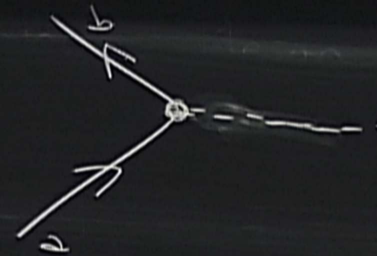
Title: 13/14 PSI - Quantum Field Theory I - Lecture 13

Date: Oct 25, 2013 03:30 PM

URL: http://pirsa.org/13100038

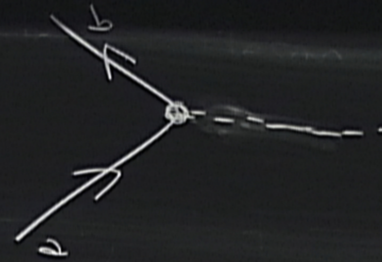
Abstract:

Interaction:



$-ig\delta_{ab}$

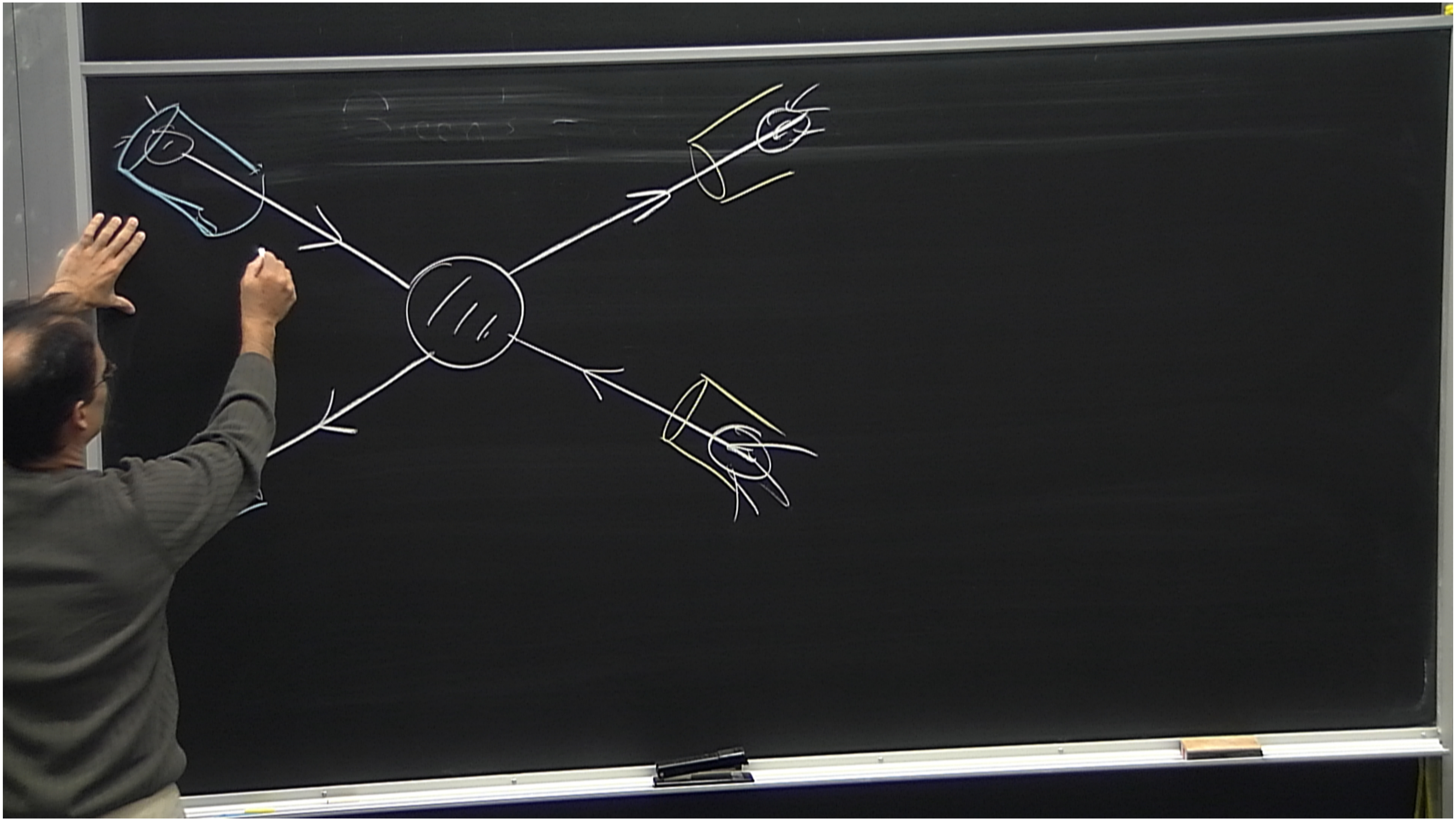
Interaction:

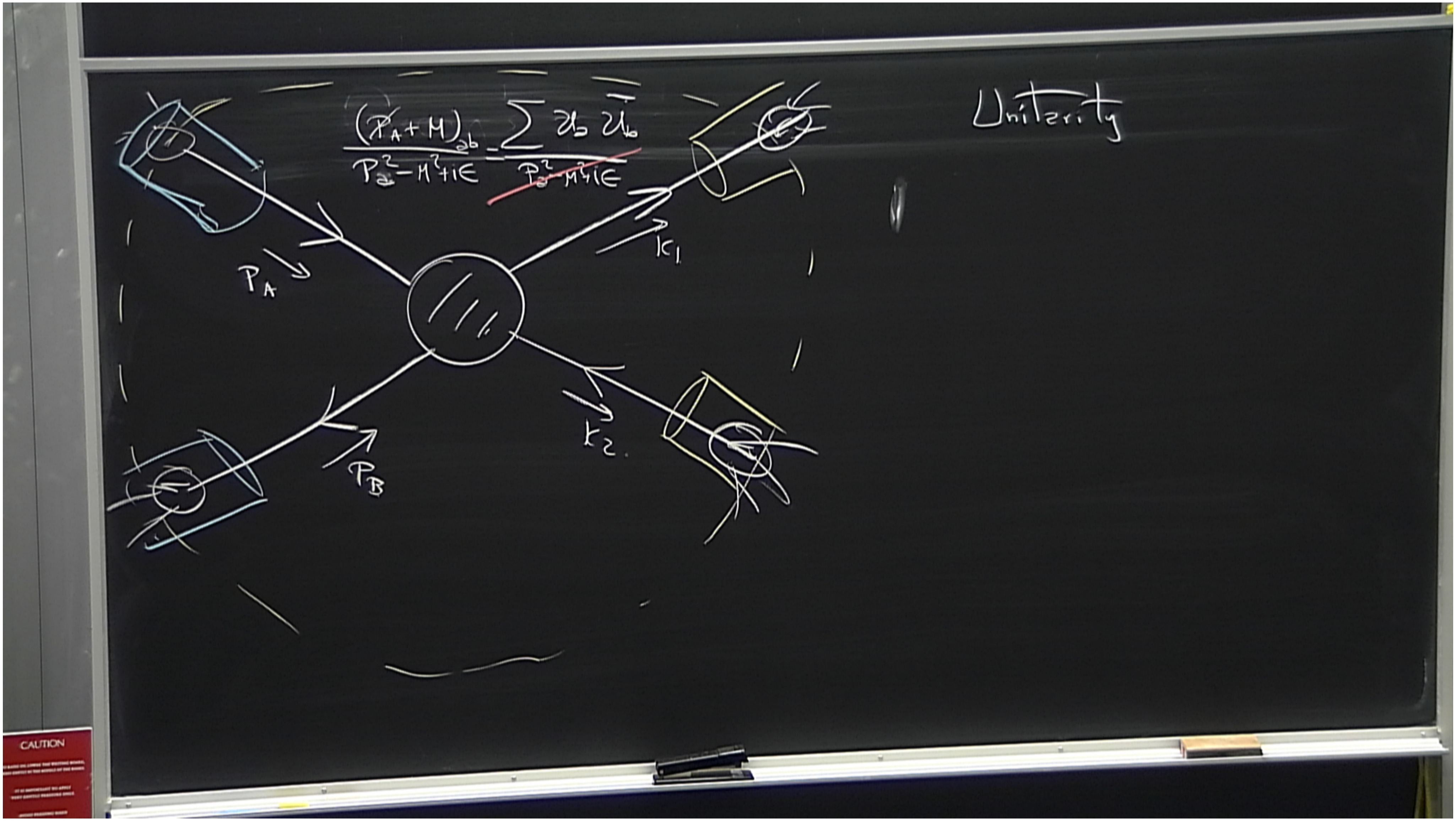


$$-ig\,\delta_{ab}$$

Rules for external nucleons & antinucleons.

$$(\not{k} + M\mathbb{1})_{ab} = \sum_{s=1}^{2} u_a^{(s)}(k)\,\bar{u}_b^{(s)}(k) \qquad (\not{k} - M\mathbb{1})_{ab} = \sum_{s=1}^{2} v_a^{(s)}(k)\,\bar{v}_b^{(s)}(k)$$




(P̸A + M)ab / (Pa² − M² + iϵ) = Σ ub ūb / (Pa² − M² + iϵ)
Unitarity
PA
PB
k1
k2

Rules:

Incoming nucleon $u^{(s)}(\vec{k})$

Outgoing nucleon $\bar{u}^{(s)}(\vec{k})$

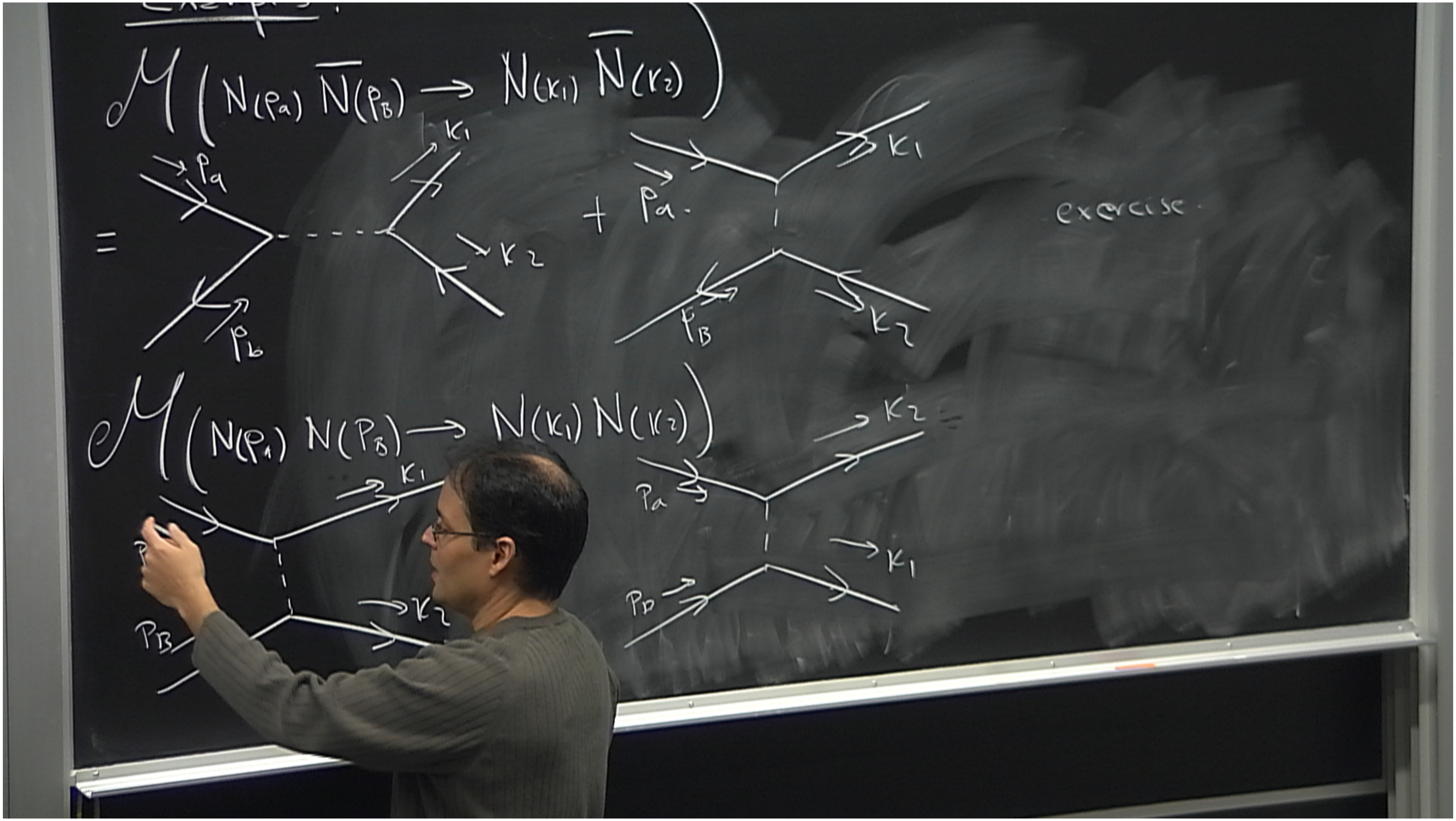
Rules:

Incoming nucleon $u^{(s)}_{(\vec{k})}$

Outgoing nucleon $\bar{u}^{(s)}_{(k)}$

Incoming antinucleon $\bar{v}^{(s)}_{(\vec{k})}$

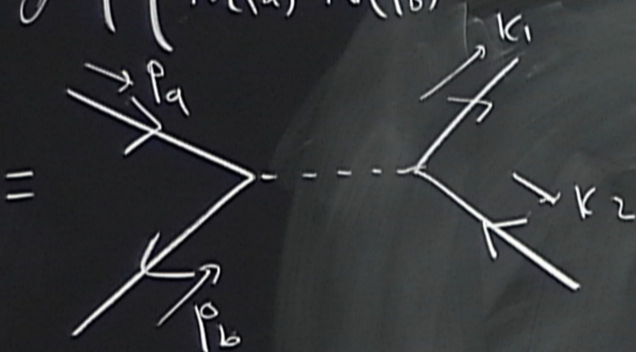
Outgoing antinucleon $v^{(s)}_{(k)}$

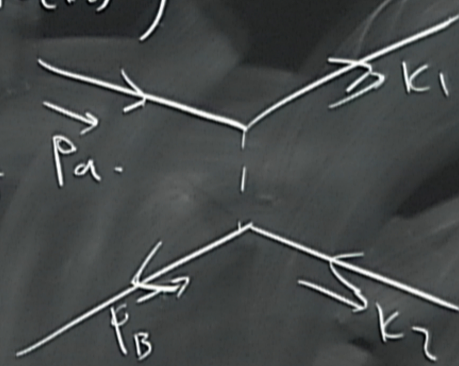


M(N(pa) N̄(pB) → N(k1) N̄(k2))
exercise
M(N(pA) N(pB) → N(k1) N(k2))

Examples:

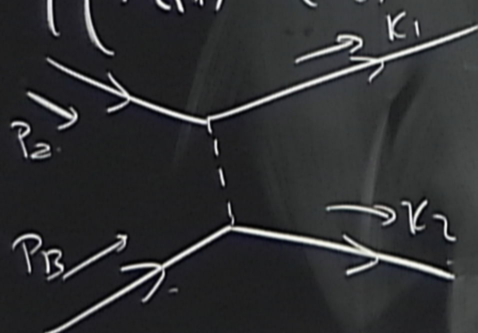
$$\mathcal{M}\left(N(p_a)\,\bar{N}(p_B) \to N(k_1)\,\bar{N}(k_2)\right)$$

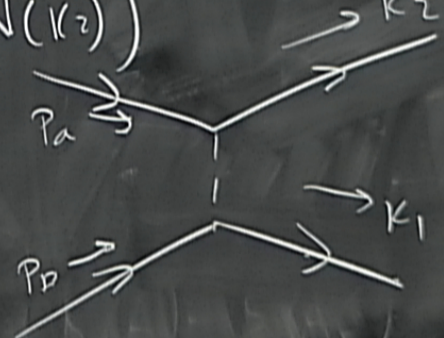
$=$ 

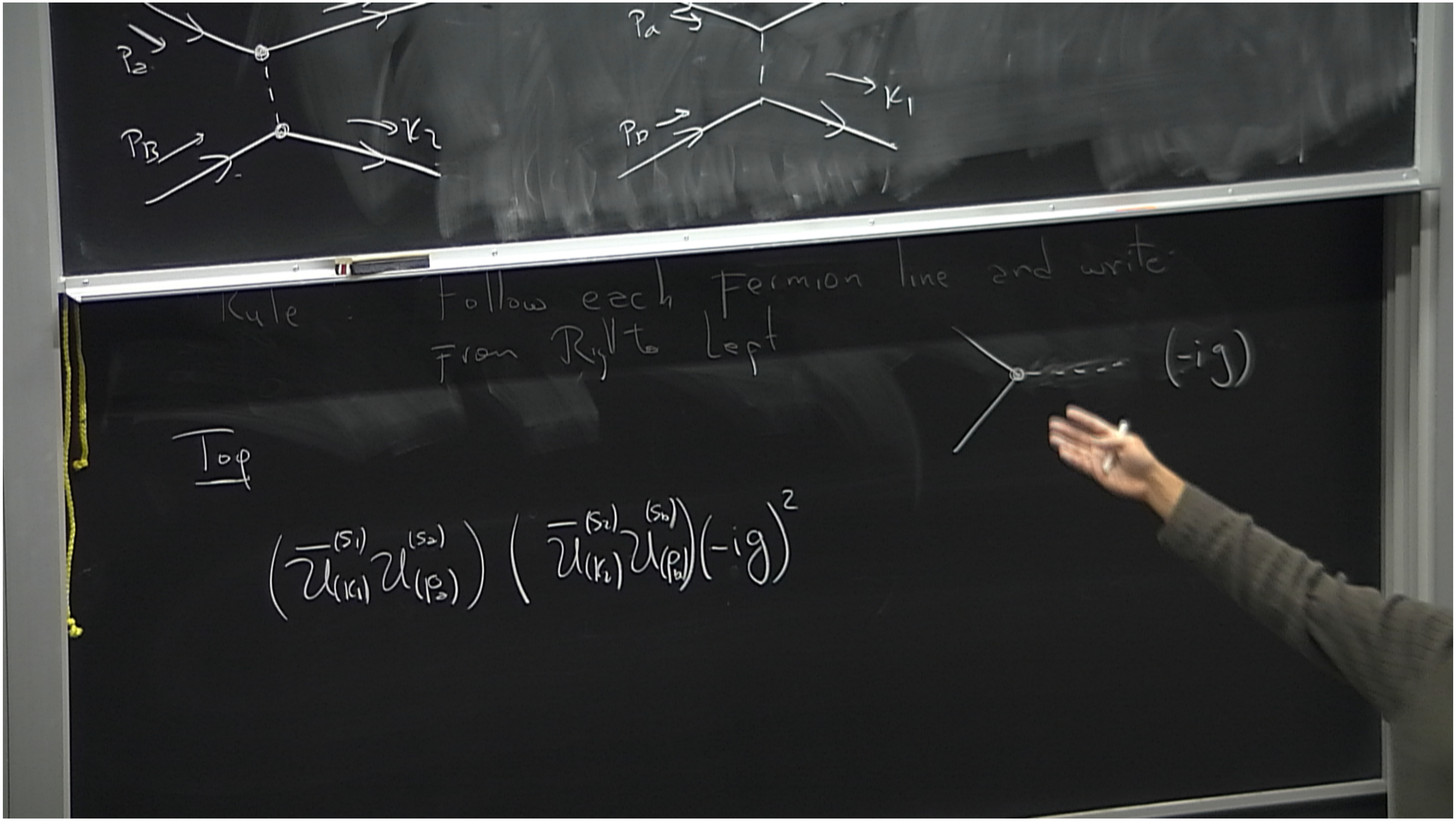
$+$ 

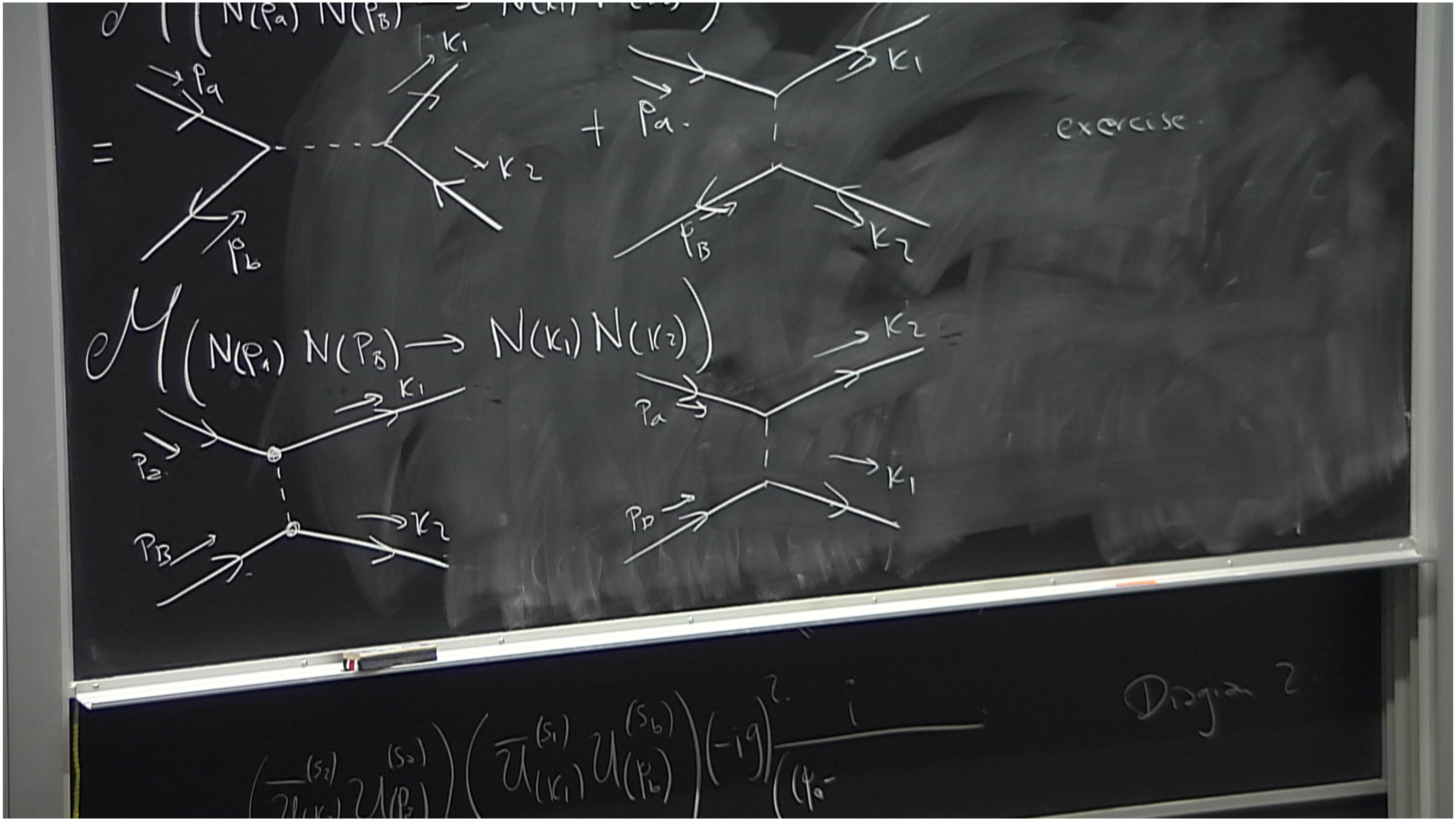
exercise.

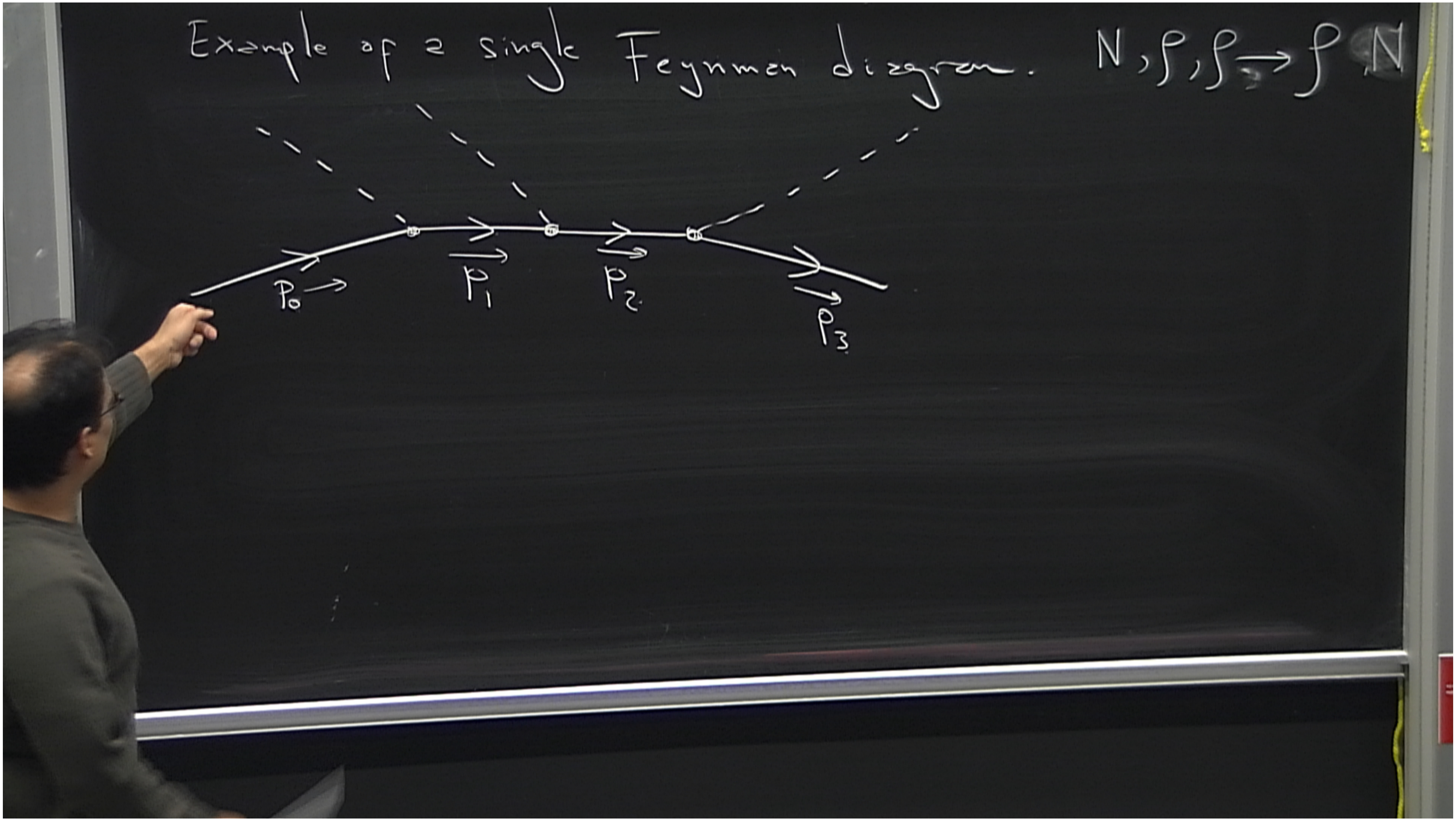
$$\mathcal{M}\left(N(p_A)\,N(p_B) \to N(k_1)\,N(k_2)\right)$$

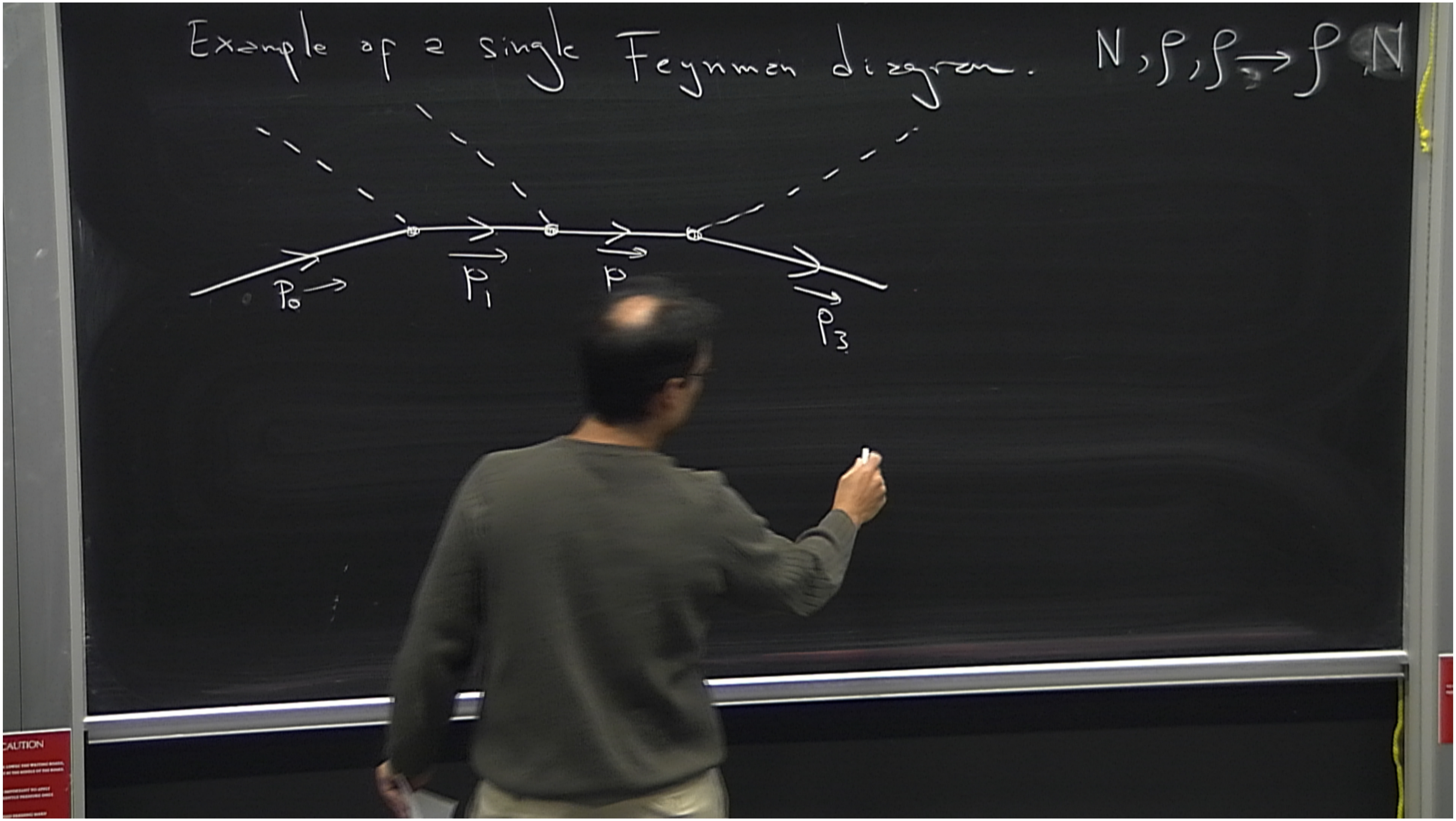


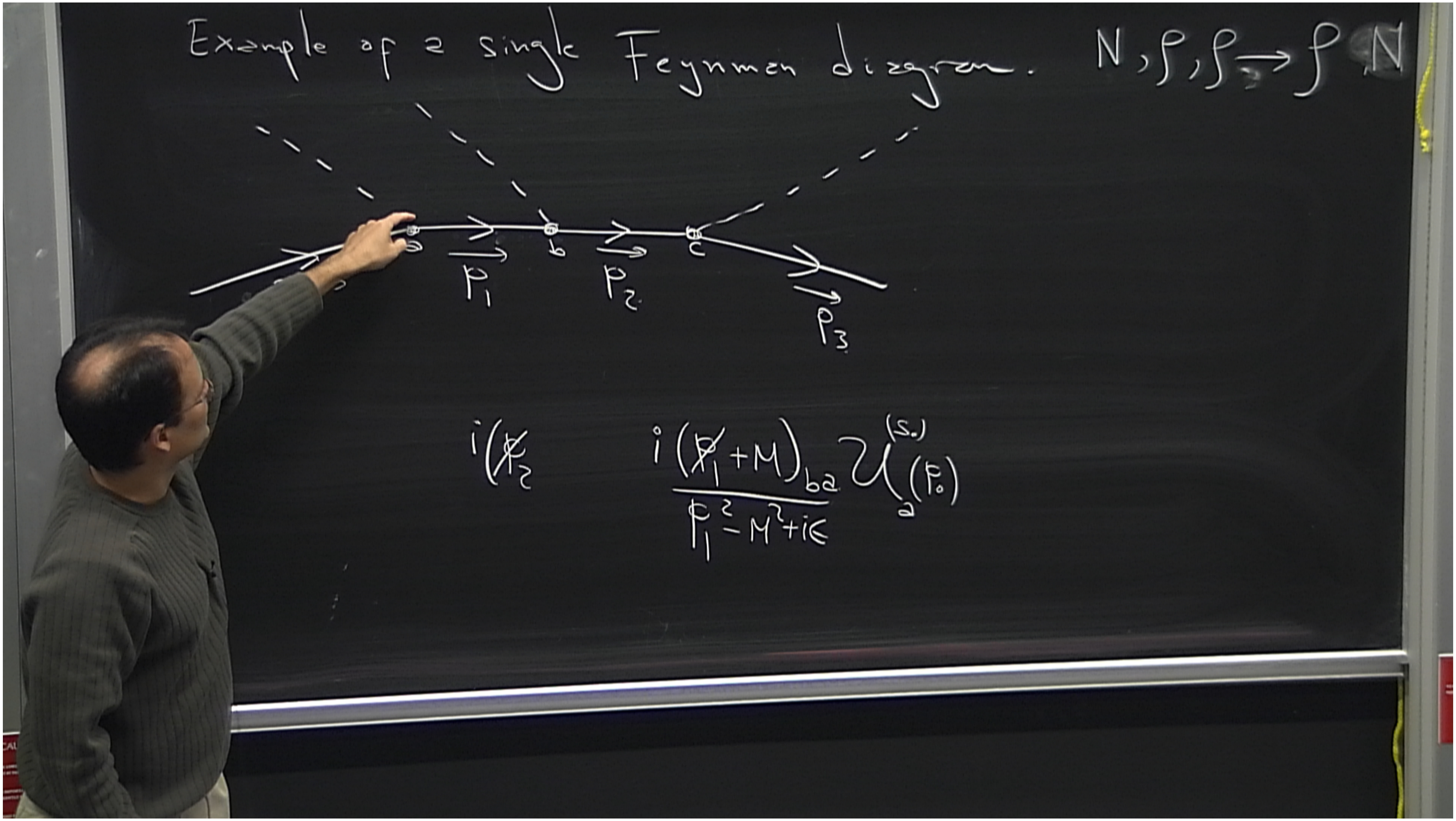



P_2
P_B
k_2
P_a
P_b
k_1
Rule : Follow each Fermion line and write
From Right to Left
$(-ig)$
Top
$\left(\bar{u}^{(s_1)}_{(k_1)} u^{(s_a)}_{(p_a)}\right)\left(\bar{u}^{(s_2)}_{(k_2)} u^{(s_b)}_{(p_b)}\right)(-ig)^2$

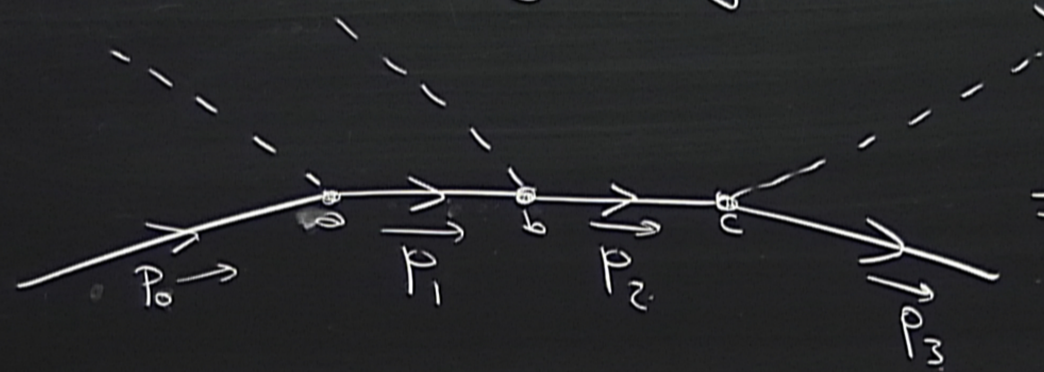

exercise
M(N(P_A) N(P_B) → N(K_1) N(K_2))
Pa
Pb
K1
K2
Diagram 2


Example of a single Feynman diagram.
p0
p1
p2
p3


Example of a single Feynman diagram.
p0
p1
p3

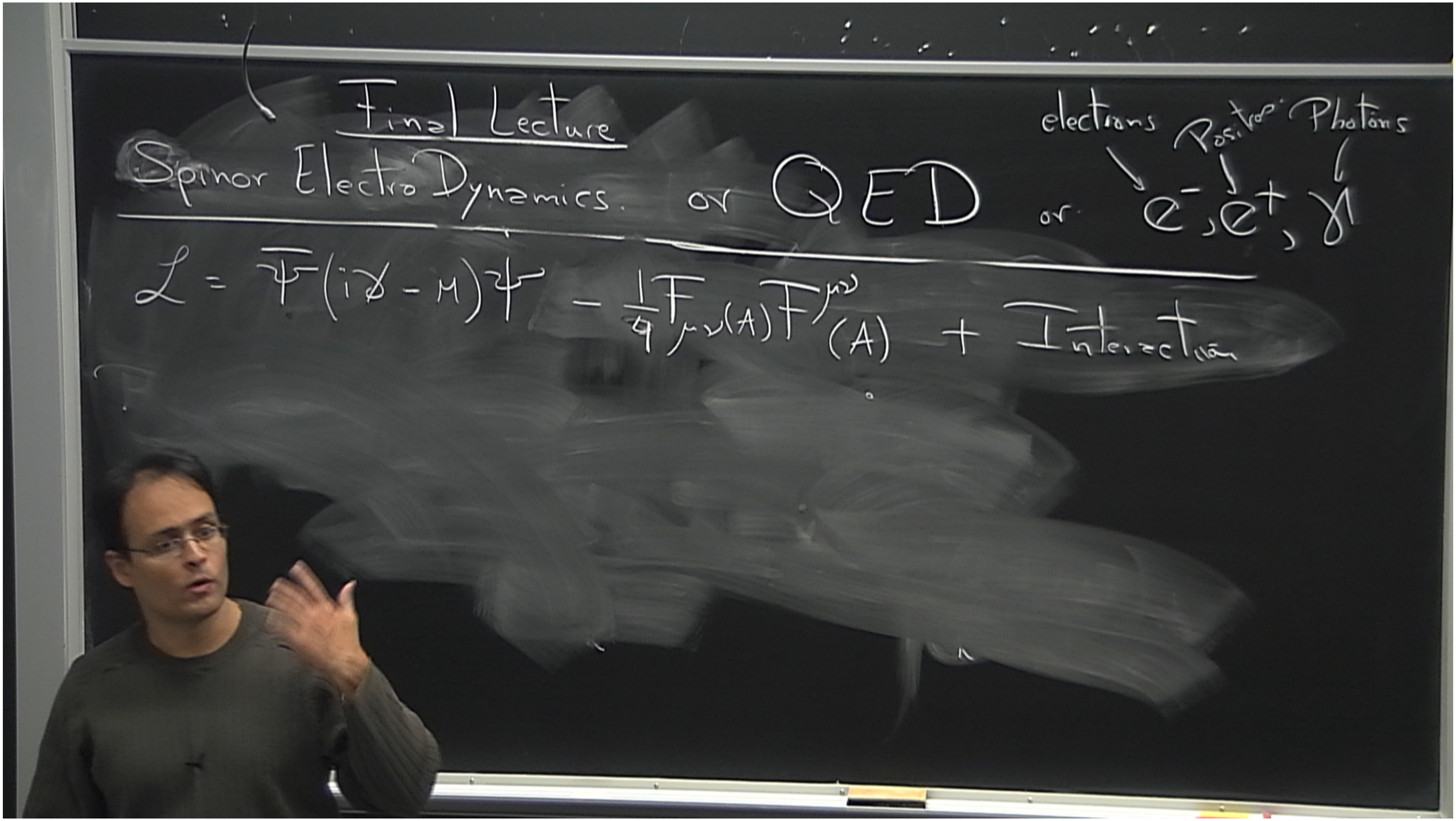

Example of a single Feynman diagram. N, ρ, ρ → ρ N
b
c
p1
p2
p3
i(p̸2
i(p̸1+M)ba / (p1² − M² + iε) u(s0)a(p0)

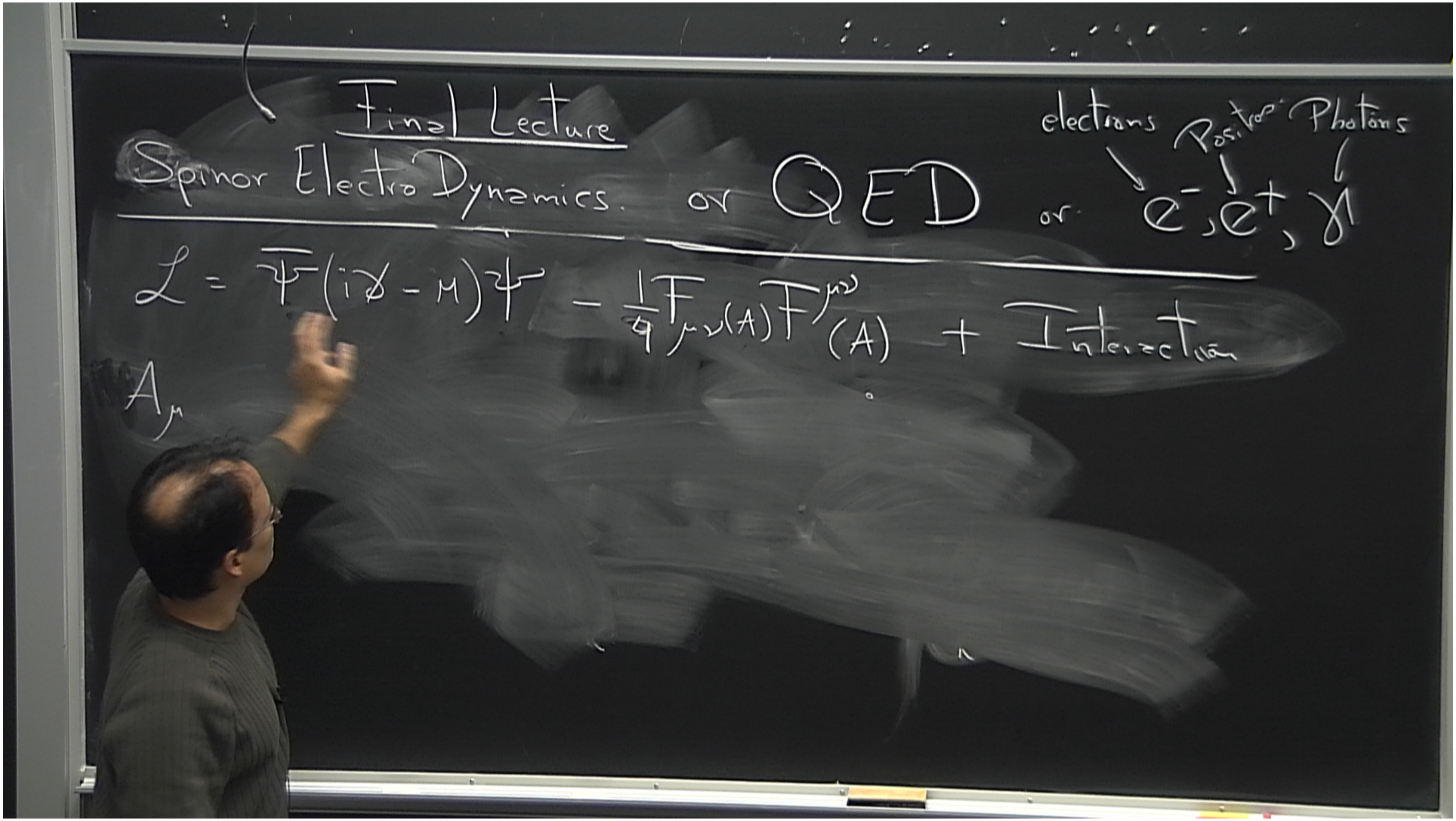
Example of a single Feynman diagram. $N, \rho, \rho \to \rho\, N$



$$= \bar{u}^{(s_3)}(p_3) \frac{i}{\not p_2 - M} \frac{i}{\not p_1 - M} u^{(s_0)}(p_0)$$

$$\bar{u}^{(s_3)}_c(p_3) \frac{i(\not p_2 + M)_{cb}}{p_2^2 - M^2 + i\epsilon} \frac{i(\not p_1 + M)_{ba}}{p_1^2 - M^2 + i\epsilon} u^{(s_0)}_a(p_0)$$


Final Lecture
electrons Positrons Photons
Spinor ElectroDynamics. or QED or e^-, e^+, γ
$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) + \text{Interaction}$


Final Lecture
electrons Positron Photons
Spinor Electro Dynamics. or QED or e^-, e^+, γ
$\mathcal{L} = \bar{\psi}(i\gamma - m)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) + \text{Interaction}$
A_μ

Need a gauge invariance

$$A_\mu \rightarrow A_\mu + \partial_\mu \omega(x)$$

Very nicely.

$$\bar{\psi}(i\not{\partial} - m)\psi$$

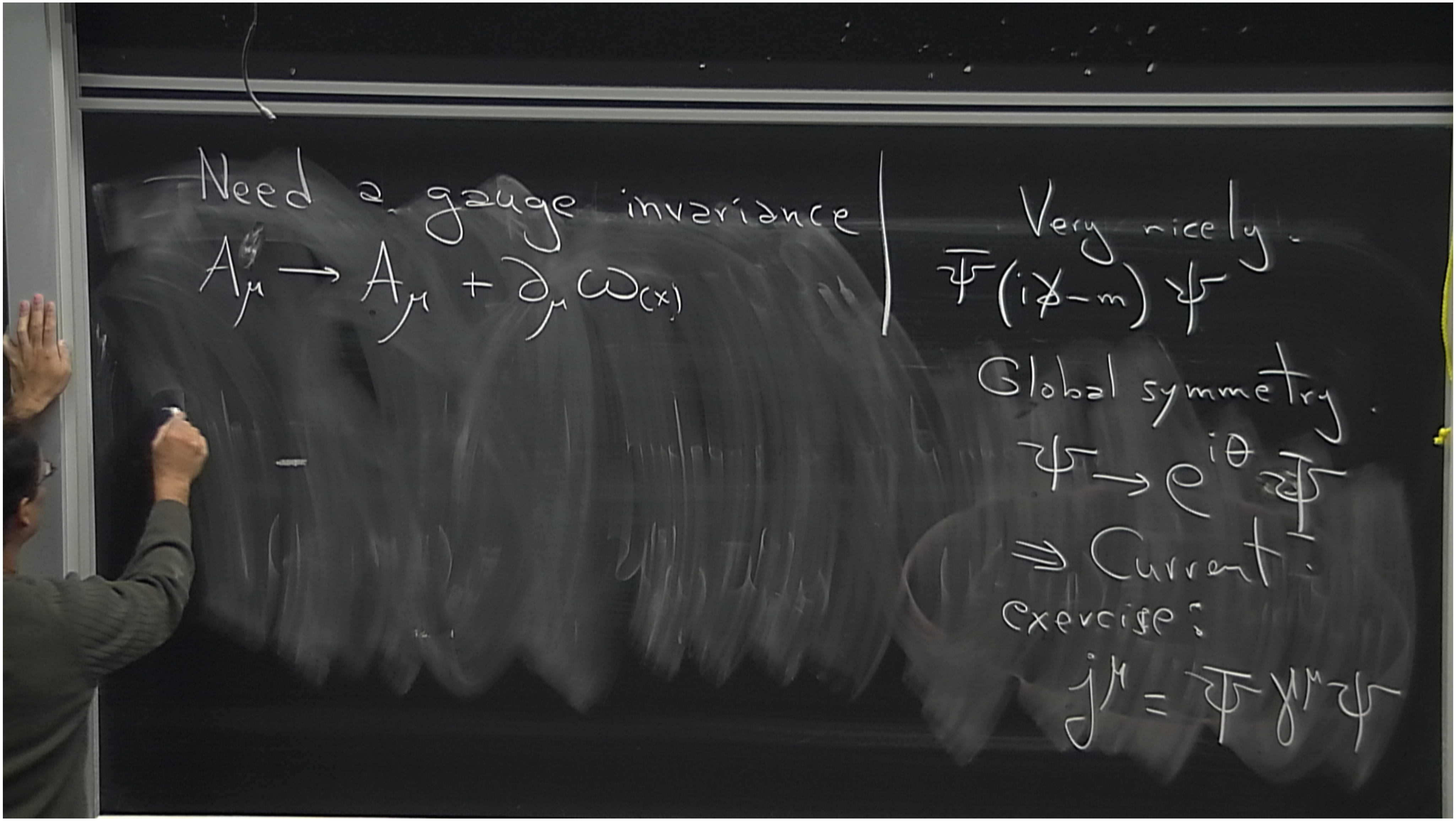
Global symmetry.

$$\psi \rightarrow e^{i\theta}\psi$$

$\Rightarrow$ Current!

exercise:

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$


Need a gauge invariance
$A_\mu \to A_\mu + \partial_\mu \omega(x)$
Very nicely.
$\bar{\Psi}(i\not{\partial} - m)\psi$
Global symmetry.
$\psi \to e^{i\theta}\psi$
$\Rightarrow$ Current!
exercise:
$j^\mu = \bar{\Psi}\gamma^\mu\psi$

Need a gauge invariance

$$A_\mu \to A_\mu + \partial_\mu \omega(x)$$

Dirac + Interaction:

$$\bar{\Psi}\gamma^\mu(i\partial_\mu + \alpha A_\mu)\psi - m\bar{\Psi}\psi$$

Very nicely.

$$\bar{\Psi}(i\not{\partial} - m)\psi$$

Global symmetry.

$$\psi \to e^{i\theta}\psi$$

$\Rightarrow$ Current!

exercise:

$$j^\mu = \bar{\Psi}\gamma^\mu\psi$$

Need a gauge invariance

$$A_\mu \to A_\mu + \partial_\mu \omega_{(x)}$$

Dirac + Interaction:

$$\bar{\Psi}\, \gamma^\mu \left(i\partial_\mu + \alpha A_\mu \right) \psi - M \bar{\Psi} \psi$$

$$\bar{\Psi}\, e^{+ie\omega_{(x)}} \gamma^\mu \left(i\partial_\mu + \alpha A_\mu \right) e^{-ie\omega_{(x)}} \psi$$

Local transformation $\theta(x) = -e\omega(x)$

Very nicely.

$$\bar{\Psi} (i\not{\partial} - m) \psi$$

Global symmetry.

$$\psi \to e^{i\theta} \psi$$

$\Rightarrow$ Current.

exercise:

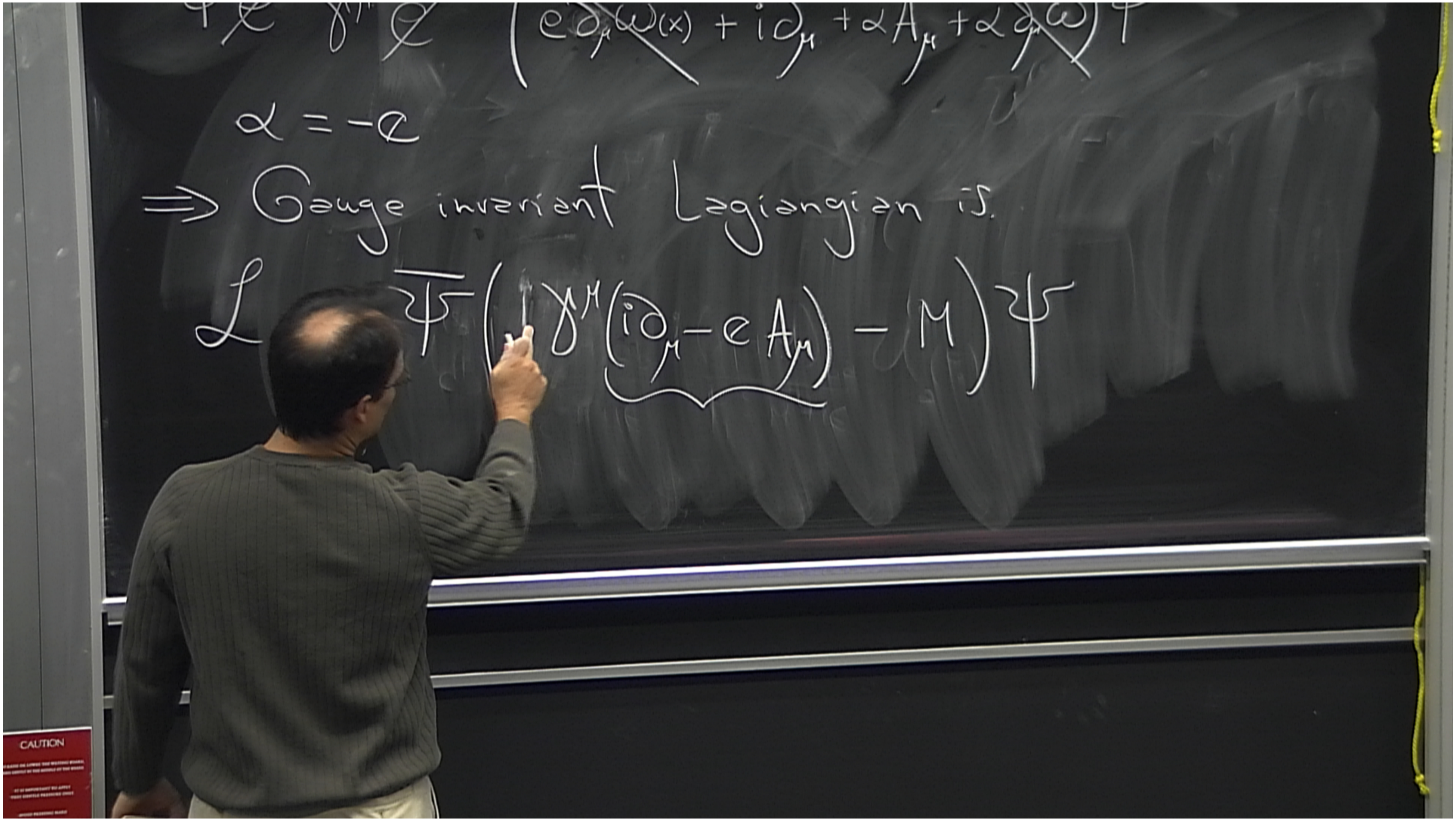
$$j^\mu = \bar{\Psi} \gamma^\mu \psi$$

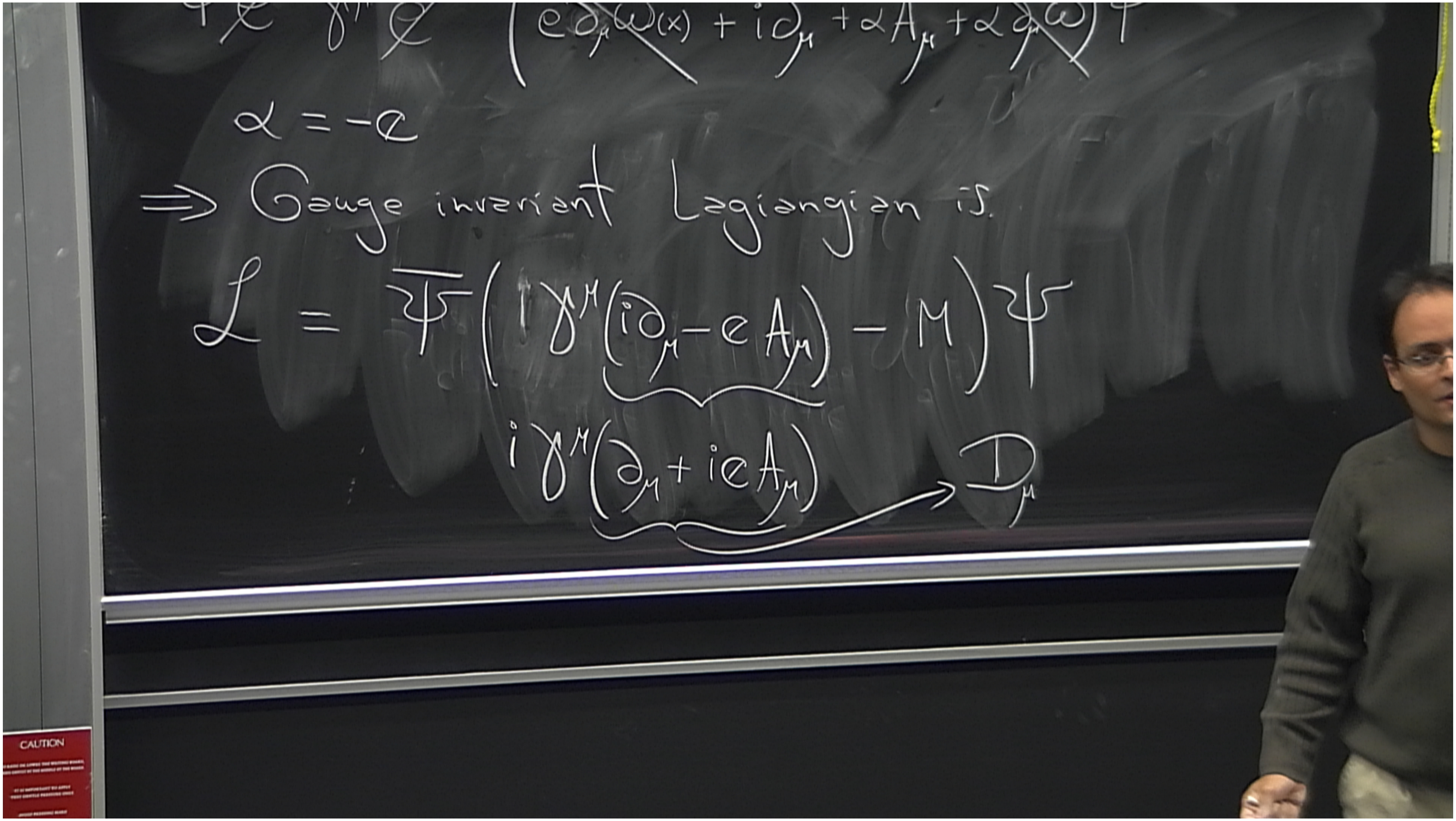
$$\bar\psi\, e\; \gamma^\mu\, e\; \left(\cancel{e\partial_\mu\omega(x)} + i\partial_\mu + \alpha A_\mu + \cancel{\alpha\partial_\mu\omega}\right)\psi$$

$$\alpha = -e$$

$\Rightarrow$ Gauge invariant Lagrangian is

$$\mathcal{L} = \bar\Psi\left(\gamma^\mu(i\partial_\mu - eA_\mu) - M\right)\Psi$$


α = -e
⇒ Gauge invariant Lagrangian is.
L = Ψ̄ (γ^μ (i∂_μ - e A_μ) - M) Ψ
CAUTION


$\alpha = -e$
$\Rightarrow$ Gauge invariant Lagrangian is.
$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\left(i\partial_\mu - eA_\mu\right) - M\right)\Psi$
$i\gamma^\mu\left(\partial_\mu + ieA_\mu\right) \to D_\mu$
CAUTION

$$\bar{\Psi}\,\cancel{e^{ie\omega(x)}}\,\gamma^\mu\,\cancel{e^{-ie\omega}}\left(\cancel{e\partial_\mu\omega(x)} + i\partial_\mu + \alpha A_\mu + \cancel{\alpha\partial_\mu\omega}\right)\Psi$$

$$\alpha = -e$$

$\Rightarrow$ Gauge invariant Lagrangian is.

$$\mathcal{L} = \bar{\Psi}\left(\gamma^\mu\underbrace{(i\partial_\mu - eA_\mu)}_{i\gamma^\mu\underbrace{(\partial_\mu + ieA_\mu)}_{\to D_\mu}} - M\right)\Psi$$

Under a gauge transform

$$D_\mu\Psi \to e^{-ie\omega(x)}D_\mu\Psi$$

$$\bar\Psi\, \cancel{e^{ie\omega(x)}}\, \gamma^\mu\, \cancel{e^{-ie\omega}} \left(\cancel{e\partial_\mu\omega(x)} + i\partial_\mu + \alpha A_\mu + \cancel{\alpha\partial_\mu\omega}\right)\Psi$$

$$\alpha = -e$$

$\Rightarrow$ Gauge invariant Lagrangian is.

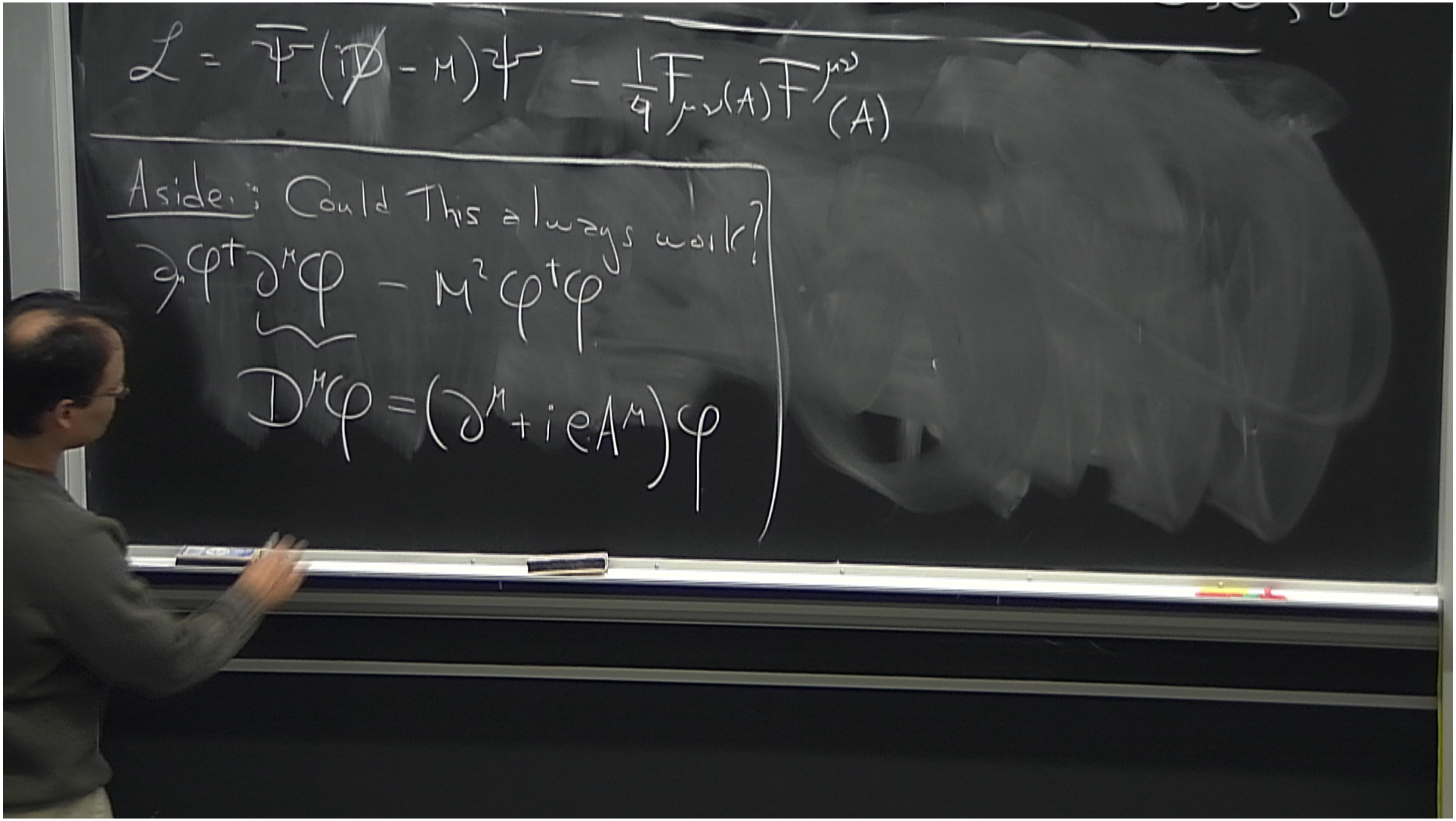
$$\mathcal{L} = \bar\Psi\left(\gamma^\mu\underbrace{(i\partial_\mu - eA_\mu)} - M\right)\Psi$$

$$i\gamma^\mu\underbrace{(\partial_\mu + ieA_\mu)} \rightarrow D_\mu$$

$e \rightarrow$ Charge.

Under a gauge transform:

$$D_\mu\Psi \rightarrow e^{-ie\omega(x)} D_\mu\Psi$$


$\mathcal{L} = \bar{\Psi}(i\not{D} - M)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$
Aside: Could This always work?
$\partial_\mu\varphi^\dagger\partial^\mu\varphi - M^2\varphi^\dagger\varphi$
$D^\mu\varphi = (\partial^\mu + ieA^\mu)\varphi$

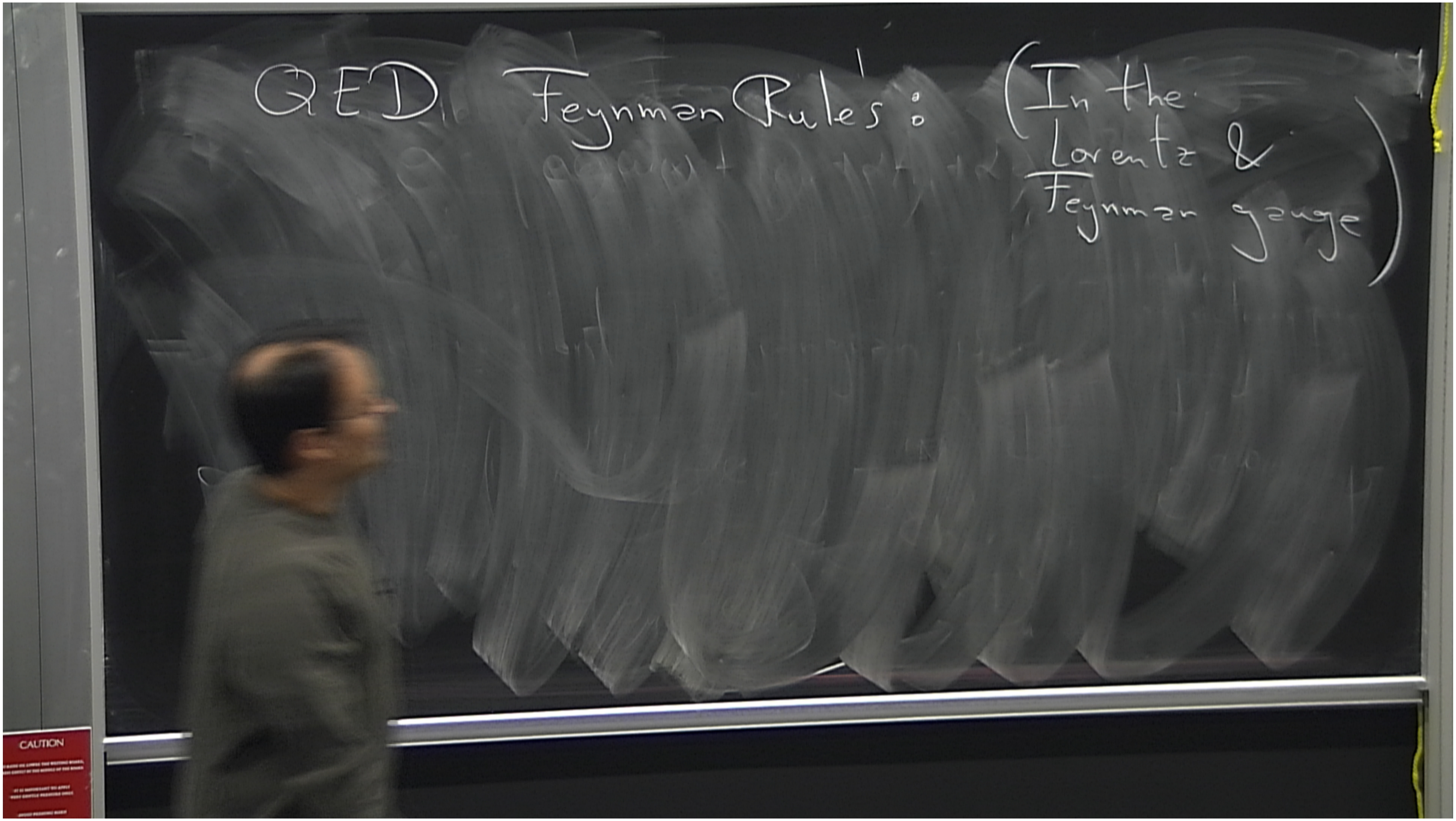
$$\mathcal{L} = \bar{\Psi}(i\not{D} - M)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Aside: Could This always work?

$$\partial_\mu\varphi^*\partial^\mu\varphi - M^2\varphi^*\varphi \longrightarrow (D_\mu\varphi)^\dagger(D^\mu\varphi)$$

$$(\partial^\mu + ieA^\mu)\varphi$$

$$(\partial_\mu - ieA_\mu)\varphi^\dagger$$


QED Feynman Rules: (In the Lorentz & Feynman gauge)
CAUTION

Final Lecture

electrons Positrons Photons

Spinor ElectroDynamics or QED or e^-, e^+, γ

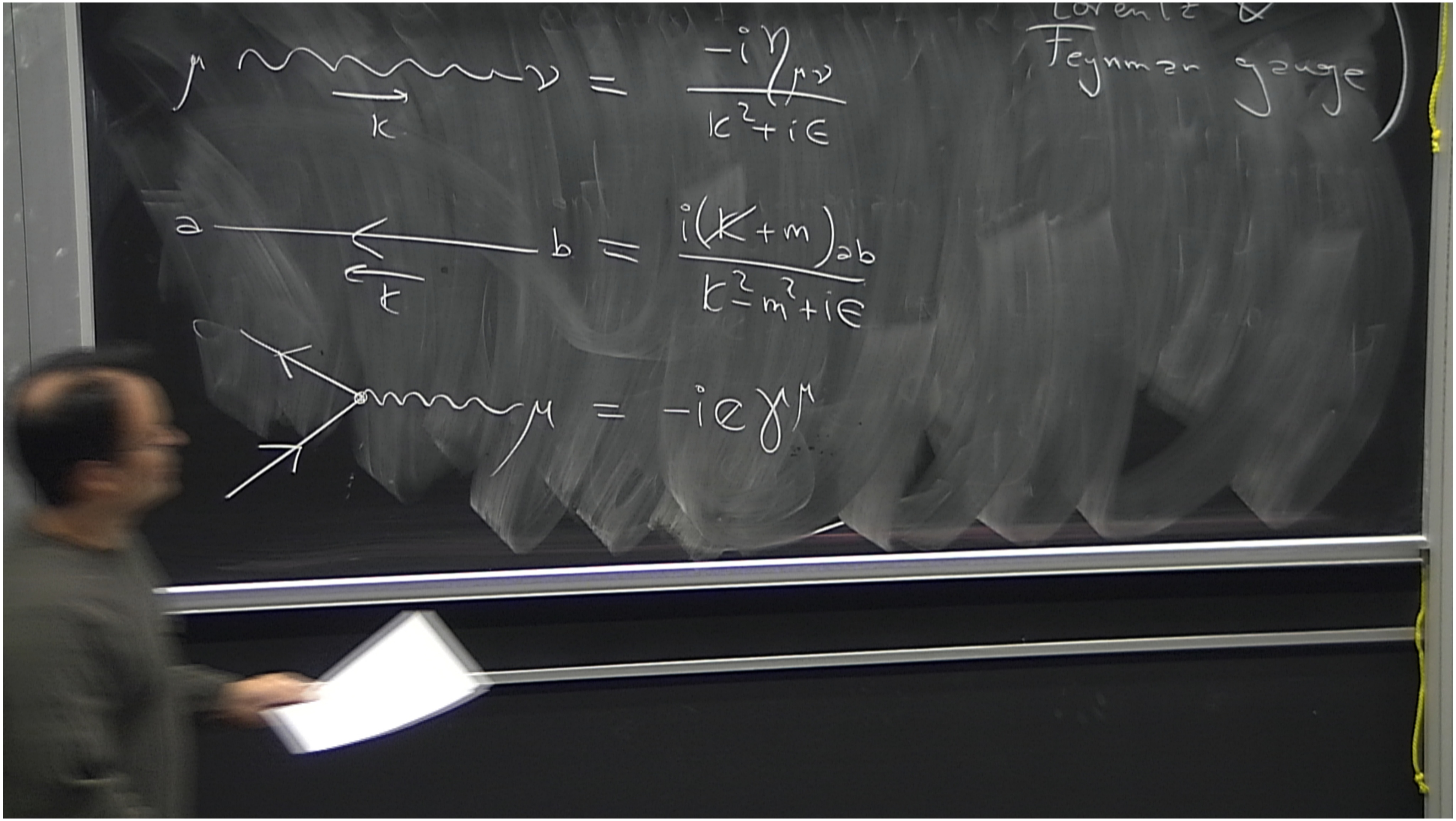
$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Aside: Could This always work?

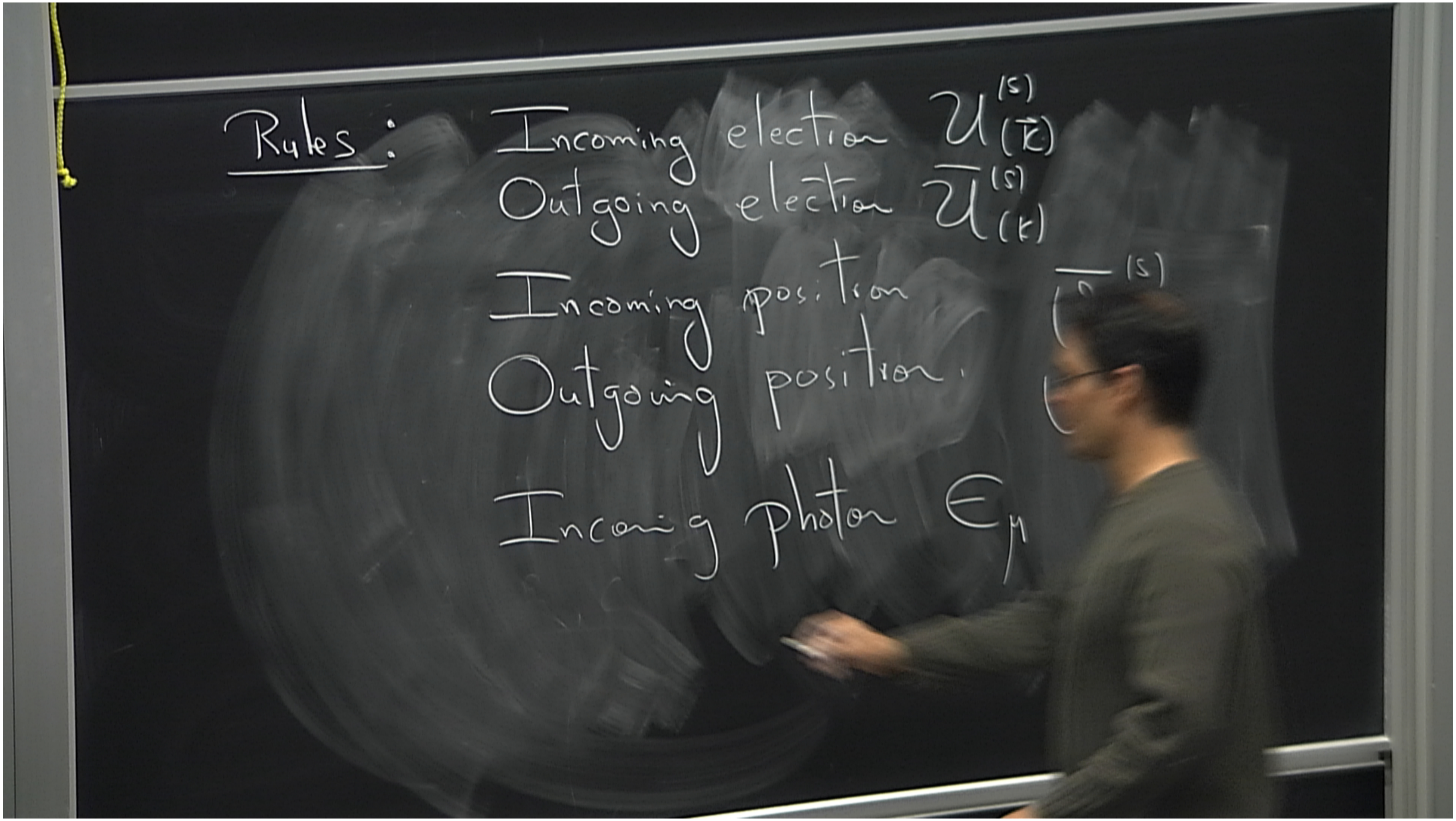
$$\partial_\mu\varphi^*\partial^\mu\varphi - M^2\varphi^*\varphi \longrightarrow \boxed{(D_\mu\varphi)^\dagger(D^\mu\varphi)}$$

$$\partial^\mu\varphi \to (\partial^\mu + ieA^\mu)\varphi$$

$$\partial_\mu\varphi^* \to (\partial_\mu - ieA_\mu)\varphi^\dagger$$



Lorentz &
Feynman gauge
$\mu \quad \nu = \frac{-i\eta_{\mu\nu}}{k^2+i\epsilon}$
k
$a \quad b = \frac{i(\not{k}+m)_{ab}}{k^2-m^2+i\epsilon}$
k
$\mu = -ie\gamma^\mu$


Rules :
Incoming election $u^{(s)}_{(\vec{k})}$
Outgoing election $\bar{u}^{(s)}_{(k)}$
Incoming positron
Outgoing positron
Incoming photon ϵ_μ

Rules:

Incoming electron $u^{(s)}_{(\vec{k})}$

Outgoing electron $\bar{u}^{(s)}_{(k)}$

Incoming positron $\bar{v}^{(s)}_{(\vec{k})}$

Outgoing positron $v^{(s)}_{(k)}$

Incoming photon ϵ_μ

Outgoing Photon ϵ^*_μ

Final Lecture

Spinor ElectroDynamics. or QED or e^-, e^+, γ

electrons, Positrons, Photons

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Aside: Could This always work?

$$\partial_\mu\varphi^*\partial^\mu\varphi - M^2\varphi^*\varphi \longrightarrow \boxed{(D_\mu\varphi)^\dagger(D^\mu\varphi)}$$

$$\partial^\mu\varphi \to (\partial^\mu + ieA^\mu)\varphi$$

$$\partial_\mu\varphi^* \to (\partial_\mu - ieA_\mu)\varphi^\dagger$$

Final Lecture

Spinor ElectroDynamics. or QED or e^-, e^+, γ

electrons Positrons Photons

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Aside: Could This always work?

$$\partial_\mu\varphi^*\partial^\mu\varphi - M^2\varphi^*\varphi \longrightarrow (D_\mu\varphi)^\dagger(D^\mu\varphi)$$

$$\partial^\mu\varphi \longrightarrow (\partial^\mu + ieA^\mu)\varphi$$

$$\partial_\mu\varphi^* \longrightarrow (\partial_\mu - ieA_\mu)\varphi^\dagger$$

Final Lecture

Spinor ElectroDynamics. or QED or $\underset{\text{electrons}}{e^-}, \underset{\text{Positrons}}{e^+}, \underset{\text{photons}}{\gamma}$

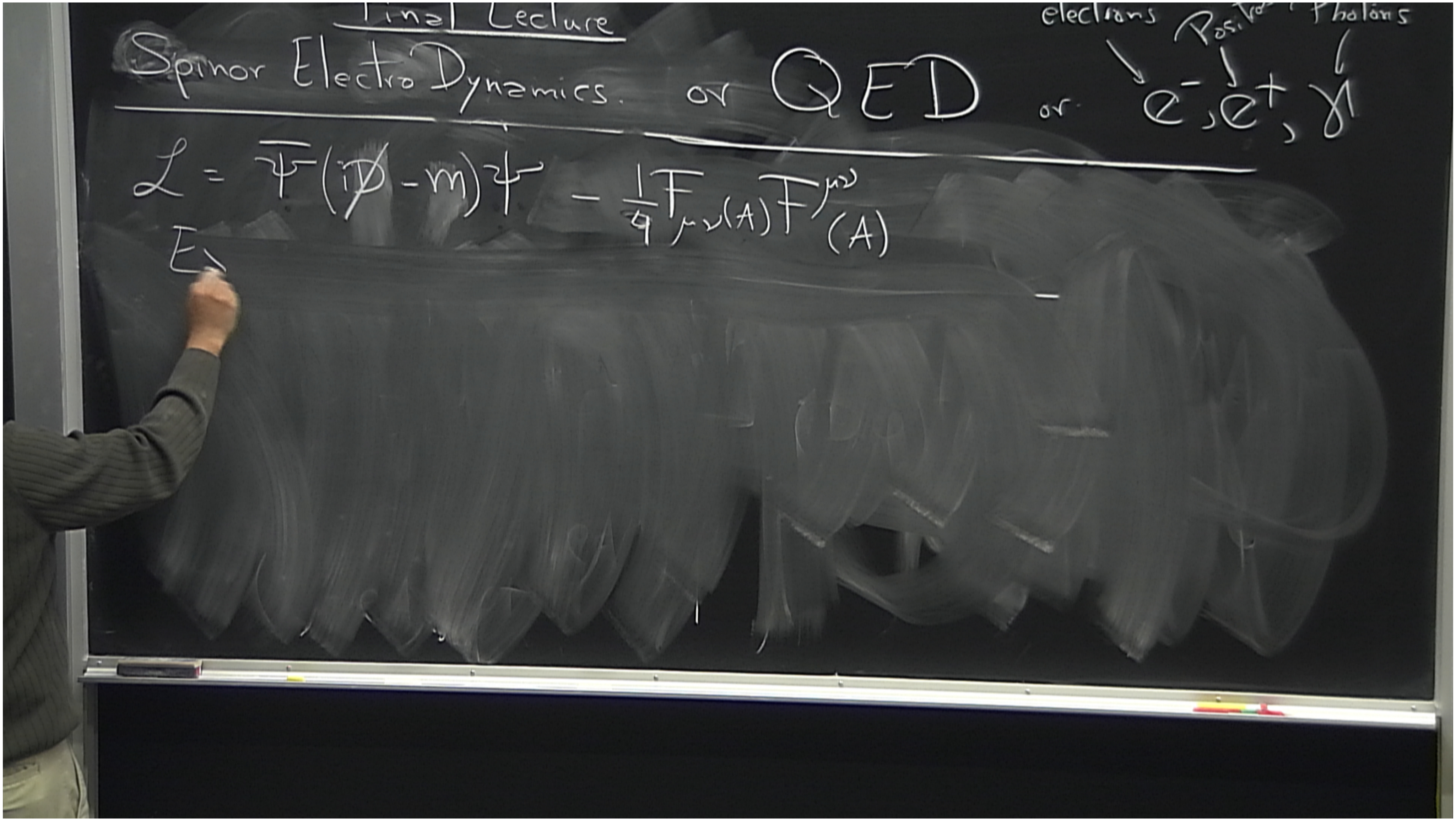
$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Aside: Could This always work?

$$\partial_\mu\varphi^*\partial^\mu\varphi - M^2\varphi^*\varphi \longrightarrow \boxed{(D_\mu\varphi)^\dagger(D^\mu\varphi)}$$

$$\partial^\mu\varphi \longrightarrow (\partial^\mu + i e A^\mu)\varphi$$

$$\partial_\mu\varphi^* \longrightarrow (\partial_\mu - i e A_\mu)\varphi^\dagger$$


Final Lecture
electrons Positrons Photons
Spinor Electro Dynamics. or QED or e^-, e^+, γ
$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Example 1:

$$\mathcal{M}\left(e^-(p_A, s_A)\, e^-(p_B, s_B) \longrightarrow e^-(k_1, s_1)\, e^-(k_2, s_2)\right)$$

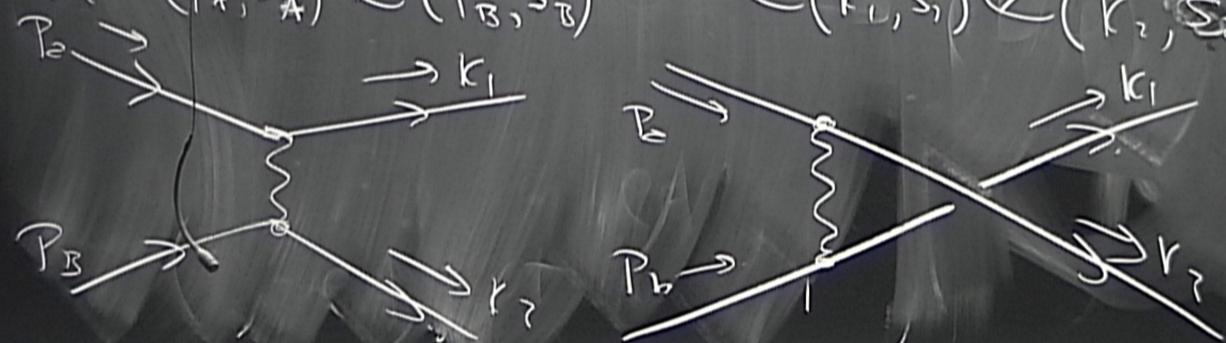
...dynamics: or QED or e, e^+, γ

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Example 1:

$$\mathcal{M}\left(e^-(p_A, s_A)\, e^-(p_B, s_B) \longrightarrow e^-(k_1, s_1)\, e^-(k_2, s_2)\right)$$

$=$



Need a gauge invariance

$$A_\mu \longrightarrow A_\mu + \partial_\mu \omega(x)$$

Very nicely.

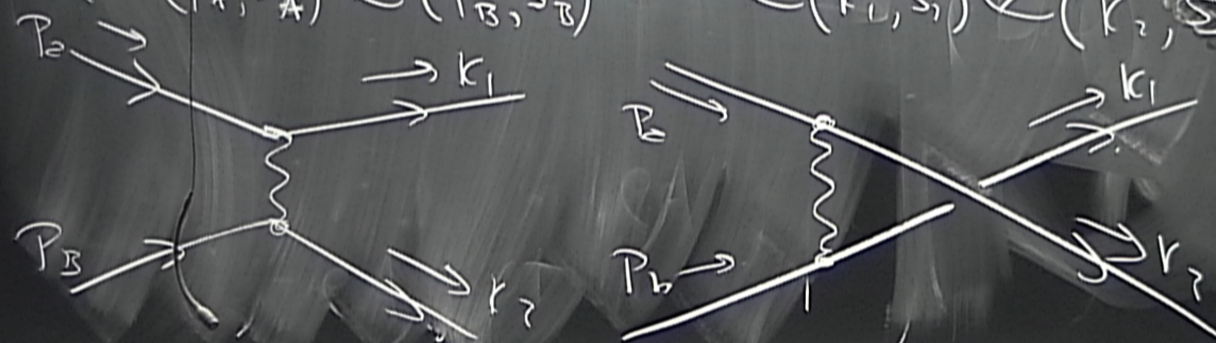
$$\bar{\psi}(i\not{\partial} - m)\psi$$

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Example 1:

$$\mathcal{M}\left(e^-(p_A, s_A)\, e^-(p_B, s_B) \longrightarrow e^-(k_1, s_1)\, e^-(k_2, s_2)\right)$$

$=$



Need a gauge invariance

$$A_\mu \longrightarrow A_\mu + \partial_\mu \omega(x)$$

Very nicely:

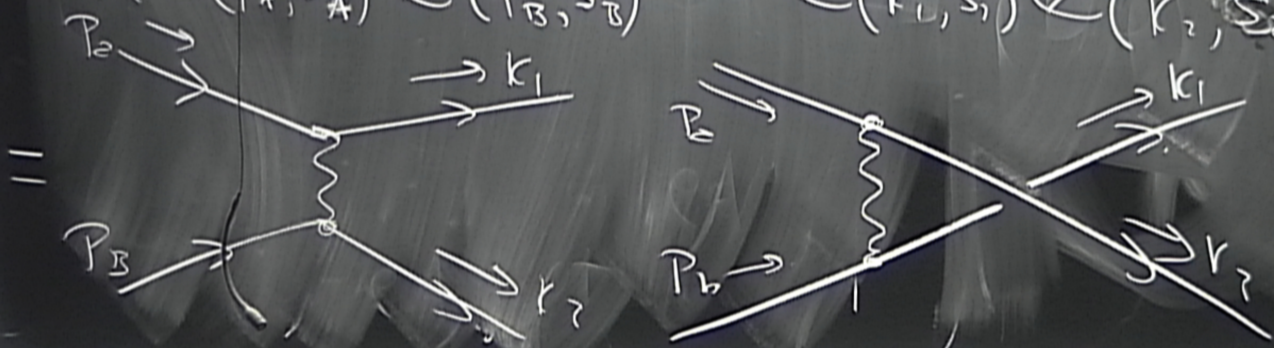
$$\bar{\psi}(i\not{\partial} - m)\psi$$

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$$

Example 1:

$$\mathcal{M}\left(e^-(p_A, s_A)\, e^-(p_B, s_B) \longrightarrow e^-(k_1, s_1)\, e^-(k_2, s_2)\right)$$

$=$



Need a gauge

$$A_\mu \longrightarrow A_\mu + \partial$$

Dirac +

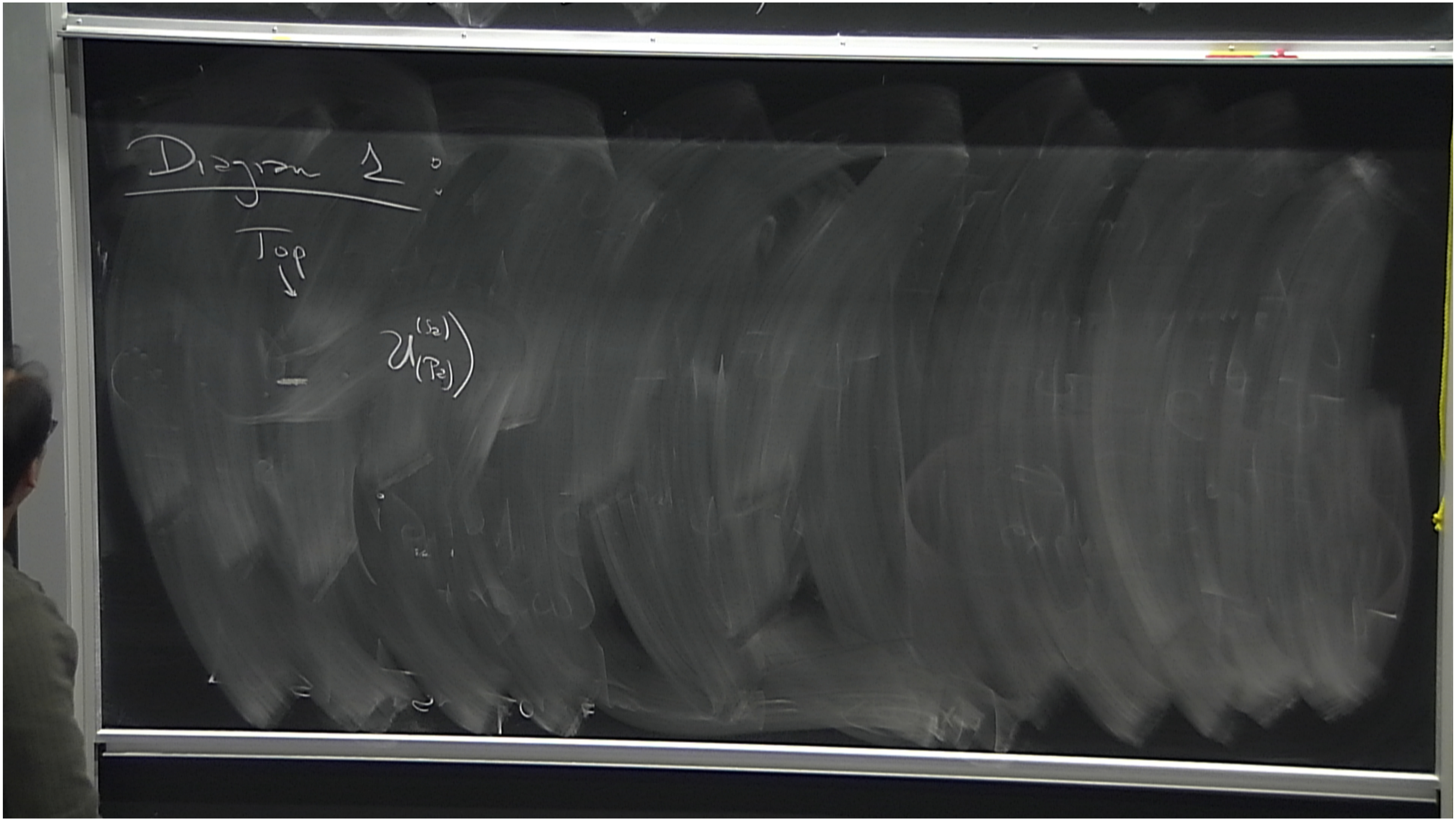


Diagram 2:
Top
$\mathcal{U}^{(S_2)}_{(P_2)}$

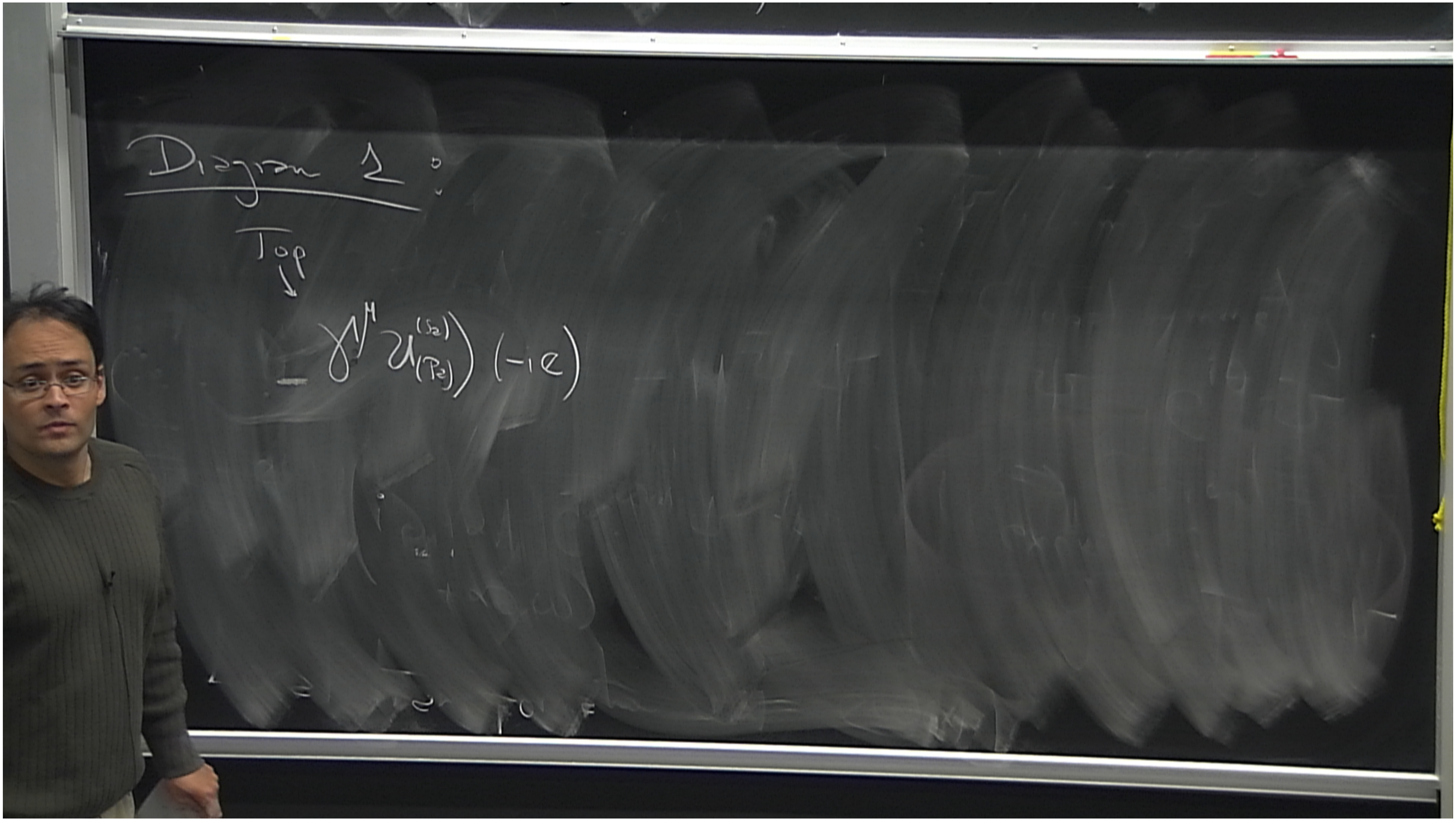


Diagram 1 :
Top
$\gamma^\mu u^{(s_2)}_{(p_2)}) (-ie)$

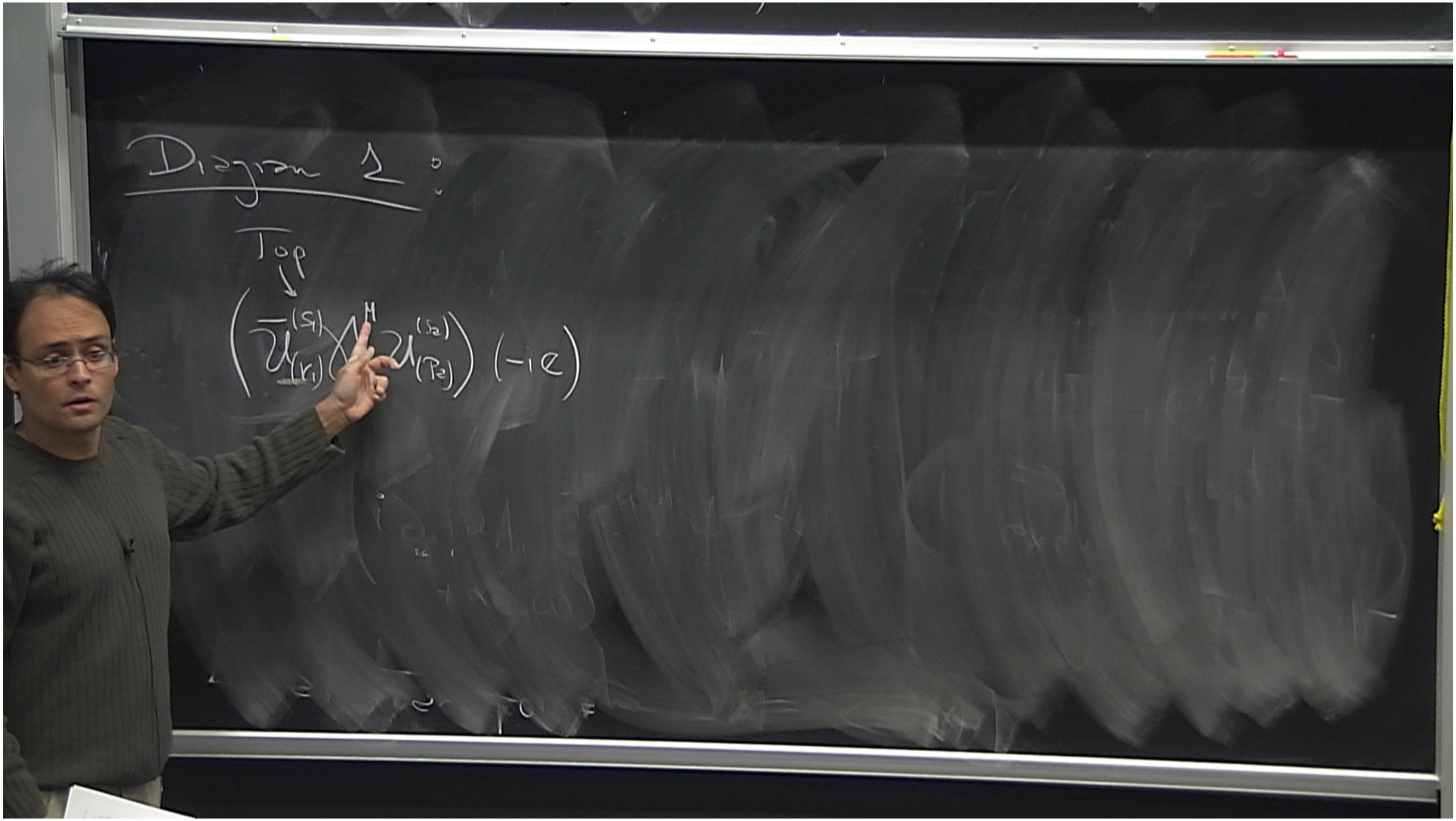


Diagram 2:
Top
$\left(\bar{u}^{(s_1)}_{(r_1)} \gamma^{\mu} u^{(s_2)}_{(p_2)}\right)(-ie)$

Diagram 1:

Top

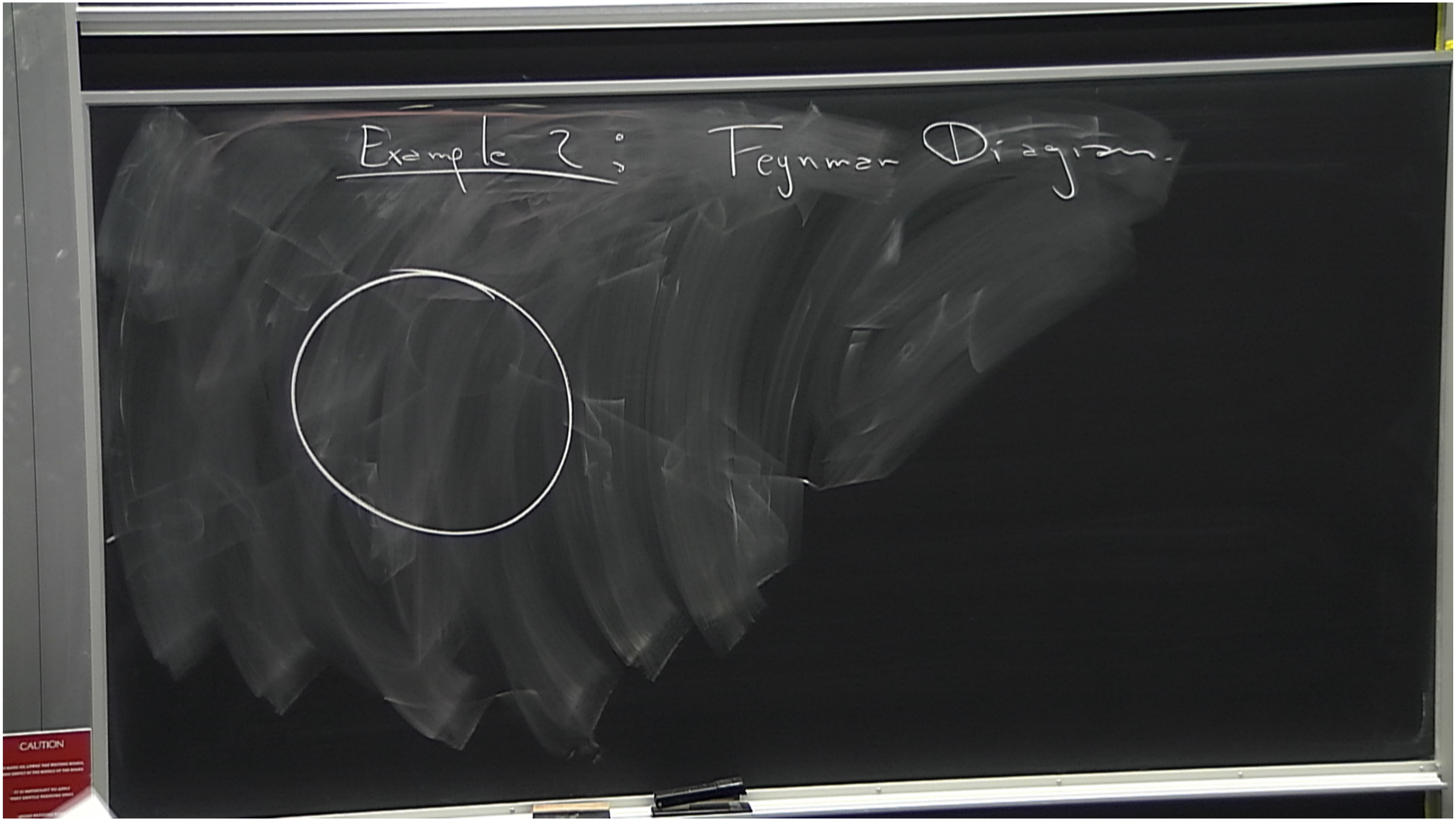
$$\left(\bar{u}^{(s_1)}_{(r_1)}\gamma^{\mu}u^{(s_2)}_{(p_2)}\right)(-ie) \qquad \left(\bar{u}^{(s_2)}_{(r_2)}\gamma^{\nu}u^{(s_b)}_{(p_3)}\right)(-ie)$$

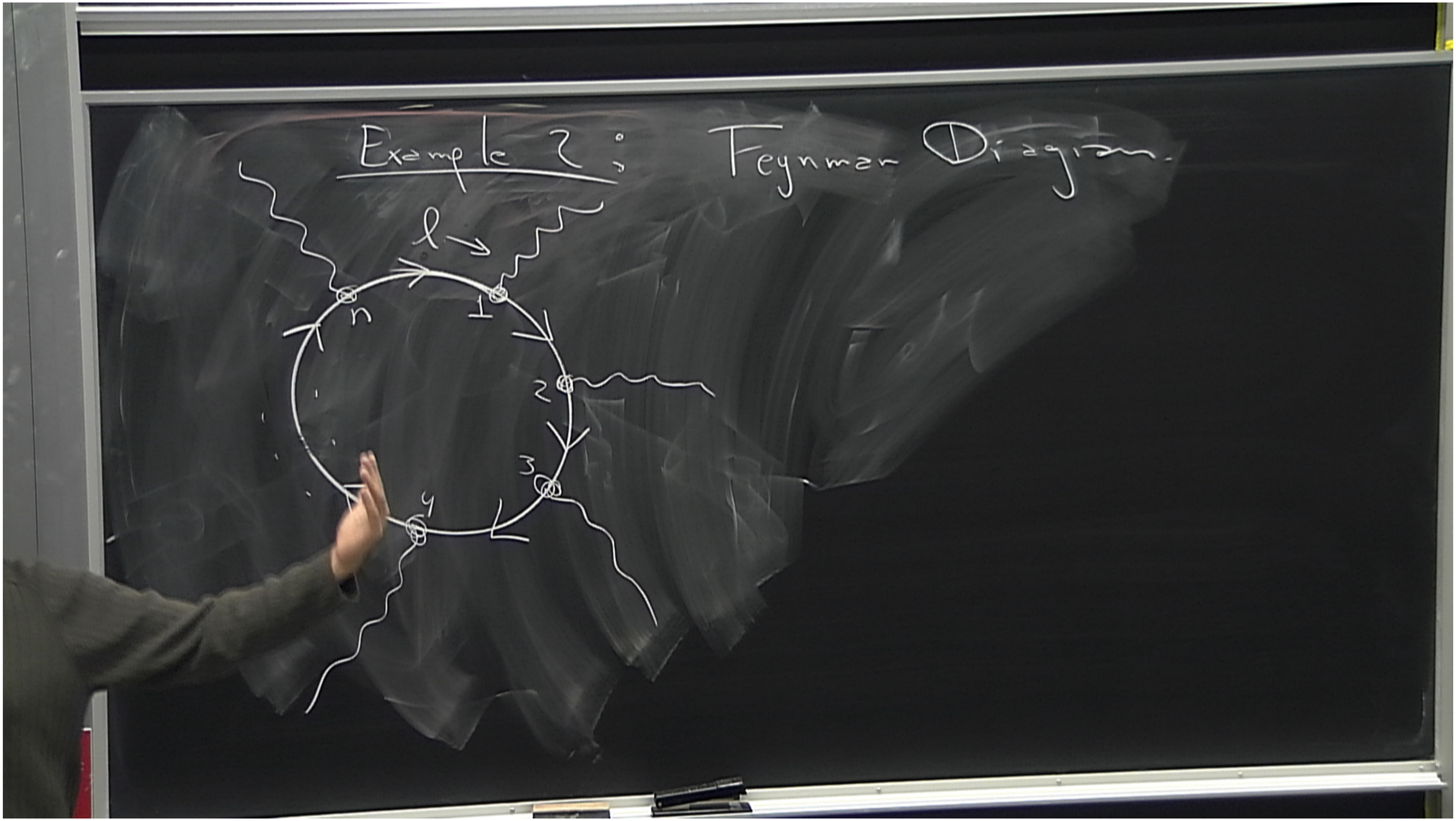
Diagram 1:

Top

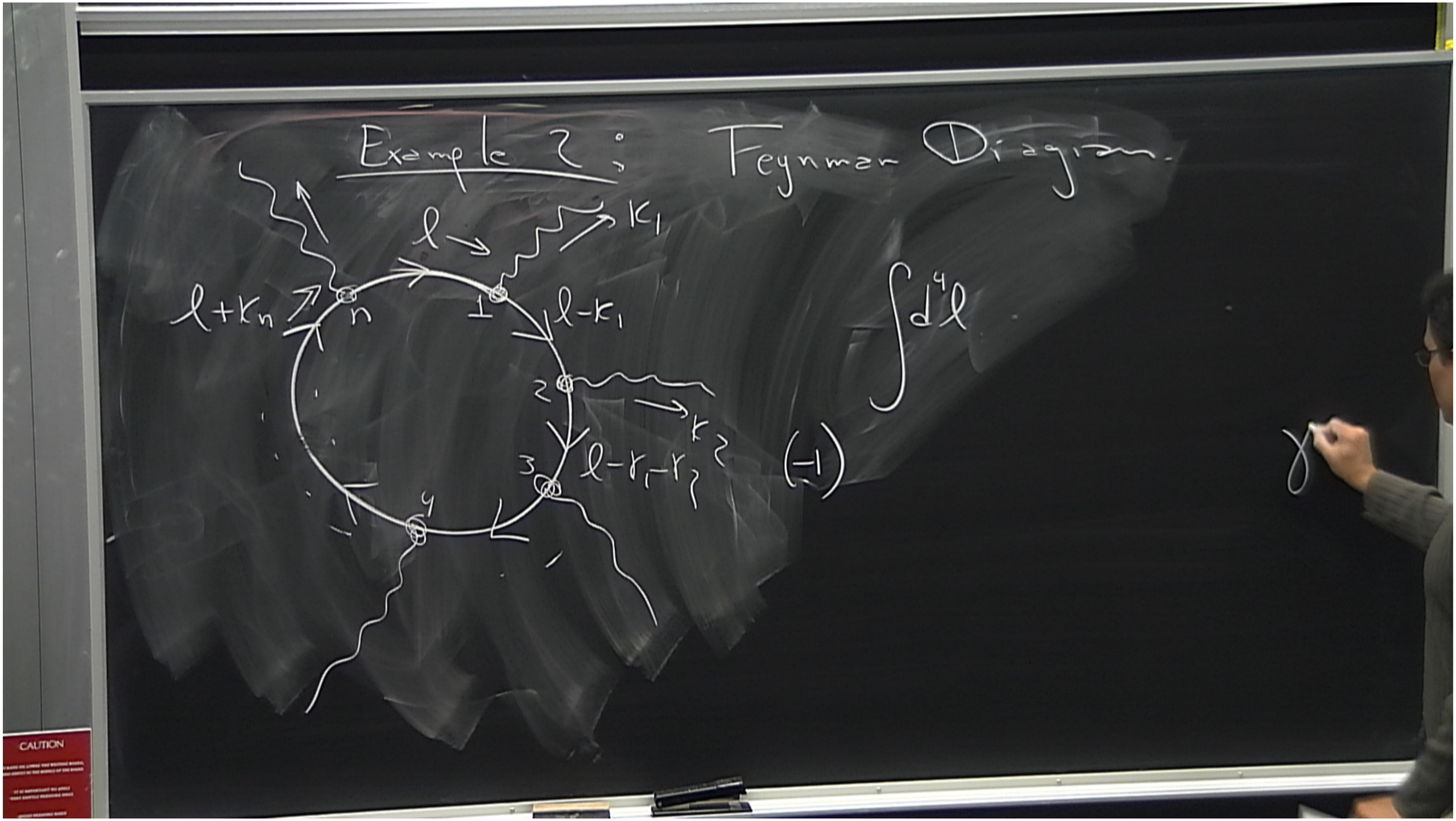
$$\left(\bar{u}^{(s_1)}_{(r_1)}\gamma^{\mu}u^{(s_2)}_{(p_2)}\right)(-ie)\frac{\left(-i\eta_{\mu\nu}\right)}{\left((p_2-r_1)^2+i\epsilon\right)}\left(\bar{u}^{(s_2)}_{(r_2)}\gamma^{\nu}u^{(s_b)}_{(p_3)}\right)(-ie)$$

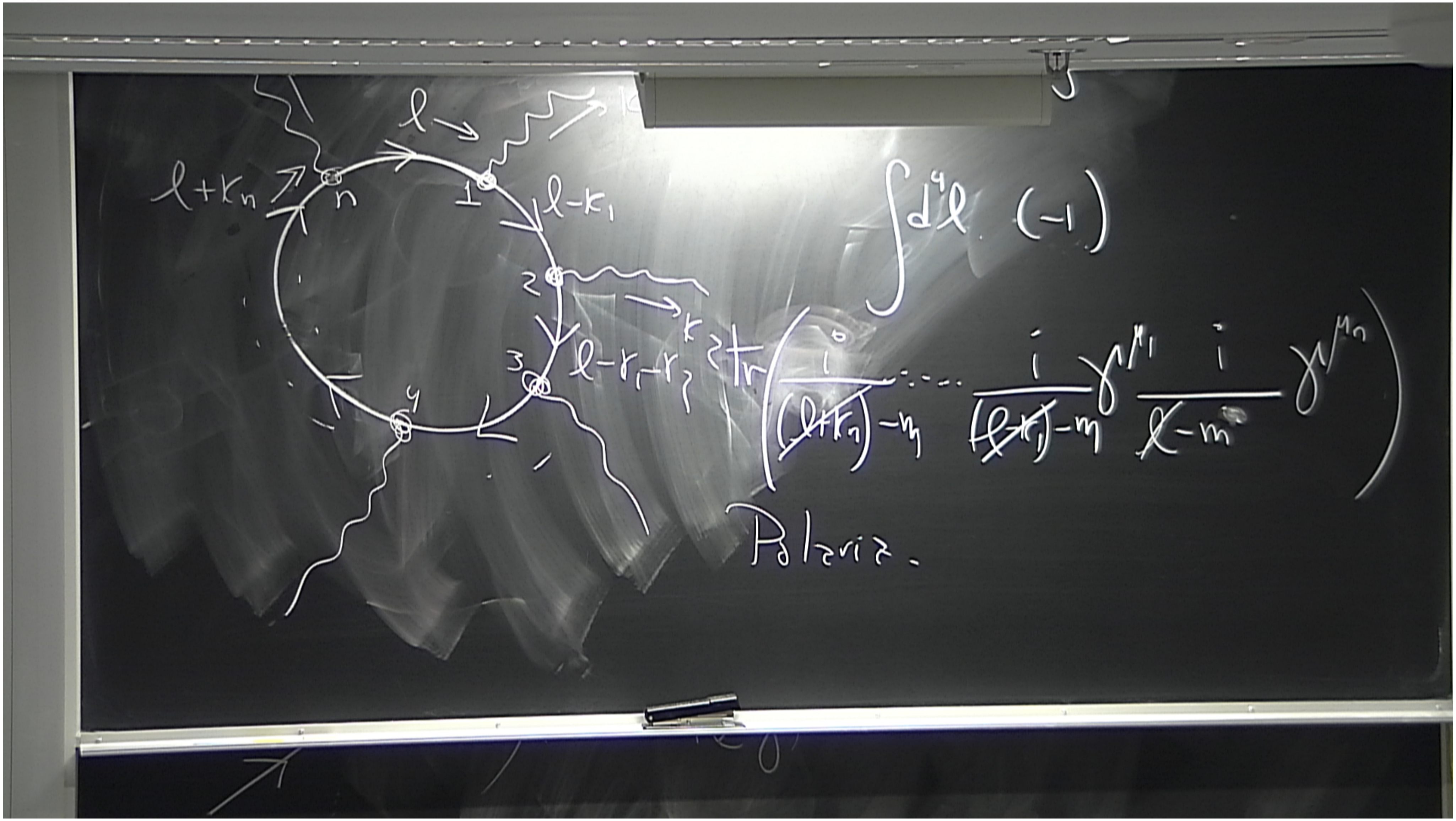
$$+(-1)(\text{Something})$$

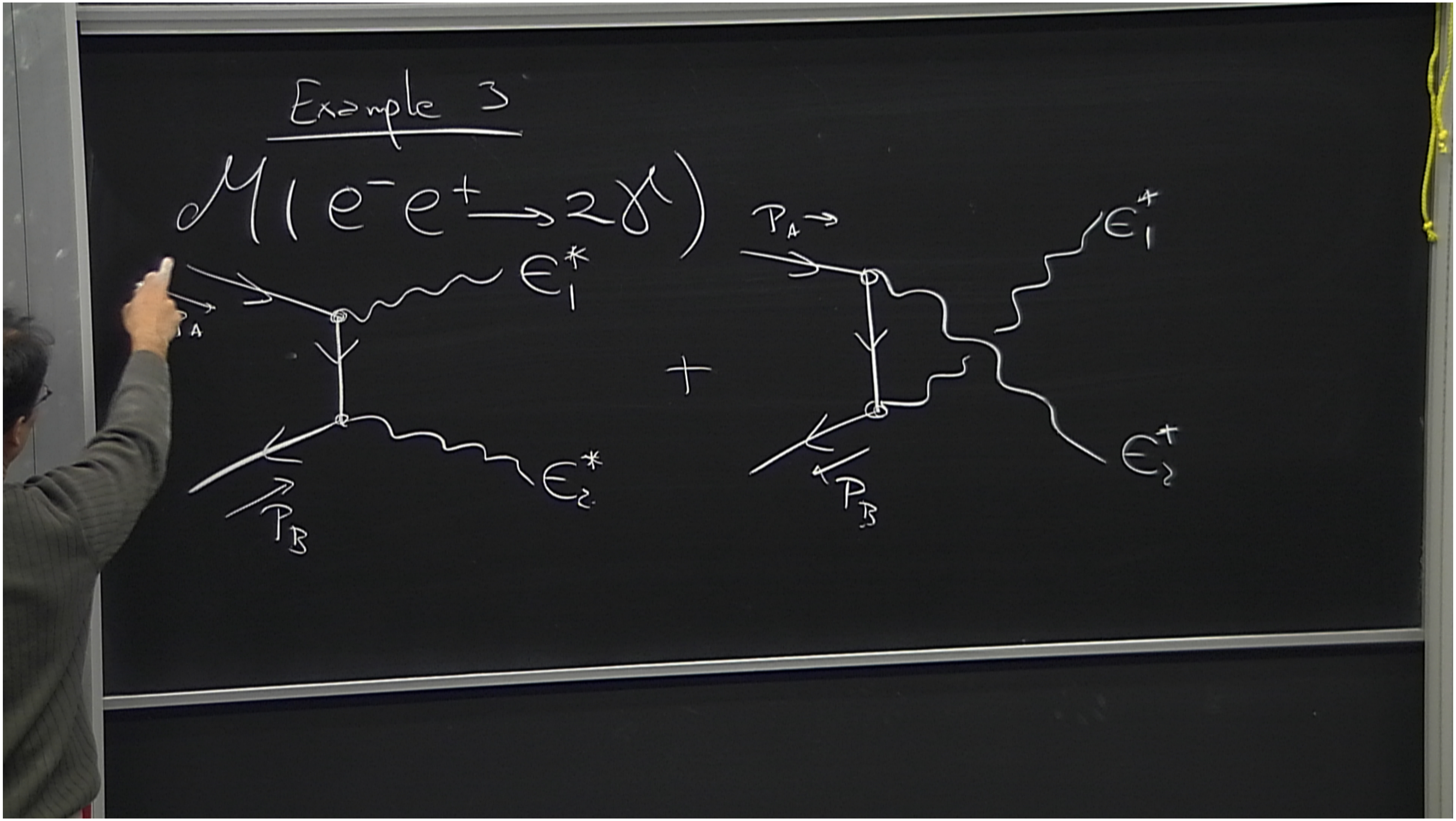

Example 2: Feynman Diagram.

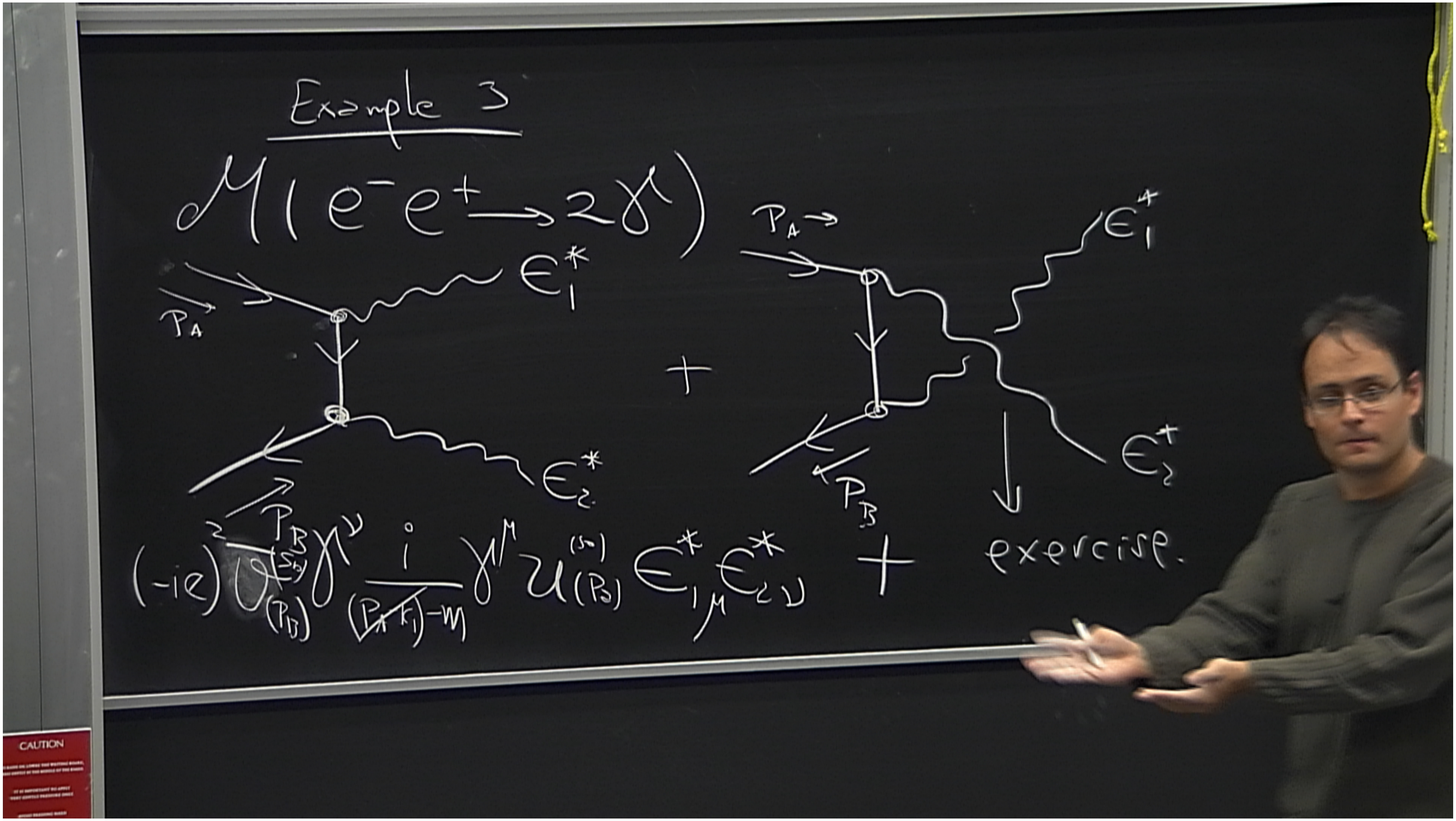


Example 2: Feynman Diagram.
ℓ
n
1
2
3
4


Example 2: Feynman Diagram.
l
k1
l+kn
n
1
l-k1
2
k2
3
l-k1-k2
4
∫d^4 l
(-1)


l →
l+k_n
n
1
l−k_1
2
k_2
3
l−k_1−k_2
4
∫d^4l (−1)
tr(i/((l+k_n)−m) ⋯ i/((l−k_1)−m) γ^μ1 i/(l−m) γ^μn)
Polariz.


Example 3
$\mathcal{M}(e^- e^+ \to 2\gamma)$
P_A
P_B
ϵ_1^*
ϵ_2^*
+
P_A
P_B
ϵ_1^*
ϵ_2^*

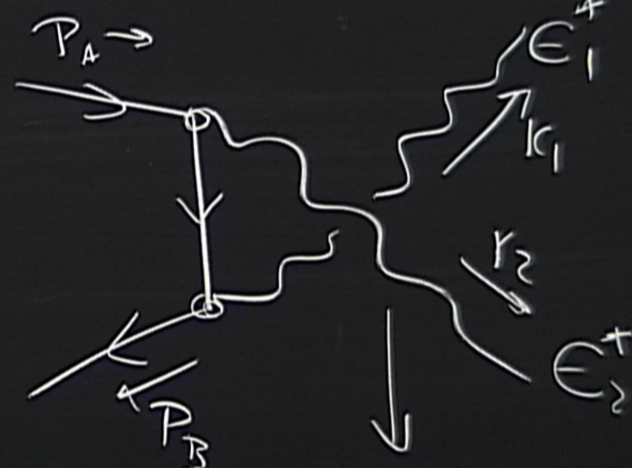

Example 3
M(e⁻e⁺ → 2γ)
P_A
ε₁*
ε₂*
P_B
+
exercise.

Example 3

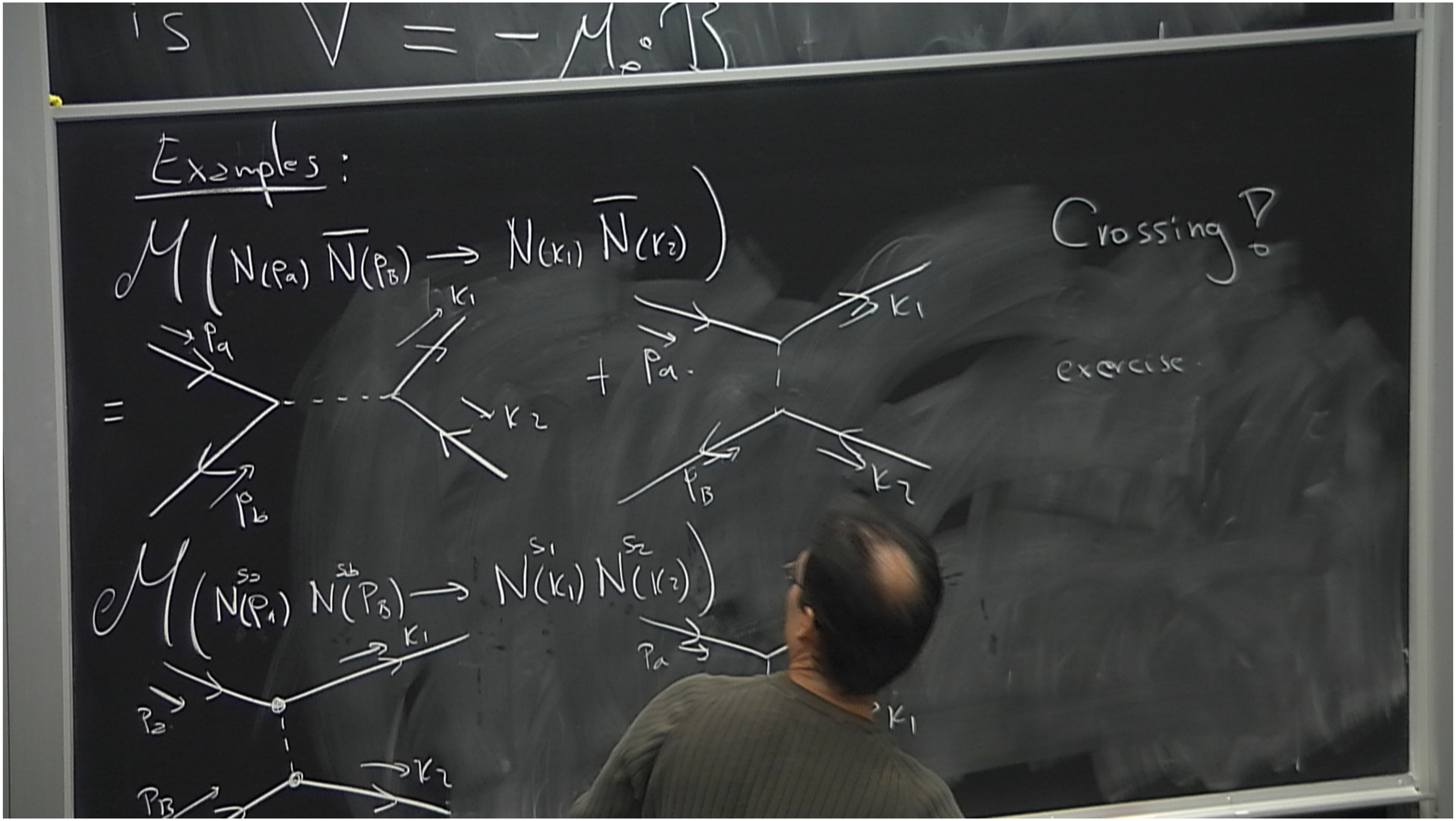
$$\mathcal{M}(e^- e^+ \to 2\gamma)$$

What happens if $\epsilon_{1\mu} \to k_{1\mu}$ $\;\to\;$ zero!

$+$



$$(-ie)^2\, \bar{v}^{(s_B)}(p_B)\, \gamma^\nu \frac{i}{(\not{p}_A - \not{k}_1) - m}\, \gamma^\mu\, u^{(s_A)}(p_A)\, \epsilon^*_{1,\mu}\, \epsilon^*_{2\nu} \; + \text{exercise.}$$


is $V = -\mu \cdot B$
Examples:
$\mathcal{M}(N(p_a)\,\bar{N}(p_B) \to N(k_1)\,\bar{N}(k_2))$
Crossing?
exercise.
$\mathcal{M}(N^{s_a}(p_a)\,N^{s_b}(p_B) \to N^{s_1}(k_1)\,N^{s_2}(k_2))$

you should be proud of being a human !?

• Recall that the potential of scattering an electron with a magnetic field $\vec{B}$.

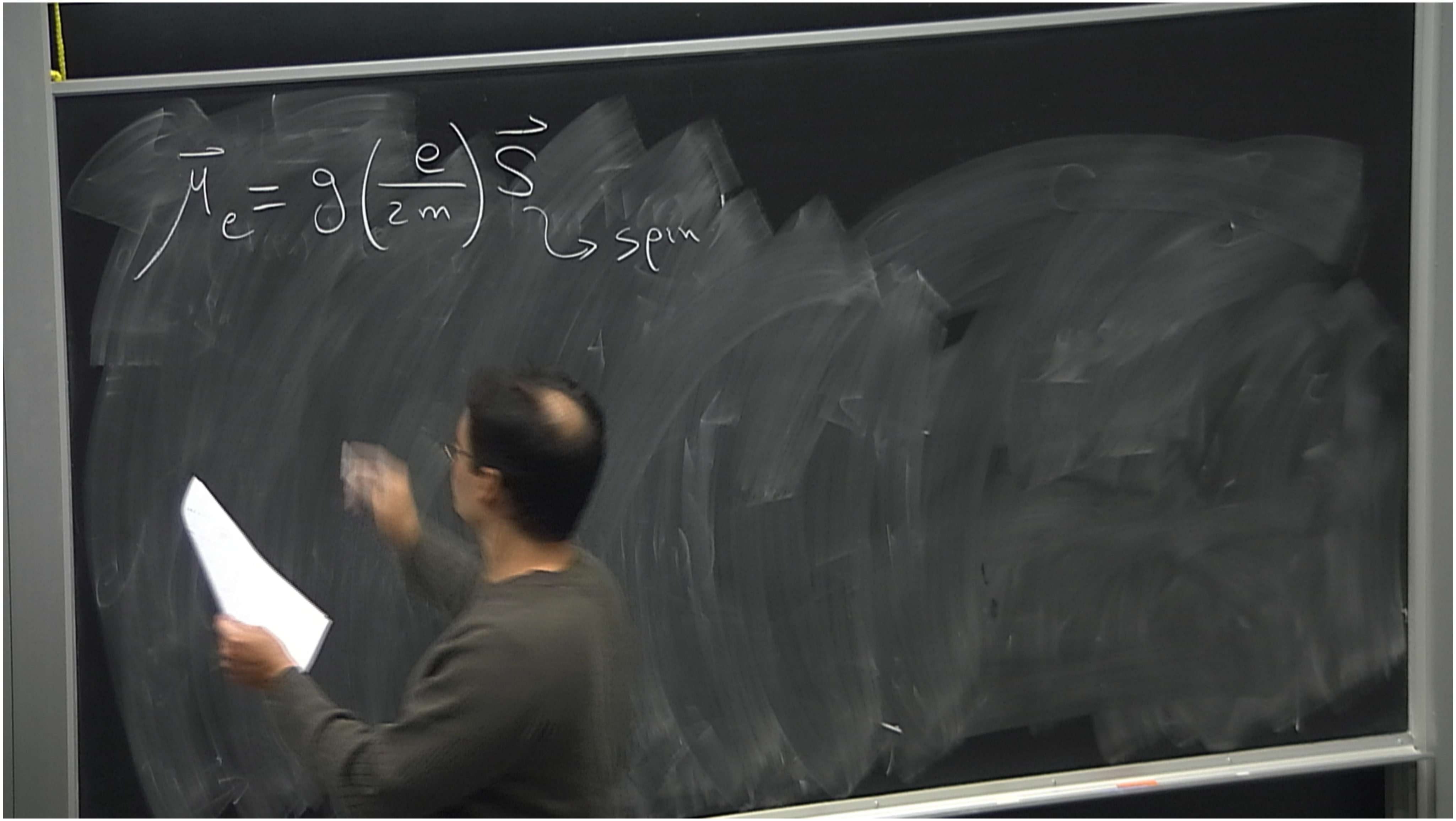
is $V = -\vec{\mu}_e \cdot \vec{B}$

$\vec{\mu}_e \longrightarrow$ Magnetic Moment.

Rule : Follow e… Fermion line and write

From Ri… Left

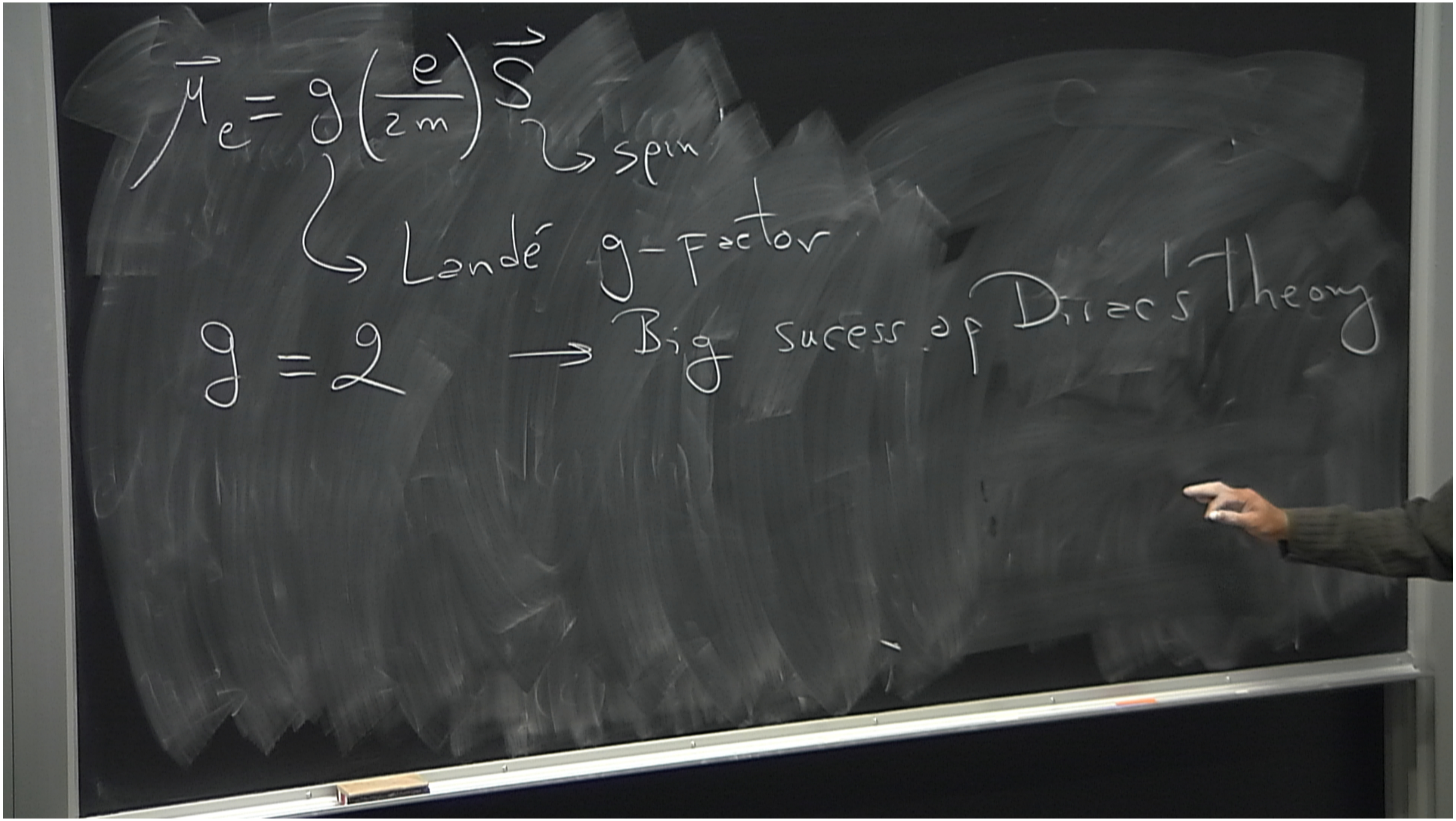
$(-ig)$

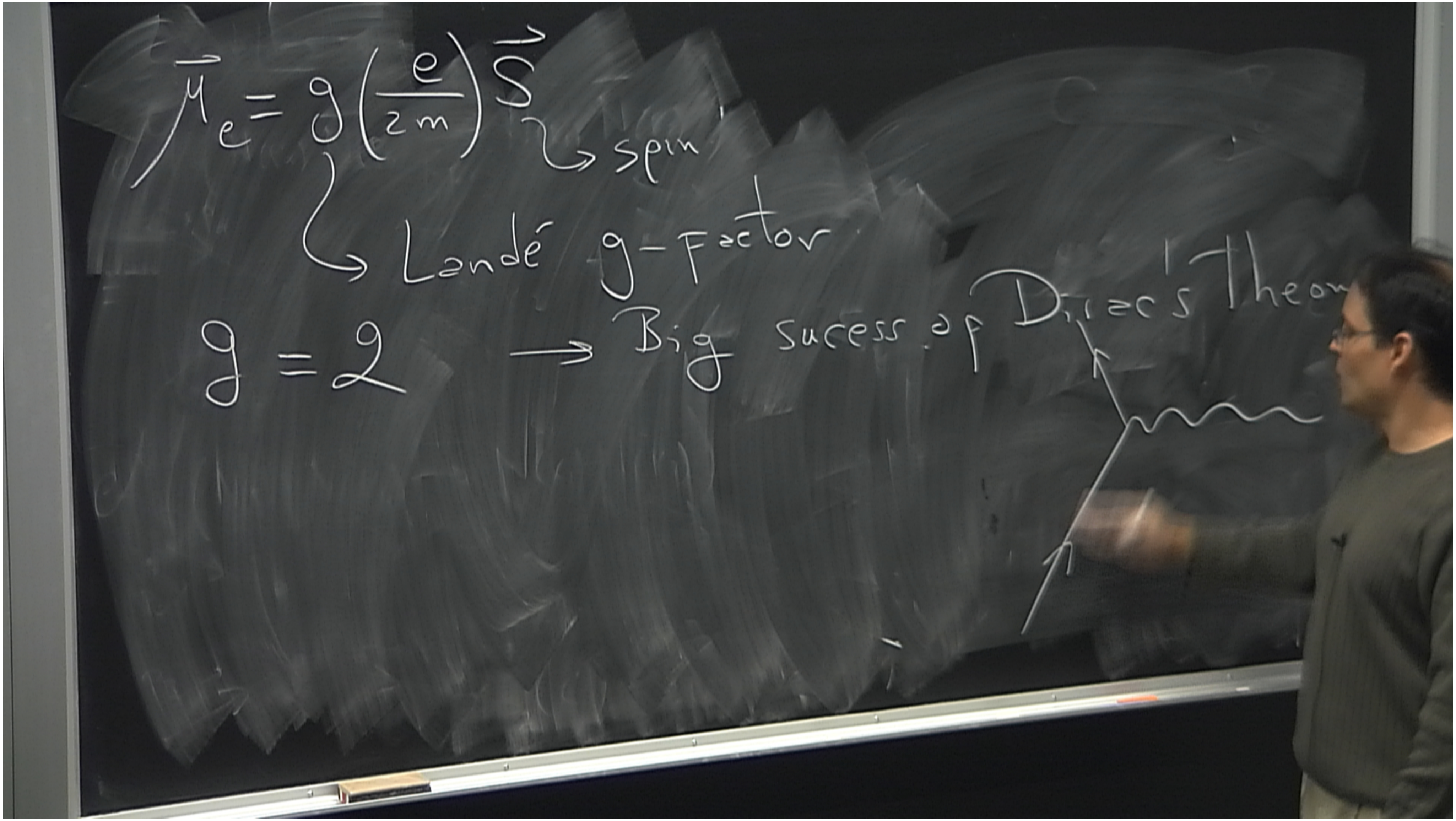

$\vec{\mu}_e = g\left(\frac{e}{2m}\right)\vec{S}$
spin

$$\vec{\mu}_e = g\left(\frac{e}{2m}\right)\vec{S}$$

$\vec{S} \to$ spin

$g \to$ Landé g-factor


$\vec{\mu}_e = g\left(\frac{e}{2m}\right)\vec{S}$
spin
Landé g-factor
$g = 2$ → Big sucess of Dirac's Theory



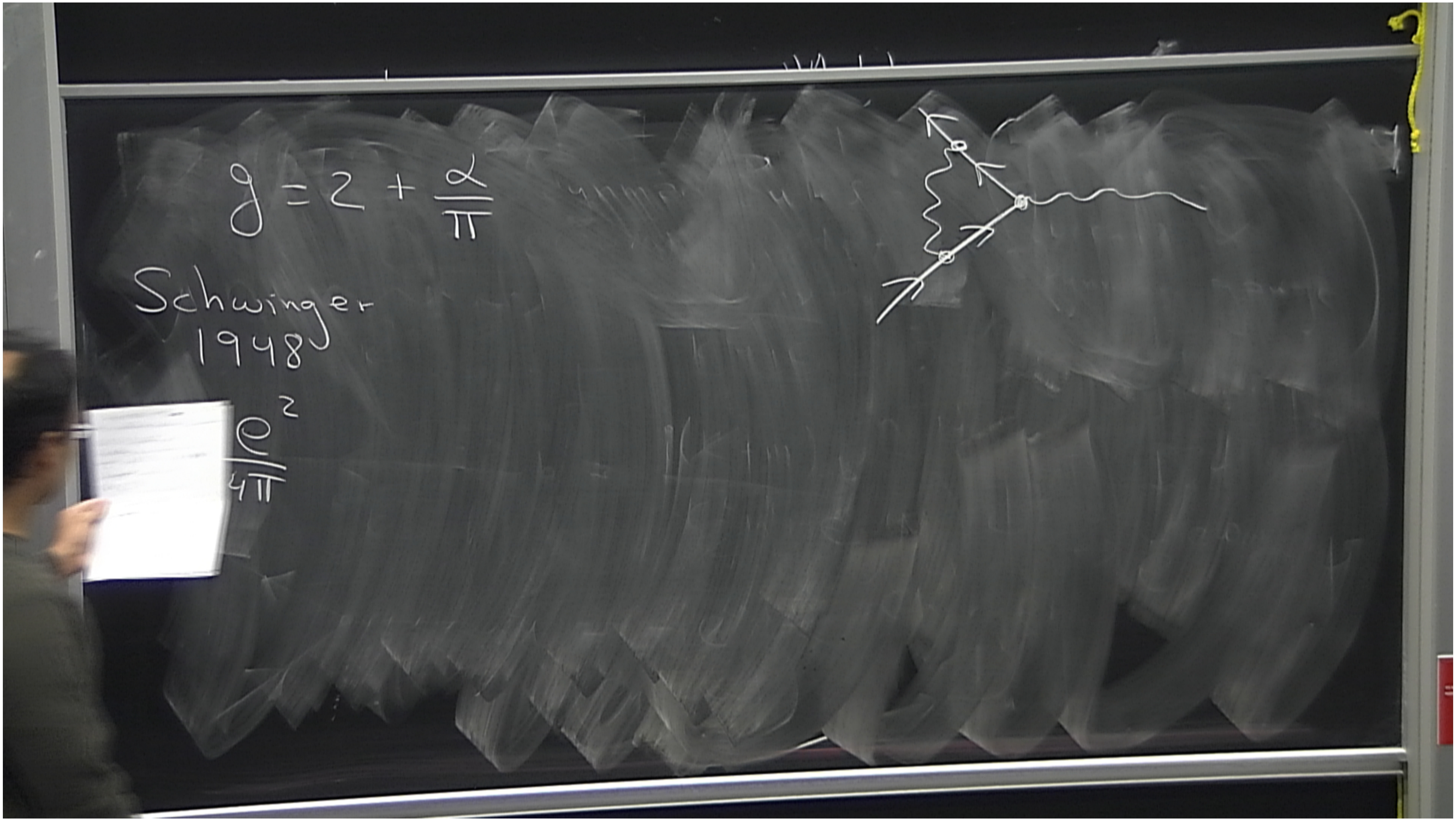
$\vec{\mu}_e = g\left(\frac{e}{2m}\right)\vec{S}$
spin
Landé g-factor
$g = 2$
Big sucess of Dirac's Theo

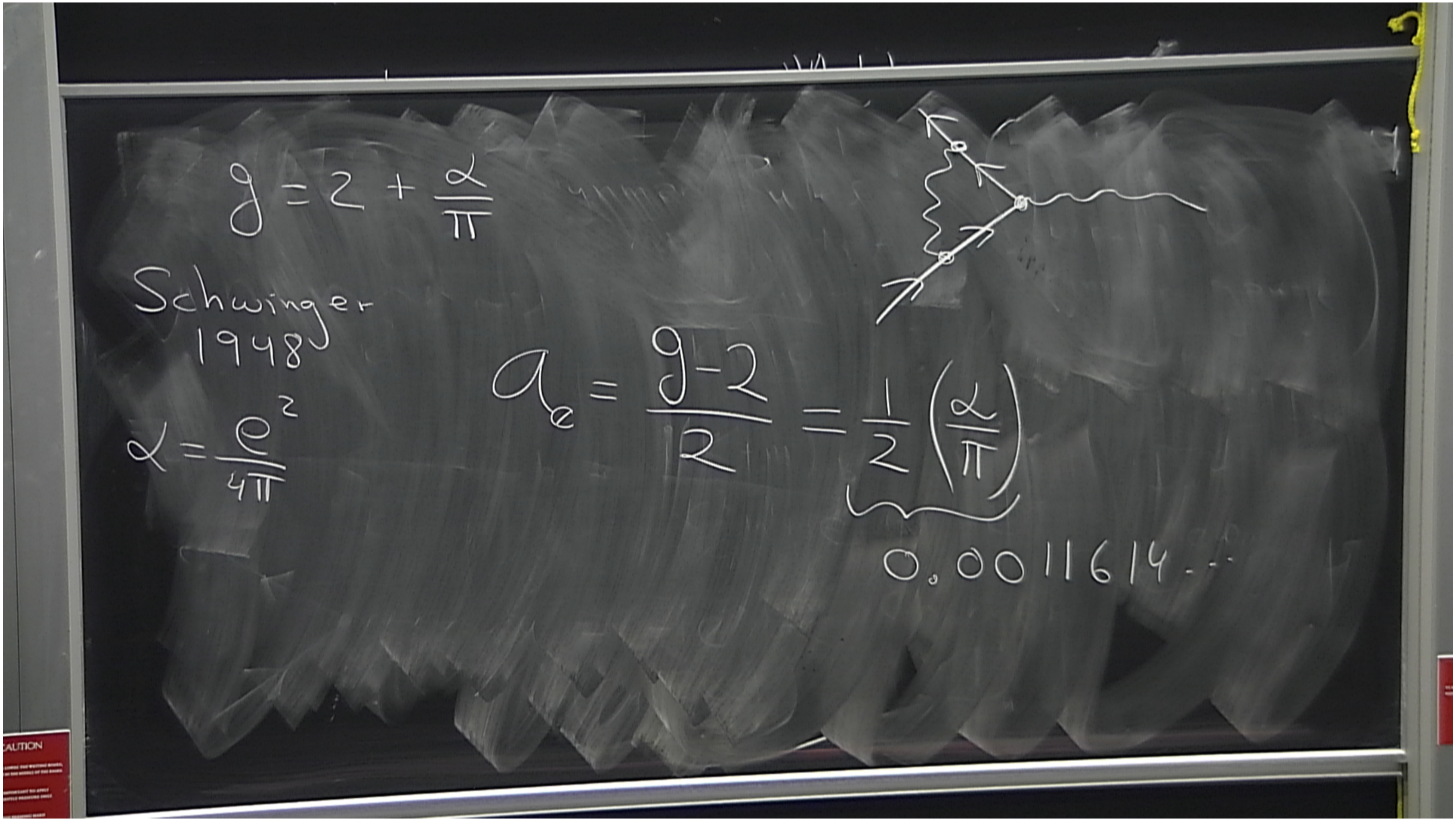
$$\vec{\mu}_e = g\left(\frac{e}{2m}\right)\vec{S}$$

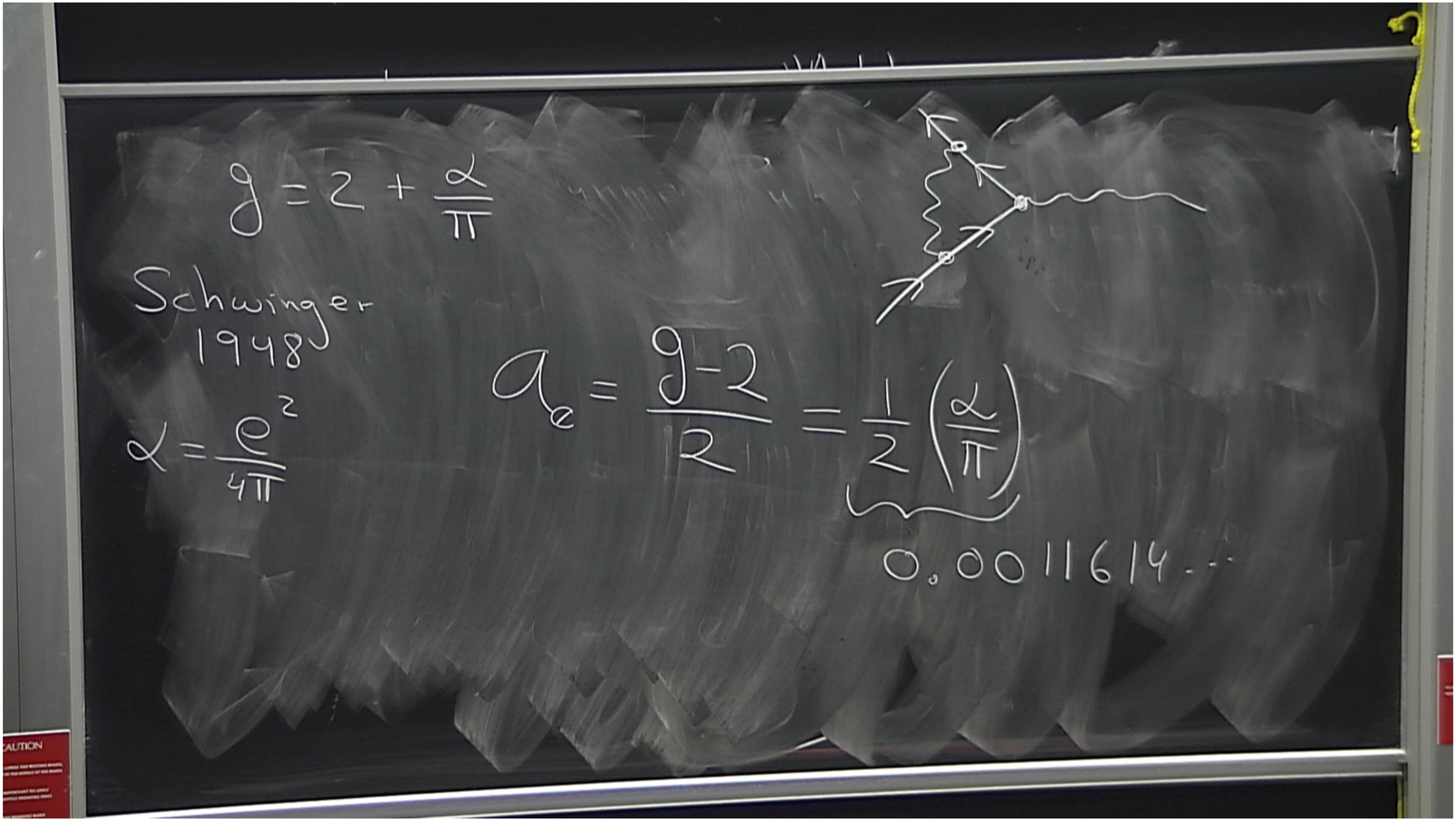
$\vec{S} \to$ spin

$g \to$ Landé g-factor

$g = 2 \longrightarrow$ Big sucess of Dirac's Theory


g = 2 + α/π
Schwinger
1948
e²/4π


$g = 2 + \frac{\alpha}{\pi}$
Schwinger 1948
$\alpha = \frac{e^2}{4\pi}$
$a_e = \frac{g-2}{2} = \underbrace{\frac{1}{2}\left(\frac{\alpha}{\pi}\right)}$
0.0011614...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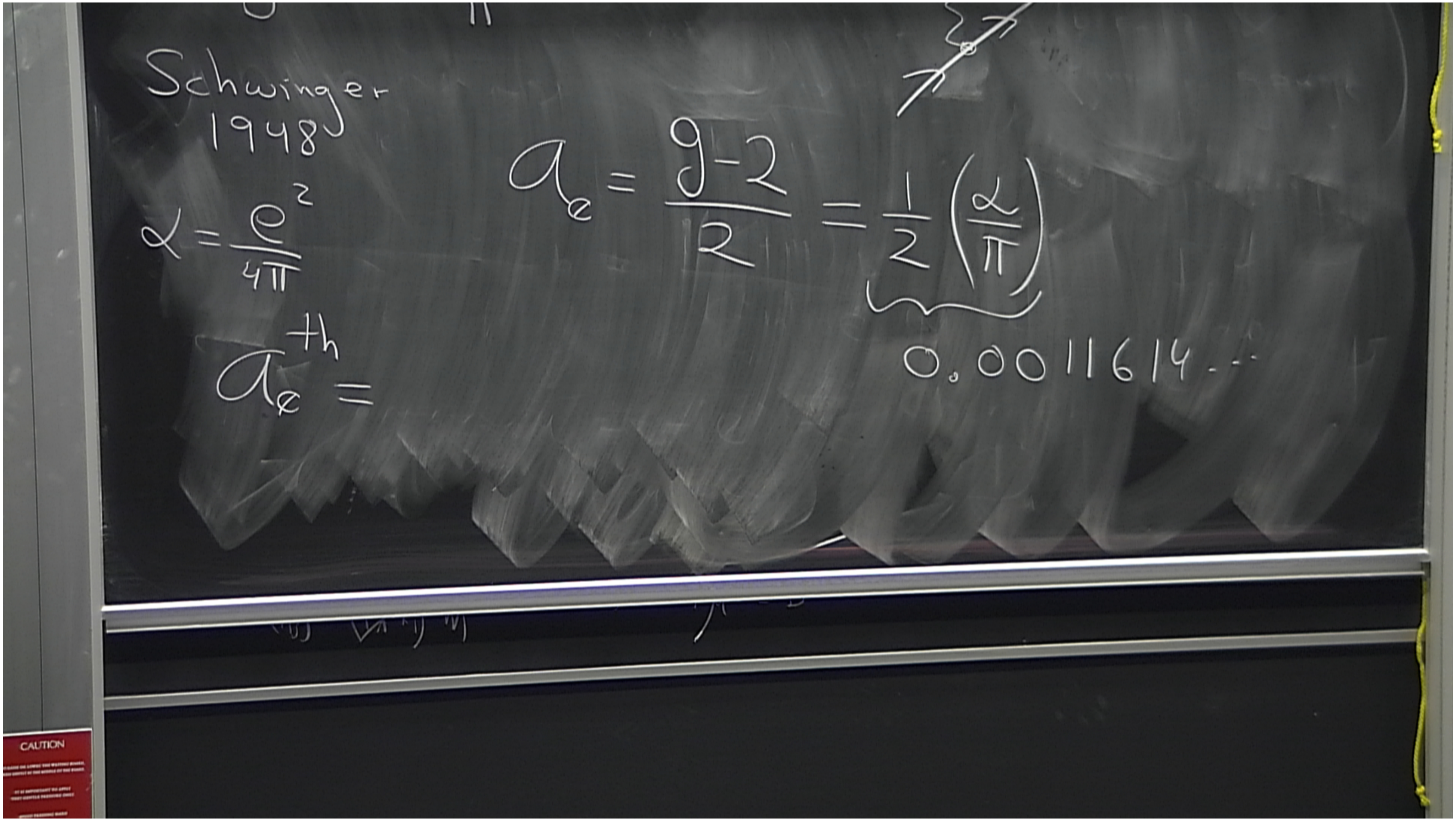
Schwinger
1948
$\alpha = \frac{e^2}{4\pi}$
$a_e = \frac{g-2}{2} = \underbrace{\frac{1}{2}\left(\frac{\alpha}{\pi}\right)}$
0.0011614...
$a_e^{th} =$
CAUTION

1948

$$\alpha = \frac{e^2}{4\pi}$$

$$a_e = \frac{g-2}{2} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)$$

$$a_e^{th} = 1,159,652,182.79(7.7)\,10^{-12}$$

$$a_e^{exp.} = 1,159,652,180.73(0.28)\times 10^{-12}$$

$$(-ie)\,\bar{u}^{(s_b)}_{(p_b)}\,\gamma^\nu \frac{i}{(\not p_a - \not k_1) - m}\,\gamma^\mu\, u^{(s_a)}_{(p_a)}\,\epsilon^*_{1,\mu}\,\epsilon^*_{2,\nu} \;+\; \text{exercise.}$$

CAUTION