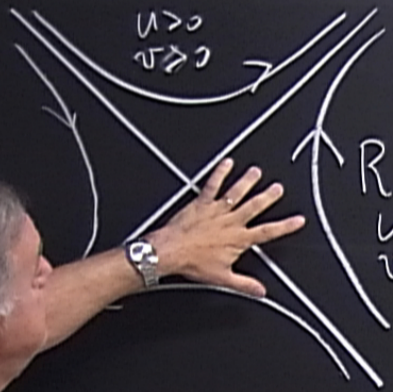


Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 9

Date: Oct 21, 2013 10:00 AM

URL: <http://pirsa.org/13100013>

Abstract:



$$t = \xi \operatorname{ch} \eta$$

$$z = \xi \operatorname{ch} \eta$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

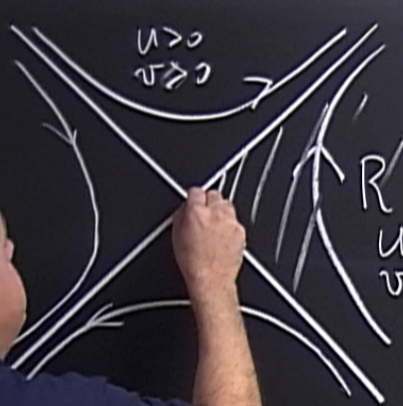
$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

$$ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\eta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\varphi)$$



$$\begin{cases} t = \xi \operatorname{ch} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$\begin{matrix} u < 0 & \xi > 0 \\ v > 0 \end{matrix}$$

$$\begin{aligned} u = t - z &= -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-u} < 0 \\ v = t + z &= \xi e^{\eta} = e^{\eta + \log \xi} = e^v > 0 \end{aligned}$$

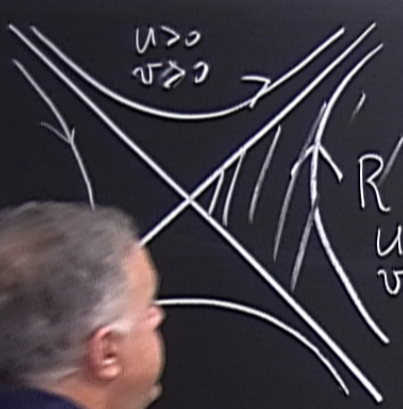
$$ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\eta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\varphi)$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
IF IT IS NECESSARY TO CLEAN
PLEASE CONTACT THE SUPPORT
SERVICE PERSONNEL



$$\begin{cases} t = \xi \operatorname{ch} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$\begin{aligned} u = t - z &= -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-u} < 0 \\ v = t + z &= \xi e^{\eta} = e^{\eta + \log \xi} = e^v > 0 \end{aligned}$$

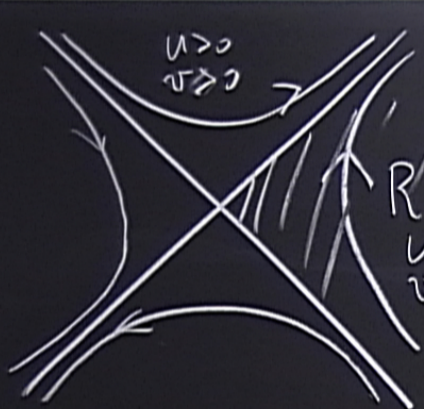
$$\begin{cases} u < 0 \\ v > 0 \end{cases} \quad \xi > 0$$

$$ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\eta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\varphi)$$



$$\begin{cases} t = \xi \operatorname{ch} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

$$u < 0 \quad \xi > 0$$

$$v > 0$$

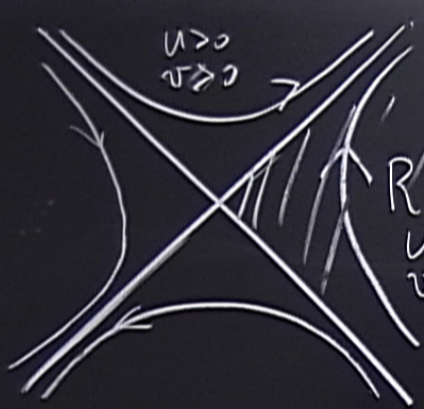
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\eta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\varphi)$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD OR THE BOARD SURFACE
IF IT IS DAMAGED BY YOU
PLEASE CONTACT THE BOARD
SERVICE CENTER



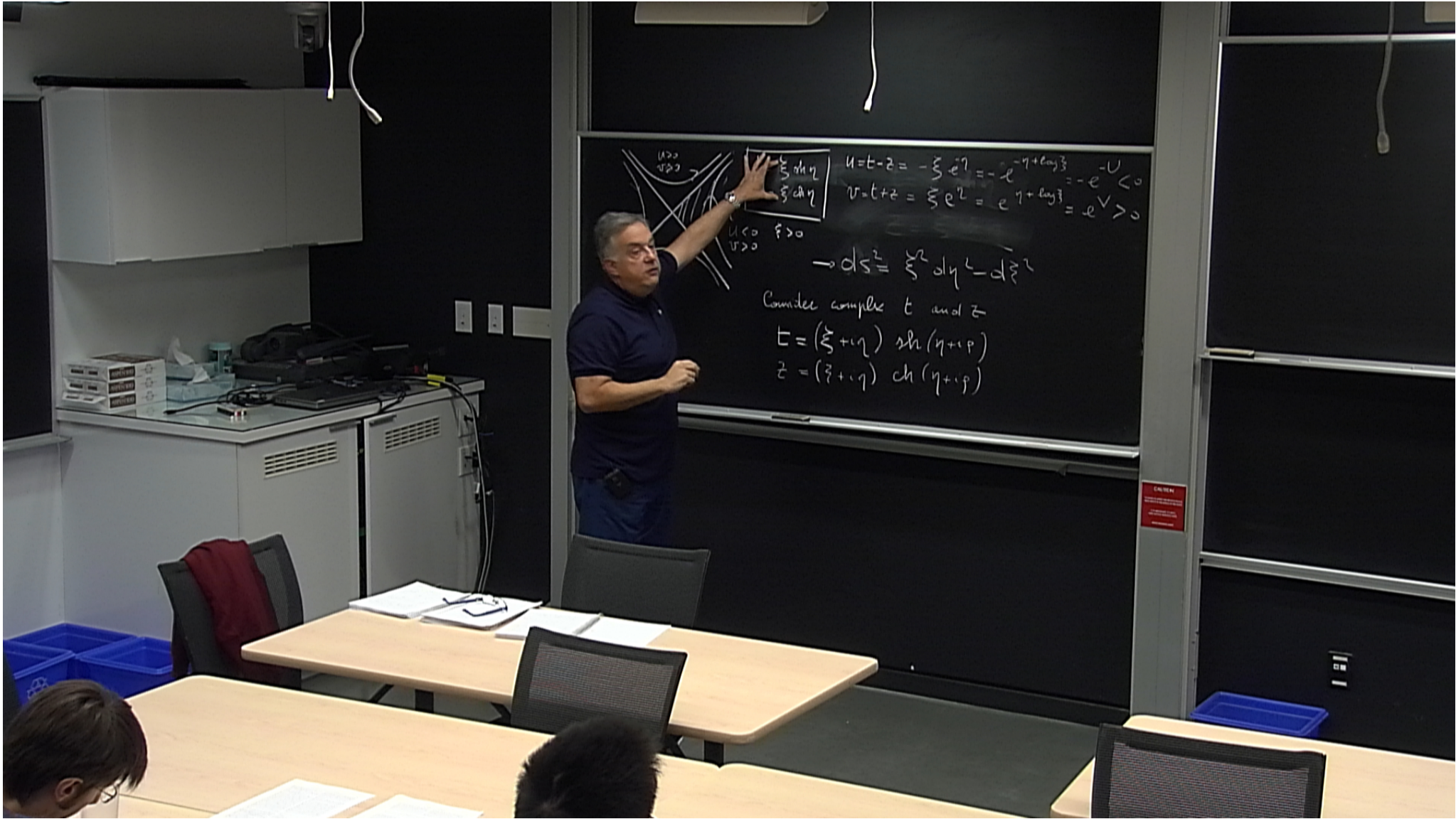
$$\begin{cases} t = \xi \operatorname{sh} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

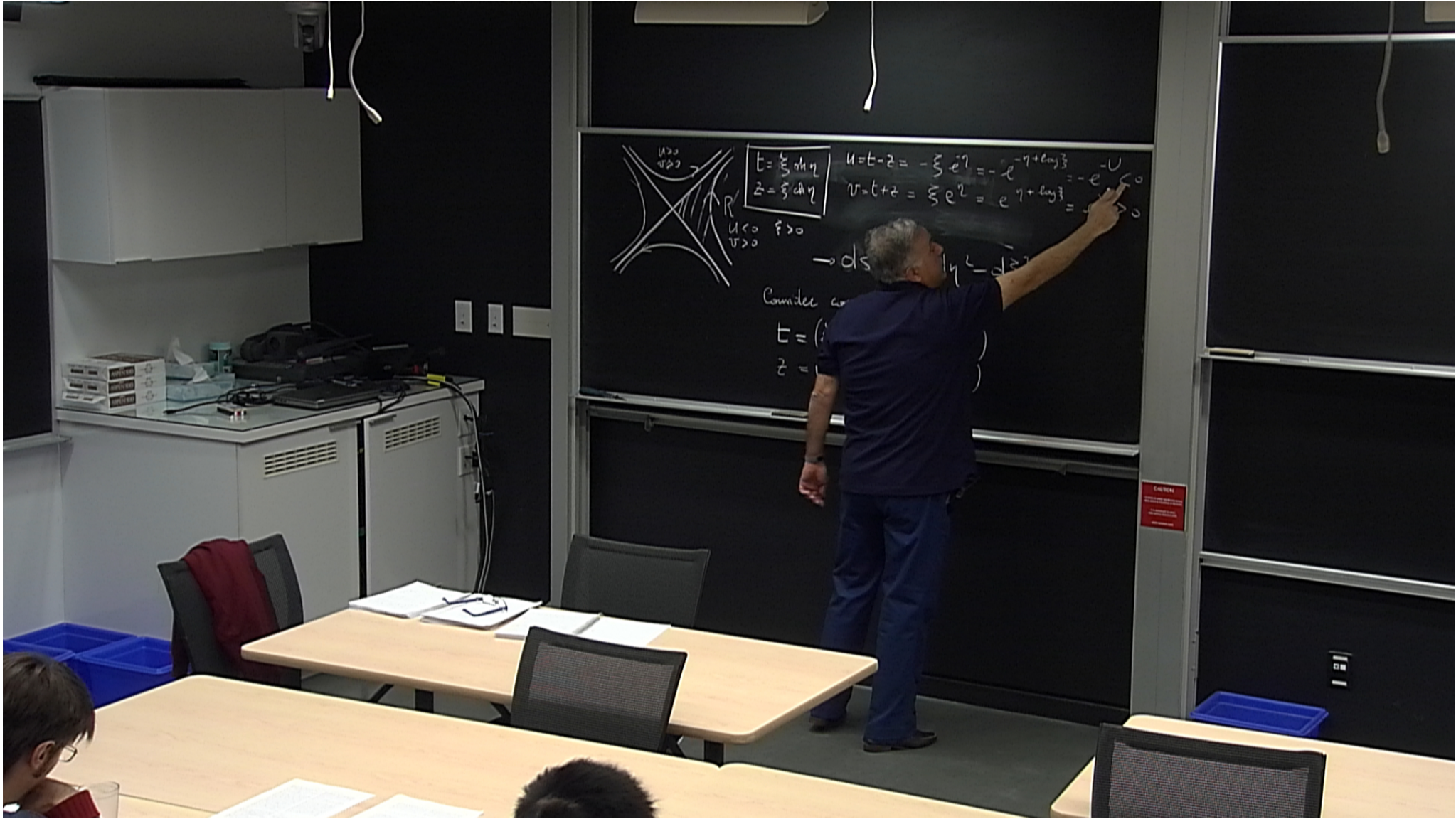
$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

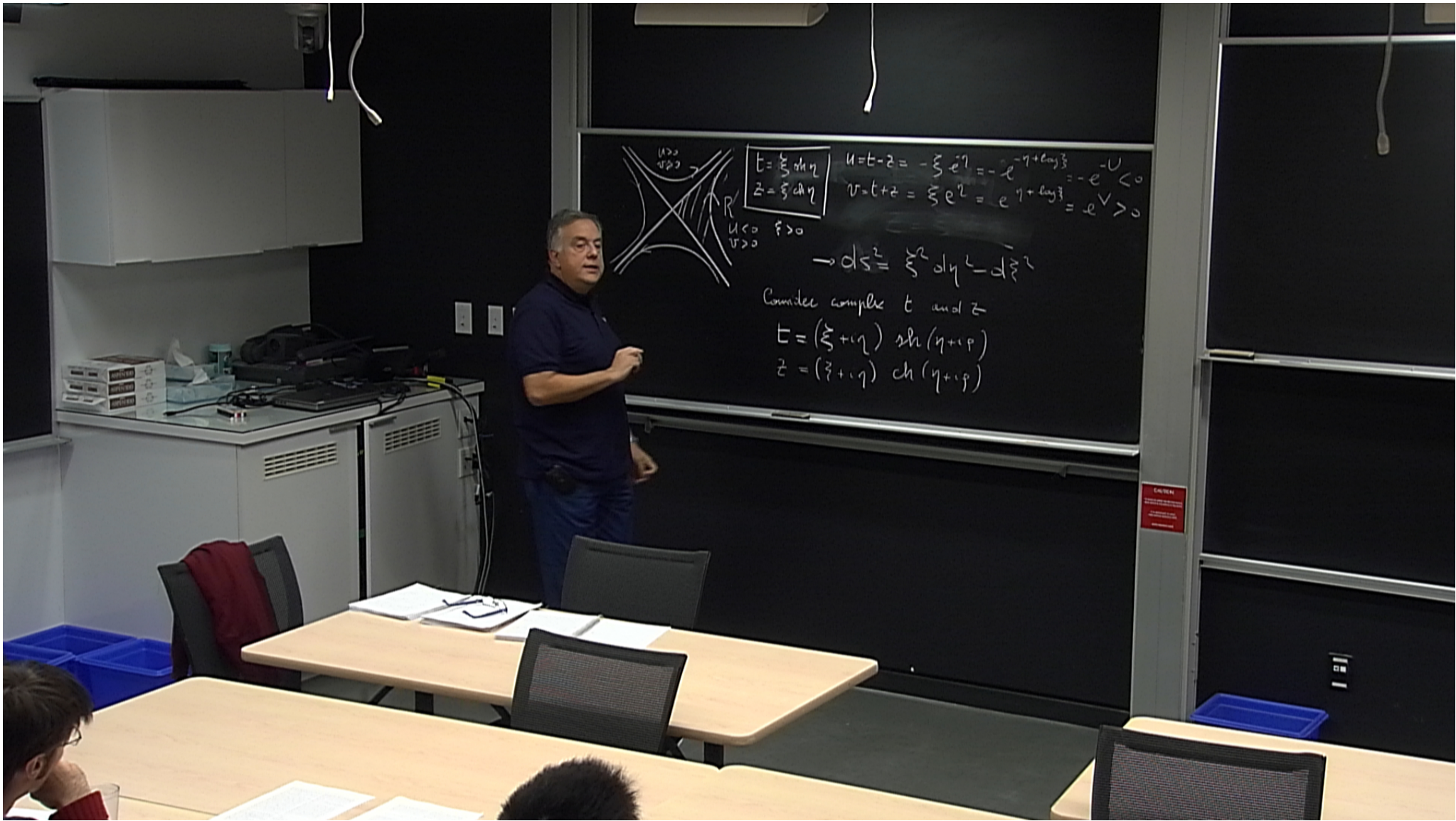
$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

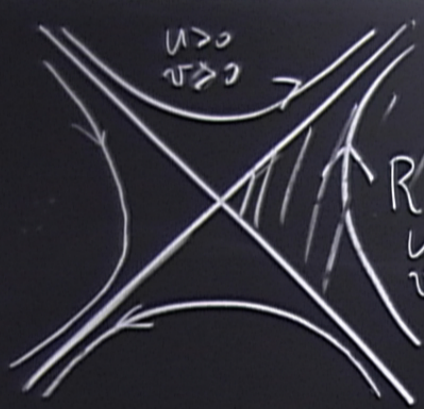
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

complex t and z
 $+i\eta) \operatorname{sh}(\eta + i\varphi)$
 $+i\eta) \operatorname{ch}(\eta + i\varphi)$









$$\begin{cases} t = \xi \operatorname{sh} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

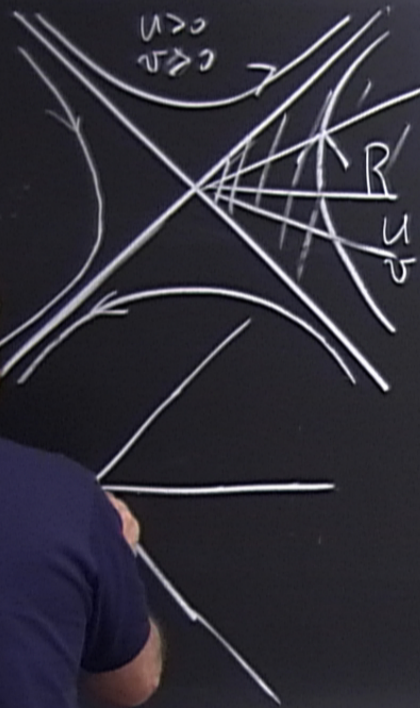
$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

$$\begin{cases} u < 0 \\ v > 0 \end{cases} \quad \xi > 0$$

$$ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$\begin{cases} \operatorname{sh}(\eta + i\varphi) \\ \operatorname{ch}(\eta + i\varphi) \end{cases}$$



$$\begin{cases} t = \xi \operatorname{sh} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

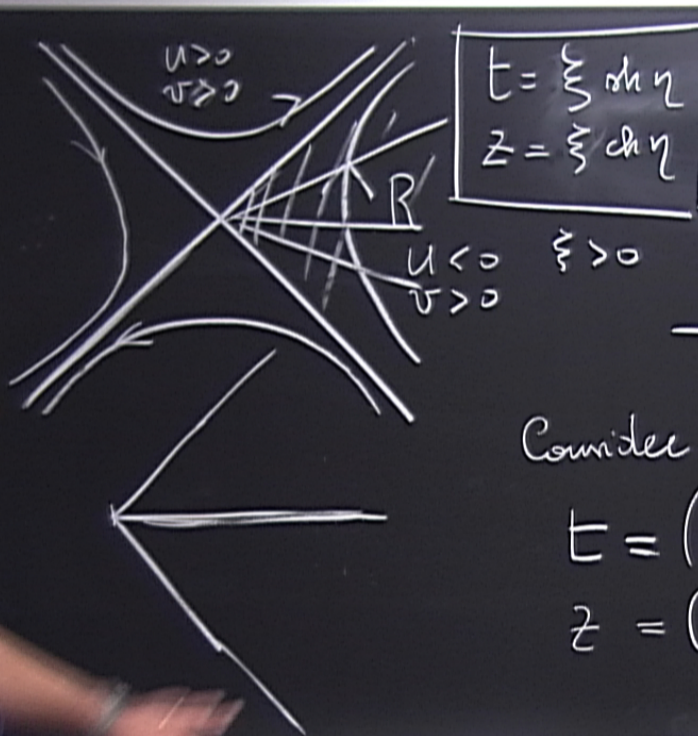
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\zeta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\zeta) \operatorname{ch}(\eta + i\varphi)$$

CAUTION
Do not lean against the screen board.
Do not touch the screen board.
Do not touch the screen board.
Do not touch the screen board.



$$\begin{cases} t = \xi \operatorname{sh} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

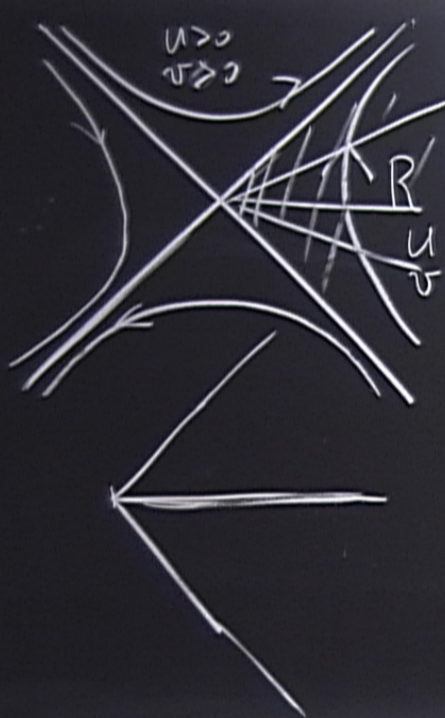
$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\zeta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\zeta + i\xi) \operatorname{ch}(\eta + i\varphi)$$



$$\begin{cases} t = \xi \operatorname{sh} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

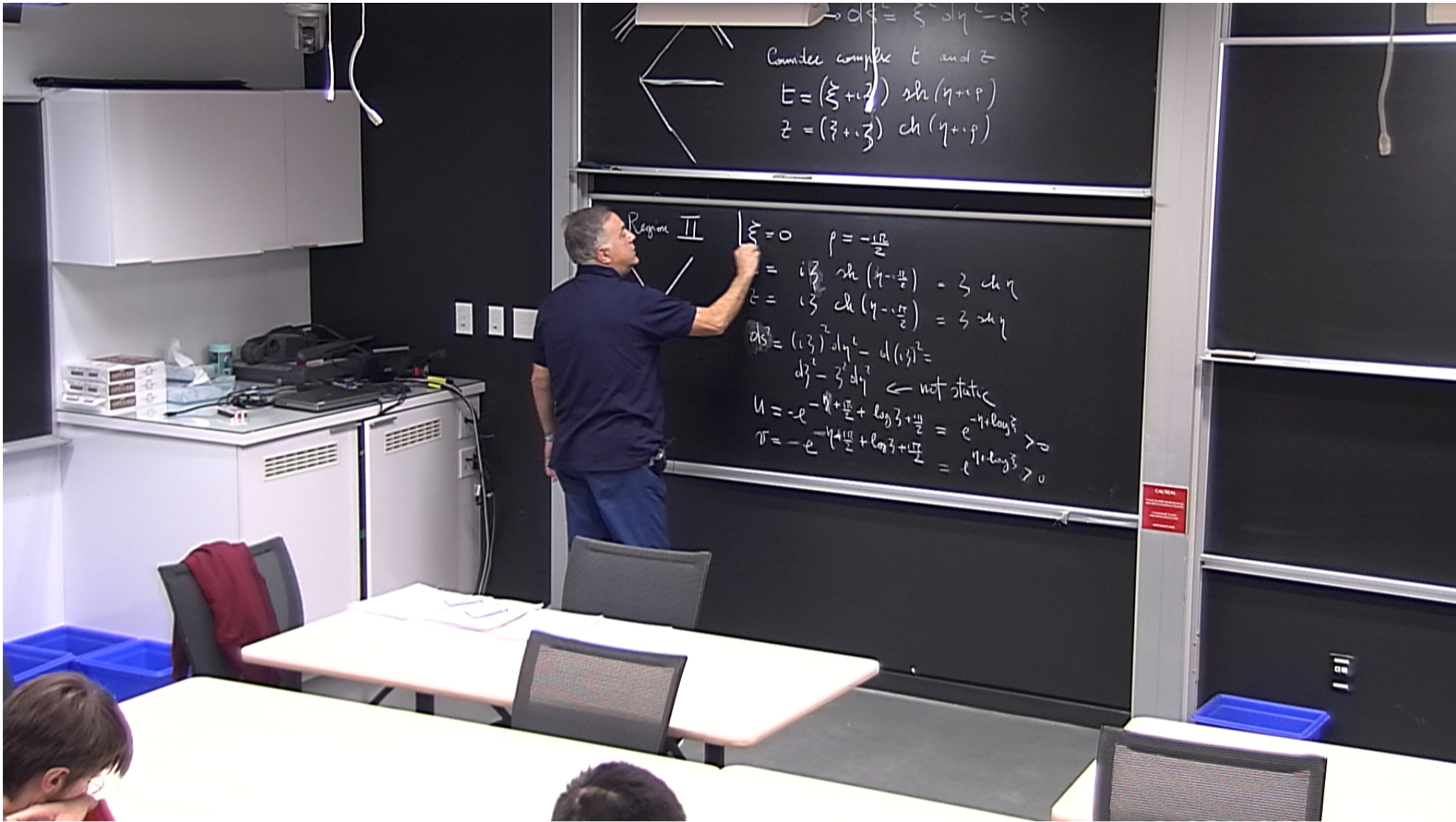
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\zeta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\xi + i\zeta) \operatorname{ch}(\eta + i\varphi)$$

CAUTION
Do not lean against the screen when
using power or the back of the screen.
It is recommended to avoid
using excessive force.
Always maintain safety.



$$\rightarrow ds = \xi dy^t - d\bar{\xi}$$

Coordinates complex t and z

$$t = (\xi + i\bar{\xi}) \operatorname{sh}(\eta + i\rho)$$

$$z = (\bar{\xi} + i\xi) \operatorname{ch}(\eta + i\rho)$$

Region II

$$\xi = 0 \quad \rho = -\frac{i\pi}{2}$$

$$t = i\bar{\xi} \operatorname{sh}\left(\eta - \frac{i\pi}{2}\right) = \bar{\xi} \operatorname{ch} \eta$$

$$z = i\xi \operatorname{ch}\left(\eta - \frac{i\pi}{2}\right) = \xi \operatorname{sh} \eta$$

$$ds^2 = (i\xi)^2 dy^t - d(i\xi)^2 =$$

$$d\bar{\xi}^2 - \xi^2 dy^t{}^2 \leftarrow \text{not static}$$

$$U = -e^{-\eta + \frac{i\pi}{2} + \log \bar{\xi} + \frac{i\pi}{2}} = e^{-\eta + \log \bar{\xi}} > 0$$

$$V = -e^{-\eta + \frac{i\pi}{2} + \log \xi + \frac{i\pi}{2}} = e^{-\eta + \log \xi} > 0$$

$$z = (3 + i3) \operatorname{ch}(\eta + i\varphi)$$

on II

$$\boxed{\xi = 0} \quad \rho = -\frac{iR}{2}$$

$$t = i3 \operatorname{sh}(\eta - i\frac{\pi}{2}) = 3 \operatorname{ch} \eta$$

$$z = i3 \operatorname{ch}(\eta - i\frac{\pi}{2}) = 3 \operatorname{sh} \eta$$

$$ds^2 = (i3)^2 d\eta^2 - d(i3)^2 =$$

$$d3^2 - 3^2 d\eta^2 \leftarrow \text{not static}$$

$$u = -e^{-\eta + \frac{\pi}{2} + \log 3 + \frac{11}{2}} = e^{-\eta + \log 3} > 0$$

$$v = -e^{-\eta + \frac{\pi}{2} + \log 3 + \frac{17}{2}} = e^{\eta + \log 3} > 0$$

$$L = (\xi + i\eta), \quad \text{with } (\xi, \eta) \text{ real}$$

$$z = (3 + i3) \operatorname{ch}(\eta + i\varphi)$$

Region II $\xi = 0$ $\rho = -\frac{iR}{z}$

$$t = i3 \operatorname{sh}(\eta - i\frac{\pi}{2}) = 3 \operatorname{ch} \eta$$

$$z = i3 \operatorname{ch}(\eta - i\frac{\pi}{2}) = 3 \operatorname{sh} \eta$$

$$dL^2 = (i3)^2 d\eta^2 - d(i3)^2 =$$

$$-3^2 - 3^2 d\eta^2 \leftarrow \text{not static}$$

$$u = -e^{-\eta + \frac{\pi}{2} + \log 3 + \frac{i\pi}{2}} = e^{-\eta + \log 3} > 0$$

$$v = -e^{-\eta + \frac{i\pi}{2} + \log 3 + \frac{i\pi}{2}} = e^{\eta + \log 3} > 0$$

$$z = (z + i\bar{z}) \operatorname{ch}(\eta + i\varphi)$$

Re II

$$\xi = 0$$

$$\rho = -\frac{iR}{z}$$

$$t = i\bar{z} \operatorname{sh}(\eta - i\frac{\pi}{2}) = z \operatorname{ch} \eta$$

$$z = i\bar{z} \operatorname{ch}(\eta - i\frac{\pi}{2}) = z \operatorname{sh} \eta$$

$$ds^2 = (i\bar{z})^2 d\eta^2 - d(i\bar{z})^2 =$$

$$\rightarrow d\bar{z}^2 - z^2 d\eta^2 \leftarrow \text{not static}$$

$$u = -e^{-\eta + \frac{i\pi}{2} + \log \bar{z} + \frac{i\pi}{2}} = e^{-\eta + \log \bar{z}} > 0$$

$$v = -e^{-\eta + \frac{i\pi}{2} + \log \bar{z} + \frac{i\pi}{2}} = e^{\eta + \log \bar{z}} > 0$$

$$z = (\xi + i\zeta) \operatorname{ch}(\eta + i\varphi)$$

Region II



$$\boxed{\xi = 0} \quad \rho = -\frac{iR}{z}$$

$$t = i\xi \operatorname{sh}(\eta - \frac{i\pi}{2}) = \xi \operatorname{ch} \eta$$

$$z = i\xi \operatorname{ch}(\eta - \frac{i\pi}{2}) = \xi \operatorname{sh} \eta$$

$$ds^2 = (i\xi)^2 d\eta^2 - d(i\xi)^2 =$$

$$\rightarrow d\xi^2 - \xi^2 d\eta^2 \quad \leftarrow \text{not static}$$

$$u = -e^{-\eta + \frac{i\pi}{2} + \log \xi + \frac{i\pi}{2}} = e^{-\eta + \log \xi} > 0$$

$$v = -e^{-\eta + \frac{i\pi}{2} + \log \xi + \frac{i\pi}{2}} = e^{\eta + \log \xi} > 0$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\rho)$$

Region L

$$\eta = 0$$

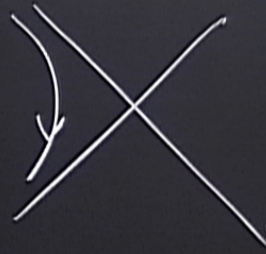
$$\rho = -i\alpha$$

$$(\rho = +i\alpha)$$

$$-\xi \operatorname{sh} \eta$$

$$-\xi \operatorname{ch} \eta$$

$\frac{\partial}{\partial \xi} L$



$$e^{-\eta + i\alpha + \log \xi} = e^{-\eta + \log \xi} > 0$$

$$e^{+\eta - i\alpha + \log \xi} = -e^{\eta + \log \xi} < 0$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\rho)$$

Region L $\xi = 0$ $\rho = -i\alpha$ ($\rho = +i\alpha$)

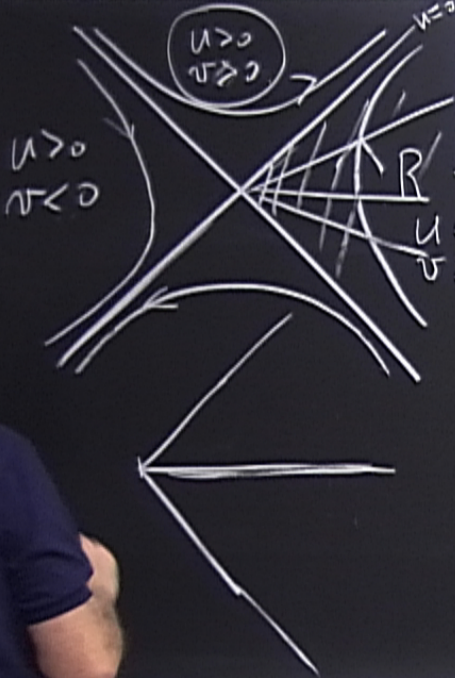
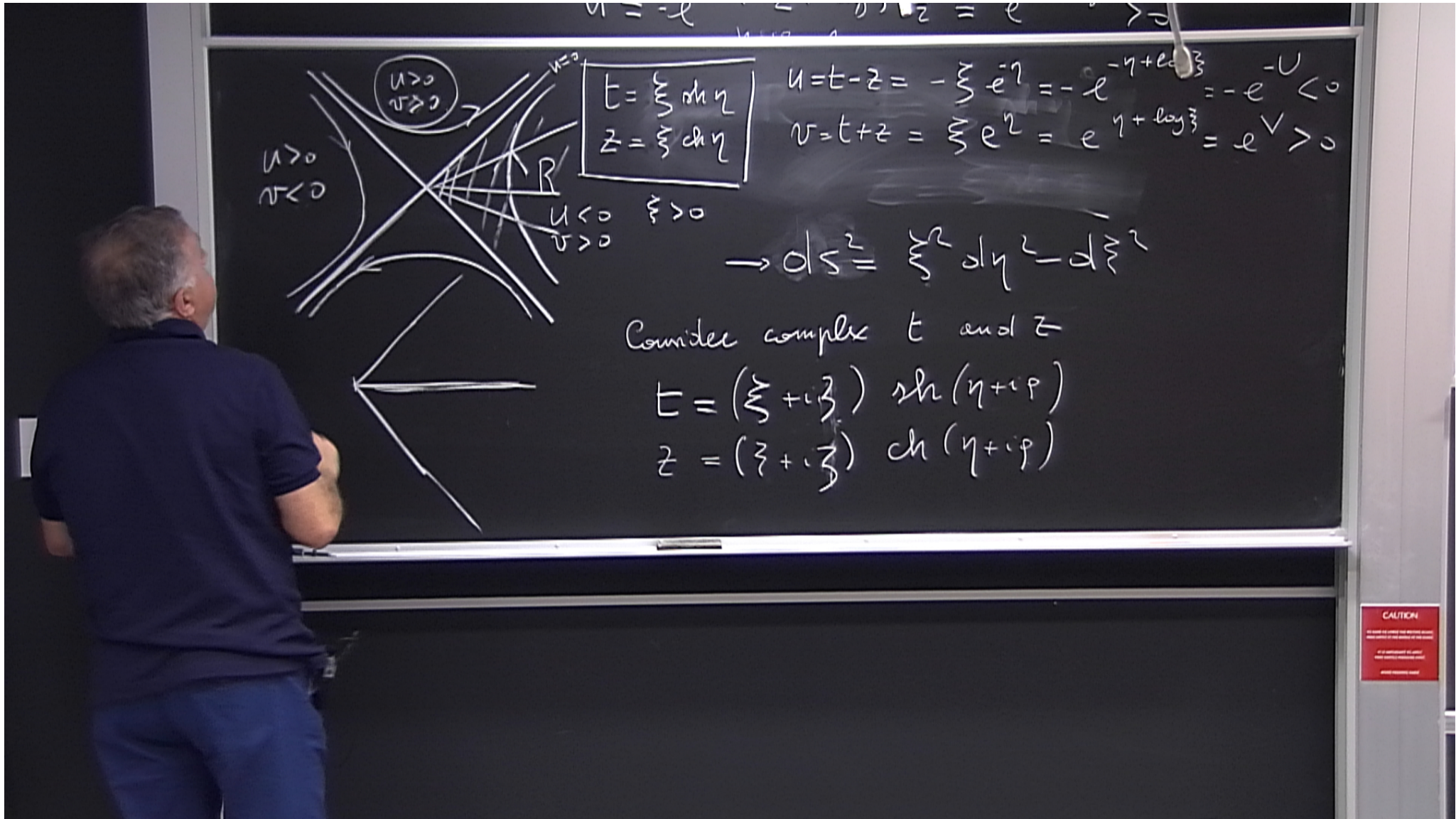
$$t = -\xi \operatorname{sh} \eta$$

$$z = -\xi \operatorname{ch} \eta$$

$$ds^2 = \xi^2 d\eta^2 - d\xi^2$$

$$u = -e^{-\eta + i\alpha + \log \xi} = e^{-\eta + \log \xi} > 0$$

$$v = e^{+\eta - i\alpha + \log \xi} = -e^{\eta + \log \xi} < 0$$



$$\begin{cases} t = \xi \operatorname{ch} \eta \\ z = \xi \operatorname{ch} \eta \end{cases}$$

$$u = t - z = -\xi e^{-\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

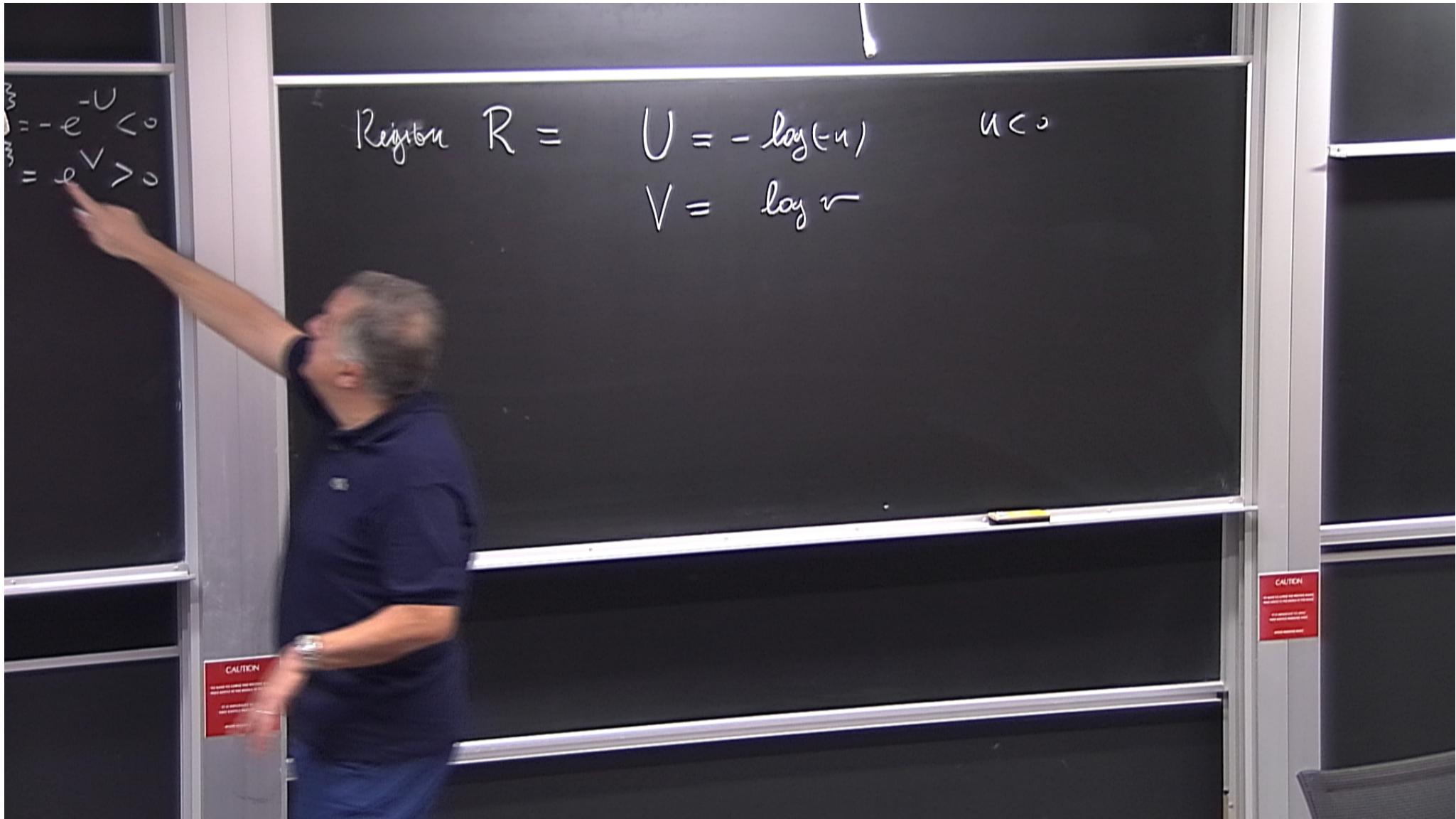
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2$$

Consider complex t and z

$$t = (\xi + i\zeta) \operatorname{sh}(\eta + i\varphi)$$

$$z = (\zeta + i\xi) \operatorname{ch}(\eta + i\varphi)$$

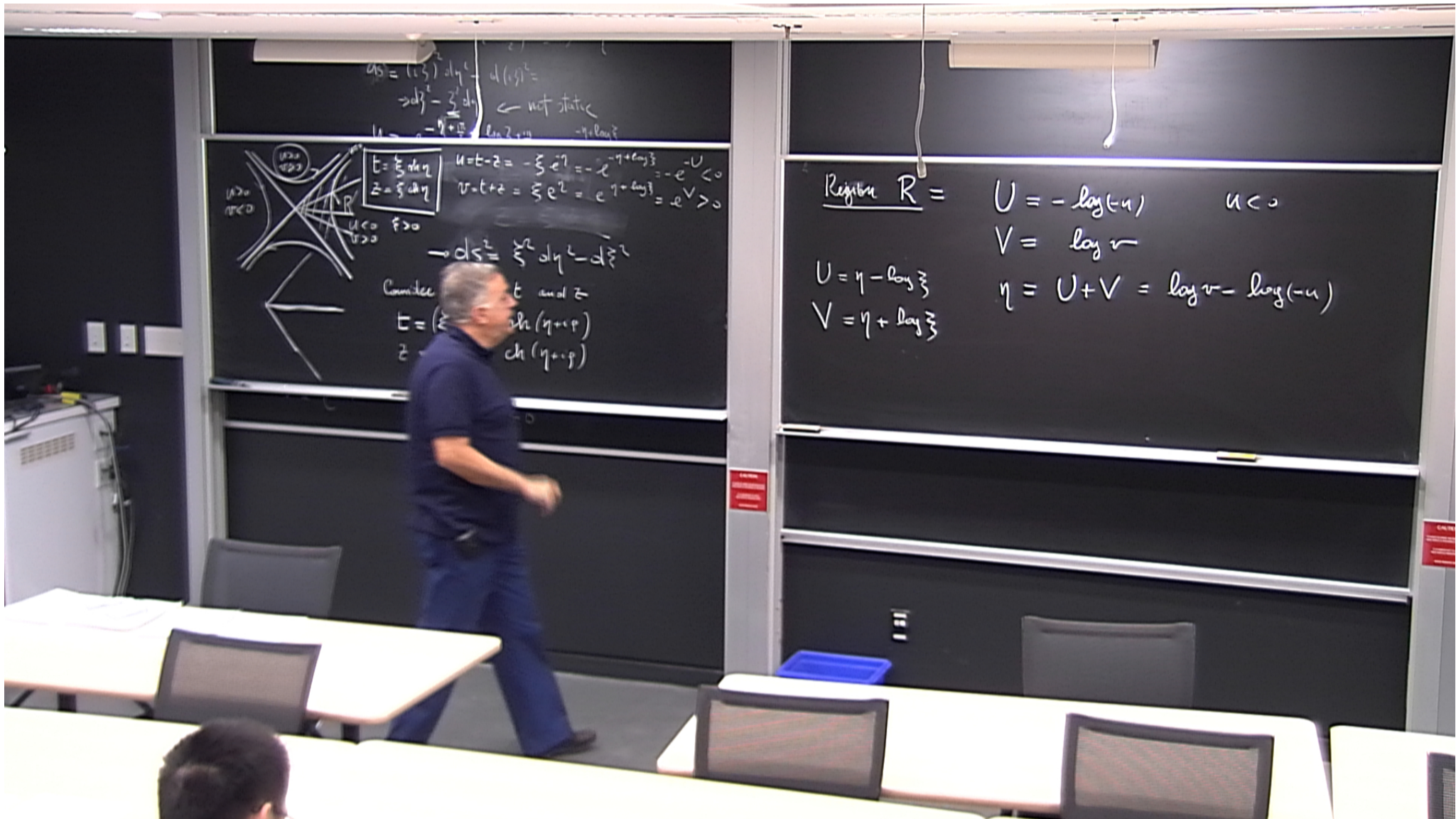
CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD FRAME
OR THE BOARD MOUNTING BRACKET
OR THE BOARD WIRING



$$\begin{aligned} z &= -e^{-U} < 0 \\ &= e^V > 0 \end{aligned}$$

$$\begin{aligned} \text{Region } R = \quad U &= -\log(-u) & u < 0 \\ V &= \log v \end{aligned}$$

$$\eta - \log \xi$$



$$ds = (t, \xi) dy^t \quad d(\xi)^2 =$$

$$\rightarrow d\xi^2 = \xi^2 dy^t - d\xi^2 \quad \leftarrow \text{not static}$$

$$u = -t - z = -\xi e^{\eta} = -e^{-\eta + \log \xi} = -e^{-U} < 0$$

$$v = t + z = \xi e^{\eta} = e^{\eta + \log \xi} = e^V > 0$$

$$t = \xi \operatorname{ch} \eta$$

$$z = \xi \operatorname{sh} \eta$$

$$\rightarrow ds^2 = \xi^2 dy^t - d\xi^2$$

Consider t and z

$$t = \xi \operatorname{ch}(\eta + p)$$

$$z = \xi \operatorname{sh}(\eta + p)$$



Riemann R =

$$U = -\log(-u) \quad u < 0$$

$$V = \log v$$

$$U = \eta - \log \xi$$

$$V = \eta + \log \xi$$

$$\eta = U + V = \log v - \log(-u)$$

Region R = $U = -\log(-u) \quad u < 0$

$V = \log v$

$U = \eta - \log \xi$

$\eta = V = \log v - \log(-u)$

$V = \eta + \log \xi$

$V = \log v$

Region L

Region R = $U = -\log(-u) \quad u < 0$

$$V = \log v$$

$$U = \eta - \log \xi$$

$$\eta = U + V = \log v - \log(-u)$$

$$V = \eta + \log \xi$$

$$V = -\log(-v)$$

Region L

$$U = \log u$$

Region R = $U = -\log(-u)$ $u < 0$

$V = \log v$

$U = \eta - \log \xi$

$\eta = U + V = \log v - \log(-u)$

$V = \eta + \log \xi$

$V = -\log(-v)$ $\eta =$

Region L

$U = \log u$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD FRAME
IF A WARNING IS SHOWN
STOP IMMEDIATELY

Region R = $U = -\log(-u)$ $u < 0$

$V = \log v$

$U = \eta - \log \xi$

$\eta = U + V = \log v - \log(-u)$

$V = \eta + \log \xi$

$V = -\log(-v)$ $\eta = \log u - \log$

Region L

$U = \log u$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD FRAME
IF IT IS DAMAGED OR DEFECTIVE
CONTACT SERVICE CENTER

Region R = $U = -\log(-u)$ $u < 0$

$$V = \log v$$

$$U = \eta - \log \xi$$

$$\frac{U+V}{2} = \frac{1}{2}(\log v - \log(-u))$$

$$V = \eta + \log \xi$$

$$-\log(-v) \quad \eta = \frac{1}{2}(\log u - \log(-v))$$

Region L

$$\log u$$

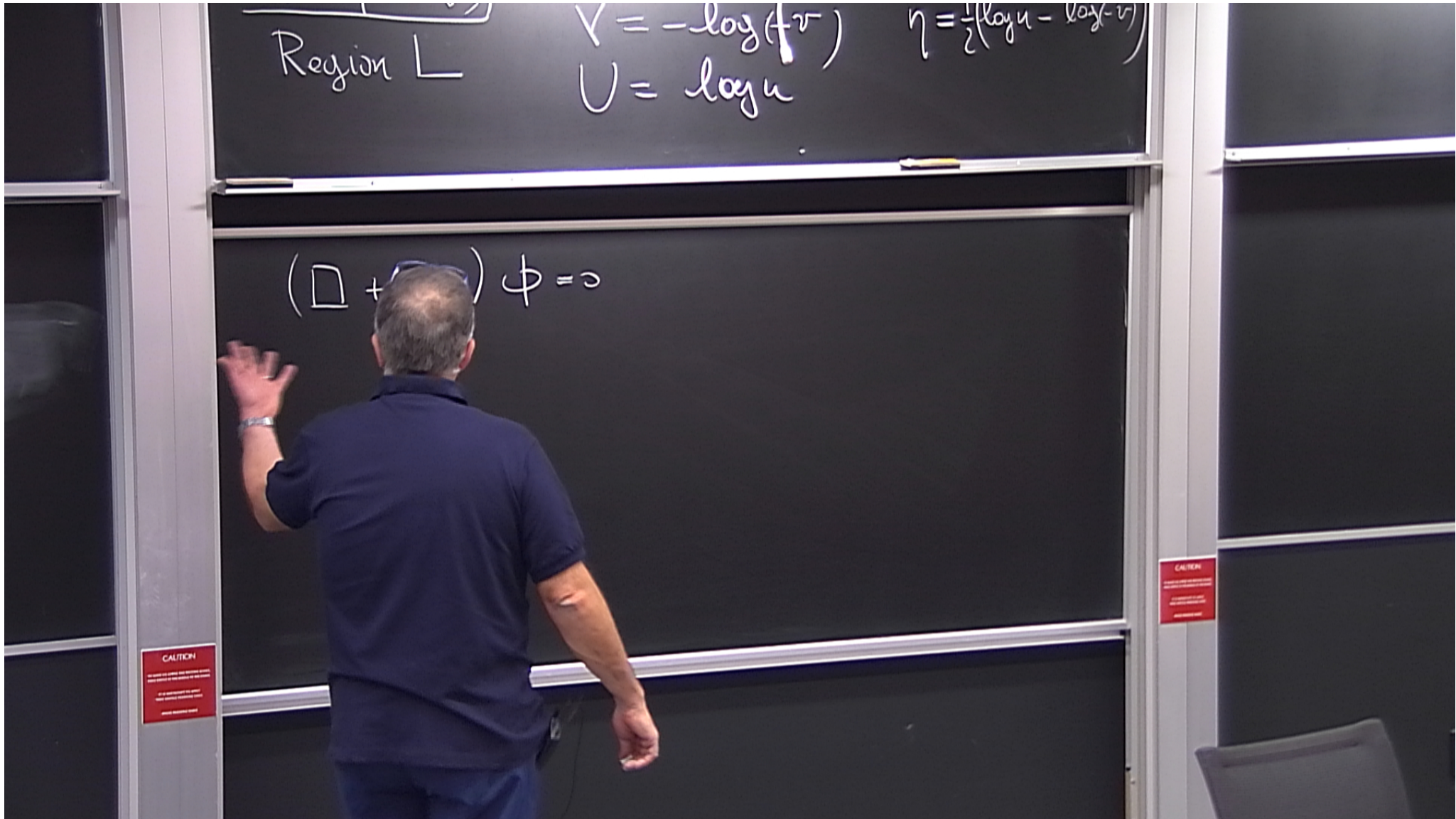
Region R = $U = -\log(-u)$ $u < 0$

$$V = \log v$$

$$U = \eta - \log \xi \quad = \quad \underline{U+V} = \frac{1}{2}(\log v - \log(-u))$$

$$\underline{V = \eta + \log \xi} = -\log(-v) \quad \eta = \frac{1}{2}(\log u - \log(-v))$$

Region L $\log u$



Region L

$$V = -\log(r)$$
$$U = \log u$$

$$\eta = \frac{1}{2}(\log u - \log v)$$

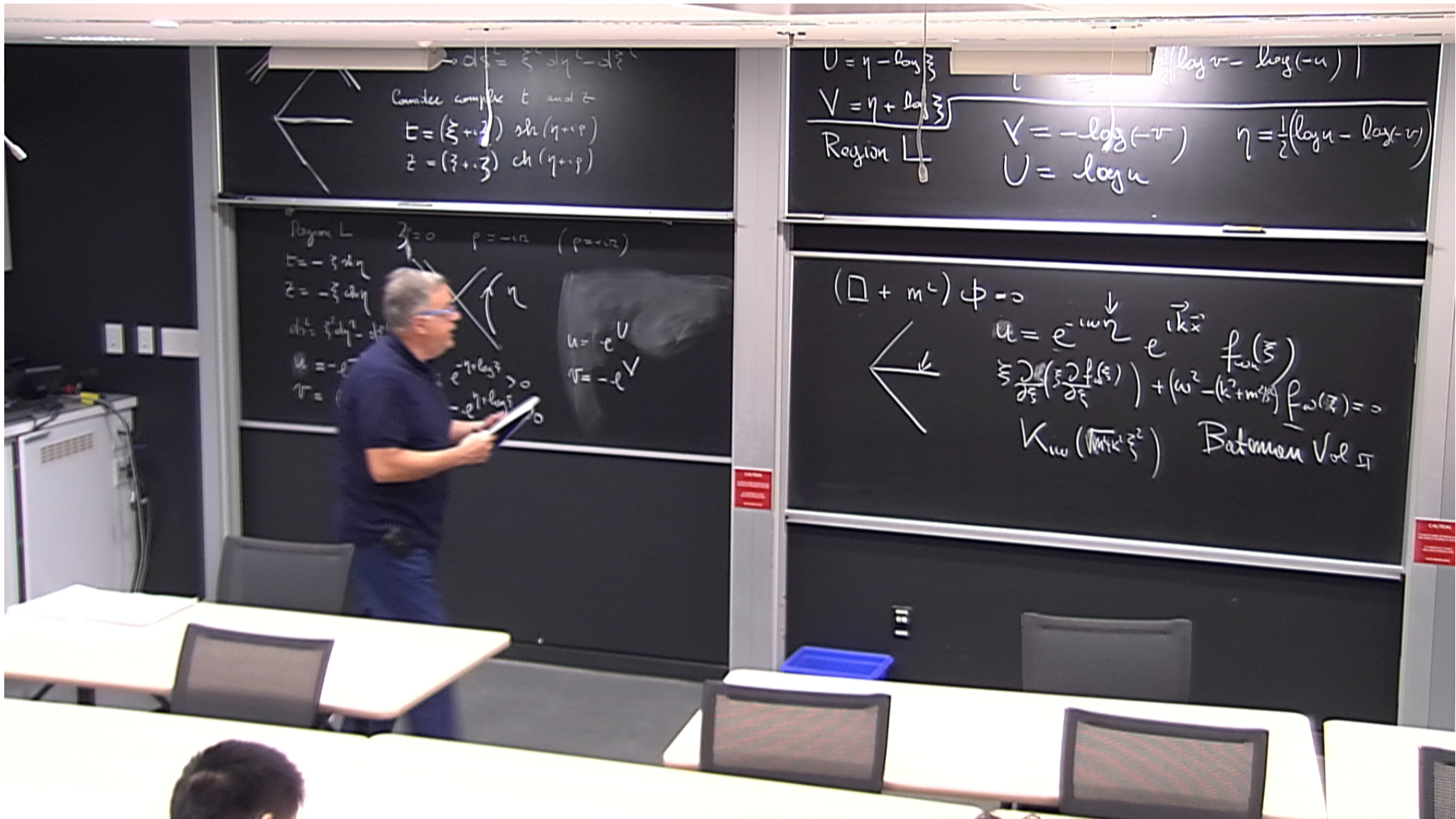
$$m^2) \phi = 0$$

$$u = e^{-i\omega\eta} e^{i\vec{k}\vec{x}} f_{\omega k}(\xi)$$

$$\xi \frac{\partial}{\partial \xi} \left(\xi \frac{\partial f_{\omega k}(\xi)}{\partial \xi} \right) + (\omega^2 - (k^2 + m^2)) f_{\omega k}(\xi) = 0$$

CAUTION

CAUTION

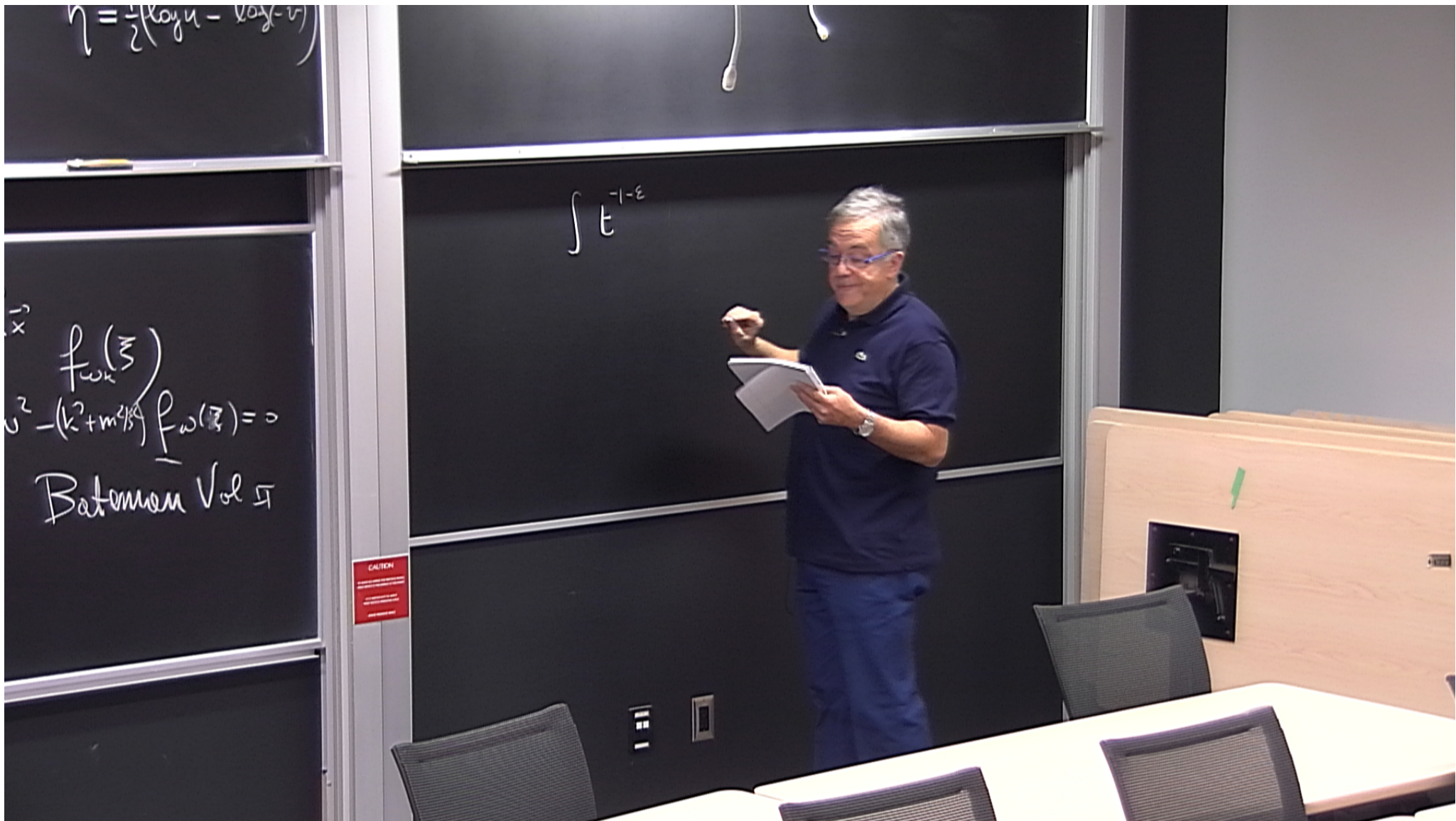


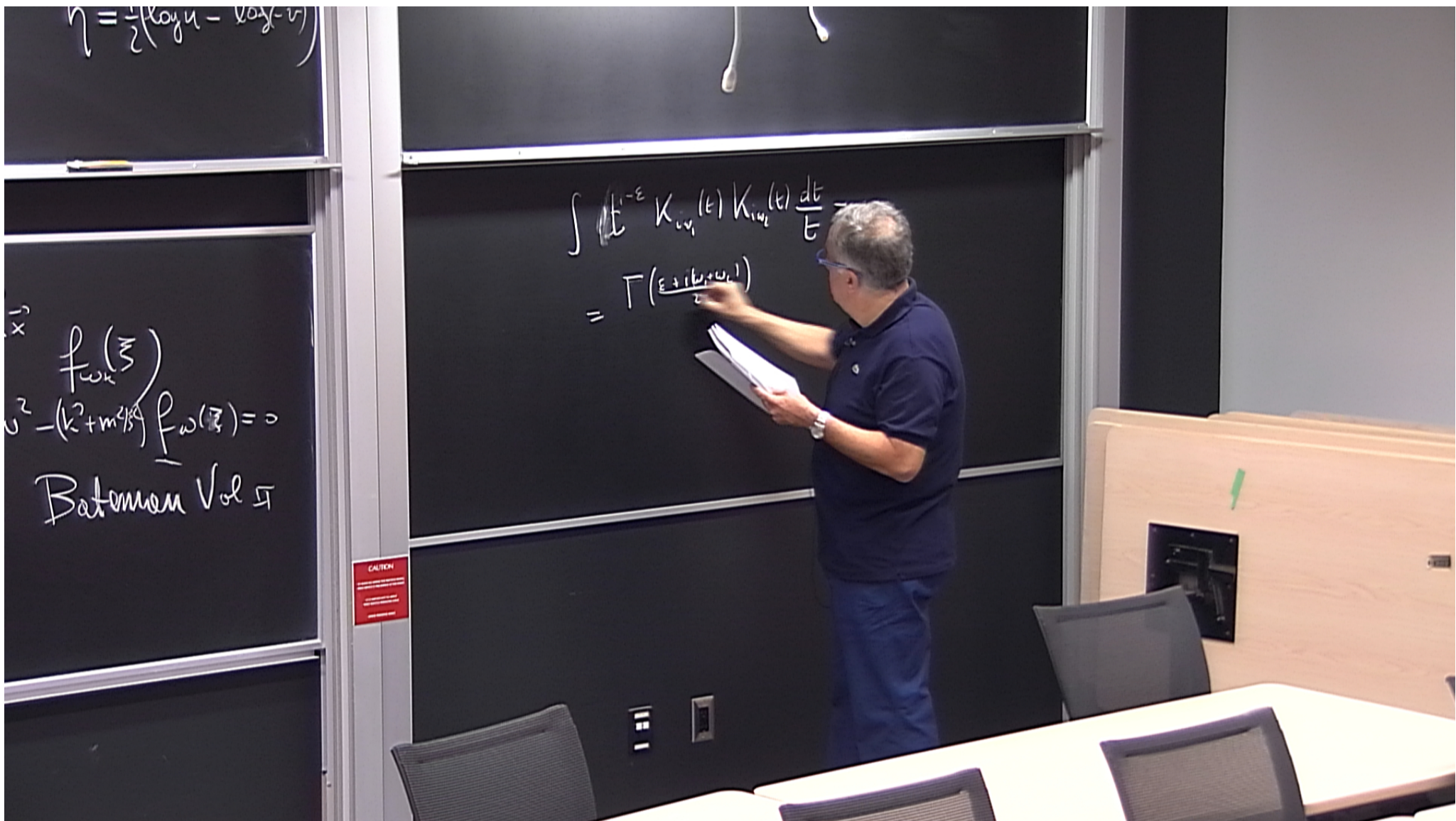
$ds^2 = \xi^2 dy^2 - dx^2$
 Consider complex t and z
 $t = (\xi + i\eta) \operatorname{sh}(\eta + i\rho)$
 $z = (\xi + i\eta) \operatorname{ch}(\eta + i\rho)$

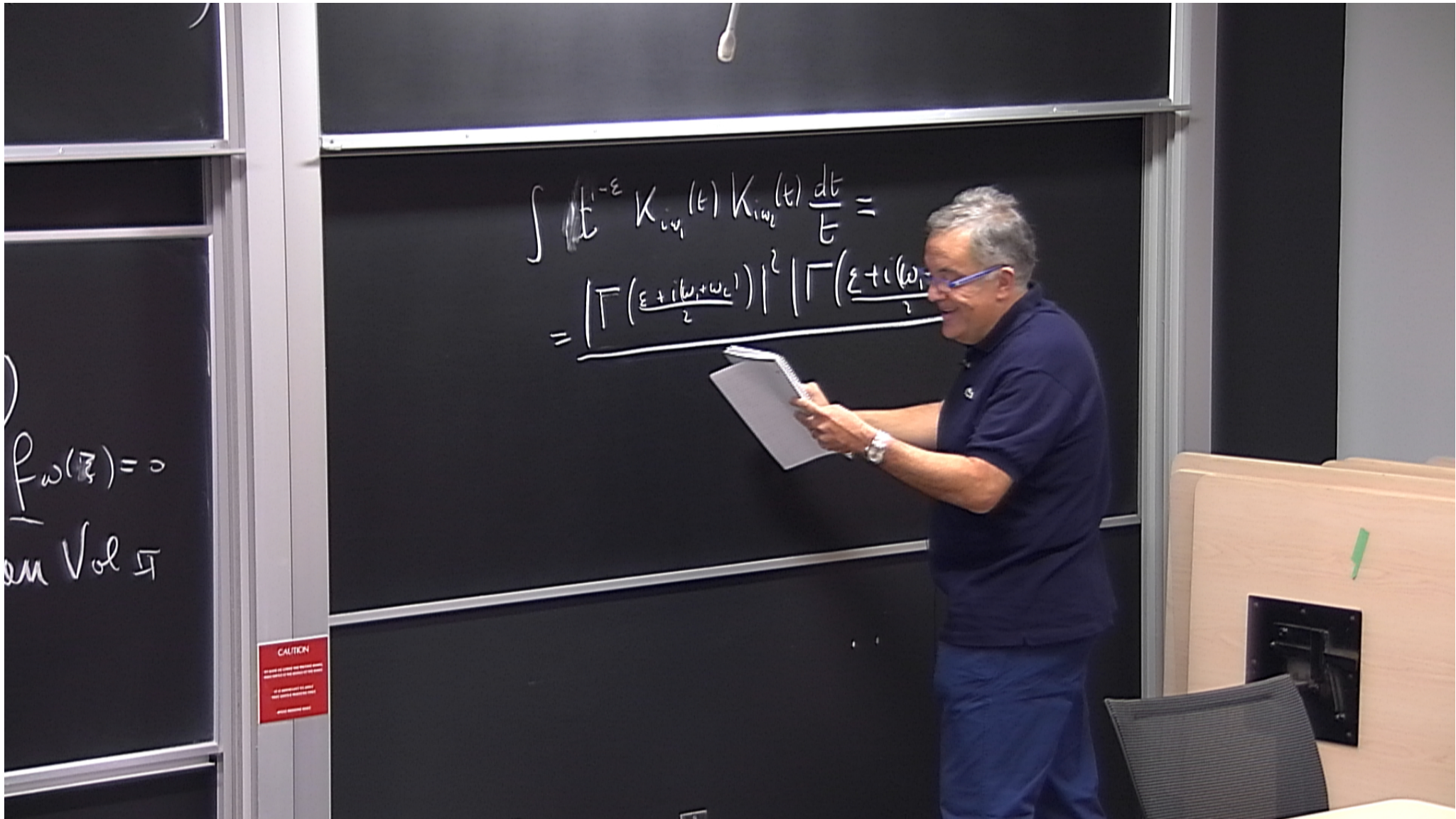
Region L $\xi = 0$ $\rho = -i\alpha$ ($\rho = i\alpha$)
 $t = -\xi \operatorname{sh} \eta$
 $z = -\xi \operatorname{ch} \eta$
 $ds^2 = \xi^2 dy^2 - dx^2$
 $u = -e^{-\eta}$
 $v = e^{\eta}$
 $e^{\eta + \log \xi} > 0$
 $-e^{\eta + \log \xi} < 0$
 $u = -e^V$
 $v = -e^U$

$U = \eta - \log \xi$
 $V = \eta + \log \xi$
 Region L $\left. \begin{array}{l} U = \log u \\ V = -\log(-v) \end{array} \right\} \eta = \frac{1}{2}(\log u - \log(-v))$

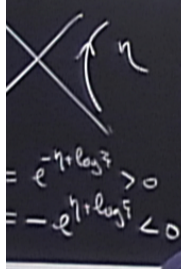
$(\square + m^2) \phi = 0$
 $u = e^{-i\omega \eta} e^{i\vec{k} \cdot \vec{x}} f_{\omega, \vec{k}}(\xi)$
 $\xi \frac{\partial}{\partial \xi} \left(\xi \frac{\partial f}{\partial \xi} \right) + (\omega^2 - (k^2 + m^2)) f = 0$
 $K_{\omega, \vec{k}}(\sqrt{m^2} \xi^2)$ Bessel J_{ν}








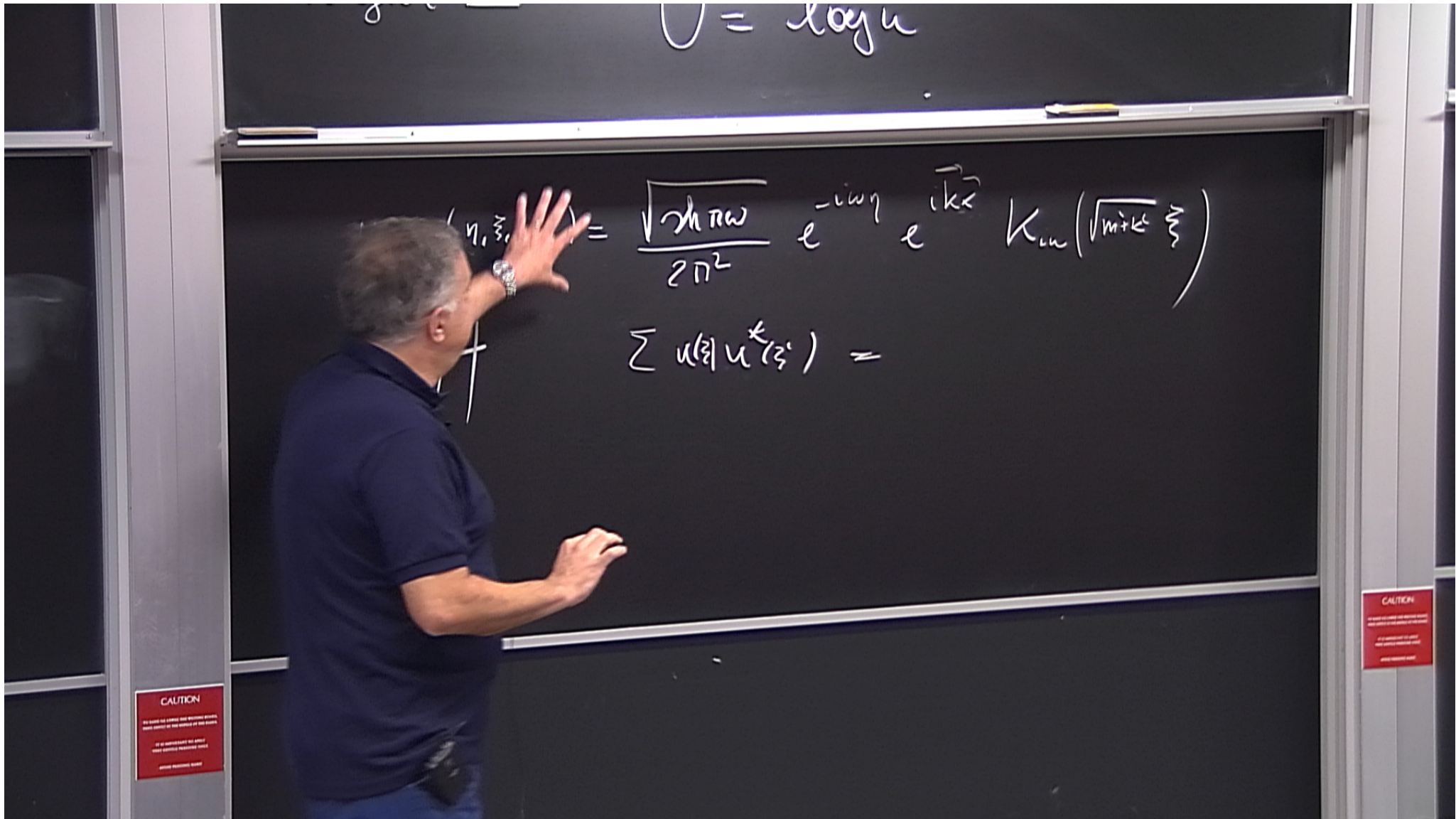
$\rightarrow d\zeta = \xi^{-1} dy^L - d\bar{\zeta}^L$
 complex t and z
 $(\xi + i\eta) \operatorname{sh}(\eta + i\rho)$
 $(\xi + i\eta) \operatorname{ch}(\eta + i\rho)$

$\rho = -i\rho \quad (\rho = +i\rho)$

 $e^{-\eta + \log \xi} > 0$
 $-e^{\eta + \log \xi} < 0$

$U = \eta - \log \xi$
 $V = \eta + \log \xi$
 Region L $\left\{ \begin{array}{l} \log v - \log(-u) \\ \log v - \log(-u) \end{array} \right.$
 $V = -\log(-v) \quad \eta = \frac{1}{2}(\log u - \log(-v))$
 $U = \log u$

$u_{w,\xi}(\eta, \bar{\xi}, v, \eta) = \frac{\sqrt{2\pi w}}{2\pi^2} e^{-iw\eta} e^{ik\bar{\xi}} K_{iw}(\sqrt{wv\xi})$


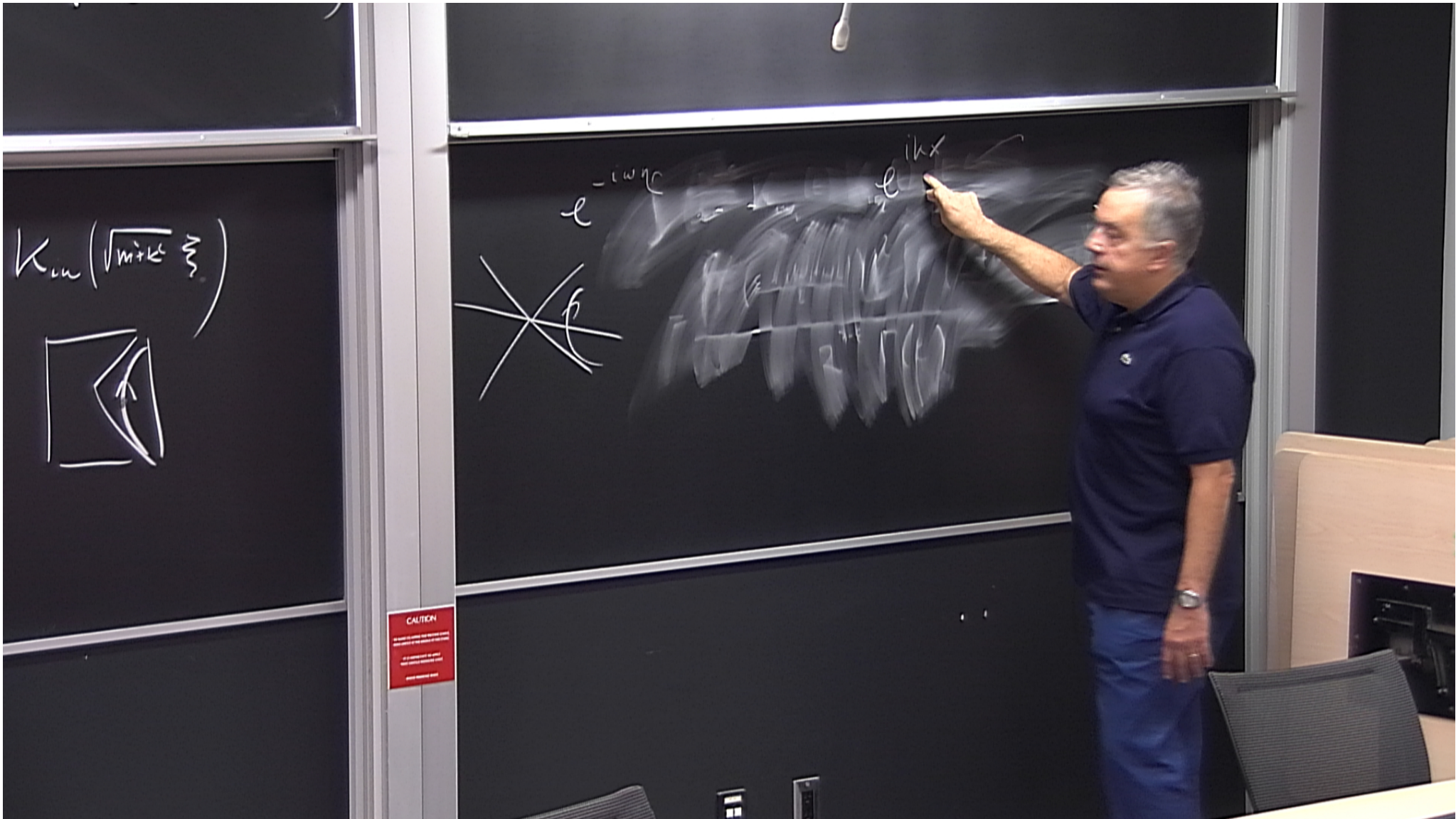
$\int_0^\infty t^{-\varepsilon} K_{iw}(t) K_{iw}(t) \frac{dt}{t} =$
 $= \frac{|\Gamma(\frac{\varepsilon + i(w + iw_c)}{2})|^2 |\Gamma(\varepsilon)|}{2^{2-\varepsilon} \Gamma(\varepsilon)}$

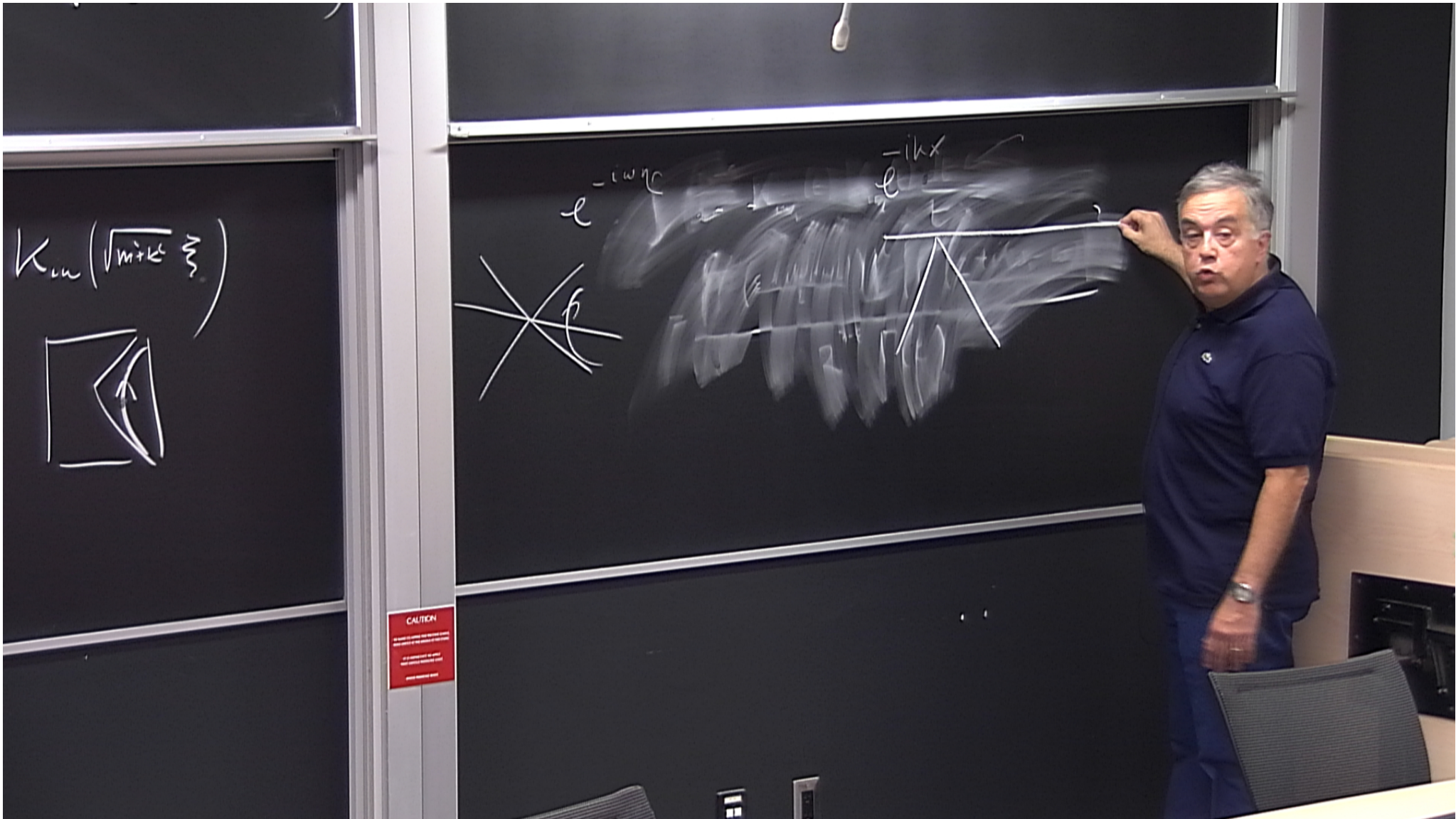


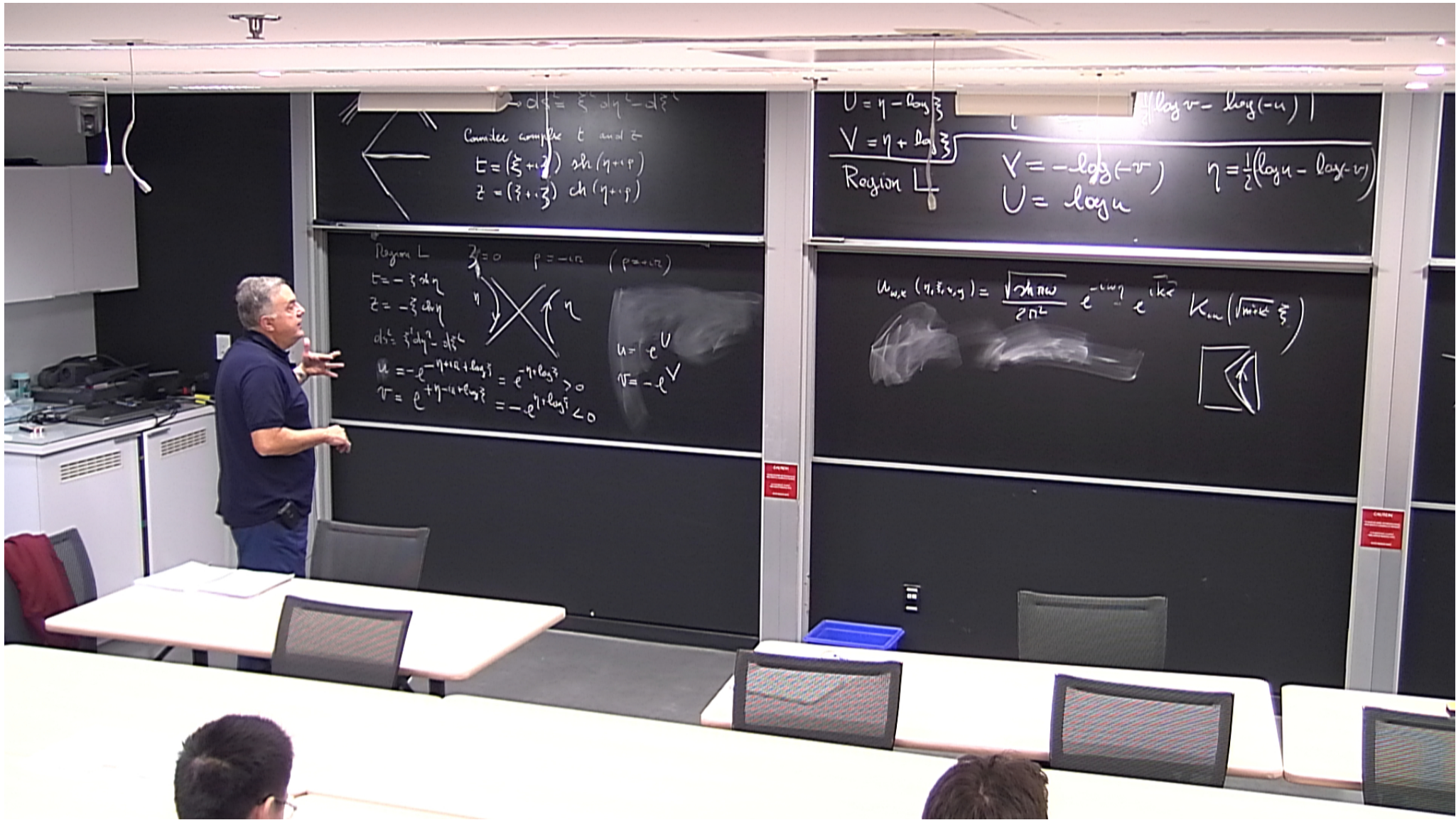
$$U = \log u$$

$$u(r, z) = \frac{\sqrt{h \pi \omega}}{2 \pi^2} e^{-i \omega t} e^{i k z} K_{\nu}(\sqrt{m^2 + k^2} z)$$

$$\sum u(r) u^*(z) =$$





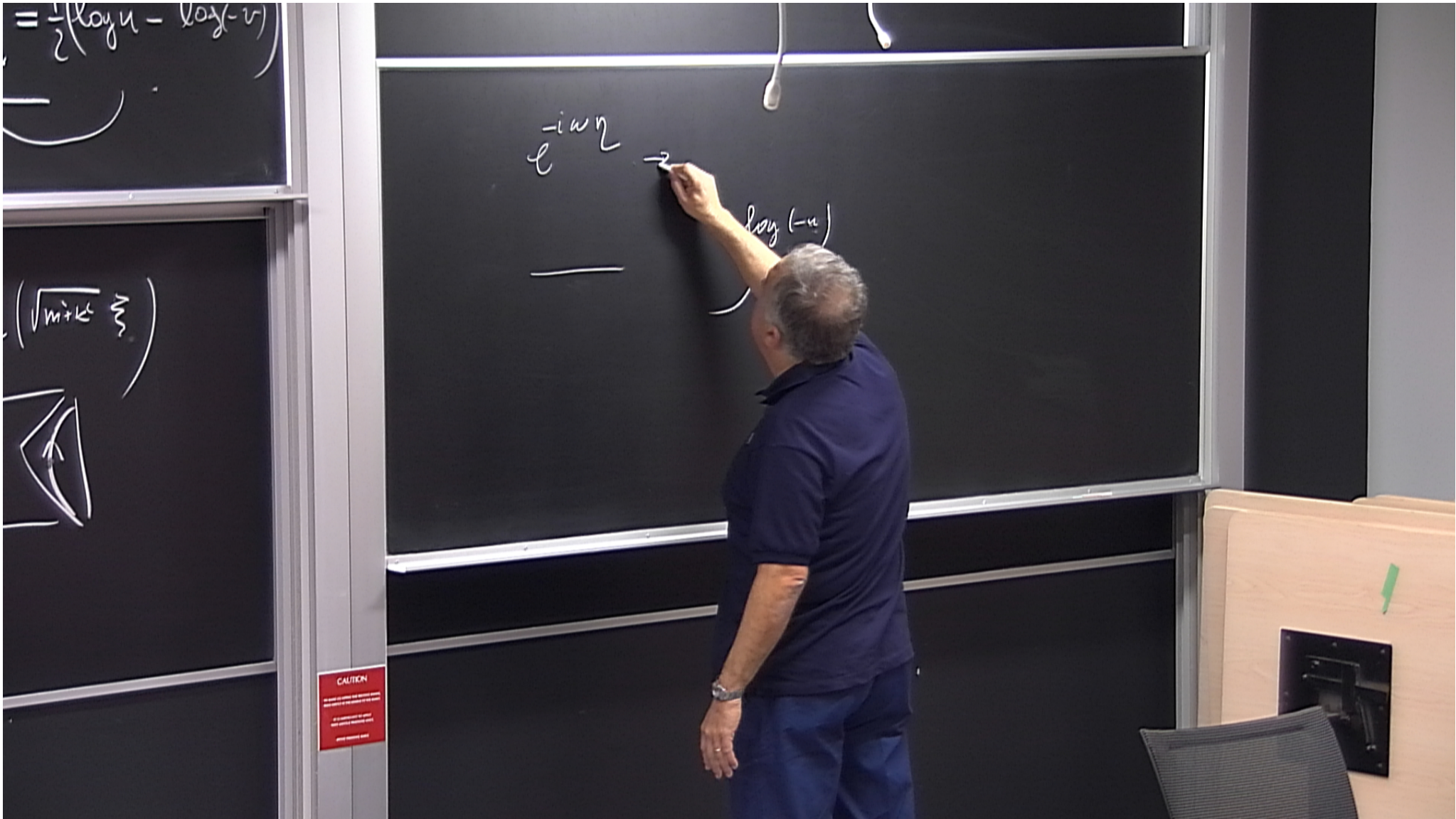


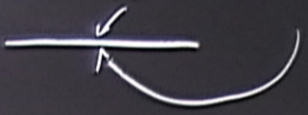
$dz = \xi d\eta - d\xi$
 Consider complex t and z
 $t = (\xi + i\eta) \operatorname{sh}(\eta + i\xi)$
 $z = (\xi + i\eta) \operatorname{ch}(\eta + i\xi)$


Region L
 $t = -\xi d\eta$
 $z = -\xi d\eta$
 $dz = \xi d\eta - d\xi$
 $u = -e^{-\eta + i\xi + \log \xi} = e^{-\eta + \log \xi} > 0$
 $v = e^{+\eta - i\xi + \log \xi} = -e^{\eta + \log \xi} < 0$
 $u = -e^u$
 $v = -e^v$

$U = \eta - \log \xi$
 $V = \eta + \log \xi$
 Region L
 $V = -\log(-v)$
 $U = \log u$
 $|\log v - \log(-u)|$
 $\eta = \frac{1}{2}(\log u - \log(-v))$

$w_{n,l}(r, \theta, \phi) = \frac{\sqrt{kappa*n*w}}{2*pi^2} e^{-i*kappa*r} e^{i*kappa*theta} K_{nu}(\sqrt(kappa*r)*xi)$



$$\eta = \frac{1}{2}(\log u - \log v)$$



$$ikx \quad K_{in}(\sqrt{m+ik} \xi)$$



$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - \rho\omega}$$


$$U_R \rightarrow \left. \begin{array}{l} \text{---} \\ \end{array} \right\} U_R$$



CAUTION
 ATTENTION
 ATTENTION

$$\eta = \frac{1}{2}(\log u - \log v)$$



$$K_{in}(\sqrt{m+k} \xi)$$



$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - \tau\omega}$$


$$U_R \rightarrow \begin{cases} U_R \\ \lambda U_L^* e^{-\tau\omega} \\ e^{-i\omega\eta} \end{cases}$$


A man in a dark blue polo shirt is standing on the right side of the chalkboard, pointing with his right hand towards the shaded region in the diagram above.

CAUTION

$$\eta = \frac{1}{2}(\log u - \log v)$$


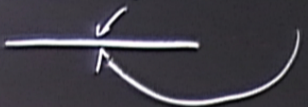
$$K_{in}(\sqrt{m+k} \xi)$$



$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - \tau\omega}$$

$$U_R \rightarrow \begin{cases} \gamma U_R \\ \lambda U_L^* e^{-\tau\omega} (1-e^{-\tau\omega})^{-1/2} \end{cases}$$




CAUTION


$$\eta = \frac{1}{2}(\log u - \log v)$$


$$K_{in}(\sqrt{m+k} \xi)$$


$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - R\omega}$$

$$U_R \rightarrow \begin{cases} U_R \\ U_L^* \end{cases} e^{-R\omega} \begin{matrix} (1-e^{-R\omega})^2 \\ (1-e^{-2R\omega})^2 \end{matrix}$$

$$e^{-i\omega\eta}$$

$$U_L$$




CAUTION

$\eta = \dots$
 \dots
 \dots
 \dots
 \dots
 $u = -e^u$
 $v = -e^v$

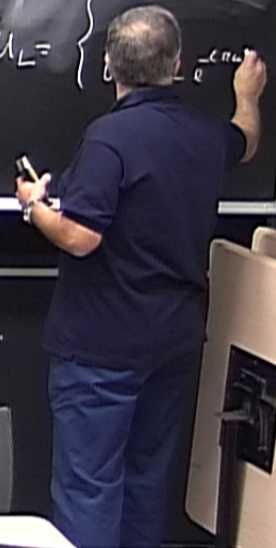
$$\begin{aligned}
 & U = \eta - \log \xi \\
 & V = \eta + \log \xi \\
 & \text{Region I} \\
 & V = -\log(-v) \quad \eta = \frac{1}{2}(\log u - \log(-v)) \\
 & U = \log u
 \end{aligned}$$

$$u_{w,r}(\eta, \bar{r}, \eta) = \frac{\sqrt{2k\pi\omega}}{2\pi^2} e^{-i\omega\eta} e^{ik\bar{r}} K_{i\omega}(\sqrt{2k\pi}\xi)$$

$u_L = e^{+i\omega\eta}$
 $u_R = e^{-i\omega\eta}$
 $u = u_R + u_L$

$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - R\omega}$$

$u_R \rightarrow \begin{cases} u_R \\ u_L^* e^{-R\omega} \end{cases}$
 $u_L = \begin{cases} u_L \\ u_R e^{-R\omega} \end{cases}$



$$u = e^U$$

$$v = -e^V$$

$$U = \eta - \log \xi$$

$$V = \eta + \log \xi$$

$$\text{Region I}$$

$$V = -\log \xi(-v) \quad \eta = \frac{1}{2}(\log u - \log(-v))$$

$$U = \log u$$

$$u_{w,r}(\eta, \bar{r}, \eta) = \frac{\sqrt{2\pi w}}{2\pi^2} e^{-w\eta} e^{ik\bar{r}} K_{iw}(\sqrt{w}k\xi)$$

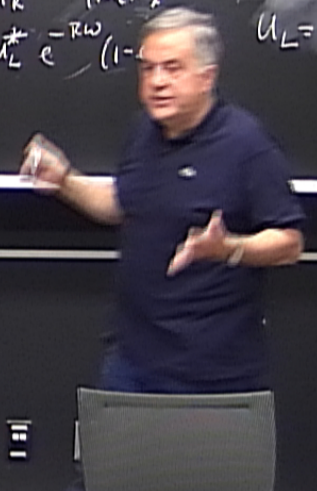
$$u_L = e^{-w\eta}$$

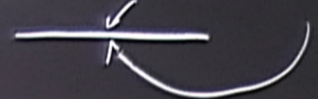
$$u_R = e^{-w\eta}$$


$$e^{-iw\eta} \rightarrow e^{-iw\eta - R\omega}$$

$$u_R \rightarrow \begin{cases} u_R \\ u_L^* e^{-R\omega} \end{cases}$$

$$u_L = \begin{cases} u_L^* e^{-R\omega} (1 - e^{-R\omega})^{-1} \\ u_L (1 - e^{-R\omega})^{-1} \end{cases}$$



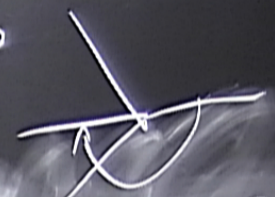
$$\eta = \frac{1}{2}(\log u - \log v)$$


$$K_{in}(\sqrt{m+k} \xi)$$


$$e^{-iky}$$

$$\text{Im} t < 0$$

$e^{-i\omega\eta} \rightarrow e^{-i\omega_L - \tau\omega}$



$$U_R \rightarrow \begin{cases} U_R \\ U_L^* e^{-\tau\omega} (1-e^{-2\tau\omega})^{-1/2} \end{cases}$$

$$U_L = \begin{cases} U_R^* e^{-\tau\omega} (1-e^{-2\tau\omega})^{-1/2} \\ U_L (1-e^{-\tau\omega})^{-1/2} \end{cases}$$

$e^{-i\omega\eta}$

CAUTION

$$\frac{1}{2}(\log u - \log(-v))$$

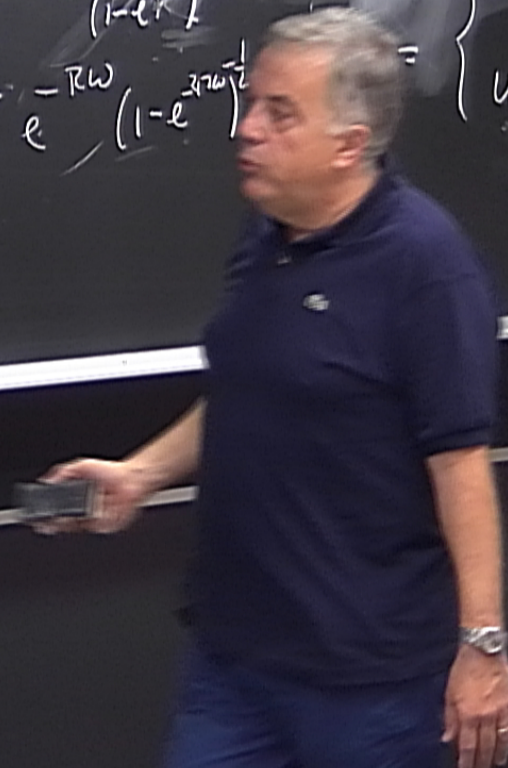
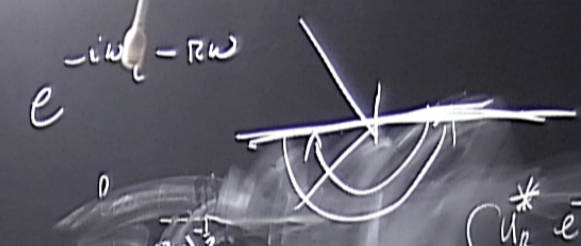
$$m+k \approx \xi$$

$$-iky$$

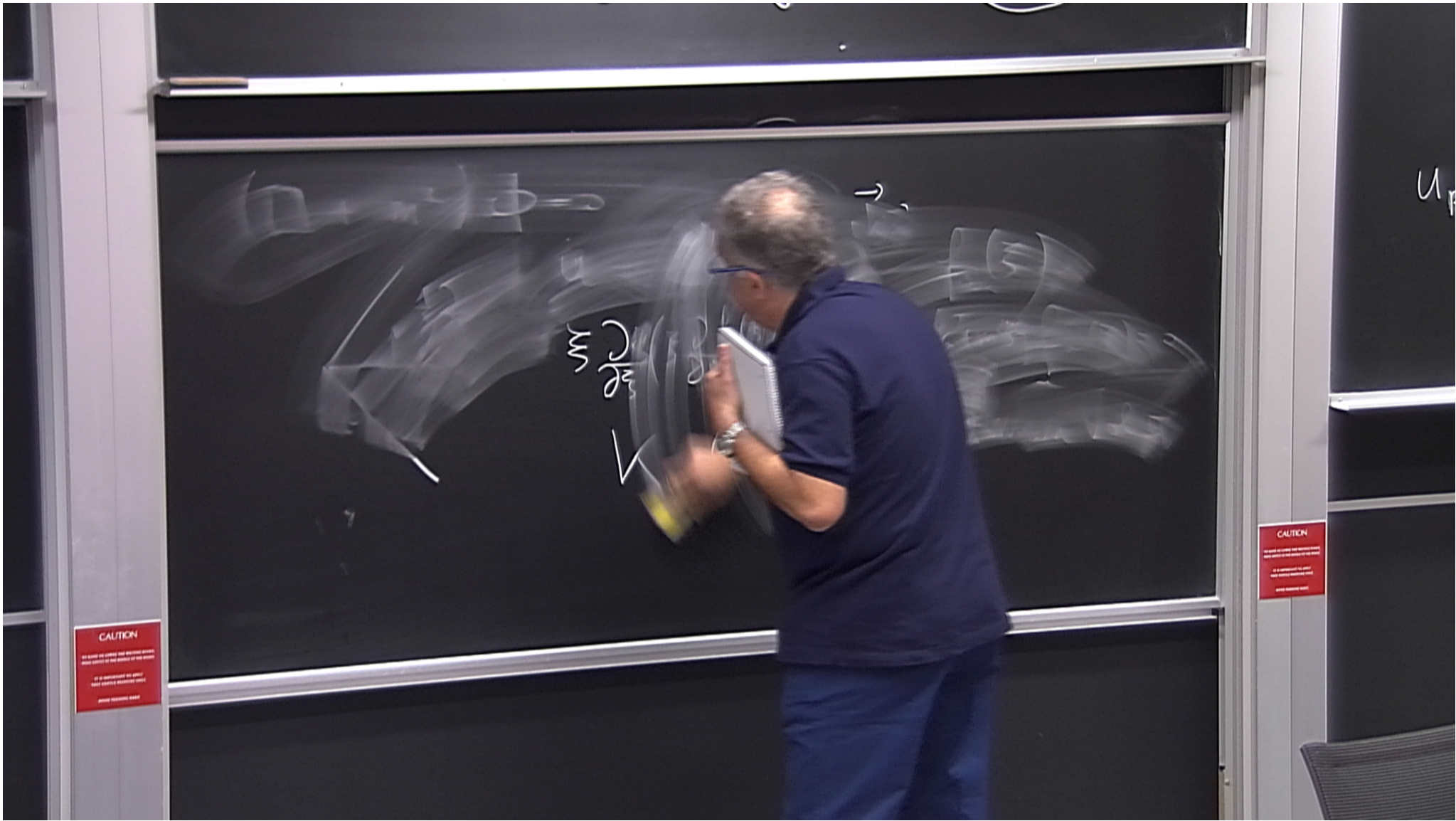
$$\int u t < 0$$

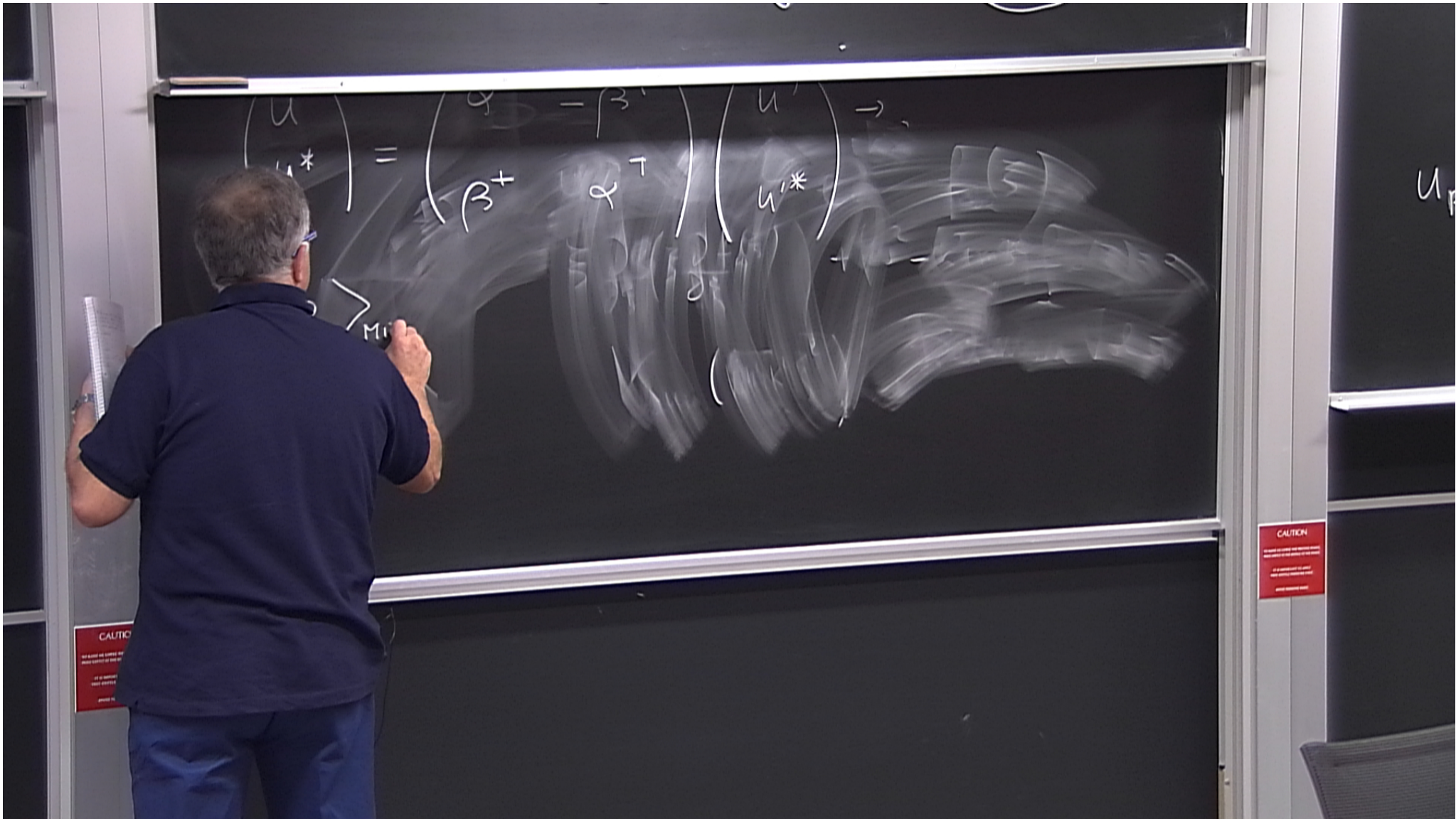
$$e^{-i\omega\eta} \rightarrow e^{-i\omega\eta - R\omega}$$

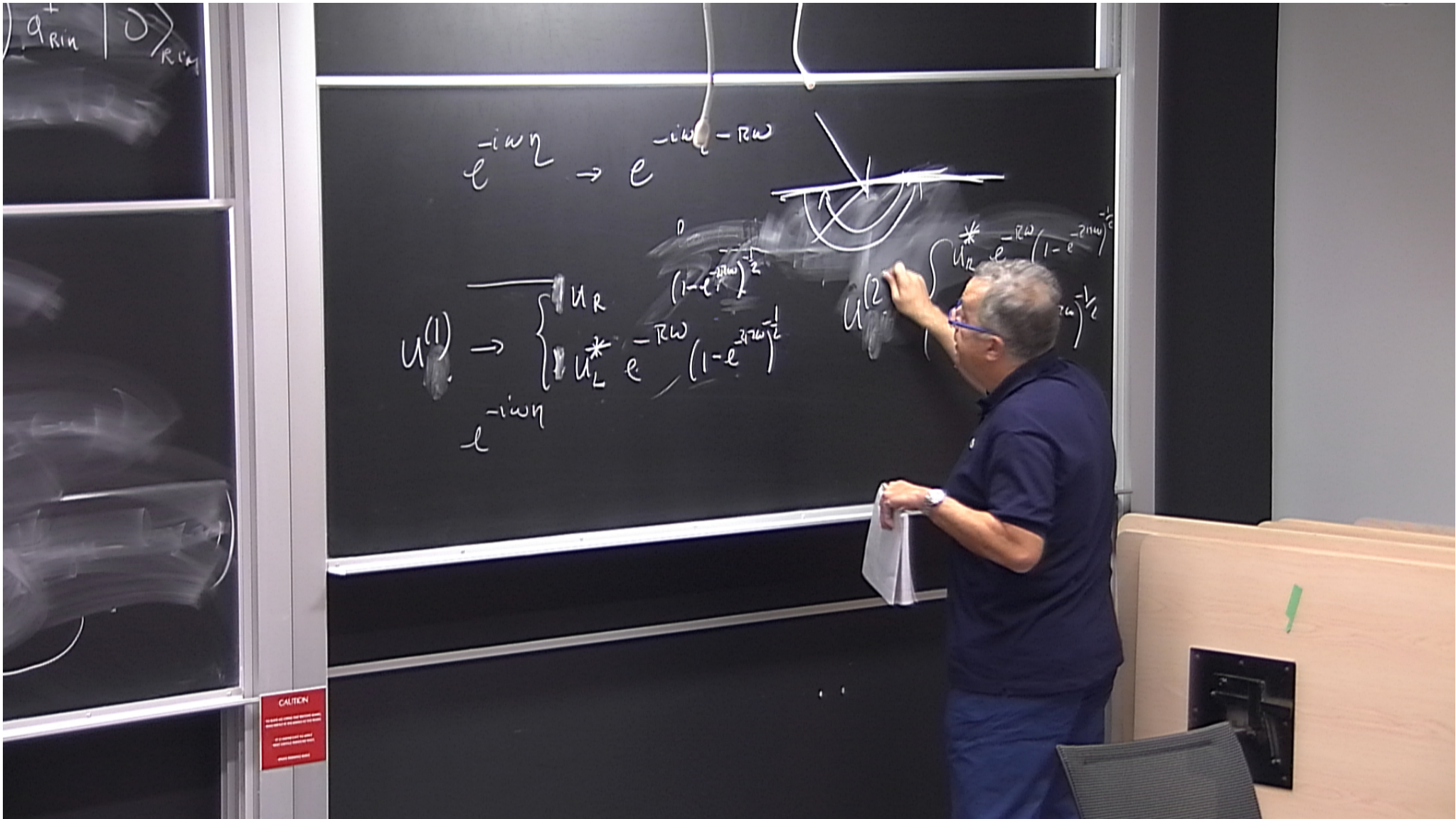
$$U_R \rightarrow \begin{cases} U_R \\ U_L^* e^{-R\omega} (1-e^{-2i\omega})^{-1/2} \end{cases} = \begin{cases} U_R^* e^{-R\omega} (1-e^{-2i\omega})^{-1/2} \\ U_L (1-e^{-2i\omega})^{-1/2} \end{cases}$$

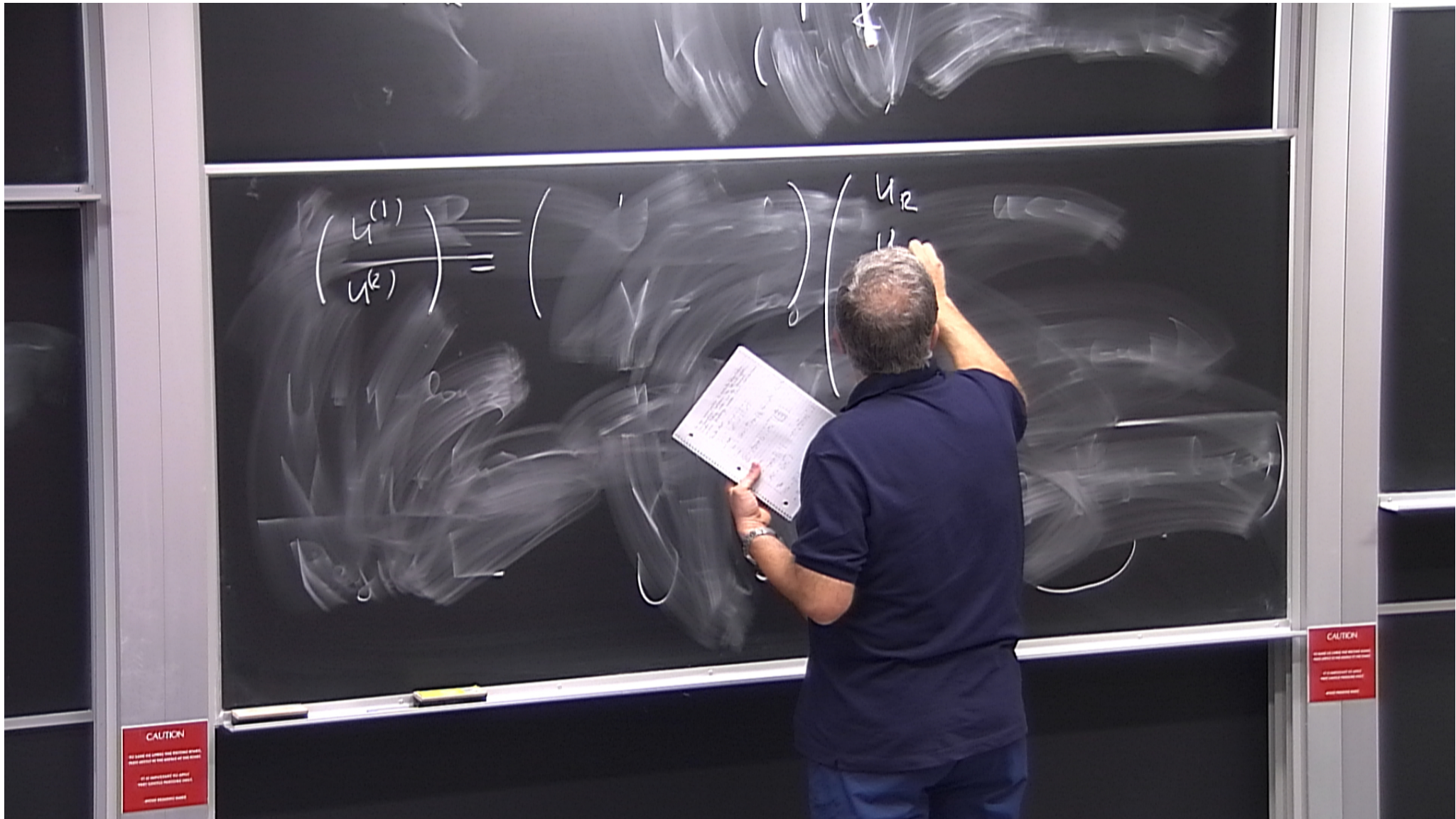


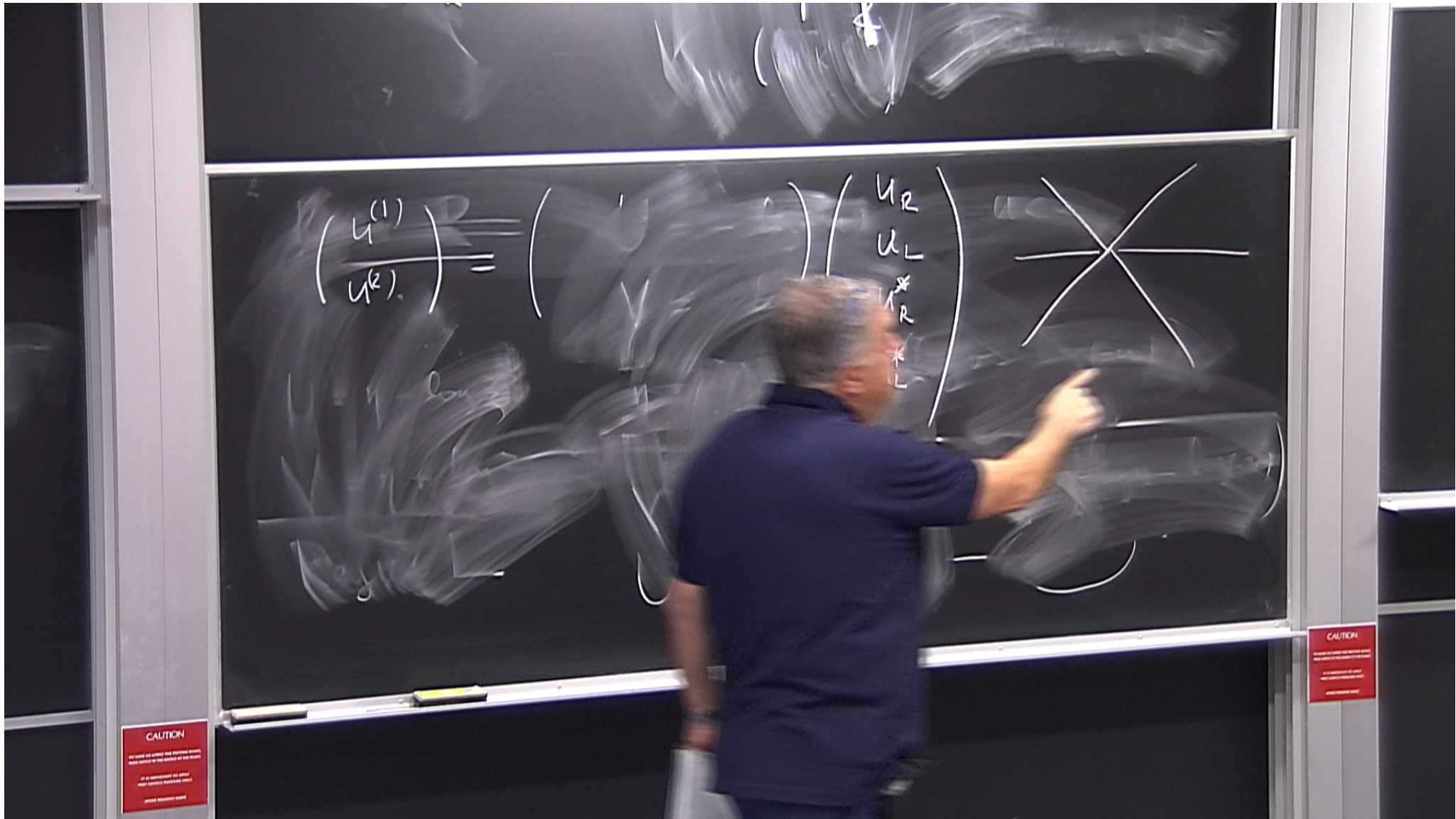
CAUTION

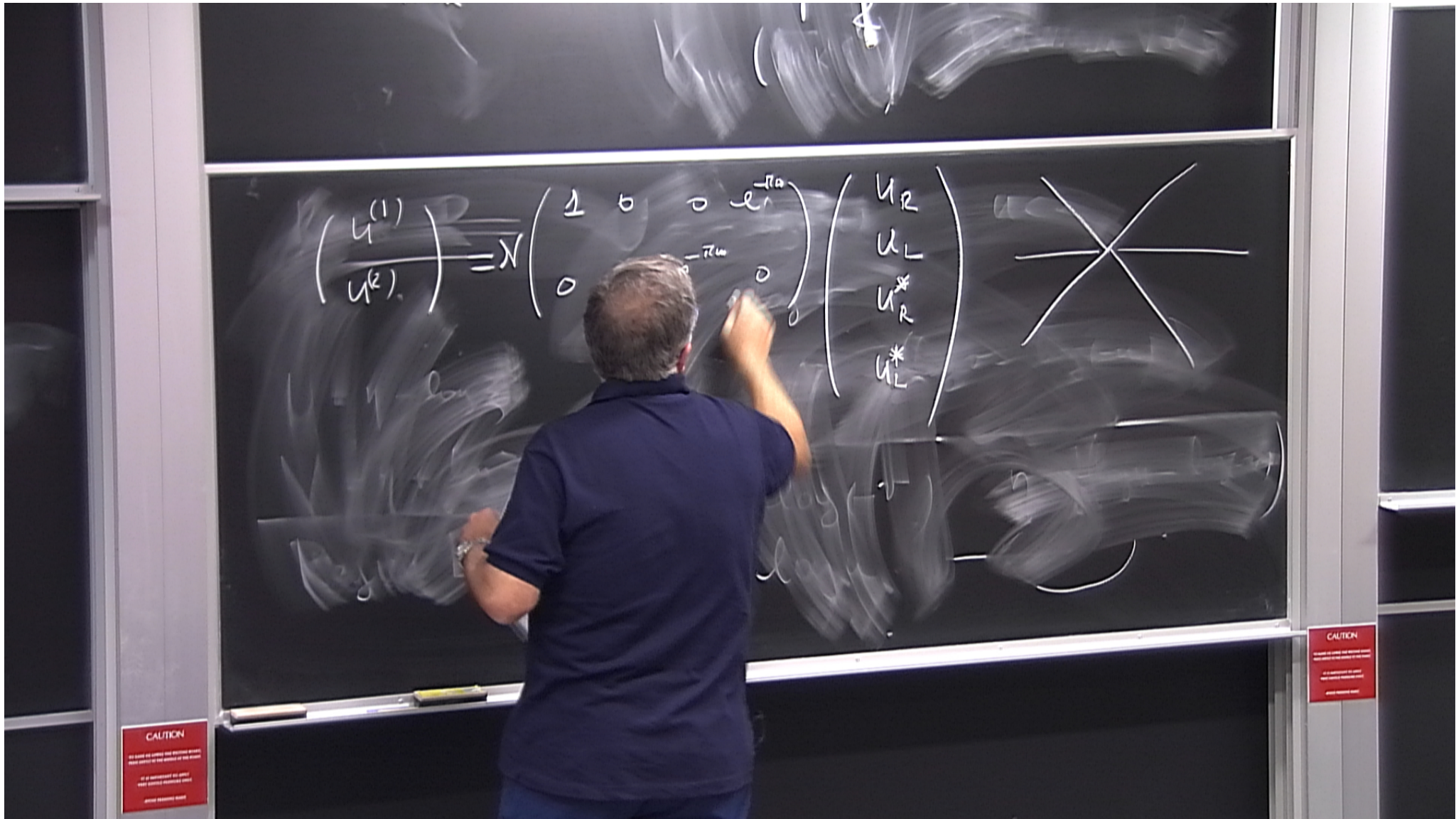


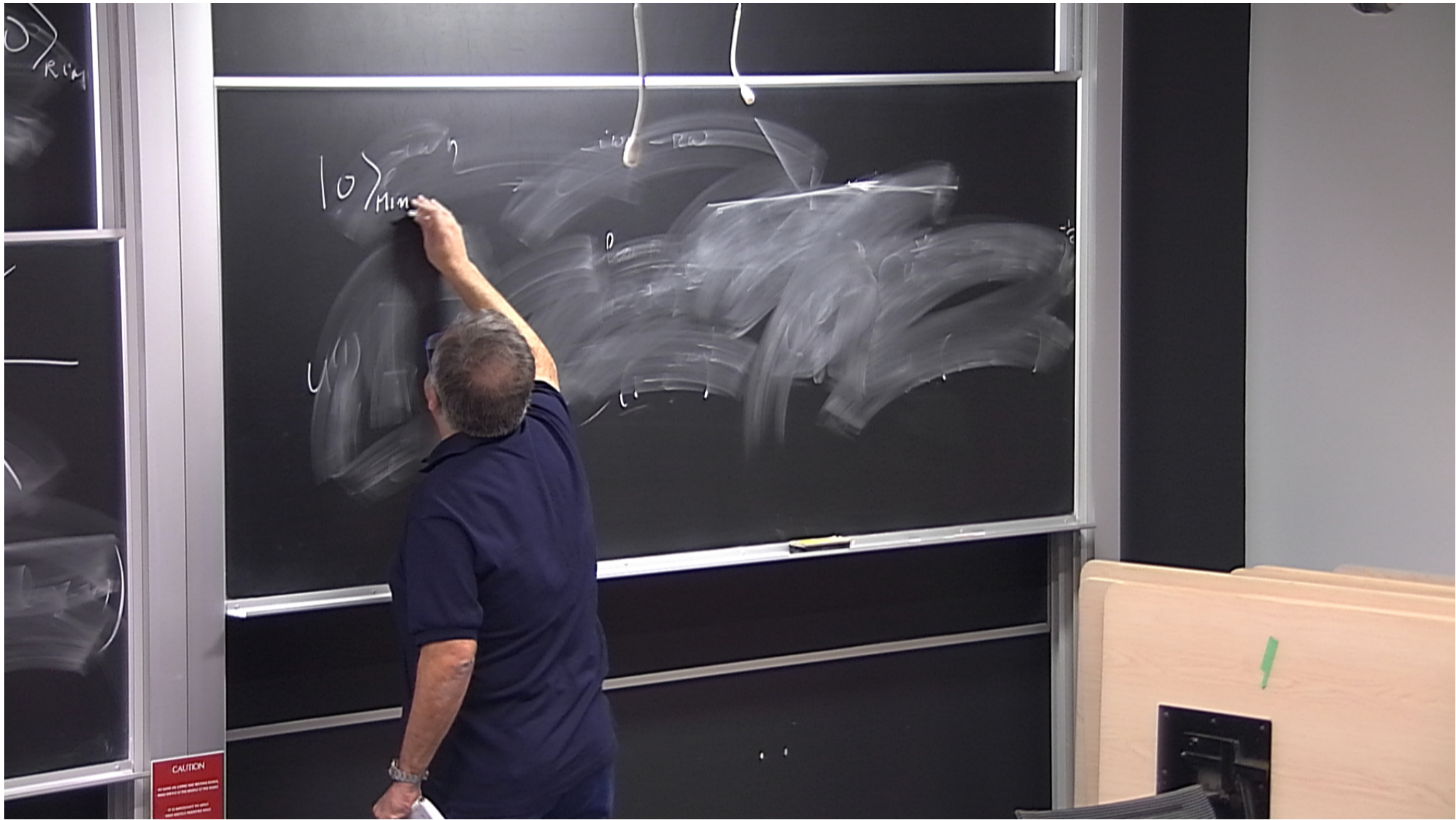


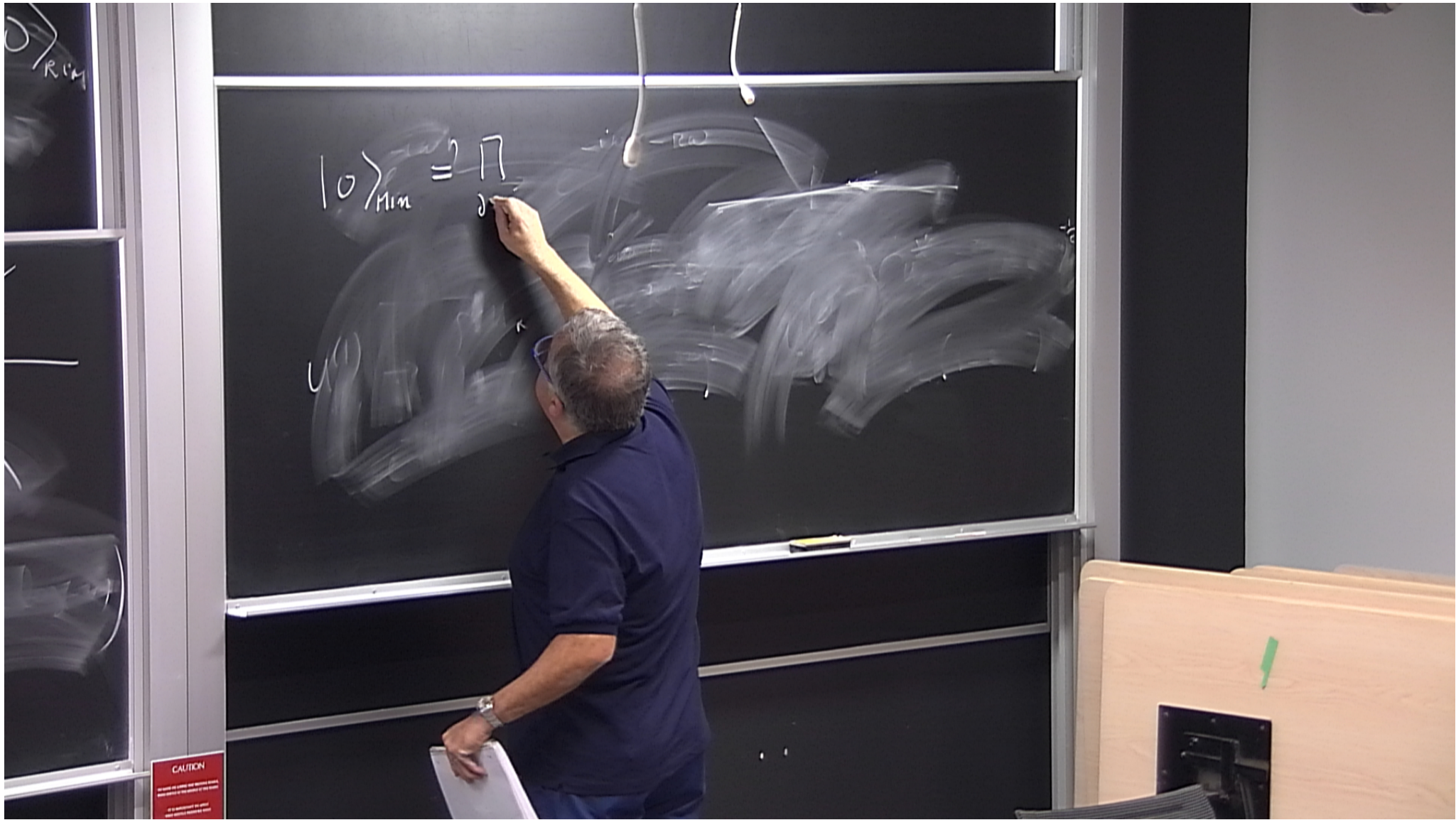


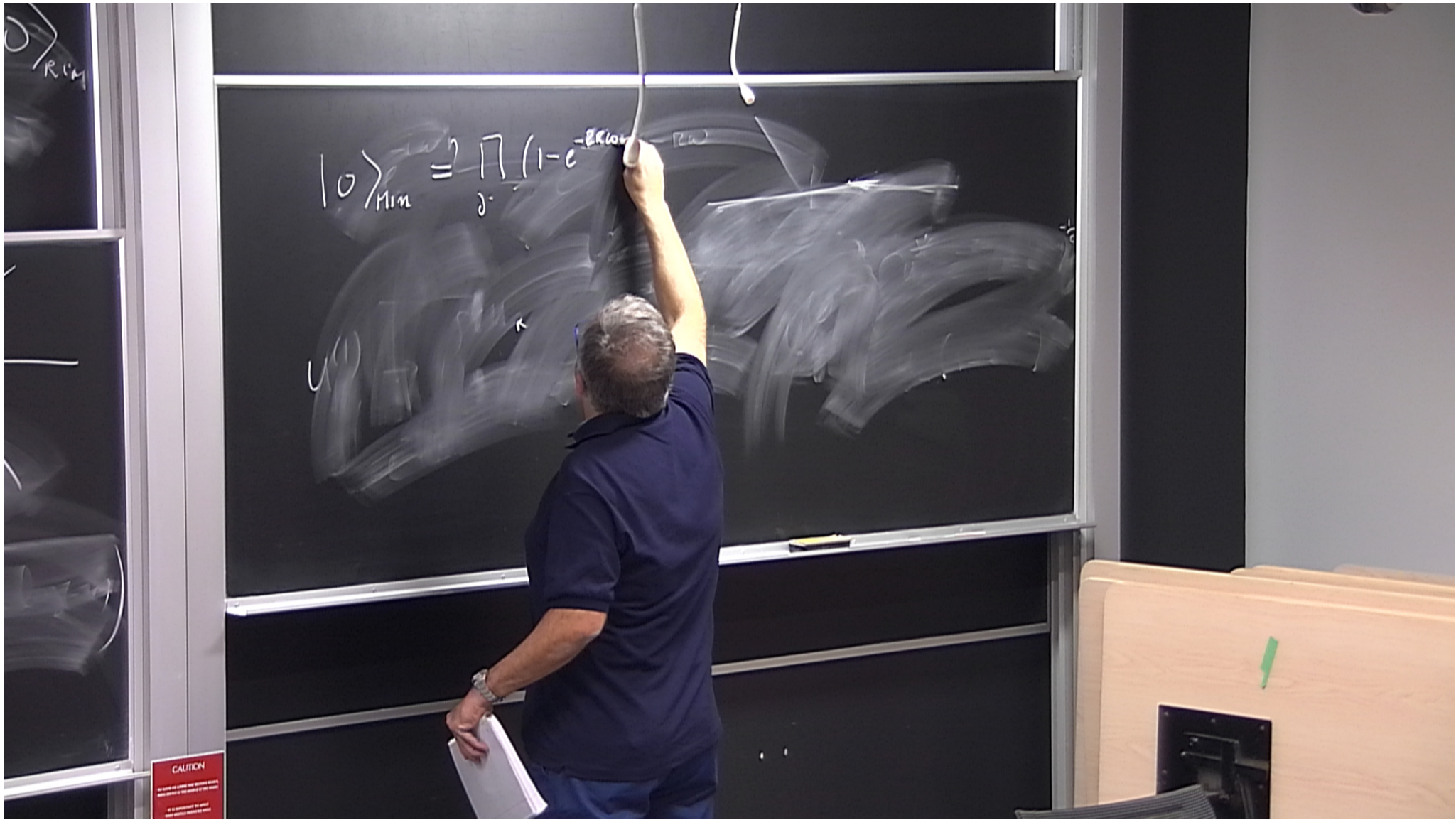


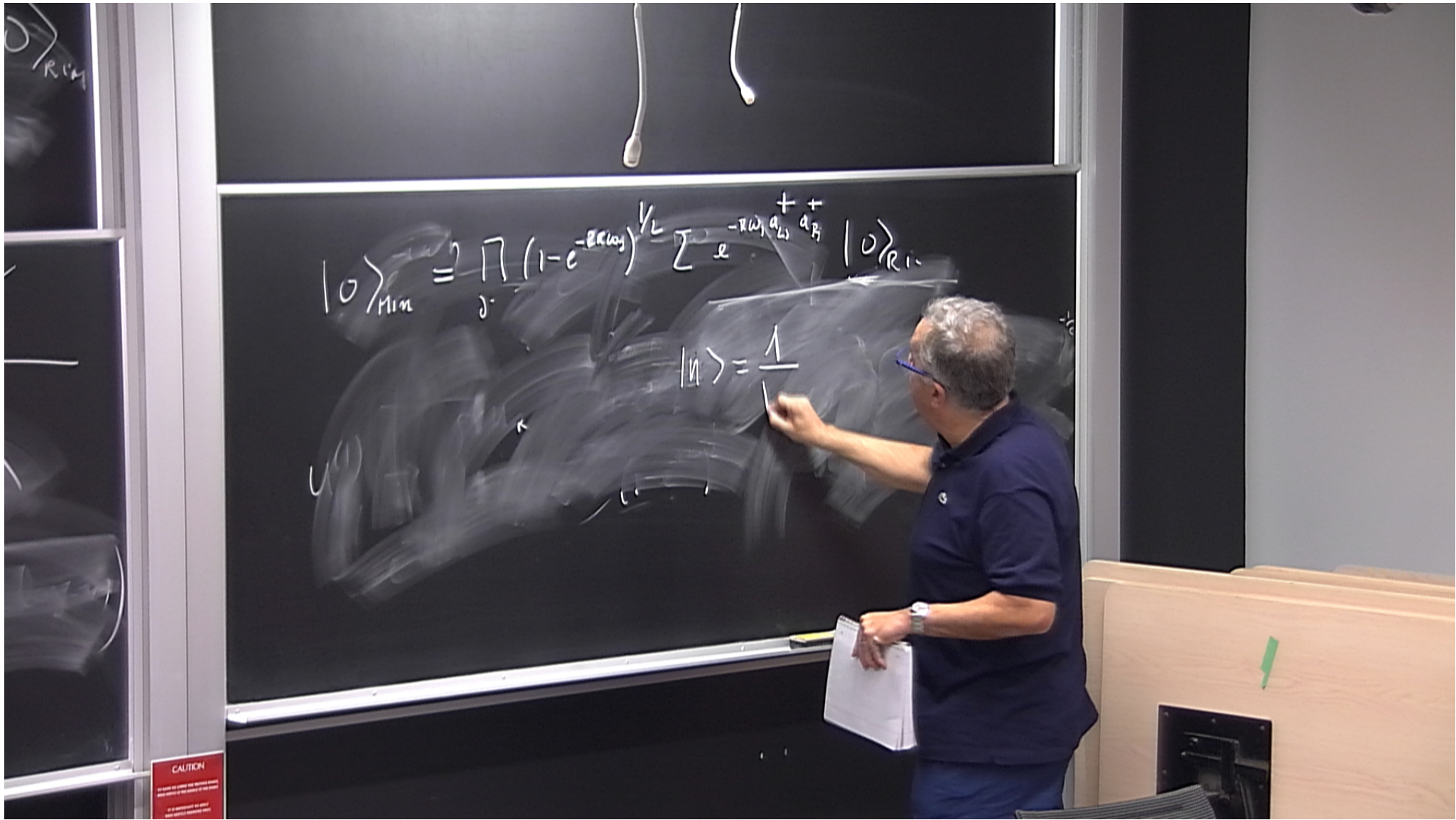


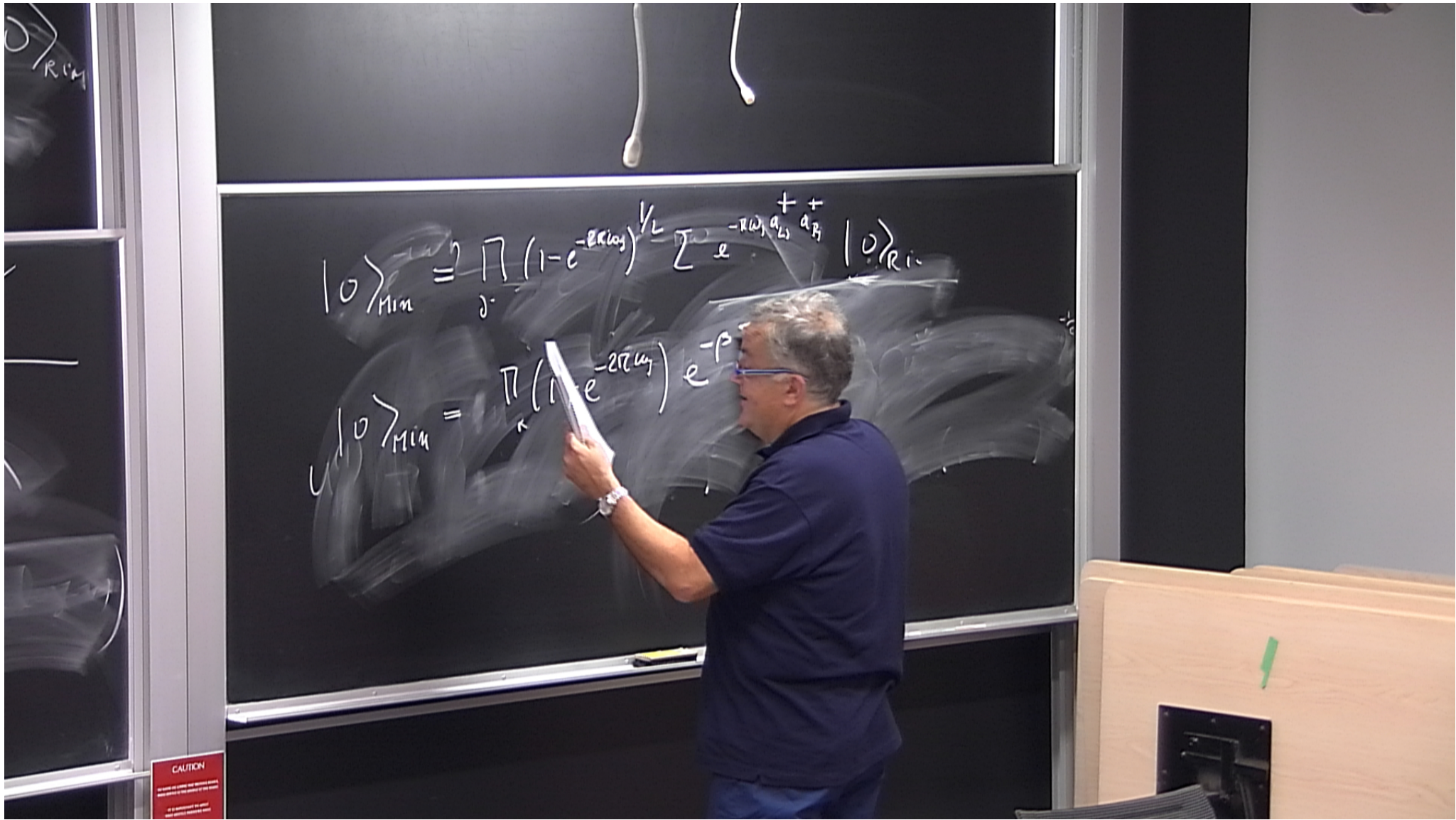


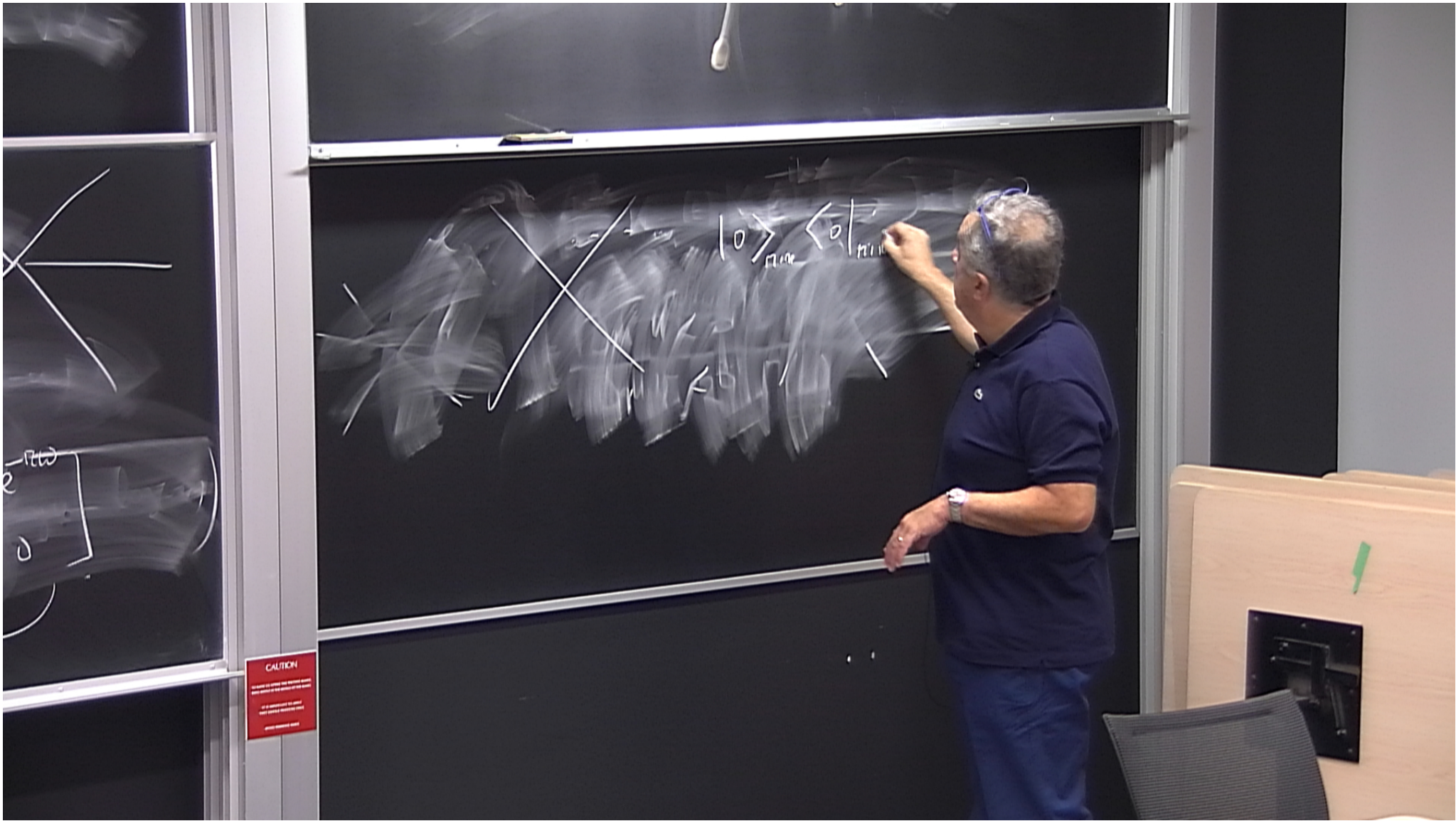


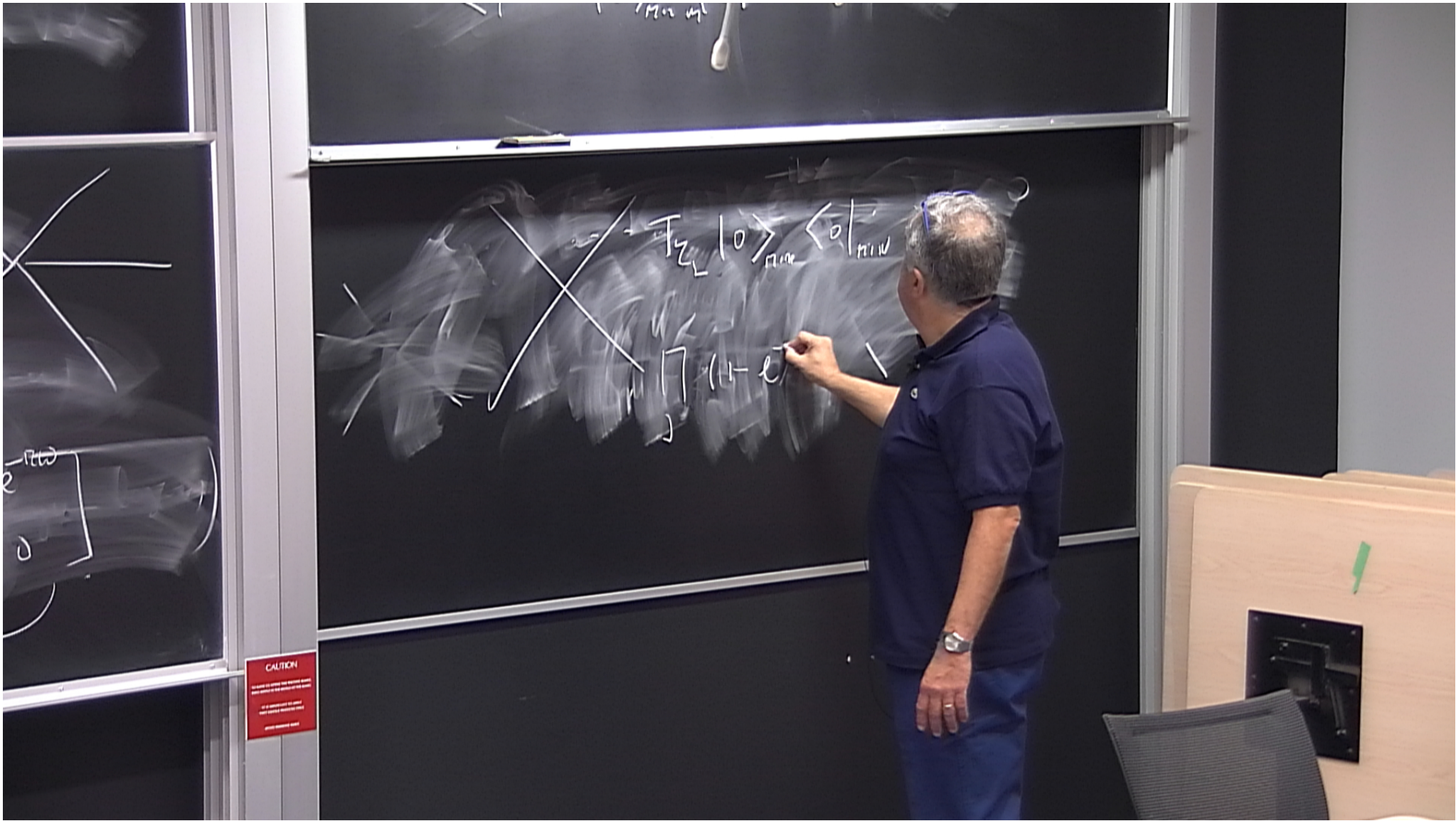


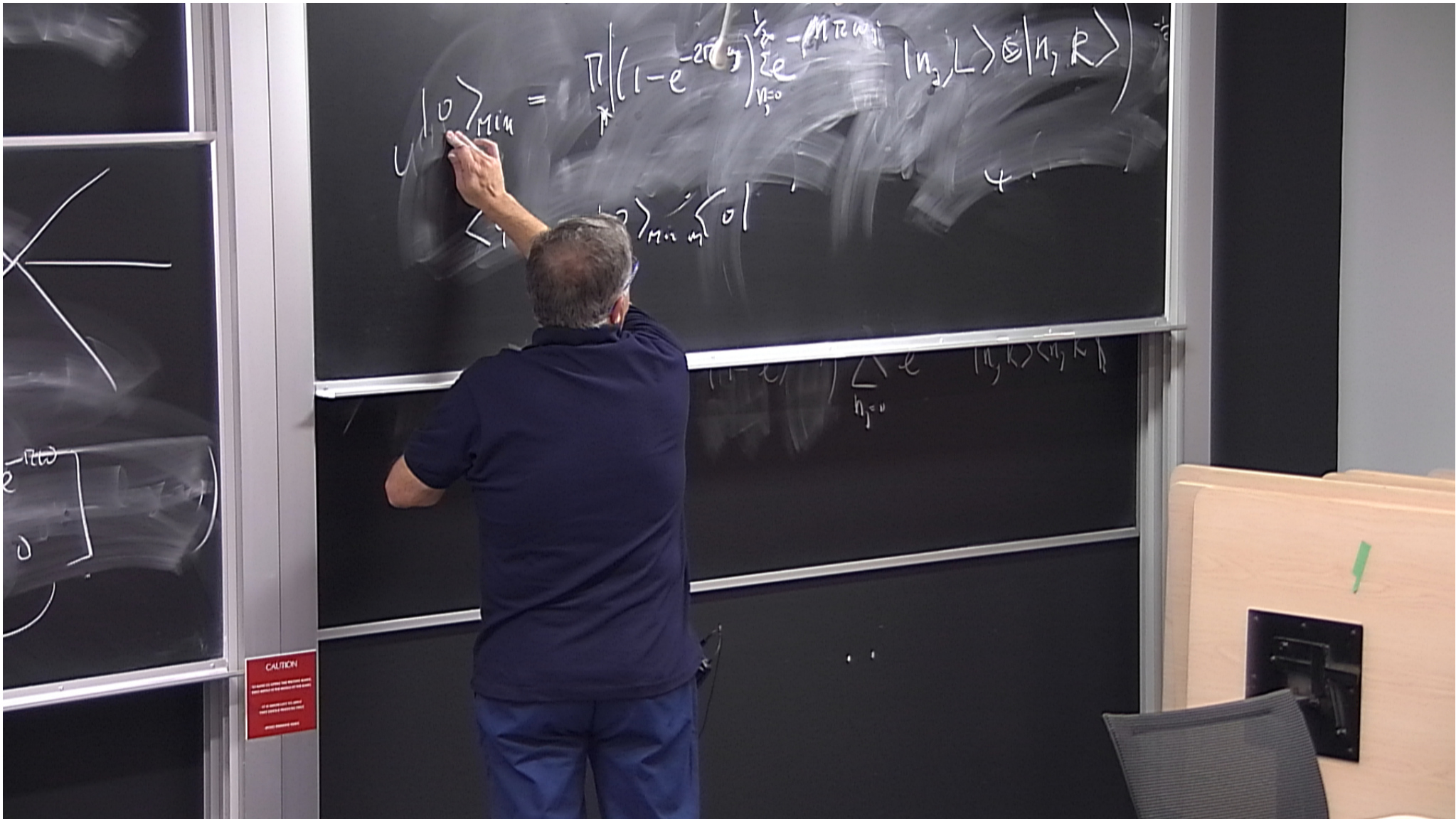


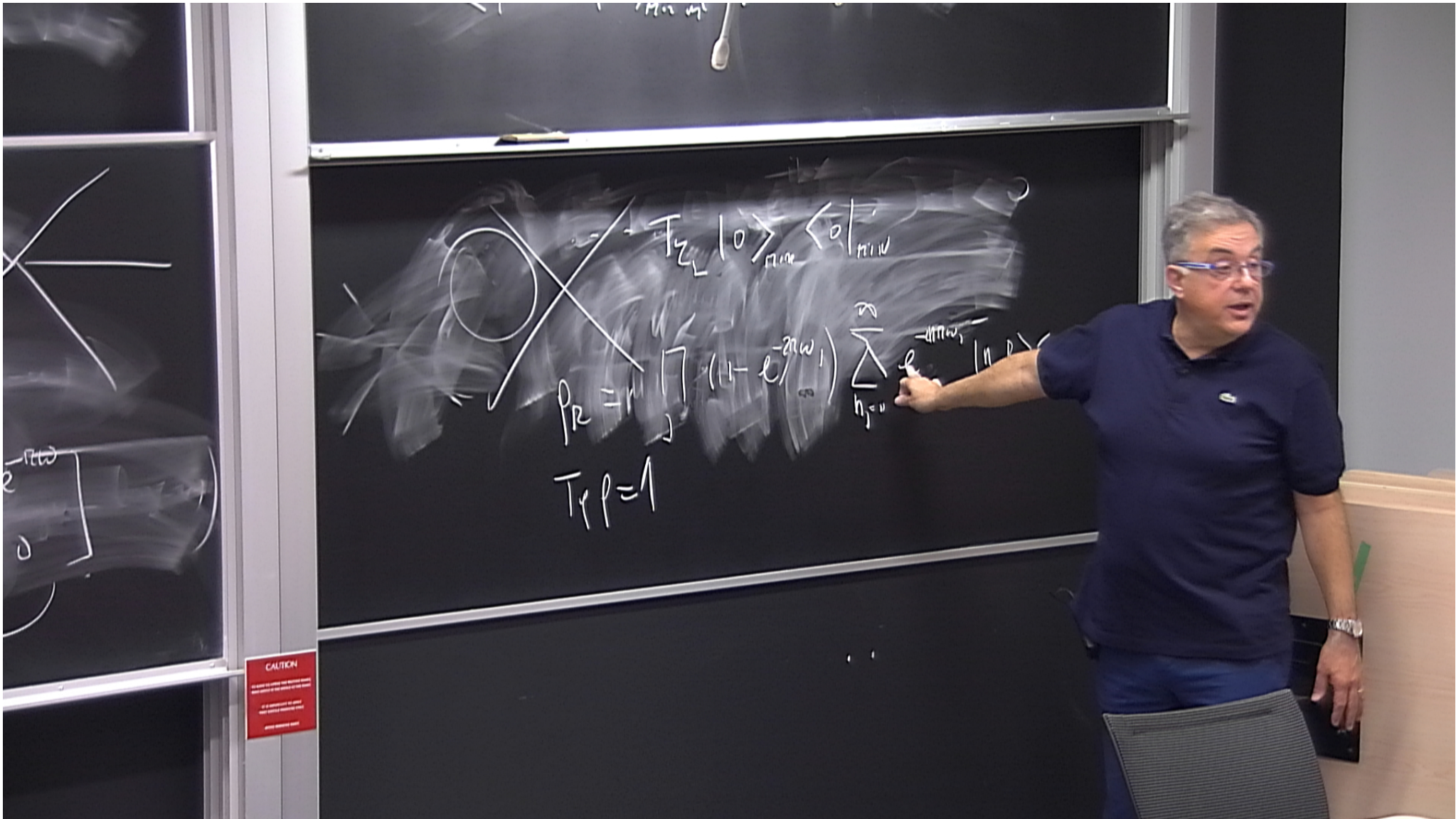


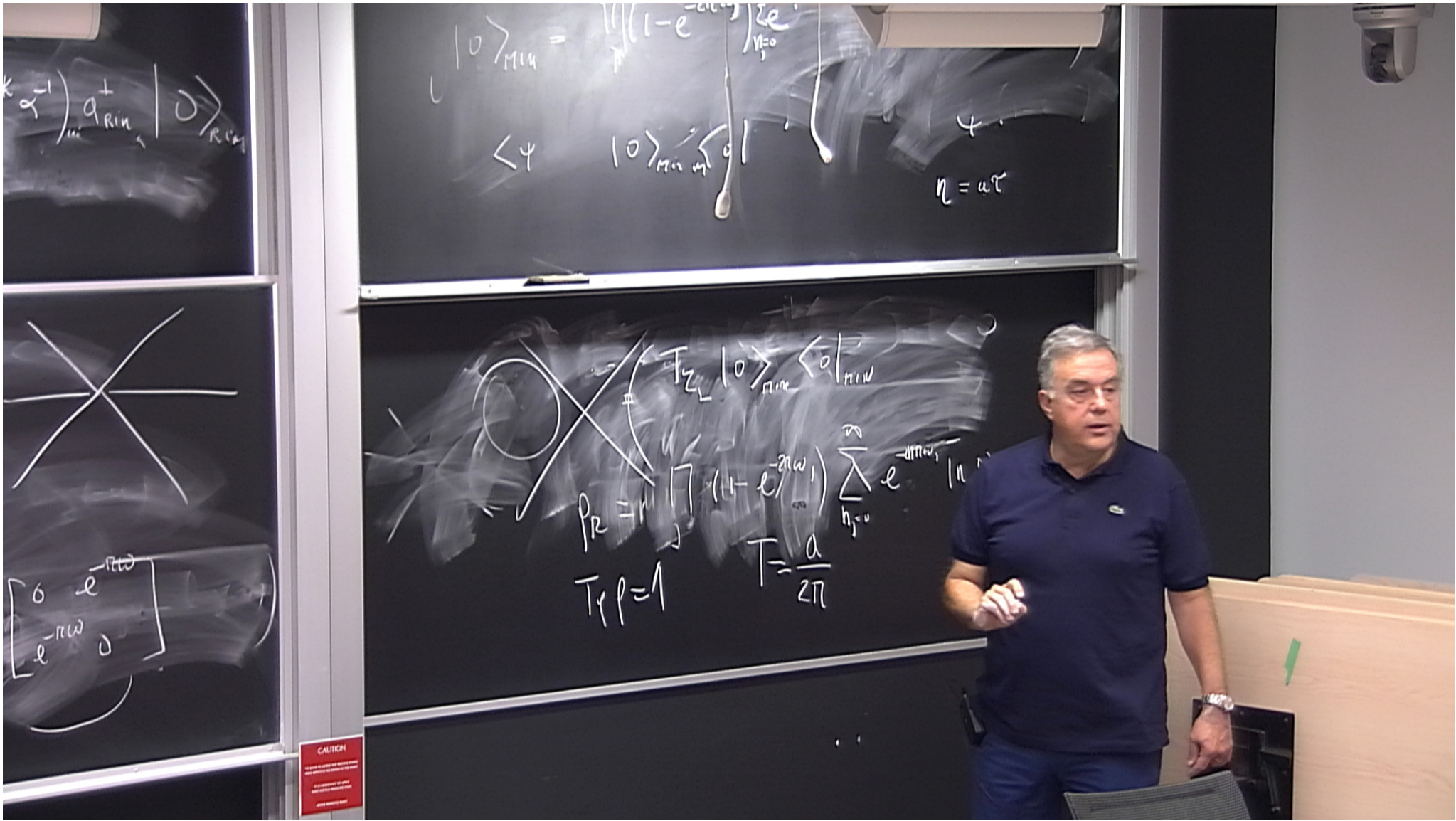


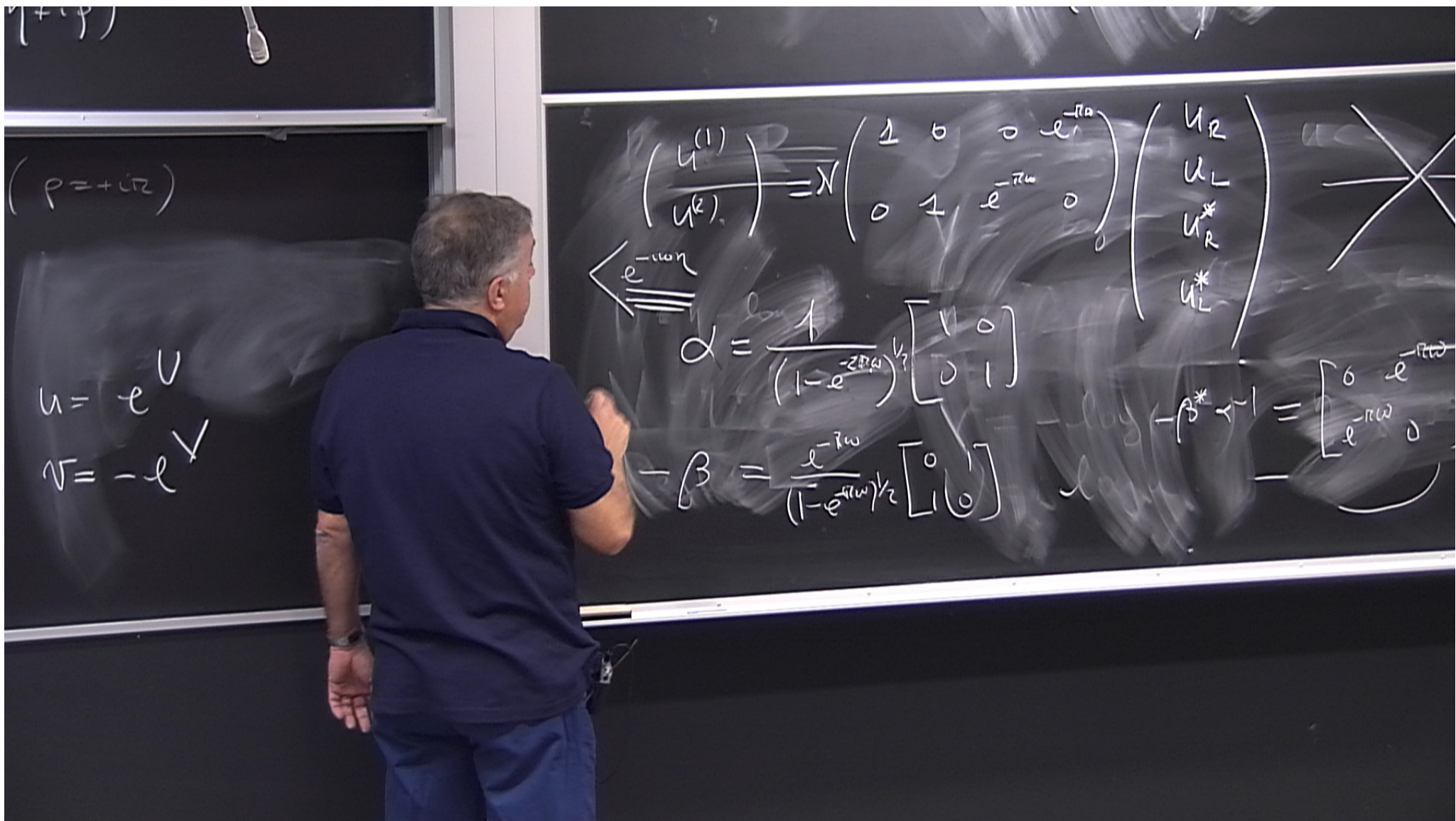












$$(p = +i\omega)$$

$$u = e^u$$

$$v = -e^v$$

$$\begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} = N \begin{pmatrix} 1 & 0 & 0 & e^{-\pi\omega} \\ 0 & 1 & e^{-\pi\omega} & 0 \end{pmatrix} \begin{pmatrix} u_R \\ u_L \\ u_R^* \\ u_L^* \end{pmatrix}$$

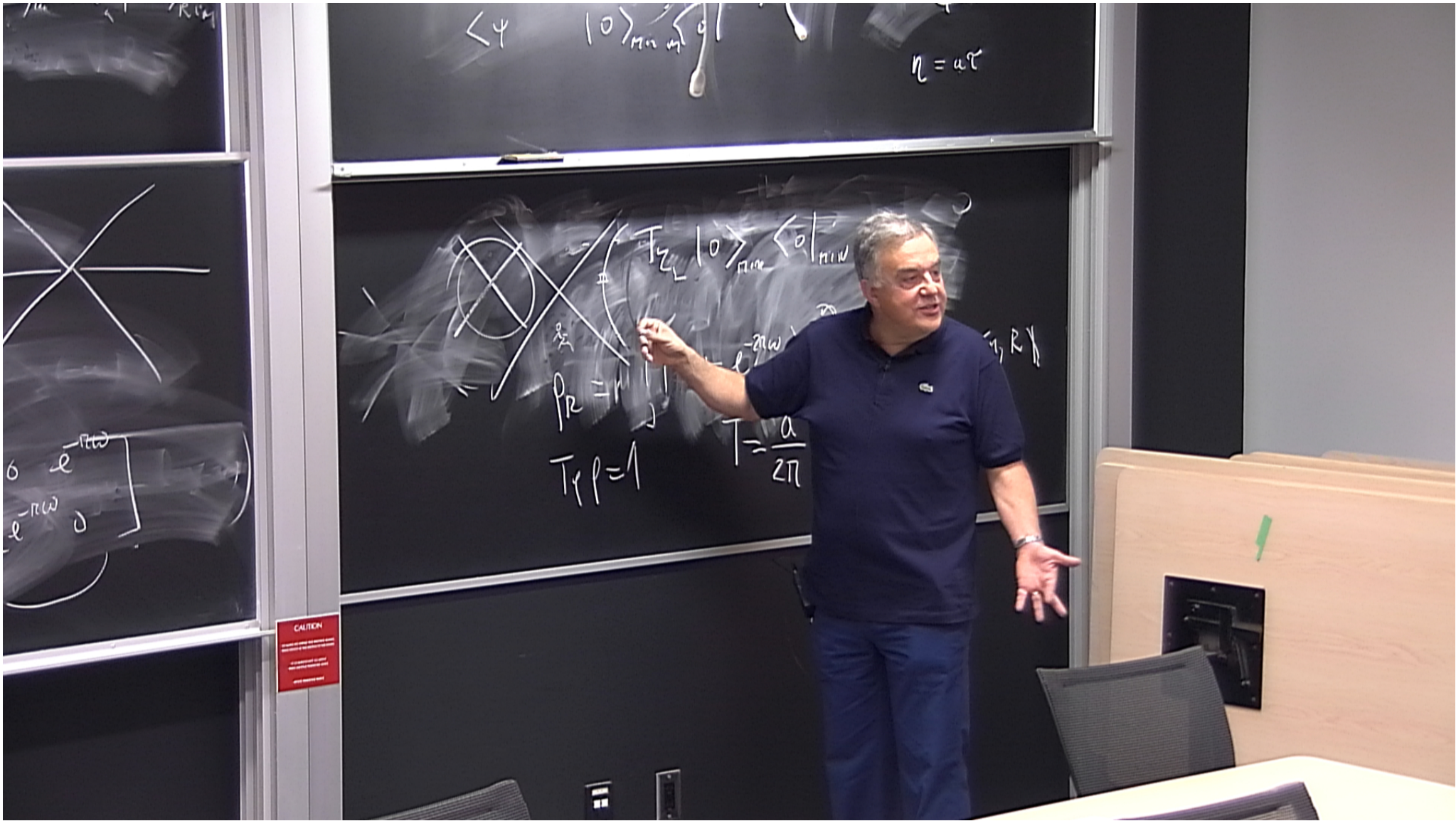
$$\leftarrow e^{-i\omega\eta}$$

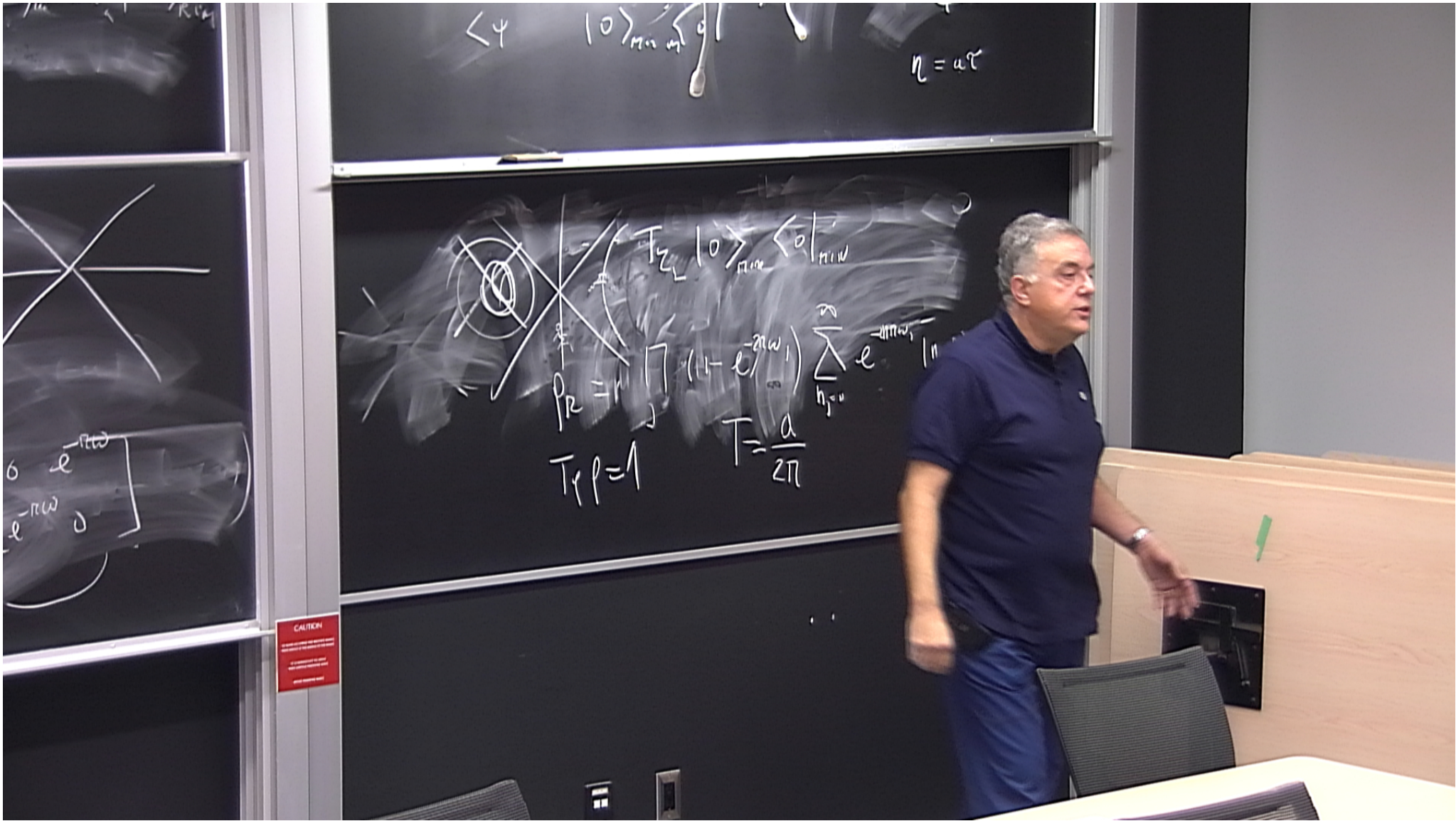
$$\alpha = \frac{1}{(1 - e^{-2\pi\omega})^{1/2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

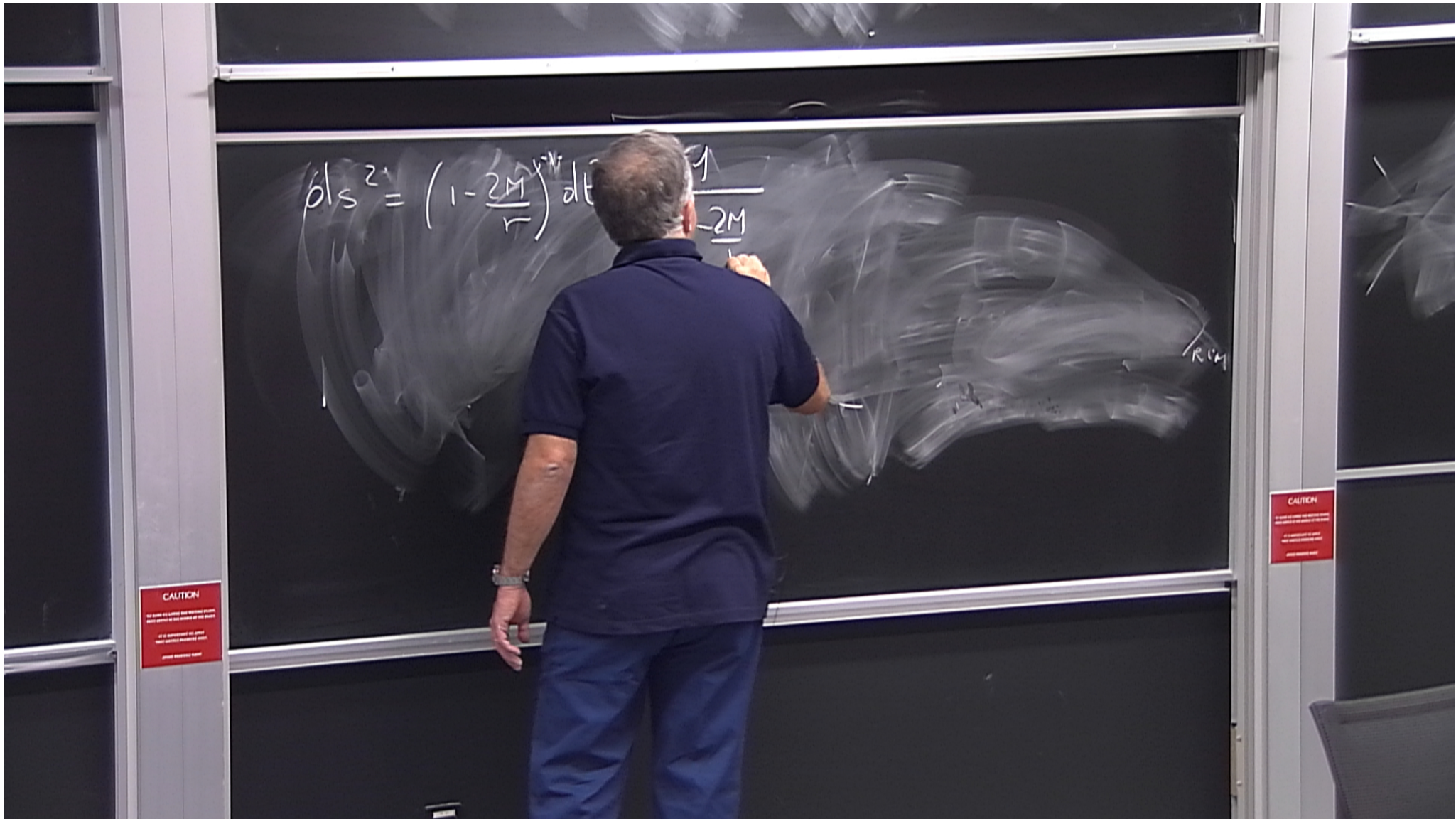
$$-\beta = \frac{e^{-\pi\omega}}{(1 - e^{-2\pi\omega})^{1/2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$-\beta^* \alpha^{-1} = \begin{bmatrix} 0 & e^{-\pi\omega} \\ e^{-\pi\omega} & 0 \end{bmatrix}$$







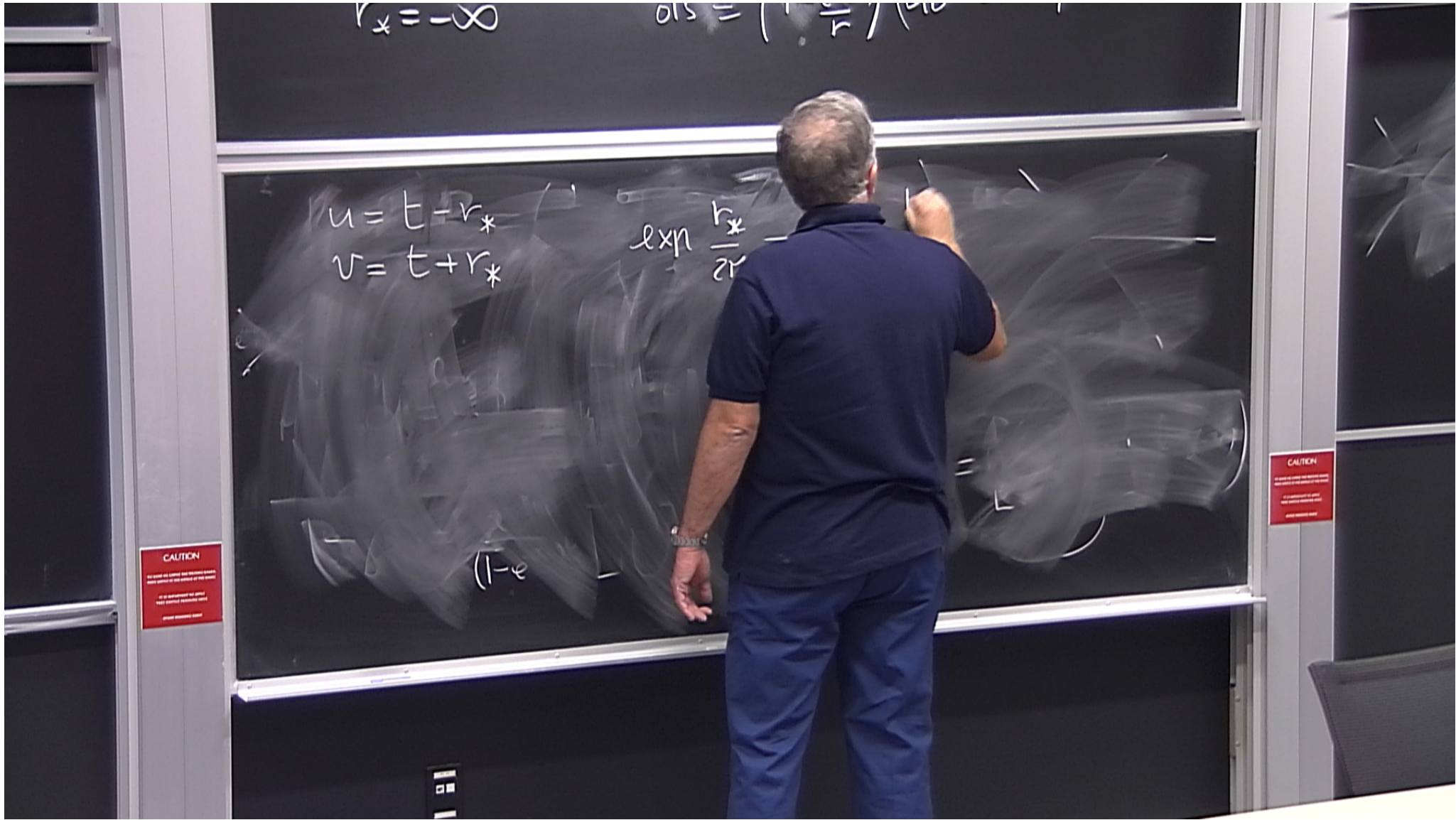


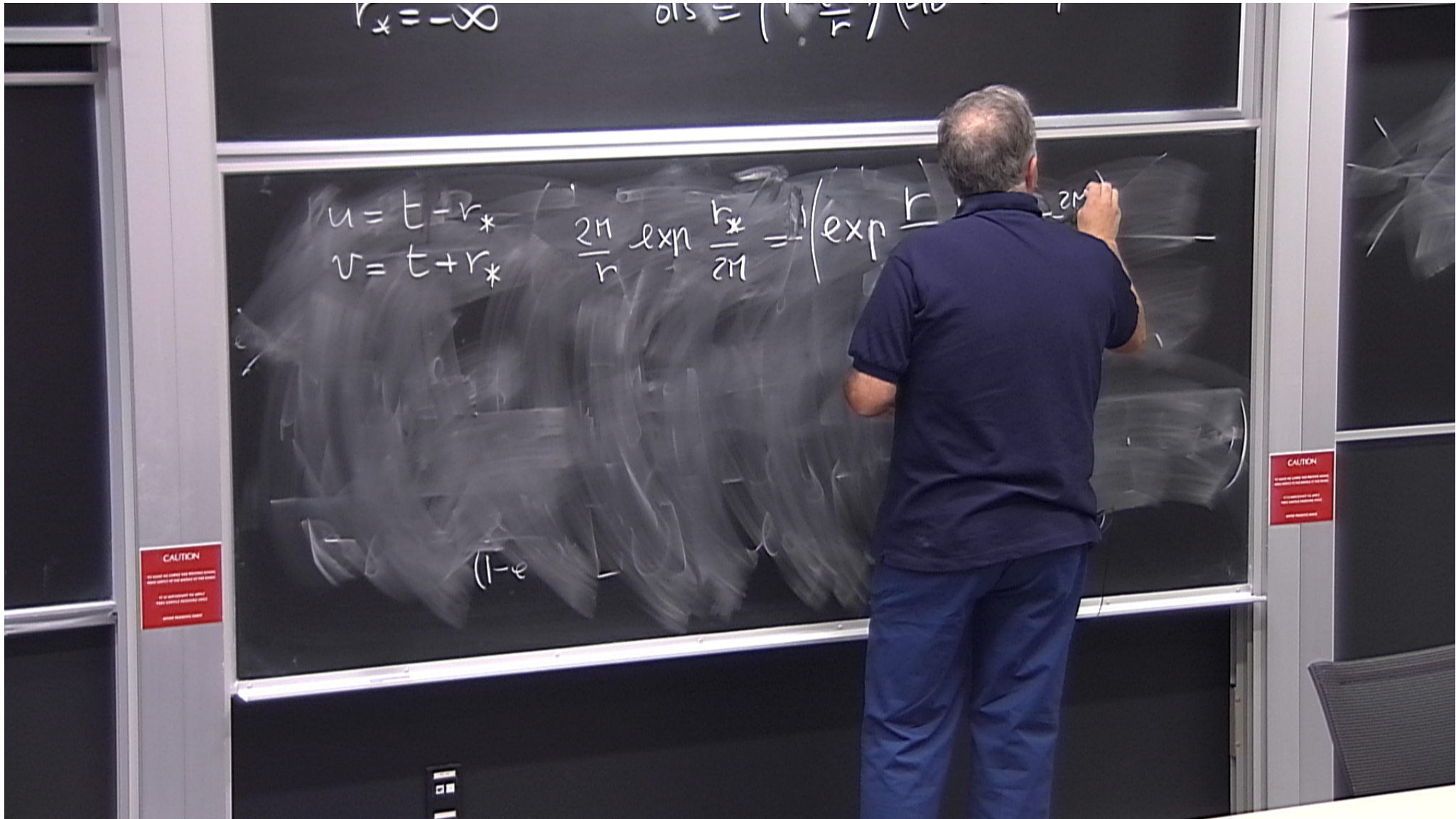
$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 - r^2 d\Omega^2$$

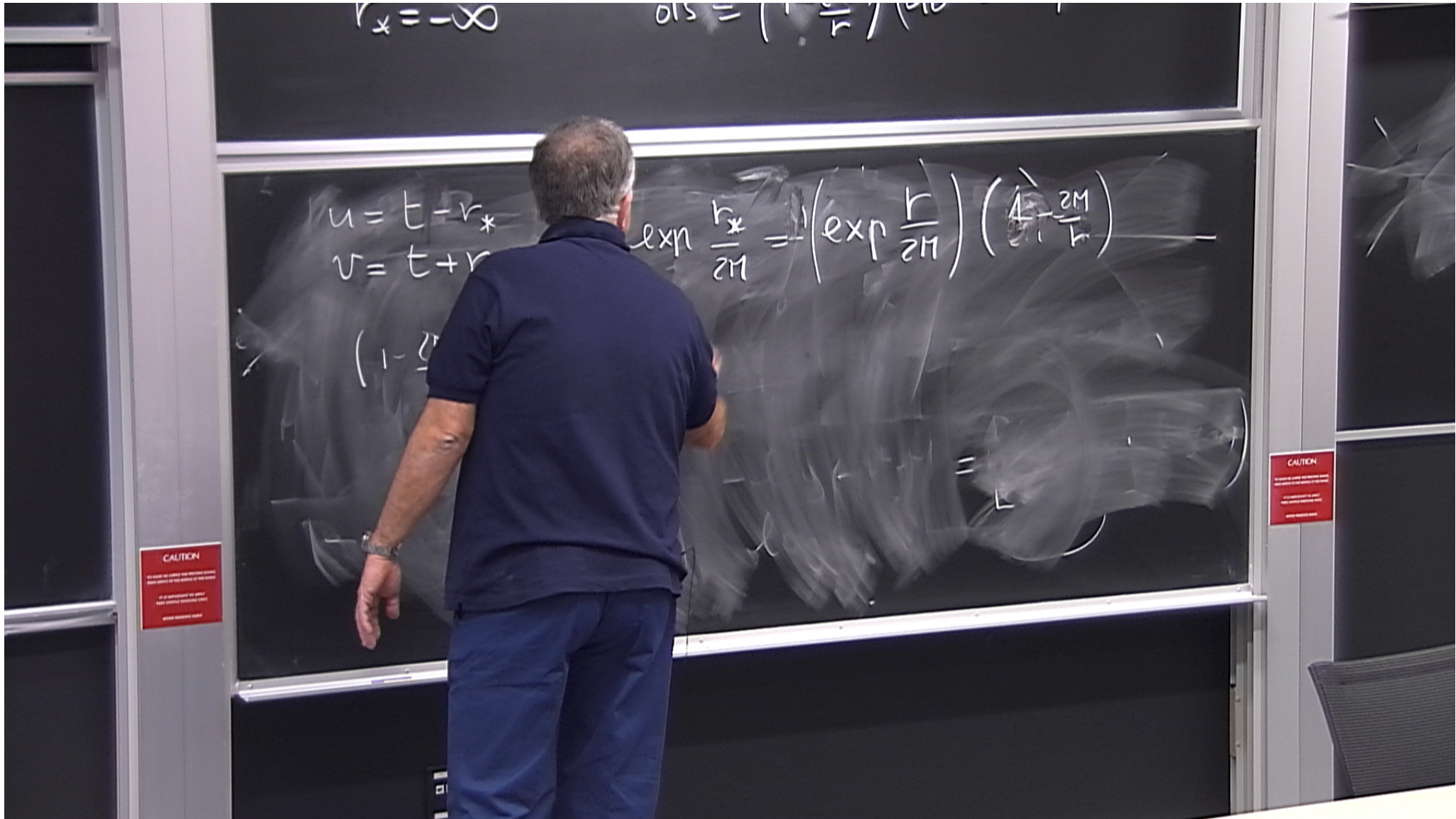
$$\left| r = 2M \right|$$

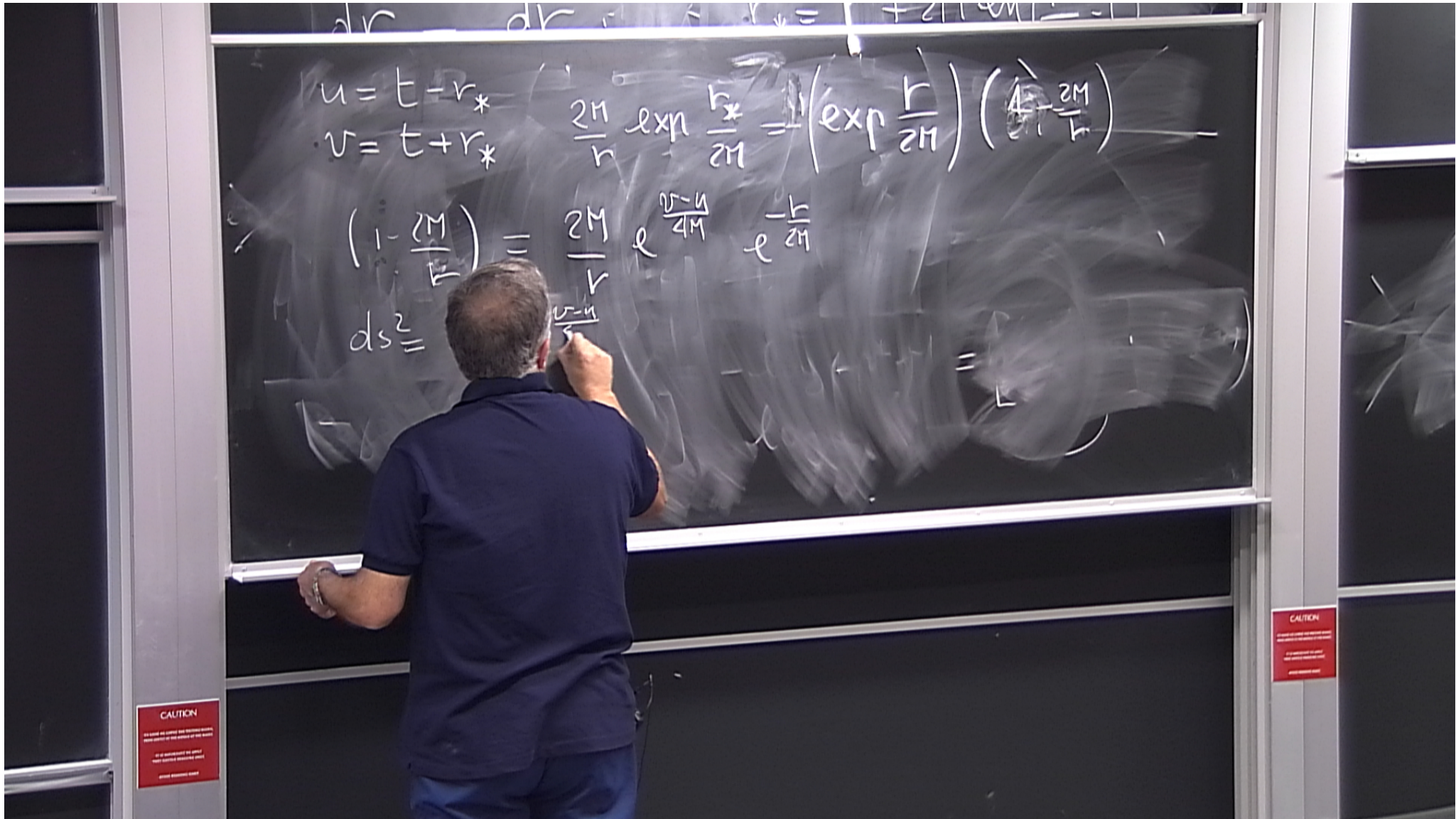
$$dr_* = \frac{dr}{\left(1 - \frac{2M}{r}\right)}$$

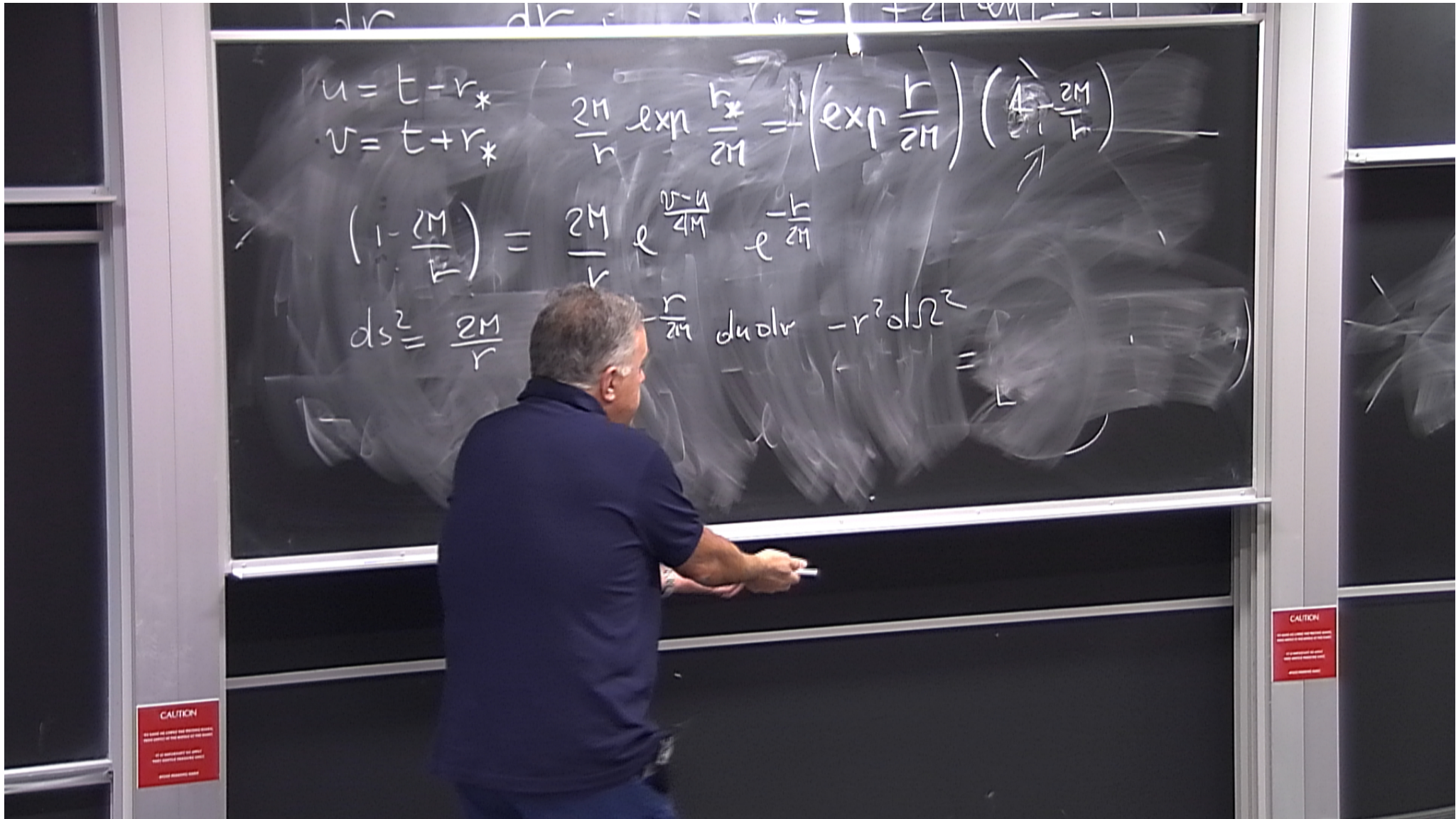
$$r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

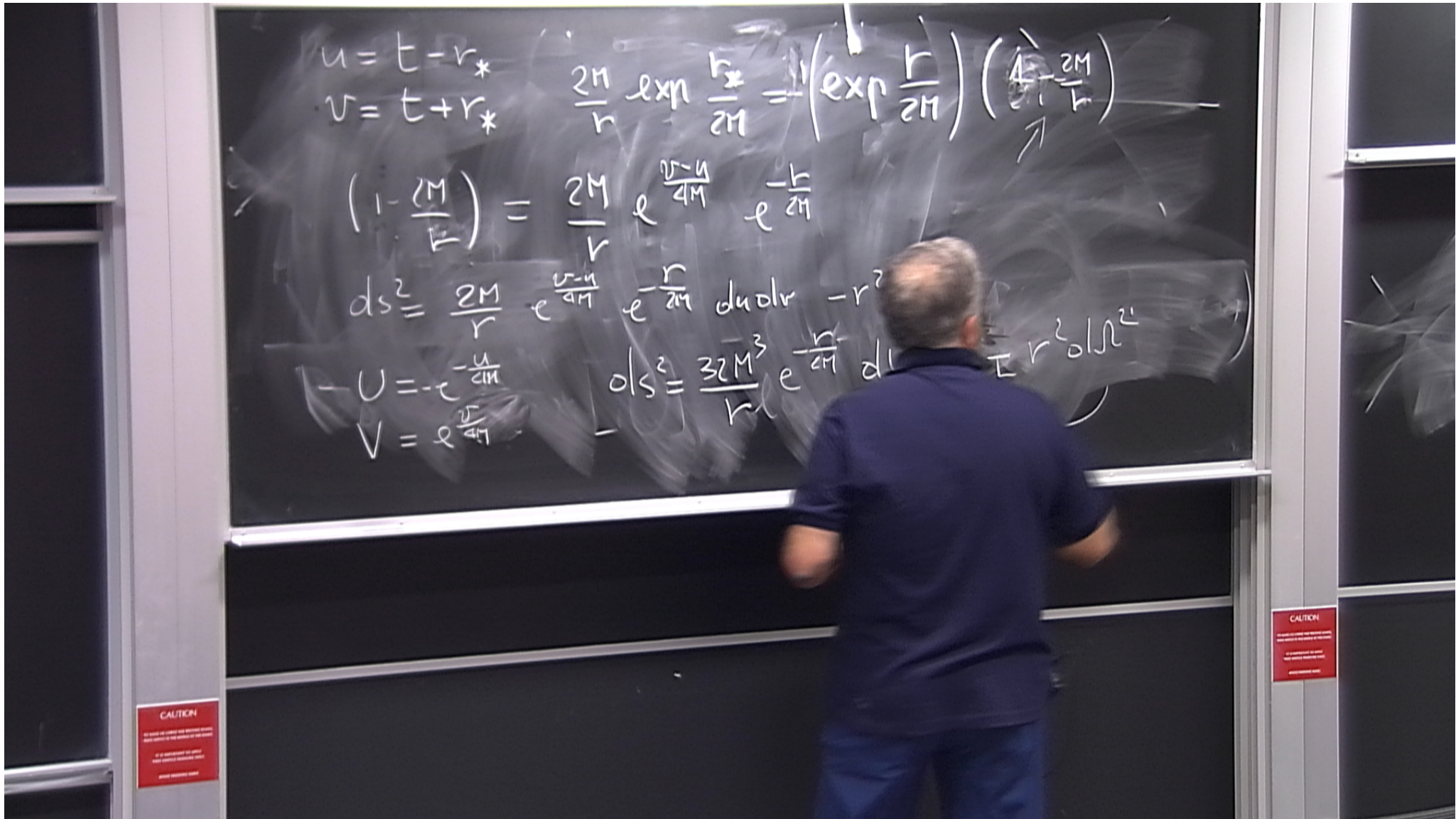












$$u = t - r_*$$

$$v = t + r_*$$

$$\frac{2M}{r} \exp \frac{r_*}{2M} = \left(\exp \frac{r}{2M} \right) \left(\frac{1 - \frac{2M}{r}}{e^{-\frac{r}{2M}}} \right)$$

$$\left(1 - \frac{2M}{r} \right) = \frac{2M}{r} e^{\frac{v-u}{4M}} e^{-\frac{r}{2M}}$$

$$r = r(u, v)$$

$$ds^2 = \frac{2M}{r} e^{\frac{v-u}{4M}} e^{-\frac{r}{2M}} du dv - r^2 d\Omega^2$$

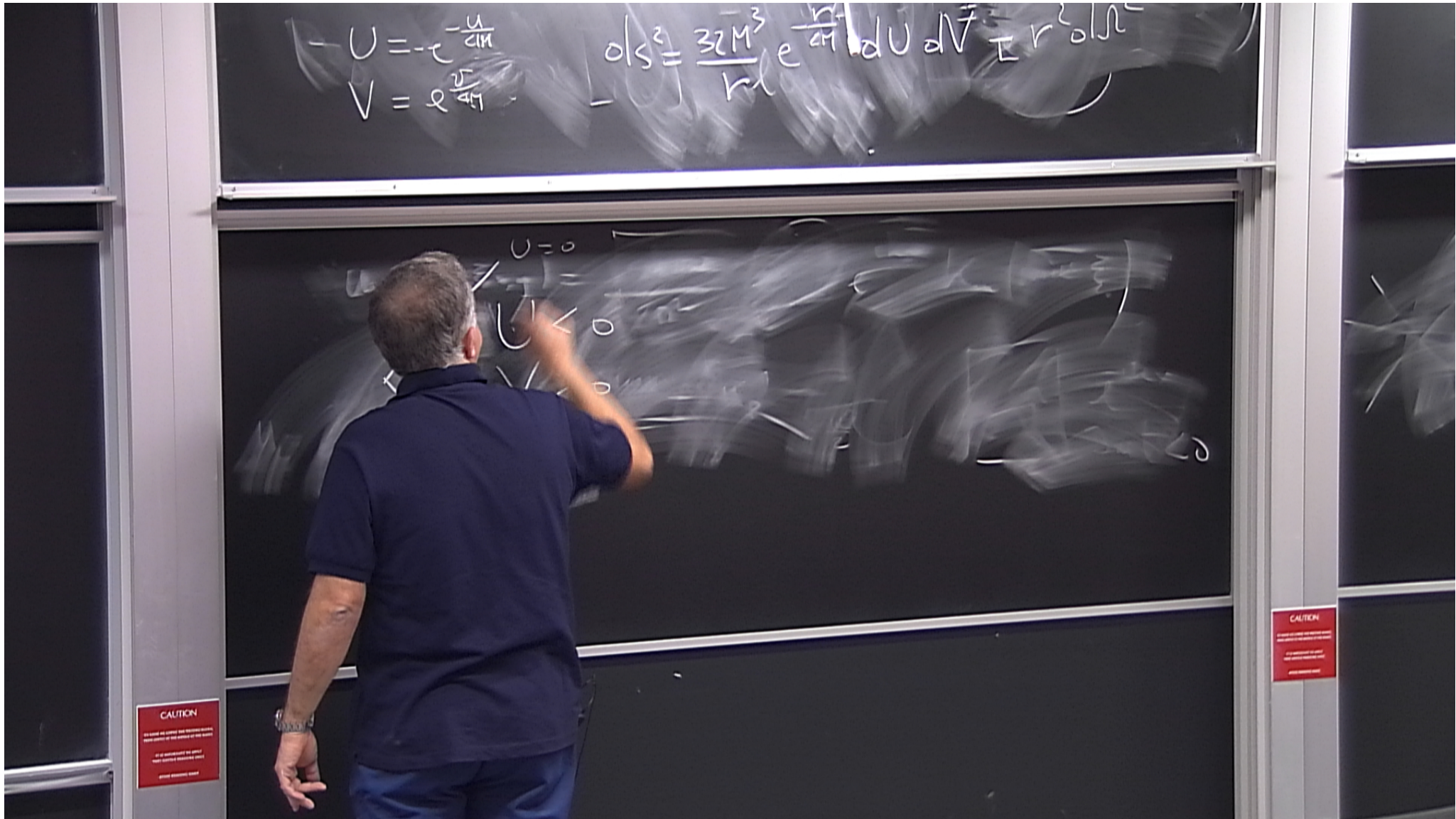
$$-U = -e^{-\frac{u}{4M}}$$

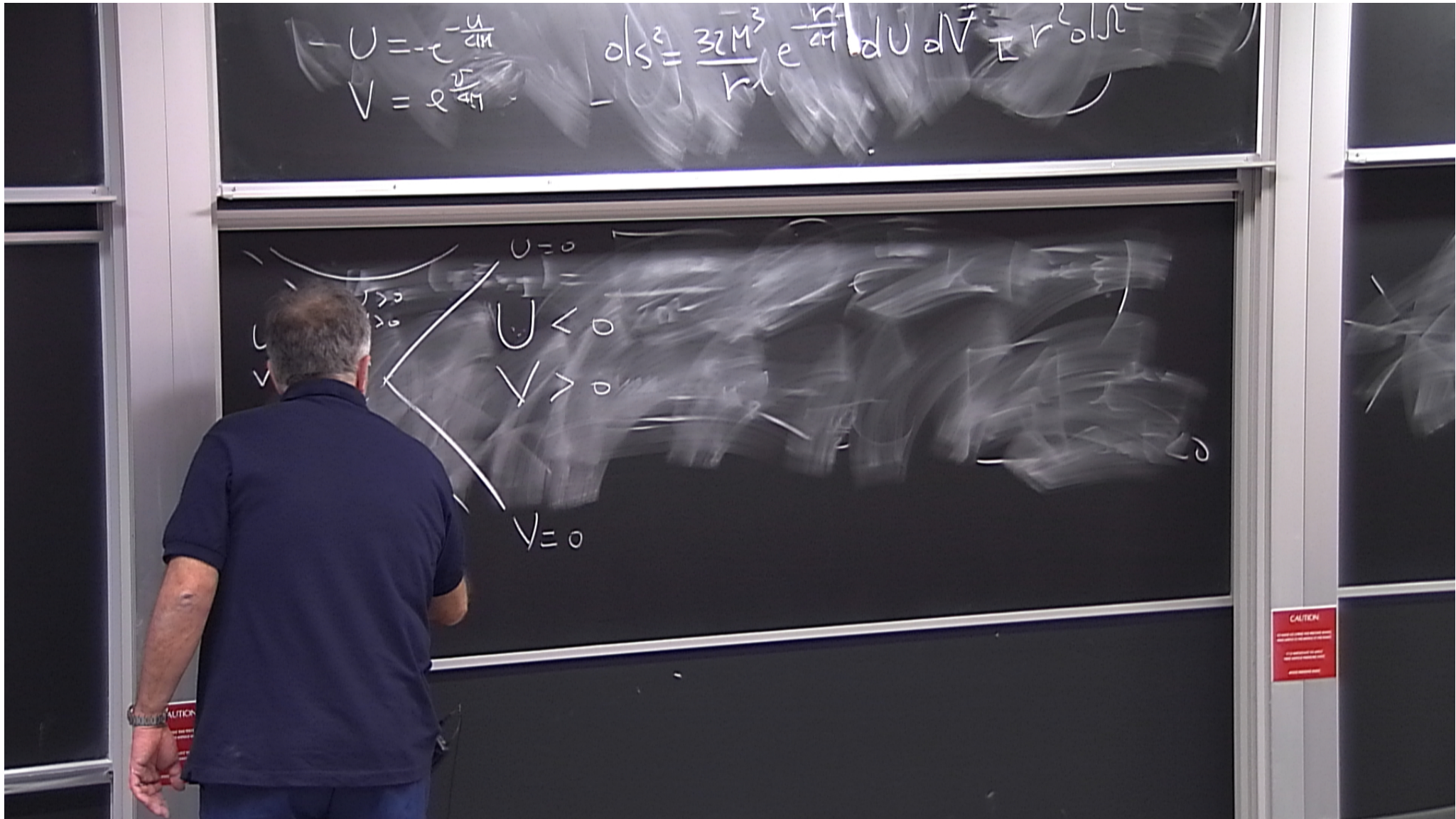
$$V = e^{\frac{v}{4M}}$$

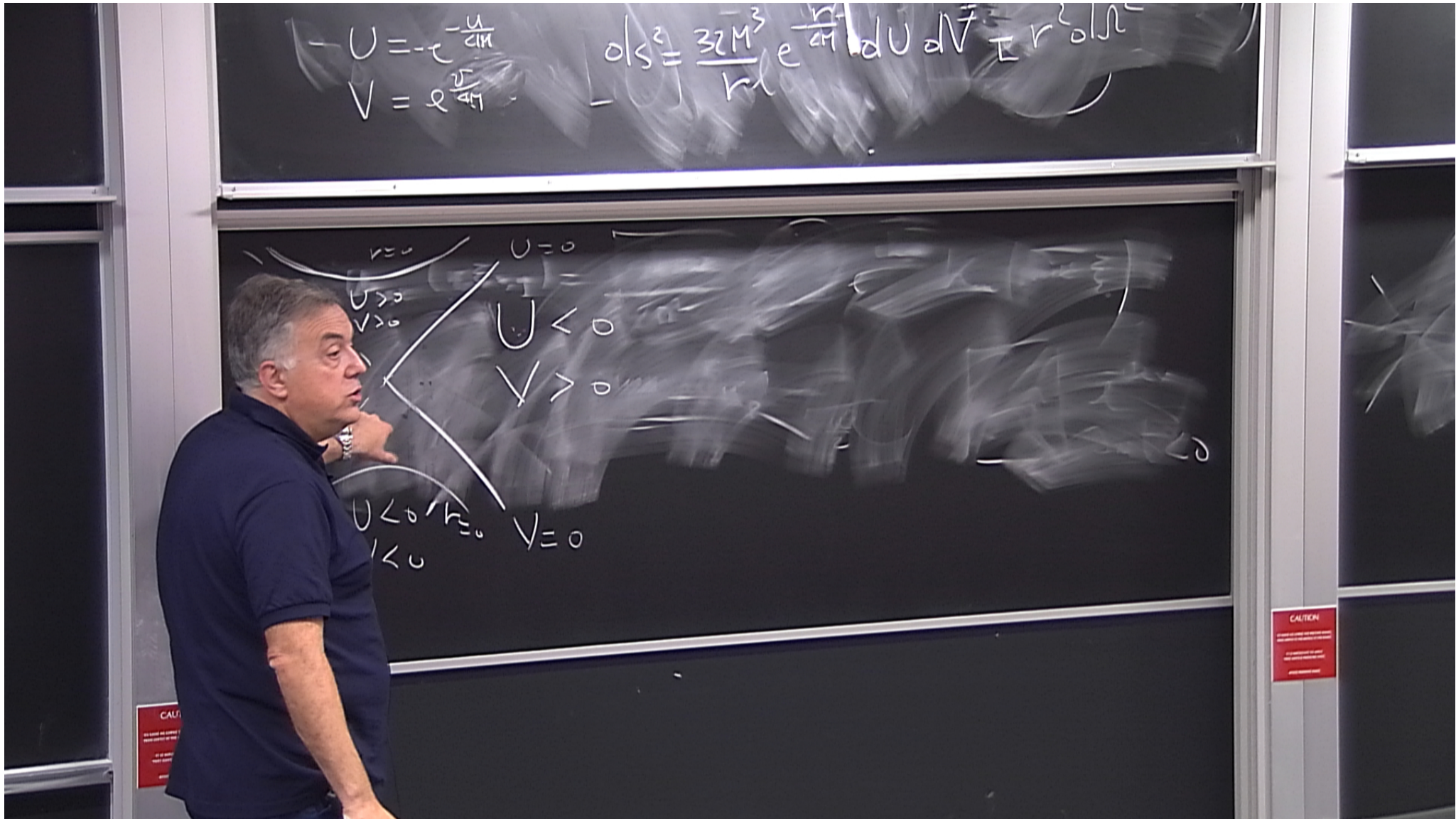
$$ds^2 = \frac{32M^3}{r^3} e^{-\frac{r}{2M}} dU dV - r^2 d\Omega^2$$

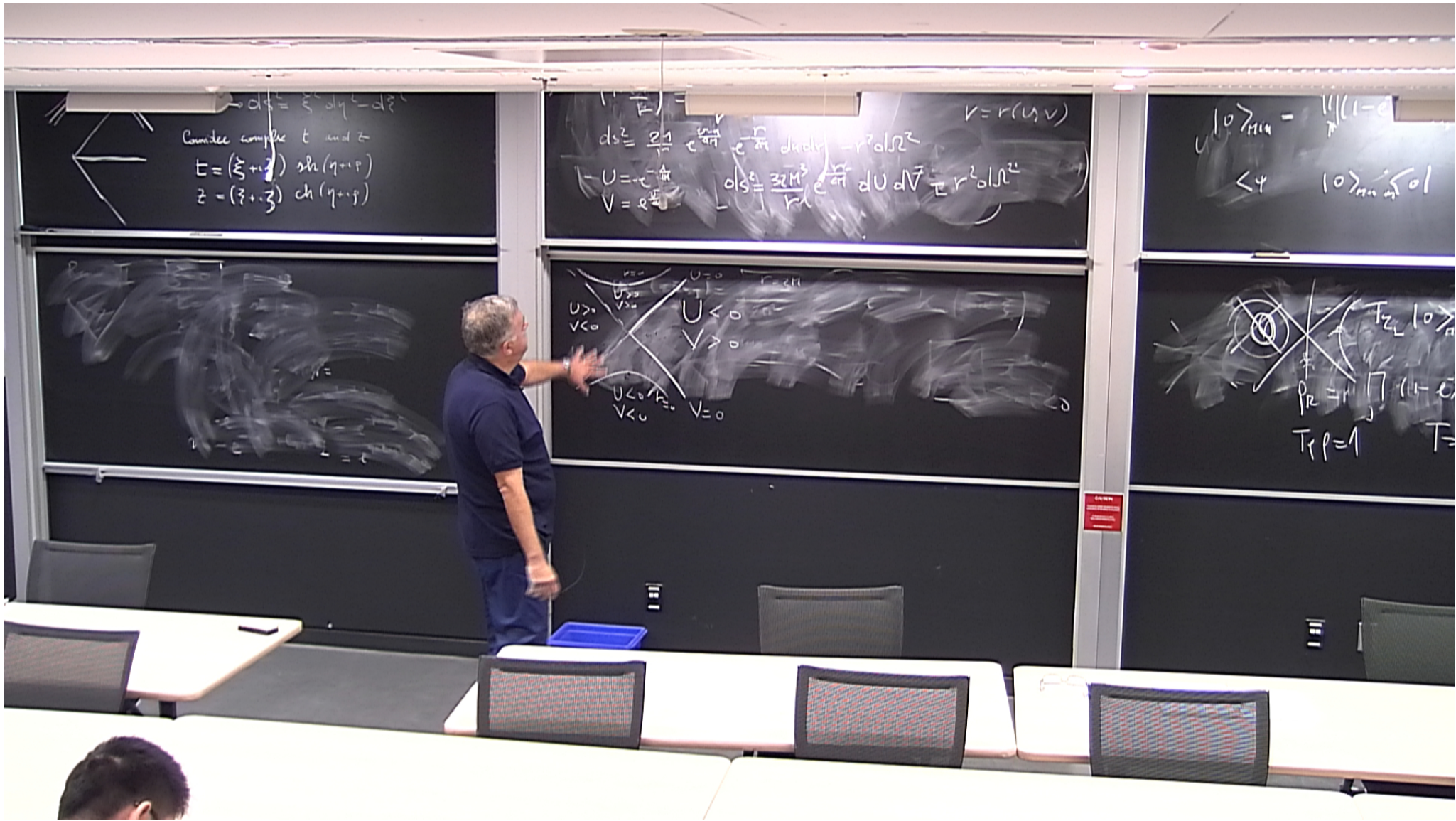
CAUTION
DO NOT TOUCH THE BOARD WHEN
IT IS BEING USED BY OTHERS
OR IT WILL BE DAMAGED

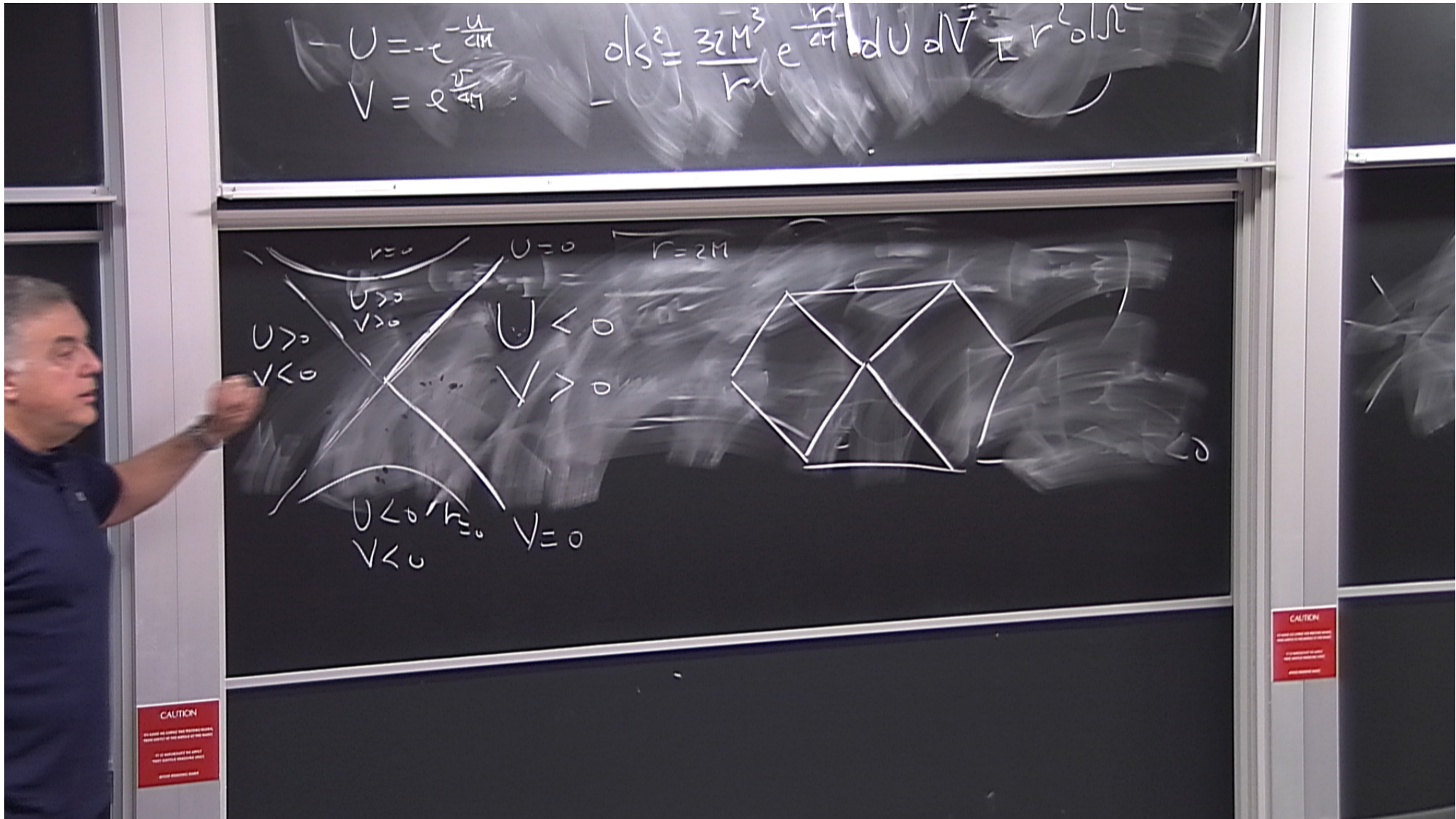
CAUTION
DO NOT TOUCH THE BOARD WHEN
IT IS BEING USED BY OTHERS
OR IT WILL BE DAMAGED

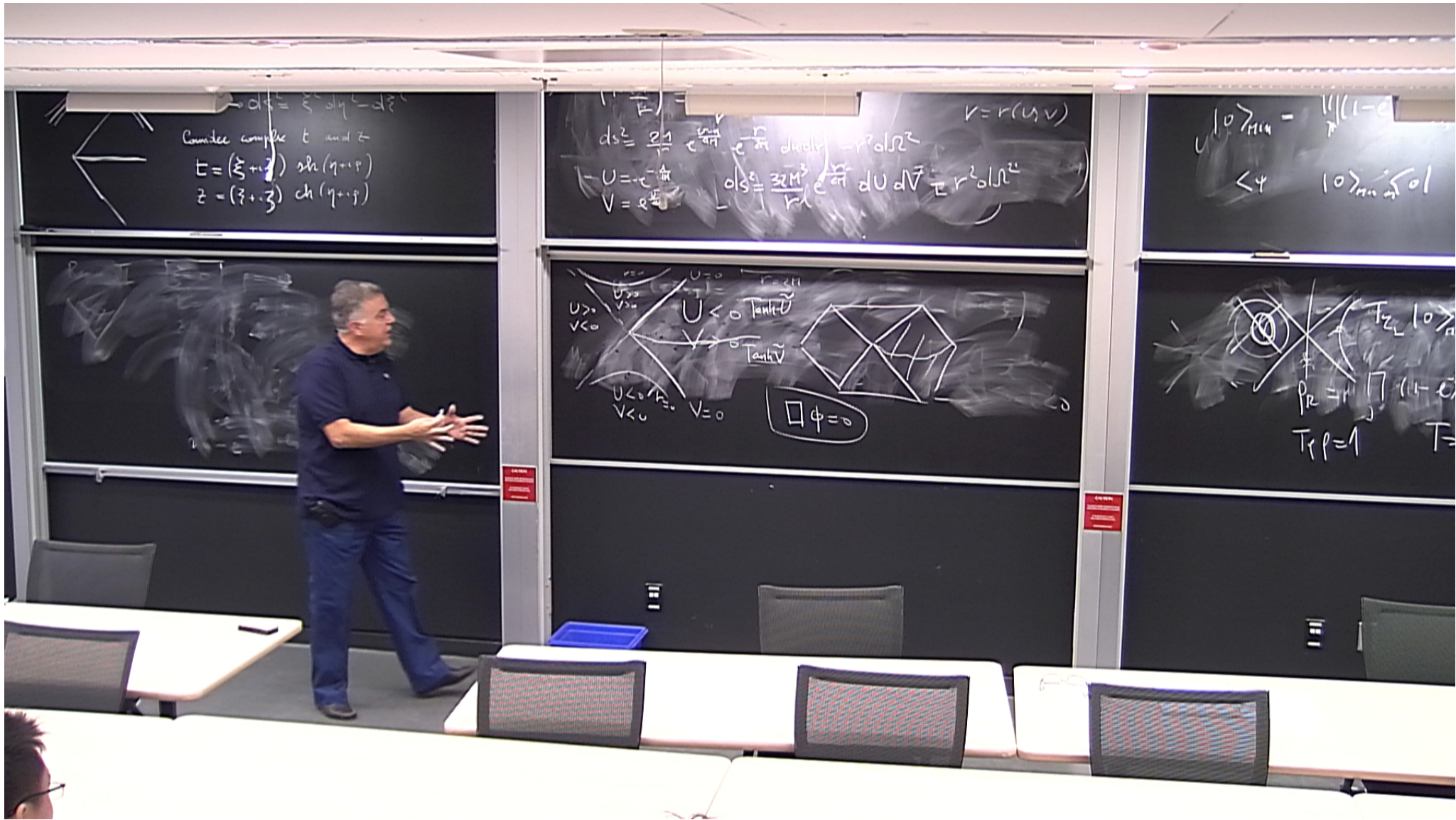










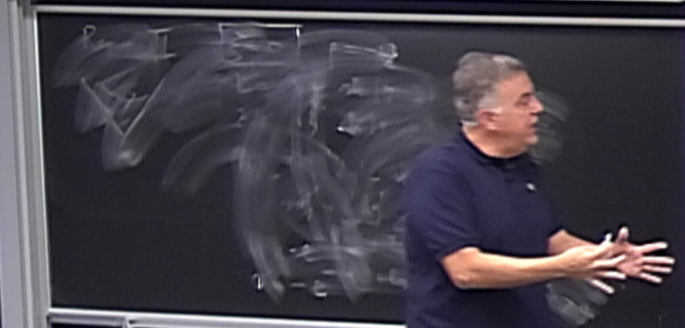


$$ds^2 = \xi^2 dy^2 - dt^2$$

Consider complex t and z

$$t = (\xi + i\eta) \operatorname{sh}(\eta + i\rho)$$

$$z = (\xi + i\eta) \operatorname{ch}(\eta + i\rho)$$



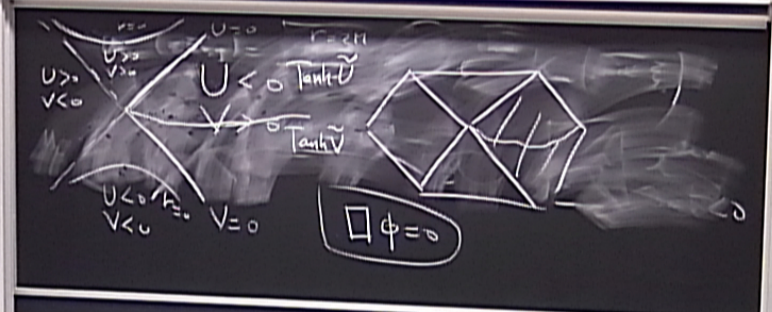
$$ds^2 = \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\Omega^2$$

$$U = -\frac{2M}{r}$$

$$V = r - 2M$$

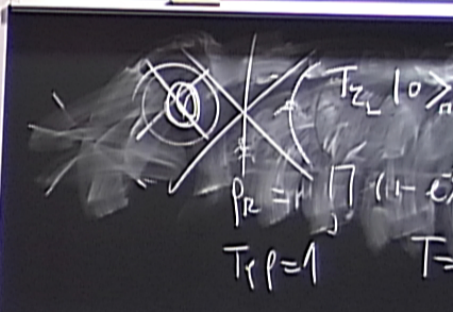
$$ds^2 = \frac{32M^2}{r} e^{-\frac{r}{2M}} dU dV - r^2 d\Omega^2$$

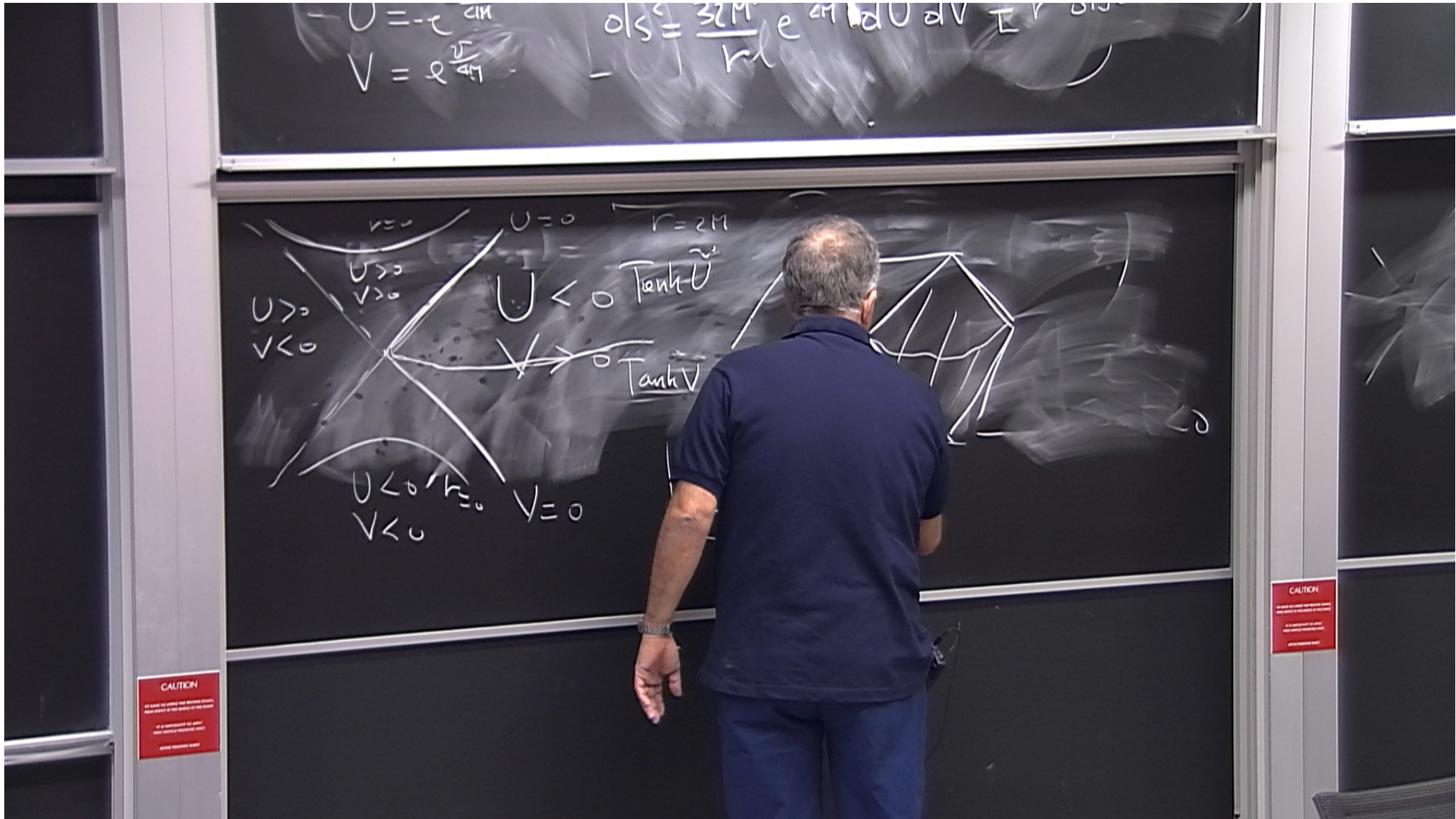
$$r = r(u, v)$$



$$|10\rangle_{\text{min}} = \frac{1}{\sqrt{2}} (|1-\rangle)$$

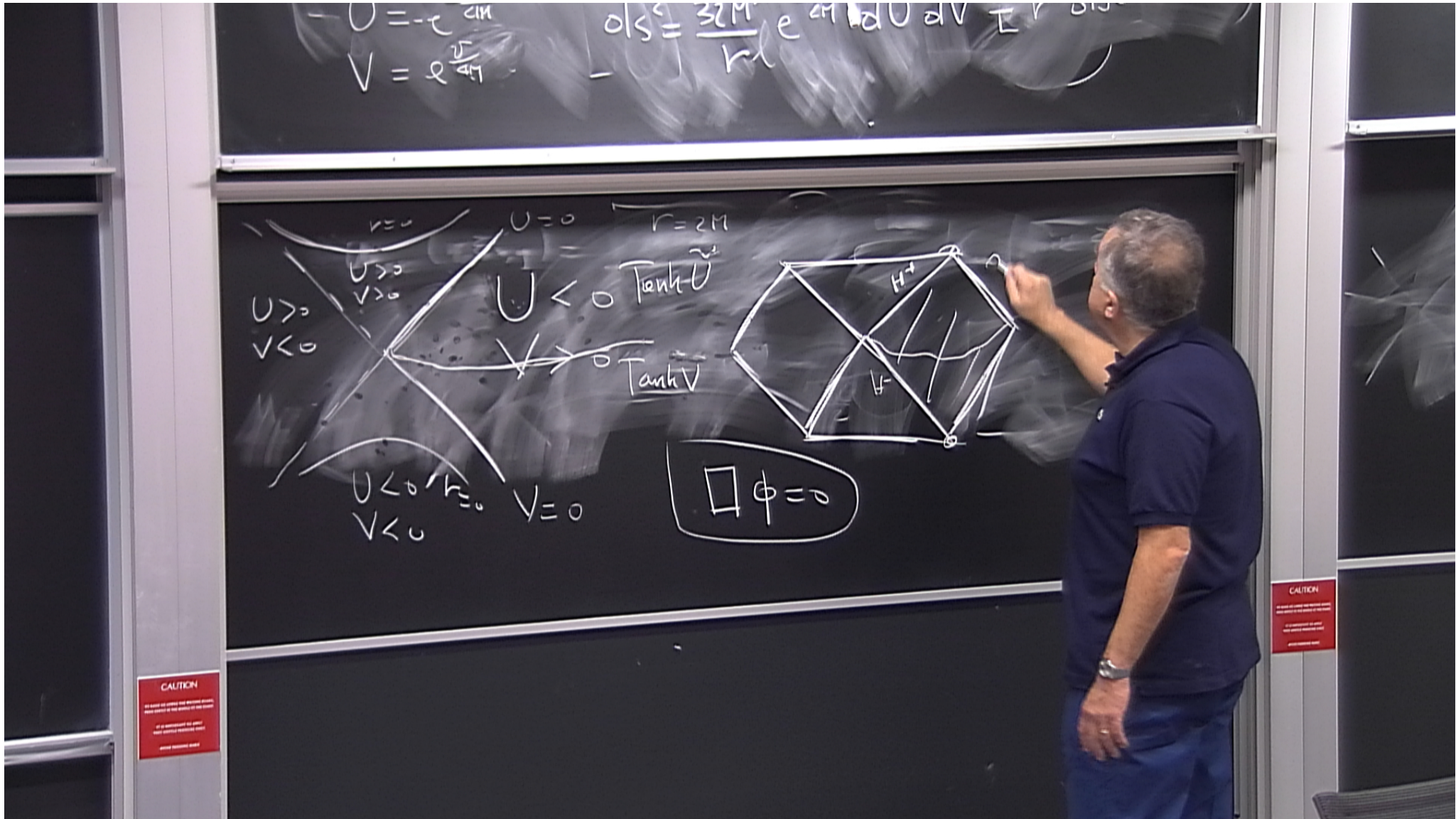
$$\langle \psi | 10\rangle_{\text{min}} \langle \psi |$$

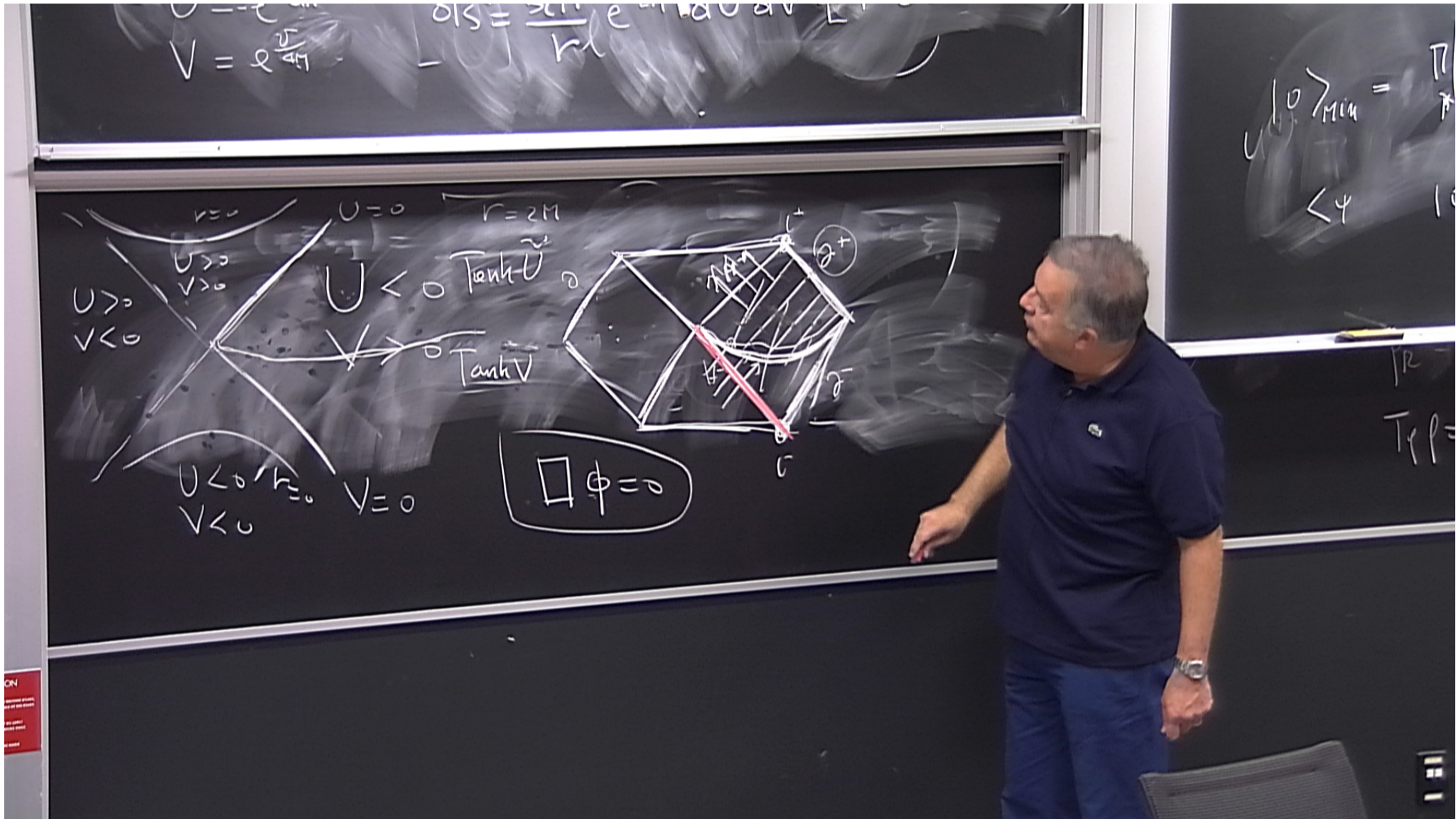


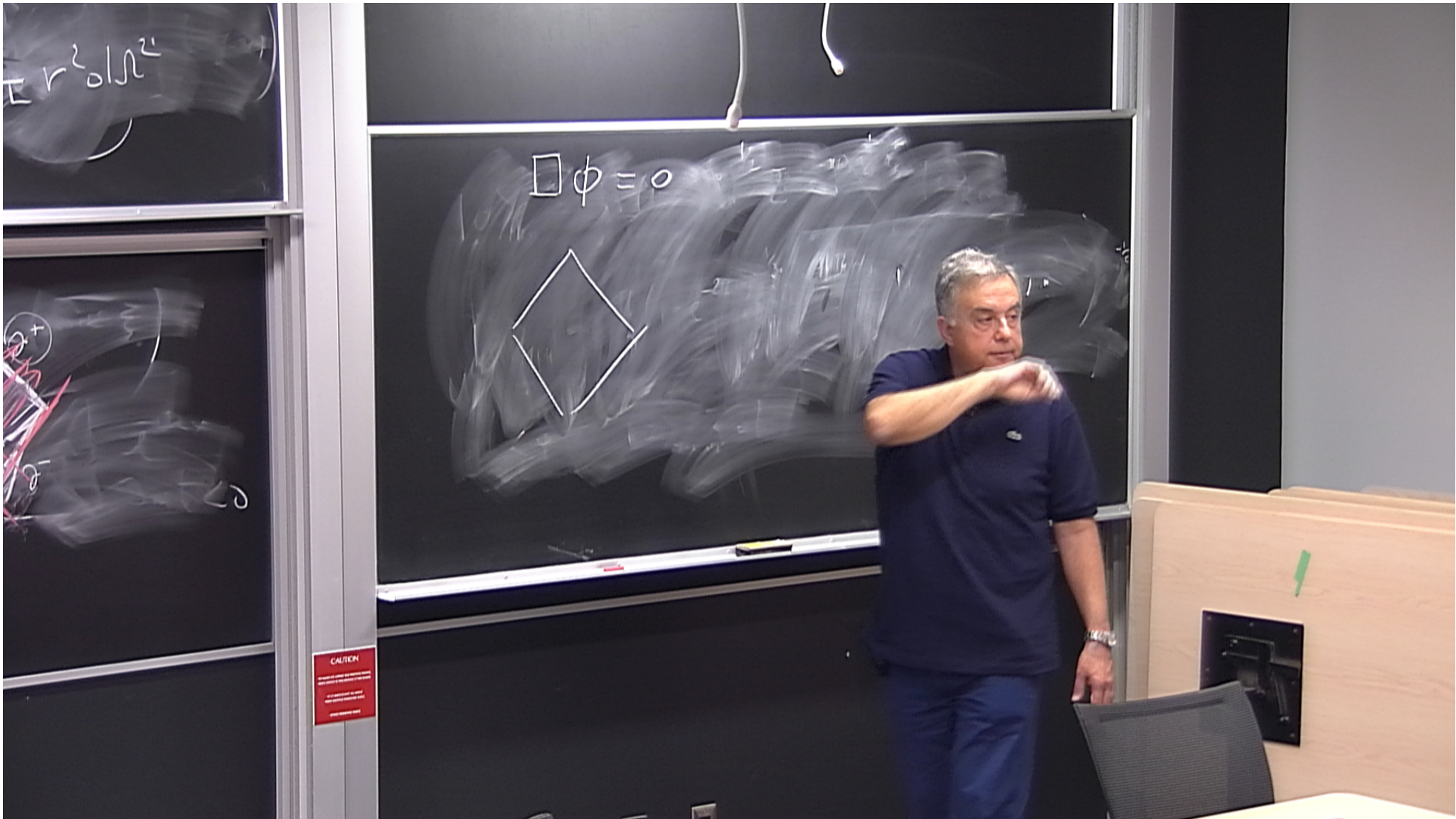


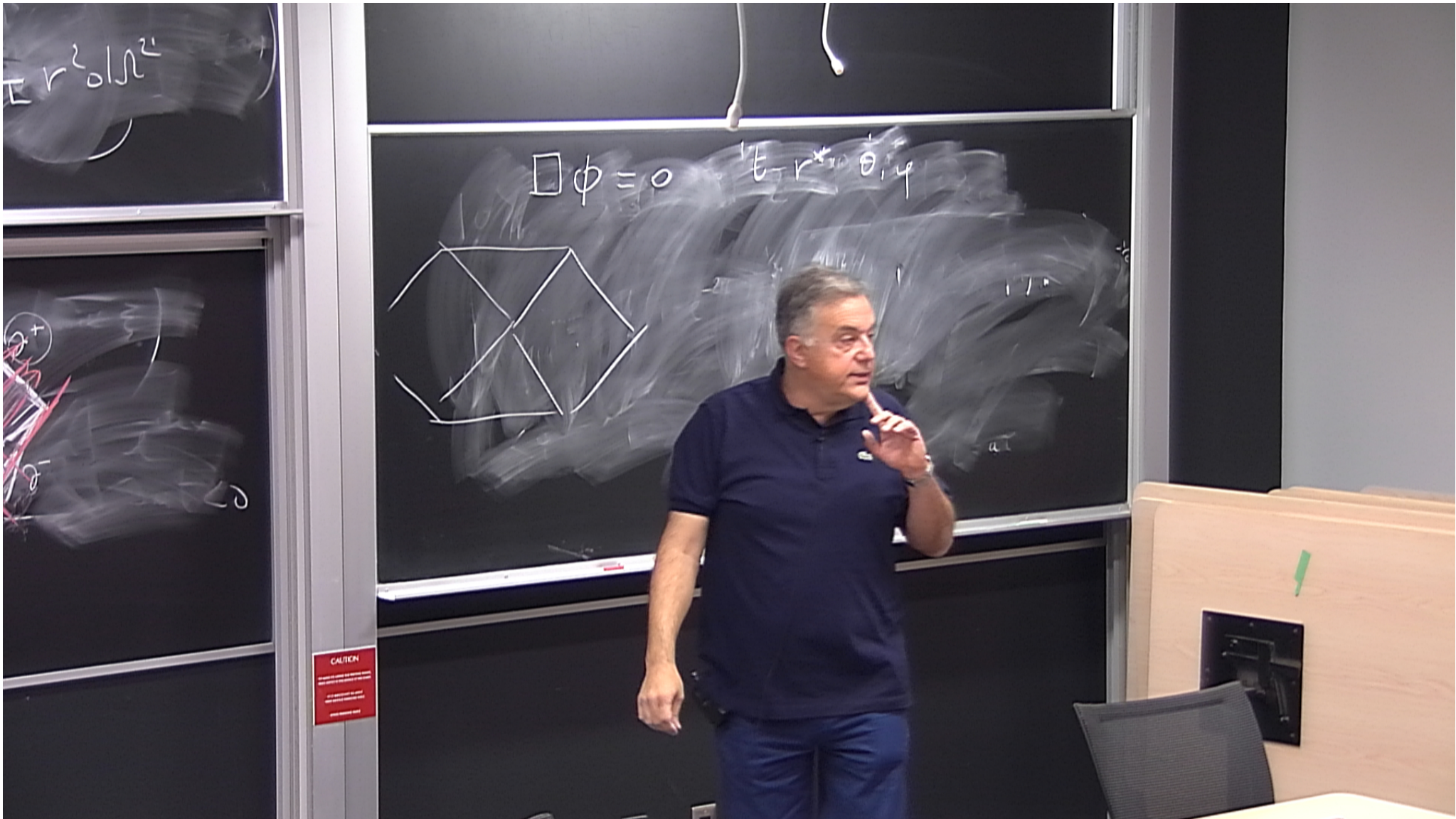
CAUTION
 Do not lean against the chalkboard.
 It is prohibited to use
 any sharp objects.

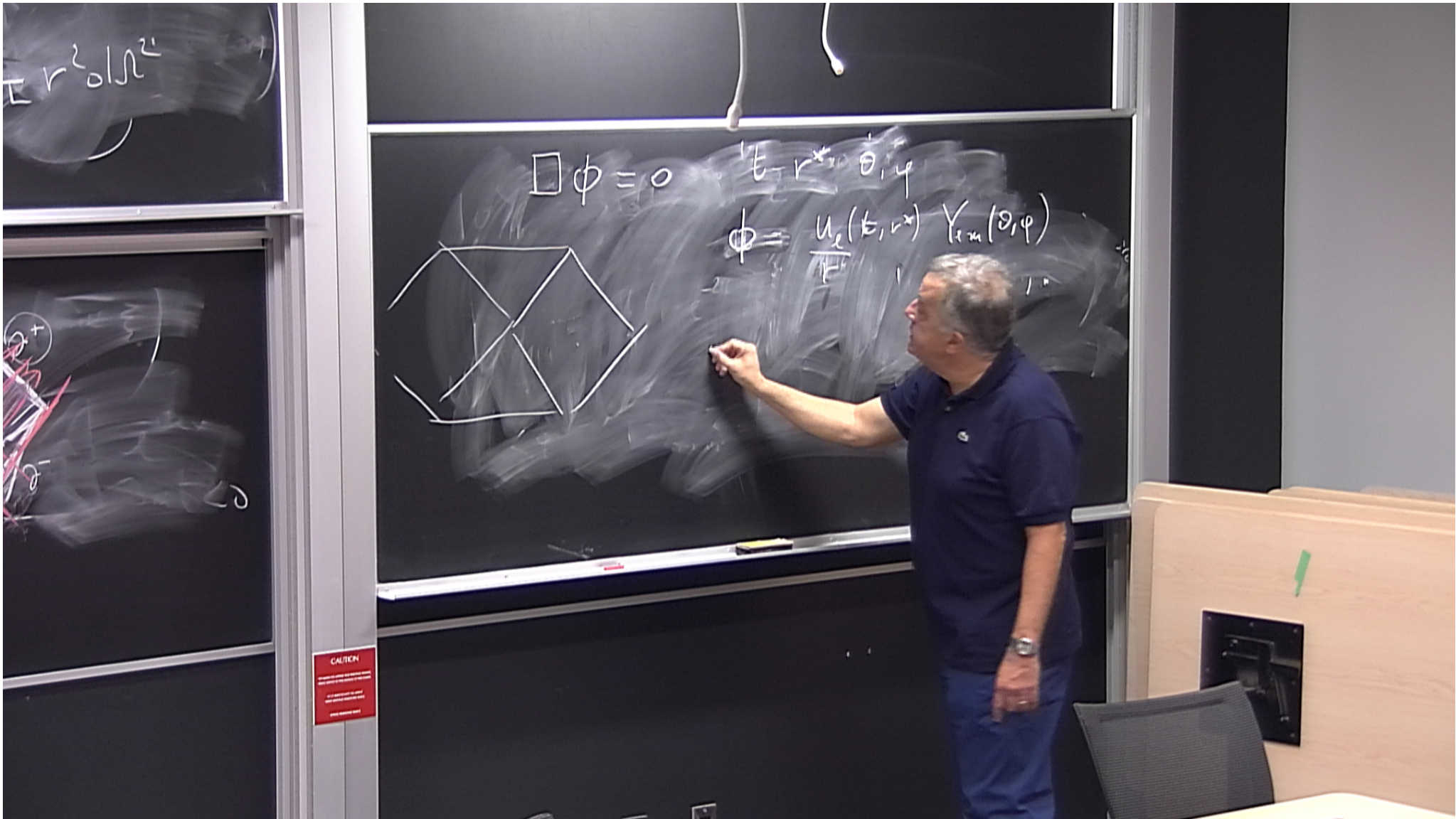
CAUTION
 Do not lean against the chalkboard.
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 any sharp objects.

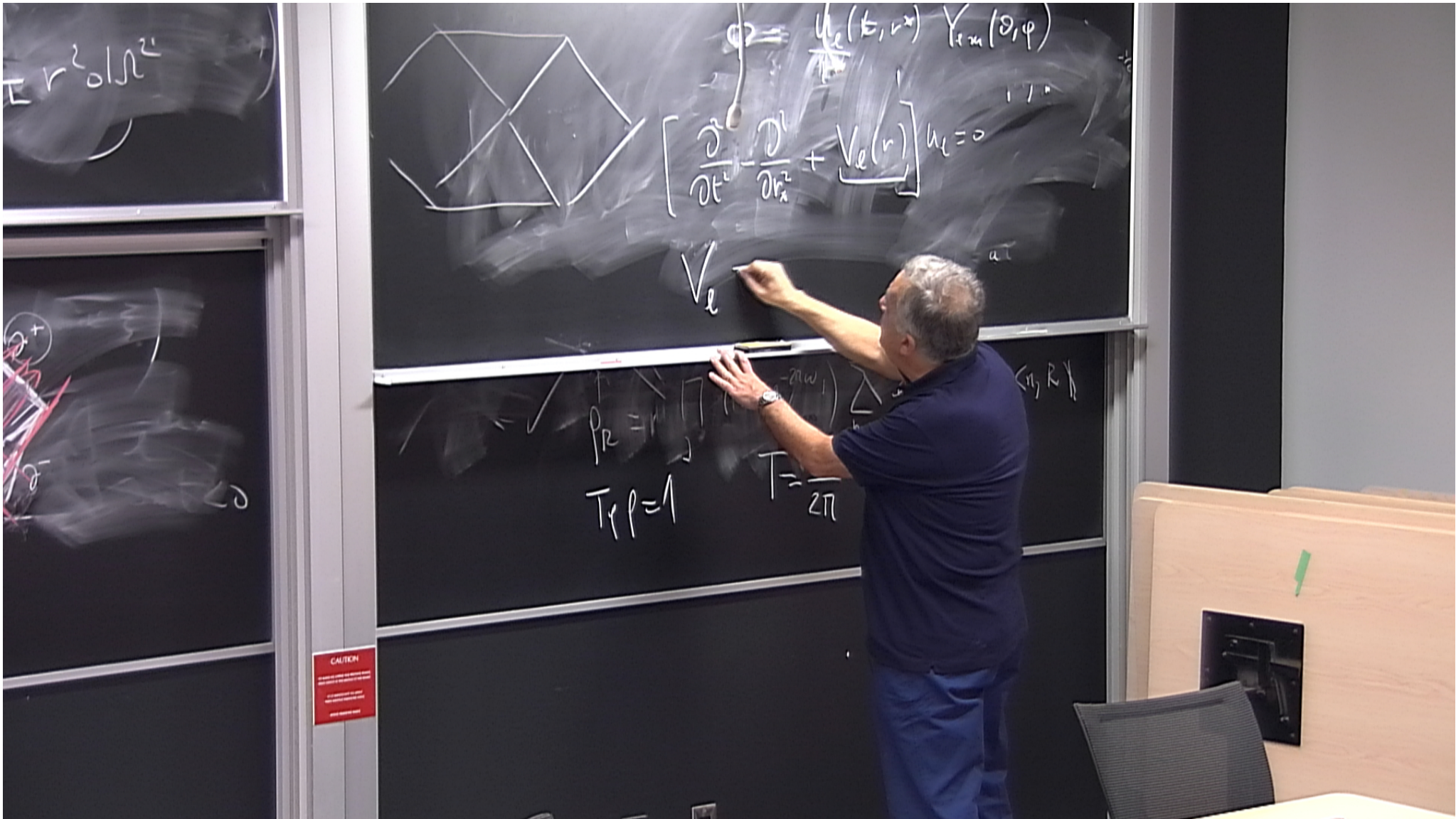


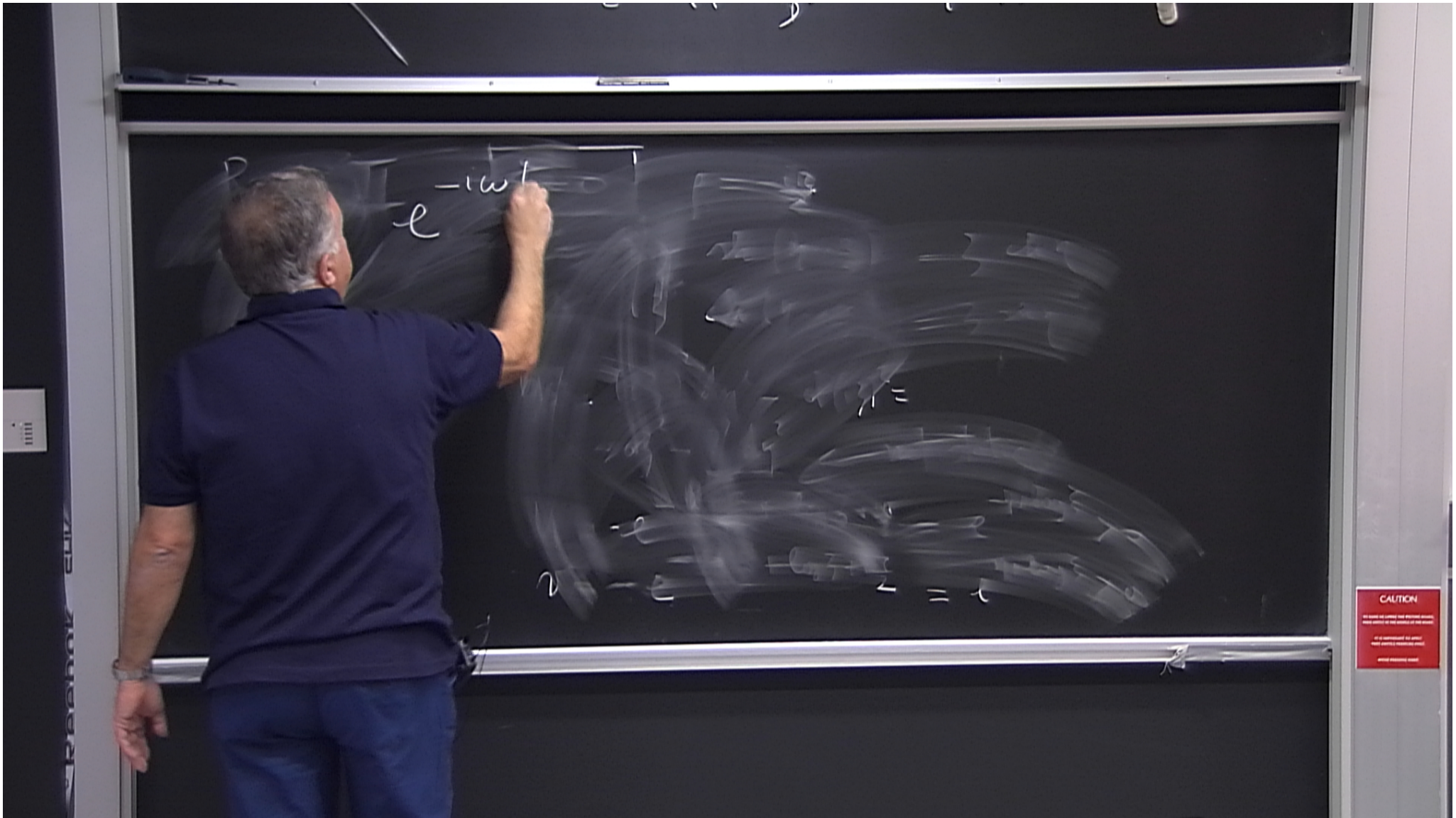


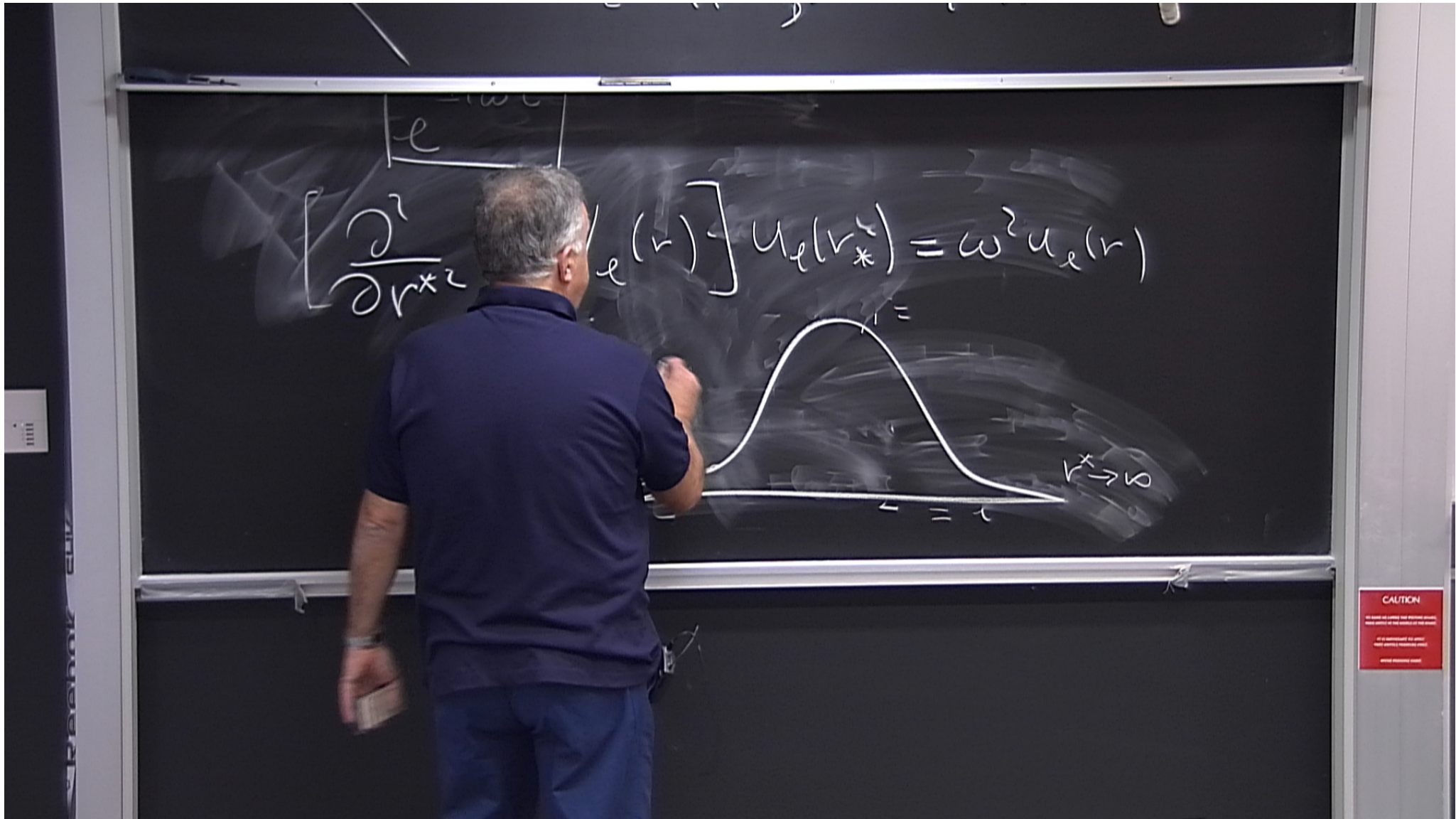


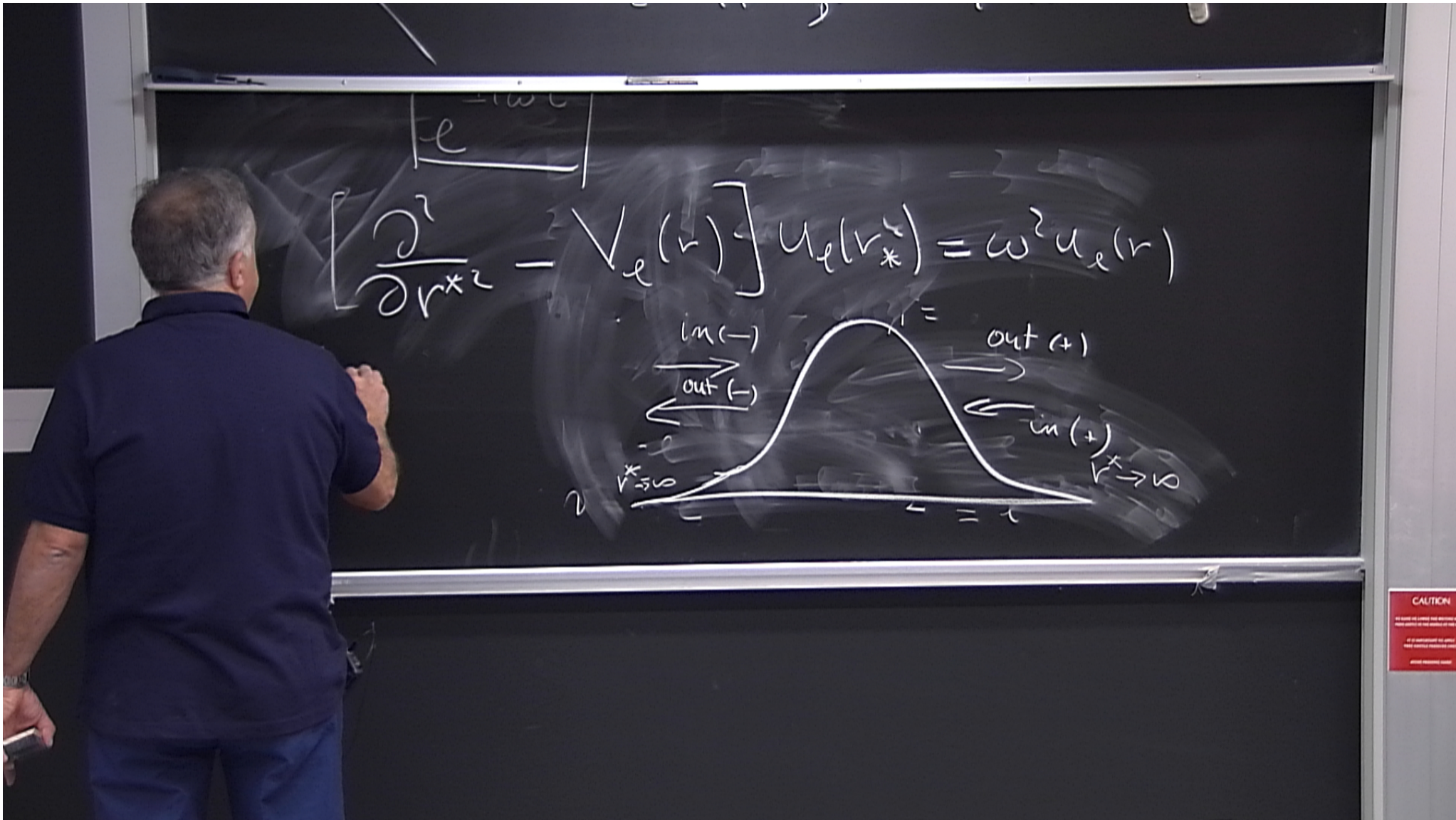


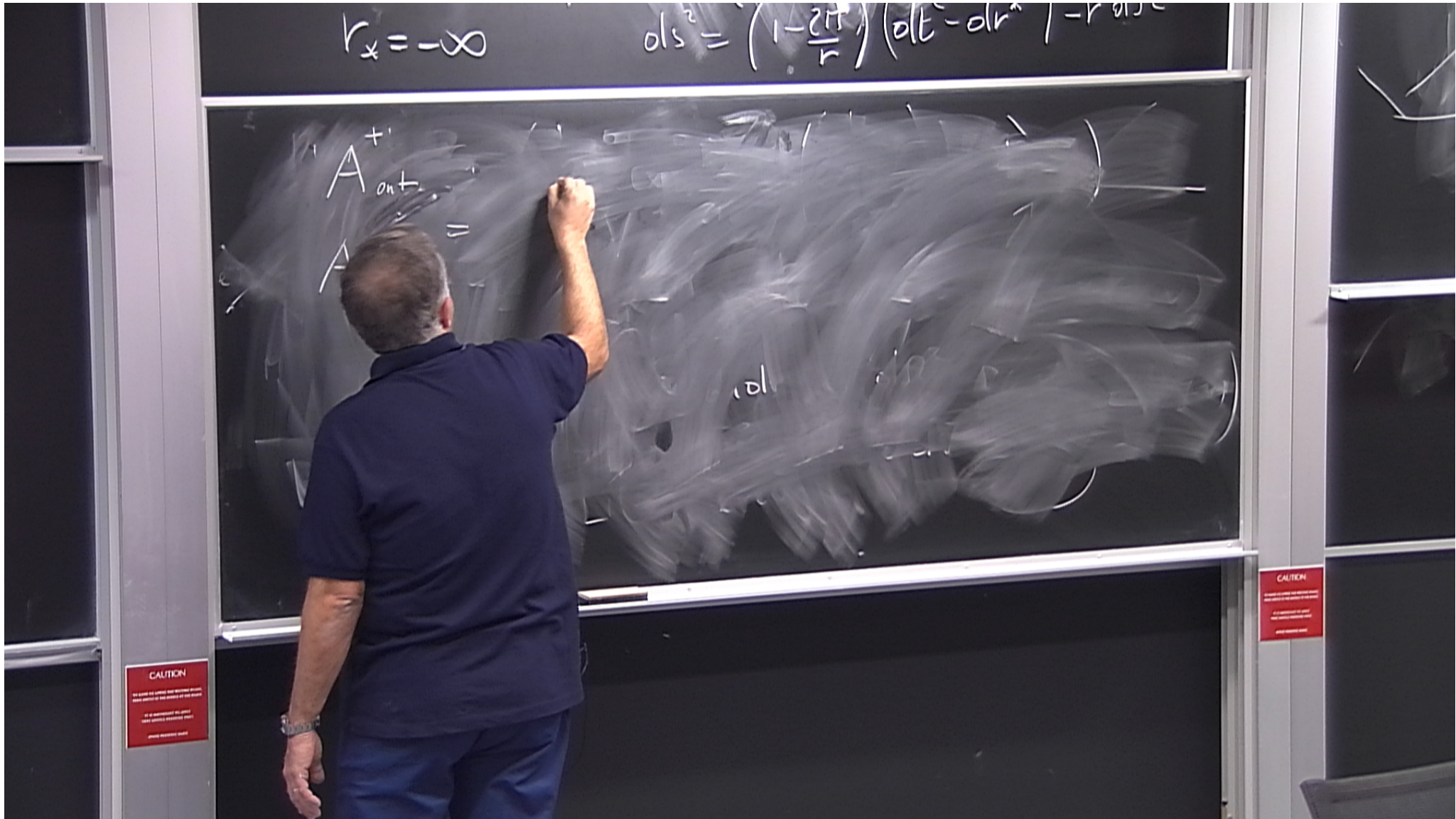












$$r_x = -\infty$$

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - dr^2 - r^2 d\Omega^2$$

$$\begin{pmatrix} A_{out}^+ \\ A_{out}^- \end{pmatrix} = S \begin{pmatrix} A_{in}^- \\ A_{in}^+ \end{pmatrix}$$
$$SS^\dagger = \mathbb{1}$$

$$z = \left(\frac{2}{3} + i\frac{1}{3}\right) \ln(\eta + i\rho)$$

$$\frac{1}{r^* z} \left[-V_e(r) \right] u_e(r^*) = \omega^2 u_e(r)$$

$r^* \sim -\infty$ $r^* \sim \infty$

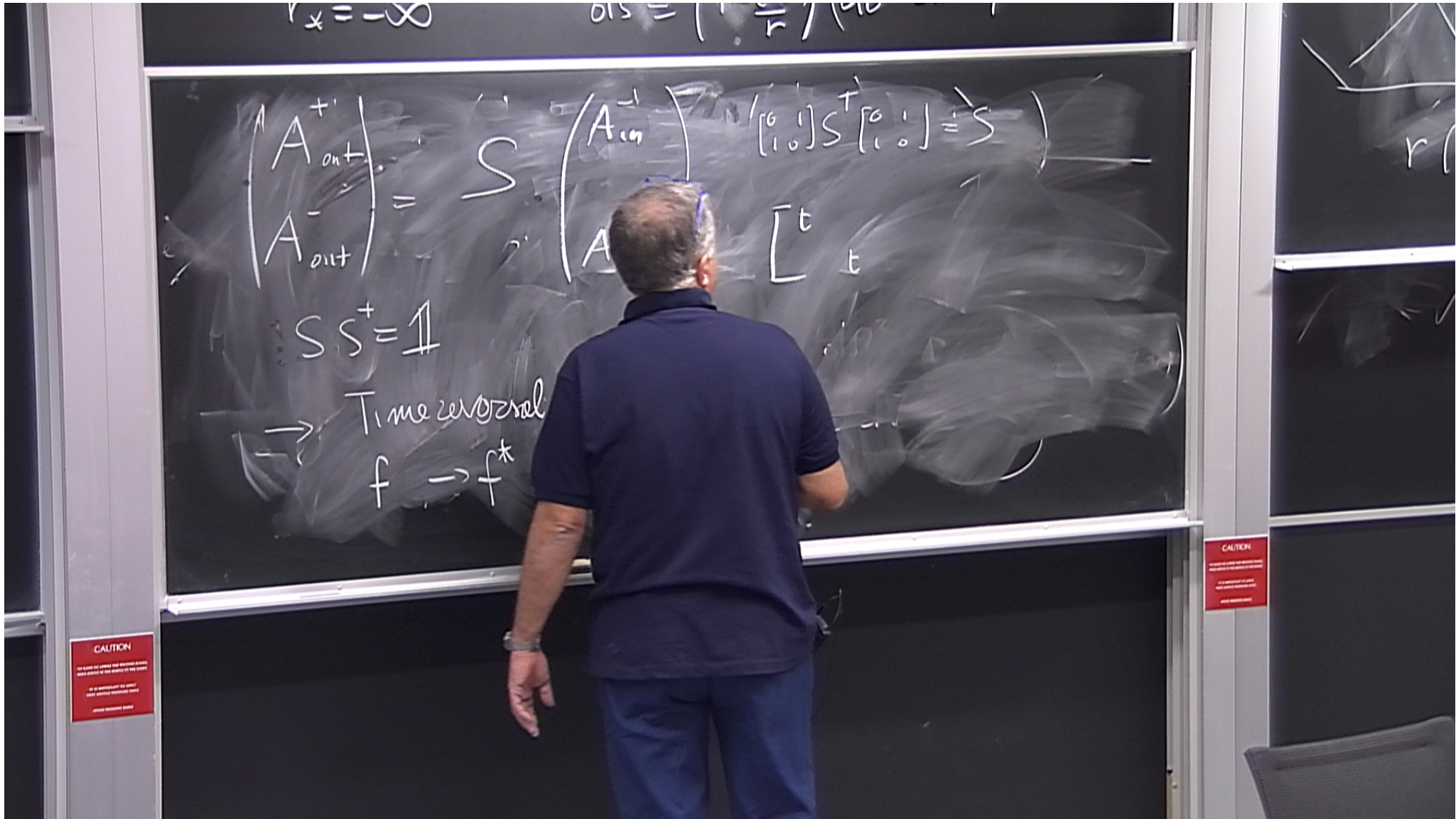
$A_{out}^+ e^{-i\omega r^*}$ $A_{in}^+ e^{-i\omega r^*}$ $A_{in}^- e^{-i\omega r^*}$

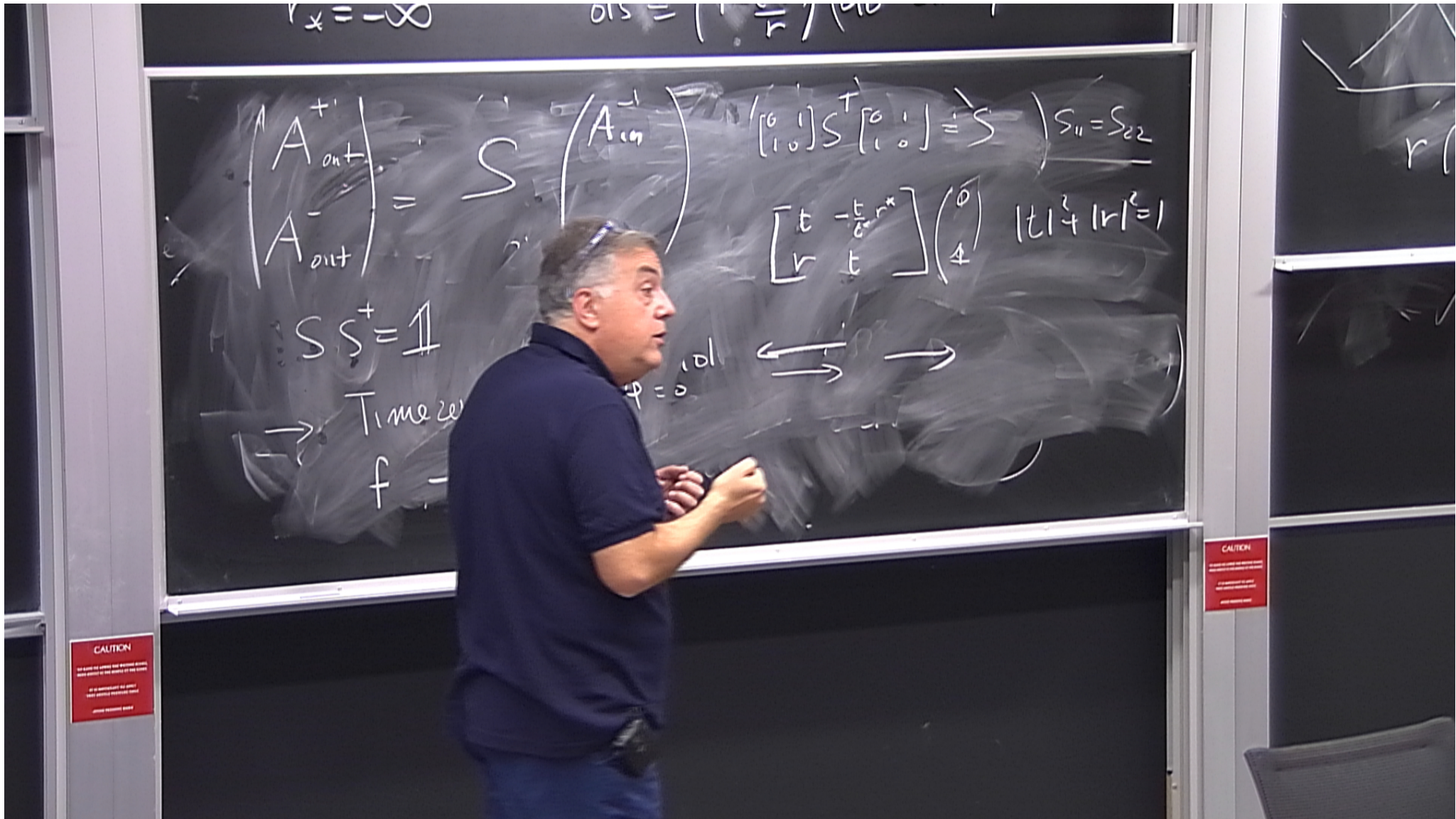
$e^{i\omega r^*}$ $e^{i\omega r^*}$

in (-) out (-) out (+) in (+)

$r^* \rightarrow \infty$

CAUTION
 All parts are cooled and become smooth.
 Please observe all the safety rules of the device.
 All in operation or under
 repair should be avoided.
 Avoid touching the





$$\begin{pmatrix} A_{out}^+ \\ A_{out}^- \end{pmatrix} = S \begin{pmatrix} A_{in}^+ \\ A_{in}^- \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = S \quad S_{11} = S_{22}$$

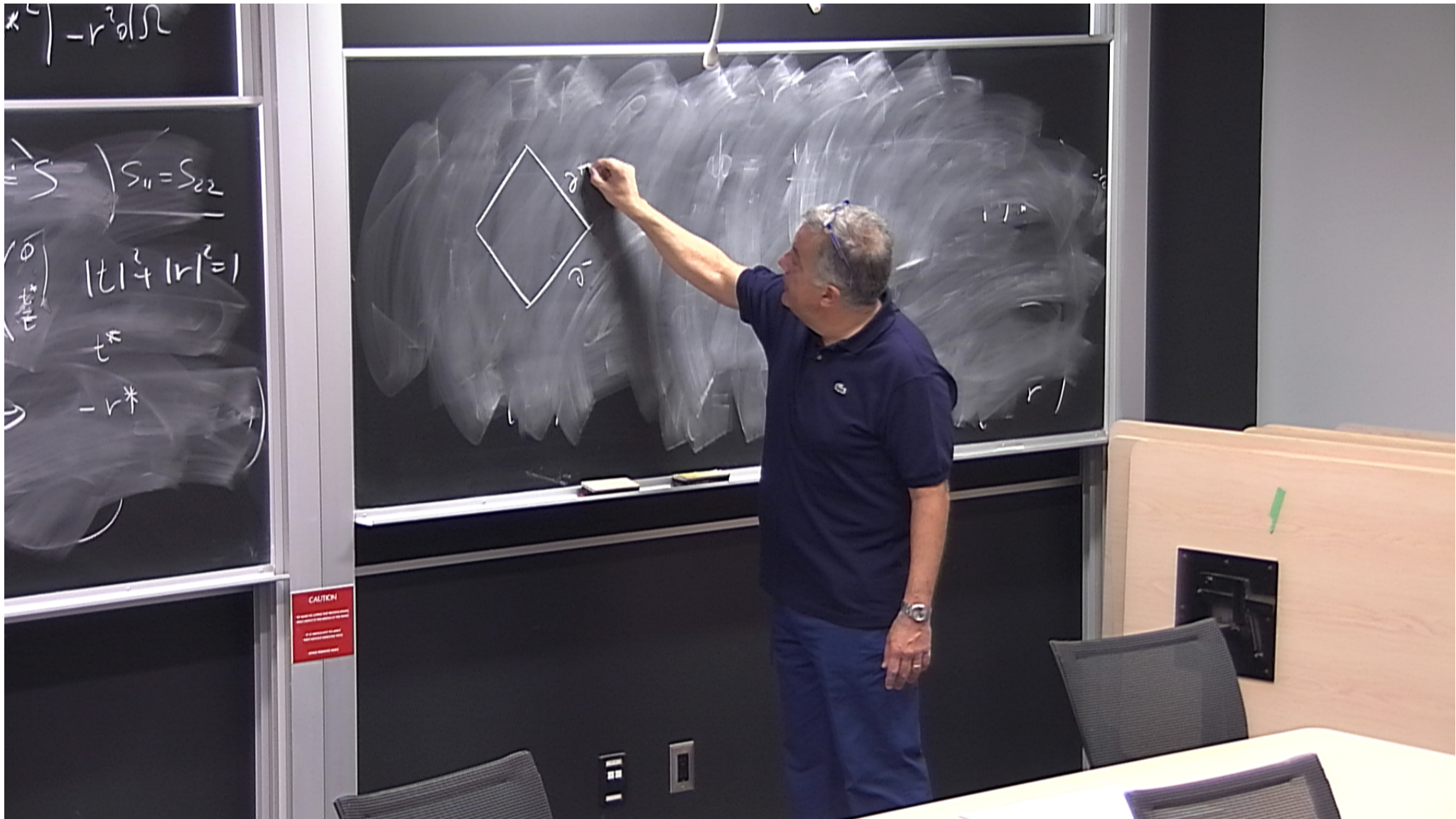
$$\begin{bmatrix} t & -\frac{t}{r} r^* \\ r & t \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |t|^2 + |r|^2 = 1$$

$$S S^+ = \mathbb{1}$$

Time reversal $\phi = 0$

CAUTION
 THE BOARD IS HOTTER THAN BREAD CRUMBS
 AND COULD BE THE SOURCE OF THE BURN.
 IT IS UNLAWFUL TO POINT
 YOUR FINGER AT THE BOARD.
 THANK YOU FOR YOUR
 ATTENTION.

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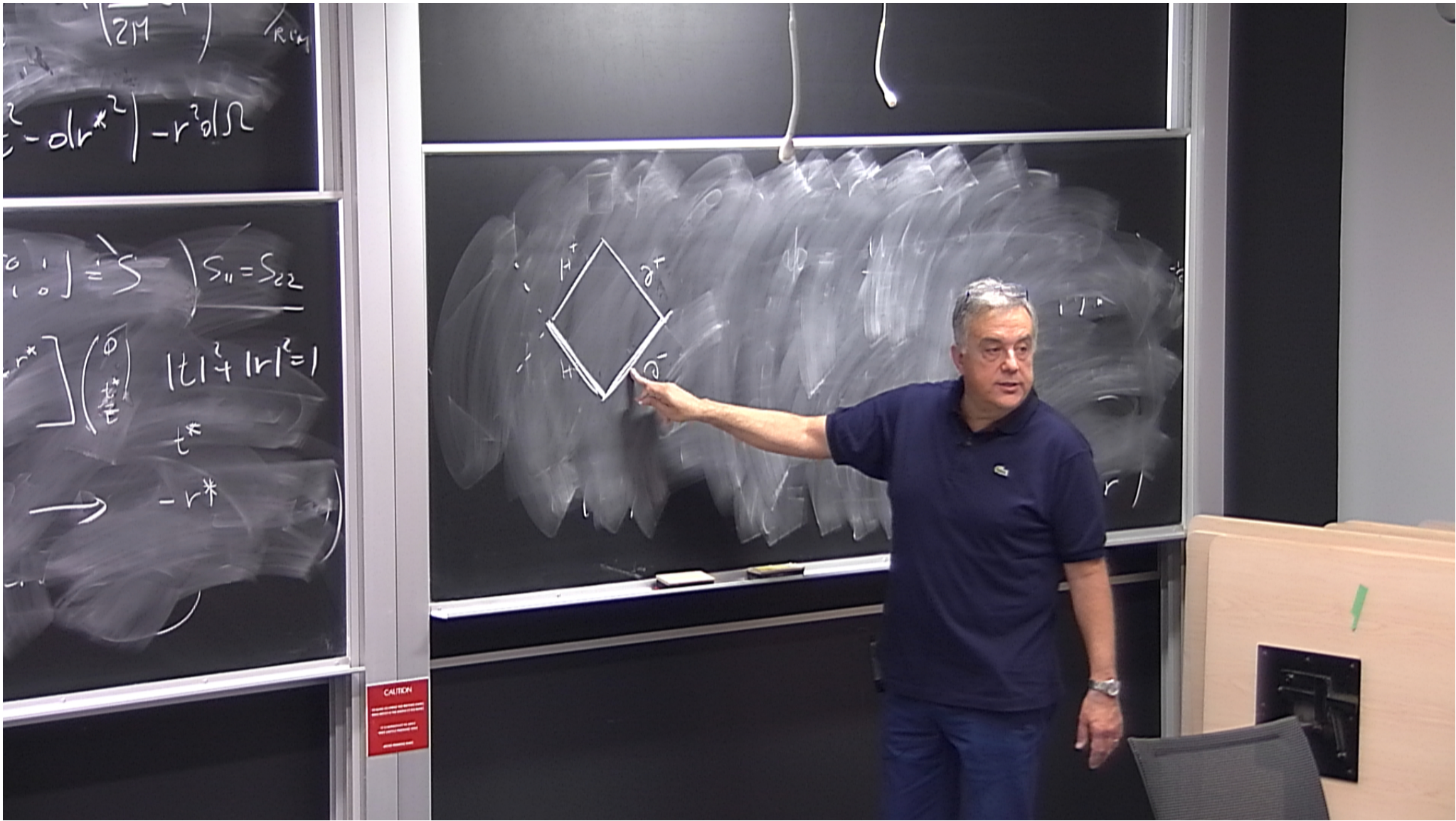
$$z = (\xi + i\zeta) \operatorname{ch}(\eta + i\varphi)$$

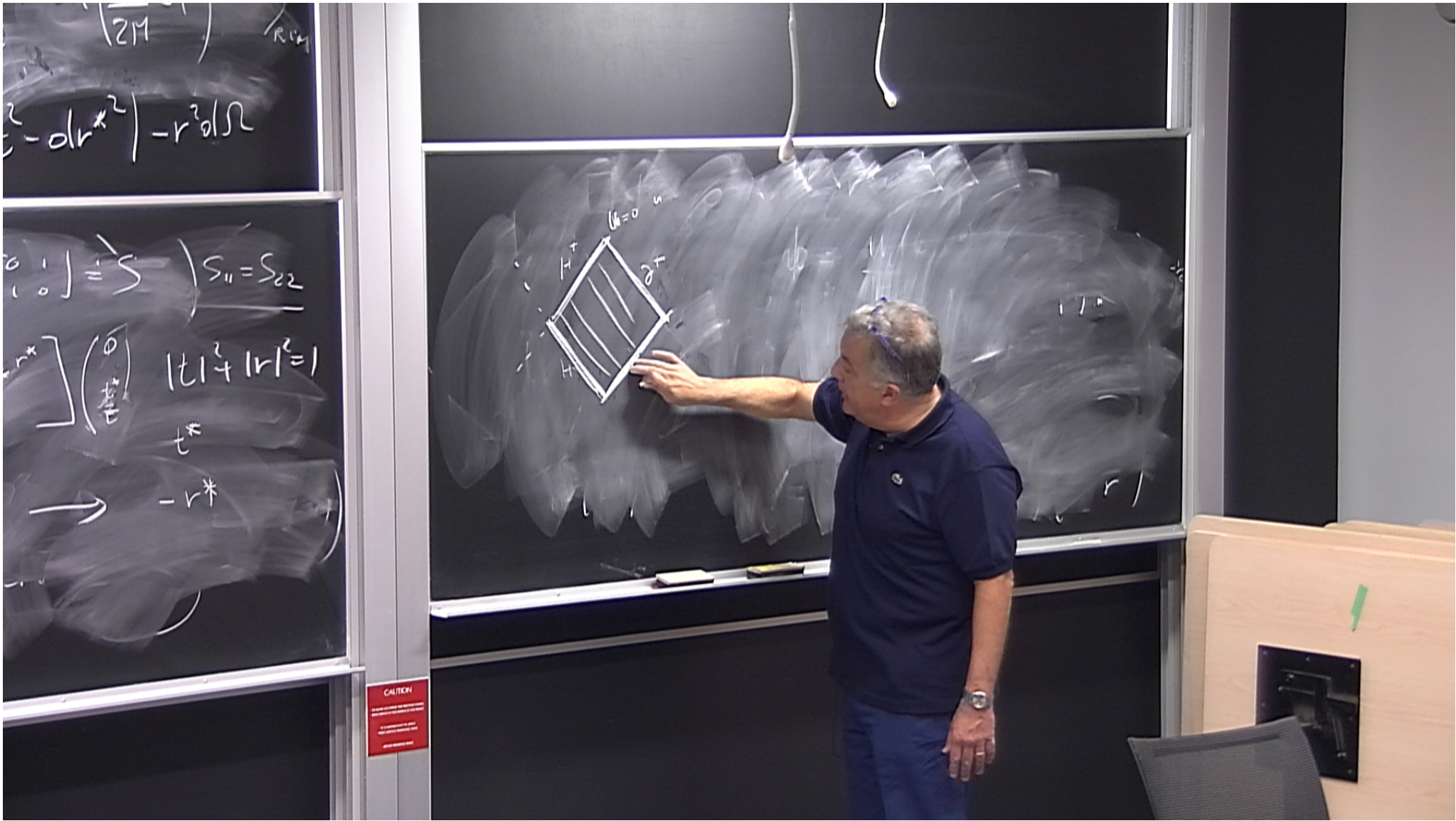
$$\left[\frac{\partial^2}{\partial r^{*2}} - V_e(r) \right] u_e(r^*) = \omega^2 u_e(r^*)$$

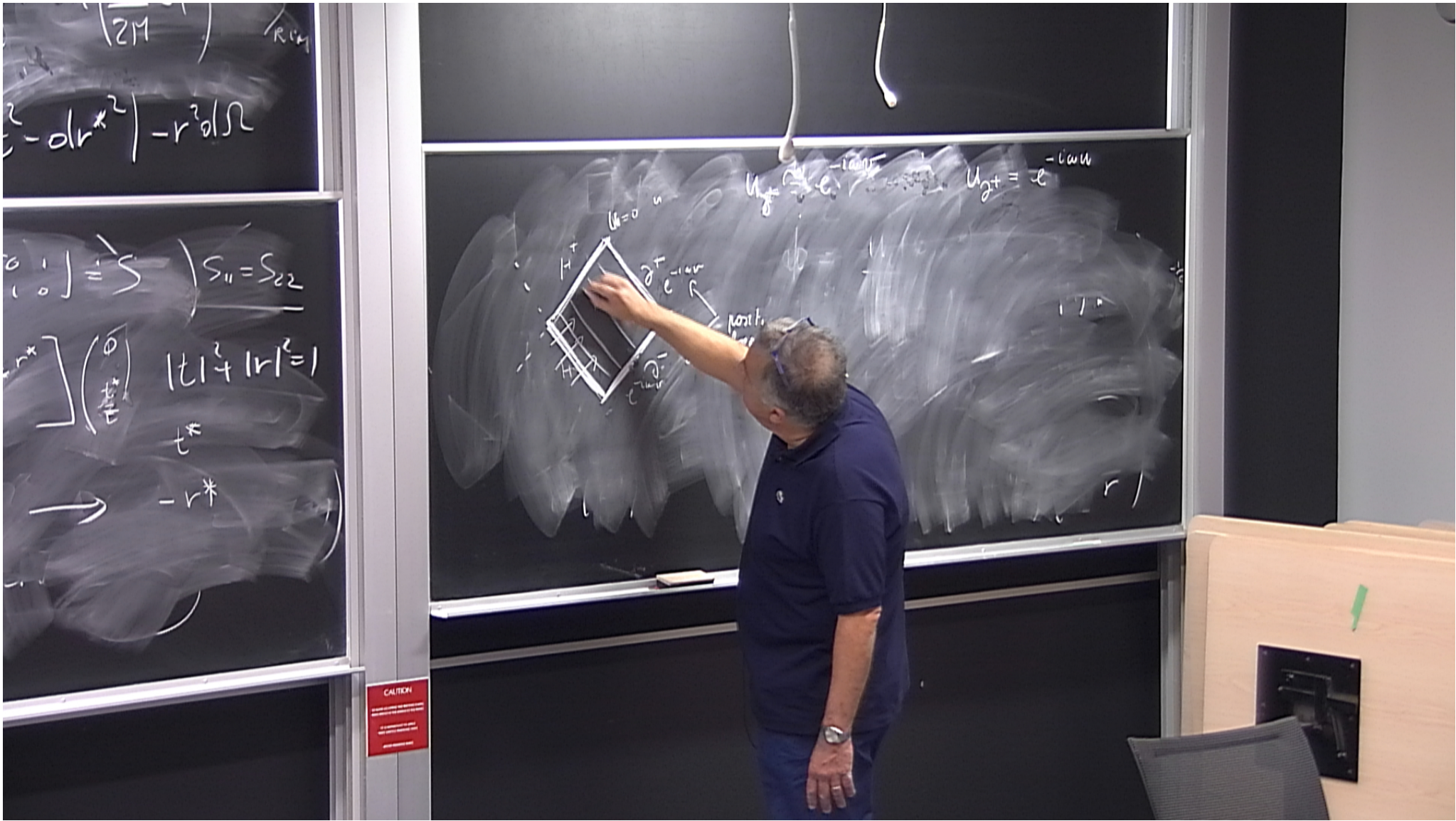
$$r^* \rightarrow -\infty \quad u_e(r^*) = A_{in}^{(-)} e^{i\omega r^*} + A_{out}^{(-)} e^{-i\omega r^*}$$

$$r^* \rightarrow +\infty \quad u_e(r^*) = A_{out}^{(+)} e^{i\omega r^*} + A_{in}^{(+)} e^{-i\omega r^*}$$









$$\frac{dr}{(1-\frac{2M}{r})} = r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

$$ds^2 = \left(1 - \frac{2M}{r}\right) (dt^2 - dr^2) - r^2 d\Omega^2$$

$$S = \int \left(A_{\mu\nu}^{-1} \dot{x}^\mu \dot{x}^\nu - V(x) \right) dt$$

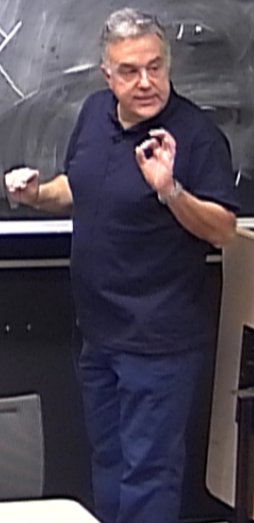
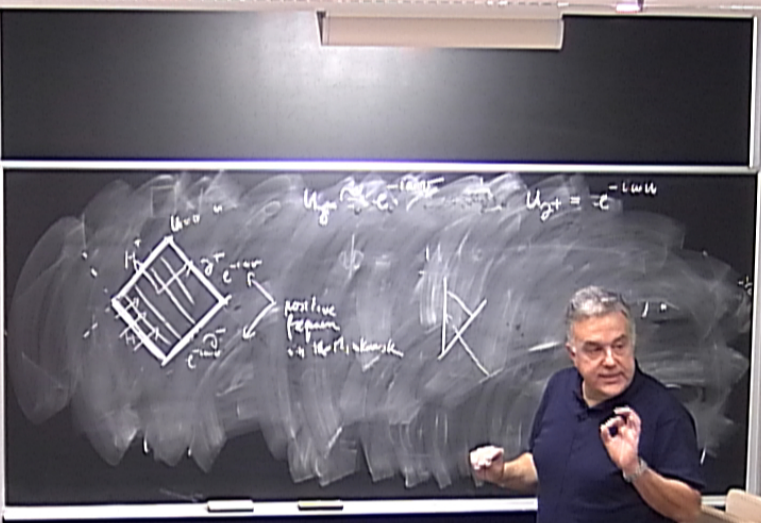
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = S$$

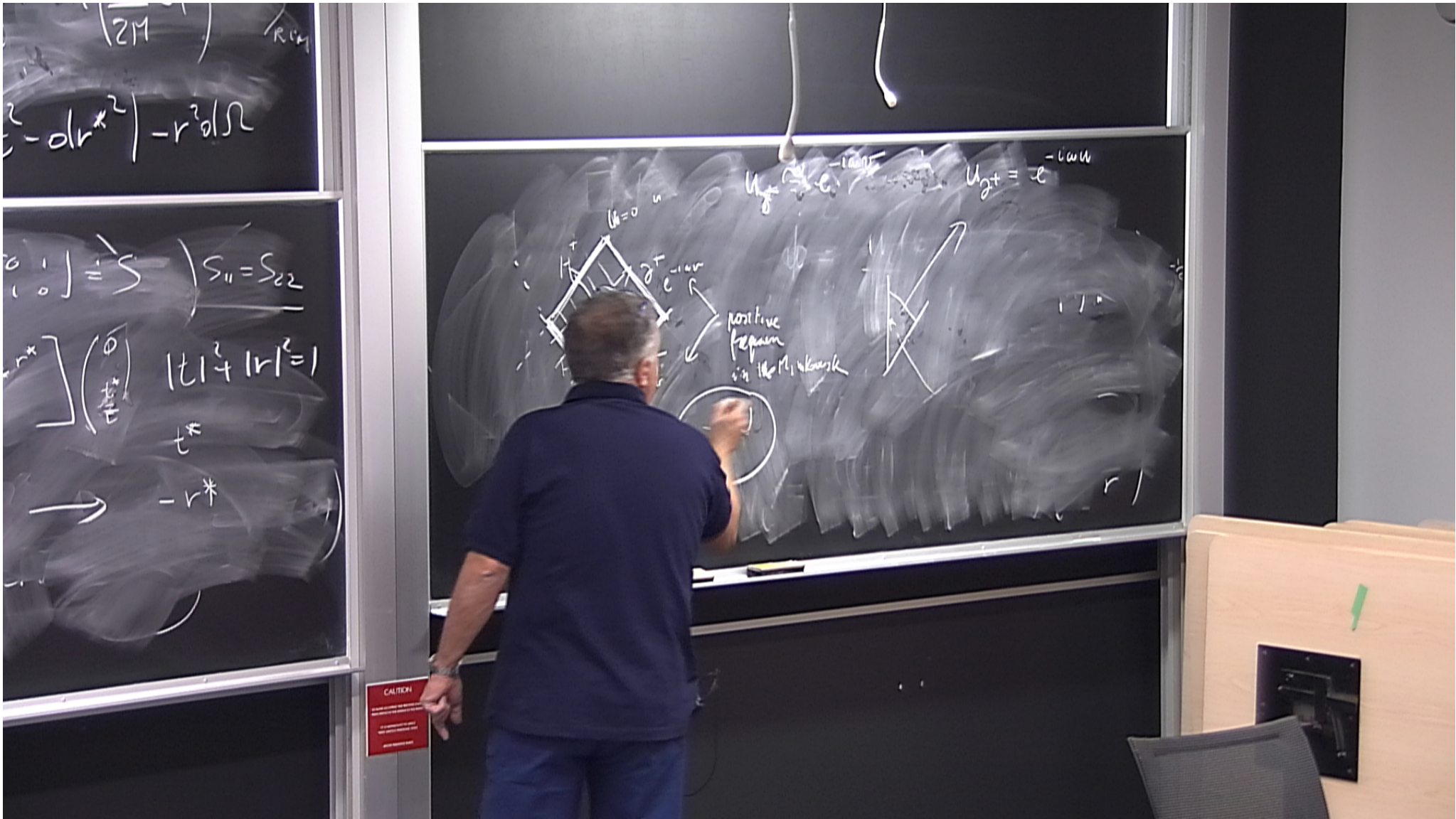
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

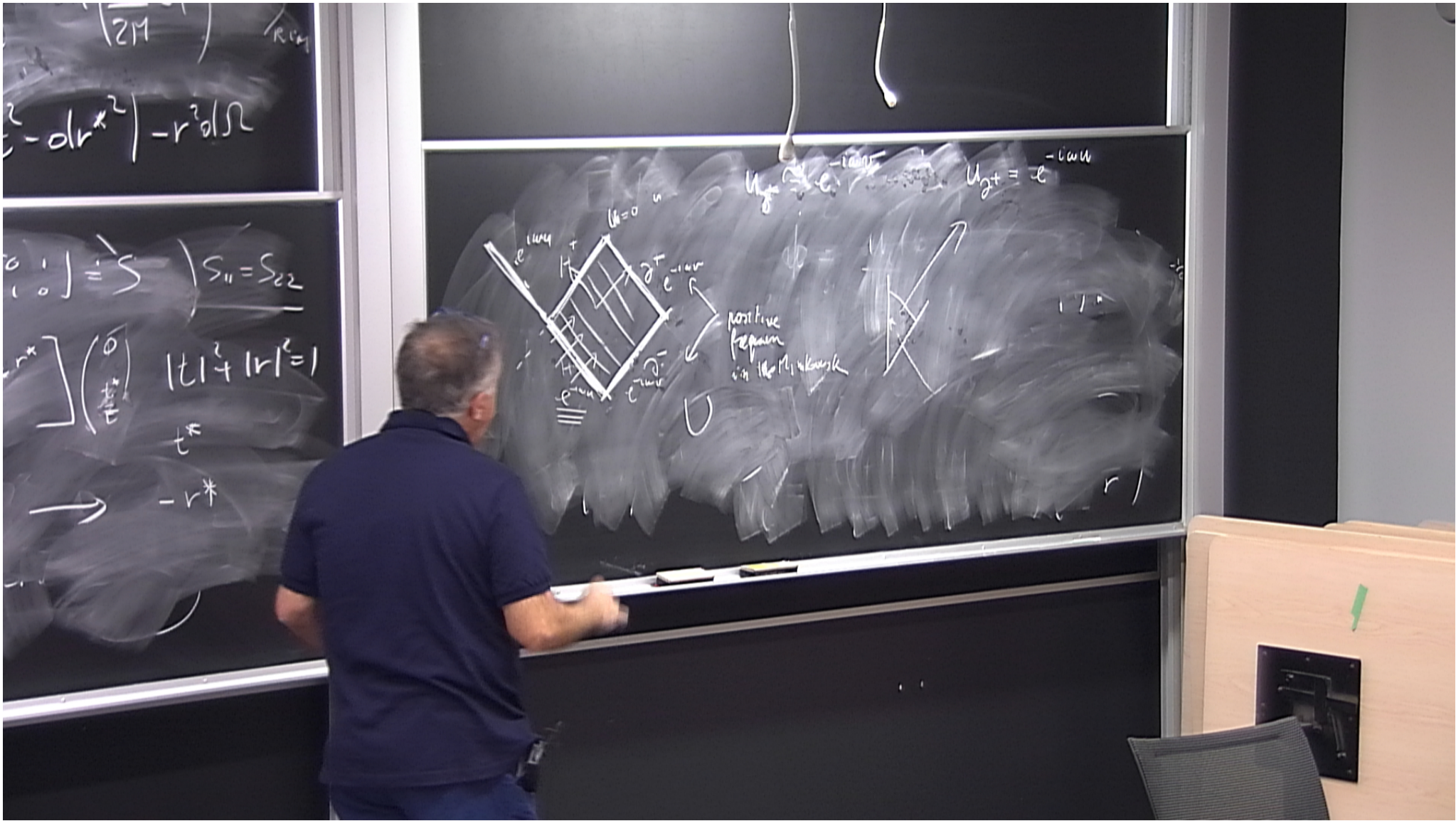
$$|L| = |r| = 1$$

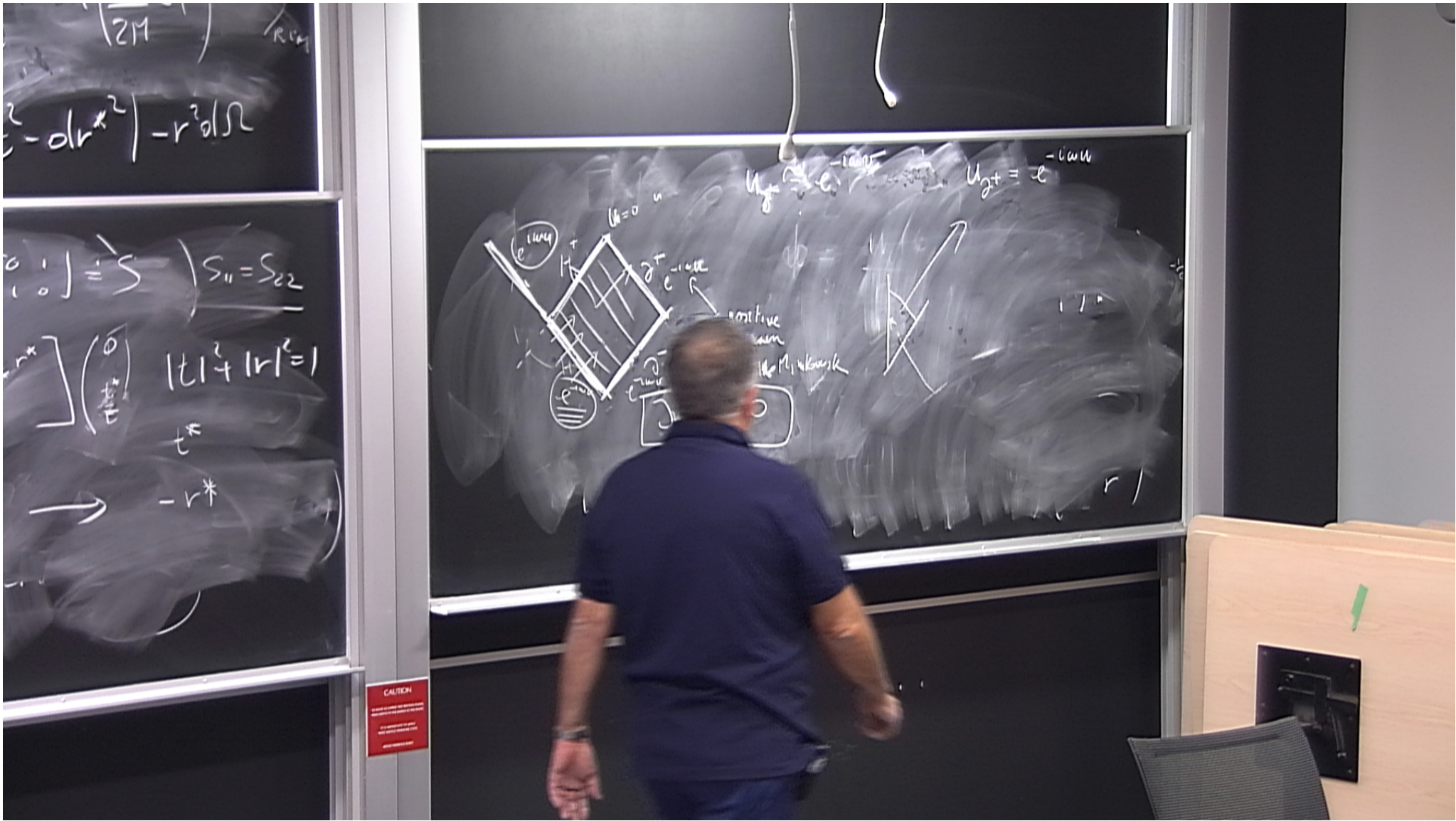
me central $\square \phi = 0 \iff \rightarrow -r^*$

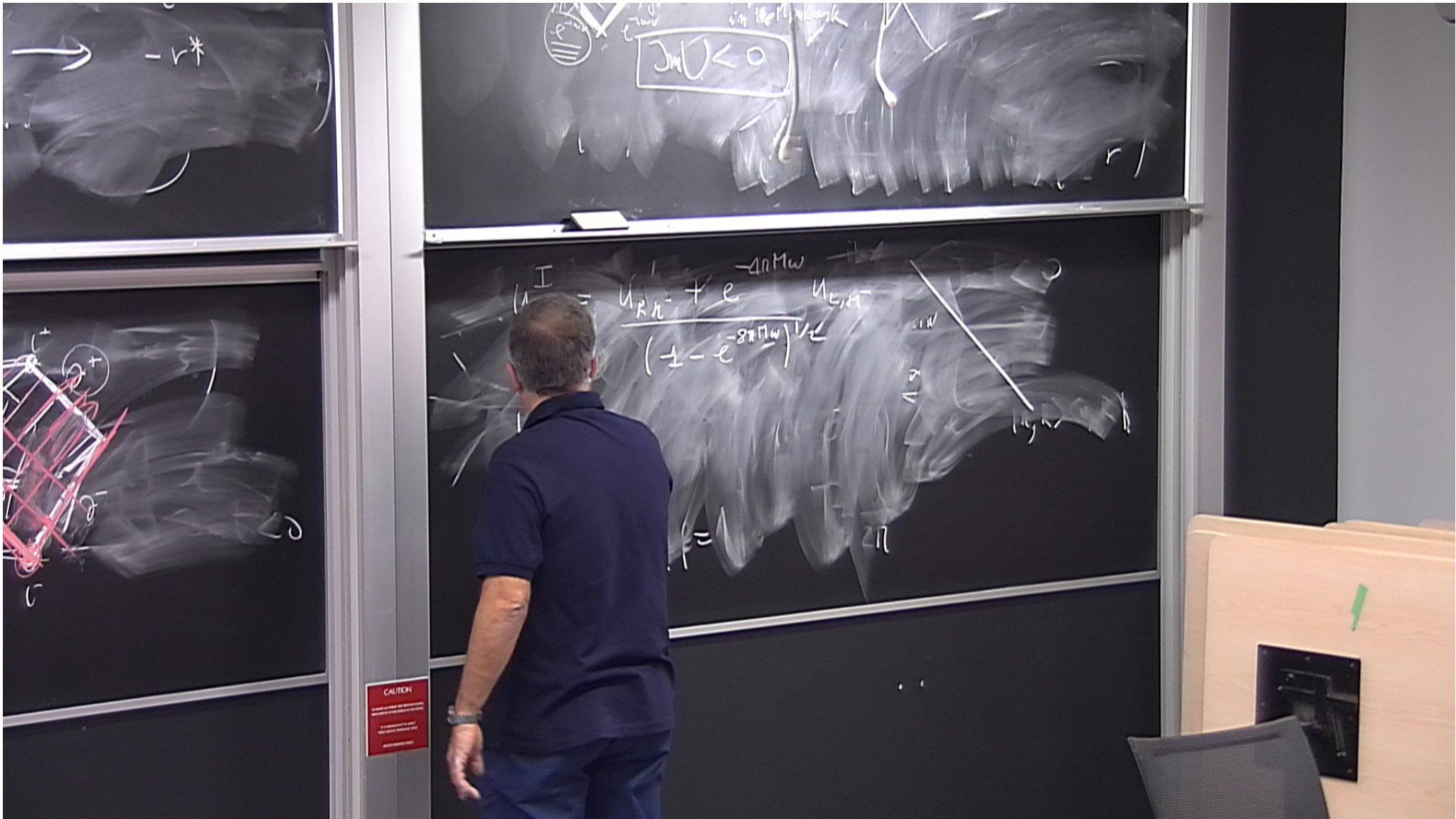
$f \rightarrow f^*$

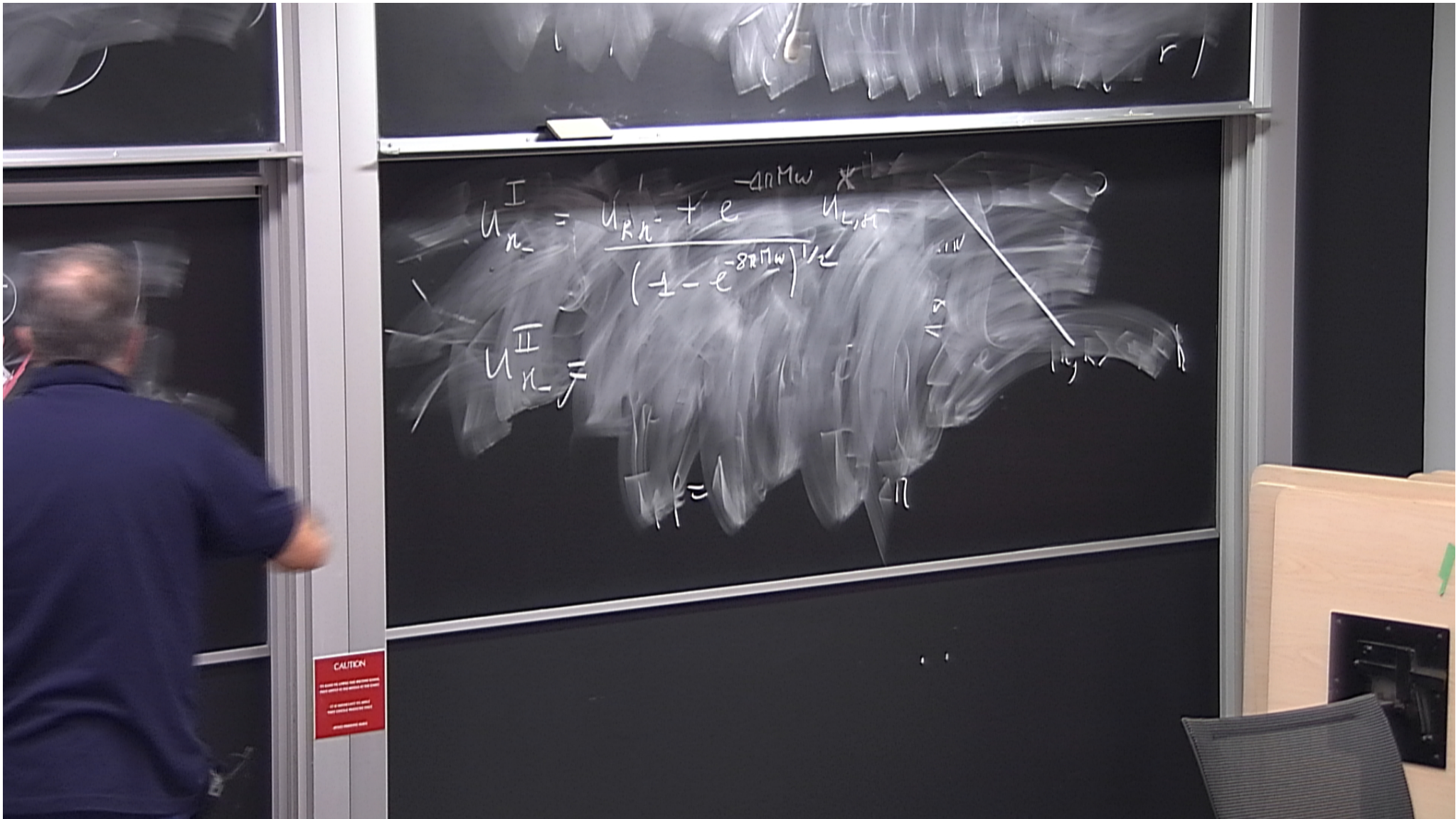


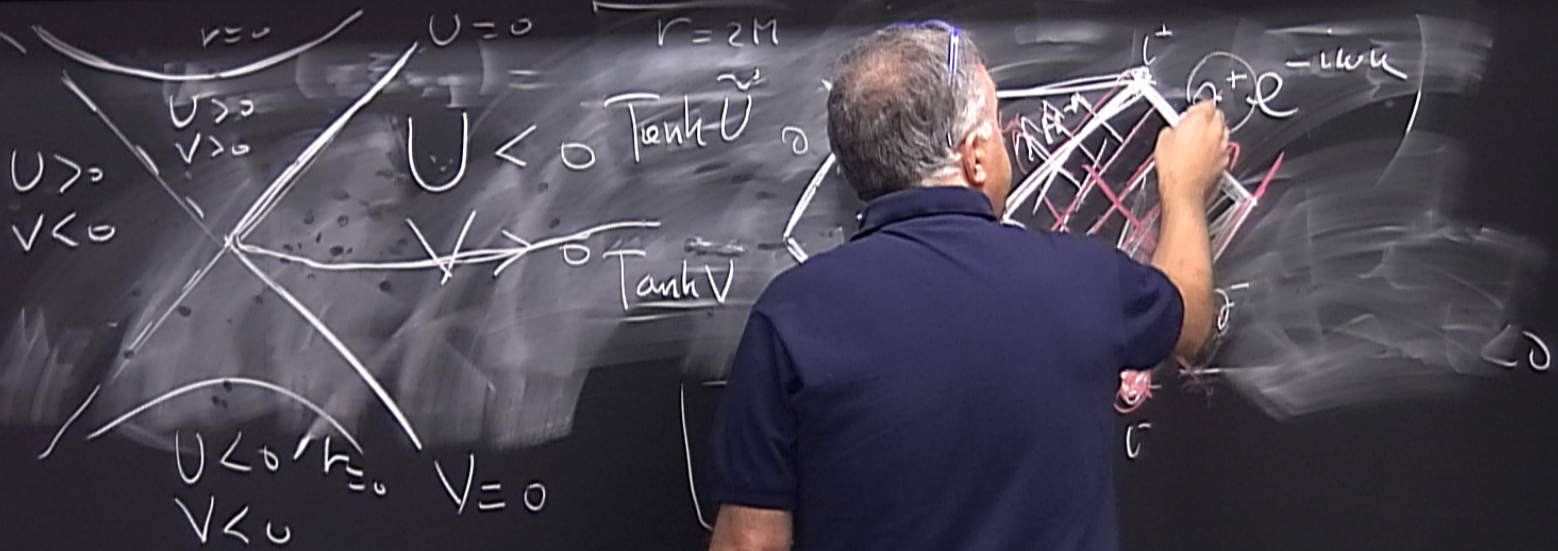






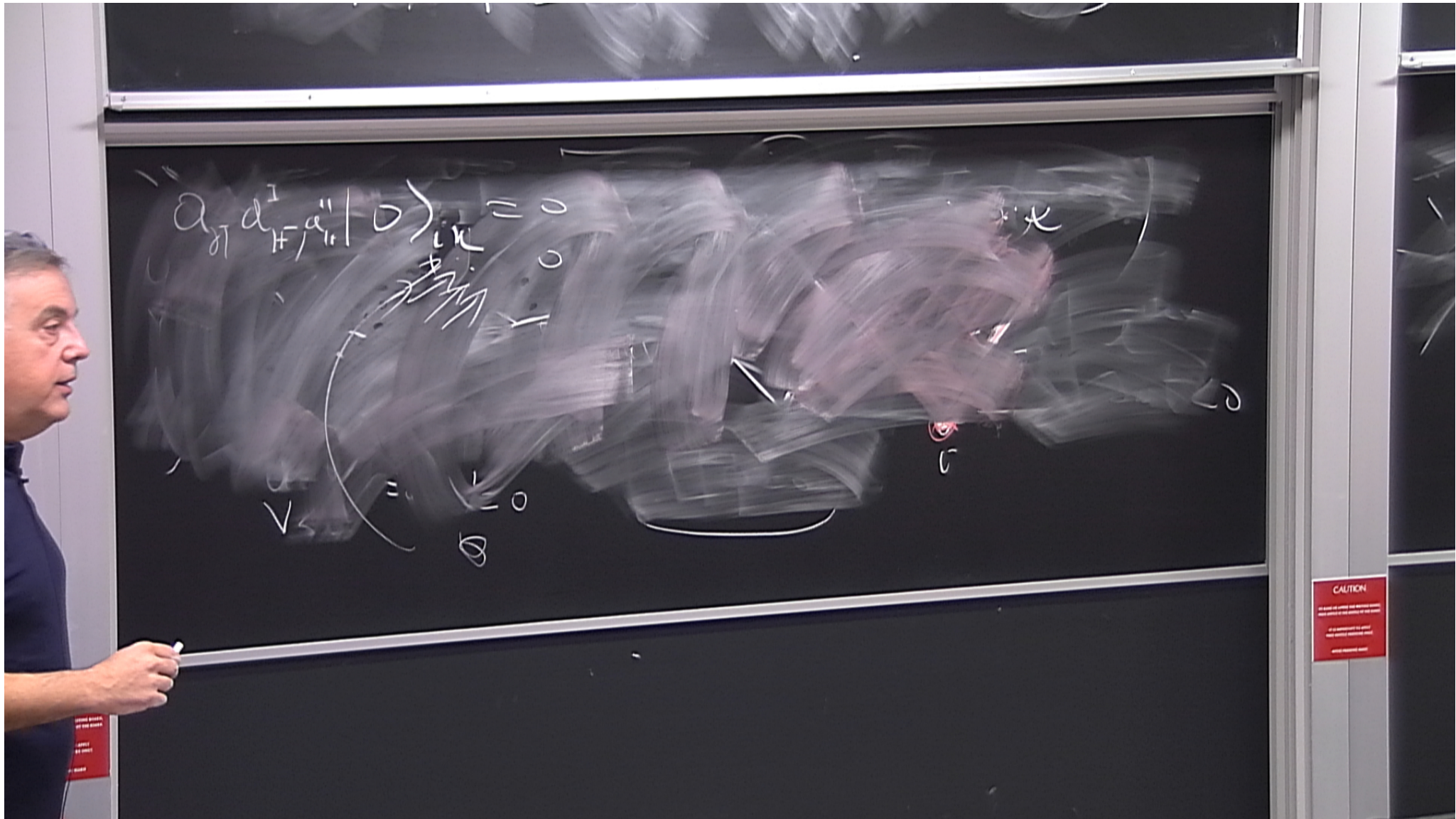


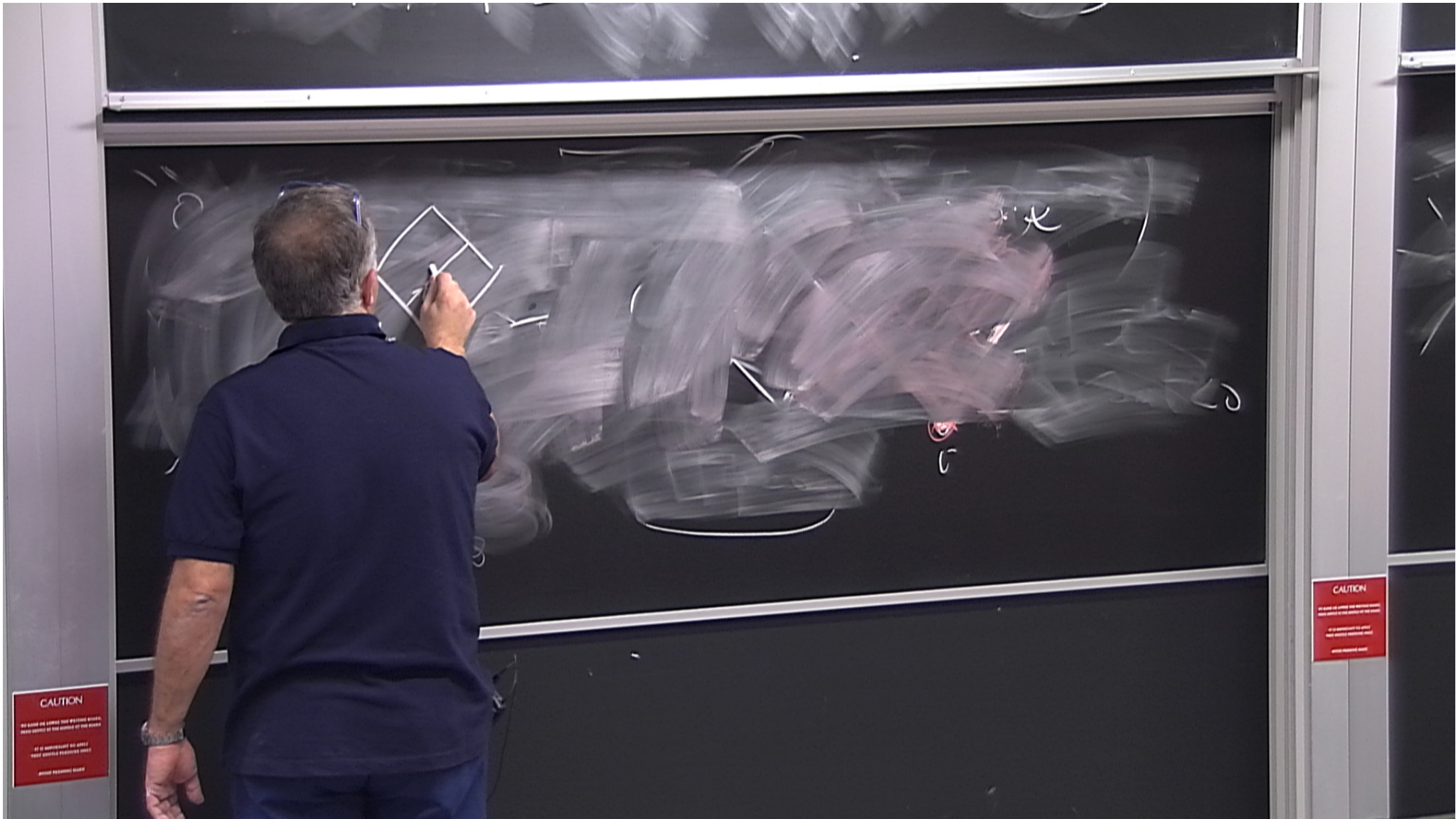


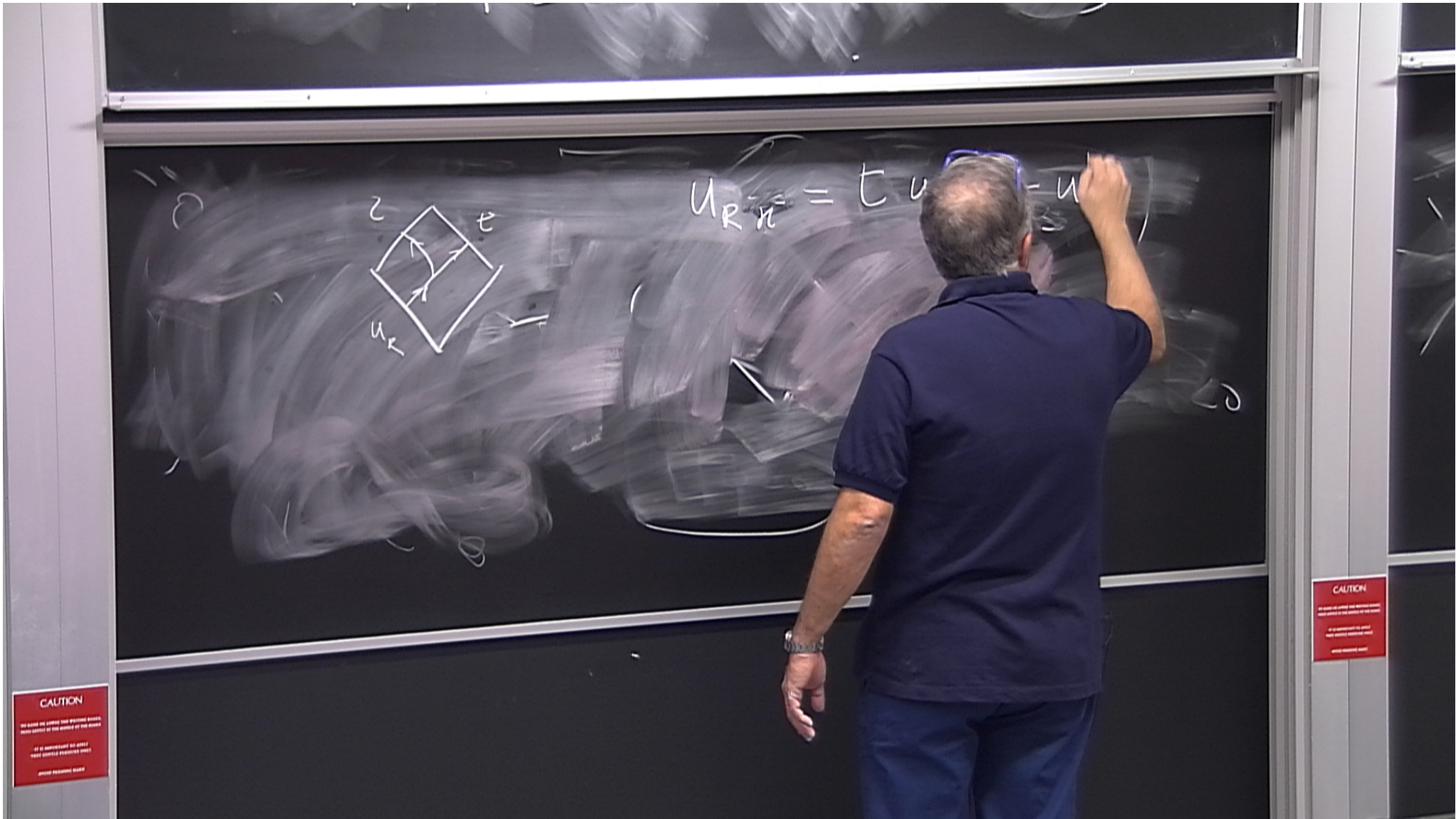


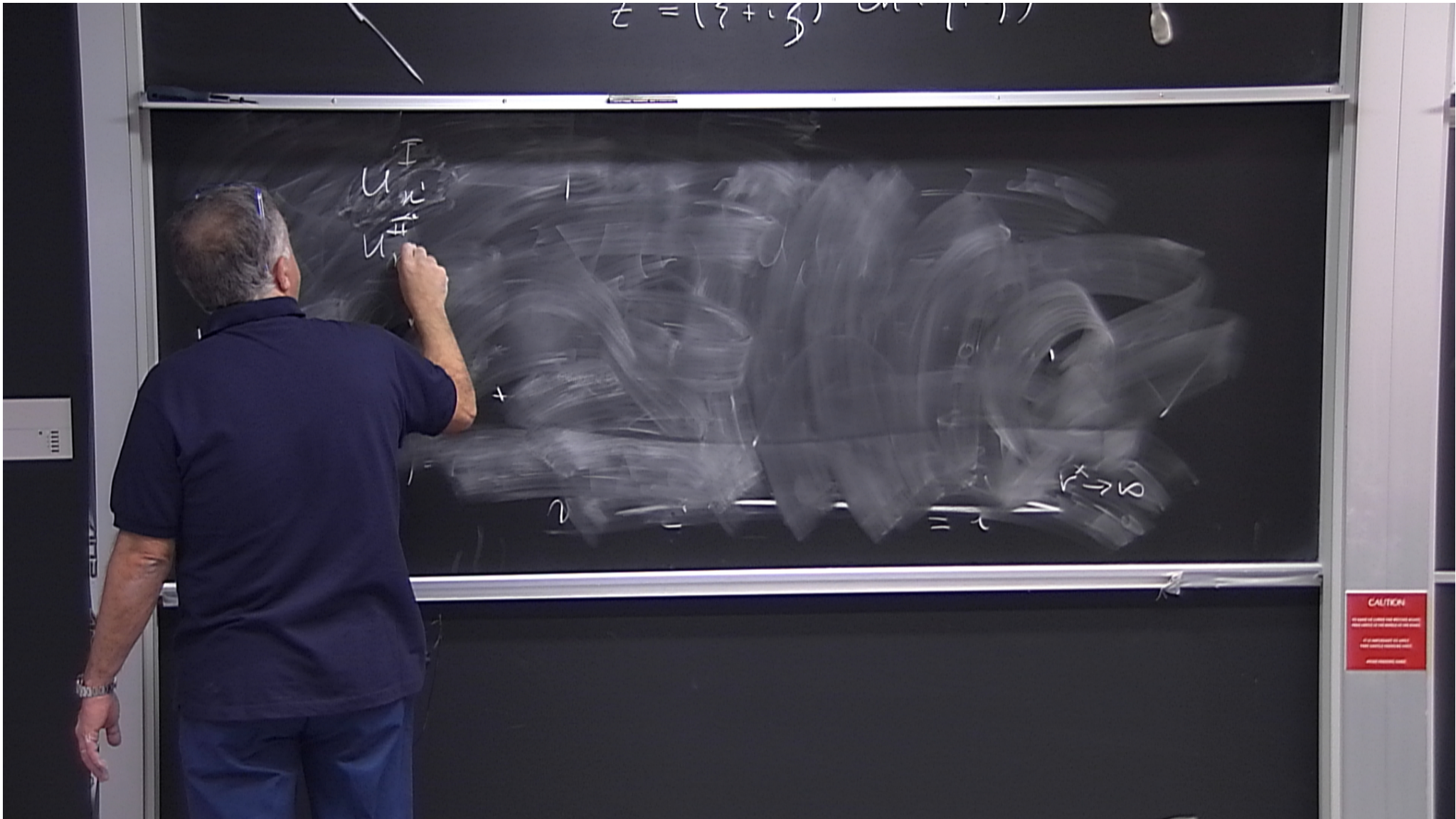
CAUTION
 THE BOARD IS LOOSE AND MAY BE DAMAGED.
 PLEASE HANDLE IT WITH CARE.
 IT IS PROHIBITED TO WRITE
 WITH ANY OTHER OBJECTS.
 THANK YOU FOR YOUR COOPERATION.

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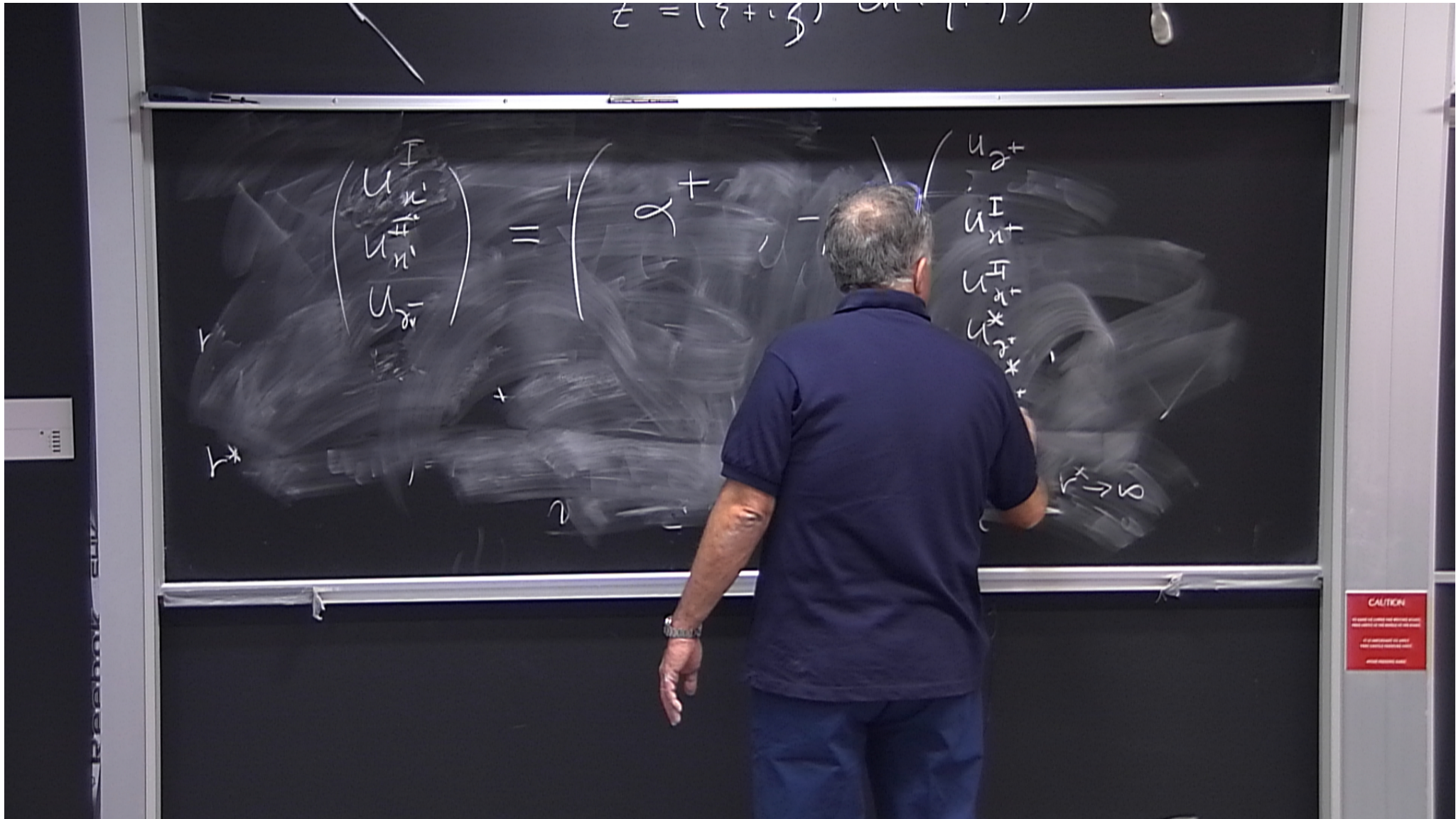


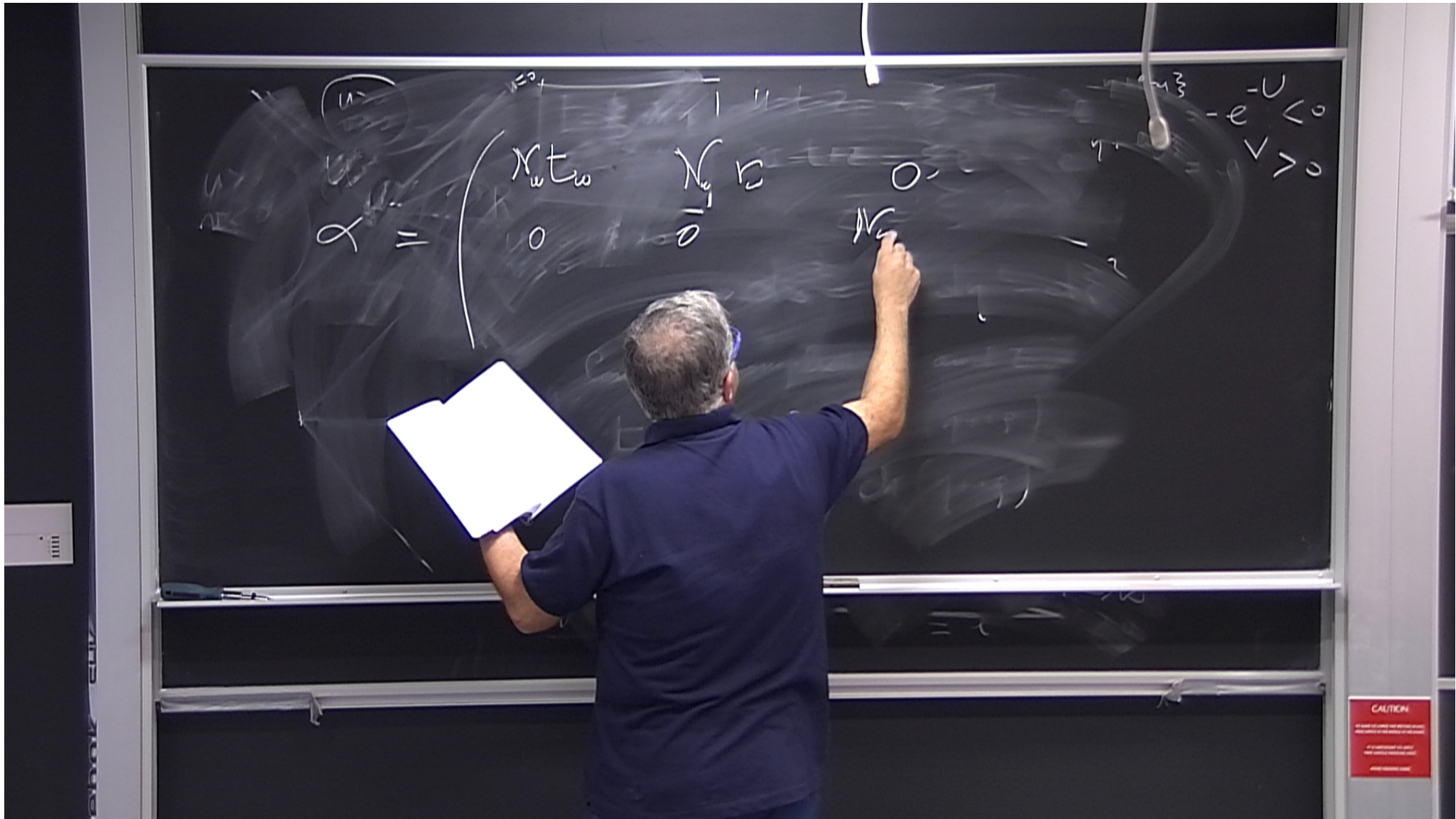


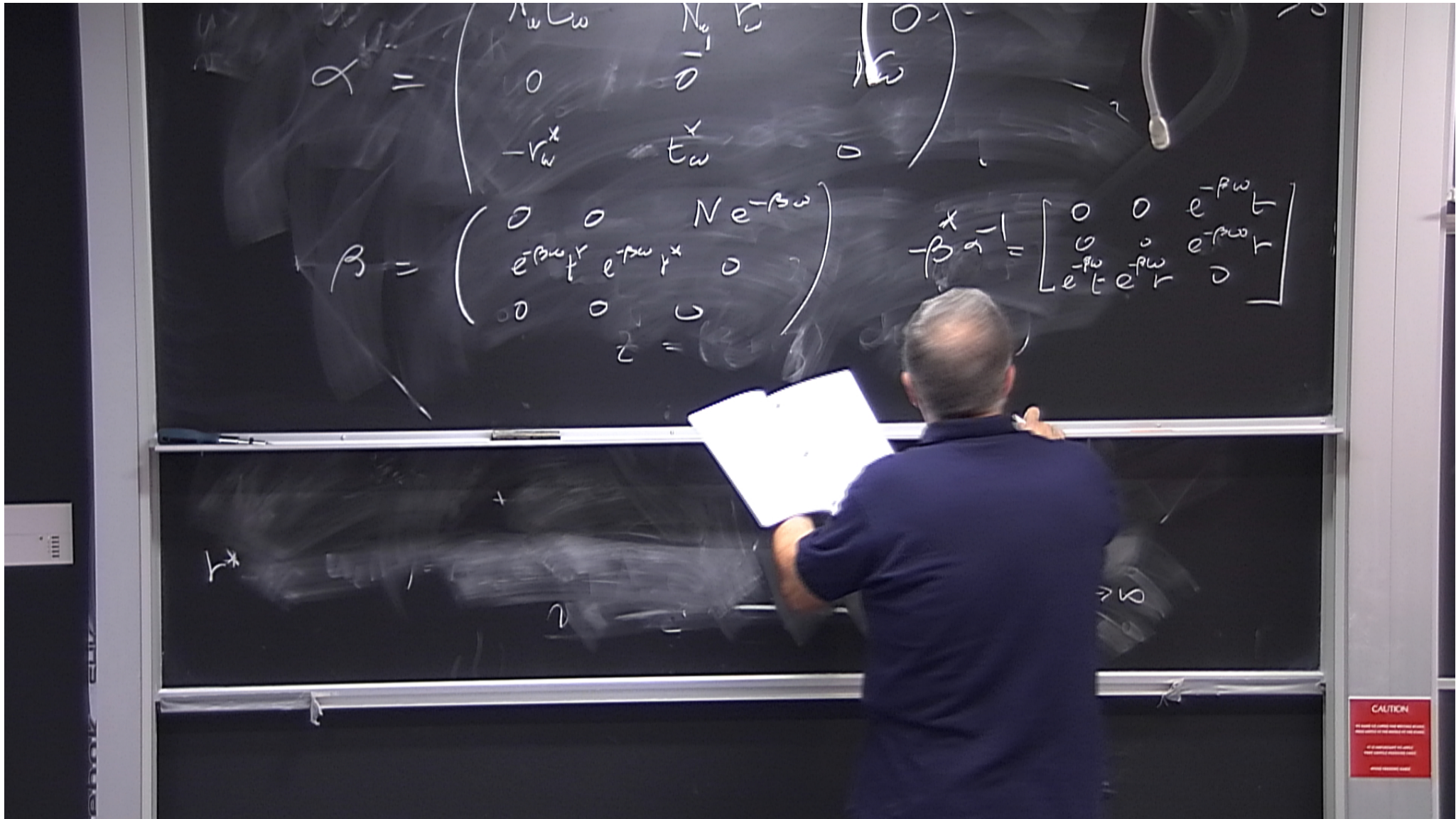




CAUTION
All electrical cables and devices should
be properly secured to the structure of the board.
It is prohibited to work
with electrical equipment while
using the board.

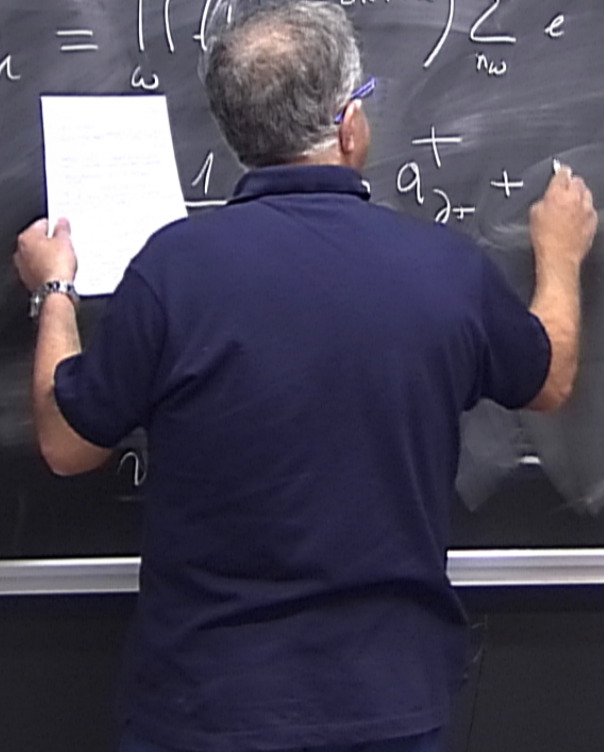






$$\beta = \begin{pmatrix} e^{-\beta\omega} & e^{-\beta\omega} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \beta^{-1} = \begin{bmatrix} 0 & 0 & e^{\beta\omega} \\ e^{\beta\omega} & e^{\beta\omega} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|0\rangle_{in} = \prod_{\omega} \left(1 - 8\pi\eta\omega \right) \sum_{n_{\omega}} e^{-8\pi\eta n_{\omega}}$$



r
 r^*

$r^* \rightarrow r$

CAUTION
 ALL WORK ON CHALKBOARD SHOULD BE COMPLETED BY THE END OF THE HOUR.
 IF NECESSARY, USE OTHER BOARD OR BOARDER.
 PLEASE REPORT DAMAGE.

$$\beta = \begin{pmatrix} e^{-\beta\omega} r & e^{-\beta\omega} r^* & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$-\beta\alpha = \begin{bmatrix} 0 & e^{-\beta\omega} r \\ -e^{-\beta\omega} r^* & 0 \end{bmatrix}$$

$$|0\rangle_{in} = \prod_{\omega} (1 - e^{-8\pi n\omega}) \sum_{n\omega} e^{-8\pi n\omega}$$

$$\frac{1}{n!} [t\omega a_{2+}^{\dagger} + r a_{2+}^{\dagger}]$$

r^*

$$\beta = \begin{pmatrix} e^{-\beta\omega} r & e^{-\beta\omega} r^* & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \beta^{-1} = \begin{bmatrix} 0 & e^{-\beta\omega} r \\ -e^{-\beta\omega} r^* & 0 \end{bmatrix}$$

$z = |0\rangle_{in}$ $\frac{1}{\text{Ident}} \frac{1}{r} e^{i(a^\dagger - \beta^{-1} a^\dagger)} |0\rangle_{in}$

$$|0\rangle_{in} = \prod_{\omega} (1 - e^{-8\pi\Gamma\omega}) \sum_{n_{\omega}} e^{-8\pi\Gamma n_{\omega}}$$

$$\frac{1}{n!} [t_{\omega} a_{\omega}^{\dagger} + r_{\omega} a_{\omega}^{\dagger}]^{n_{\omega}} |0\rangle_{out} \langle 0|$$

ρ_{0+}

$\omega \rightarrow \omega$

CAUTION
 Do not touch the blackboard panel
 directly or indirectly with your hands.
 If you need to use the blackboard panel,
 please use the handle.

