Title: 13/14 PSI - Quantum Field Theory I - Lecture 12

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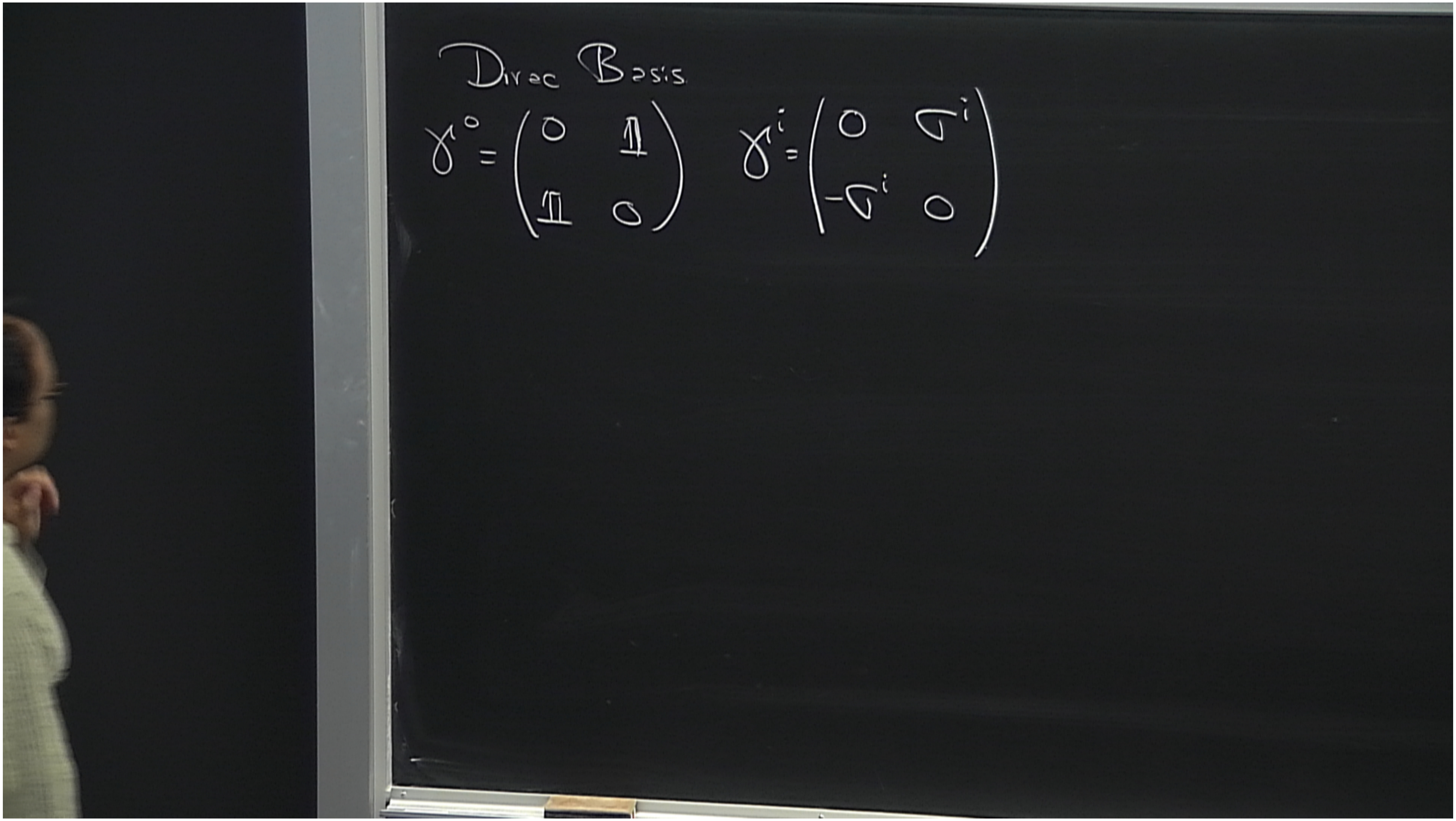
Abstract:


Dirac Basis
$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$
$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$

Dirac Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$$

Weyl Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$$

$$\not{\partial} = \gamma^\mu \partial_\mu$$

Weyl Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi \qquad \not{\partial} = \gamma^\mu \partial_\mu$$

Solve Dirac's Theory : Plane waves.

$$\psi^{(+)}(t,x) = \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}}\, u(\vec{k})\, e^{-ik\cdot x}$$

$$a,b = 1,2,3,4$$

$$\gamma^\mu_{ab}$$

$$k^0 = \omega_k > 0$$

$$(i\not\partial - m)\psi = 0$$

Field equation:

$$\Rightarrow (\not k - m\mathbb{1})\, u(k) = 0$$

$(i\not\partial - m)\Psi = 0 \quad \Rightarrow \quad (\not k - m\mathbb{1})\, u(\vec k) = 0$

Interpretation for m.

$-(\not k + m)(\not k - m)\, u(\vec k) = 0 \quad \rightarrow \quad (k^2 - m^2)\mathbb{1}\, u(\vec k) = 0$

$(\not k \not k - m^2)\, u(\vec k) = 0$

If $u \neq 0$

need $k^2 = m^2$

$$\not k \not k = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \underbrace{\{\gamma^\mu, \gamma^\nu\}}_{2\eta^{\mu\nu}\mathbb{1}} = k^2 \mathbb{1}$$

Want to Solve $(\not{K} - m)\, u(\vec{k}) = 0$

Rest frame $K^\mu = (m, \vec{0})$

$$m(\gamma^0 - \mathbb{1})\, u(k^*) = 0$$

Want to Solve $(\not{k} - m)\,u(k) = 0$

Rest frame $k^\mu = (m, \vec{0})$

$$m(\gamma^0 - \mathbb{1})\,u(k^*) = 0 \quad \Rightarrow \quad \begin{pmatrix} -\mathbb{1} & \mathbb{1} \\ \mathbb{1} & -\mathbb{1} \end{pmatrix} u(k^*) = 0$$

Want to Solve $(\not{k}-m)\,u(k)=0$

Rest frame $k^\mu=(m,\vec{0})$

$$m(\gamma^0-\mathbb{1})\,u(k^*)=0 \quad\Rightarrow\quad \begin{pmatrix}-\mathbb{1}_{2\times2} & \mathbb{1}_{2\times2}\\ \mathbb{1}_{2\times2} & -\mathbb{1}_{2\times2}\end{pmatrix}u(k^*)=0$$

Rank is 2.

Want to Solve $(\not{K} - m)\, u(\vec{k}) = 0$

Rest frame $K^{*\mu} = (m, \vec{0})$

$$m(\gamma^0 - \mathbb{1})\, u(k^*) = 0 \Rightarrow \begin{pmatrix} -\mathbb{1}_{2\times2} & \mathbb{1}_{2\times2} \\ \mathbb{1}_{2\times2} & -\mathbb{1}_{2\times2} \end{pmatrix} u(k^*) = 0$$

rest frame

Rank is 2 $\Rightarrow$ corank is 2. Number of eigenvectors with zero eigenvalue.

Want to Solve $(\not{k}-m)\,u(k)=0$

Rest frame $k^{*\mu}=(m,\vec{0})$ — rest frame

$$m(\gamma^0-\mathbb{1})\,u(k^*)=0 \Rightarrow \begin{pmatrix} -\mathbb{1}_{2\times 2} & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & -\mathbb{1}_{2\times 2} \end{pmatrix} u(k^*)=0$$

Rank is 2 $\Rightarrow$ corank is 2. Number of eigenvectors with zero eigenvalue.

$$u \in \left\{ \underbrace{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}}_{u^1},\ \underbrace{\begin{pmatrix}0\\1\\0\\1\end{pmatrix}}_{u^2} \right\} \Rightarrow u^s(k^*)=\sqrt{m}\begin{pmatrix}\xi^1\\ \xi^1\end{pmatrix} \qquad \xi^1=\begin{pmatrix}1\\0\end{pmatrix}\ \ \xi^2=\begin{pmatrix}0\\1\end{pmatrix}$$

Boost to get a general solution:

Exercise: $k^{*\mu} \longrightarrow k^{\mu}$.

$$u^s(k) = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\xi^s \\ \sqrt{k\cdot\tilde{\sigma}}\,\xi^s \end{pmatrix}$$

Boost to get a general solution:

Exercise: $k^{*\mu} \longrightarrow k^{\mu}$.

$$u^s(k) = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\xi^s \\ \sqrt{k\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$$

$$\bar{u}^s(k)\,u^r(k) = 2m\,\delta^{sr}$$

$$\bar{u}(k) = u^{\dagger}(k)\,\gamma^0$$

Boost to get a general solution:

Exercise: $k^{*\mu} \longrightarrow k^{\mu}$.

$$u^s(k) = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\xi^s \\ \sqrt{k\cdot\tilde{\sigma}}\,\xi^s \end{pmatrix}$$

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$$\bar{u}(k) = u^\dagger(k)\,\gamma^0$$

$$u^s \bar{u}^s$$

Boost to get a general solution:

Exercise: $k^{*\mu} \longrightarrow k^{\mu}$.

$$u^s(\vec{k}) = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\xi^s \\ \sqrt{k\cdot\tilde{\sigma}}\,\xi^s \end{pmatrix}$$

$$\bar{u}^s(k)\,u^r(k) = 2m\,\delta^{sr}$$

$$\bar{u}(k) = u^\dagger(k)\,\gamma^0$$

$$\sum_{s=1}^{2} u^s_a(k)\,\bar{u}^s_b(k) = (\not{k} + m)_{ab}$$

Boost to get a general solution:

Exercise: $k^{*\mu} \longrightarrow k^{\mu}$.

$$u^s(k) = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\xi^s \\ \sqrt{k\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$$

$$\bar{u}^s(k)\, u^r(k) = 2m\,\delta^{sr}$$

$$\bar{u}(k) = u^\dagger(k)\,\gamma^0$$

$$\boxed{\sum_{s=1}^{2} u^s_a(k)\,\bar{u}^s_b(k) = (\not{k} + m)_{ab}}$$

Negative energy solution:

$$\psi^{(-)}_{(t,x)} = \int \frac{d^3\bar{k}}{\sqrt{2\omega_k}}\, v_{(k)}\, e^{ik\cdot x}$$

$$v^s_{(k)} = \begin{pmatrix} \sqrt{k\cdot\sigma}\,\eta^s \\ -\sqrt{k\cdot\bar{\sigma}}\,\eta^s \end{pmatrix} \qquad \eta^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \eta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\bar{v}^s_{(k)}\, v^r_{(k)} = -2m\,\delta^{rs}$$

$$\boxed{\sum_{s=1}^{2} v^s_{a(k)}\, \bar{v}^s_{b(k)} = (\not{k} - m)_{ab}}$$

Quantization (Canonical)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi = \underbrace{\bar{\psi}\, i\gamma^0}\partial_0\psi + \bar{\psi}(i\gamma^i\partial_i - m)\psi$$

$$= \frac{1}{2} i\psi^\dagger\dot{\psi} - \frac{1}{2} i\dot{\psi}^\dagger\psi \qquad i\psi^\dagger\partial_0\psi$$

$$+ \bar{\psi}(i\gamma^i\partial_i - m)\psi$$

(Canonical)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi = \bar{\psi} i\gamma^0\partial_0\psi + \bar{\psi}(i\gamma^i\partial_i - m)\psi$$

$$= \frac{1}{2} i\psi^\dagger\dot{\psi} - \frac{1}{2} i\dot{\psi}^\dagger\psi \qquad i\psi^\dagger\partial_0\psi$$

$$+ \bar{\psi}(i\gamma^i\partial_i - m)\psi$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\psi}} = \frac{1}{2} i\psi^\dagger = \Pi_\psi \qquad \frac{\partial\mathcal{L}}{\partial\dot{\psi}^\dagger} = -\frac{1}{2} i\psi = \Pi_{\psi^\dagger}$$

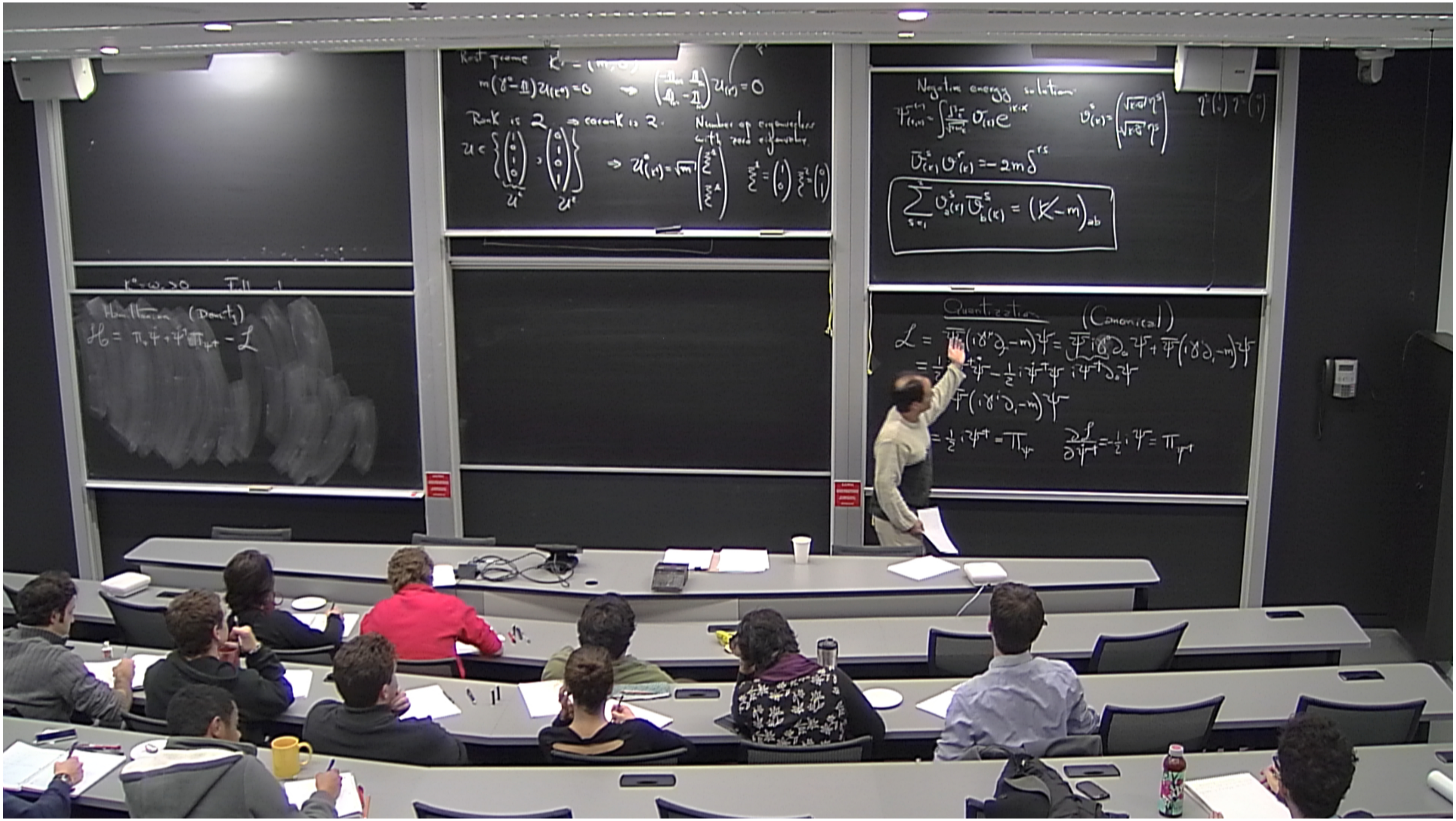
Quantization (Canonical)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi = \underbrace{\bar{\psi}\, i\gamma^0 \partial_0 \psi}_{i\psi^\dagger \partial_0 \psi} + \bar{\psi}(i\gamma^i \partial_i - m)\psi$$

$$= \frac{1}{2} i \psi^\dagger \dot{\psi} - \frac{1}{2} i \dot{\psi}^\dagger \psi$$

$$+ \bar{\psi}(i\gamma^i \partial_i - m)\psi$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{1}{2} i \psi^\dagger = \Pi_\psi \qquad \frac{\partial \mathcal{L}}{\partial \dot{\psi}^\dagger} = -\frac{1}{2} i \psi = \Pi_{\psi^\dagger}$$



$k^0 = \omega_k > 0$ Field

Hamiltonian (Density)

$$\mathcal{H} = \Pi_\psi \dot{\psi} + \dot{\psi}^\dagger \Pi_{\psi^\dagger} - \mathcal{L}$$

Hamiltonian (Density)

$$\mathcal{H} = \Pi_\psi \dot{\psi} + \dot{\psi}^\dagger \Pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi}(-i\gamma^i \partial_i + m)\psi$$

Equal time commutation relations

$$\left[\psi_a(t,\vec{x}),\ \Pi_b(t,\vec{y})\right] = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

$$\Pi_b(t,\vec{y}) \to \psi^\dagger_b(t,\vec{y})$$

Hamiltonian (Density)

$$\mathcal{H} = \pi_\psi \dot{\psi} + \dot{\psi}^\dagger \pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi}(-i\gamma^i\partial_i + m)\psi$$

Equal time commutation relations

$$\Big[\psi_a(t,\vec{x}),\ \underset{\displaystyle\downarrow\atop\displaystyle\psi^\dagger_b(t,\vec{y})}{\pi_b(t,\vec{y})}\Big] = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

$$[\psi_a, \psi_b] = 0$$

$$[\psi_a^\dagger, \psi_b^\dagger] = 0$$

$\psi_b(t,\vec{y})$

$[\psi_a, \psi_b] = 0$

$[\psi_a^+, \psi_b^+] = 0$

$$\psi_a^{(+)}(t,\vec{x}) = \int u\, a\, e \cdots \qquad \psi^{(-)}(t,\vec{x}) = \int \cdots v\, a^+ \cdots$$

$$[a(\vec{k}), a(\vec{k}')] = 0 \qquad [a^+(\vec{k}), a^+(\vec{k}')] = 0.$$

$\psi_b(t,\vec{y})$ $\quad [\psi_a, \psi_b] = 0$

$[\psi_a^\dagger, \psi_b^\dagger] = 0$

$\psi_a^{(+)}(t,\vec{x}) = \int u\, a\, e \cdots \qquad \psi^{(-)}(t,\vec{x}) = \int \cdots v\, a^\dagger \cdots$

$[a(\vec{k}), a(\vec{k}')] = 0 \qquad [a^\dagger(\vec{k}), a^\dagger(\vec{k}')] = 0.$

$a^\dagger_{(\vec{k}_1)} a^\dagger_{(\vec{k}_2)} |0\rangle = |\vec{k}_1, \vec{k}_2\rangle = |\vec{k}_2\, \vec{k}_1\rangle$

Exercise: Keep going $\Longrightarrow$ Violates causality!

Replace commutators by anticommutators.

$$\{\psi_a(t,\vec{x}),\psi_b^\dagger(t,\vec{y})\} = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

$$\{\psi_a(t,\vec{x}),\psi_b(t,\vec{y})\} = 0$$

Replace commutators by anticommutators.

$$\{\psi_a(t,\vec{x}),\ \psi_b^\dagger(t,\vec{y})\} = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

$$\{\psi_a(t,\vec{x}),\ \psi_b(t,\vec{y})\} = 0 \qquad \{\psi_a^\dagger(t,\vec{x}),\ \psi_b^\dagger(t,\vec{y})\} = 0$$

Replace commutators by anticommutators.

$$\{\psi_a(t,\vec{x}), \psi_b^\dagger(t,\vec{y})\} = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

$$\{\psi_a(t,\vec{x}), \psi_b(t,\vec{y})\} = 0 \qquad \{\psi_a^\dagger(t,\vec{x}), \psi_b^\dagger(t,\vec{y})\} = 0$$

Causality $\Rightarrow$ $\mathcal{O}_{(t,\vec{x})}$ = function of an even number of fields.

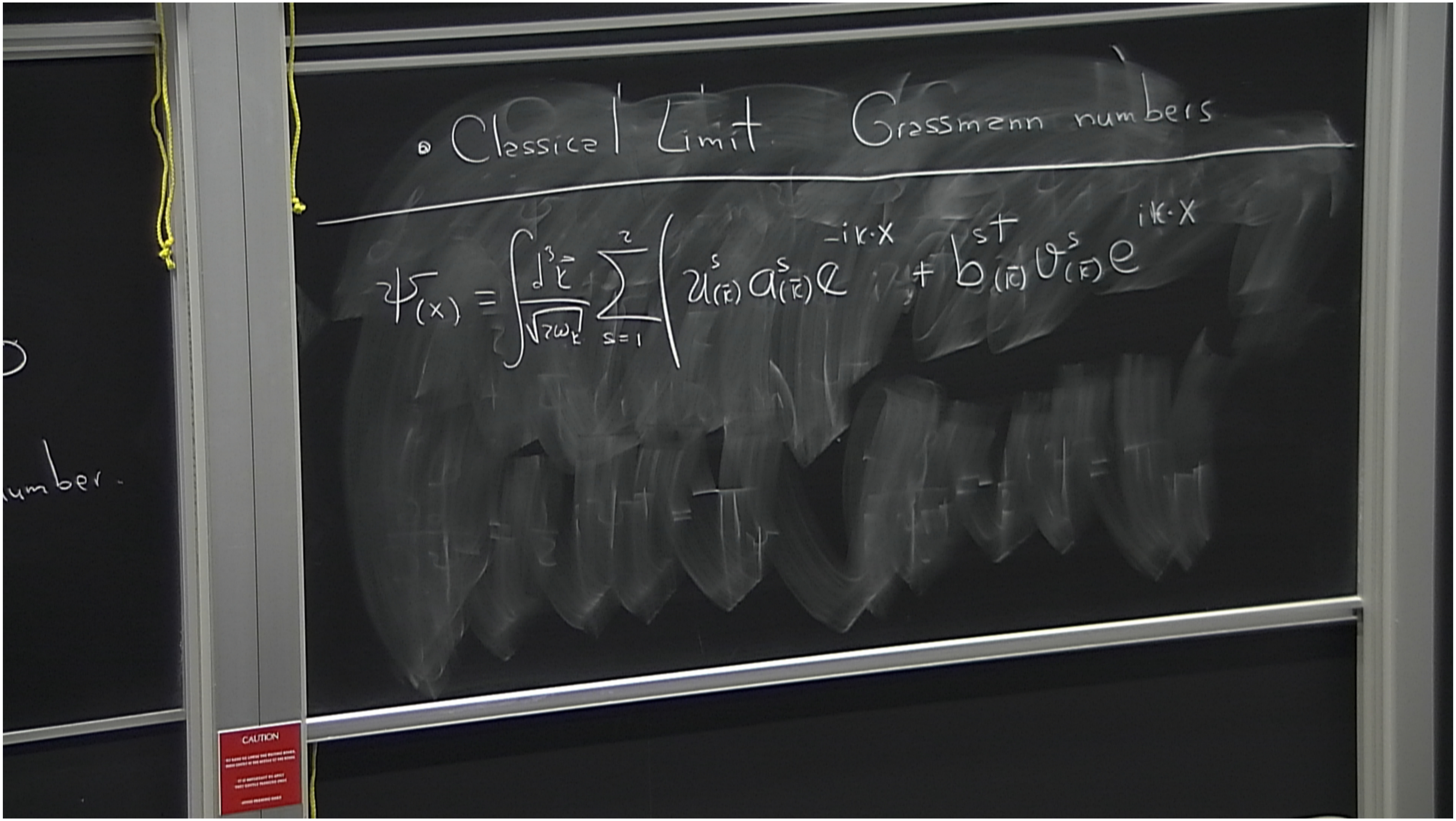
Replace commutators by anticommutators.

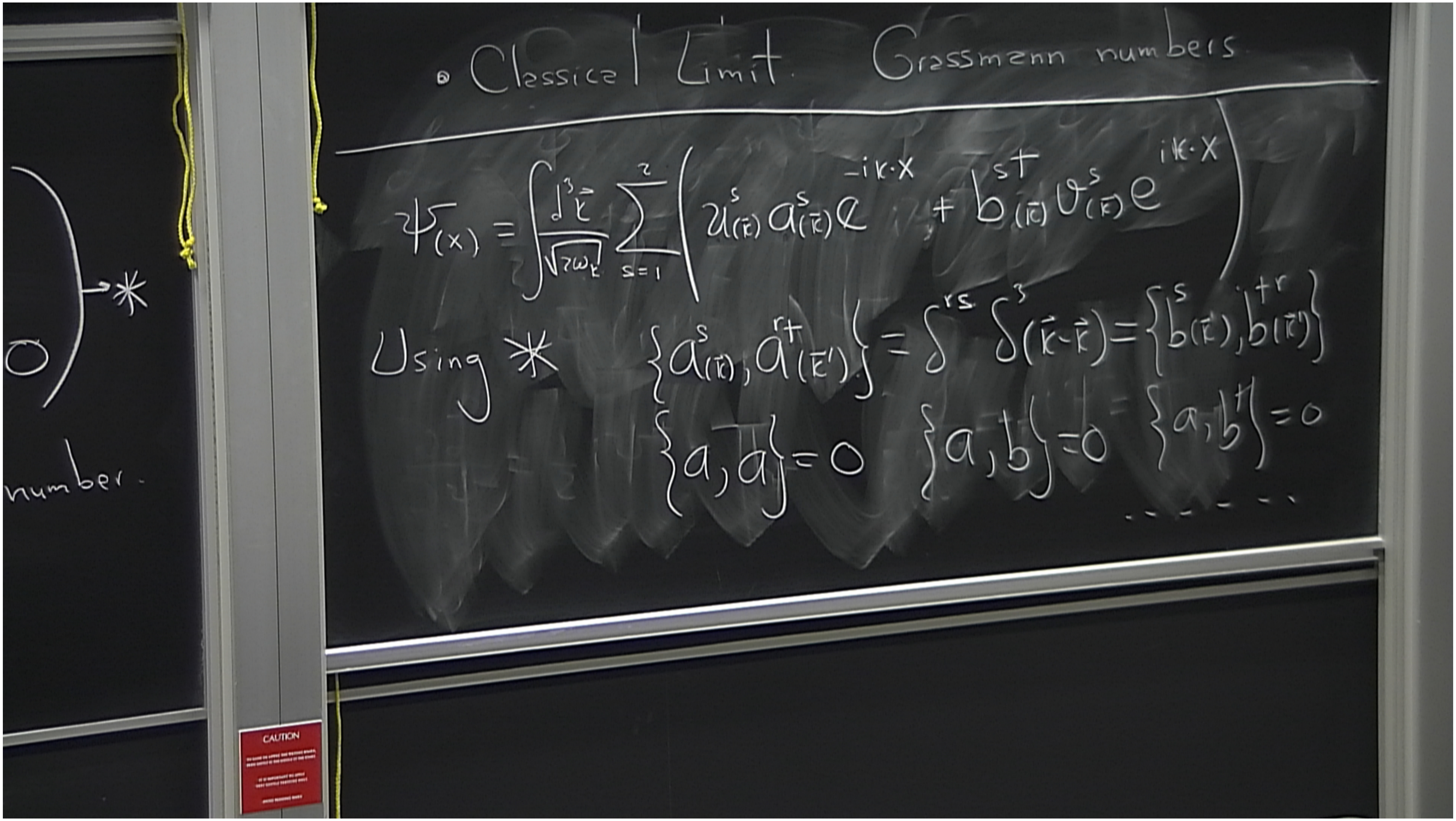
$$\{\psi_a(t,\vec{x}), \psi_b^\dagger(t,\vec{y})\} = \delta^3(\vec{x}-\vec{y})\,\delta_{ab}$$

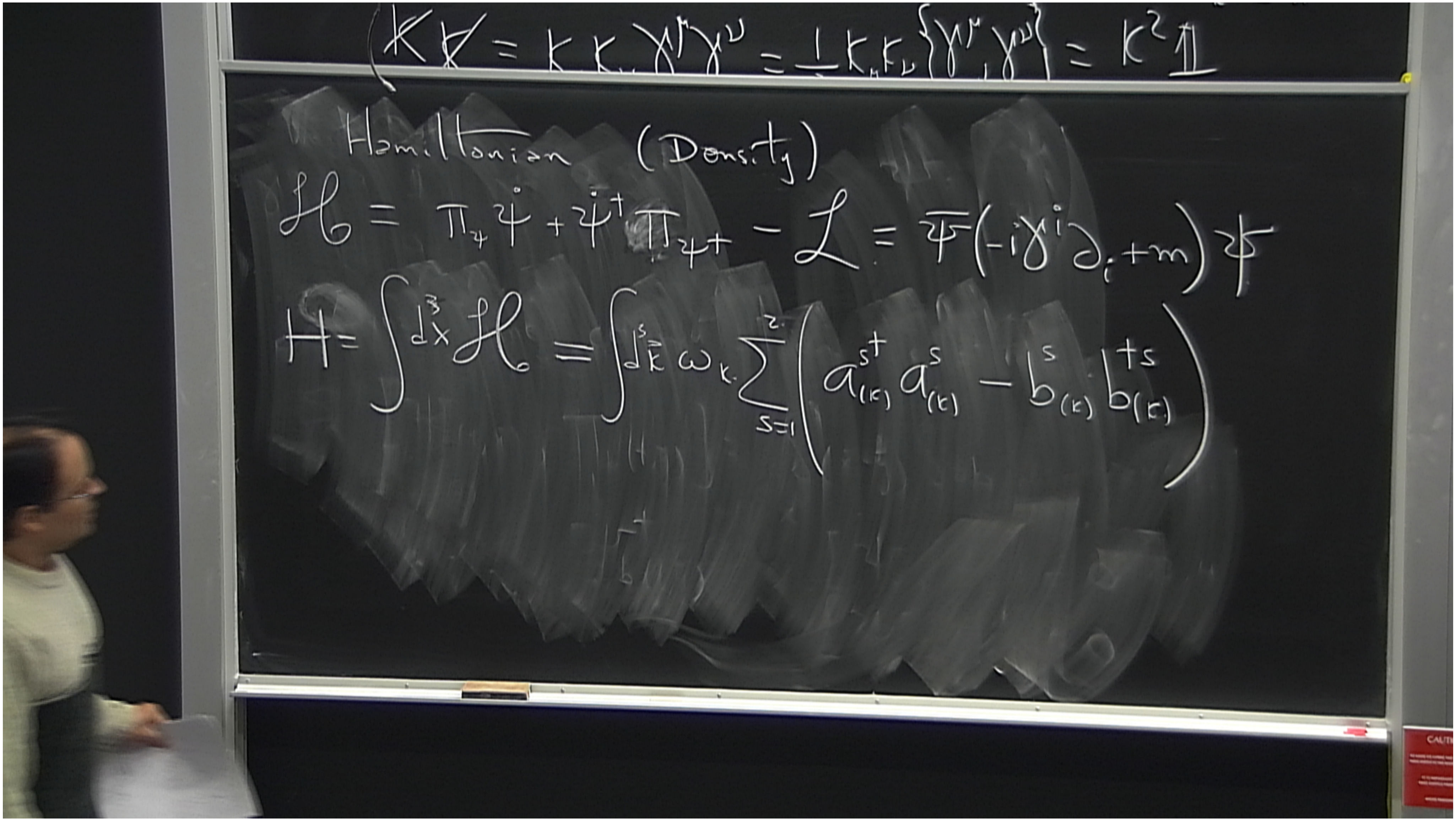
$$\{\psi_a(t,\vec{x}), \psi_b(t,\vec{y})\} = 0 \qquad \{\psi_a^\dagger(t,\vec{x}), \psi_b^\dagger(t,\vec{y})\} = 0$$

Causality $\Rightarrow$ $\mathcal{O}_{(t,\vec{x})}$ = function of an even number of fields. (ψ)

$$[\mathcal{O}_{(t,\vec{x})}, \tilde{\mathcal{O}}_{(t,\vec{y})}] = 0$$


• Classical Limit Grassmann numbers
$\psi_{(x)} = \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}} \sum_{s=1}^{2} \left(u^s_{(\vec{k})} a^s_{(\vec{k})} e^{-ik\cdot x} + b^{s\dagger}_{(\vec{k})} v^s_{(\vec{k})} e^{ik\cdot x} \right)$


• Classical Limit. Grassmann numbers
$\psi_{(x)} = \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}} \sum_{s=1}^{2} \left(u^s_{(\vec k)}\, a^s_{(\vec k)}\, e^{-ik\cdot x} + b^{s\dagger}_{(\vec k)}\, v^s_{(\vec k)}\, e^{ik\cdot x} \right)$
Using * $\{a^s_{(\vec k)}, a^{r\dagger}_{(\vec k')}\} = \delta^{rs}\,\delta^3(\vec k - \vec k') = \{b^s_{(\vec k)}, b^{\dagger r}_{(\vec k')}\}$
$\{a,a\}=0$ $\{a,b\}=0$ $\{a,b^\dagger\}=0$ ......
$\rightarrow *$
number.


$\not{k}\not{k} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \{\gamma^\mu, \gamma^\nu\} = k^2 \mathbb{1}$
Hamiltonian (Density)
$\mathcal{H} = \pi_\psi \dot{\psi} + \dot{\psi}^\dagger \pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi}(-i\gamma^i \partial_i + m)\psi$
$H = \int d^3x\, \mathcal{H} = \int d^3k\, \omega_k \sum_{s=1}^{2} \left(a^{s\dagger}_{(k)} a^{s}_{(k)} - b^{s}_{(k)} b^{\dagger s}_{(k)} \right)$

Hamiltonian (Density)

$$\mathcal{H} = \Pi_\psi \dot{\psi} + \dot{\psi}^\dagger \Pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi}\left(-i\gamma^i \partial_i + m\right)\psi$$

$$H = \int d^3x\, \mathcal{H} = \int d^3k\, \omega_k \sum_{s=1}^{2} \left(a^{s\dagger}_{(k)} a^{s}_{(k)} - b^{s}_{(k)} b^{\dagger s}_{(k)} \right)$$

Define $|0\rangle$ / $a^s_{(k)}|0\rangle = 0$ $\quad b^s_{(k)}|0\rangle = 0$

$$\psi^{(-)}_{(t,\vec{x})} = \int v\, a^\dagger$$

$$-b^{s}_{(k)}\, b^{\dagger s}_{(k')} = b^{\dagger s}_{(k')}\, b^{s}_{(k)} - \delta^{3}(k-k')$$

$$-b^{s}(k)\, b^{\dagger s}(k') = b^{\dagger s}(k')\, b^{s}(k) - \delta^{3}(k-k') \rightarrow -\delta^{3}(0) = -\int d^{3}x$$

$$H = \int d^{3}\vec{k}\, \omega_{k} \left(\sum_{s} a^{\dagger} a + b^{\dagger} b \right) - \int d^{3}\vec{k}\, \omega_{k}\, \delta^{(3)}(0)$$

$$-b^{s}_{(k)}\, b^{\dagger s}_{(k')} = b^{\dagger s}_{(k')}\, b^{s}_{(k)} - \delta^{3}(k-k') \rightarrow -\delta^{3}(0) = -\int d^{3}x$$

$$H = \int d^{3}\vec{k}\, \omega_{k} \left(\sum_{s} a^{\dagger} a + b^{\dagger} b \right) - 2 \int d^{3}\vec{k}\, \omega_{k}\, \delta^{(3)}(0)$$

Perhaps some sort of cancellation between Bosons & Fermions solves the CC problem?

Hilbert Space:

$$a^{\dagger s_1}_{(\bar{k}_1)} a^{\dagger s_2}_{(\bar{k}_2)} |0\rangle \equiv |e(\bar{k}_1, s_1), e(\bar{k}_2, s_2)\rangle$$

$$= -|e(\bar{k}_2, s_2), e(\bar{k}_1, s_1)\rangle$$

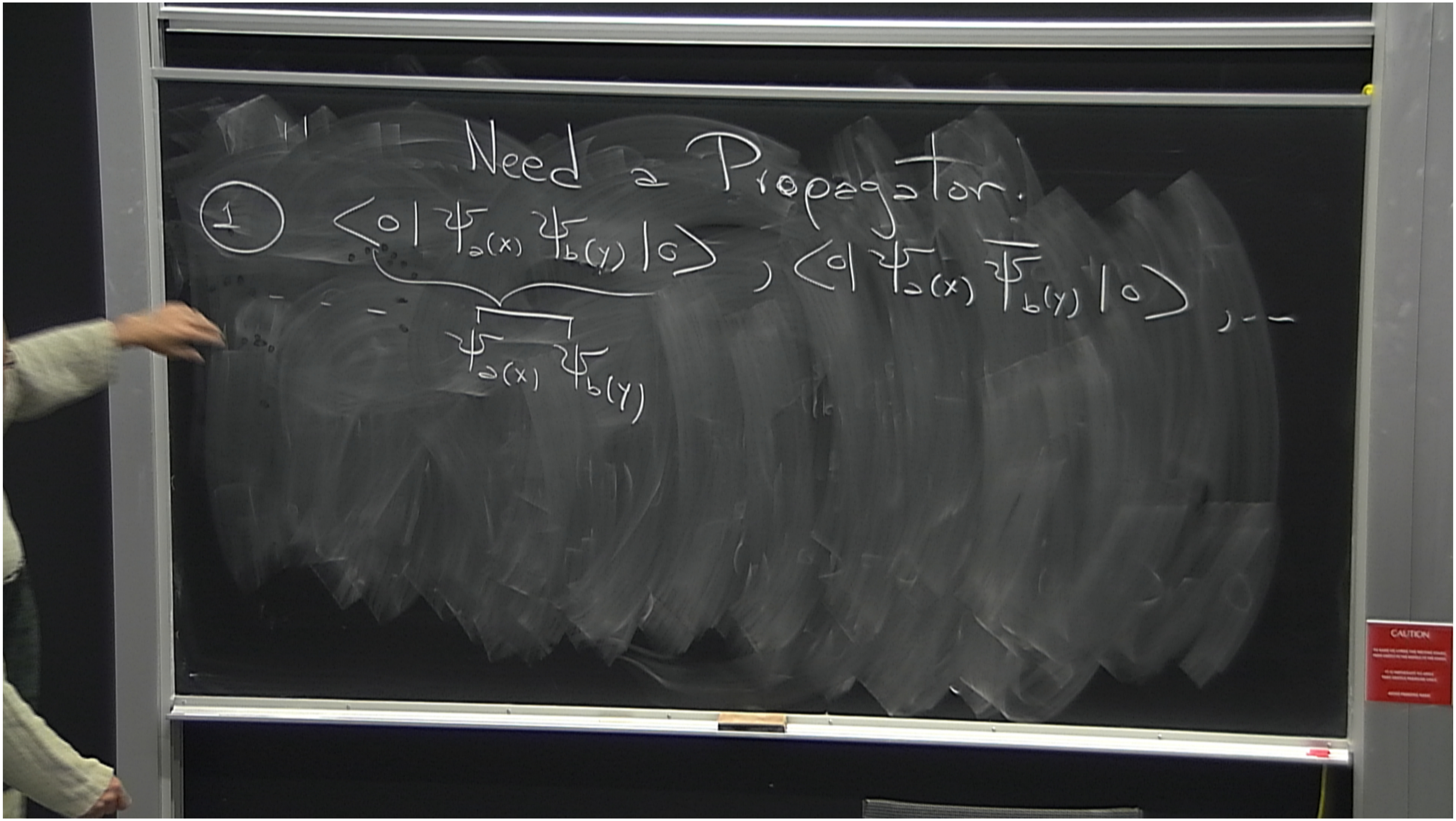
• Classical Limit Grassmann numbers

$$\psi(x) = \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}} \sum_{s=1}^{2} \left(u^s(\vec{k})\, a^s(\vec{k})\, e^{-ik\cdot x} + b^{s\dagger}(\vec{k})\, v^s(\vec{k})\, e^{ik\cdot x} \right)$$

Using * $\{a^s(\vec{k}), a^{r\dagger}(\vec{k}')\} = \delta^{rs}\, \delta^3(\vec{k}-\vec{k}') = \{b^s(\vec{k}), b^{\dagger r}(\vec{k}')\}$

$\{a, a\} = 0 \quad \{a, b\} = 0 \quad \{a, b^\dagger\} = 0$

......



Need a Propagator!
① $\langle 0|\psi_a(x)\psi_b(y)|0\rangle$, $\langle 0|\psi_a(x)\overline{\psi}_b(y)|0\rangle$, ...
$\psi_a(x)\,\psi_b(y)$

Need a Propagator.

(1) $\langle 0|\psi_a(x)\,\psi_b(y)|0\rangle\,,\ \langle 0|\psi_a(x)\,\bar{\psi}_b(y)|0\rangle\,,\ \ldots$

$\langle 0|T(\quad)|0\rangle \qquad \psi_a(x)\,\psi_b(y)$

$\psi_a(x)\,\bar{\psi}_b(y) = \langle 0|T(\psi_a(x)\,\bar{\psi}_b(y))|0\rangle$

$k^0 = \omega_k > 0$

Field equation.

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad \Rightarrow \quad (\cancel{k} - m\mathbb{1})\, u(\vec{k}) = 0$$

...rpretation for m.

$$(\cancel{k} + m)(\cancel{k} - m)\, u(\vec{k}) = 0 \longrightarrow (k^2 - m^2)\,\mathbb{1}\, u(\vec{k}) = 0$$

$$(\cancel{k}\cancel{k} - m^2)\, u(\vec{k}) = 0$$

If $u \neq 0$ need $k^2 = m^2$

$$\cancel{k}\cancel{k} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \overbrace{\{\gamma^\mu, \gamma^\nu\}}^{2\eta^{\mu\nu}\mathbb{1}} = k^2\, \mathbb{1}$$

Green's functions

① $(\partial_\mu \partial^\mu + m^2)\,\phi(x) = 0$

$$(\partial_\mu \partial^\mu + m^2)\, G(x-y) = -i\,\delta^4(x-y)$$

$$G(x-y) = \int d^4k\, \tilde{G}(k)\, e^{-ik\cdot x} \implies (-k^2+m^2)\,\tilde{G}(k) = -i$$

$$\tilde{G}(k) = \frac{i}{k^2-m^2}$$

$$\boxed{G_F(x-y) = \int d^4k\, \frac{i\, e^{-ik\cdot x}}{k^2-m^2+i\epsilon}}$$

Green's functions

$I = 1, 2, \ldots$

① $$(\partial_\mu \partial^\mu + m^2)\,\phi^{(I)}(x) = 0$$

$$(\partial_\mu \partial^\mu + m^2)\,G^{(IJ)}(x-y) = -i\,\delta^4(x-y)$$

$$G(x-y) = \int d^4k\, \tilde{G}(k)\, e^{-ik\cdot x} \implies (-k^2+m^2)\,\tilde{G}(k) = -i$$

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$$\boxed{G_F(x-y) = \int d^4k\, \frac{i\, e^{-ik\cdot x}}{k^2-m^2+i\epsilon}}$$

Green's functions

$$I = 1, 2, \dots$$

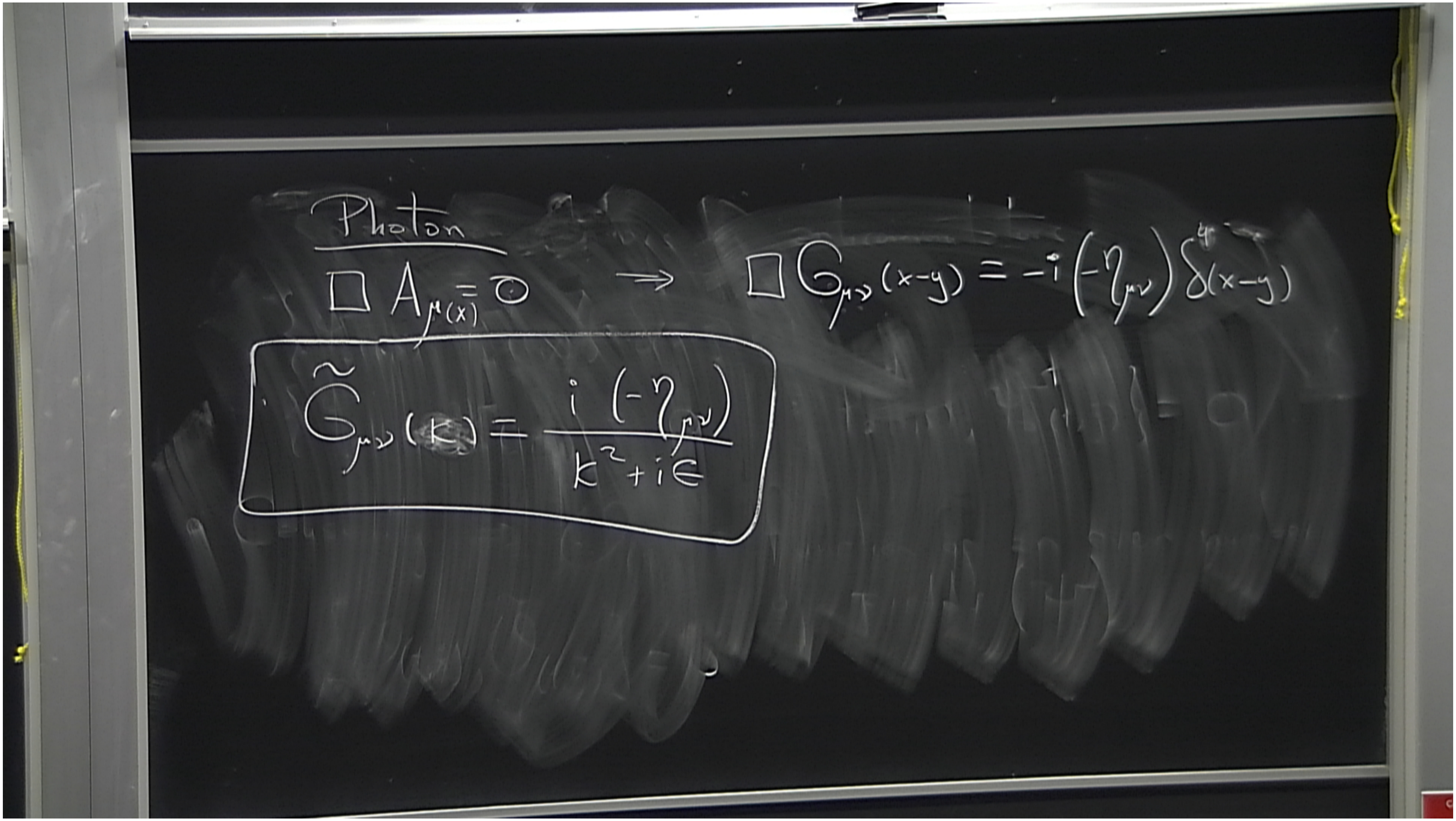
① $$(\partial_\mu \partial^\mu + m^2)\, \phi^{(I)}(x) = 0$$

$$(\partial_\mu \partial^\mu + m^2)\, G^{(IJ)}(x-y) = -i\, \delta^4(x-y)\, \delta^{IJ}$$

$$G^{(IJ)}(x-y) = \int d^4k\, \tilde{G}(k)\, e^{-ik\cdot x}\, \delta^{IJ} \Rightarrow (-k^2+m^2)\,\tilde{G}^{IJ}(k) = -i\, \delta^{IJ}$$

$$\tilde{G}(k) = \frac{i}{k^2-m^2}$$

$$\boxed{G_F^{IJ}(x-y) = \int d^4k\, \frac{\delta^{IJ}\, i\, e^{-ik\cdot x}}{k^2-m^2+i\epsilon}}$$


Photon
$\Box A_\mu(x) = 0 \rightarrow \Box G_{\mu\nu}(x-y) = -i(-\eta_{\mu\nu})\delta^4(x-y)$
$\boxed{\tilde{G}_{\mu\nu}(k) = \frac{i(-\eta_{\mu\nu})}{k^2+i\epsilon}}$

Dirac Field

$$(i\not{\partial} - m)\,\psi(x) = 0 \qquad (i\not{\partial} - m\mathbb{1})_{ab}\, S_{bc}(x-y) = i\,\delta^4(x-y)\,\delta_{ac}$$

Momentum space : $(\not{k} - m\mathbb{1})_{ab}\,\tilde{S}_{bc}(k) = i\,\delta_{ac}$

$$\tilde{S}(k) = \frac{i}{\not{k} - m\mathbb{1}}$$

Dirac Field

$$(i\not\partial - m)\psi(x) = 0 \qquad (i\not\partial - m\mathbb{1})_{ab} S_{bc}(x-y) = i\delta^4(x-y)\delta_{ac}$$

Momentum space: $(\not k - m\mathbb{1})_{ab}\tilde{S}_{bc}(k) = i\delta_{ac}$

$$\tilde{S}(k) = \frac{i}{\not k - m\mathbb{1}} = i(\not k - m\mathbb{1})^{-1}$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad \Rightarrow \quad (\not{k} - m\mathbb{1})\, u(k) = 0$$

...rpretation for m.

$$(\not{k} + m)(\not{k} - m) = (\not{k}\not{k} - m^2) = (k^2 - m^2)\,\mathbb{1}_{4\times4} \longrightarrow (k^2 - m^2)\,\mathbb{1}\, u(k) = 0$$

If $u \neq 0$ need. $k^2 = m^2$

$$\not{k}\not{k} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \overbrace{\{\gamma^\mu, \gamma^\nu\}}^{2\eta^{\mu\nu}\mathbb{1}} = k^2\,\mathbb{1}$$

Need a Propagator.

① $\langle 0|\psi_a(x)\,\psi_b(y)|0\rangle$, $\langle 0|\psi_a(x)\,\bar{\psi}_b(y)|0\rangle$, ...

$$\overbracket{\psi_a(x)\,\psi_b(y)}$$

$$\langle 0|T(\qquad)|0\rangle$$

$$\overbracket{\psi_a(x)\,\bar{\psi}_b(y)}$$

$$= \langle 0|T(\psi_a(x)\,\bar{\psi}_b(y))|0\rangle$$

Want

$$= \int d^4k\, \frac{(\not{k}+m)_{ab}\, e^{-ik\cdot(x-y)}}{k^2-m^2+i\epsilon}$$

$k^2 - m^2 + i\epsilon$

Nucleon – Meson dynamics. (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho - \frac{1}{2} m^2 \rho^2$$

$\cdots \; k^2 - m^2 + i\epsilon$

Nucleon – Meson dynamics. (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - \frac{1}{2}m^2\rho^2 - g\,\underbrace{\bar{\psi}_{(x)}\psi_{(x)}\rho_{(x)}}_{\text{Yukawa coupling}}$$

Feynman rules:

$\xrightarrow{\quad} k$

$$k^2 - m^2 + i\epsilon$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho - \frac{1}{2} m^2 \rho^2 - g\, \underbrace{\bar{\psi}_{(x)} \psi_{(x)} \rho_{(x)}}_{\text{Yukawa coupling}}$$

Feynman rules for Scattering Amplitudes?

$$\overset{k}{\longrightarrow} = \frac{i}{k^2 - m^2 + i\epsilon} \longleftarrow \rho_{(x)} \rho_{(y)}$$

Yukawa coupling

Wick's Thm.

$k^2 - m^2 + i\epsilon$

Nucleon – Meson dynamics. (Yukawa Theory)

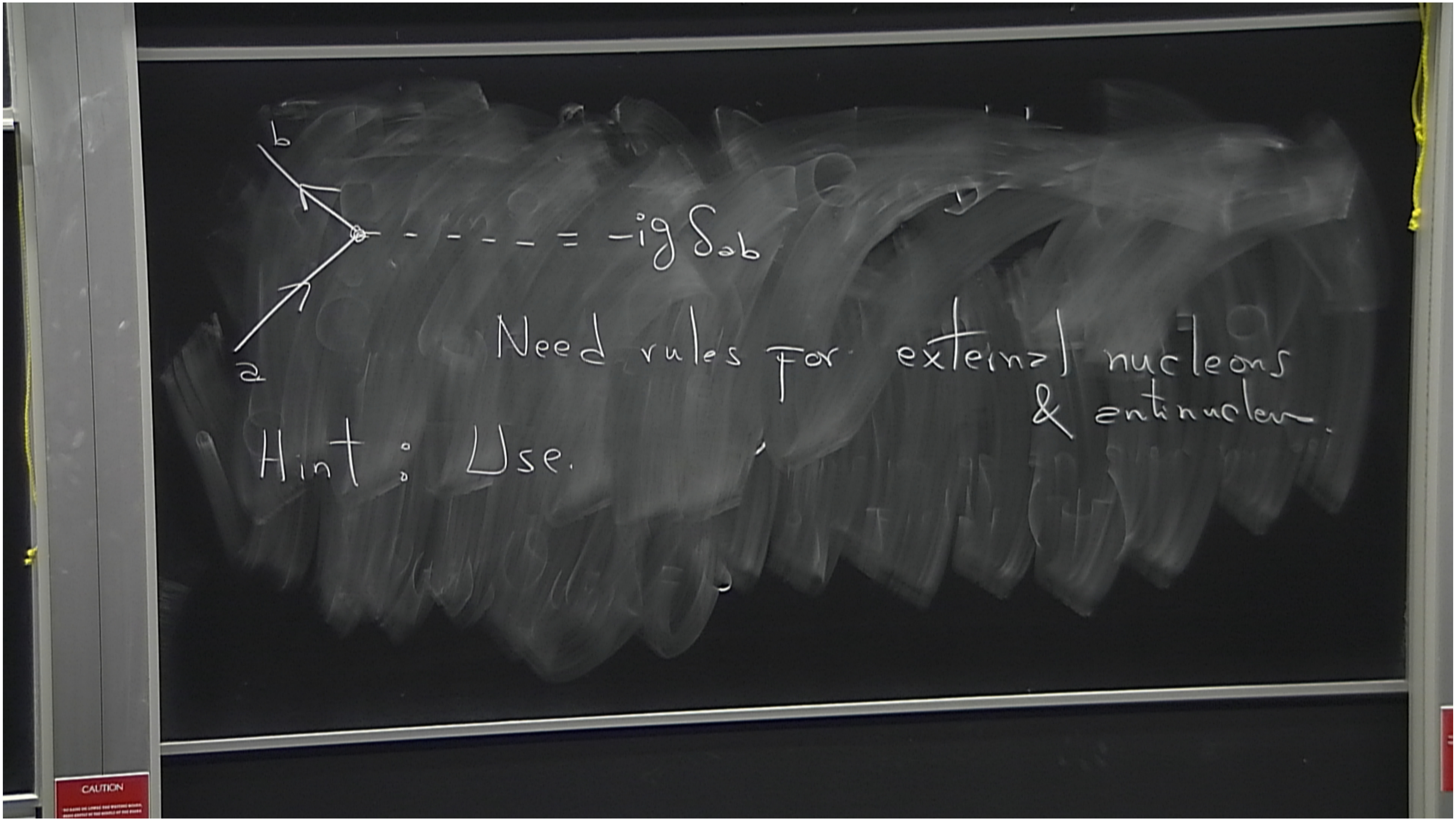
$$\mathcal{L} = \bar{\psi}(i\cancel{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho - \frac{1}{2} m^2 \rho^2 - g\, \underbrace{\bar{\psi}_{(x)} \psi_{(x)} \rho_{(x)}}_{\text{Yukawa coupling}}$$

Feynman rules for Scattering Amplitude?

$$- - - - - - - \overset{k}{\longrightarrow} = \frac{i}{k^2 - m^2 + i\epsilon} \longleftarrow \rho_{(x)} \rho_{(y)}$$

$$a \longrightarrow b = \frac{i(\cancel{k} + m)_{ab}}{k^2 - M^2 + i\epsilon} \longleftarrow \psi_a(x)\, \bar{\psi}_b(y)$$

(Wick's Thm.)


b
a
= −ig δab
Need rules for external nucleons
& antinucleons.
Hint: Use.
CAUTION

$$k^2 - m^2 + i\epsilon$$

Nucleon - Meson dynamics (Yukawa Theory)

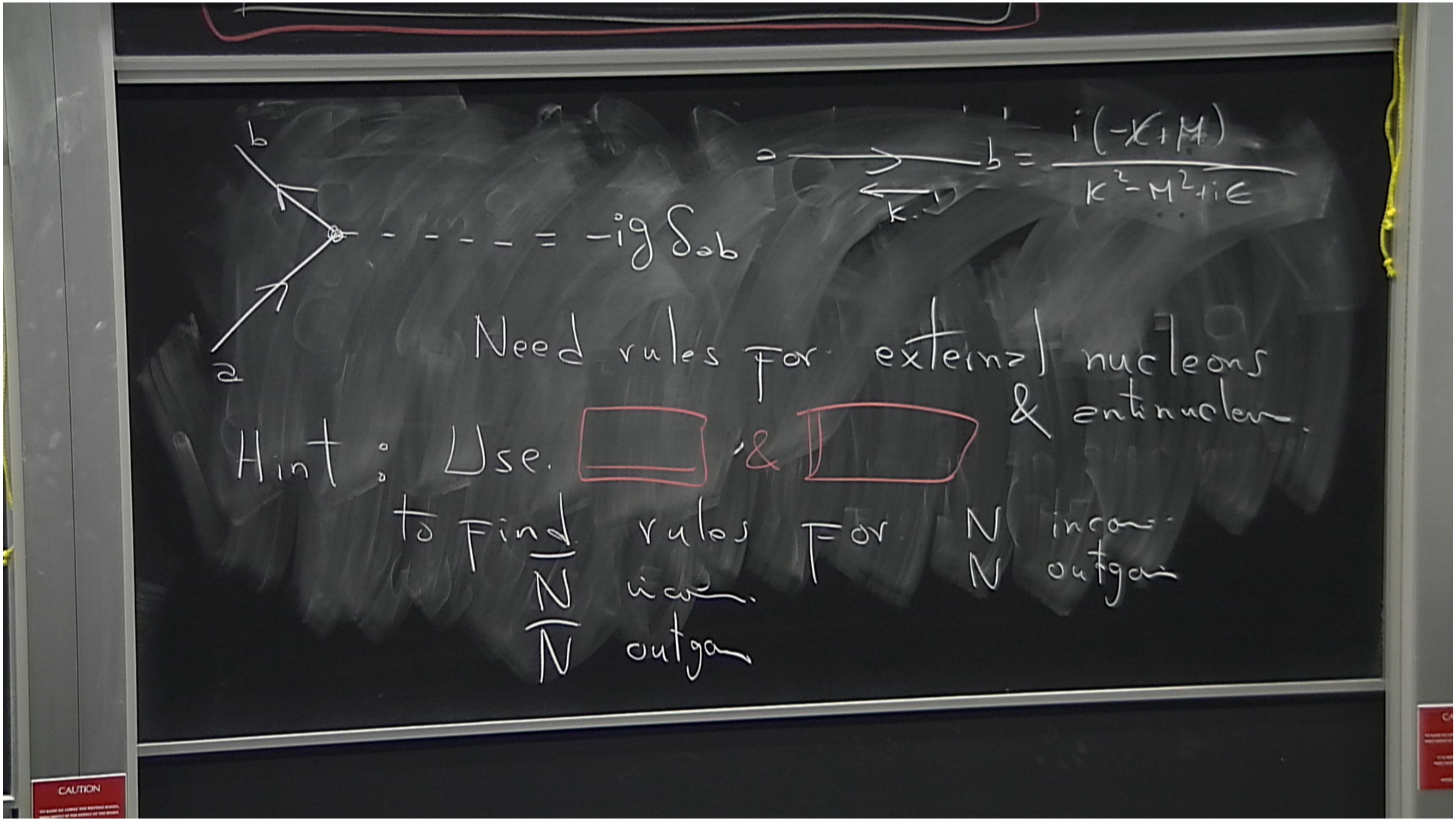
$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - \frac{1}{2}m^2\rho^2 - g\,\underbrace{\bar{\psi}_{(x)}\psi_{(x)}\rho_{(x)}}_{\text{Yukawa coupling}}$$

Feynman rules for Scattering Amplitudes:

$$\xrightarrow{\;k\;} \quad = \frac{i}{k^2 - m^2 + i\epsilon} \longleftarrow \rho_{(x)}\rho_{(y)}$$

$$a \xrightarrow{\;k\;} b = \frac{i(\not{k} + m)_{ab}}{k^2 - M^2 + i\epsilon} \longleftarrow \psi_{a(x)}\bar{\psi}_{b(y)}$$

Wick's Thm.


b
a
$= -ig\delta_{ab}$
$a \longrightarrow b = \frac{i(-\not{k} + M)}{k^2 - M^2 + i\epsilon}$
k
Need rules for external nucleons
& antinucleons.
Hint: Use &
to find rules for N incom
N outgo
$\bar{N}$ incom.
$\bar{N}$ outgo