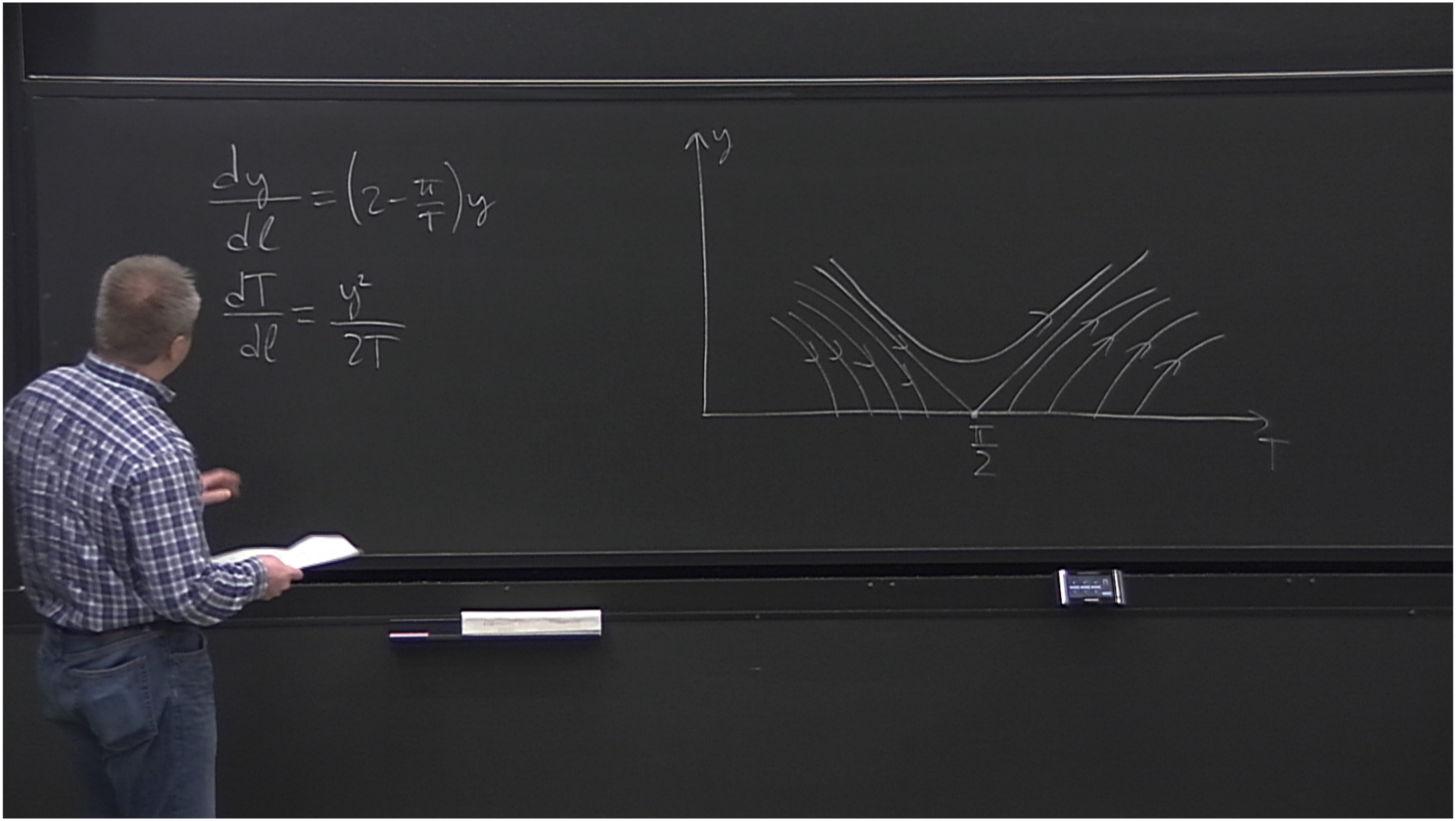
URL: http://pirsa.org/13100062

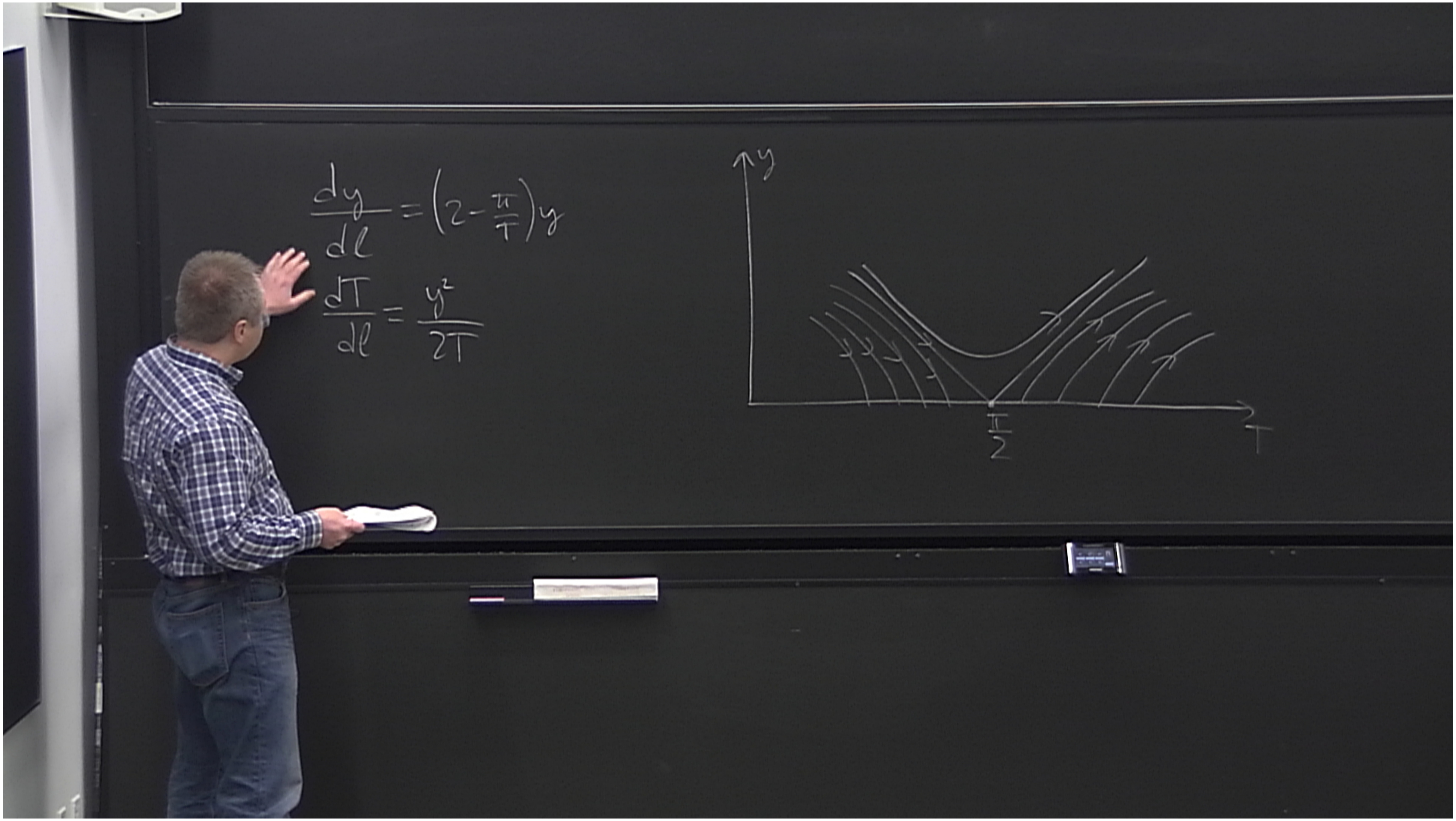
Abstract:

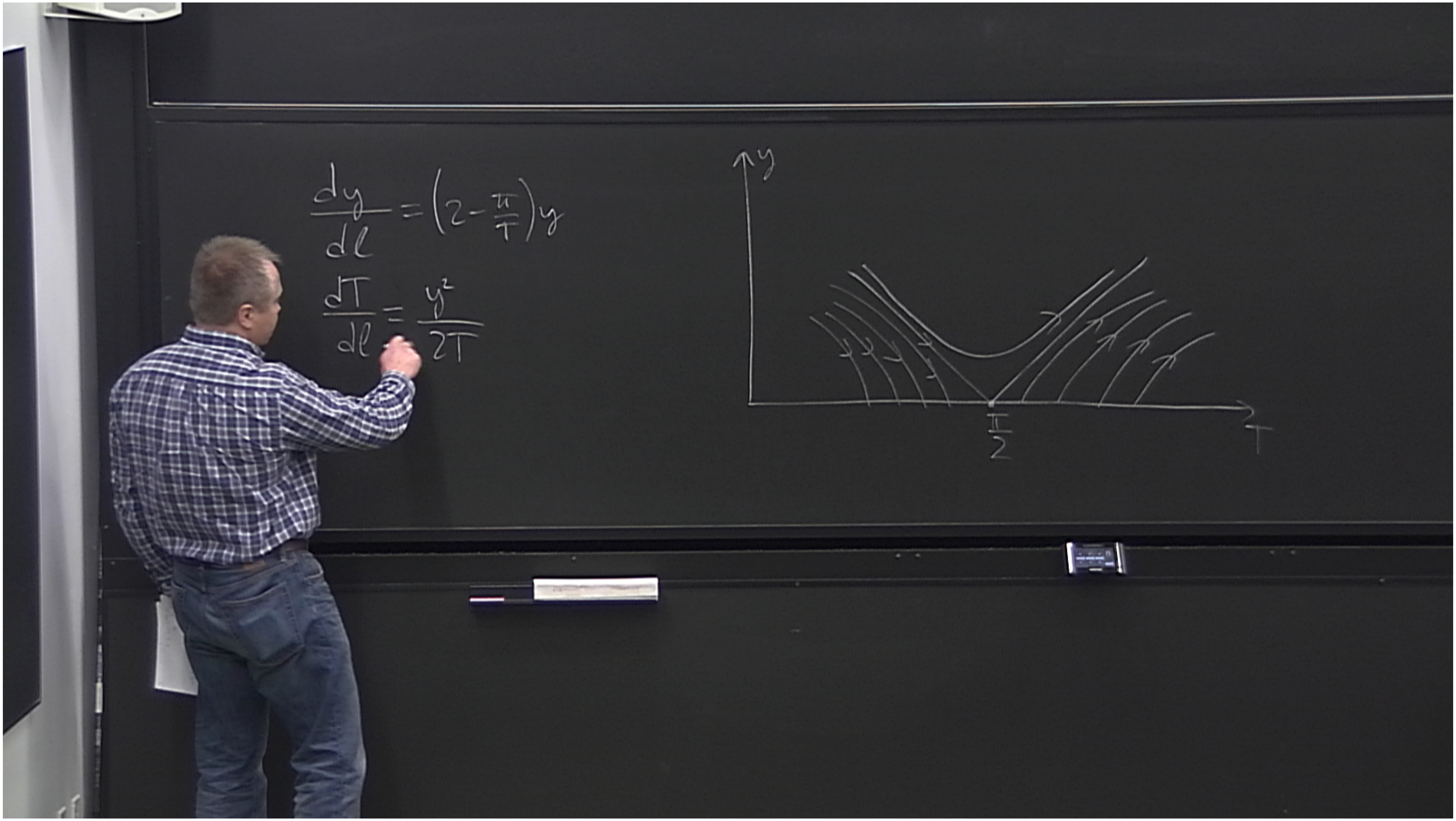

$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right) y$
$\frac{dT}{dl} = \frac{y^2}{2T}$
y

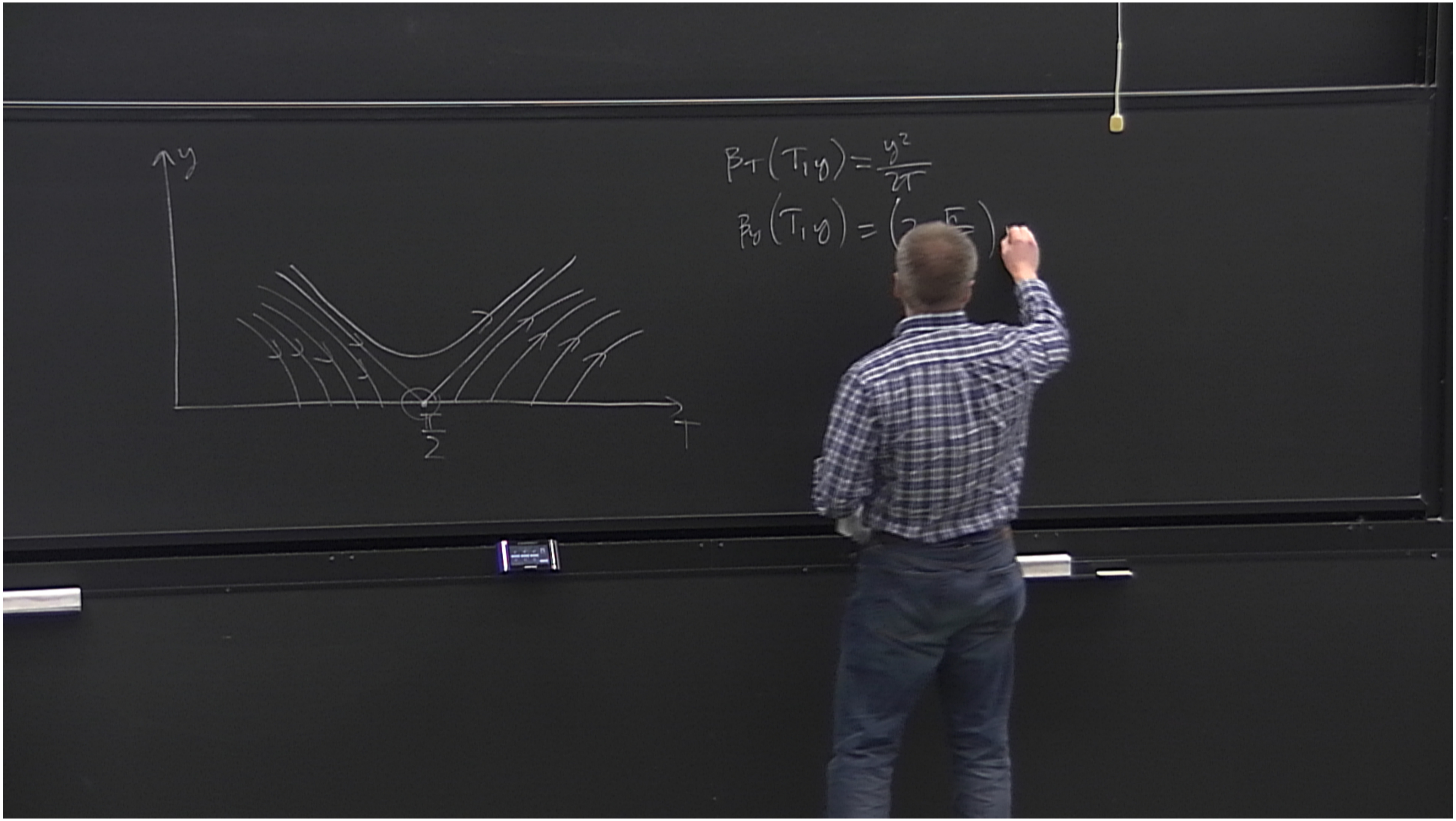

$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right) y$
$\frac{dT}{dl} = \frac{y^2}{2T}$
y
T
$\frac{\pi}{2}$

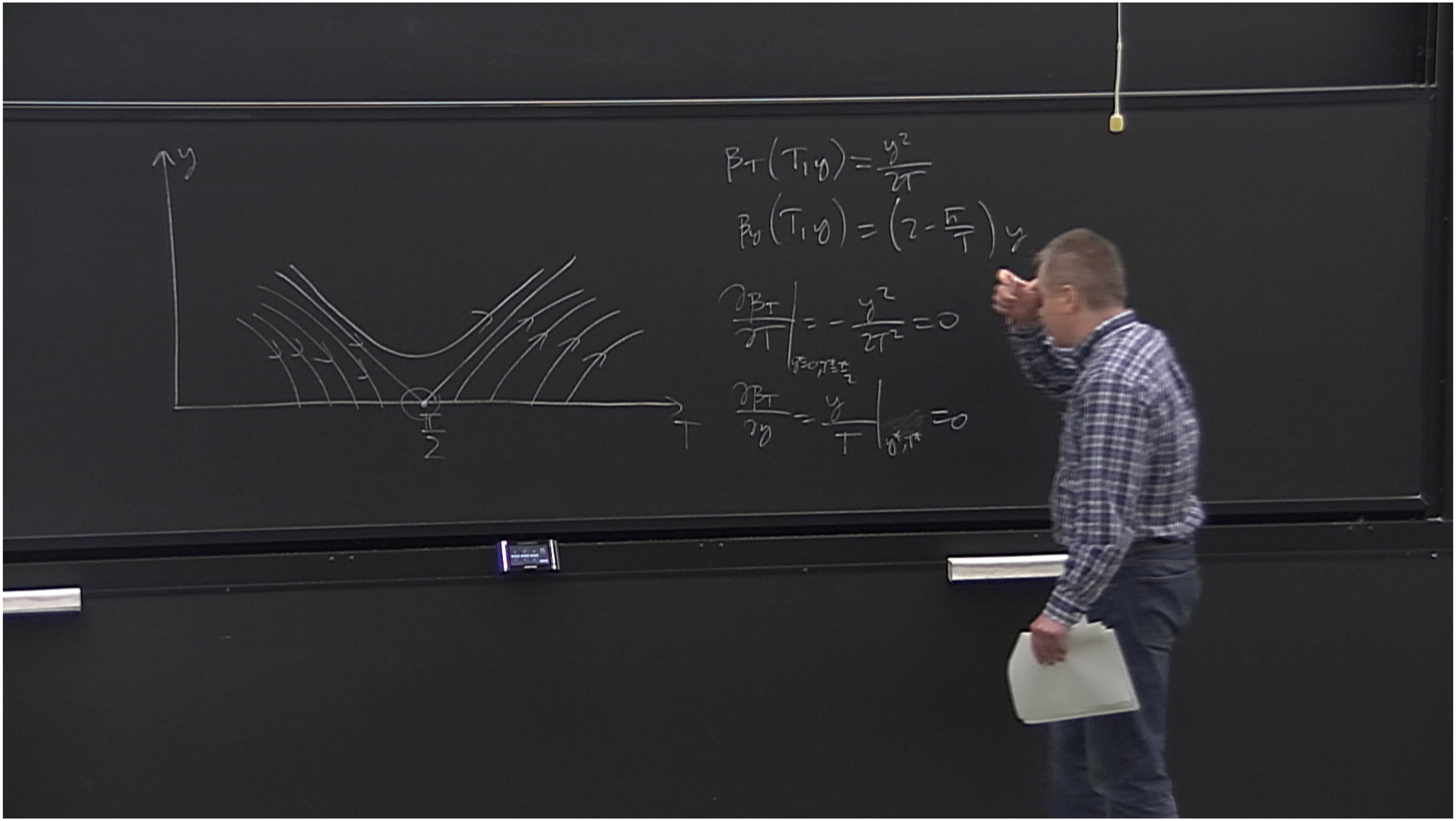

$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right) y$
$\frac{y^2}{2T}$
y
T
$\frac{\pi}{2}$

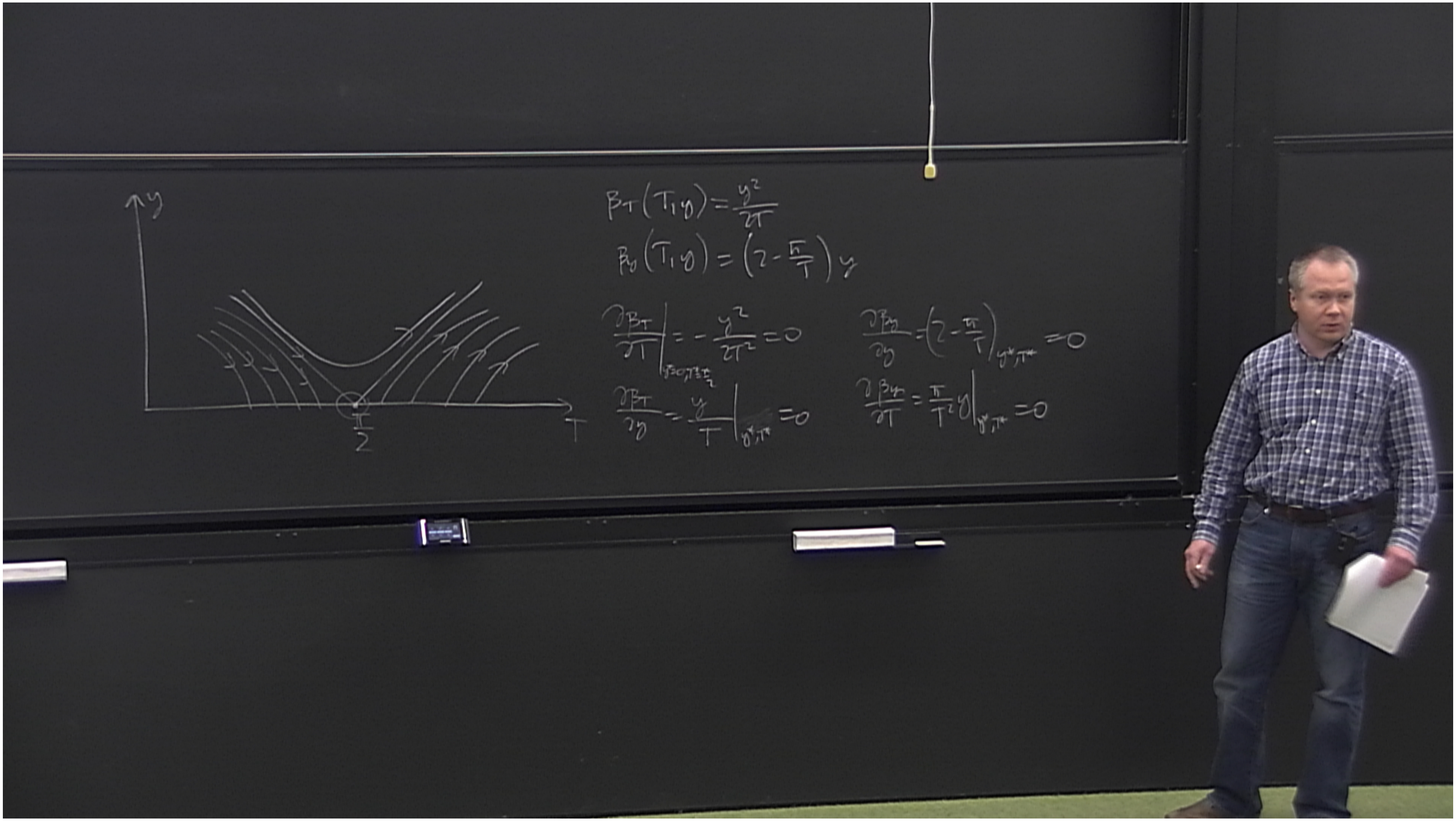

$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right) y$
$\frac{dT}{dl} = \frac{y^2}{2T}$
y
T
$\frac{\pi}{2}$

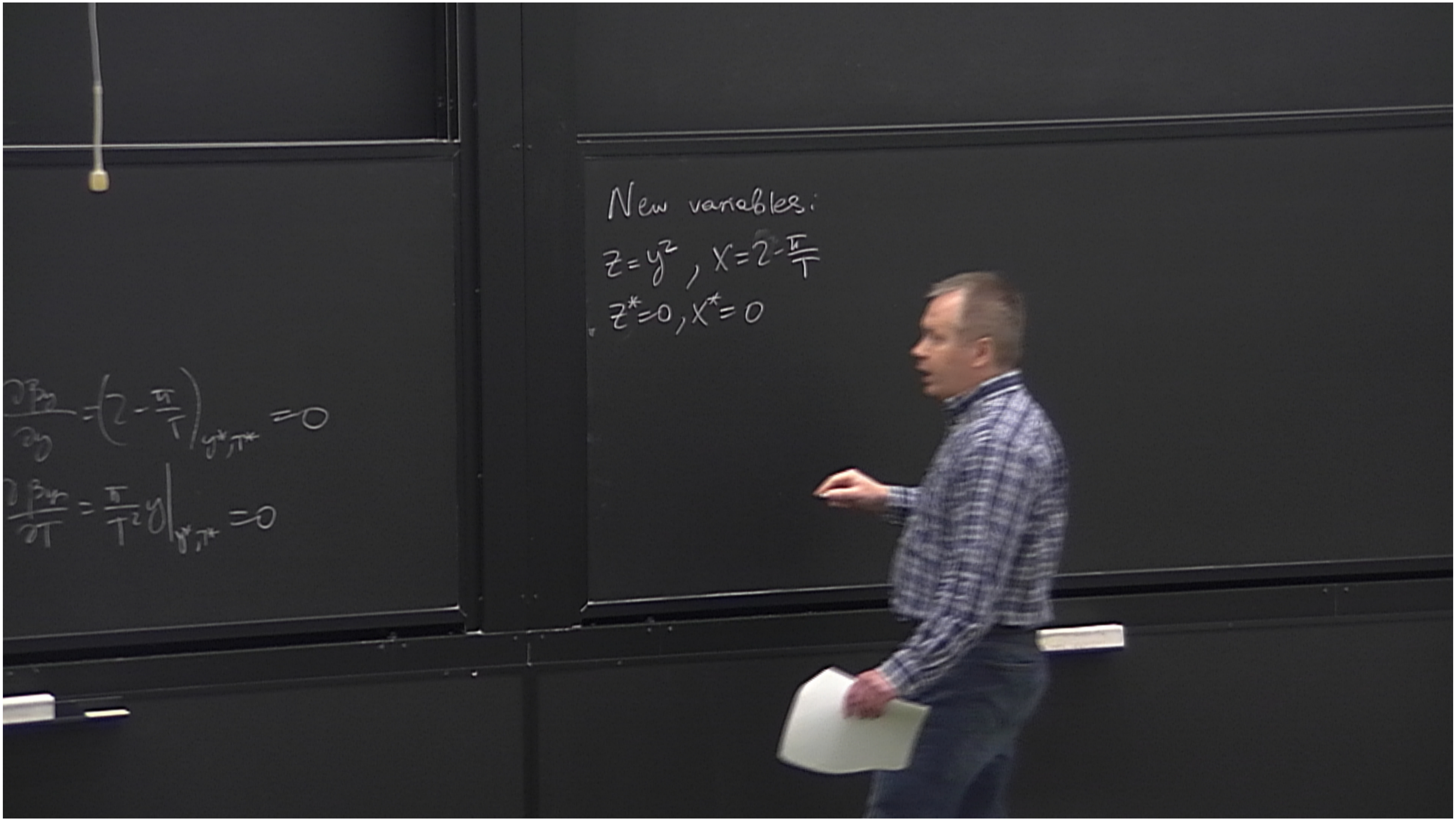

$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right) y$
$\frac{dT}{dl} = \frac{y^2}{2T}$
y
T
$\frac{\pi}{2}$

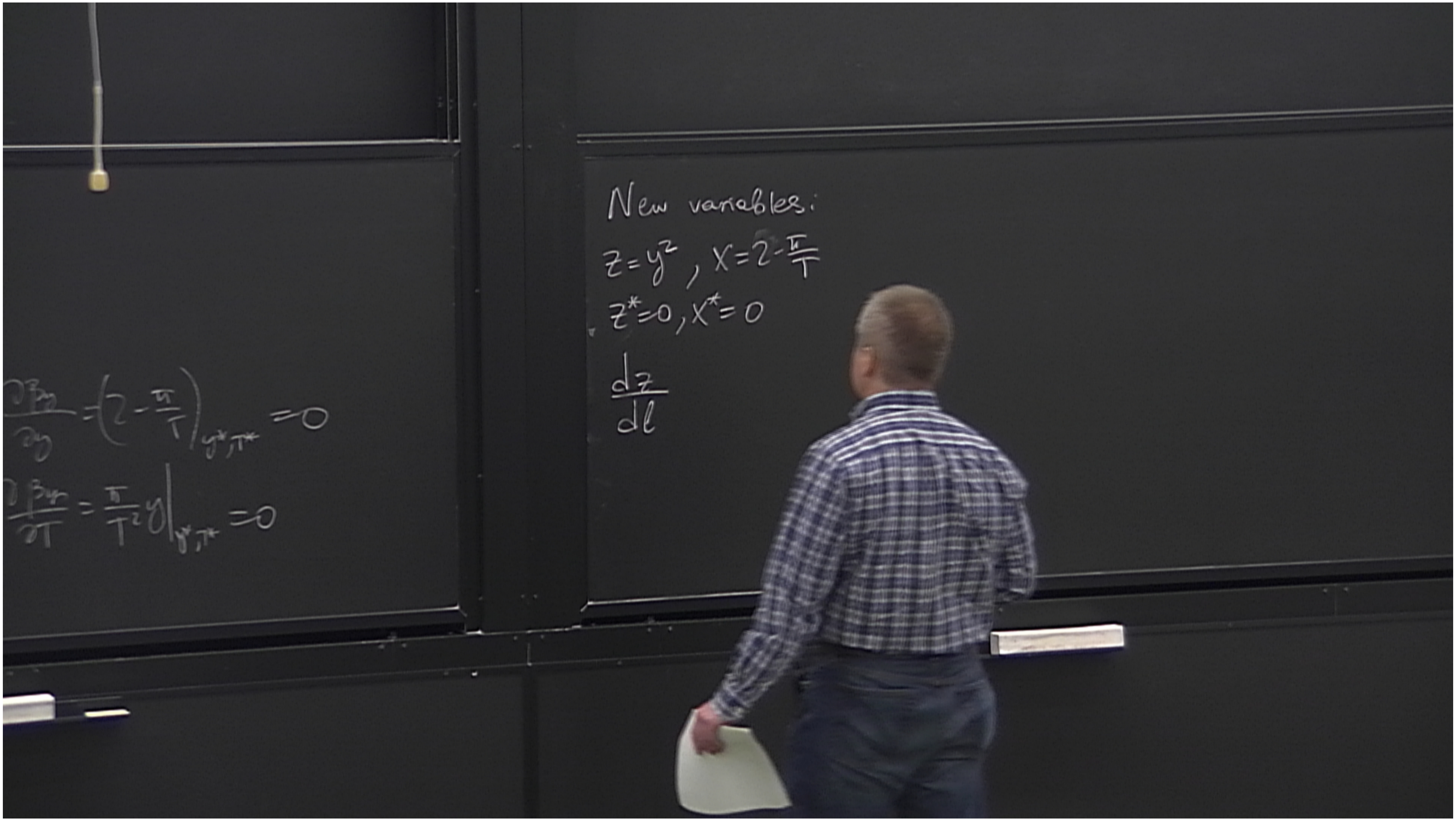

dy/dl = (2 - π/T) y
dT/dl = y²/2T
y
T
π/2

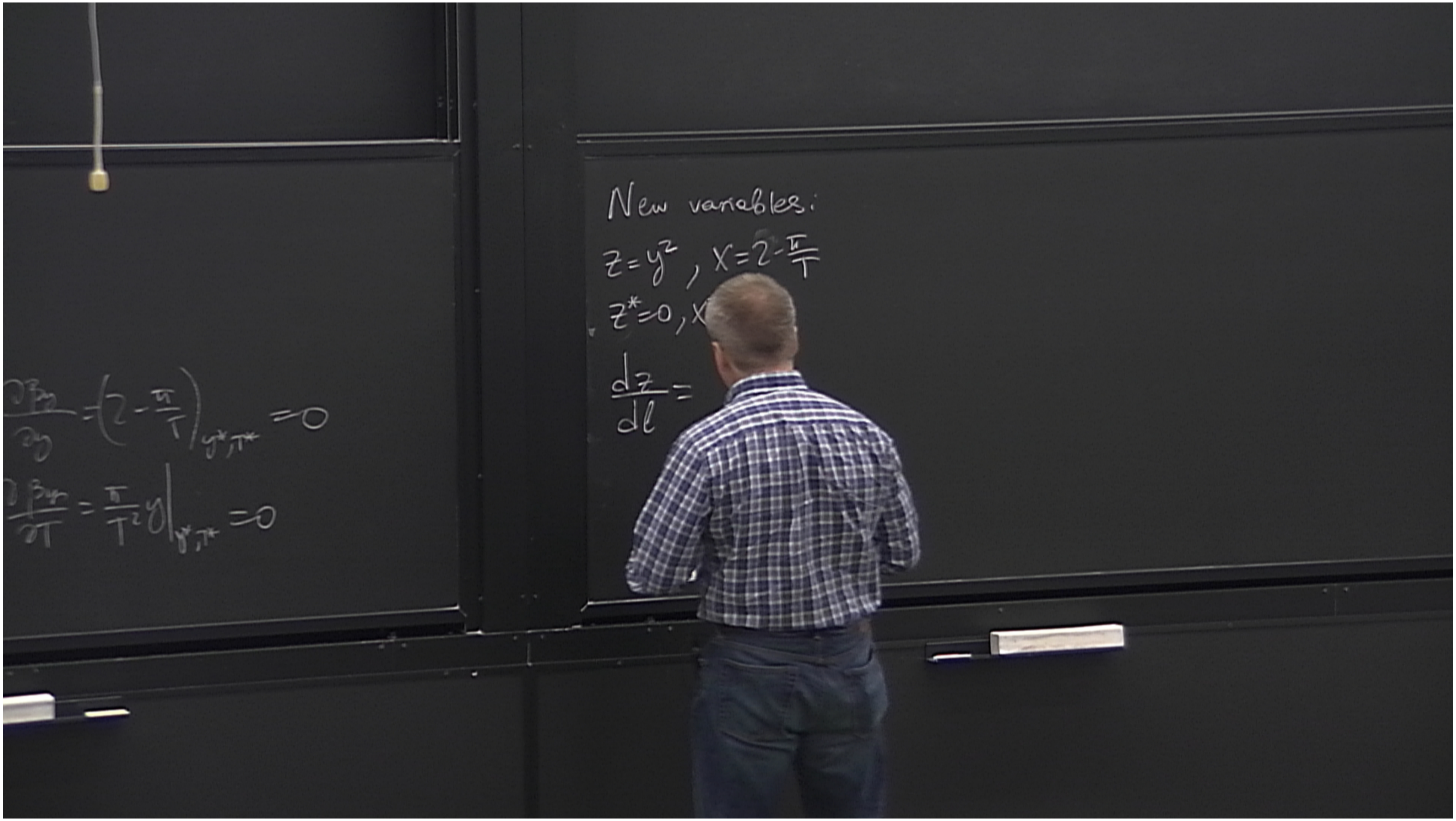

y
T
π/2
βT(T,y) = y²/2T
βy(T,y) = (

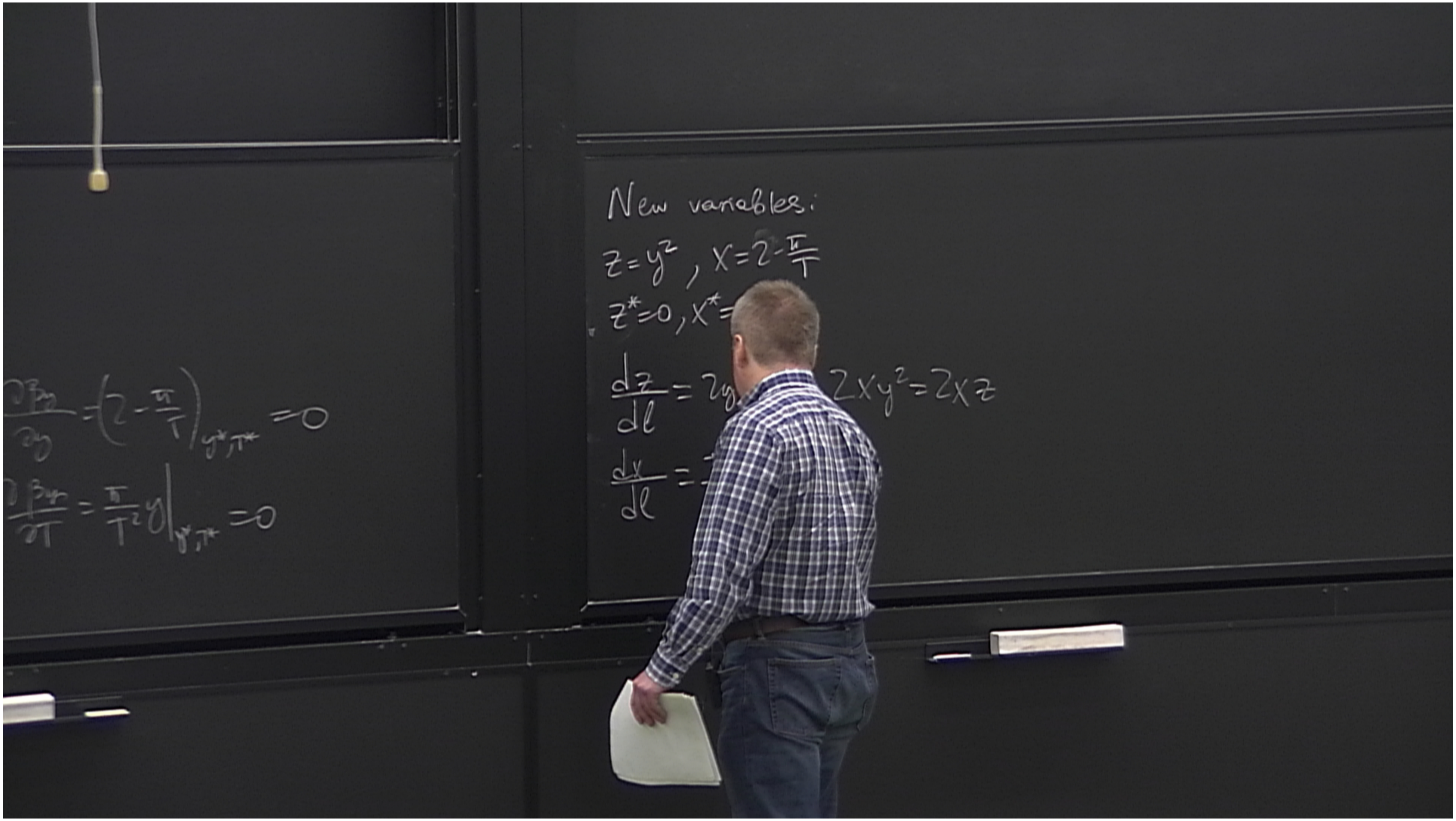

y
T
π/2
β_T(T, y) = y²/2T
β_y(T, y) = (2 - π/T) y
∂β_T/∂T = - y²/2T² = 0
∂β_T/∂y = y/T = 0


y
T
π/2
β_T(T, y) = y²/2T
β_y(T, y) = (2 - π/T) y
∂β_T/∂T | = - y²/2T² = 0
∂β_T/∂y = y/T | y*,T* = 0
∂β_y/∂y = (2 - π/T) | y*,T* = 0
∂β_y/∂T = π/T² y | y*,T* = 0


New variables:
z = y^2 , x = 2 - π/T
z* = 0 , x* = 0


New variables:
$z = y^2$, $X = 2 - \frac{\pi}{T}$
$z^* = 0$, $X^* = 0$
$\frac{dz}{dl}$


New variables:
z = y², x = 2 - π/T
z* = 0, x
dz/dl =
∂β_y/∂y = (2 - π/T)|_{y*,T*} = 0
∂β_y/∂T = π/T² y|_{y*,T*} = 0


∂β_y/∂y = (2 - π/T)|_{y*,π*} = 0
∂β_y/∂T = π/T² y|_{y*,π*} = 0
New variables:
z = y², x = 2 - π/T
z* = 0, x* =
dz/dl = 2y … 2xy² = 2xz
dx/dl =

$$\frac{\partial \beta_y}{\partial y} = \left(2 - \frac{\pi}{T}\right)\Big|_{y^*,T^*} = 0$$

$$\frac{\partial \beta_y}{\partial T} = \frac{\pi}{T^2} y\Big|_{y^*,T^*} = 0$$

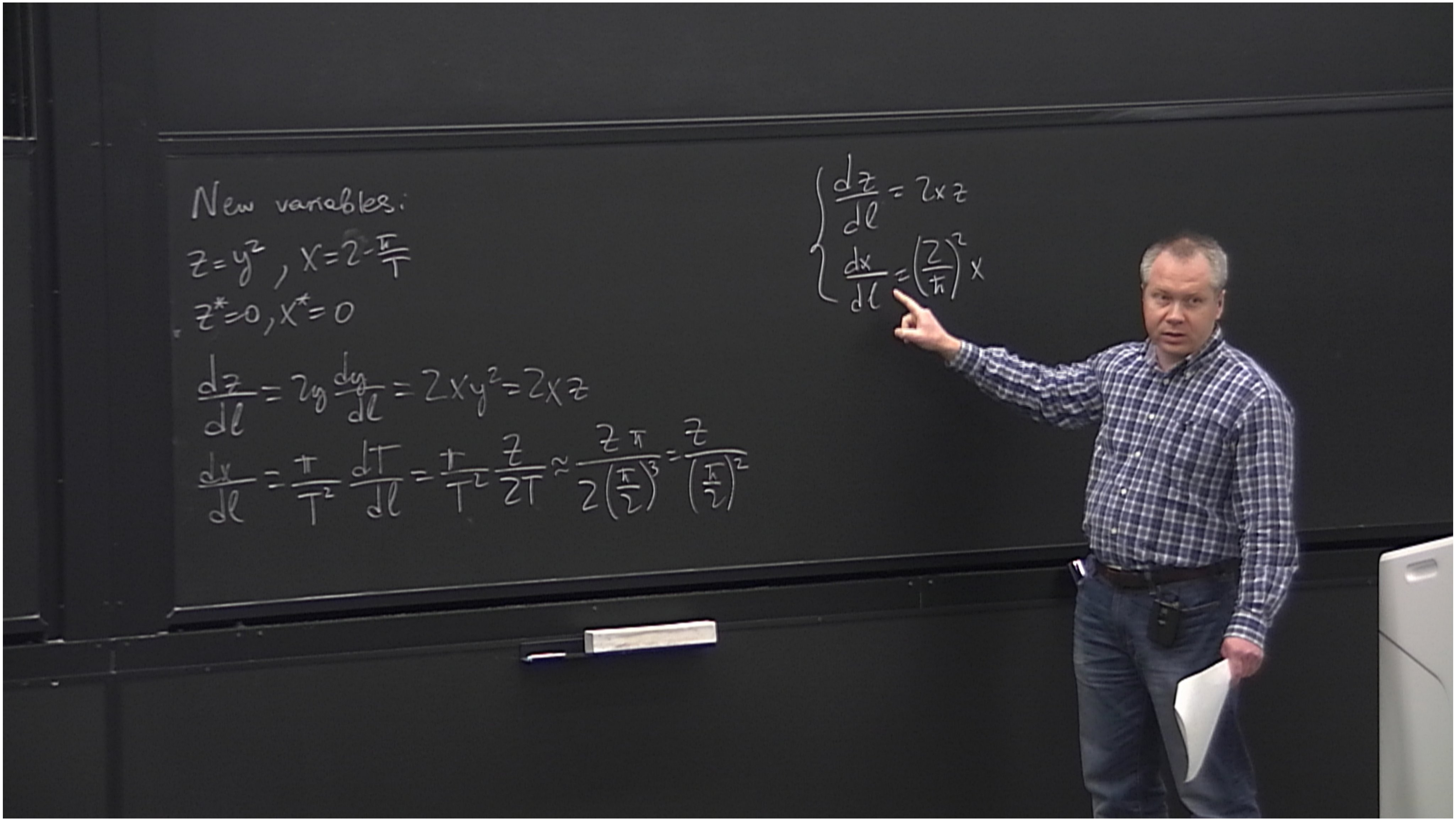
New variables:

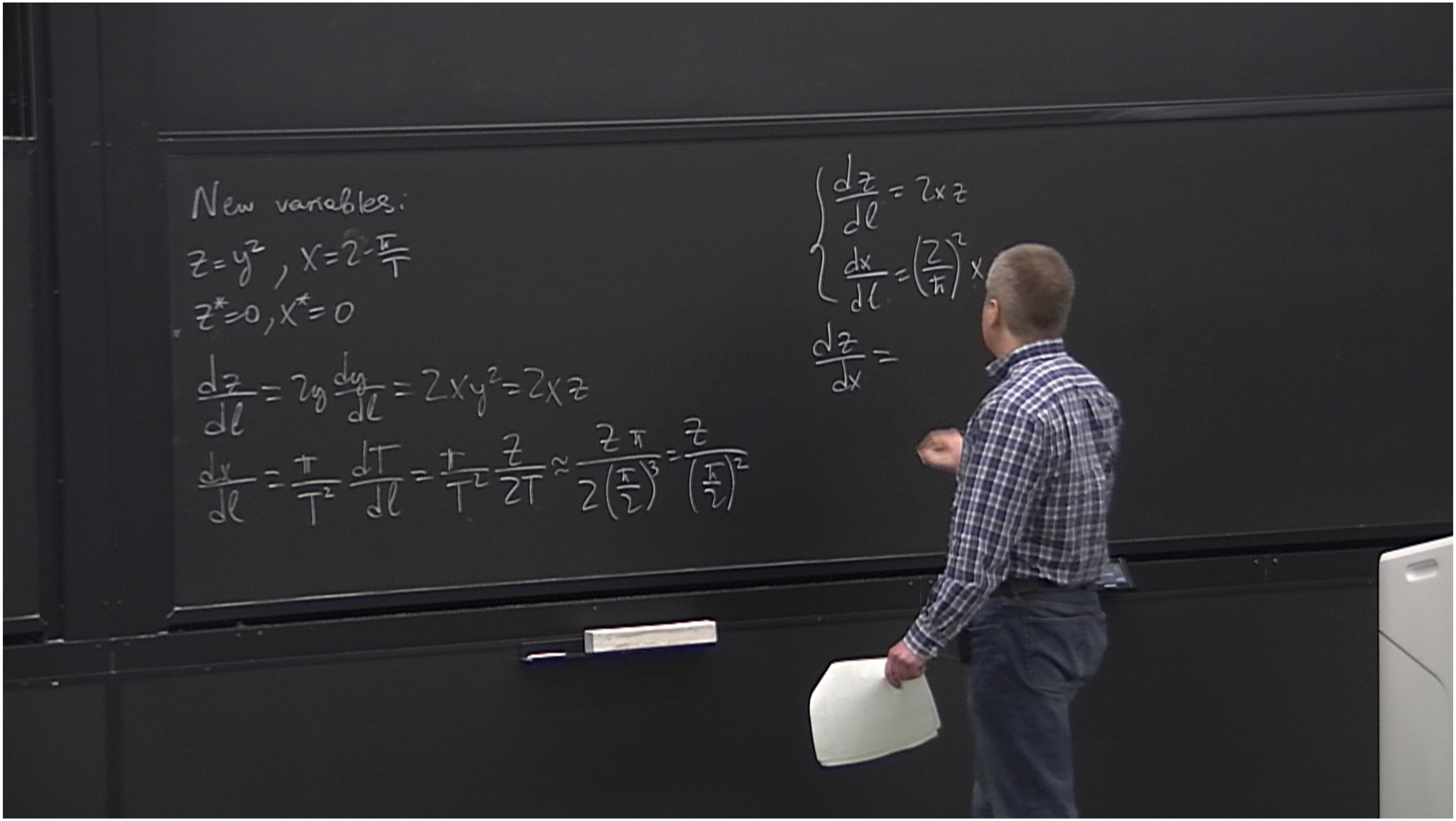
$z = y^2$, $x = 2 - \frac{\pi}{T}$

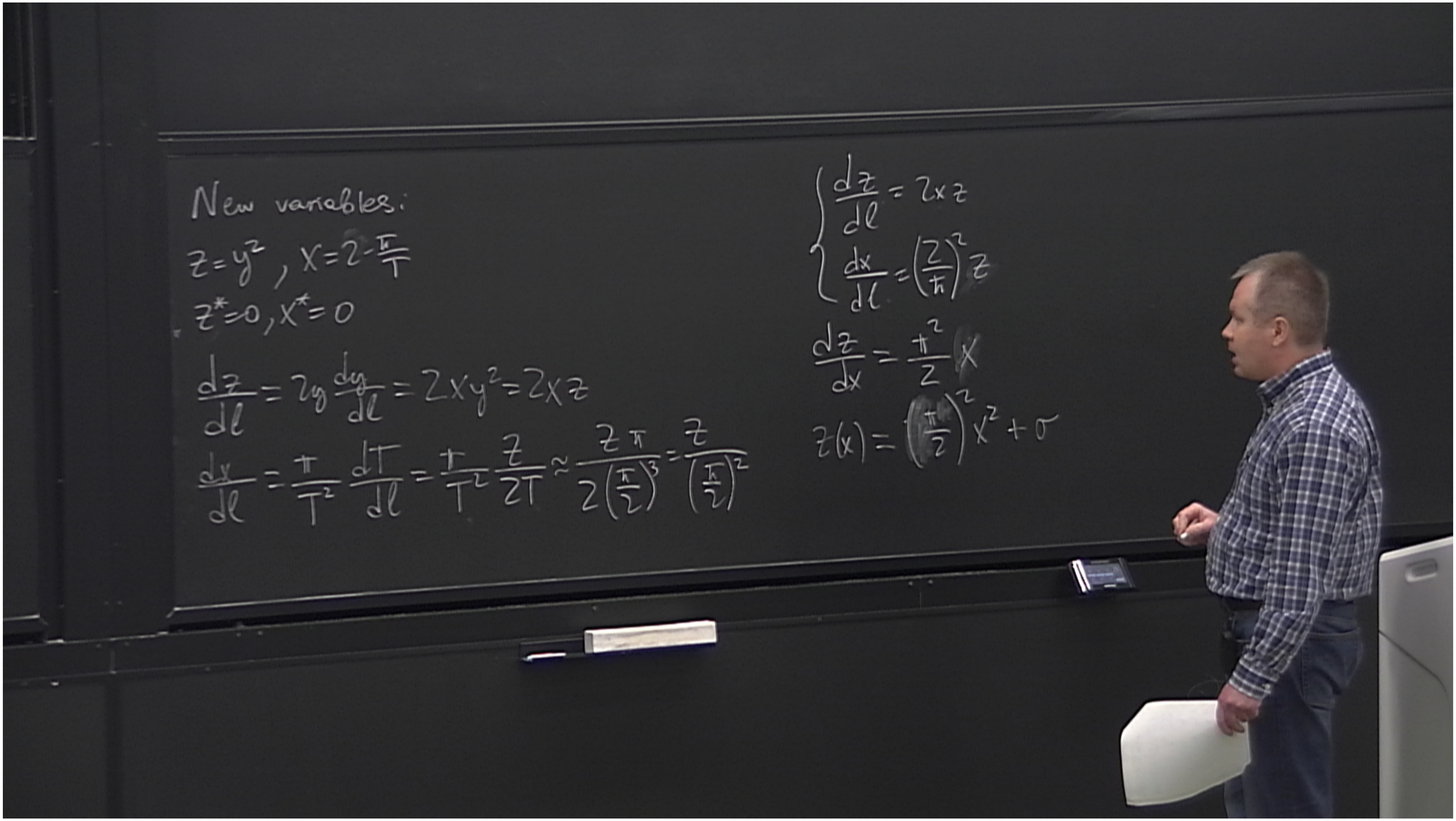
$z^* = 0$, $x^* = 0$

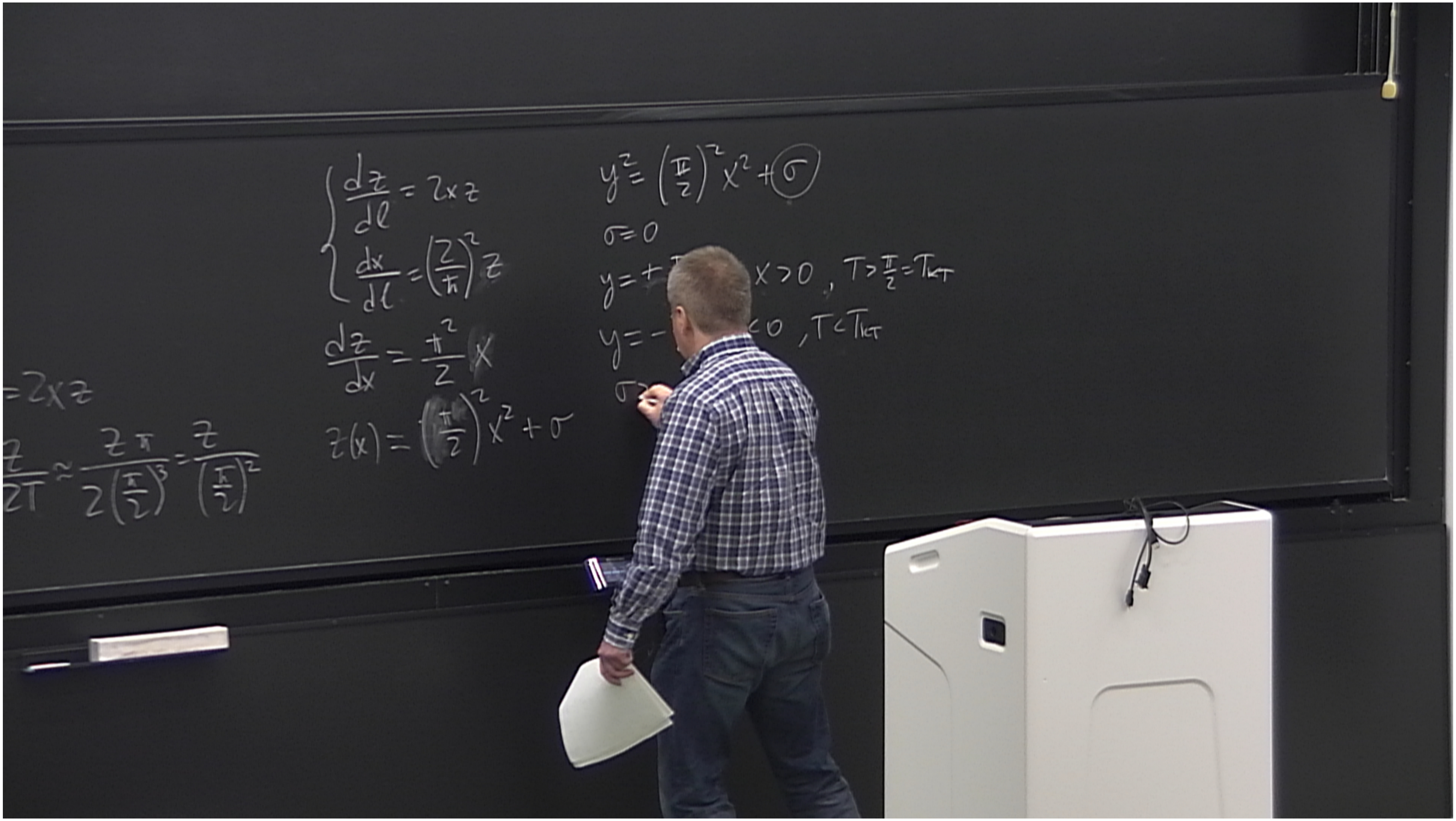
$$\frac{dz}{dl} = 2y\frac{dy}{dl} = 2xy^2 = 2xz$$

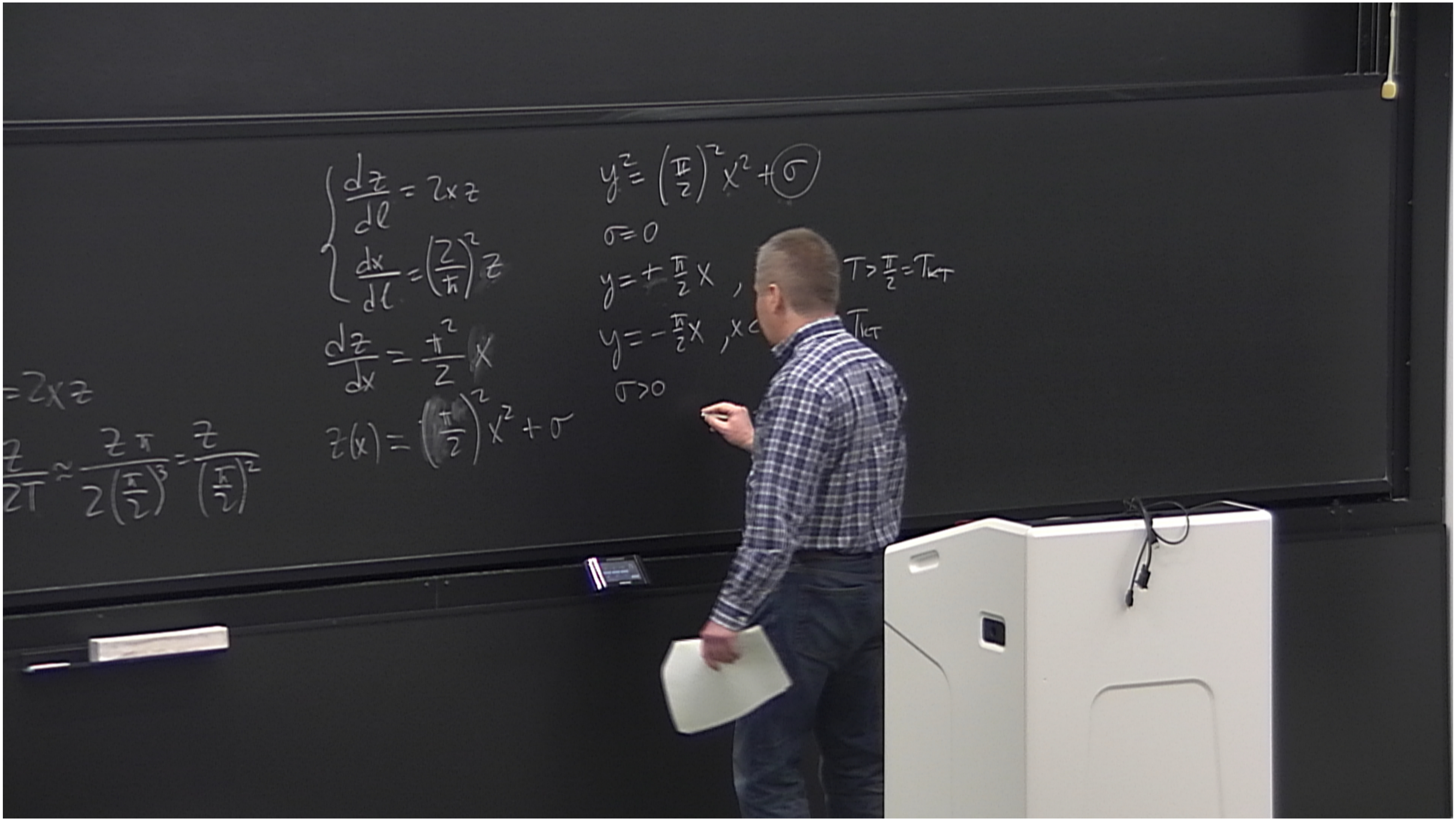
$$\frac{dx}{dl} = \frac{\pi}{T^2}\frac{dT}{dl} = \frac{\pi}{T^2}\frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$$

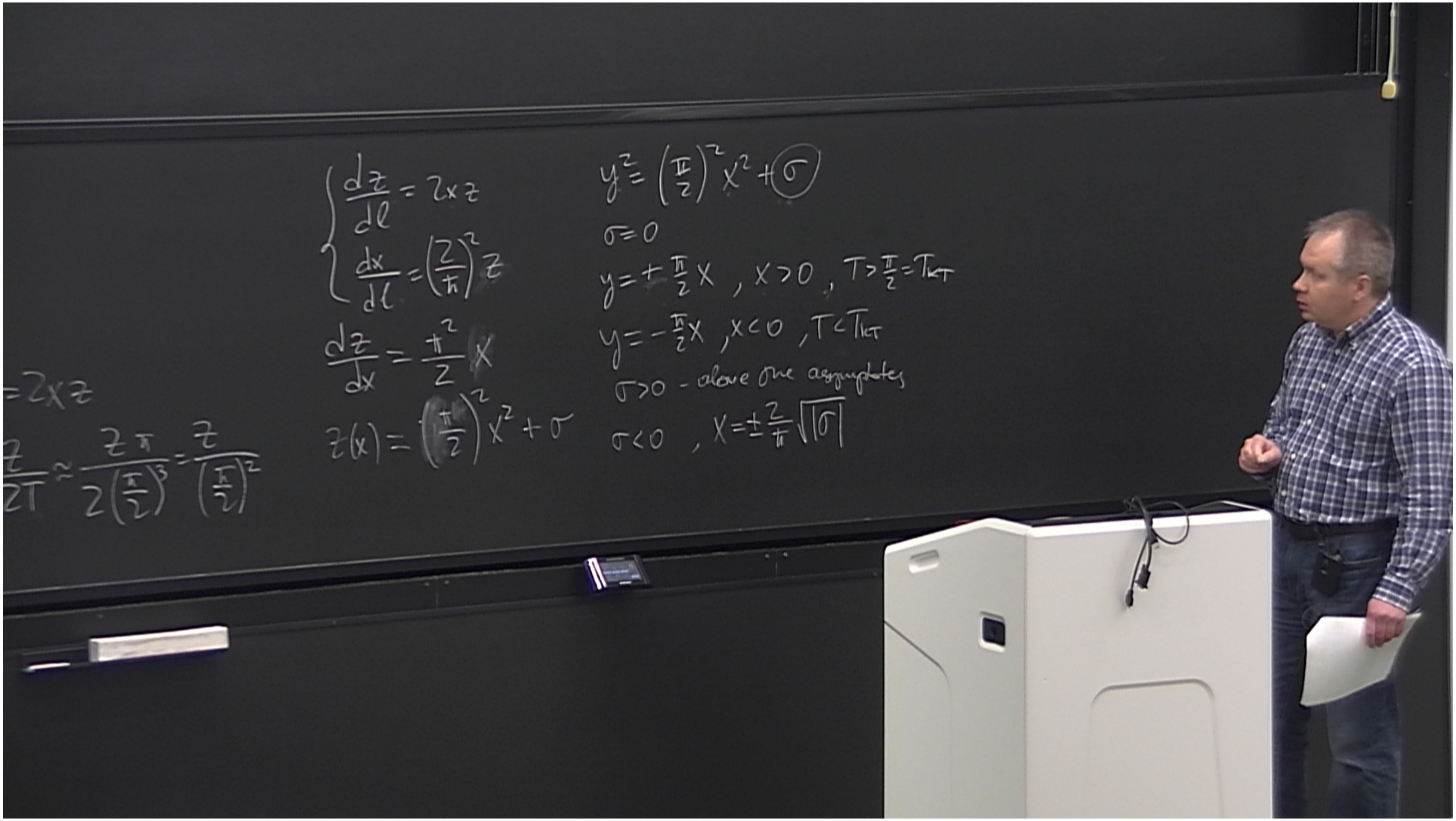

New variables:
$z = y^2$, $x = 2 - \frac{\pi}{T}$
$z^* = 0$, $x^* = 0$
$\frac{dz}{dl} = 2y\frac{dy}{dl} = 2xy^2 = 2xz$
$\frac{dx}{dl} = \frac{\pi}{T^2}\frac{dT}{dl} = \frac{\pi}{T^2}\frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$
$\frac{dz}{dl} = 2xz$
$\frac{dx}{dl} = \left(\frac{2}{\pi}\right)^2 x$

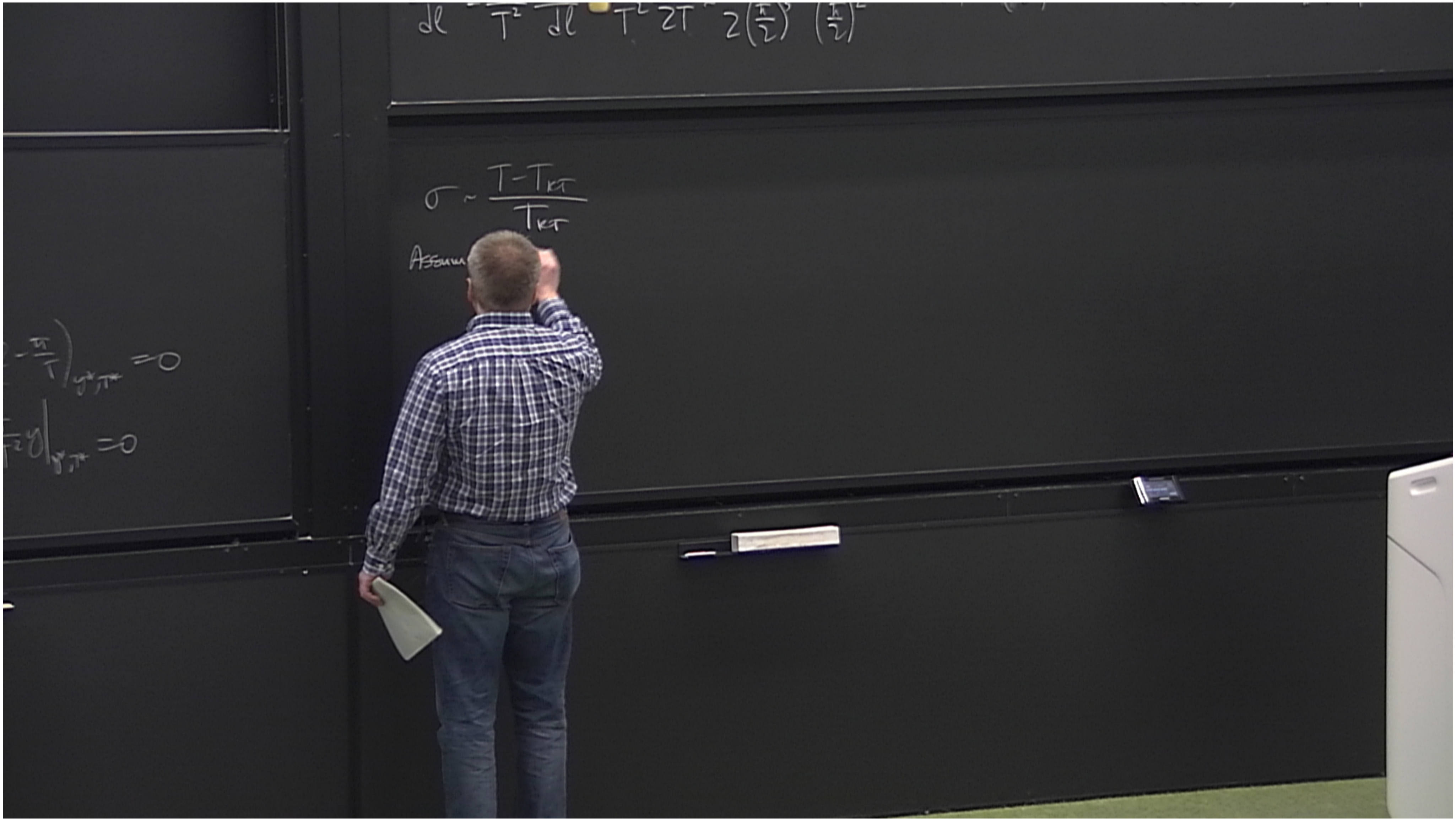

New variables:
z = y^2 , x = 2 - π/T
z* = 0, x* = 0
dz/dl = 2y dy/dl = 2xy^2 = 2xz
dx/dl = π/T^2 dT/dl = π/T^2 z/2T ≈ zπ/2(π/2)^3 = z/(π/2)^2
dz/dl = 2xz
dx/dl = (2/π)^2 x
dz/dx =

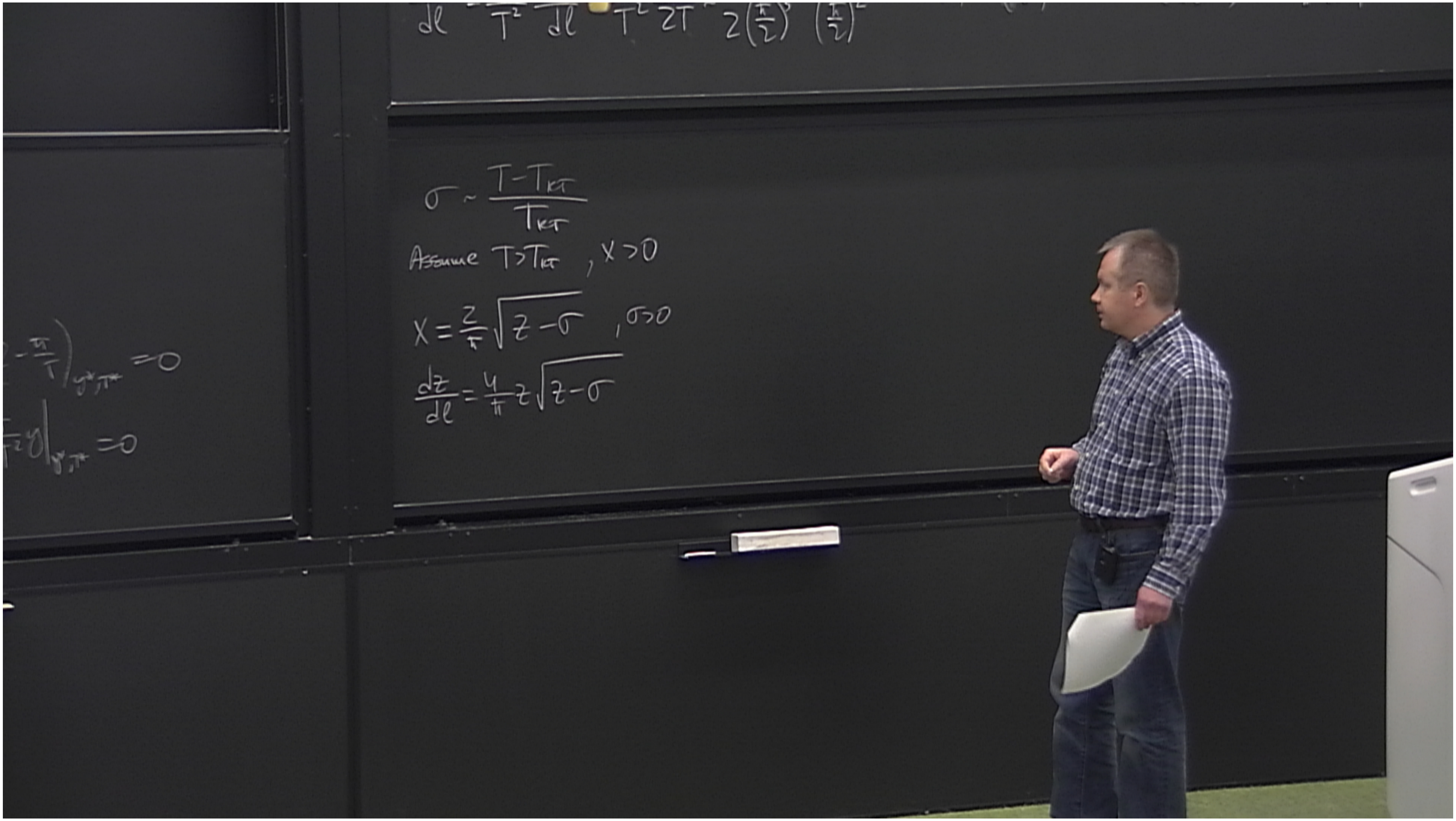

New variables:
$z = y^2, \; x = 2 - \frac{\pi}{T}$
$z^* = 0, \; x^* = 0$
$\frac{dz}{dl} = 2y\frac{dy}{dl} = 2xy^2 = 2xz$
$\frac{dx}{dl} = \frac{\pi}{T^2}\frac{dT}{dl} = \frac{\pi}{T^2}\frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$
$\frac{dz}{dl} = 2xz$
$\frac{dx}{dl} = \left(\frac{2}{\pi}\right)^2 z$
$\frac{dz}{dx} = \frac{\pi^2}{2} x$
$z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma$

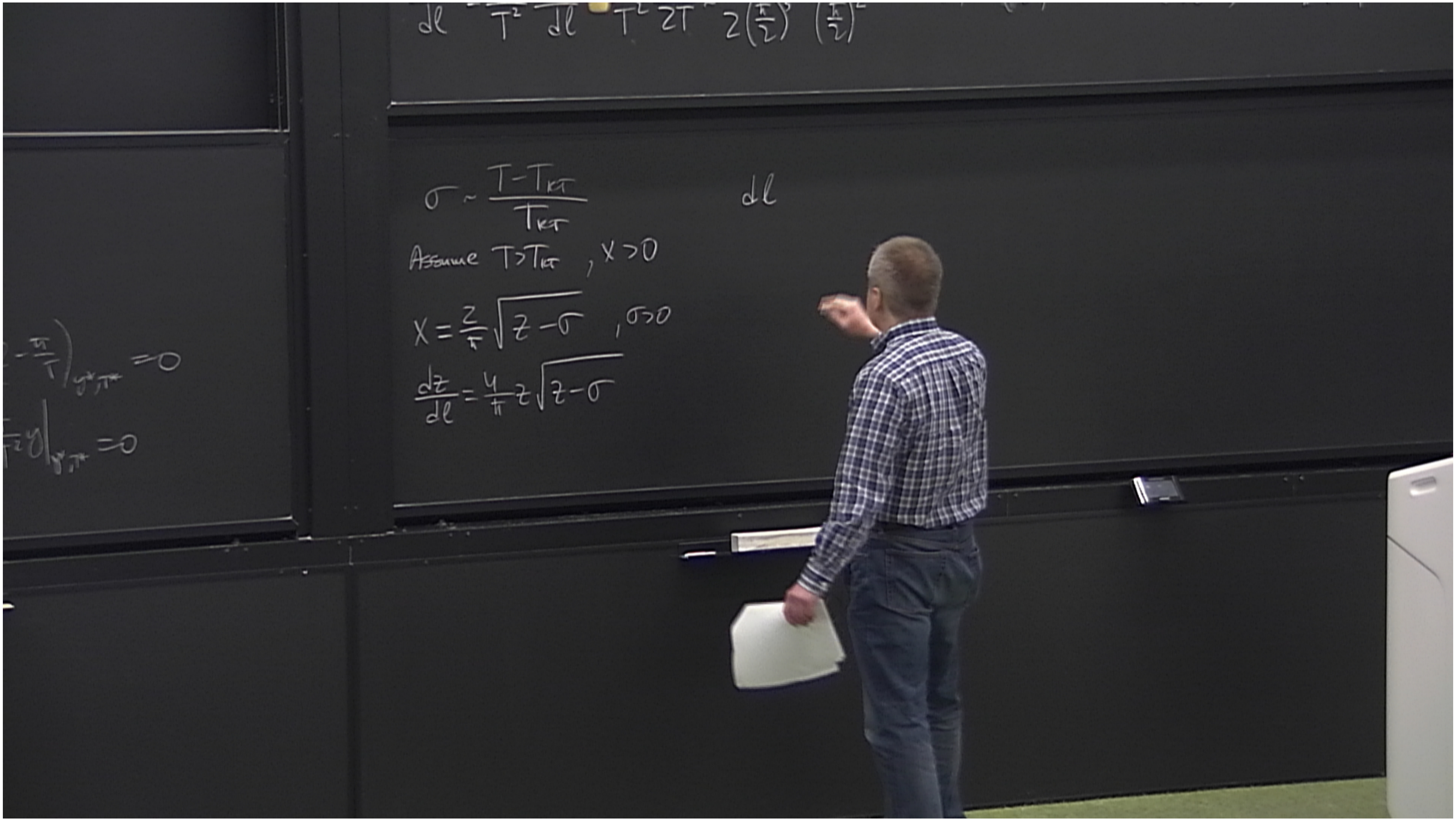

dz/dl = 2xz
dx/dl = (2/π)² z
dz/dx = π²/2 x
z(x) = (π/2)² x² + σ
y² = (π/2)² x² + σ
σ = 0
x > 0 , T > π/2 = T_KT
< 0 , T < T_KT
= 2xz

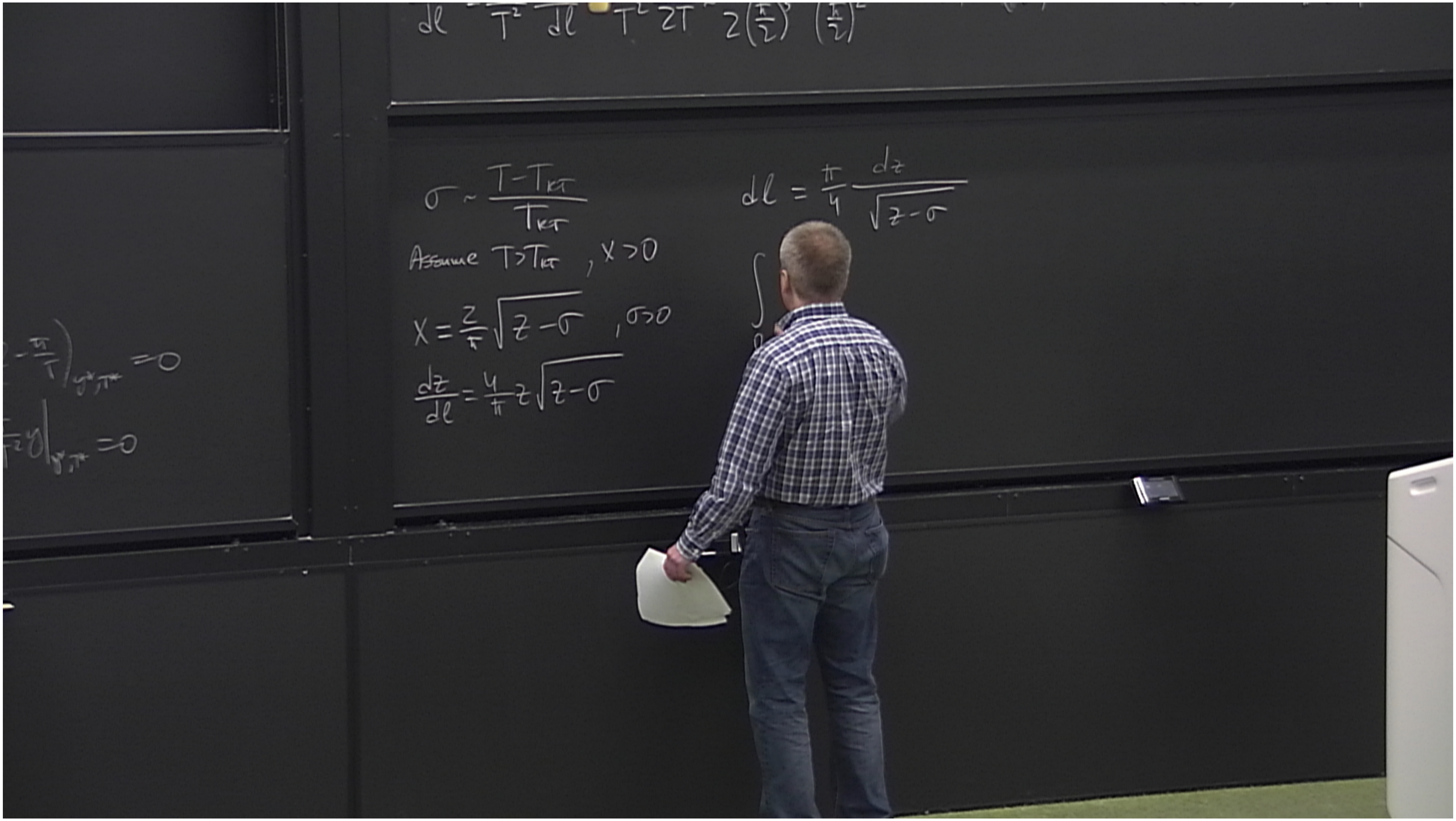

dz/dl = 2xz
dx/dl = (2/π)^2 z
dz/dx = π^2/2 x
z(x) = (π/2)^2 x^2 + σ
y^2 = (π/2)^2 x^2 + σ
σ = 0
y = + π/2 x ,
T > π/2 = T_KT
y = − π/2 x , x
T_KT
σ > 0
= 2xz
≈ zπ / 2(π/2)^3 = z / (π/2)^2

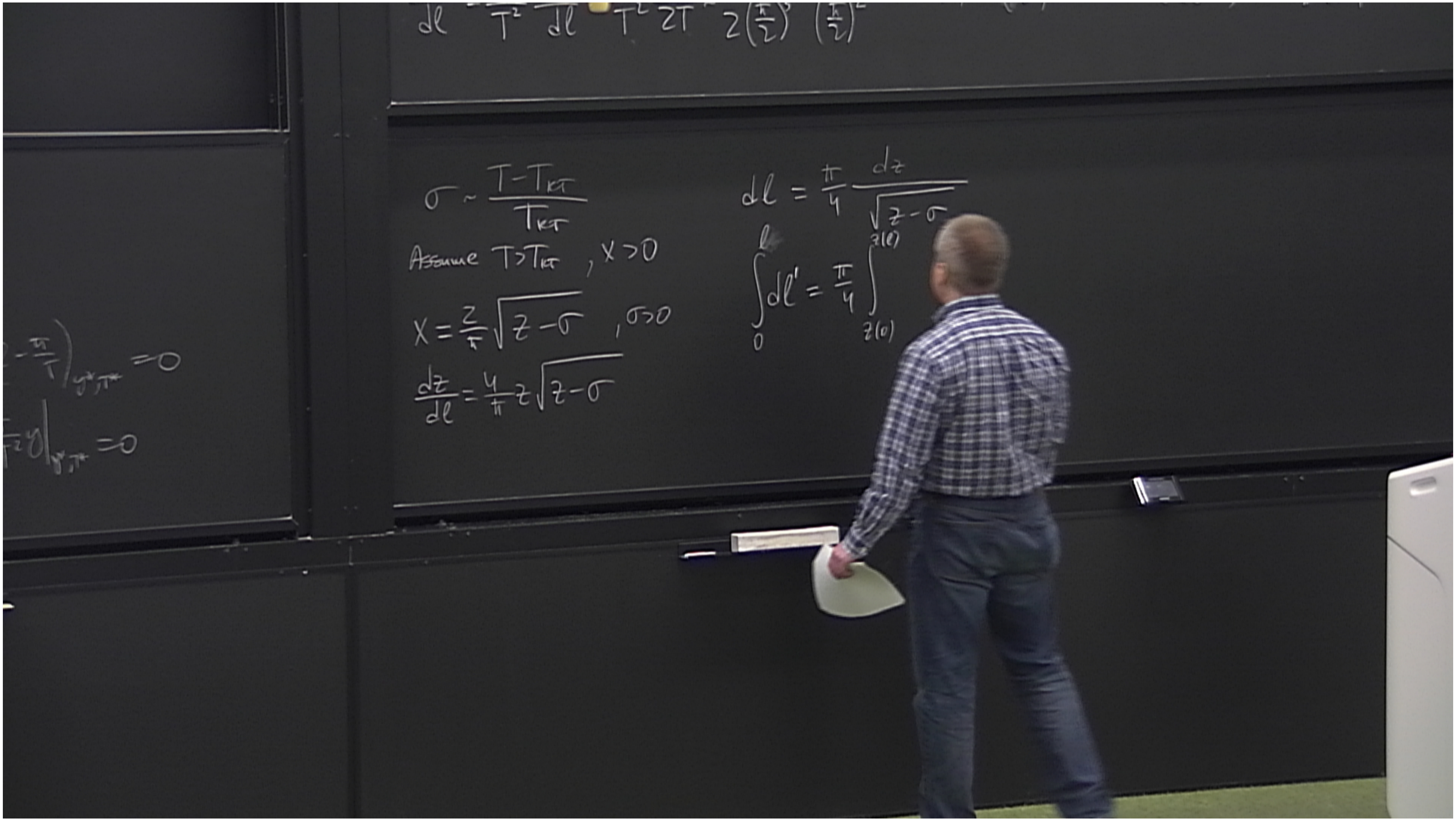

dz/dl = 2xz
dx/dl = (2/π)^2 z
dz/dx = π^2/2 x
z(x) = (π/2)^2 x^2 + σ
y^2 = (π/2)^2 x^2 + σ
σ=0
y = + π/2 x , x>0 , T > π/2 = T_KT
y = − π/2 x , x<0 , T < T_KT
σ>0 - above the asymptotes
σ<0 , x = ± 2/π √|σ|

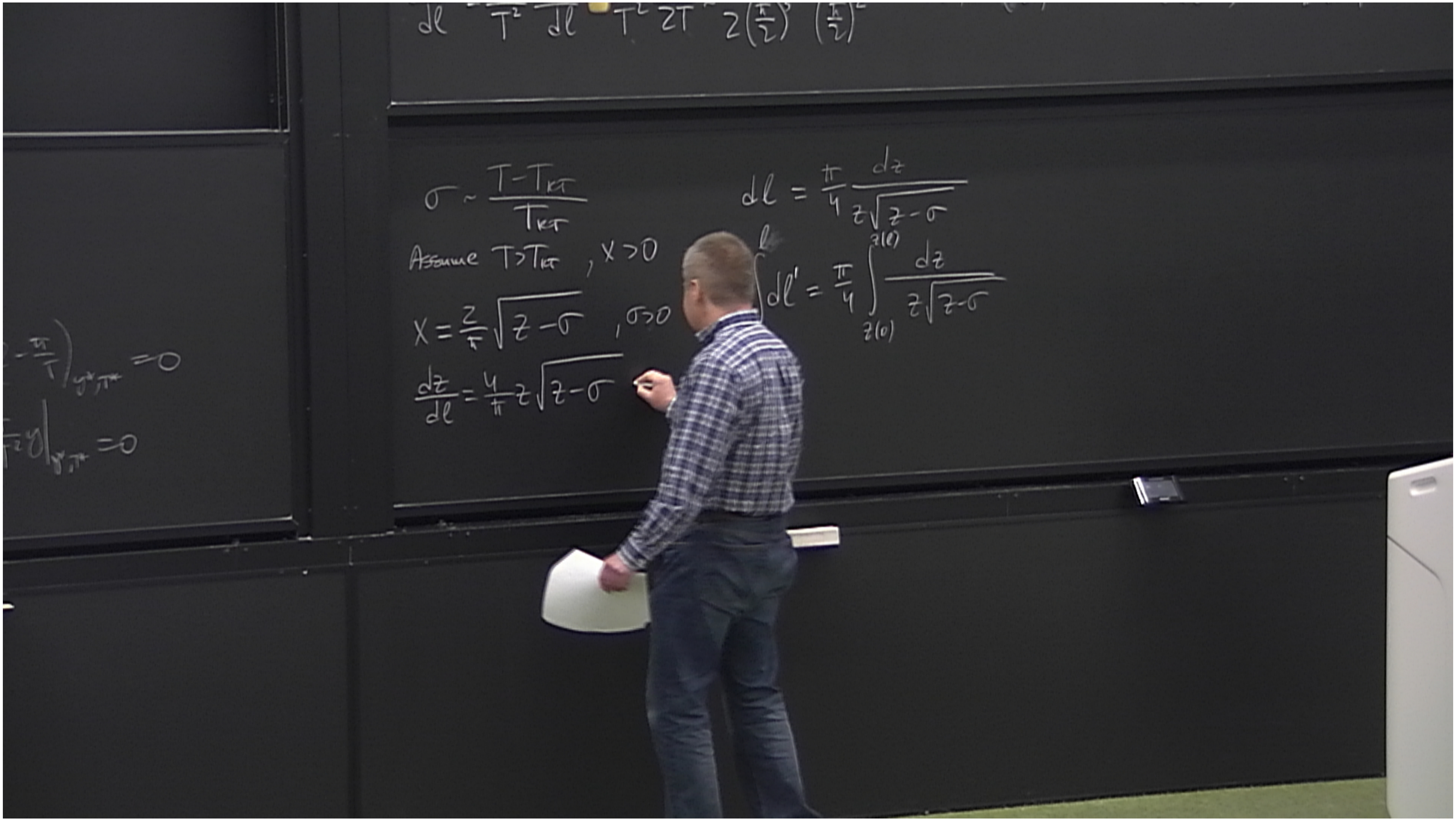


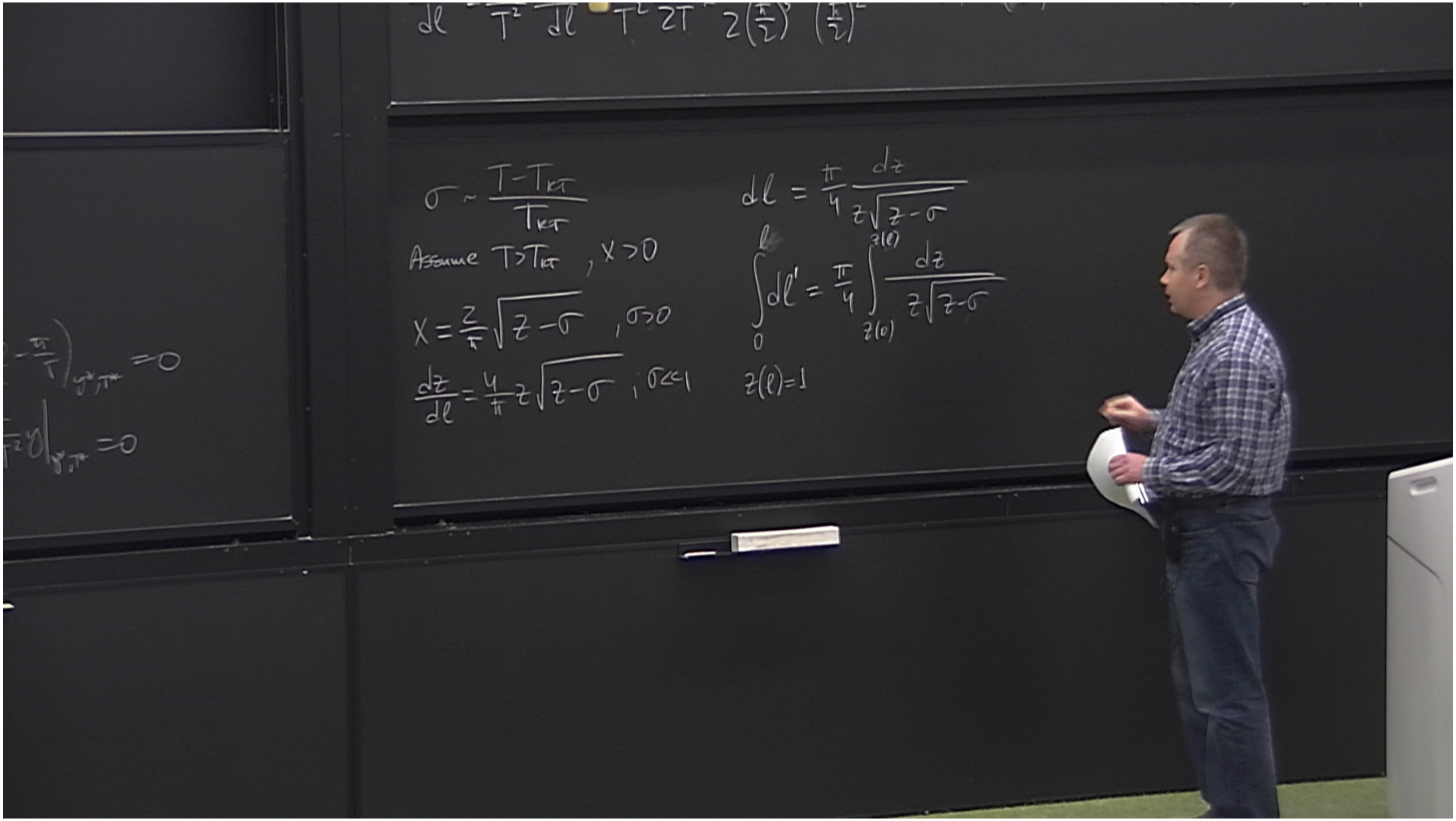

σ ~ (T − T_KT) / T_KT
Assume T > T_KT , x > 0
x = (2/π) √(z − σ) , σ > 0
dz/dl = (4/π) z √(z − σ)

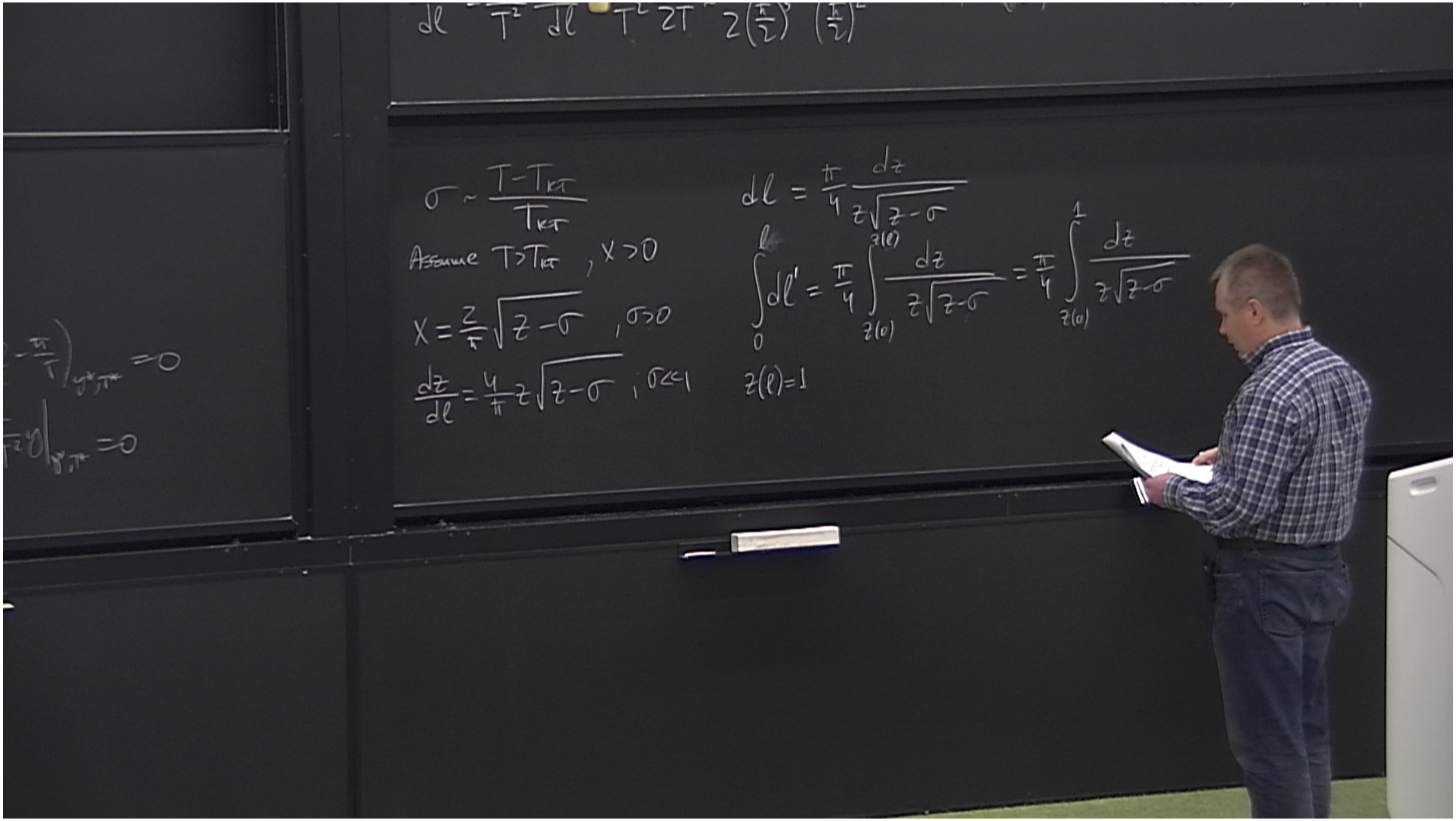

σ ~ (T−T_KT)/T_KT
dl
Assume T>T_KT , x>0
x = (2/π)√(z−σ) , σ>0
dz/dl = (4/π) z √(z−σ)

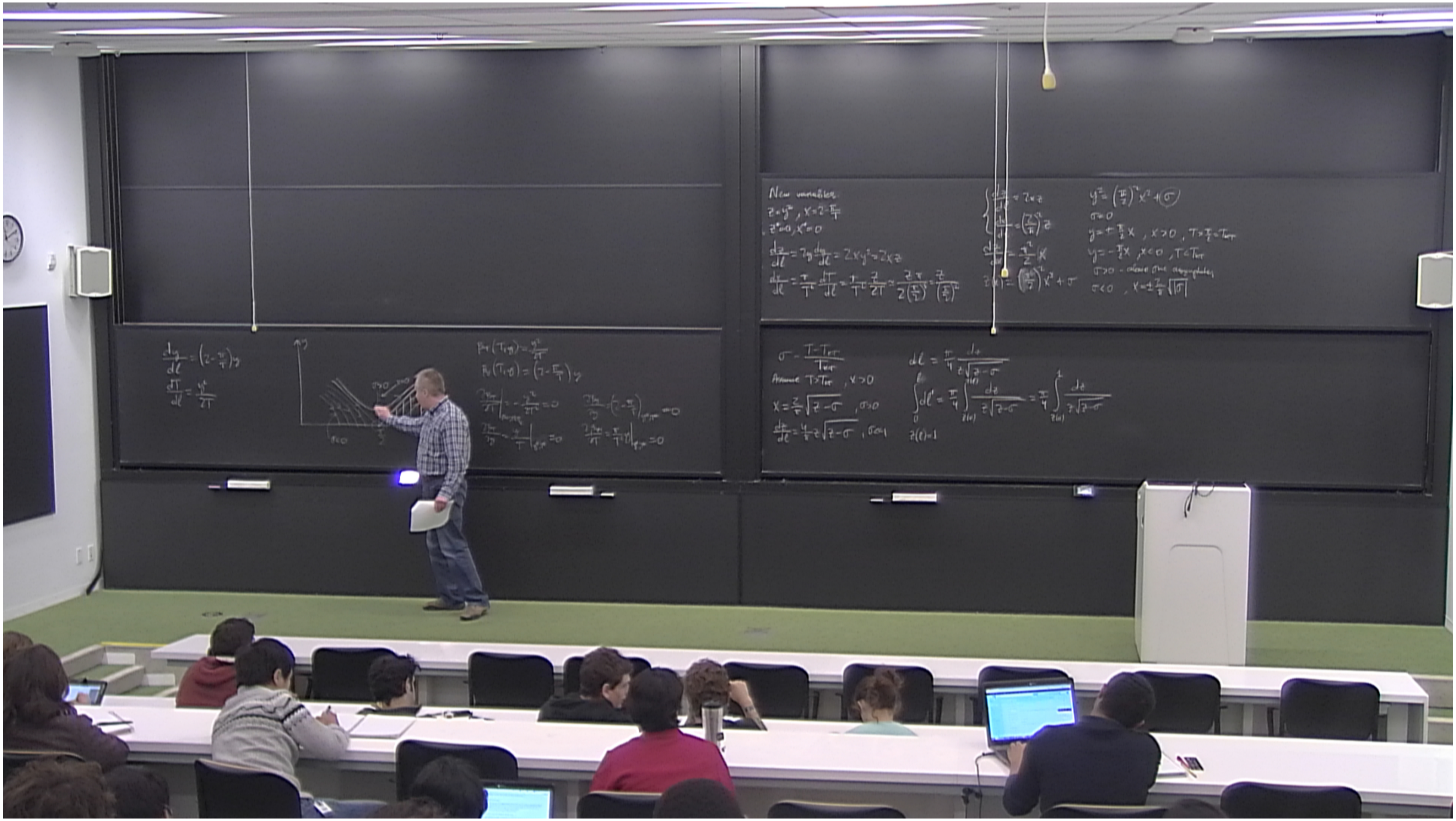

σ ~ (T − T_KT) / T_KT
Assume T > T_KT , x > 0
x = (2/π) √(z − σ) , σ > 0
dz/dl = (4/π) z √(z − σ)
dl = (π/4) dz / √(z − σ)

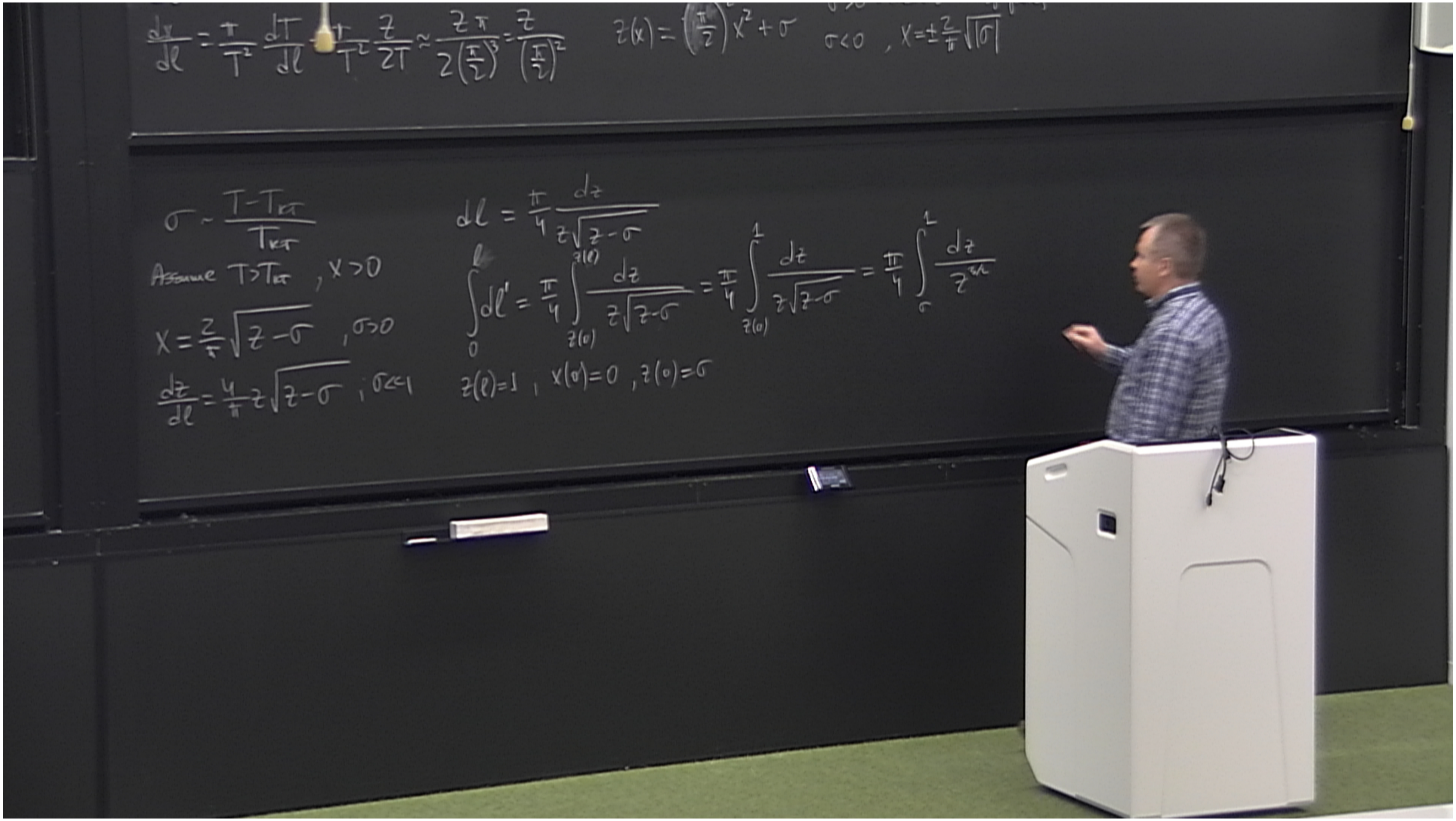

σ ~ (T − T_KT) / T_KT
Assume T > T_KT , x > 0
x = 2/π √(z − σ) , σ > 0
dz/dl = 4/π z √(z − σ)
dl = π/4 dz / √(z − σ)
∫₀^l dl' = π/4 ∫_{z(0)}^{z(l)}

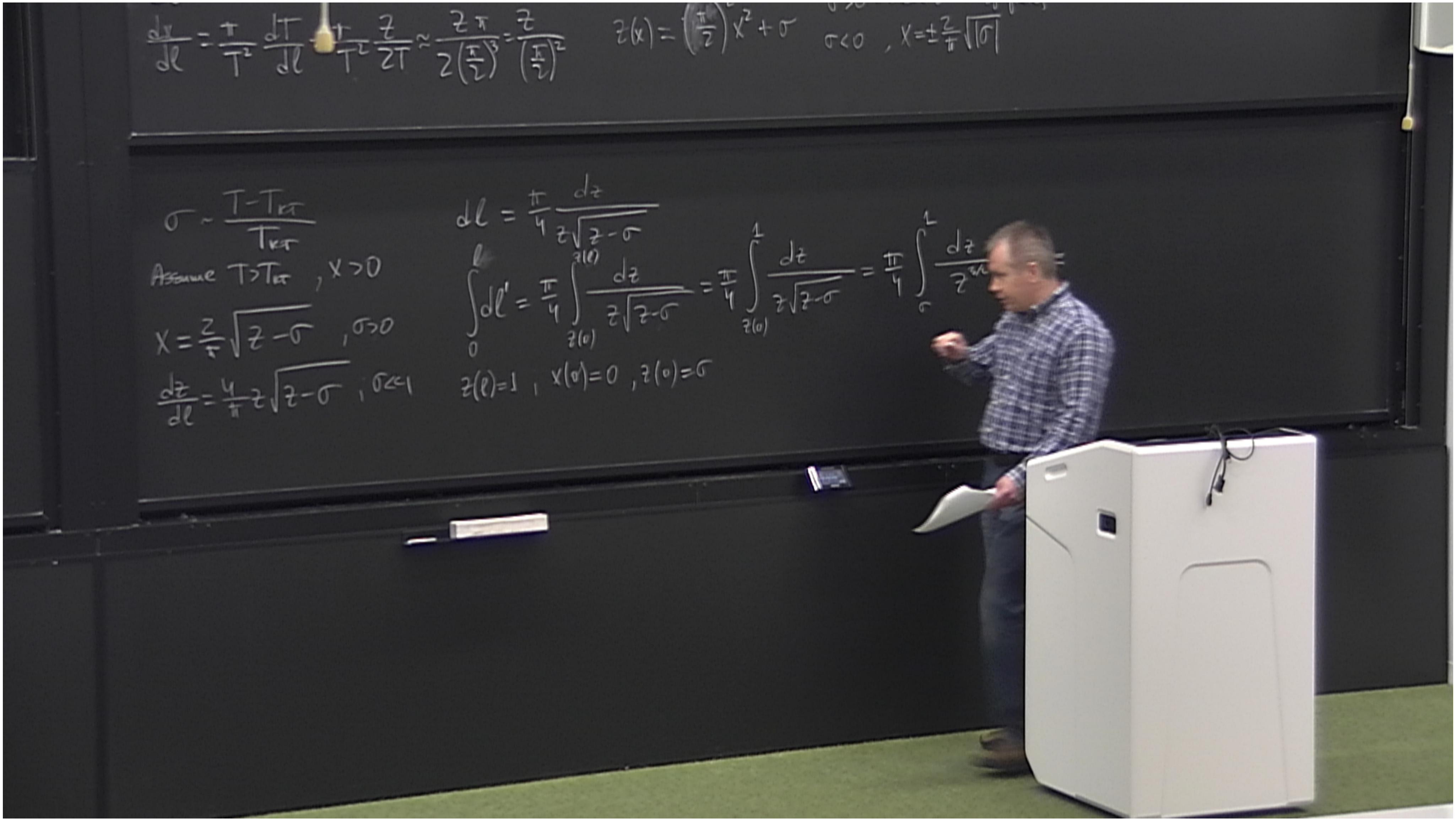

σ ~ (T−T_KT)/T_KT
Assume T>T_KT , x>0
x = (2/π)√(z−σ) , σ>0
dz/dl = (4/π) z√(z−σ)
dl = (π/4) dz/(z√(z−σ))
∫ dl' = (π/4) ∫_{z(0)}^{z(l)} dz/(z√(z−σ))

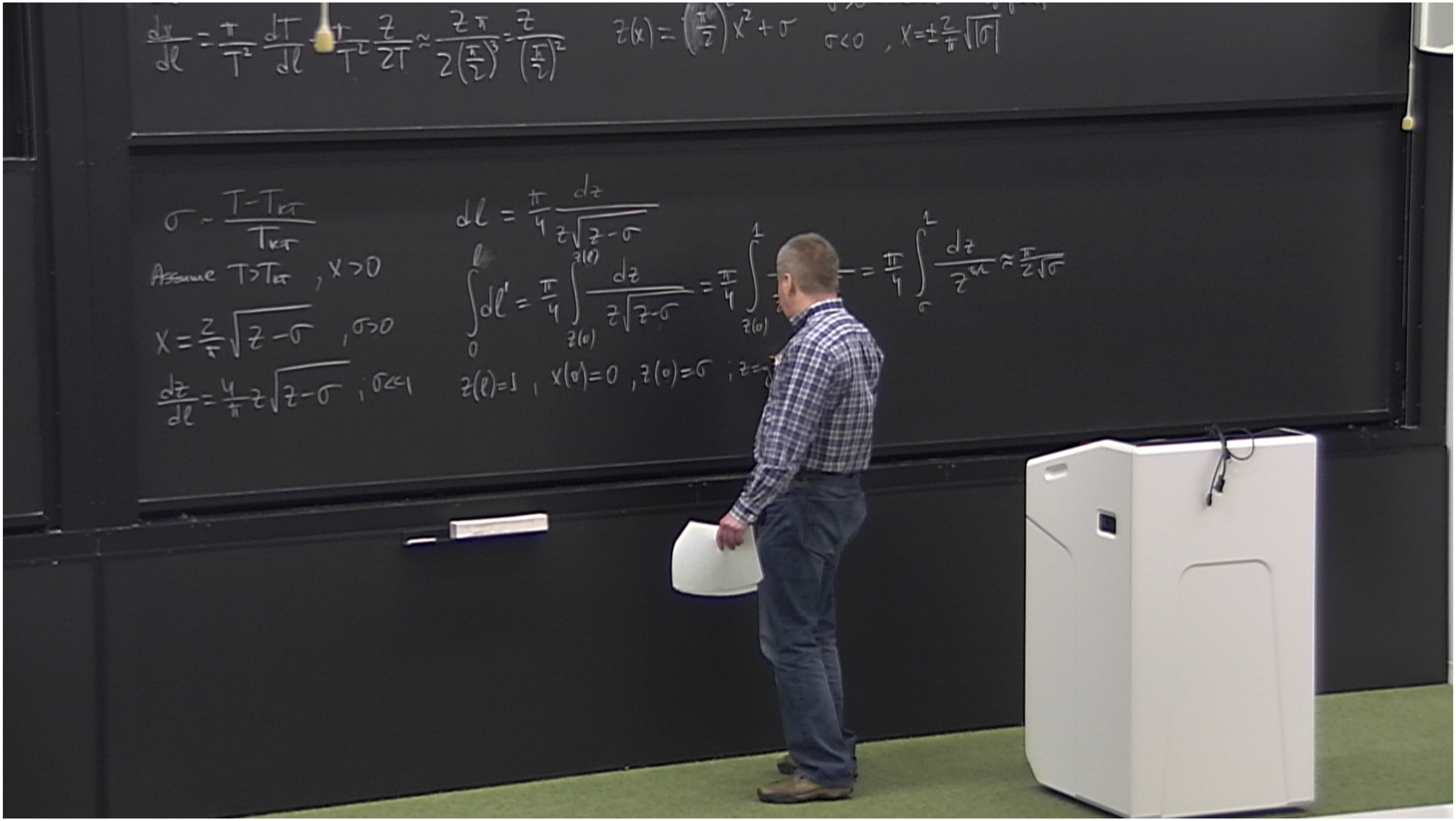

σ ~ (T − T_KT) / T_KT
Assume T > T_KT , x > 0
x = (2/π) z √(z − σ) , σ > 0
dz/dl = (4/π) z √(z − σ) ; σ << 1
dl = (π/4) dz / (z √(z − σ))
∫₀^l dl′ = (π/4) ∫_{z(0)}^{z(l)} dz / (z √(z − σ))
z(l) = 1

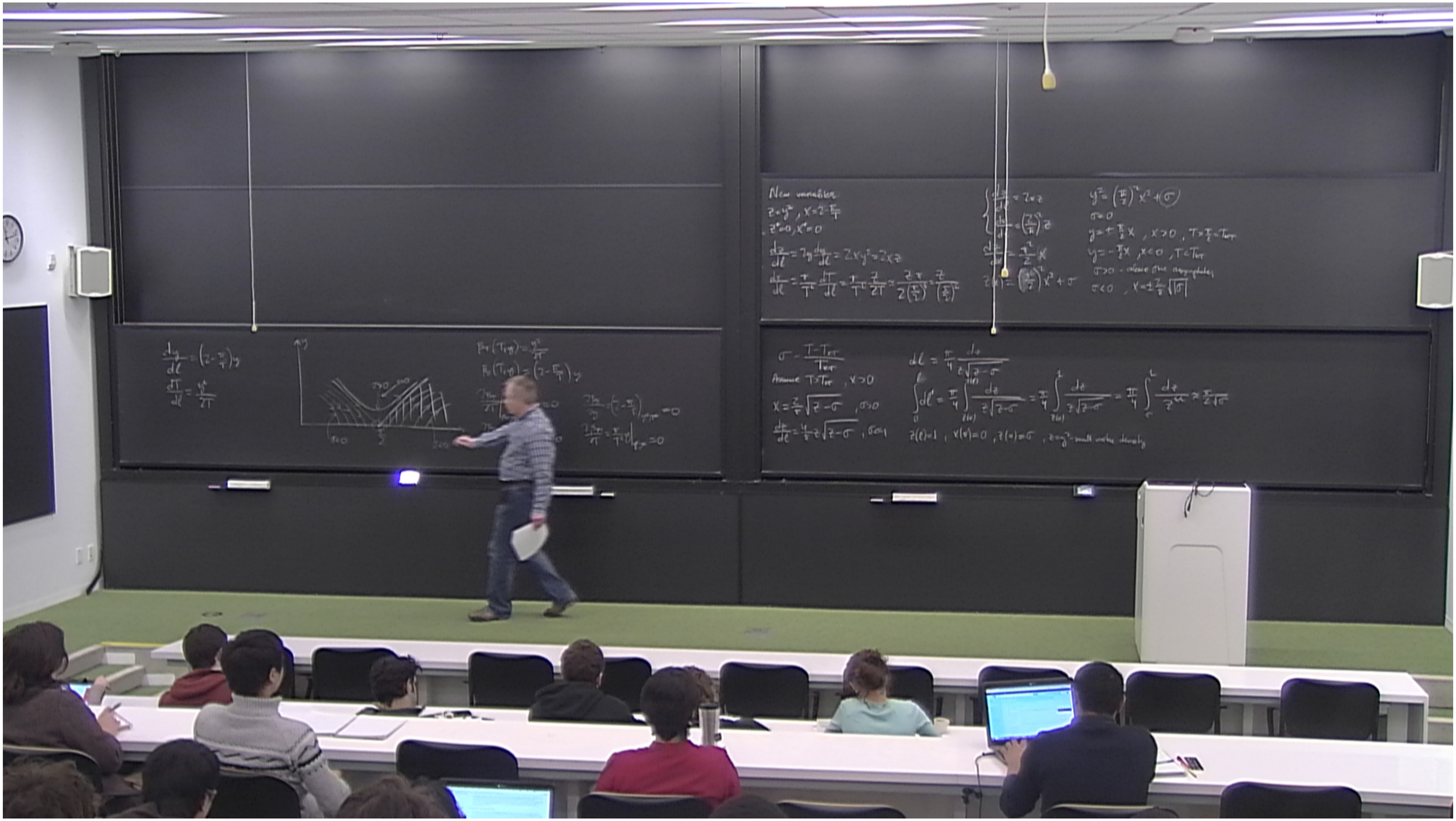

$\sigma \sim \frac{T - T_{KT}}{T_{KT}}$
Assume $T > T_{KT}$, $x > 0$
$x = \frac{2}{\pi}\sqrt{z - \sigma}$, $\sigma > 0$
$\frac{dz}{d\ell} = \frac{4}{\pi} z \sqrt{z - \sigma}$; $\sigma \ll 1$
$d\ell = \frac{\pi}{4} \frac{dz}{z\sqrt{z-\sigma}}$
$\int_0^{\ell} d\ell' = \frac{\pi}{4} \int_{z(0)}^{z(\ell)} \frac{dz}{z\sqrt{z-\sigma}} = \frac{\pi}{4} \int_{z(0)}^{1} \frac{dz}{z\sqrt{z-\sigma}}$
$z(\ell) = 1$

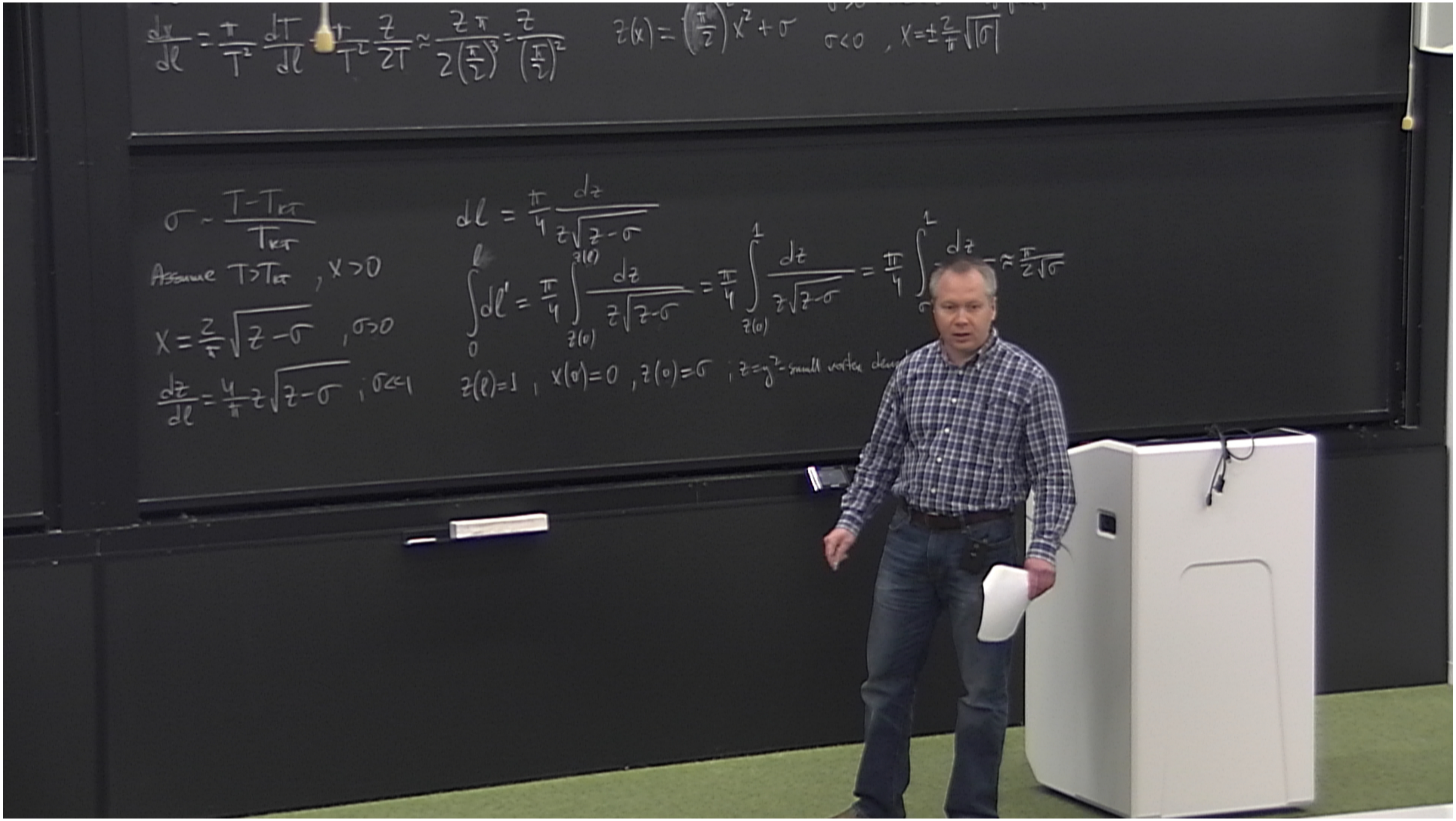



z(x) = (π/2) x^2 + σ
σ<0 , x = ± 2/π √|σ|
σ ~ (T − T_KT)/T_KT
Assume T>T_KT , x>0
x = 2/π √(z − σ) , σ>0
dz/dl = 4/π z √(z − σ) ; σ<<1
dl = π/4 dz/(z √(z − σ))
z(l)=1 , x(0)=0 , z(0)=σ

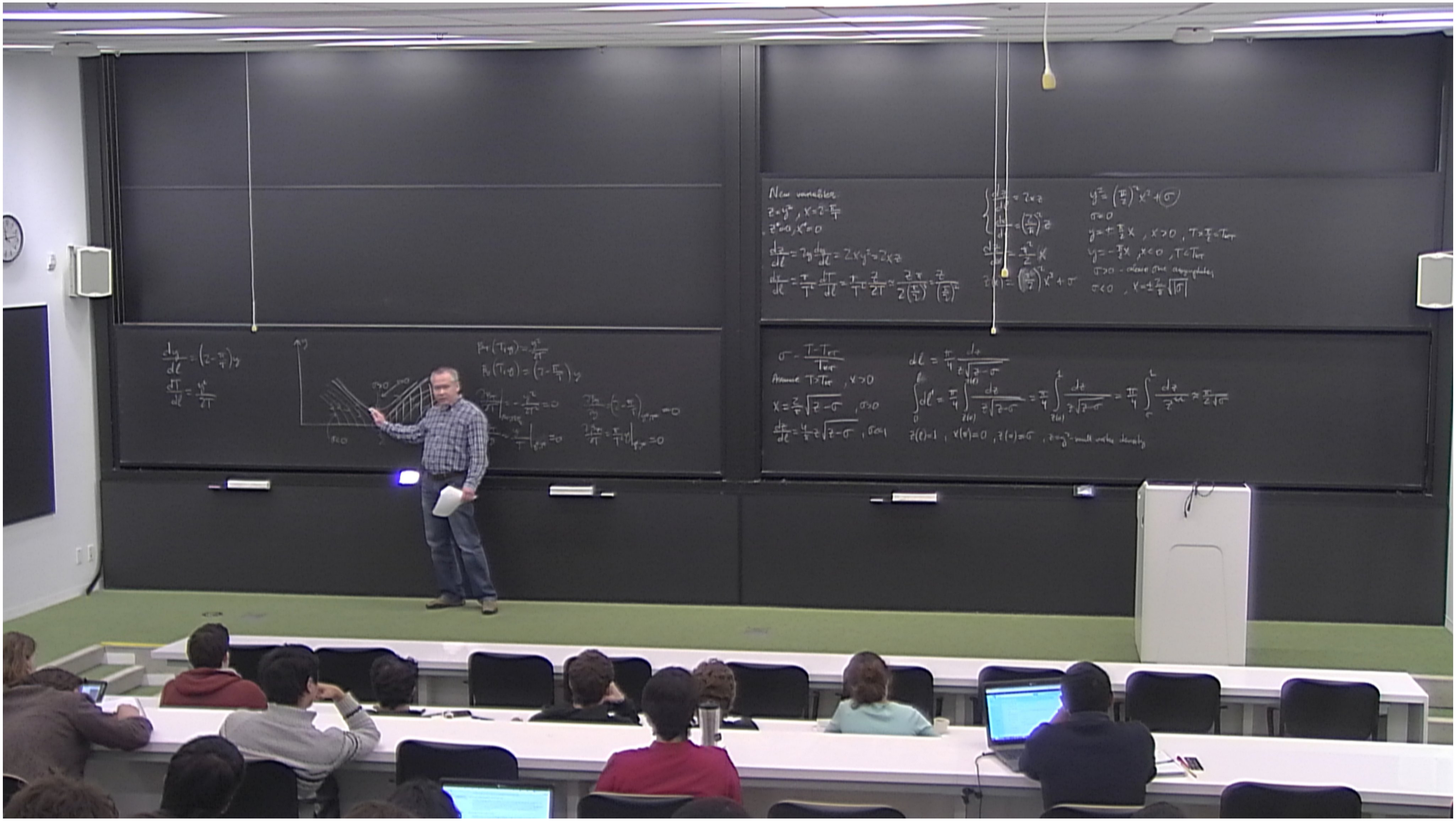


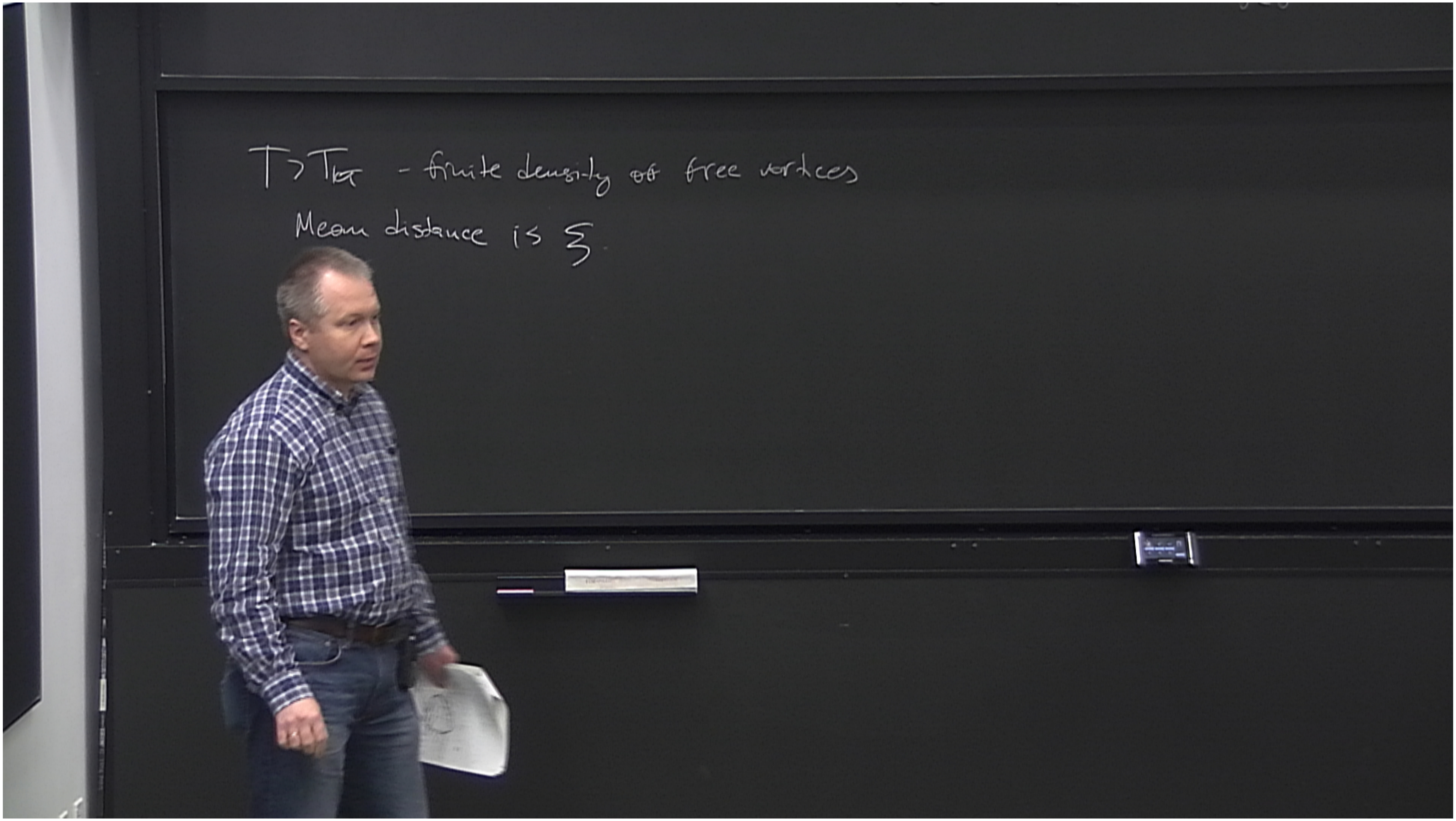

σ ~ (T − T_KT)/T_KT
Assume T > T_KT , x > 0
x = (2/π) √(z − σ) , σ > 0
dz/dl = (4/π) z √(z − σ) ; σ << 1
dl = (π/4) dz / (z √(z − σ))
z(l) = 1 , x(0) = 0 , z(0) = σ
σ < 0 , x = ± (2/π) √|σ|

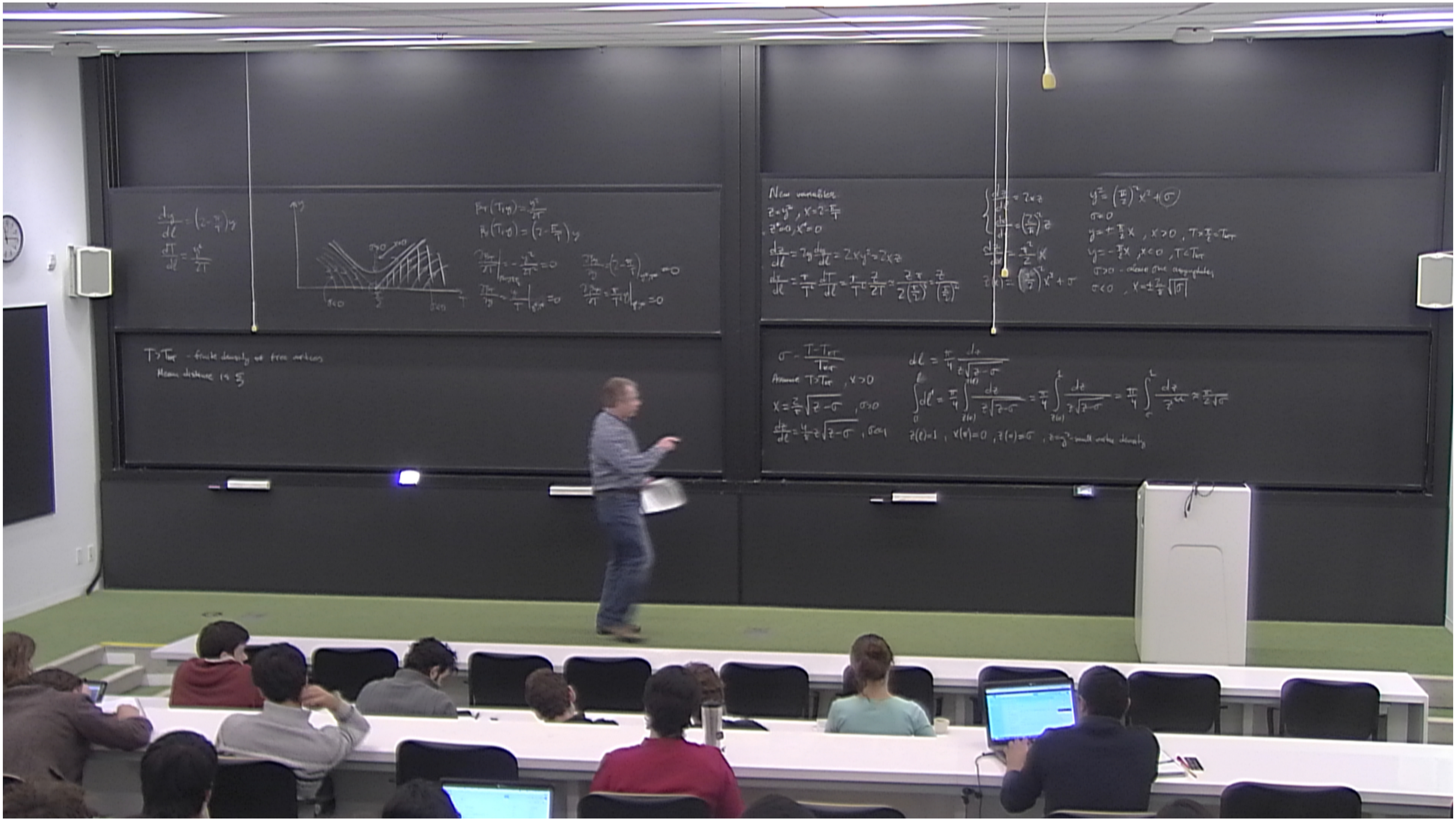


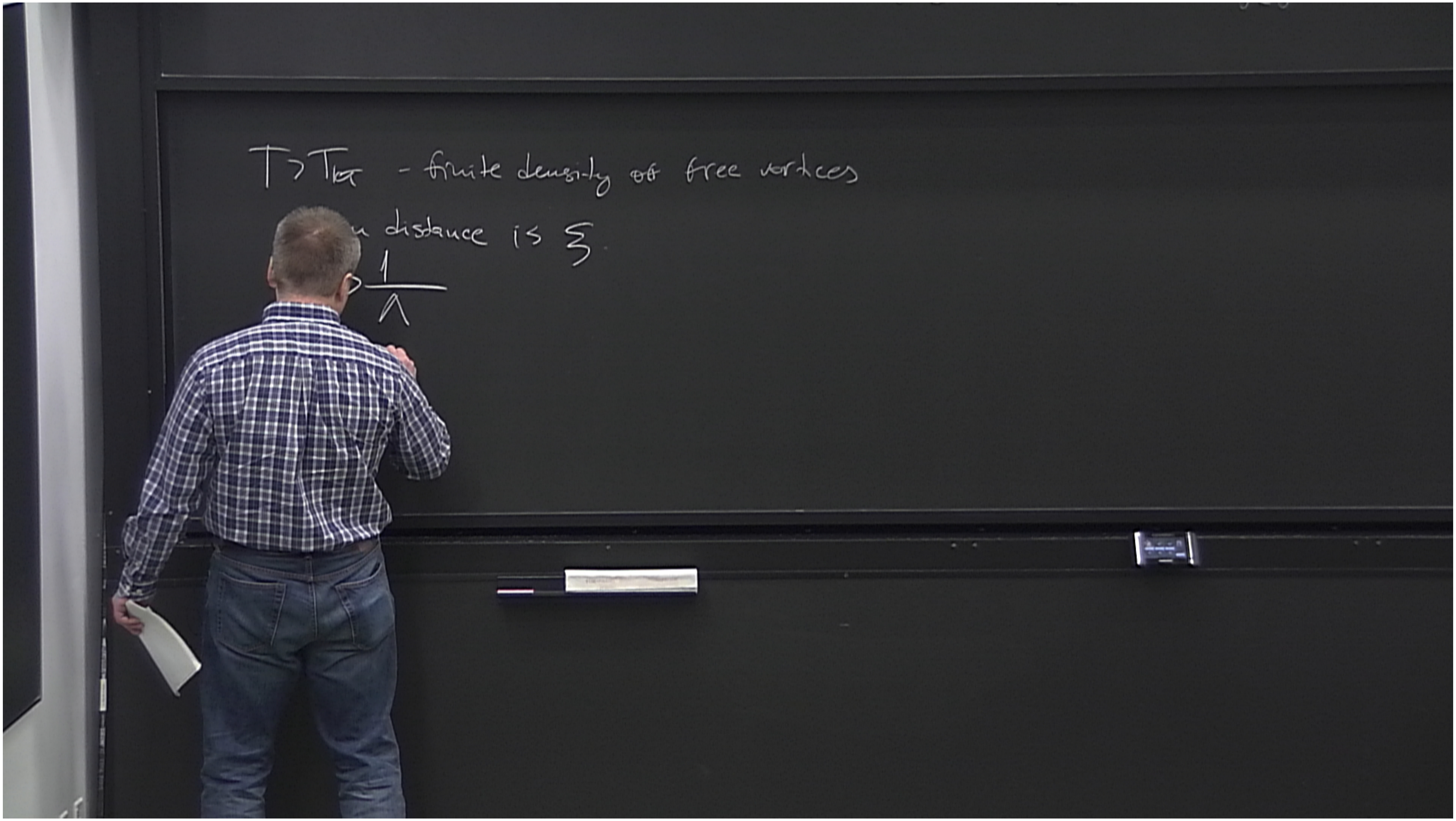


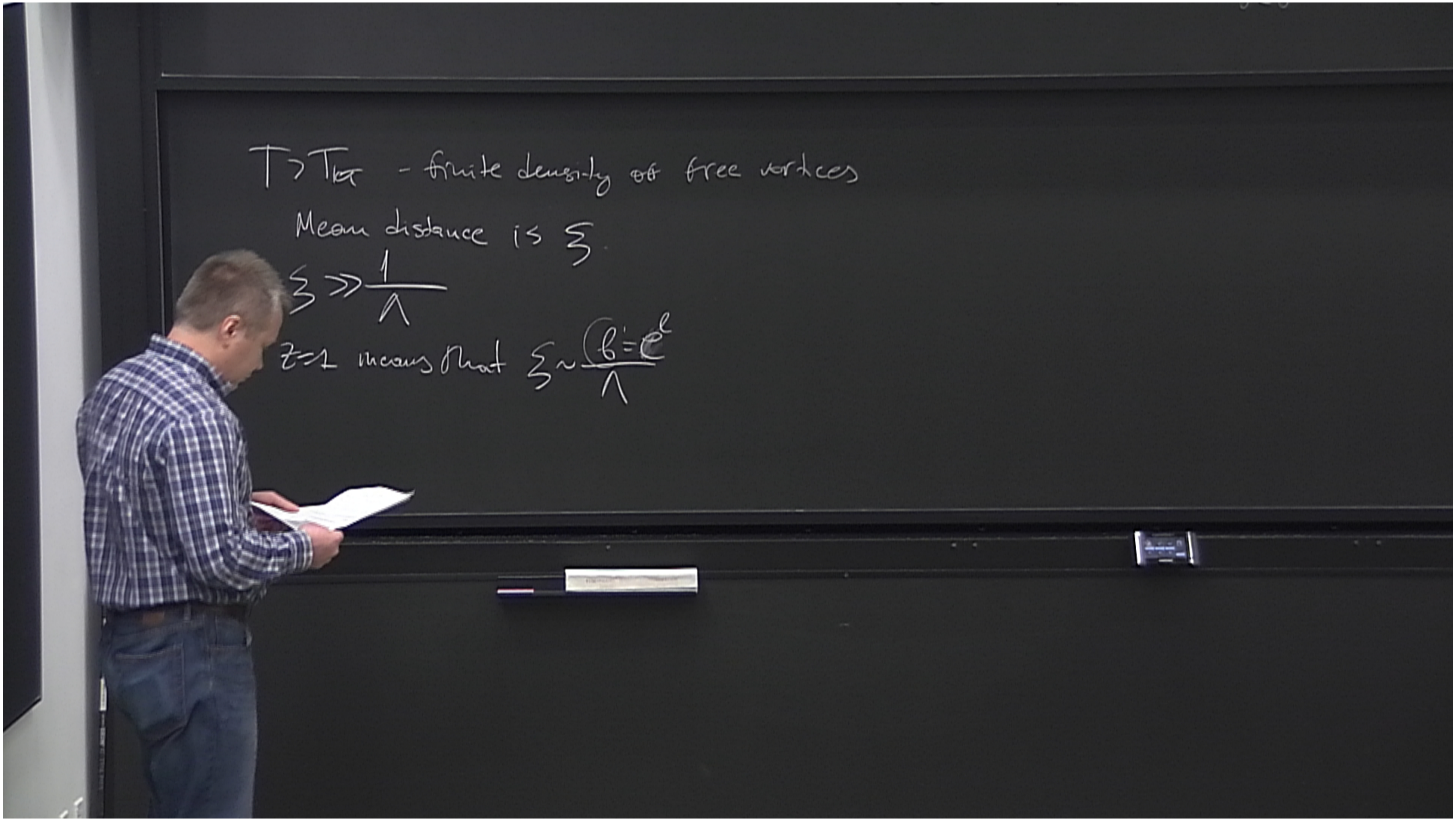
σ ~ (T − T_KT) / T_KT
Assume T > T_KT , x > 0
x = 2/π √(z − σ) , σ > 0
dz/dl = 4/π z √(z − σ) ; σ << 1
dl = π/4 dz / (z √(z − σ))
z(l) = 1 , x(σ) = 0 , z(0) = σ ; z = y² - small vortex dens
σ < 0 , x = ± 2/π √|σ|

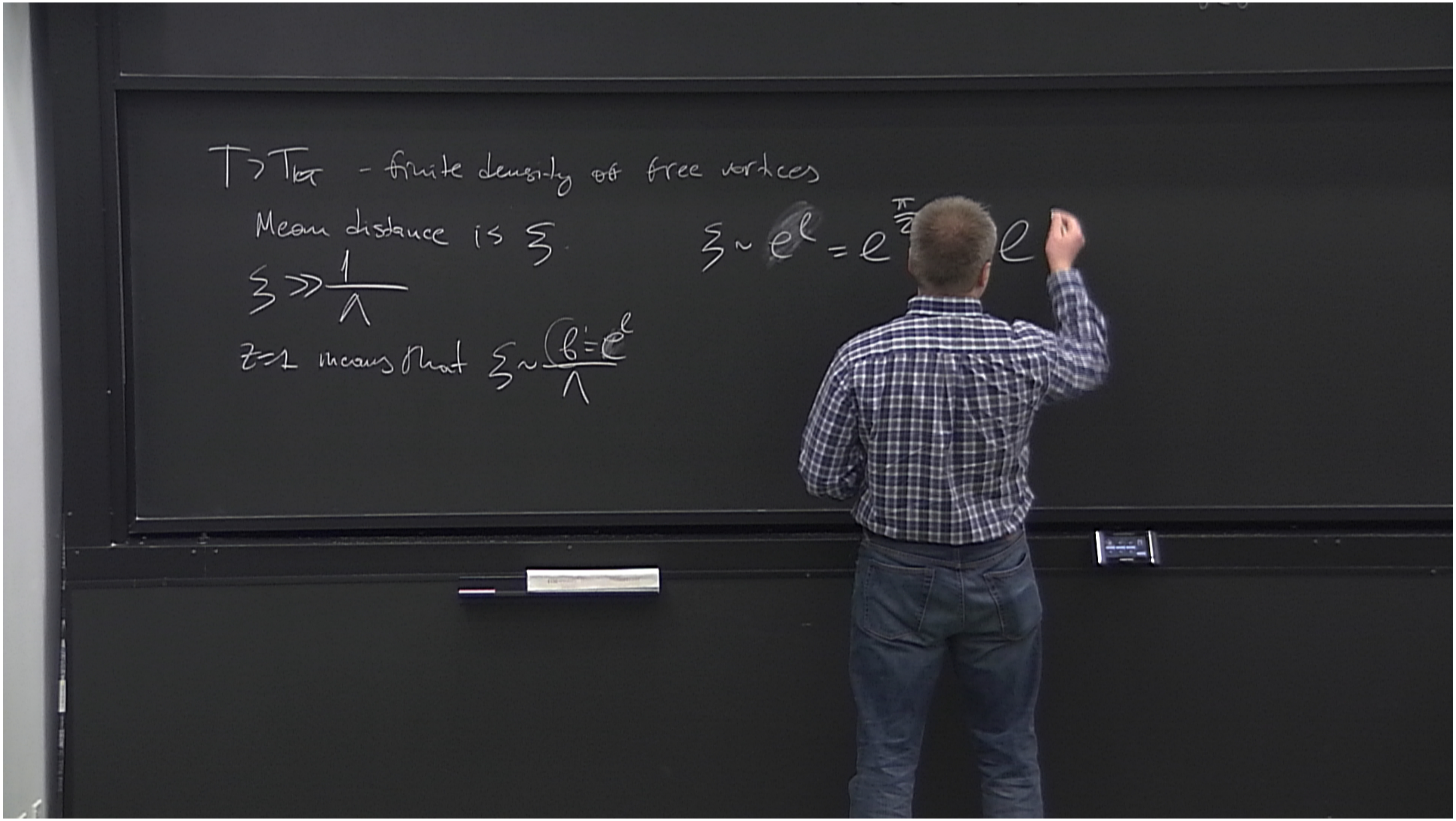


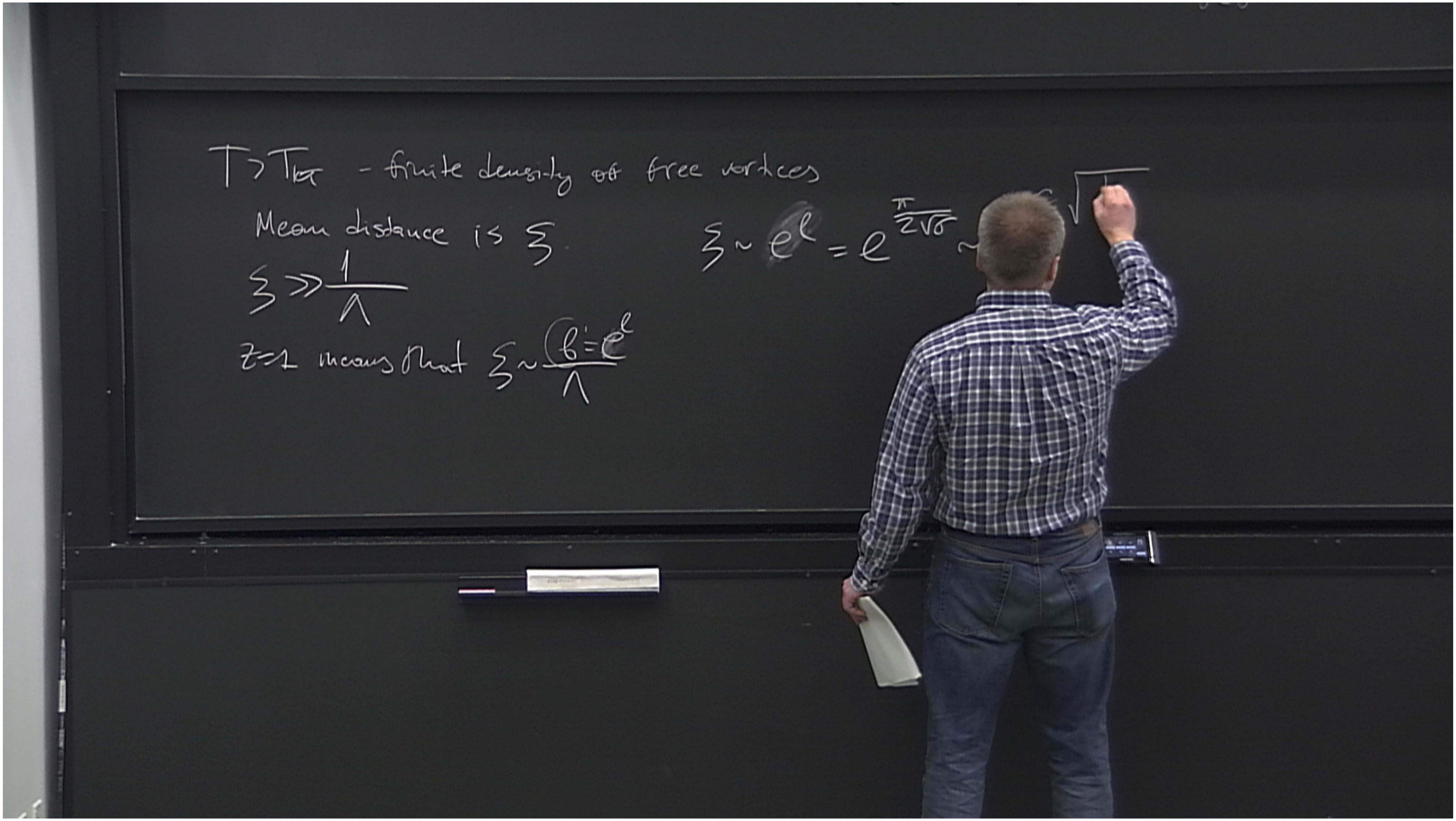

T > T_KT - finite density of free vortices
Mean distance is ξ.

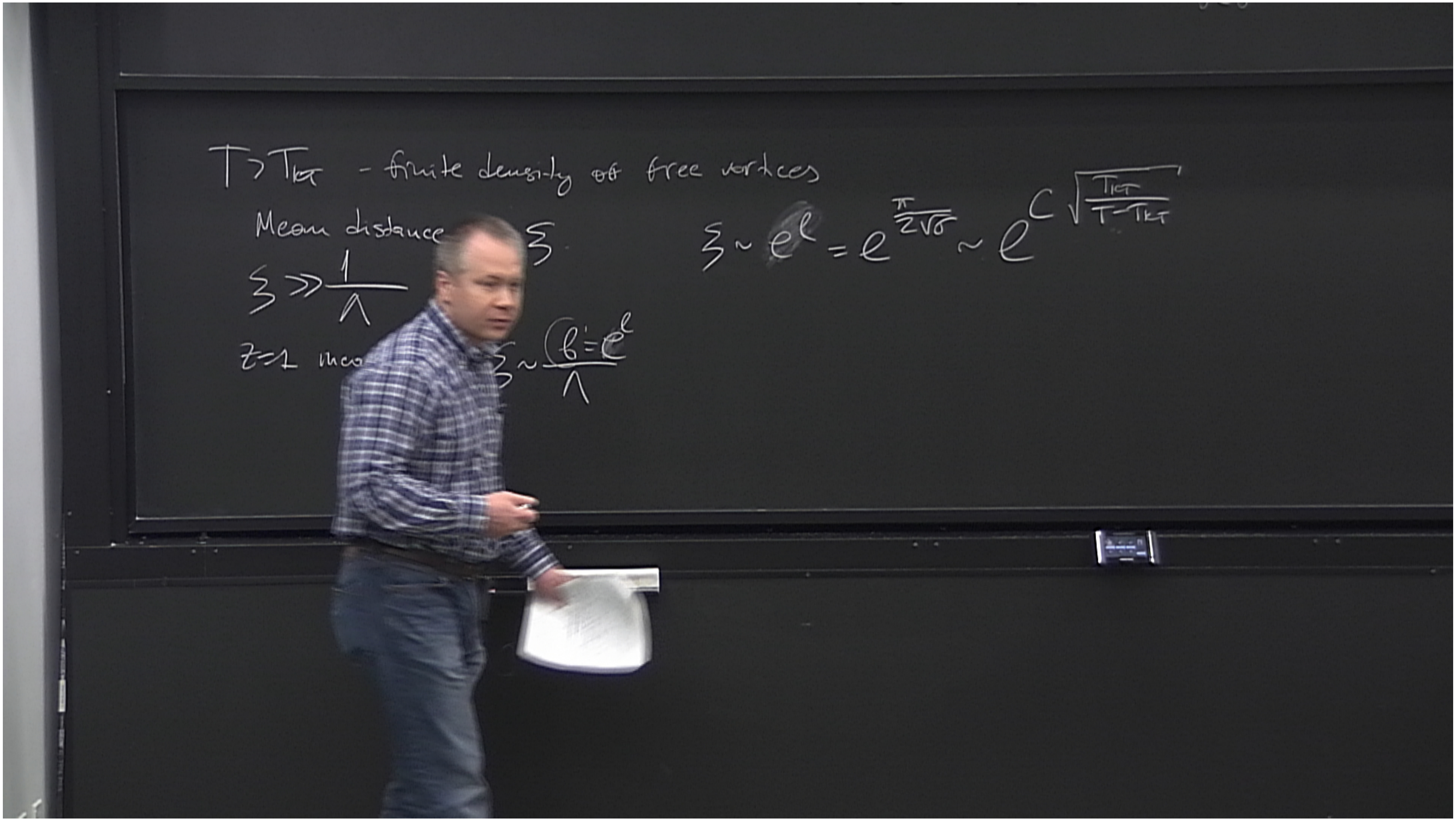


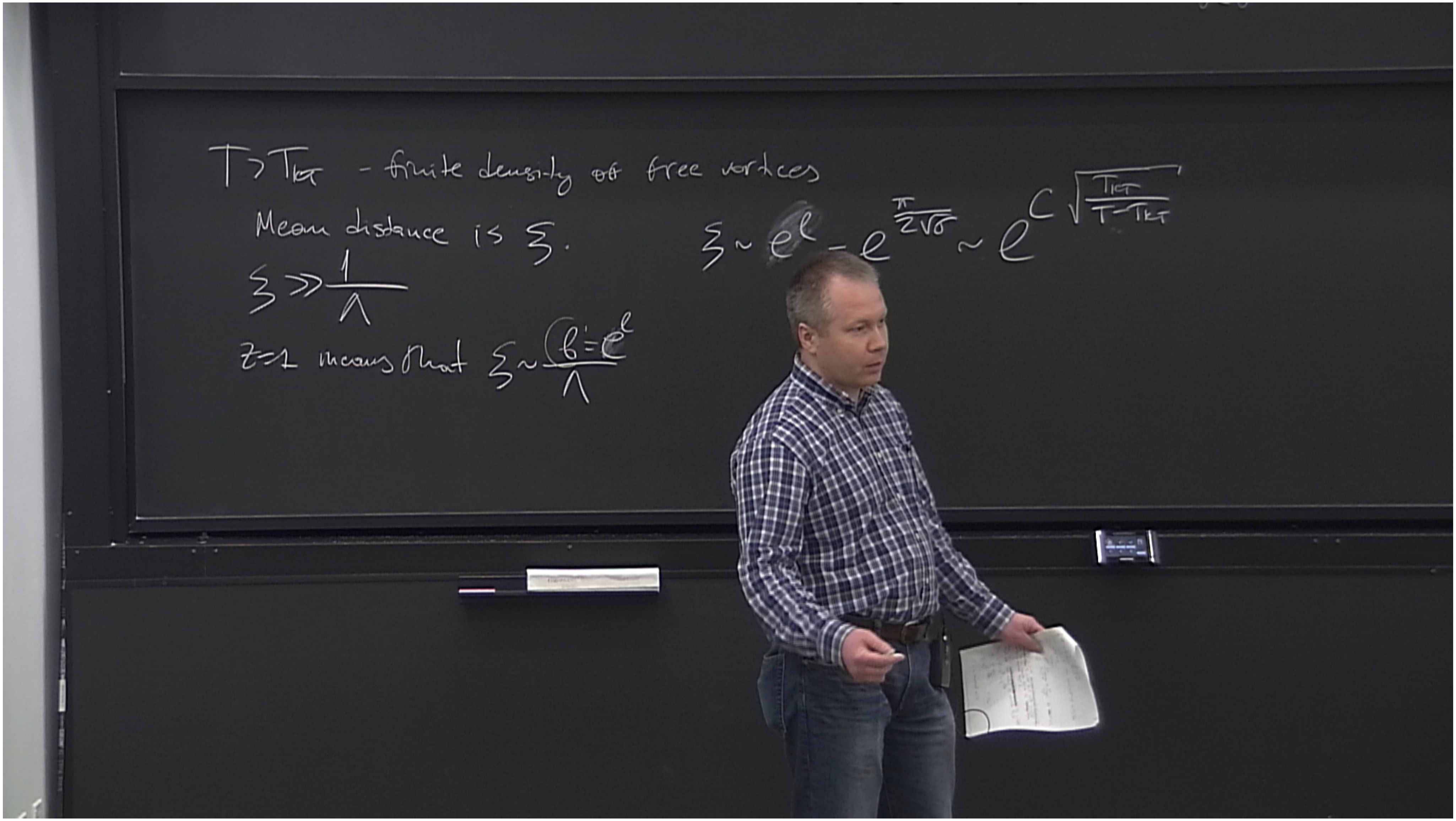

T > T_KT - finite density of free vortices
distance is ξ
1/Λ

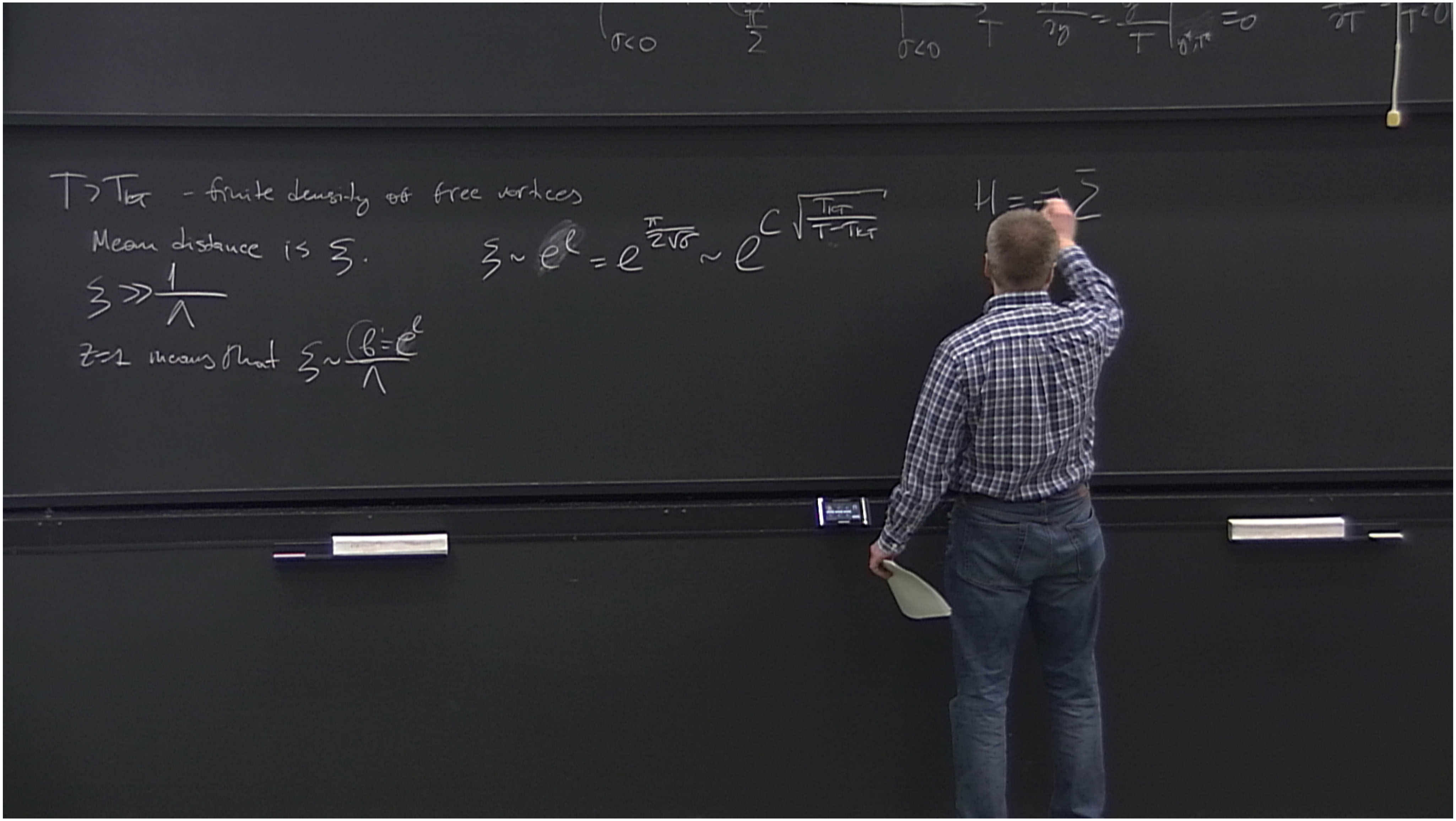

T > T_KT - finite density of free vortices
Mean distance is ξ.
ξ >> 1/Λ
z=1 means that ξ ~ b'=e^l / Λ

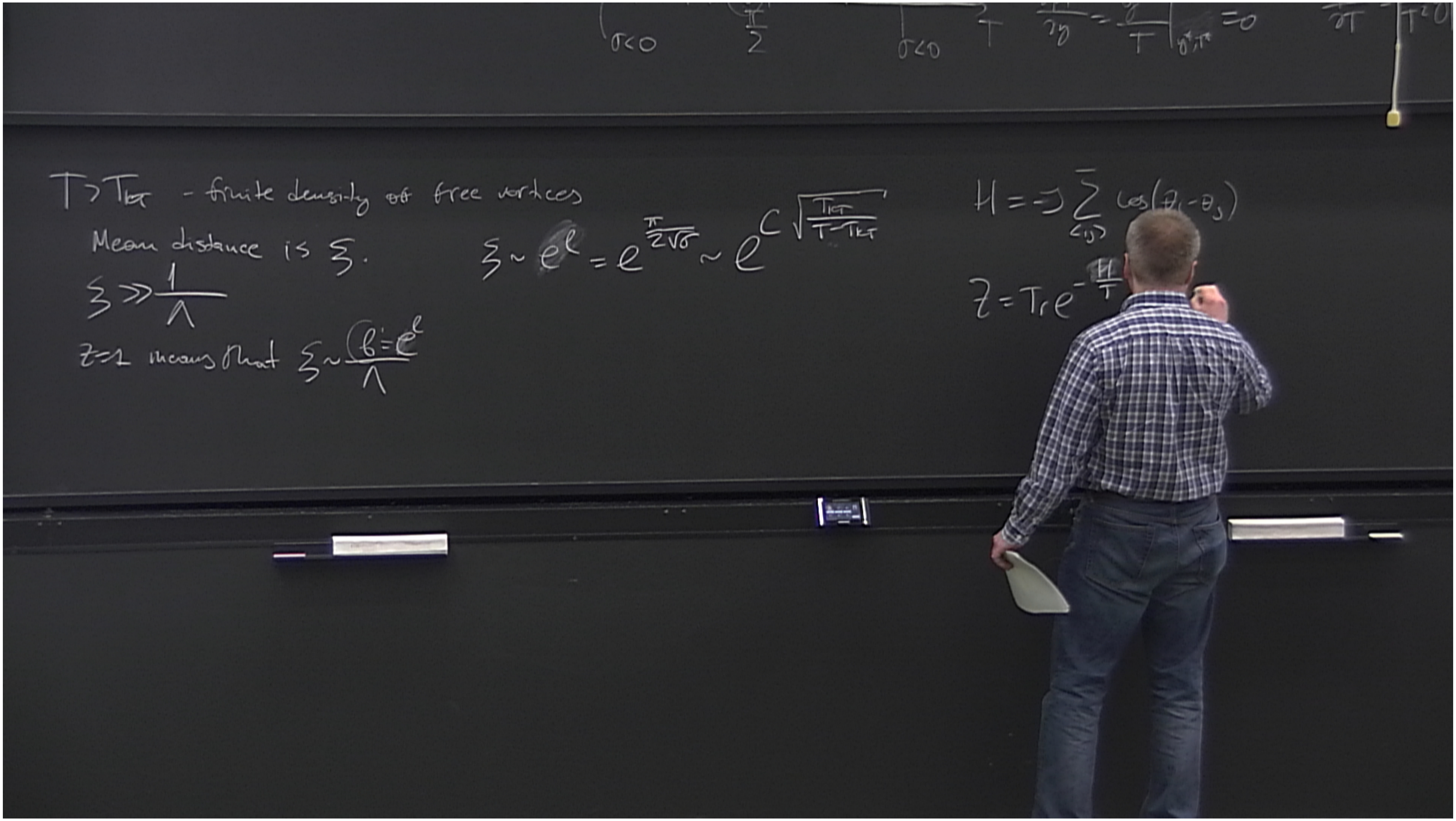

T > T_KT - finite density of free vortices
Mean distance is ξ
ξ >> 1/Λ
z=1 means that ξ ~ (b' = e^l)/Λ
ξ ~ e^l = e

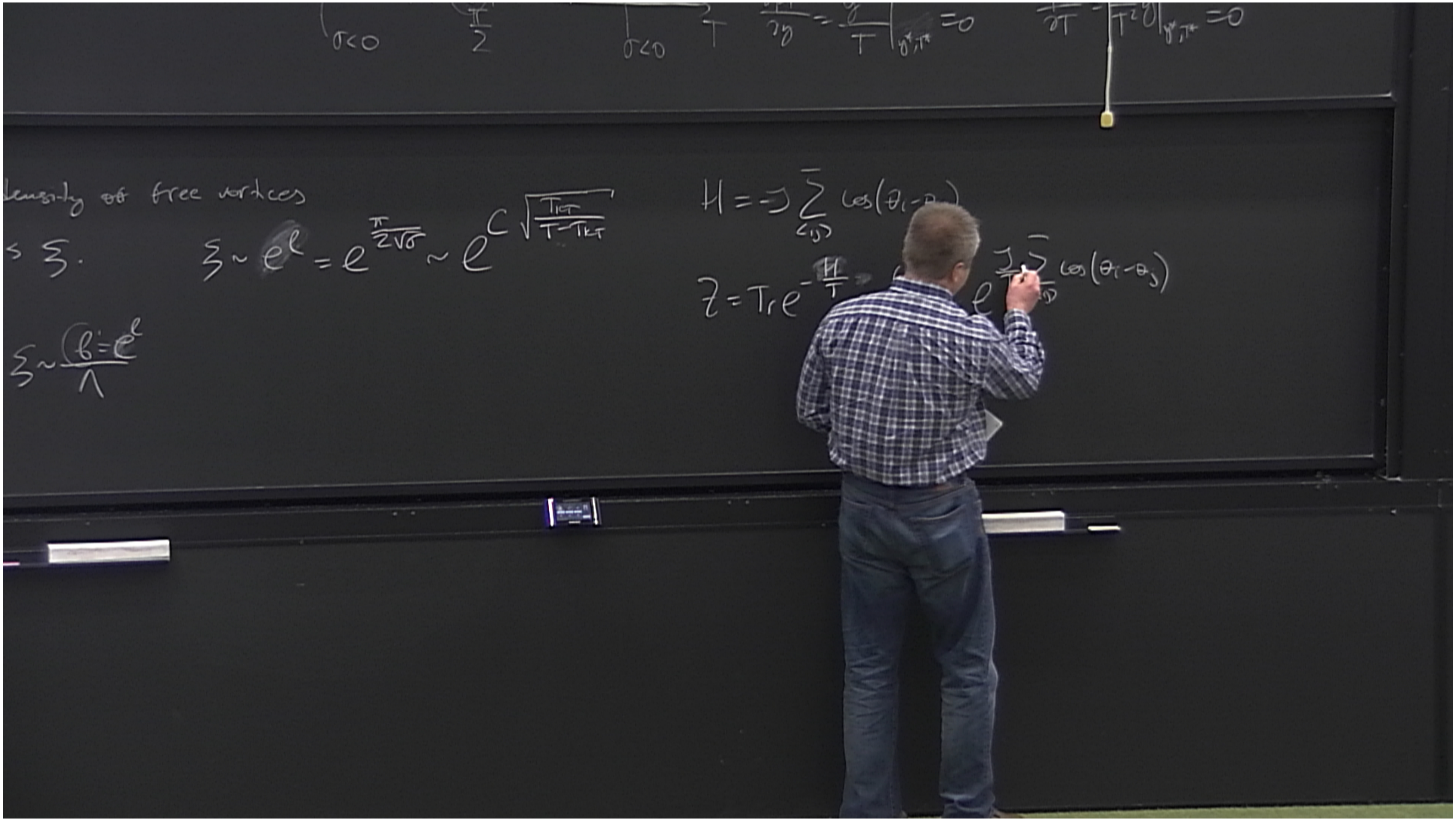

T > T_KT - finite density of free vortices
Mean distance is ξ
ξ >> 1/Λ
z=1 means that ξ ~ b=e^l / Λ
ξ ~ e^l = e^(π/2√σ) ~

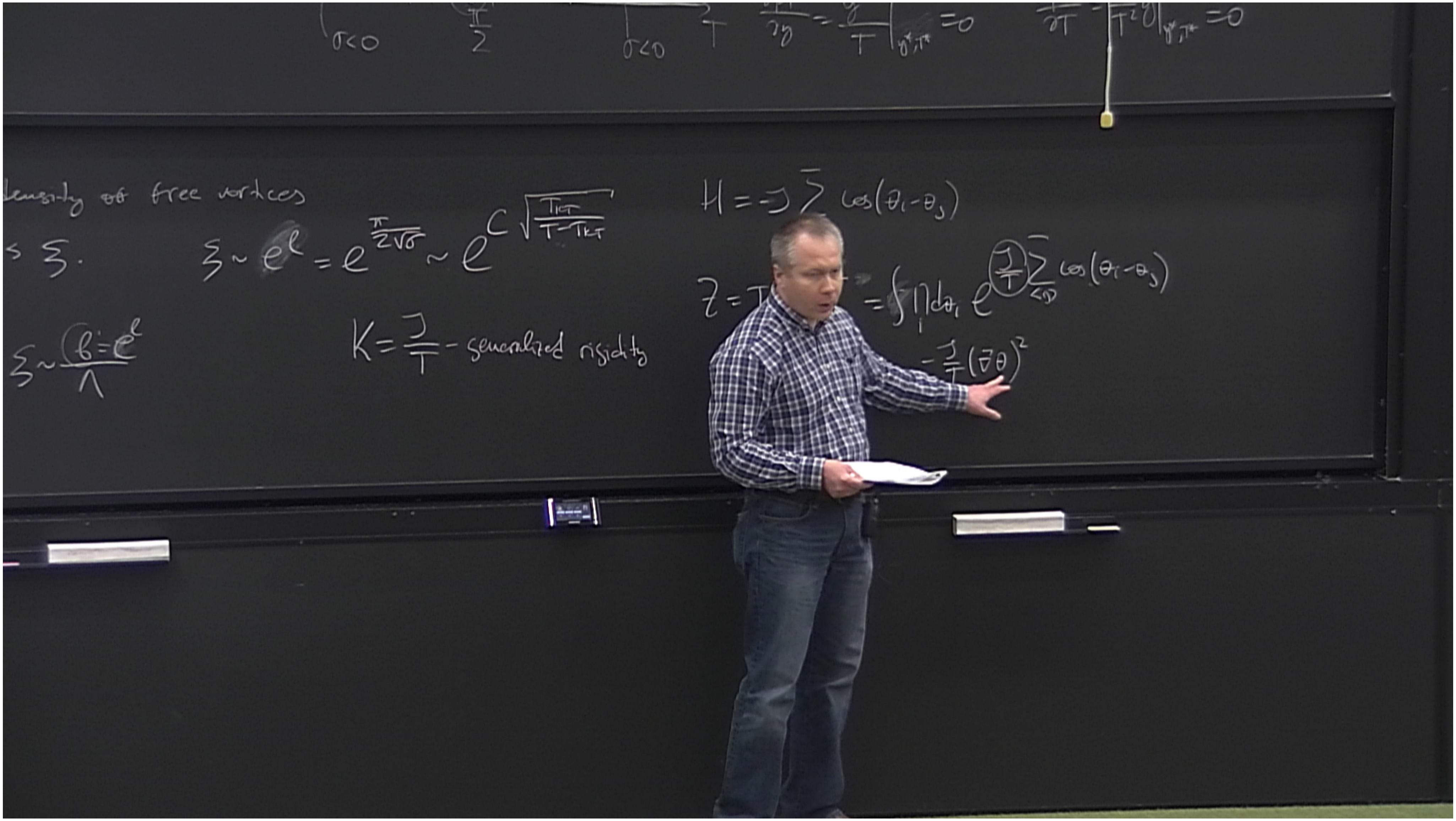

T > T_KT - finite density of free vortices
Mean distance ξ
ξ >> 1/Λ
ξ ~ e^l = e^(π/2√σ) ~ e^(C √(T_KT/(T−T_KT)))
z=1
ξ ~ (b'=e^l)/Λ

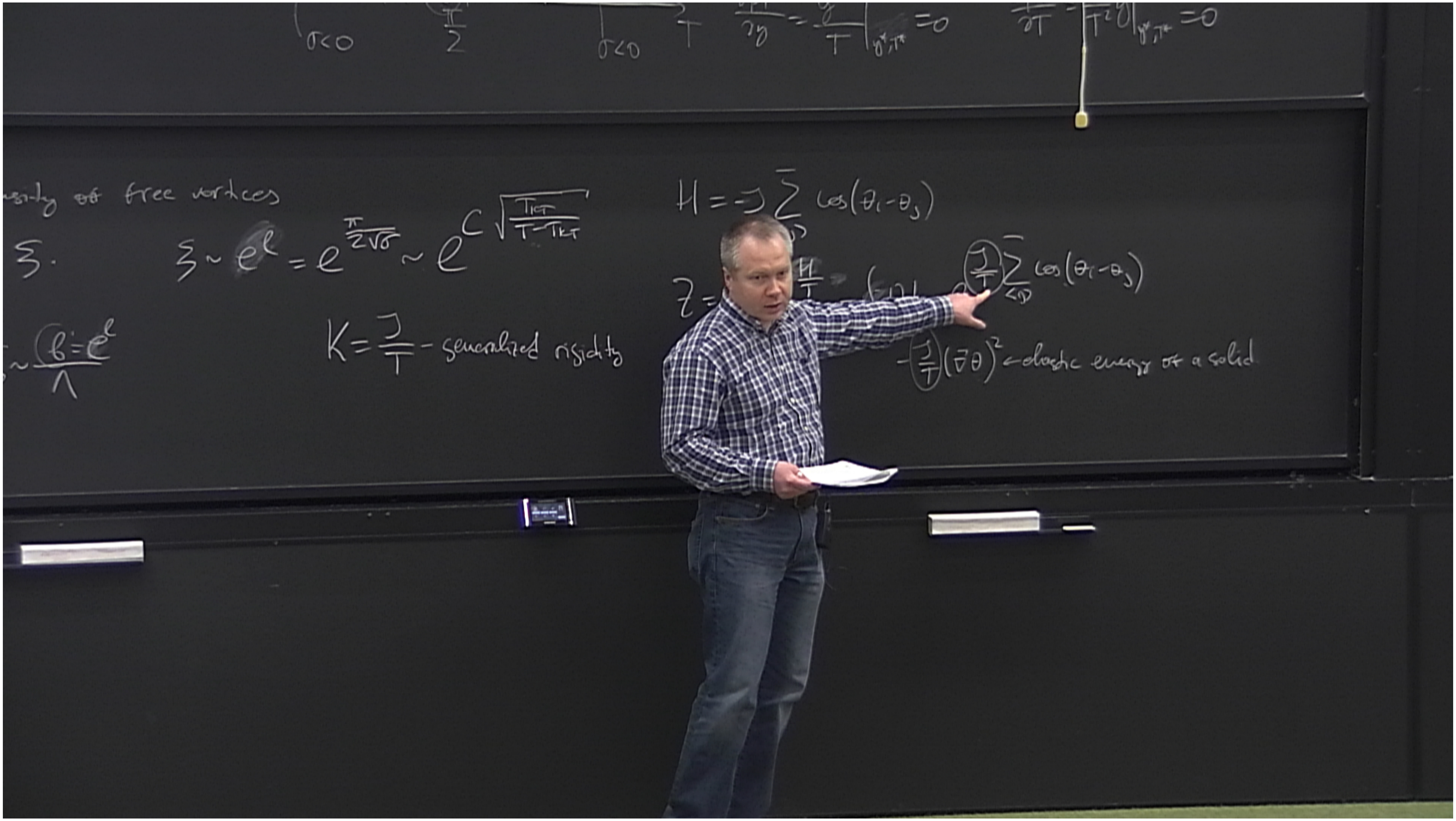

T > T_KT - finite density of free vortices
Mean distance is ξ.
ξ >> 1/Λ
z=1 means that ξ ~ (b = e^l)/Λ
ξ ~ e^l ~ e^(π/2√σ) ~ e^(C √(T_KT/(T−T_KT)))

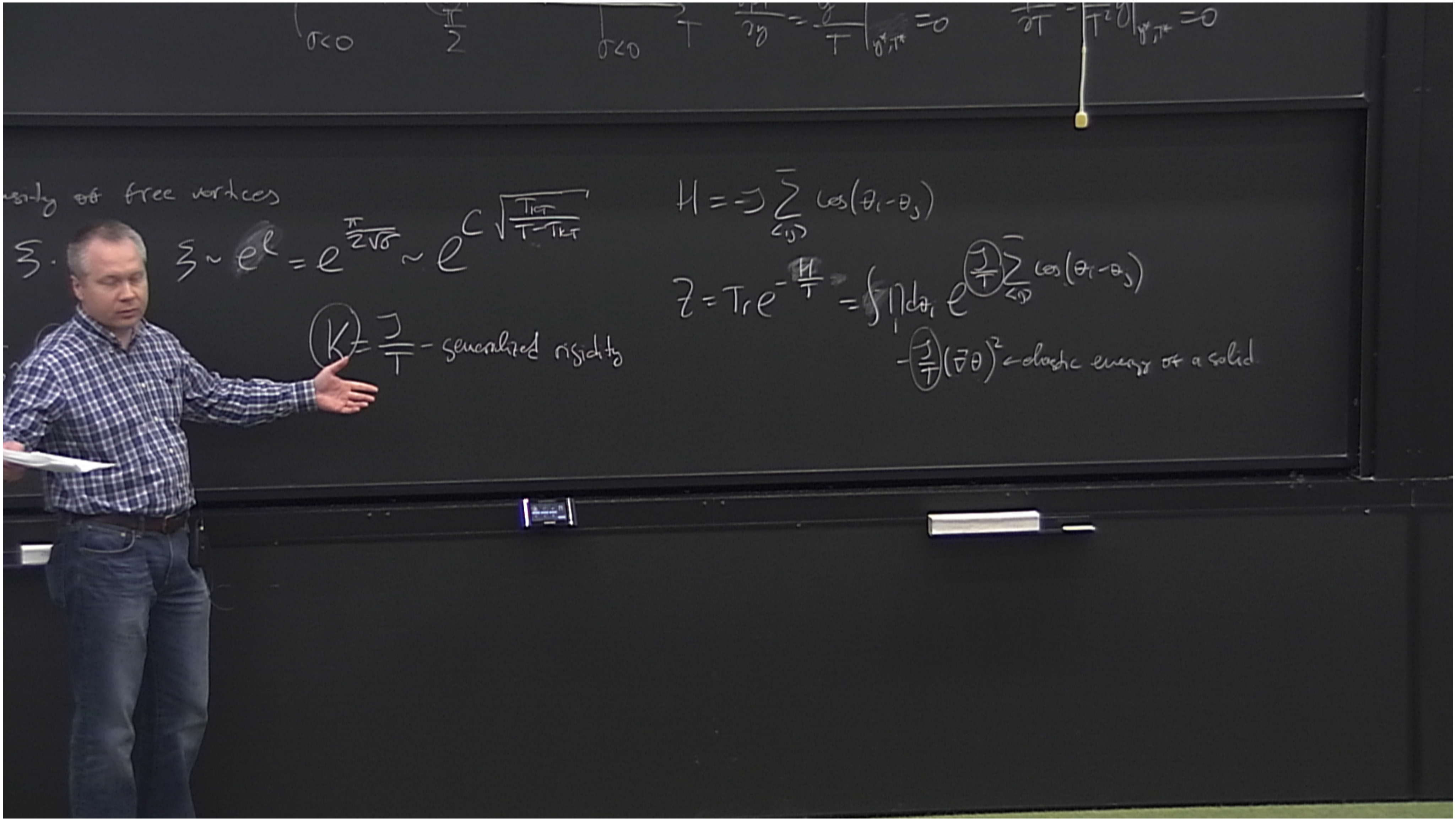

T > T_KT - finite density of free vortices
Mean distance is ξ.
ξ ~ e^l = e^{π/(2√σ)} ~ e^{C √(T_KT/(T−T_KT))}
ξ >> 1/Λ
z=1 means that ξ ~ (b=e^l)/Λ

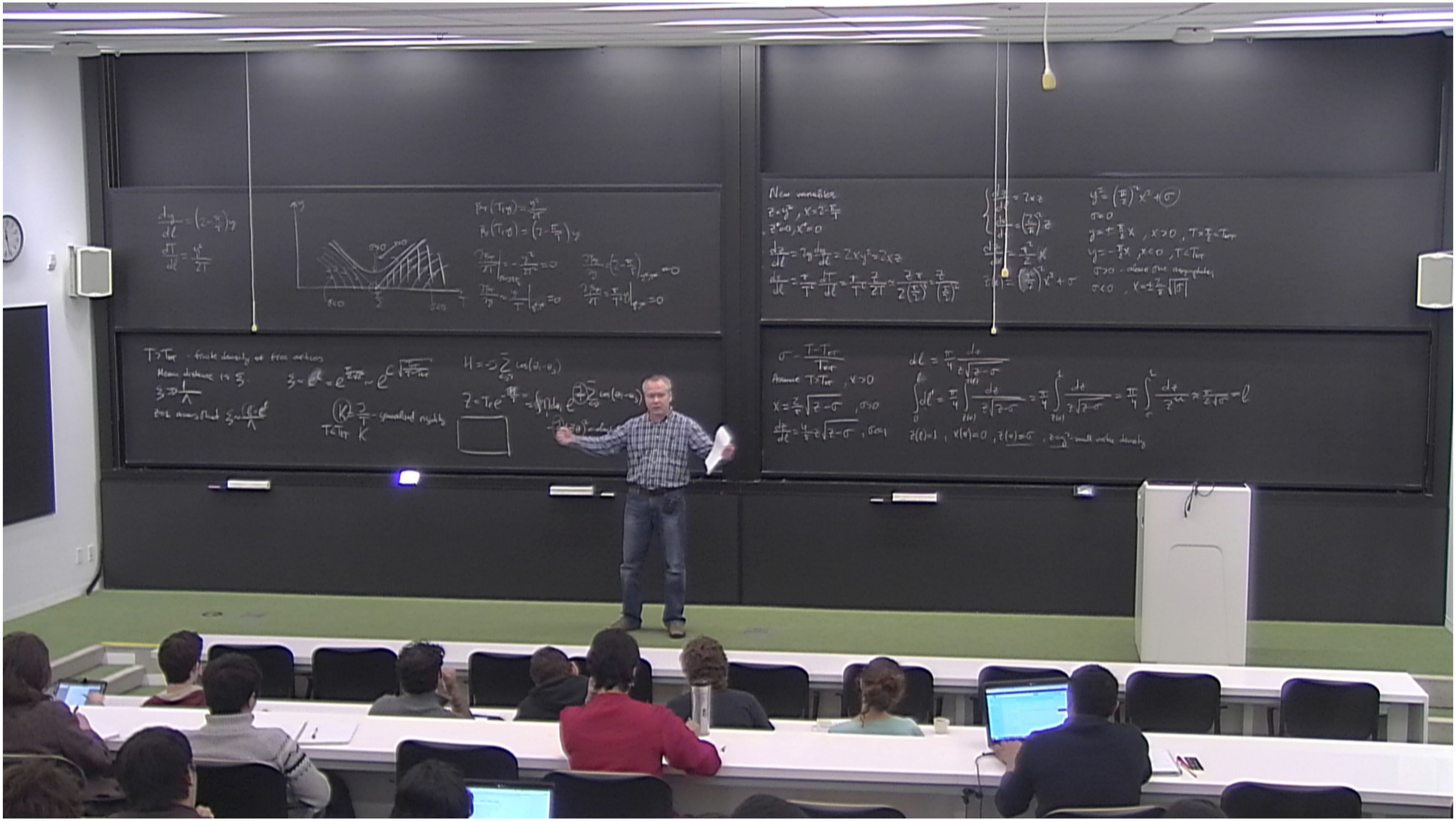

T > T_KT - finite density of free vortices
Mean distance is ξ.
ξ >> 1/Λ
z=1 means that ξ ~ (b = e^l)/Λ
ξ ~ e^l = e^(π/(2√σ)) ~ e^(C √(T_KT/(T−T_KT)))
H = −J Σ_⟨ij⟩ cos(θ_i − θ_j)
Z = Tr e^(−H/T)

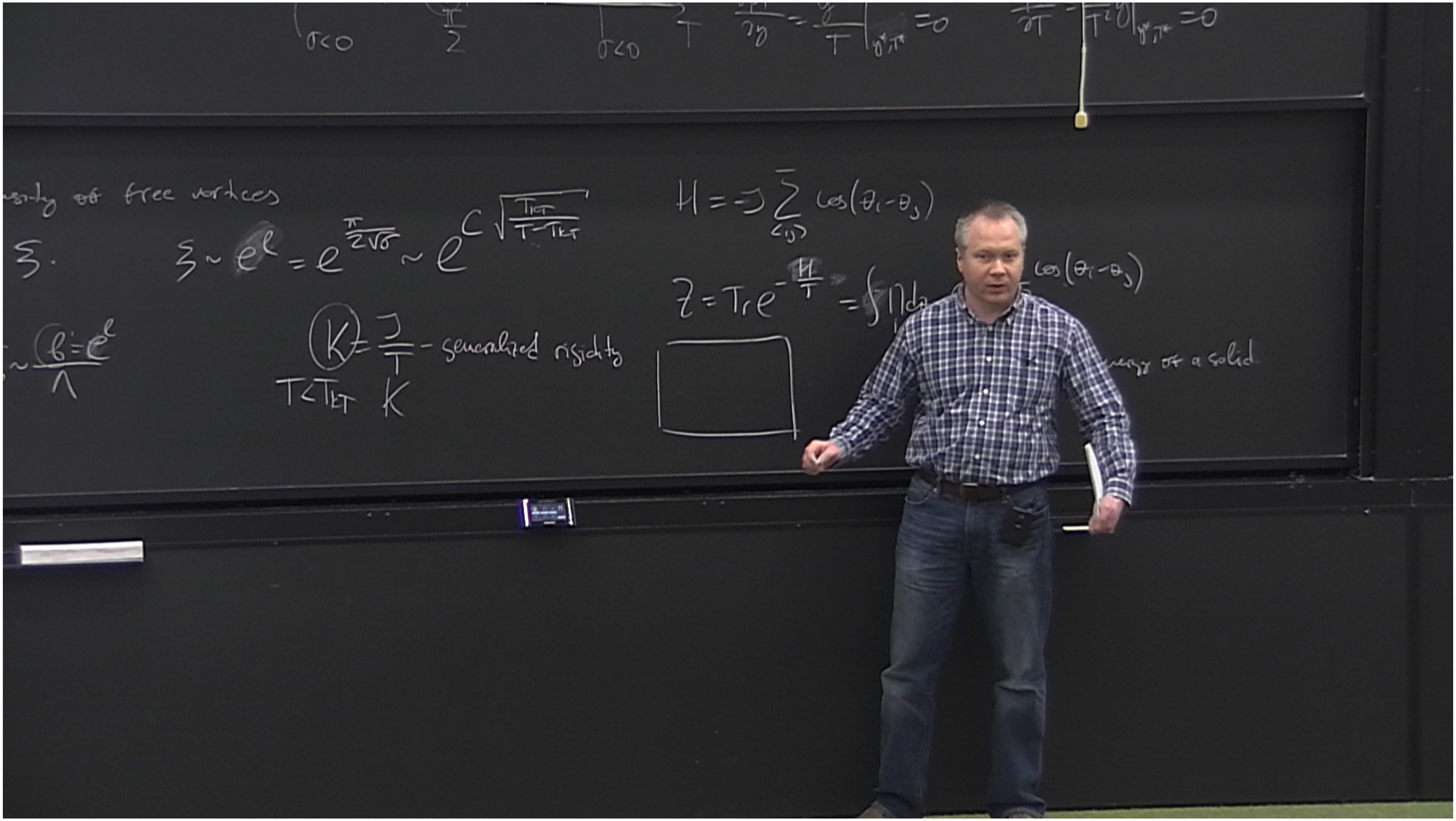

density of free vortices

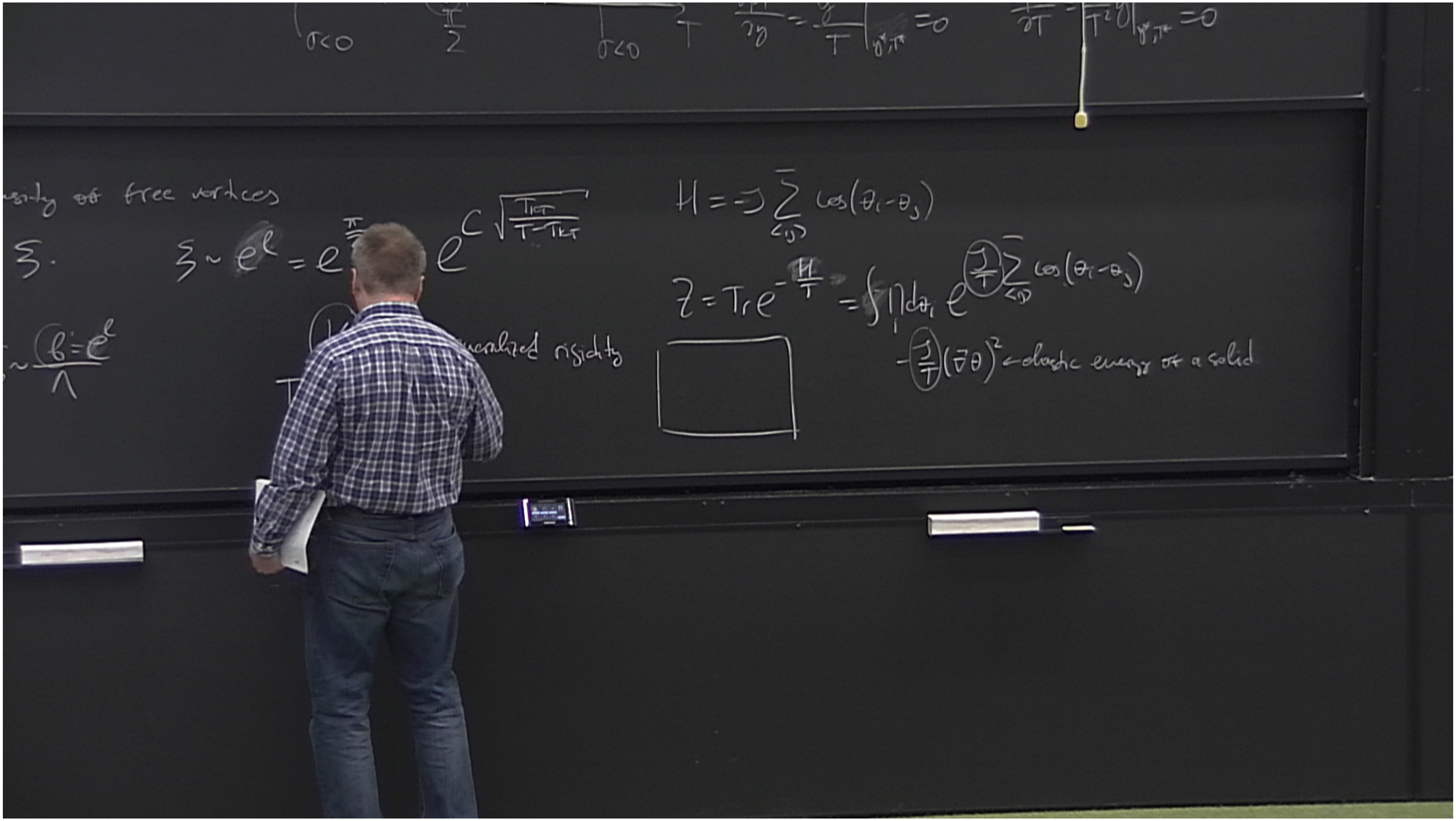

free vortices
K = J/T - generalized rigidity
H = -J Σ cos(θi - θj)


free vortices
K = J/T - generalized rigidity
H = -J Σ cos(θi - θj)
elastic energy of a solid

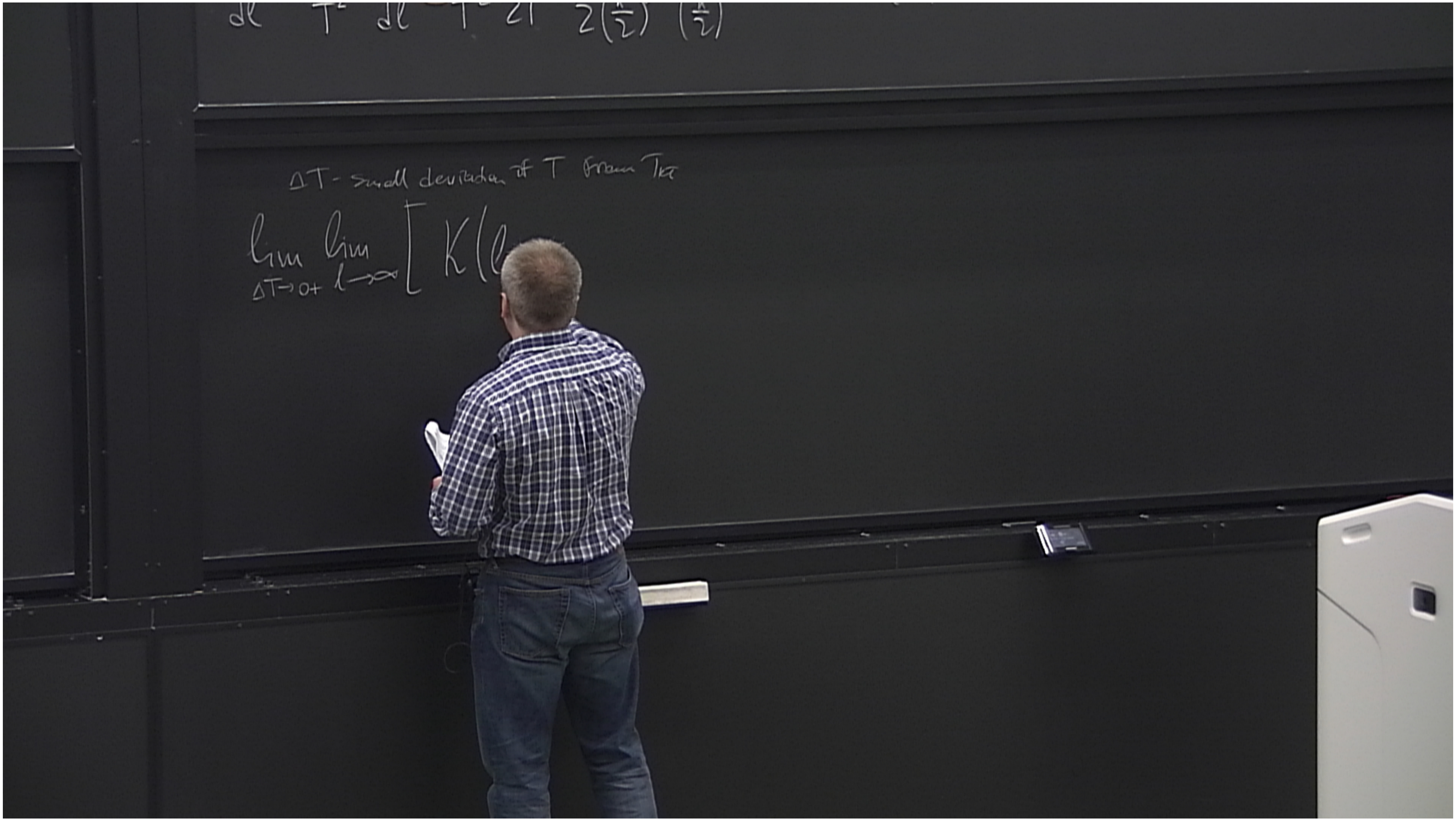

of free vortices
ξ ~ e^l = e^{π/2√σ} ~ e^{C √(T_KT/(T−T_KT))}
K = J/T − generalized rigidity
H = −J Σ_<ij> cos(θ_i − θ_j)
Z = Tr e^{−H/T} = ∫ Π_i dθ_i e^{J/T Σ_<ij> cos(θ_i − θ_j)}
−J/T (∇θ)^2 ← elastic energy of a solid

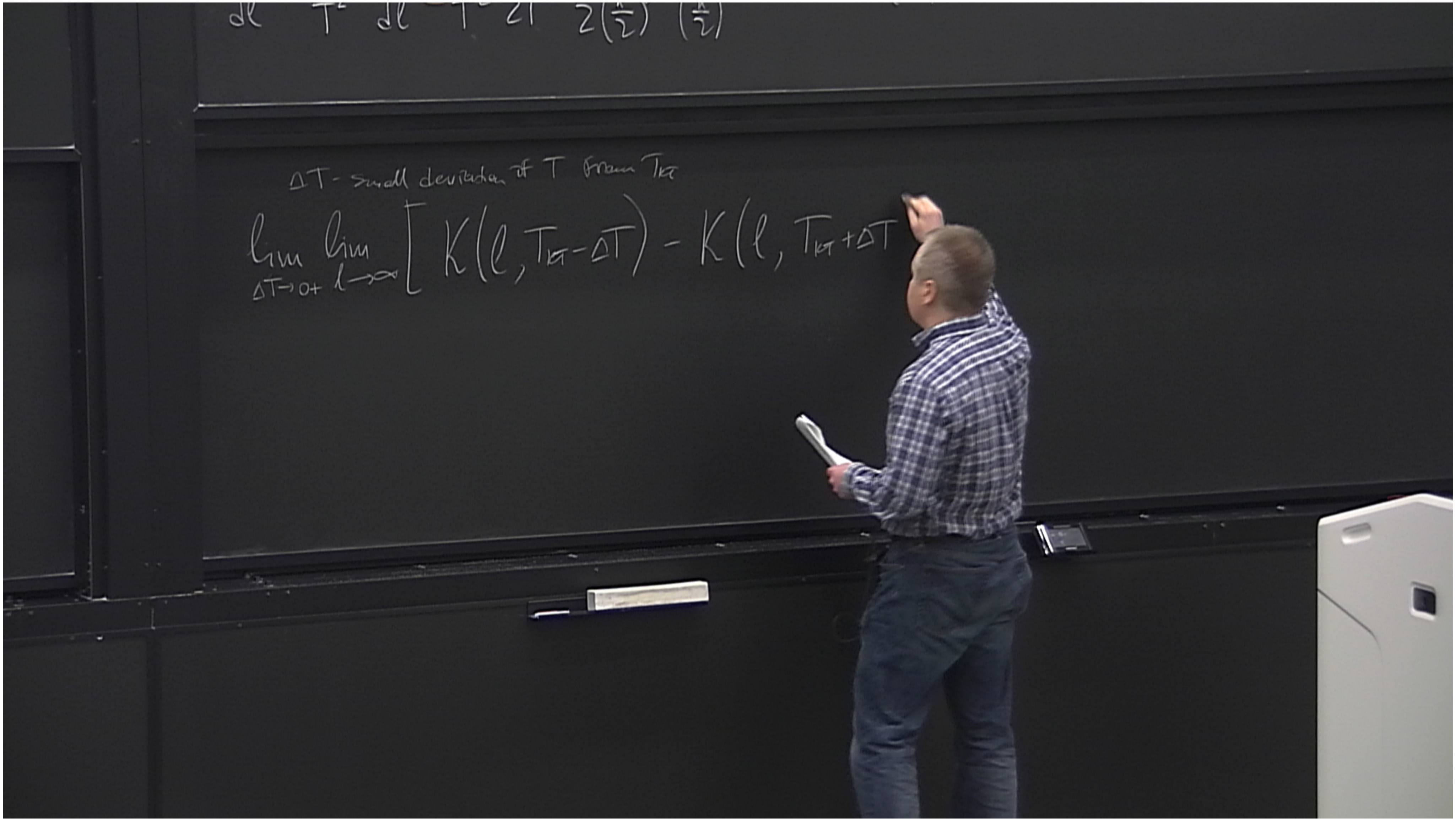


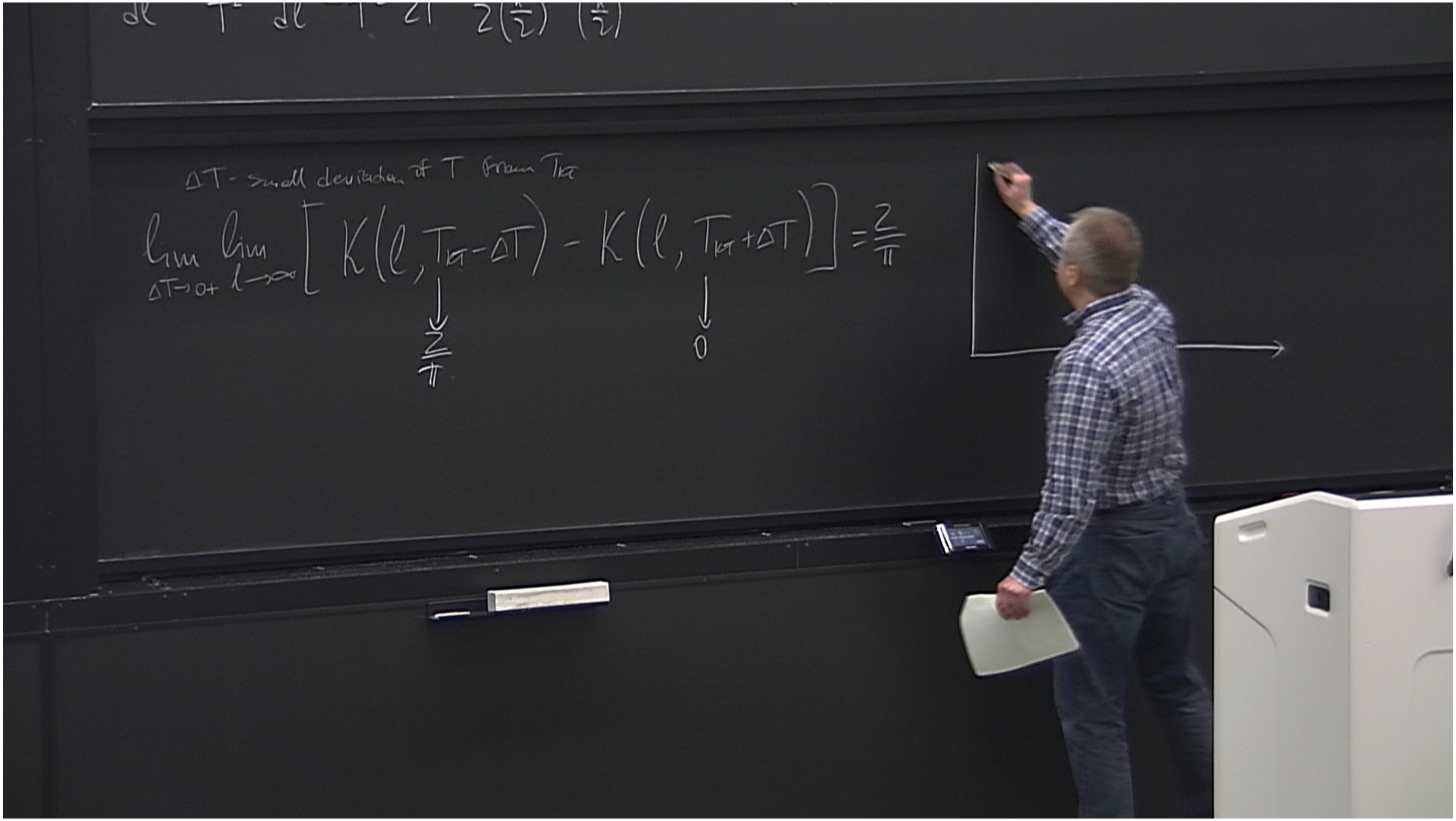

free vortices
generalized rigidity
of a solid

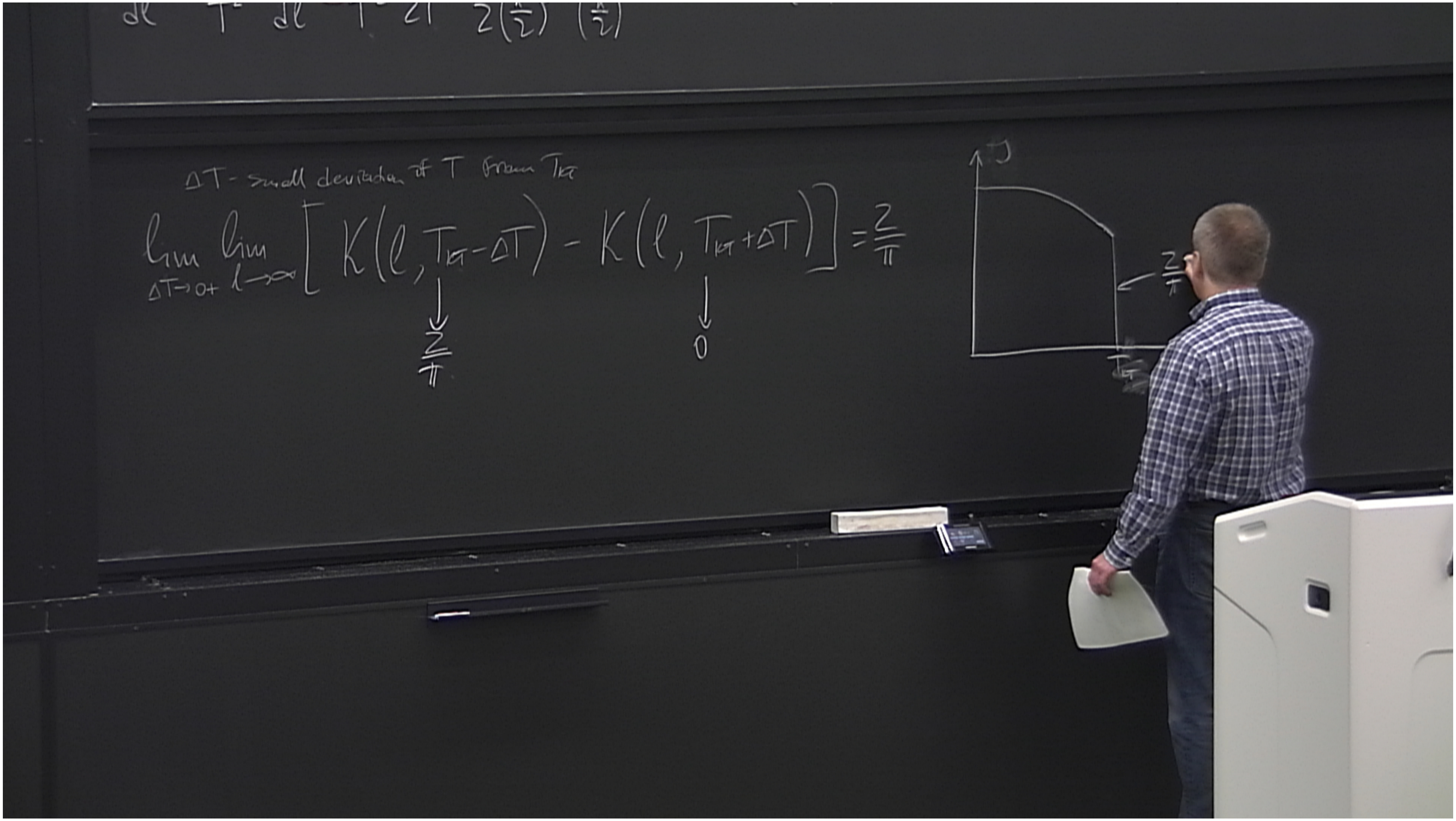

free vortices
ralized rigidity
H = -J Σ cos(θi - θj)
Z = Tr e^(-H/T)
elastic energy of a solid

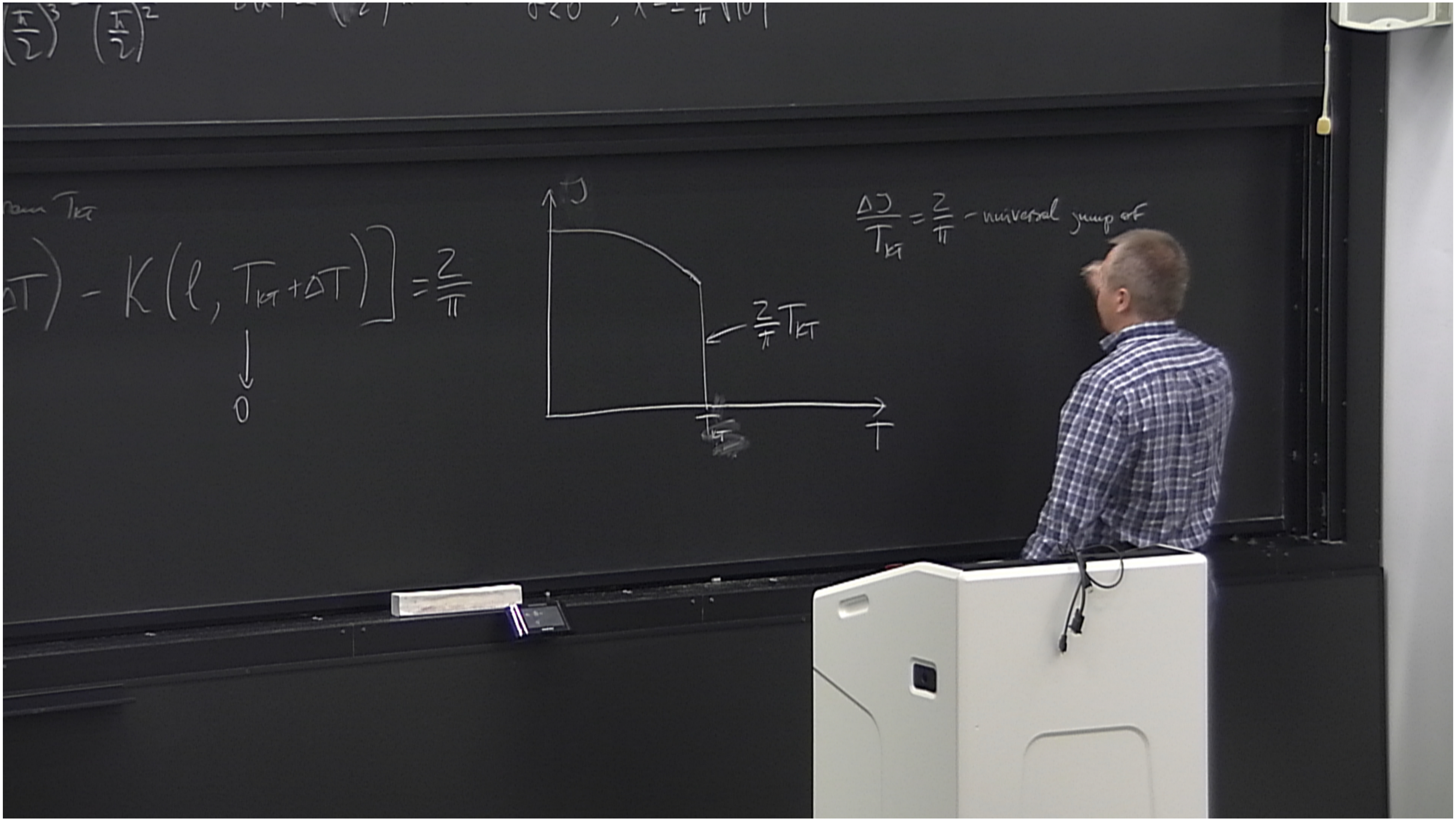

z(l)=1 , x(0)=0 , z(0)=σ , z=y² - small vortex density

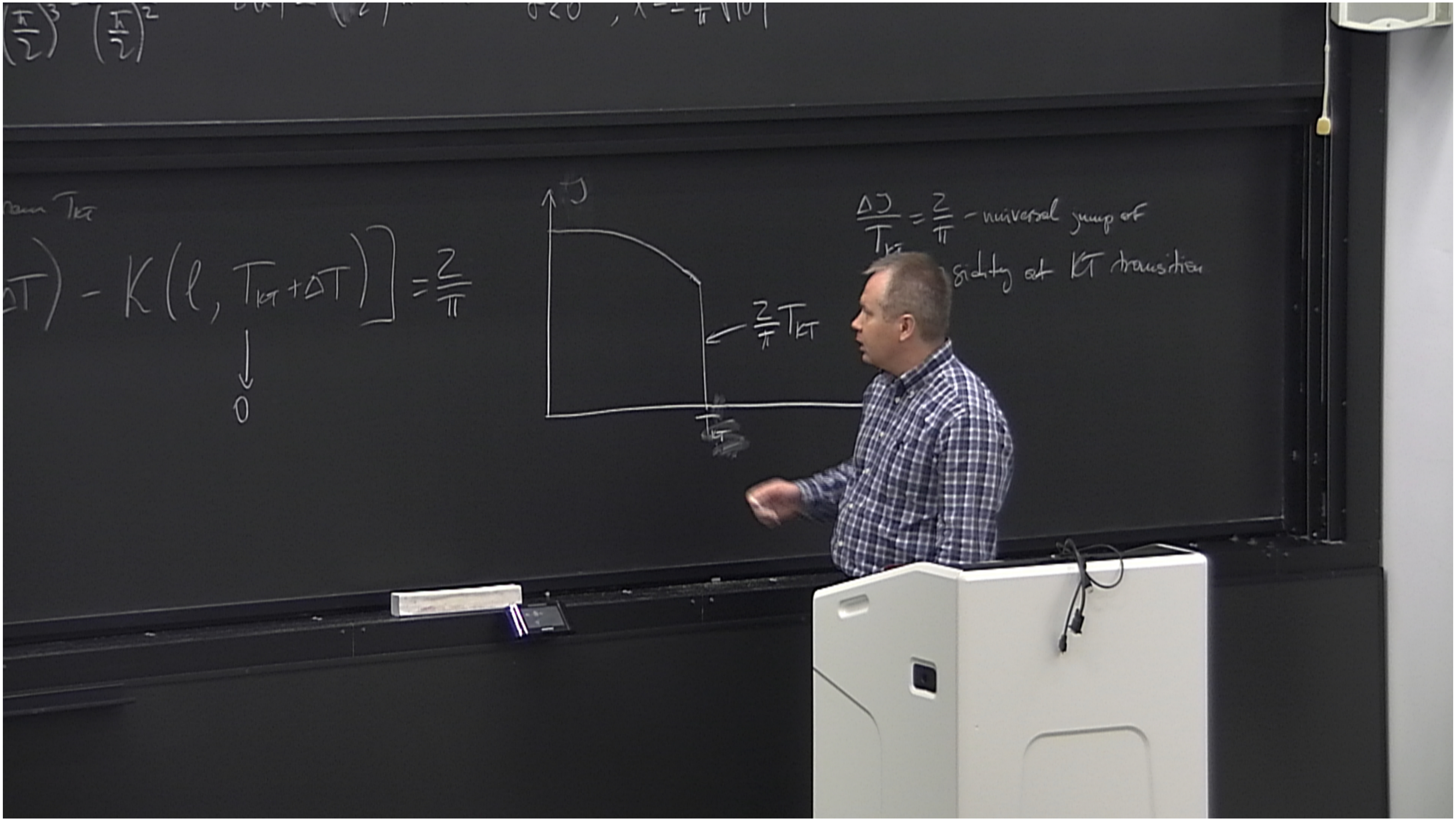

ΔT - small deviation of T from


ΔT - small deviation of T from T_KT
lim lim [K(l, T_KT - ΔT) - K(l, T_KT + ΔT
ΔT→0+ l→∞

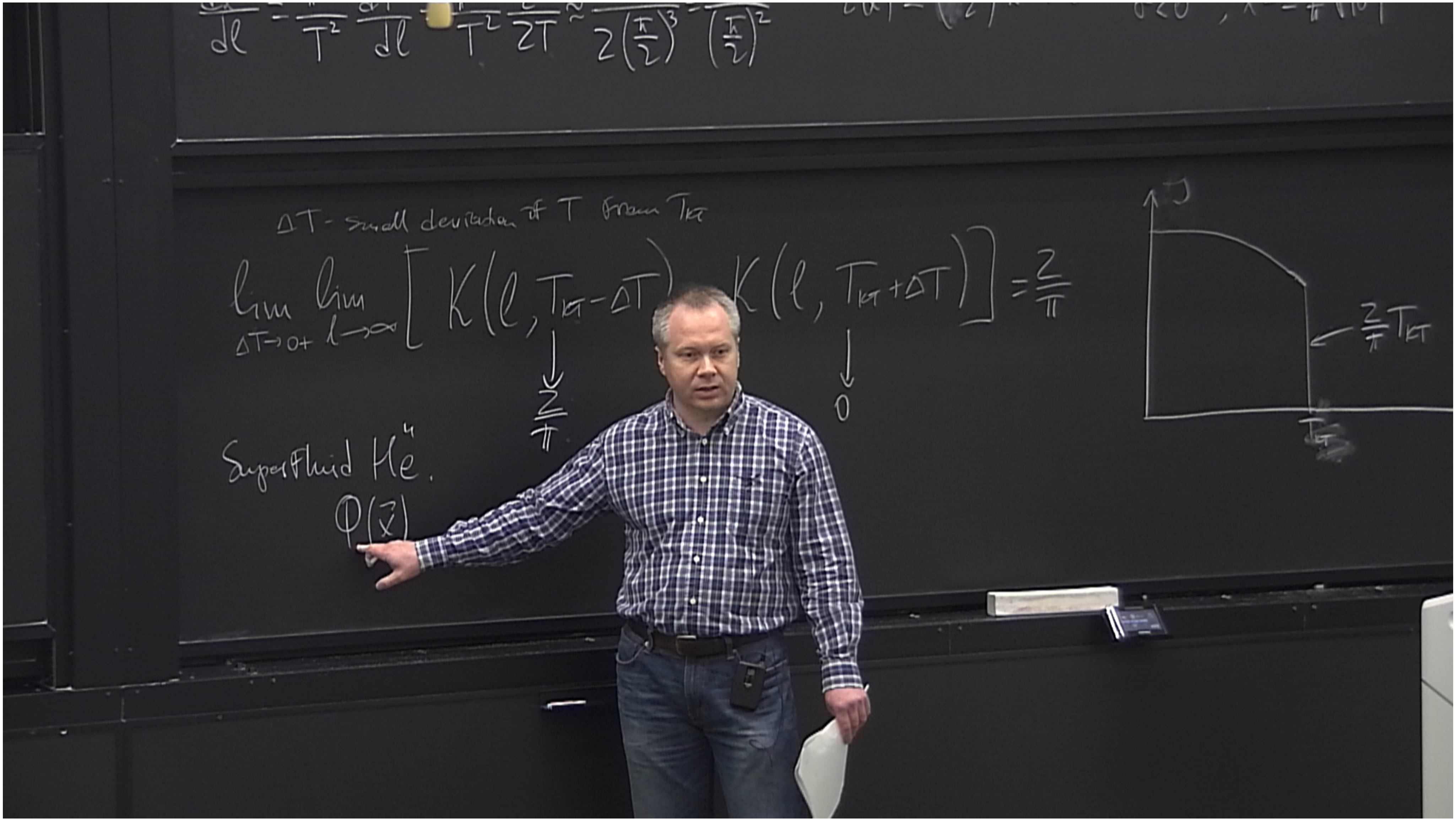

ΔT - small deviation of T from T_KT
lim lim [K(l, T_KT - ΔT) - K(l, T_KT + ΔT)] = 2/π
ΔT→0+ l→∞
2/π
0

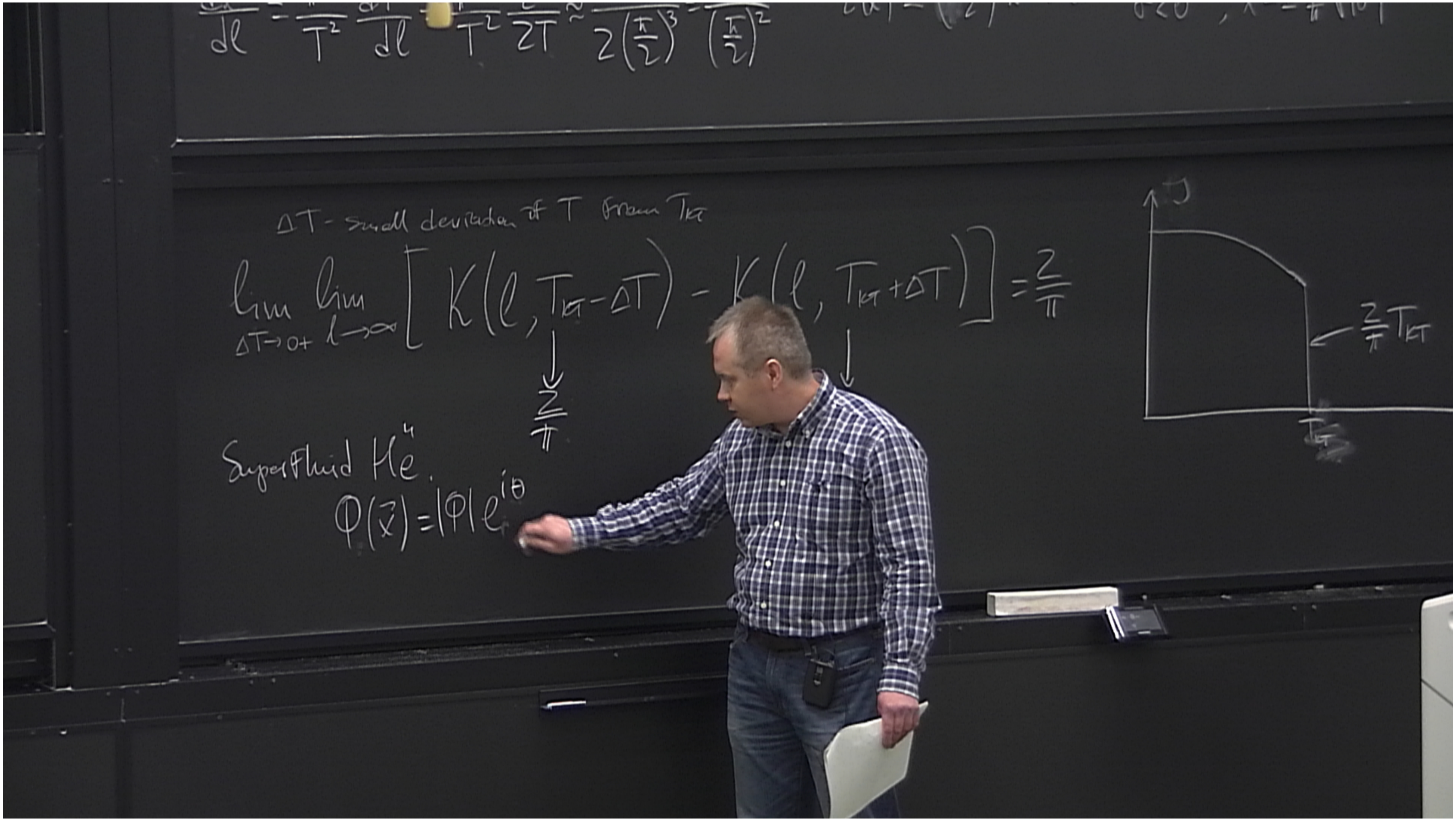

ΔT - small deviation of T from T_KT
lim_{ΔT→0+} lim_{l→∞} [K(l, T_KT − ΔT) − K(l, T_KT + ΔT)] = 2/π
2/π
0
J
2/π
T_KT

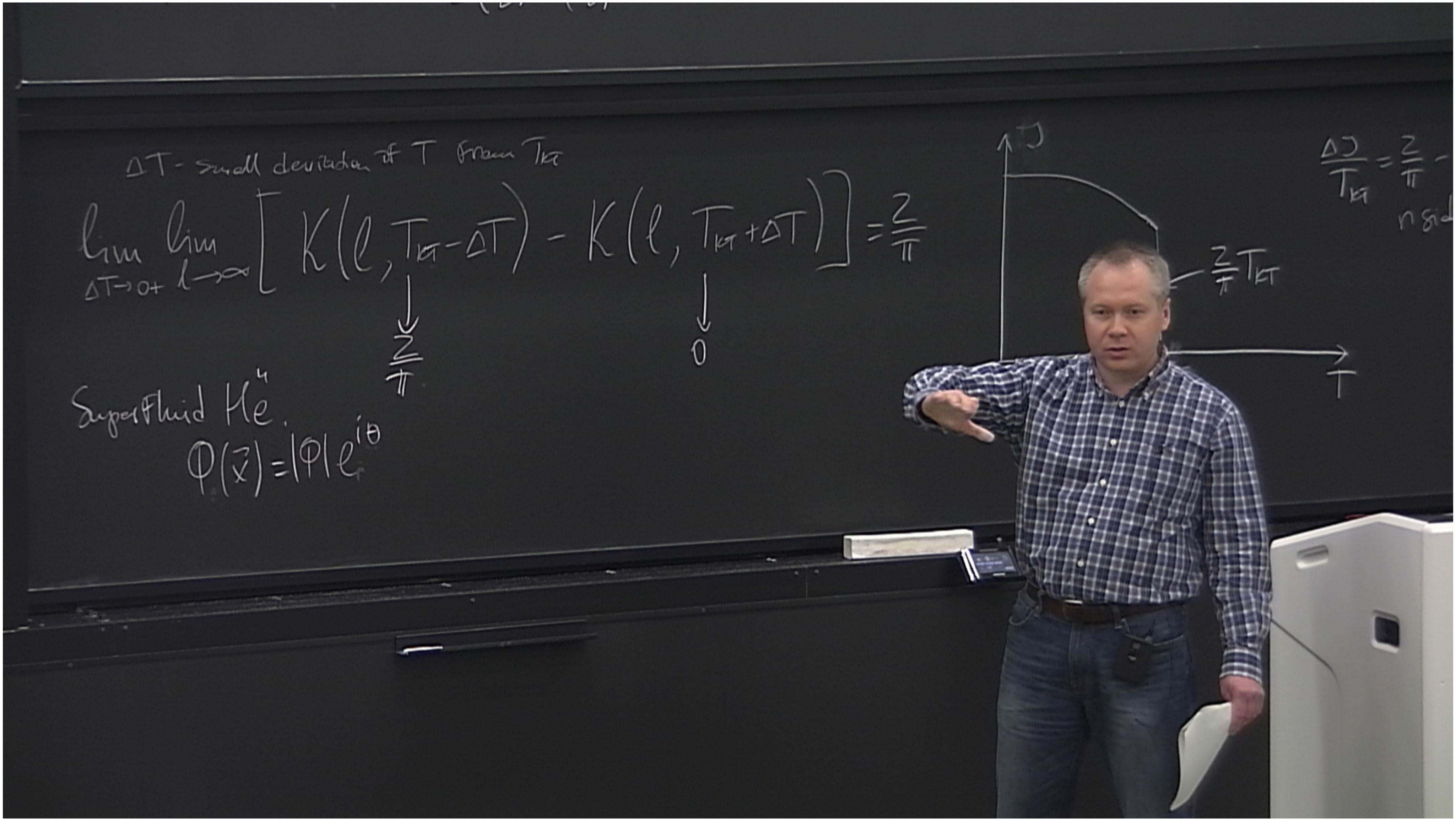

T_KT
K(l, T_KT + ΔT)] = 2/π
0
J
2/π T_KT
T_KT
T
ΔJ/T_KT = 2/π – universal jump of



$\frac{\Delta J}{T_{KT}} = \frac{2}{\pi}$ - universal jump of
sidity at KT transition
$\frac{2}{\pi} T_{KT}$




ΔT - small deviation of T from TKT
lim lim [K(l, TKT - ΔT) - K(l, TKT + ΔT)] = 2/π
ΔT→0+ l→∞
2/π
Superfluid He4
φ(x) = |φ| e^{iθ}
2/π TKT



ΔT - small deviation of T from T_{KT}
$\lim_{\Delta T\to 0+} \lim_{l\to\infty} \left[K(l, T_{KT}-\Delta T) - K(l, T_{KT}+\Delta T) \right] = \frac{2}{\pi}$
$\frac{2}{\pi}$
0
Superfluid He^4.
$\varphi(\vec{x}) = |\varphi| e^{i\theta}$
J
$\frac{2}{\pi} T_{KT}$
T
$\frac{\Delta J}{T_{KT}} = \frac{2}{\pi}$