

Title: 13/14 PSI - Quantum Field Theory I - Lecture 12

Date: Oct 24, 2013 03:30 PM

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Abstract:

Dirac Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \gamma^i \\ -\gamma^i & 0 \end{pmatrix}$$



Dirac Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m) \psi$$



Weyl Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m) \psi$$

$$\not{\partial} = \gamma^\mu \partial_\mu$$



Weyl Basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m) \psi$$

$$\not{\partial} = \gamma^\mu \partial_\mu$$

Solve Dirac's Theory. : Plane waves.

$$\psi^{(+)}(t, x) = \int \frac{d^3 \vec{k}}{(2\pi)^3} u(\vec{k}) e^{-ik \cdot x}$$

$a, b = 1, 2, 3, 4$

$$\gamma_{ab}^{\mu}$$



$$k^0 = \omega_k > 0$$
$$(i\cancel{\not{X}} - m)\cancel{\psi} = 0$$

Field equation.

$$\Rightarrow (\cancel{X} - m\mathbb{1})\mathcal{U}(E) = 0$$



$$(i\cancel{\not{D}} - m)\psi = 0 \quad \Rightarrow \quad (\cancel{K} - m\mathbb{1})\mathcal{U}(\vec{k}) = 0$$

Interpretation for  $m$ .

$$-(\cancel{K} + m)(\cancel{K} - m)\mathcal{U}(\vec{k}) = 0$$

$$(\cancel{K}\cancel{K} - m^2)\mathcal{U}(\vec{k}) = 0$$

$$(K^2 - m^2)\mathbb{1}\mathcal{U}(\vec{k}) = 0$$

If  $\mathcal{U} \neq 0$   
need  $K^2 = m^2$

$$\cancel{K}\cancel{K} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \underbrace{\{\gamma^\mu, \gamma^\nu\}}_{2\eta^{\mu\nu}\mathbb{1}} = K^2 \mathbb{1}$$



Want to solve  $(K-m)U(\mathbf{p})=0$

Rest frame  $K^M = (m, \vec{0})$

$$m(\gamma^0 - \underline{\mathbb{1}})U(\mathbf{k}^*)=0$$

CAUTION

DO NOT TOUCH THE BOARD OR THE BOARDER  
IF YOU HAVE ANY QUESTIONS  
PLEASE ASK THE INSTRUCTOR

Want to Solve  $(K-m)u_{(K)} = 0$

Rest frame  $K^M = (m, \vec{0})$

$$m(\gamma^0 - \underline{\underline{1}})u_{(K^*)} = 0 \quad \Rightarrow \quad \begin{pmatrix} -\underline{\underline{1}} & \underline{\underline{1}} \\ \underline{\underline{1}} & -\underline{\underline{1}} \end{pmatrix} u_{(K^*)} = 0$$

CAUTION  
DO NOT TOUCH THE BOARD OR THE BOARDER  
IF YOU HAVE ANY QUESTIONS  
PLEASE ASK THE INSTRUCTOR

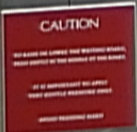


Want to solve  $(K-m)U_{(K)} = 0$

Rest frame  $K^M = (m, \vec{0})$

$$m(\gamma^0 - \underline{\mathbb{1}})U_{(K^*)} = 0 \quad \Rightarrow \quad \begin{pmatrix} -\underline{\mathbb{1}}_{2 \times 2} & \underline{\mathbb{1}}_{2 \times 2} \\ \underline{\mathbb{1}}_{2 \times 2} & -\underline{\mathbb{1}}_{2 \times 2} \end{pmatrix} U_{(K^*)} = 0$$

Rank is 2





Want to solve  $(K - m)U(R) = 0$

Rest frame  $K^* = (m, \vec{0})$

$$m(\gamma^0 - \underline{1})U(K^*) = 0$$

$$\Rightarrow \begin{pmatrix} -\underline{1}_{2 \times 2} & \underline{1}_{2 \times 1} \\ \underline{1}_{2 \times 1} & -\underline{1}_{2 \times 2} \end{pmatrix} U(K^*) = 0$$

Rank is 2  $\Rightarrow$  corank is 2.

rest frame  
Number of eigenvectors with zero eigenvalue.



Want to solve  $(K-m)U_{(K)} = 0$

Rest frame  $K^{\mu} = (m, \vec{0})$

$$m(\gamma^0 - \underline{1})U_{(K^*)} = 0$$

$$\Rightarrow \begin{pmatrix} -\underline{1}_{2 \times 2} & \underline{1}_{2 \times 1} \\ \underline{1}_{2 \times 1} & -\underline{1}_{2 \times 2} \end{pmatrix} U_{(K^*)} = 0$$

rest frame

Rank is 2  $\Rightarrow$  corank is 2.

Number of eigenvectors with zero eigenvalue.

$$U \in \left\{ \underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}}_{U^1}, \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}}_{U^2} \right\}$$

$$\Rightarrow U_{(K^*)}^s = \sqrt{m} \begin{pmatrix} \zeta^1 \\ \zeta^A \end{pmatrix}$$

$$\zeta^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \zeta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Boost to get a general solution:

Exercise:  $K^* \mu \rightarrow K \mu$

$$\mathcal{U}^s(\mathbf{k}) = \begin{pmatrix} \sqrt{k \cdot \tilde{\mathbf{g}}} \zeta^s \\ \sqrt{k \cdot \tilde{\mathbf{g}}} \zeta^s \end{pmatrix}$$



Boost to get a general solution.

Exercise:  $k^* \mu \rightarrow k \mu$ .

$$U_{(k)}^s = \begin{pmatrix} \sqrt{k \cdot \vec{\sigma}} \begin{matrix} \uparrow \\ \downarrow \end{matrix}^s \\ \sqrt{k \cdot \vec{\sigma}} \begin{matrix} \uparrow \\ \downarrow \end{matrix}^s \end{pmatrix}$$

$$\bar{U}_{(k)}^s U_{(k)}^r = 2m \delta^{sr}$$

$$\bar{U}_{(k)} = U_{(k)}^\dagger \gamma^0$$



Boost to get a general solution.

Exercise:  $k^* \mu \rightarrow k \mu$

$$U_{(k)}^s = \begin{pmatrix} \sqrt{k \cdot \vec{\sigma}} \begin{matrix} \uparrow \\ \downarrow \end{matrix}^s \\ \sqrt{k \cdot \vec{\sigma}} \begin{matrix} \uparrow \\ \downarrow \end{matrix}^s \end{pmatrix}$$

$$\bar{U}_{(k)}^s U_{(k)}^r = 2m \delta^{sr}$$

$$\bar{U}_{(k)} = U_{(k)}^\dagger \gamma^0$$

$$U^s \quad \bar{U}^s$$



Boost to get a general solution.

Exercise:  $k^\mu \rightarrow k^\mu$

$$U^s(k) = \begin{pmatrix} \sqrt{k \cdot \sigma} \zeta^s \\ \sqrt{k \cdot \bar{\sigma}} \zeta^s \end{pmatrix}$$

$$\bar{U}^s(k) U^r(k) = 2m \delta^{sr}$$

$$\bar{U}(k) = U^\dagger(k) \gamma^0$$

$$\sum_{s=1}^2 U_a^s(k) \bar{U}_b^s(k) = (\not{k} + m)_{ab}$$



Boost to get a general solution.

Exercise:  $k^* \mu \rightarrow k \mu$ .

$$U^s(k) = \begin{pmatrix} \sqrt{k \cdot \bar{\sigma}} \zeta^s \\ \sqrt{k \cdot \sigma} \zeta^s \end{pmatrix}$$

$$\bar{U}^s(k) U^r(k) = 2m \delta^{sr}$$

$$\bar{U}(k) = U^\dagger(k) \gamma^0$$

$$\sum_{s=1}^2 U_a^s(k) \bar{U}_b^s(k) = (K + m)_{ab}$$



Negative energy solution,

$$\psi_{(+,x)}^{(-)} = \int \frac{d^3k}{\sqrt{2\omega_k}} \psi_{(k)} e^{ik \cdot x}$$

$$\psi_{(k)}^s = \begin{pmatrix} \sqrt{k \cdot \sigma} \eta^s \\ -\sqrt{k \cdot \bar{\sigma}} \eta^s \end{pmatrix}$$

$$\eta^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \eta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\bar{\psi}_{(k)}^s \psi_{(k)}^r = -2m \delta^{rs}$$

$$\sum_{s=1}^2 \psi_{a(k)}^s \bar{\psi}_{b(k)}^s = (\not{k} - m)_{ab}$$



## Quantization (Canonical)

$$\begin{aligned}\mathcal{L} &= \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi = \underbrace{\bar{\psi} i\gamma^0 \partial_0 \psi}_{i\psi^\dagger \dot{\psi}} + \bar{\psi} (i\gamma^i \partial_i - m) \psi \\ &= \frac{1}{2} i\psi^\dagger \dot{\psi} - \frac{1}{2} i\dot{\psi}^\dagger \psi + i\psi^\dagger \partial_0 \psi \\ &\quad + \bar{\psi} (i\gamma^i \partial_i - m) \psi\end{aligned}$$



(Canonical)

$$\mathcal{L} = \bar{\psi} (\gamma^\mu \partial_\mu - m) \psi = \underbrace{\bar{\psi} i \gamma^0 \partial_0 \psi}_{\text{kinetic}} + \bar{\psi} (i \gamma^i \partial_i - m) \psi$$

$$= \frac{1}{2} i \dot{\psi}^\dagger \psi - \frac{1}{2} i \psi^\dagger \dot{\psi} + \bar{\psi} (i \gamma^i \partial_i - m) \psi$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{1}{2} i \psi^\dagger = \pi_\psi \quad \frac{\partial \mathcal{L}}{\partial \dot{\psi}^\dagger} = -\frac{1}{2} i \psi = \pi_{\psi^\dagger}$$

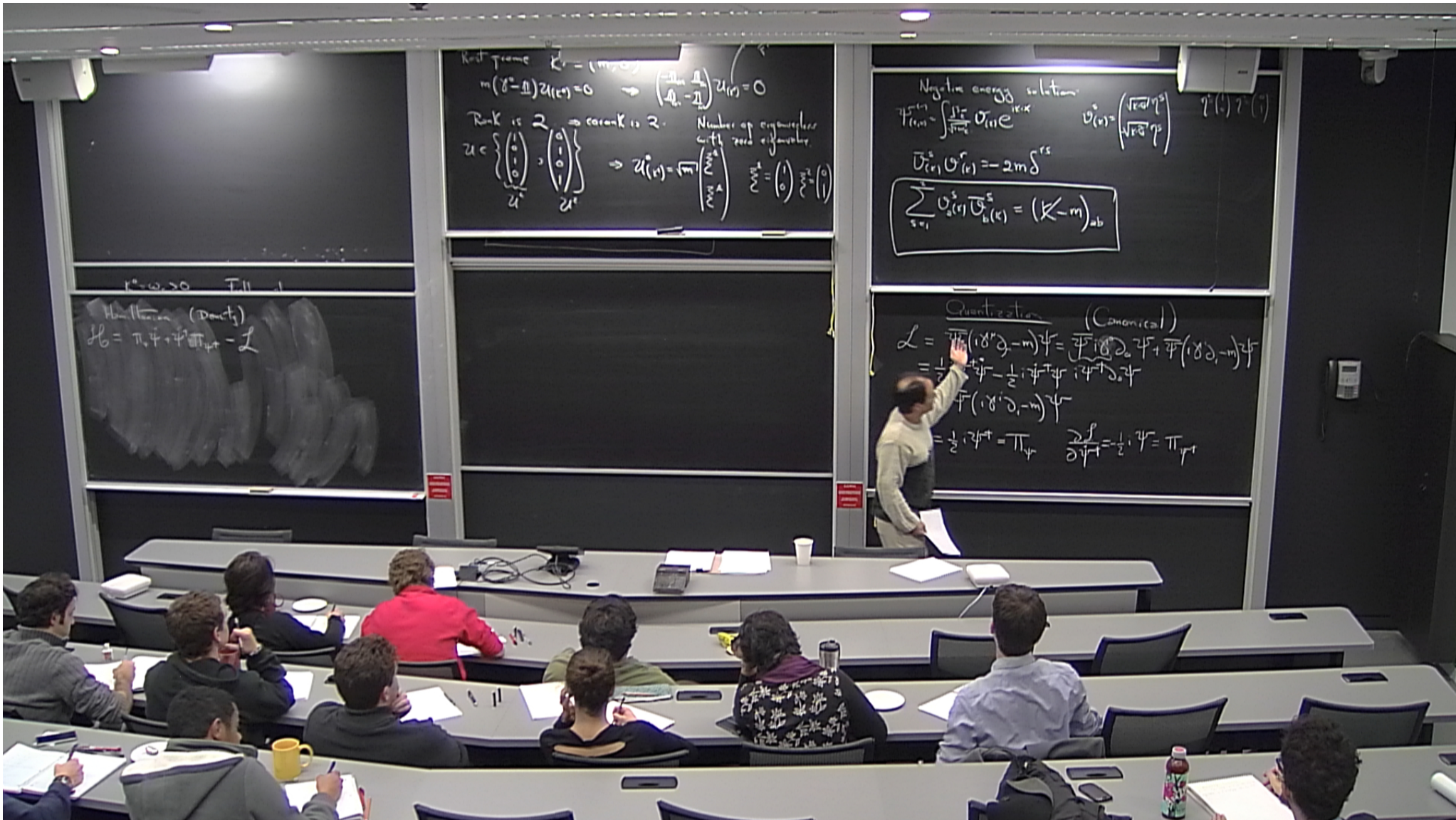


## Quantization (Canonical)

$$\begin{aligned}\mathcal{L} &= \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi = \underbrace{\bar{\psi} i\gamma^0 \partial_0 \psi}_{\text{time derivative}} + \bar{\psi} (i\gamma^i \partial_i - m) \psi \\ &= \frac{1}{2} i \dot{\psi}^\dagger \psi - \frac{1}{2} i \dot{\psi}^\dagger \psi + i \psi^\dagger \partial_0 \psi \\ &\quad + \bar{\psi} (i\gamma^i \partial_i - m) \psi\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{1}{2} i \psi^\dagger = \pi_\psi \quad \frac{\partial \mathcal{L}}{\partial \dot{\psi}^\dagger} = -\frac{1}{2} i \psi = \pi_{\psi^\dagger}$$







$$k^0 = \omega_r > 0$$

Fiedl -1

Hamiltonian (Density)

$$\mathcal{H} = \pi_{\psi} \dot{\psi} + \dot{\psi}^{\dagger} \pi_{\psi^{\dagger}} - \mathcal{L}$$



Hamiltonian (Density)

$$\mathcal{H} = \pi_\psi \dot{\psi} + \dot{\psi}^\dagger \pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi} (-i\gamma^i \partial_i + m) \psi$$

Equal time commutation relations

$$[\psi_a(t, \vec{x}), \pi_b(t, \vec{y})] = \delta^3(\vec{x} - \vec{y}) \delta_{ab}$$

$\downarrow$   
 $\psi_b^\dagger(t, \vec{y})$



Hamiltonian (Density)

$$\mathcal{L} = \pi_\psi \dot{\psi} + \dot{\psi}^\dagger \pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi} (-i\gamma^i \partial_i + m) \psi$$

Equal time commutation relations

$$[\psi_a(t, \vec{x}), \pi_b(t, \vec{y})] = \int^3 \delta(\vec{x} - \vec{y}) \delta_{ab}$$

$$\downarrow$$
$$\psi_b^\dagger(t, \vec{y})$$

$$[\psi_a, \psi_b] = 0$$

$$[\psi_a^\dagger, \psi_b^\dagger] = 0$$



$$b(t, \vec{y})$$

$$[\psi_a, \psi_b] = 0$$
$$[\psi_a^+, \psi_b^+] = 0$$

$$\psi_a^{(+)}(t, \vec{x}) = \int u a e$$

$$\psi_a^{(-)}(t, \vec{x}) = \int v a^+$$

$$[a(\vec{r}), a(\vec{r}')] = 0$$

$$[a^+(\vec{r}), a^+(\vec{r}')] = 0$$



$$|b(t, \vec{y})\rangle$$

$$[T_a, T_b] = 0$$
$$[\psi_a^\dagger, \psi_b^\dagger] = 0$$

$$\psi_z^{(+)}(t, \vec{x}) = \int u a e$$

$$\psi_z^{(-)}(t, \vec{x}) = \int v a^\dagger$$

$$[a(\vec{k}), a(\vec{k}')] = 0 \quad [a^\dagger(\vec{k}), a^\dagger(\vec{k}')] = 0$$

$$a_{(\vec{k}_1)}^\dagger a_{(\vec{k}_2)}^\dagger |0\rangle = |\vec{k}_1, \vec{k}_2\rangle = |\vec{k}_2, \vec{k}_1\rangle$$

Exercise: keep going  $\Rightarrow$  Violates causality!  $\nabla$



Replace commutators by anticommutators.

$$\left\{ \psi_a^-(t, \vec{x}), \psi_b^+(t, \vec{y}) \right\} = \int^3 \delta(\vec{x} - \vec{y}) \delta_{ab}$$

$$\left\{ \psi_a^-(t, \vec{x}), \psi_b^-(t, \vec{y}) \right\} = 0$$



Replace commutators by anticommutators.

$$\left\{ \psi_a(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = \int^3 \delta(\vec{x} - \vec{y}) \delta_{ab}$$

$$\left\{ \psi_a(t, \vec{x}), \psi_b(t, \vec{y}) \right\} = 0 \quad \left\{ \psi_a^\dagger(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = 0$$



Replace commutators by anticommutators.

$$\left\{ \psi_a(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = \delta^3(\vec{x} - \vec{y}) \delta_{ab}$$

$$\left\{ \psi_a(t, \vec{x}), \psi_b(t, \vec{y}) \right\} = 0 \quad \left\{ \psi_a^\dagger(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = 0$$

Causality  $\Rightarrow \mathcal{O}_{(t, \vec{x})} =$  function of an even number of fields.



Replace commutators by anticommutators.

$$\left\{ \psi_a(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = \delta^3(\vec{x} - \vec{y}) \delta_{ab}$$

$$\left\{ \psi_a(t, \vec{x}), \psi_b(t, \vec{y}) \right\} = 0 \quad \left\{ \psi_a^\dagger(t, \vec{x}), \psi_b^\dagger(t, \vec{y}) \right\} = 0$$

Causality  $\Rightarrow \mathcal{O}_{(t, \vec{x})} =$  function of an even number  
of fields. ( $\psi$ )

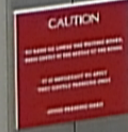
$$[\mathcal{O}_{(t, \vec{x})}, \tilde{\mathcal{O}}_{(t, \vec{y})}] = 0$$



• Classical Limit Grassmann numbers

$$\psi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3}} \sum_{s=1}^2 \left( u_{(k)}^s a_{(k)}^s e^{-ik \cdot x} + v_{(k)}^s b_{(k)}^s e^{ik \cdot x} \right)$$

number.





• Classical Limit. Grassmann numbers

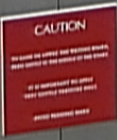
$$\psi(x) = \int \frac{d^3k}{\sqrt{2\omega_k}} \sum_{s=1}^2 \left( u_{(k)}^s a_{(k)}^s e^{-ik \cdot x} + b_{(k)}^{st} v_{(k)}^s e^{ik \cdot x} \right)$$

Using \*  $\{a_{(k)}^s, a_{(k')}^{rt}\} = \int \delta(\vec{k}-\vec{k}') = \{b_{(k)}^s, b_{(k')}^{rt}\}$

$\{a, a\} = 0$      $\{a, b\} = 0$      $\{a, b^{\dagger}\} = 0$

.....

number \*





$$K^2 = k_\mu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu \gamma_\nu \{\gamma^\mu, \gamma^\nu\} = k^2 \mathbb{1}$$

Hamiltonian (Density)

$$\mathcal{H} = \pi_\psi \dot{\psi} + \dot{\psi}^\dagger \pi_{\psi^\dagger} - \mathcal{L} = \bar{\psi} (-i\gamma^i \partial_i + m) \psi$$

$$H = \int d^3x \mathcal{H} = \int d^3k \omega_k \sum_{s=1}^2 \begin{pmatrix} a_{(k)}^{st} a_{(k)}^s - b_{(k)}^s b_{(k)}^{ts} \end{pmatrix}$$



Hamiltonian (Density)

$$\mathcal{H} = \pi_{\psi} \dot{\psi} + \bar{\psi}^{\dagger} \dot{\psi} - \mathcal{L} = \bar{\psi} (-i\gamma^i \partial_i + m) \psi$$

$$H = \int d^3x \mathcal{H} = \int d^3k \omega_k \sum_{s=1}^2 \left( a_{(k)}^{s\dagger} a_{(k)}^s - b_{(k)}^s b_{(k)}^{s\dagger} \right)$$

Define  $|0\rangle$  /  $a_{(k)}^s |0\rangle = 0$   $b_{(k)}^s |0\rangle = 0$

$$\psi^{\dagger}(\vec{x}, t) = \int \dots \psi a^{\dagger} \dots$$



$$-b^s(k) b^{ts}(k') = b^{ts}(k') b^s(k) = \delta^3(k-k')$$



$$-b^s(\mathbf{k})b^{\dagger s}(\mathbf{k}') = b^{\dagger s}(\mathbf{k}')b^s(\mathbf{k}) - \delta^3(\mathbf{k}-\mathbf{k}') \rightarrow -S^{(0)} = -\int d^3x$$

$$H = \int d^3\mathbf{k} \omega_{\mathbf{k}} \left( \sum_{\mathbf{s}} a^{\dagger} a + b^{\dagger} b \right) - \int d^3\mathbf{k} \omega_{\mathbf{k}} S^{(0)}$$



$$-b^s(k) b^{ts}(k') = b^{ts}(k') b^s(k) = \delta^3(k-k') \rightarrow -\dot{S}^{(0)} = -\int d^3x$$

$$H = \int d^3\vec{k} \omega_k \left( \sum_s a^{\dagger} a + b^{\dagger} b \right) - 2 \int d^3\vec{k} \omega_k \dot{S}^{(0)}$$

Perhaps some sort of cancellation between  
Bosons & Fermions solves the CC problem.



Hilbert Space :

$$\begin{aligned} a(\vec{k}_1)^{+s_1} a(\vec{k}_2)^{+s_2} |0\rangle &= |\mathcal{Q}(\vec{k}_1, s_1), \mathcal{Q}(\vec{k}_2, s_2)\rangle \\ &= -|\mathcal{Q}(\vec{k}_2, s_2), \mathcal{Q}(\vec{k}_1, s_1)\rangle \end{aligned}$$



• Classical Limit. Grassmann numbers

$$\psi(x) = \int \frac{d^3k}{\sqrt{2\omega_k}} \sum_{s=1}^2 \left( u_{(k)}^s a_{(k)}^s e^{-ik \cdot x} + b_{(k)}^{st} v_{(k)}^s e^{ik \cdot x} \right)$$

Using \*

$$\{a_{(k)}^s, a_{(k')}^{r\dagger}\} = \int d^3r \delta(\vec{k}-\vec{k}') = \{b_{(k)}^s, b_{(k')}^{r\dagger}\}$$

$$\{a, a\} = 0 \quad \{a, b\} = 0 \quad \{a, b^\dagger\} = 0$$

.....



Need a Propagator.

①  $\langle 0 | \psi_a(x) \psi_b(y) | 0 \rangle, \langle 0 | \psi_a(x) \bar{\psi}_b(y) | 0 \rangle, \dots$

$\psi_a(x) \psi_b(y)$

CAUTION  
Do not touch the surface of the board.  
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Need a Propagator.

①

$$\langle 0 | \psi_{a(x)} \psi_{b(y)} | 0 \rangle, \langle 0 | \psi_{a(x)} \bar{\psi}_{b(y)} | 0 \rangle, \dots$$

$$\begin{array}{c} \psi_{a(x)} \psi_{b(y)} \\ \downarrow \\ \langle 0 | T( \quad ) | 0 \rangle \end{array}$$

$$\begin{array}{c} \psi_{a(x)} \bar{\psi}_{b(y)} \\ \downarrow \\ = \langle 0 | T( \psi_{a(x)} \bar{\psi}_{b(y)} ) | 0 \rangle \end{array}$$



$$k^0 = \omega_k > 0$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Field equation

$$\Rightarrow (\not{k} - m)\mathcal{U}(k) = 0$$

preparation for m.

$$-(\not{k} + m)(\not{k} - m)\mathcal{U}(k) = 0$$

$$(\not{k}^2 - m^2)\mathcal{U}(k) = 0$$

$$(k^2 - m^2)\mathbb{1}\mathcal{U}(k) = 0$$

If  $\mathcal{U} \neq 0$   
need  $k^2 - m^2$

$$\not{k}\not{k} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \{\gamma^\mu, \gamma^\nu\} = k^2 \mathbb{1}$$



## Green's functions

$$\textcircled{1} (\partial_\mu \partial^\mu + m^2) \phi(x) = 0$$

$$(\partial_\mu \partial^\mu + m^2) G(x-y) = -i \delta^4(x-y)$$

$$G(x-y) = \int d^4k \tilde{G}(k) e^{-ik \cdot x}$$

$$\Rightarrow (-k^2 + m^2) \tilde{G}(k) = -i$$

$$\tilde{G}(k) = \frac{i}{k^2 - m^2}$$

$$G_F(x-y) = \int d^4k \frac{i e^{-ik \cdot x}}{k^2 - m^2 + i\epsilon}$$



## Green's functions

$I=1, 2, \dots$

$$\textcircled{1} (\partial_\mu \partial^\mu + m^2) \phi^{(I)}(x) = 0$$

$$(\partial_\mu \partial^\mu + m^2) G^{(IJ)}(x-y) = -i \delta^4(x-y)$$

$$G(x-y) = \int d^4k \tilde{G}(k) e^{-ik \cdot x}$$

$$\Rightarrow (-k^2 + m^2) \tilde{G}(k) = -i$$

$$\tilde{G}(k) = \frac{i}{k^2 - m^2}$$

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# Green's functions

$$\textcircled{1} (\partial_\mu \partial^\mu + m^2) \phi^{(I)}(x) = 0$$

$I=1, 2, \dots$

$$(\partial_\mu \partial^\mu + m^2) G^{(IJ)}(x-y) = -i \delta^4(x-y) \delta^{IJ}$$

$$G^{(IJ)}(x-y) = \int d^4k \tilde{G}^{IJ}(k) e^{-ik \cdot x} \delta^{IJ} \Rightarrow (-k^2 + m^2) \tilde{G}^{IJ}(k) = -i \delta^{IJ}$$

$$\tilde{G}^{IJ}(k) = \frac{i}{k^2 - m^2}$$

$$G_F^{IJ}(x-y) = \int d^4k \frac{\delta^{IJ} i e^{-ik \cdot x}}{k^2 - m^2 + i\epsilon}$$



Photon

$$\square A_\mu(x) = 0 \quad \Rightarrow$$

$$\square G_{\mu\nu}(x-y) = -i(-\eta_{\mu\nu})\delta(x-y)$$

$$\tilde{G}_{\mu\nu}(k) = \frac{i(-\eta_{\mu\nu})}{k^2 + i\epsilon}$$



## Dirac Field

$$(i\not{\partial} - m)\psi(x) = 0$$

$$(i\not{\partial} - m)_{ab} S_{bc}(x-y) = i\int^4 \delta(x-y) \delta_{ac}$$

Momentum space:  $(\not{k} - m)_{ab} \tilde{S}_{bc}(k) = i\delta_{ac}$

$$\tilde{S}(k) = \frac{i}{\not{k} - m}$$



## Dirac Field

$$(i\not{\partial} - m)\psi(x) = 0$$

$$(i\not{\partial} - m)_{ab} S_{bc}(x-y) = i\delta^4(x-y)\delta_{ac}$$

Momentum space:  $(\not{k} - m\mathbb{1})_{ab} \tilde{S}_{bc}(k) = i\delta_{ac}$

$$\tilde{S}(k) = \frac{i}{\not{k} - m\mathbb{1}} = i(\not{k} - m\mathbb{1})^{-1}$$



field equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \Rightarrow (\not{K} - m\mathbb{1})\mathcal{U}(E) = 0$$

representation for  $m$ .

$$\begin{aligned}
 & -(\not{K} + m)(\not{K} - m) \\
 &= (\not{K}\not{K} - m^2) = (k^2 - m^2)\mathbb{1}_{4 \times 4}
 \end{aligned}
 \rightarrow (k^2 - m^2)\mathbb{1}\mathcal{U}(E) = 0$$

If  $\mathcal{U} \neq 0$   
need  $k^2 = m^2$

$$\not{K}\not{K} = k_\mu k_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} k_\mu k_\nu \{\gamma^\mu, \gamma^\nu\} = k^2 \mathbb{1}$$



Need a Propagator.

$$① \langle 0 | \psi_{a(x)} \psi_{b(y)} | 0 \rangle, \langle 0 | \psi_{a(x)} \bar{\psi}_{b(y)} | 0 \rangle, \dots$$

$$\begin{array}{c} \psi_{a(x)} \psi_{b(y)} \\ \downarrow \\ \langle 0 | T( \quad ) | 0 \rangle \end{array}$$

Want

$$\begin{aligned} & \langle 0 | T( \psi_{a(x)} \bar{\psi}_{b(y)} ) | 0 \rangle \\ & \equiv \int d^4 k \frac{(K+m)_{ab} e^{-ik \cdot (x-y)}}{k^2 - m^2 + i\epsilon} \end{aligned}$$



$$k = m + i\epsilon$$

# Nucleon-Meson dynamics. (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_{\mu}\rho\partial^{\mu}\rho - \frac{1}{2}m^2\rho^2$$

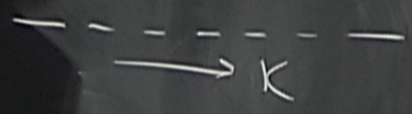


$$k = m + i\epsilon$$

# Nucleon-Meson dynamics (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{1}{2}m^2 \rho^2 - g \bar{\psi}(x)\psi \rho(x)$$

Feynman rules:



Yukawa coupling



$$k = m + i\epsilon$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - g\bar{\psi}\psi\phi$$

Feynman rules for Scattering Amplitude

$$\text{---} \xrightarrow{k} \text{---} = \frac{i}{k^2 - m^2 + i\epsilon}$$

$$\int \psi(x)\psi(y)$$

Yukawa coupling  
Wick's Thm.



$$k^2 - m^2 + i\epsilon$$

# Nucleon-Meson dynamics (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{1}{2}m^2 \rho^2 - g \bar{\psi}(x)\psi(x)\rho(x)$$

Feynman rules for Scattering Amplitude

$$\begin{aligned} \text{---} \xrightarrow{k} \text{---} &= \frac{i}{k^2 - m^2 + i\epsilon} \\ a \xrightarrow{b} &= \frac{i(\not{k} + M)_{ab}}{k^2 - M^2 + i\epsilon} \end{aligned}$$

$\int(x)\rho(y)$   
 Yukawa coupling  
 Wick's Thm.  
 $\psi_a(x) \bar{\psi}_b(y)$





Need rules for external nucleons  
& antinucleons.

Hint: Use.



$$k^2 - m^2 + i\epsilon$$

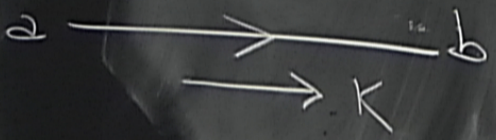
# Nucleon-Meson dynamics (Yukawa Theory)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{1}{2}m^2 \rho^2 - g \bar{\psi}(x)\psi(x)\rho(x)$$

Feynman rules for Scattering Amplitude



$$= \frac{i}{k^2 - m^2 + i\epsilon}$$



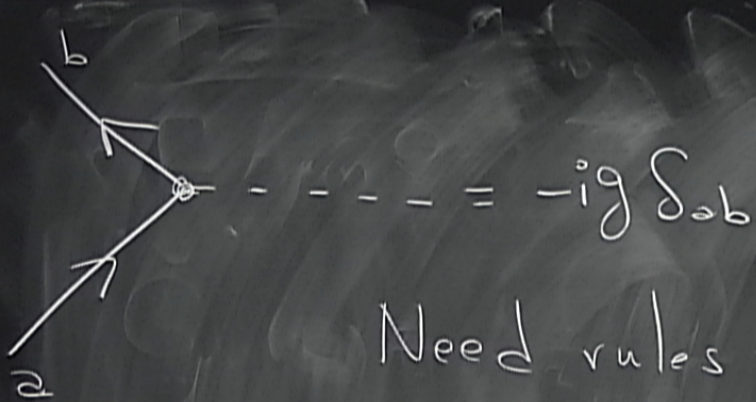
$$= \frac{i(\not{k} + m)_{ab}}{k^2 - M^2 + i\epsilon}$$

$$\int \psi(x)\psi(y)$$

$$\psi_a(x) \bar{\psi}_b(y)$$

Yukawa coupling  
Wick's Thm.


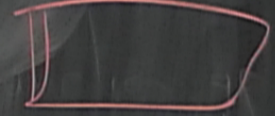




$$b = \frac{i(-k + M)}{k^2 - M^2 + i\epsilon}$$

A Feynman diagram showing a propagator with momentum 'k' and a particle 'b'.

Need rules for external nucleons & antinucleons.

Hint: Use  &  to find rules for N incoming, N outgoing."/>