

Title: 13/14 PSI - Statistical Mechanics - Lecture 14

Date: Oct 25, 2013 10:45 AM

URL: <http://pirsa.org/13100062>

Abstract:

$$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right)y$$

$$\frac{dT}{dl} = \frac{y^2}{2T}$$



$$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right)y$$

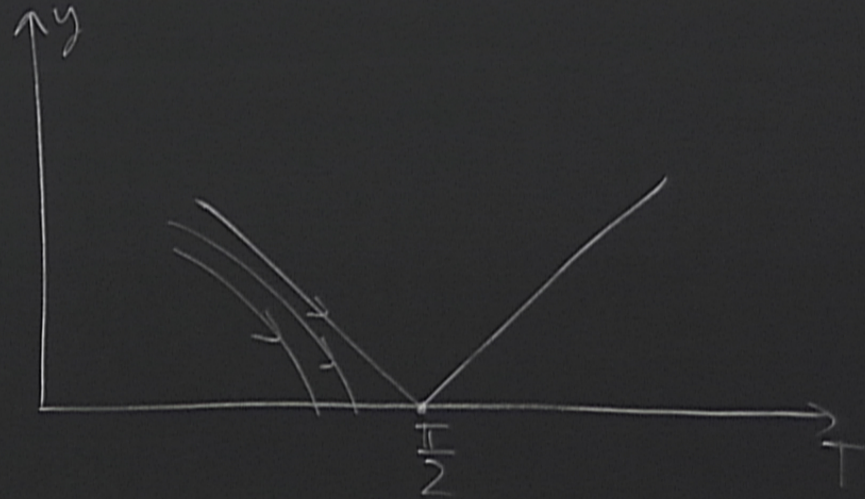
$$\frac{dT}{dl} = \frac{y^2}{2T}$$

y

T

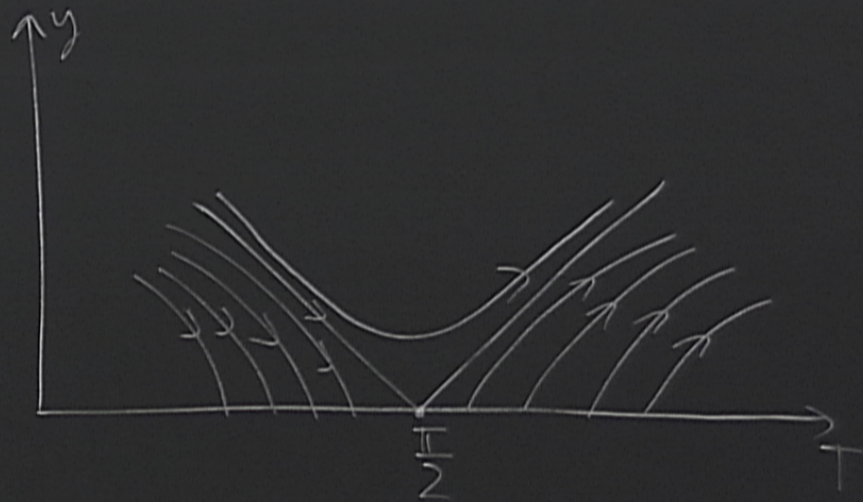
$$\frac{dy}{dt} = \left(2 - \frac{\pi}{T}\right)y$$

$$\frac{y^2}{2T}$$



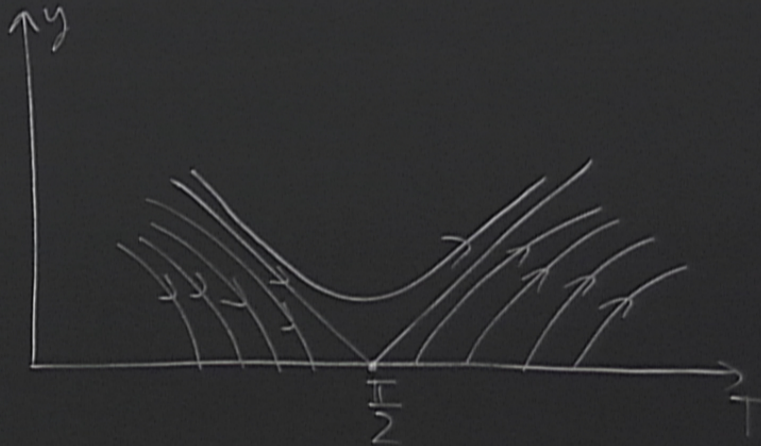
$$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right)y$$

$$\frac{dT}{dl} = \frac{y^2}{2T}$$



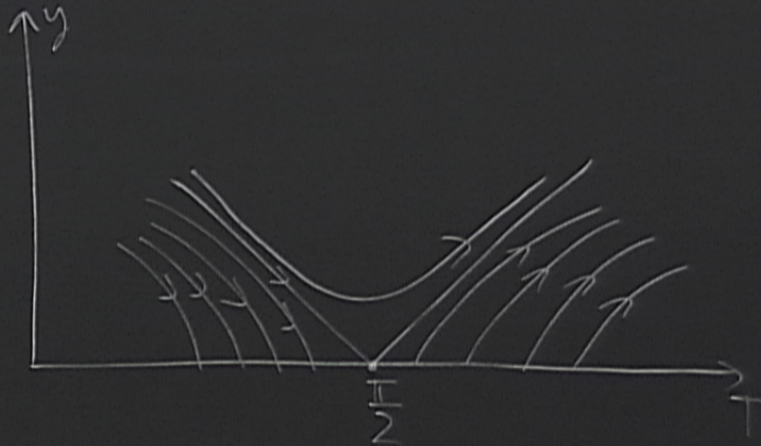
$$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right)y$$

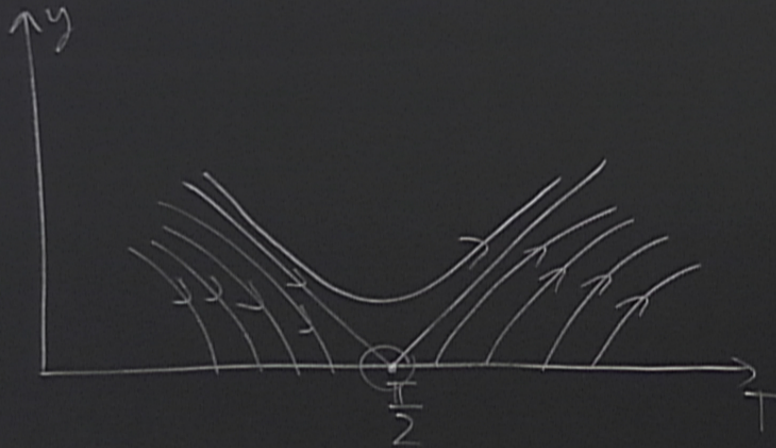
$$\frac{dT}{dl} = \frac{y^2}{2T}$$



$$\frac{dy}{dl} = \left(2 - \frac{\pi}{T}\right)y$$

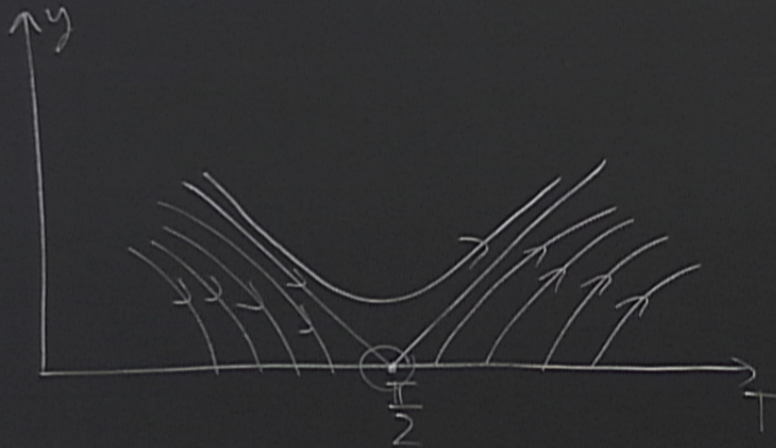
$$\frac{dT}{dl} = \frac{y^2}{2T}$$





$$P_T(T, y) = \frac{y^2}{2T}$$

$$P_y(T, y) = \left(2 - \frac{y}{T}\right)$$

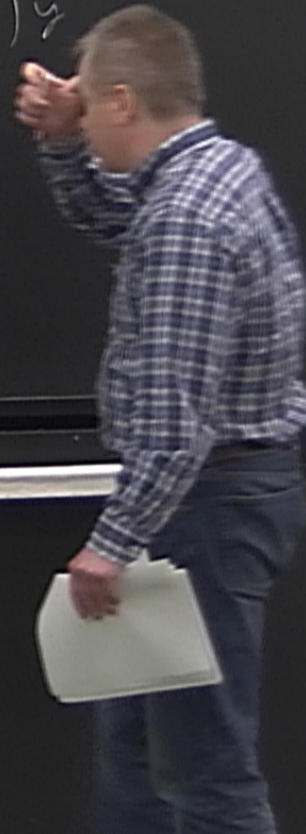


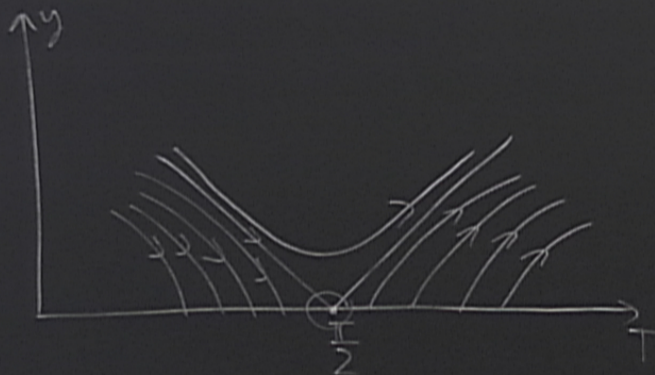
$$P_T(T, y) = \frac{y^2}{2T}$$

$$P_y(T, y) = \left(2 - \frac{y}{T}\right) y$$

$$\left. \frac{\partial P_T}{\partial T} \right|_{y=0, T=T/2} = -\frac{y^2}{2T^2} = 0$$

$$\left. \frac{\partial P_T}{\partial y} \right|_{y=0, T=T/2} = \frac{y}{T} = 0$$





$$P_T(T, y) = \frac{y^2}{2T}$$

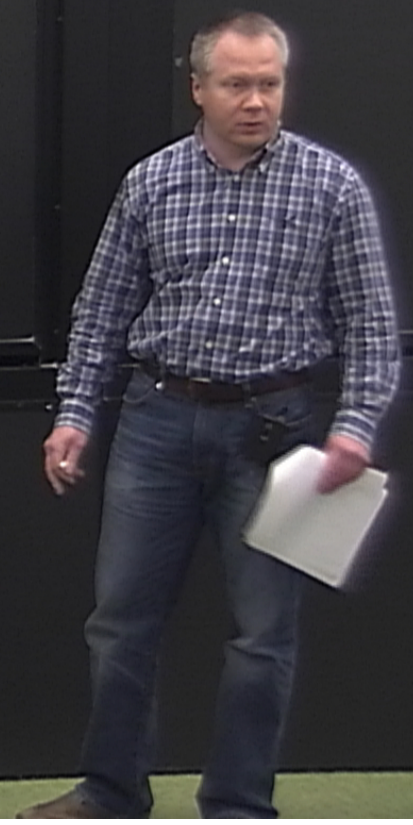
$$P_y(T, y) = \left(2 - \frac{T}{T}\right) y$$

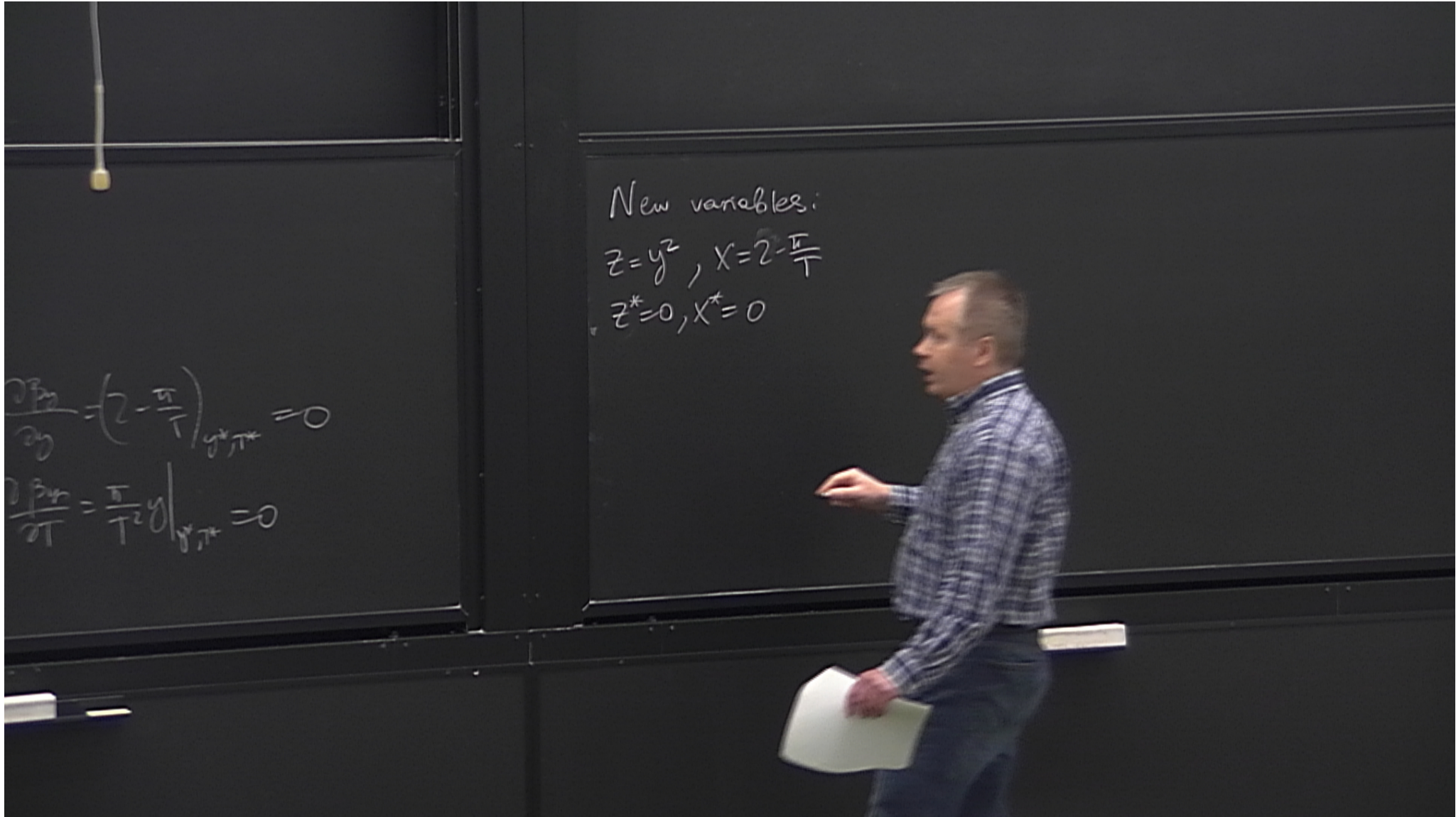
$$\frac{\partial P_T}{\partial T} \Big|_{y=0, T=H/2} = -\frac{y^2}{2T^2} = 0$$

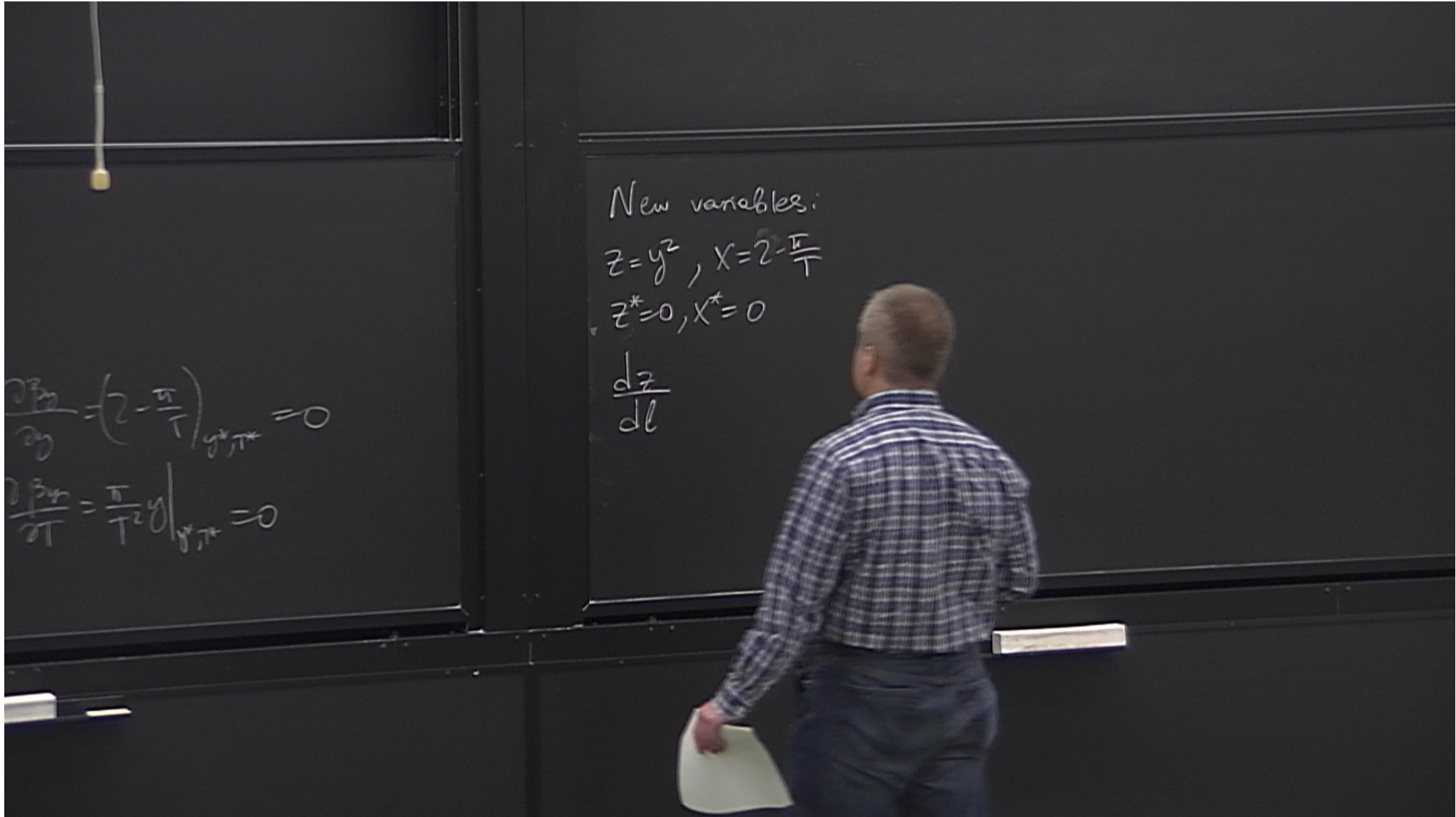
$$\frac{\partial P_y}{\partial y} = \left(2 - \frac{T}{T}\right) y \Big|_{y=0, T=H/2} = 0$$

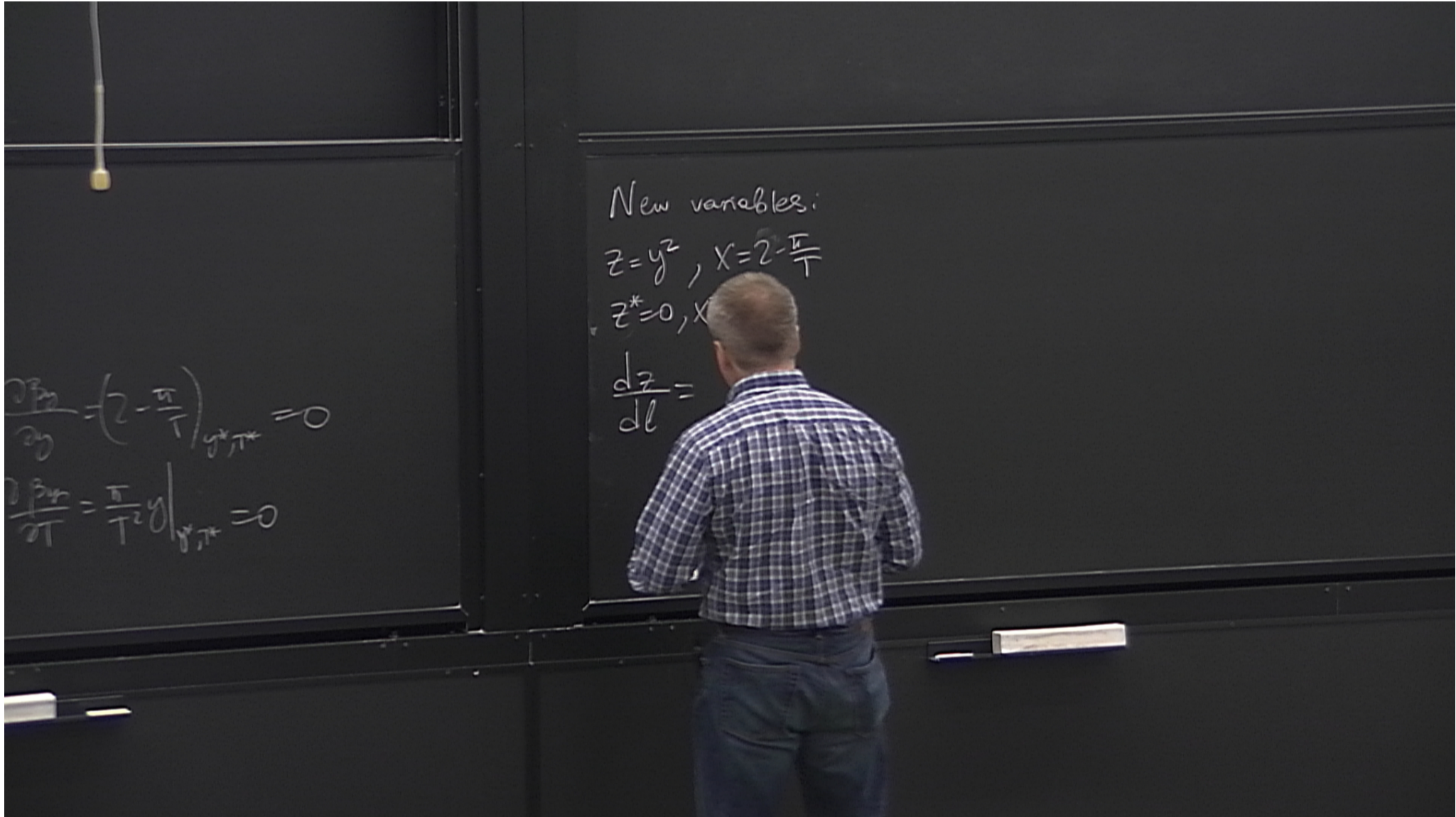
$$\frac{\partial P_T}{\partial y} = \frac{y}{T} \Big|_{y=0, T=H/2} = 0$$

$$\frac{\partial P_y}{\partial T} = \frac{T}{T^2} y \Big|_{y=0, T=H/2} = 0$$









New variables:

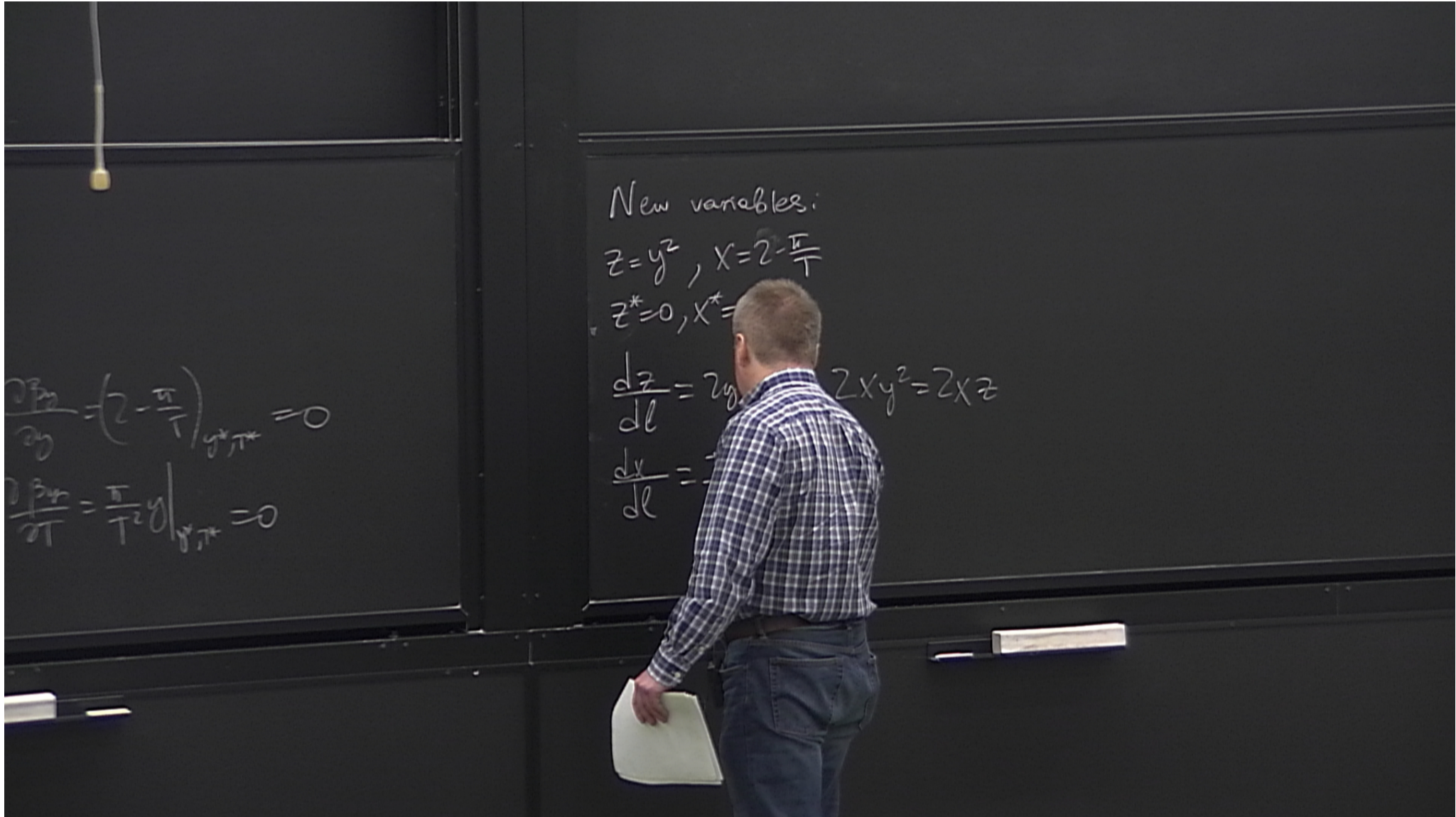
$$z = y^2, \quad X = 2 - \frac{\pi}{T}$$

$$z^* = 0, \quad X^*$$

$$\frac{dz}{dt} =$$

$$\frac{dP}{dy} = \left(2 - \frac{\pi}{T}\right) y^{x, T^*} = 0$$

$$\frac{dP}{dT} = \frac{\pi}{T^2} y \Big|_{y^*, T^*} = 0$$



$$\frac{\partial \beta_2}{\partial y} = \left(2 - \frac{\pi}{T}\right) \Big|_{y^*, T^*} = 0$$
$$\frac{\partial \beta_4}{\partial T} = \frac{\pi}{T^2} y \Big|_{y^*, T^*} = 0$$

New variables:
 $z = y^2, x = 2 - \frac{\pi}{T}$
 $z^* = 0, x^* =$

$$\frac{dz}{dt} = 2y \cdot 2xy^2 = 2xz$$
$$\frac{dx}{dt} =$$

$$\frac{\partial F_2}{\partial y} = \left(2 - \frac{\pi}{T}\right) y^2 \Big|_{y^*, T^*} = 0$$
$$\frac{\partial F_2}{\partial T} = \frac{\pi}{T^2} y \Big|_{y^*, T^*} = 0$$

New variables:

$$z = y^2, \quad X = 2 - \frac{\pi}{T}$$

$$z^* = 0, \quad X^* = 0$$

$$\frac{dz}{dt} = 2y \frac{dy}{dt} = 2Xy^2 = 2Xz$$

$$\frac{dX}{dt} = \frac{\pi}{T^2} \frac{dT}{dt} = \frac{\pi}{T^2} \frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$$

New variables:

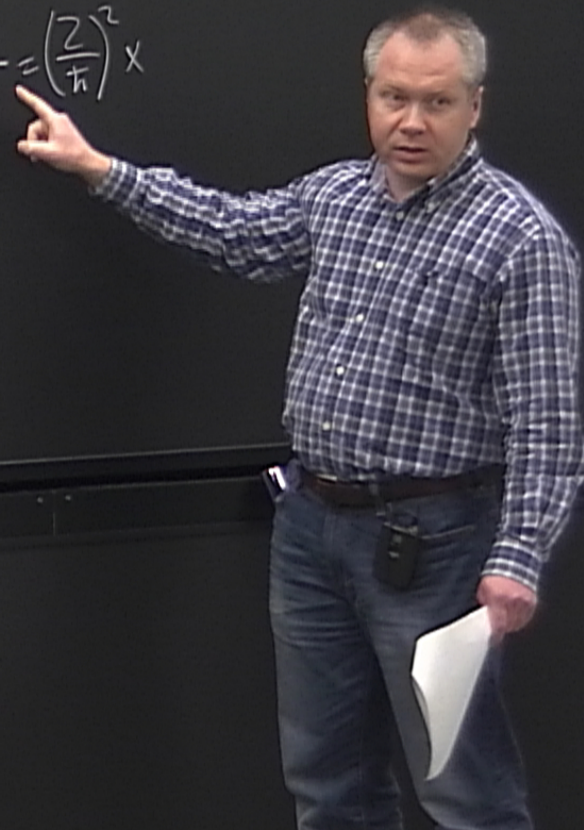
$$z = y^2, X = 2 \frac{\pi}{T}$$

$$z^* = 0, X^* = 0$$

$$\frac{dz}{dl} = 2y \frac{dy}{dl} = 2Xy^2 = 2Xz$$

$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} = \frac{\pi}{T^2} \frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{z}\right)^3} = \frac{z}{\left(\frac{\pi}{z}\right)^2}$$

$$\begin{cases} \frac{dz}{dl} = 2Xz \\ \frac{dx}{dl} = \left(\frac{z}{\pi}\right)^2 X \end{cases}$$



New variables:

$$z = y^2, \quad x = 2 \frac{\pi}{T}$$

$$z^* = 0, \quad x^* = 0$$

$$\frac{dz}{dl} = 2y \frac{dy}{dl} = 2xy^2 = 2xz$$

$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} = \frac{\pi}{T^2} \frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$$

$$\begin{cases} \frac{dz}{dl} = 2xz \\ \frac{dx}{dl} = \left(\frac{2}{\pi}\right)^2 x \end{cases}$$
$$\frac{dz}{dx} =$$

New variables:

$$z = y^2, X = 2 \frac{\pi}{T}$$

$$z^* = 0, X^* = 0$$

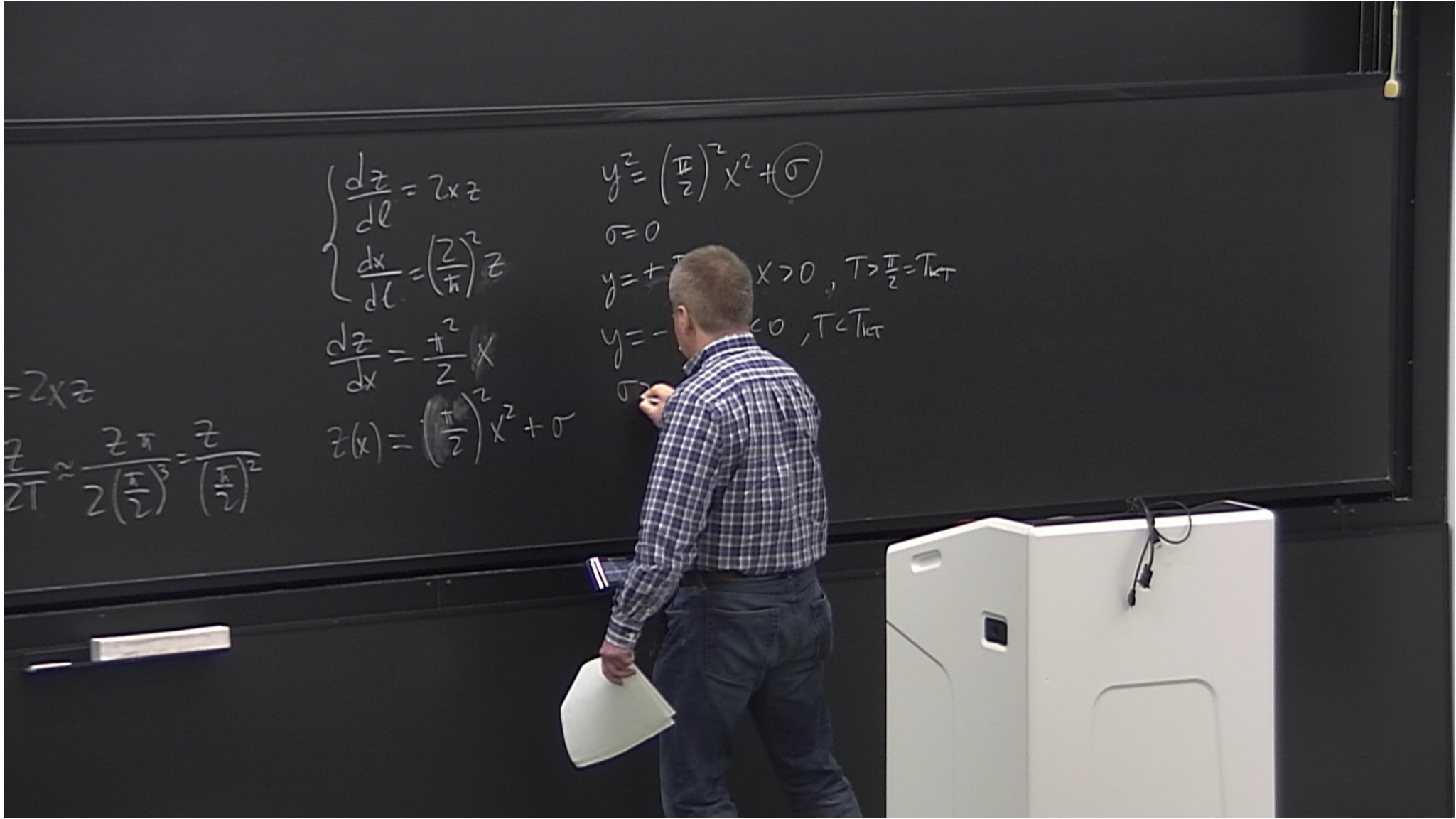
$$\frac{dz}{dl} = 2y \frac{dy}{dl} = 2Xy^2 = 2Xz$$

$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} = \frac{\pi}{T^2} \frac{z}{2T} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$$

$$\begin{cases} \frac{dz}{dl} = 2Xz \\ \frac{dx}{dl} = \left(\frac{2}{\pi}\right)^2 z \end{cases}$$

$$\frac{dz}{dx} = \frac{\pi^2}{2} X$$

$$z(x) = \left(\frac{\pi}{2}\right)^2 X^2 + \sigma$$



$$\begin{cases} \frac{dz}{dl} = z \times z \\ \frac{dx}{dl} = \left(\frac{z}{h}\right)^2 z \end{cases}$$

$$\frac{dz}{dx} = \frac{\pi^2}{z} x$$

$$z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma$$

$$y^2 = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma$$

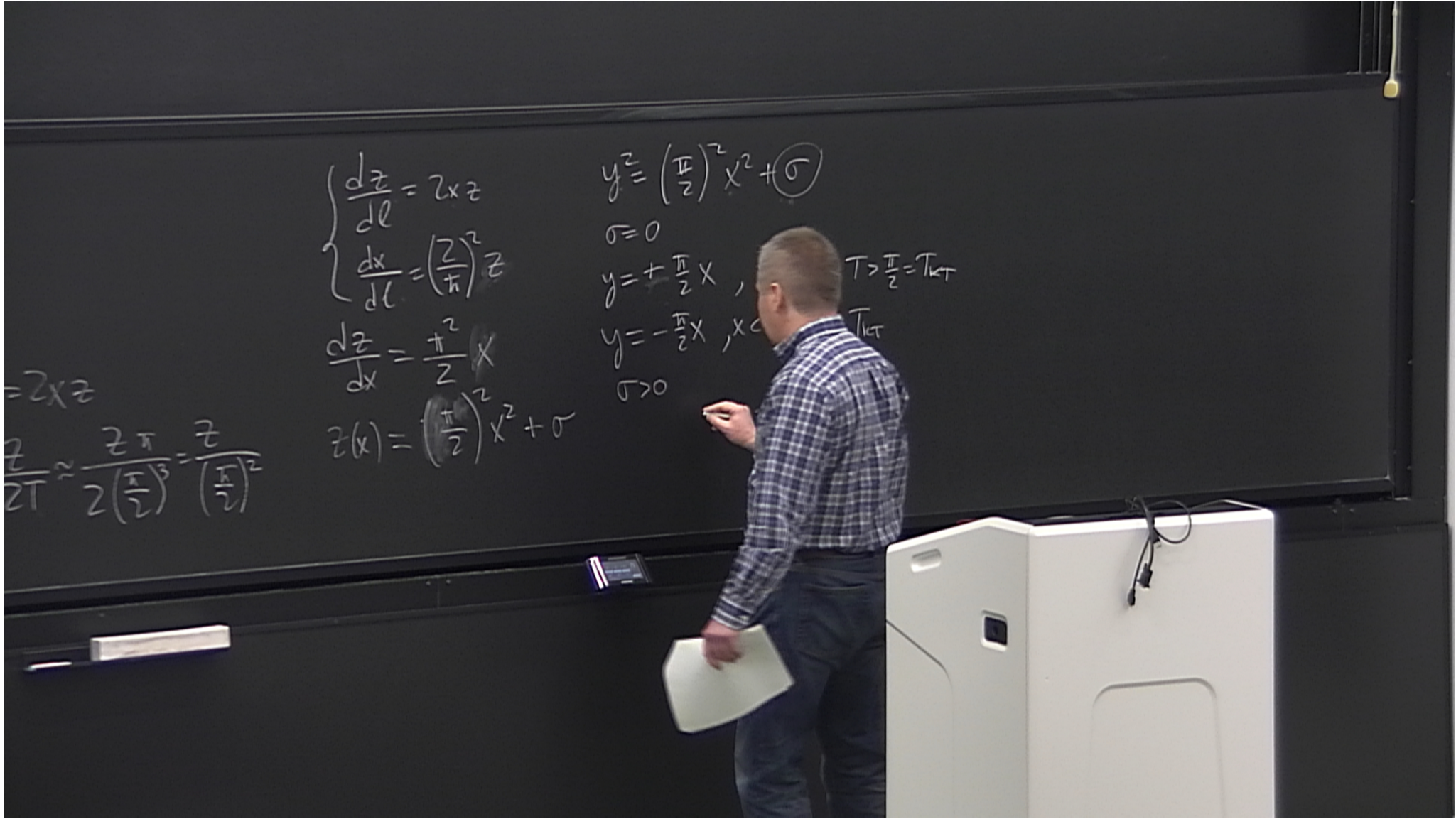
$$\sigma = 0$$

$$y = +T \quad x > 0, T > \frac{\pi}{2} = T_{\text{crit}}$$

$$y = -T \quad x < 0, T < T_{\text{crit}}$$

$$= z \times z$$

$$\frac{z}{2T} \approx \frac{z \pi}{2 \left(\frac{\pi}{2}\right)^3} = \frac{z}{\left(\frac{\pi}{2}\right)^2}$$



$$\begin{cases} \frac{dz}{dl} = z \times z \\ \frac{dx}{dl} = \left(\frac{z}{h}\right)^2 z \end{cases}$$

$$\frac{dz}{dx} = \frac{z^2}{z} X$$

$$z(x) = \left(\frac{h}{2}\right)^2 X^2 + \sigma$$

$$y^2 = \left(\frac{h}{2}\right)^2 X^2 + \sigma$$

$$\sigma = 0$$

$$y = +\frac{h}{2} X, X > 0, T > \frac{h}{2} = T_{\text{cut}}$$

$$y = -\frac{h}{2} X, X < 0, T < T_{\text{cut}}$$

$$\sigma > 0 - \text{above one asymptote}$$

$$\sigma < 0, X = \pm \frac{2}{h} \sqrt{|\sigma|}$$

$$\frac{z}{2T} \approx \frac{z \pi}{2 \left(\frac{h}{2}\right)^3} = \frac{z}{\left(\frac{h}{2}\right)^2}$$



$$dL = T^2 dL + T^2 ZT + 2\left(\frac{T}{Z}\right) \left(\frac{T}{Z}\right)^2$$

$$\sigma = \frac{T - T_{kr}}{T_{kr}}$$

Assum

$$\left. \frac{-\pi}{T} \right|_{y^*, T^*} = 0$$
$$\left. F_2 y \right|_{y^*, T^*} = 0$$

$$dl = \frac{1}{T^2} dl = T^2 zT \cdot z \left(\frac{1}{z}\right)^2 \left(\frac{1}{z}\right)^2$$

$$\sigma \sim \frac{T - T_{KT}}{T_{KT}}$$

Assume $T > T_{KT}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}$$

$$\left. \frac{d}{dl} \left(\frac{1}{T} \right) \right|_{y, T^*} = 0$$
$$\left. \frac{d}{dl} \left(\frac{1}{T} \right) \right|_{y, T^*} = 0$$



$$dl = T^2 \frac{dT}{T^3} = T^2 \cdot 2T^{-3} dT = 2 \left(\frac{T}{T}\right)^2 \left(\frac{dT}{T}\right)$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}} \quad dl$$

Assume $T > T_{cr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}$$

$$\left. \frac{dz}{dl} \right|_{z, T_{cr}} = 0$$

$$\left. \frac{dz}{dl} \right|_{z, T_{cr}} = 0$$

$$dl = \frac{T^2}{T^2} dl = T^2 z T^2 z \left(\frac{\pi}{z}\right)^2 \left(\frac{\pi}{z}\right)^2$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}$$

$$dl = \frac{\pi}{4} \frac{dz}{\sqrt{z - \sigma}}$$

$$\int$$

$$\left. \frac{-\pi}{T} \right|_{y, T} = 0$$

$$\left. \frac{z}{\pi} \right|_{y, T} = 0$$

$$dl = \frac{T^2}{T} dl = T^2 ZT \quad z\left(\frac{\pi}{2}\right) \quad \left(\frac{\pi}{2}\right)^2$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}$$

$$dl = \frac{\pi}{4} \frac{dz}{\sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)}$$

$$\left. \frac{-\pi}{T} \right|_{y, T^*} = 0$$

$$\left. \frac{z}{\pi} \right|_{y, T^*} = 0$$

$$dl = \frac{T^2}{T^2} dl = T^2 zT \quad z\left(\frac{\pi}{2}\right) \quad \left(\frac{\pi}{2}\right)^2$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\left. \frac{-\pi}{T} \right|_{y, T^*} = 0$$

$$\left. \frac{z}{y} \right|_{y, T^*} = 0$$

$$dl = \frac{T^2}{T^2} dl = T^2 zT \quad z\left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right)^2$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

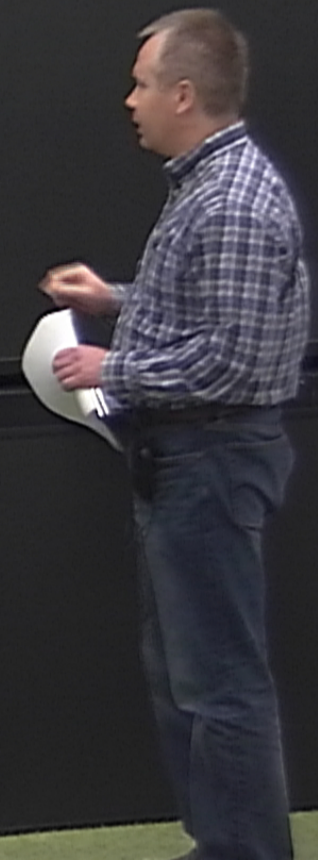
$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}}$$

$$z(l) = 1$$

$$\left. \frac{dz}{dl} \right|_{y, T^*} = 0$$

$$\left. z \right|_{y, T^*} = 0$$



$$dl = \frac{T^2}{T} dl = T^2 zT \quad z\left(\frac{\pi}{2}\right) \left(\frac{\pi}{2}\right)^2$$

$$\sigma \sim \frac{T - T_{kr}}{T_{kr}}$$

Assume $T > T_{kr}$, $X > 0$

$$X = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

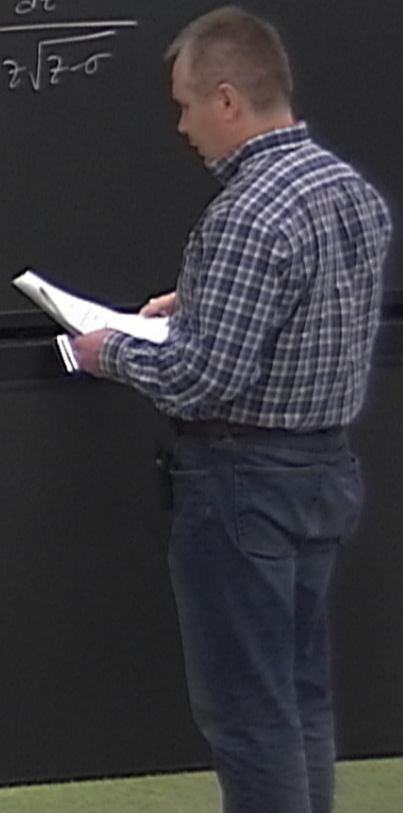
$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

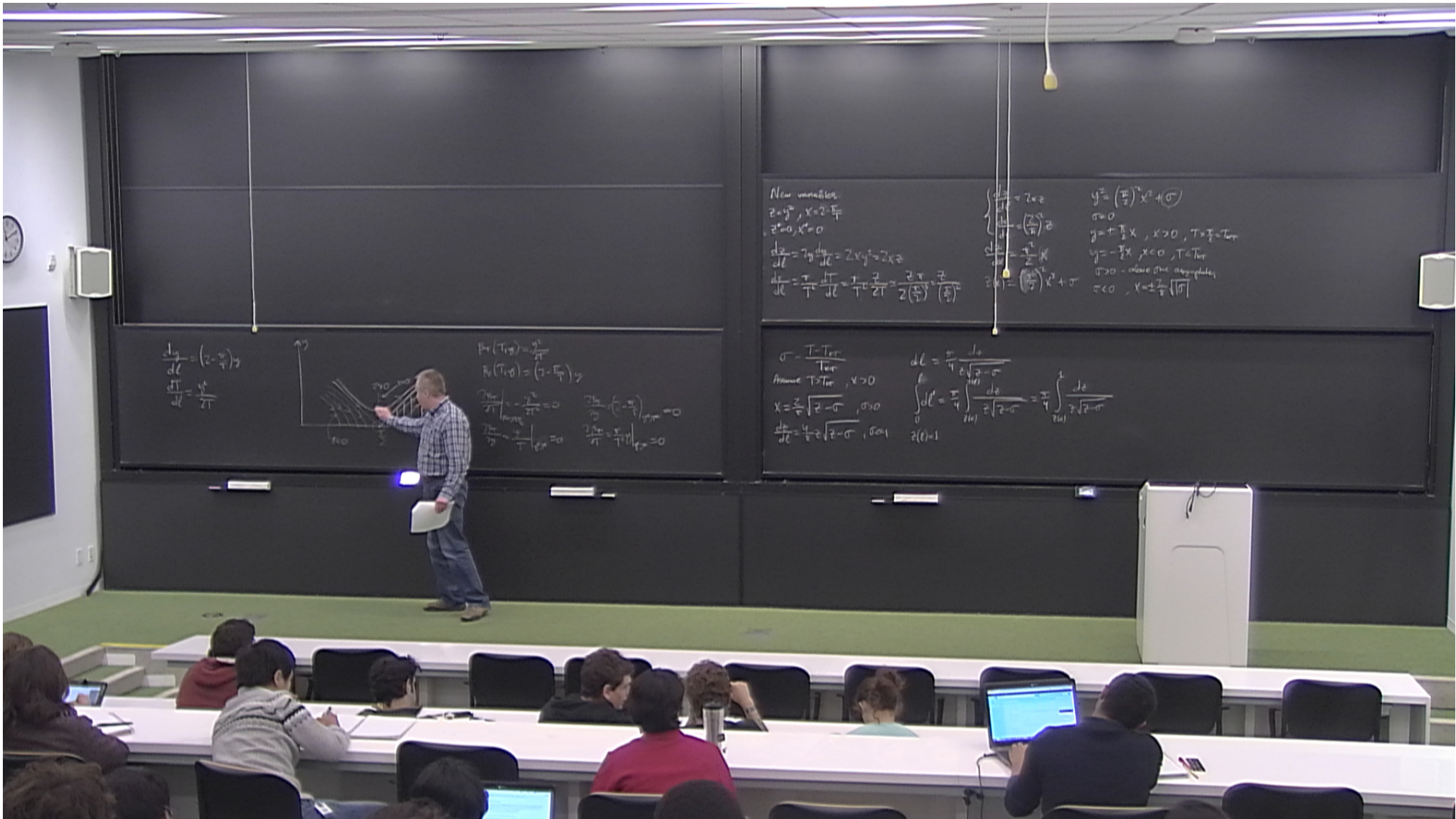
$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{z(0)}^1 \frac{dz}{z \sqrt{z - \sigma}}$$

$$z(l) = 1$$

$$\left. \frac{dz}{dl} \right|_{y, T^*} = 0$$

$$\left. \frac{dz}{dl} \right|_{y, T^*} = 0$$





$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} + \frac{\pi}{T^2} \frac{z}{zT} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^2} = \frac{z}{\left(\frac{\pi}{2}\right)^2} \quad z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma \quad \sigma < 0, \quad x = \pm \frac{z}{\pi} \sqrt{|\sigma|}$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $x > 0$

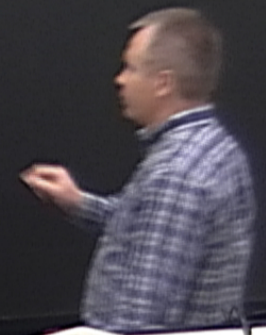
$$x = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{z(0)}^1 \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{\sigma}^1 \frac{dz}{z^2}$$

$z(l) = 1, \quad x(0) = 0, \quad z(0) = \sigma$



$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} + \frac{\pi}{T^2} \frac{z}{zT} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^2} = \frac{z}{\left(\frac{\pi}{2}\right)^2} \quad z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma \quad \sigma < 0, \quad x = \pm \frac{z}{\pi} \sqrt{|\sigma|}$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $x > 0$

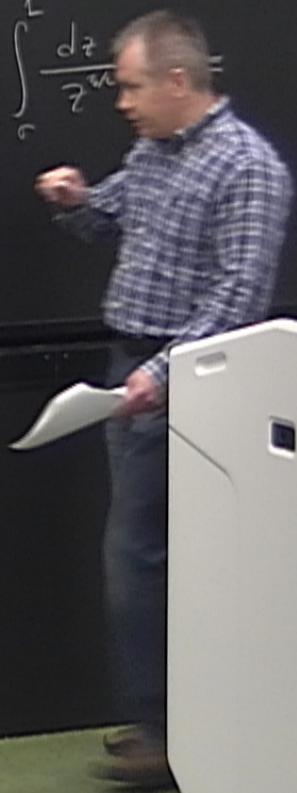
$$x = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{z(0)}^1 \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{\sigma}^1 \frac{dz}{z^2}$$

$$z(l) = 1, \quad x(0) = 0, \quad z(0) = \sigma$$



$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} + \frac{\pi}{T^2} \frac{z}{zT} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^2} = \frac{z}{\left(\frac{\pi}{2}\right)^2} \quad z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma \quad \sigma < 0, \quad x = \pm \frac{z}{\pi} \sqrt{|\sigma|}$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $x > 0$

$$x = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

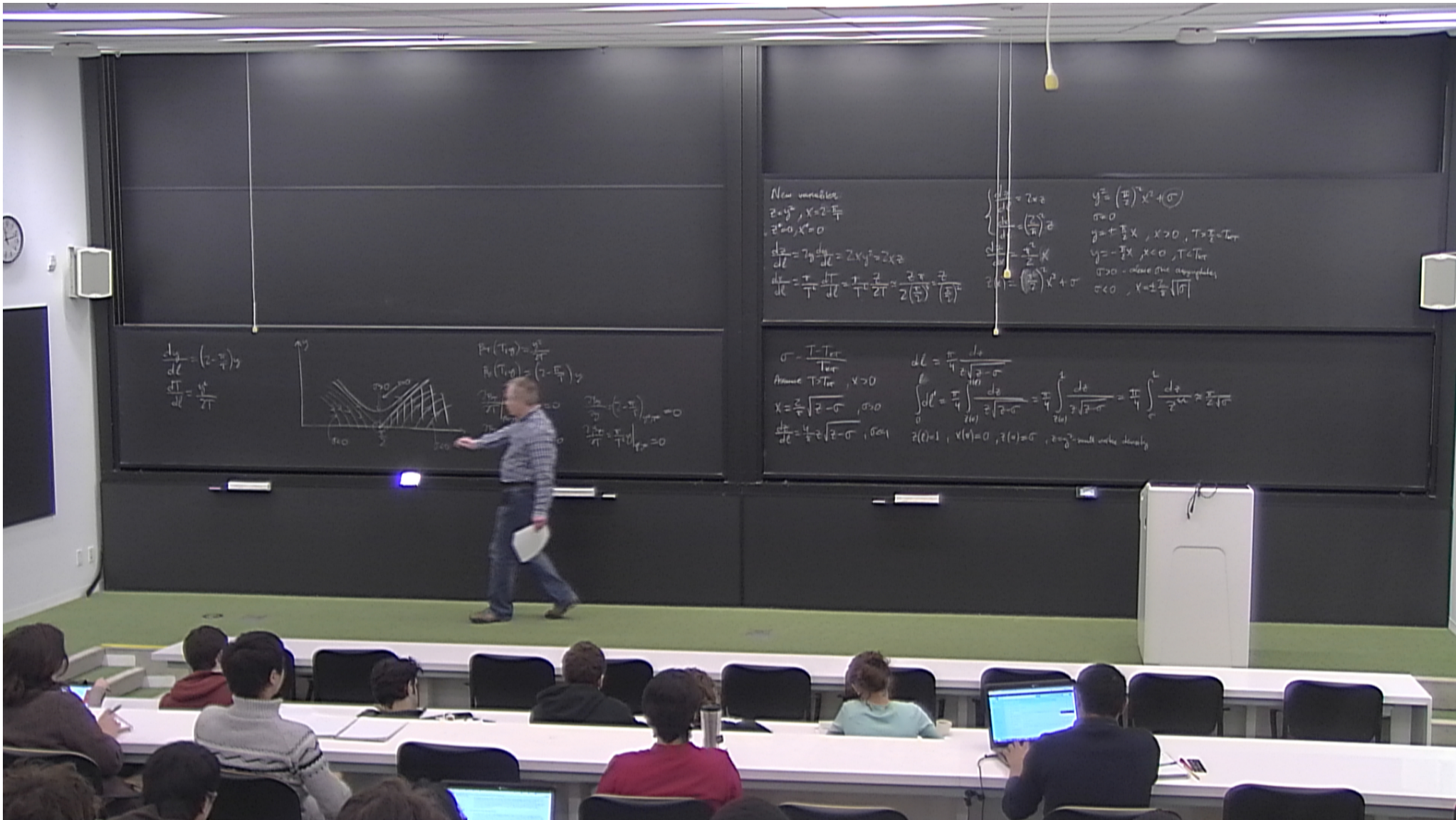
$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{z(0)}^1 \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{\sigma}^1 \frac{dz}{z \sqrt{z - \sigma}} \approx \frac{\pi}{2\sqrt{\sigma}}$$

$z(l) = 1, \quad x(0) = 0, \quad z(0) = \sigma, \quad z = \dots$





$$\frac{dx}{dl} = \frac{\pi}{T^2} \frac{dT}{dl} + \frac{\pi}{T^2} \frac{z}{zT} \approx \frac{z\pi}{2\left(\frac{\pi}{2}\right)^2} = \frac{z}{\left(\frac{\pi}{2}\right)^2} \quad z(x) = \left(\frac{\pi}{2}\right)^2 x^2 + \sigma \quad \sigma < 0, \quad x = \pm \frac{z}{\pi} \sqrt{|\sigma|}$$

$$\sigma \sim \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}$, $x > 0$

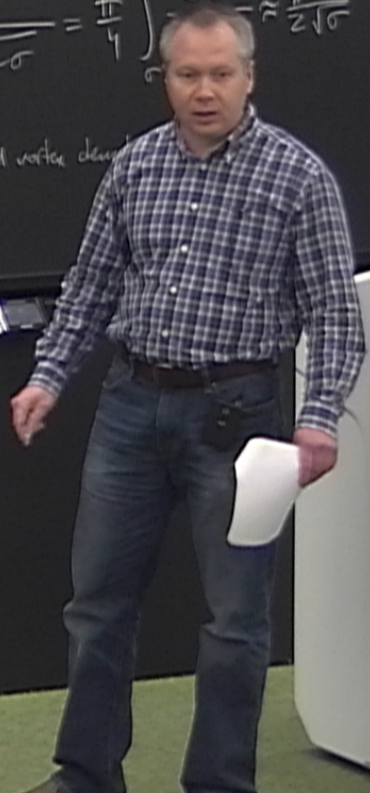
$$x = \frac{z}{\pi} \sqrt{z - \sigma}, \quad \sigma > 0$$

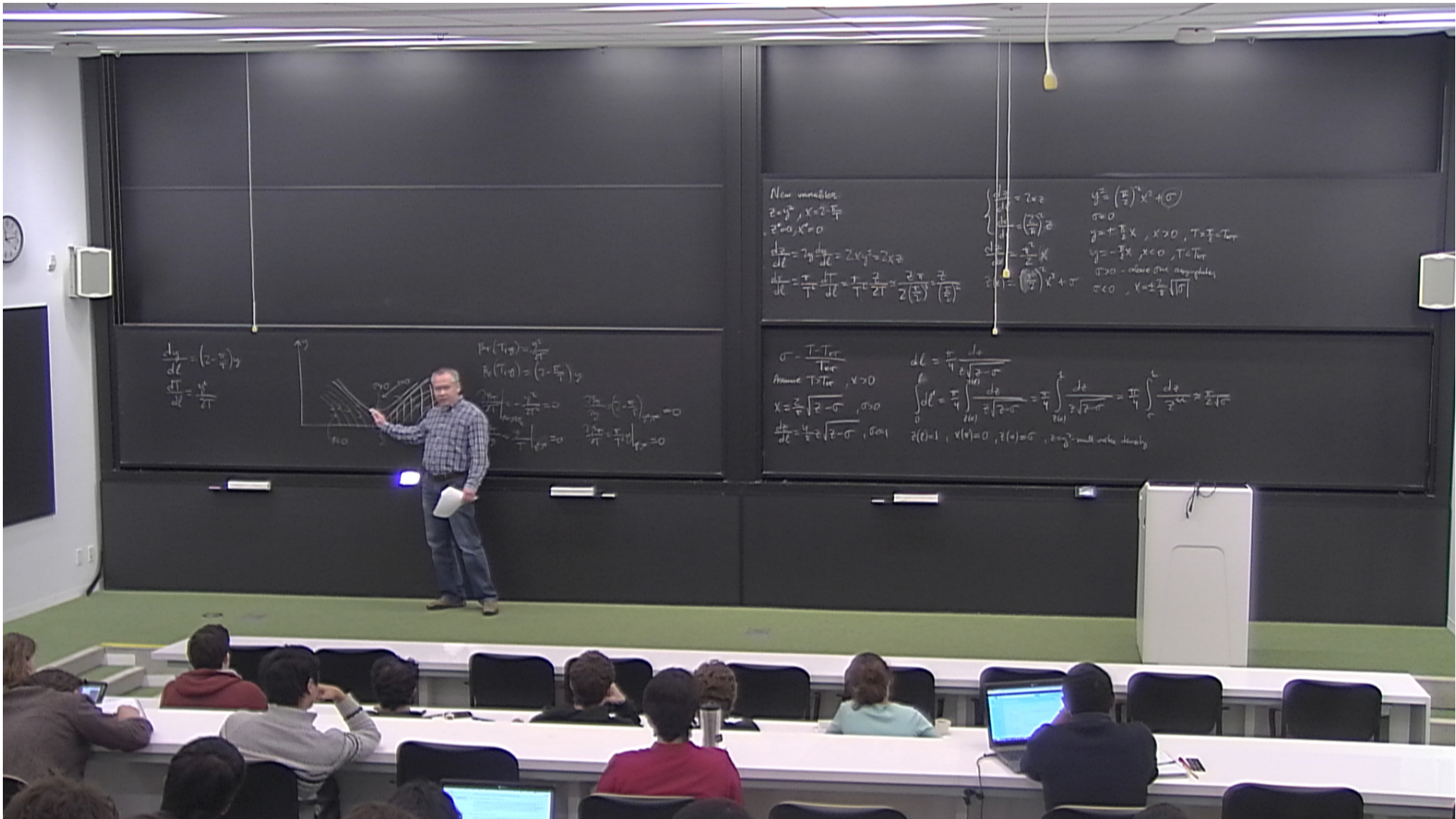
$$\frac{dz}{dl} = \frac{4}{\pi} z \sqrt{z - \sigma}, \quad \sigma < 1$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{z - \sigma}}$$

$$\int_0^l dl' = \frac{\pi}{4} \int_{z(0)}^{z(l)} \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{z(0)}^1 \frac{dz}{z \sqrt{z - \sigma}} = \frac{\pi}{4} \int_{\sigma}^1 \frac{dz}{z \sqrt{z - \sigma}} \approx \frac{\pi}{2\sqrt{\sigma}}$$

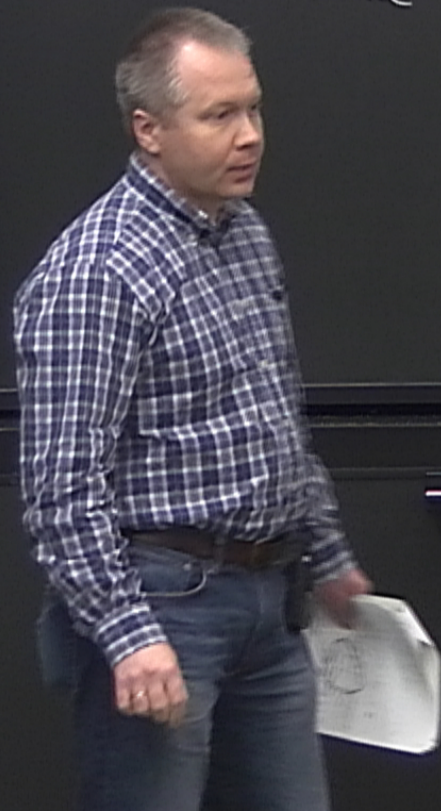
$z(l) = 1, \quad x(0) = 0, \quad z(0) = \sigma; \quad z = y^2 = \text{small vortex density}$

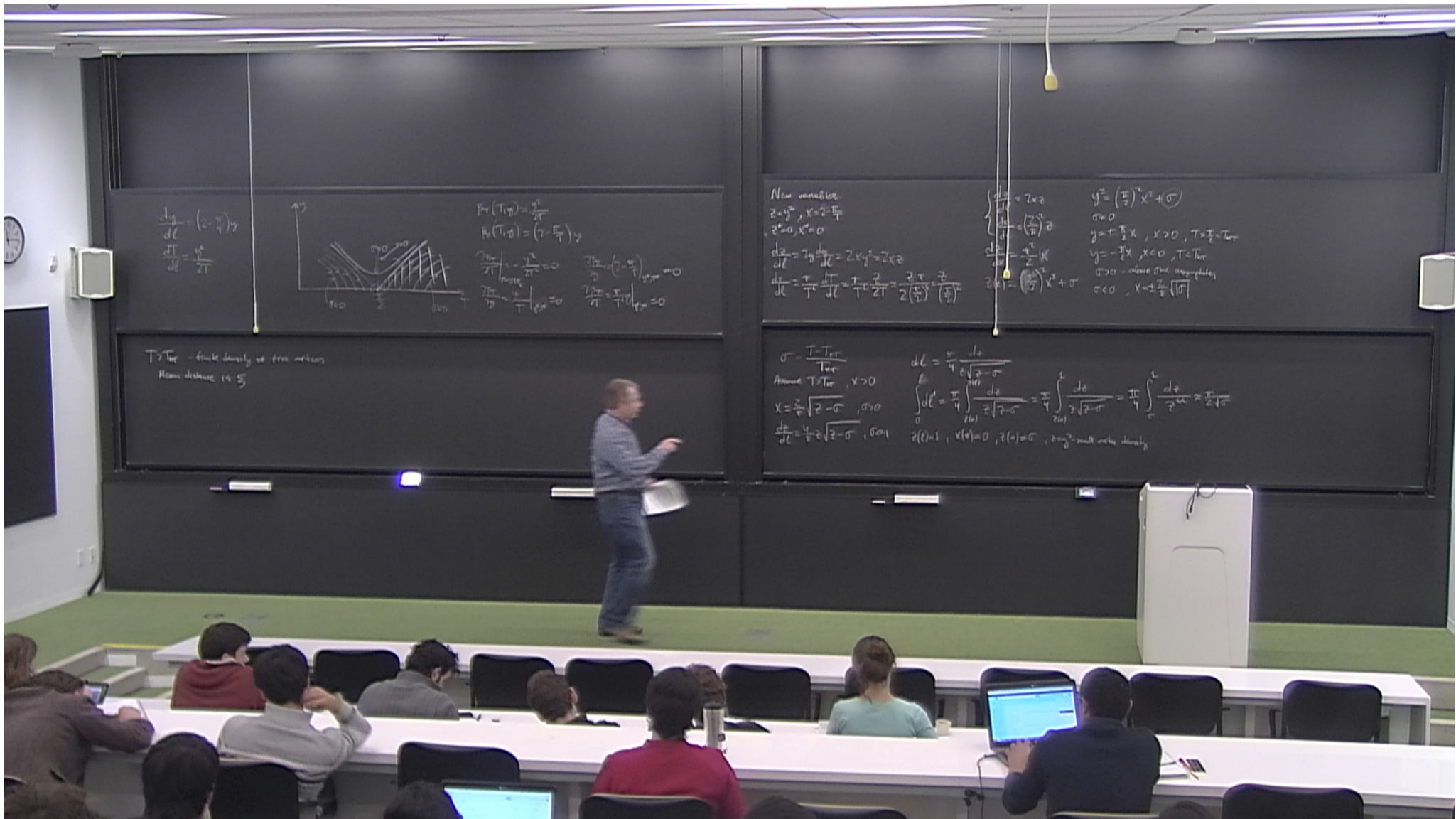




$T > T_{cr}$ - finite density of free vortices

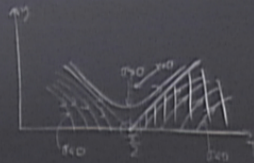
Mean distance is ξ





$$\frac{dy}{dl} = \left(1 - \frac{y}{l}\right)y$$

$$\frac{dT}{dl} = \frac{y}{2l}$$



$$F_l(T, y) = \frac{y^2}{2l}$$

$$F_l(T, 0) = (1 - F_l) y$$

$$\frac{\partial F_l}{\partial l} = -\frac{y^2}{2l^2} = 0 \quad \frac{\partial F_l}{\partial y} = \frac{y}{l} = 0$$

$$\frac{\partial F_l}{\partial y} = \frac{y}{l} = 0 \quad \frac{\partial F_l}{\partial l} = -\frac{y^2}{2l^2} = 0$$

$T > T_{cr}$ - finite density of free particles
 Mean distance is $\frac{1}{y}$

New variables
 $z = y^2, X = z - F_l$
 $z^* = 0, X^* = 0$

$$\frac{dz}{dl} = 2y \frac{dy}{dl} = 2X y^2 = 2X z$$

$$\frac{dX}{dl} = \frac{y}{l} - \frac{dy}{dl} = \frac{y}{l} - \left(1 - \frac{y}{l}\right)y = \frac{y}{l} - y + \frac{y^2}{l} = \frac{y^2 - y(l-y)}{l} = \frac{y^2 - yz}{l}$$

$$\begin{cases} \frac{dz}{dl} = 2X z \\ \frac{dX}{dl} = \frac{y}{l} - \frac{y^2}{l} \end{cases} \quad y^2 = \left(\frac{z}{2}\right)^2 + \sigma$$

$$\sigma = 0 \quad y = \pm \frac{z}{2}, X > 0, T > \frac{z}{2} = T_{cr}$$

$$y = -\frac{z}{2}, X < 0, T < T_{cr}$$

$$\sigma > 0 \text{ - above free asymptote}$$

$$\sigma < 0, X = \pm \frac{z}{2} \sqrt{1 - \frac{\sigma}{z}}$$

$$\sigma = \frac{T - T_{cr}}{T_{cr}}$$

Assume $T > T_{cr}, y > 0$

$$X = \frac{z}{2} \sqrt{1 - \frac{\sigma}{z}}, \sigma > 0$$

$$\frac{dz}{dl} = \frac{y}{l} z \sqrt{1 - \frac{\sigma}{z}}, \sigma < 1$$

$$dl = \frac{\pi}{4} \frac{dz}{z \sqrt{1 - \frac{\sigma}{z}}}$$

$$\int_0^l dl = \frac{\pi}{4} \int_0^l \frac{dz}{z \sqrt{1 - \frac{\sigma}{z}}} = \frac{\pi}{4} \int_r^L \frac{dz}{z \sqrt{1 - \frac{\sigma}{z}}} \approx \frac{\pi}{4} \ln \frac{L}{r}$$

$z(l) = 1, y(l) = 0, z(0) = \sigma, z = \sigma^2$ - small velocity density

$T > T_{cr}$ - finite density of free vortices

mean distance is ξ
 $\xi \sim \frac{1}{\Lambda}$

$T > T_{cr}$ - finite density of free vertices

Mean distance is ξ .

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b=e^l}{\lambda}$

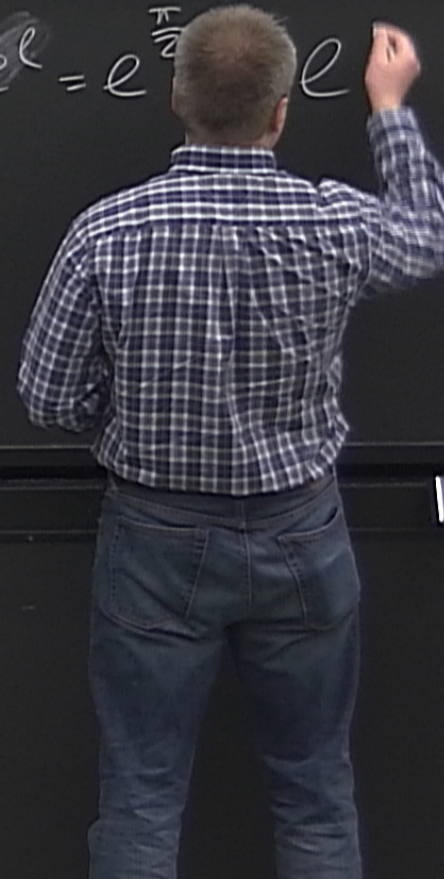
$T > T_{cr}$ - finite density of free vertices

Mean distance is ξ .

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b=e^l}{\lambda}$

$$\xi \sim e^l = e^{\frac{\pi}{2} l}$$



$T > T_{cr}$ - finite density of free vertices

Mean distance is ξ .

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b=e^l}{\lambda}$

$$\xi \sim e^l = e^{\frac{\pi}{2\sqrt{g}}}$$

$$\sqrt{\quad}$$

$T > T_{KT}$ - finite density of free vertices

Mean distance ξ

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ mean $\xi \sim \frac{b=e^l}{\lambda}$

$$\xi \sim e^l = e^{\frac{\pi}{2\sqrt{\delta}}} \sim e^C \sqrt{\frac{T_{KT}}{T - T_{KT}}}$$

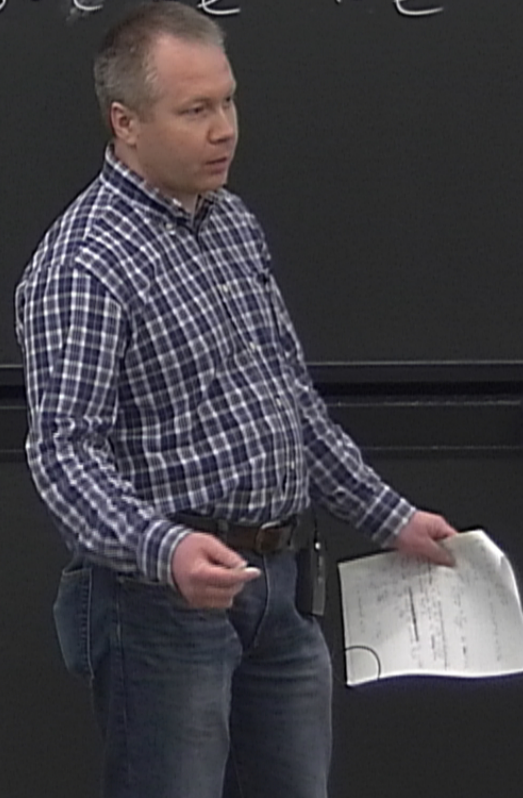
$T > T_{cr}$ - finite density of free vertices

Mean distance is ξ .

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b=e^l}{\lambda}$

$$\xi \sim e^l - e^{\frac{\pi}{2\sqrt{\delta}}} \sim e^{C \sqrt{\frac{T_{cr}}{T - T_{cr}}}}$$



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial T} \left| \frac{\partial}{\partial T} = 0 \right. \quad \frac{\partial}{\partial T} \quad T < 0$$

$T > T_{cr}$ - finite density of free vertices

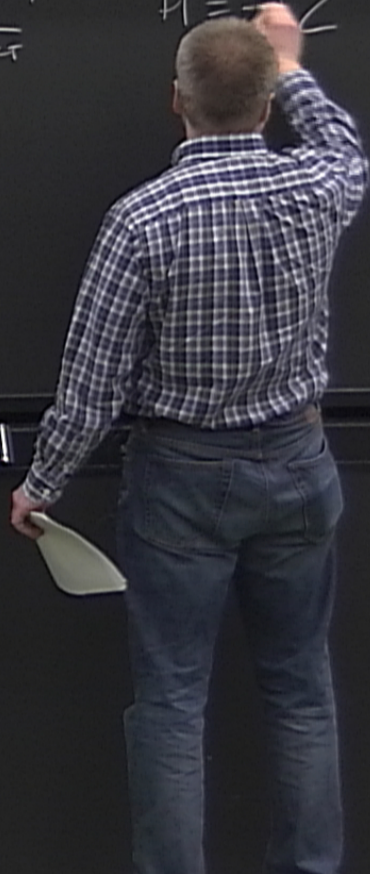
Mean distance is ξ .

$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b = e^l}{\lambda}$

$$\xi \sim e^l = e^{\frac{\pi}{2\sqrt{\sigma}} l} \sim e^{C \sqrt{\frac{T_{cr}}{T - T_{cr}}}}$$

$$H = -\frac{1}{\xi}$$



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad T \quad \frac{1}{2\sigma} = \frac{1}{T} \quad \frac{\partial}{\partial T} = 0 \quad \frac{\partial}{\partial T} \quad T < 0$$

$T > T_{KT}$ - finite density of free vertices

Mean distance is ξ .

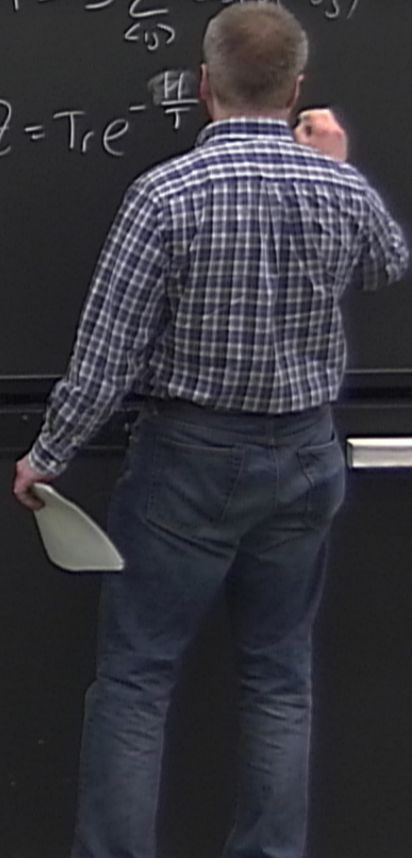
$$\xi \gg \frac{1}{\lambda}$$

$z=1$ means that $\xi \sim \frac{b=e^l}{\lambda}$

$$\xi \sim e^l = e^{\frac{\pi}{2\sqrt{\sigma}} l} \sim e^{C \sqrt{\frac{T_{KT}}{T-T_{KT}}}}$$

$$H \Rightarrow \sum_{\phi_j} \cos(\theta_i - \theta_j)$$

$$Z = \text{Tr} e^{-\frac{H}{T}}$$



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad \frac{1}{T} = \frac{1}{T} \Big|_{\theta=0} = 0 \quad \frac{1}{\partial T} - \frac{1}{T^2} \Big|_{\theta=0} = 0$$

density of free fermions

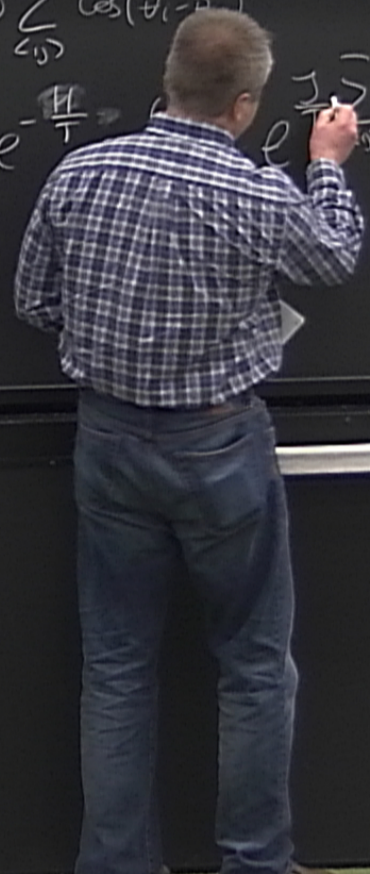
$$\sim \xi.$$

$$\xi \sim e^{\ell} = e^{\frac{\pi}{2\sqrt{v}} \ell} \sim e^{C \sqrt{\frac{T_{\text{eff}}}{T - T_{\text{eff}}}}}$$

$$\xi \sim \frac{b \cdot e^{\ell}}{\Lambda}$$

$$H \Rightarrow \sum_{i,j} \cos(\theta_i - \theta_j)$$

$$Z = \text{Tr} e^{-\frac{H}{T}} \Rightarrow e^{-\frac{1}{T} \sum_{i,j} \cos(\theta_i - \theta_j)}$$



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad \frac{\partial \psi}{\partial \theta} = \frac{\partial \psi}{\partial \theta} \Big|_{\theta=0} = 0 \quad \frac{\partial \psi}{\partial \theta} = \frac{\partial \psi}{\partial \theta} \Big|_{\theta=\pi} = 0$$

density of free vertices

$$\sim \xi.$$

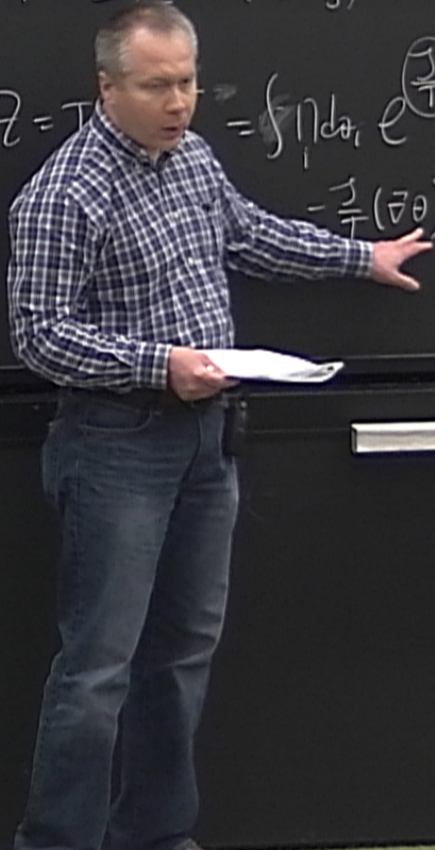
$$\xi \sim e^{\ell} = e^{\frac{\pi}{2\sqrt{\sigma}} \ell} \sim e^{C \sqrt{\frac{T_{\text{eff}}}{T - T_{\text{eff}}}}}$$

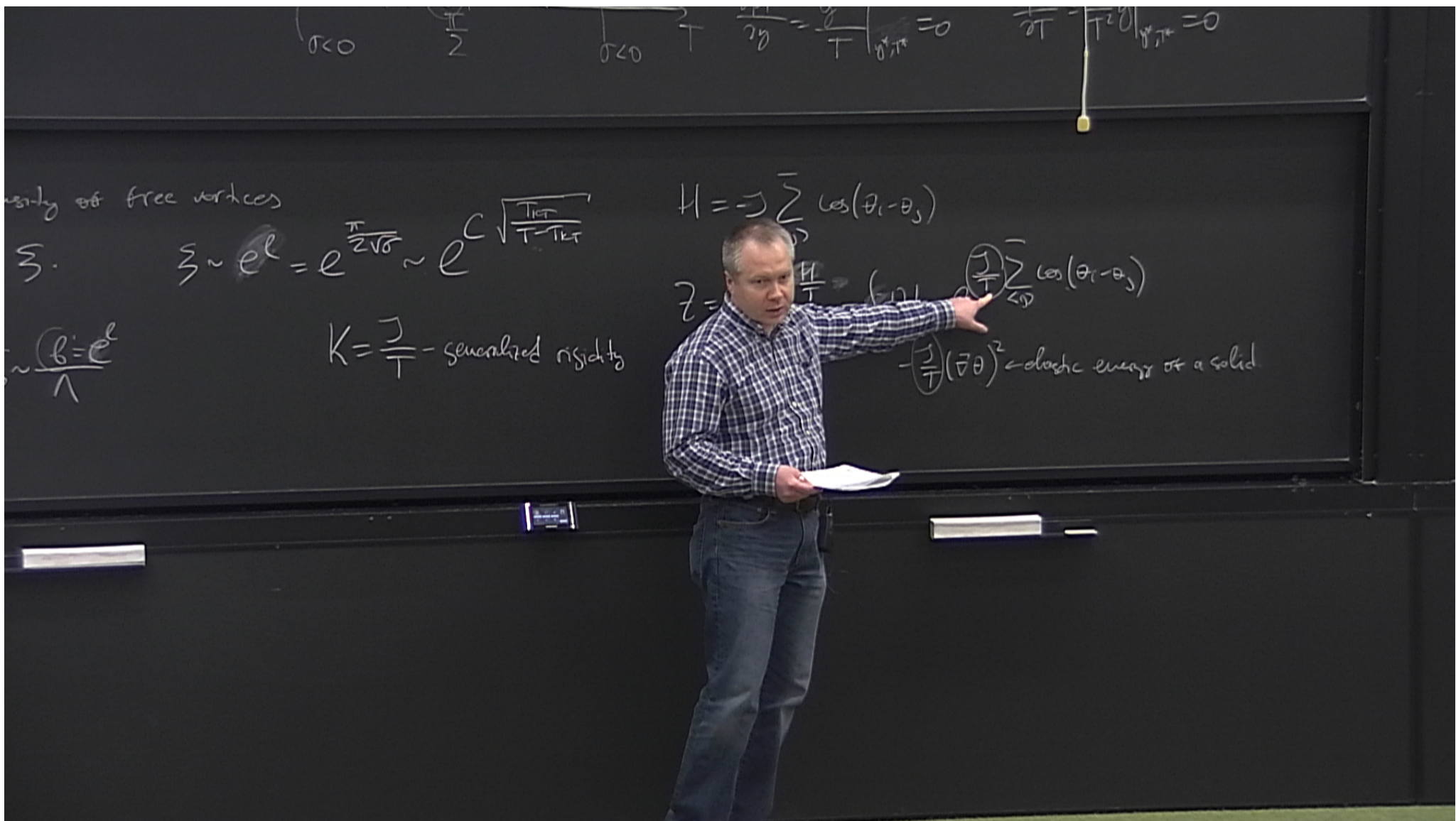
$$\xi \sim \frac{b = e^{\ell}}{\lambda}$$

$$K = \frac{J}{T} - \text{generalized rigidity}$$

$$H = \sum \cos(\theta_i - \theta_j)$$

$$Z = T^N \int \prod d\theta_i e^{\left(\frac{J}{T}\right) \sum \cos(\theta_i - \theta_j)} = \frac{J}{T} (\overline{\cos \theta})^2$$





rigidity of free vortices

$$\xi \sim e^l = e^{\frac{\pi}{2\sqrt{\sigma}}} \sim e^{C\sqrt{\frac{T_{KT}}{T-T_{KT}}}}$$

$$\sim \frac{b=e^l}{\lambda}$$

$$K = \frac{J}{T} - \text{generalized rigidity}$$

$$H = -J \sum_{ij} \cos(\theta_i - \theta_j)$$

$$Z = e^{\frac{H}{T}} = e^{-\frac{J}{T} \sum_{ij} \cos(\theta_i - \theta_j)}$$

$$-\frac{J}{T} (\delta\theta)^2 \leftarrow \text{elastic energy of a solid}$$

rigidity of free vortices

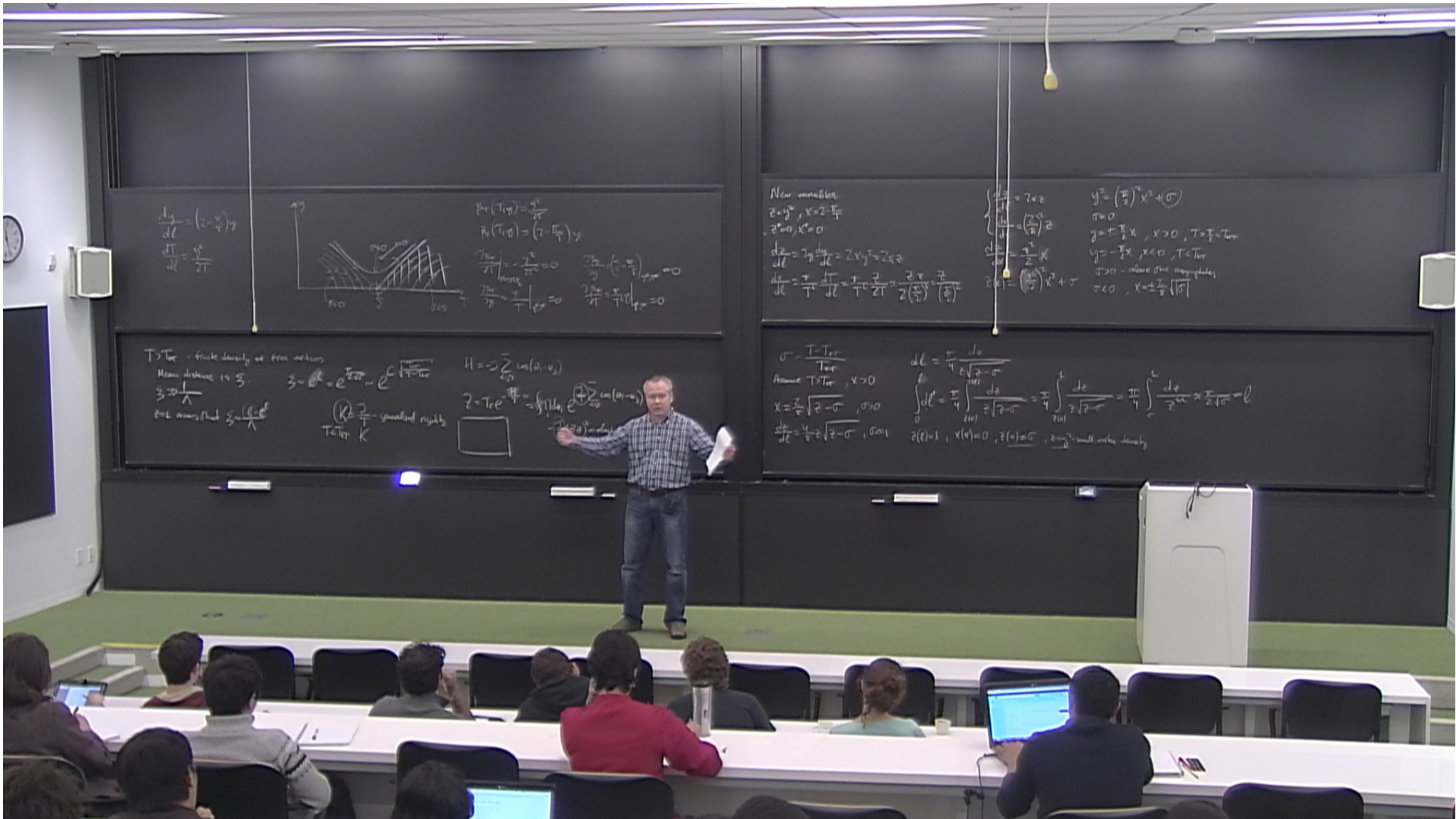
$$\xi \sim e^{\ell} = e^{\frac{\pi}{2\sqrt{\sigma}} \ell} \sim e^{C \sqrt{\frac{T_{eff}}{T - T_{KT}}}}$$

$$K = \frac{J}{T} - \text{generalized rigidity}$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

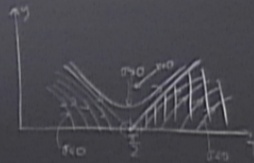
$$Z = \text{Tr} e^{-\frac{H}{T}} = \int \prod_i d\theta_i e^{-\frac{J}{T} \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)}$$

$$-\frac{J}{T} (\nabla \theta)^2 \leftarrow \text{elastic energy of a solid}$$



$$\frac{dy}{dt} = (z - \frac{y}{T})y$$

$$\frac{dT}{dt} = \frac{y}{2T}$$



$$P = (T - \frac{y}{2})^2$$

$$P(T, 0) = (T - \frac{y}{2})^2$$

$$\frac{\partial P}{\partial T} = 2(T - \frac{y}{2}) = 0 \Rightarrow T = \frac{y}{2}$$

$$\frac{\partial P}{\partial y} = -\frac{y}{T} = 0 \Rightarrow y = 0$$

New variables

$$z = y^2, X = z - \frac{T}{2}$$

$$z^* = 0, X^* = 0$$

$$\frac{dz}{dt} = 2y \frac{dy}{dt} = 2Xy^2 = 2Xz$$

$$\frac{dT}{dt} = \frac{y}{2T} \Rightarrow \frac{dT}{T} = \frac{y}{2T^2} dt = \frac{\sqrt{z}}{2T^2} dt = \frac{\sqrt{z}}{2(\frac{z}{2})^2} dt = \frac{\sqrt{z}}{z^2} dt = \frac{1}{z^{3/2}} dt$$

$$\frac{dz}{dt} = 2Xz$$

$$\frac{dX}{dt} = \frac{d}{dt}(z - \frac{T}{2}) = \frac{dz}{dt} - \frac{dT}{dt} = 2Xz - \frac{1}{z^{3/2}}$$

$$z(t) = (\frac{T}{2})^2 X^2 + \sigma$$

$$\sigma = 0$$

$$y = \pm \sqrt{z}, X > 0, T > \frac{T}{2} = T_{sep}$$

$$y = -\sqrt{z}, X < 0, T < T_{sep}$$

$$\sigma > 0 \text{ - slow flow asymptote}$$

$$\sigma < 0, X = \pm \frac{\sqrt{\sigma}}{\sqrt{2}}$$

$T \rightarrow T_{sep}$ - finite density of free workers

Mean distance is $\frac{L}{2}$

$$z = e^{-\frac{y}{T}} = e^{-\frac{\sqrt{z}}{T}}$$

$\frac{K}{T} z$ - specialized ability

$$H = \sum_{i=1}^n \ln(\alpha_i - \alpha_i)$$

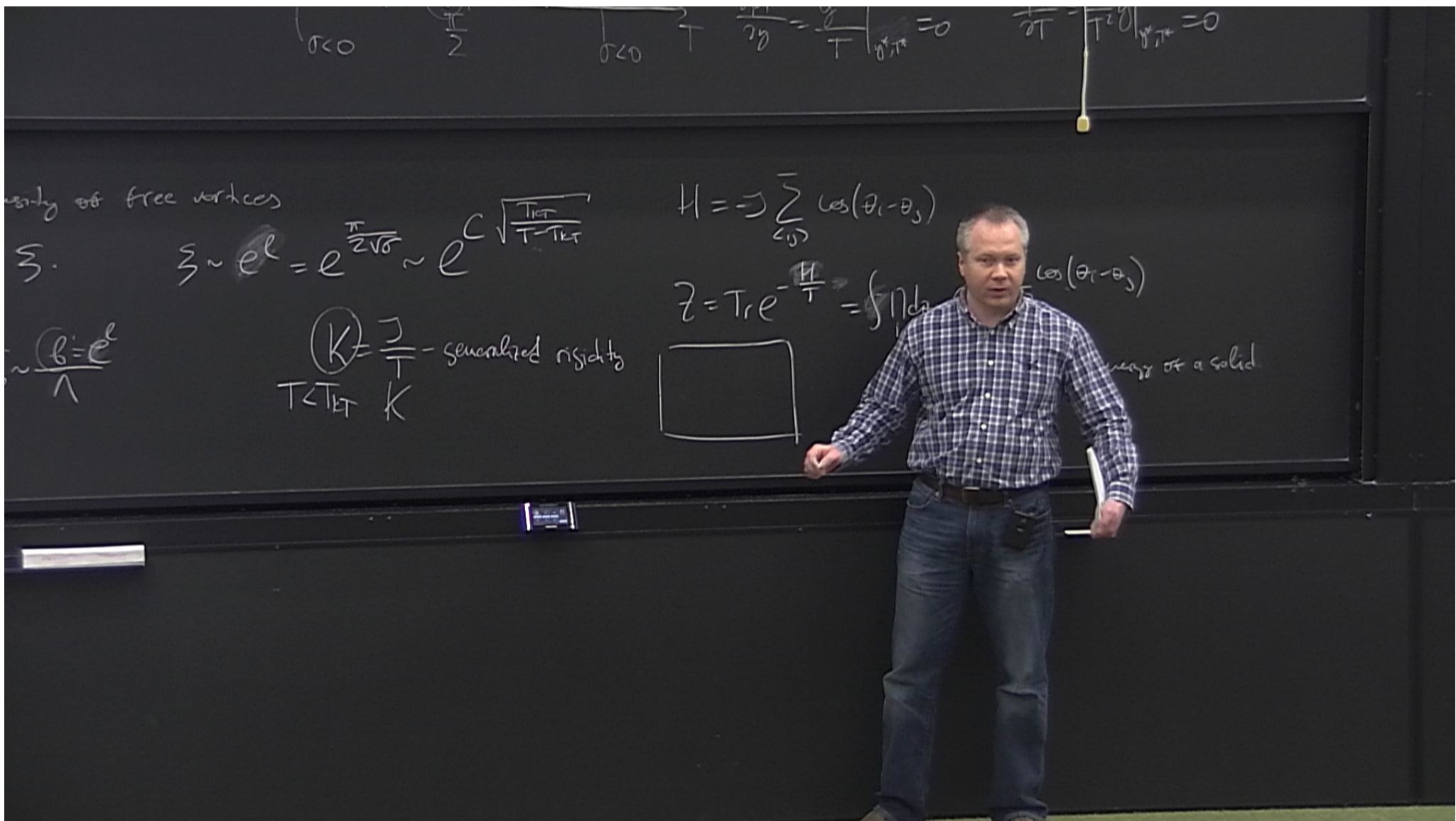
$$Z = T e^{-\frac{y}{T}} = \int_0^L \frac{1}{z} e^{-\frac{\sqrt{z}}{T}} dz$$

$$\sigma = T - T_{sep}$$

$$dL = \frac{dT}{2} \frac{dT}{T} = \frac{dT}{2T}$$

$$\frac{dL}{dT} = \frac{1}{2T}$$

$$L = \frac{1}{2} \ln \frac{T}{T_{sep}}$$



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad T \quad \frac{\partial H}{\partial \theta} = \frac{\partial}{\partial T} \left(\frac{1}{T} \right) = 0 \quad \frac{\partial}{\partial T} \left(\frac{1}{T} \right) \Big|_{\theta, T^*} = 0$$

energy of free vertices

$$\xi \sim e^{\ell} = e^{\frac{\pi}{2\sqrt{\sigma}} \ell} \sim e^{C \sqrt{\frac{T_{KT}}{T - T_{KT}}}}$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

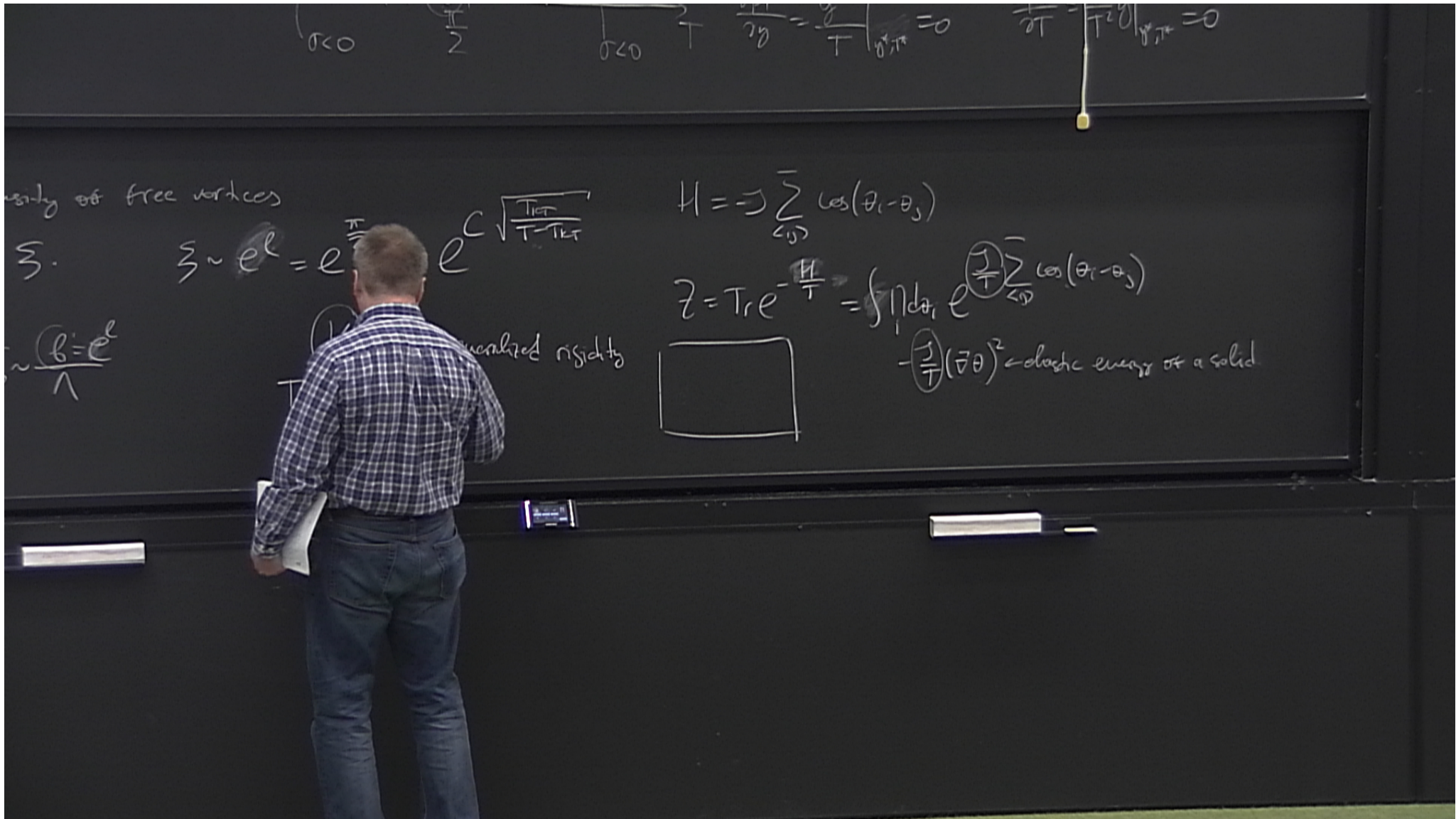
$$Z = \text{Tr} e^{-\frac{H}{T}} = \int \prod d\theta_i e^{-\beta H} = \int \prod d\theta_i e^{\beta J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)}$$

$$\sim \frac{e^{\ell}}{\ell}$$

$$\left(\frac{K}{T} \right) = \frac{J}{T} - \text{generalized rigidity}$$



energy of a solid



$$\sigma < 0 \quad \frac{\pi}{2} \quad \sigma < 0 \quad T \quad \frac{\partial \ln Z}{\partial \theta} = \frac{\partial}{\partial T} \ln Z = 0 \quad \frac{\partial \ln Z}{\partial T} = \frac{\partial}{\partial T} \ln Z = 0$$

rigidity of free vertices

$$\xi \sim e^{\ell} = e^{\frac{\pi}{2} C \sqrt{\frac{T_{eff}}{T - T_{eff}}}}$$

$$\sim \frac{b = e^{\ell}}{\lambda}$$

generalized rigidity

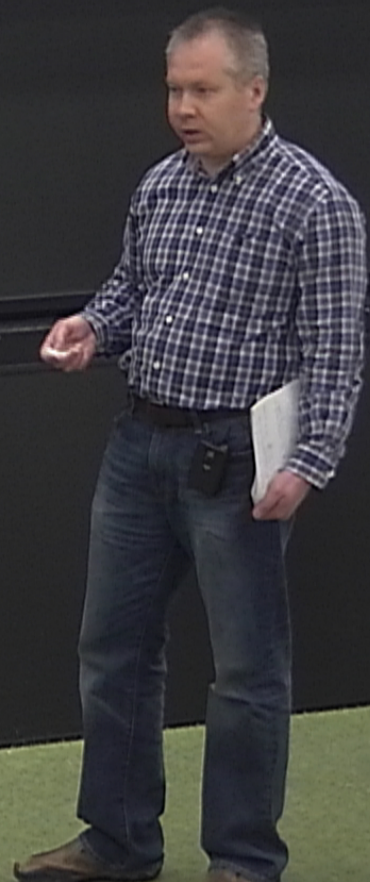
$$H = - \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$Z = \text{Tr} e^{-\frac{H}{T}} = \int \prod d\theta_i e^{-\frac{J}{T} \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)}$$



$-\frac{J}{T} (\nabla \theta)^2 \leftarrow$ elastic energy of a solid

$$\frac{dz}{dt} = \frac{1}{\pi} z \sqrt{z-0}, \quad 0 < z < 1 \quad z(l)=1, \quad X(0)=0, \quad z(0)=0, \quad z=y^2 \text{ - small vortex density}$$



$$d\ell = T^{-1} d\ell = \frac{1}{2} \left(\frac{d\ell}{T} \right) \left(\frac{d\ell}{T} \right)$$

ΔT - small deviation of T from T_0

$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} [K(l)]$$

$$d\ell \quad T = d\ell \quad T = \frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$$

ΔT - small deviation of T from T_{cr}

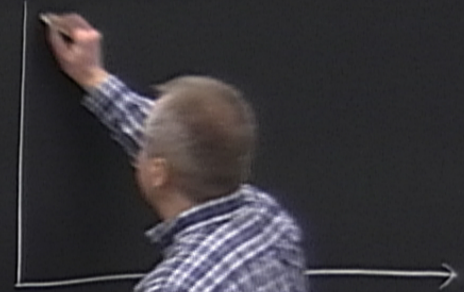
$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} [K(l, T_{cr} - \Delta T) - K(l, T_{cr} + \Delta T)]$$

$dl \quad T = dl \quad T = 2l \quad 2(\frac{l}{2}) \quad (\frac{l}{2})$

ΔT - small deviation of T from T_{cr}

$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} \left[K(l, T_{cr} - \Delta T) - K(l, T_{cr} + \Delta T) \right] = \frac{2}{\pi}$$

\downarrow $\frac{2}{\pi}$ \downarrow 0

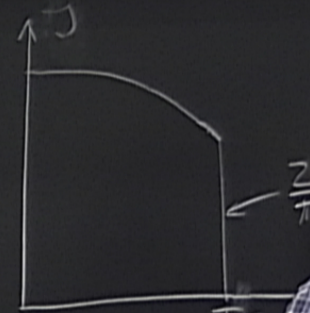


$d\ell \quad T = d\ell \quad T = 2\left(\frac{A}{\ell}\right) \left(\frac{A}{\ell}\right)$

ΔT - small deviation of T from T_{cr}

$$\lim_{\Delta T \rightarrow 0^+} \lim_{\ell \rightarrow \infty} \left[K(\ell, T_{cr} - \Delta T) - K(\ell, T_{cr} + \Delta T) \right] = \frac{2}{\pi}$$

\downarrow $\frac{2}{\pi}$ \downarrow 0

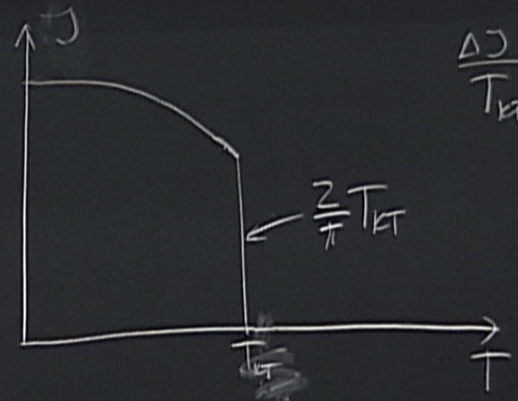


$$\left(\frac{\hbar}{2}\right)^3 \quad \left(\frac{\hbar}{2}\right)^2 \quad \dots \quad \hbar < 0, \quad \lambda = \pm \sqrt{\hbar |0|}$$

from T_{KT}

$$K(l, T_{KT} + \Delta T) - K(l, T_{KT}) = \frac{2}{\pi}$$

↓
0



$$\frac{\Delta J}{T_{KT}} = \frac{2}{\pi} \text{ - universal jump of}$$

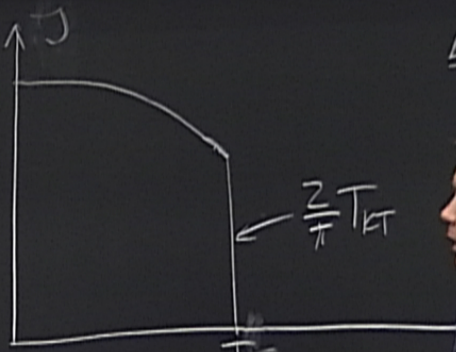


$$\left(\frac{\hbar}{2}\right)^3 \quad \left(\frac{\hbar}{2}\right)^2 \quad \dots \quad \chi = \pm \sqrt{10}$$

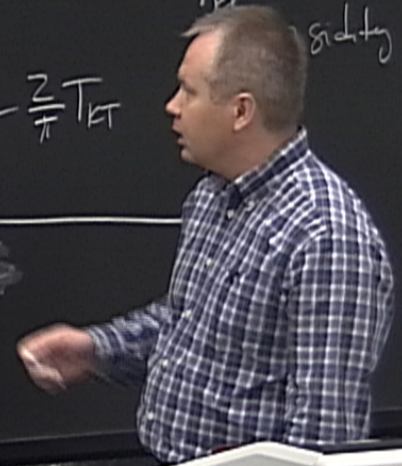
near T_{KT}

$$K(l, T_{KT} + \Delta T) - K(l, T_{KT}) = \frac{2}{\pi}$$

↓
0



$\frac{\Delta J}{T_{KT}} = \frac{2}{\pi}$ - universal jump of rigidity at KT transition



$$\frac{d\lambda}{dl} = \frac{1}{T^2} \frac{dT}{dl} = \frac{1}{T^2} \frac{dT}{dT} \approx \frac{2 \left(\frac{\kappa}{2}\right)^3 - \left(\frac{\kappa}{2}\right)^2}{2 \left(\frac{\kappa}{2}\right)^3 - \left(\frac{\kappa}{2}\right)^2}$$

ΔT - small deviation of T from T_{KT}

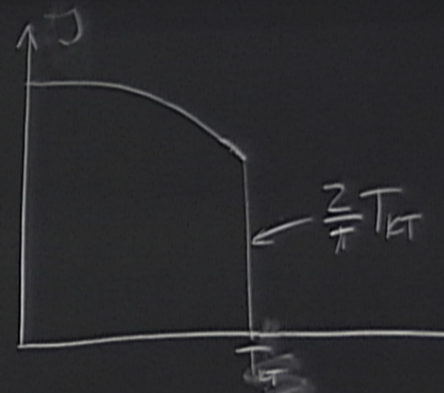
$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} [K(l, T_{KT} - \Delta T) - K(l, T_{KT} + \Delta T)] = \frac{2}{\pi}$$

\downarrow
 $\frac{2}{\pi}$

\downarrow
0

Superfluid ^4He .

$\Phi(\vec{x})$



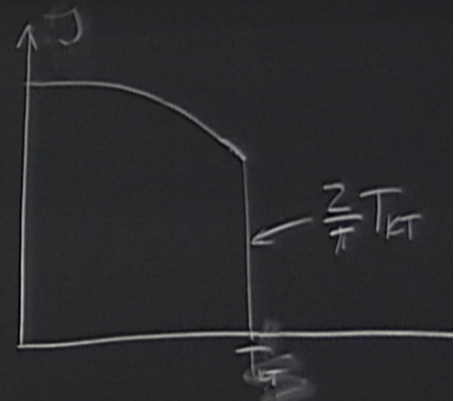
$$\frac{d\lambda}{dl} = \frac{1}{T^2} \frac{dT}{dl} = \frac{1}{T^2} \frac{dT}{dT} \approx \frac{1}{2} \left(\frac{\kappa}{T}\right)^3 - \left(\frac{\kappa}{T}\right)^2$$

ΔT - small deviation of T from T_{KT}

$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} [K(l, T_{KT} - \Delta T) - K(l, T_{KT} + \Delta T)] = \frac{2}{\pi}$$

Superfluid ^4He
 $\Phi(\vec{r}) = |\Phi| e^{i\theta}$

$\frac{2}{\pi}$



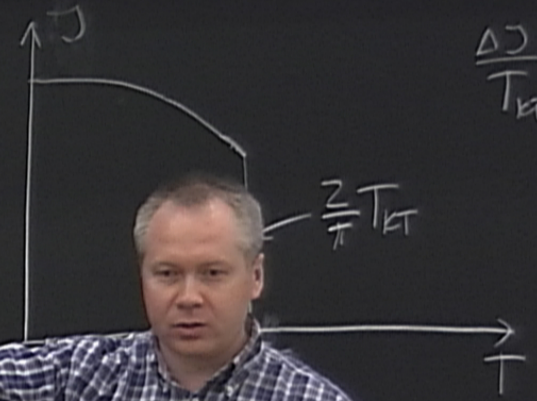
ΔT - small deviation of T from T_{KT}

$$\lim_{\Delta T \rightarrow 0^+} \lim_{l \rightarrow \infty} [K(l, T_{KT} - \Delta T) - K(l, T_{KT} + \Delta T)] = \frac{2}{\pi}$$

Superfluid He^4
 $\Phi(\vec{x}) = |\Phi| e^{i\theta}$

$\frac{2}{\pi}$

0



$$\frac{\Delta J}{T_{KT}} = \frac{2}{\pi} \dots$$

$\frac{2}{\pi} T_{KT}$