Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 8

Date: Oct 16, 2013 10:00 AM

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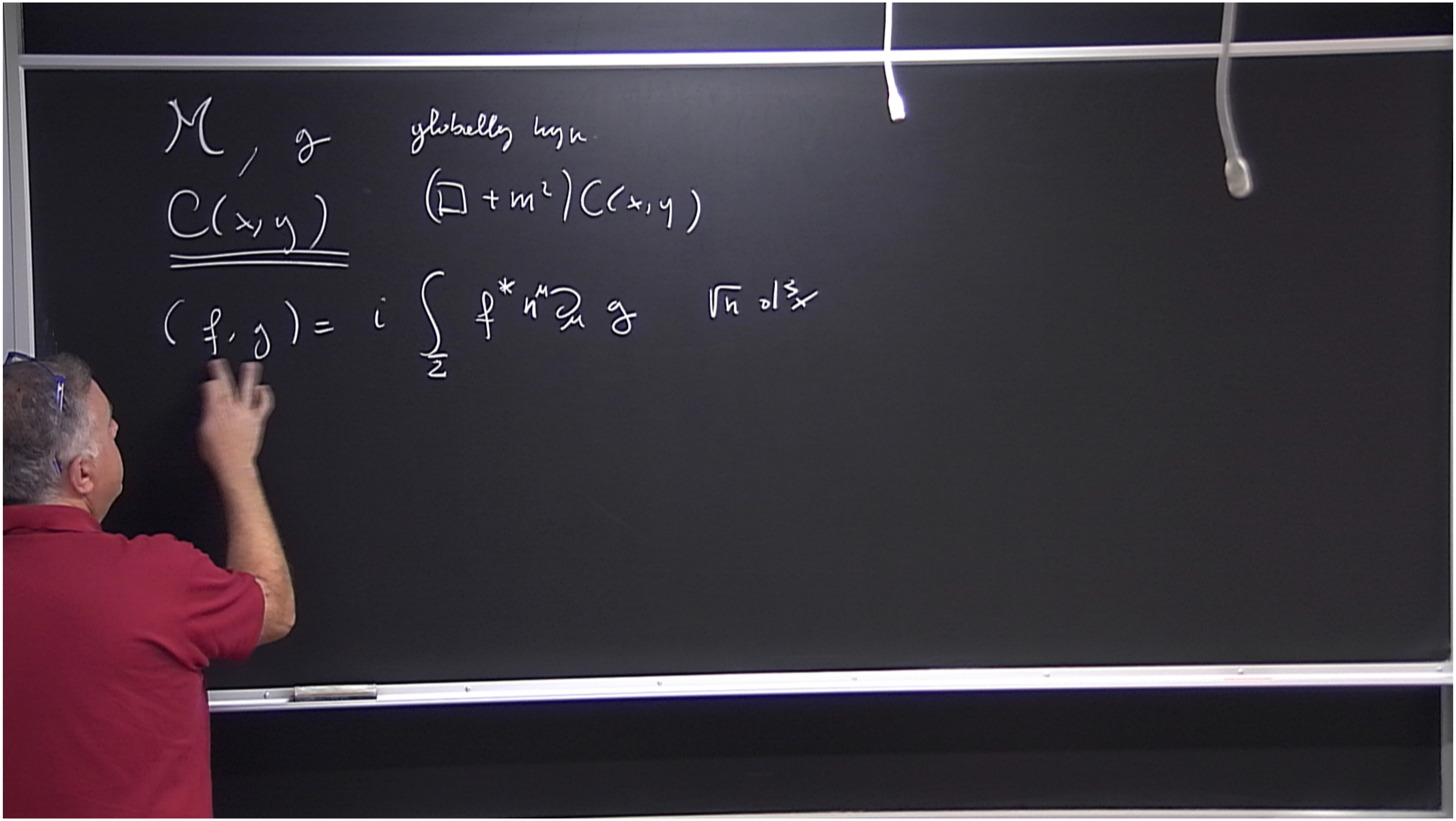
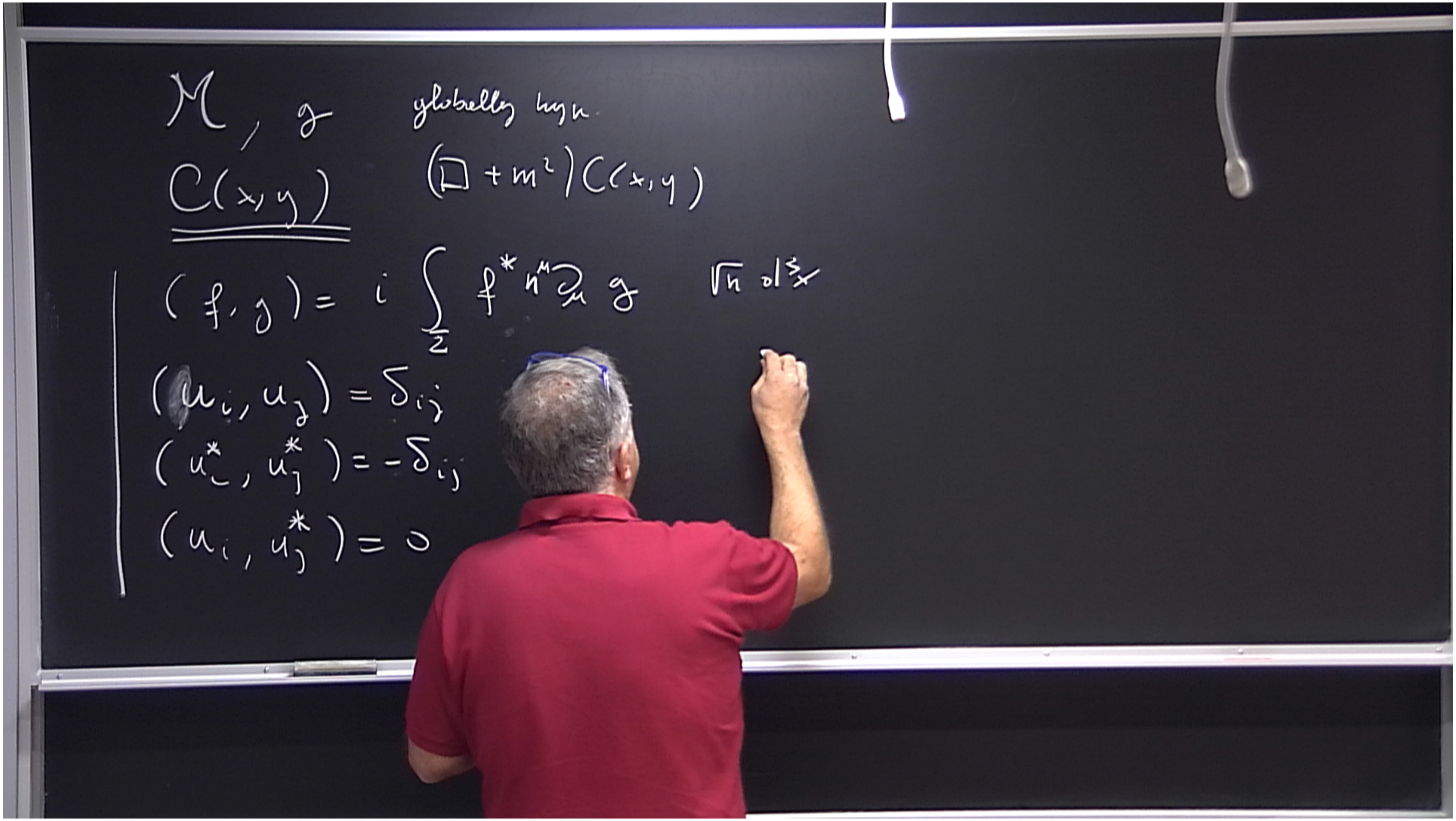
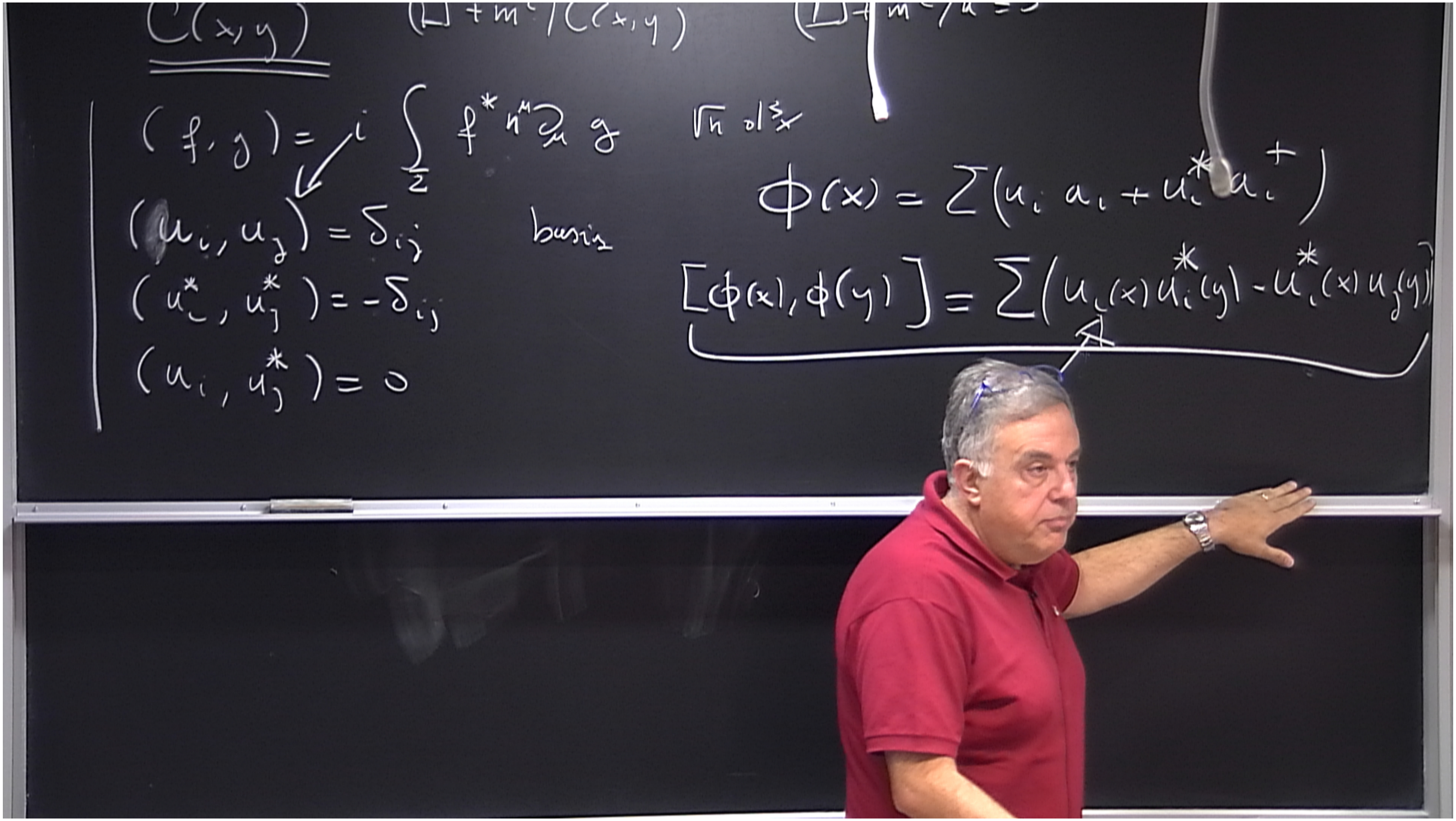
Abstract:


M, g globally hyp
$C(x,y)$
$(\Box + m^2) C(x,y)$
$(f,g) = i \int_\Sigma f^* n^\mu \partial_\mu g \sqrt{h}\, d^3x$


$\mathcal{M}, g$ globally hyp.
$\underline{\underline{C(x,y)}}$
$(\Box + m^2) C(x,y)$
$(f, g) = i \int_\Sigma f^* n^\mu \partial_\mu g \sqrt{h}\, d^3x$
$(u_i, u_j) = \delta_{ij}$
$(u_i^*, u_j^*) = -\delta_{ij}$
$(u_i, u_j^*) = 0$



$C(x,y)$
$(\Box + m^2)C(x,y)$
$(\Box + m^2)u = 0$
$(f, g) = i \int_\Sigma f^* n^\mu \partial_\mu g \quad \sqrt{h}\, d^3x$
$(u_i, u_j) = \delta_{ij}$ basis
$(u_i^*, u_j^*) = -\delta_{ij}$
$(u_i, u_j^*) = 0$
$\phi(x) = \sum (u_i a_i + u_i^* a_i^\dagger)$
$[\phi(x), \phi(y)] = \sum (u_i(x) u_i^*(y) - u_i^*(x) u_i(y))$

$$(u_i^*, u_j^*) = -\delta_{ij}$$

$$(u_i, u_j^*) = 0$$

$$[\phi(x), \phi(y)] = \sum \left(u_i(x) u_i^*(y) - u_i^*(x) u_j(y) \right)$$

$$C(x,y) = W(x,y) - W(y,x)$$

$$W(x,y) = \sum u_i(x) u_i^*(y)$$

$$a_i |0\rangle = 0$$

$$u'_i = \alpha_{ij} u_j + \beta_{ij} u_j^*$$

$$u_i^* = \beta_{ij}^* u_j + \alpha_{ij}^* u_j^*$$

$$(u'_i, u'_j) =$$

$$u'_i = \alpha_{ij} u_j + \beta_{ij} u_j^* \quad \Longleftarrow \quad \text{canonical transformation}$$

$$u_i'^* = \beta_{ij}^* u_j + \alpha_{ij}^* u_j^*$$

$$(u'_i, u'_j) = \left(\alpha_{ik} u_k + \beta_{ik} u_k^*,\ \alpha_{jl} u_l + \beta_{jl} u_l^* \right)$$

$$\alpha_{ik}^* \alpha_{jk} - \beta_{ik}^* \beta_{jk} = \delta_{ij}$$

$$\alpha \alpha^\dagger - \beta \beta^\dagger = \mathbb{1}$$

$$\begin{pmatrix} u' \\ u'^* \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} u \\ u^* \end{pmatrix}$$

$$\alpha\alpha^\dagger - \beta\beta^\dagger = \mathbb{1}$$
$$\alpha\beta^T - \beta\alpha^T = 0$$

$$\begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} \alpha^\dagger & -\beta^\dagger \\ -\beta^\dagger & \alpha^\dagger \end{pmatrix} = \mathbb{1}$$

$$\begin{pmatrix} u' \\ u'^* \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} u \\ u^* \end{pmatrix}$$

$$\begin{pmatrix} u \\ u^* \end{pmatrix} = \begin{pmatrix} \alpha^\dagger & -\beta^T \\ -\beta^\dagger & \alpha^T \end{pmatrix} \begin{pmatrix} u' \\ u'^* \end{pmatrix}$$

$$\alpha\alpha^\dagger - \beta\beta^\dagger = \mathbb{1}$$

$$\alpha\beta^T - \beta\alpha^T = 0$$

$$-\beta^\dagger\alpha + \alpha^T\beta^* = 0$$

$$-\beta^\dagger\beta + \alpha^T\alpha^* = \mathbb{1}$$

$$\phi = \sum \left(u_i a_i + u_i^* a_i^\dagger \right)$$

$$= \sum \left(u_i' a_i' + u_i'^* a_i'^\dagger \right)$$

$$\begin{pmatrix} u' \\ u'^* \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} u \\ u^* \end{pmatrix}$$

$$\begin{pmatrix} u \\ u^* \end{pmatrix} = \begin{pmatrix} \alpha^\dagger & -\beta^T \\ -\beta^\dagger & \alpha^T \end{pmatrix} \begin{pmatrix} u' \\ u'^* \end{pmatrix}$$

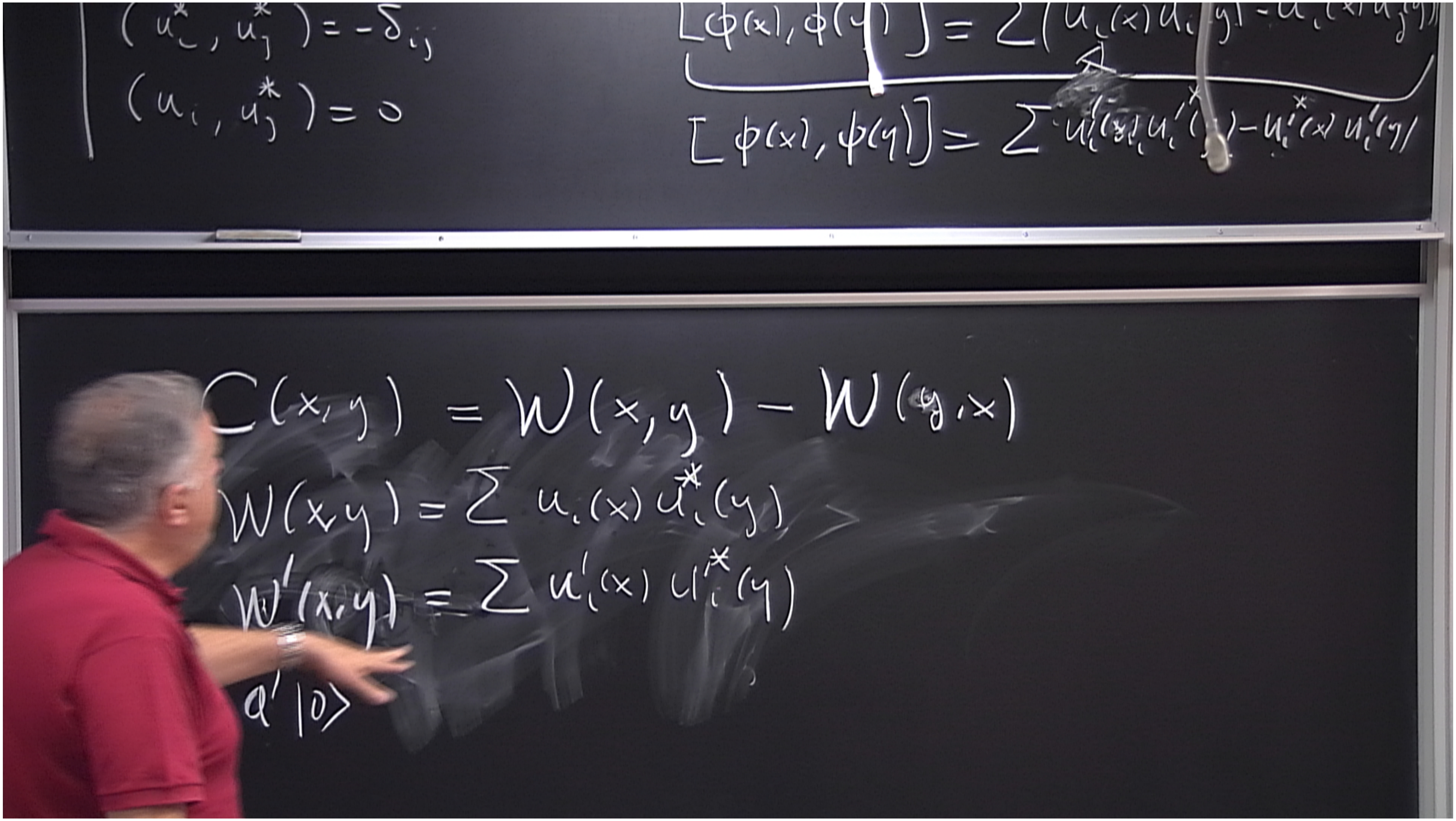
$$\begin{pmatrix} a' \\ a'^\dagger \end{pmatrix} = \begin{pmatrix} \alpha^* & -\beta^* \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} a \\ a^\dagger \end{pmatrix}$$

$$\alpha\alpha^\dagger - \beta\beta^\dagger = \mathbb{1}$$

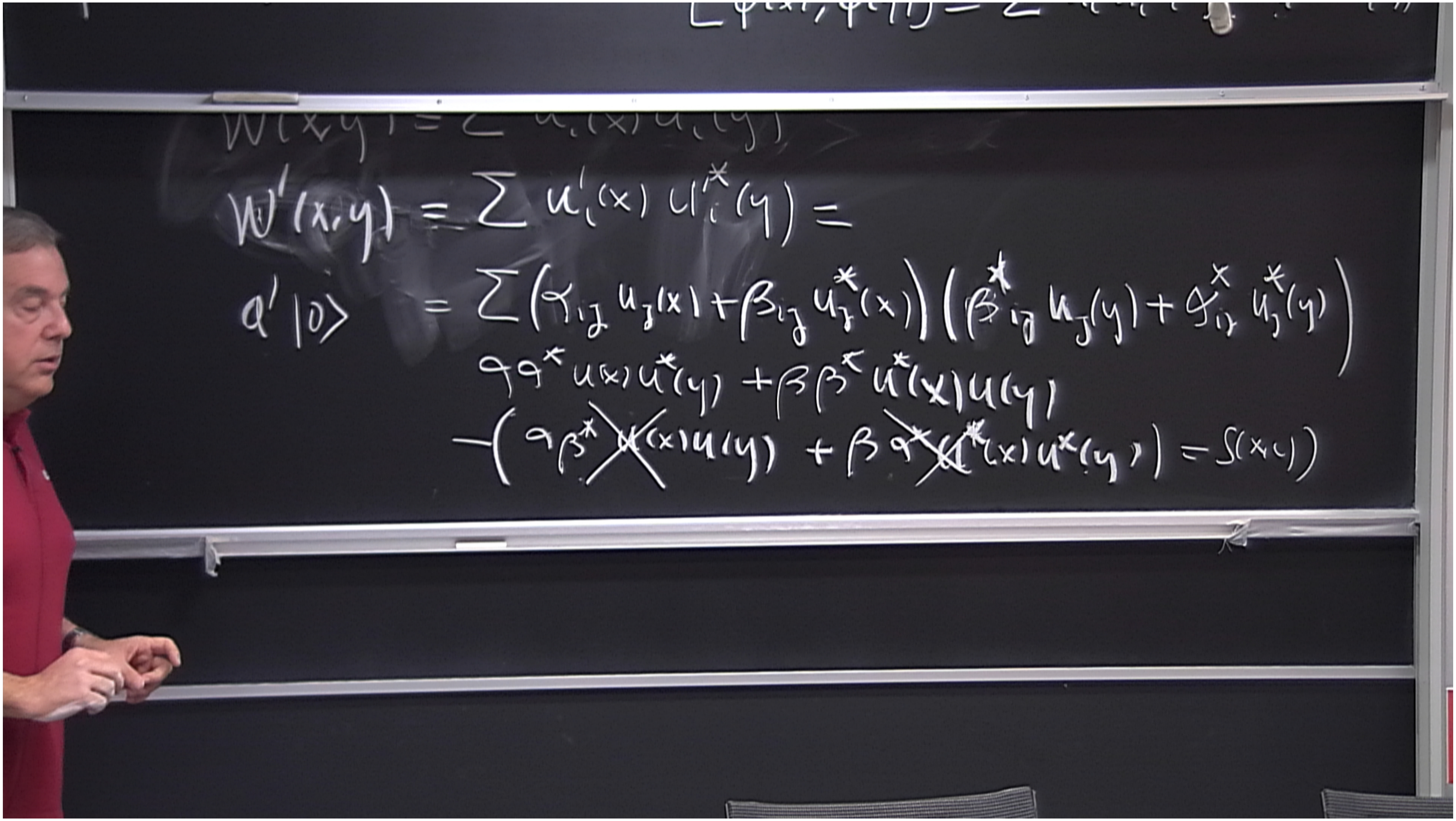
$$\alpha\beta^T - \beta\alpha^T = 0$$

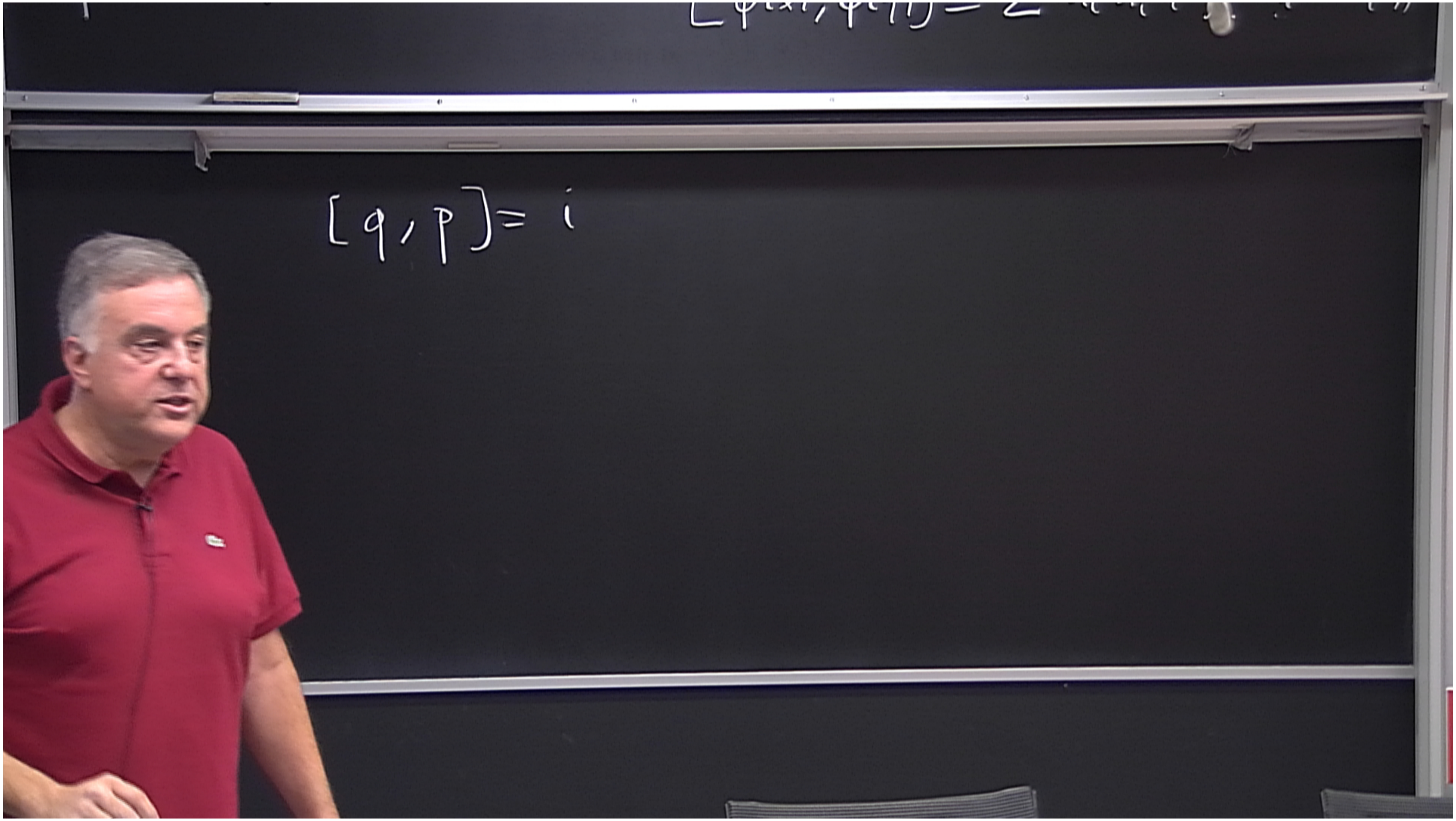
$$-\beta^\dagger\alpha + \alpha^T\beta^* = 0$$

$$-\beta^\dagger\beta + \alpha^T\alpha^* = \mathbb{1}$$



$(u_i^*, u_j^*) = -\delta_{ij}$
$(u_i, u_j^*) = 0$
$[\phi(x), \phi(y)] = \sum (u_i(x) u_i^*(y) - u_i^*(x) u_j(y))$
$[\phi(x), \phi(y)] = \sum u_i'(x) u_i'^*(y) - u_i'^*(x) u_i'(y)$
$C(x,y) = W(x,y) - W(y,x)$
$W(x,y) = \sum u_i(x) u_i^*(y)$
$W'(x,y) = \sum u_i'(x) u_i'^*(y)$
$a'|0\rangle$


$W'(x,y) = \sum u'_i(x)\, u'^*_i(y) =$
$a'|0\rangle = \sum(\alpha_{ij} u_j(x) + \beta_{ij} u_j^*(x))(\beta^*_{ij} u_j(y) + \alpha^*_{ij} u_j^*(y))$
$\alpha\alpha^* u(x) u^*(y) + \beta\beta^* u^*(x) u(y)$
$-(\alpha\beta^* \cancel{u}(x) u(y) + \beta\alpha^* \cancel{u^*}(x) u^*(y)) = \delta(x,y))$


$[q, p] = i$

$$[\phi(x), \phi(y)] = \sum \ldots$$

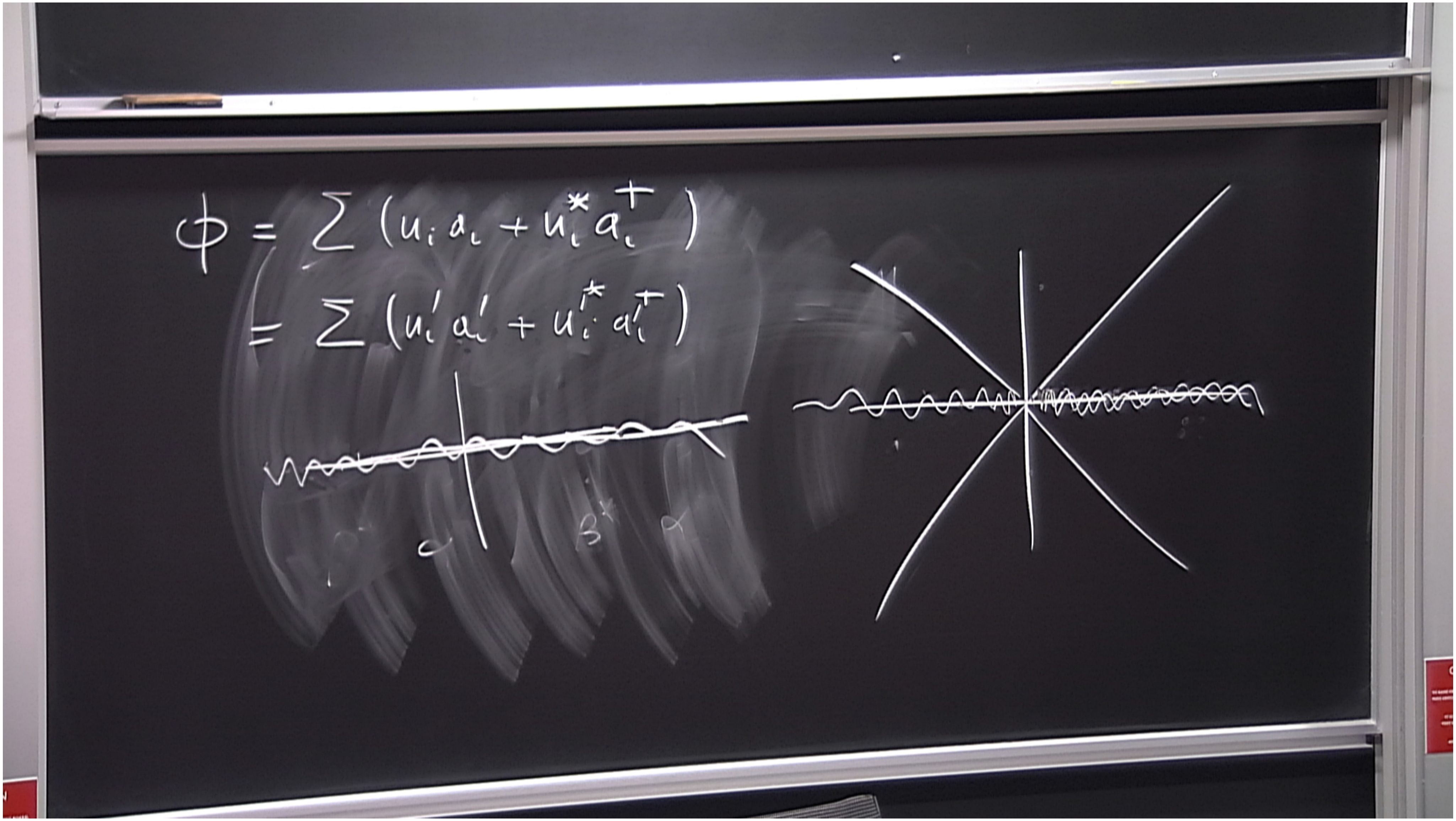
$$W'(x,y) = \sum u_i(x)\, u'_i(y) =$$

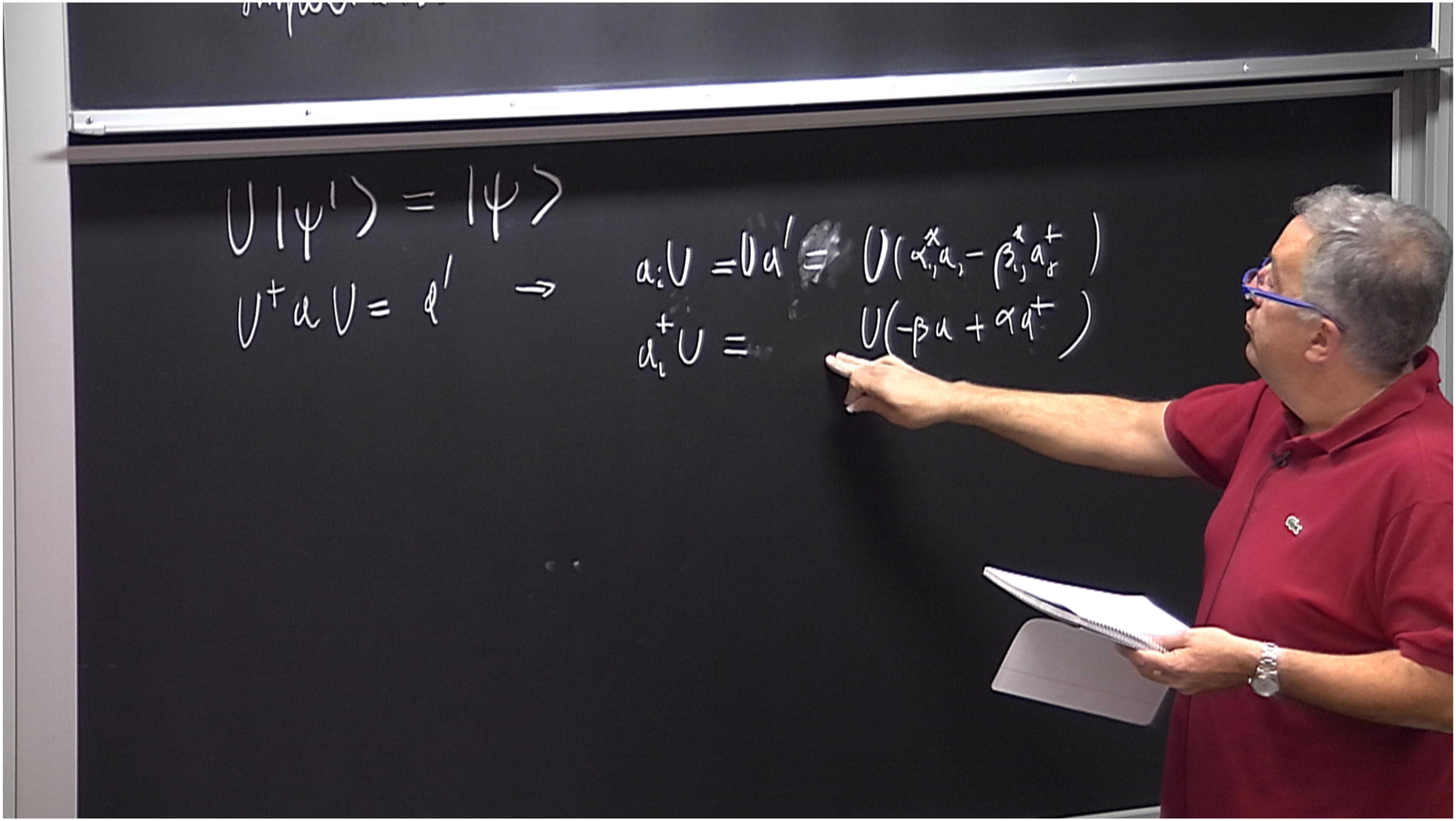
$$a'|0\rangle = \sum \left(\alpha_{ij} u_j(x) + \beta_{ij} u_j^*(x)\right)\left(\beta^*_{ij} u_j(y) + \alpha^*_{ij} u_j^*(y)\right)$$

$$\to \alpha\alpha^* u(x) u^*(y) + \beta\beta^* u^*(x) u(y)$$

$$-\left(\cancel{\alpha\beta^* u(x)} u(y) + \cancel{\beta\alpha^* u^*(x)} u^*(y)\right) + S(x,y))$$

$$a = \frac{(\mathrm{ch}\gamma\, a + \mathrm{sh}\gamma\, b) + (\mathrm{ch}\gamma\, a' + \mathrm{sh}\gamma\, b')}{\sqrt{2}}$$

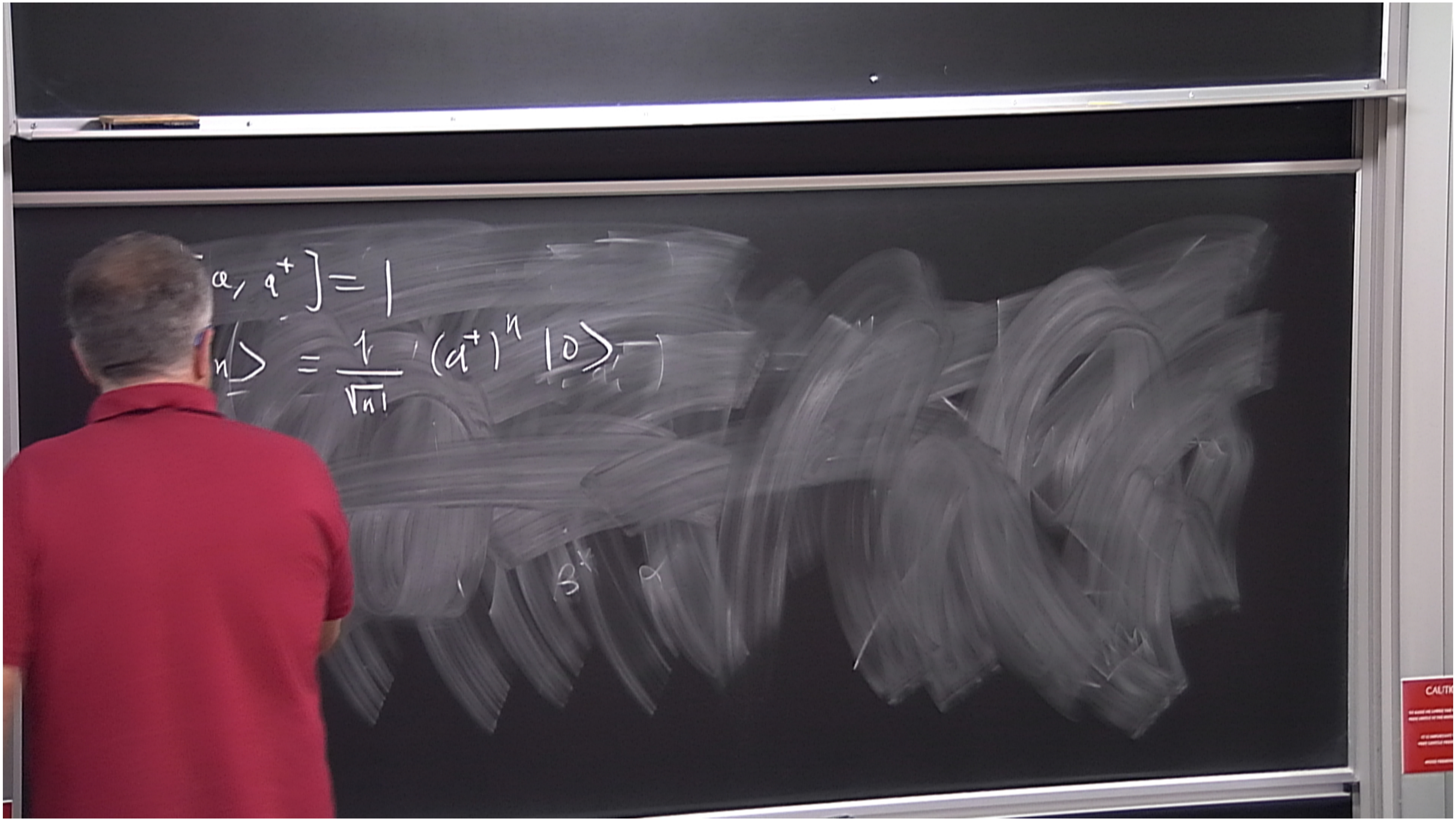

φ = Σ (u_i a_i + u_i* a_i†)
= Σ (u_i' a_i' + u_i'* a_i'†)
α
β*
α

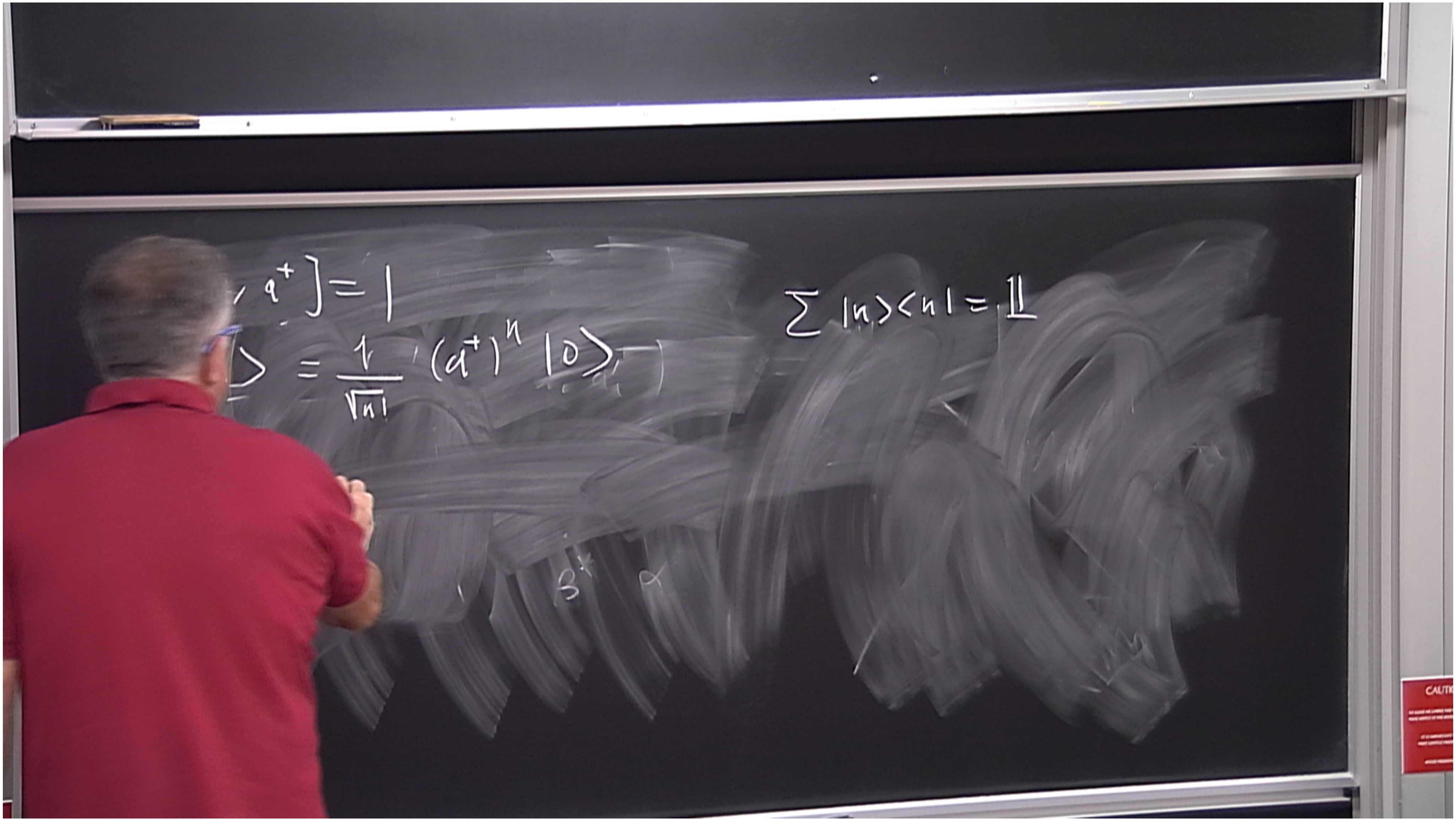

$U|\psi'\rangle = |\psi\rangle$
$U^{+} a U = a' \Rightarrow$
$a_i U = U a' = U(\alpha^{*}_{ij} a_j - \beta^{*}_{ij} a_j^{+})$
$a_i^{+} U = U(-\beta a + \alpha a^{+})$

$$U|\psi'\rangle = |\psi\rangle$$

$$U^\dagger a U = a' \quad \Rightarrow \quad a_i U = U a' = U(\alpha^*_{ij} a_j - \beta^*_{ij} a_j^\dagger)$$

$$a_i^\dagger U = U(-\beta a + \alpha a^\dagger)$$


[a, a⁺] = 1
n⟩ = 1/√n! (a⁺)ⁿ |0⟩



$, a^{+}] = 1$
$\rangle = \frac{1}{\sqrt{n!}} (a^{+})^{n} |0\rangle$
$\sum |n\rangle\langle n| = \mathbb{1}$

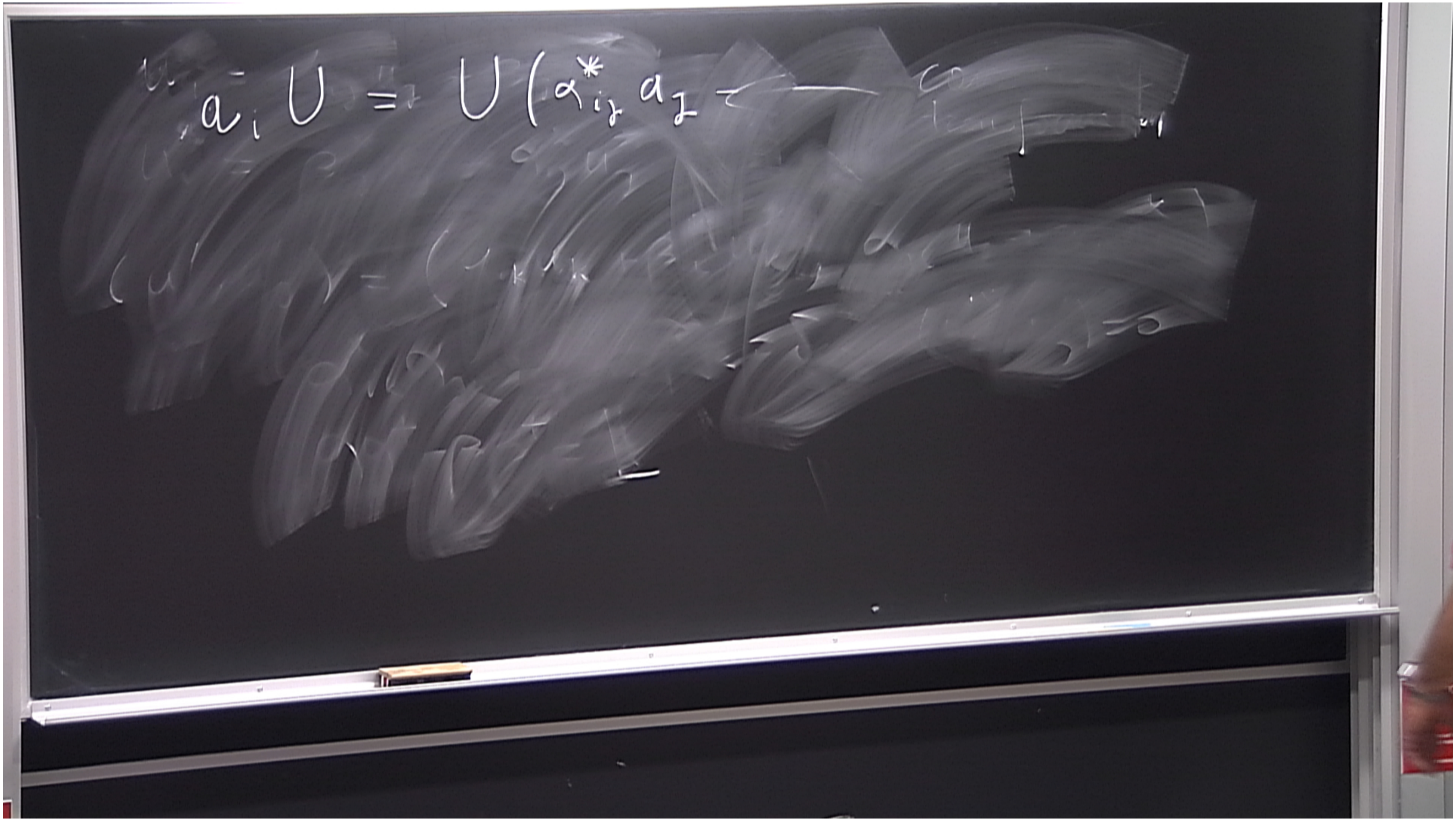
$$\sum |n\rangle\langle n| = \mathbb{1}$$

$$|n\rangle = \frac{1}{\sqrt{n!}} (a_i^+)^n |0\rangle$$

$$\int |z\rangle\langle z| \, \frac{dz^* \wedge dz}{2\pi i} \, e^{-|z|^2} = \mathbb{1}$$

$$|z\rangle = \exp z a^+ \, |0\rangle = \sum \frac{z^n a^{+n}}{n!} |0\rangle = \sum \frac{z^n}{\sqrt{n!}} |n\rangle$$

$$a|z\rangle = z|z\rangle$$


$a_i U = U(a^*_i, a_I$

$$a_i U = U(\alpha^*_{ij} a_j - \beta^*_{ij} a_j^+)$$

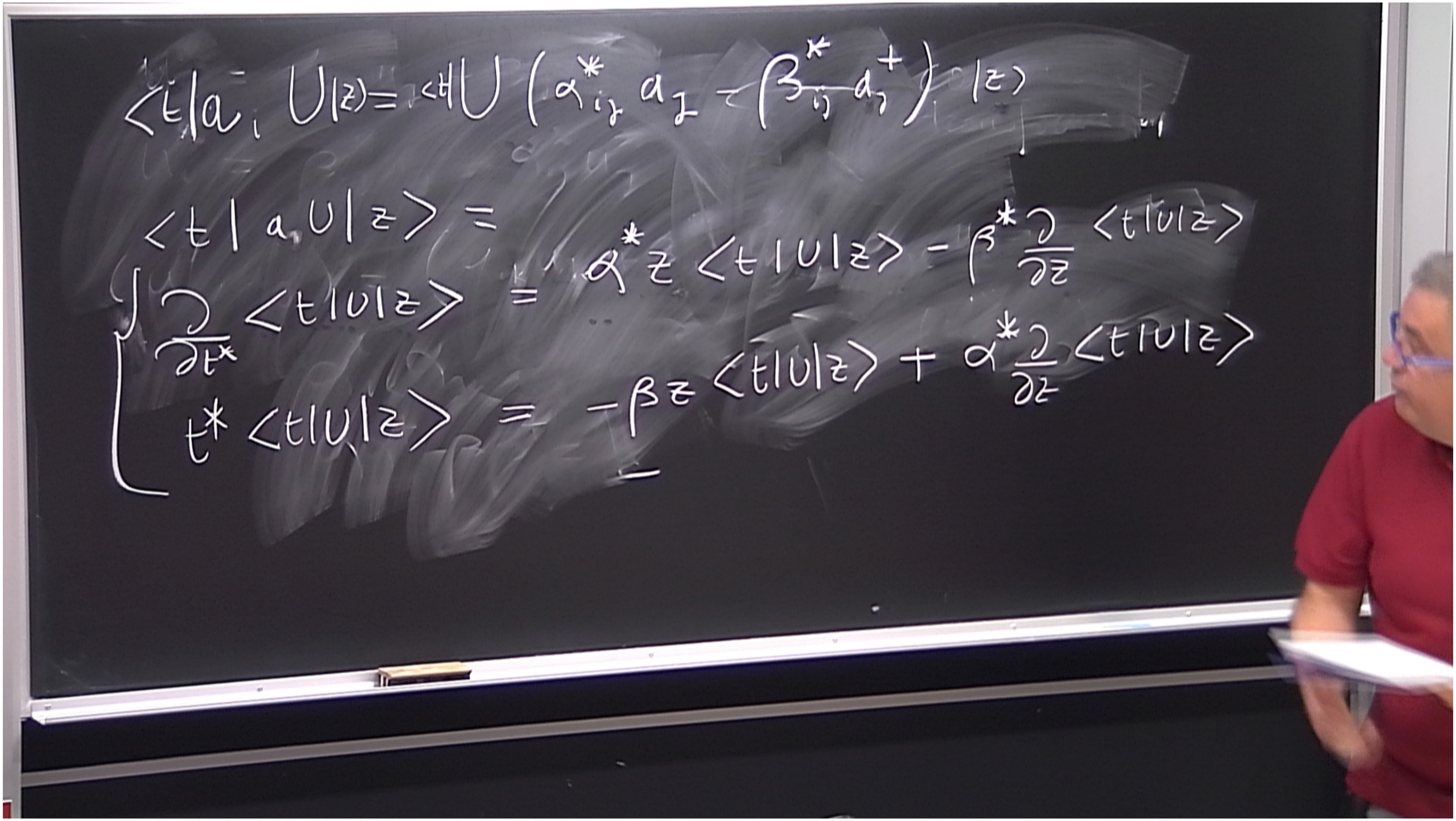
$$\langle t | a_i U | z \rangle =$$

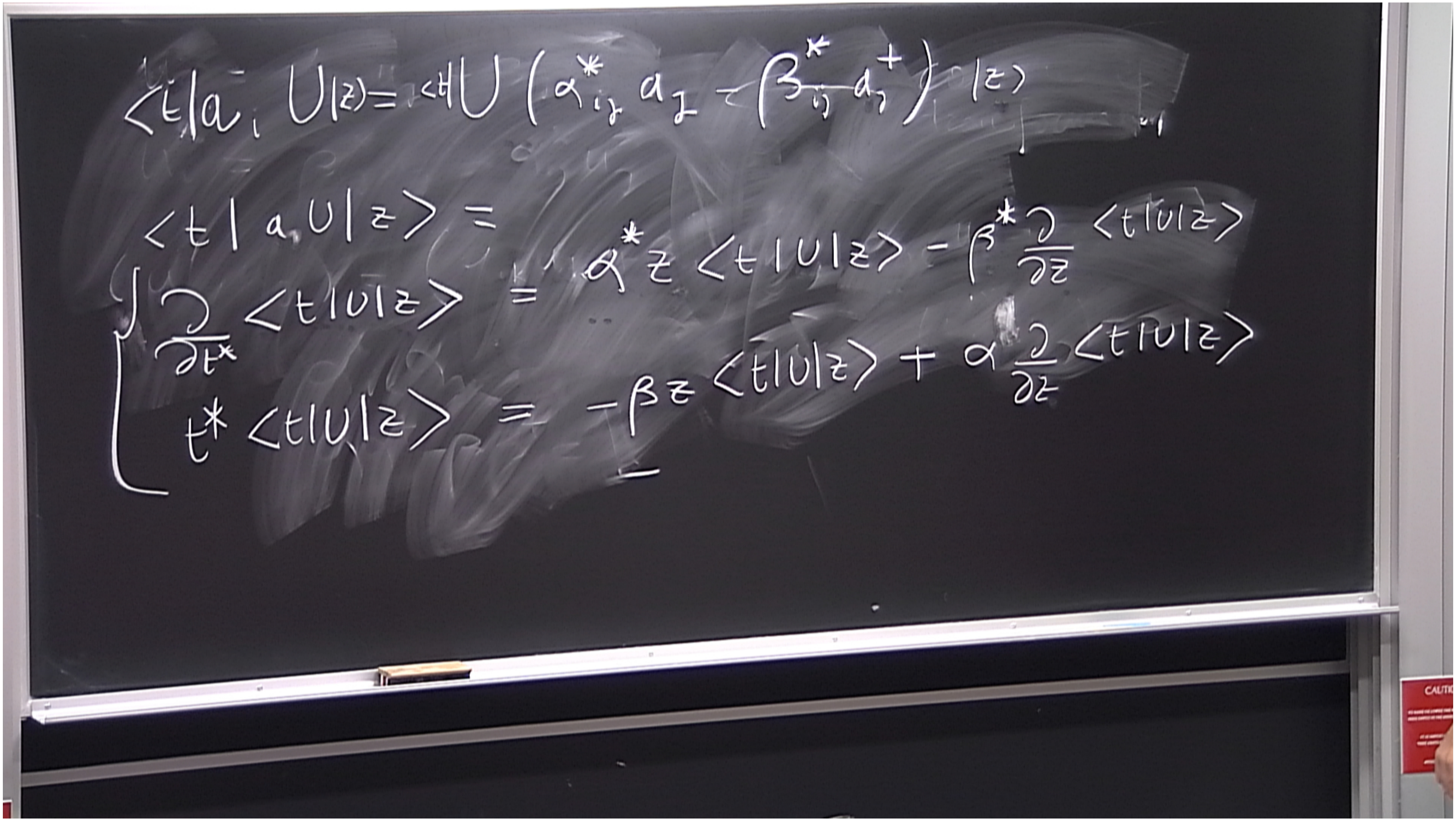
$$= \frac{\partial}{\partial t^*} \langle t | U | z \rangle$$

$$\langle t | a_i U | z \rangle = \langle t | U \left(\alpha^*_{ij} a_j - \beta^*_{ij} a_j^+ \right) | z \rangle$$

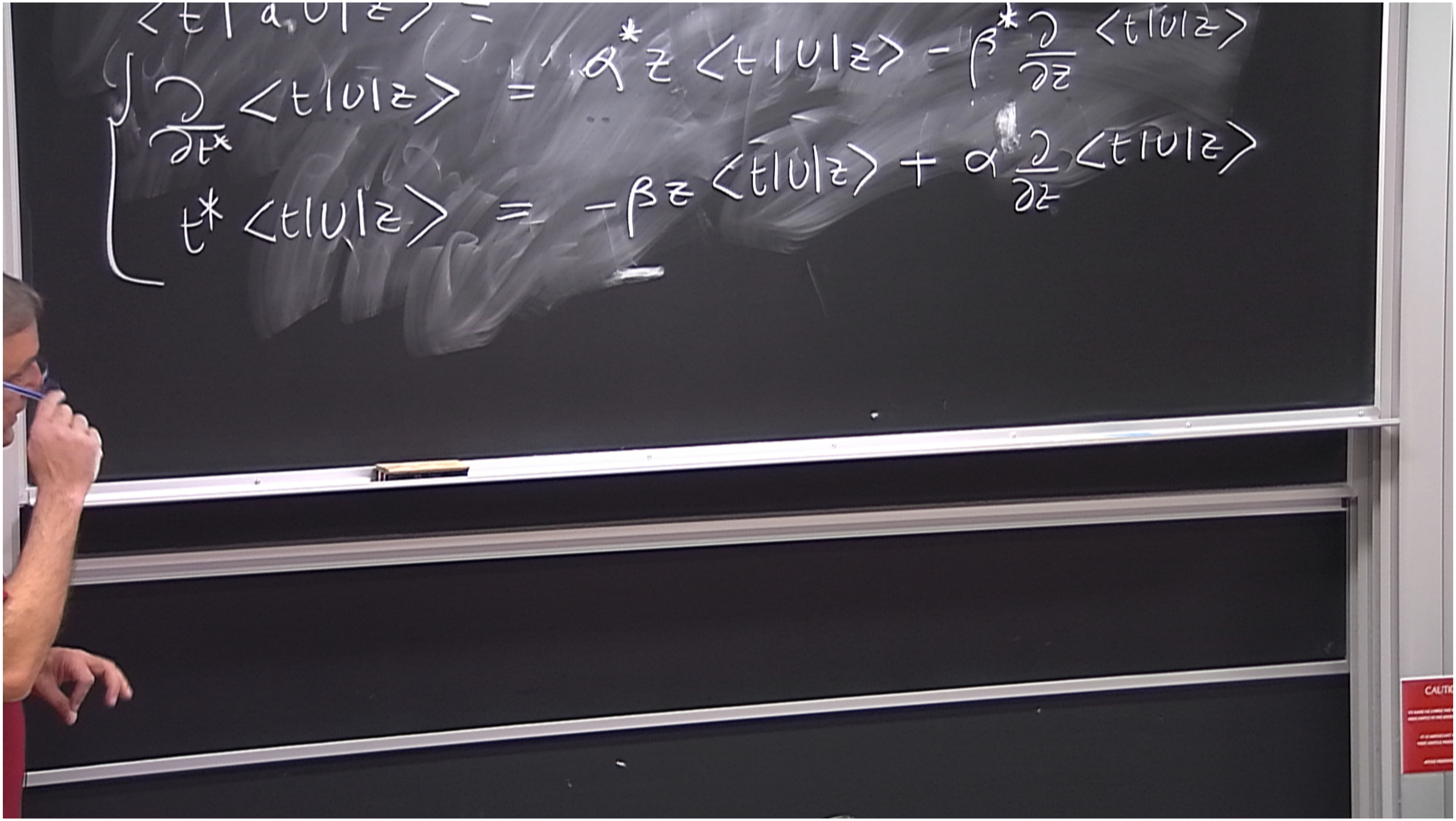
$$\langle t | a U | z \rangle = \alpha^* z \langle t | U | z \rangle - \beta^* \frac{\partial}{\partial z} \langle t | U | z \rangle$$

$$\begin{cases} \dfrac{\partial}{\partial t^*} \langle t | U | z \rangle = \alpha^* z \langle t | U | z \rangle - \beta^* \dfrac{\partial}{\partial z} \langle t | U | z \rangle \\ t^* \langle t | U | z \rangle = -\beta z \langle t | U | z \rangle + \alpha^* \dfrac{\partial}{\partial z} \langle t | U | z \rangle \end{cases}$$


$\langle t|a_i U|z\rangle = \langle t|U(\alpha^*_{ij} a_j - \beta^*_{ij} a_j^\dagger)|z\rangle$
$\langle t|a U|z\rangle =$
$\alpha^* z\langle t|U|z\rangle - \beta^*\frac{\partial}{\partial z}\langle t|U|z\rangle$
$\frac{\partial}{\partial t^*}\langle t|U|z\rangle =$
$-\beta z\langle t|U|z\rangle + \alpha^*\frac{\partial}{\partial z}\langle t|U|z\rangle$
$t^*\langle t|U|z\rangle =$



$\langle t|a_i U|z\rangle = \langle t|U(\alpha^*_{ij} a_j - \beta^*_{ij} a_j^\dagger)|z\rangle$
$\langle t|aU|z\rangle =$
$\frac{\partial}{\partial t^*}\langle t|U|z\rangle = \alpha^* z\langle t|U|z\rangle - \beta^*\frac{\partial}{\partial z}\langle t|U|z\rangle$
$t^*\langle t|U|z\rangle = -\beta z\langle t|U|z\rangle + \alpha\frac{\partial}{\partial z}\langle t|U|z\rangle$


$\langle t | a\, U | z \rangle =$
$\frac{\partial}{\partial t^*} \langle t|U|z\rangle = \alpha^* z \langle t|U|z\rangle - \beta^* \frac{\partial}{\partial z} \langle t|U|z\rangle$
$t^* \langle t|U|z\rangle = -\beta z \langle t|U|z\rangle + \alpha \frac{\partial}{\partial z} \langle t|U|z\rangle$

$$U|\psi'\rangle = |\psi\rangle$$

$$U^\dagger a U = a' \Rightarrow \quad a_i U = U a' = U(\alpha^*_{ij} a_j - \beta^*_{ij} a^\dagger_j)$$

$$a_i^\dagger U = U(-\beta a + \alpha a^\dagger)$$

$$\alpha\alpha^\dagger = 1 + \beta\beta^\dagger$$

$$\langle\psi|\alpha\alpha^\dagger|\psi\rangle = \|\psi\|^2 + \|\beta^\dagger\psi\|^2$$

$$\|\alpha^\dagger\psi\|^2 = \|\psi\|^2$$

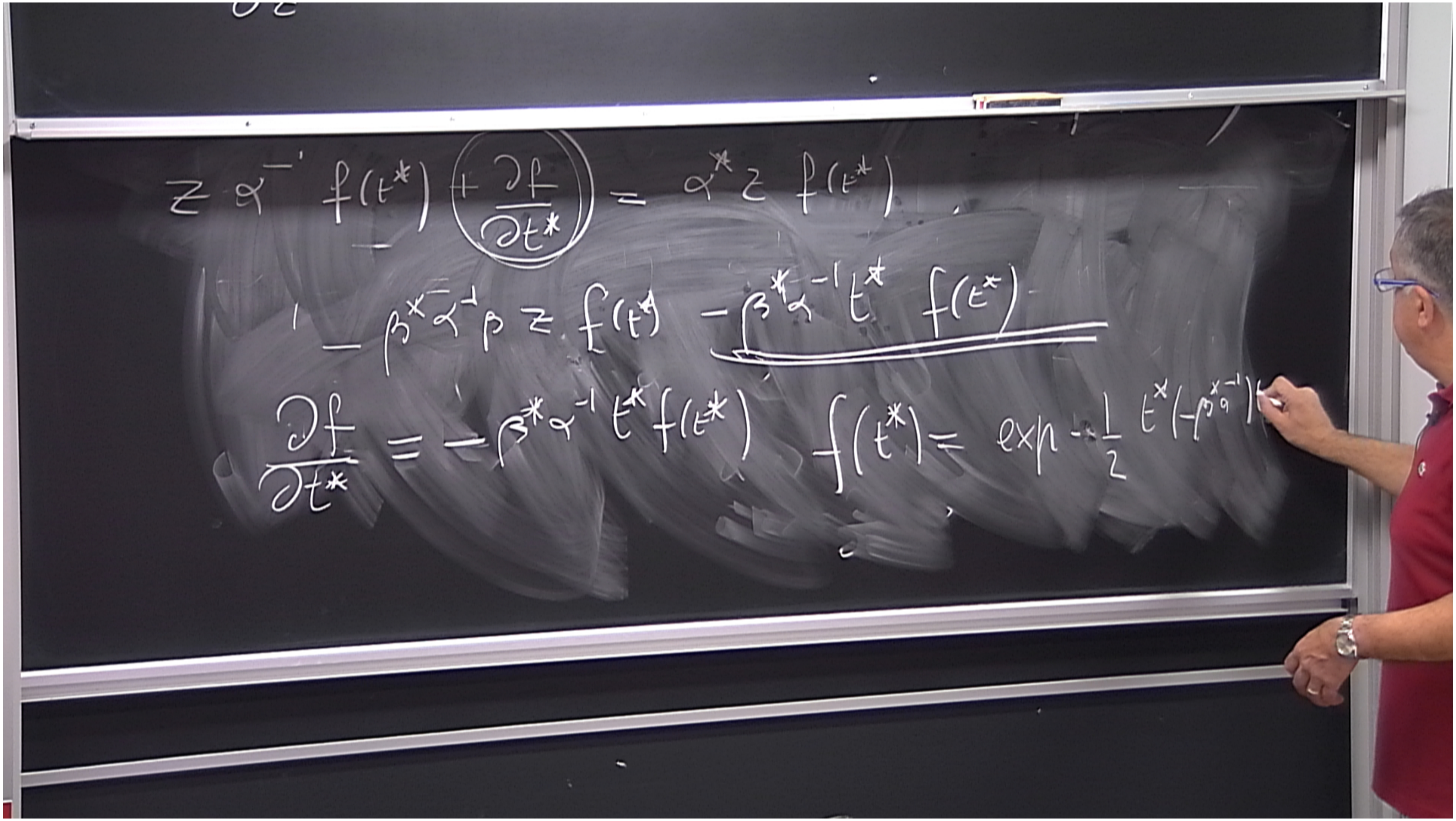
$$\langle t|U|z\rangle = \left(\exp z\alpha^{-1}t^{*} + \frac{1}{2} z\alpha^{-1}\beta z\right) f(t^{*})$$

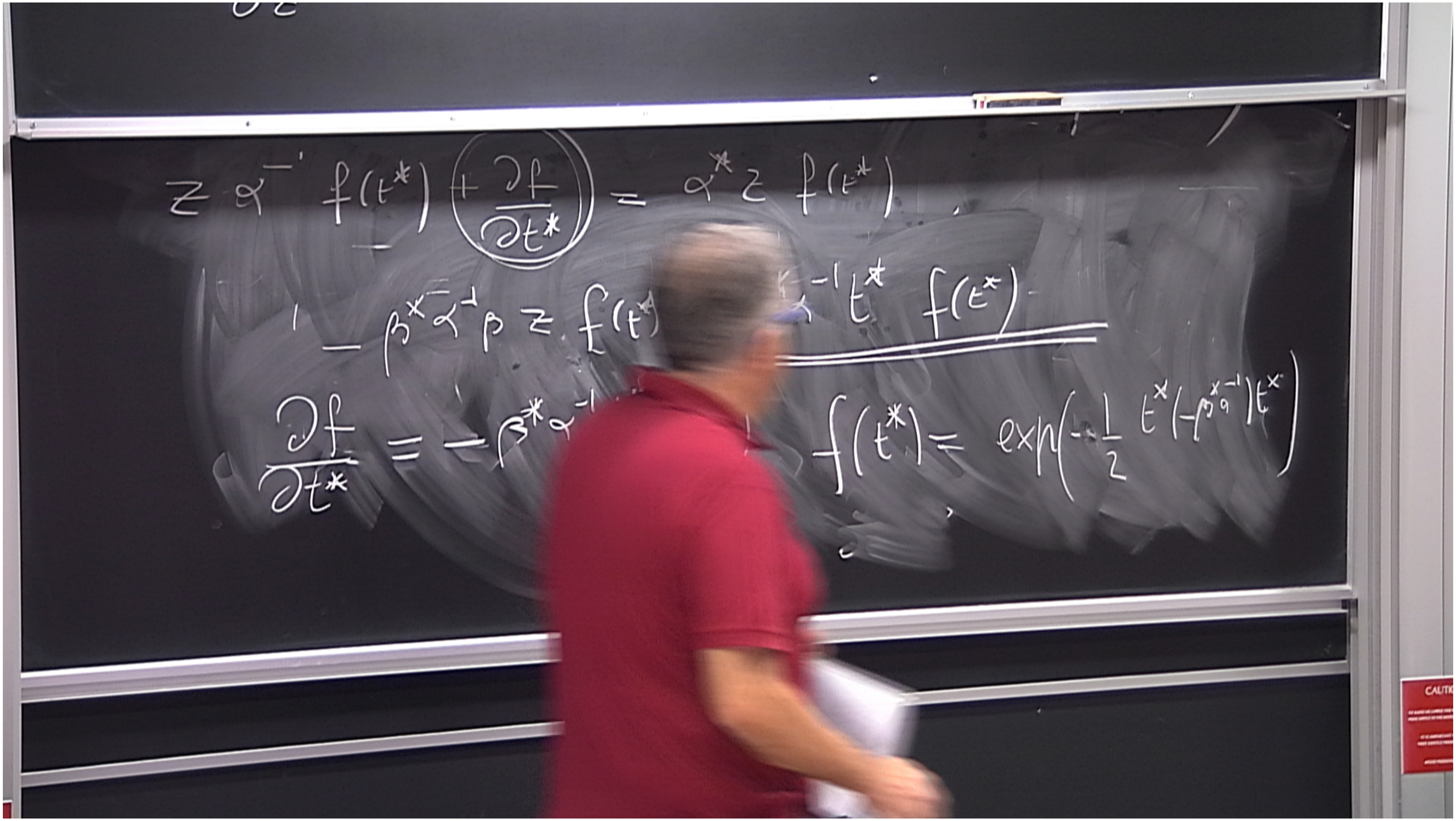
$$\langle t|U|z\rangle = \left(\exp\ z\alpha^{-1}t^* + \frac{1}{2} z \underbrace{\alpha^{-1}\beta}\, z\right) f(t^*)$$

$$\beta^T = \alpha^{-1}\beta\,\alpha^T$$

$$\alpha\beta^T = \beta\alpha^T$$

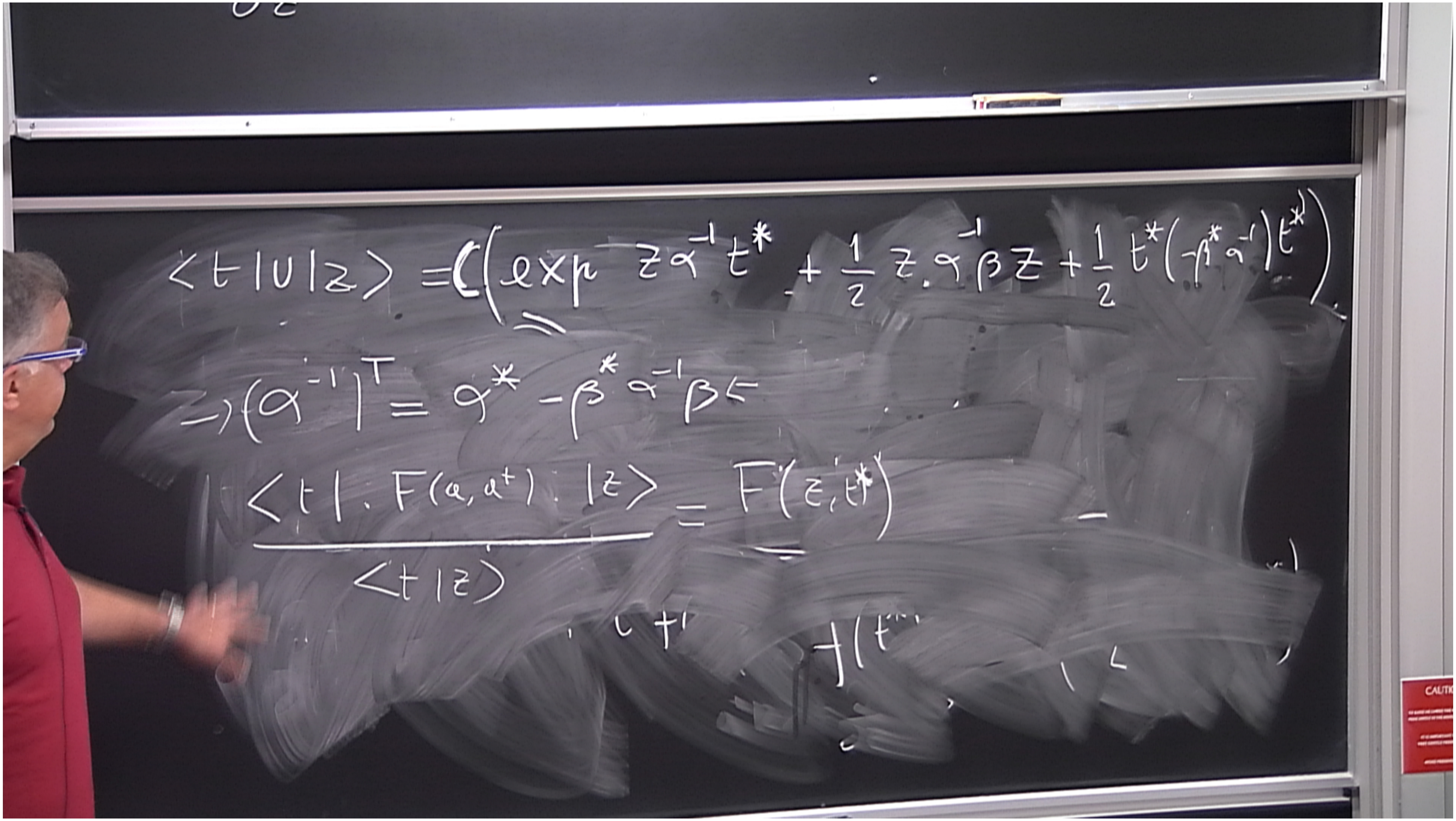
$$(\alpha^{-1}\beta)^T = \beta^T(\alpha^{-1})^T \qquad \alpha^{-1}\beta\,\cancel{\alpha^T}\,\cancel{\alpha^{-1T}}$$

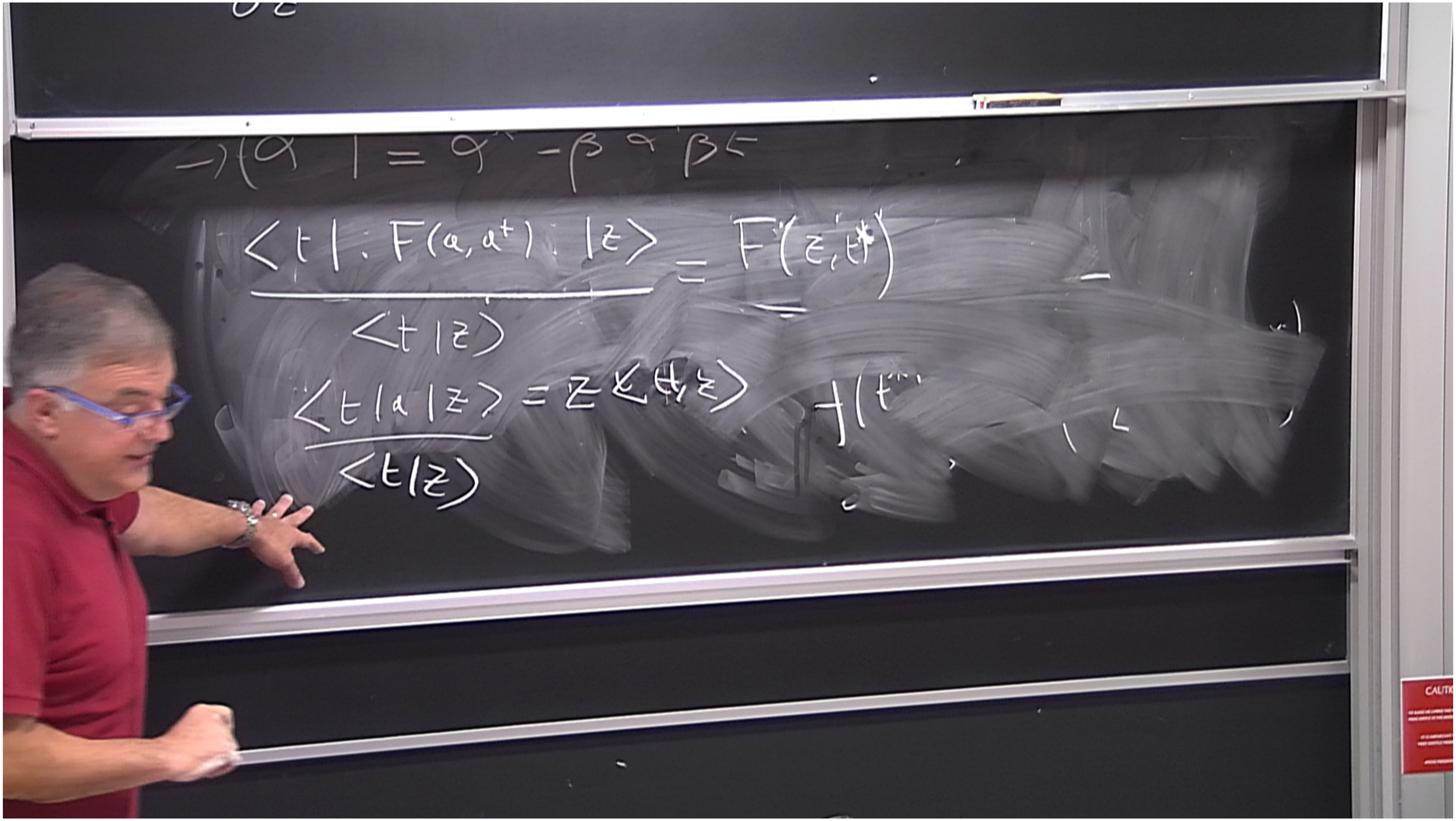

$z\alpha^{-1} f(t^*) + \frac{\partial f}{\partial t^*} = \alpha^* z f(t^*)$
$-\beta^*\alpha^{-1}\beta z f(t^*) - \beta^*\alpha^{-1} t^* f(t^*)$
$\frac{\partial f}{\partial t^*} = -\beta^*\alpha^{-1} t^* f(t^*)$
$f(t^*) = \exp -\frac{1}{2} t^*(-\beta^*\alpha^{-1})$


$z\,\alpha^{-1} f(t^*) + \frac{\partial f}{\partial t^*} = \alpha^* z\, f(t^*)$
$f(t^*) = \exp\left(-\frac{1}{2} t^* (-\beta^* \alpha^{-1}) t^*\right)$

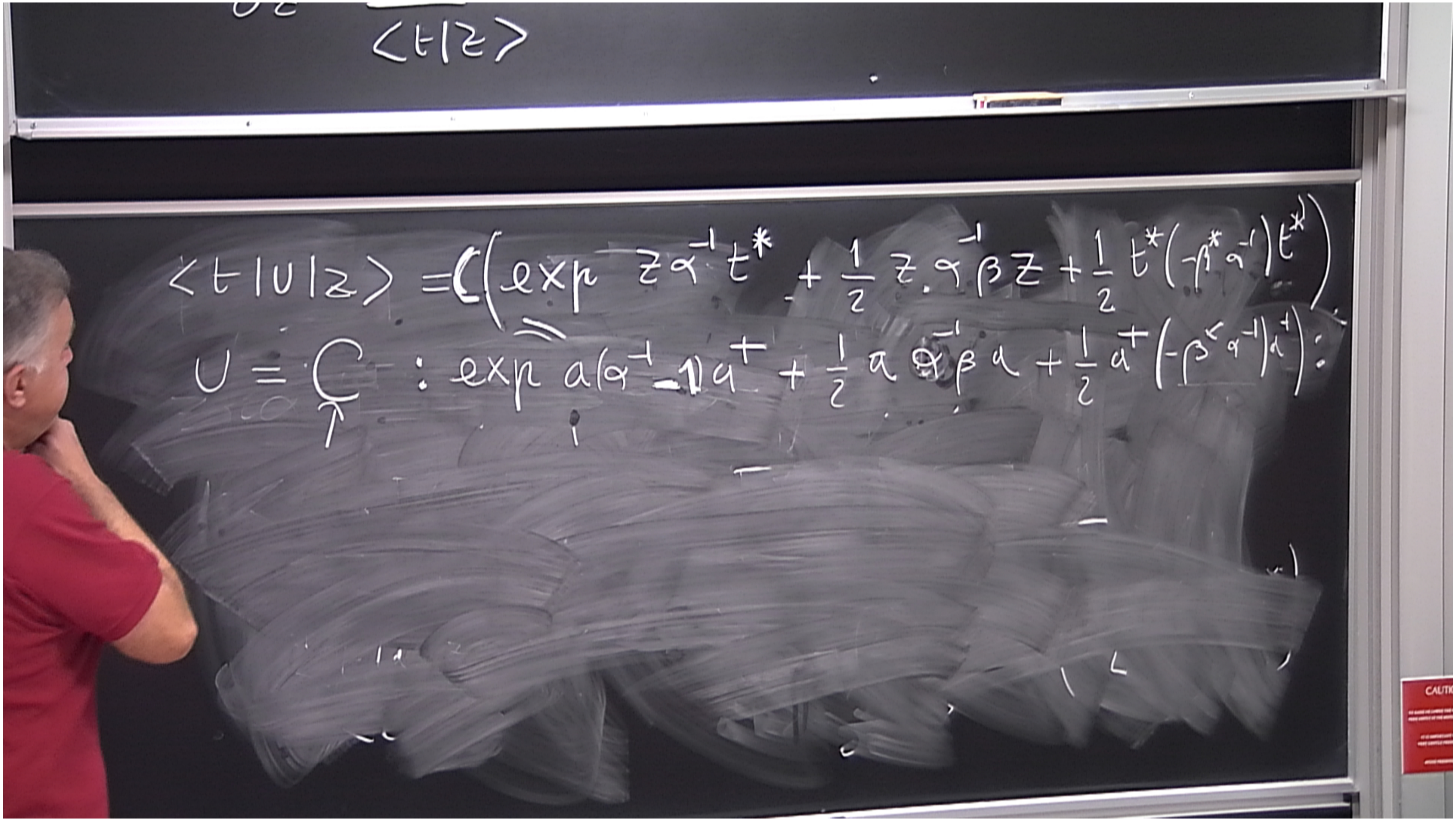
$$\langle t|U|z\rangle = C\left(\exp\ z\alpha^{-1}t^* + \frac{1}{2} z\,\alpha^{-1}\beta z + \frac{1}{2} t^*(-\beta^*\alpha^{-1})t^*\right)$$

$$\Rightarrow (\alpha^{-1})^T = \alpha^* - \beta^*\alpha^{-1}\beta$$


$\langle t|U|z\rangle = C\left(\exp\, z\alpha^{-1}t^* + \frac{1}{2} z\alpha^{-1}\beta z + \frac{1}{2} t^*(-\beta^*\alpha^{-1})t^*\right)$
$\Rightarrow (\alpha^{-1})^T = \alpha^* - \beta^*\alpha^{-1}\beta$
$\frac{\langle t|\, F(a,a^\dagger)\, |z\rangle}{\langle t|z\rangle} = F(z,t^*)$



$\frac{\langle t| :F(a,a^+): |z\rangle}{\langle t|z\rangle} = F(z,t^*)$
$\frac{\langle t|a|z\rangle}{\langle t|z\rangle} = z$


⟨t|z⟩
⟨t|U|z⟩ = C (exp zα⁻¹t* + ½ zα⁻¹βz + ½ t*(−β*α⁻¹)t*)
U = C :exp a(α⁻¹−1)a⁺ + ½ aα⁻¹βa + ½ a⁺(−β*α⁻¹)a⁺:

$$\langle t|z\rangle$$

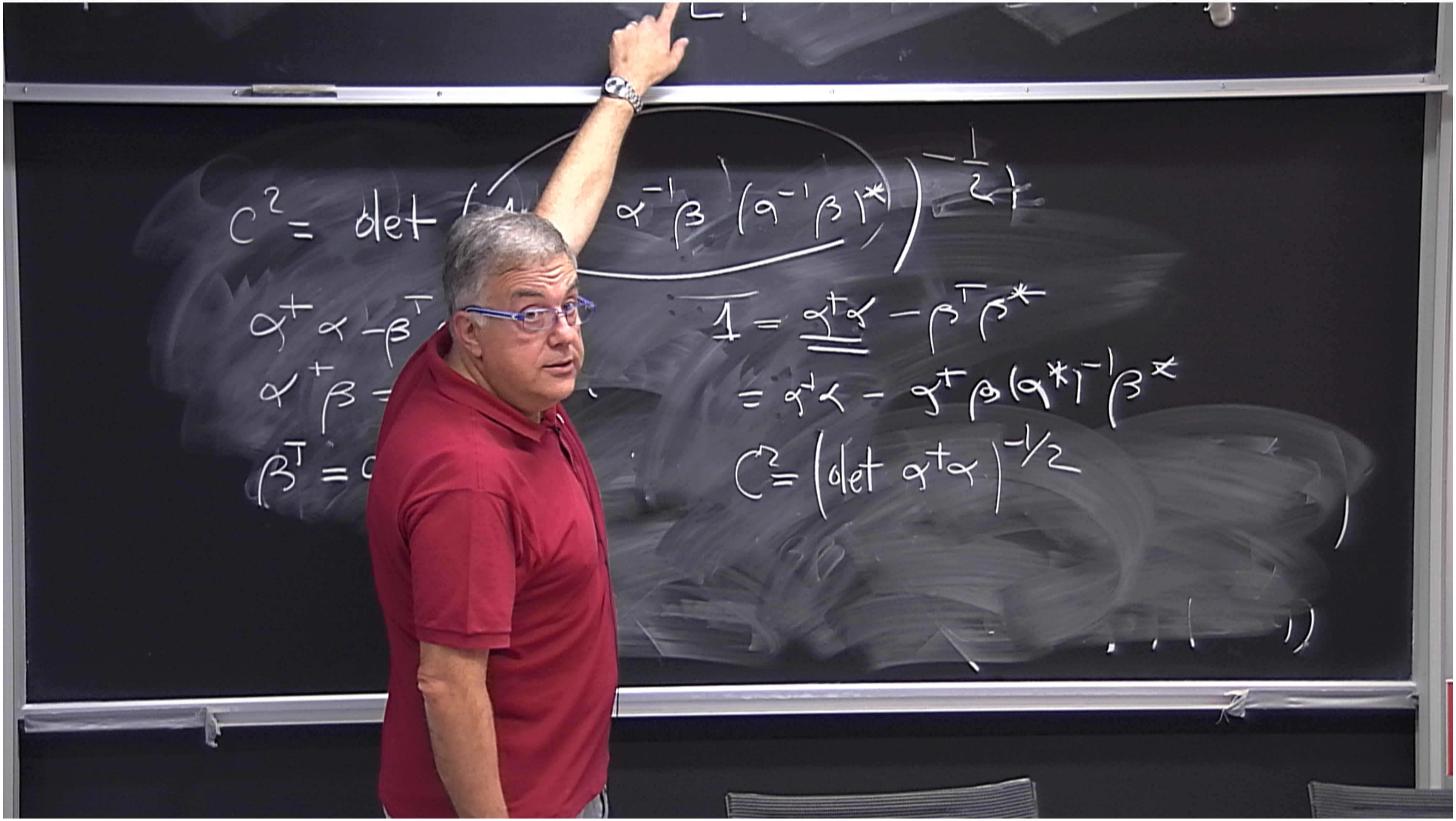
$$\langle t|U|z\rangle = C\left(\exp\, z\alpha^{-1}t^* + \frac{1}{2} z\,\alpha^{-1}\beta z + \frac{1}{2} t^*\left(-\beta^*\alpha^{-1}\right)t^*\right)$$

$$U = C \,:\exp\, a(\alpha^{-1}-1)a^{+} + \frac{1}{2} a\,\alpha^{-1}\beta a + \frac{1}{2} a^{+}\left(-\beta^*\alpha^{-1}\right)a^{+}:$$

$$\langle 0|U\, U^{+}|0\rangle = |C|^2$$

$$c^2 = \det\left(\mathbb{1} - \alpha^{-1}\beta\,(\alpha^{-1}\beta)^*\right)^{-\frac{1}{2}}$$

$$\alpha^\dagger \alpha - \beta^T \beta^* = \mathbb{1}$$


$1 = \alpha^\dagger\alpha - \beta^T\beta^*$
$= \alpha^\dagger\alpha - \alpha^\dagger\beta(\alpha^*)^{-1}\beta^*$
$C^2 = (\det \alpha^\dagger\alpha)^{-1/2}$

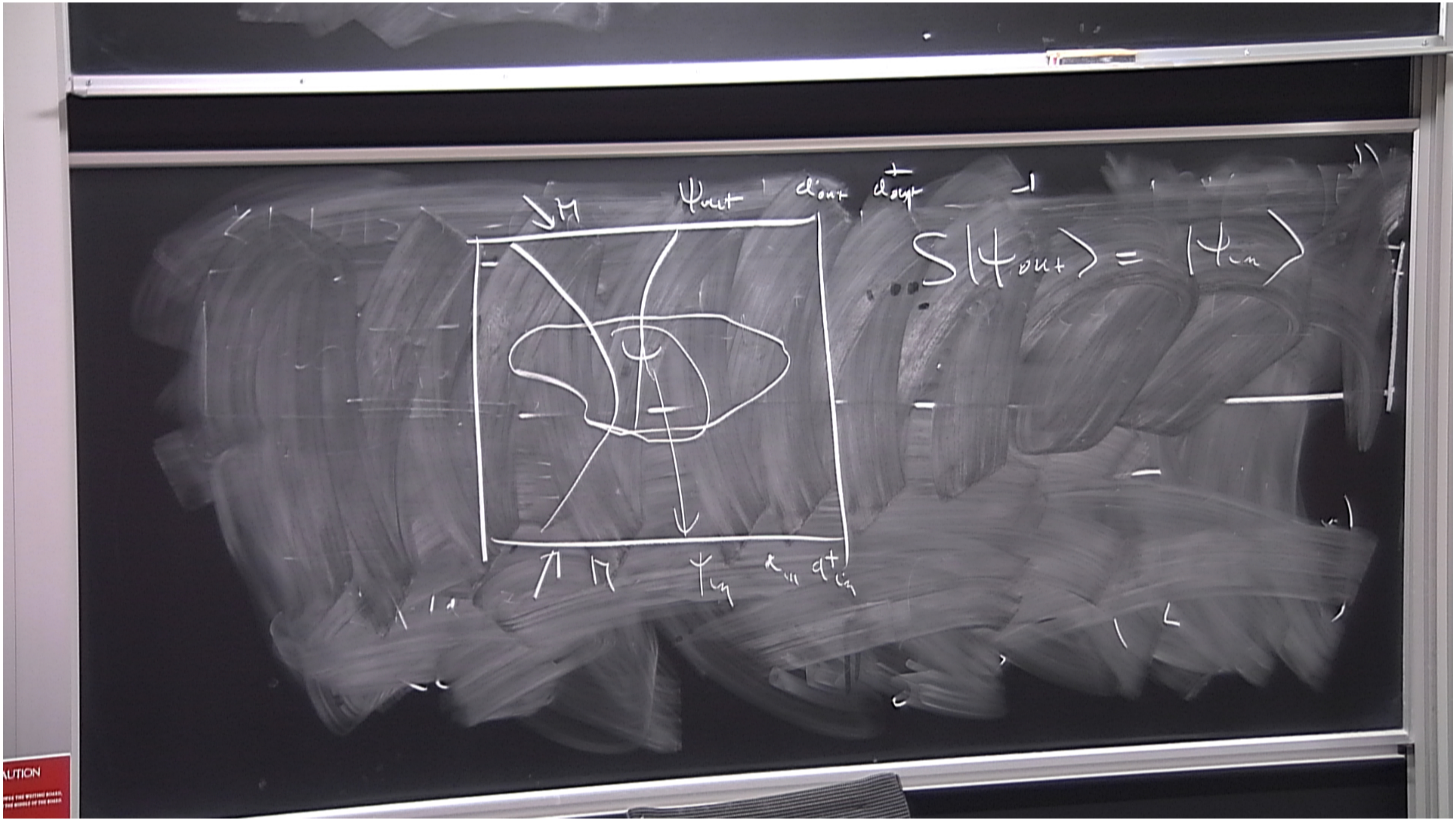
$$\rightarrow \alpha\beta - \beta\alpha = 0$$

$$\begin{pmatrix} u \\ u^* \end{pmatrix} = \begin{pmatrix} \alpha^\dagger & -\beta^T \\ -\beta^\dagger & \alpha^T \end{pmatrix}\begin{pmatrix} u' \\ u'^* \end{pmatrix}$$

$$\begin{cases} -\beta^T a + \alpha^T \beta^* = 0 \\ -\beta^\dagger \beta + \alpha^T \alpha^* = \mathbb{1} \end{cases}$$

$$\begin{pmatrix} a' \\ a'^\dagger \end{pmatrix} = \begin{pmatrix} \alpha^* & -\beta^* \\ -\beta & \alpha \end{pmatrix}\begin{pmatrix} a \\ a^\dagger \end{pmatrix} \longleftarrow$$

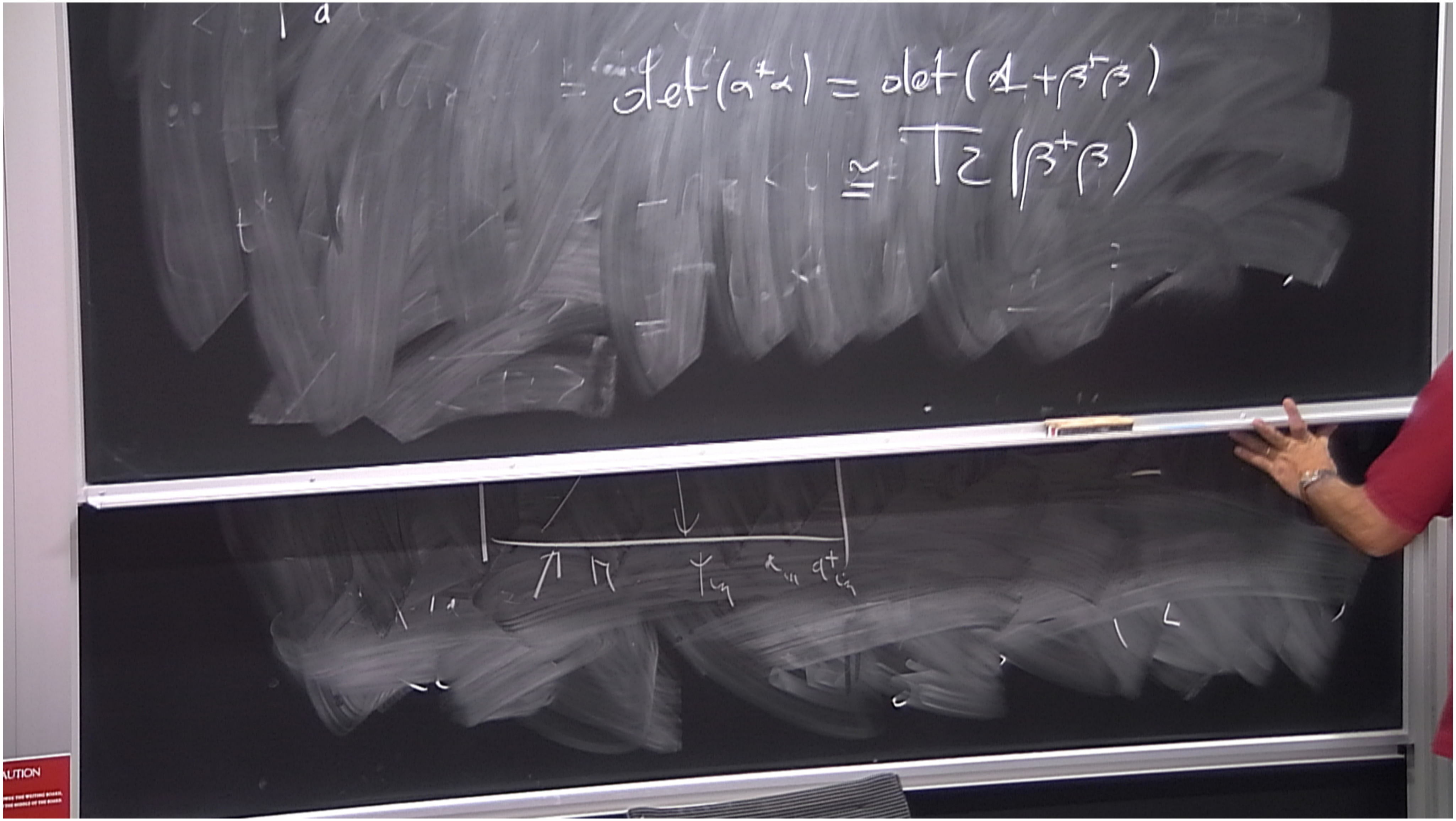
$$U = \frac{1}{(\det \alpha^\dagger \alpha)^{1/4}} :\exp\left[a(\alpha^{-1} - \mathbb{1})a^\dagger + \frac{1}{2} a \alpha^{-1} \beta a + \frac{1}{2} a^\dagger (-\beta^* \alpha^{-1}) a^\dagger \right]:$$

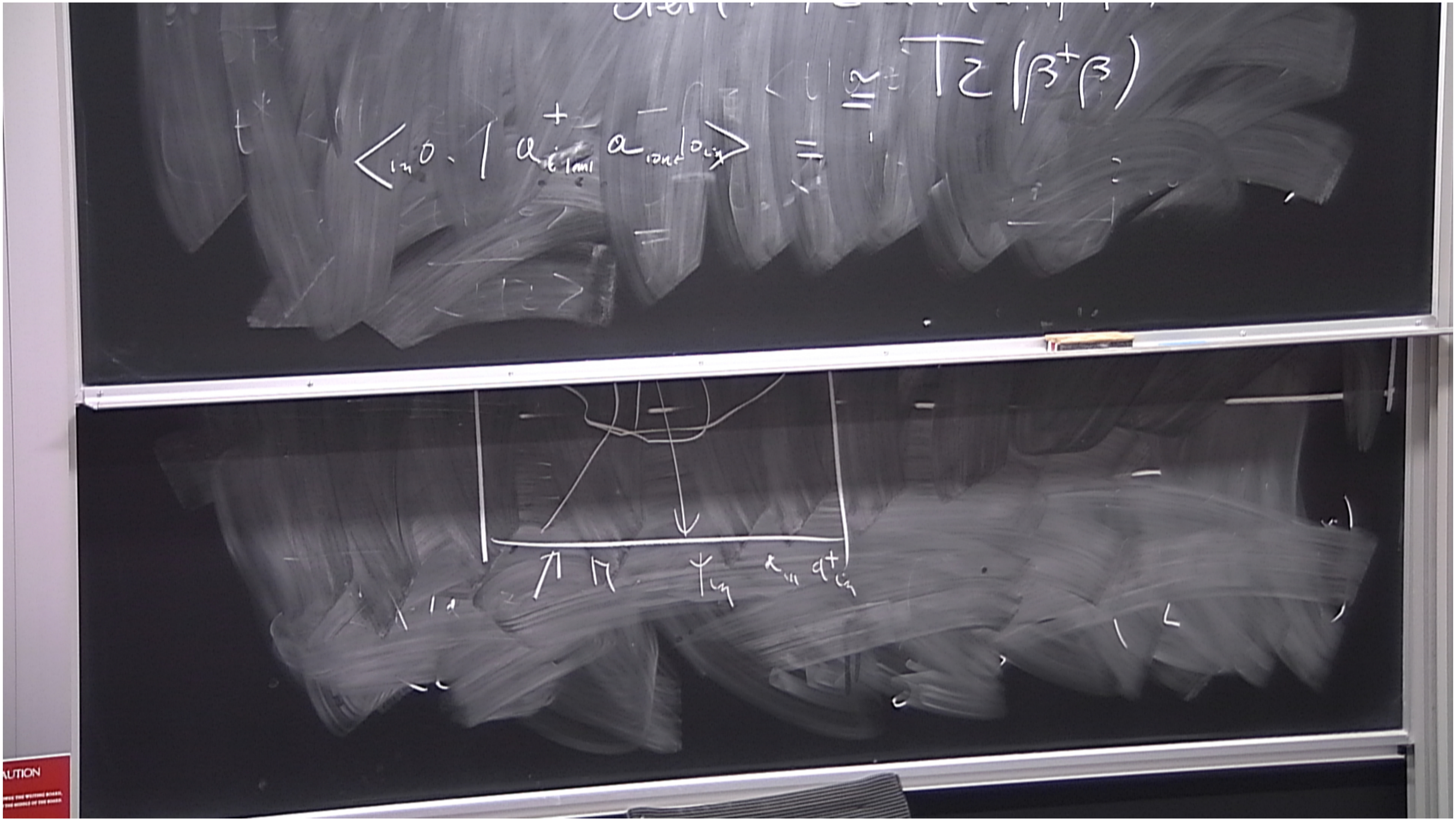


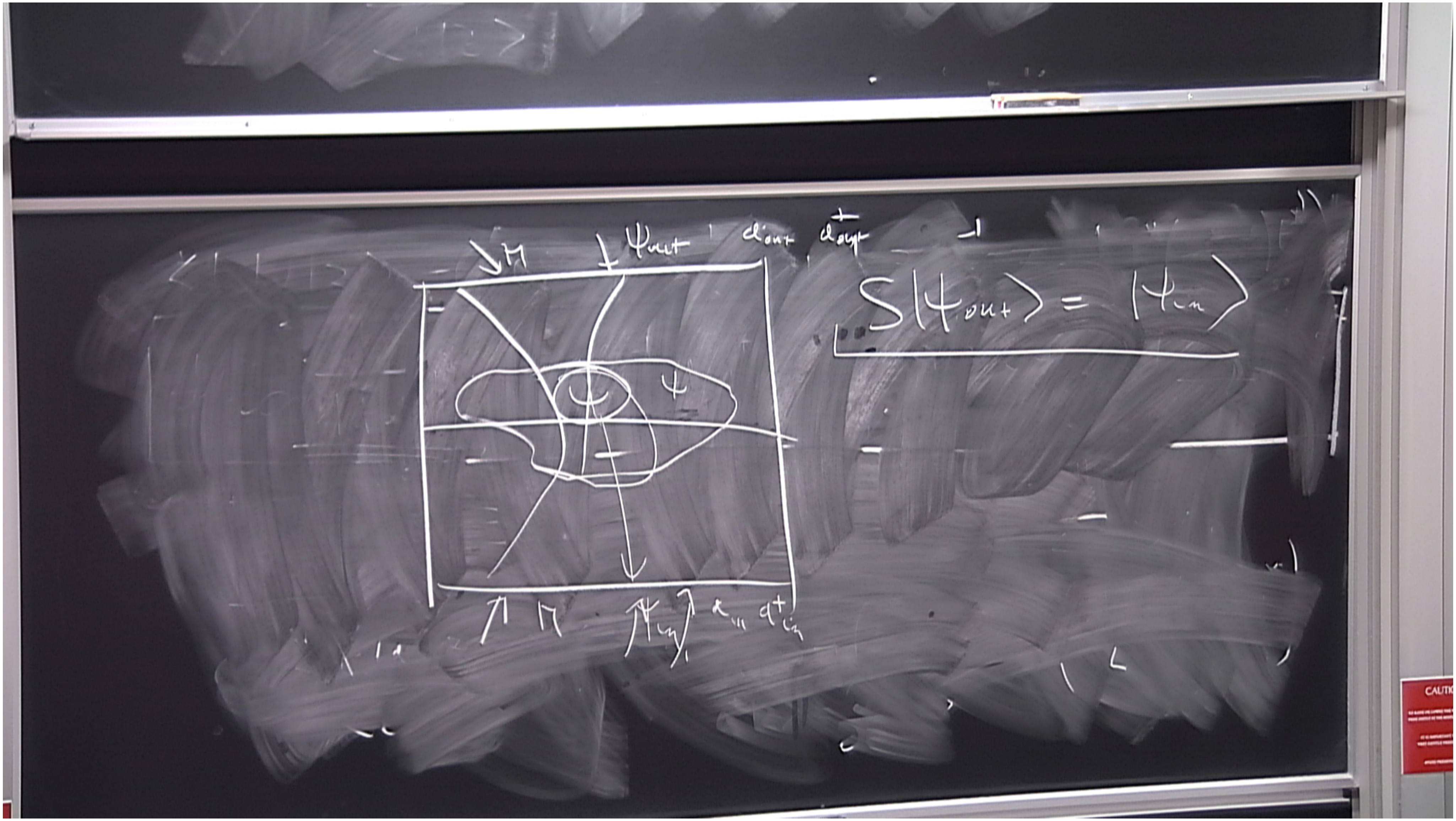
Ψ_{out}
a_{out} $a^\dagger_{out}$
$\mathcal{M}$
$S|\Psi_{out}\rangle = |\Psi_{in}\rangle$
Ψ_{in}
a_{in} $a^\dagger_{in}$

$$= \det(\alpha^{+}\alpha) = \det(\mathbb{1} + \beta^{+}\beta)$$

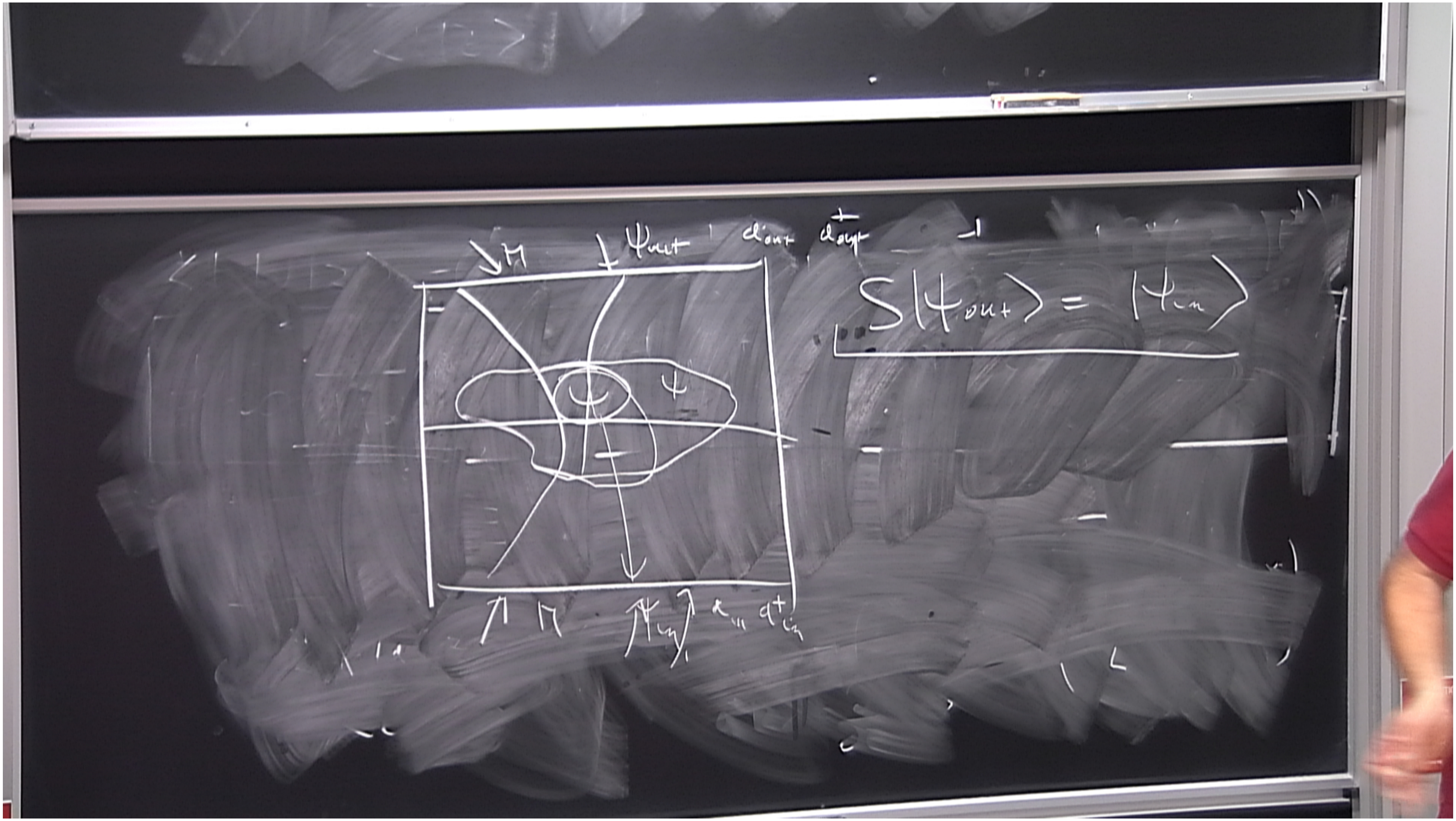
$$\simeq \mathrm{Tr}(\beta^{+}\beta)$$

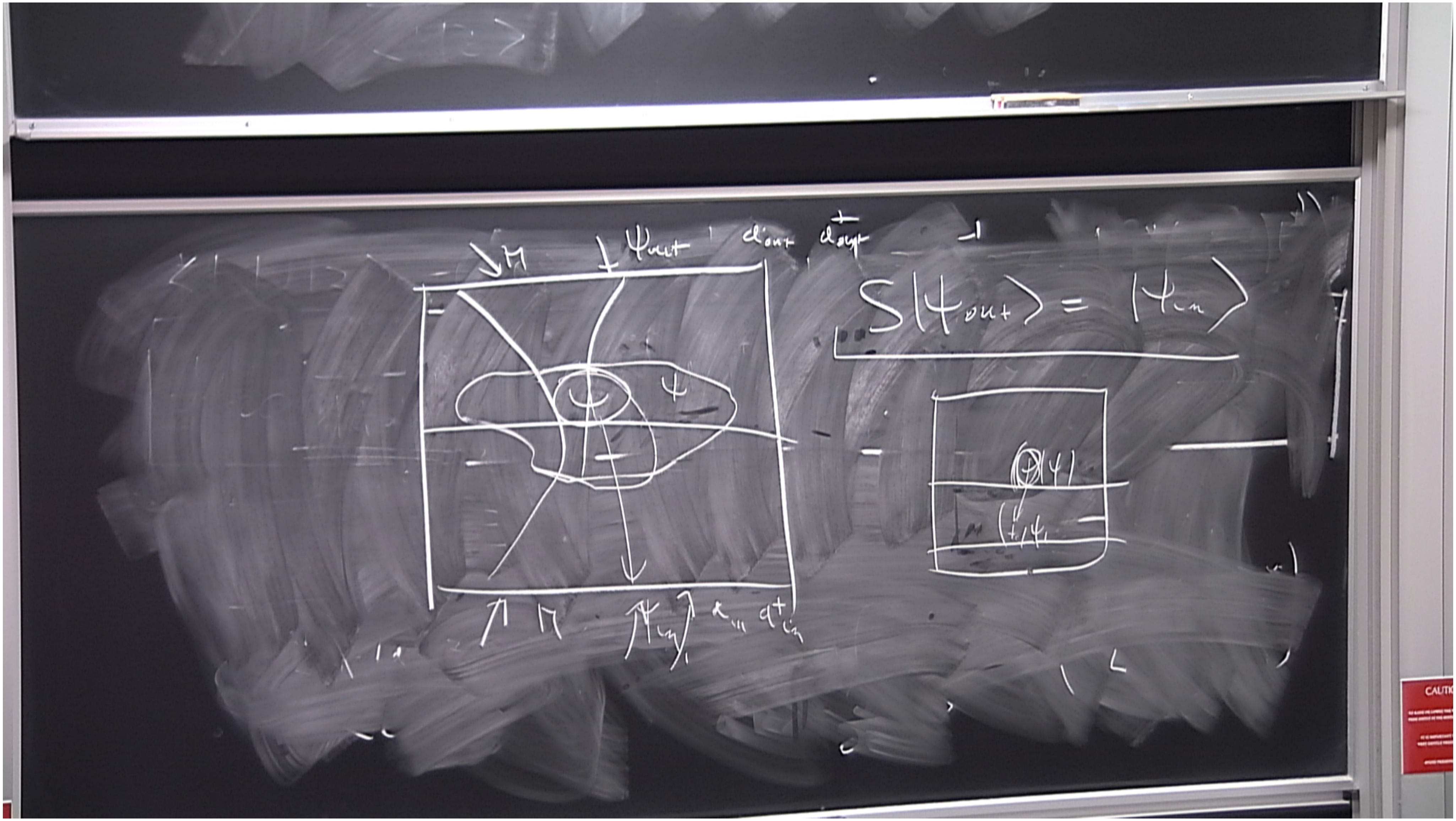

det(α†α) = det(1 + β†β)
≅ Tr(β†β)



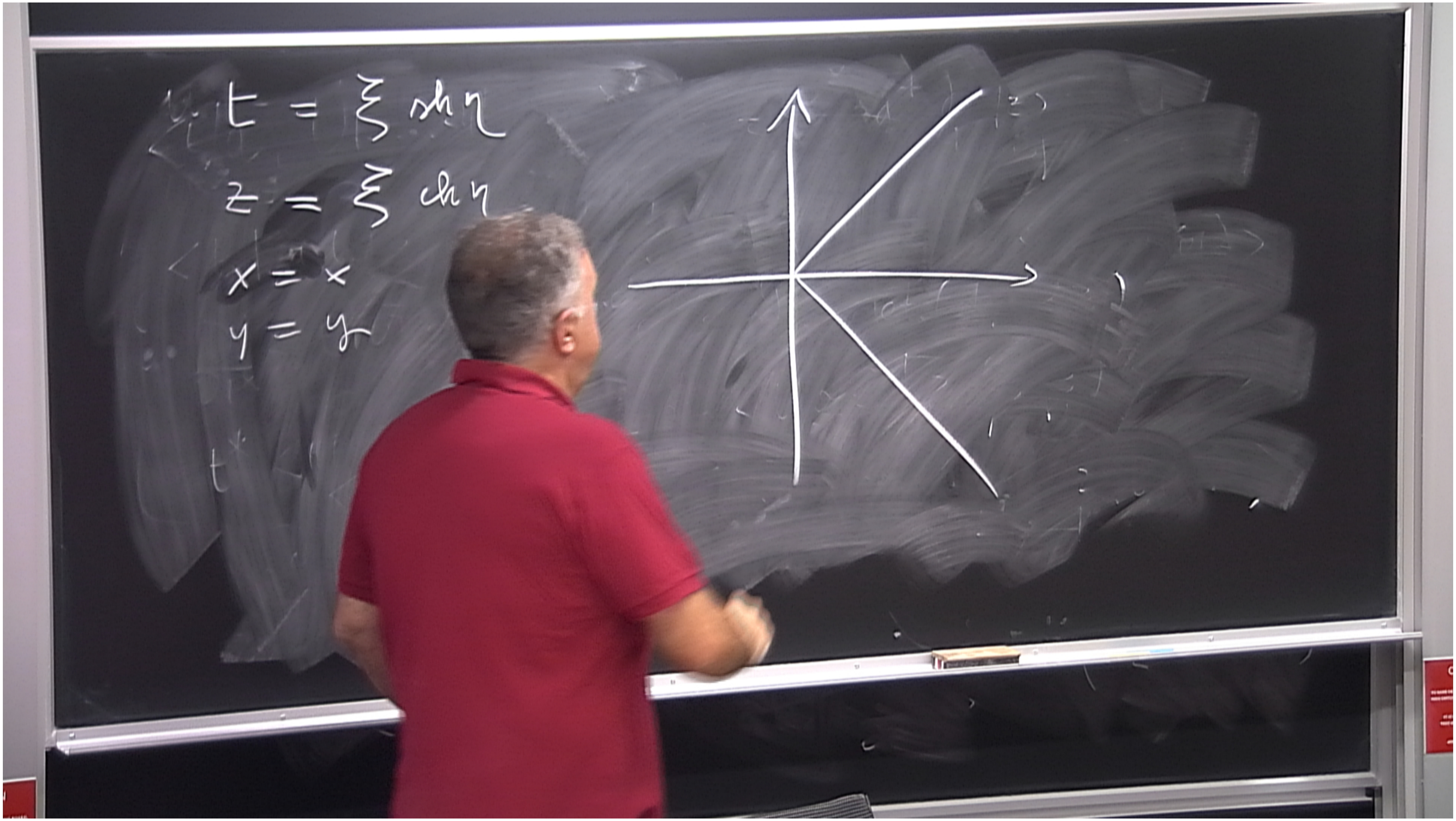


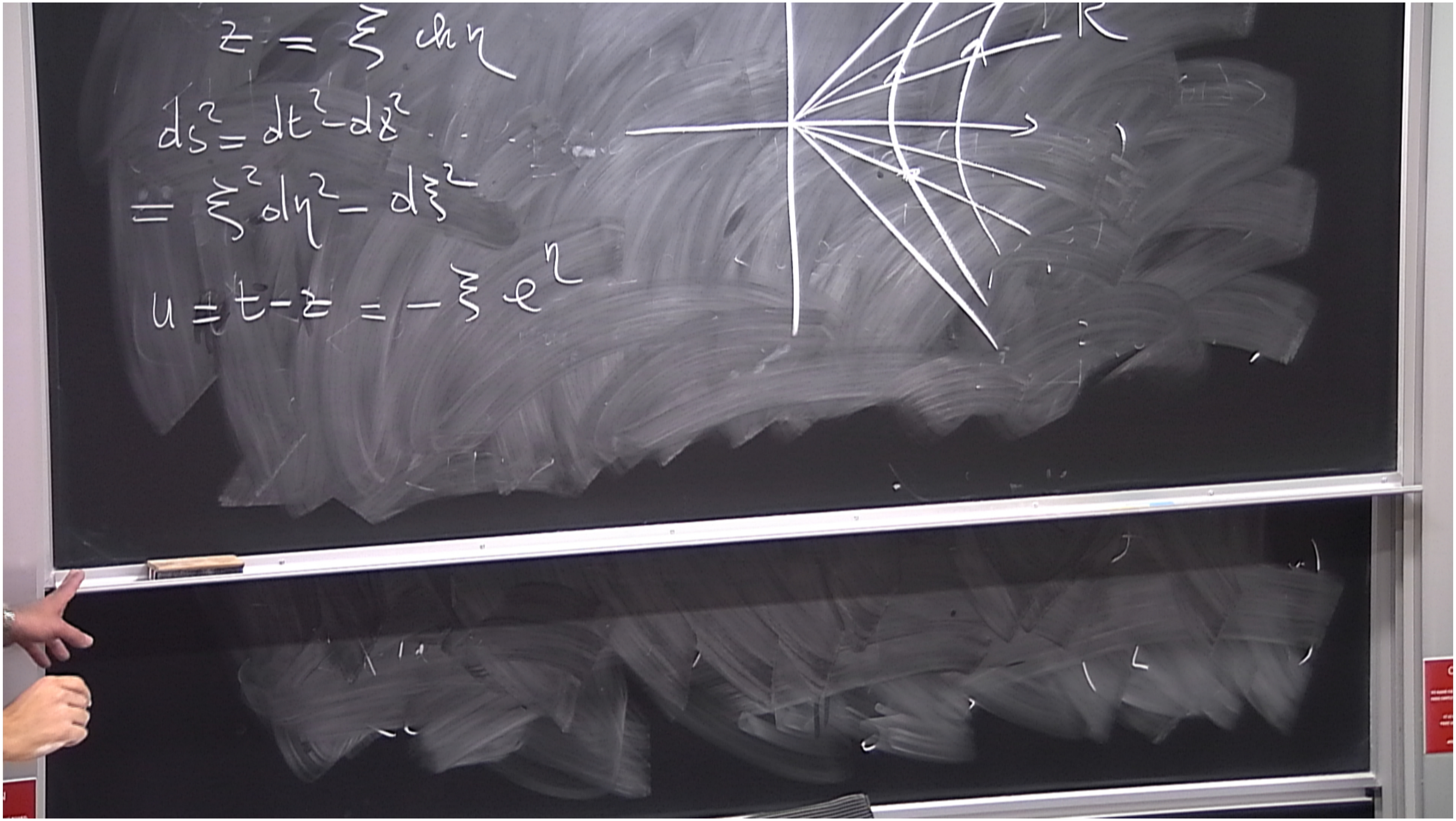
M
Ψout
a out a† out
S|Ψout⟩ = |Ψin⟩
Ψ
M
Ψin
a in a† in


$S|\psi_{out}\rangle = |\psi_{in}\rangle$
ψ_{out}
a_{out} $a^{\dagger}_{out}$
ψ
ψ_{in}
a_{in} $a^{\dagger}_{in}$



$S|\Psi_{out}\rangle = |\Psi_{in}\rangle$


$t = \xi \operatorname{sh} \eta$
$z = \xi \operatorname{ch} \eta$
$x = x$
$y = y$


$z = \xi \operatorname{ch} \eta$
$ds^2 = dt^2 - dz^2$
$= \xi^2 d\eta^2 - d\xi^2$
$u = t - z = -\xi e^{\eta}$

$$z = \xi \, \mathrm{ch}\, \eta$$

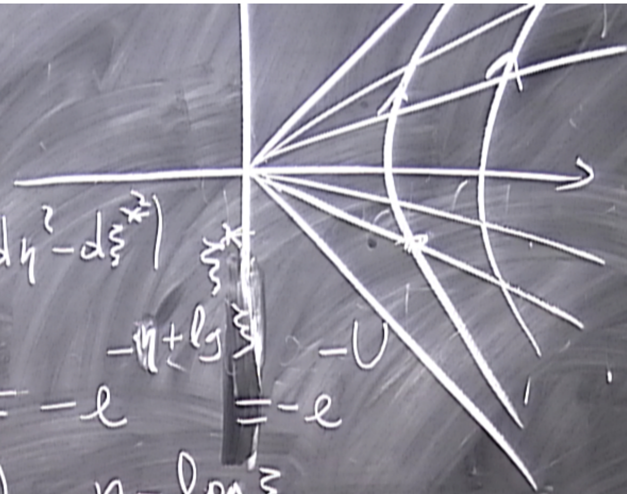
$$ds^2 = dt^2 - dz^2$$

$$= \xi^2 d\eta^2 - d\xi^2 = \xi^2 (d\eta^2 - d\xi^{*2}) \quad \xi^*$$

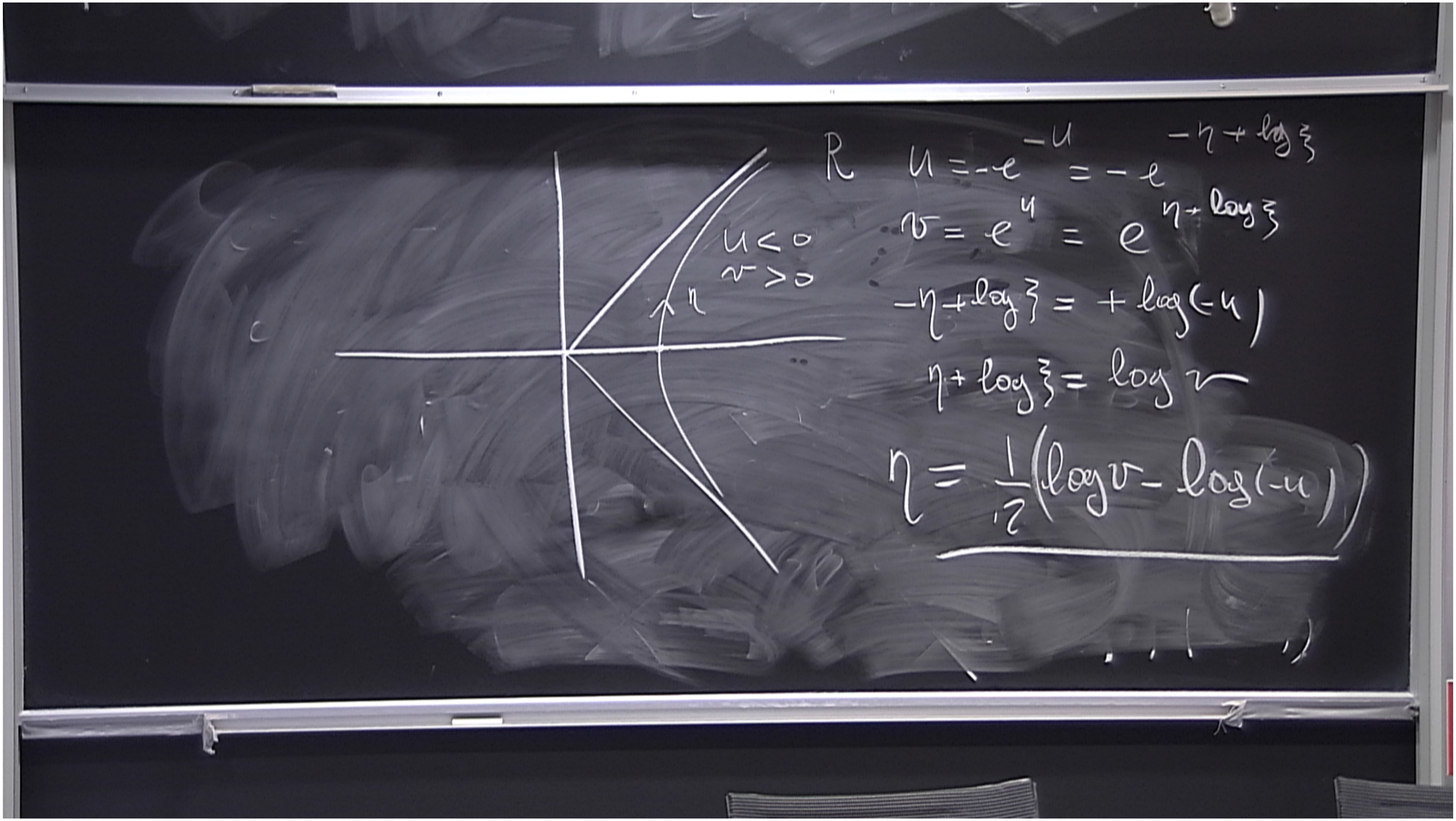
$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U}$$

$$v = t + z = e^{\eta + \log \xi} \qquad U = \eta - \log \xi$$

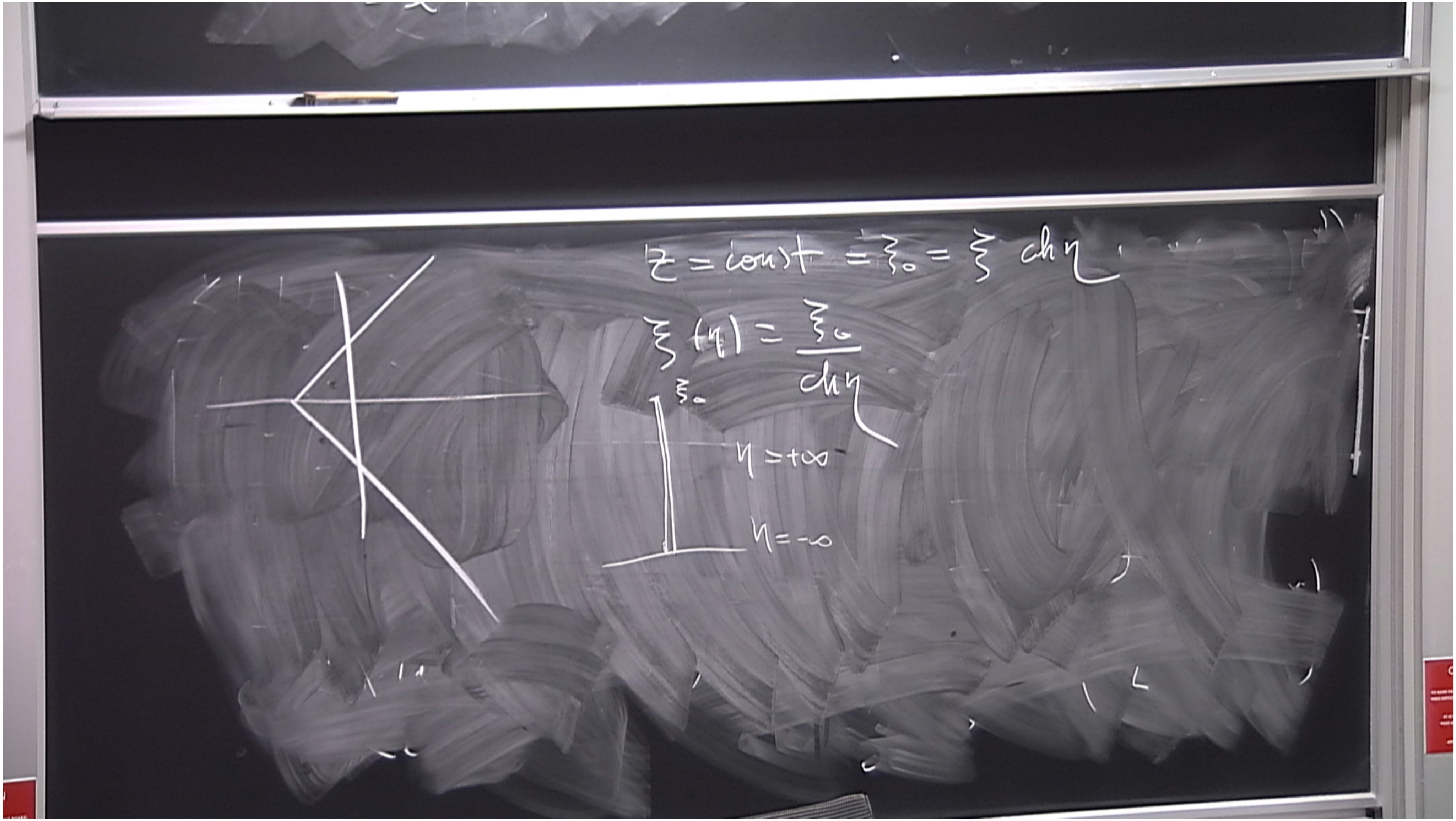
$$= e^{V}$$



$$\boxed{\begin{array}{l} u < 0 \\ v > 0 \end{array}}$$



$u<0$
$v>0$
η
R
$u = -e^{-u} = -e^{-\eta + \log\xi}$
$v = e^{u} = e^{\eta + \log\xi}$
$-\eta + \log\xi = +\log(-u)$
$\eta + \log\xi = \log v$
$\eta = \frac{1}{2}\left(\log v - \log(-u)\right)$


$z = \text{const} = \xi_0 = \xi \operatorname{ch}\eta$
$\xi(\eta) = \frac{\xi_0}{\operatorname{ch}\eta}$
ξ_0
$\eta = +\infty$
$\eta = -\infty$

$$t = \xi \,\mathrm{sh}\,\eta$$

$$z = \xi \,\mathrm{ch}\,\eta$$

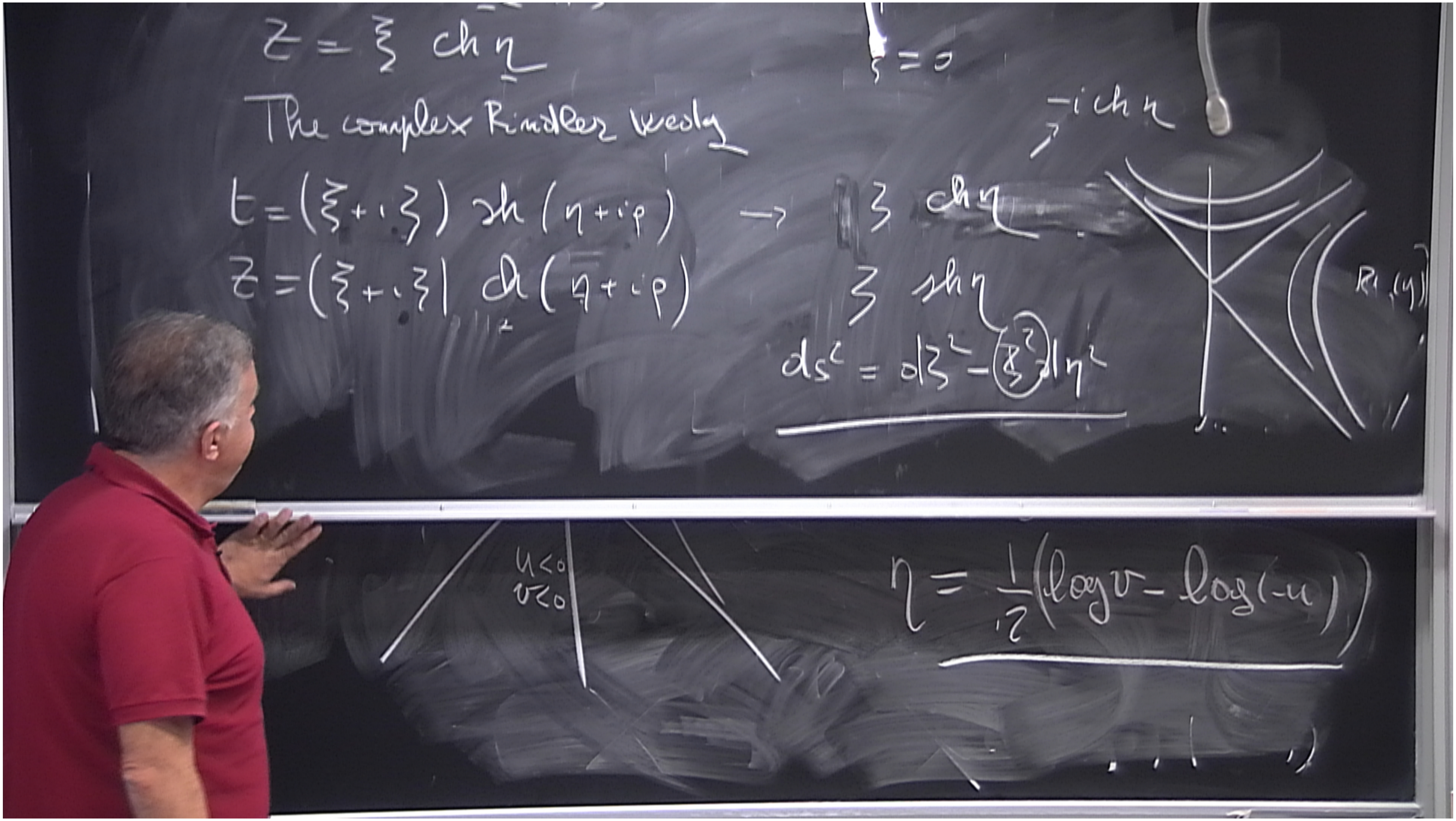
$$\rho = -\frac{i\pi}{2}$$

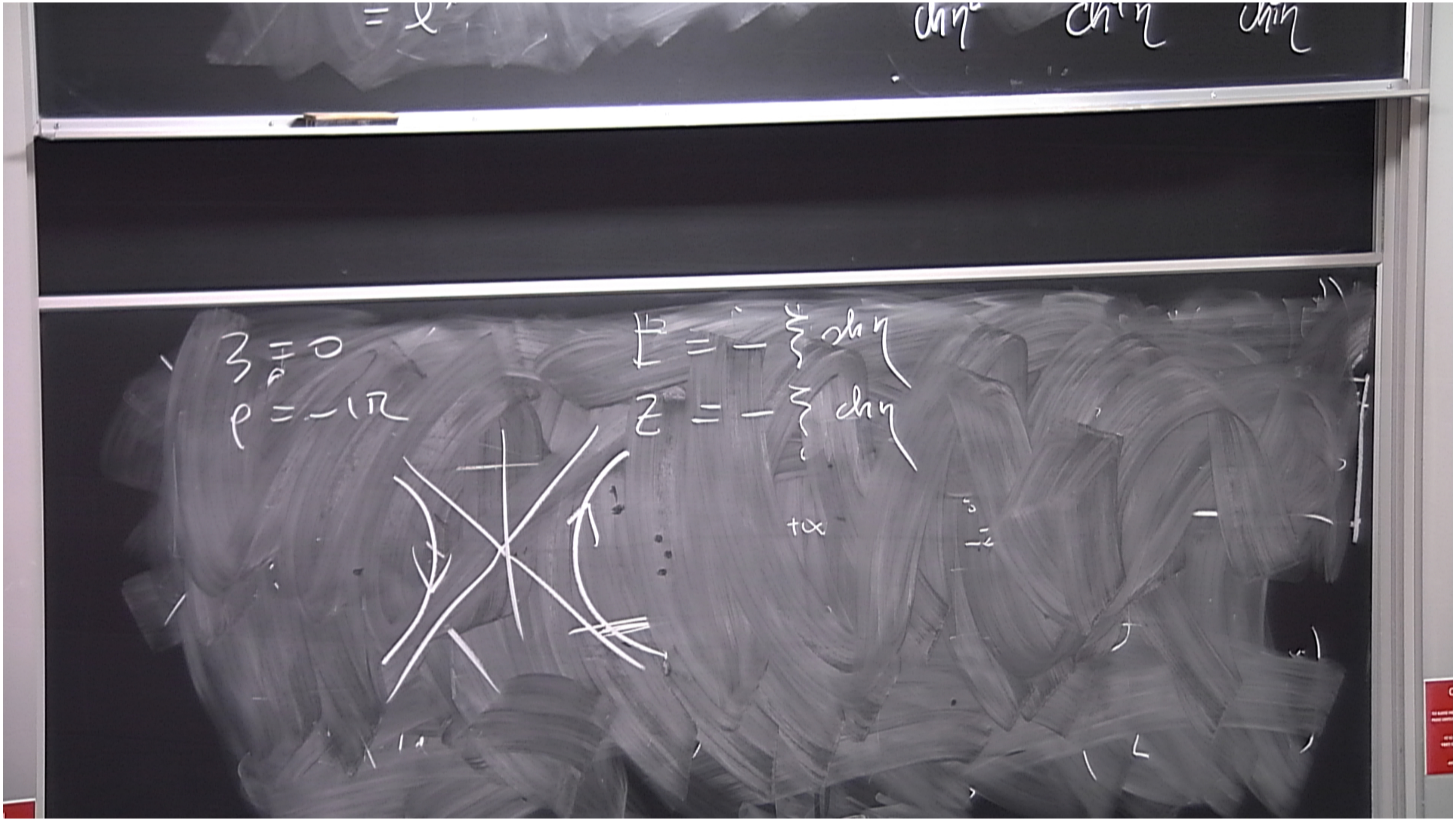
$$\xi = 0$$

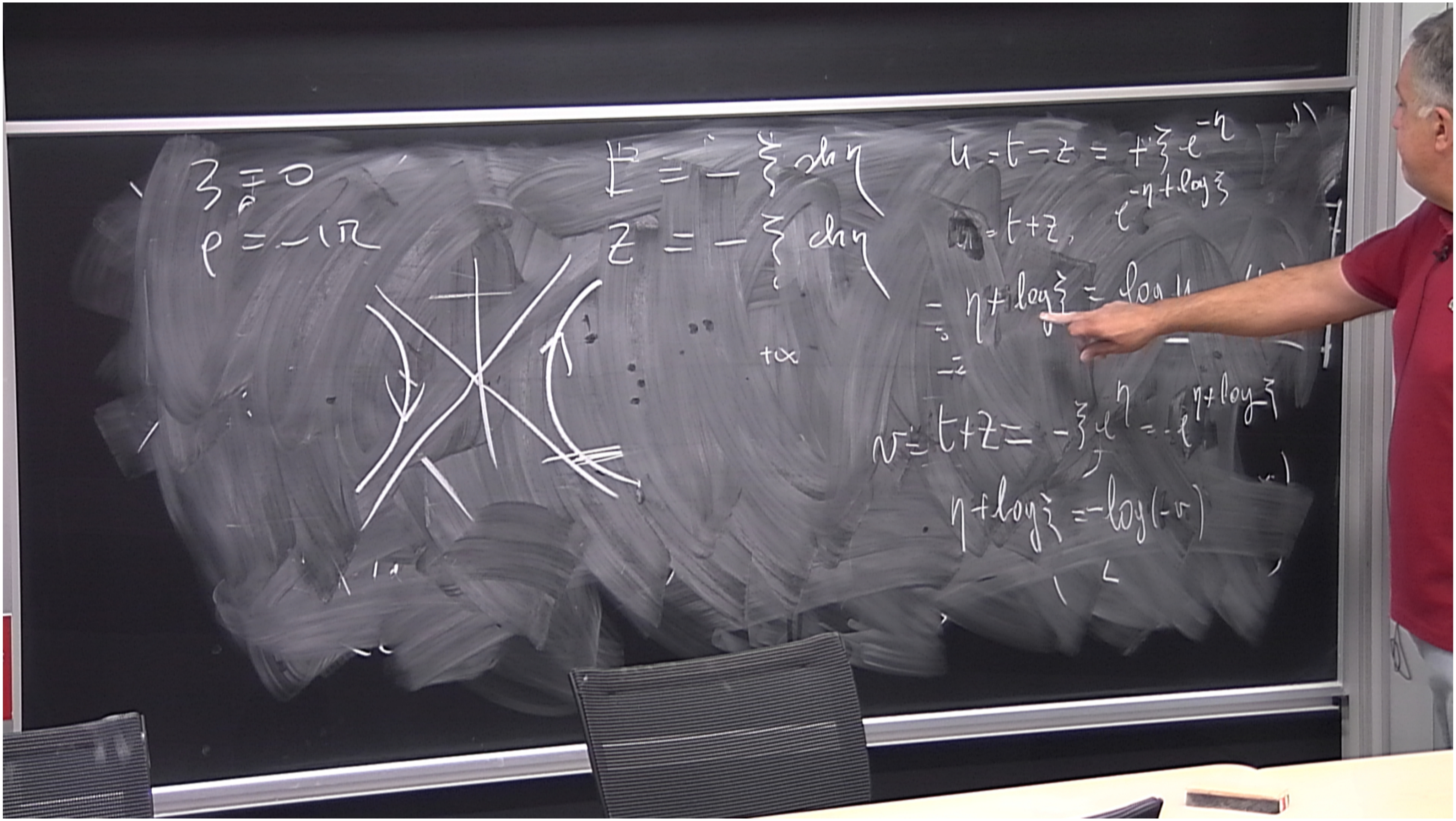
The complex Rindler wedge

$$t = (\xi + i\zeta)\,\mathrm{sh}(\eta + i\rho)$$

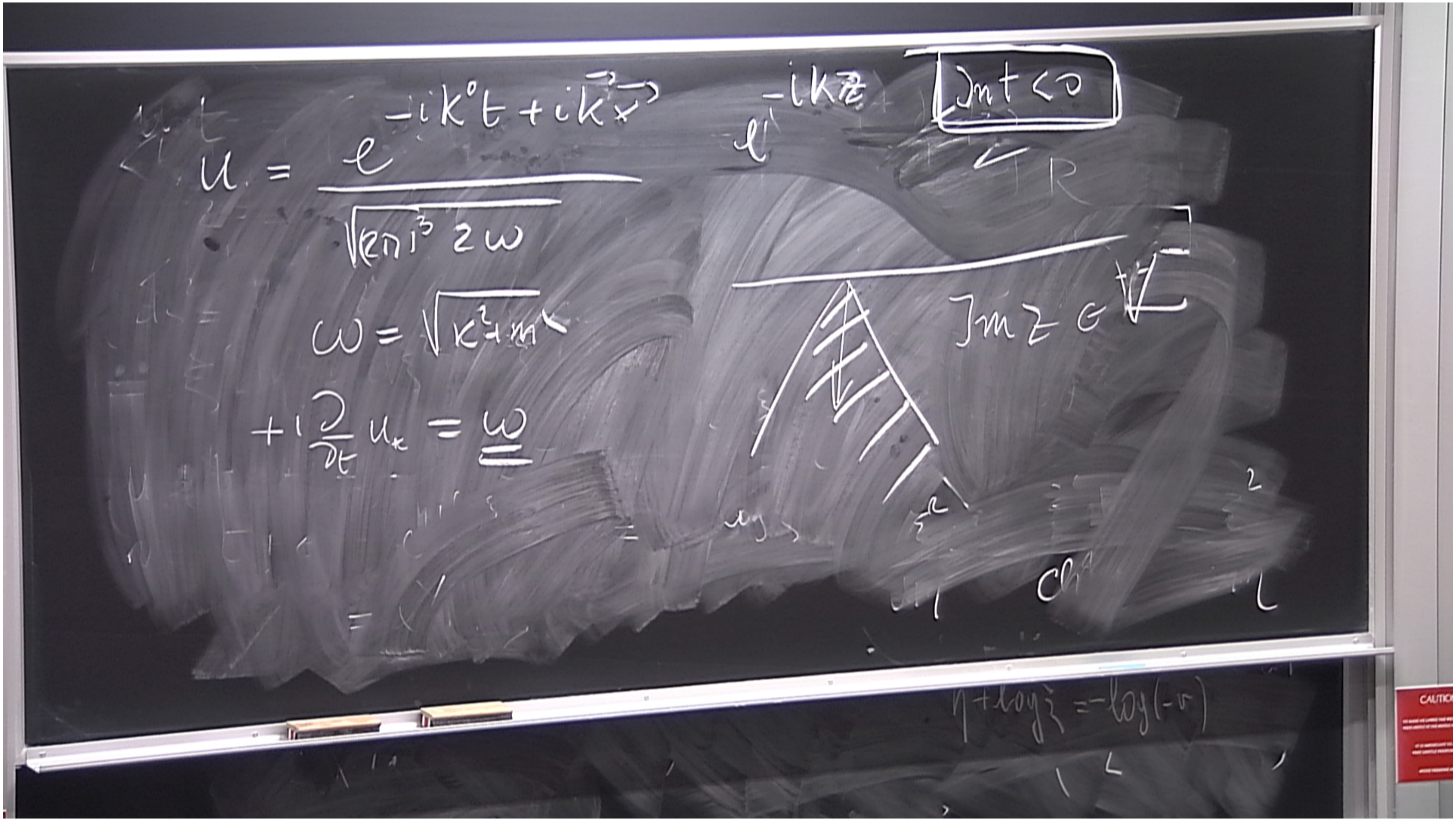
$$z = (\xi + i\zeta)\,\mathrm{ch}(\eta + i\rho)$$

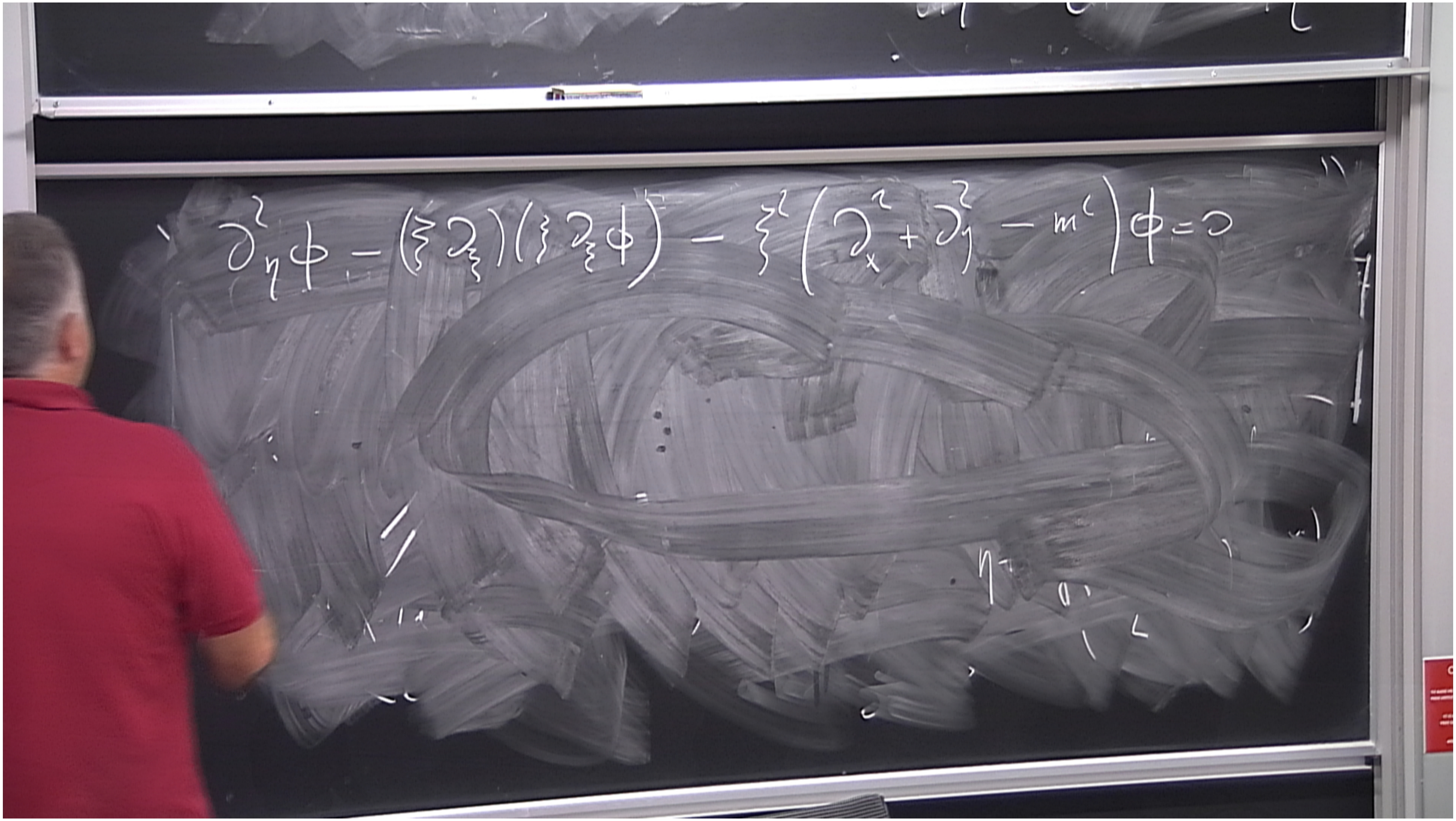

z = ξ ch η
ξ = 0
The complex Rindler wedge
−i ch η
t = (ξ + i ζ) sh (η + iρ) → ζ ch η
z = (ξ + i ζ) ch (η + iρ)
ζ sh η
ds² = dζ² − ξ² dη²
u<0
v<0
η = ½(log v − log(−u))

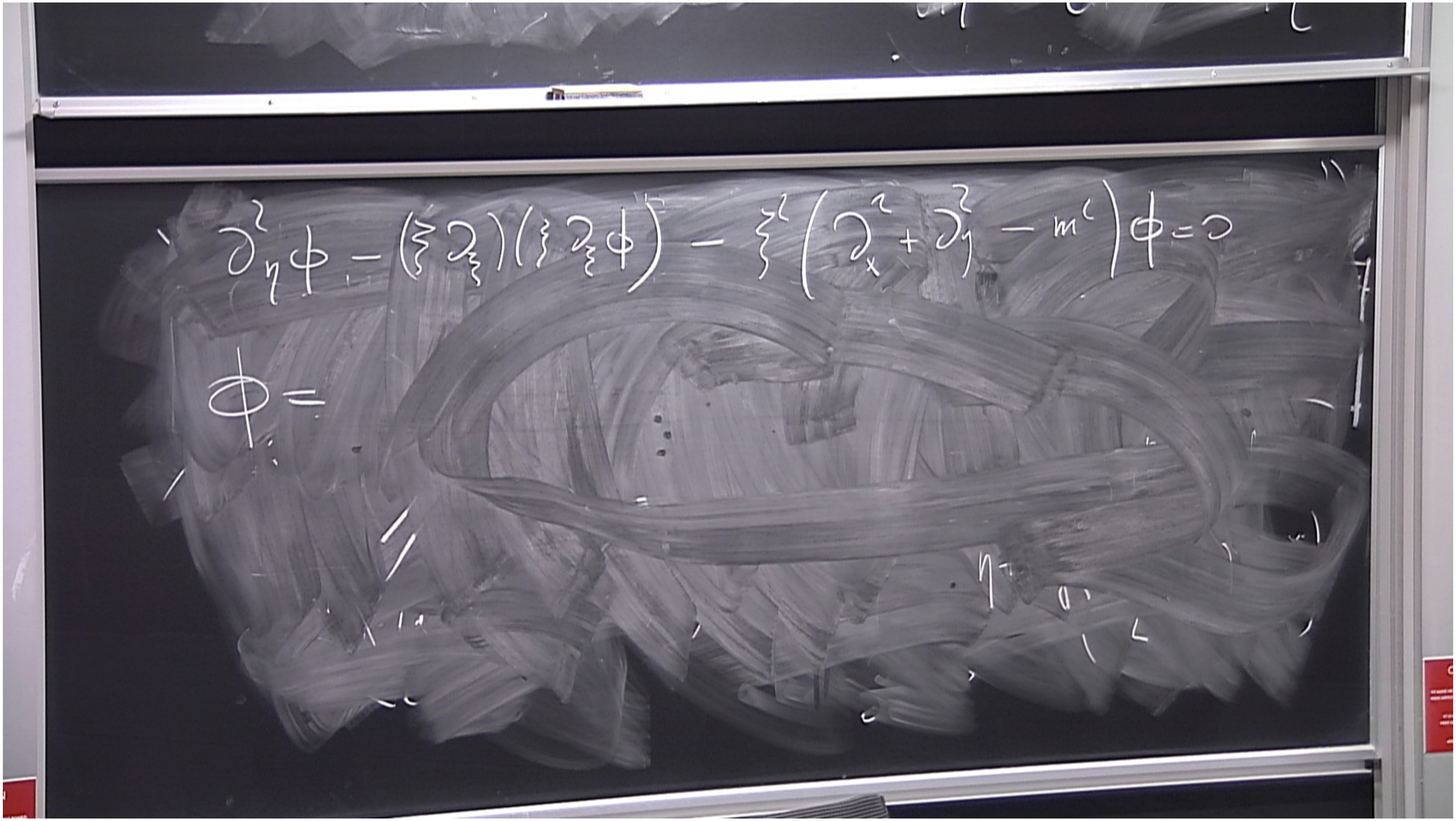

ζ = 0
ρ = −iπ
+∞



t = −ξ sh η
z = −ξ ch η
u = t − z = +ξ e^{−η}
e^{−η + log ξ}
η + log ξ = log u
v = t + z = −ξ e^{η} = −e^{η + log ξ}
η + log ξ = −log(−v)


$u = \frac{e^{-ik^0 t + i\vec{k}\vec{x}}}{\sqrt{(2\pi)^3 2\omega}}$
$\omega = \sqrt{k^2+m^2}$
$+i\frac{\partial}{\partial t} u_k = \omega$
e^{-ikz}
$\boxed{\operatorname{Im} t < 0}$
$\operatorname{Im} z \in$
$\eta + \log \xi = -\log(-v)$

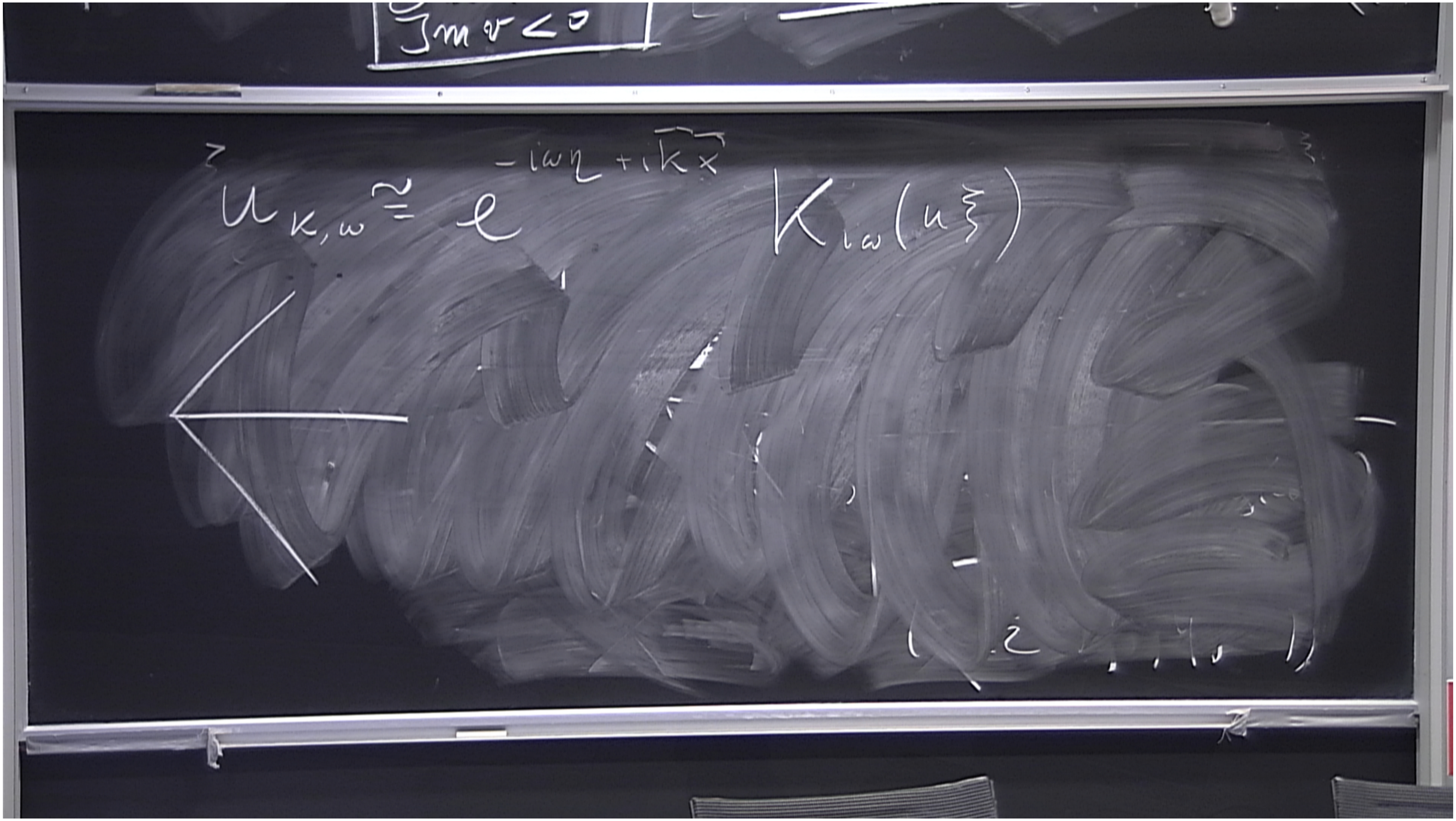

$\partial_\eta^2\phi - (\xi\partial_\xi)(\xi\partial_\xi\phi) - \xi^2(\partial_x^2 + \partial_y^2 - m^2)\phi = 0$


$\partial_\eta^2\phi - (\xi\partial_\xi)(\xi\partial_\xi\phi) - \xi^2(\partial_x^2+\partial_y^2-m^2)\phi = 0$
$\phi =$

$$\partial_\eta^2 \phi - (\xi\partial_\xi)(\xi\partial_\xi\phi) - \xi^2\left(\partial_x^2+\partial_y^2-m^2\right)\phi = 0$$

$$\phi = e^{-i\omega\eta + ik_x x + ik_y y}\, f_{k,\omega}(\xi)$$

$$(\xi\partial_\xi)(\xi\partial_\xi) f + \left(\omega^2 - \xi^2\left(k_x^2+k_y^2+m^2\right)\right) f_{k,\omega}(\xi) = 0$$



$\operatorname{Im} v < 0$
$u_{k,\omega} \cong e^{-i\omega\eta + i\vec{k}\vec{x}} K_{i\omega}(\kappa\xi)$