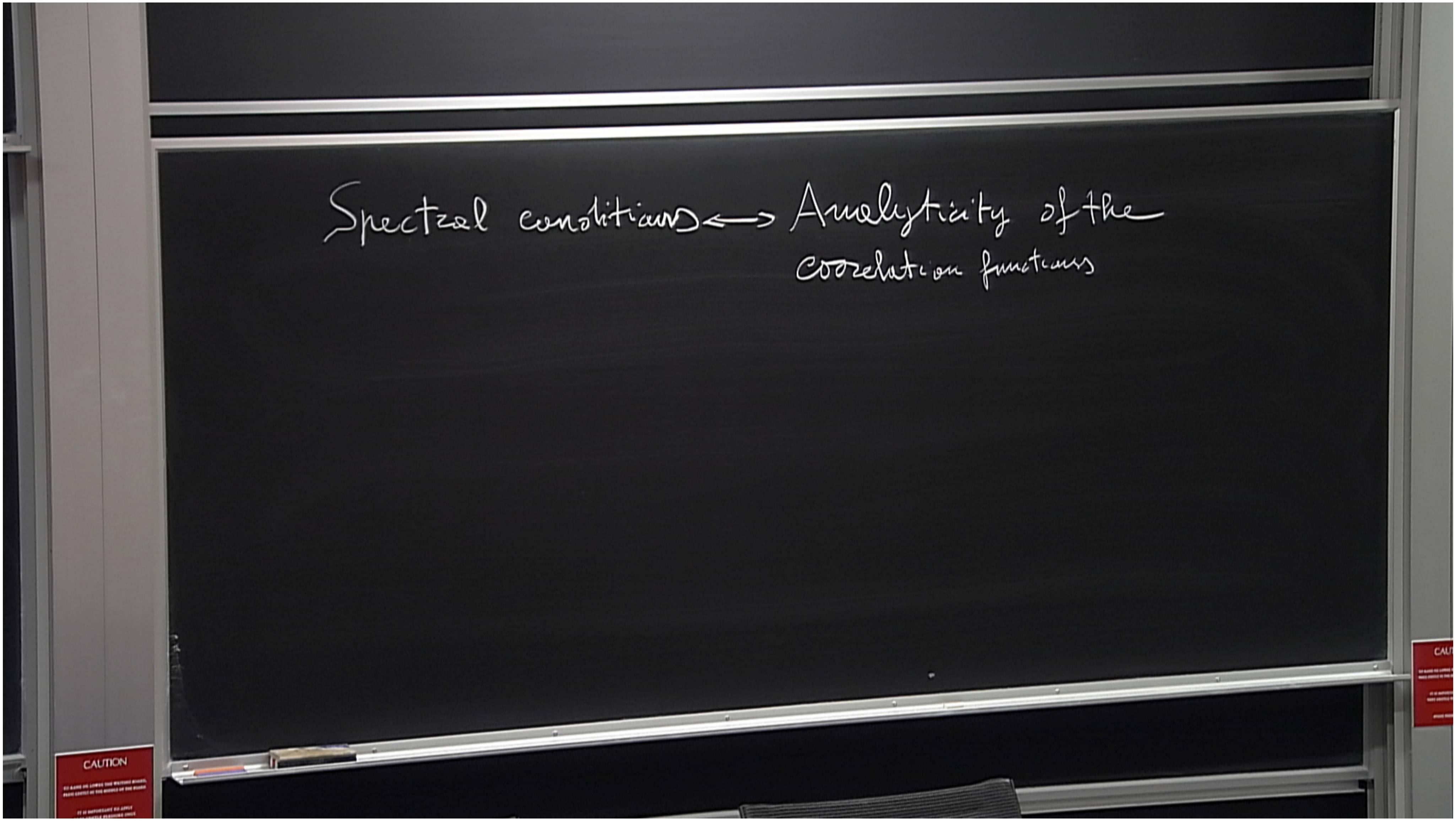
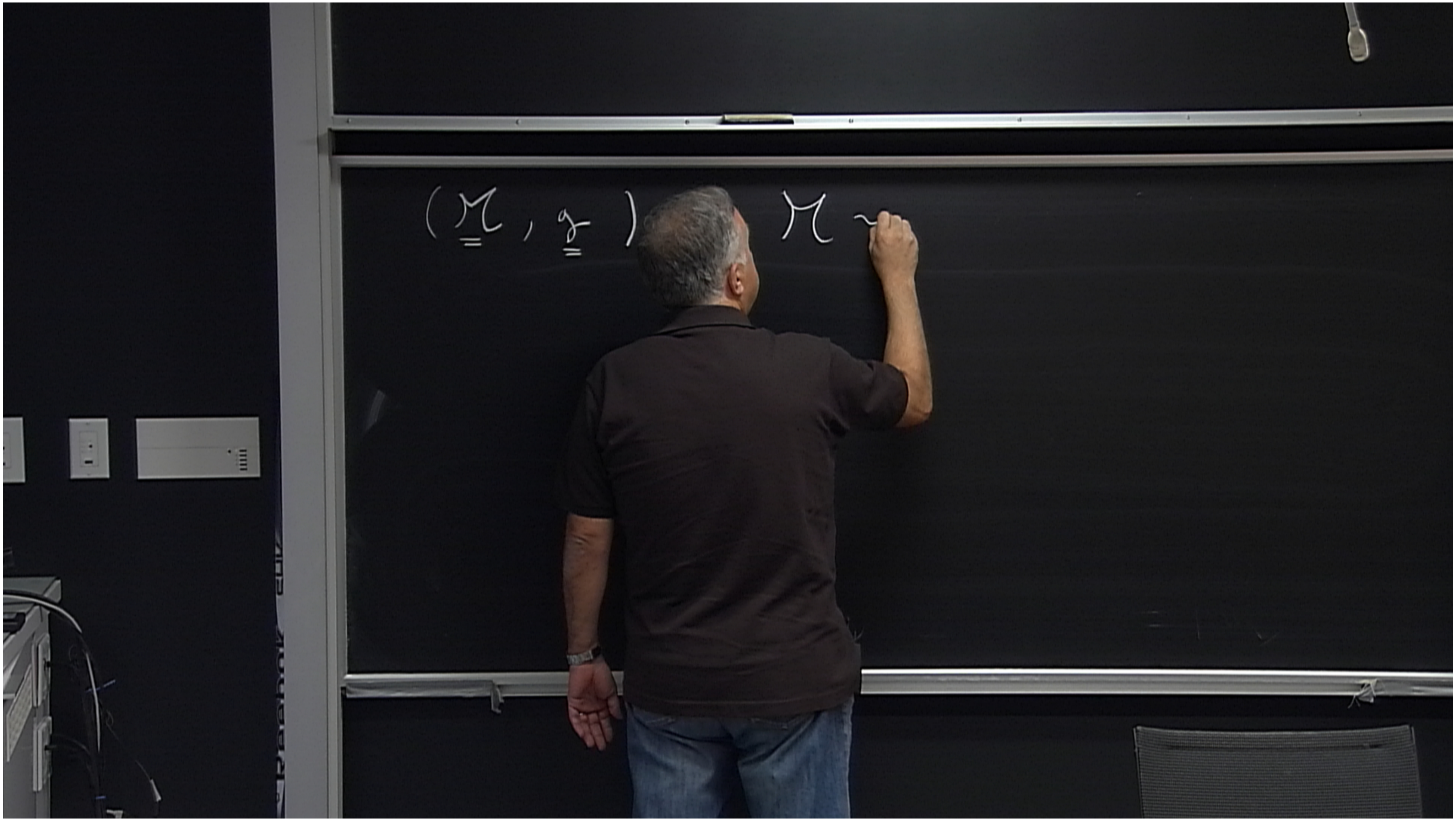
Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 6

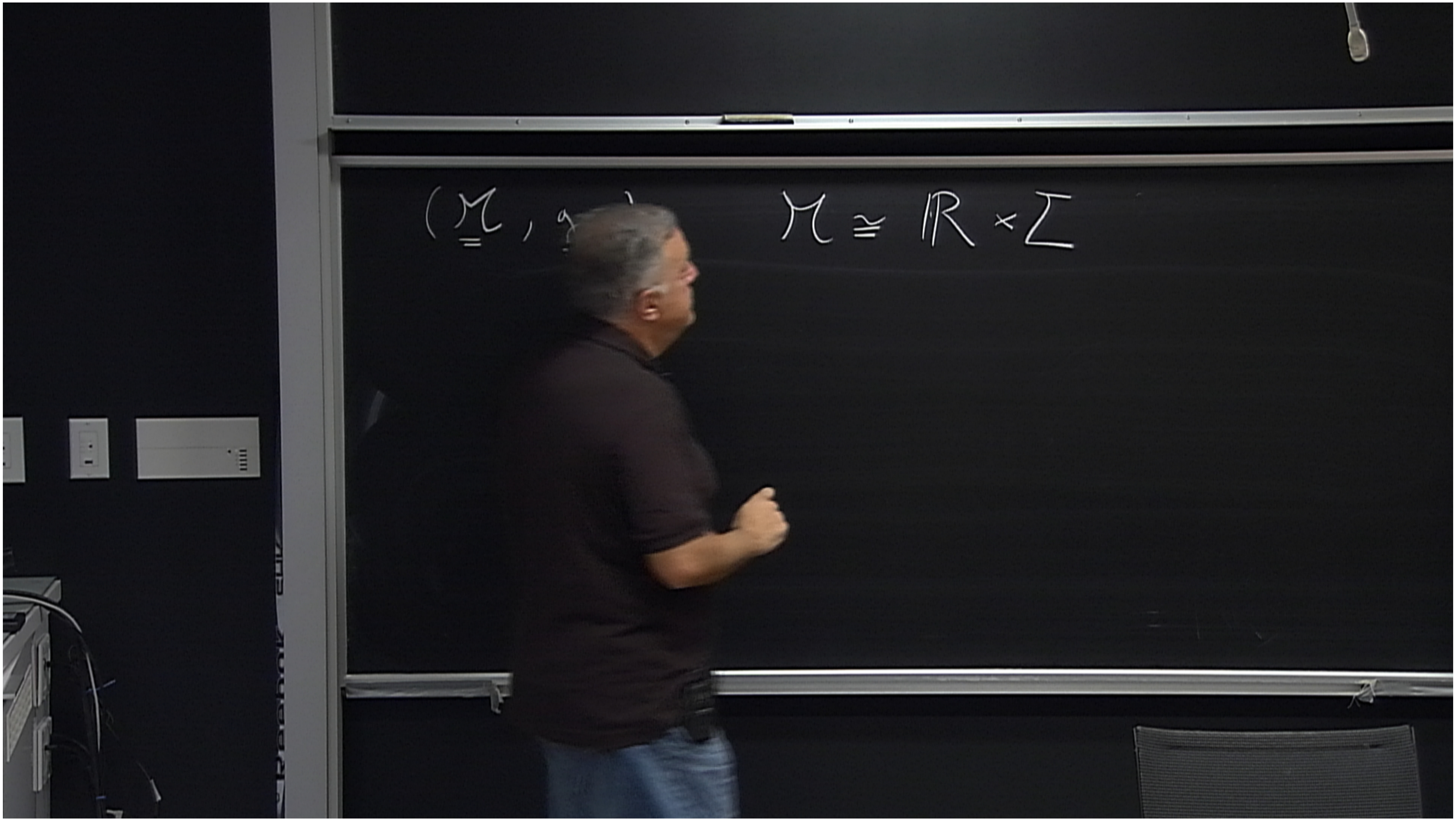
Date: Oct 09, 2013 10:00 AM

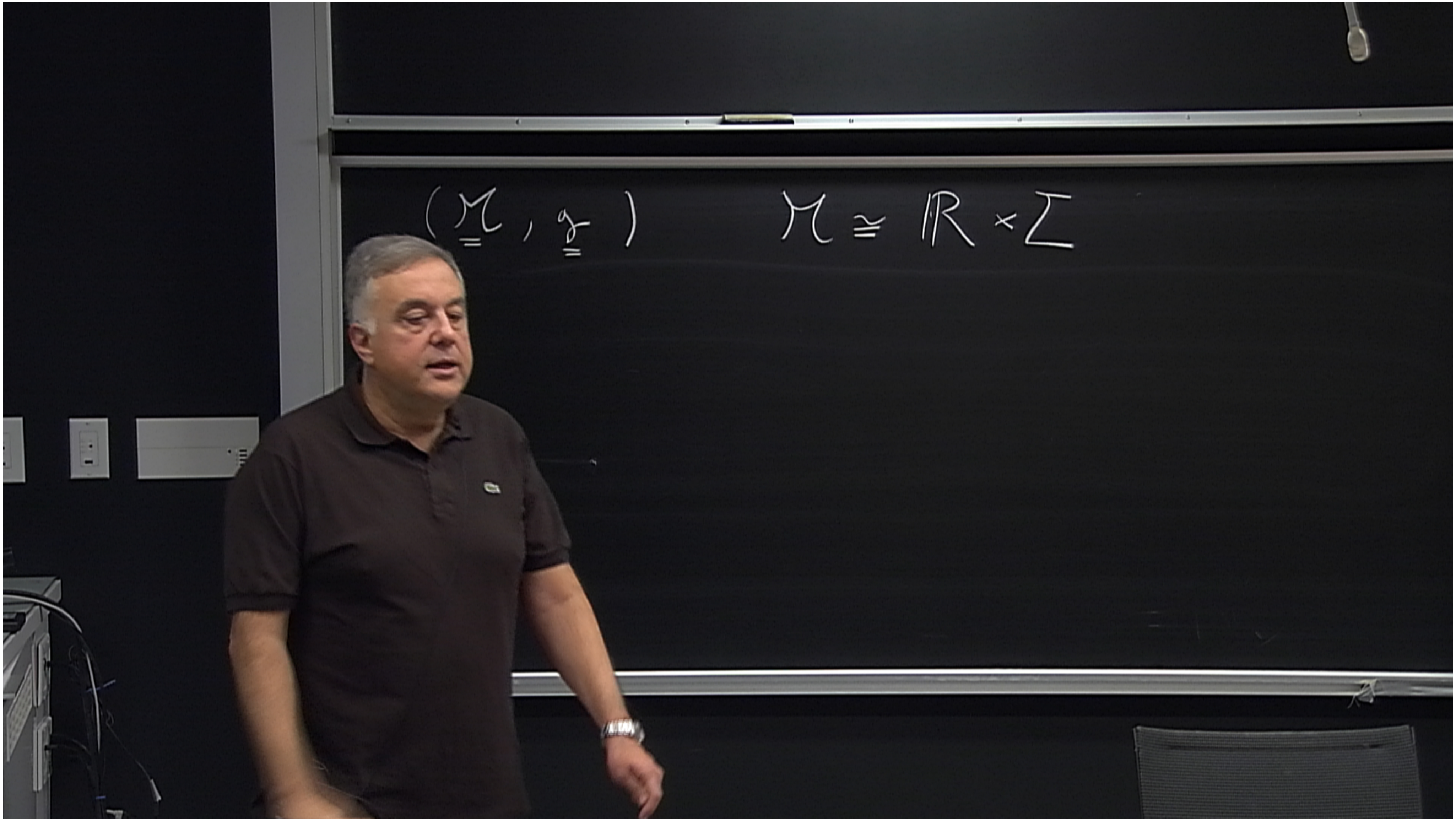
URL: http://pirsa.org/13100016

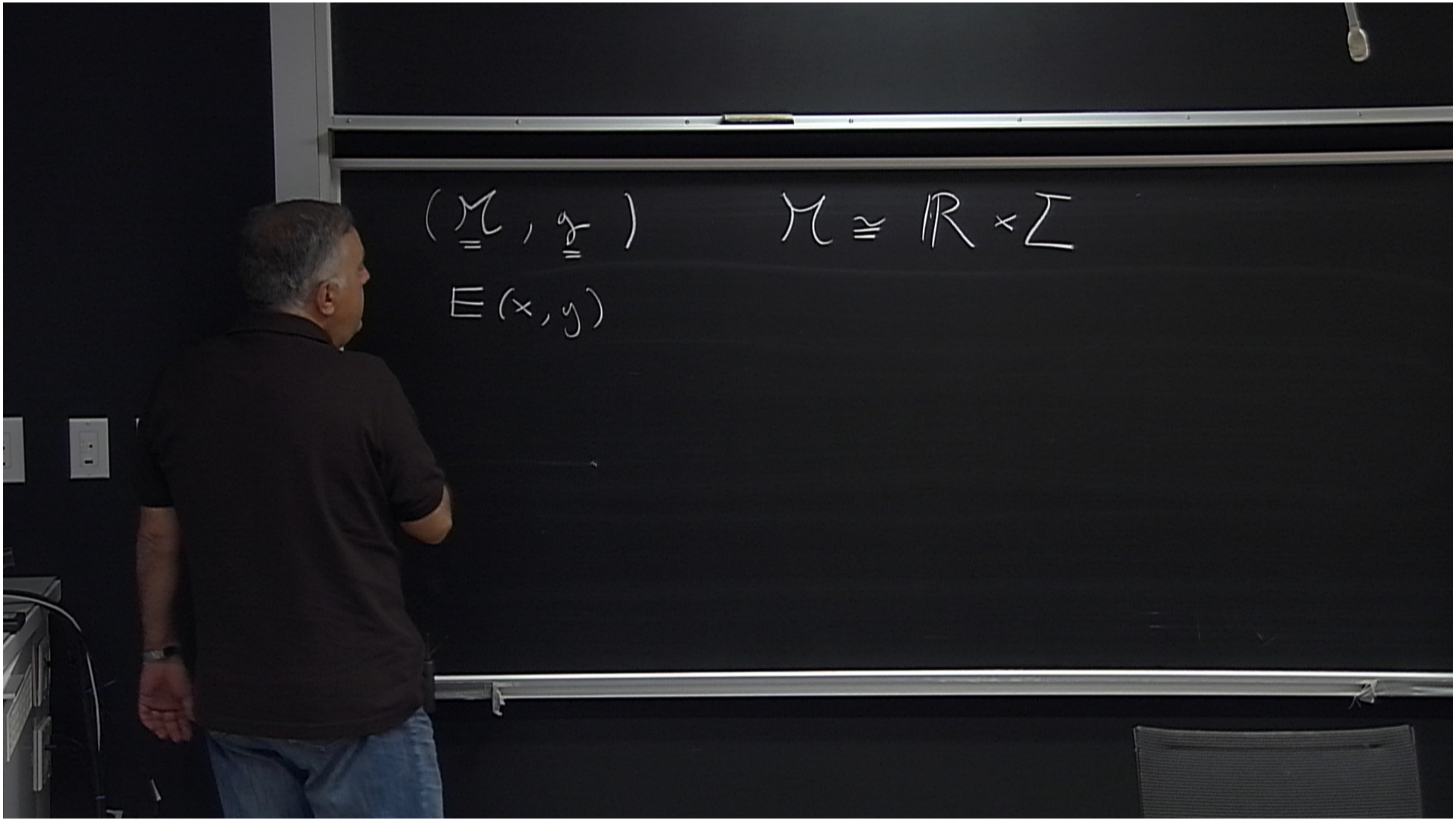
Abstract:

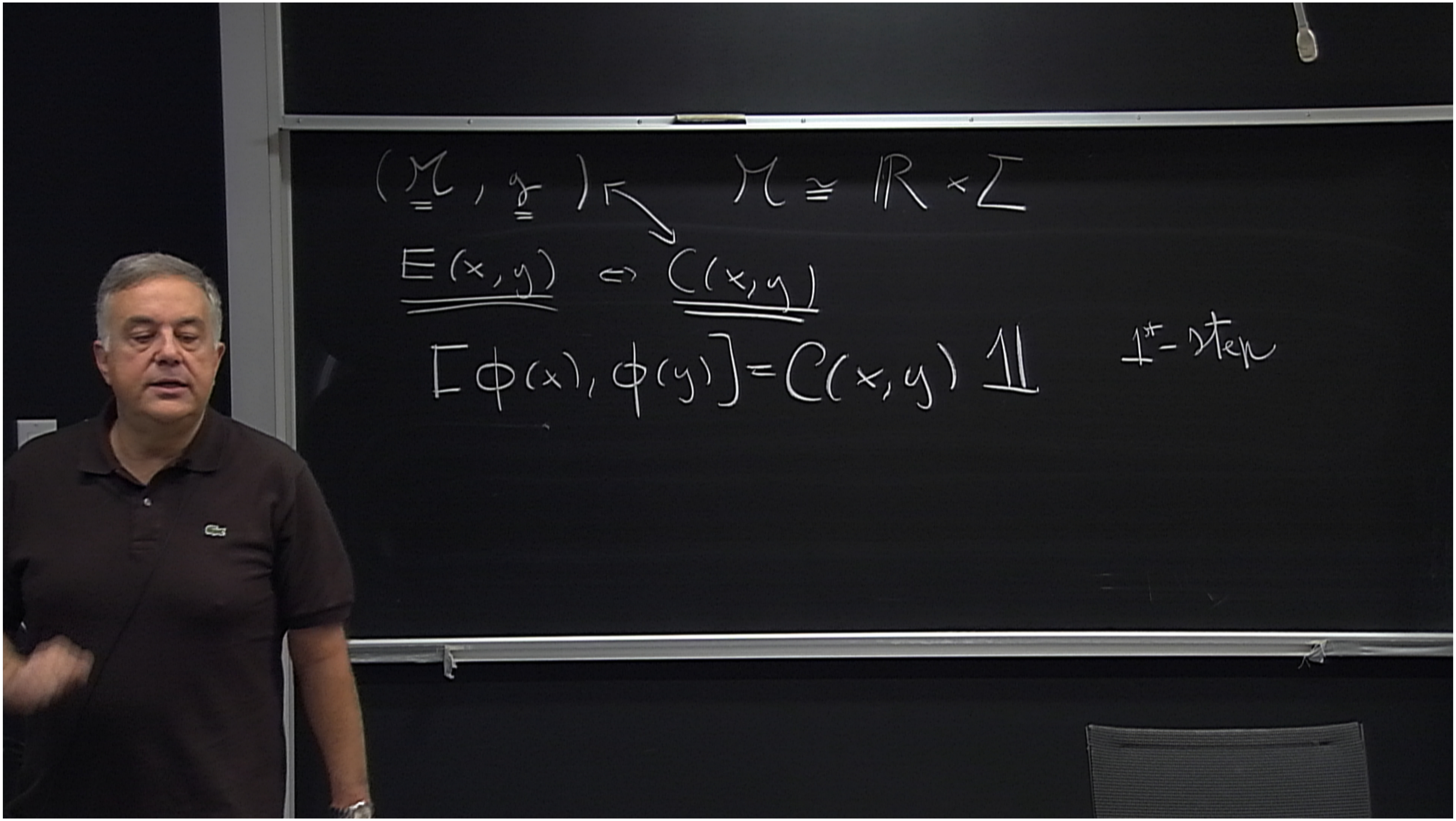

Spectral conditions ⟷ Analyticity of the
correlation functions
CAUTION

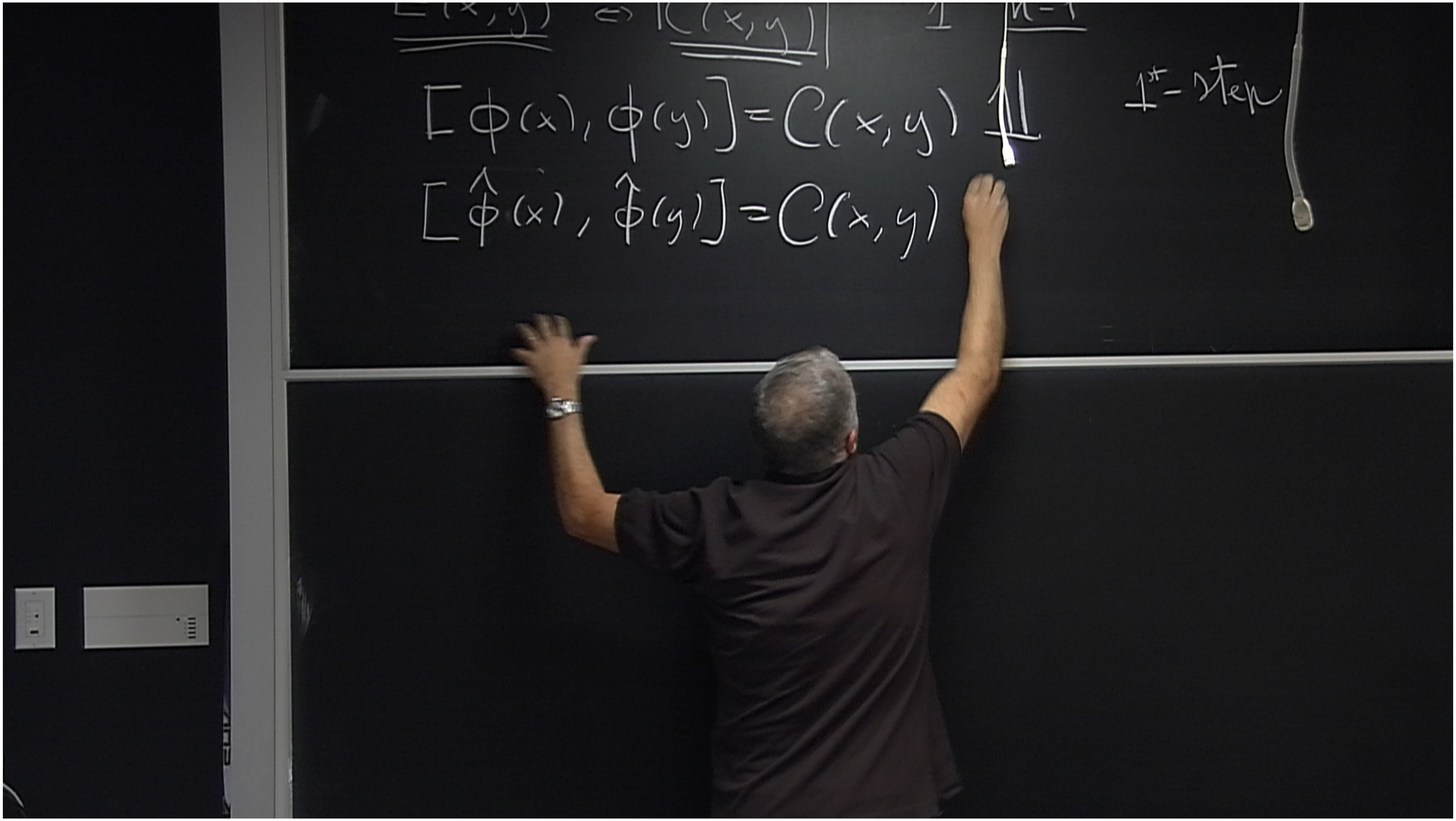

(M , g)
M

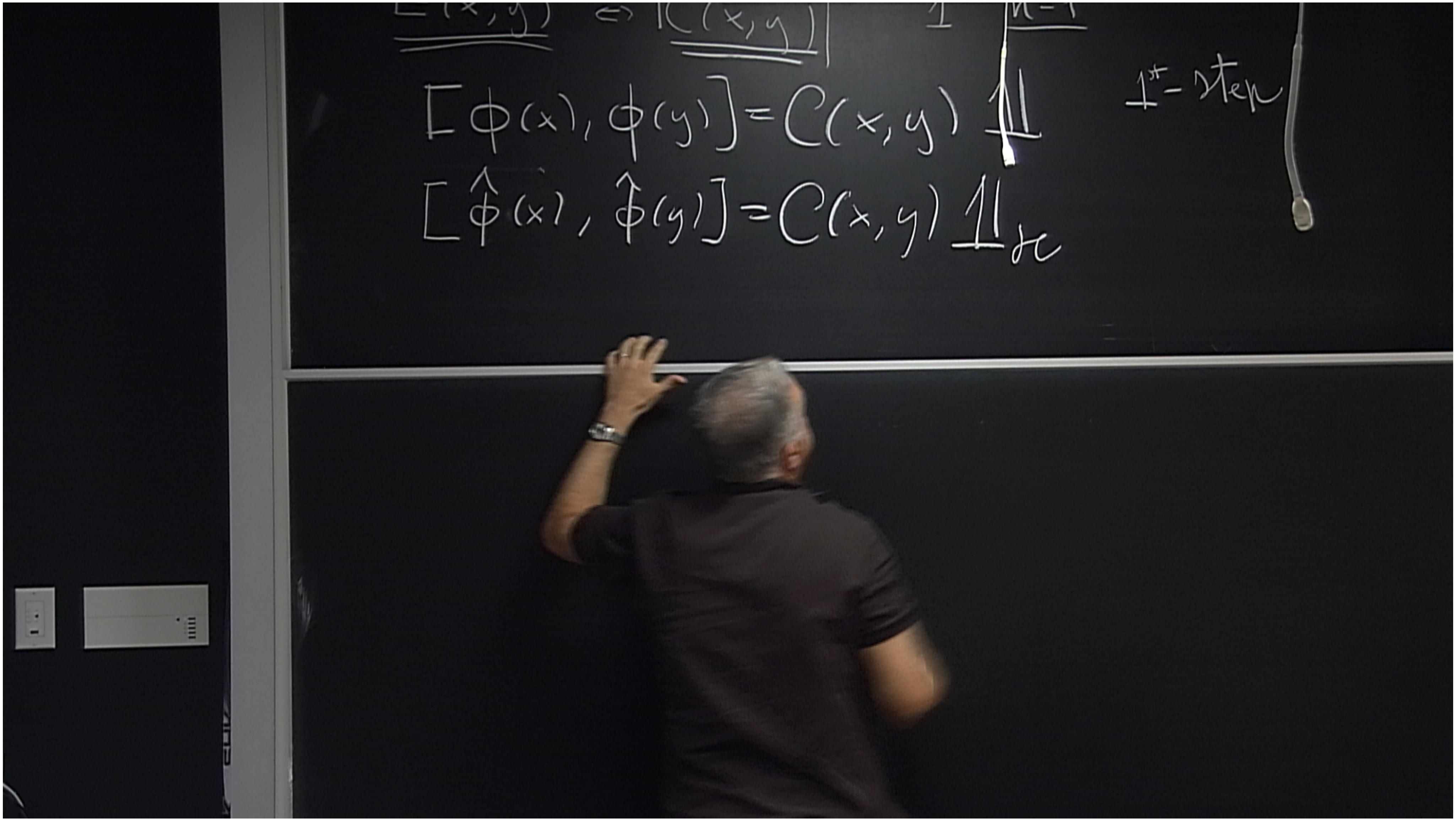

M ≅ ℝ × Σ

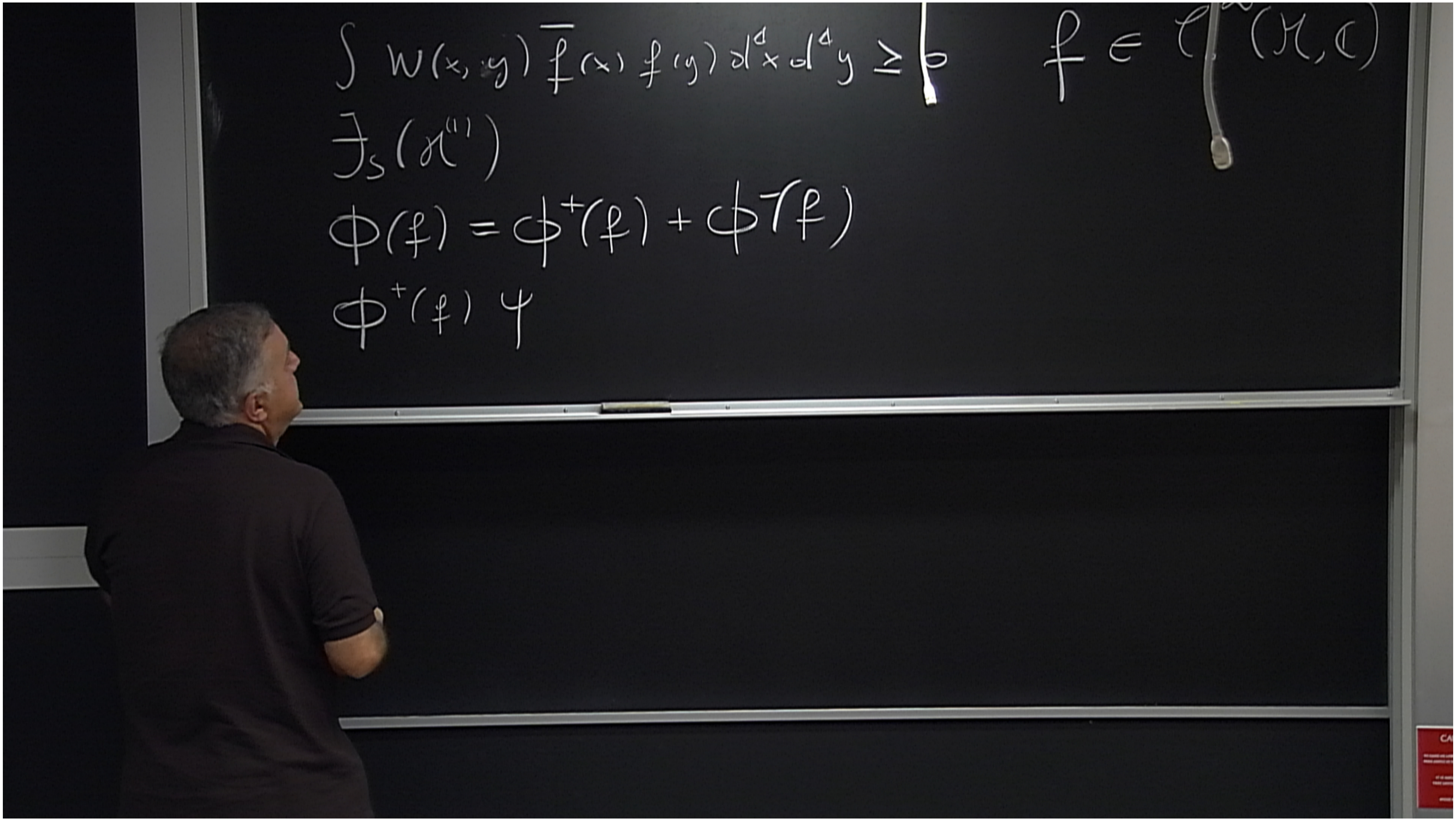

(M, g)
$M \cong \mathbb{R} \times \Sigma$

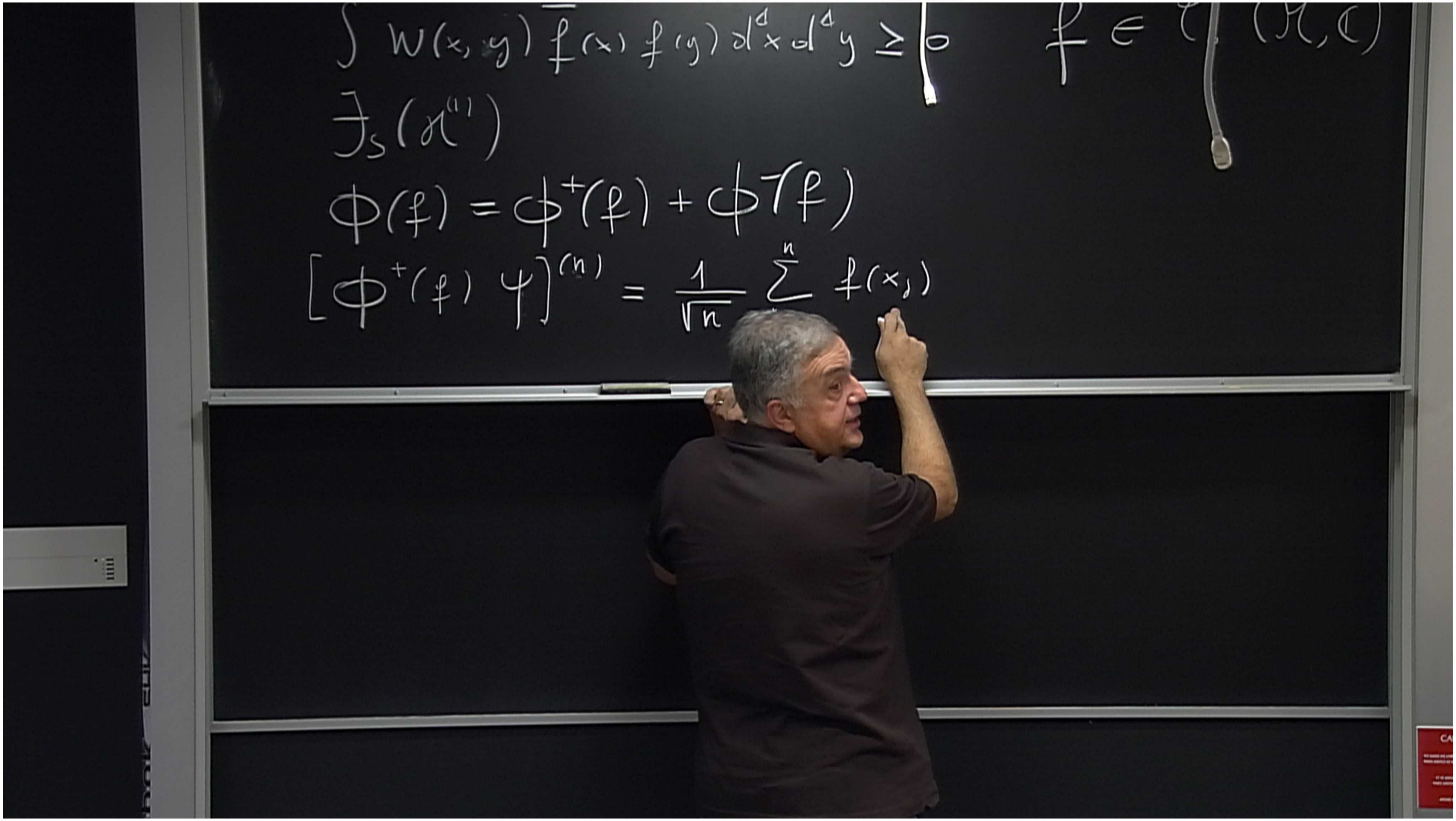

(M, g)
M ≅ ℝ × Σ
E(x, y)

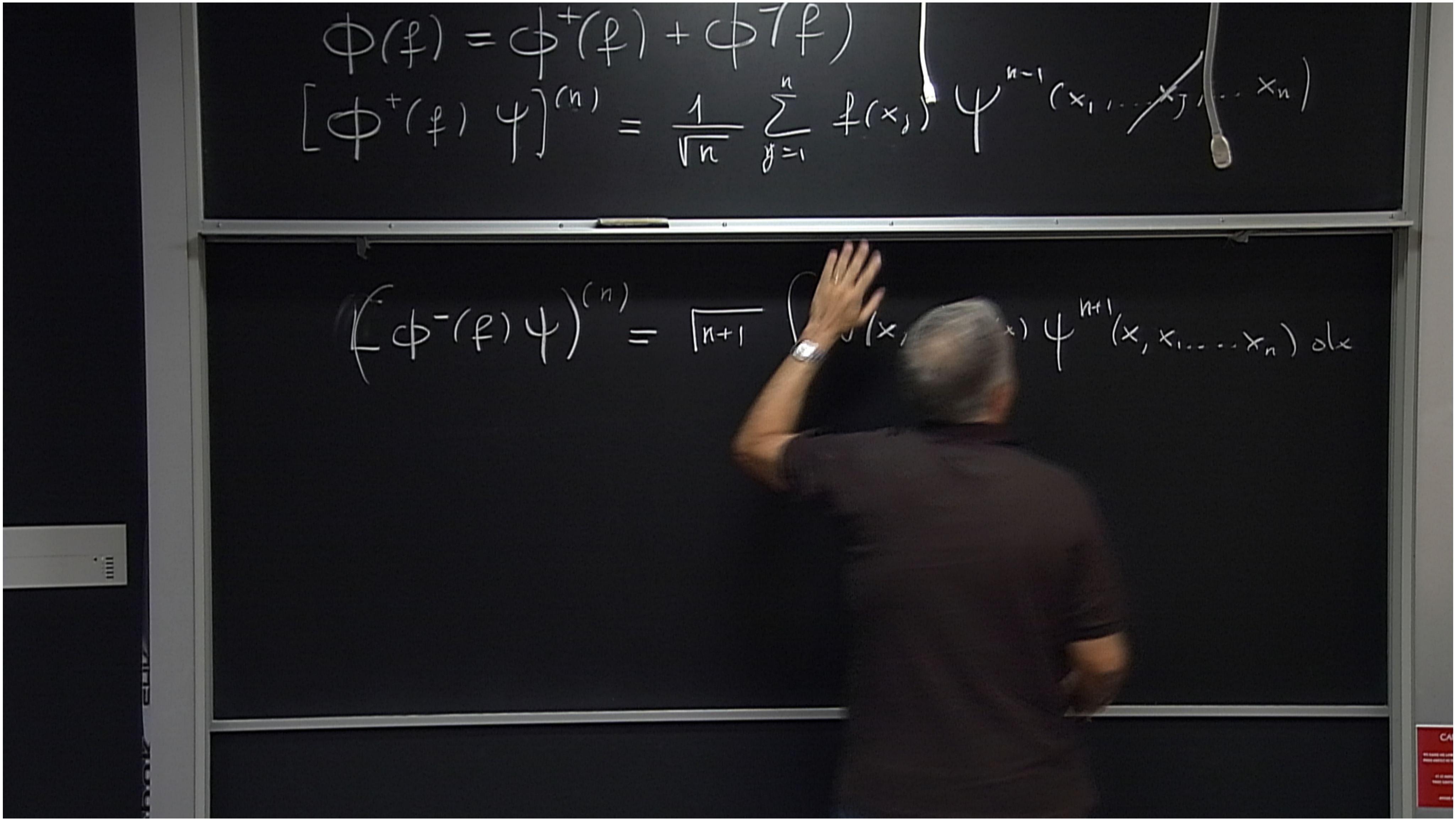

(M, g)
M ≅ ℝ × Σ
E(x, y) ⇔ C(x, y)
[φ(x), φ(y)] = C(x, y) 𝟙
1st - step

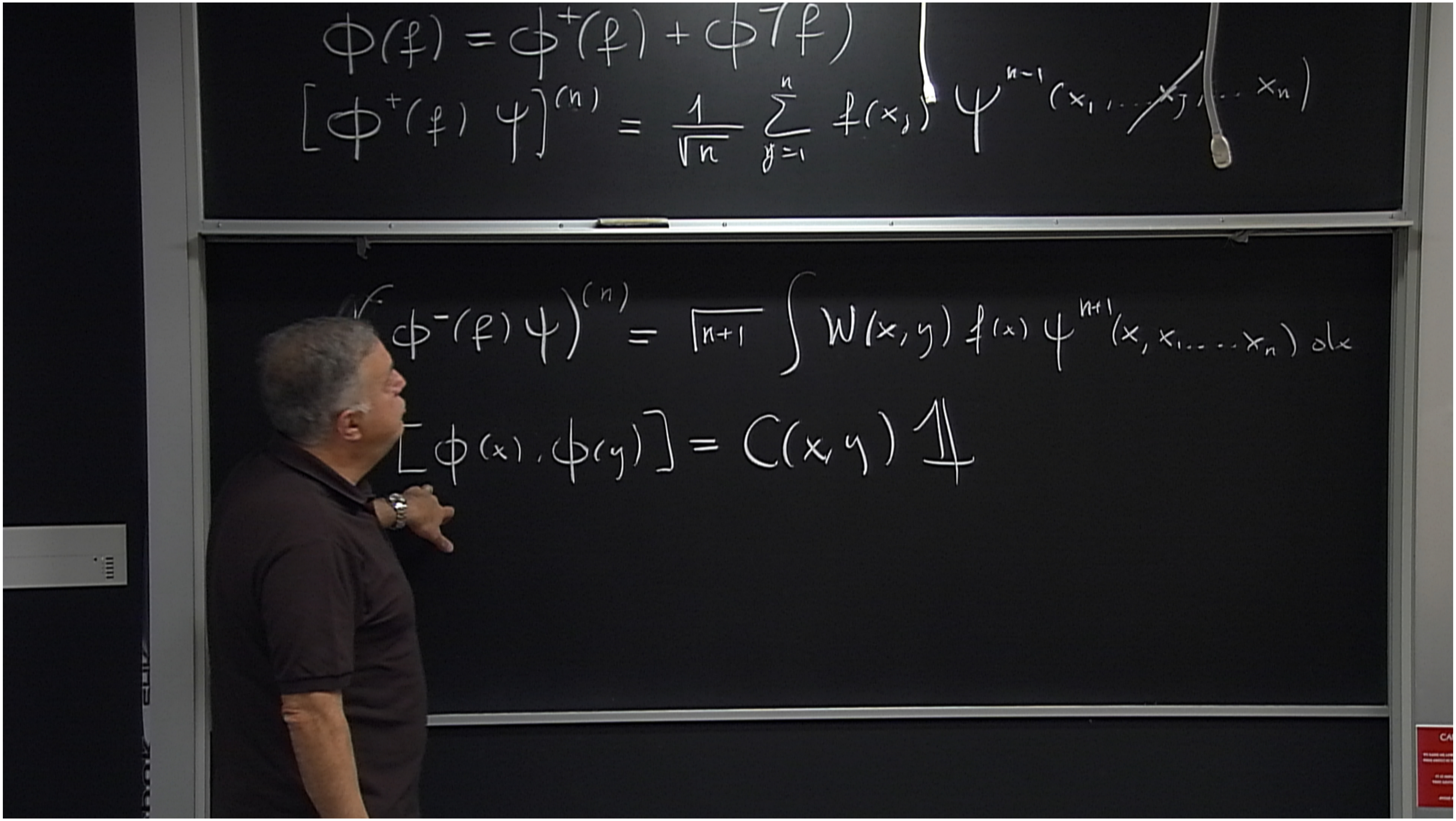

[φ(x), φ(y)] = C(x,y) 𝟙
[φ̂(x), φ̂(y)] = C(x,y)
1st - step

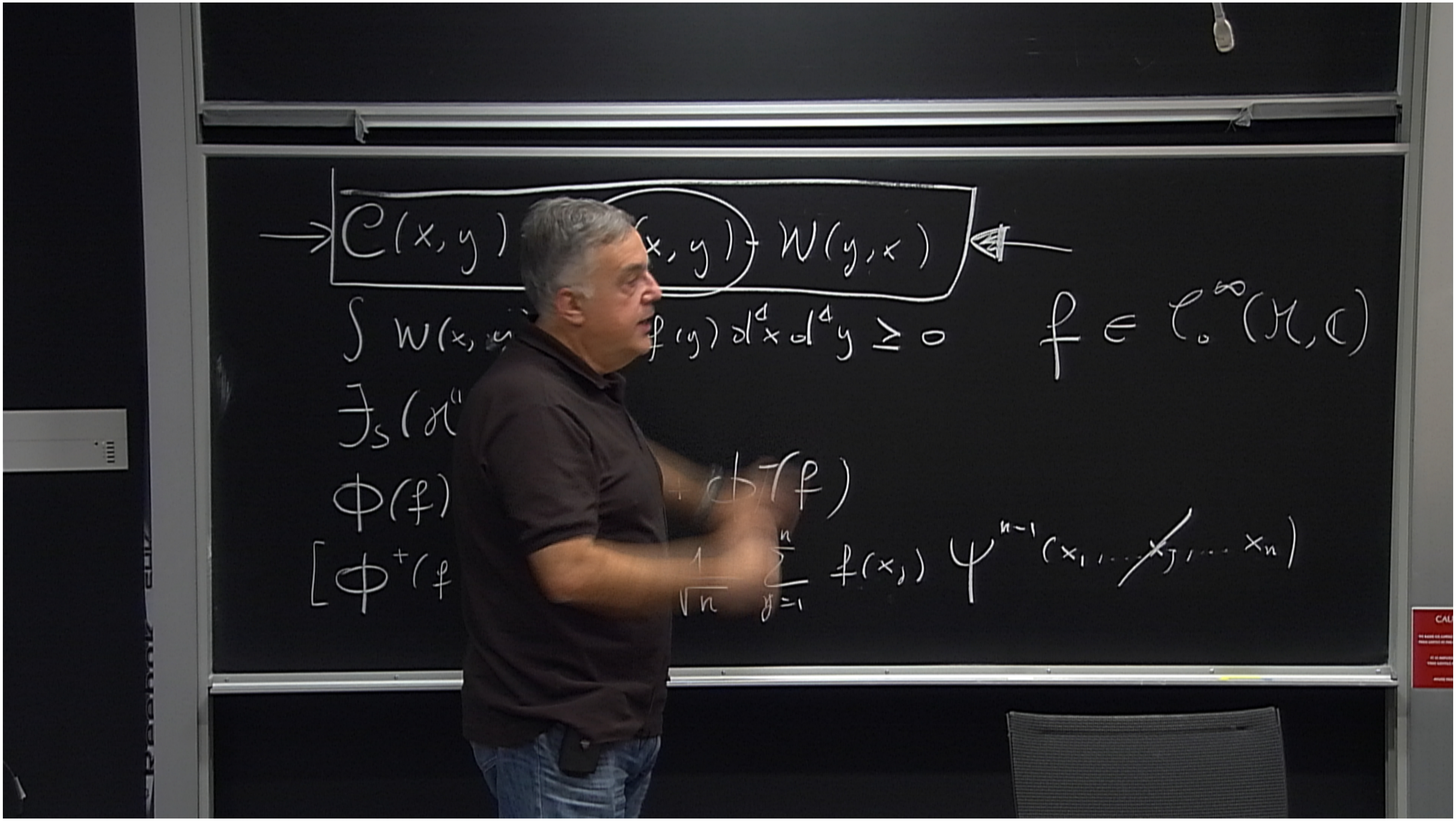

[φ(x), φ(y)] = C(x,y) 𝟙
[φ̂(x), φ̂(y)] = C(x,y) 𝟙_H
1st step

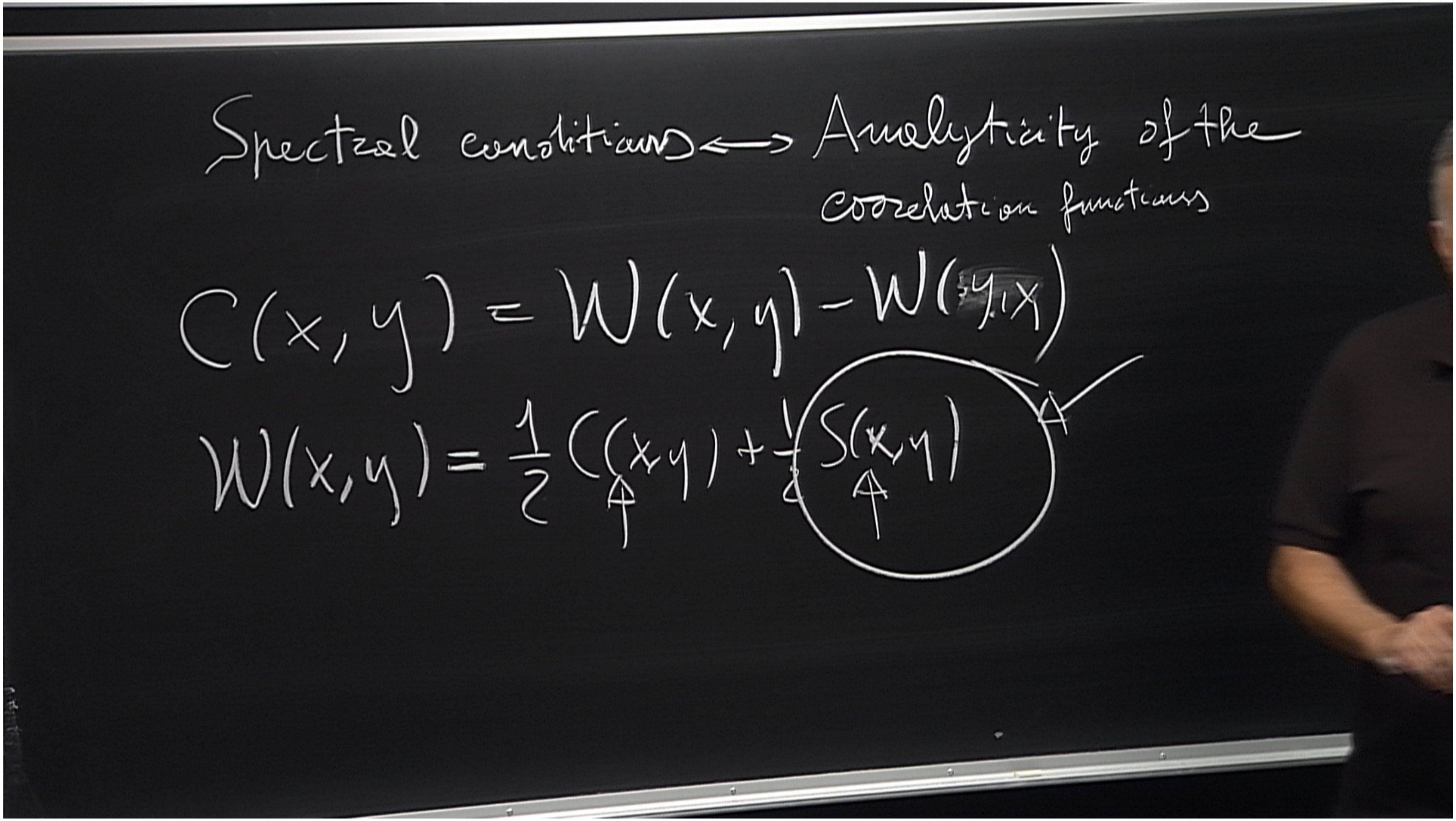

$\int W(x,y)\,\overline{f}(x)\,f(y)\,d^4x\,d^4y \geq 0$
$f \in C^{\infty}(M, \mathbb{C})$
$\mathcal{F}_s(\mathcal{H}^{(1)})$
$\phi(f) = \phi^+(f) + \phi^-(f)$
$\phi^+(f)\,\Psi$


∫ W(x,y) f̄(x) f(y) d^4x d^4y ≥ 0
Φ(f) = Φ+(f) + Φ−(f)
[Φ+(f) ψ](n) = 1/√n Σ^n f(xj)

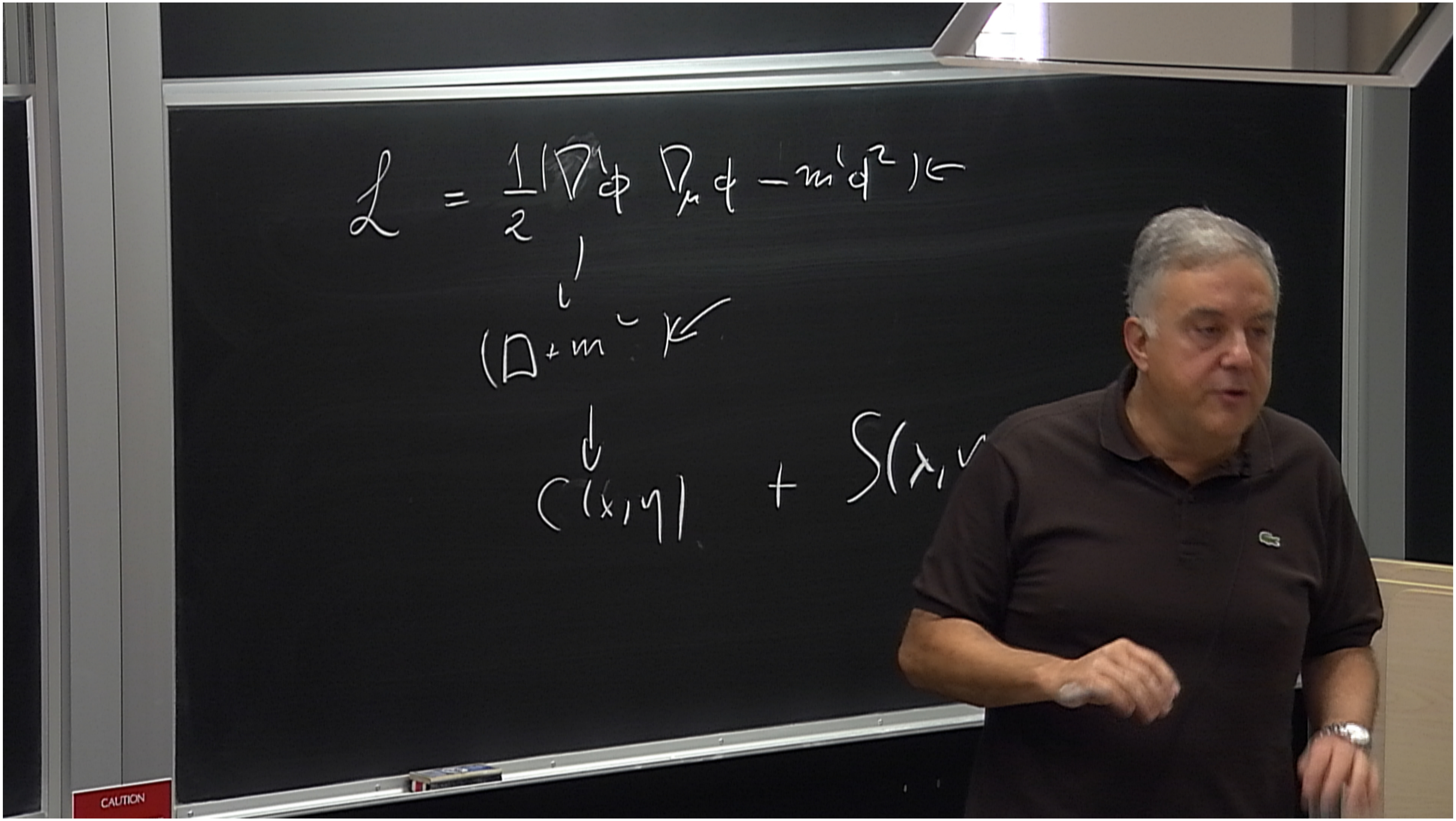

$\Phi(f) = \phi^{+}(f) + \phi^{-}(f)$
$[\phi^{+}(f)\ \psi]^{(n)} = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} f(x_j)\ \psi^{n-1}(x_1, \dots, x_j, \dots x_n)$
$(\phi^{-}(f)\psi)^{(n)} = \sqrt{n+1}$
$\psi^{n+1}(x, x_1 \dots x_n)\ dx$

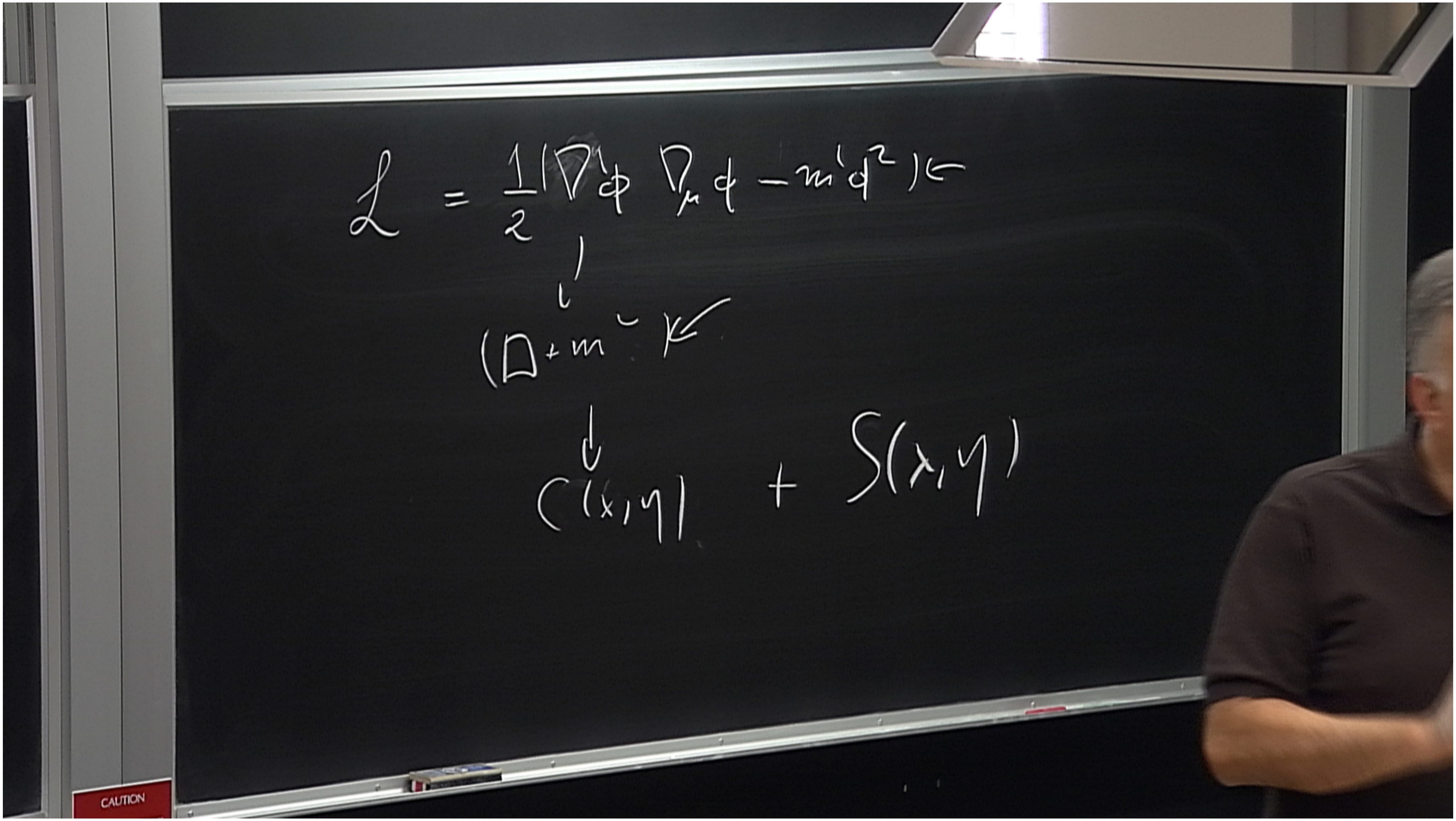

$\Phi(f) = \phi^+(f) + \phi^-(f)$
$[\phi^+(f)\,\psi]^{(n)} = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} f(x_j)\, \psi^{n-1}(x_1, \ldots \not{x_j}, \ldots x_n)$
$(\phi^-(f)\,\psi)^{(n)} = \sqrt{n+1} \int W(x,y)\, f(x)\, \psi^{n+1}(x, x_1 \ldots x_n)\, dx$
$[\phi(x), \phi(y)] = C(x,y)\, \mathbb{1}$

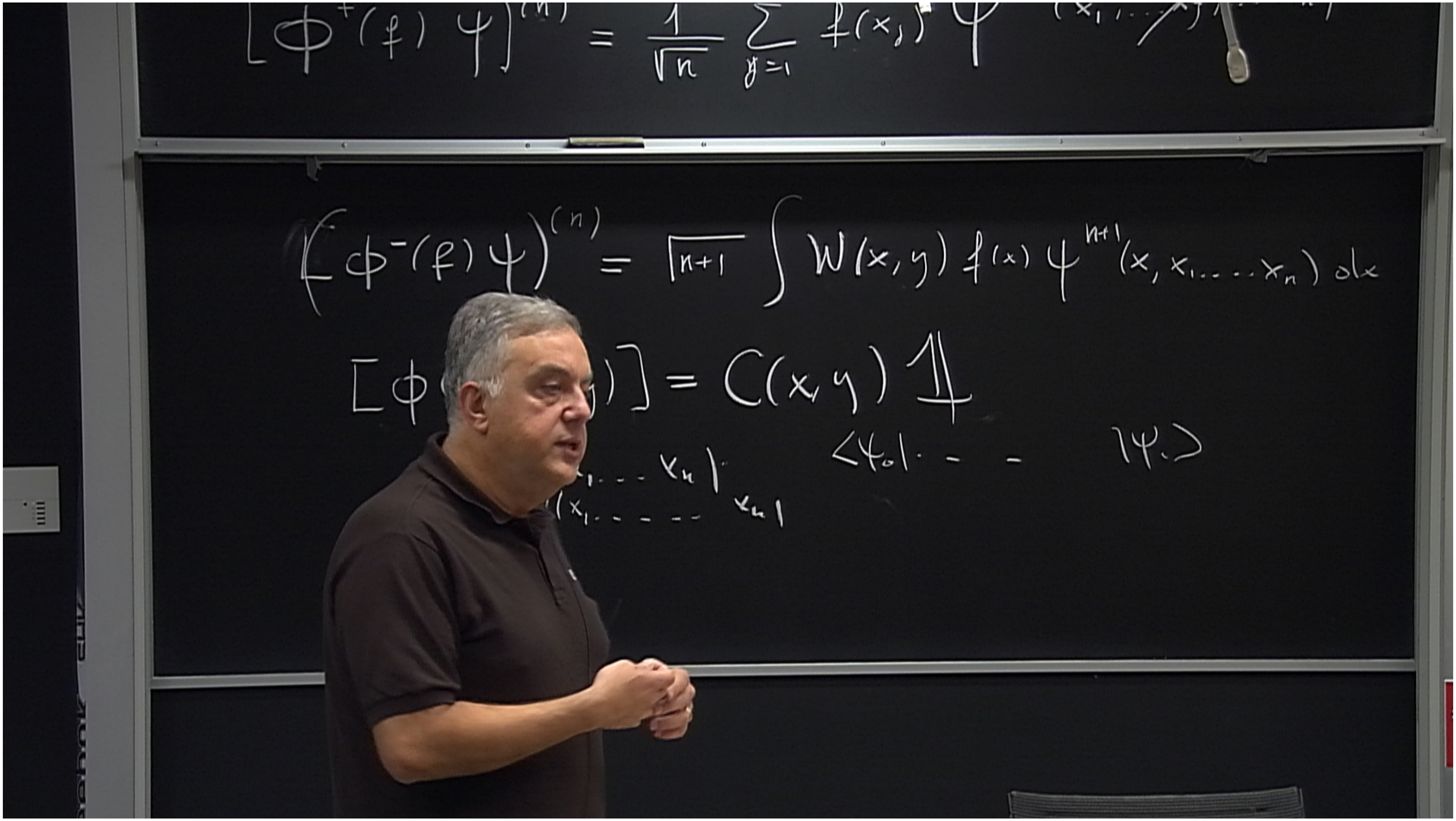

f ∈ C₀^∞(M, ℂ)

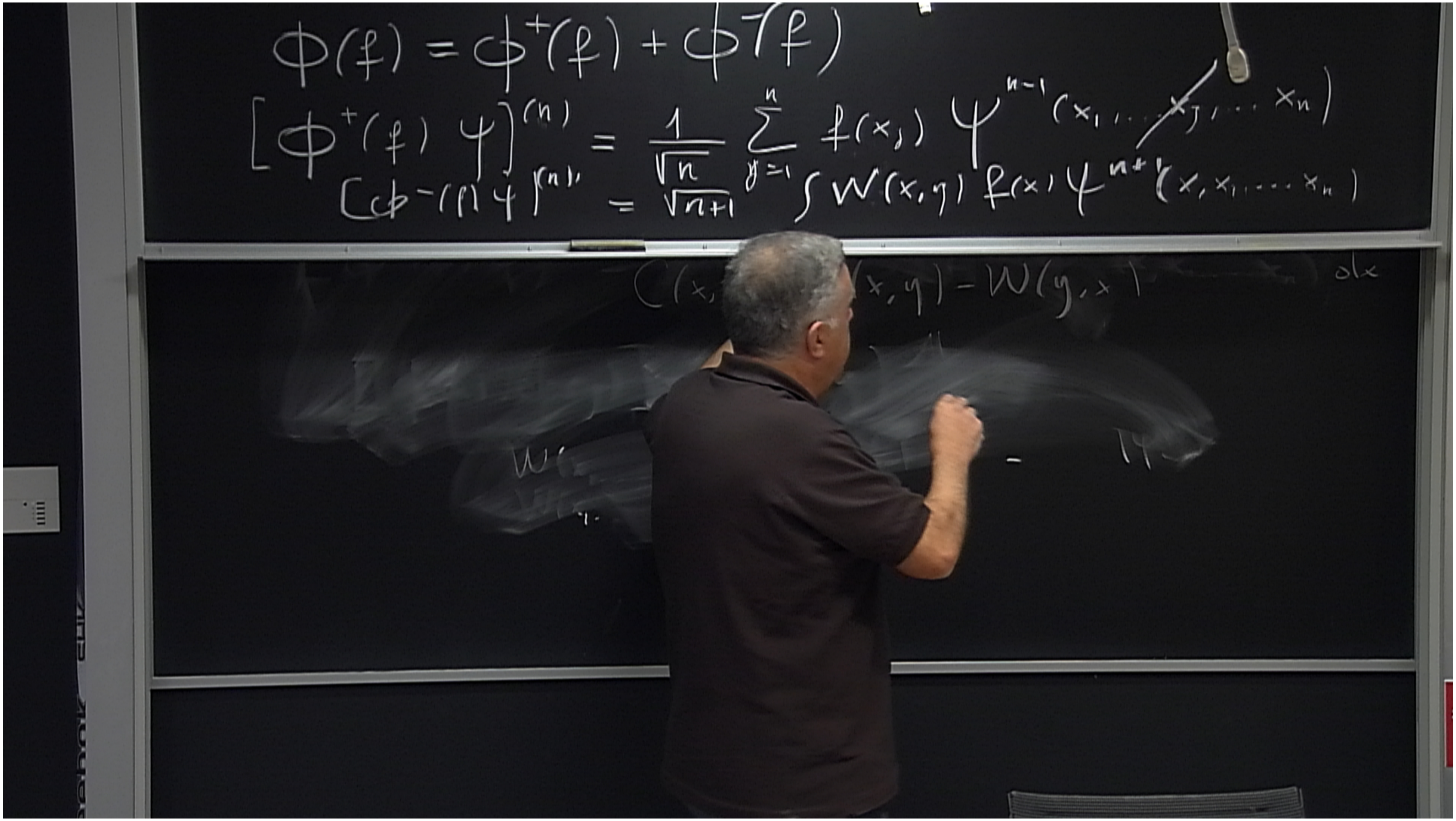


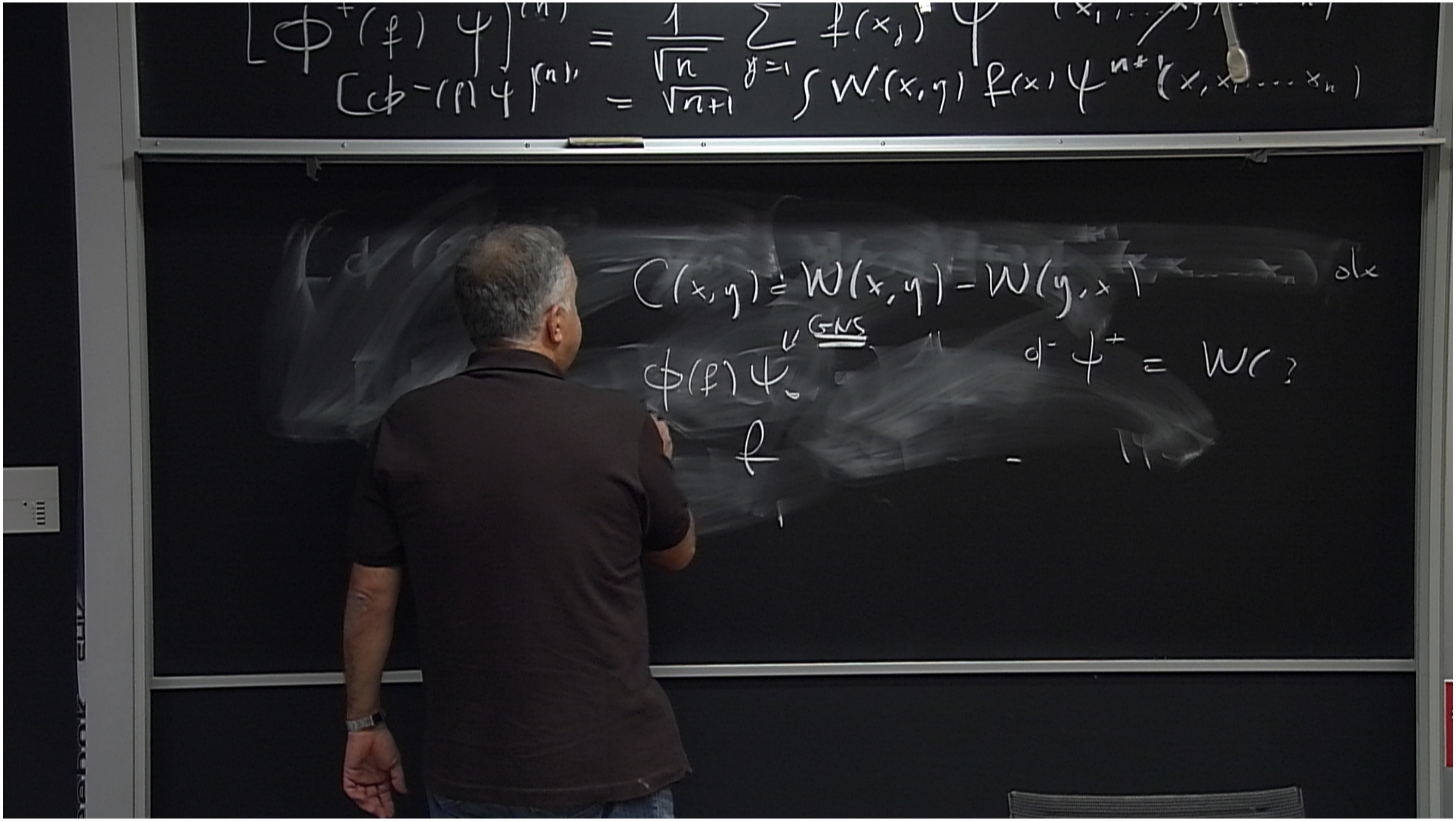
Spectral conditions ⟷ Analyticity of the correlation functions
C(x, y) = W(x, y) - W(y, x)
W(x, y) = 1/2 C(x, y) + 1/2 S(x, y)

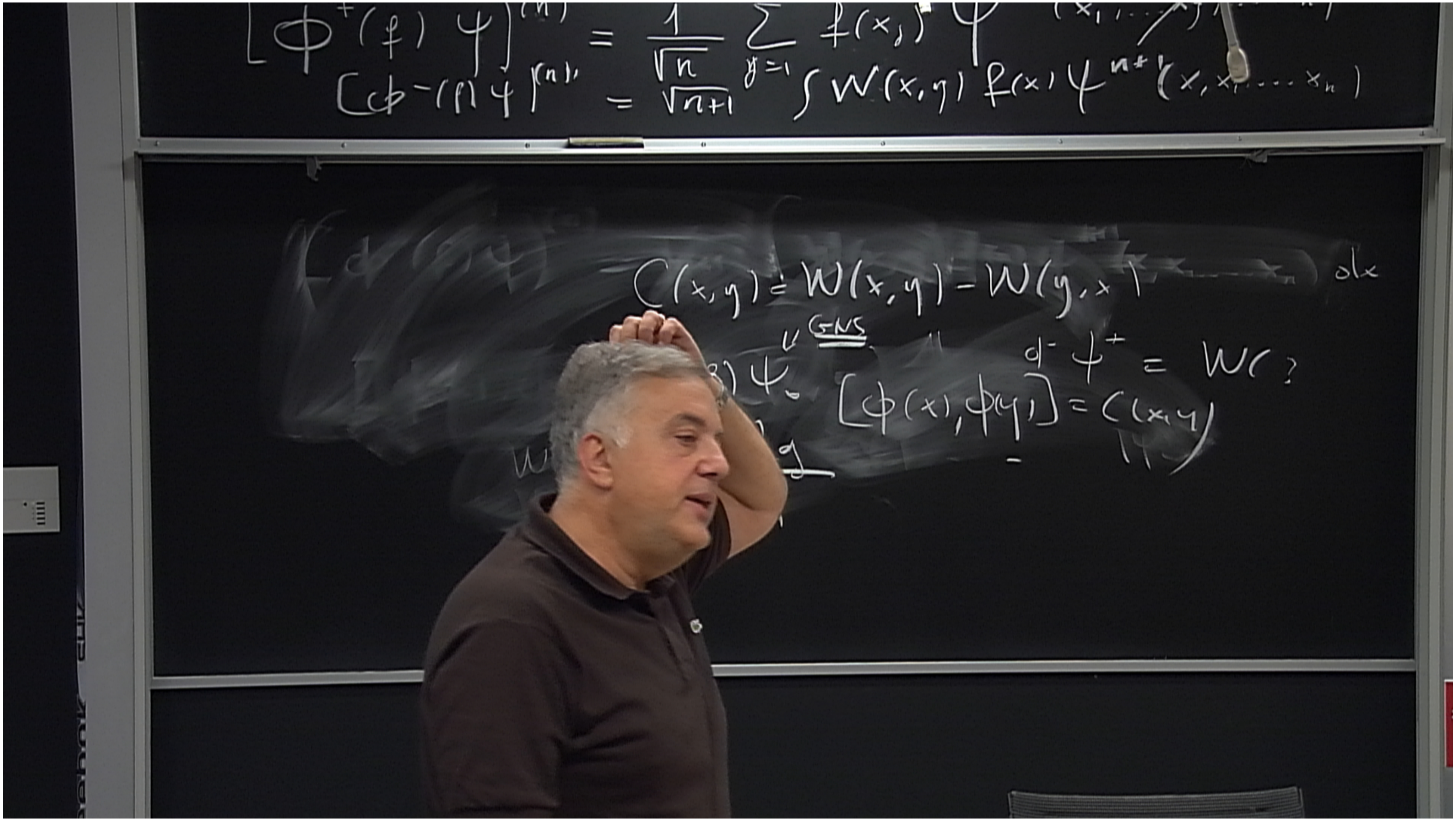

$\mathcal{L} = \frac{1}{2}(\nabla^\mu \phi \nabla_\mu \phi - m^2 \phi^2)$
$(\Box + m^2)$
$G(x,y) + S(x,y$
CAUTION

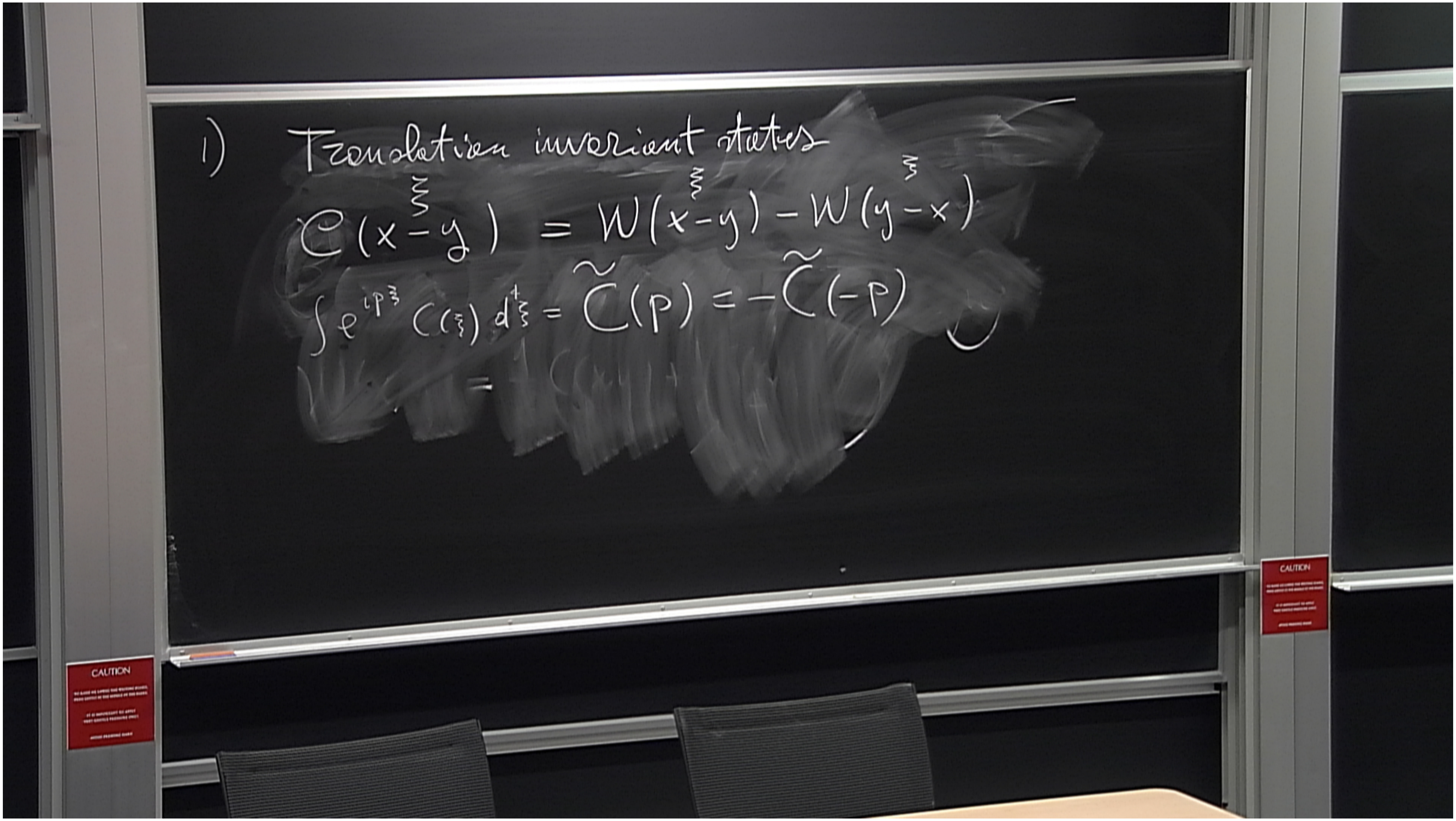

CAUTION

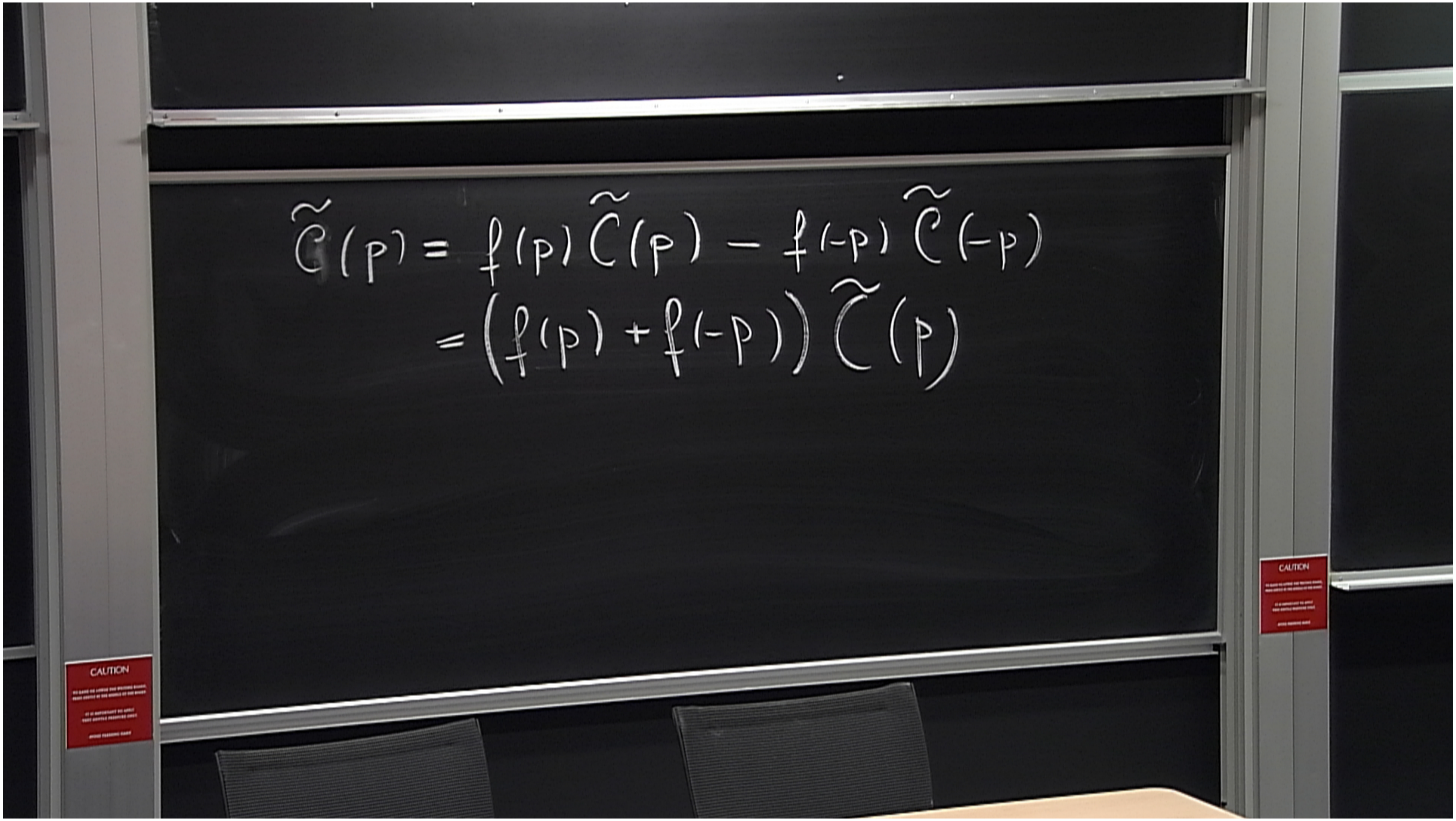

(Φ⁻(f) ψ)^(n) = √(n+1) ∫ W(x,y) f(x) ψ^(n+1)(x, x₁ . . . . xₙ) dx
= C(x, y) 1
<ψ₀| . . .
|ψ₀>

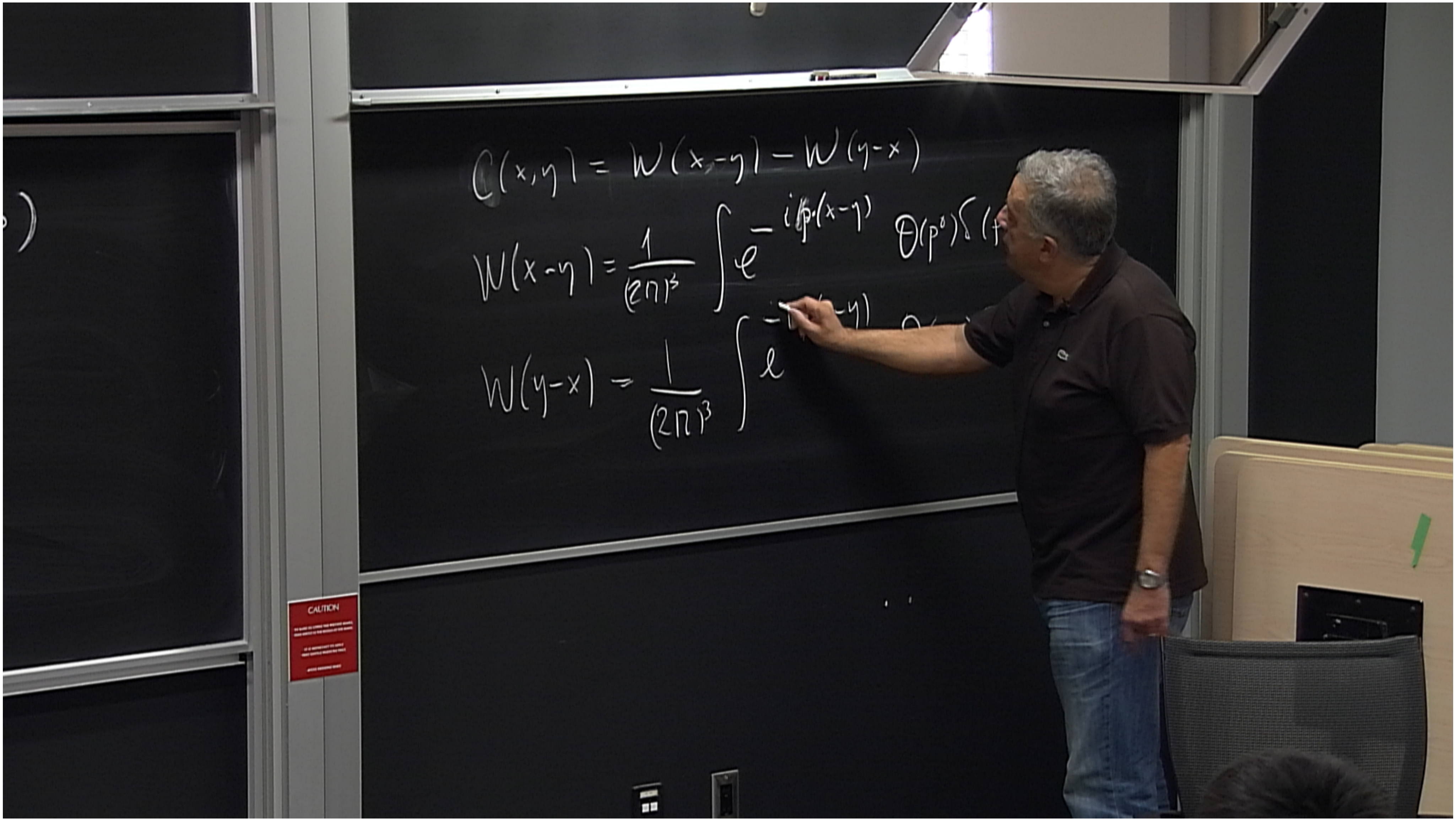

Φ(f) = φ⁺(f) + φ⁻(f)
[φ⁺(f) ψ]^(n) = 1/√n Σ_{j=1}^n f(x_j) ψ^(n-1)(x_1, ... x_j, ... x_n)
[φ⁻(f) ψ]^(n) = √(n+1) ∫ W(x,y) f(x) ψ^(n+1)(x, x_1, ... x_n) dx

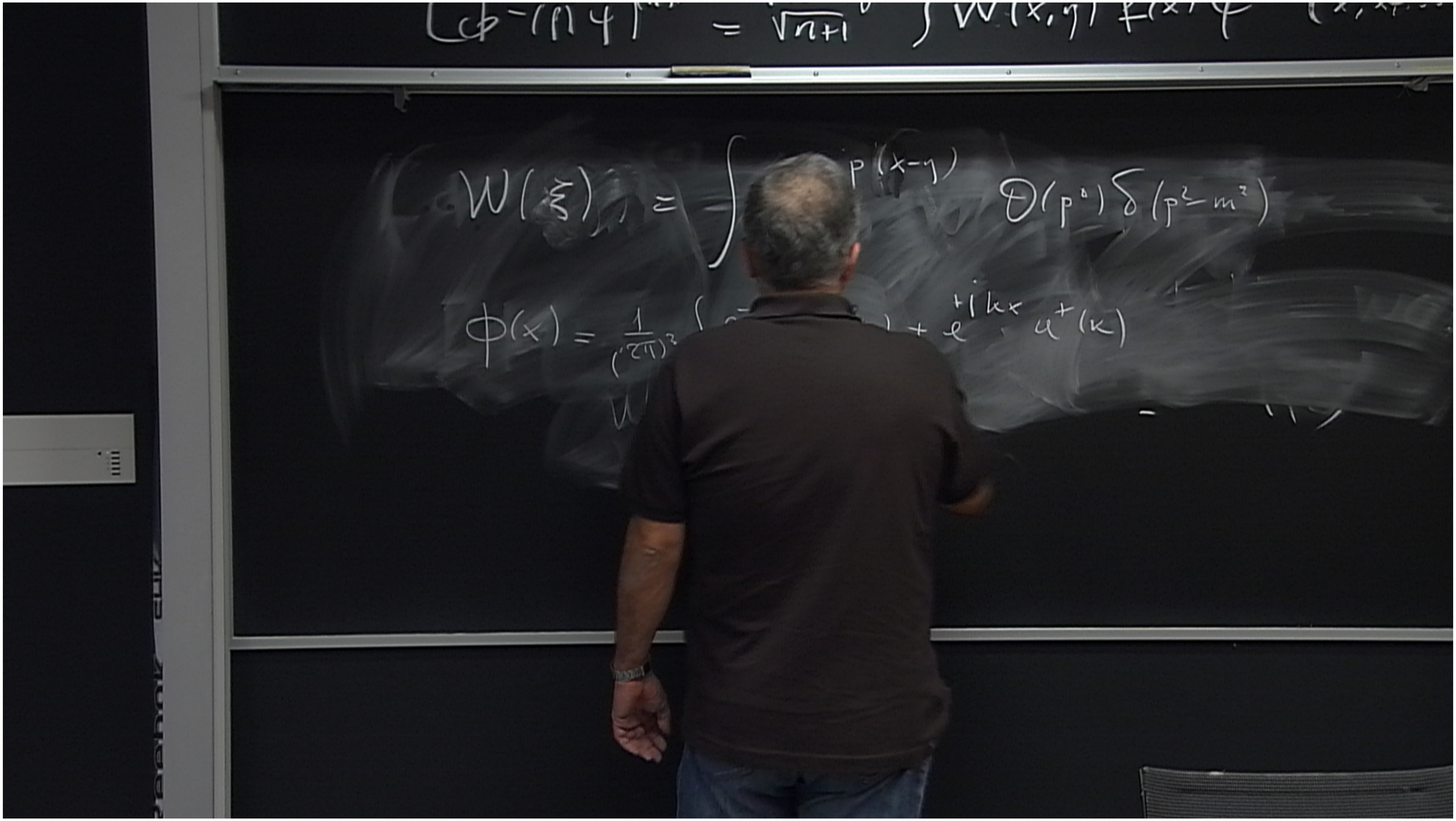

C(x,y) = W(x,y) - W(y,x)
GNS

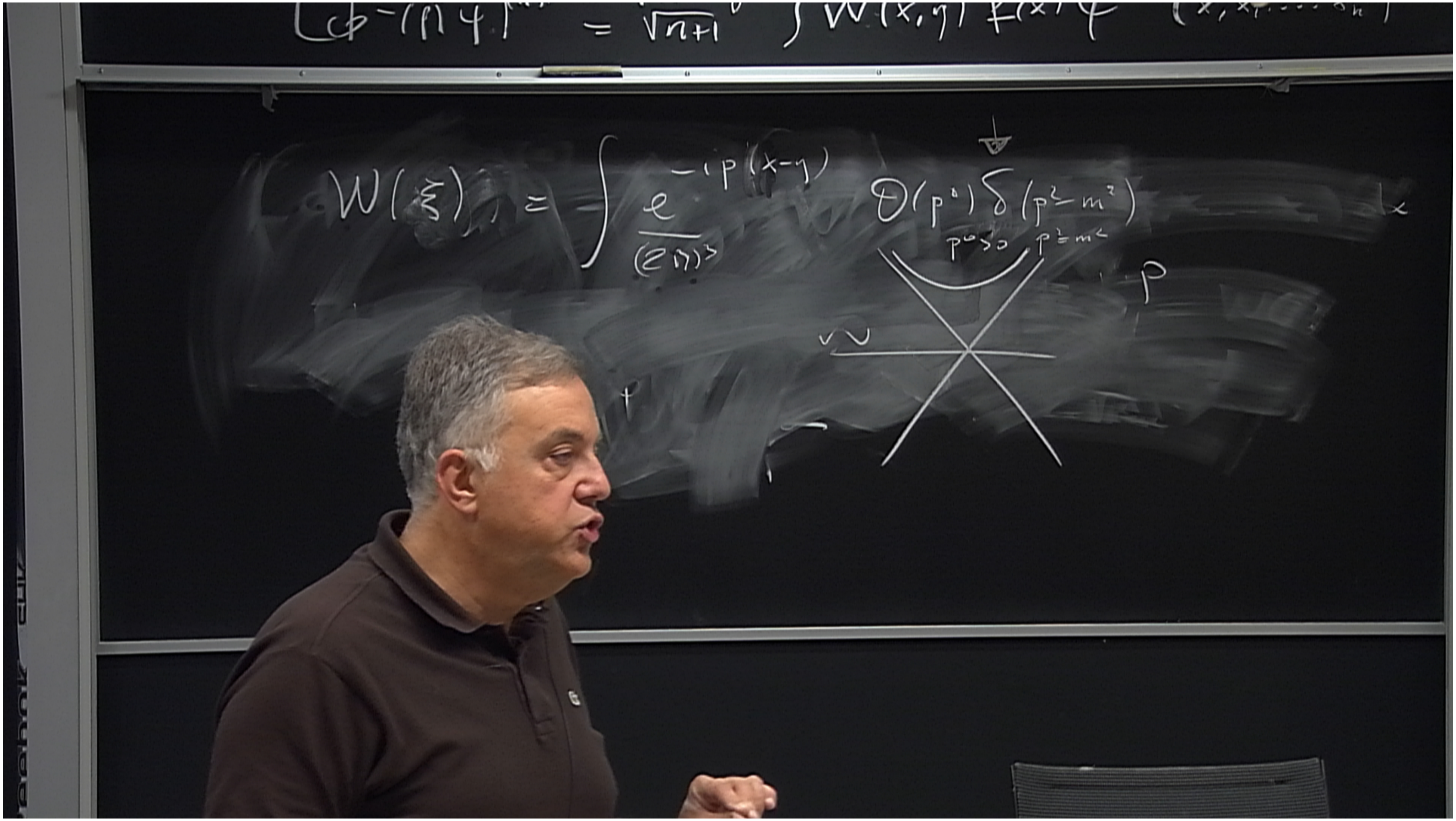

C(x,y) = W(x,y) − W(y,x)
GNS
[φ(x), φ(y)] = C(x,y)

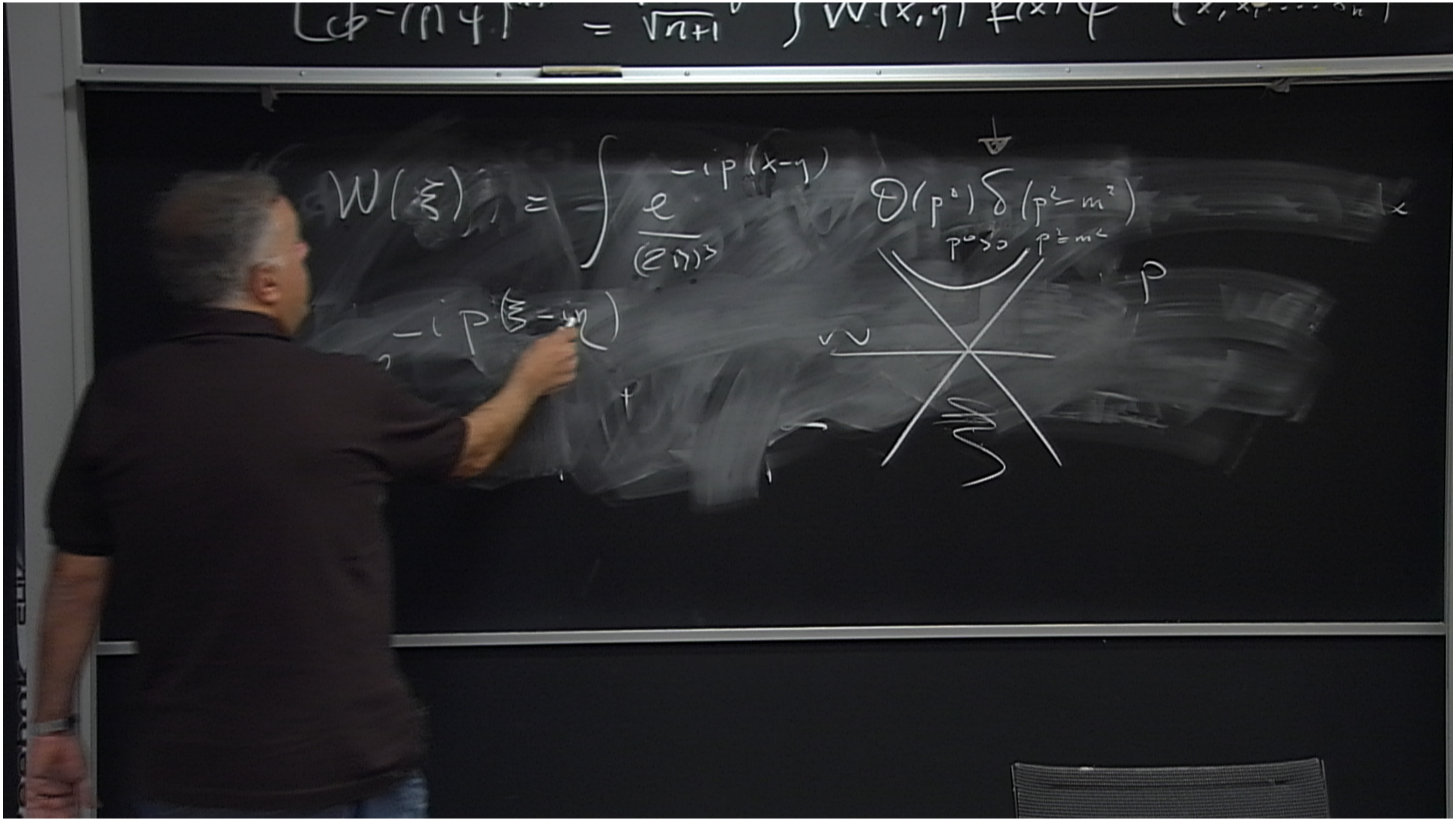

1) Translation invariant states
$C(x-y) = W(x-y) - W(y-x)$
$\int e^{ip\xi} C(\xi)\, d^4\xi = \tilde{C}(p) = -\tilde{C}(-p)$
CAUTION
CAUTION

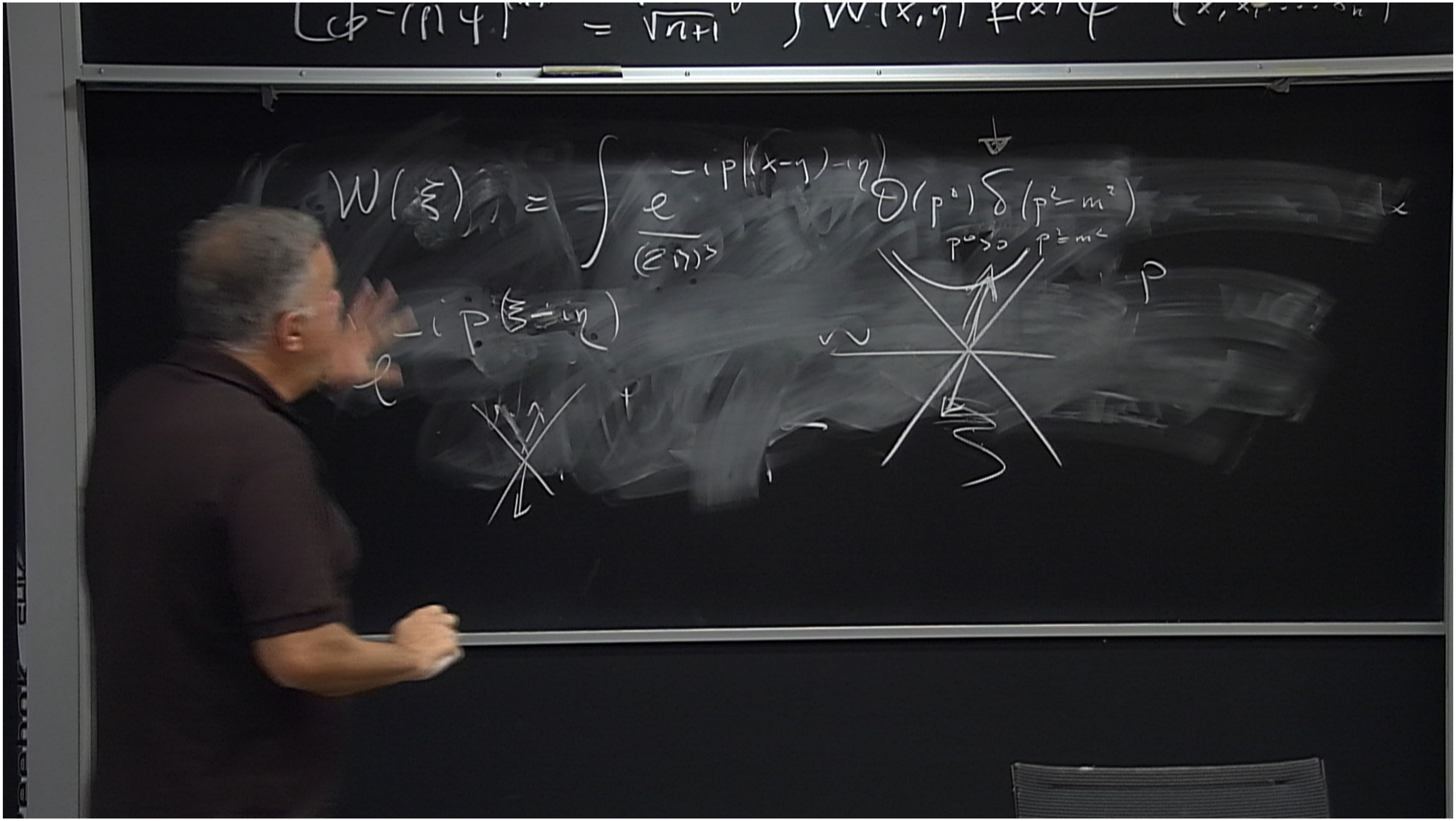

$\tilde{C}(p) = f(p)\,\tilde{C}(p) - f(-p)\,\tilde{C}(-p)$
$= \left(f(p) + f(-p)\right)\tilde{C}(p)$
CAUTION
CAUTION

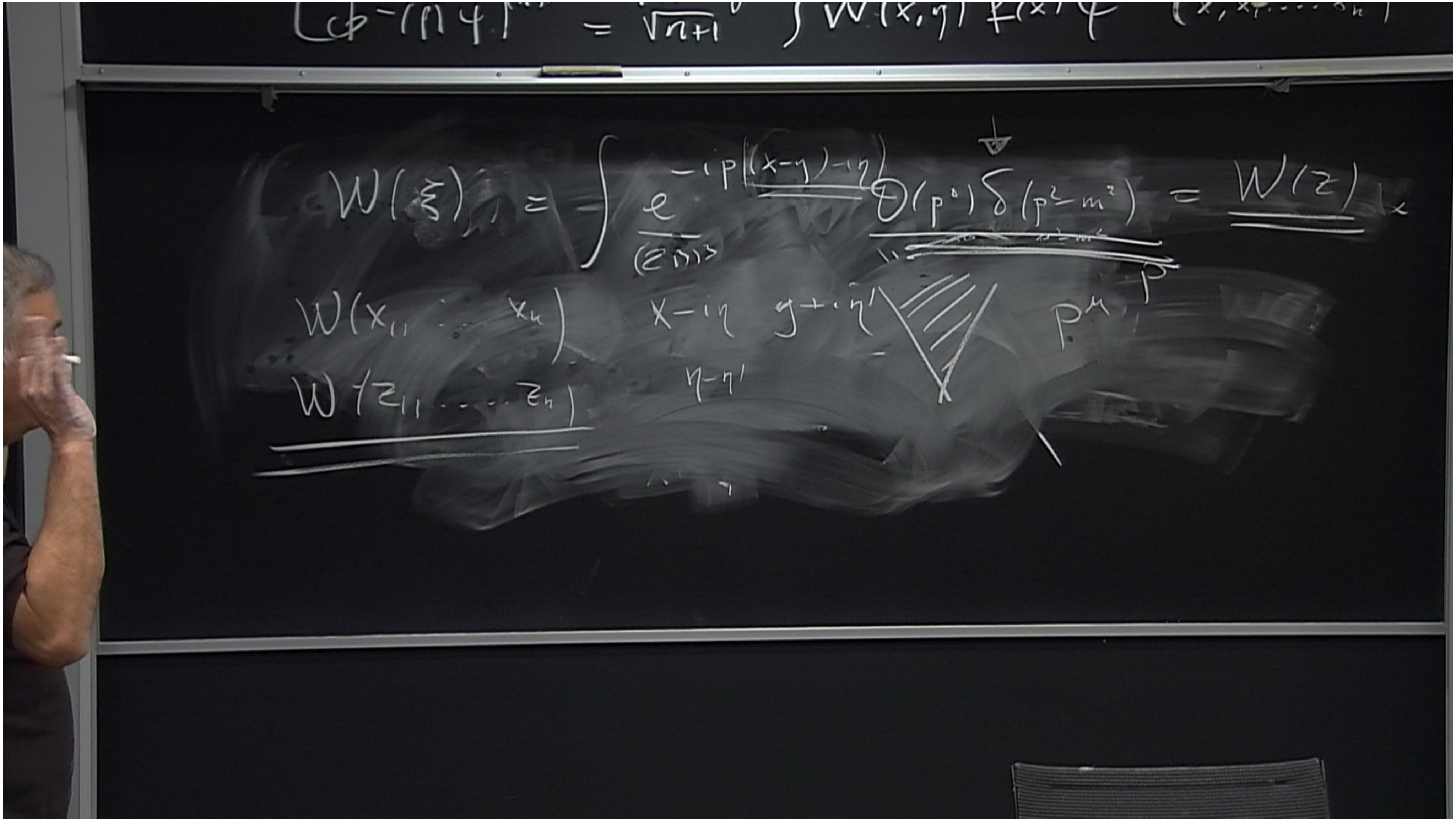

C(x,y) = W(x-y) - W(y-x)
W(x-y) = 1/(2π)^3 ∫ e^{-ip·(x-y)} θ(p^0)δ(
W(y-x) = 1/(2π)^3 ∫ e
CAUTION

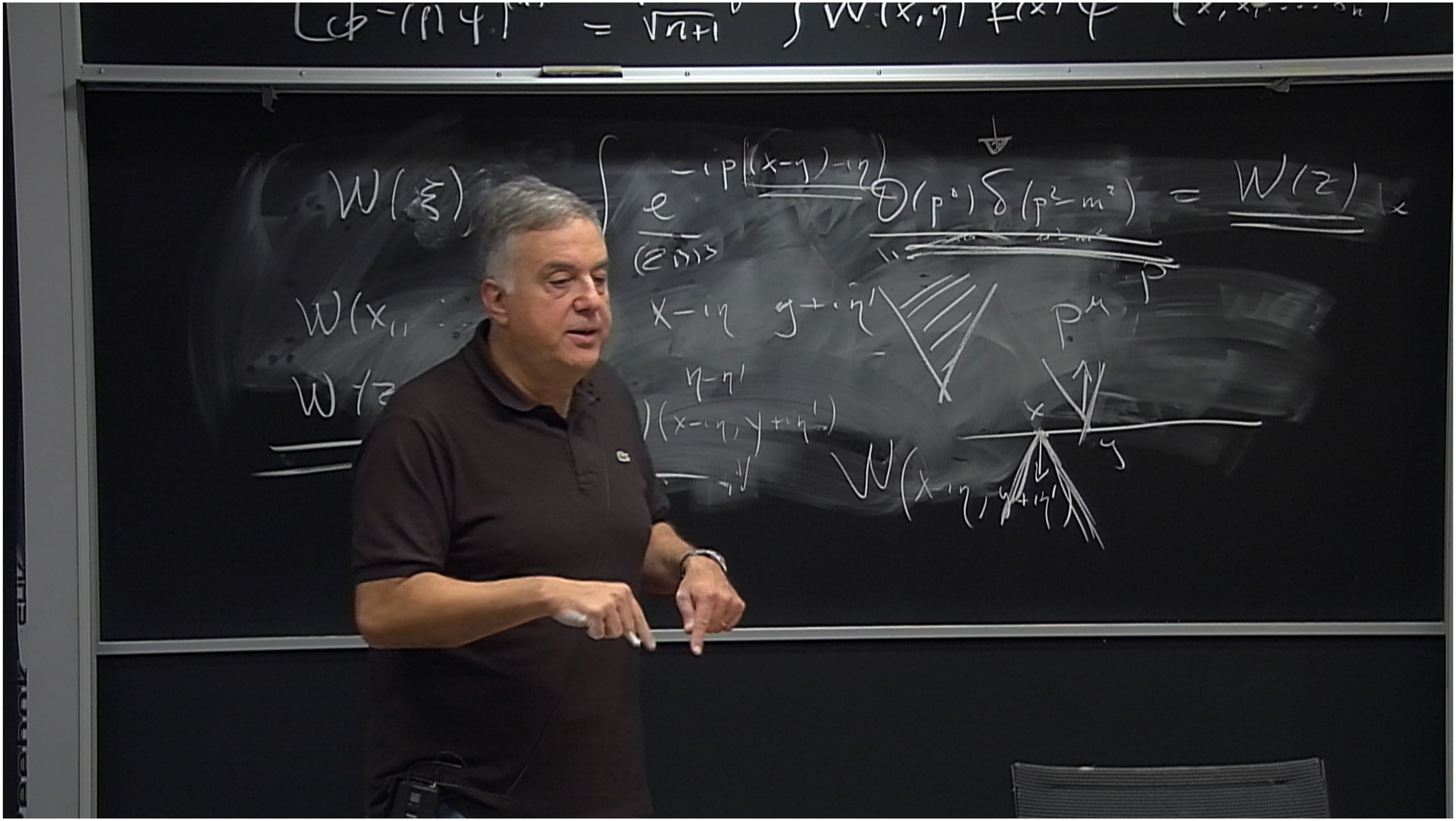


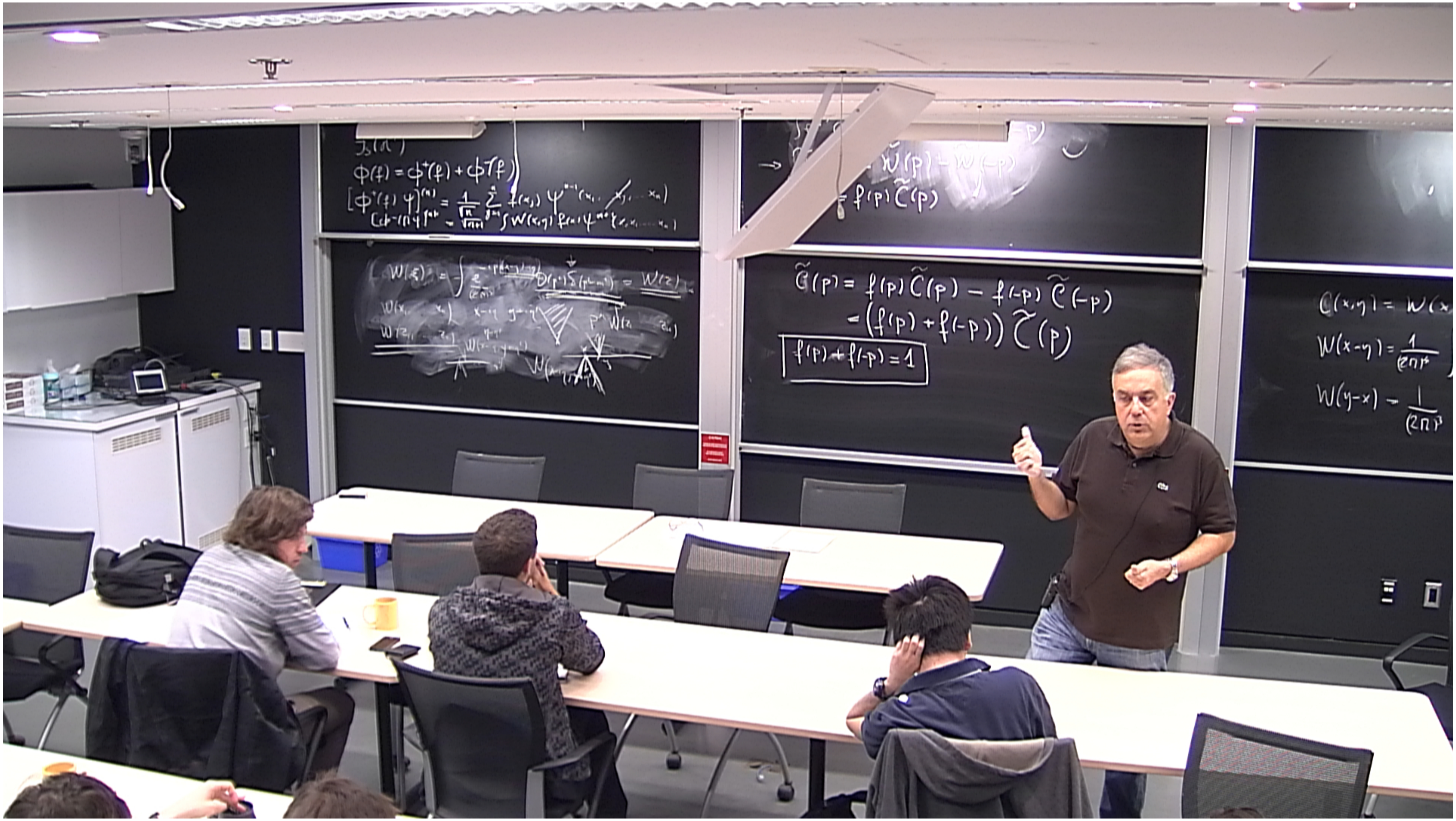

W(ξ) = ∫ e^{-ip(x-y)} / (2π)^3 θ(p^0) δ(p^2 - m^2)
p^0 > 0   p^2 = m^2


W(ξ) = ∫ e^{-ip(x-y)} / (2π)^3 θ(p^0) δ(p^2 - m^2)
p^0 > 0
p^2 = m^2
p

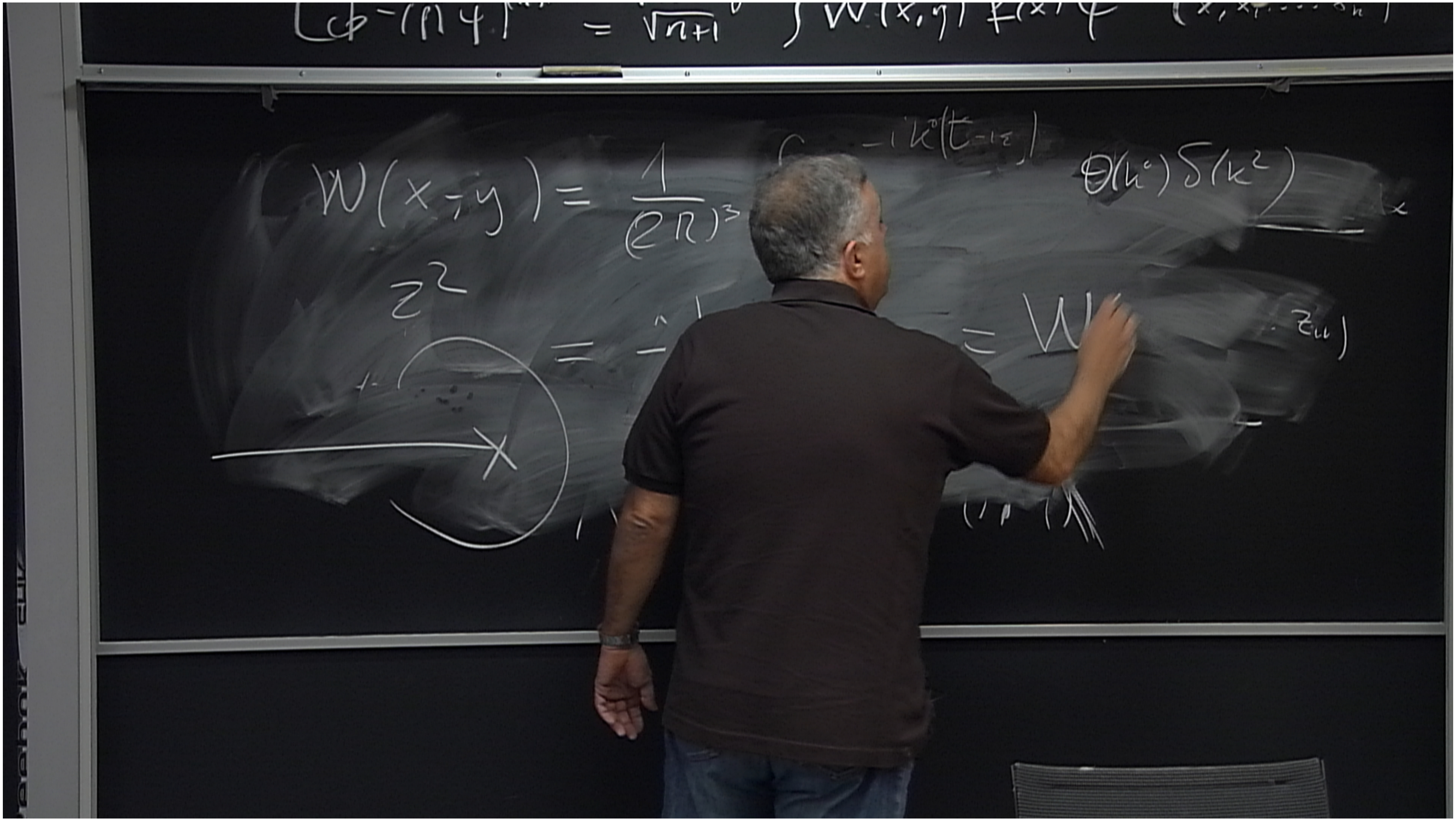

W(ξ) = ∫ e^{-ip((x-y)-iη)} / (2π)^3 θ(p^0) δ(p^2 - m^2)
p^0>0   p^2=m^2

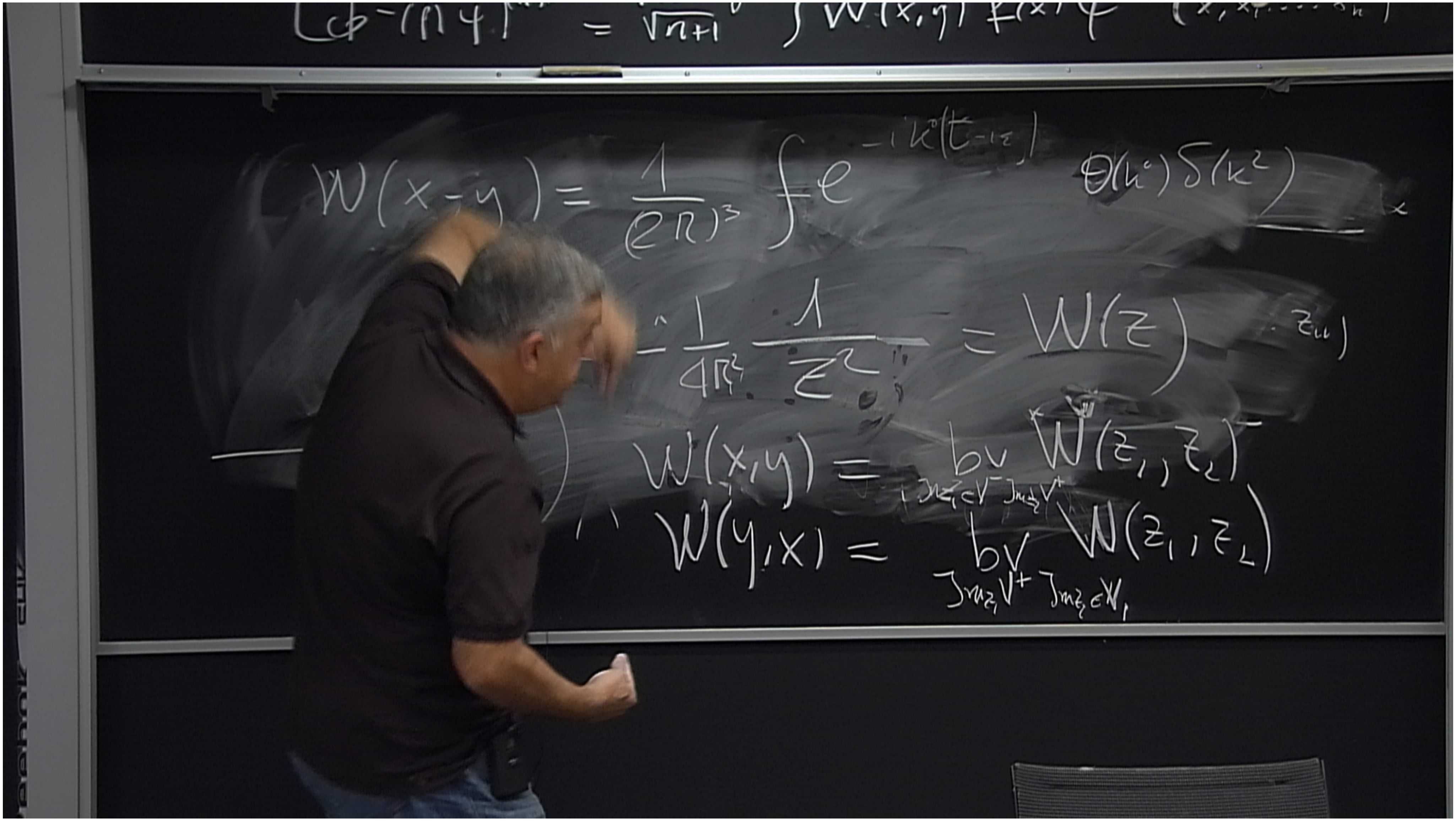

W(ξ) = ∫ e^{-ip((x-y)-iη)} / (2π)^3 θ(p^0) δ(p^2 - m^2) = W(z)
W(x_1, ... x_n)
x - iη   y + iη'
η - η'
W(z_1, ... z_n)
p^μ

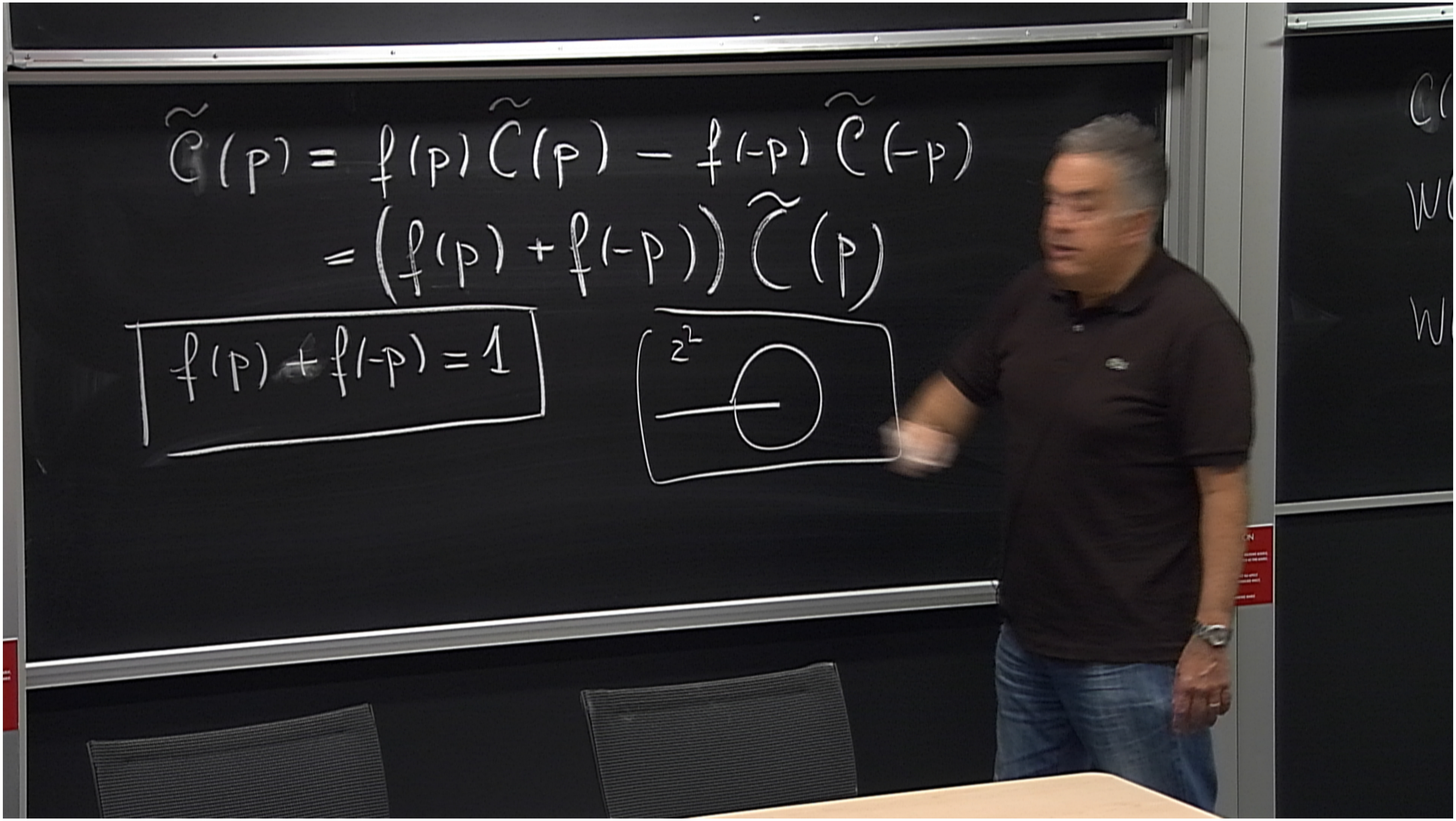


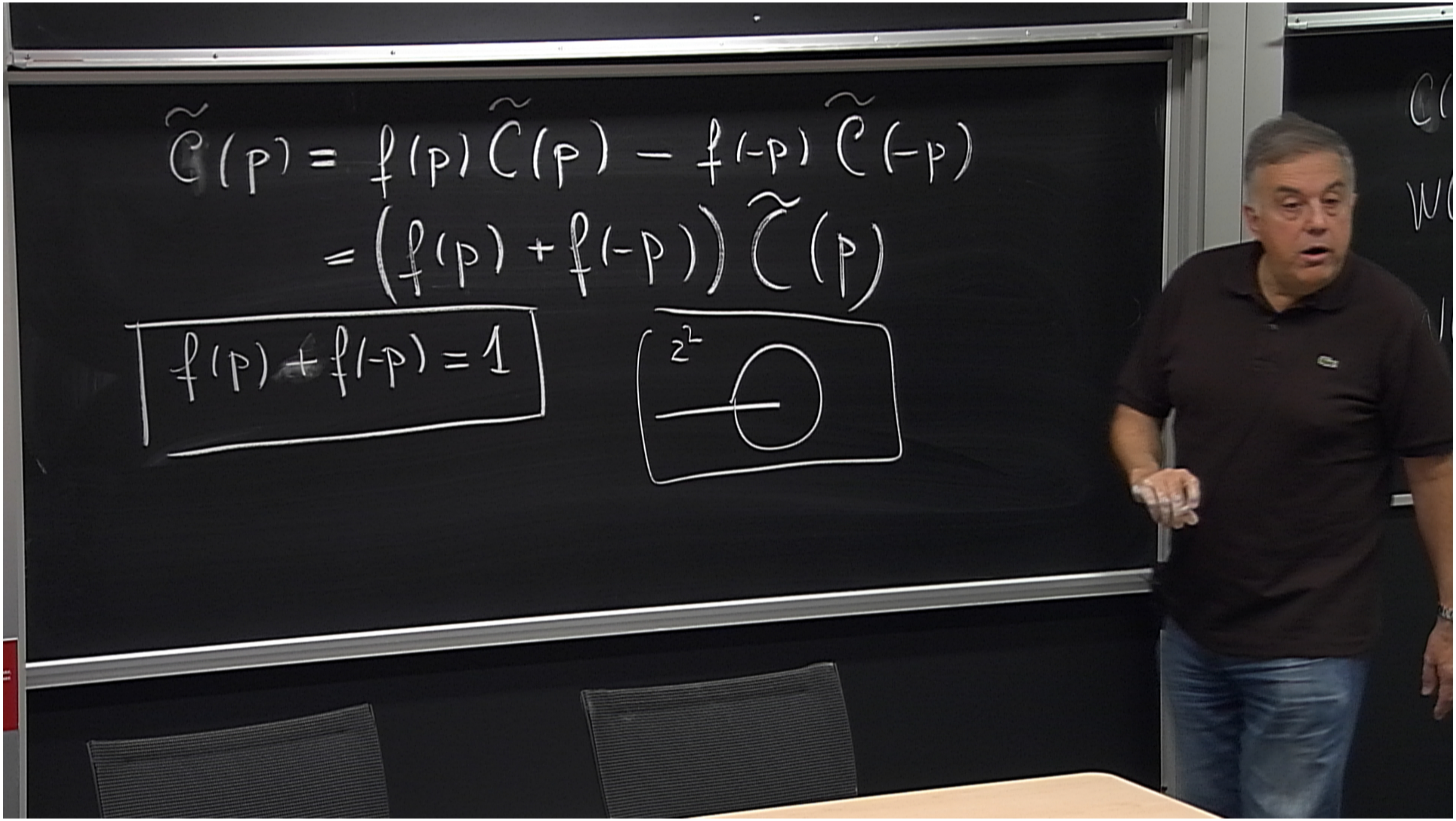


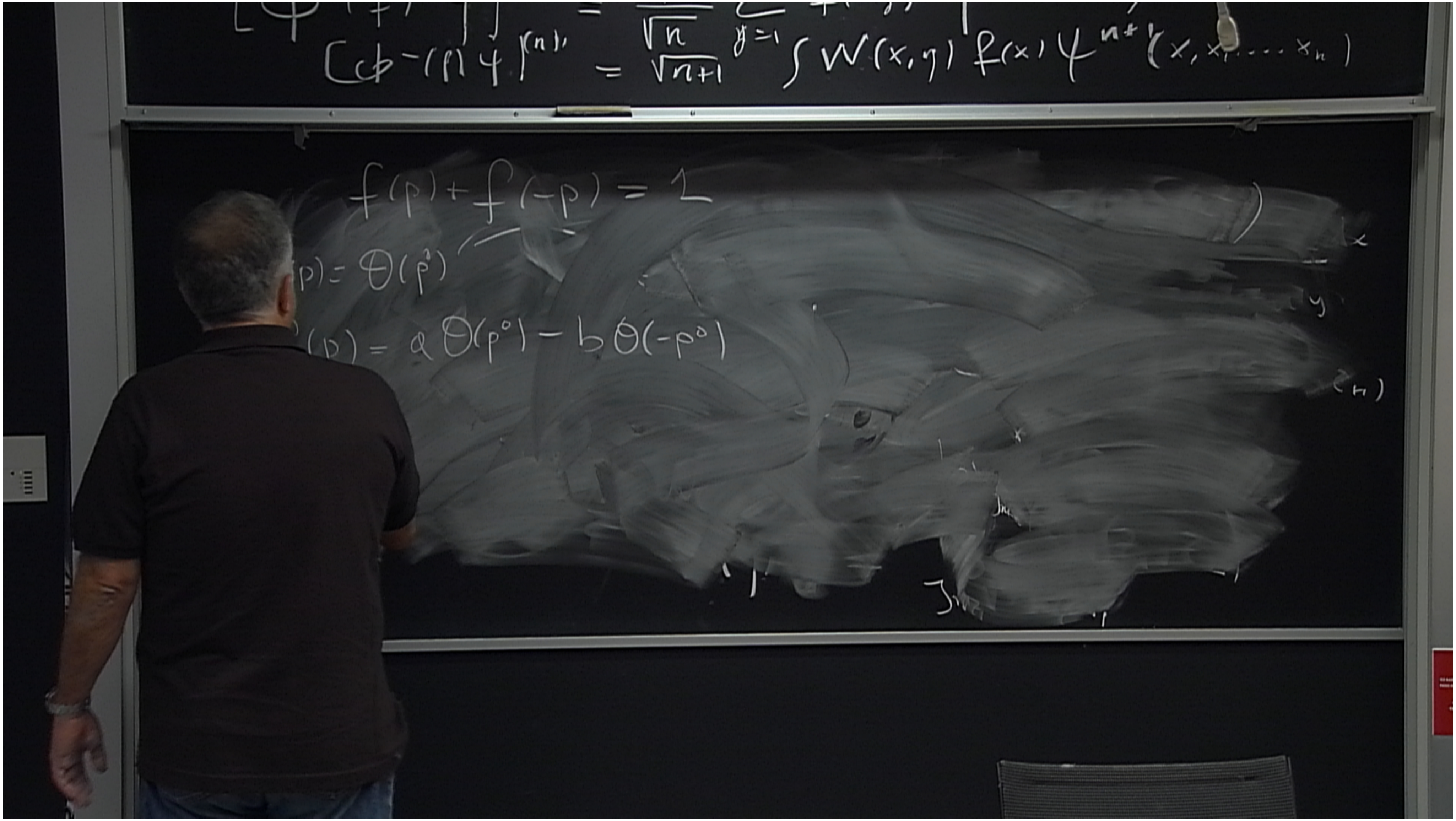
$\tilde{G}(p) = f(p)\tilde{C}(p) - f(-p)\tilde{C}(-p)$
$= (f(p) + f(-p))\tilde{C}(p)$
$f(p) + f(-p) = 1$

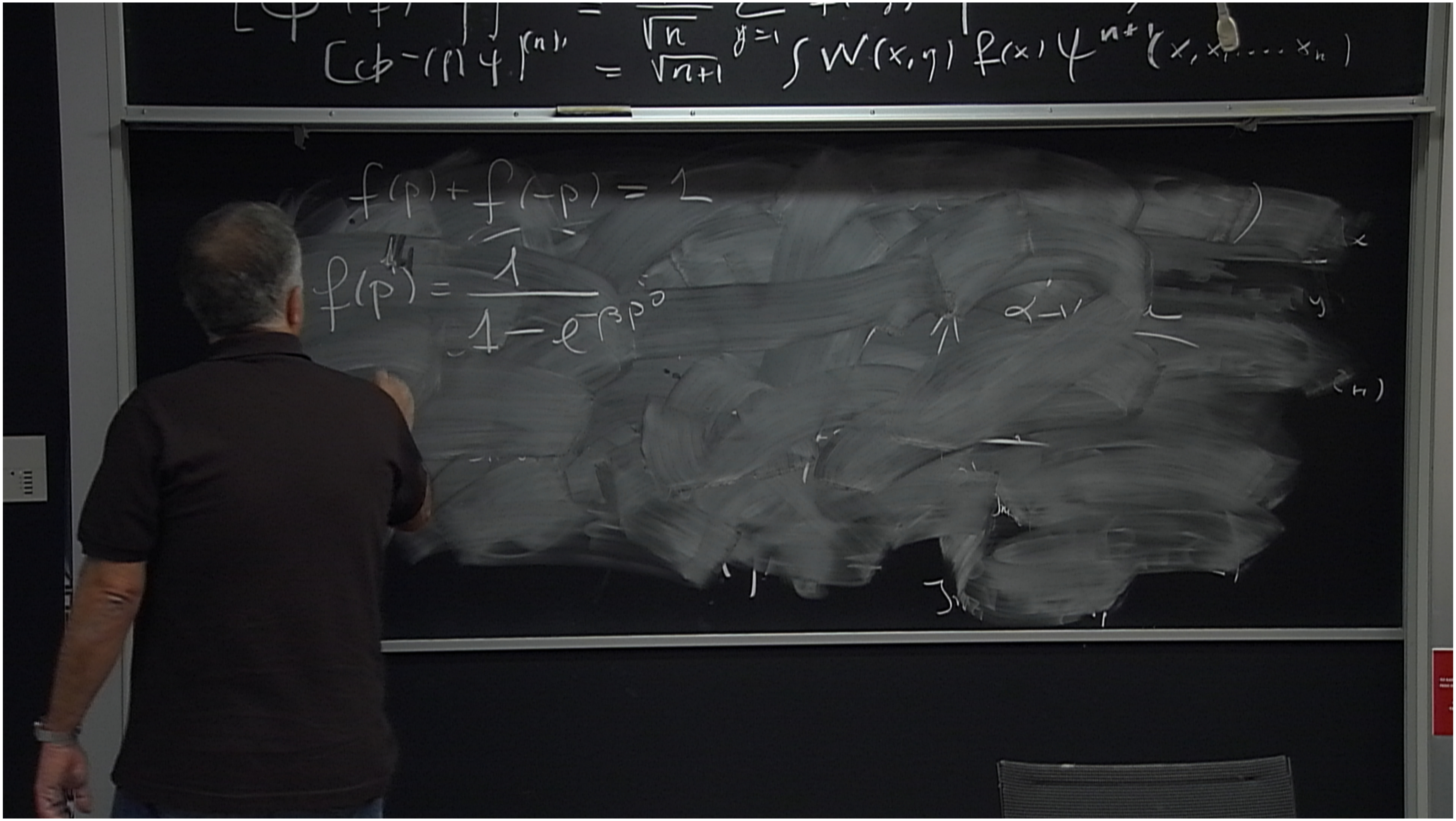


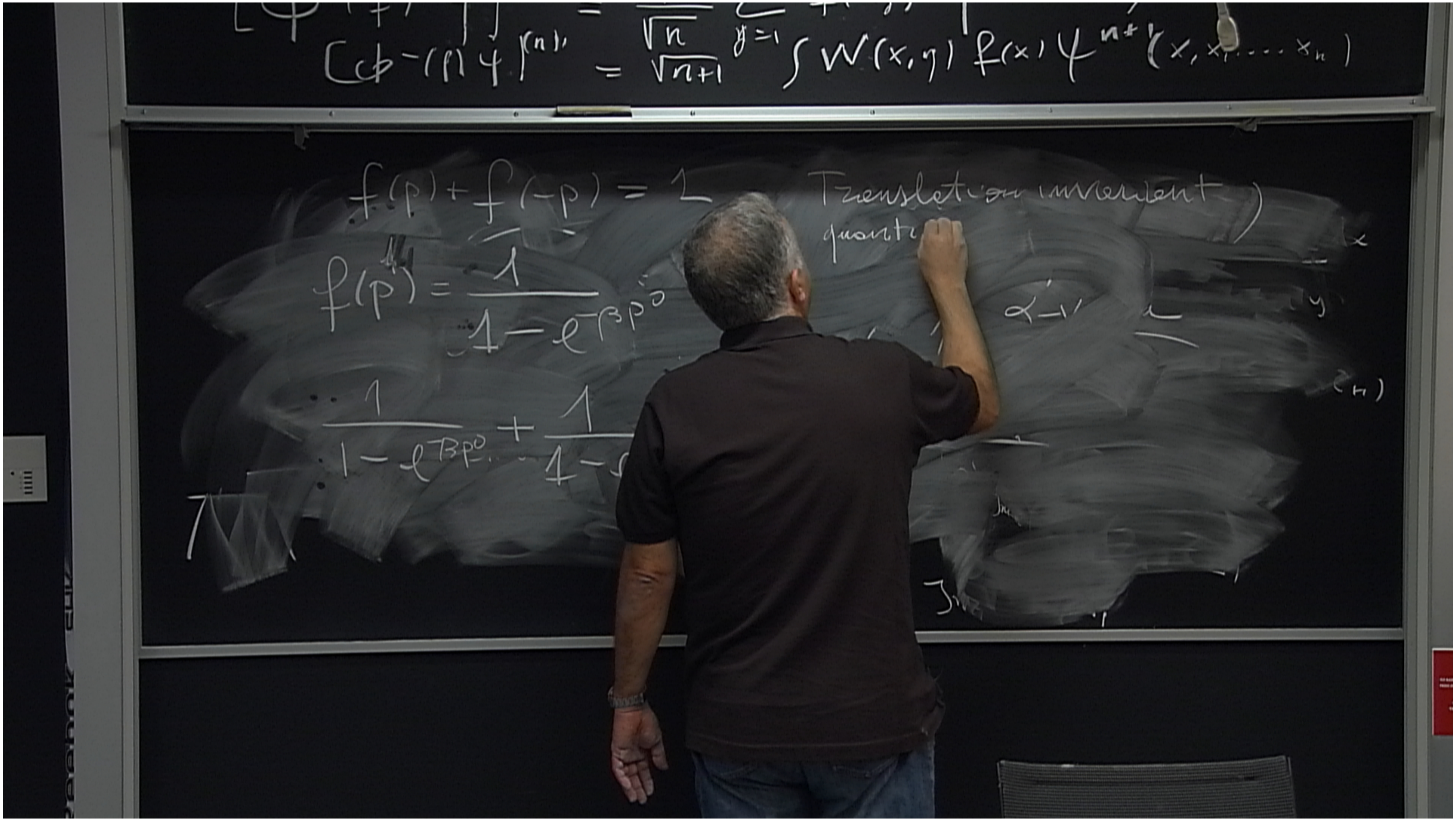

W(x-y) = 1/(2π)^3 ∫ e^{-ik^0(t-iε)} θ(k^0) δ(k^2)
= 1/4π^2 1/z^2 = W(z)
W(x,y) = bv W(z_1, z_2)
W(y,x) = bv W(z_1, z_2)

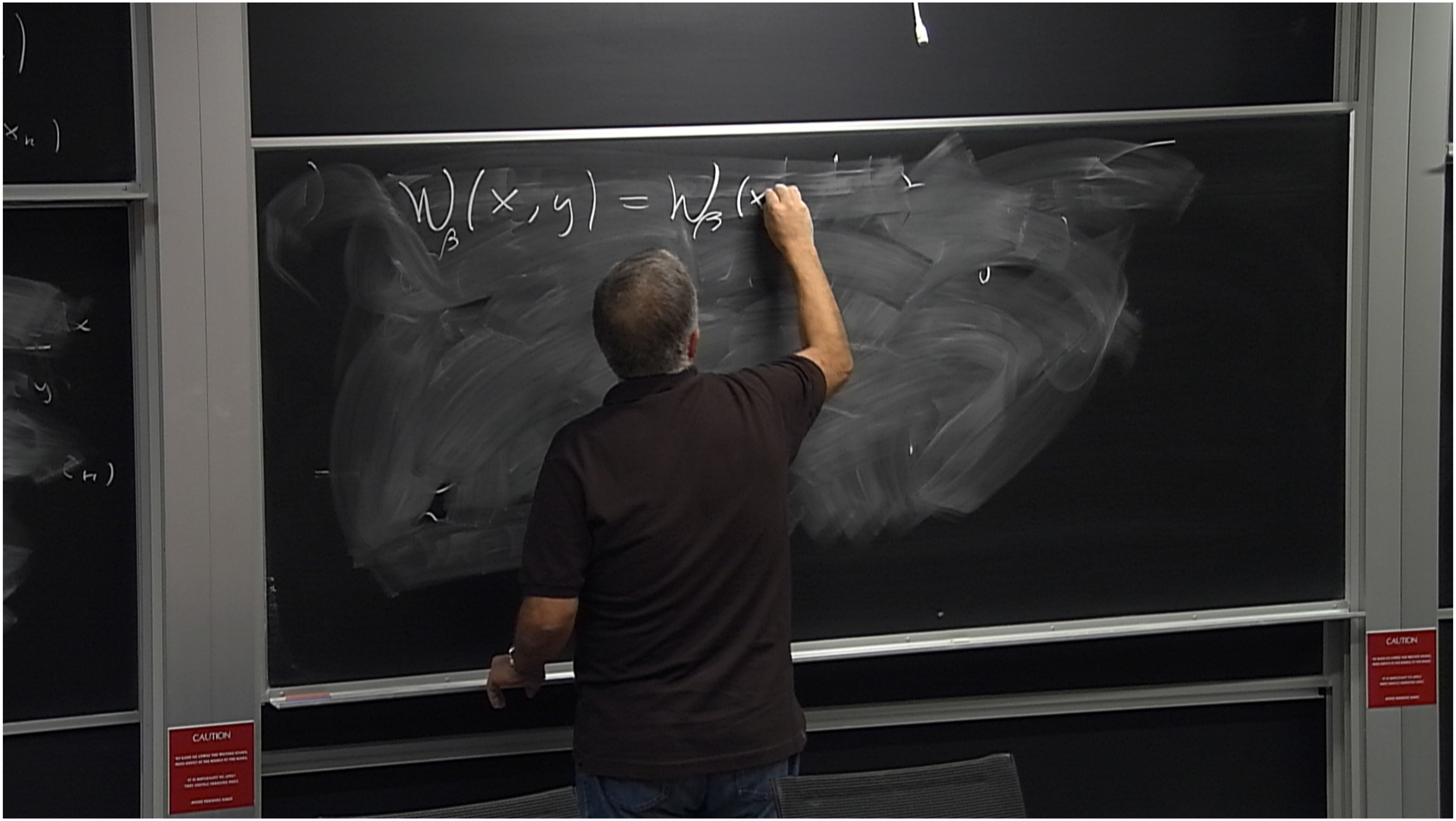

$\tilde{C}(p) = f(p)\tilde{C}(p) - f(-p)\tilde{C}(-p)$
$= (f(p) + f(-p))\tilde{C}(p)$
$f(p) + f(-p) = 1$
z^2

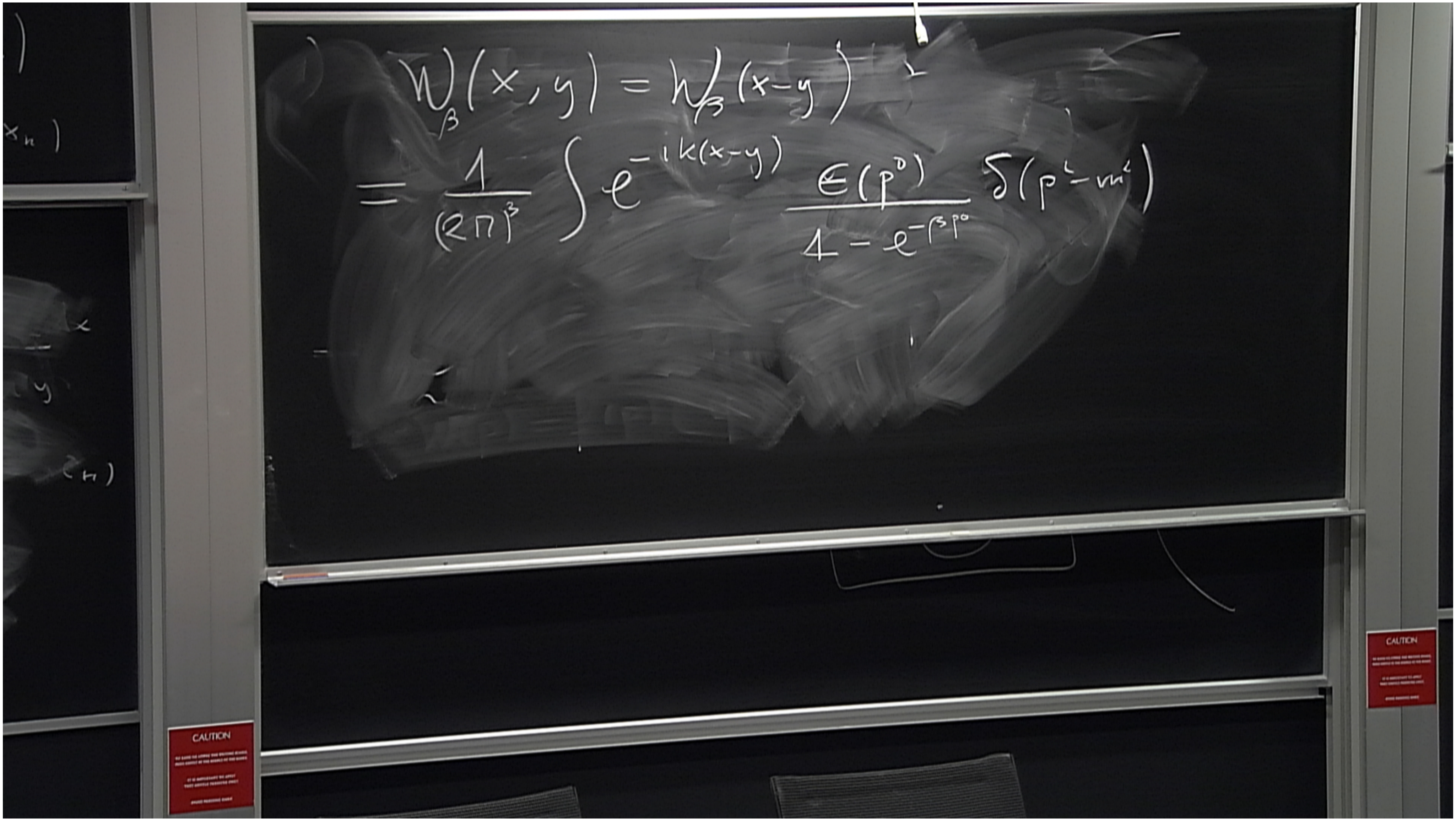

$\tilde{C}(p) = f(p)\,\tilde{C}(p) - f(-p)\,\tilde{C}(-p)$
$= \left(f(p) + f(-p)\right)\tilde{C}(p)$
$f(p) + f(-p) = 1$
z^2

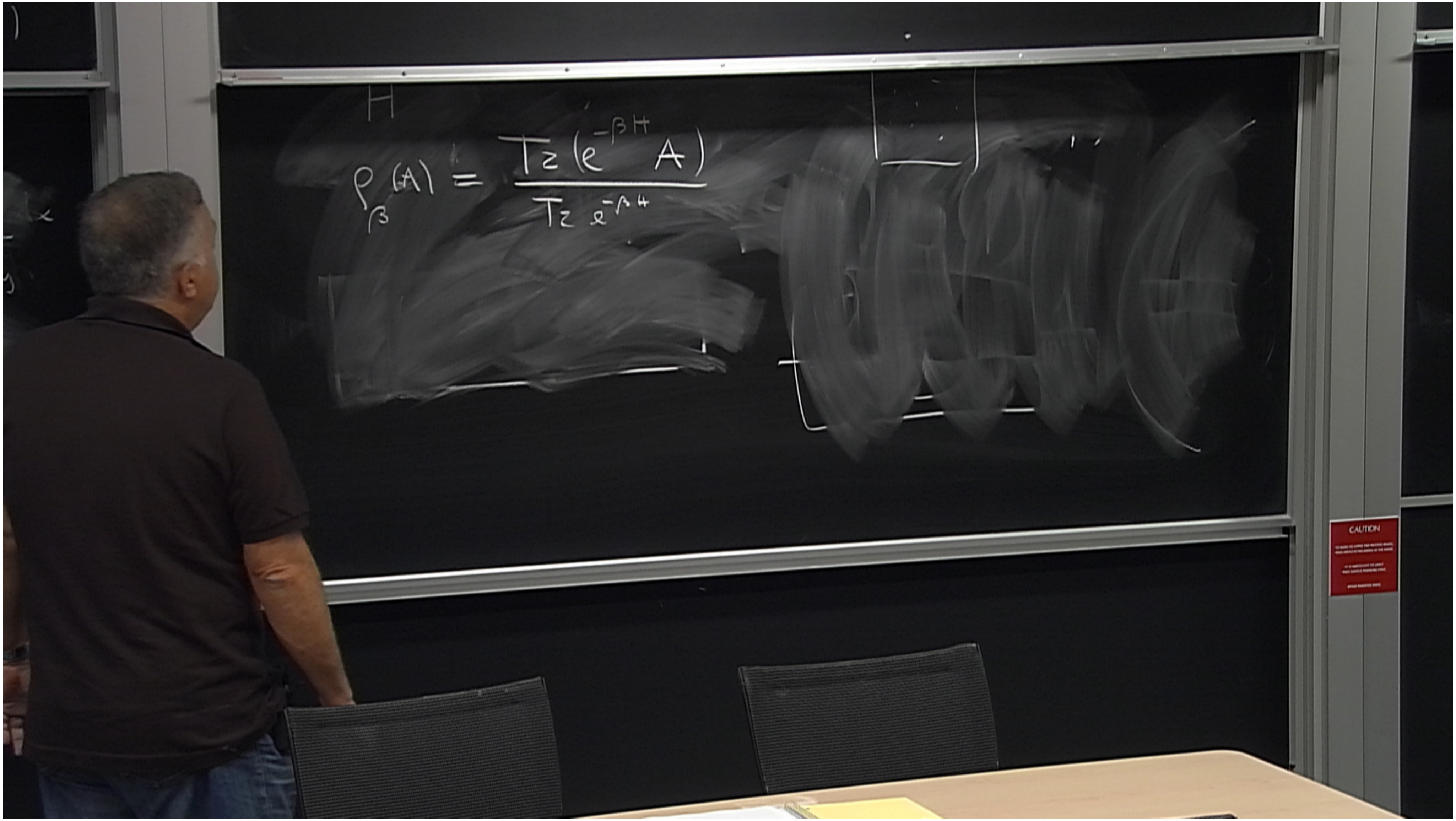

f(p) + f(-p) = 1
(p) = θ(p°)
(p) = a θ(p°) − b θ(−p°)

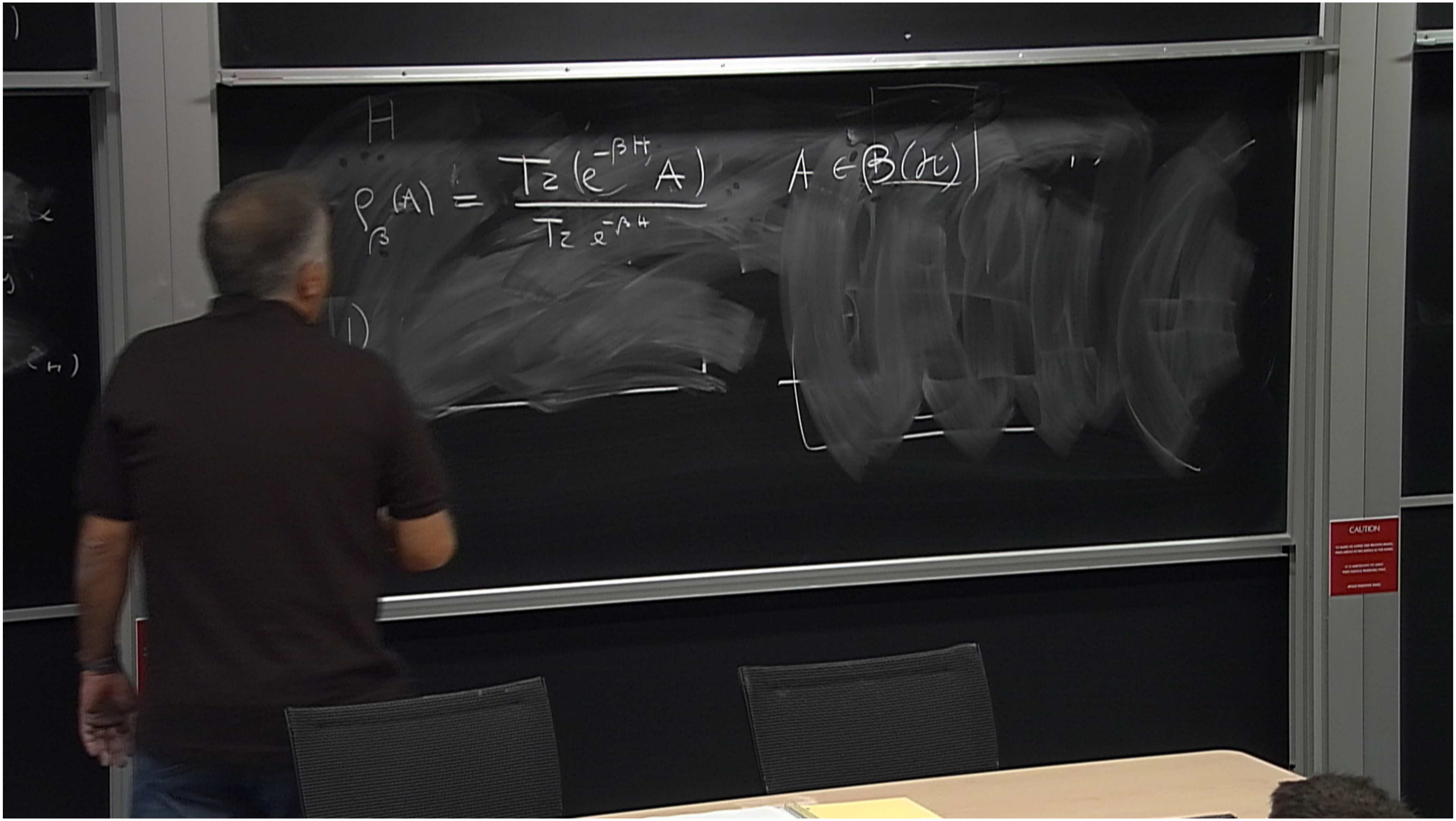

f(p) + f(-p) = 1
f(p) = 1 / (1 - e^{-βp^0})

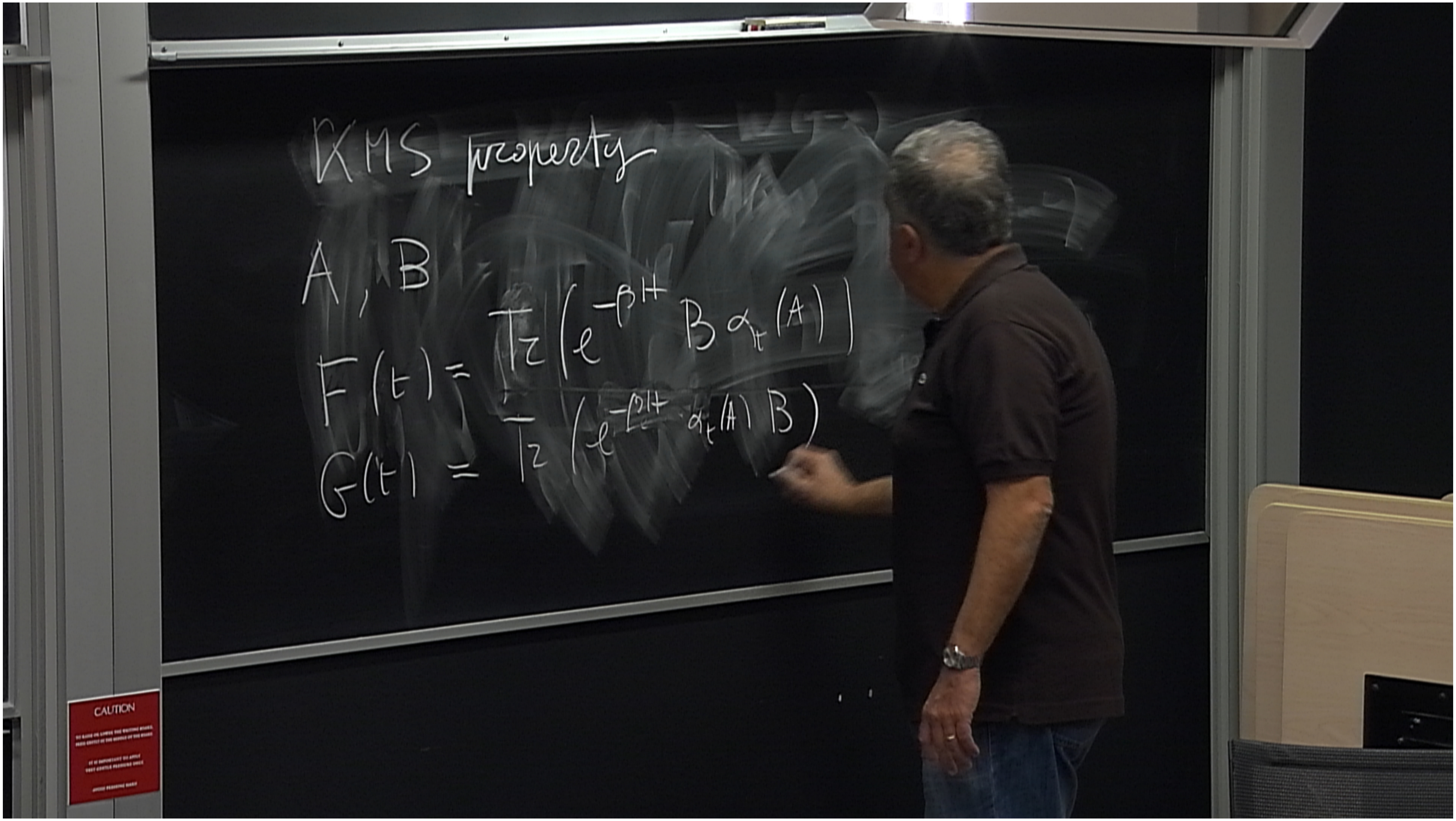

f(p) + f(-p) = 1
Translation invariant

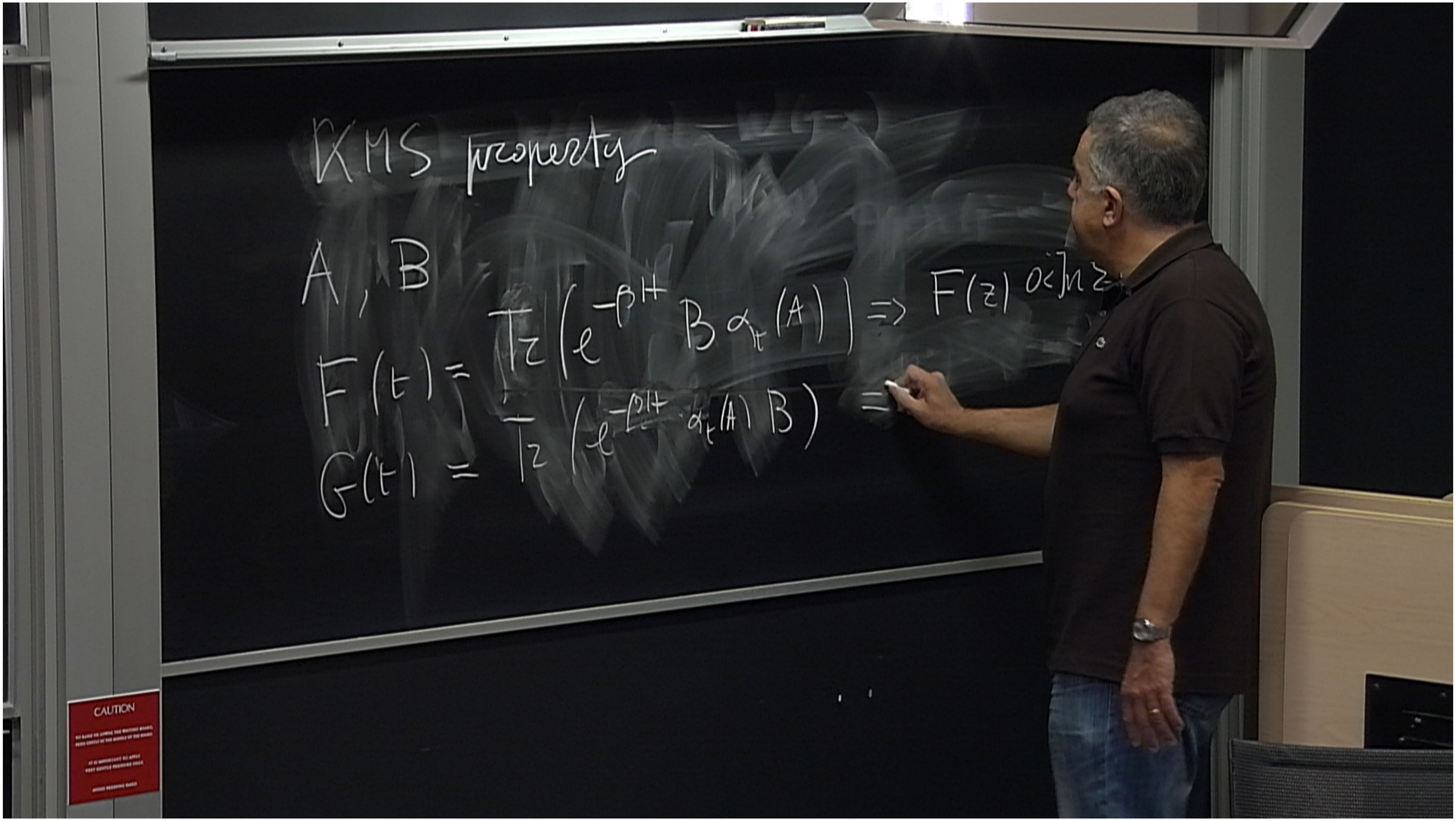

$W_\beta(x, y) = W_\beta(x$

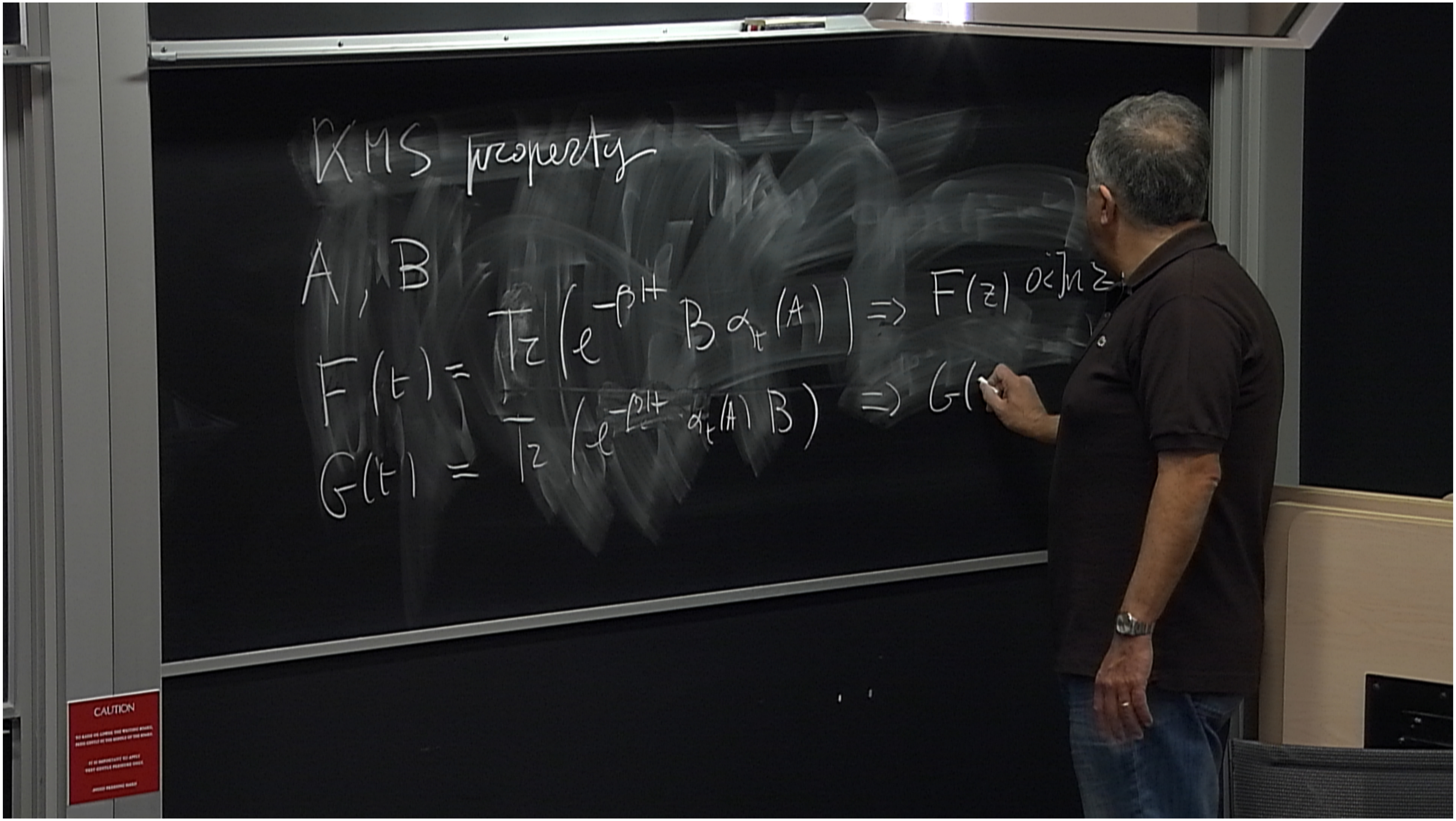

$W_\beta(x,y) = W_\beta(x-y)$
$= \frac{1}{(2\pi)^3} \int e^{-ik(x-y)} \frac{\epsilon(p^0)}{1-e^{-\beta p^0}} \delta(p^2-m^2)$
CAUTION
CAUTION

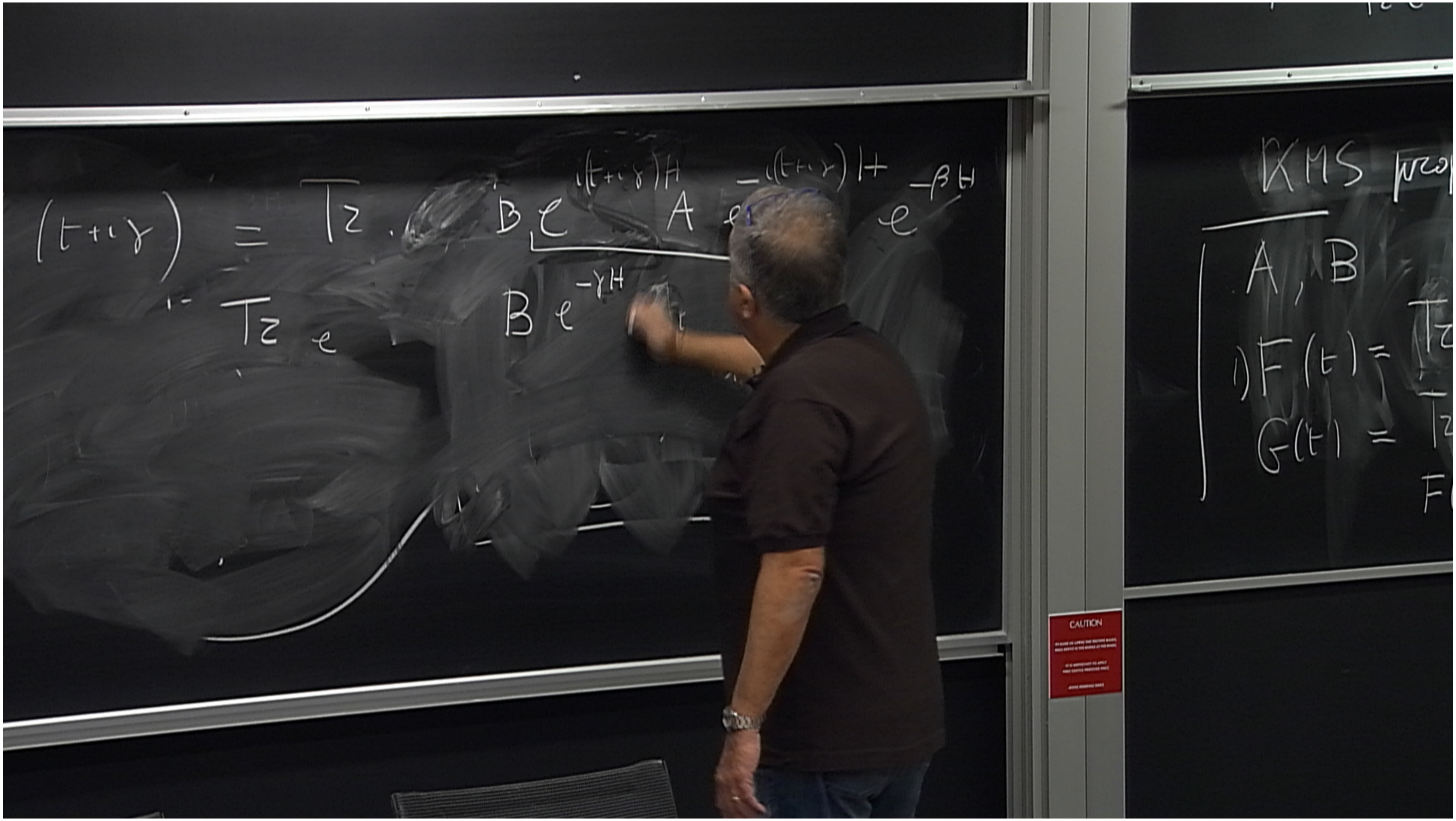

H
$\rho_\beta(A) = \frac{\mathrm{Tr}(e^{-\beta H} A)}{\mathrm{Tr}\, e^{-\beta H}}$
CAUTION

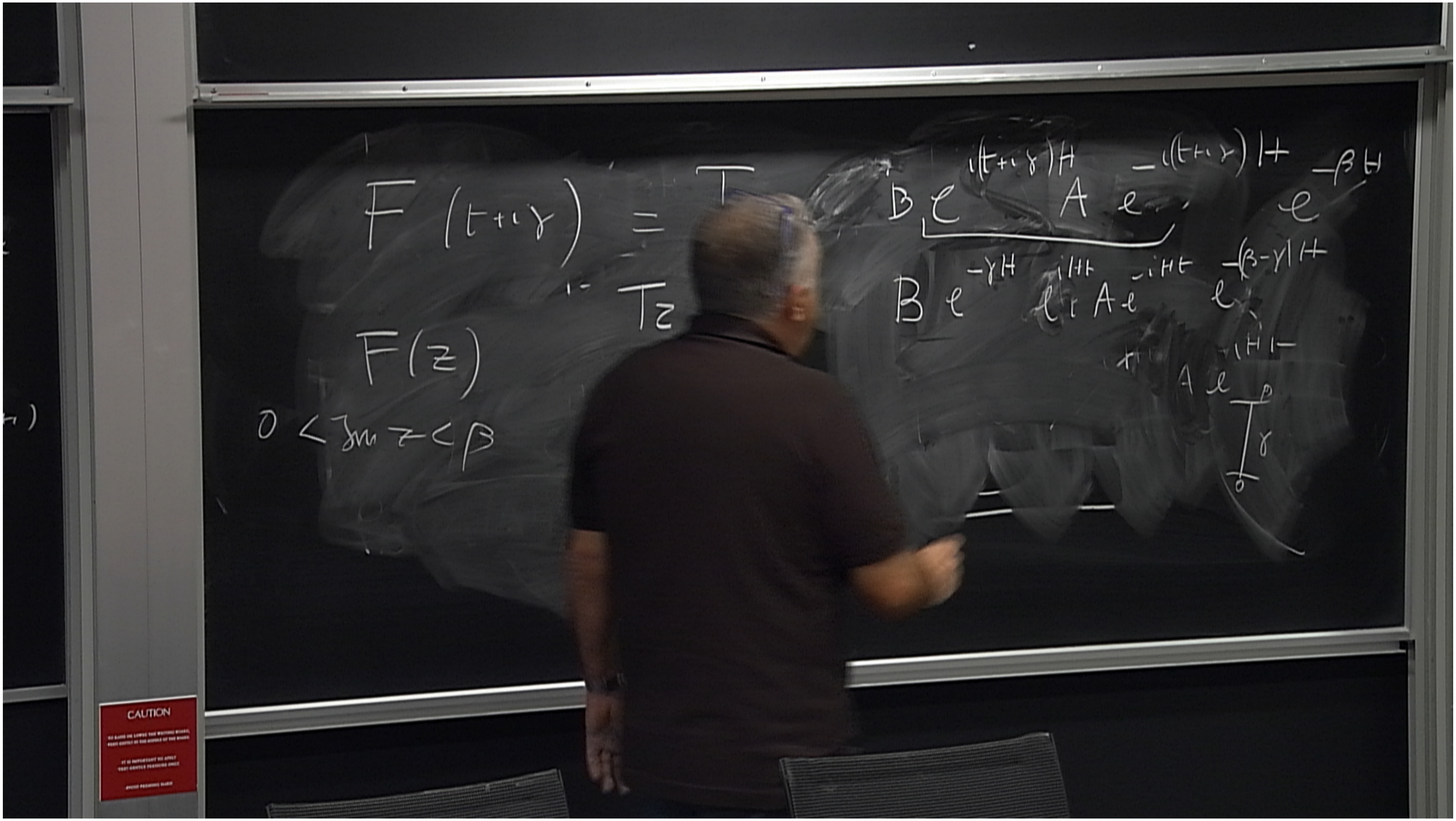

H
ρβ(A) = Tr(e^{-βH} A) / Tr e^{-βH}
A ∈ B(H)
CAUTION

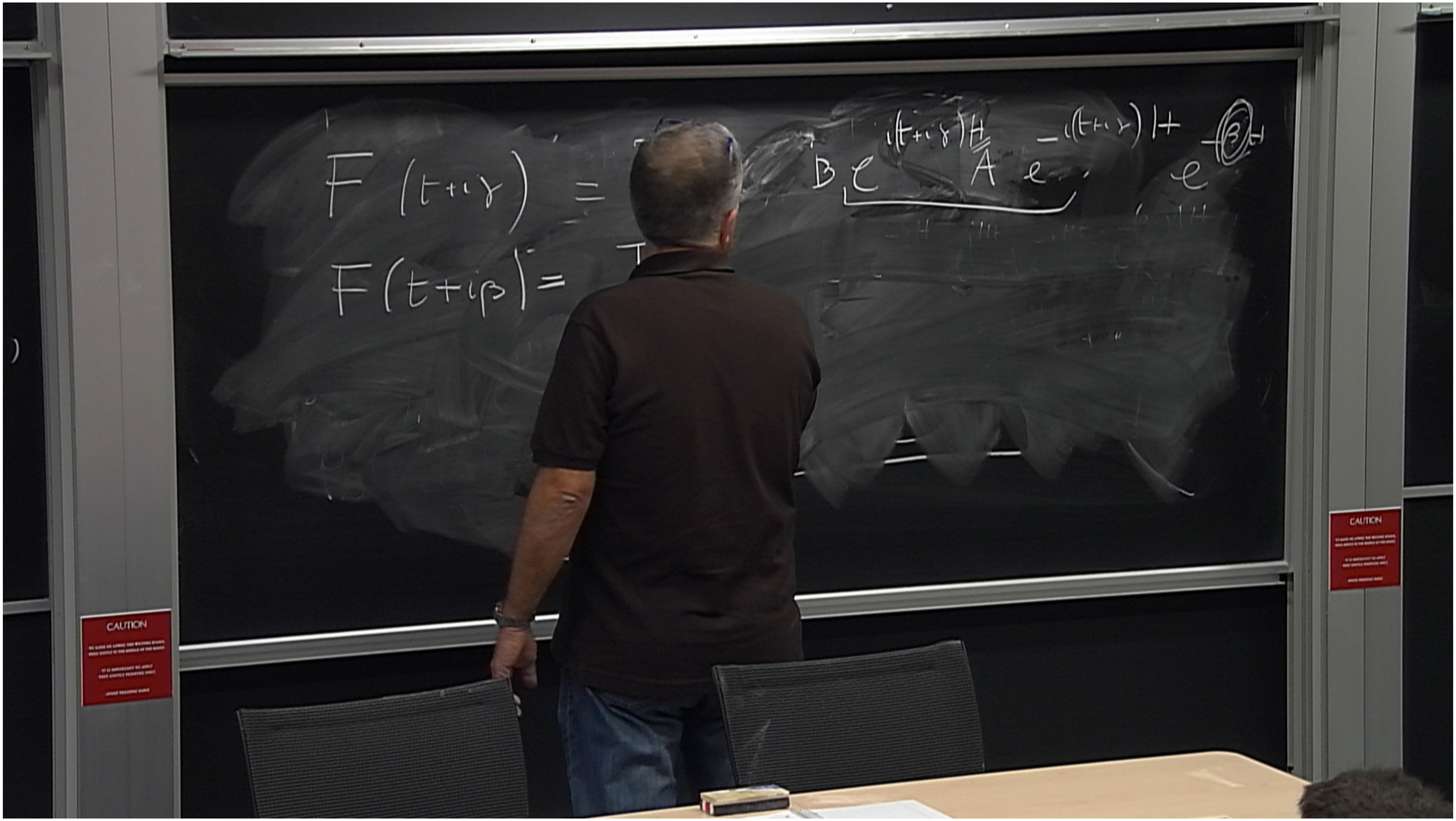

KMS property
A, B
$F(t) = \mathrm{Tr}(e^{-\beta H} B \alpha_t(A))$
$G(t) = \mathrm{Tr}(e^{-\beta H} \alpha_t(A) B)$
CAUTION

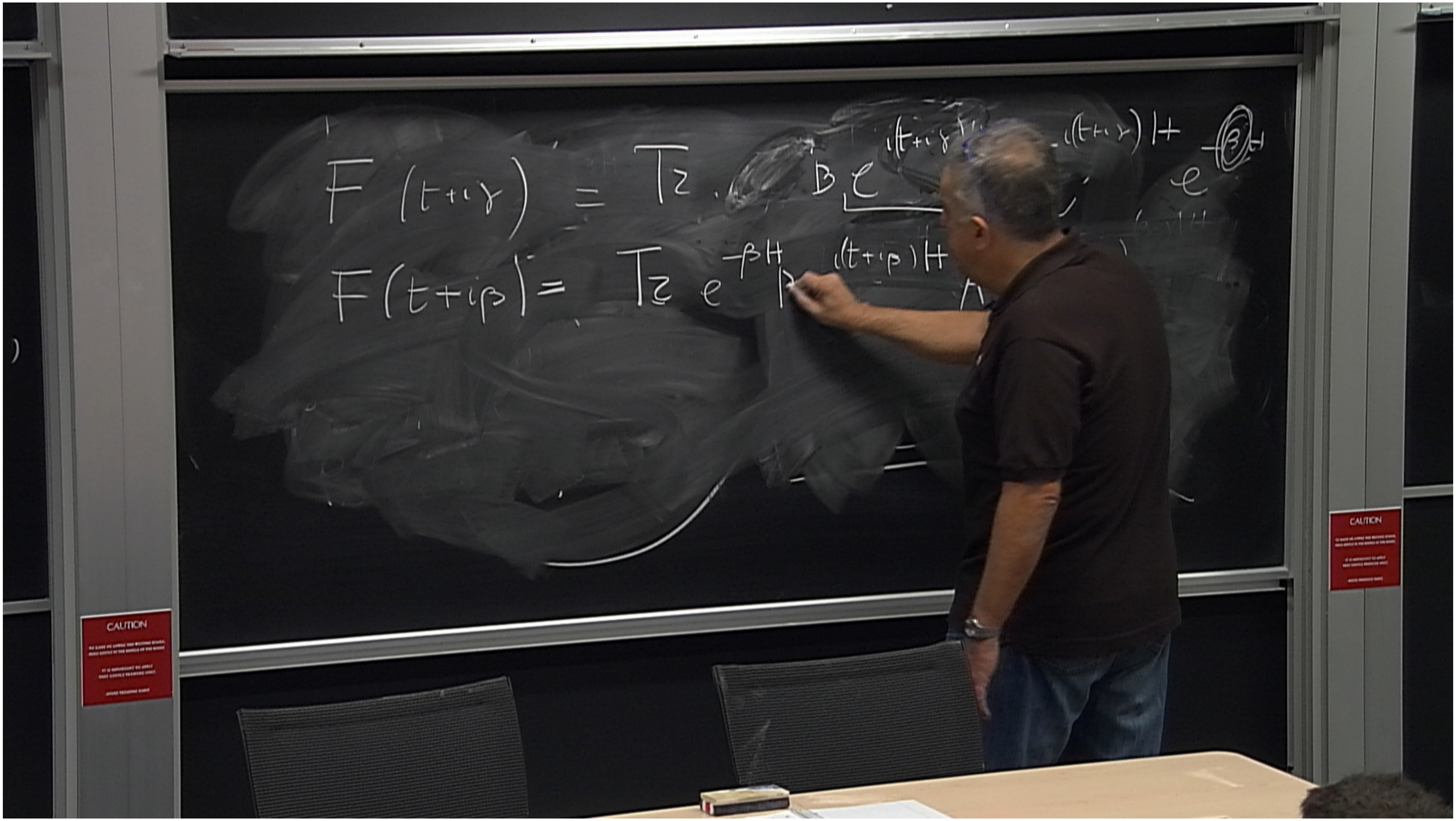

KMS property
A, B
F(t) = Tr(e^{-βH} B α_t(A)) ⇒ F(z)
G(t) = Tr(e^{-βH} α_t(A) B) =
CAUTION

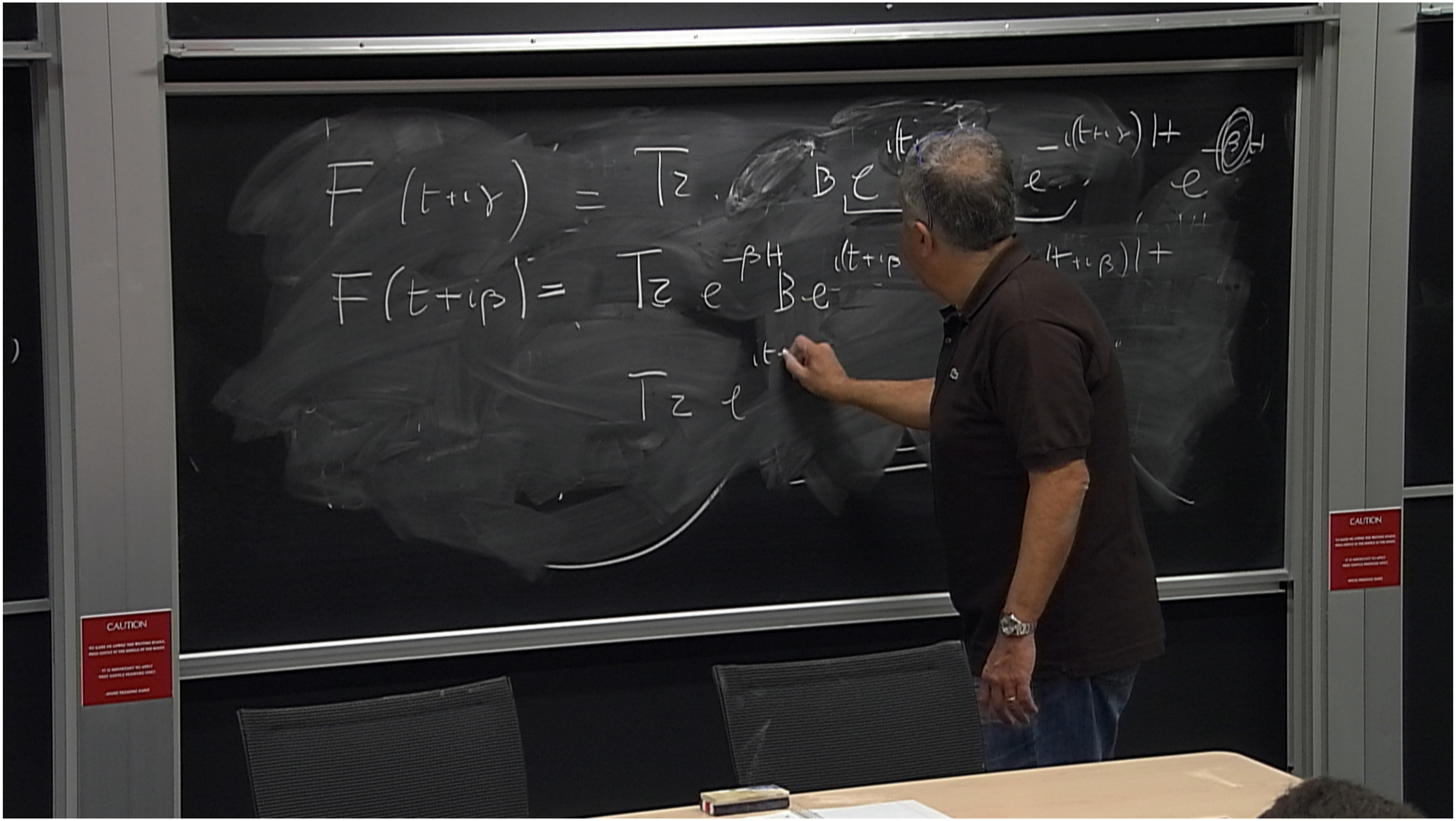

KMS property
A, B
F(t) = Tr(e^{-βH} B α_t(A)) ⇒ F(z)
G(t) = Tr(e^{-βH} α_t(A) B) ⇒ G(
CAUTION

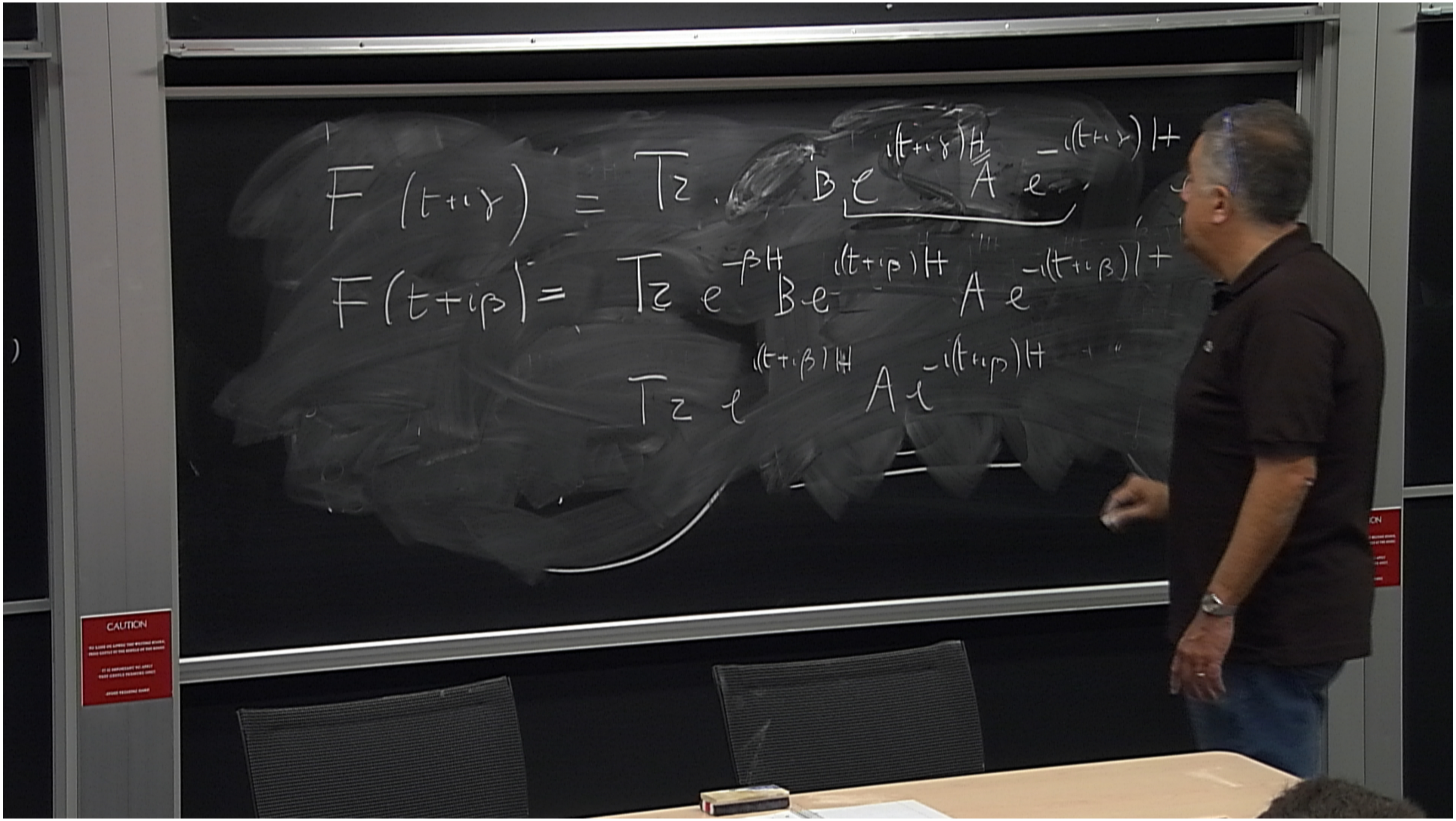

KMS
A, B
F(t) =
G(t) =


F(t+iγ) =
F(z)
0 < Im z < β
CAUTION

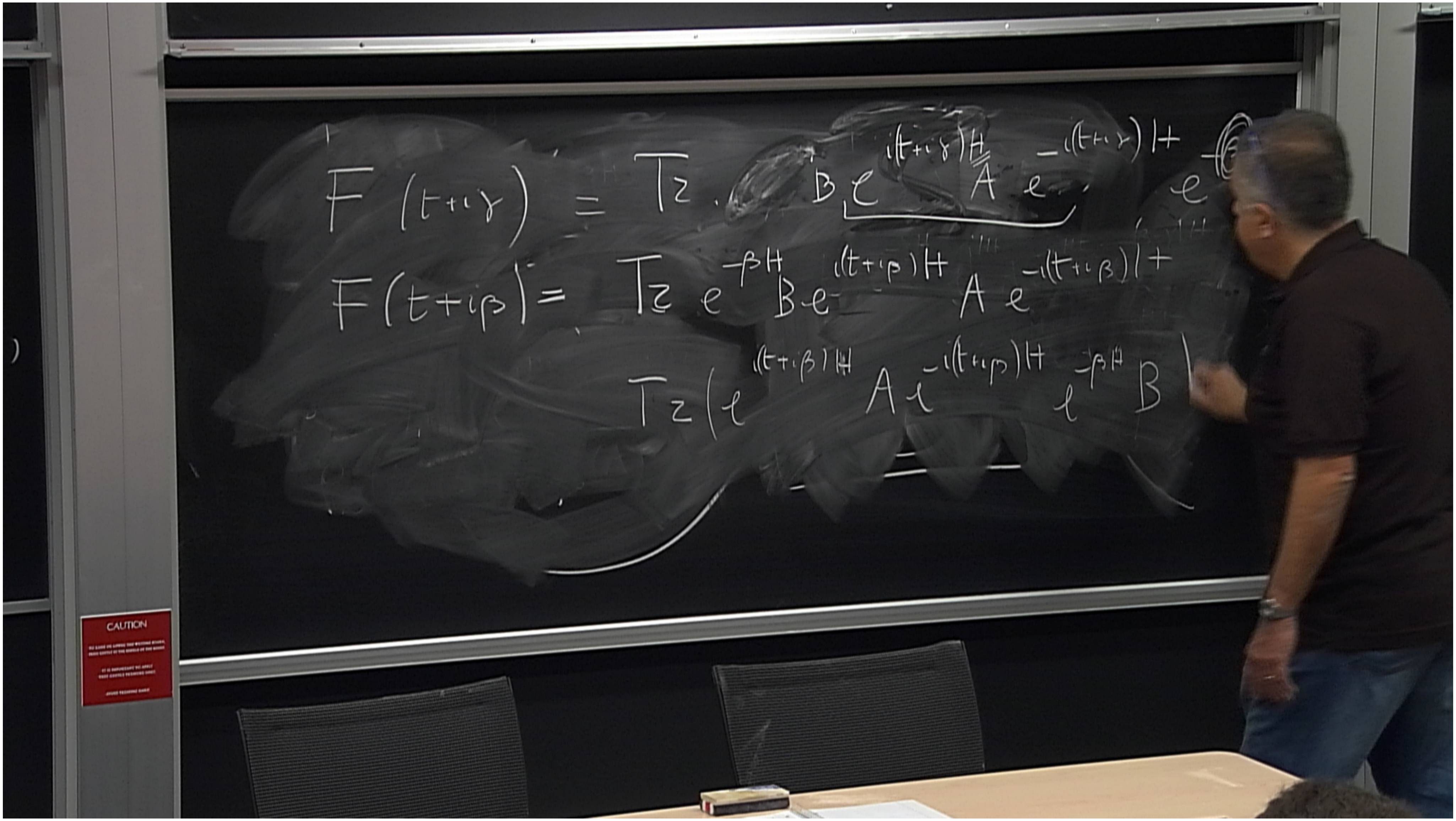


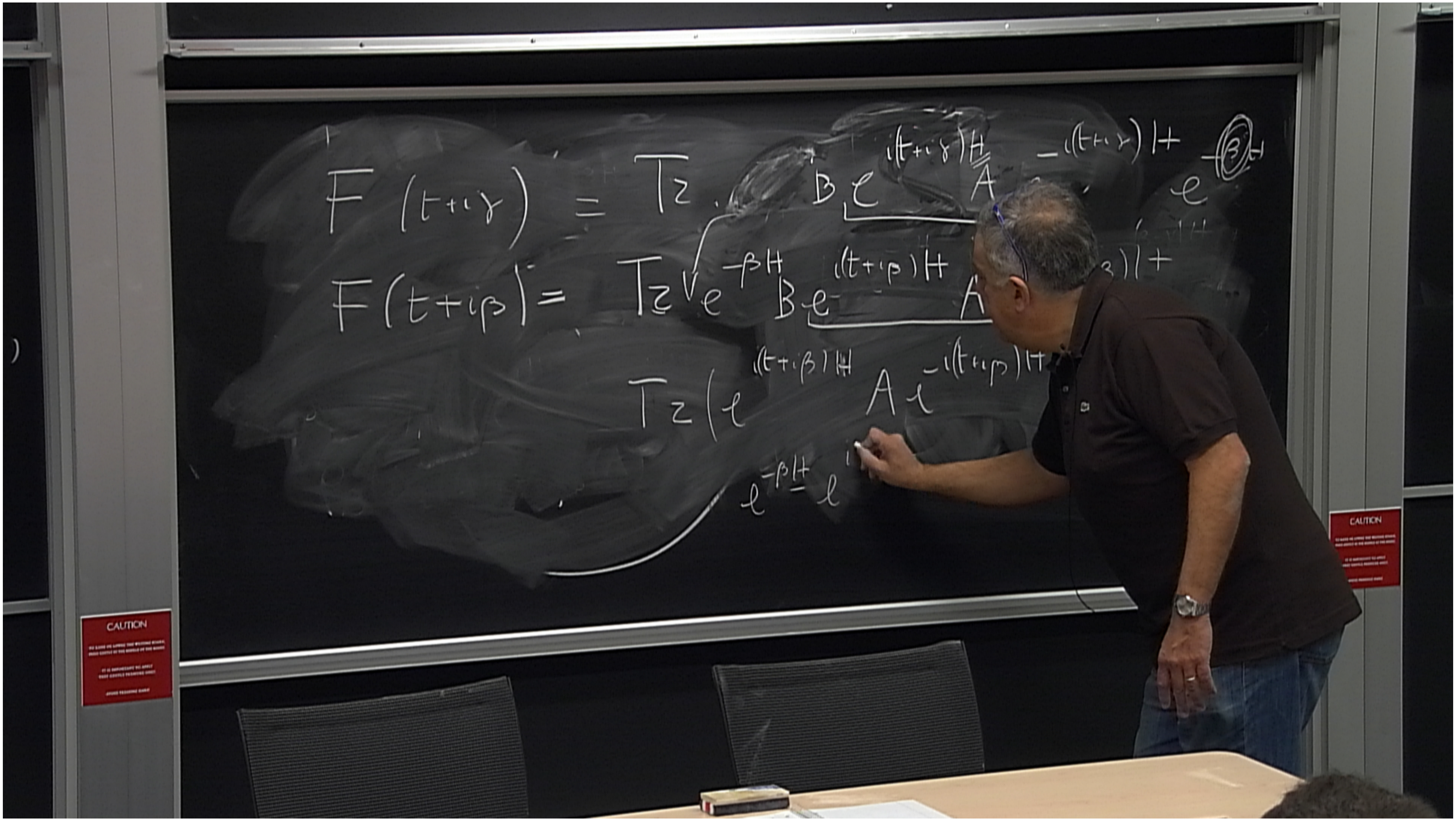

F (t+iγ) = Tr
F(t+iβ) = Tr e −βH
CAUTION
CAUTION

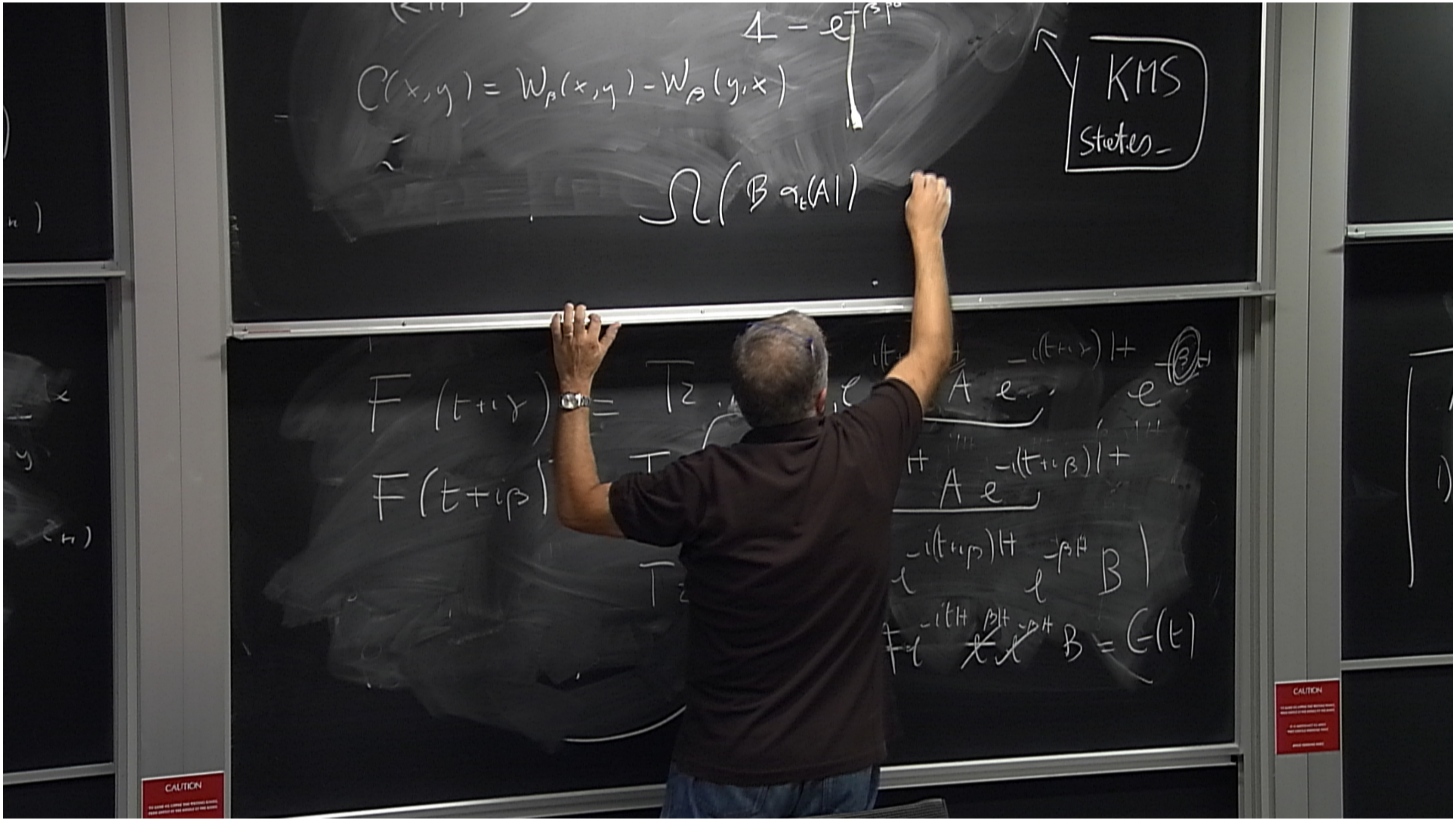


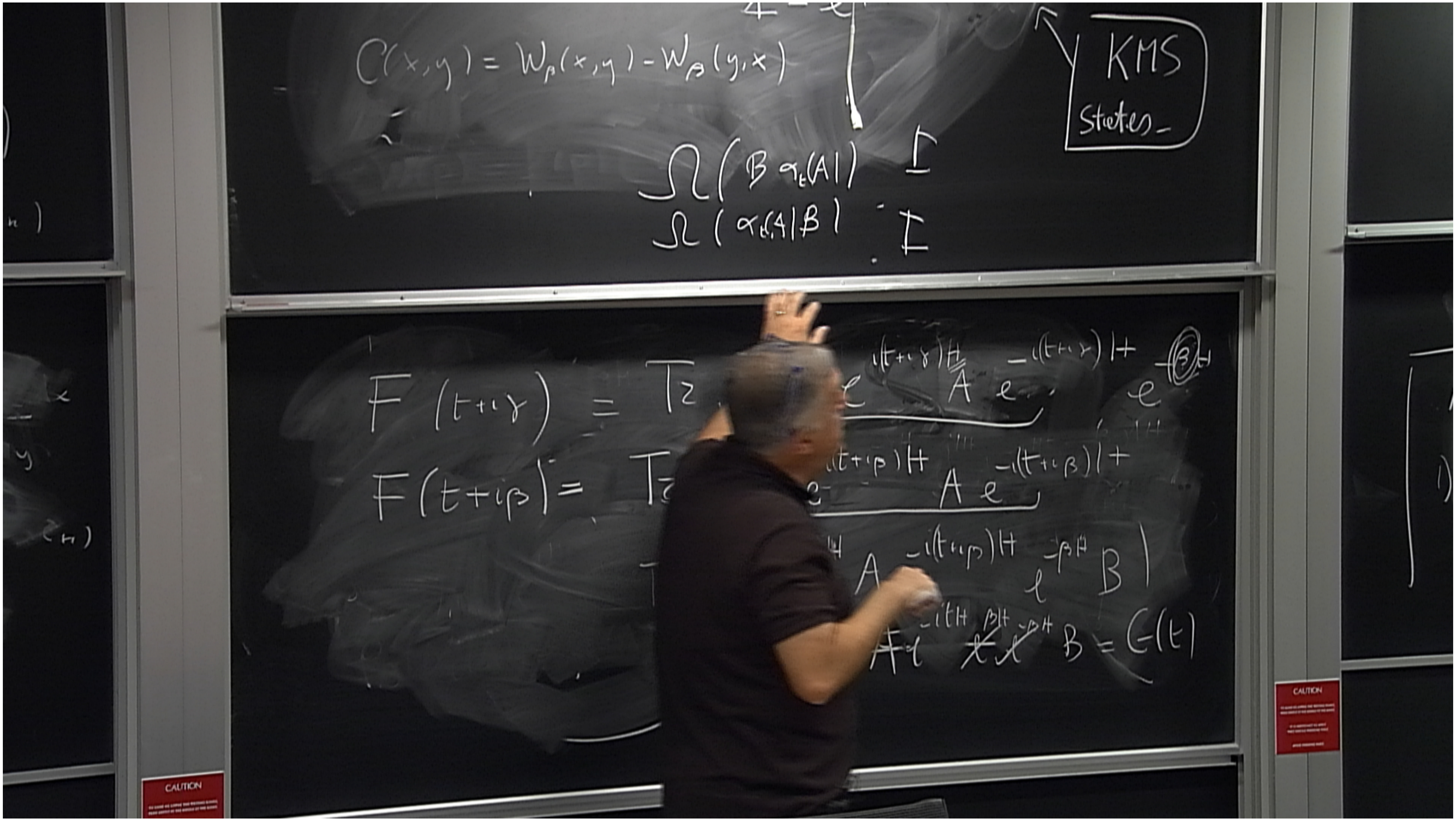


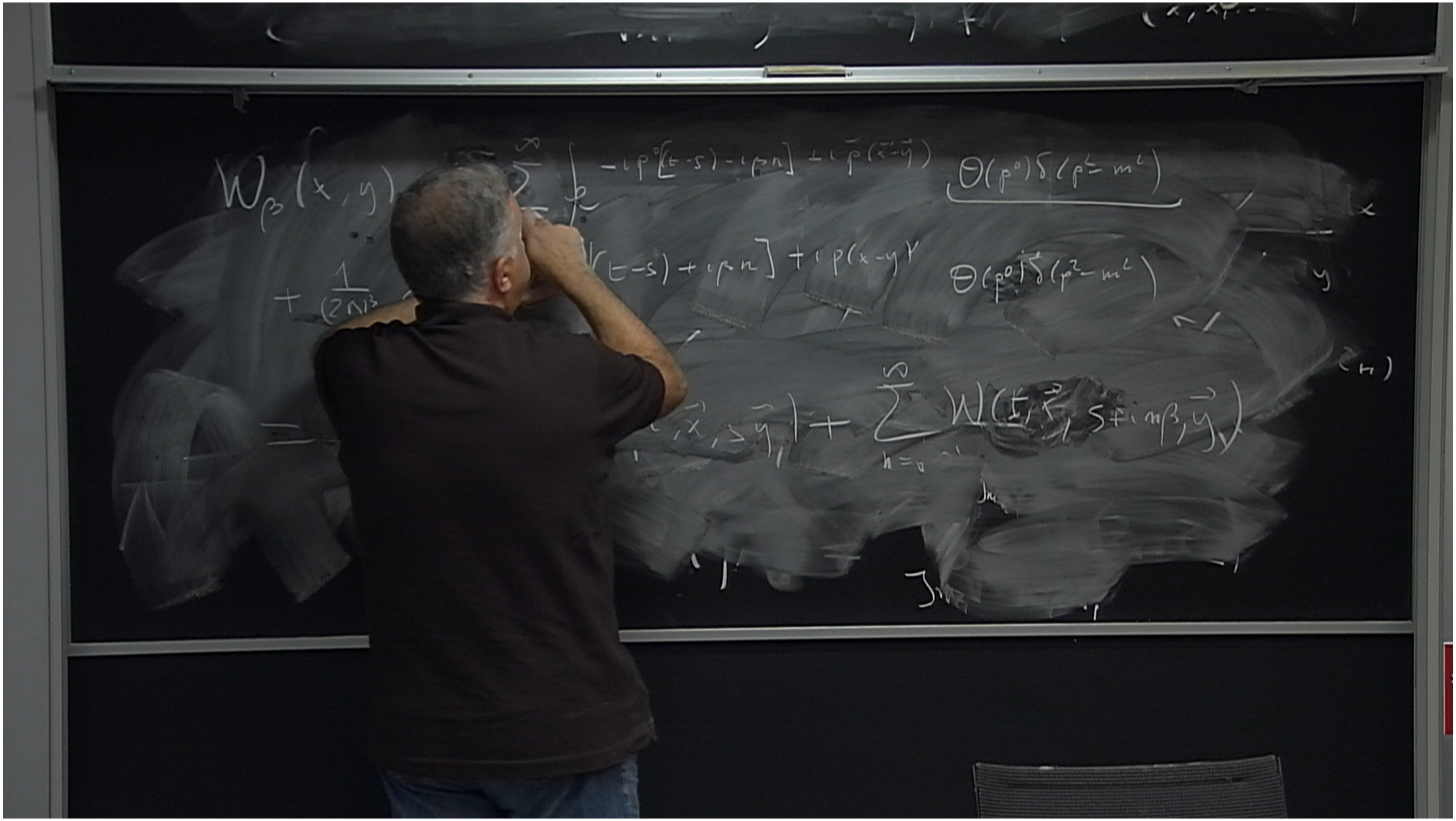
F(t+iγ) = Tr B e^{i(t+iγ)H} A e^{-i(t+iγ)H}
F(t+iβ) = Tr e^{-βH} B e^{i(t+iβ)H} A e^{-i(t+iβ)H}
Tr e^{i(t+iβ)H} A e^{-i(t+iβ)H}
CAUTION

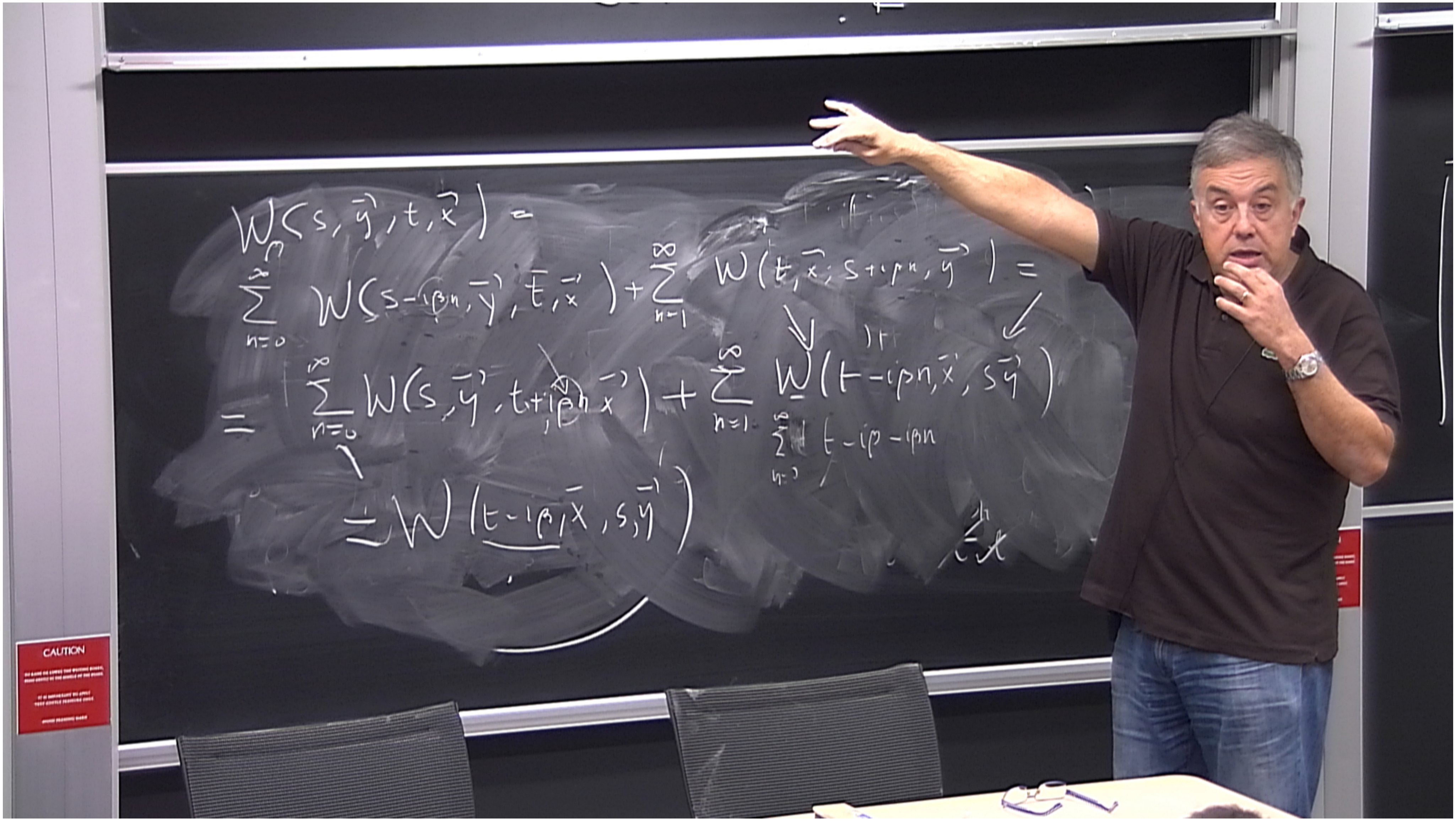

F(t+iγ) = Tr B e^{i(t+iγ)H} A e^{-i(t+iγ)H}
F(t+iβ) = Tr e^{-βH} B e^{i(t+iβ)H} A e^{-i(t+iβ)H}
Tr(e^{i(t+iβ)H} A e^{-i(t+iβ)H} e^{-βH} B)
CAUTION

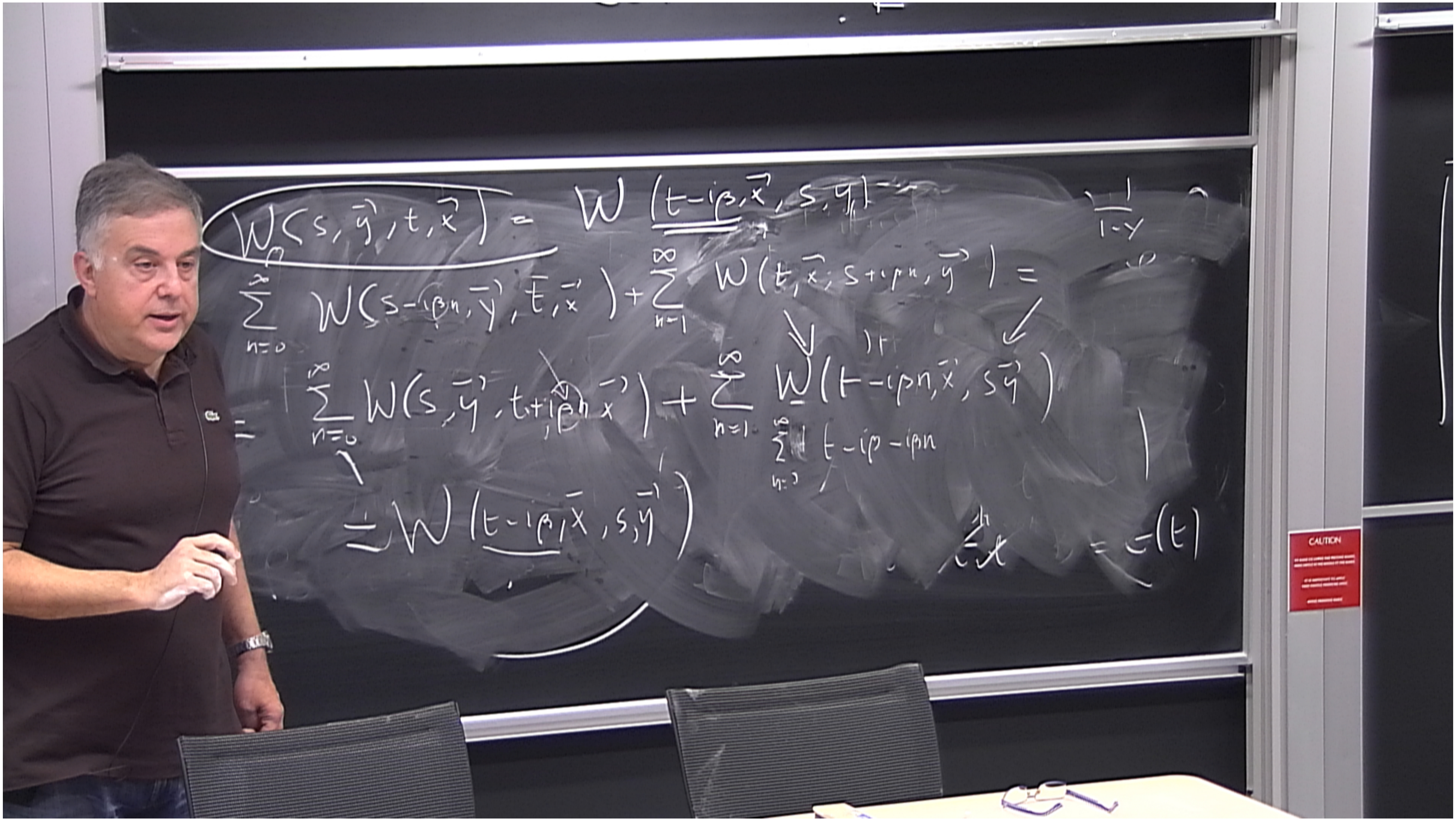

CAUTION
CAUTION

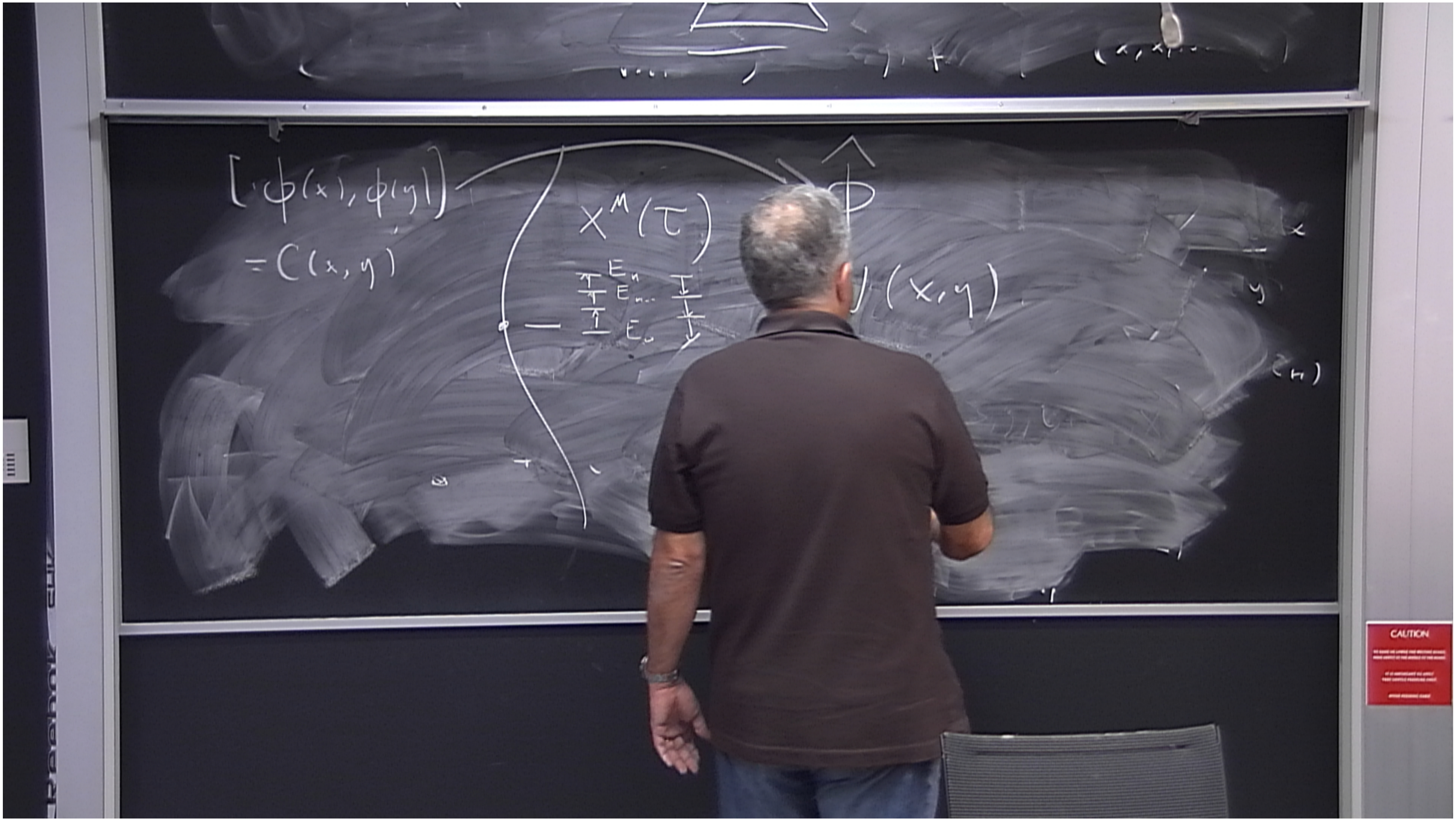

C(x,y) = Wβ(x,y) - Wβ(y,x)
KMS
States
F(t+iγ)
F(t+iβ)
B = G(t)

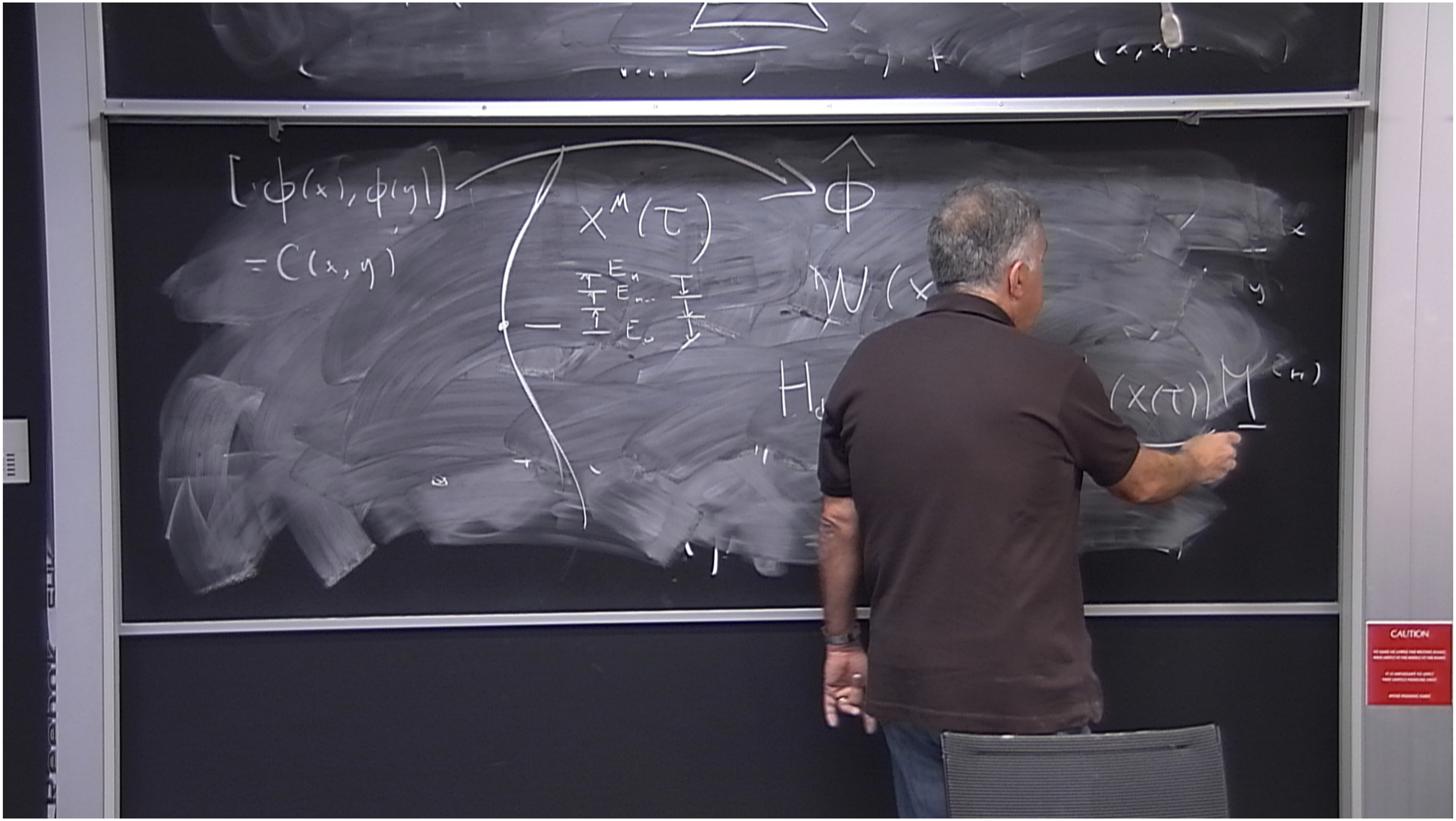

C(x,y) = W_β(x,y) - W_β(y,x)
KMS
States
Ω(B α_t(A))
Ω(α_t(A) B)
F(t+iγ) =
F(t+iβ) =
= G(t)
CAUTION
CAUTION

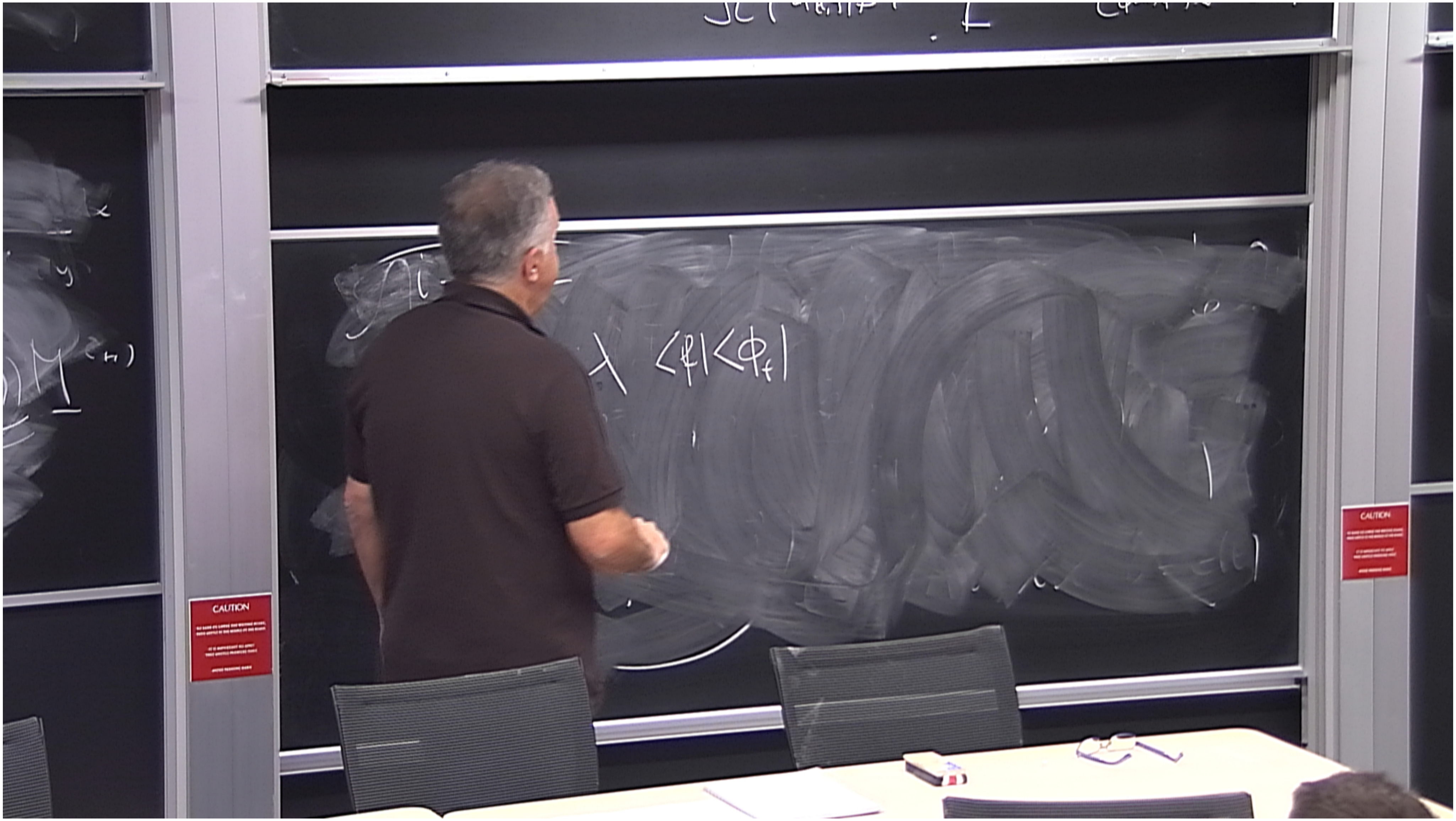


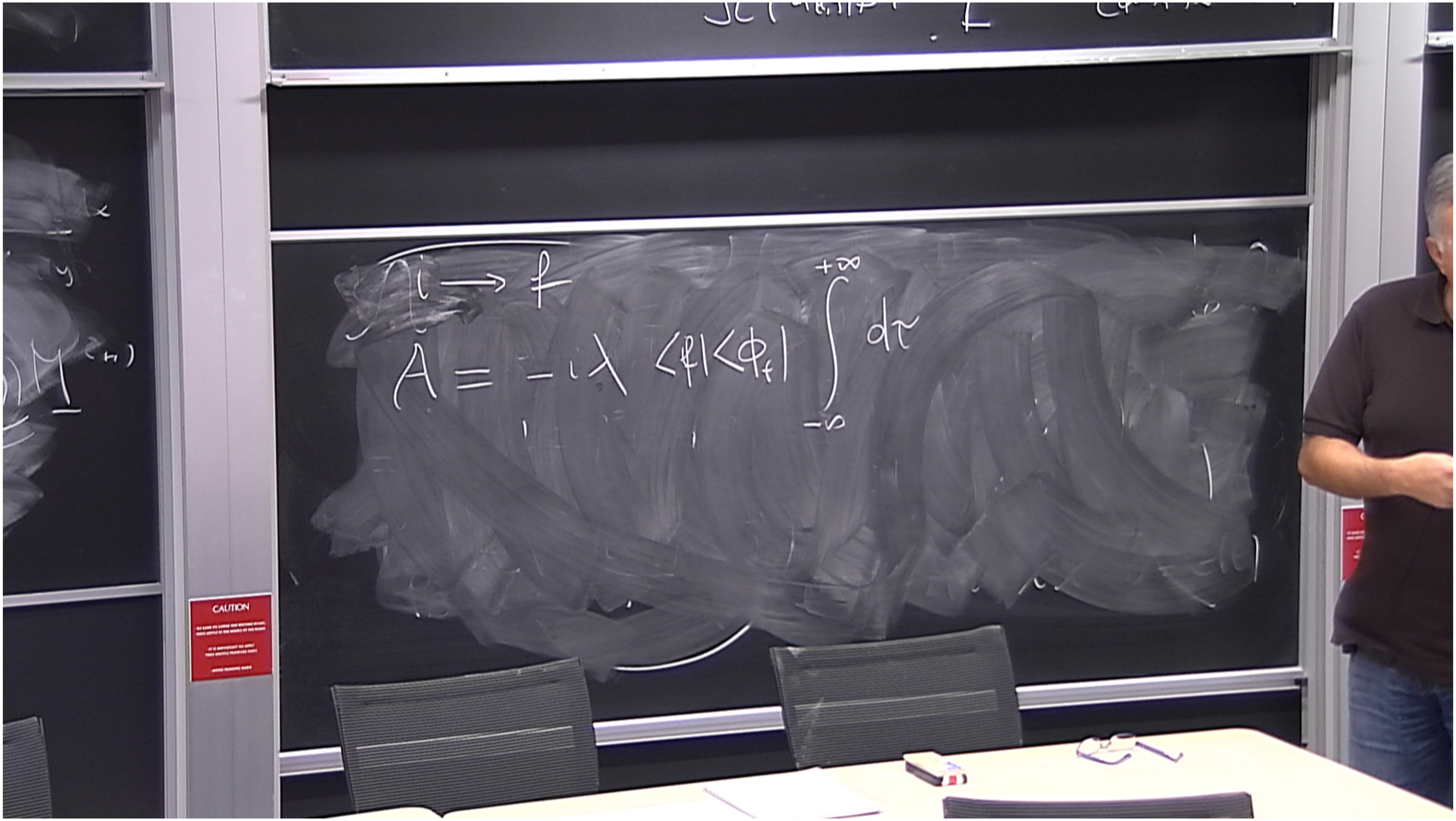

CAUTION

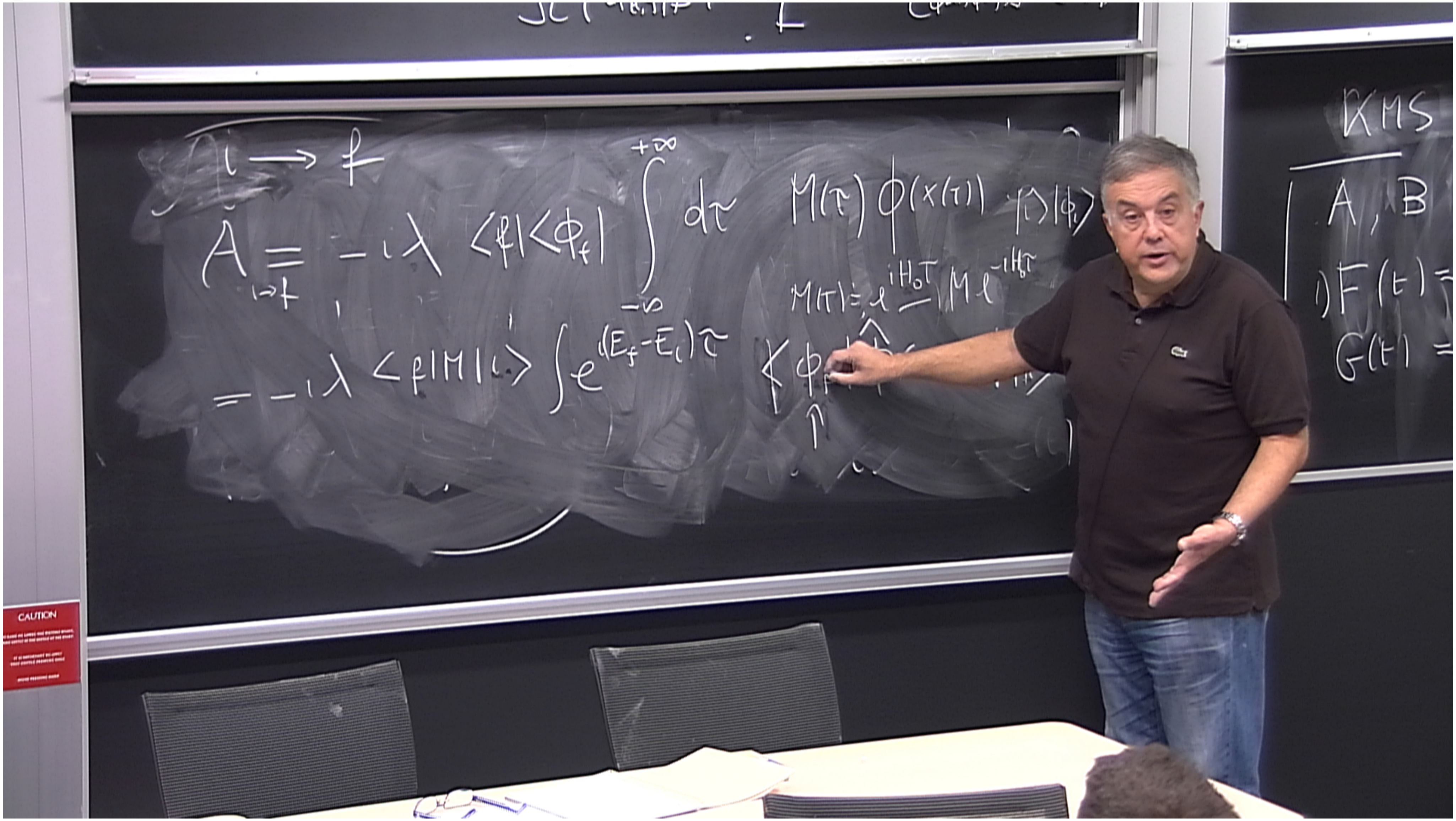

CAUTION

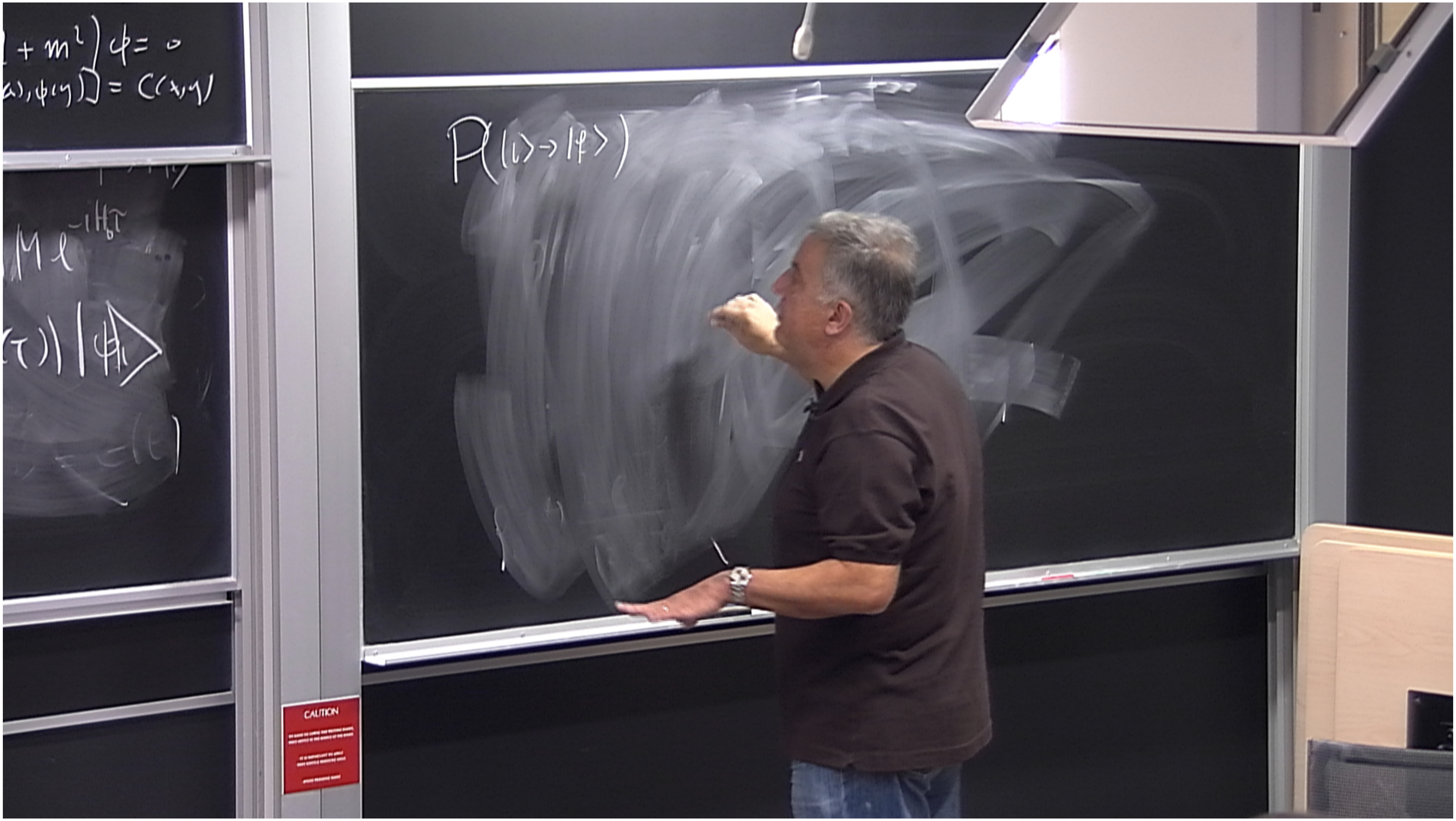

[φ(x), φ(y)]
= C(x, y)
X^μ(τ)
E_n
E_0
(x, y)
CAUTION

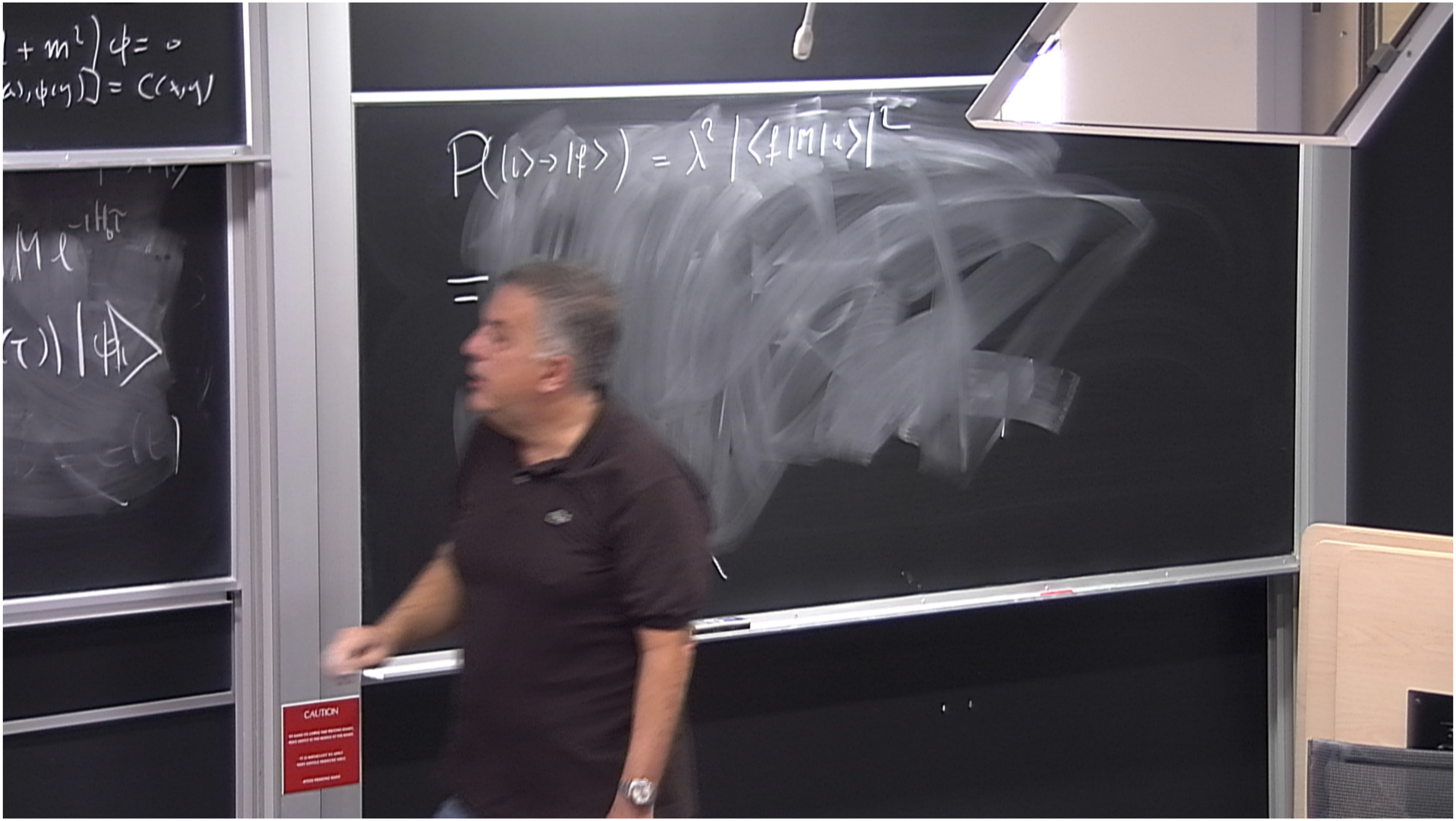

[φ(x), φ(y)]
= C(x, y)
x^μ(τ)
CAUTION

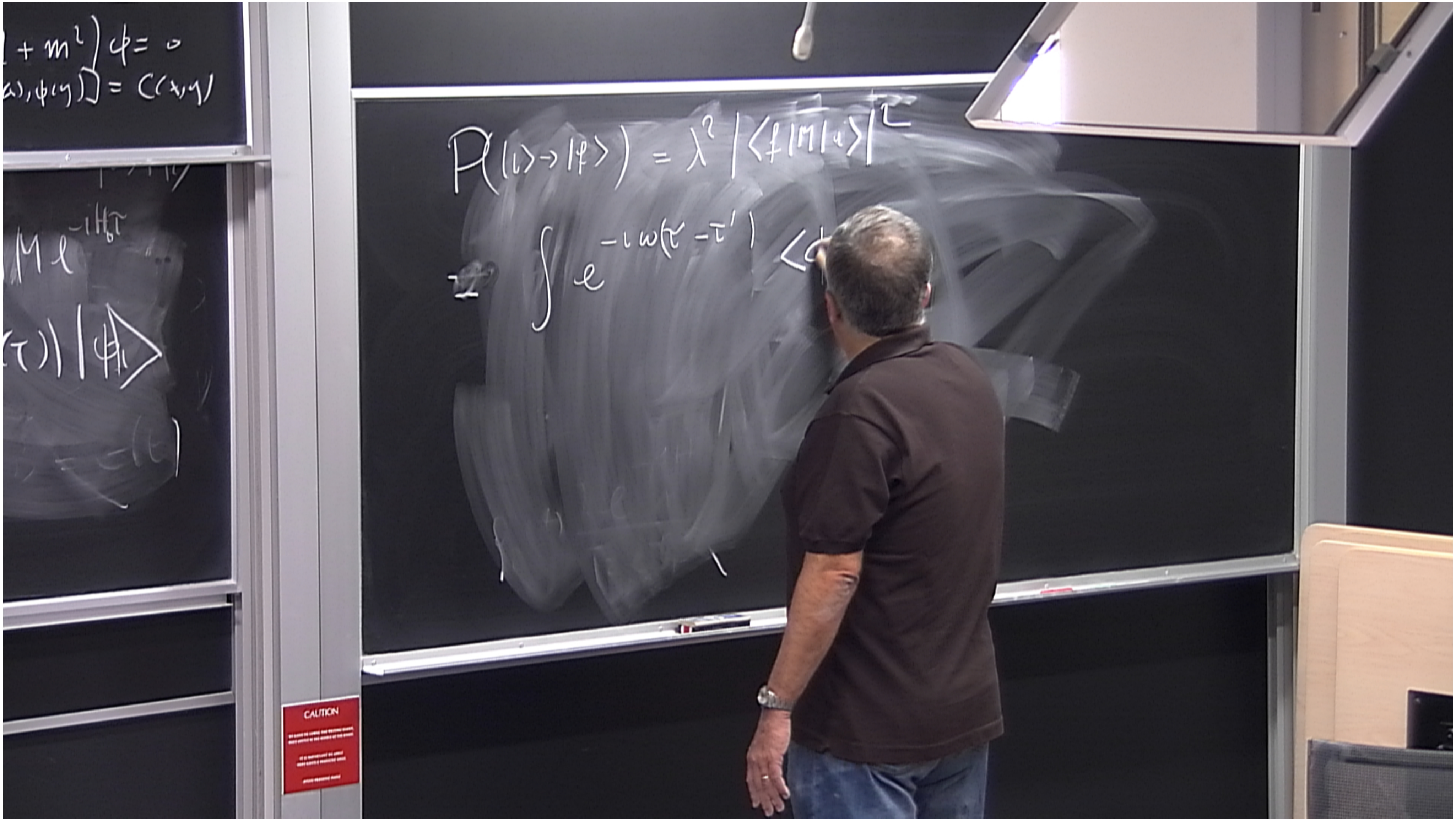

CAUTION

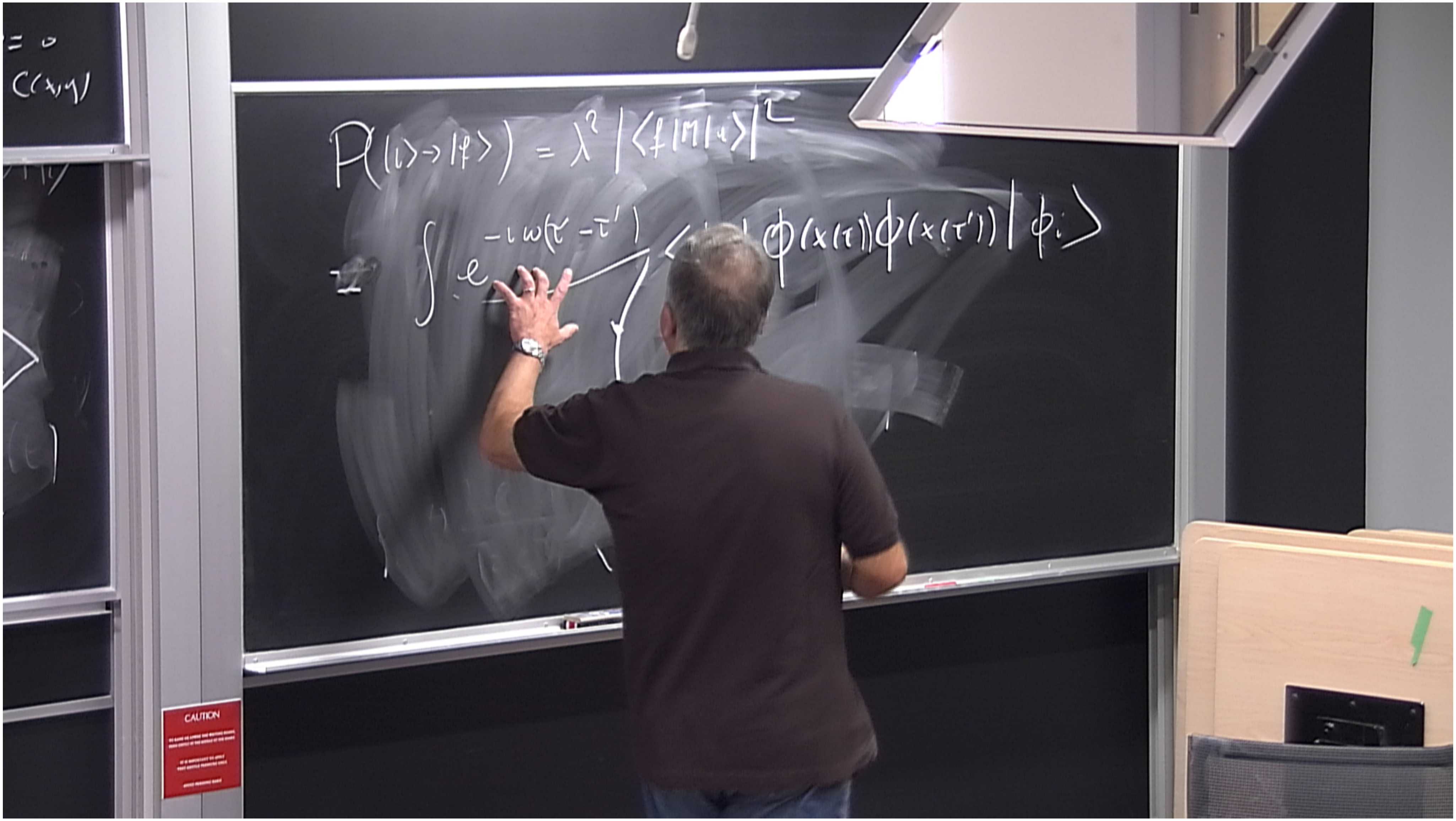

i → f
A = −iλ ⟨f|⟨Φ_f| ∫_{−∞}^{+∞} dτ
CAUTION


i ⟶ f
KMS
A , B

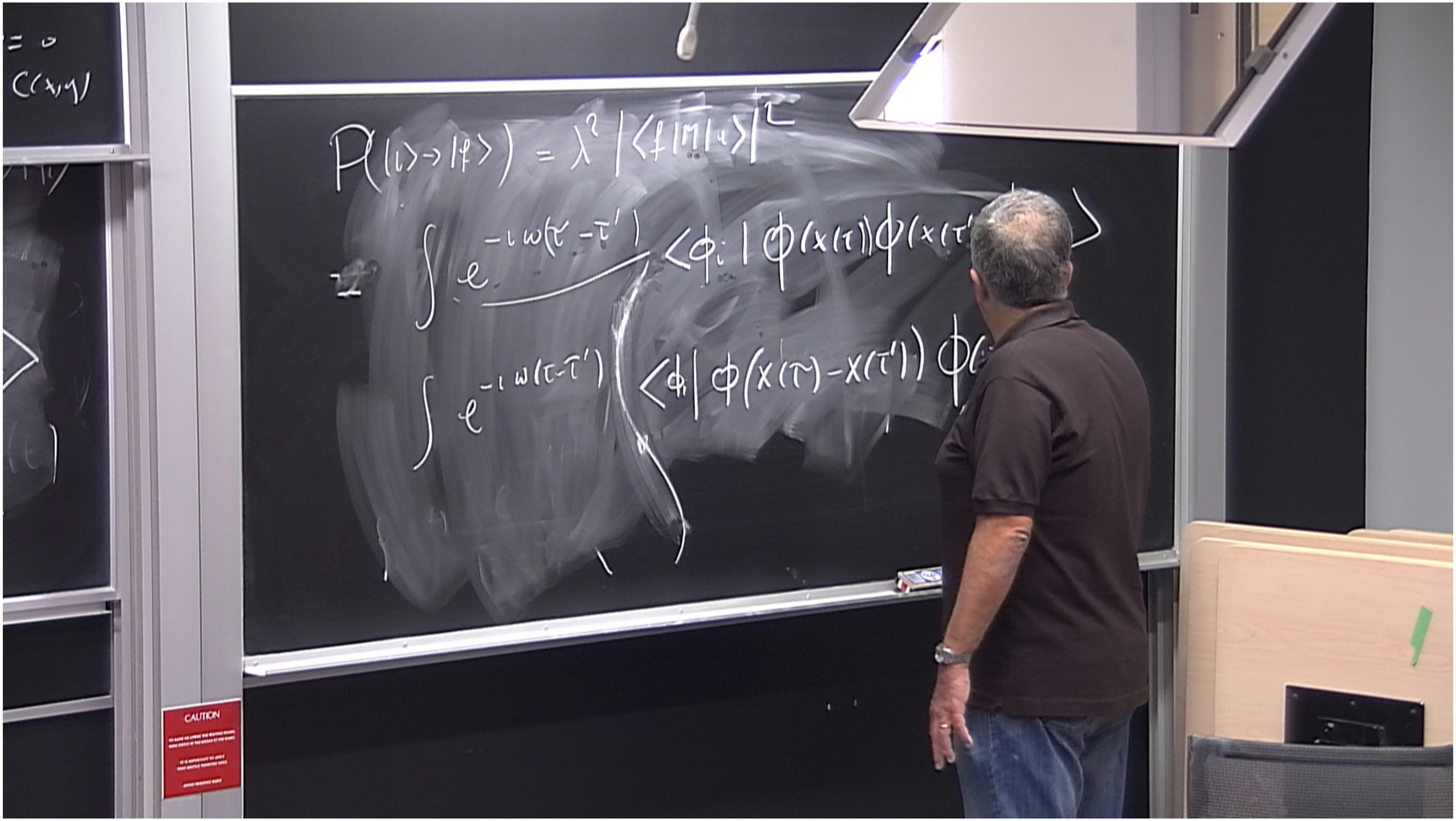

P(|i> -> |f>)
CAUTION

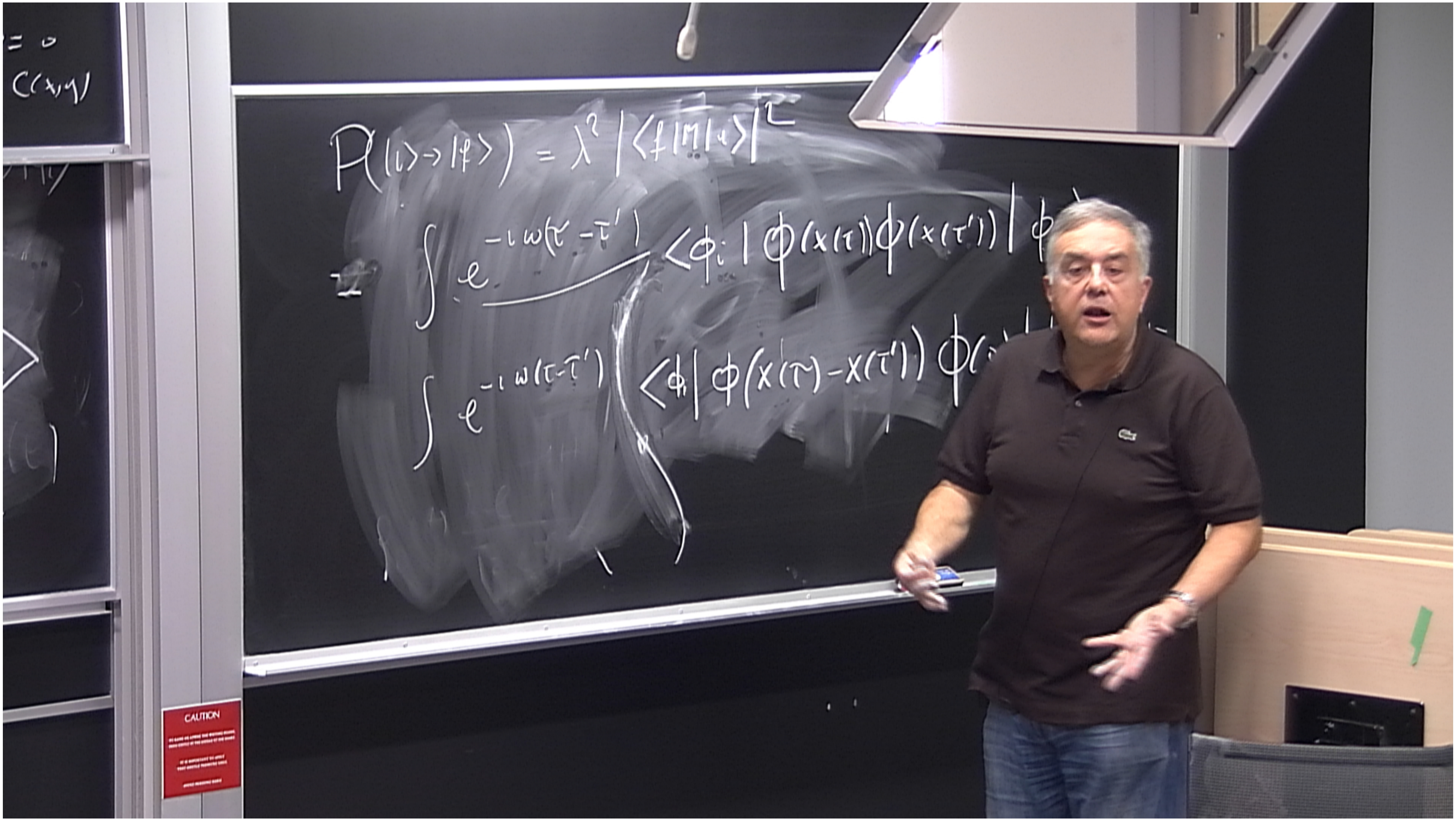


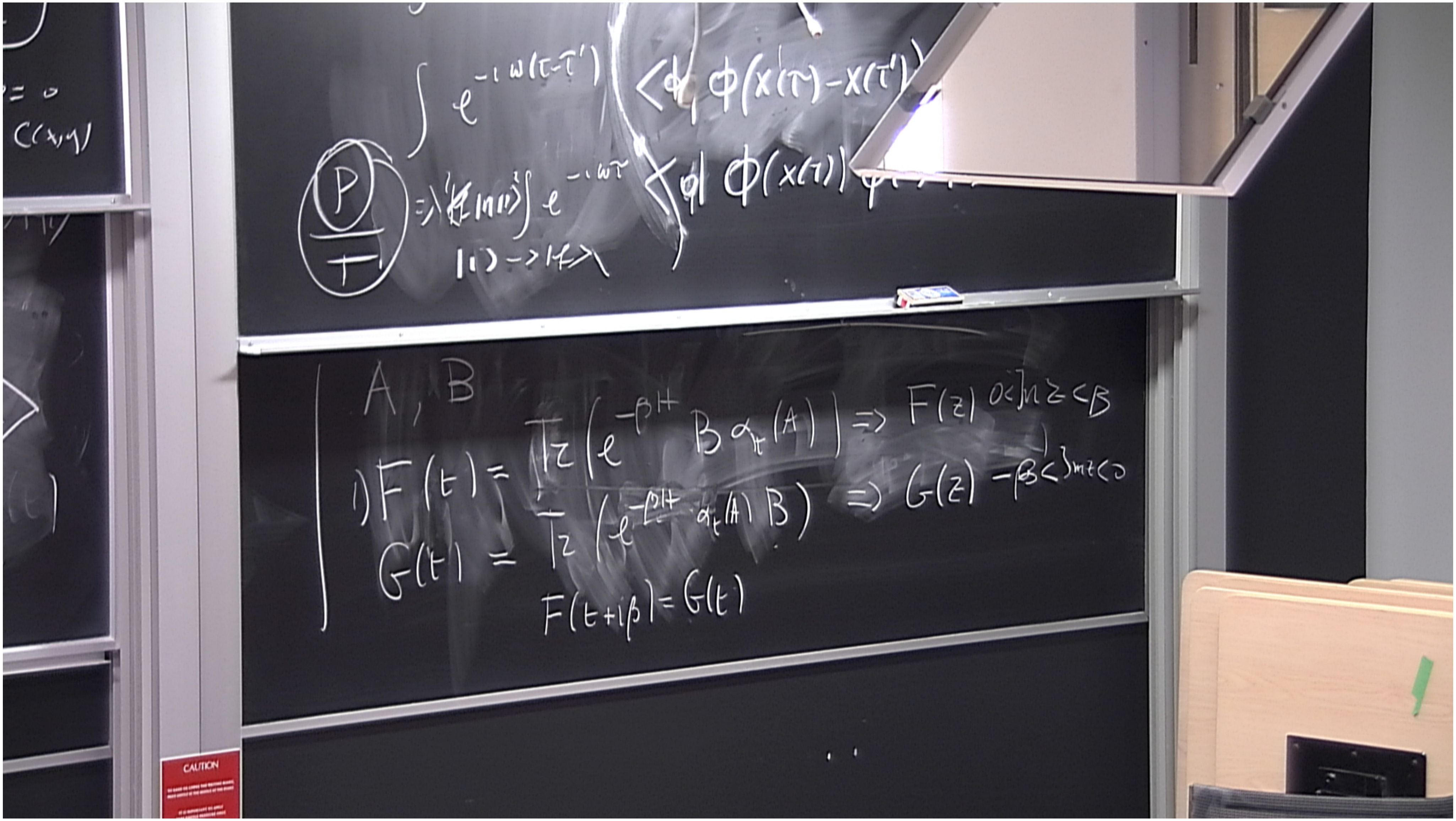

P(|i>→|f>) = λ²|<f|M|i>|²
∫ e^{-iω(τ-τ')} <
CAUTION



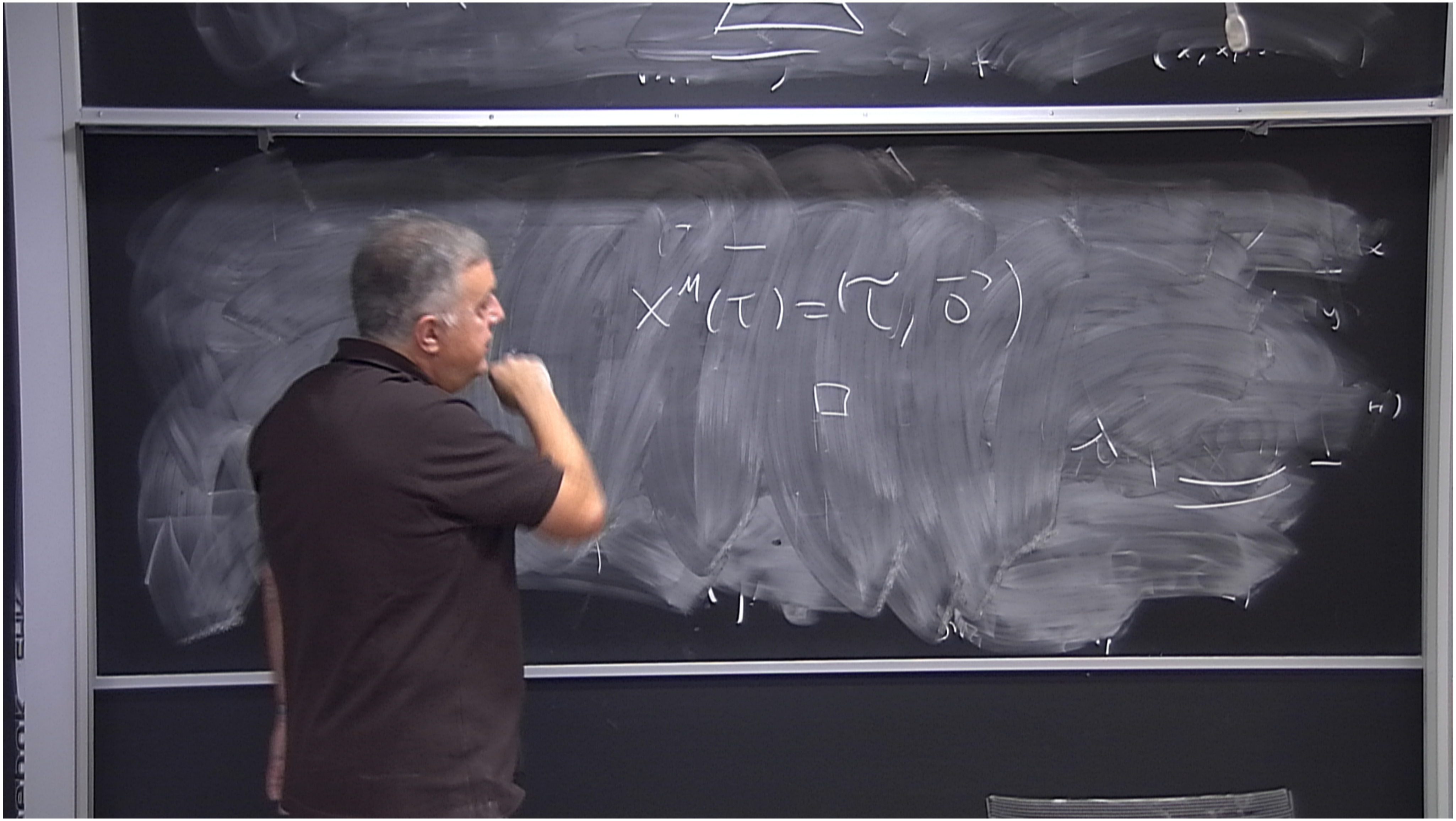
P(|i> -> |f>) = λ² |<f|M|i>|²
CAUTION

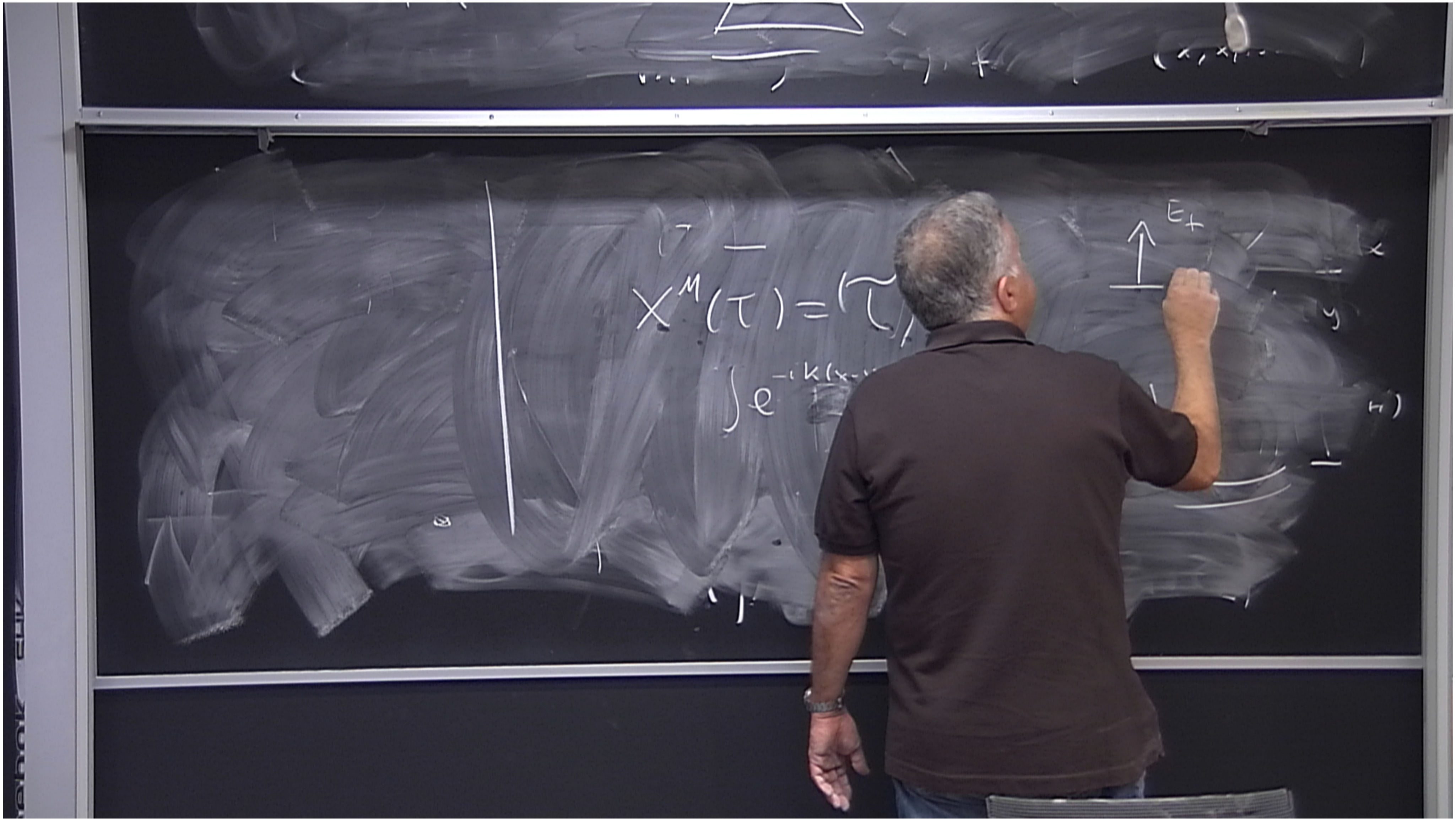

P(|i>→|f>) = λ²|<f|M|i>|²
CAUTION

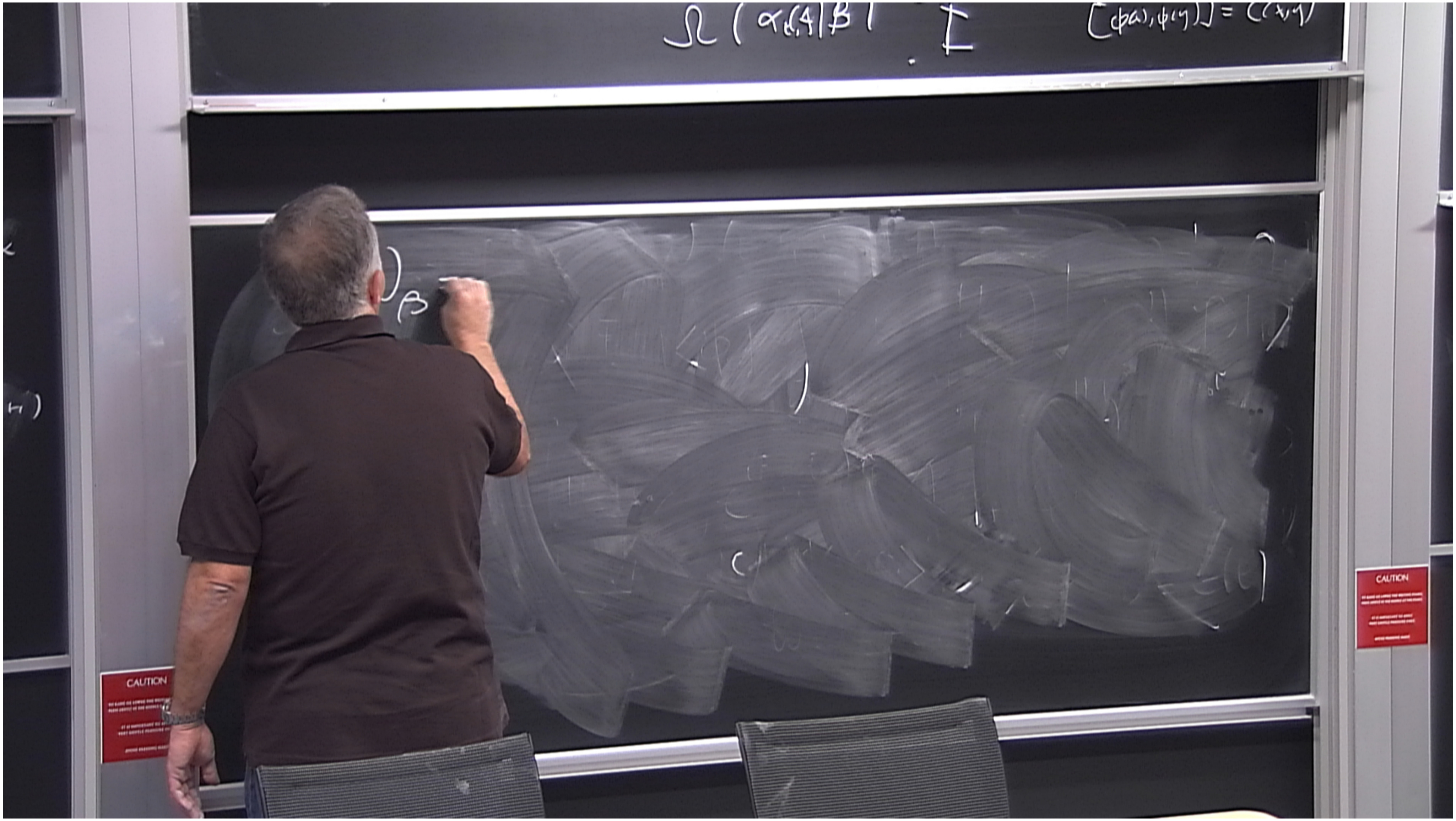

P(|i> -> |f>) = λ² |<f|M|i>|²
CAUTION

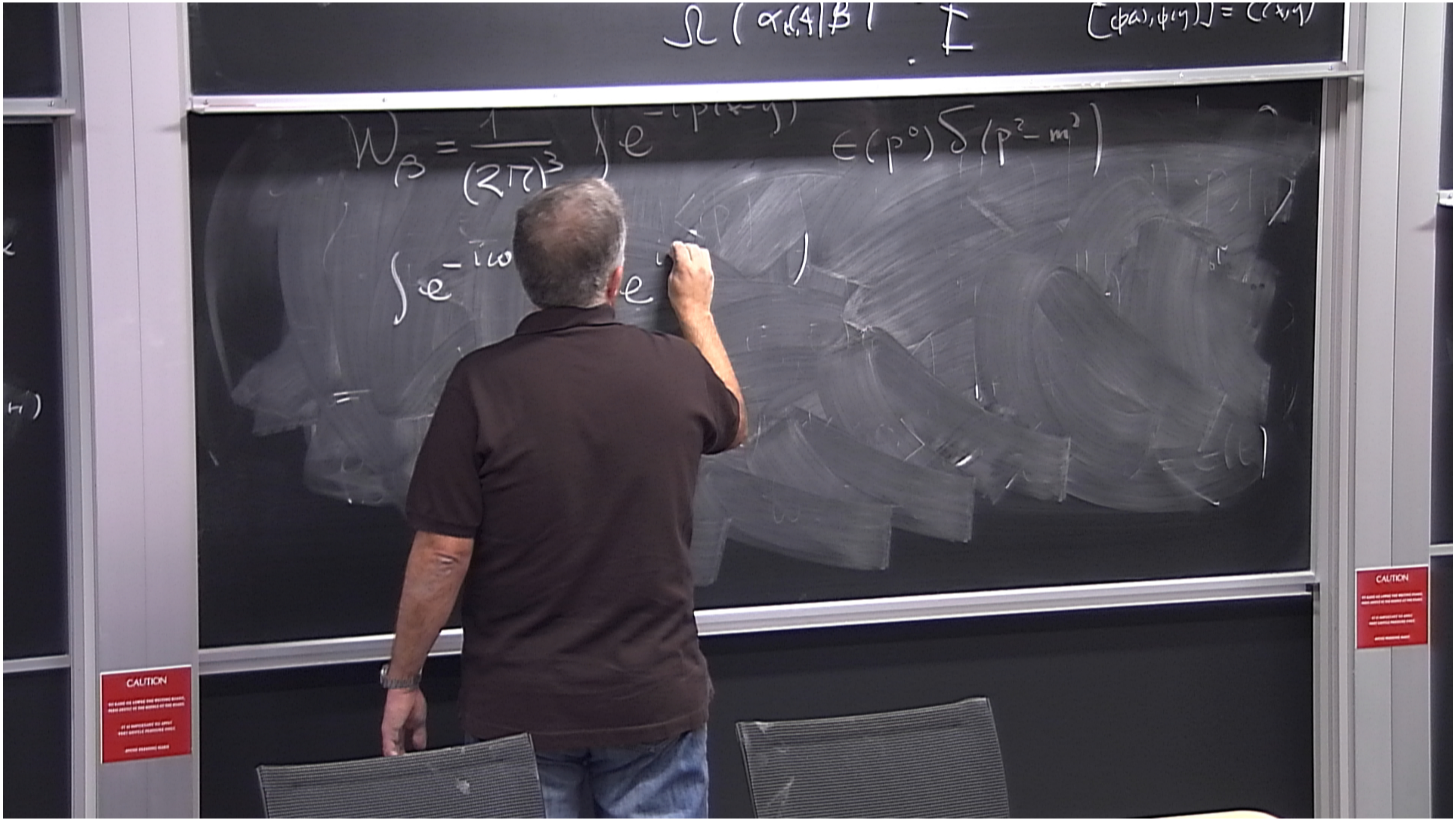


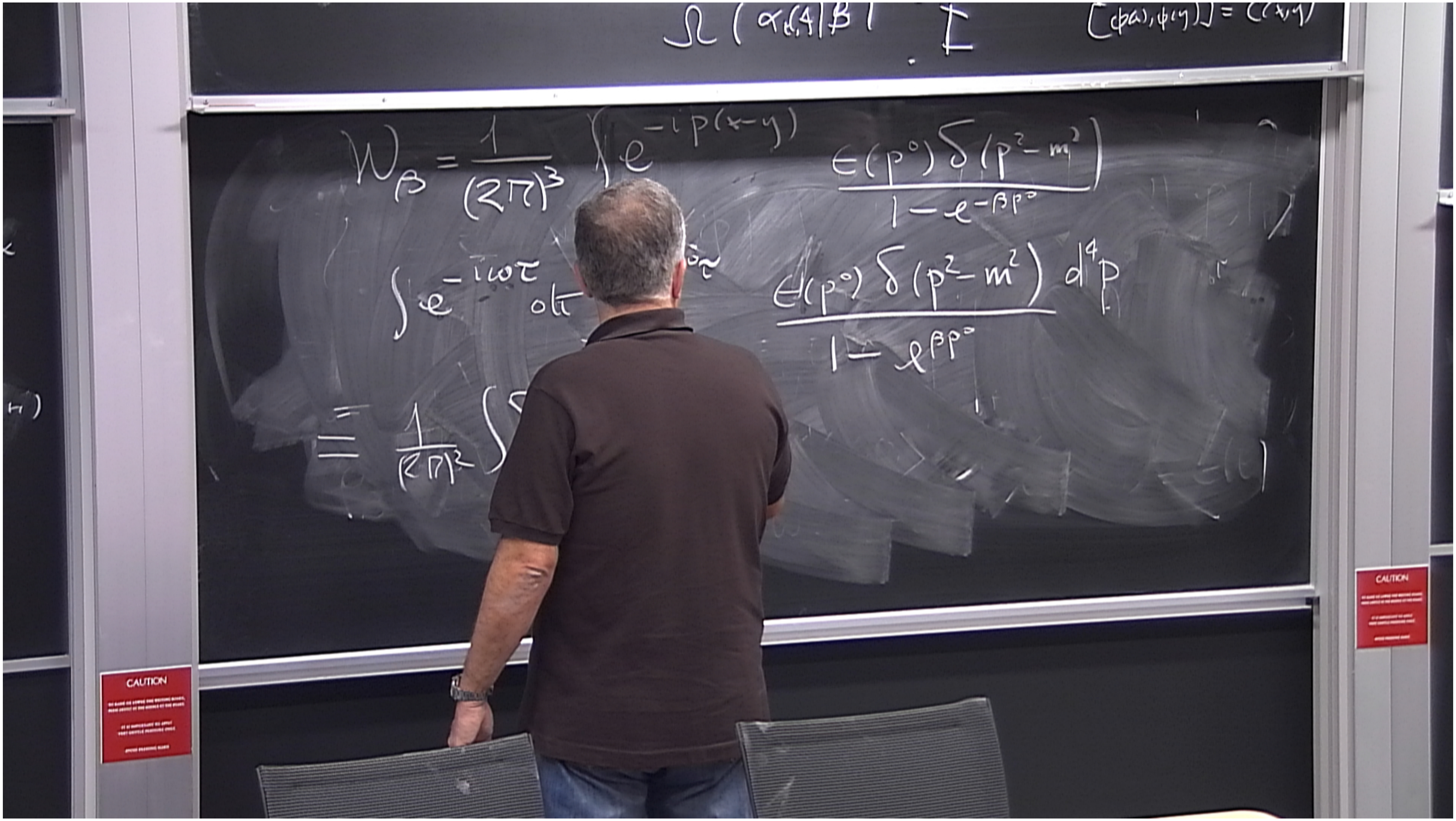
∫ e^{-iω(τ-τ')} <φ| Φ(x(τ)-x(τ'))
|i> -> |f>
A, B
1) F(t) = Tr(e^{-βH} B α_t(A)) => F(z) 0 < Im z < β
G(t) = Tr(e^{-βH} α_t(A) B) => G(z) -β < Im z < 0
F(t+iβ) = G(t)


X^μ(τ) = (τ, σ)

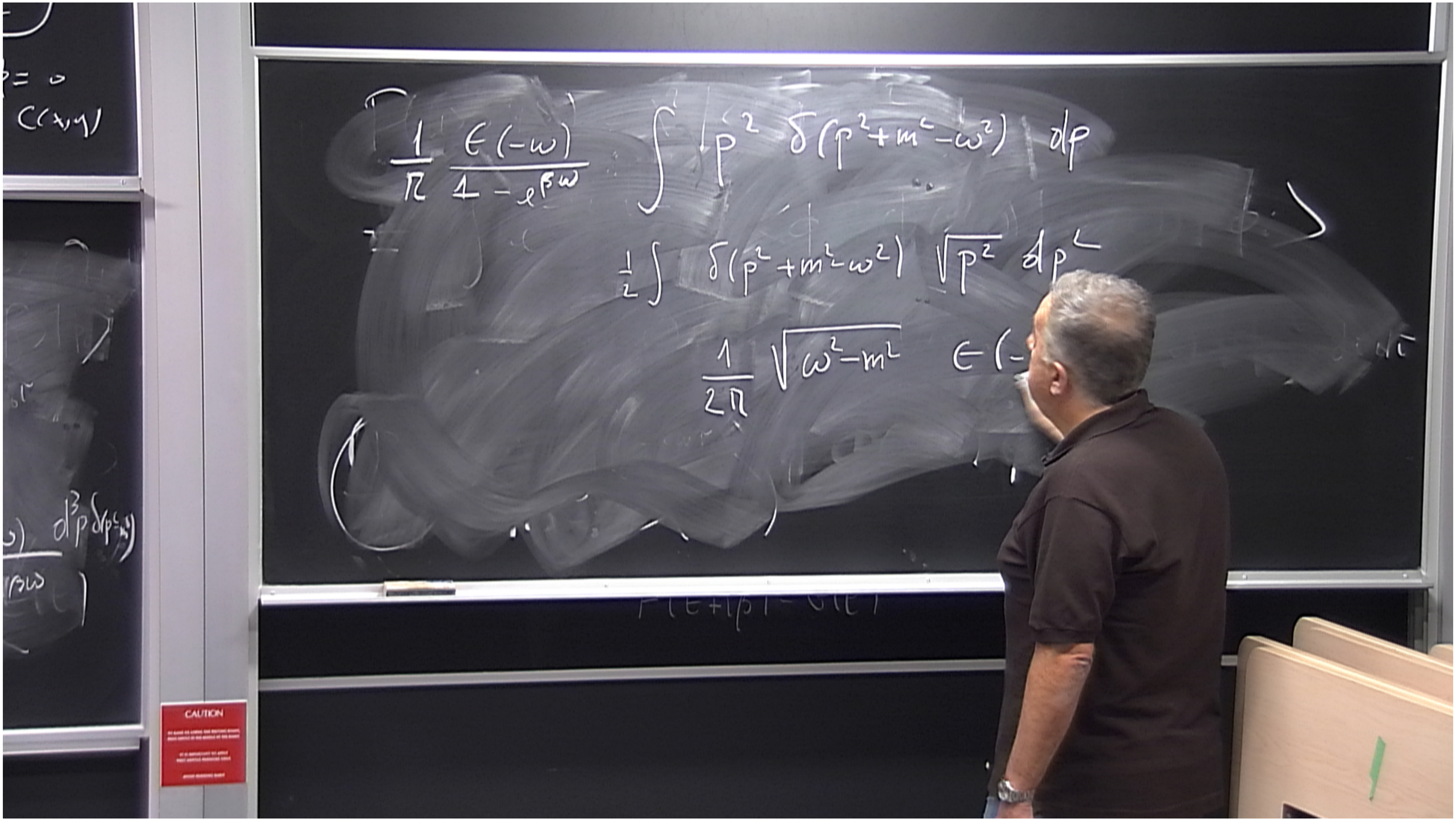


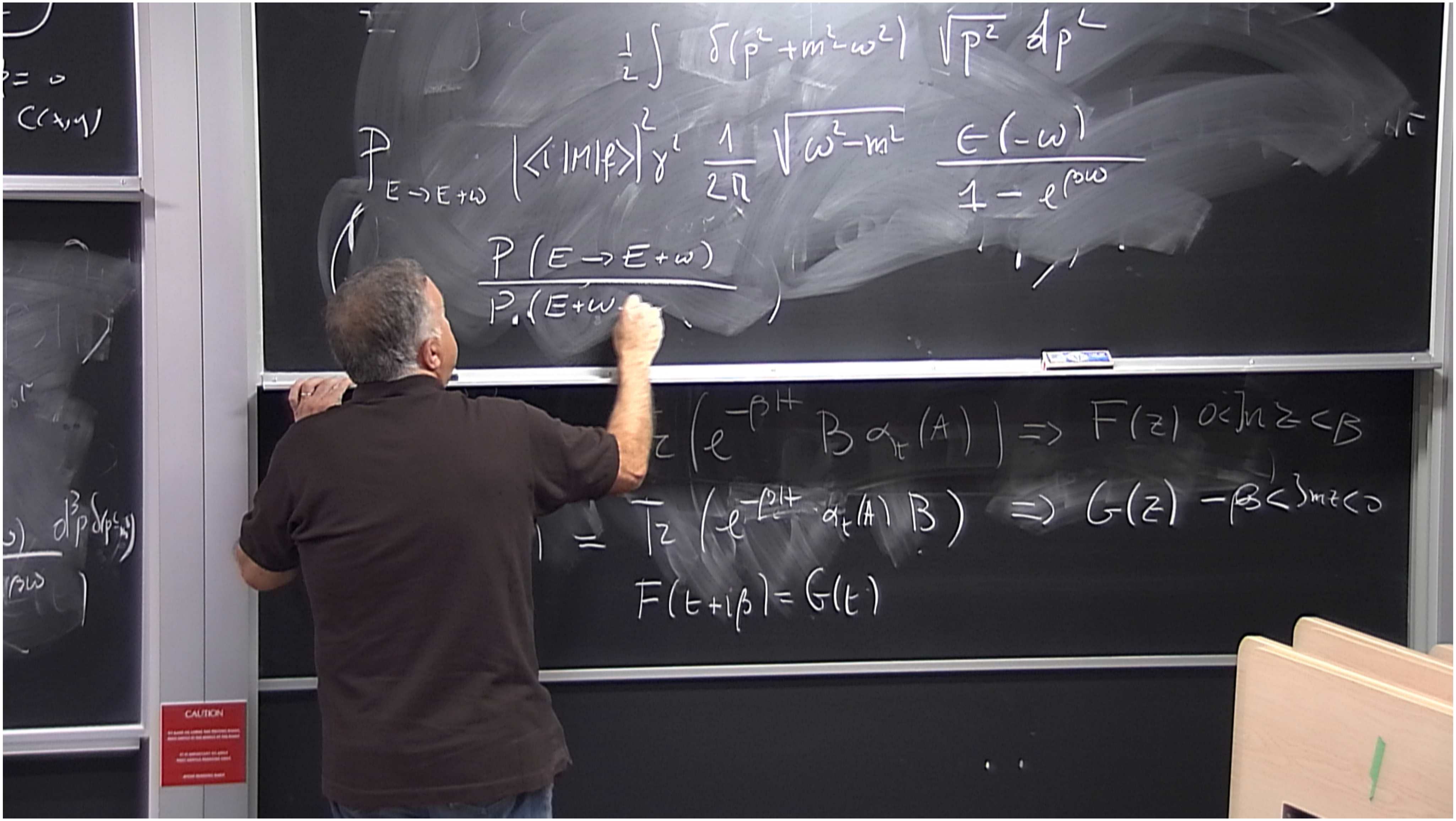

CAUTION
CAUTION

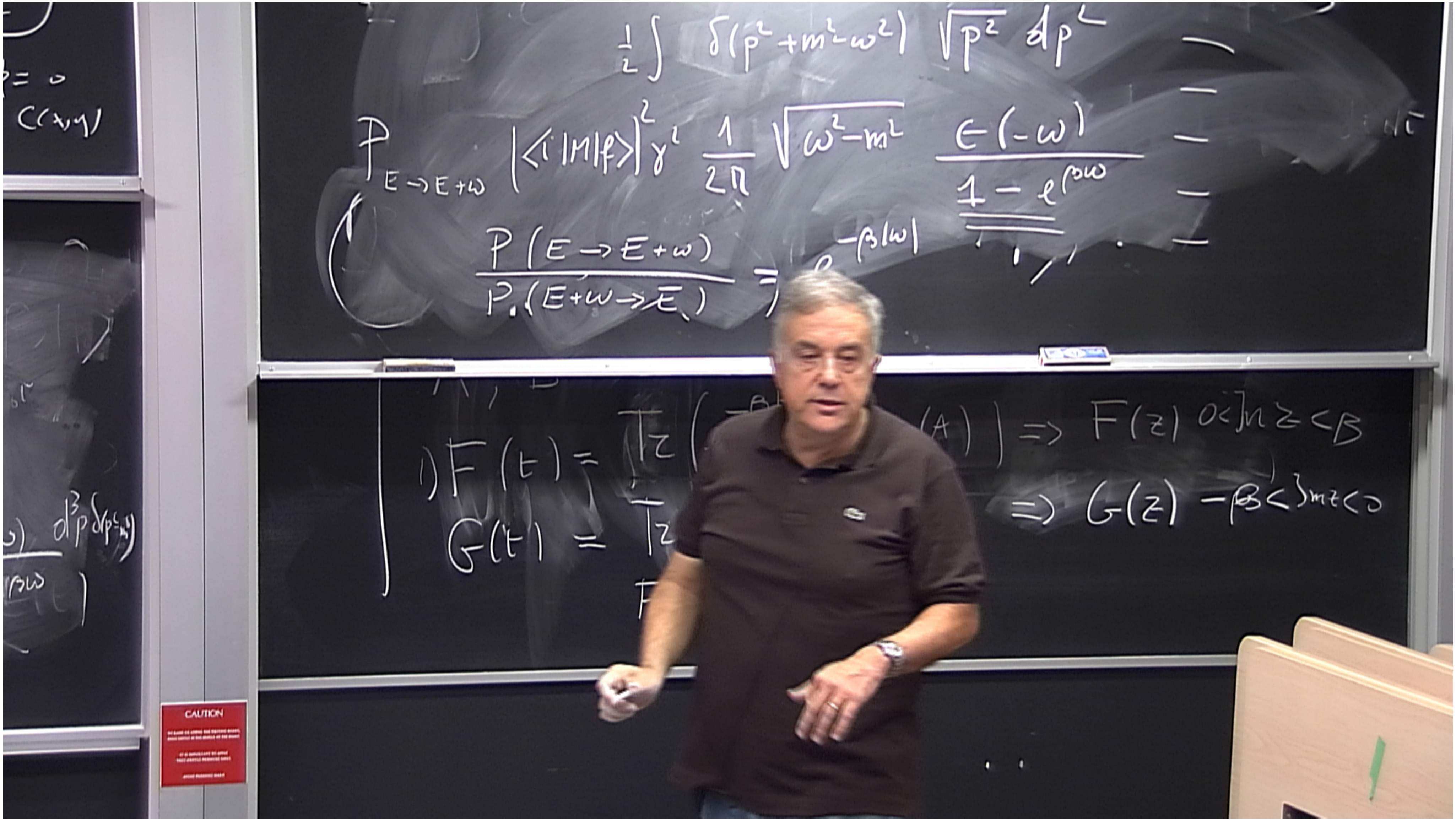

CAUTION
CAUTION

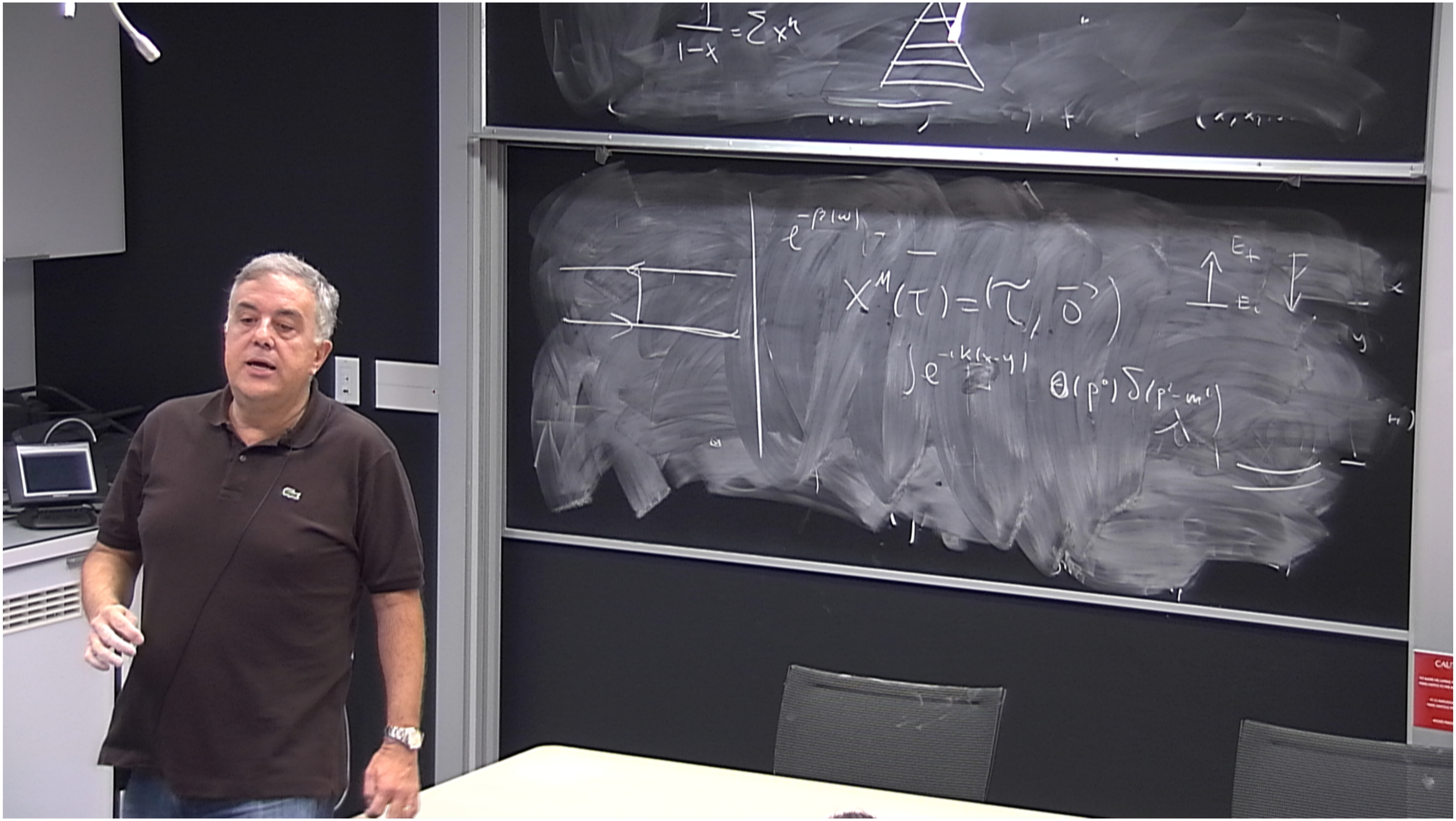


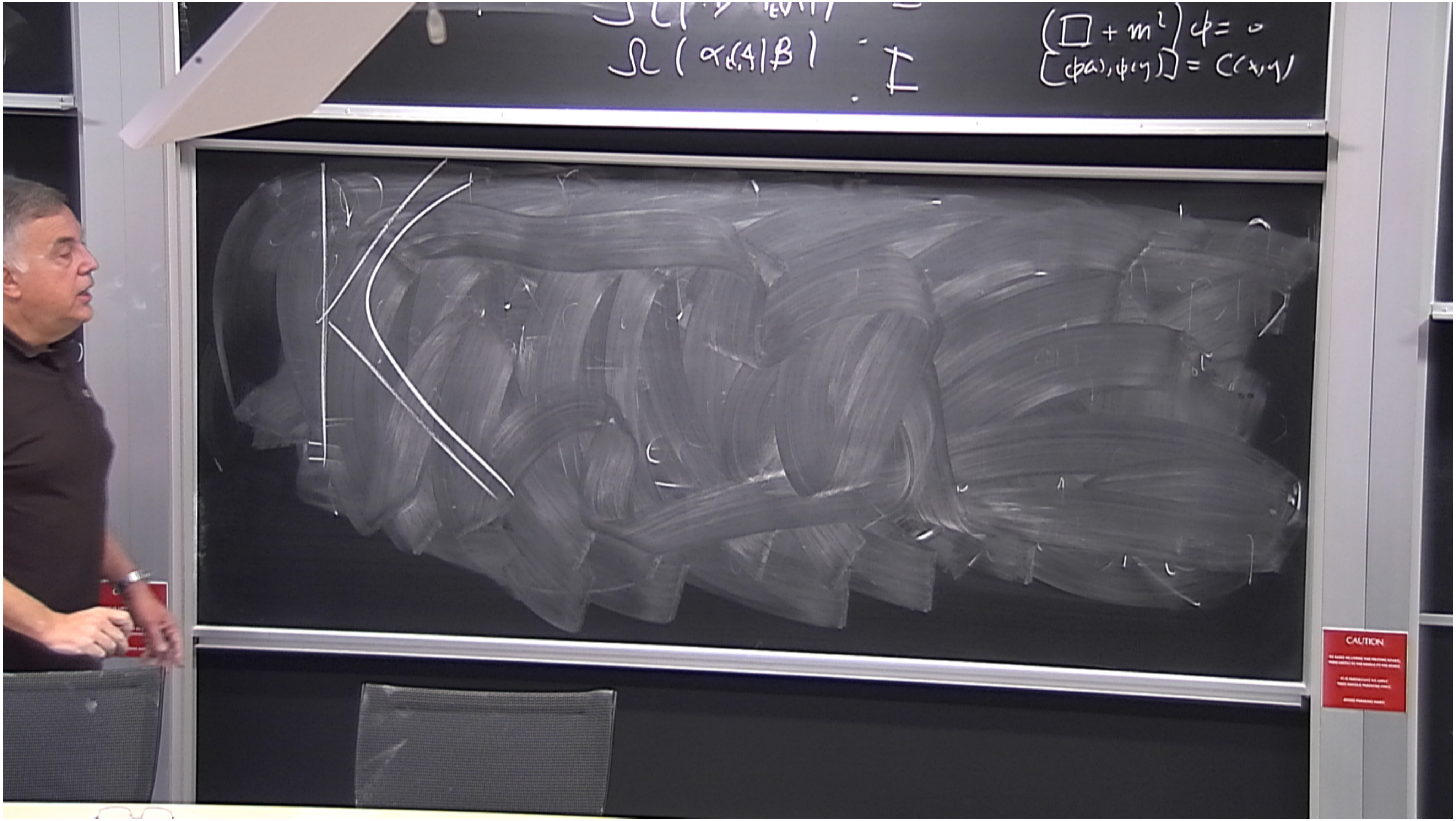
CAUTION
CAUTION

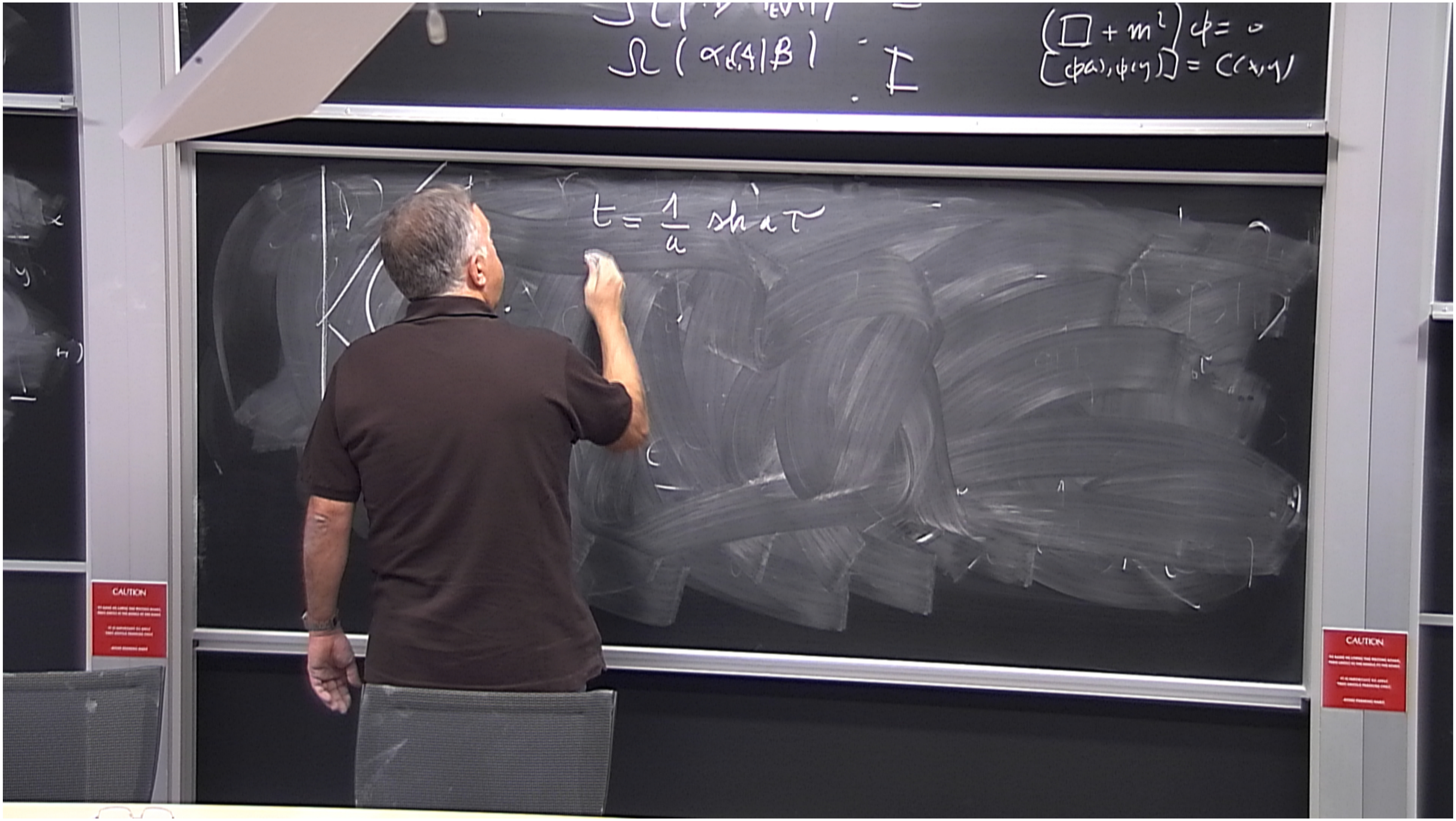

$\frac{1}{\pi}\frac{\epsilon(-\omega)}{1-e^{\beta\omega}} \int p^2 \, \delta(p^2+m^2-\omega^2) \, dp$
$\frac{1}{2}\int \delta(p^2+m^2-\omega^2) \sqrt{p^2} \, dp^2$
$\frac{1}{2\pi}\sqrt{\omega^2-m^2} \quad \epsilon(-$
CAUTION

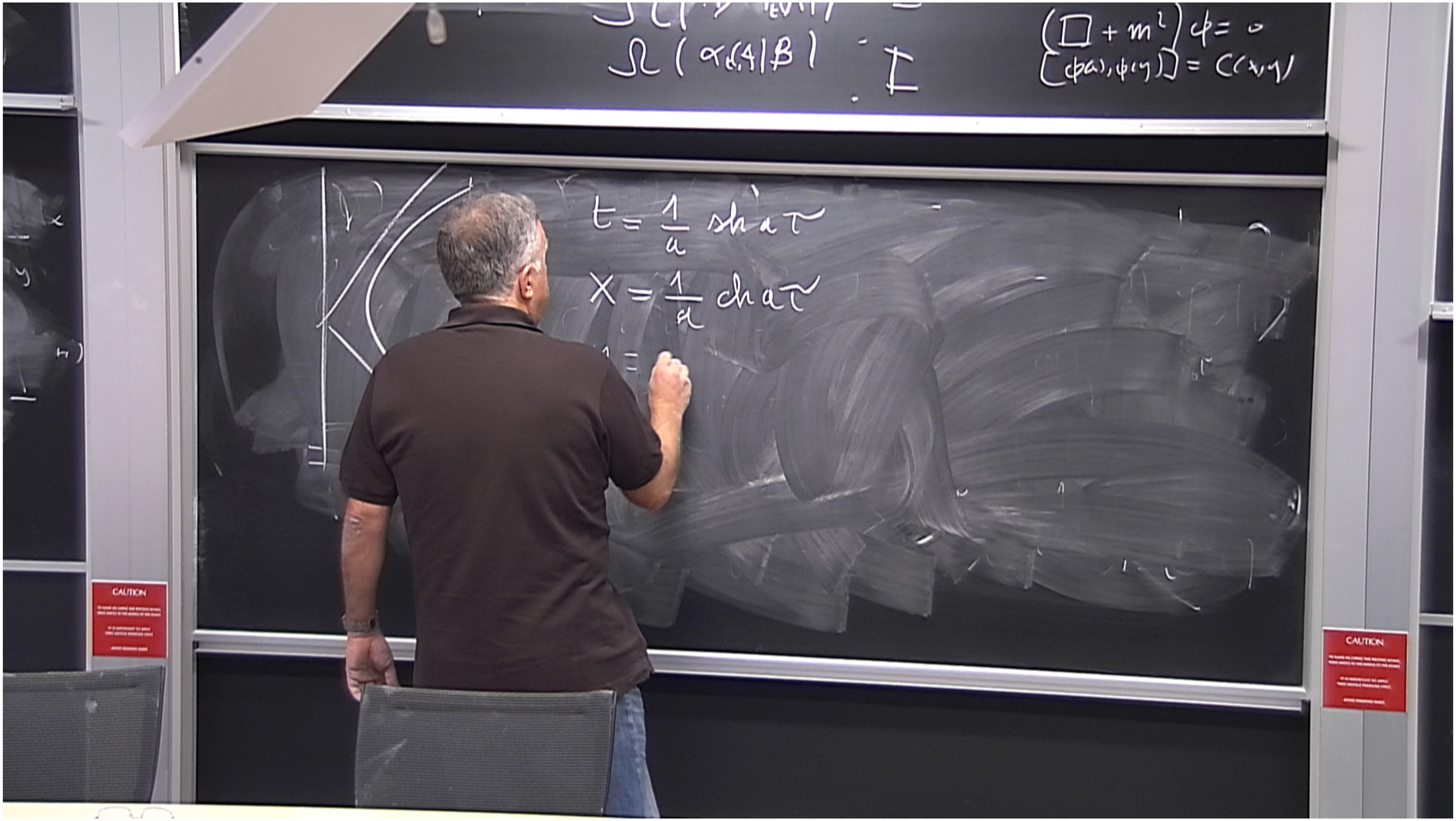

P (E → E+ω)
F(t+iβ) = G(t)
CAUTION


CAUTION

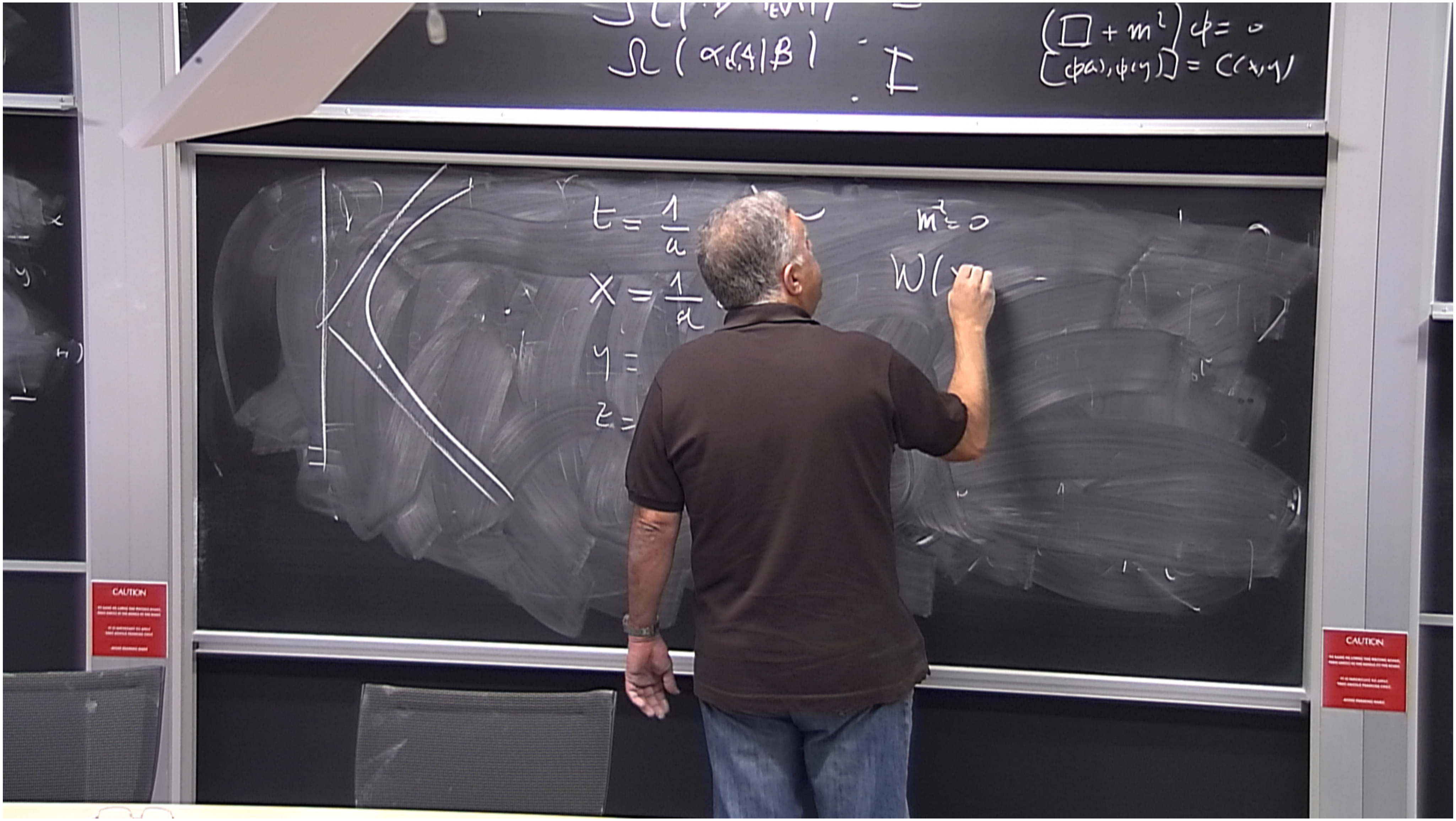

1/(1−x) = Σ x^n
X^μ(τ) = (τ, 0⃗)
∫ e^{−ik(x−y)}
θ(p⁰) δ(p² − m²)
E+
Ei

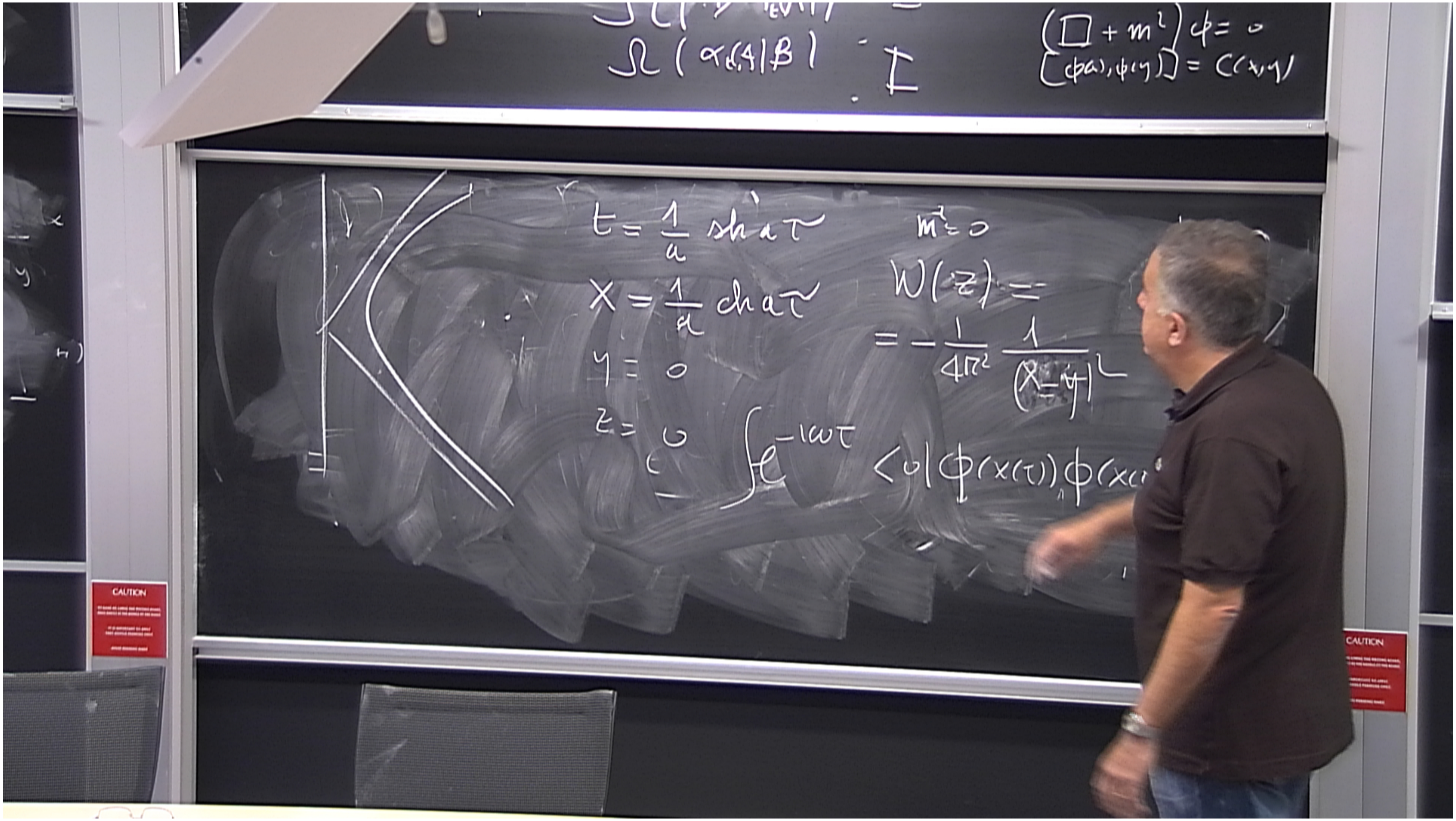

(□ + m²) φ = 0
[φ(x), φ(y)] = C(x,y)
CAUTION


t = 1/a sh aτ
(□ + m²) φ = 0
[φ(x), φ(y)] = C(x,y)
CAUTION
CAUTION

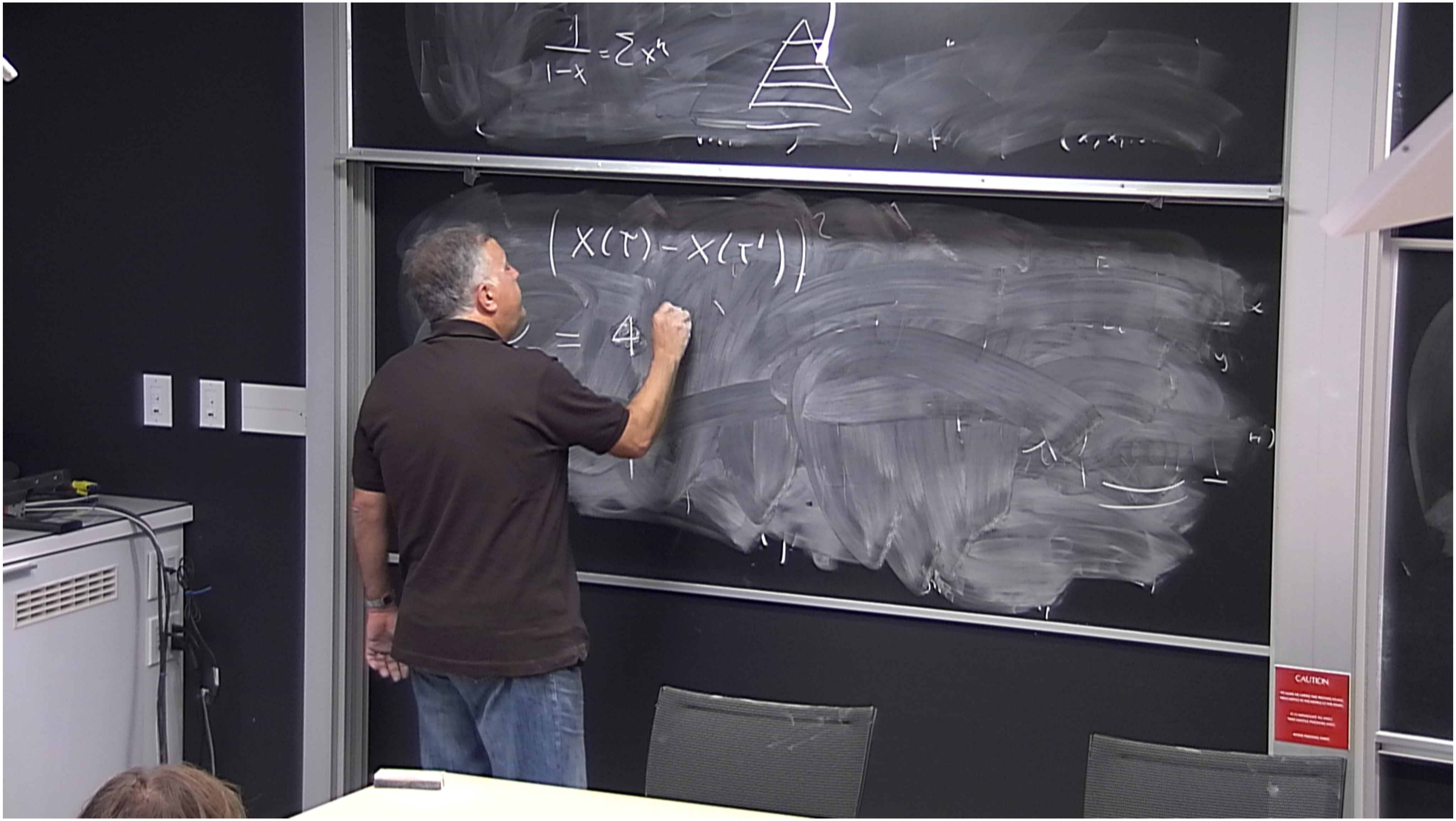


(□ + m²) φ = 0
[φ(x), φ(y)] = C(x,y)
t = 1/a sh aτ
x = 1/a ch aτ
CAUTION
CAUTION

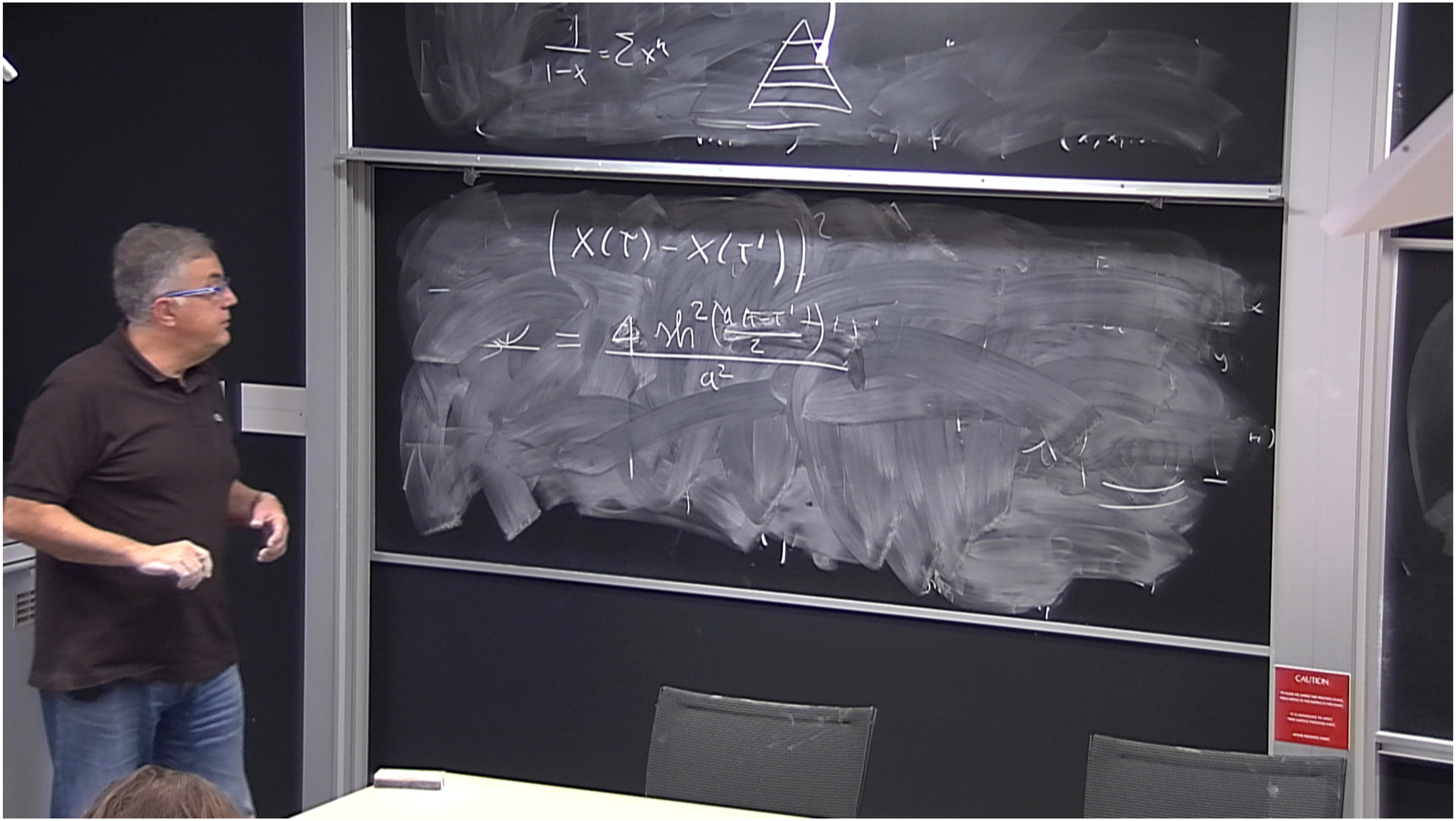

(□ + m²) φ = 0
[φ(x), φ(y)] = C(x,y)
t = 1/u
x = 1/u
m² = 0



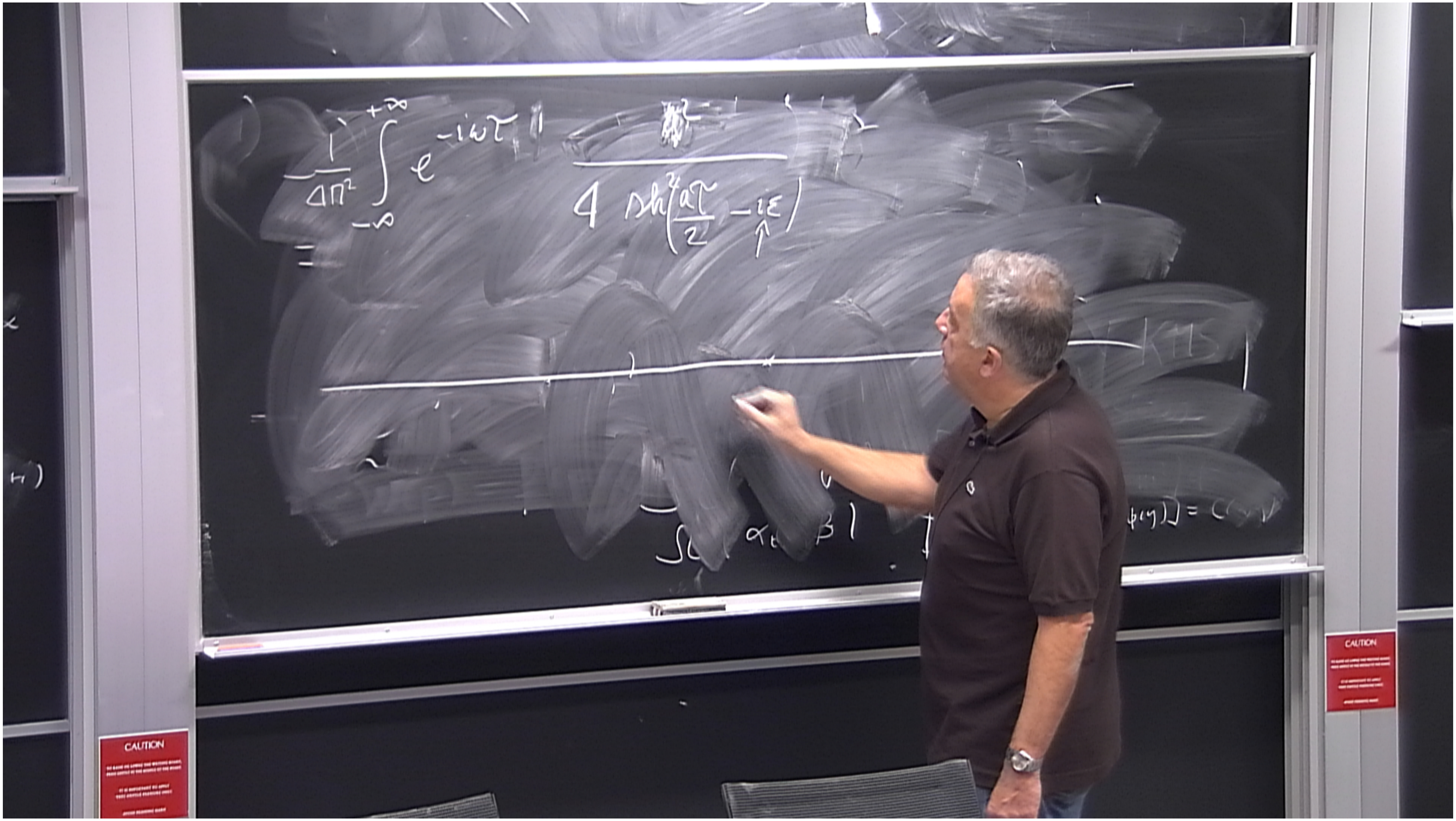
(□ + m²) φ = 0
[φ(x), φ(y)] = C(x,y)
t = 1/a sh aτ
x = 1/a ch aτ
y = 0
z = 0
m² = 0
W(z) =
= − 1/4π² 1/(x−y)²
CAUTION

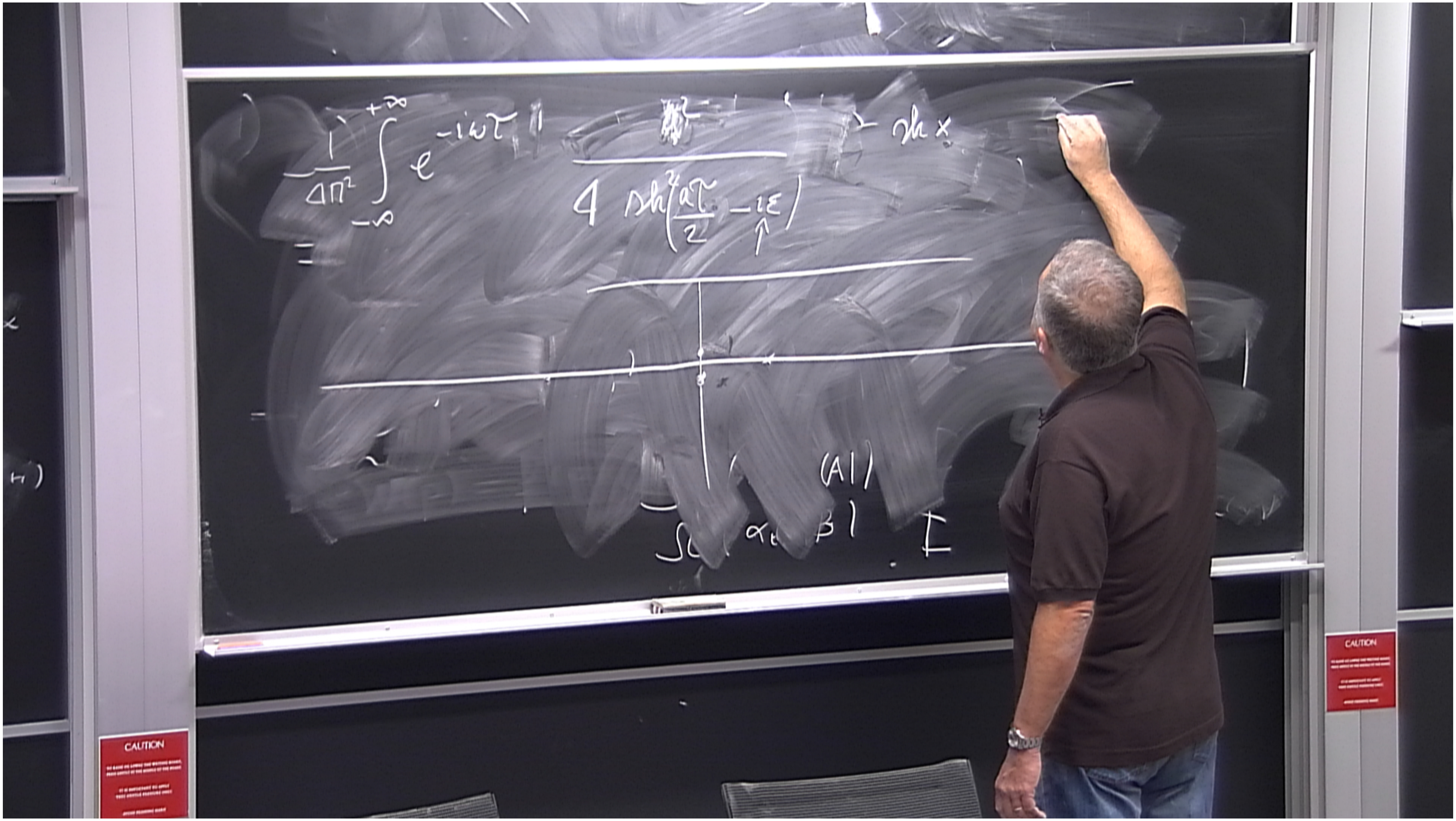


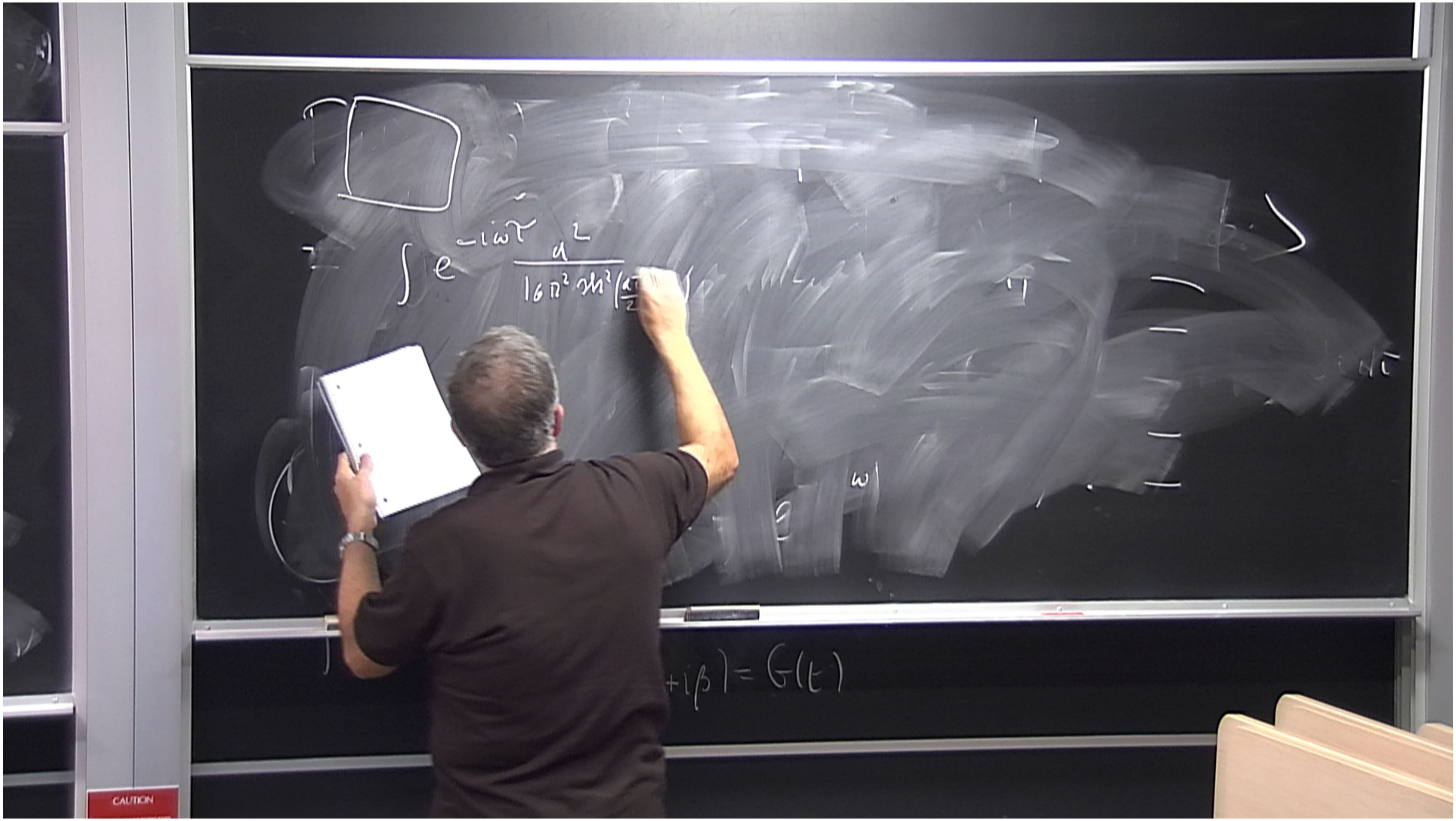
$\frac{1}{1-x} = \sum x^n$
$(X(\tau) - X(\tau'))^2$
= 4



1/(1−x) = Σxⁿ
(X(τ) − X(τ'))²
= 4 sh²(a(τ−τ')/2) / a²
CAUTION


CAUTION
CAUTION


(A|)
CAUTION
CAUTION


=G(t)