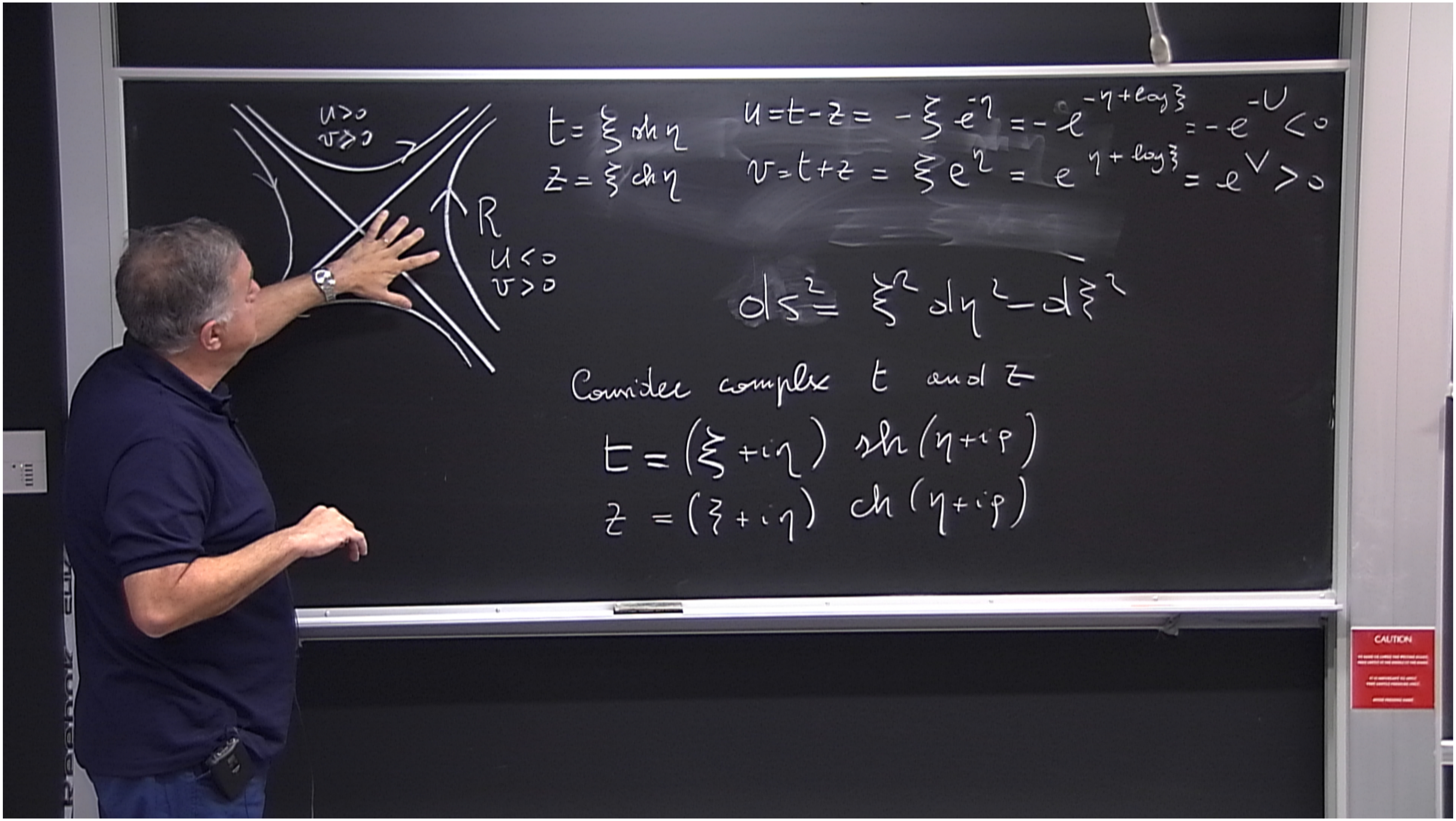
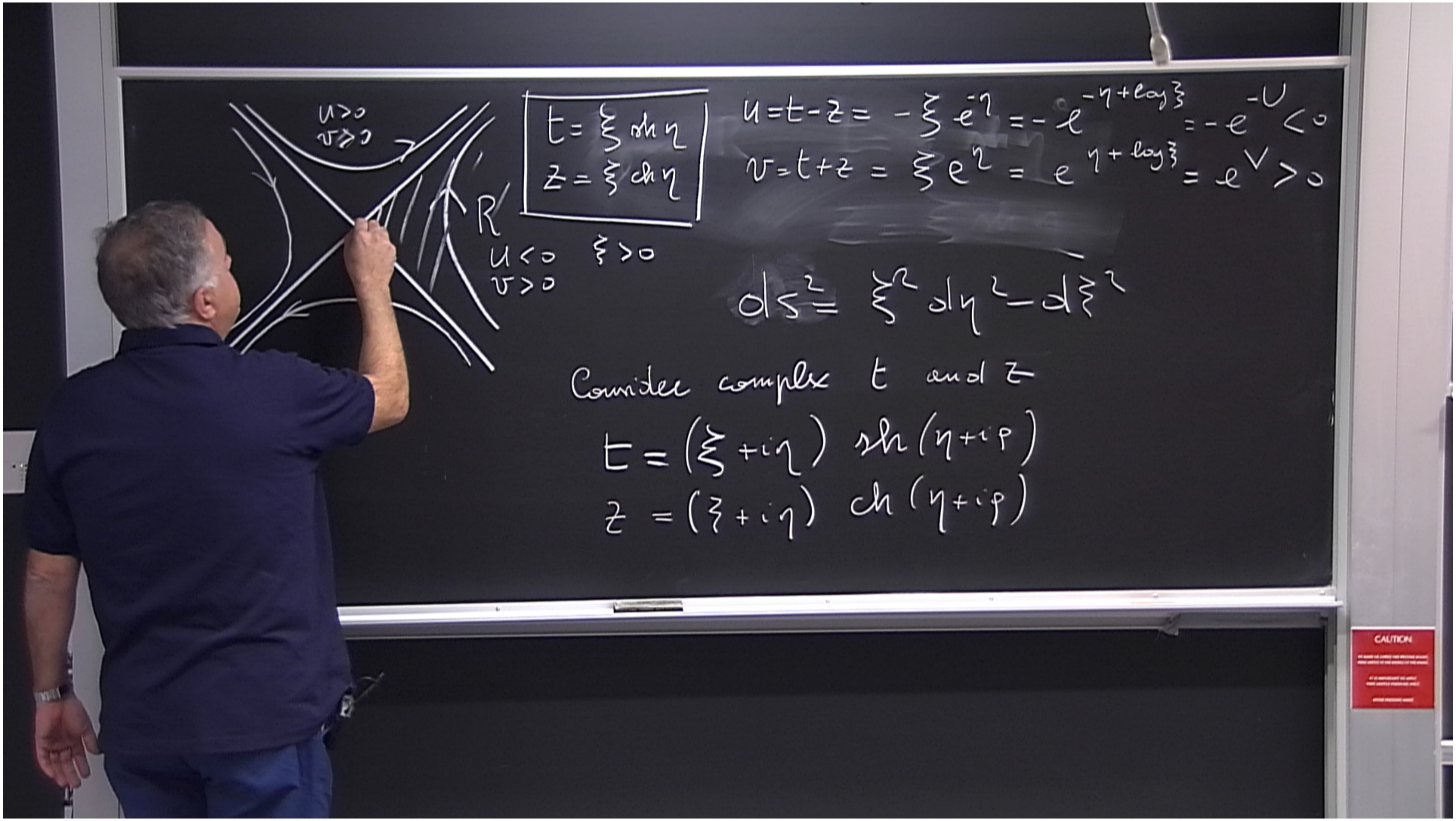
Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 9

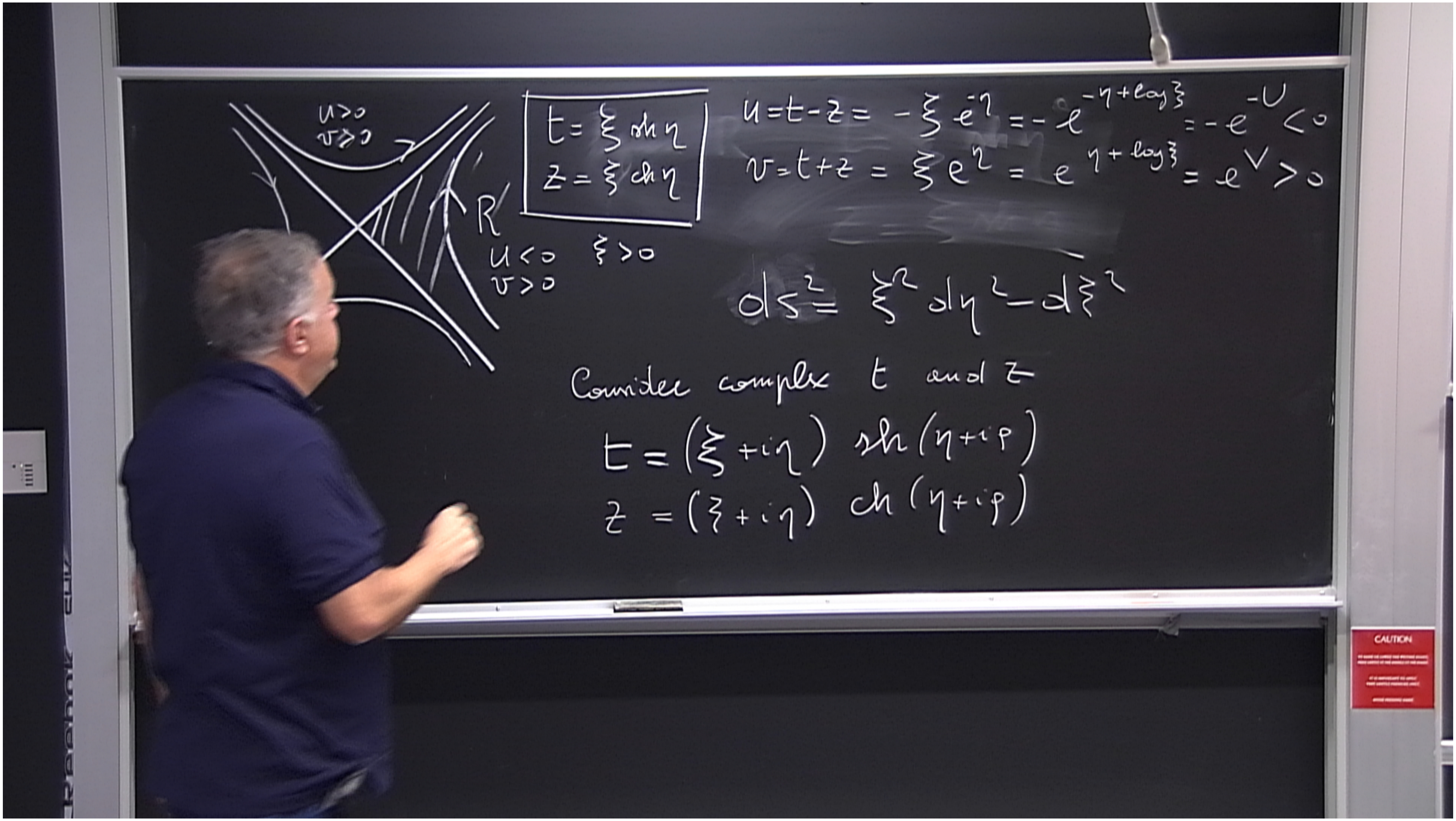
Date: Oct 21, 2013 10:00 AM

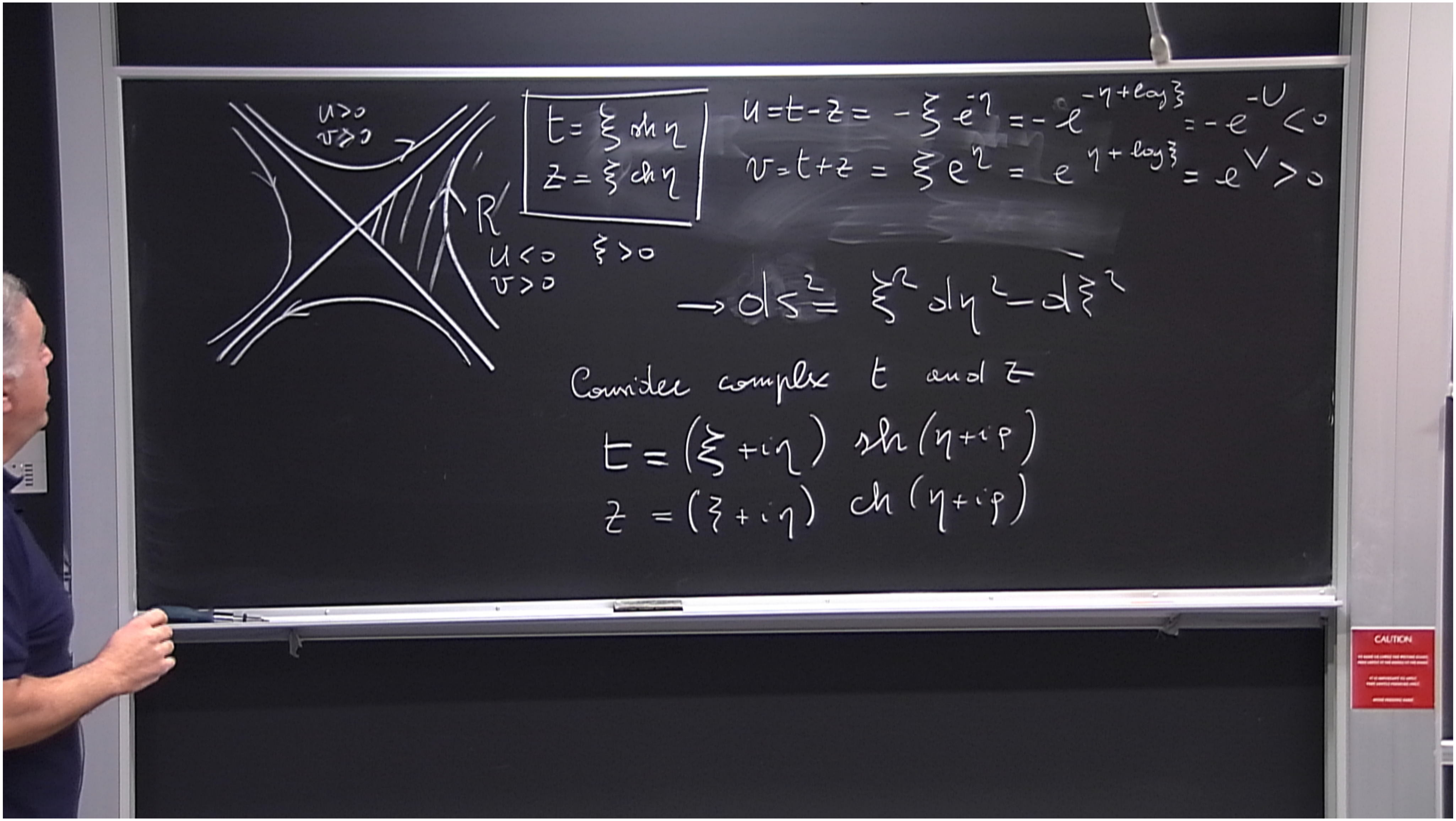
URL: http://pirsa.org/13100013

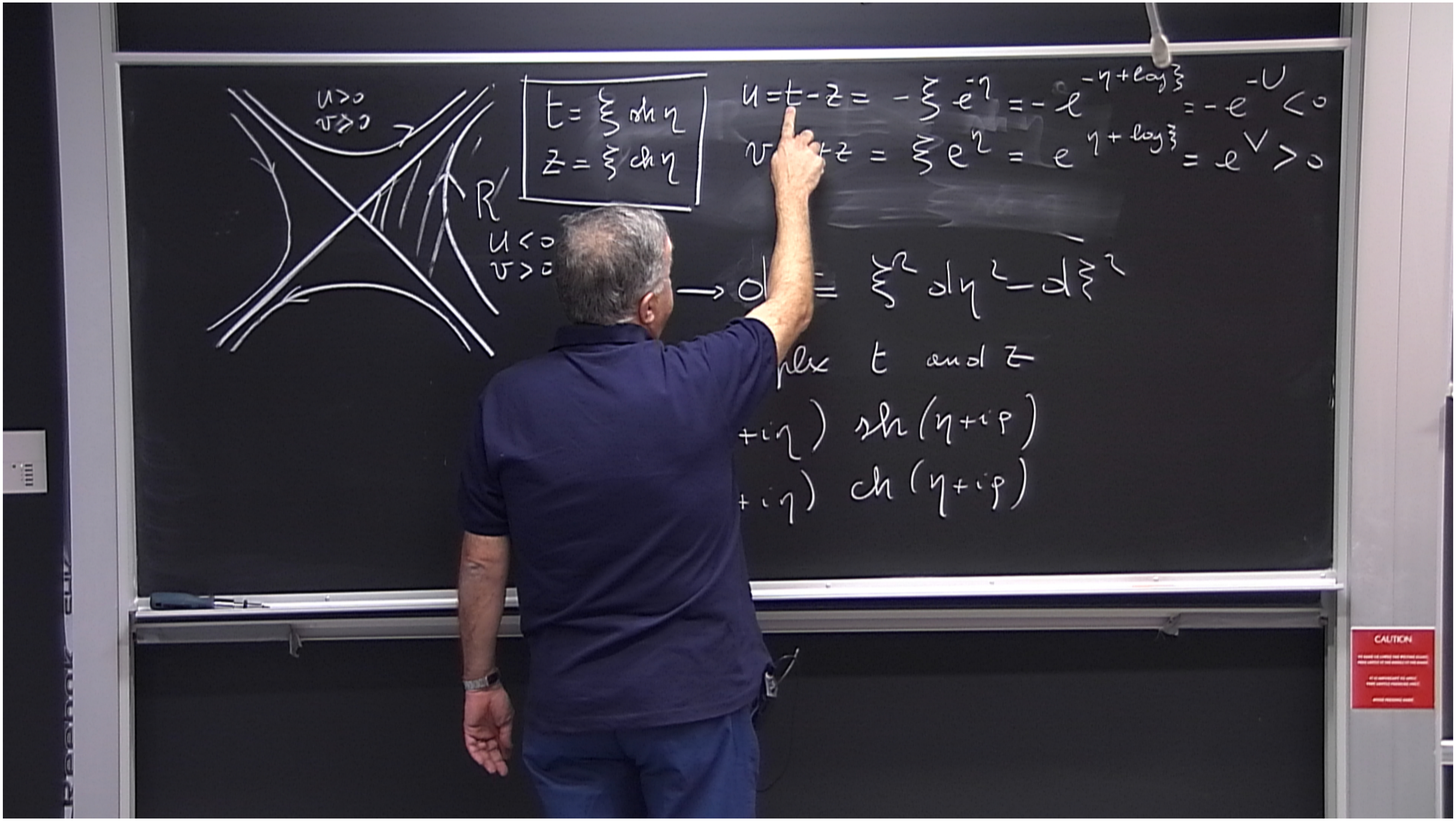
Abstract:

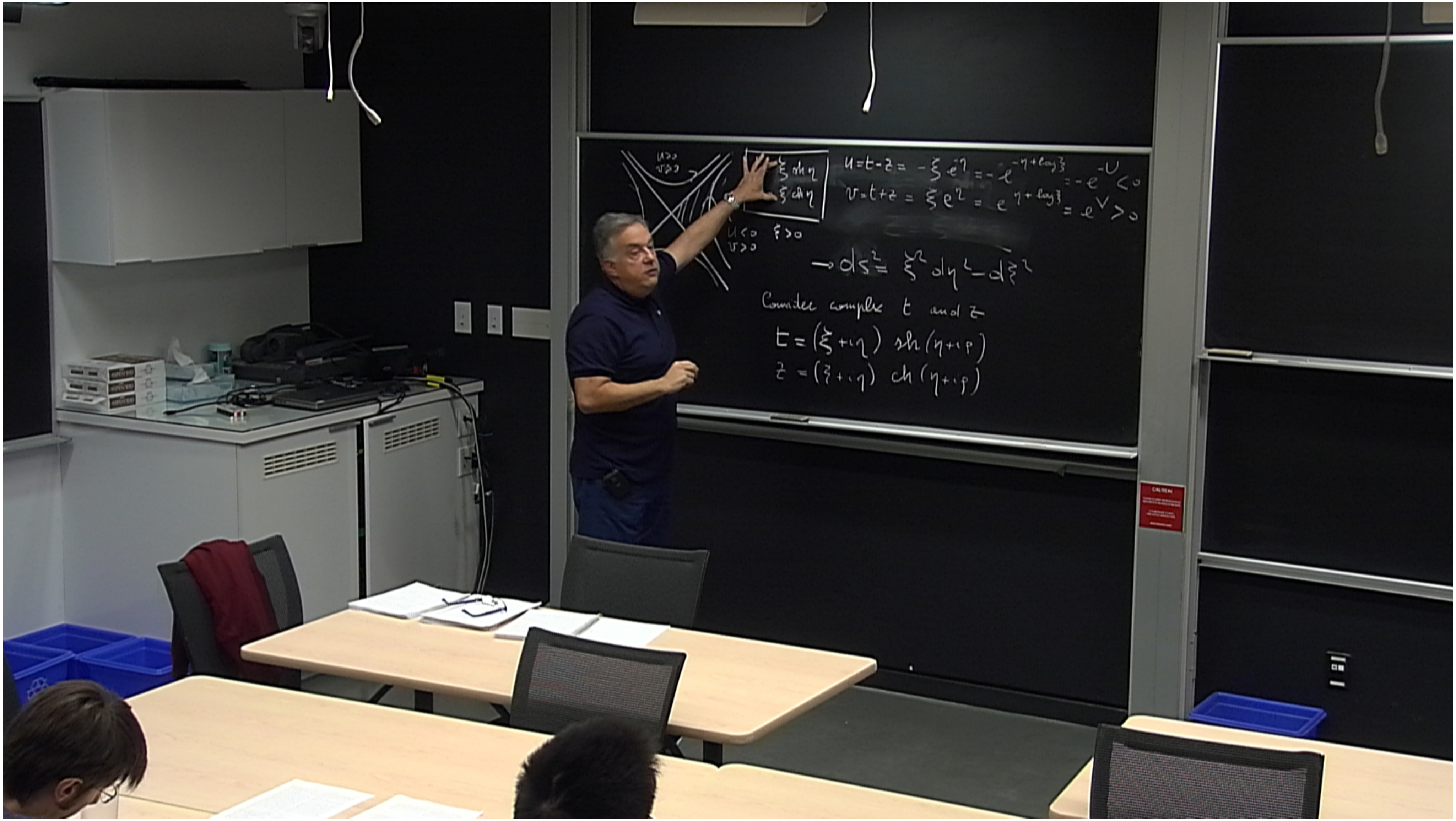

u>0
v>0
R
u<0
v>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η} = − e^{−η + log ξ} = − e^{−U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iη) sh (η + iφ)
z = (ξ + iη) ch (η + iφ)
CAUTION

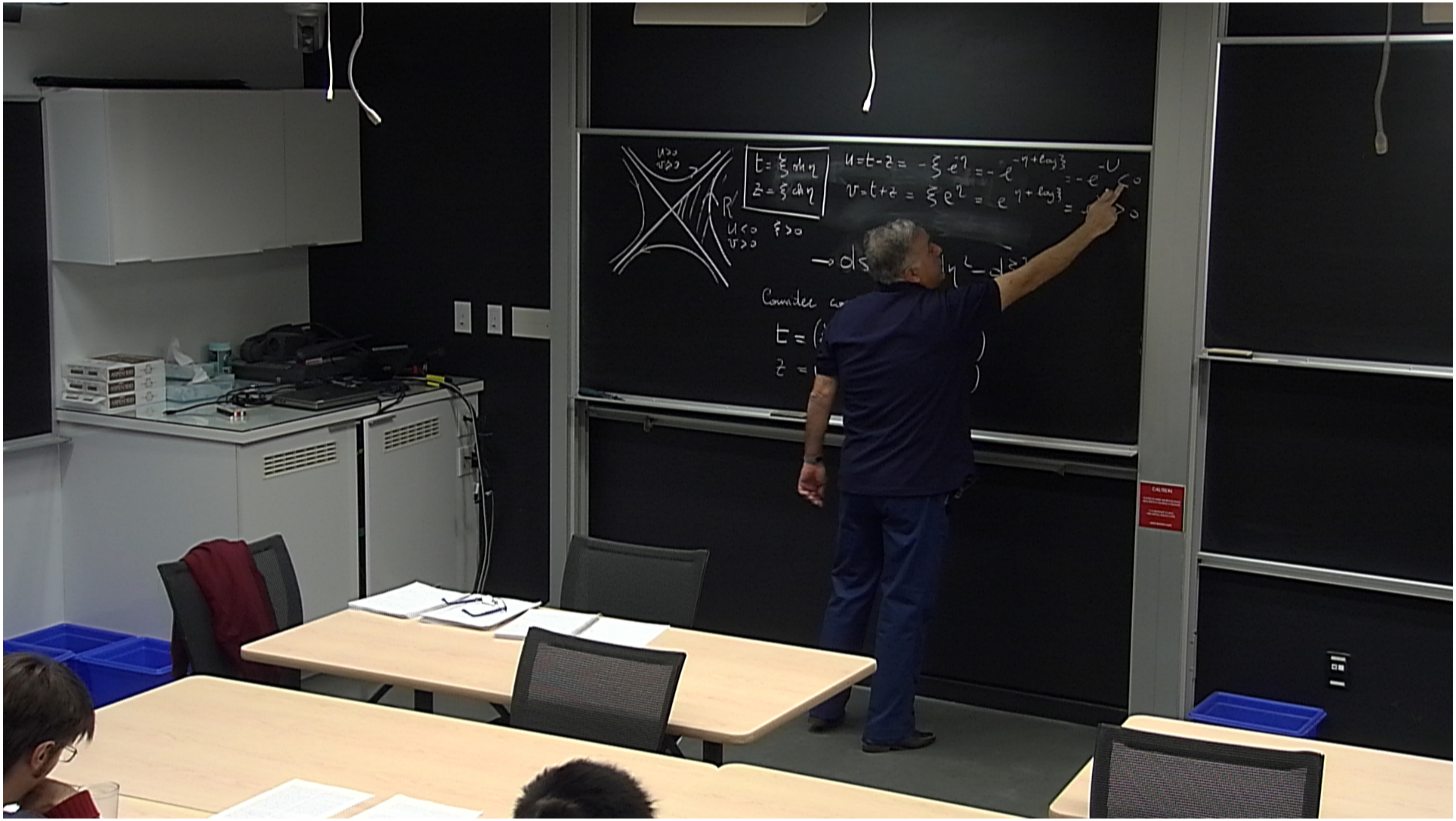

u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η} = − e^{−η + log ξ} = − e^{−U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iη) sh (η + iρ)
z = (ξ + iη) ch (η + iρ)

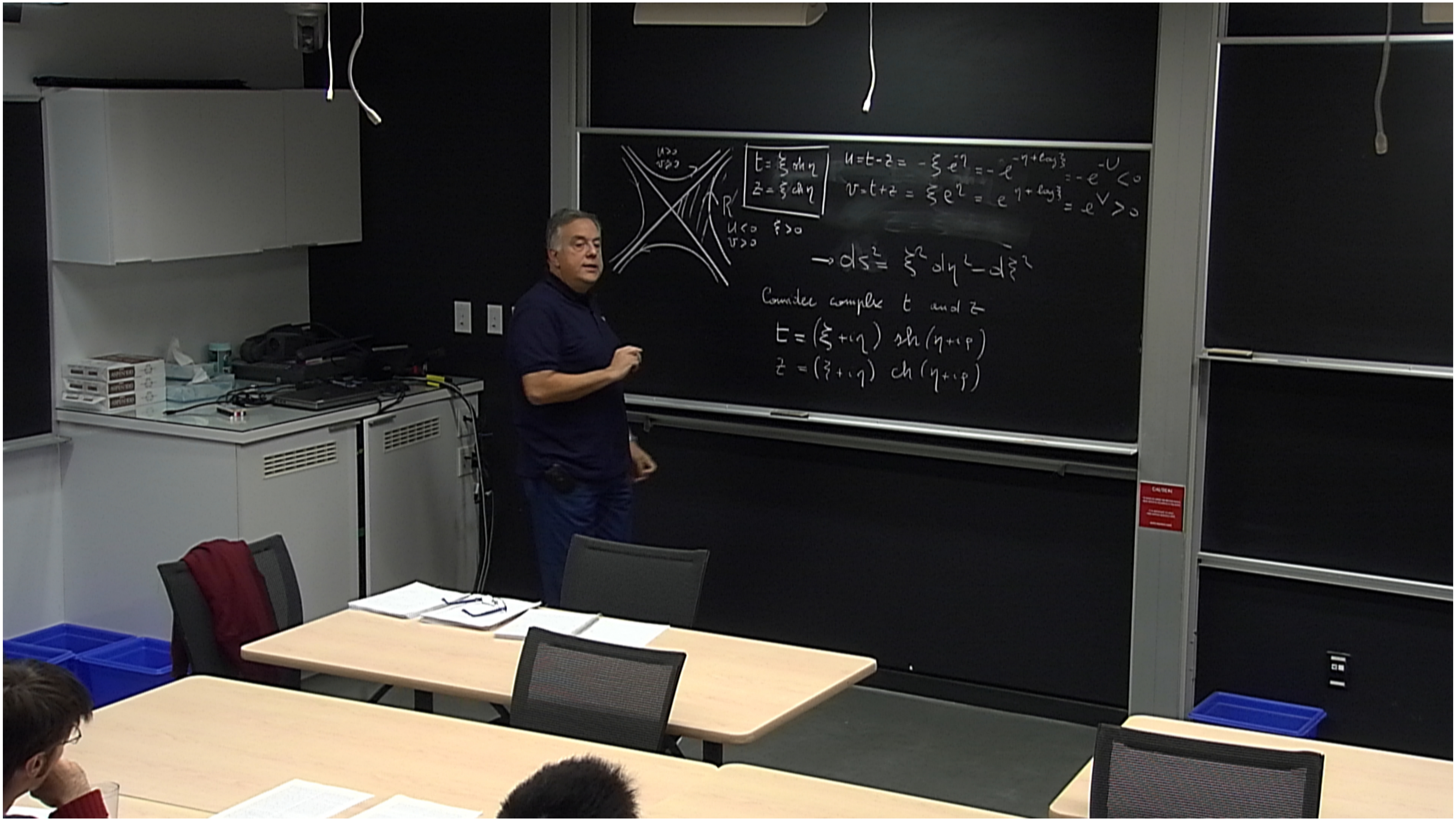

u > 0
v ≥ 0
R
u < 0   ξ > 0
v > 0
t = ξ sh η
z = ξ ch η
u = t − z = −ξ e^−η = −e^(−η + log ξ) = −e^−U < 0
v = t + z = ξ e^η = e^(η + log ξ) = e^V > 0
ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iη) sh (η + iρ)
z = (ξ + iη) ch (η + iρ)
CAUTION


u>0
v>0
R
u<0
v>0
ξ>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η} = − e^{−η + log ξ} = − e^{−U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
→ ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iη) sh(η + iρ)
z = (ξ + iη) ch(η + iρ)

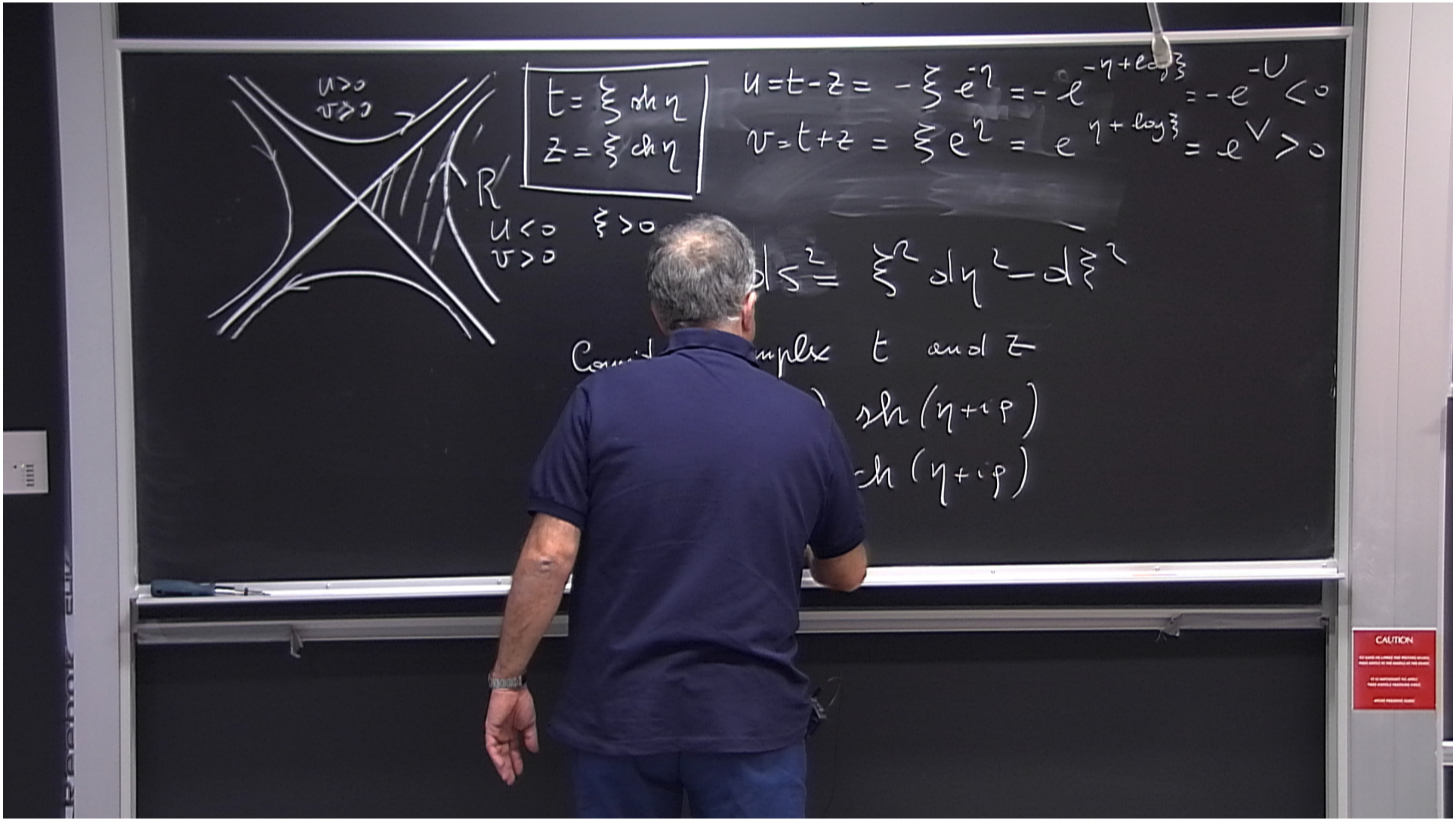

u>0
v>0
R
u<0
v>0
t = ξ sh η
z = ξ ch η
u = t - z = - ξ e^{-η} = - e^{-η + log ξ} = - e^{-U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
= ξ² dη² - dξ²
t and z
+iη) sh(η+iφ)
+iη) ch(η+iφ)


Consider complex t and z

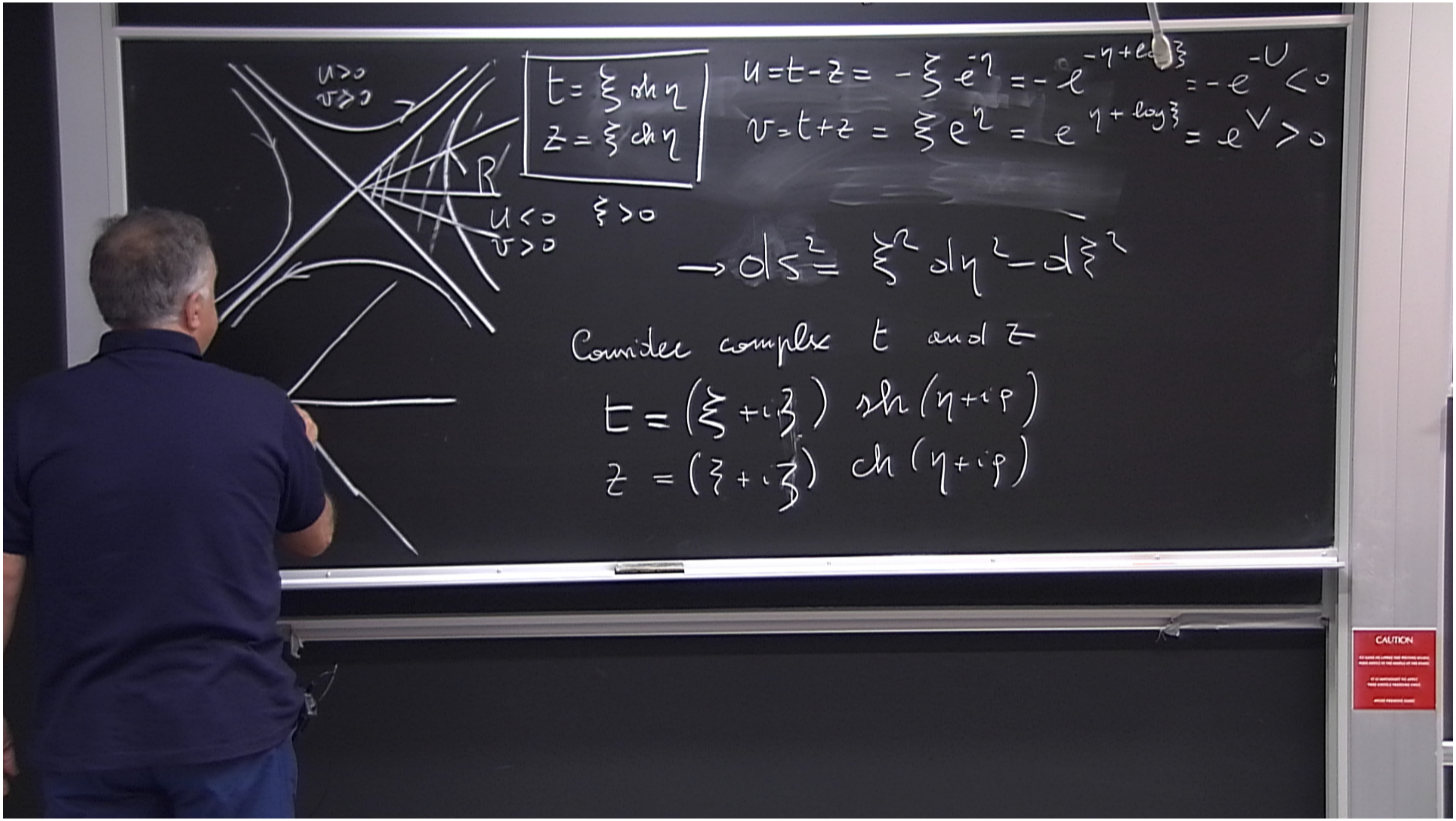


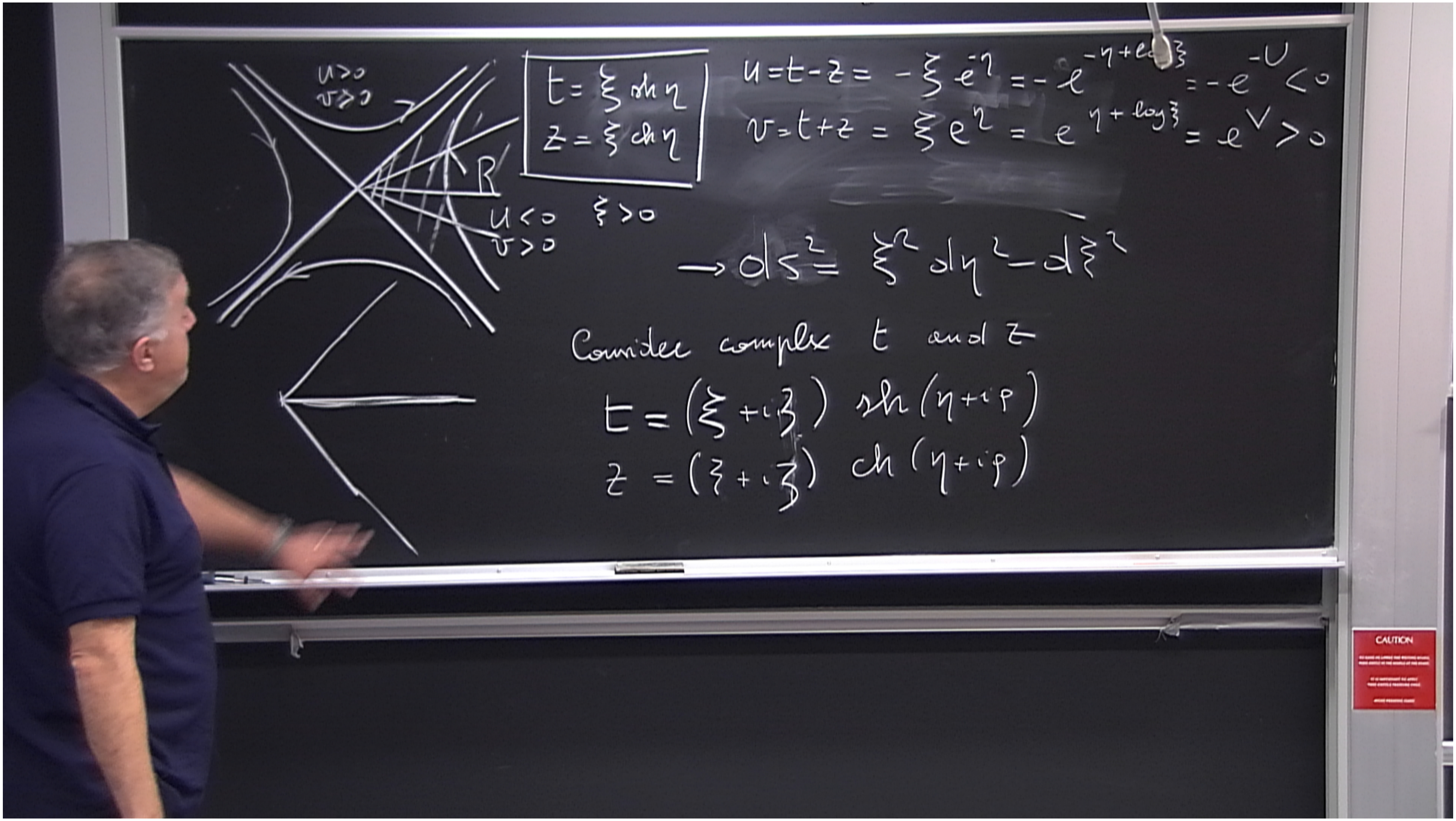


u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t - z = - ξ e^{-η} = - e^{-η + log ξ} = - e^{-U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
→ ds² = ξ² dη² - dξ²
Consider complex t and z
t = (ξ + iη) sh (η + iρ)
z = (ξ + iη) ch (η + iρ)



u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t - z = - ξ e^-η = - e^-η + log ξ = - e^-U < 0
v = t + z = ξ e^η = e^η + log ξ = e^V > 0
ds² = ξ² dη² - dξ²
t and z
sh (η + iρ)
ch (η + iρ)
CAUTION


u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{-η} = − e^{-η + log ξ} = − e^{-U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
→ ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iζ) sh(η + iφ)
z = (ξ + iζ) ch(η + iφ)
CAUTION


u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t − z = −ξ e^{−η} = −e^{−η+log ξ} = −e^{−U} < 0
v = t + z = ξ e^{η} = e^{η+log ξ} = e^{V} > 0
→ ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iζ) sh(η + iφ)
z = (ξ + iζ) ch(η + iφ)
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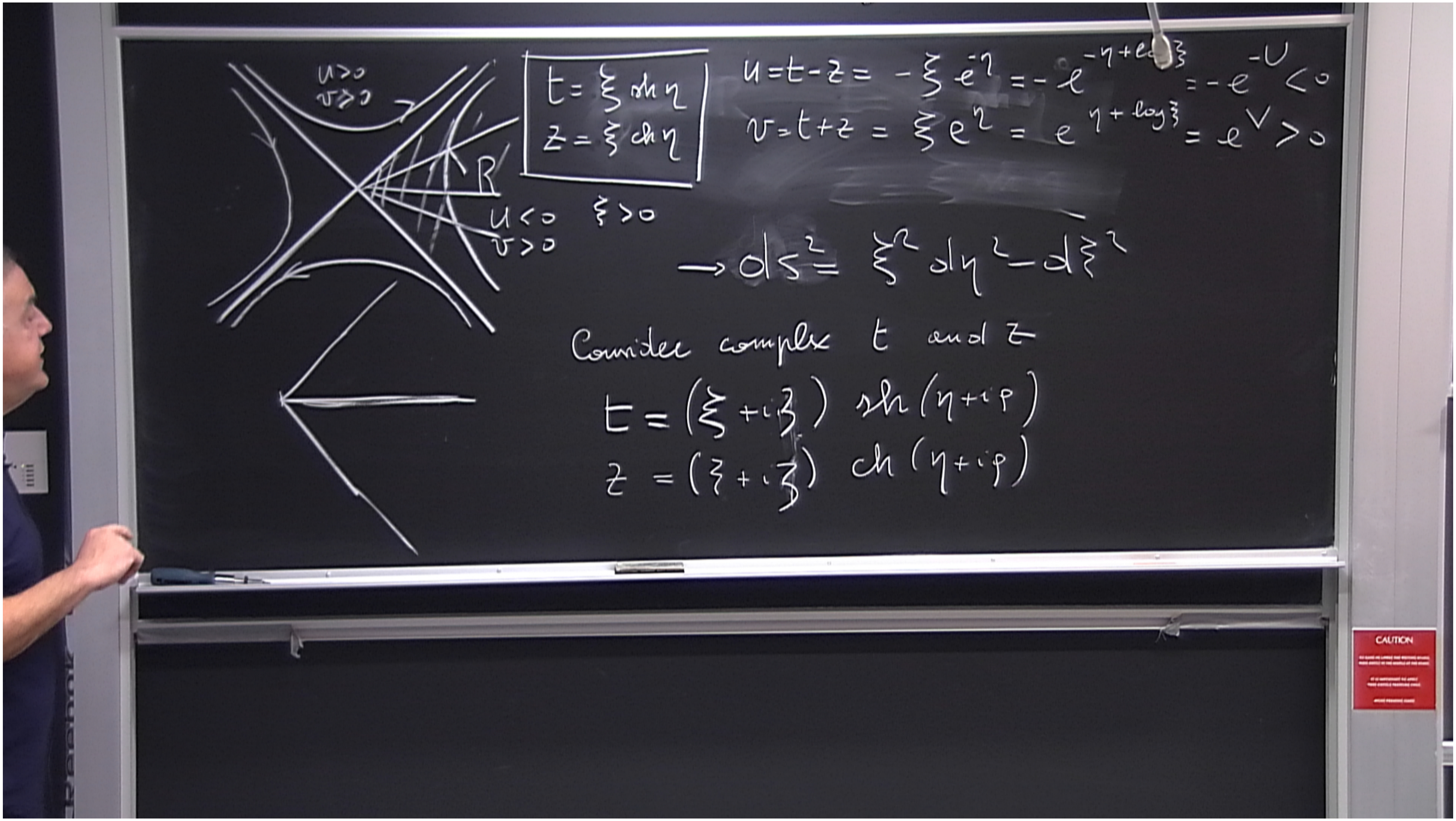
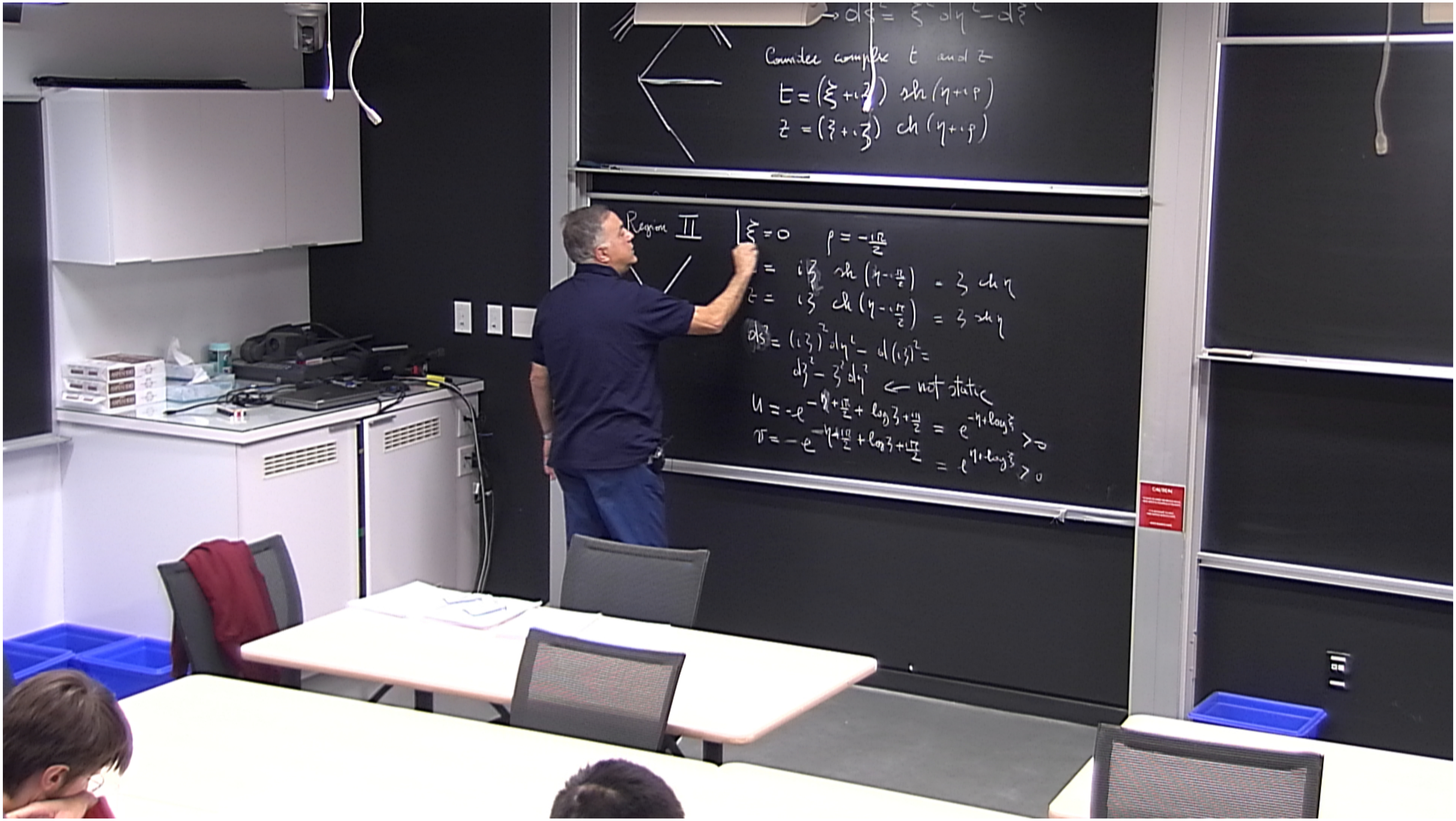
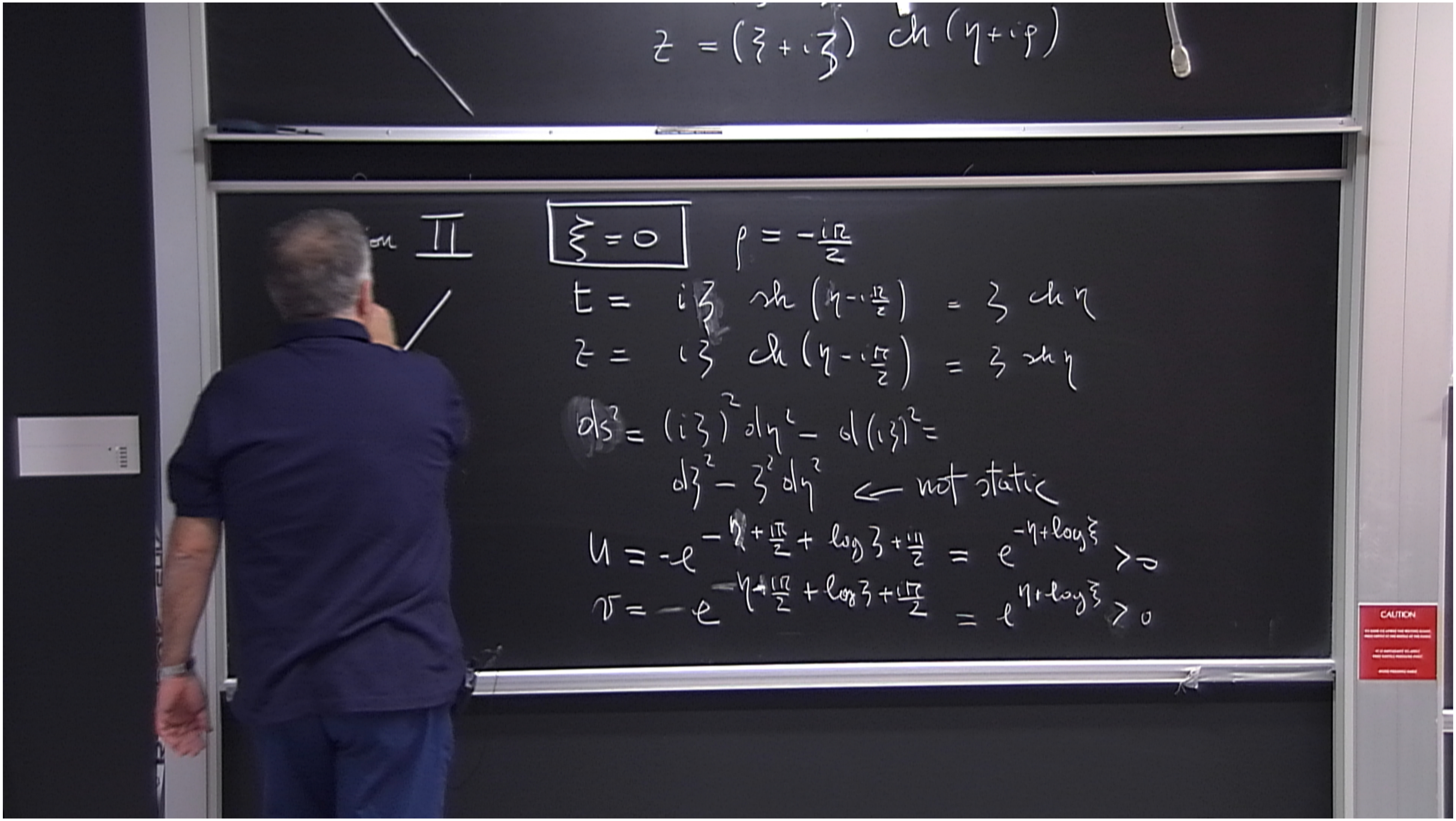
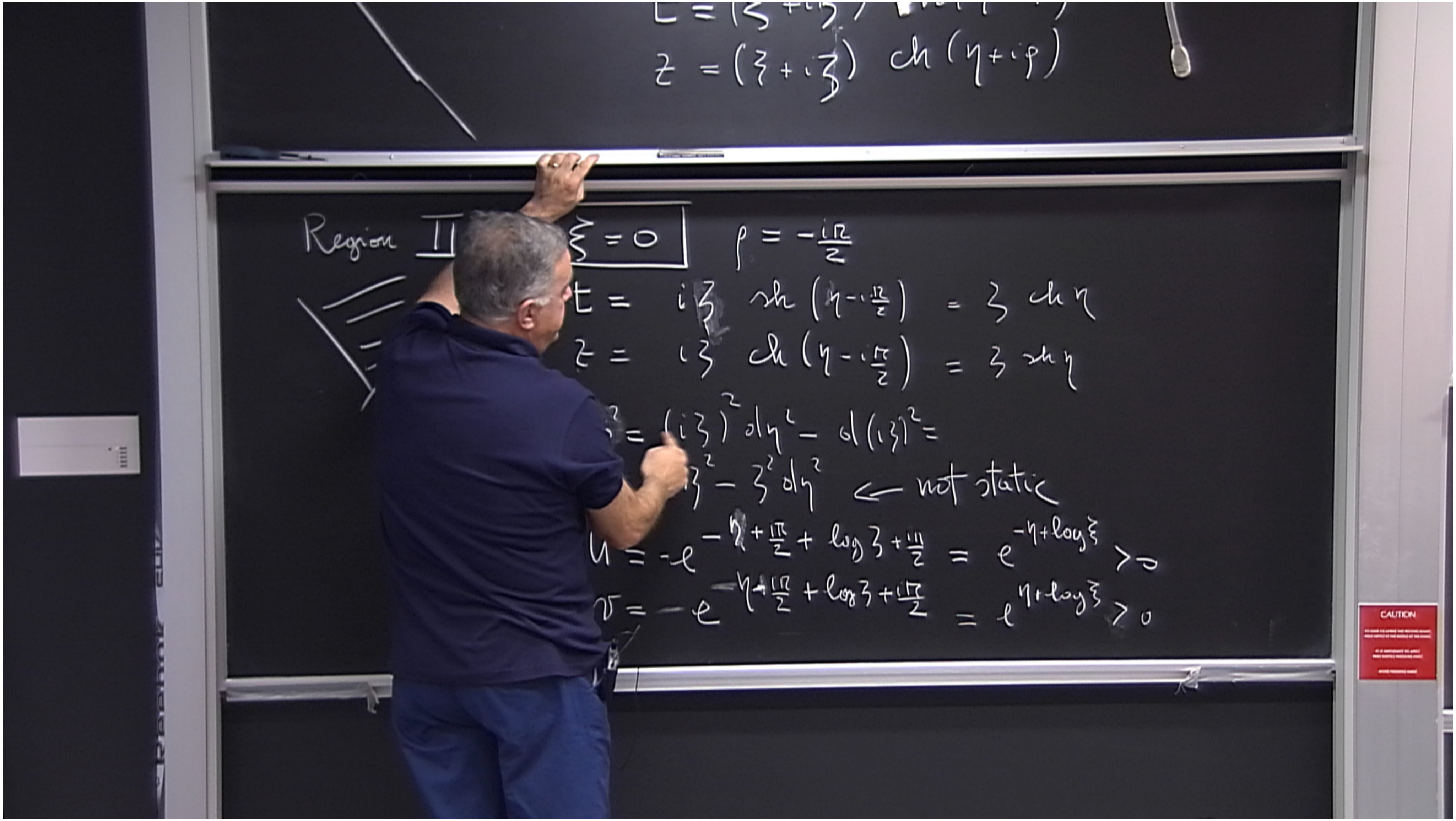

u>0
v>0
R
u<0 ξ>0
v>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η} = − e^{−η + log ξ} = − e^{−U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
→ ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iζ) sh(η + iφ)
z = (ξ + iζ) ch(η + iφ)

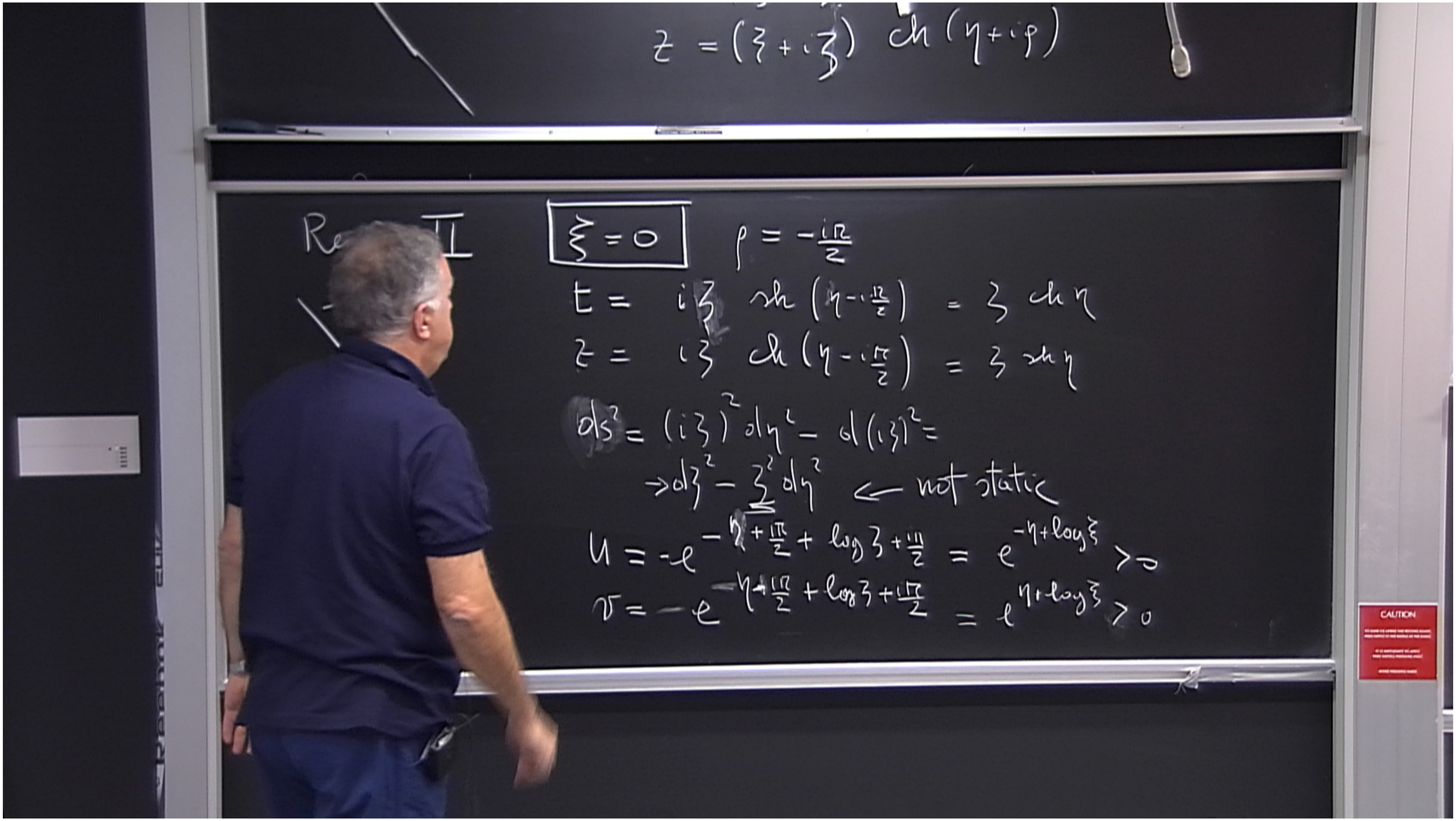


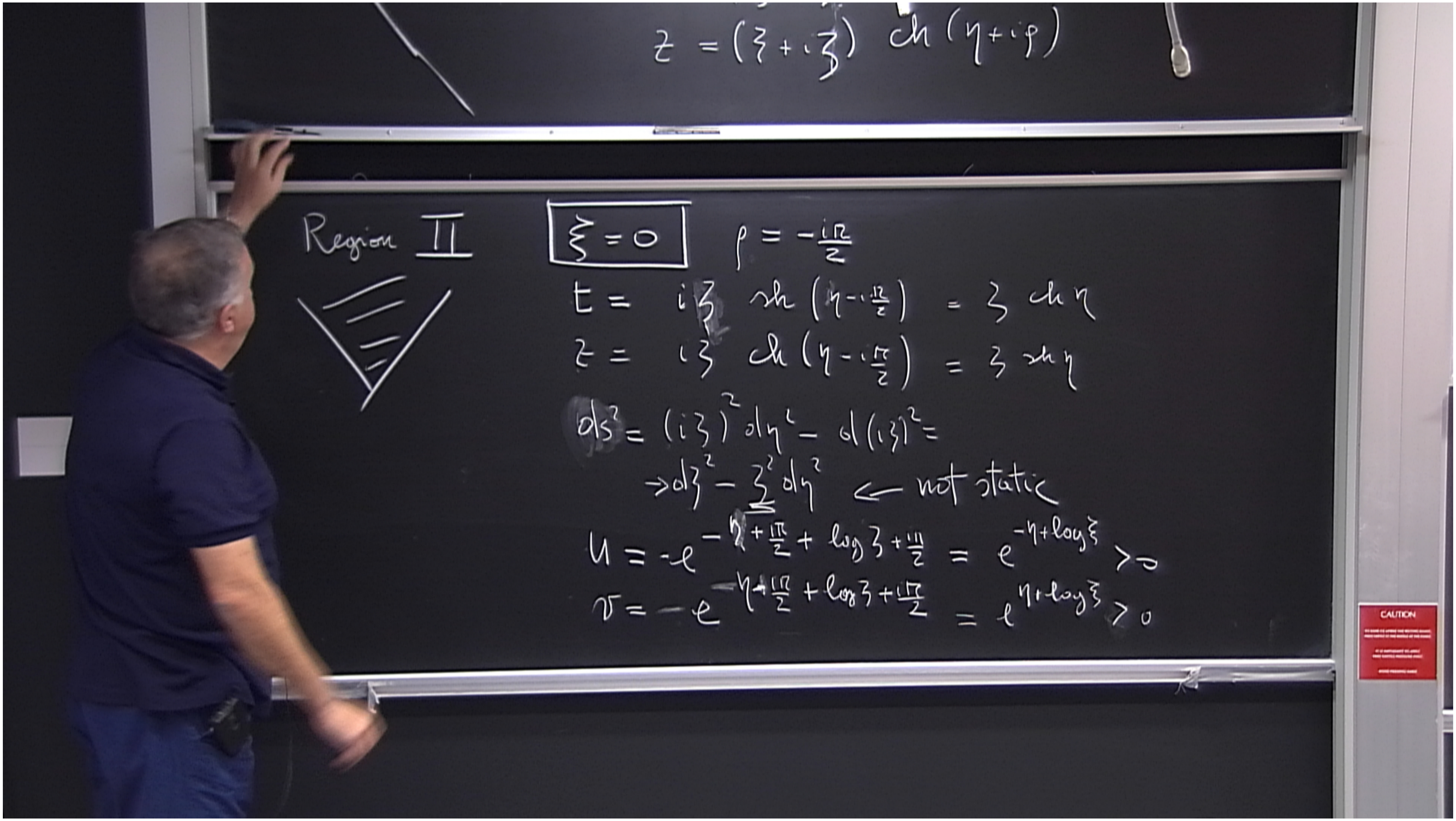
Consider complex t and z
Region II


z = (ξ + iζ) ch(η + iρ)
on II
ξ = 0   ρ = −iπ/2
t = iζ sh(η − iπ/2) = ζ ch η
z = iζ ch(η − iπ/2) = ζ sh η
ds² = (iζ)² dη² − d(iζ)² =
dζ² − ζ² dη² ← not static
u = −e^(−η + iπ/2 + log ζ + iπ/2) = e^(−η + log ξ) > 0
v = −e^(η + iπ/2 + log ζ + iπ/2) = e^(η + log ξ) > 0
CAUTION

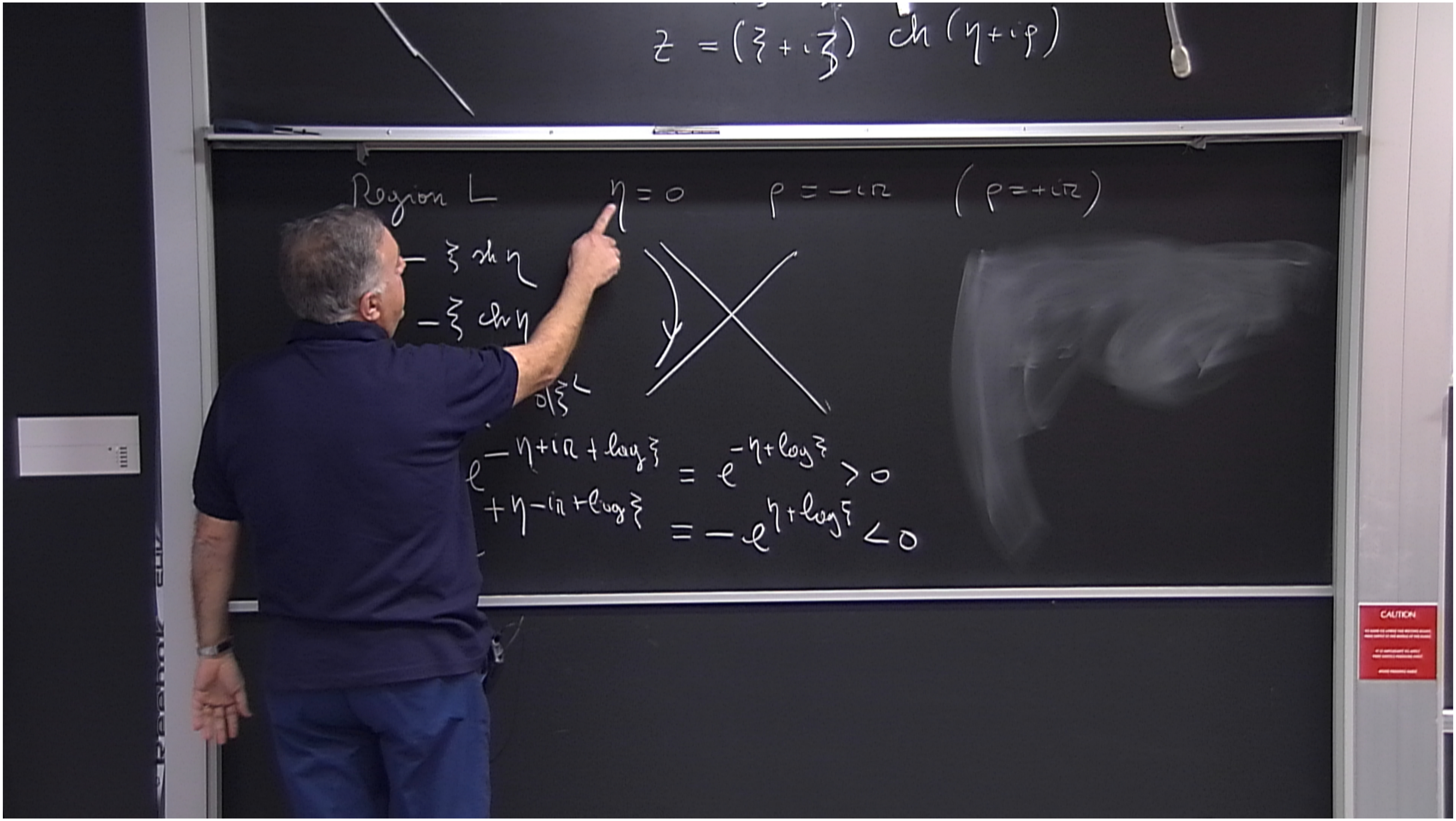


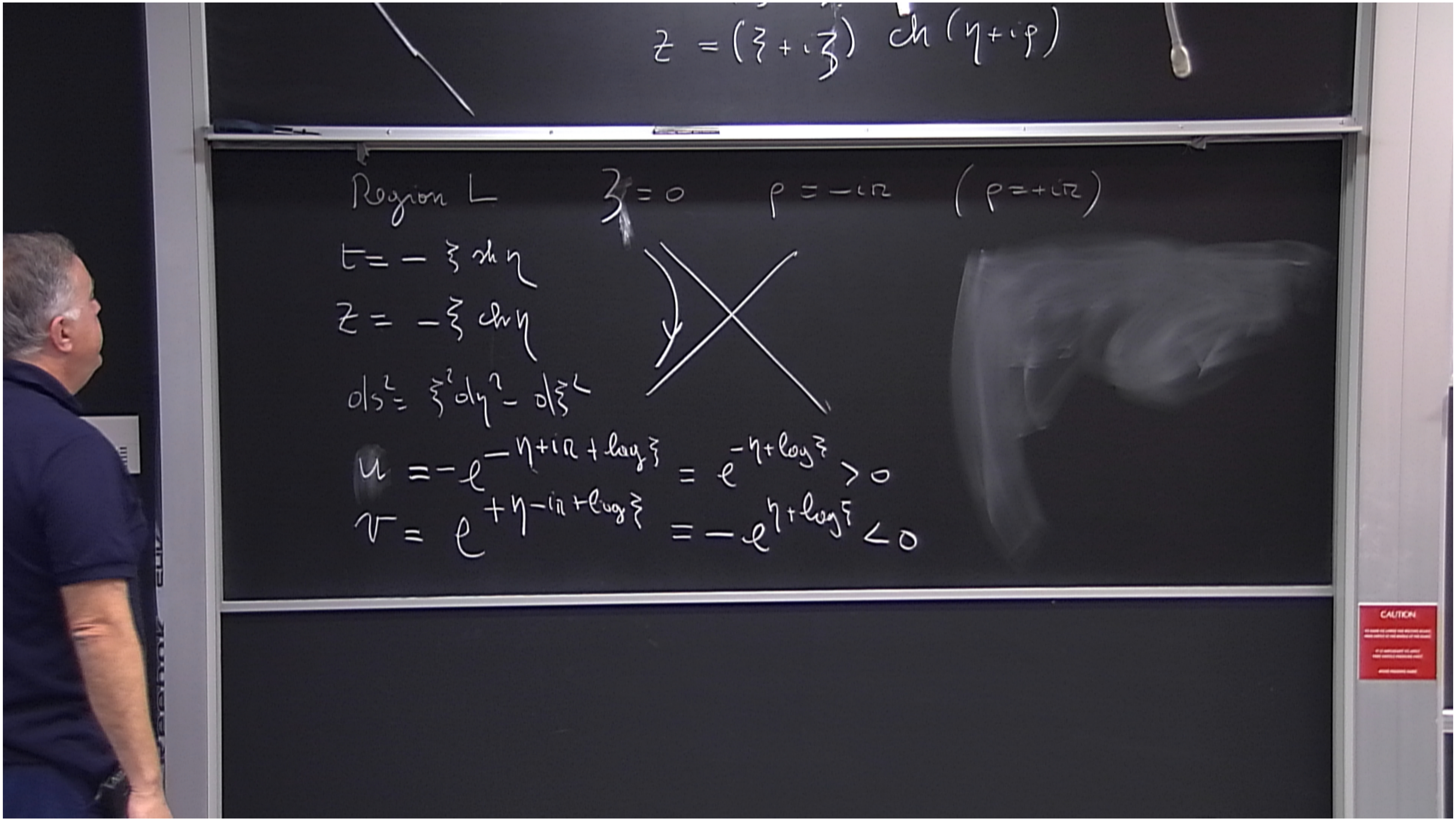
z = (ξ + iζ) ch(η + iφ)
Region II
ξ = 0
ρ = -iπ/2
t = iζ sh(η - iπ/2) = ζ ch η
z = iζ ch(η - iπ/2) = ζ sh η
= (iζ)² dη² − d(iζ)² =
ζ² − ζ² dη² ← not static
u = -e^{-η + iπ/2 + log ζ + iπ/2} = e^{-η + log ξ} > 0
v = -e^{η + iπ/2 + log ζ + iπ/2} = e^{η + log ξ} > 0
CAUTION

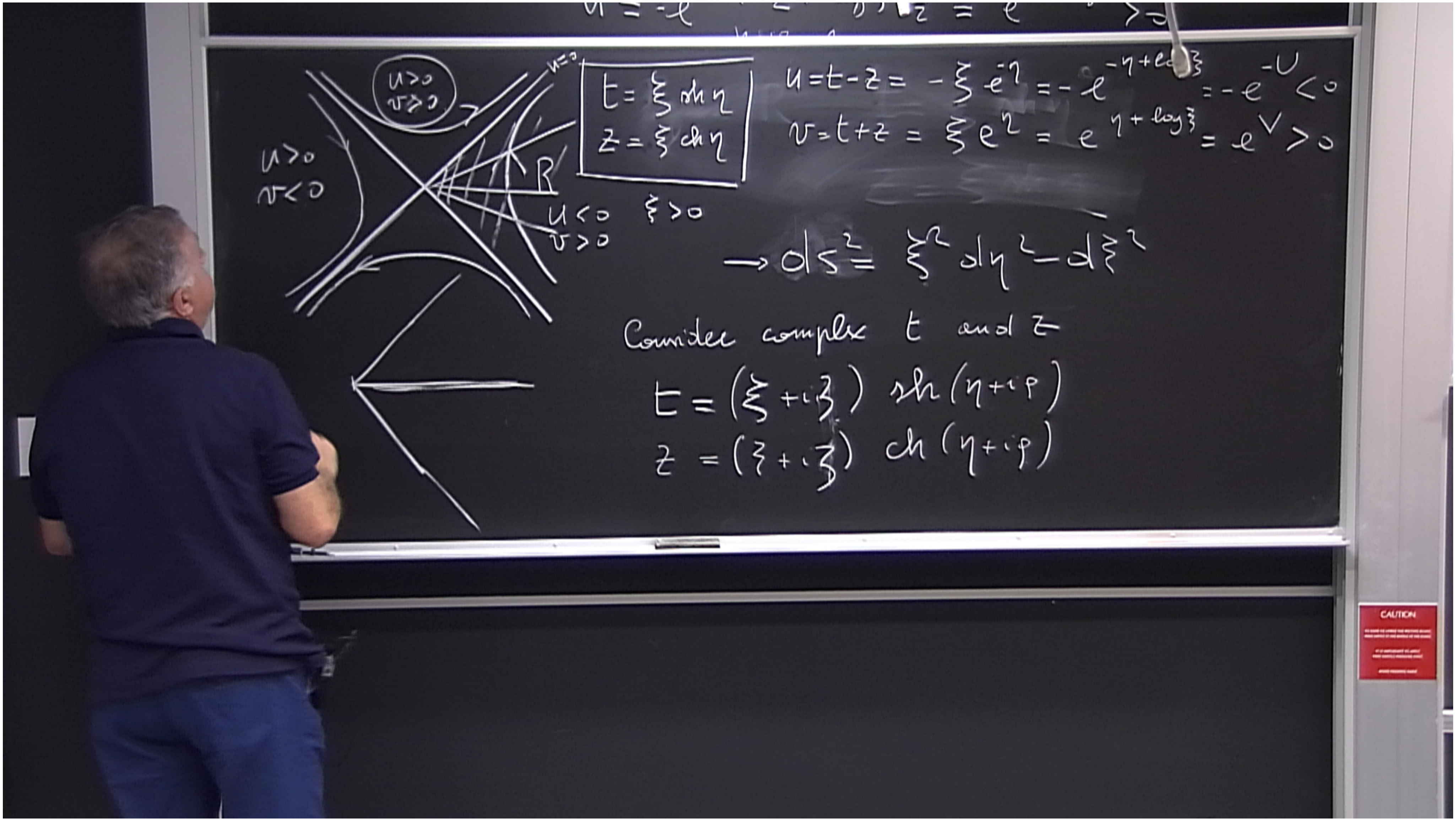

z = (ξ + i ζ) ch (η + i ρ)
ξ = 0
ρ = − iπ/2
t = i ζ sh (η − iπ/2) = ζ ch η
z = i ζ ch (η − iπ/2) = ζ sh η
ds² = (iζ)² dη² − d(iζ)² =
→ dζ² − ζ² dη² ← not static



z = (ξ + iζ) ch(η + iρ)
Region II
ξ = 0   ρ = −iπ/2
t = iζ sh(η − iπ/2) = ζ ch η
z = iζ ch(η − iπ/2) = ζ sh η
ds² = (iζ)² dη² − d(iζ)² =
→ dζ² − ζ² dη² ← not static
u = −e^{−η + iπ/2 + log ζ + iπ/2} = e^{−η + log ξ} > 0
v = −e^{η + iπ/2 + log ζ + iπ/2} = e^{η + log ξ} > 0
CAUTION


z = (ξ + iζ) ch(η + iρ)
Region L
η = 0
ρ = −iπ
(ρ = +iπ)
− ξ sh η
− ξ ch η
e^{−η+iπ+log ξ} = e^{−η+log ξ} > 0
+η−iπ+log ξ = −e^{η+log ξ} < 0
CAUTION


$z = (\xi + i\zeta)\, \mathrm{ch}(\eta + i\rho)$
Region L
$\zeta = 0$
$\rho = -i\pi$
$(\rho = +i\pi)$
$t = -\xi\, \mathrm{sh}\, \eta$
$z = -\xi\, \mathrm{ch}\, \eta$
$ds^2 = \xi^2 d\eta^2 - d\xi^2$
$u = -e^{-\eta + i\pi + \log \xi} = e^{-\eta + \log \xi} > 0$
$v = e^{+\eta - i\pi + \log \xi} = -e^{\eta + \log \xi} < 0$
CAUTION


u>0 v>0
u>0 v<0
u<0 v>0
R
ξ>0
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η} = − e^{−η + log ξ} = − e^{−U} < 0
v = t + z = ξ e^{η} = e^{η + log ξ} = e^{V} > 0
→ ds² = ξ² dη² − dξ²
Consider complex t and z
t = (ξ + iζ) sh(η + iφ)
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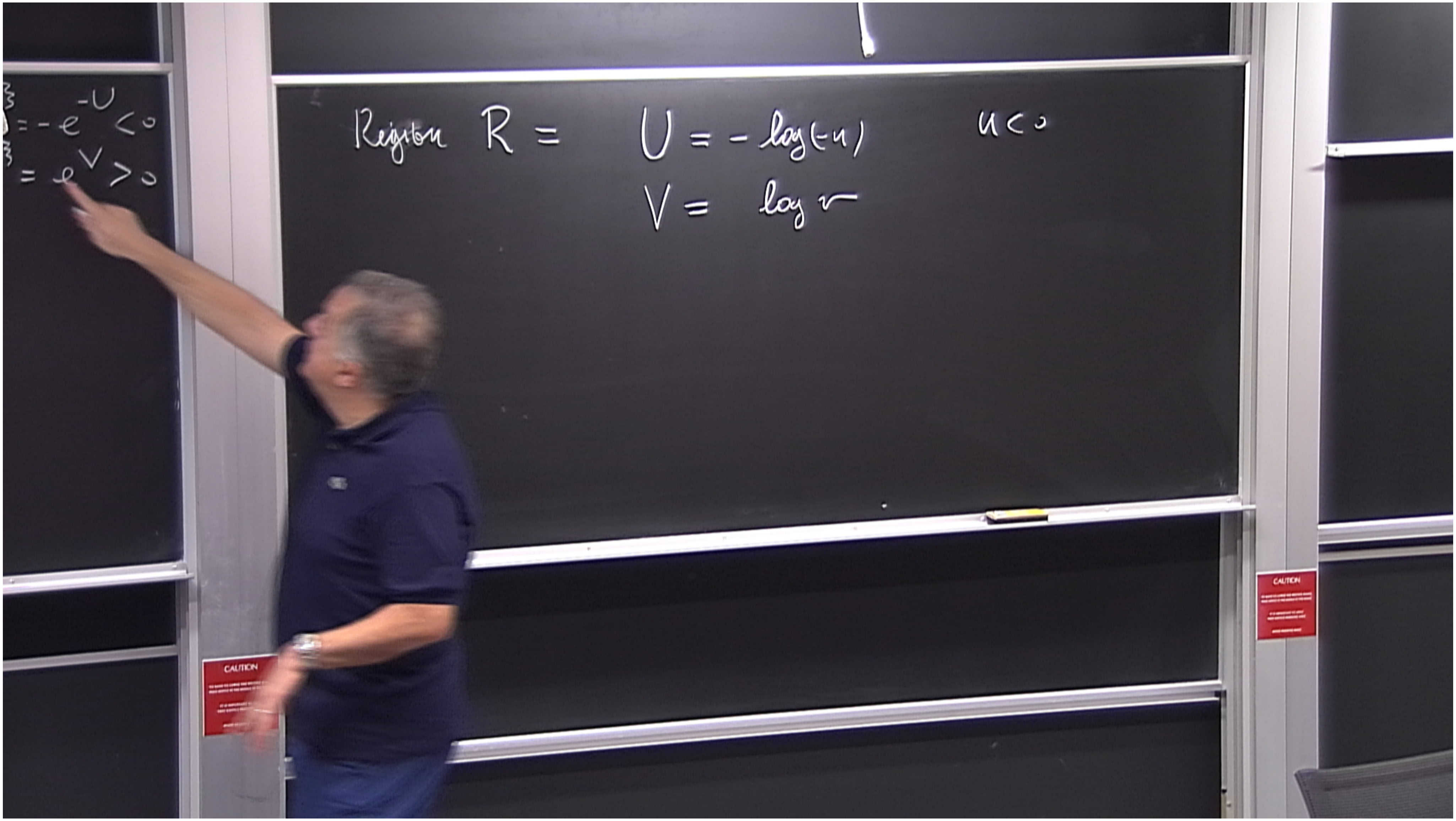
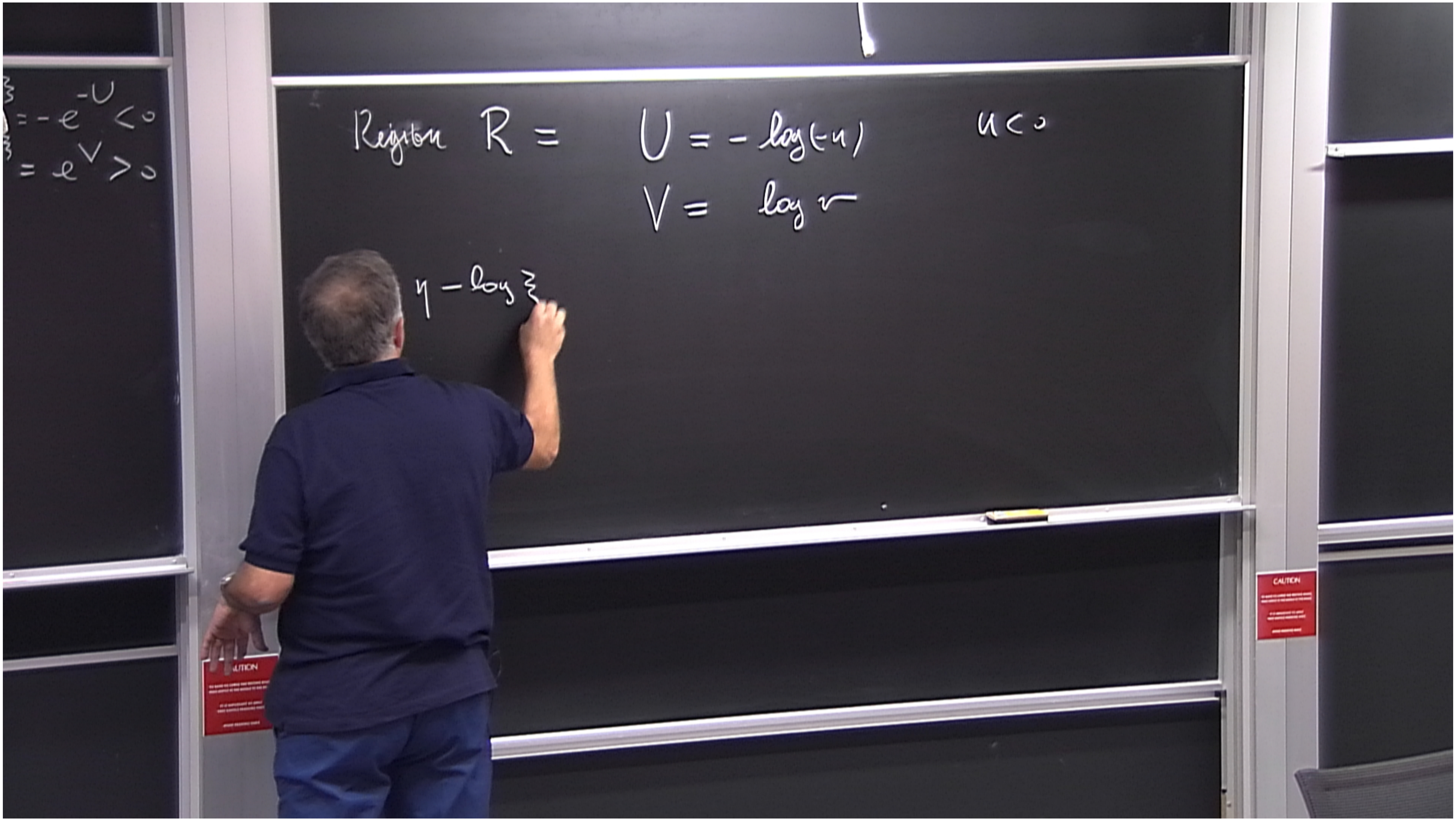
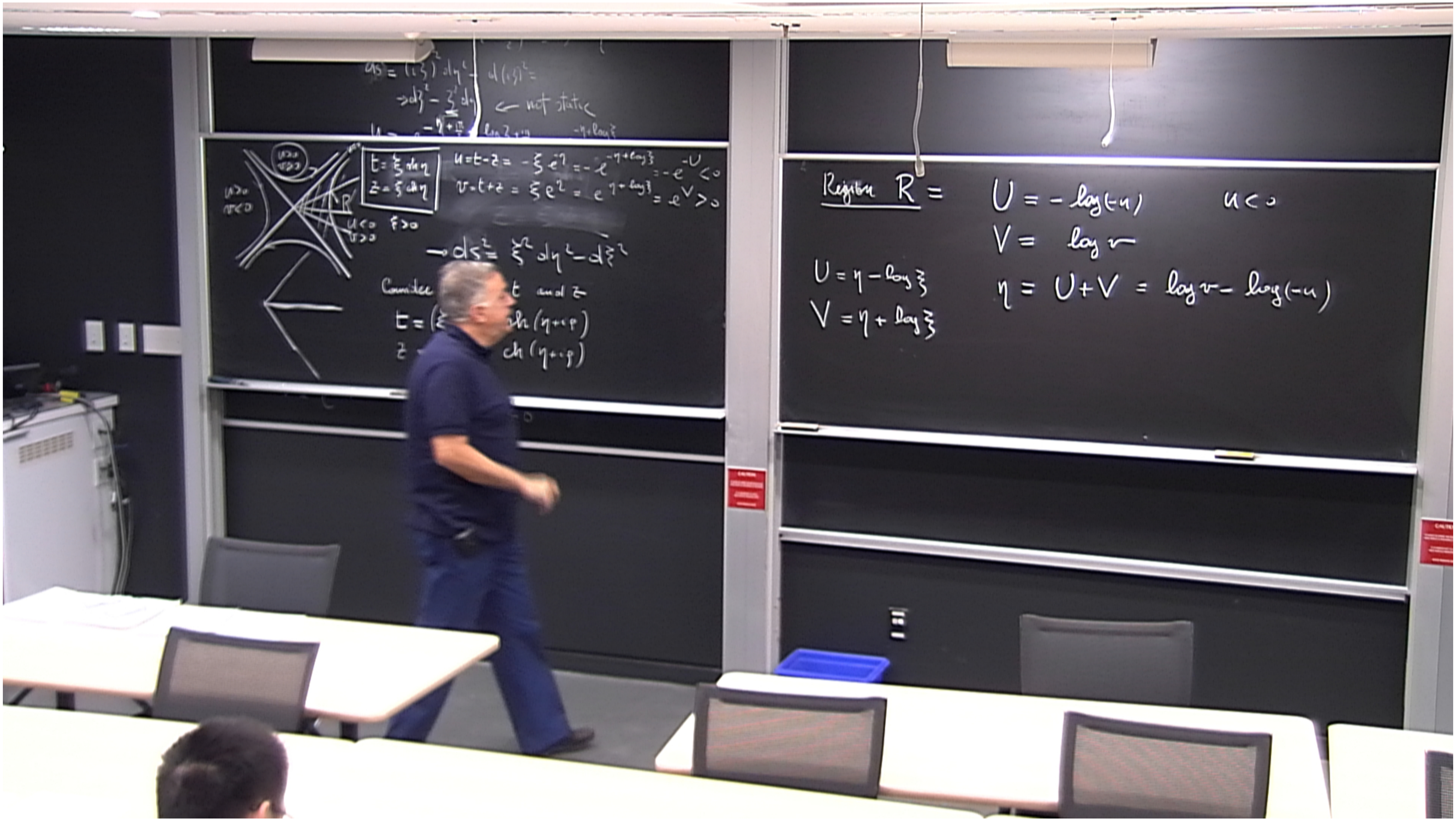
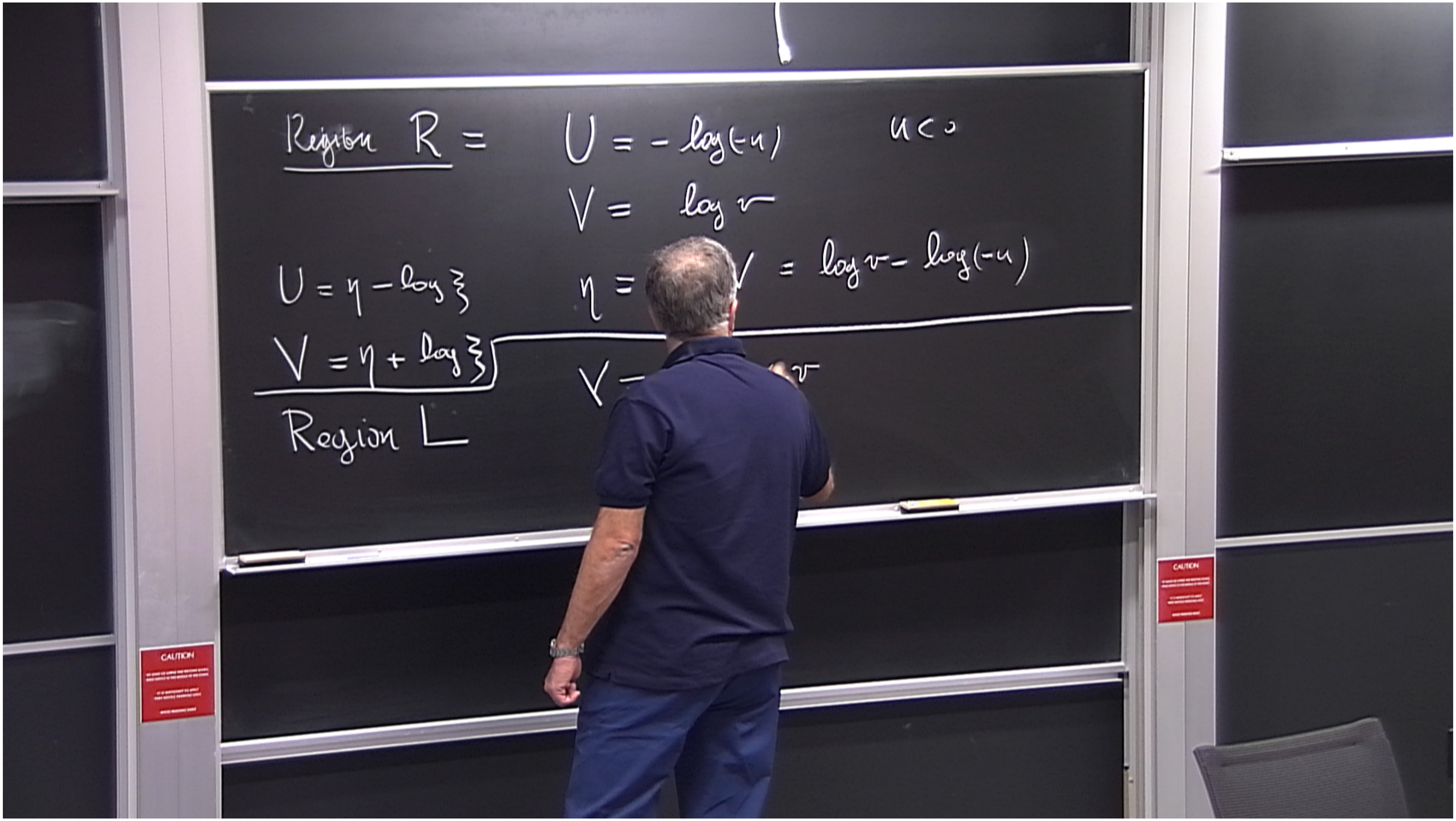
CAUTION

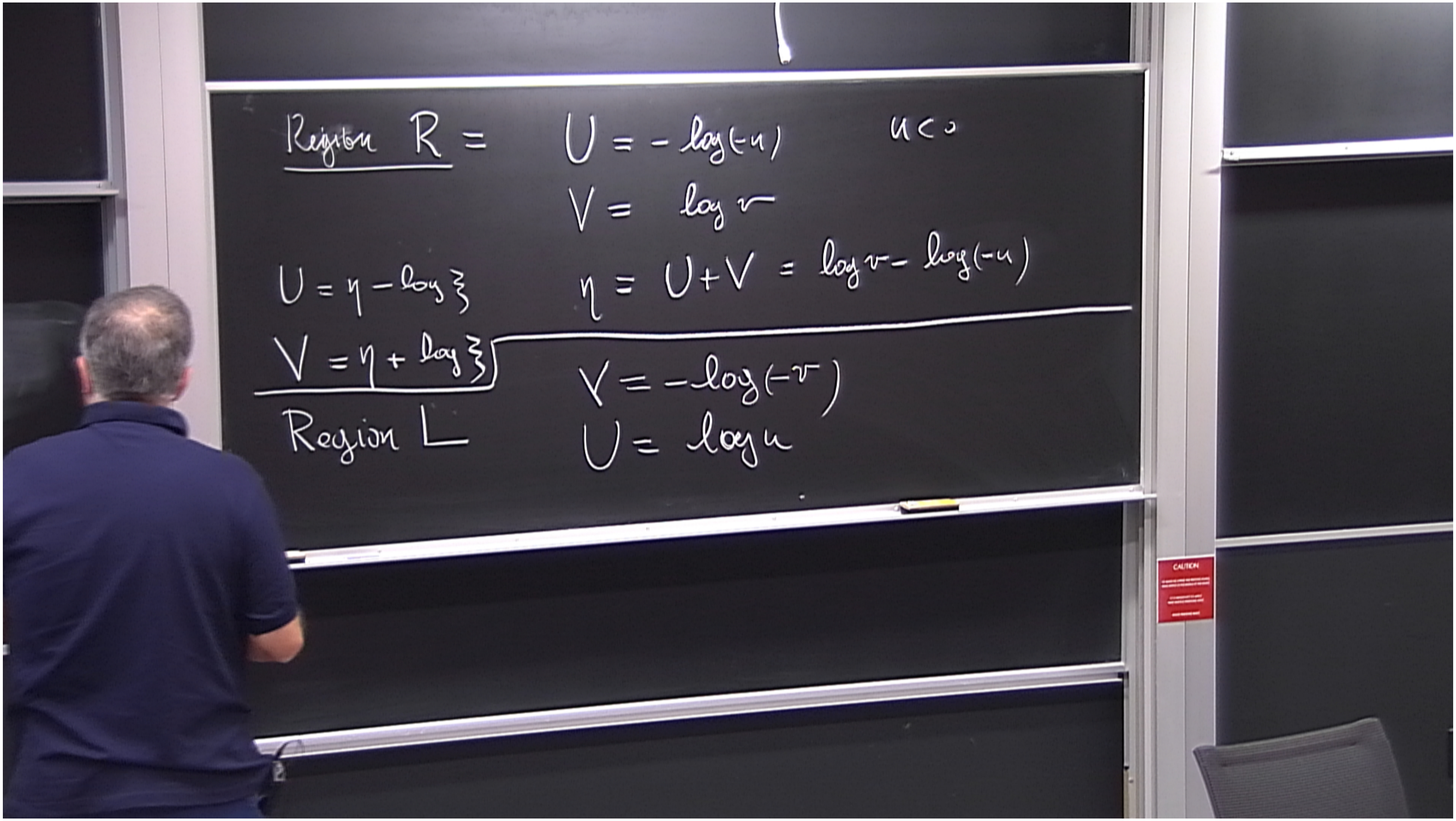

= - e^{-U} < 0
= e^{V} > 0
Region R =
U = - log(-u)
u < 0
V = log v
CAUTION
CAUTION

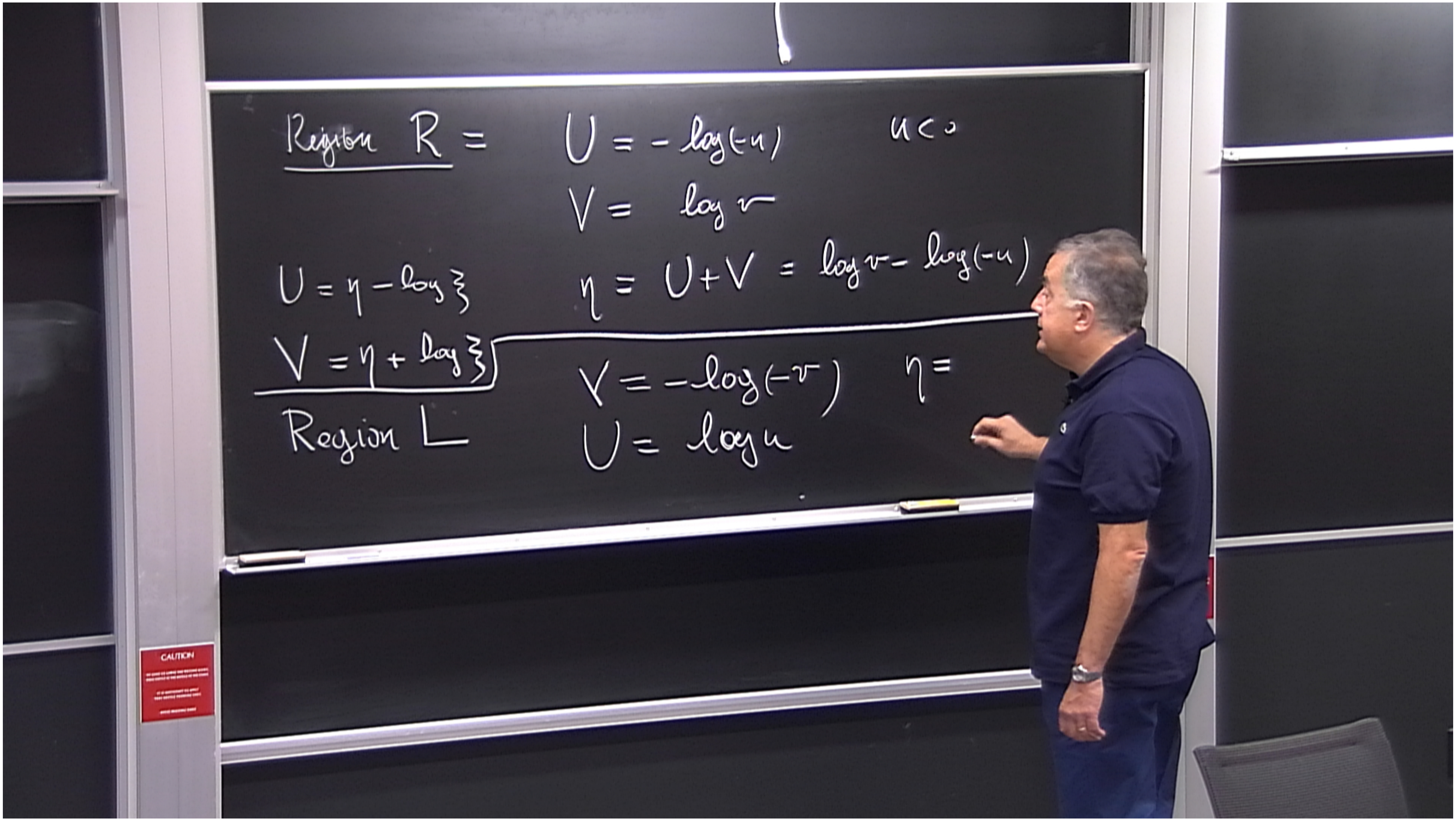

= - e^-U < 0
= e^V > 0
Region R =
U = - log(-u)
u < 0
V = log v
η - log ξ

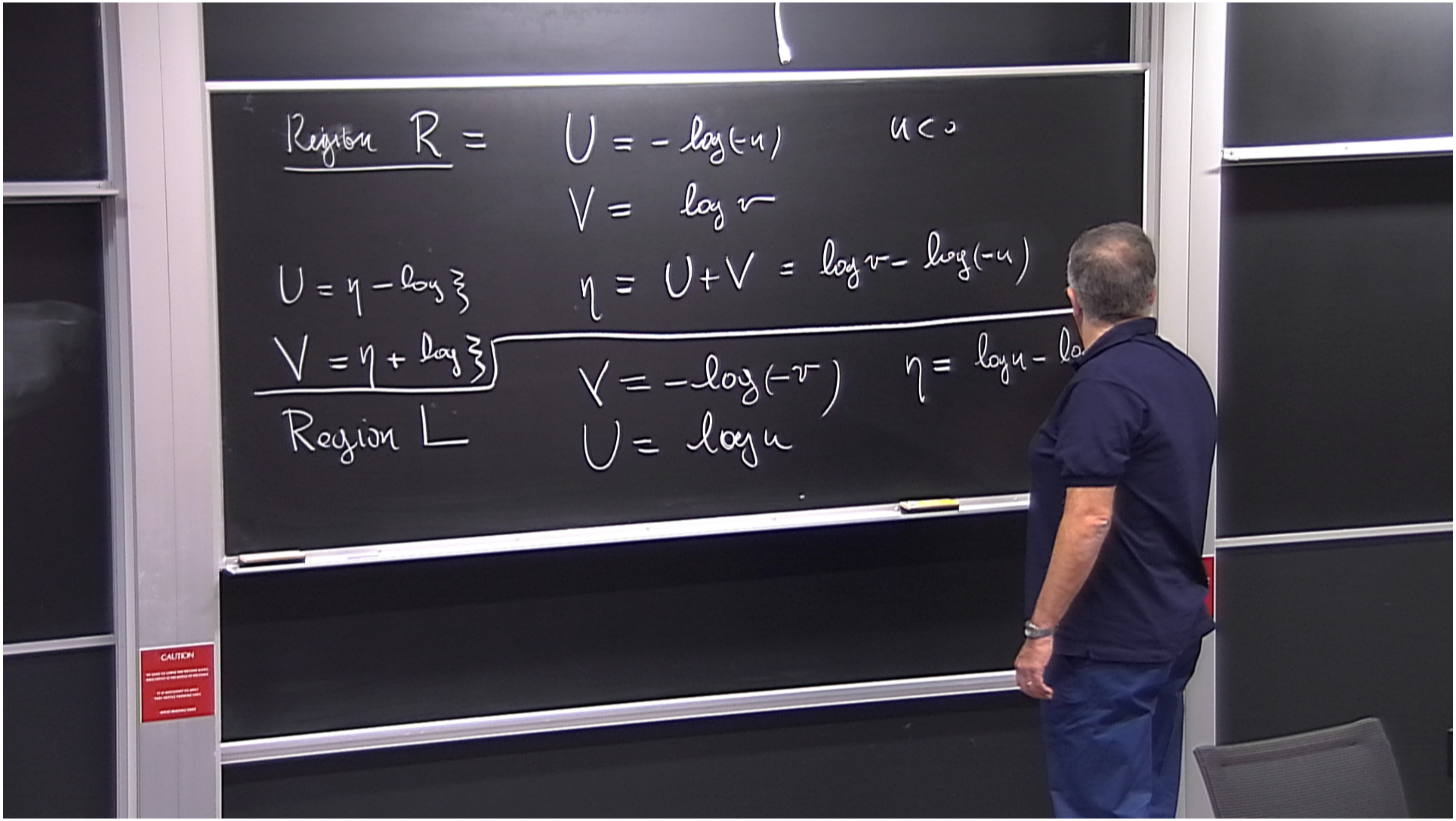


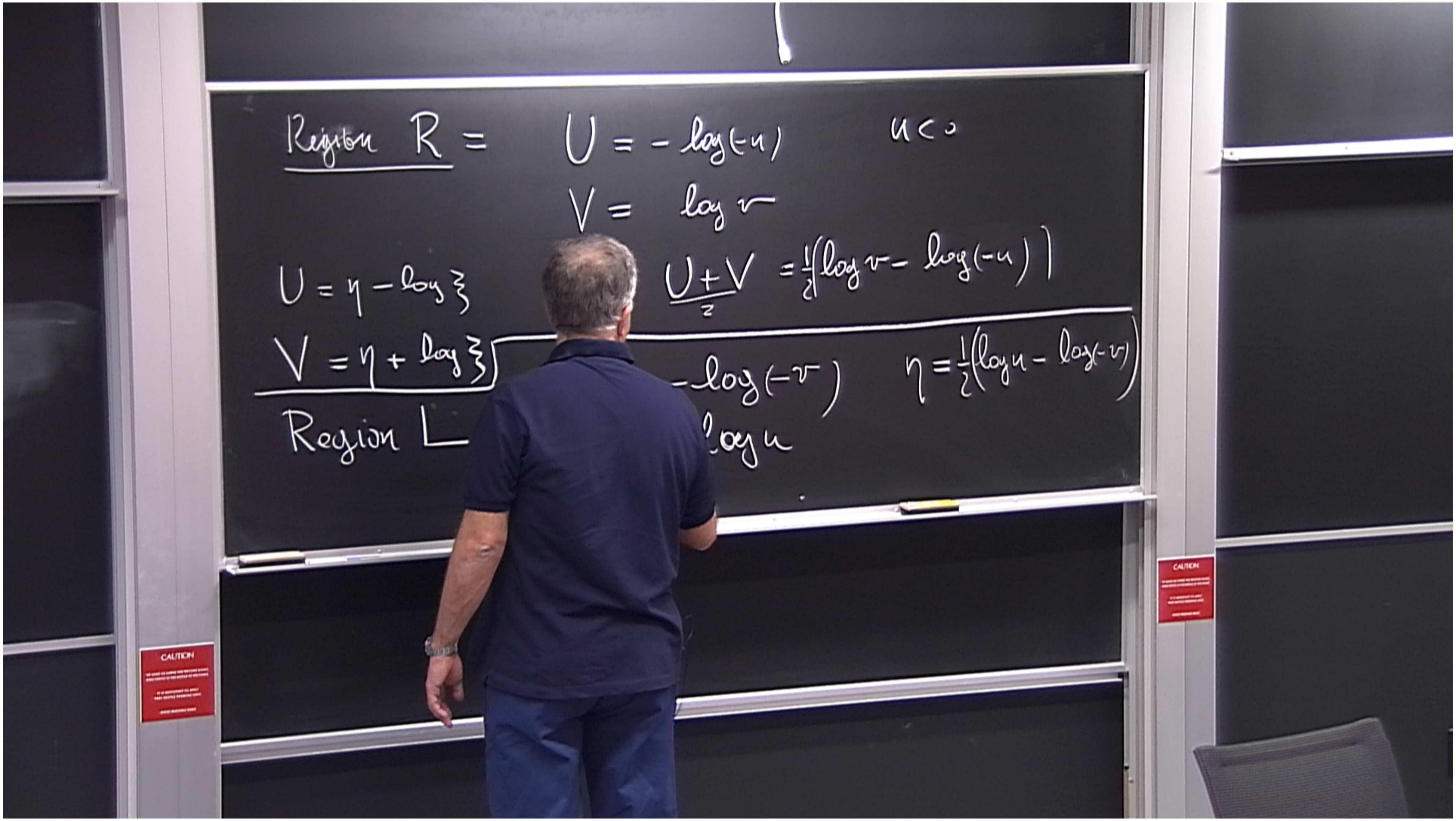
Region R =
U = − log(−u)
u < 0
V = log v
U = η − log ξ
V = η + log ξ
η = U + V = log v − log(−u)
← not static
t = ξ sh η
z = ξ ch η
u = t − z = − ξ e^{−η}
v = t + z = ξ e^{η}
u < 0
ξ > 0
→ ds² = ξ² dη² − dξ²
Consider


Region R =
U = - log(-u)
u < 0
V = log v
U = η - log ξ
= log v - log(-u)
V = η + log ξ
Region L

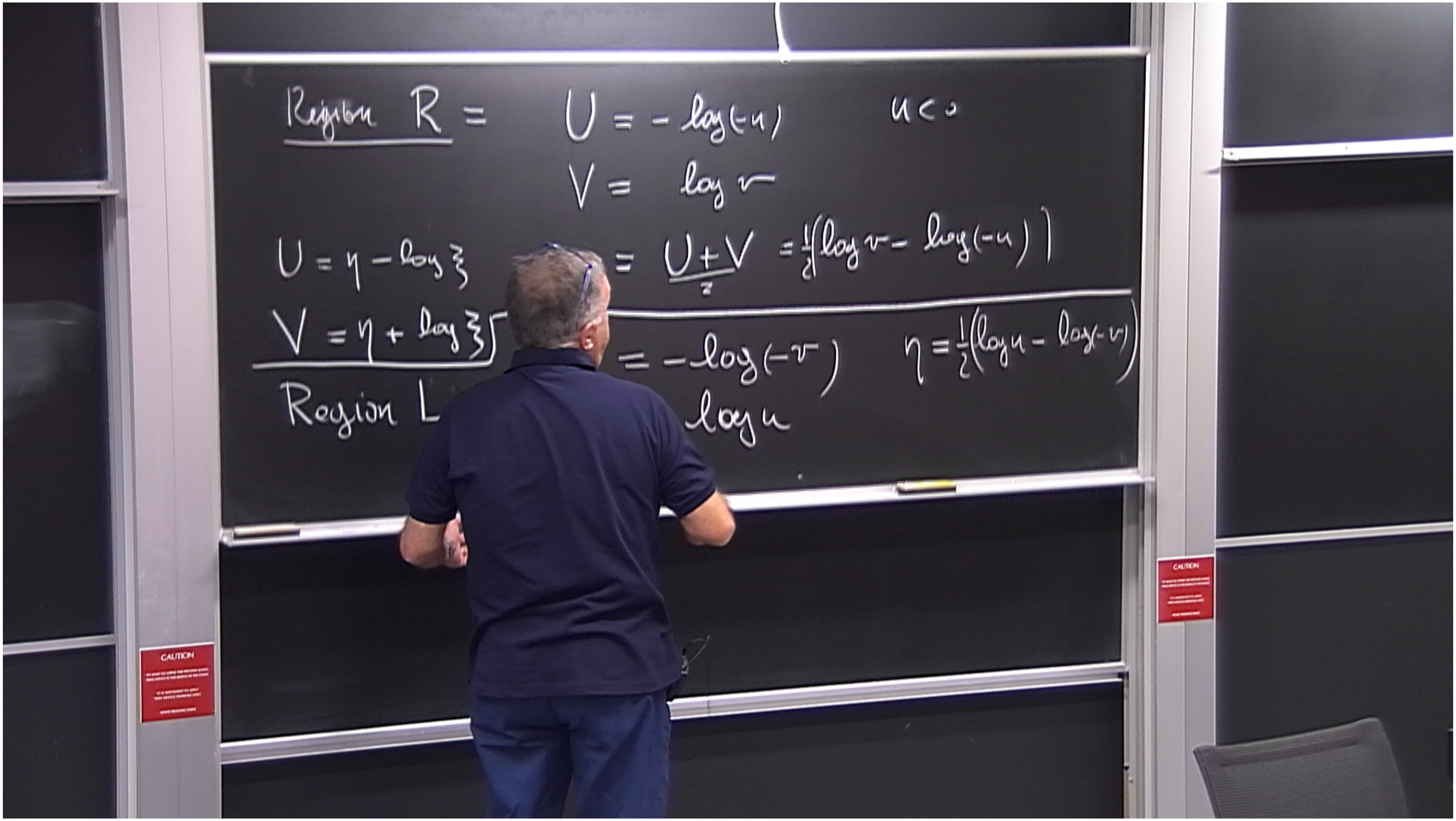

Region R =
U = - log(-u)
u < 0
V = log v
U = η - log ξ
η = U + V = log v - log(-u)
V = η + log ξ
Region L
V = -log(-v)
U = log u

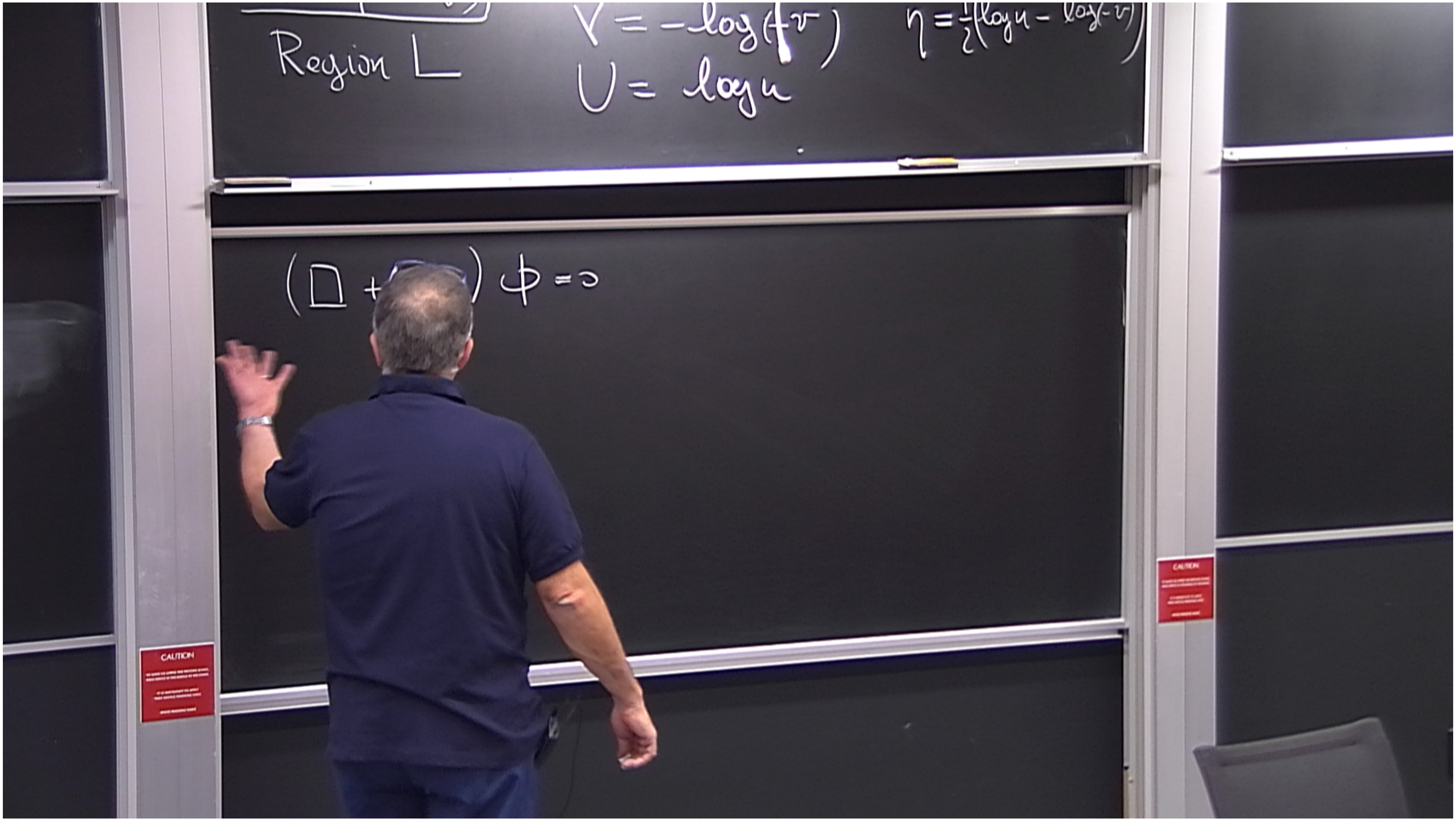

Region R =
U = - log(-u)
u < 0
V = log v
U = η - log ξ
η = U + V = log v - log(-u)
V = η + log ξ
V = - log(-v)
η =
Region L
U = log u

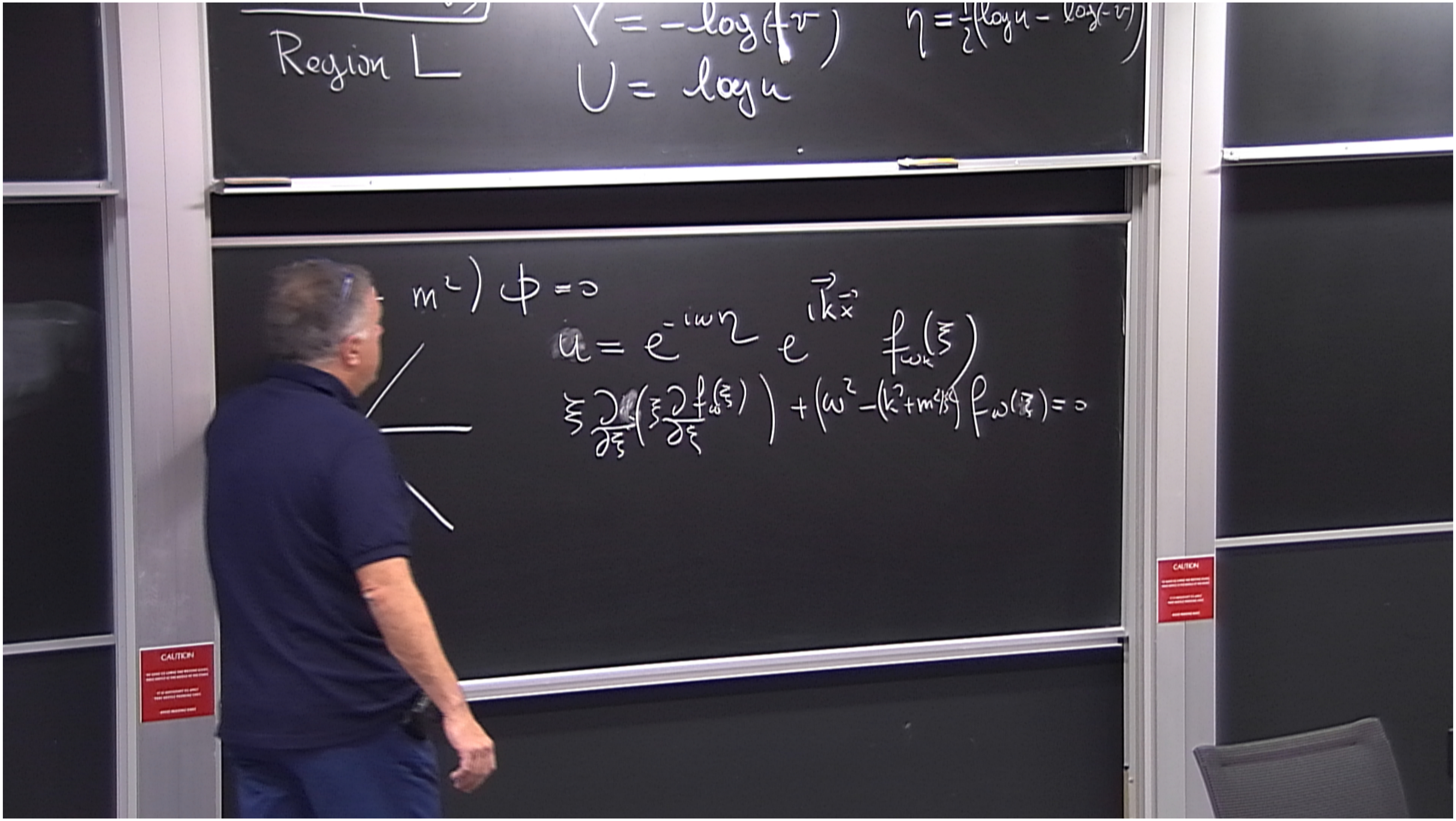

Region R =
U = - log(-u)
u < 0
V = log v
U = η - log ξ
η = U + V = log v - log(-u)
V = η + log ξ
V = -log(-v)
η = log u - lo
Region L
U = log u

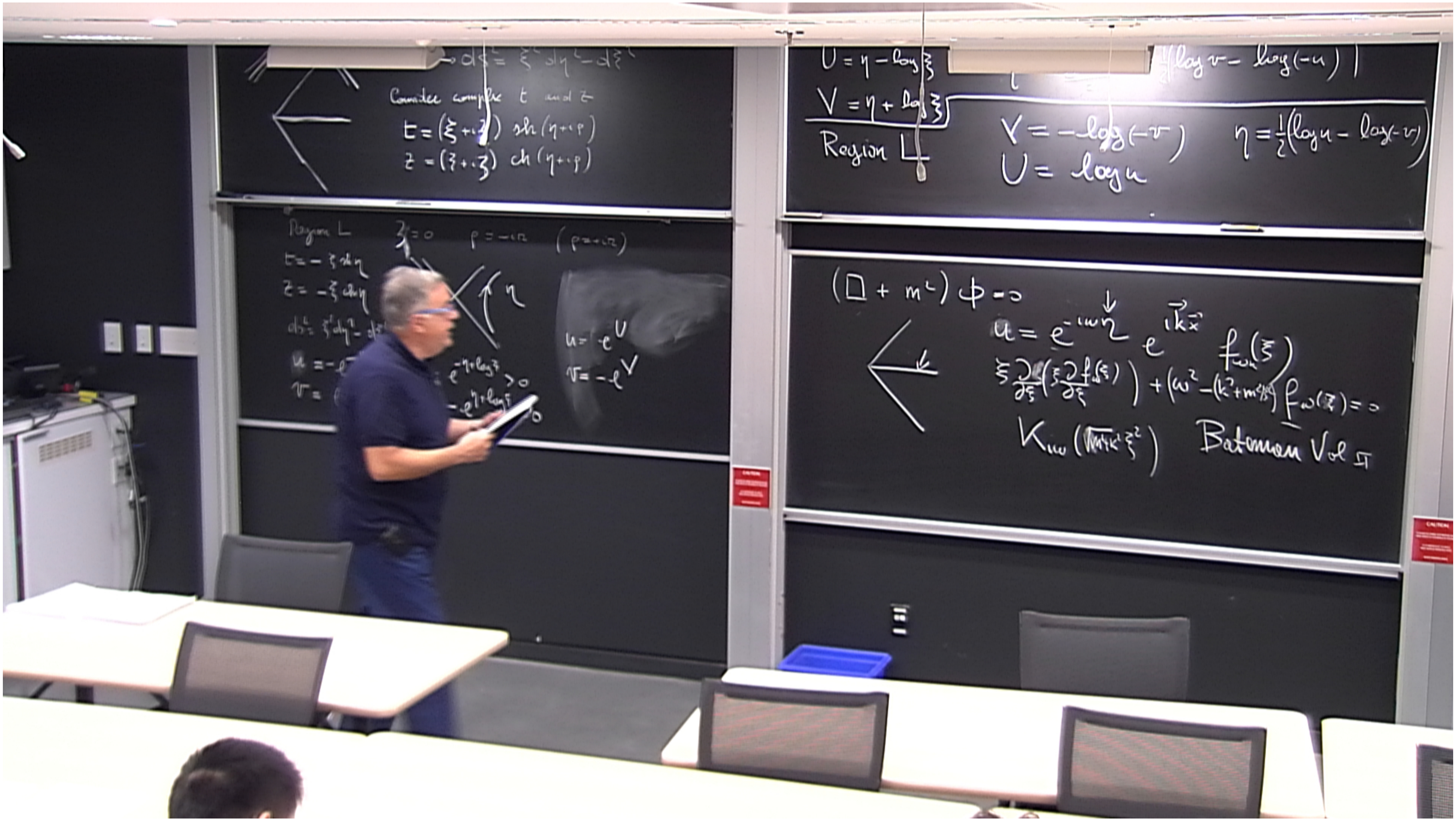


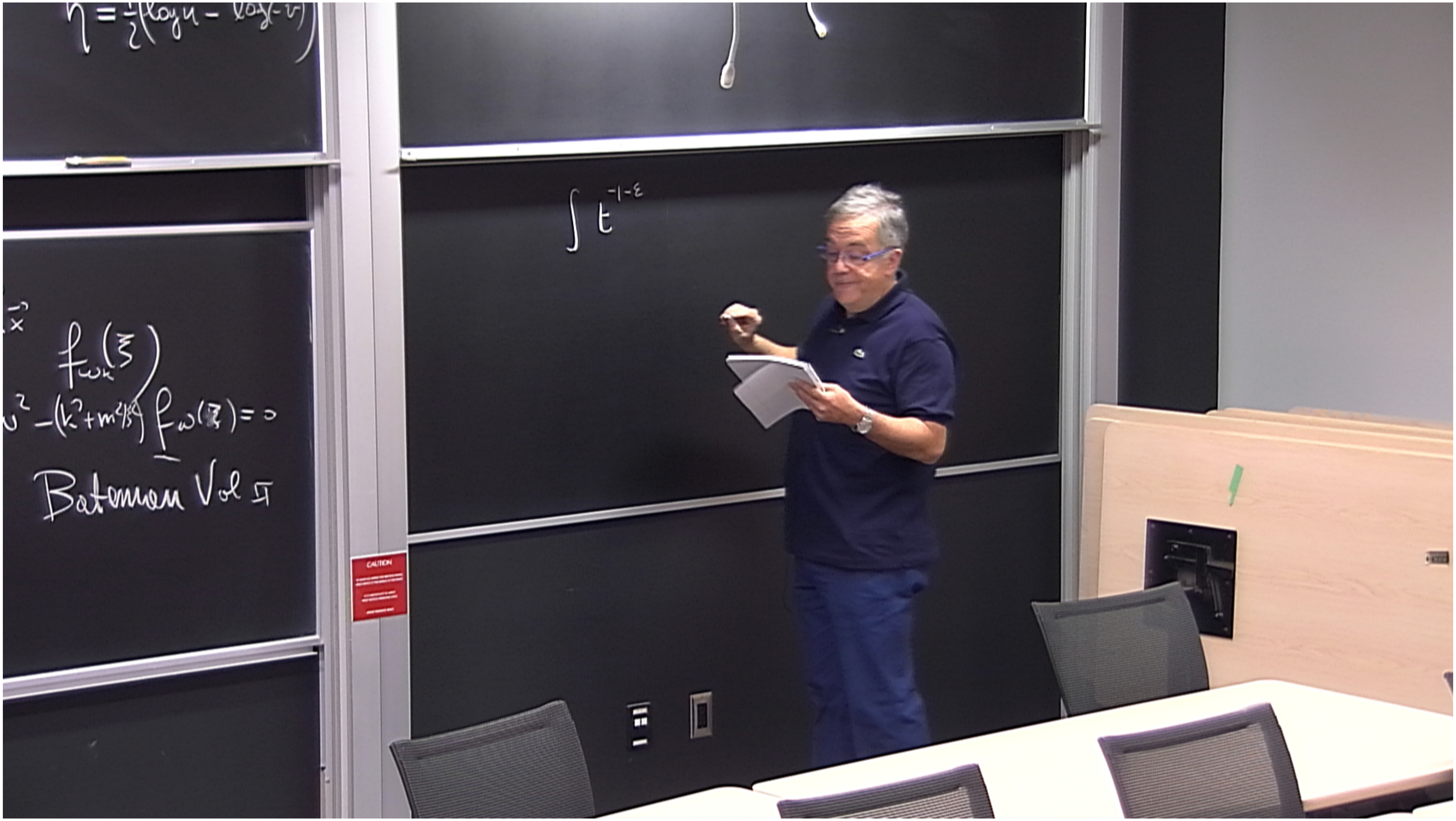
Region R =
U = - log(-u)
u < 0
V = log v
U = η - log ξ
V = η + log ξ
(U+V)/2 = 1/2(log v - log(-u))
-log(-v)
η = 1/2(log u - log(-v))
Region L
log u
CAUTION

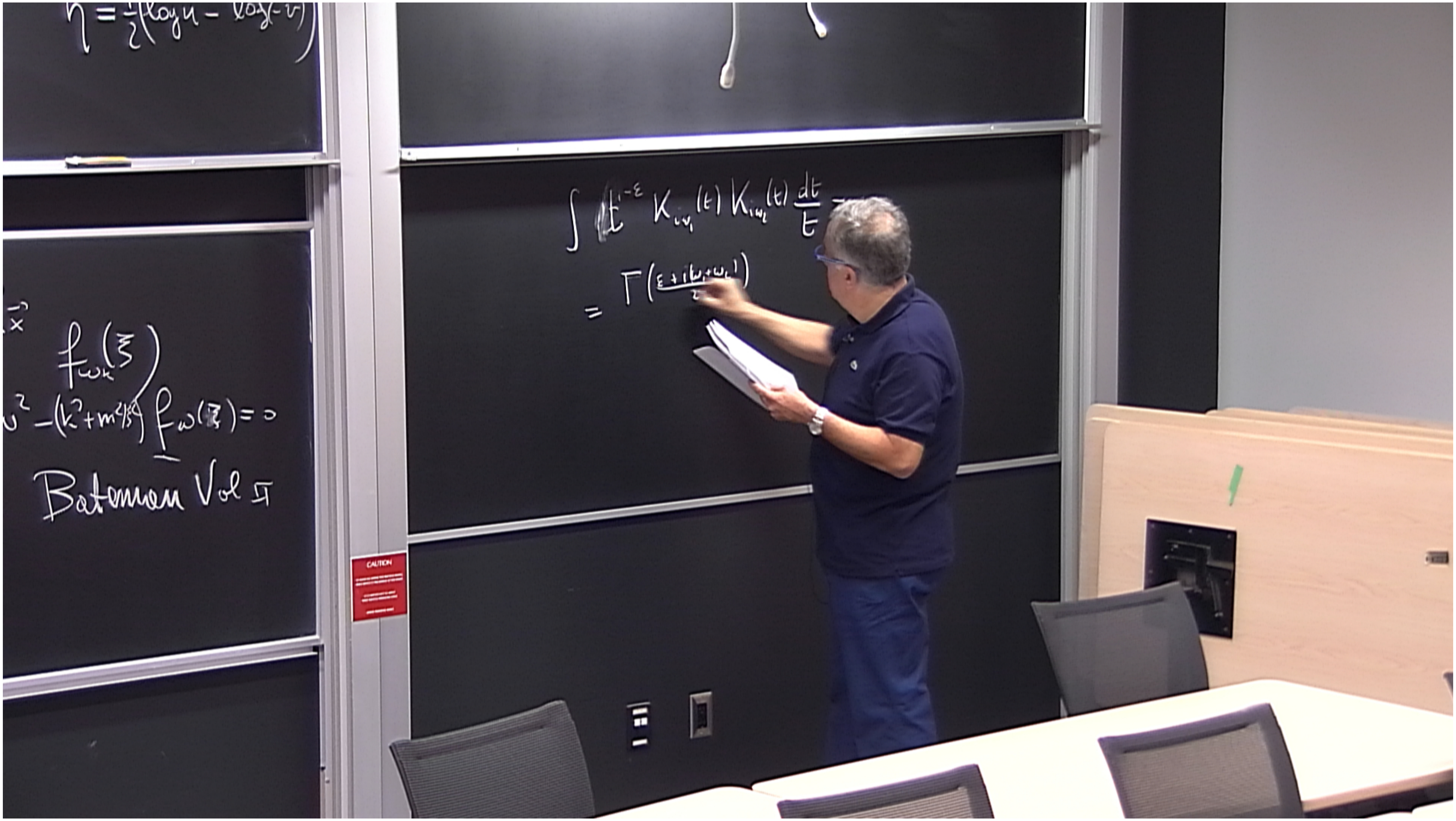

Region R = U = - log(-u) u<0
V = log v
U = η - log ξ
= U+V / 2 = ½(log v - log(-u))
V = η + log ξ
Region L
= - log(-v) η = ½(log u - log(-v))
log u


Region L
V = -log(-v)
U = log u
η = ½(log u - log(-v))
(□ +) φ = 0
CAUTION

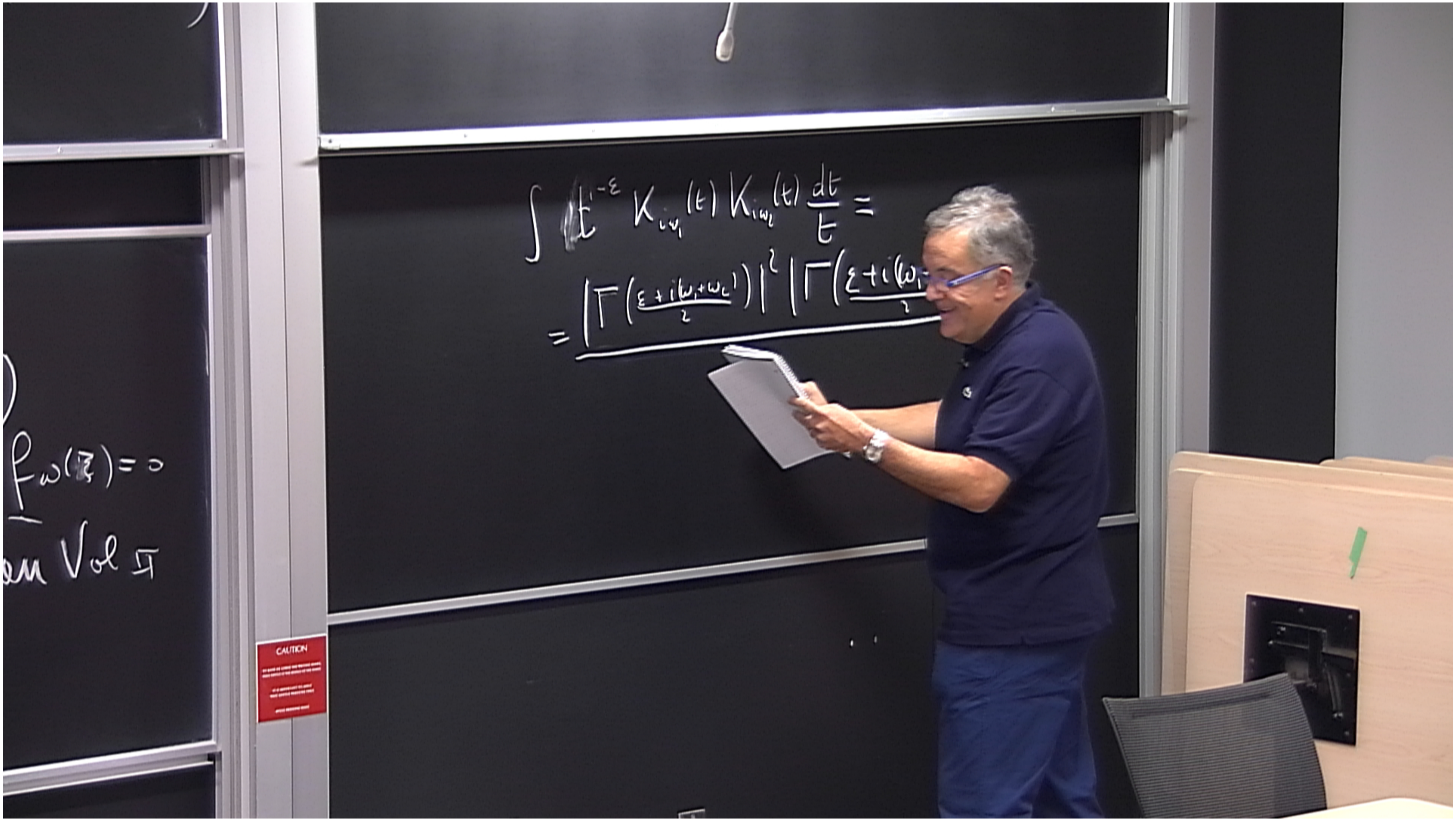

Region L
U = log u
m²) φ = 0
CAUTION
CAUTION

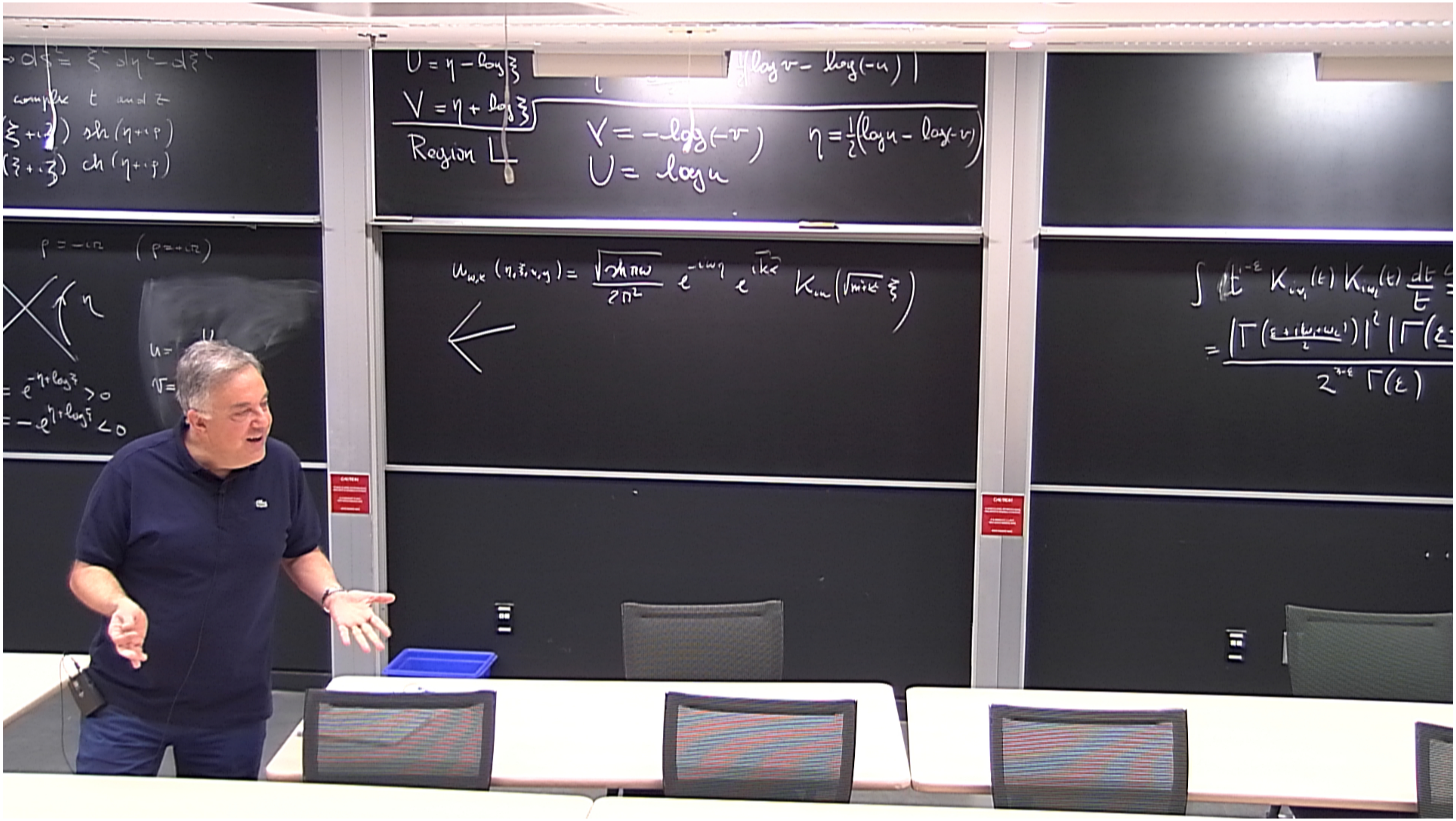

Consider complex t and z
Region L
Region L
(□ + m²) φ = 0
Bateman Vol II

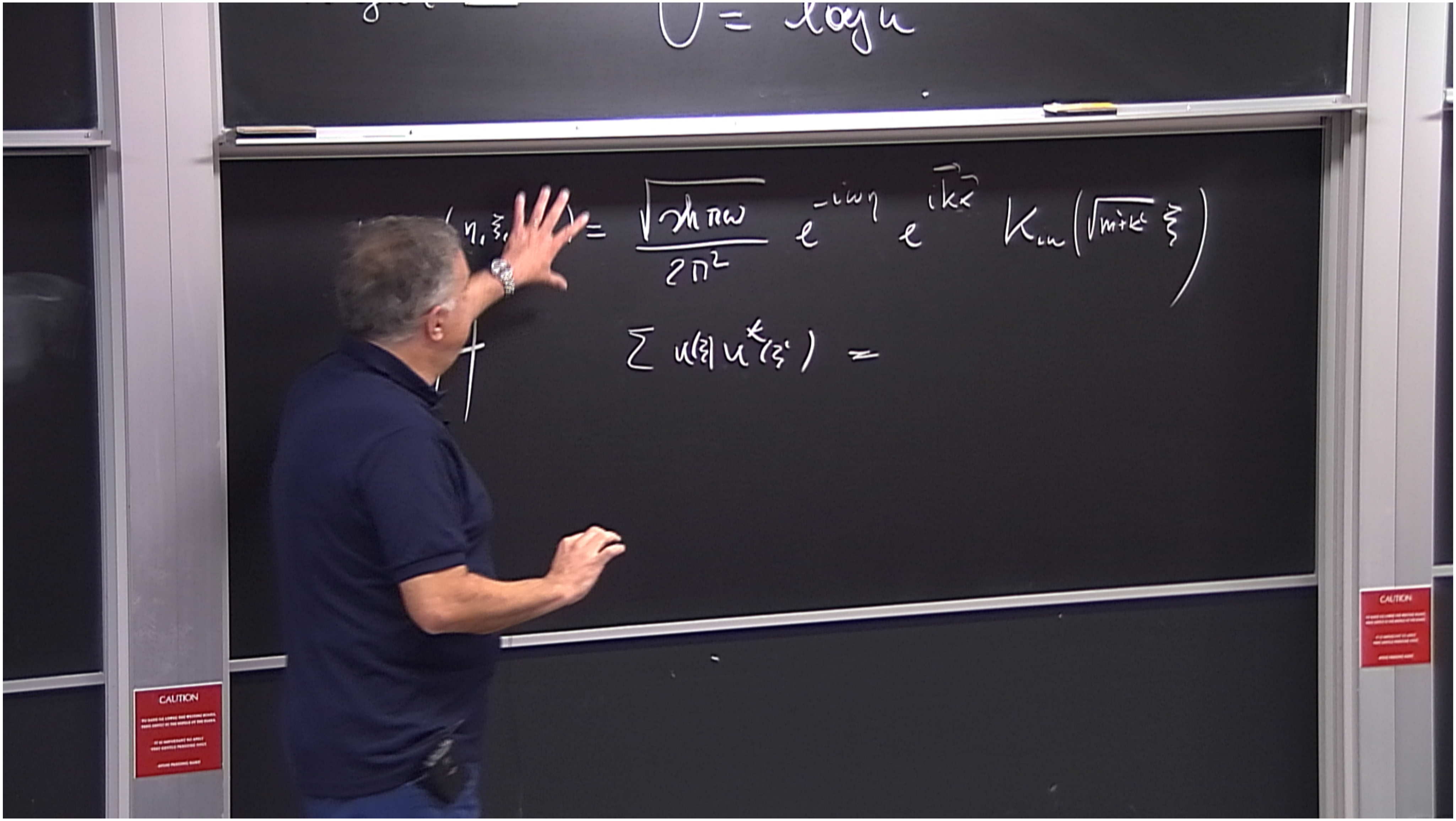

Bateman Vol II

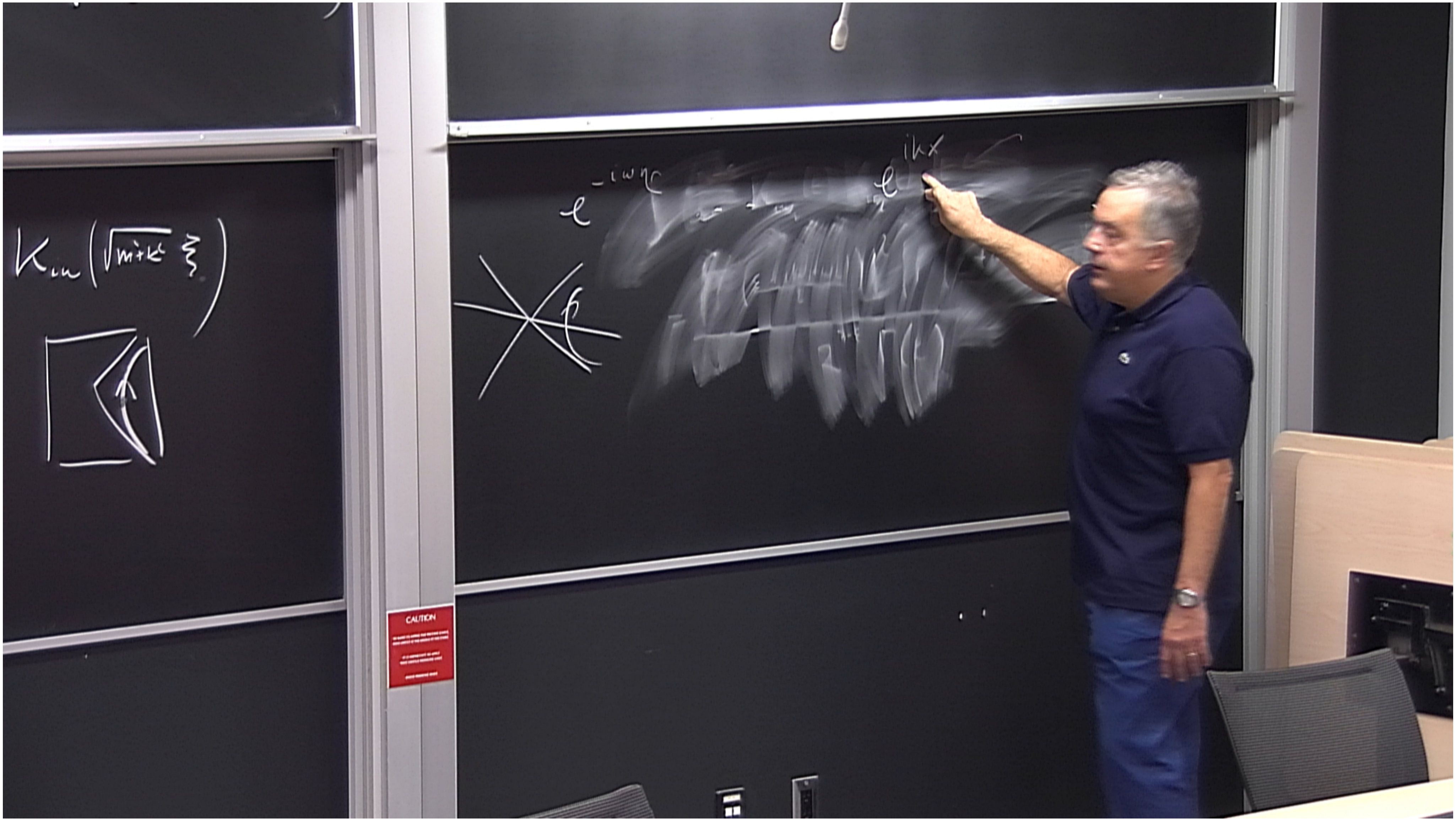


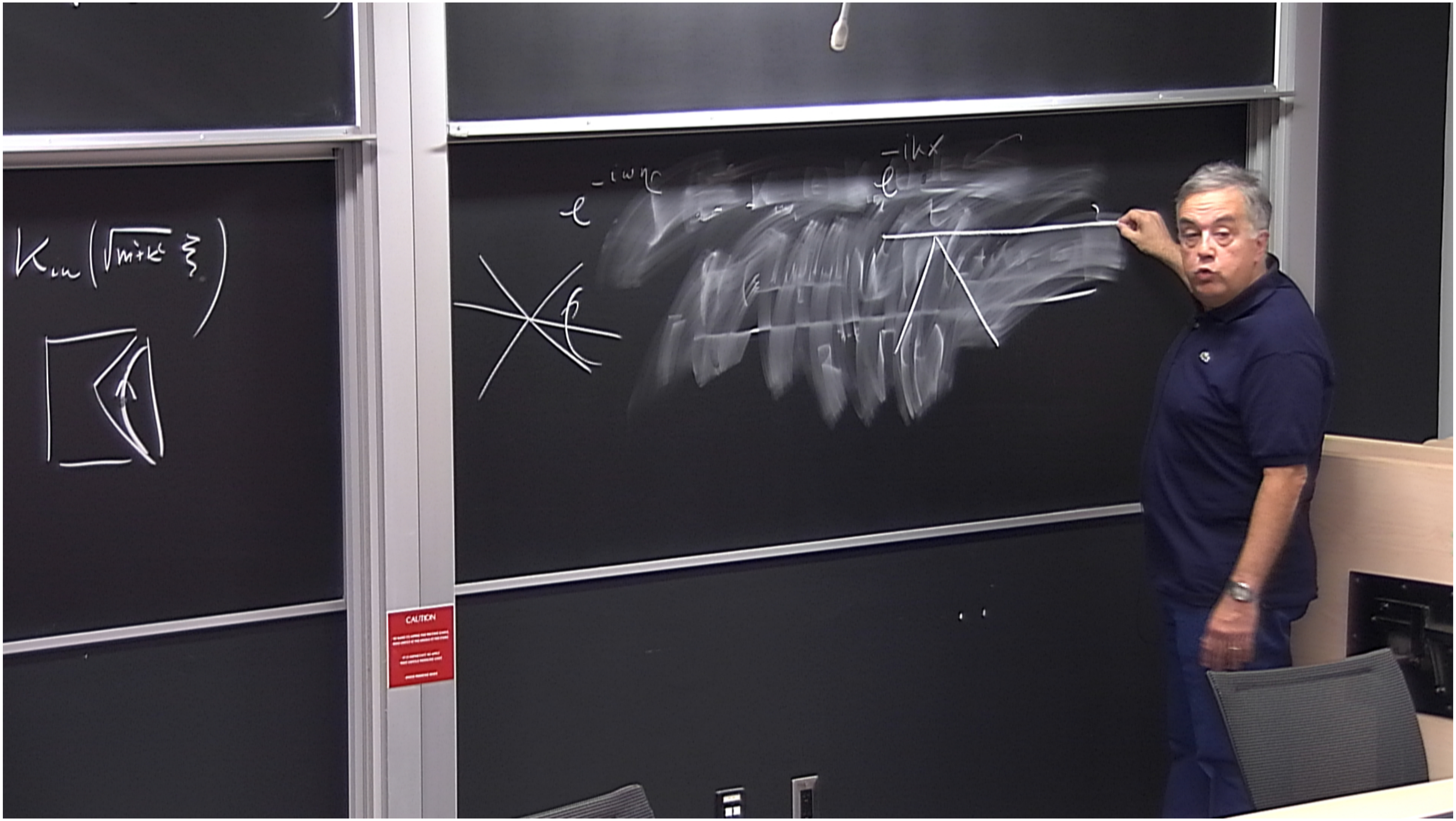
Bateman Vol II

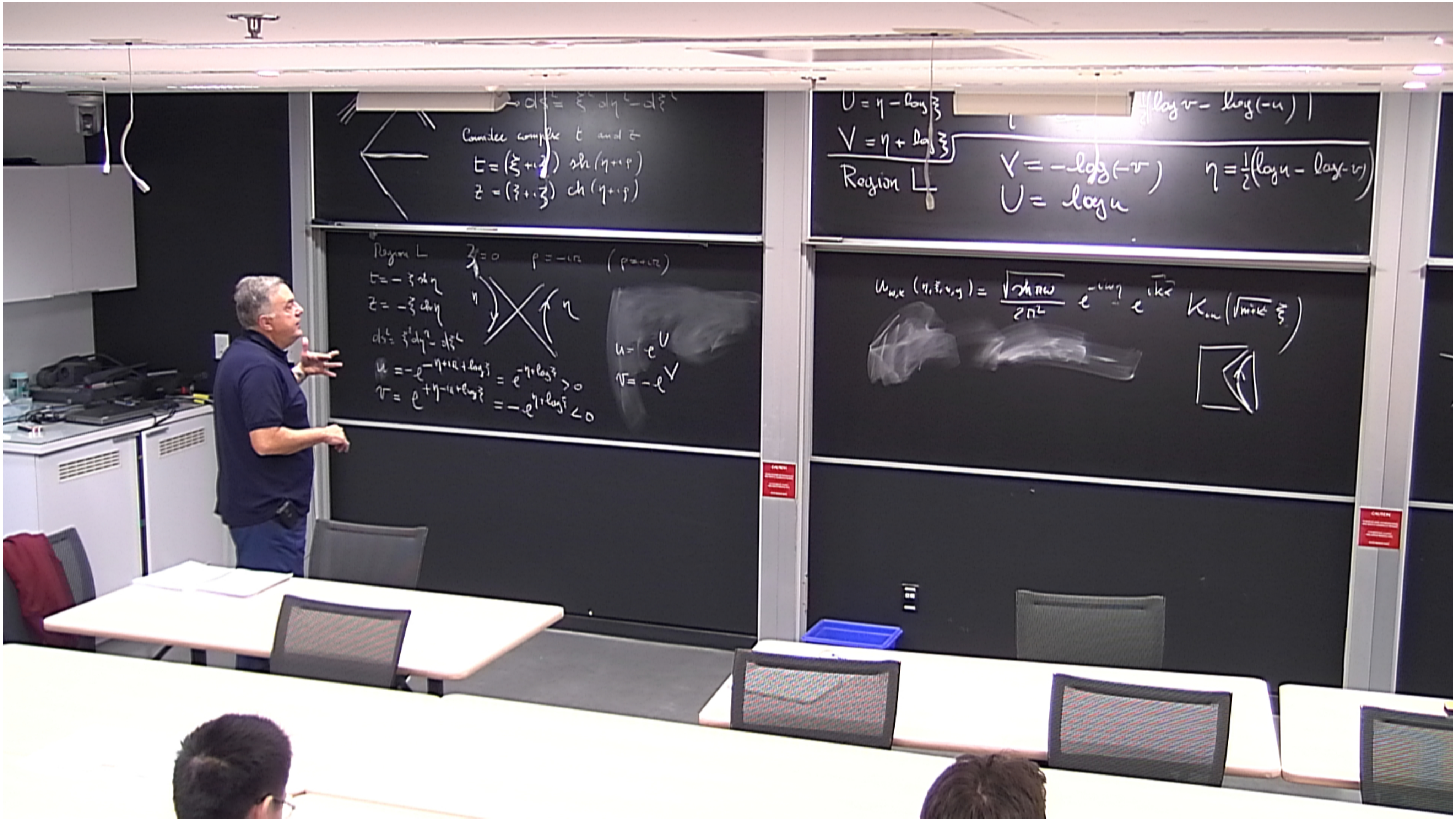


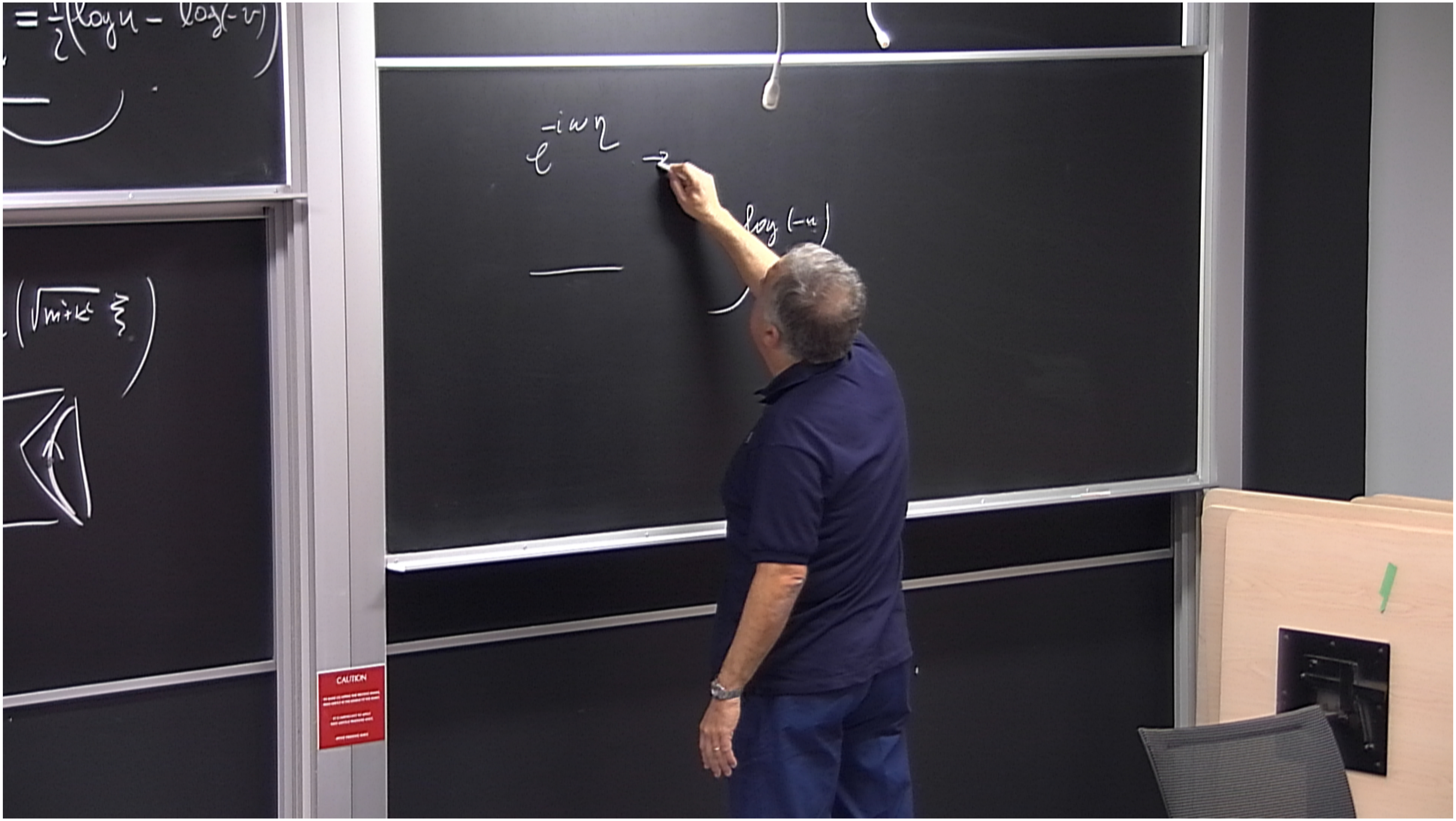


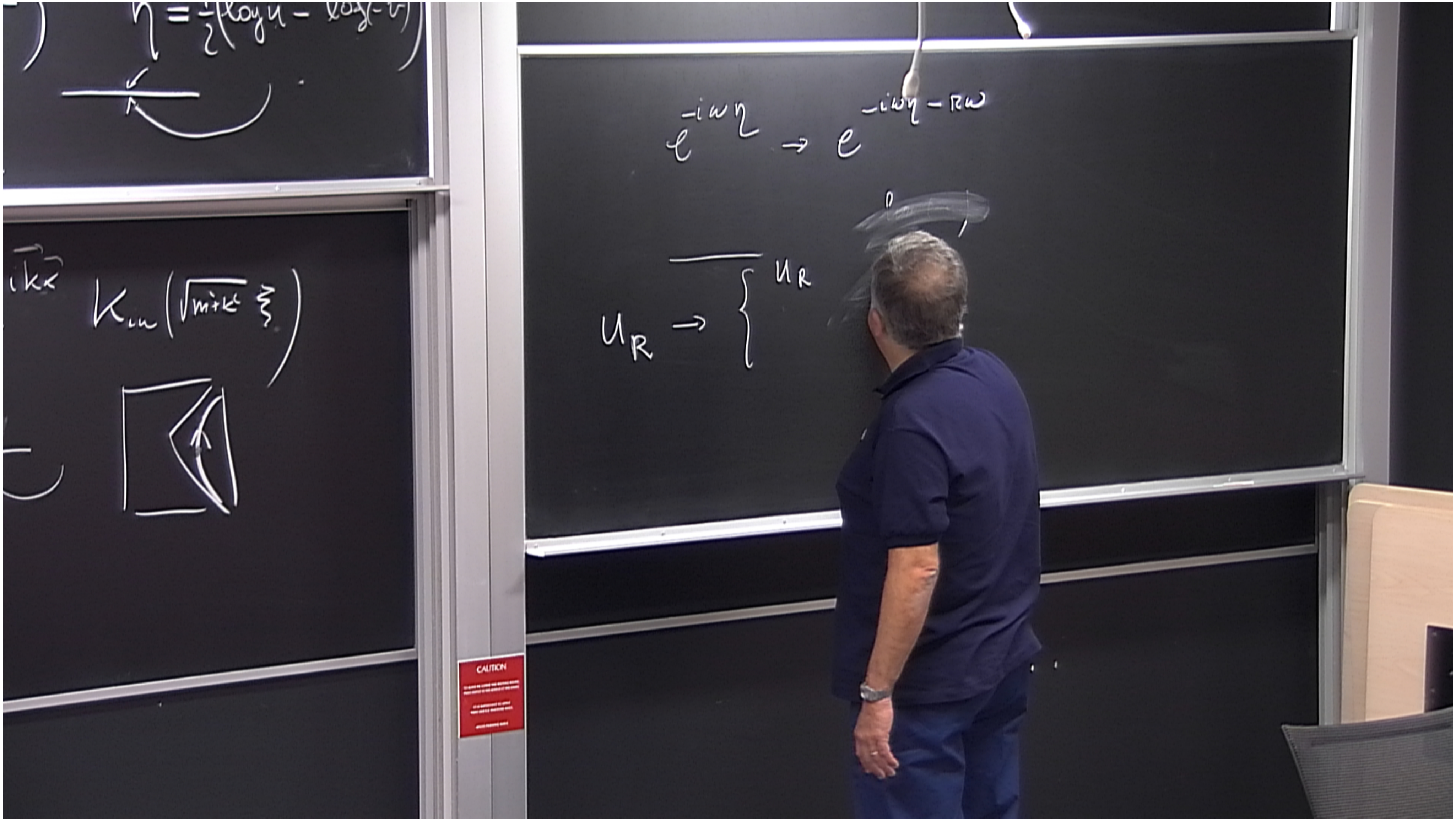

CAUTION
CAUTION

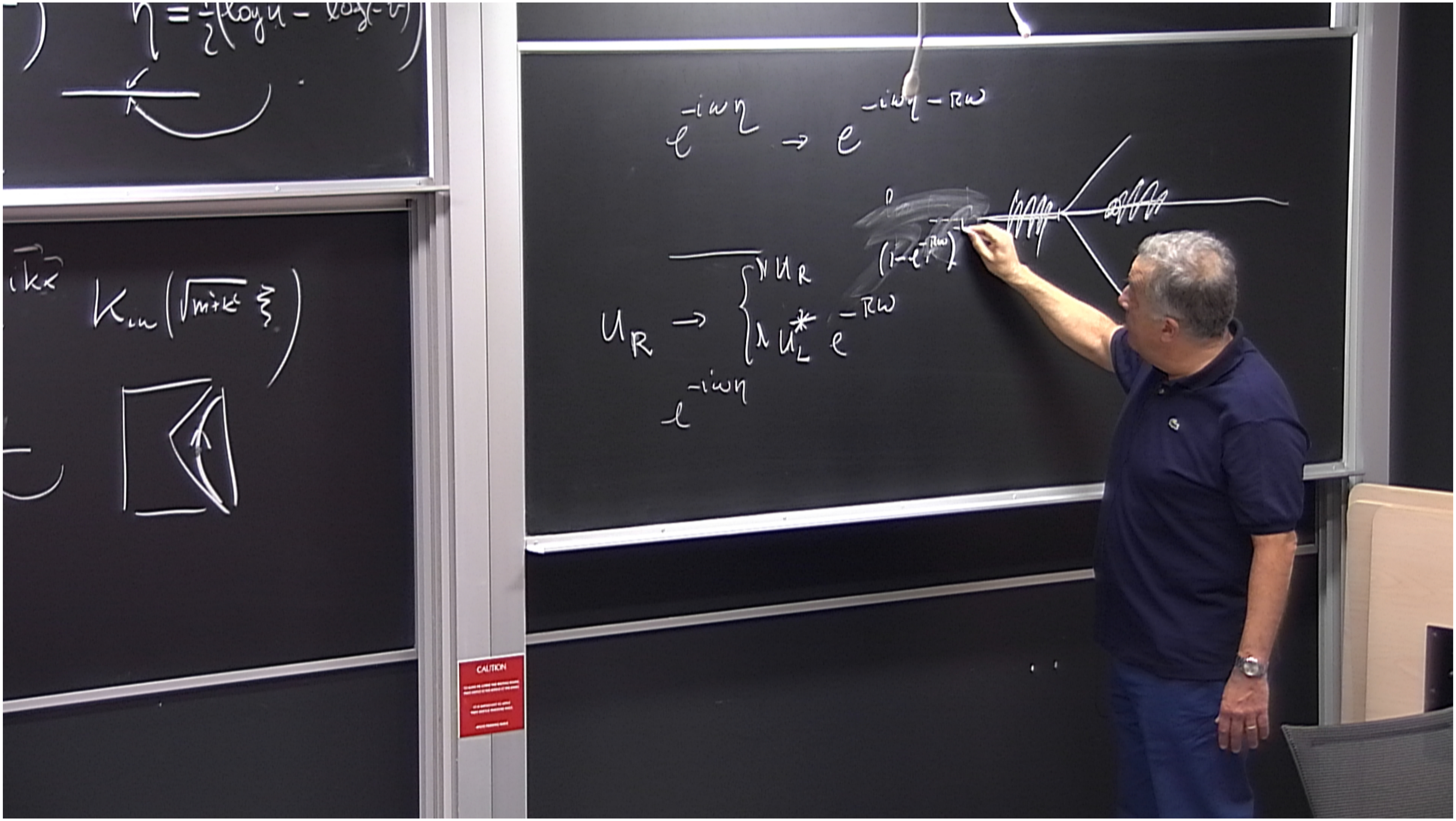

$e^{-i\omega\eta}$
e^{ikx}
$K_{i\omega}\left(\sqrt{m^2+k^2}\,\xi\right)$
CAUTION

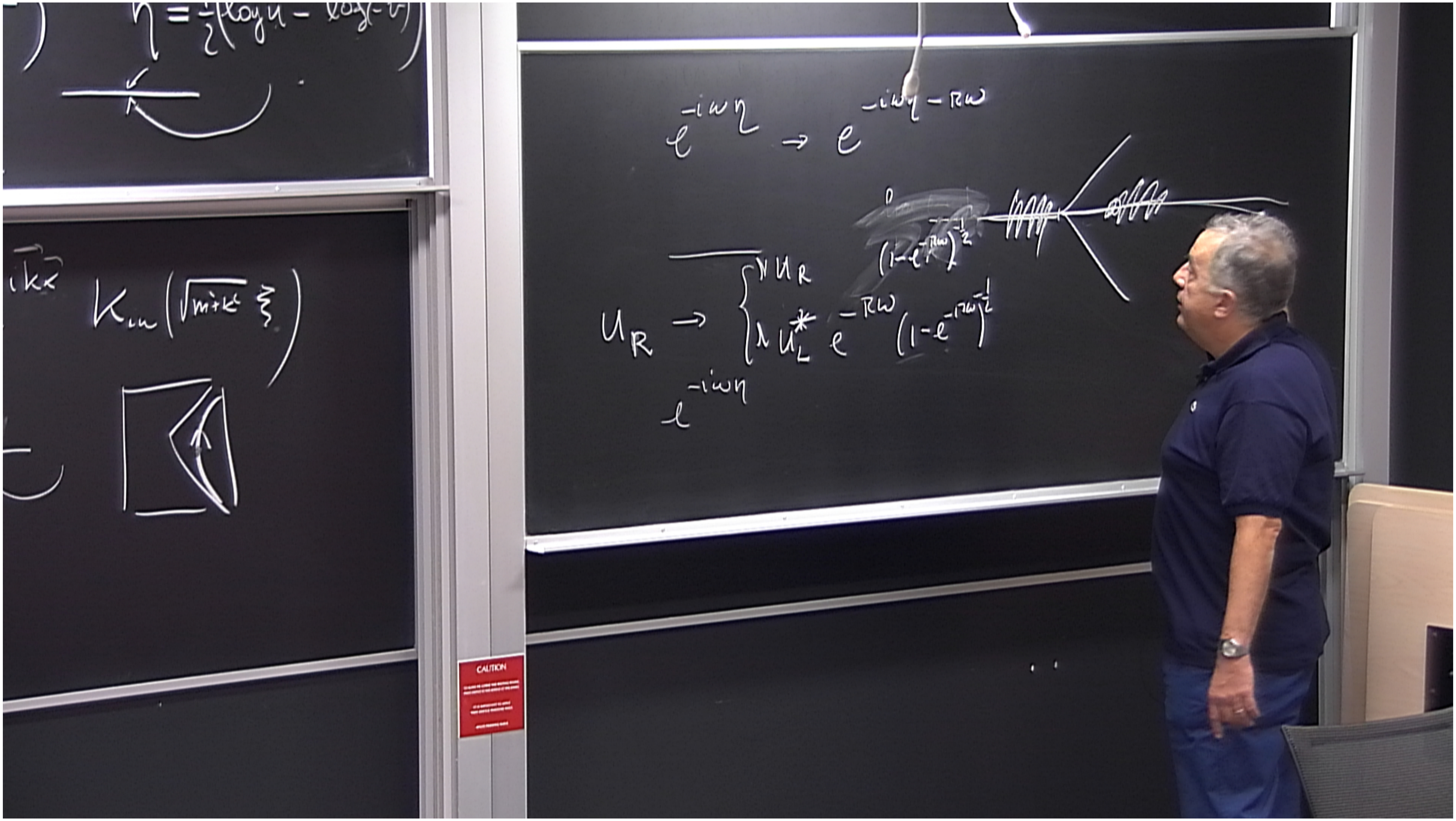

CAUTION

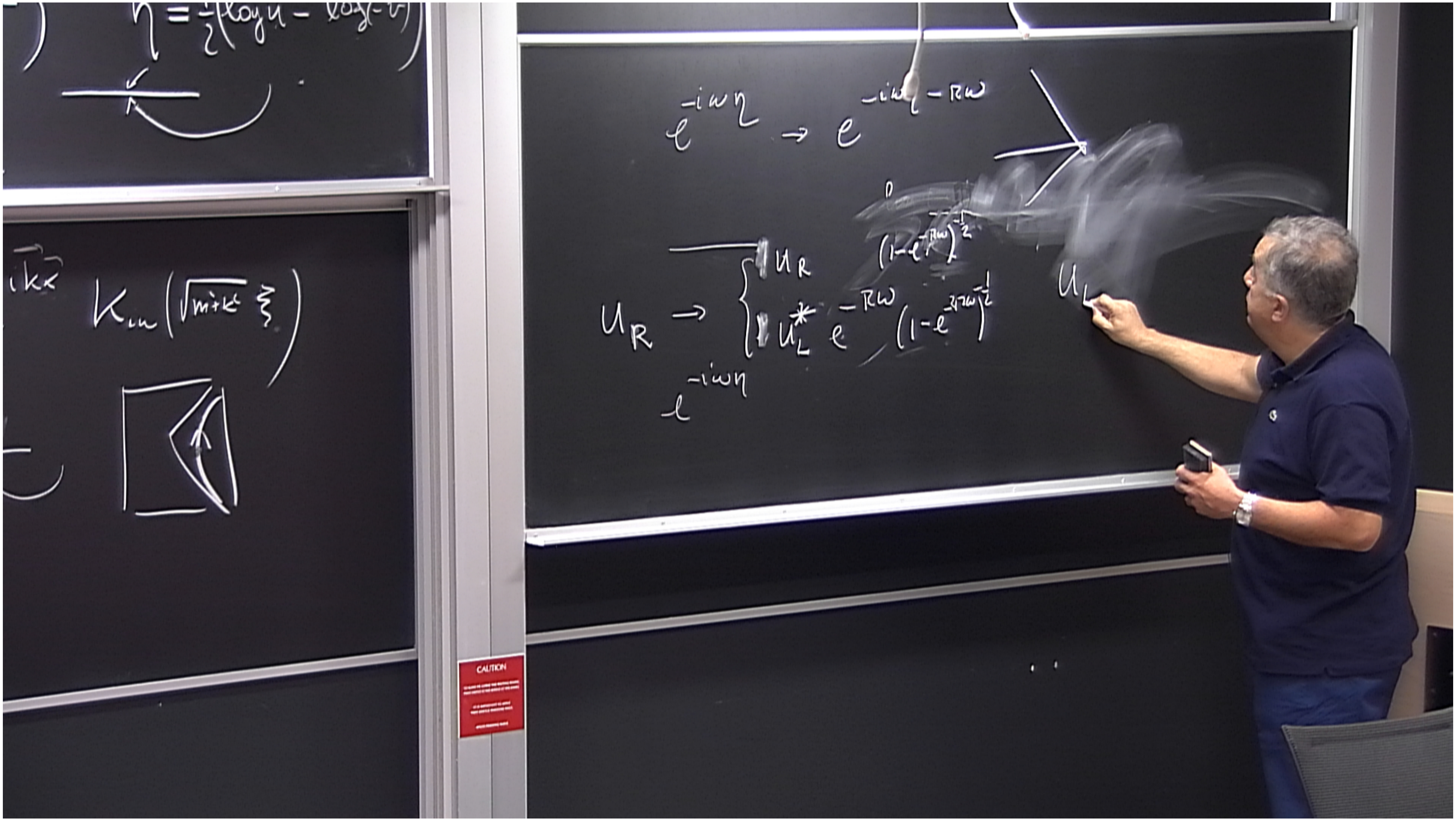


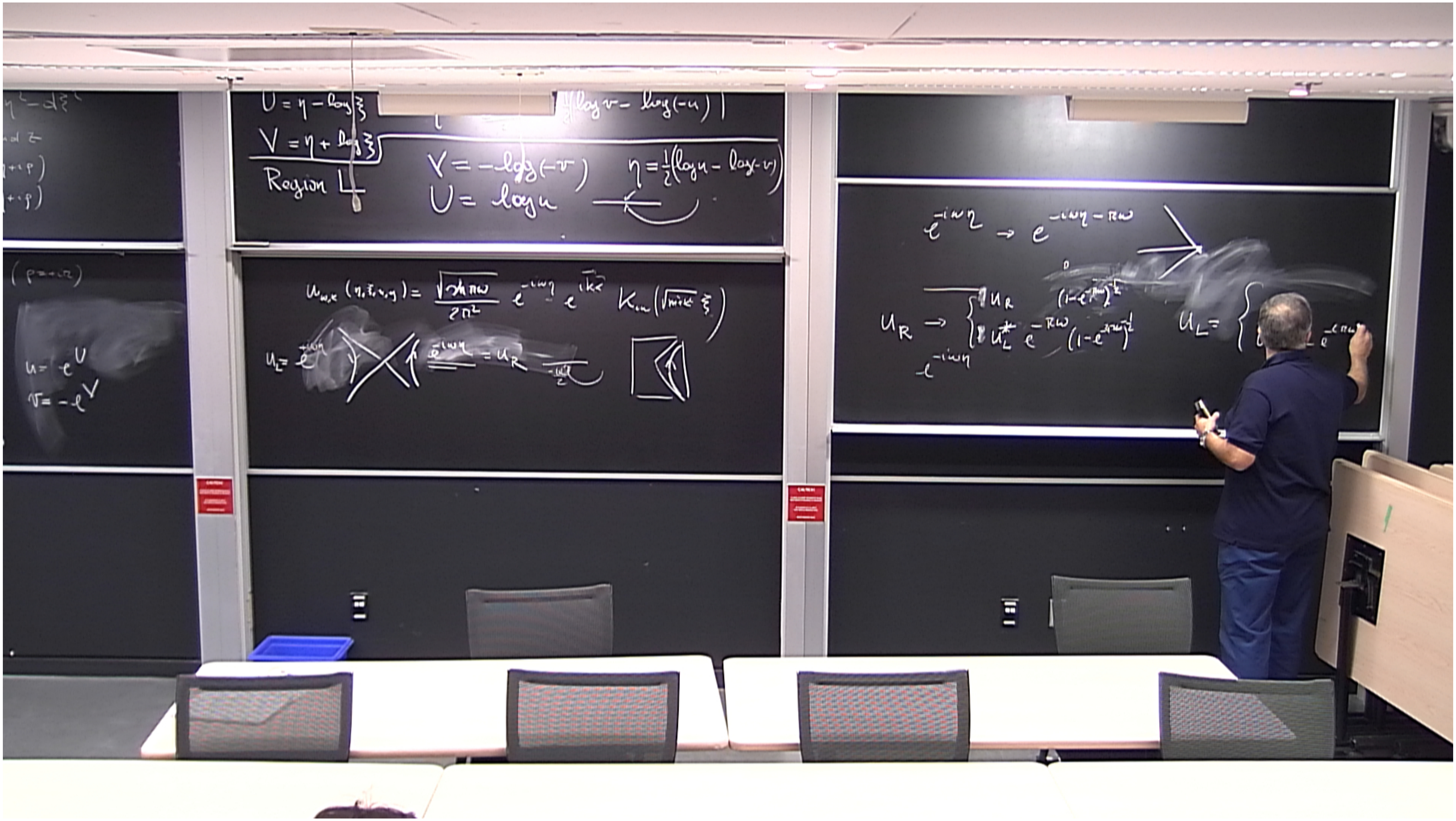

CAUTION

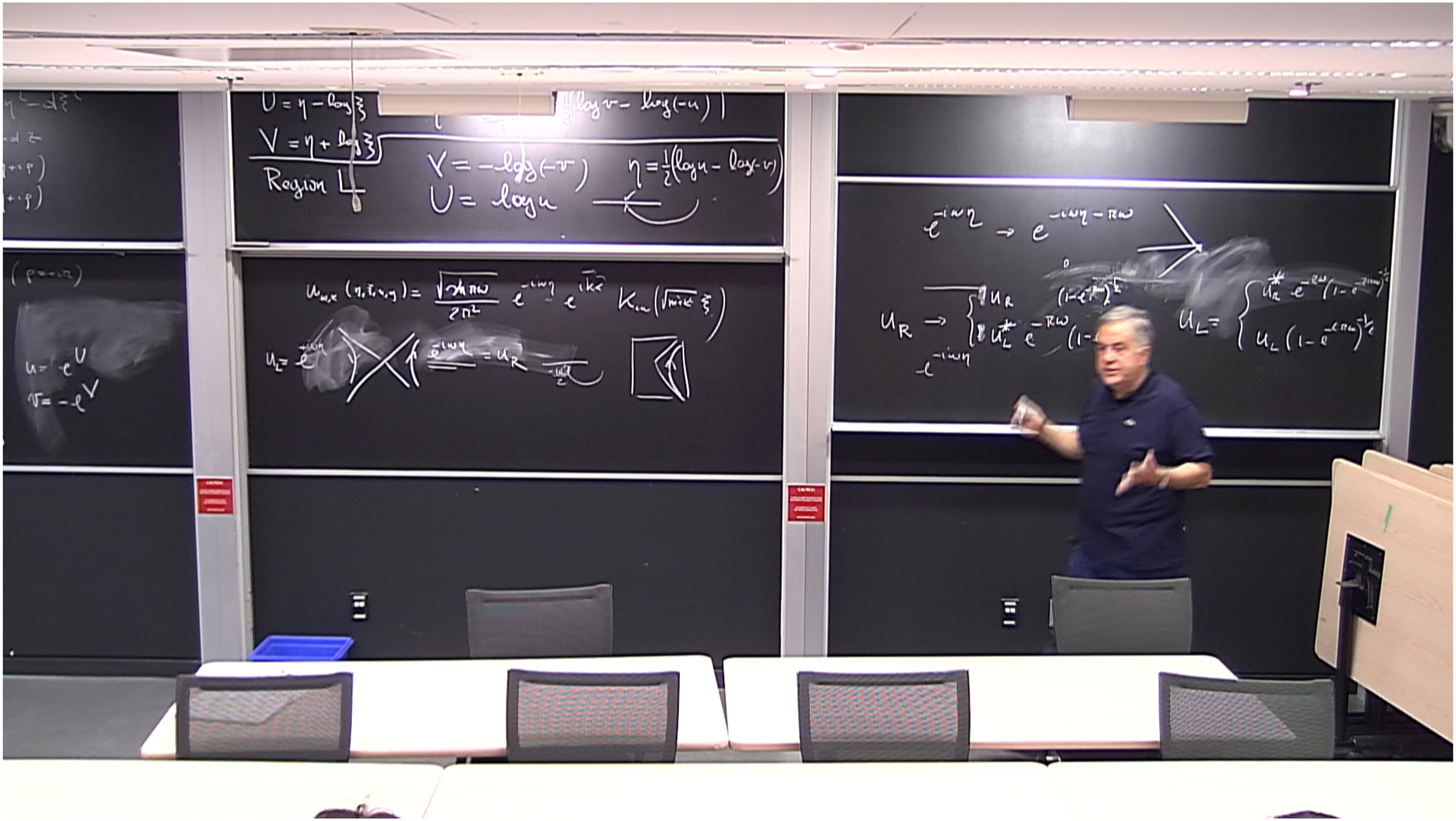

$e^{-i\omega\eta} \to e^{-i\omega\eta - \pi\omega}$
$u_R \to \{ u_R$
$K_{i\omega}(\sqrt{m^2+k^2}\,\xi)$
CAUTION

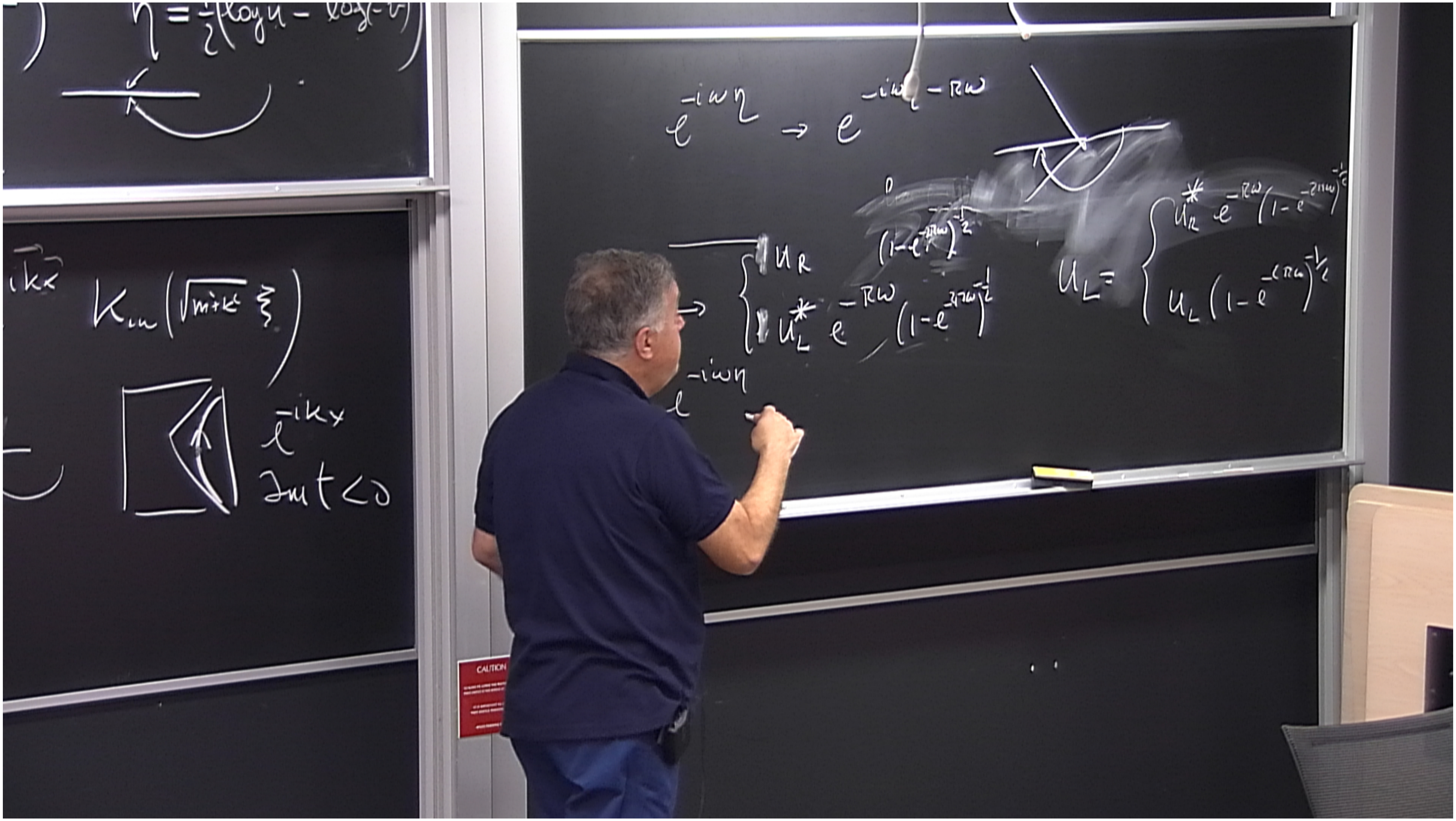


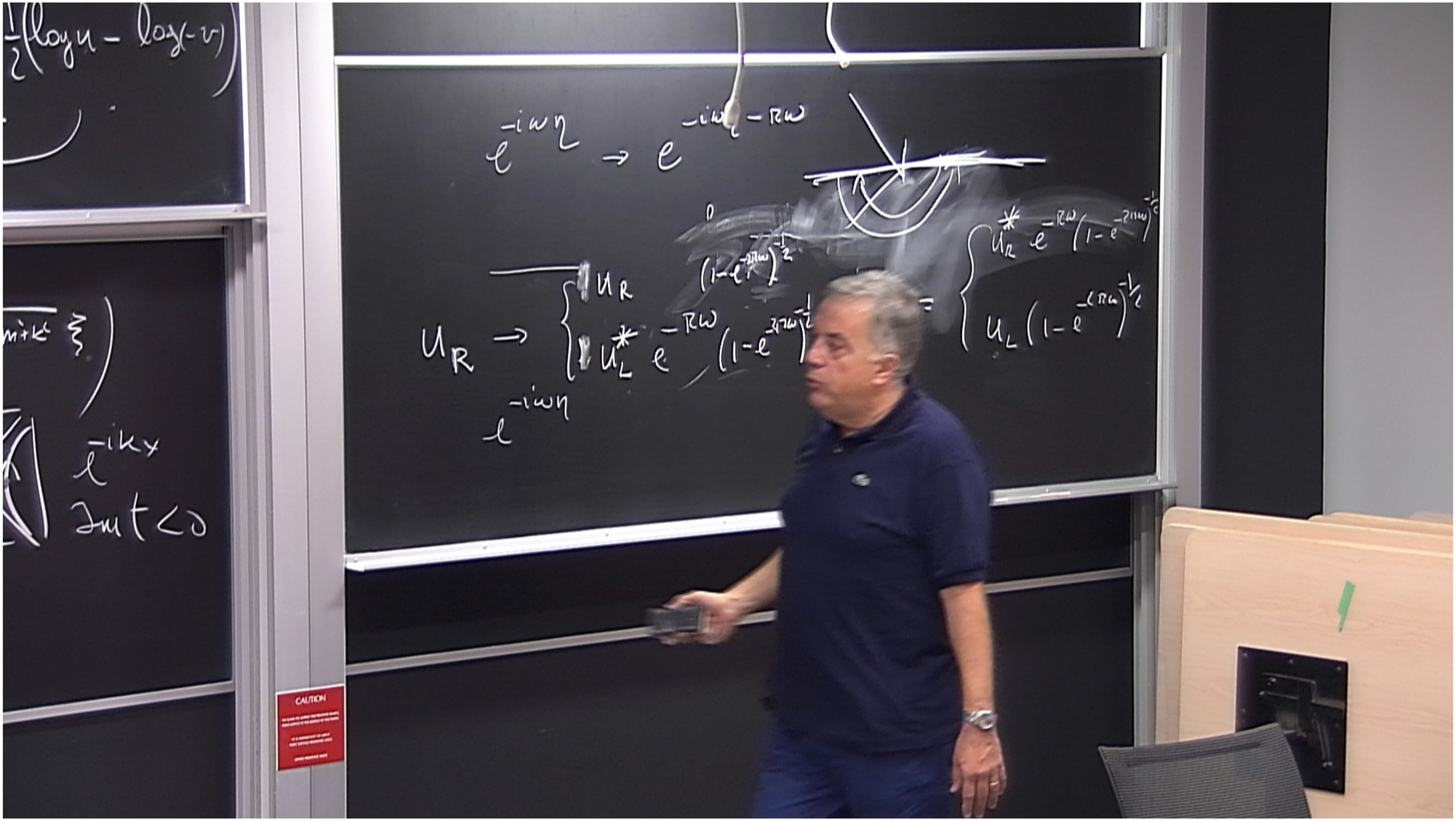








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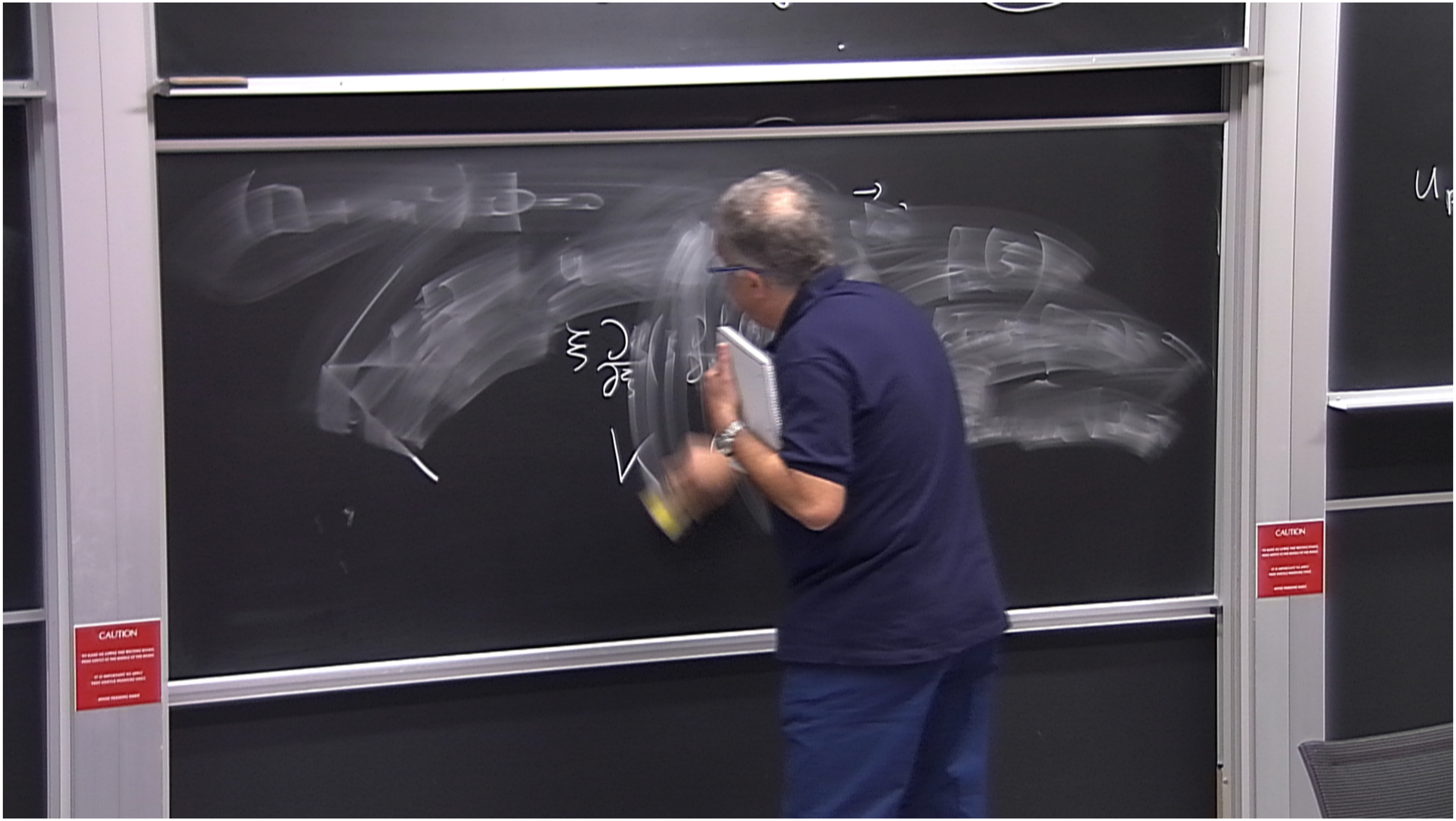
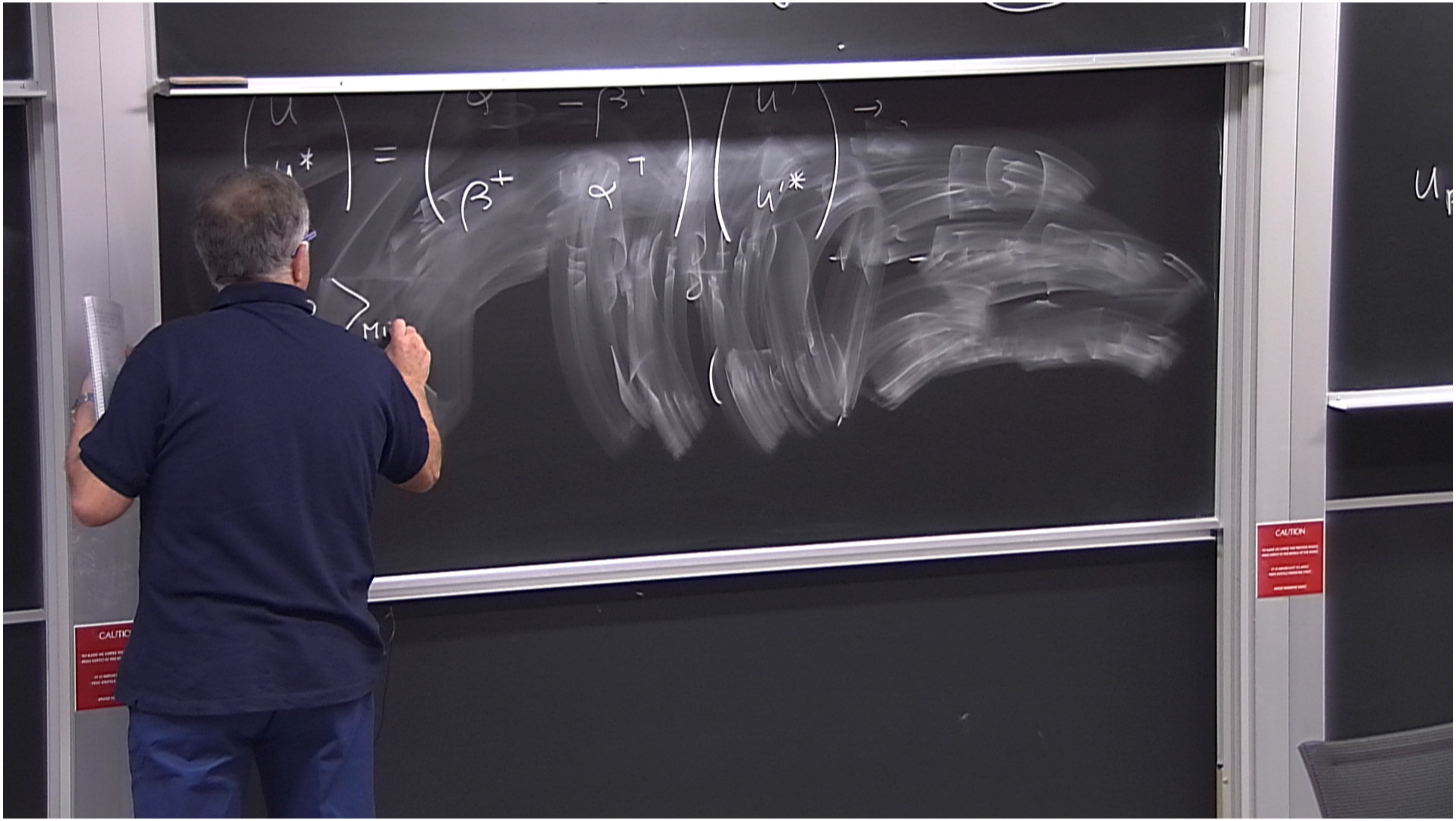
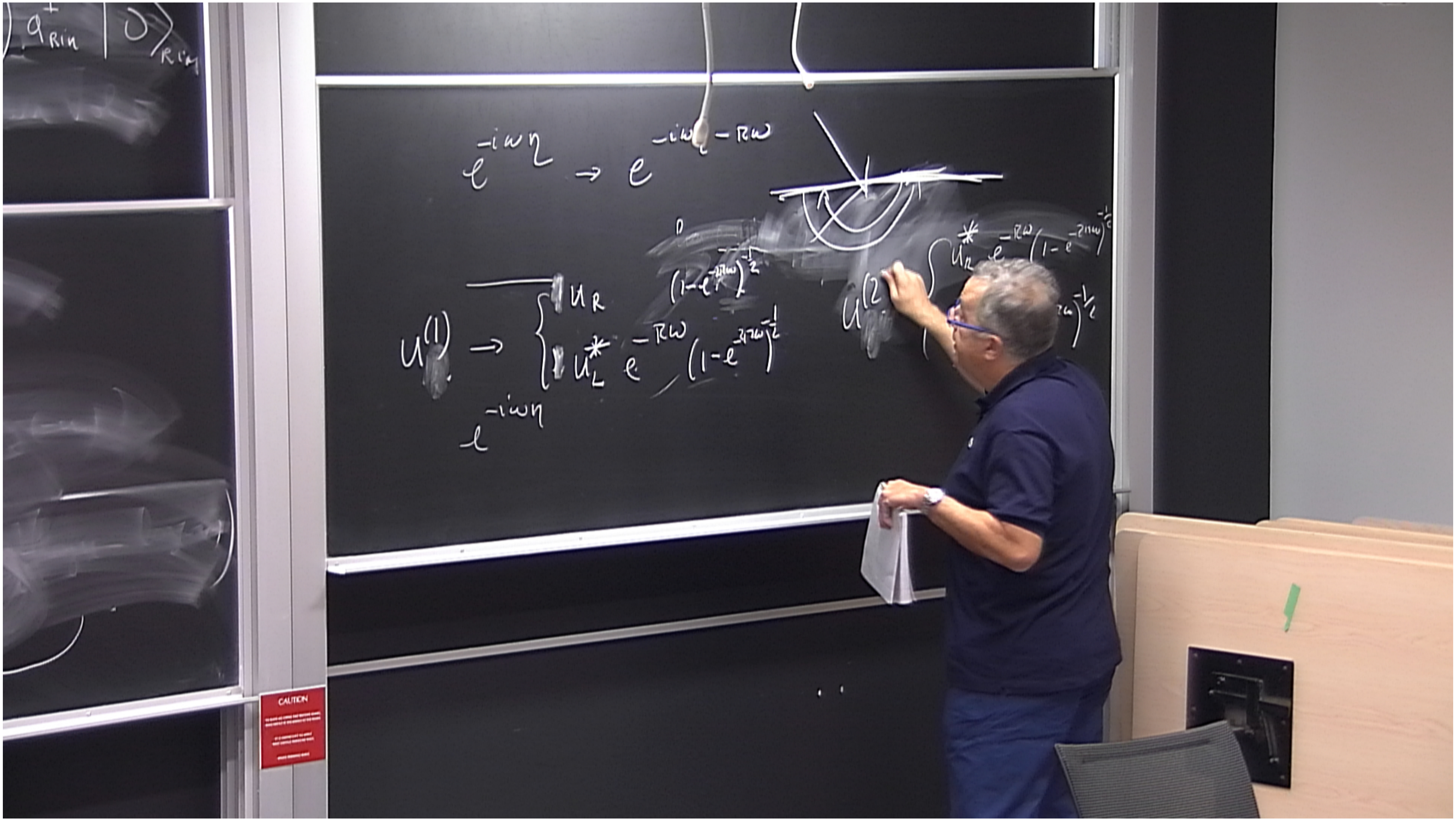
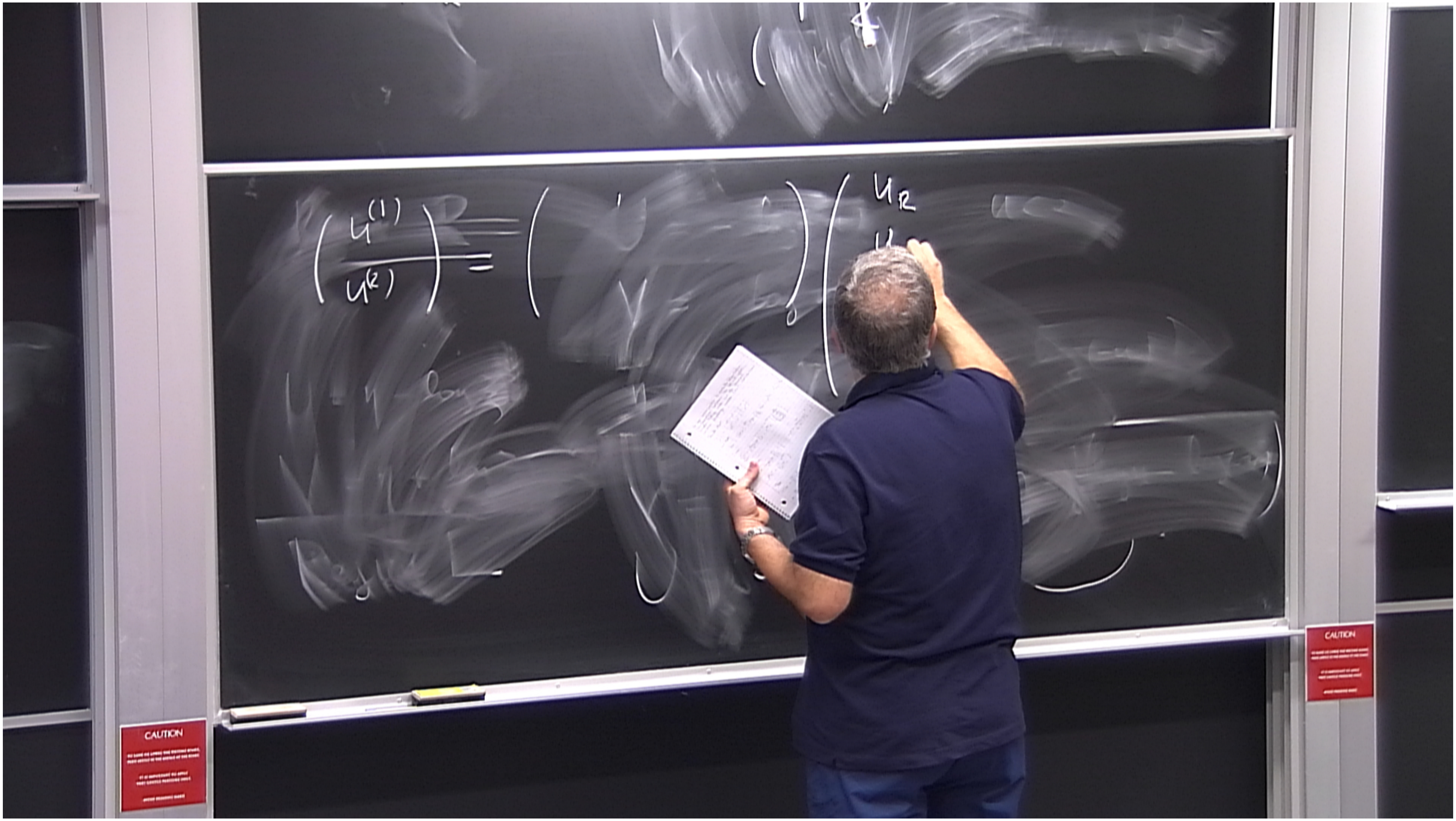
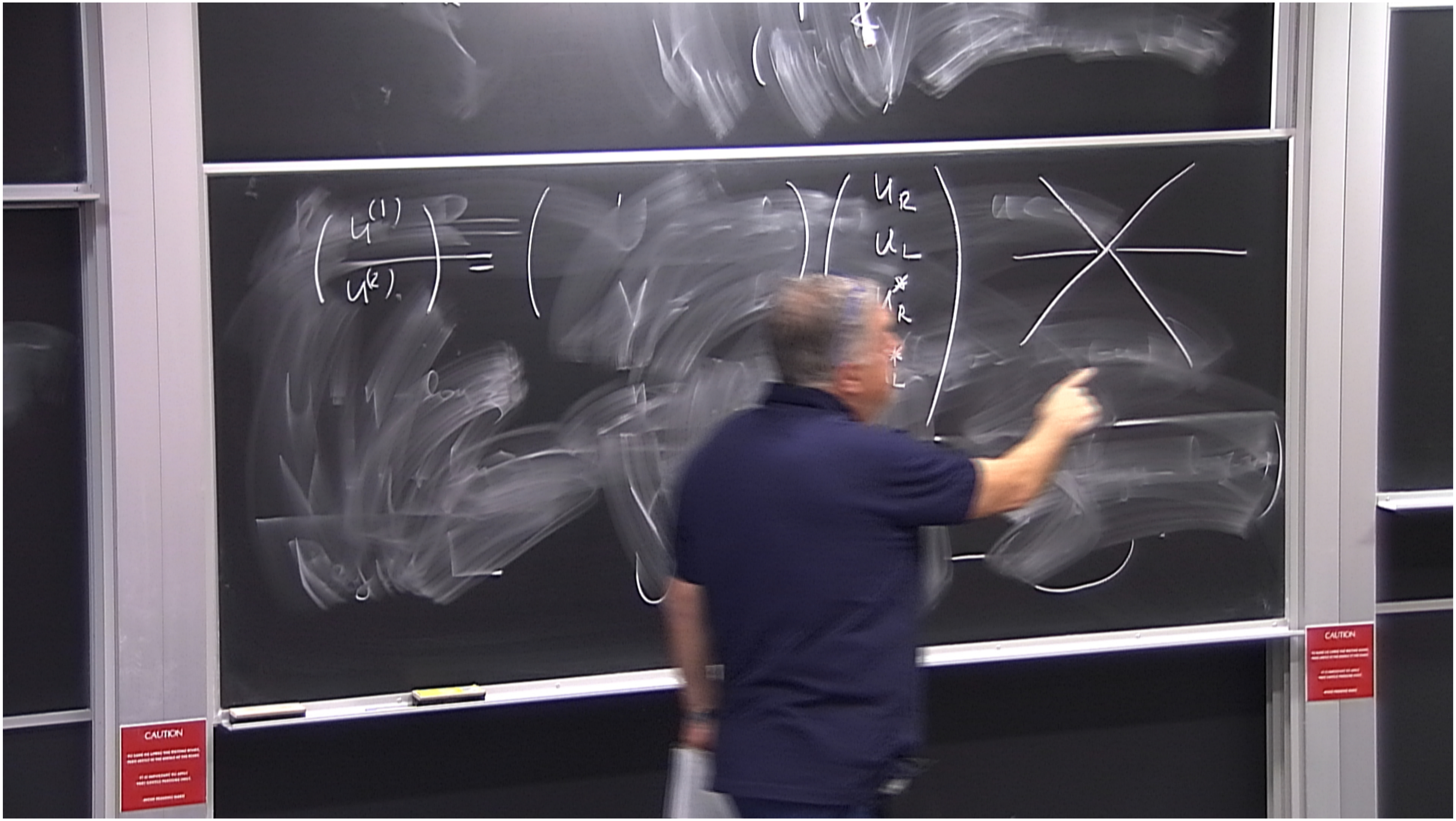



CAUTION




CAUTION
CAUTION



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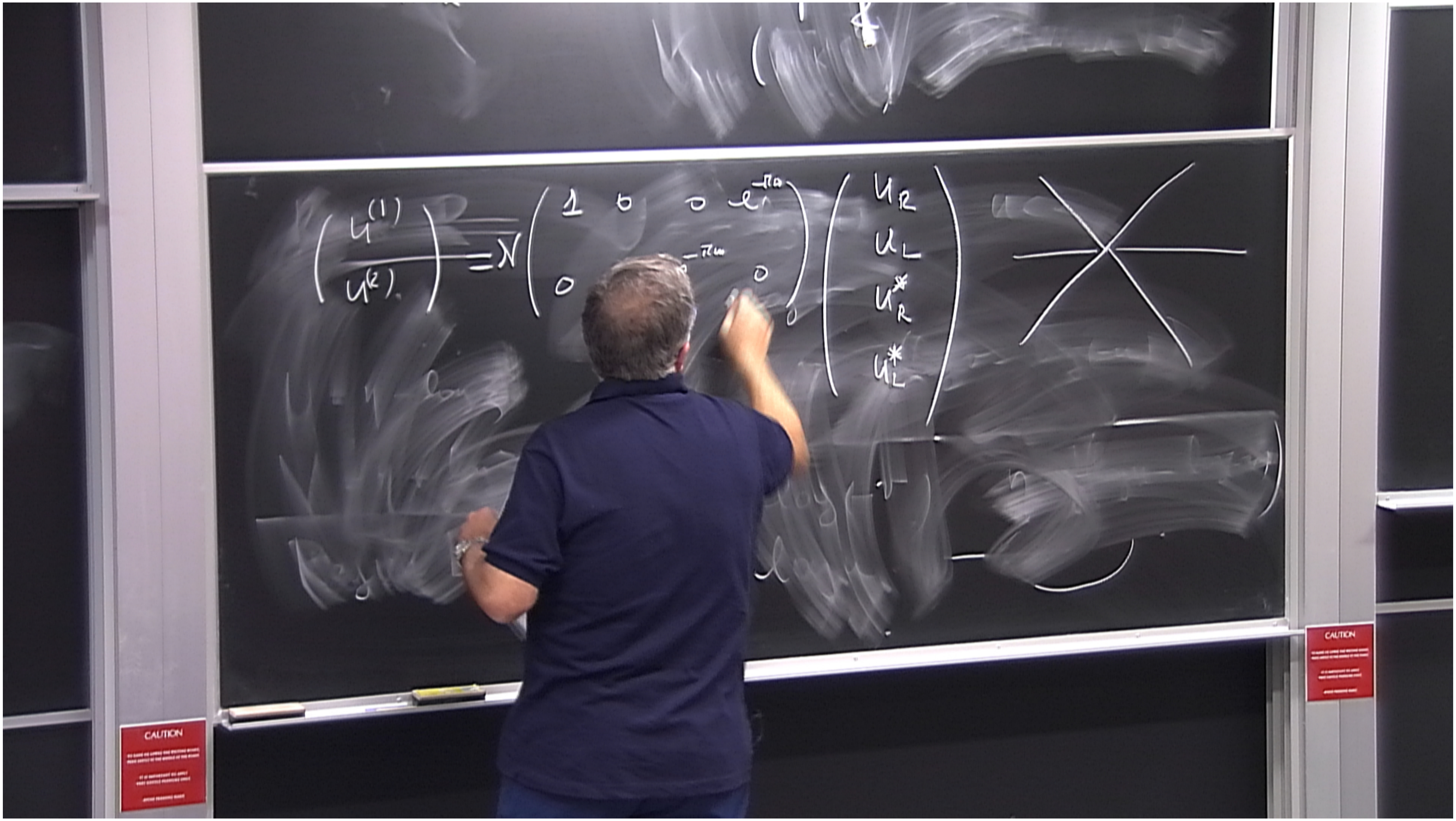
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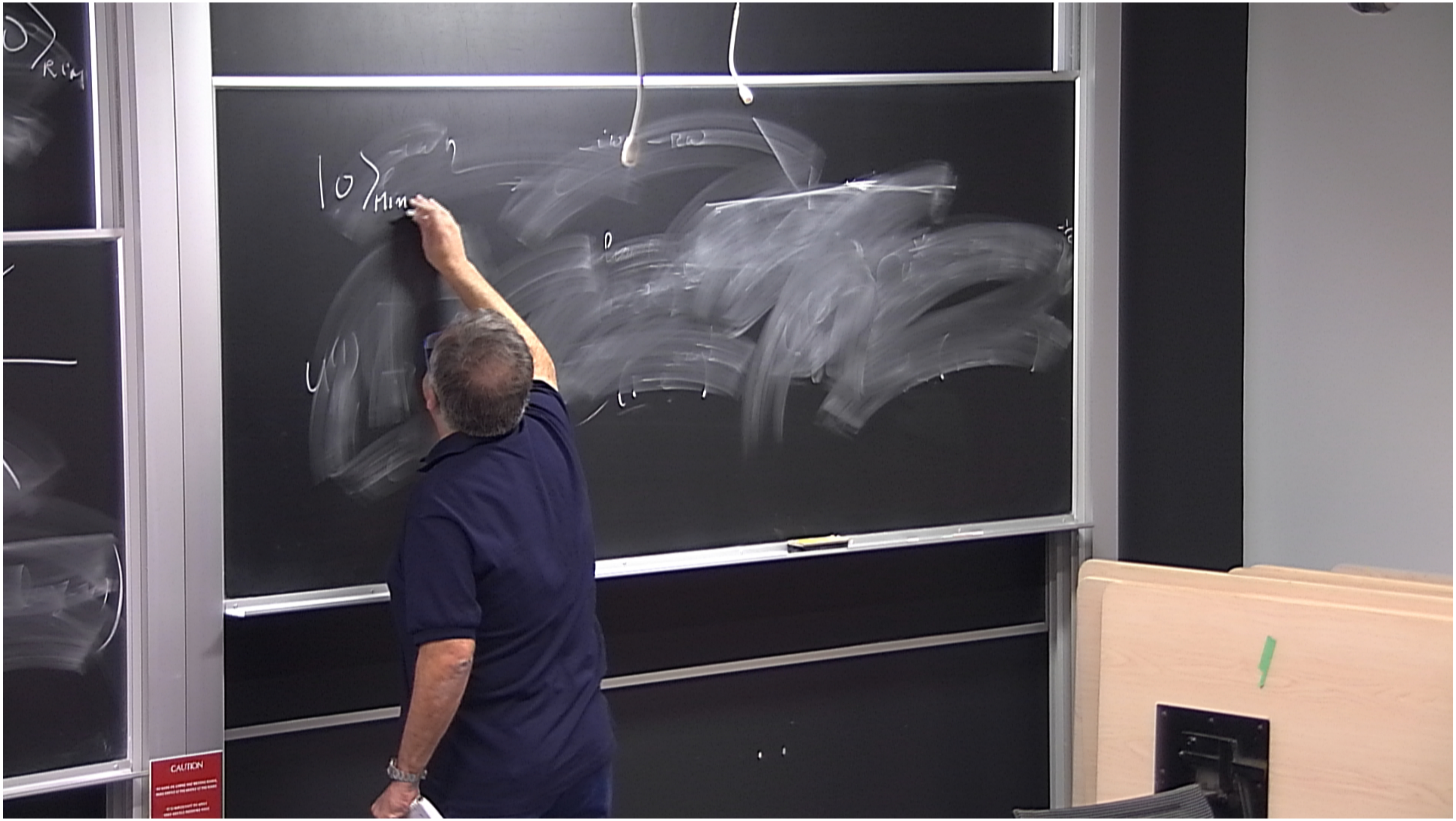
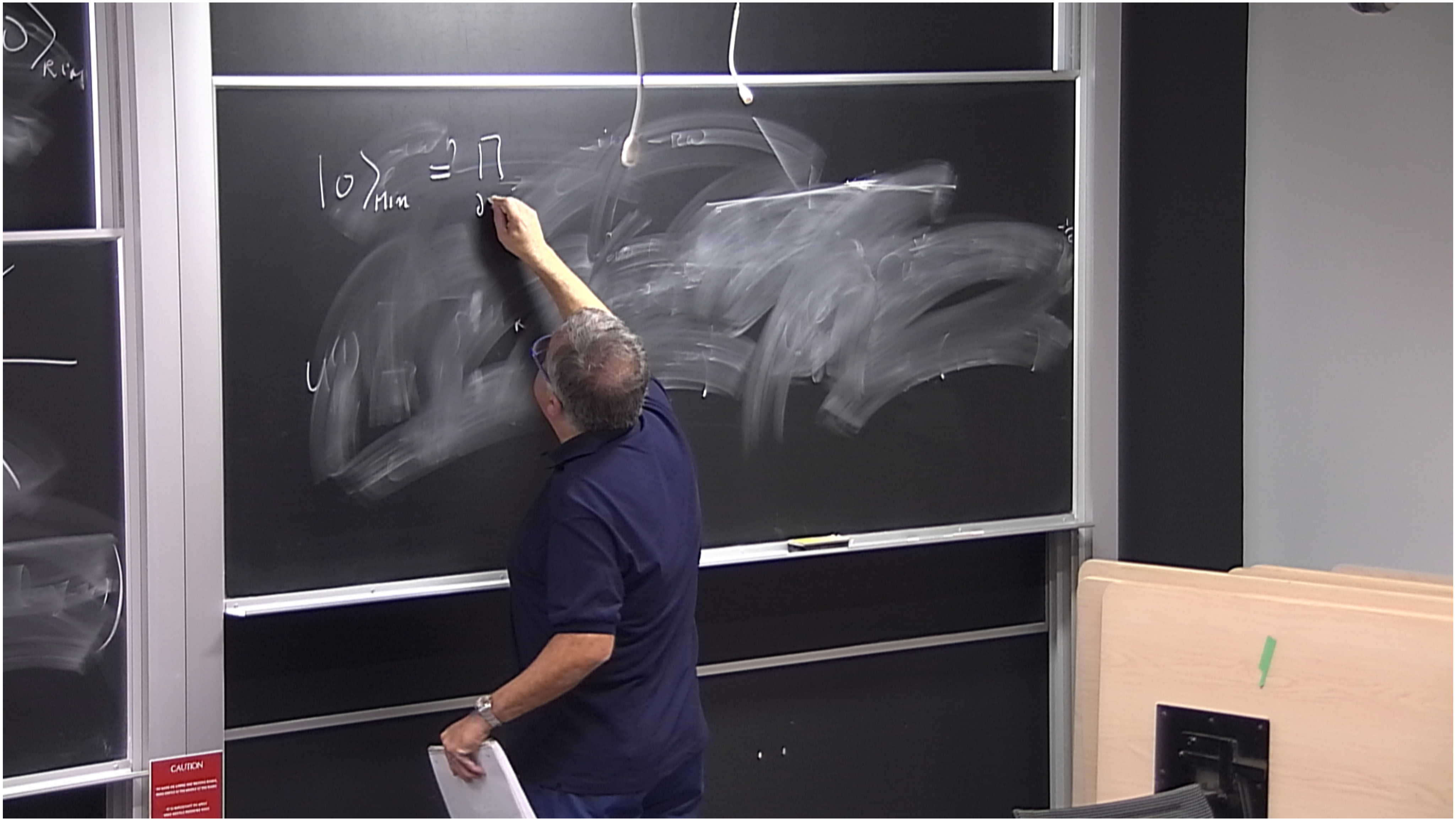
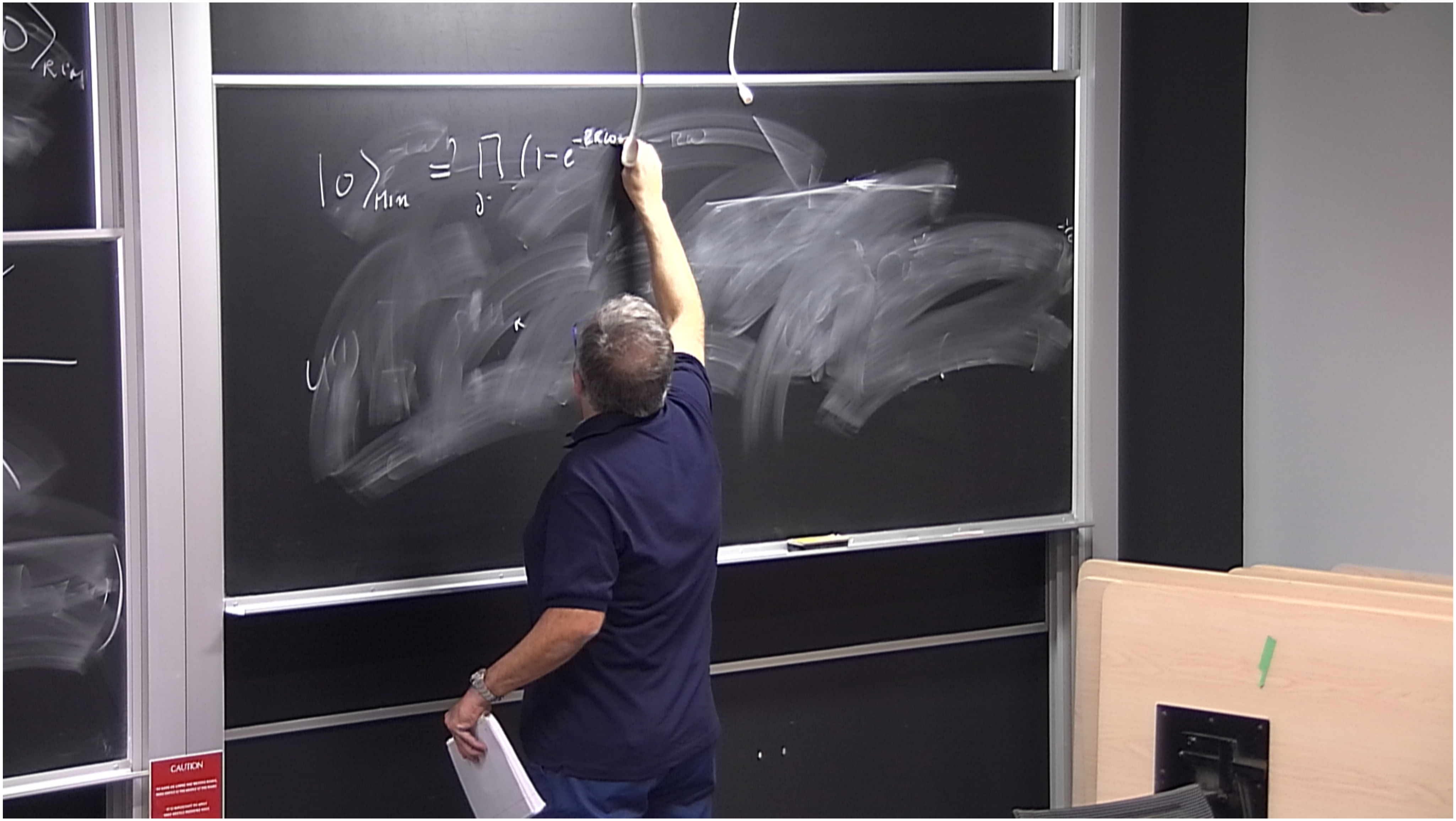
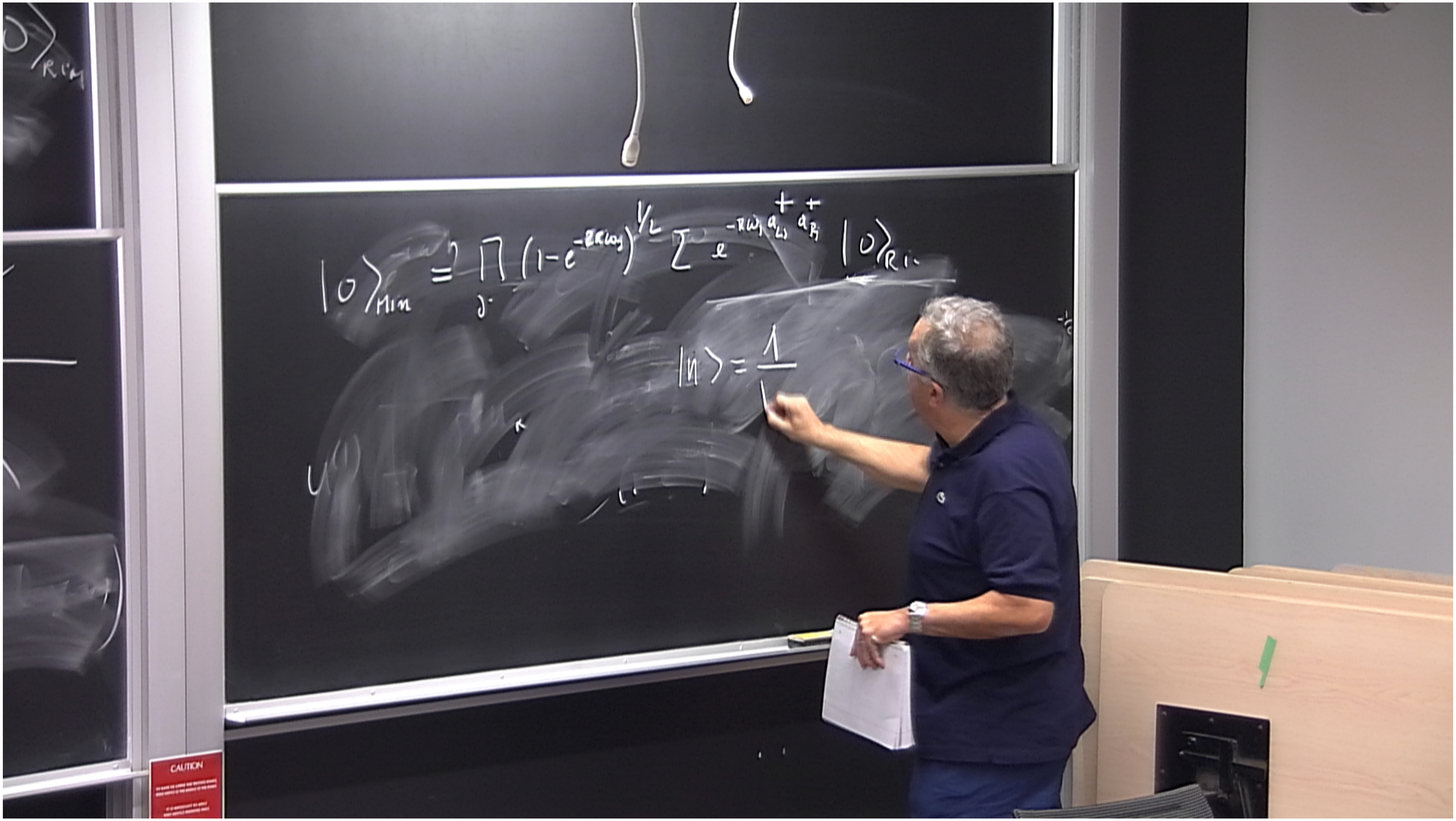
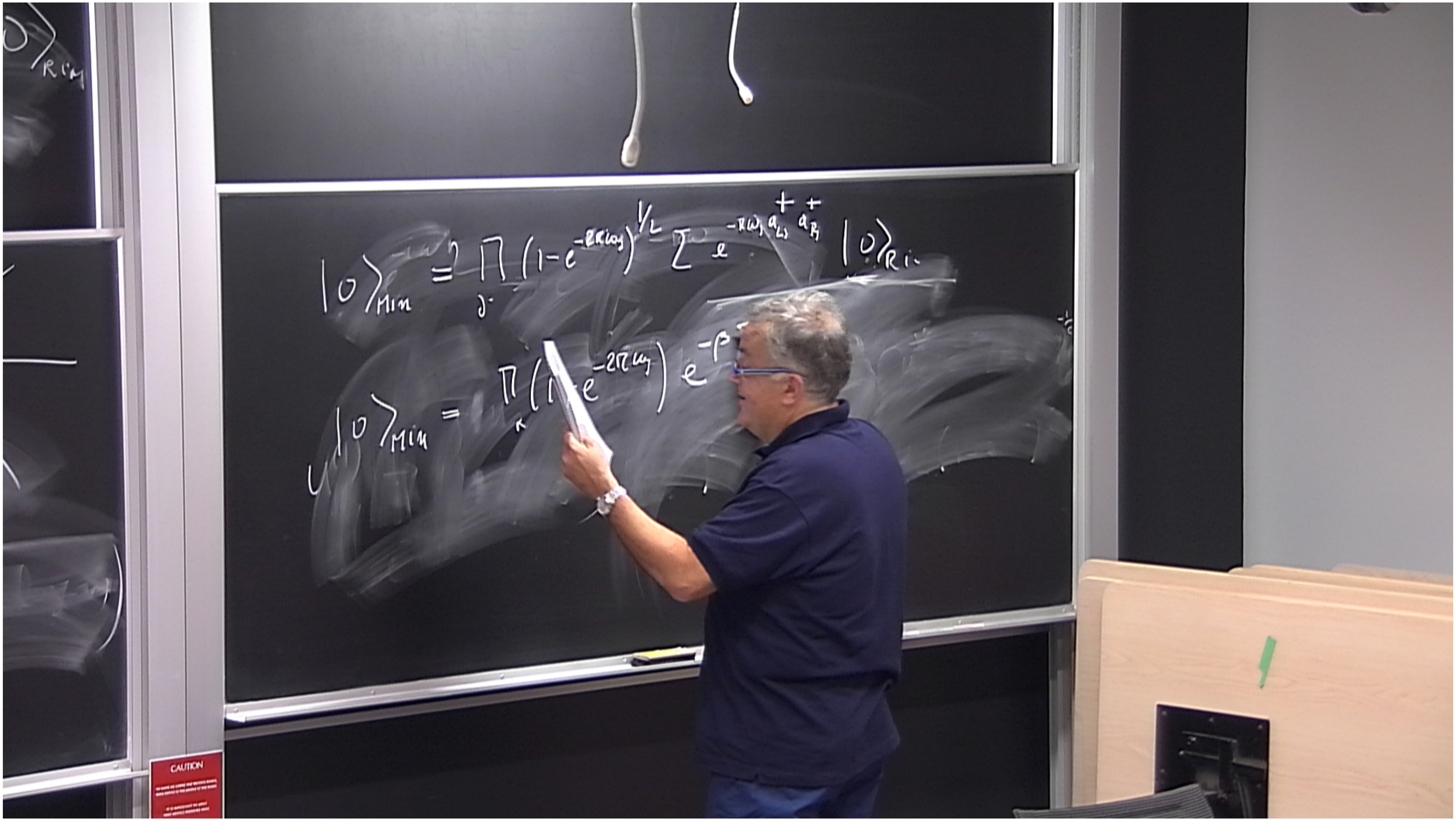
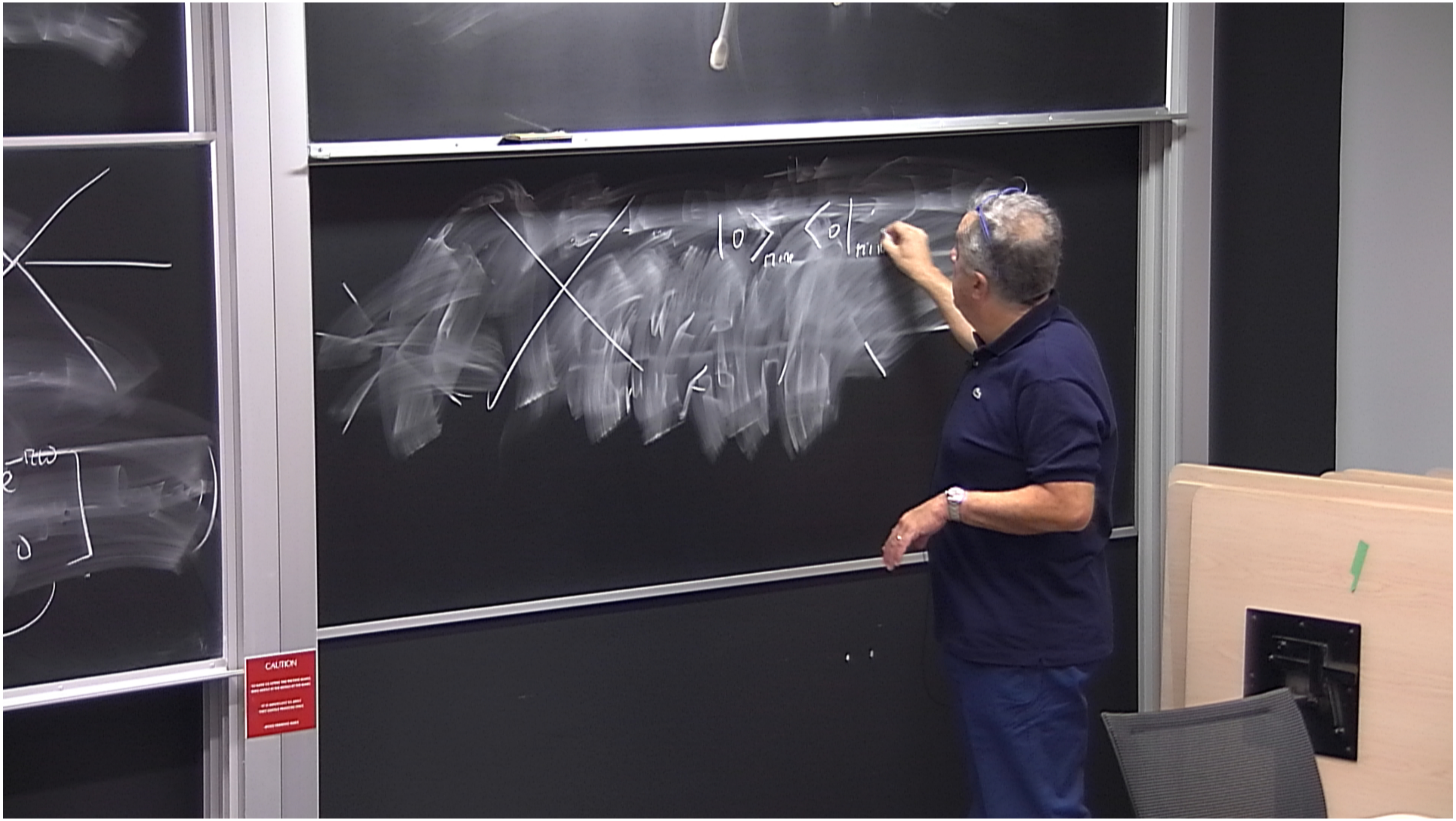

|0⟩Min
CAUTION

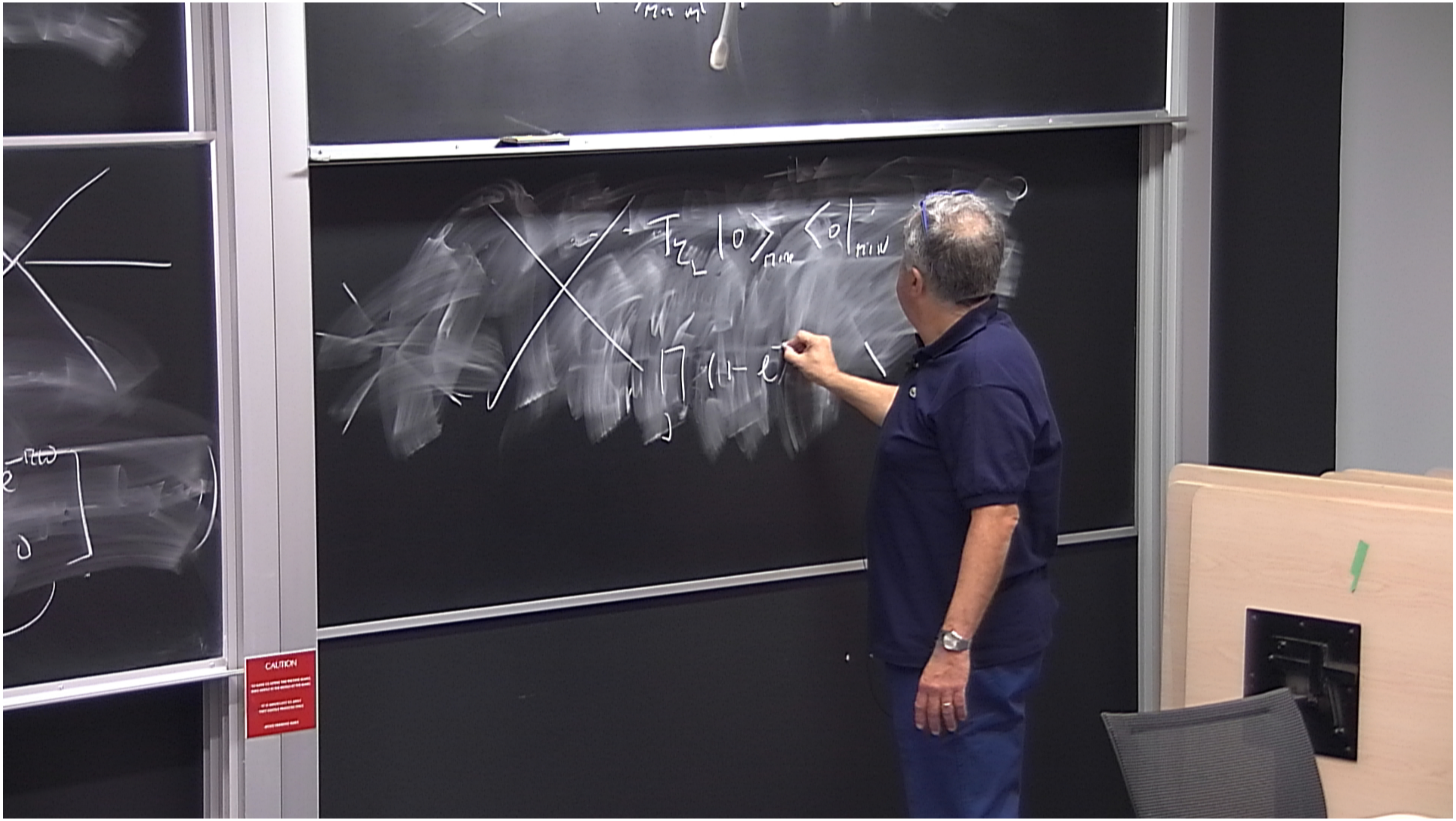

CAUTION

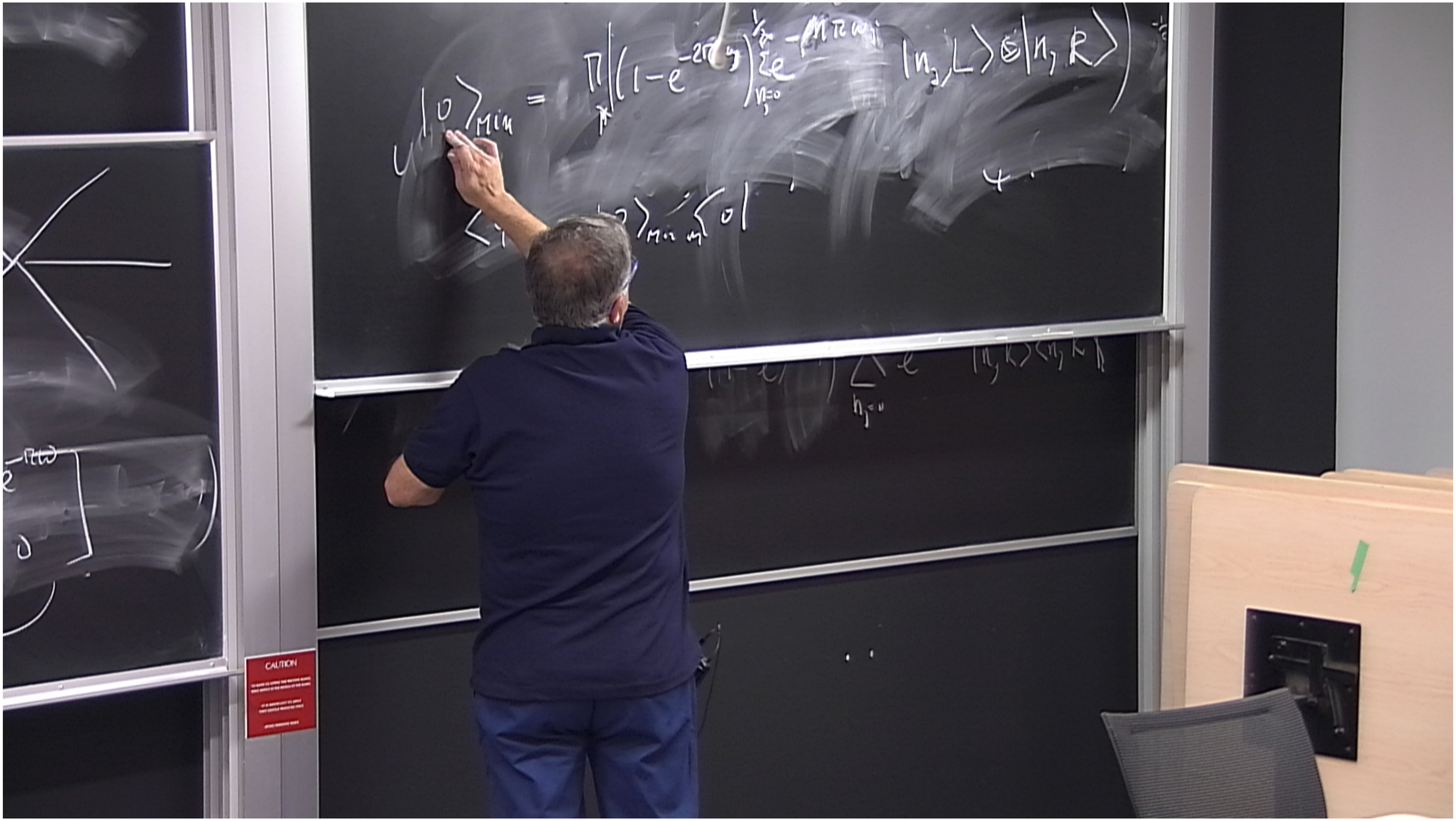

CAUTION

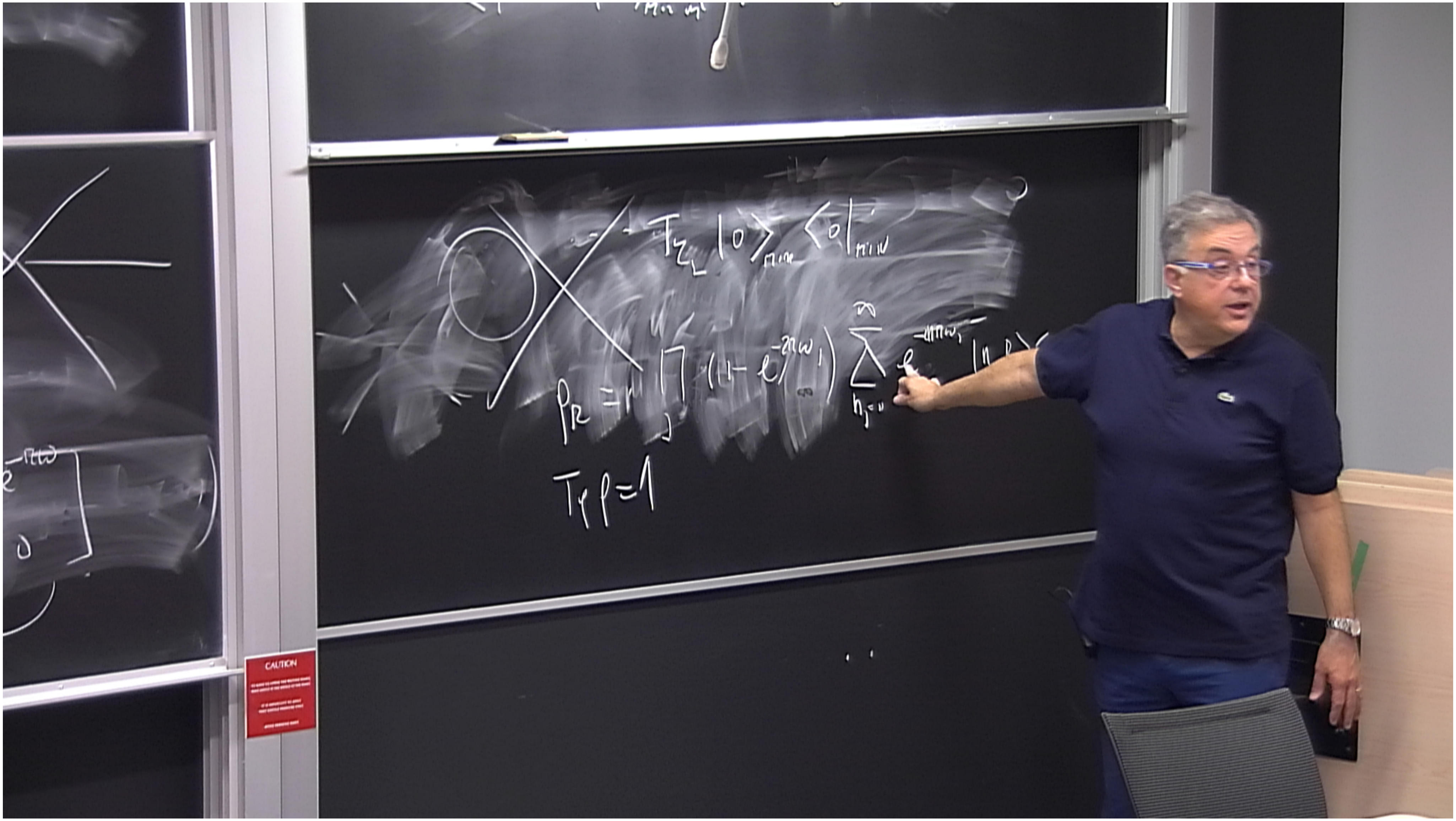



CAUTION

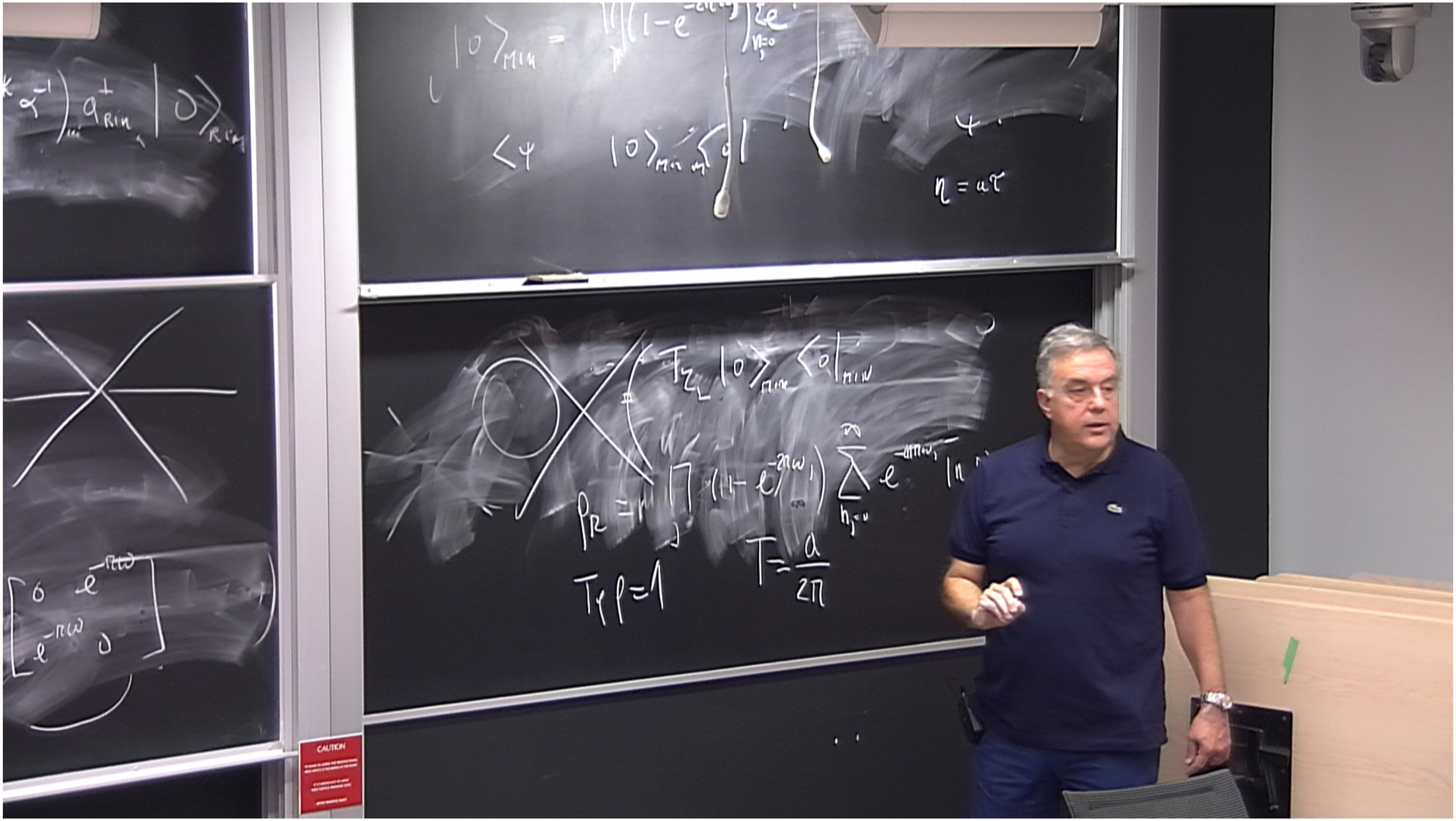

CAUTION

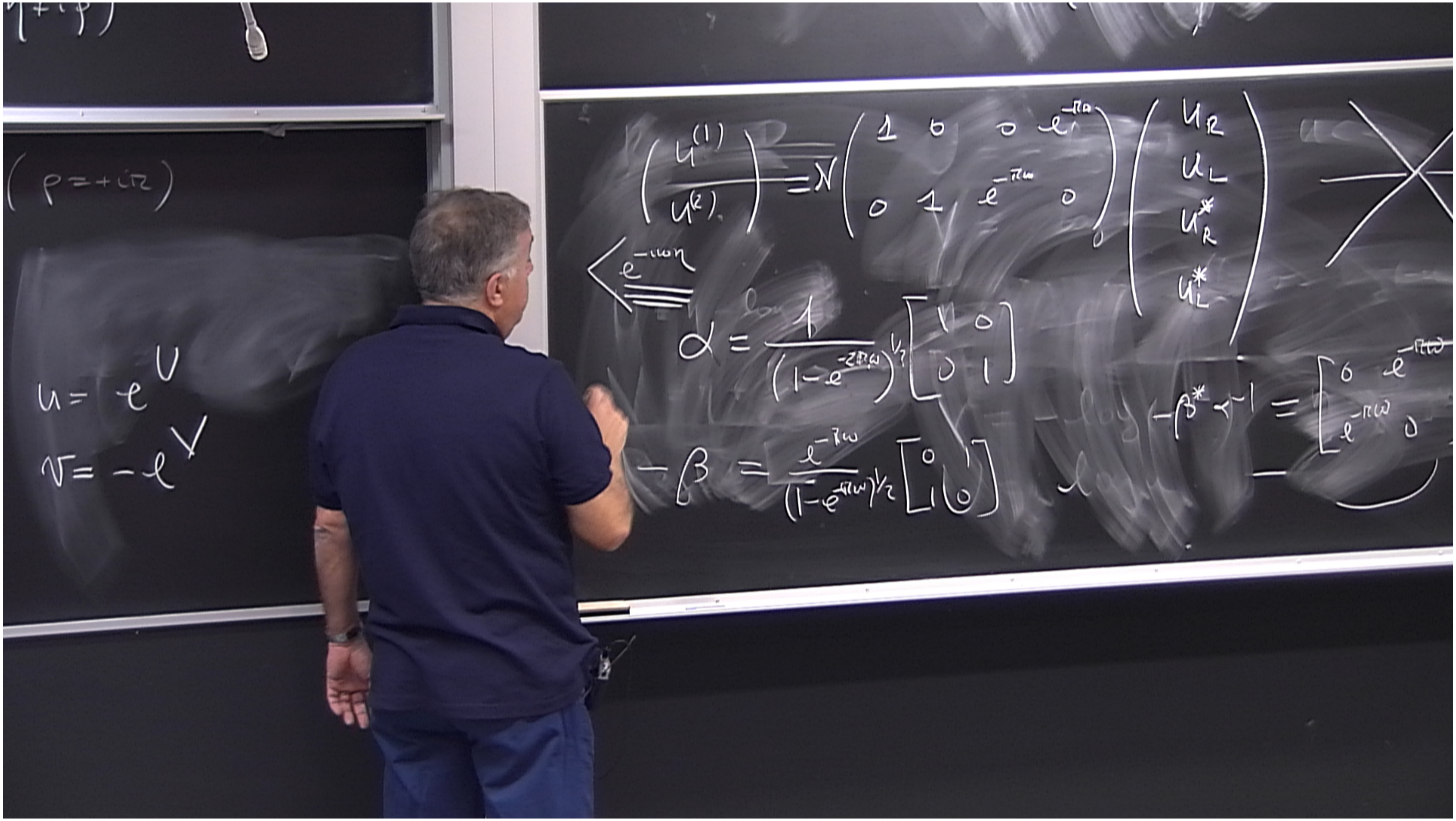

CAUTION

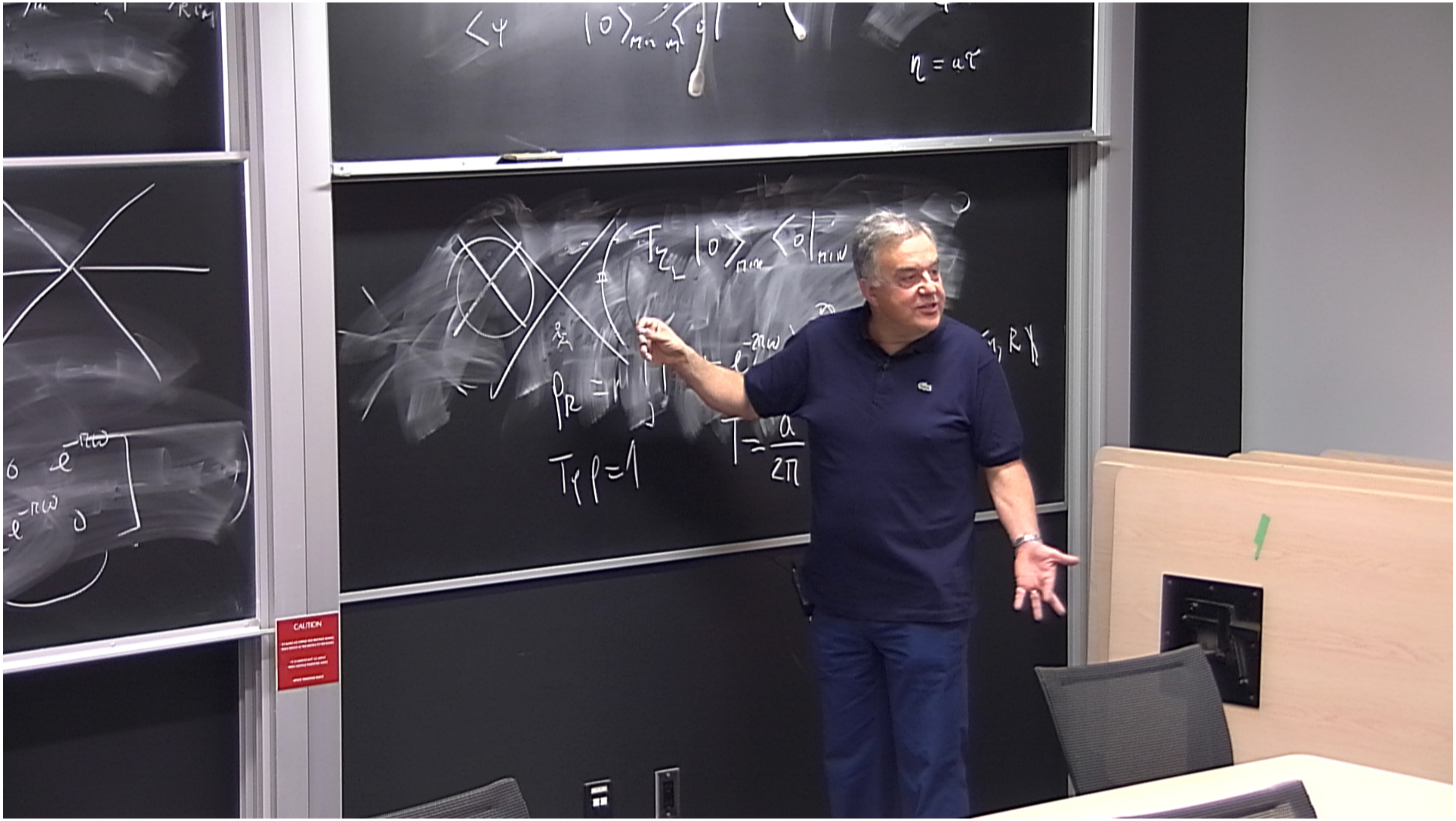

CAUTION

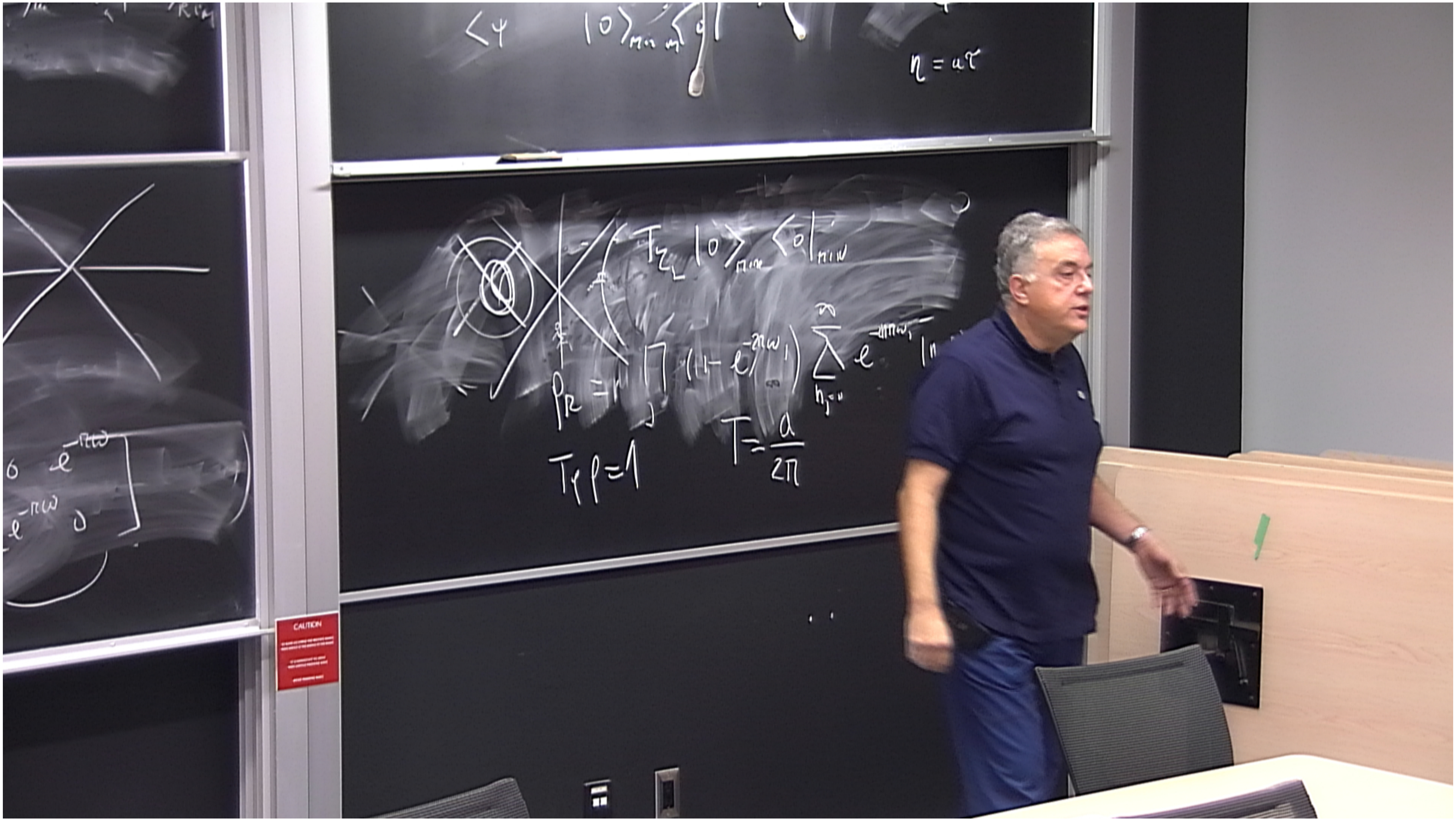


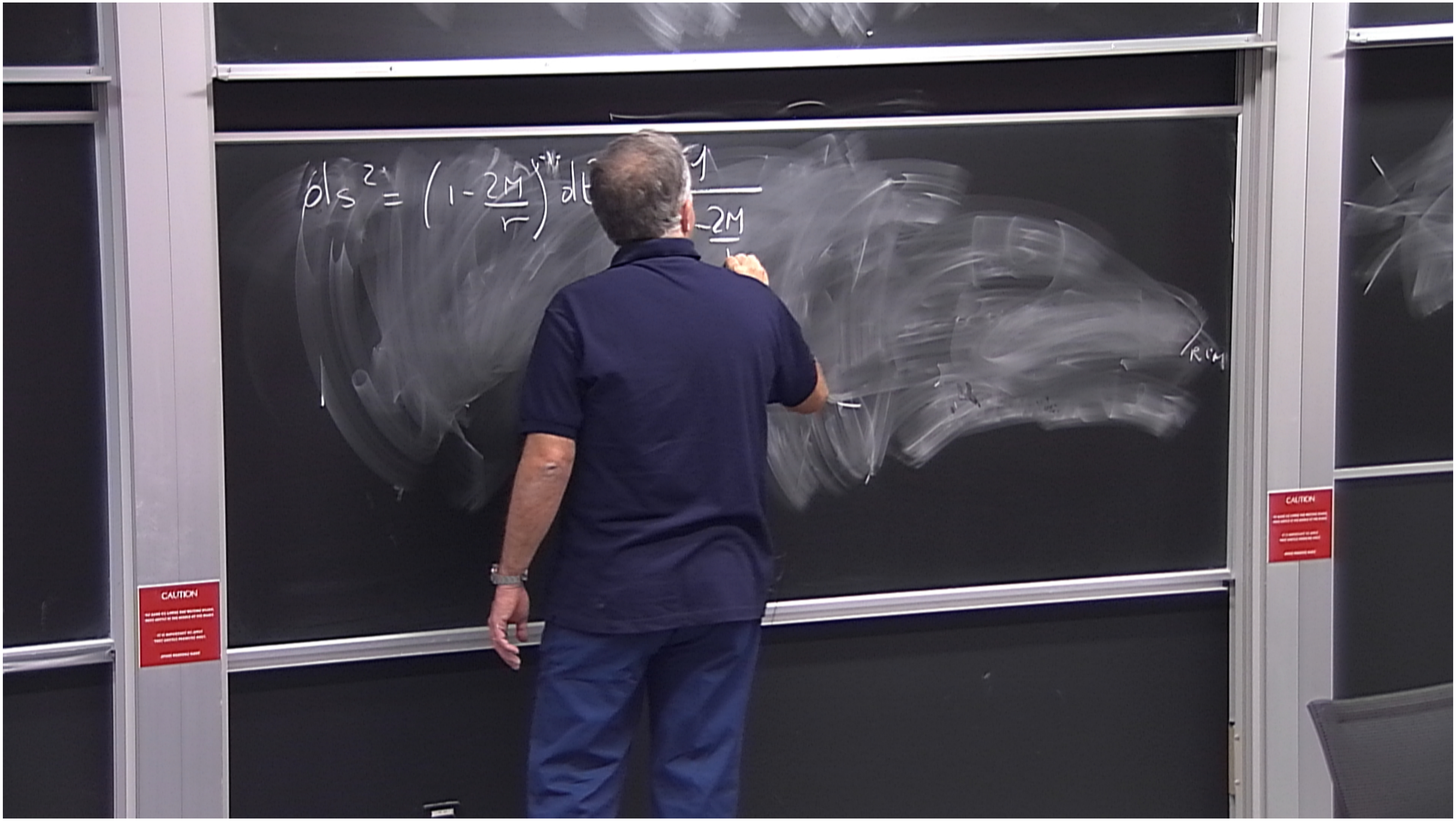
CAUTION

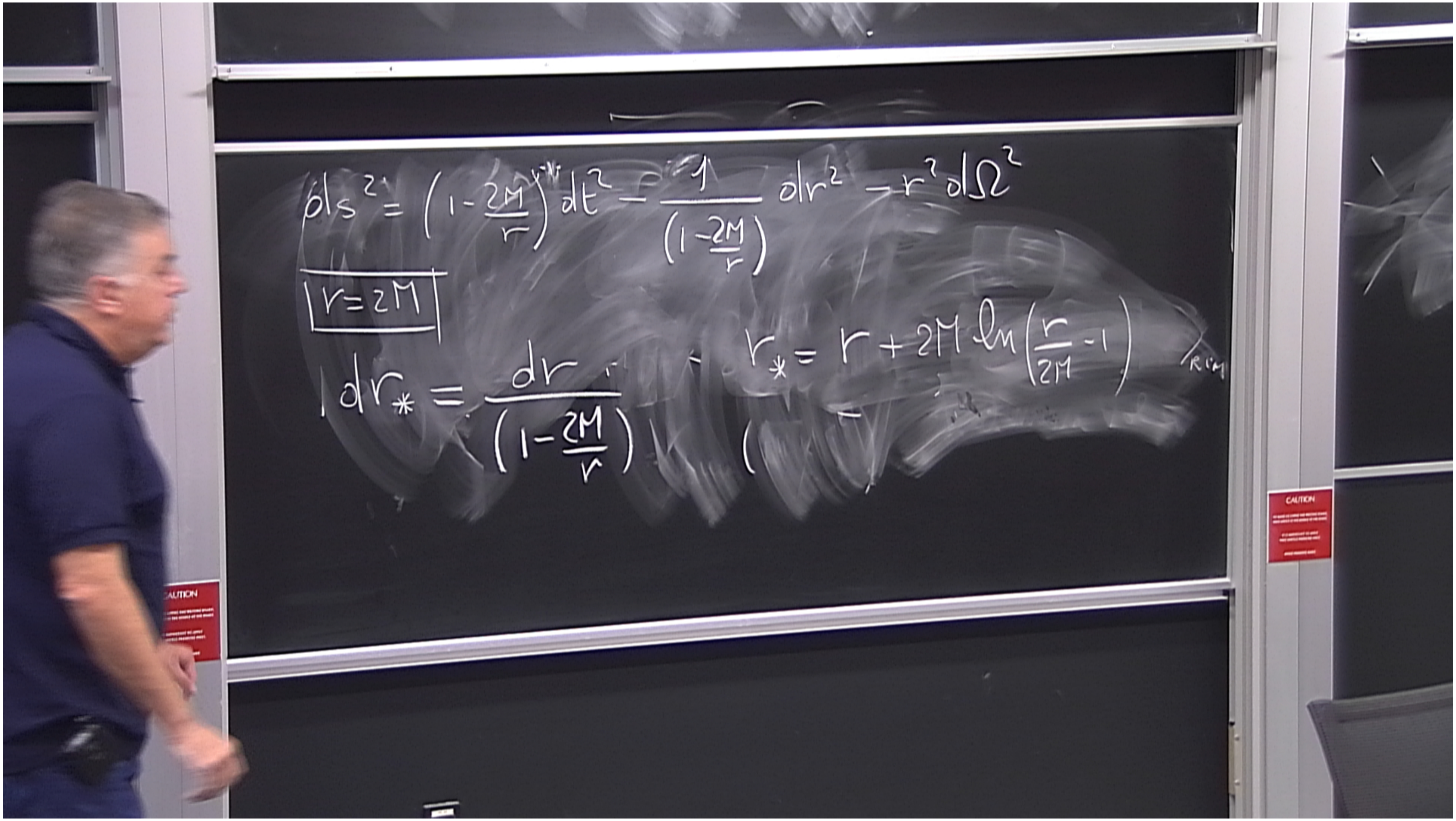


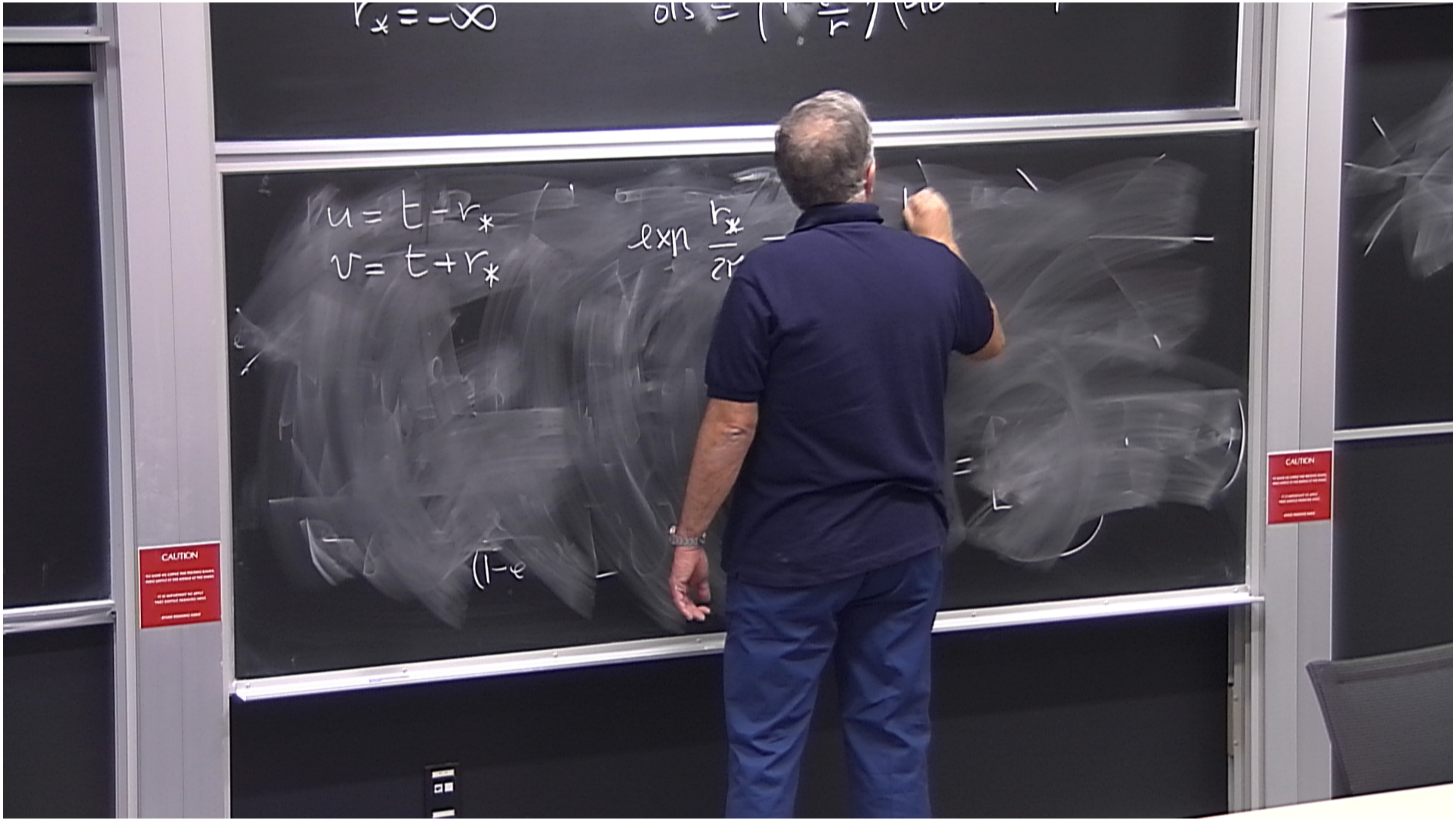

(ρ = +iz)
u = e^U
v = -e^V
α = 1/(1 - e^{-2πω})^{1/2} [1 0; 0 1]
-β = e^{-πω}/(1 - e^{-2πω})^{1/2} [0 1; 1 0]

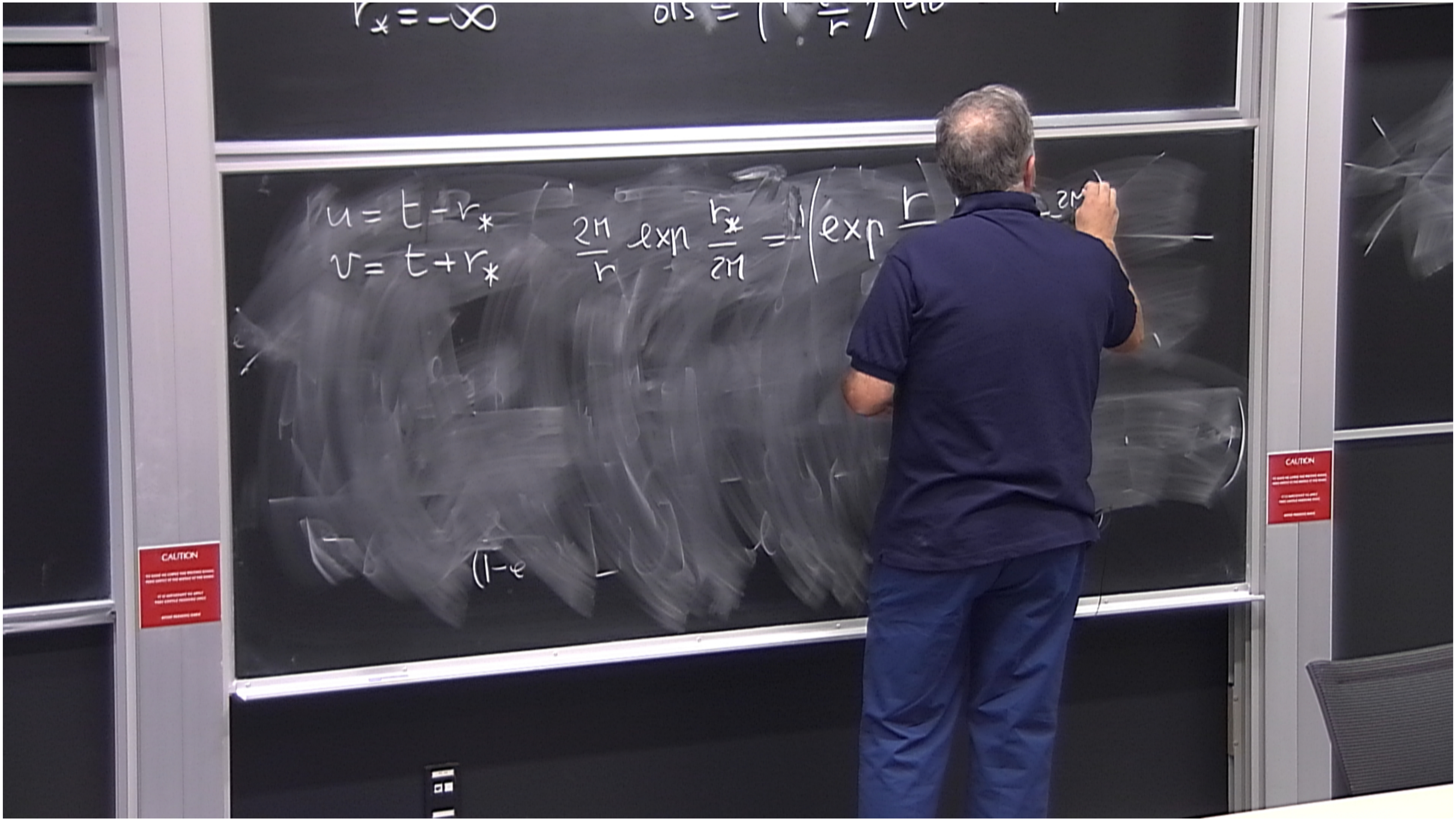

$\eta = a\tau$
$T_{\Sigma_L} |0\rangle_{MIN} \langle 0|_{MIN}$
$\rho_R =$
$T_\Sigma \rho = 1$
$T = \frac{a}{2\pi}$
CAUTION

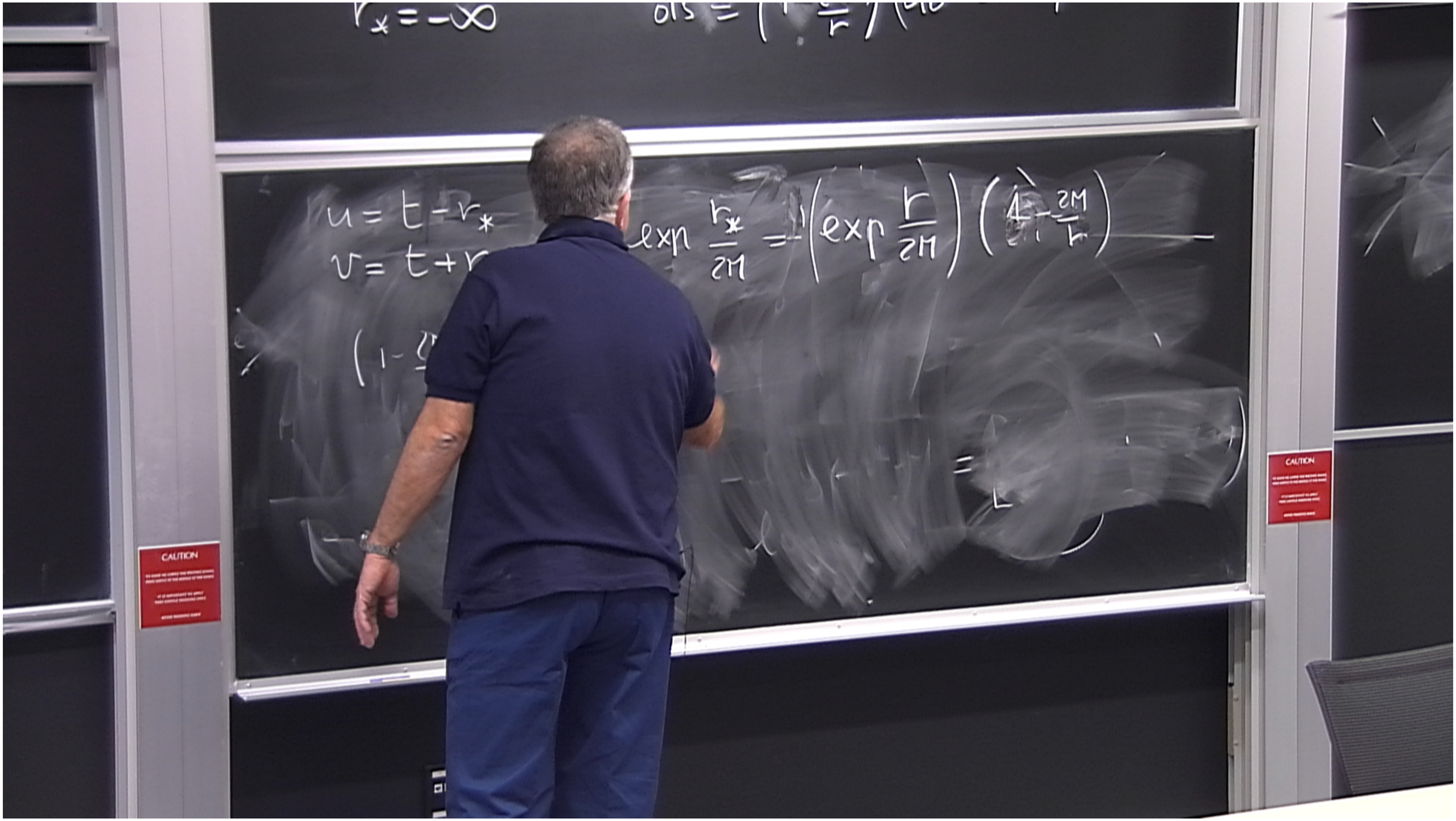

η = uτ
Tr ρ = 1
T = a/2π
CAUTION

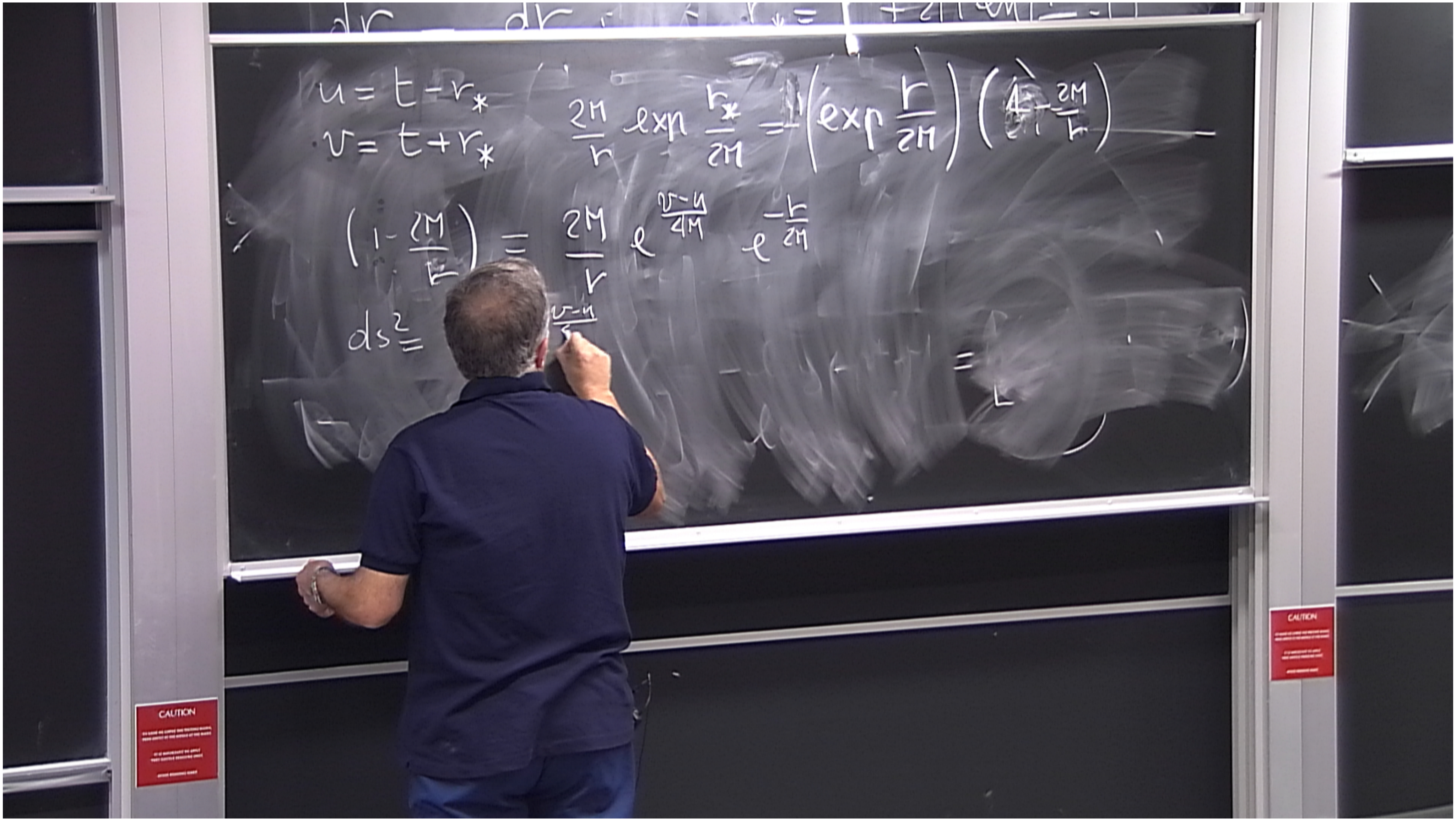


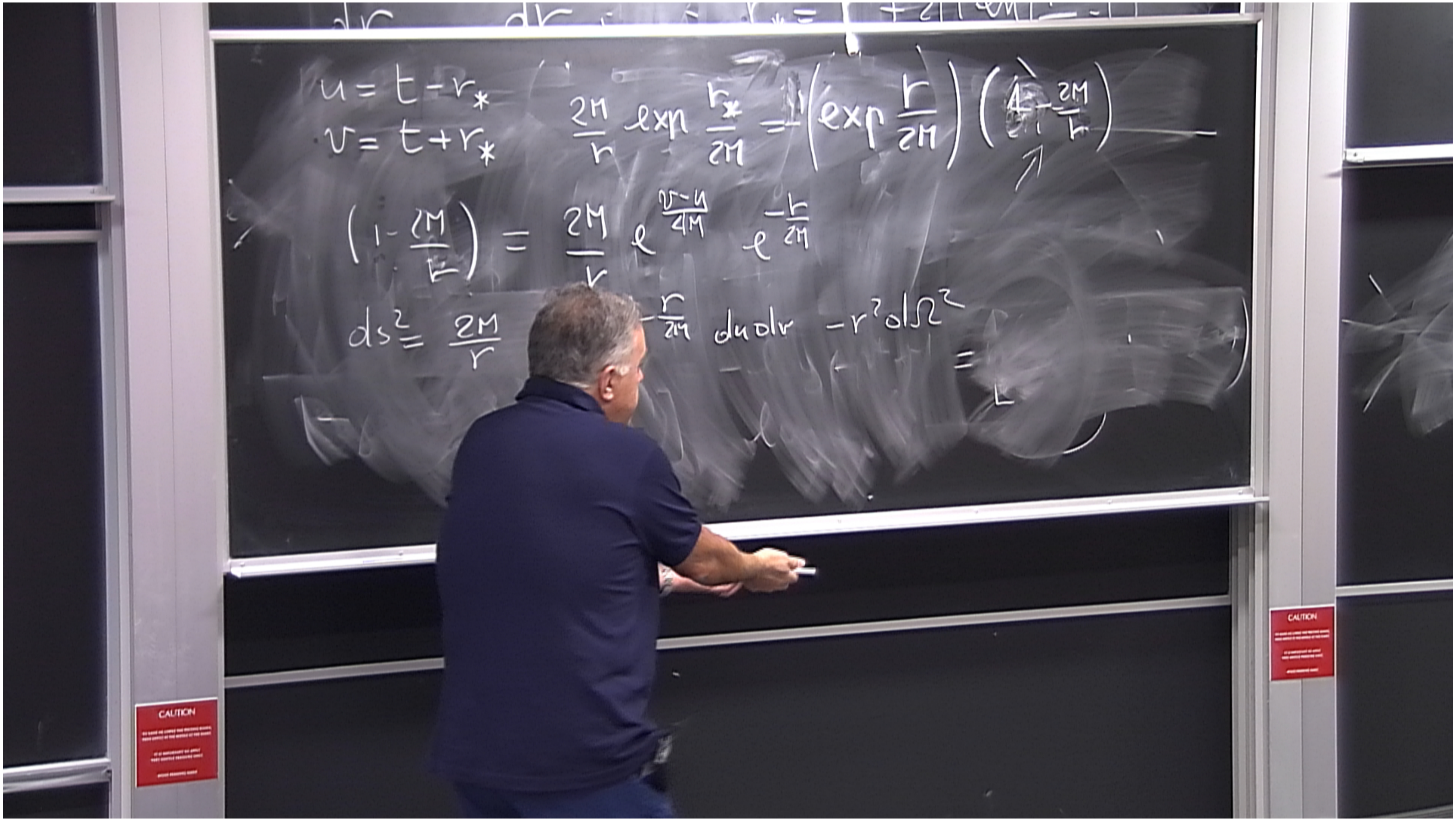

$ds^2 = \left(1-\frac{2M}{r}\right) dt^2 - \frac{1}{\left(1-\frac{2M}{r}\right)} dr^2 - r^2 d\Omega^2$
$r = 2M$
$dr_* = \frac{dr}{\left(1-\frac{2M}{r}\right)}$
$r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$
CAUTION

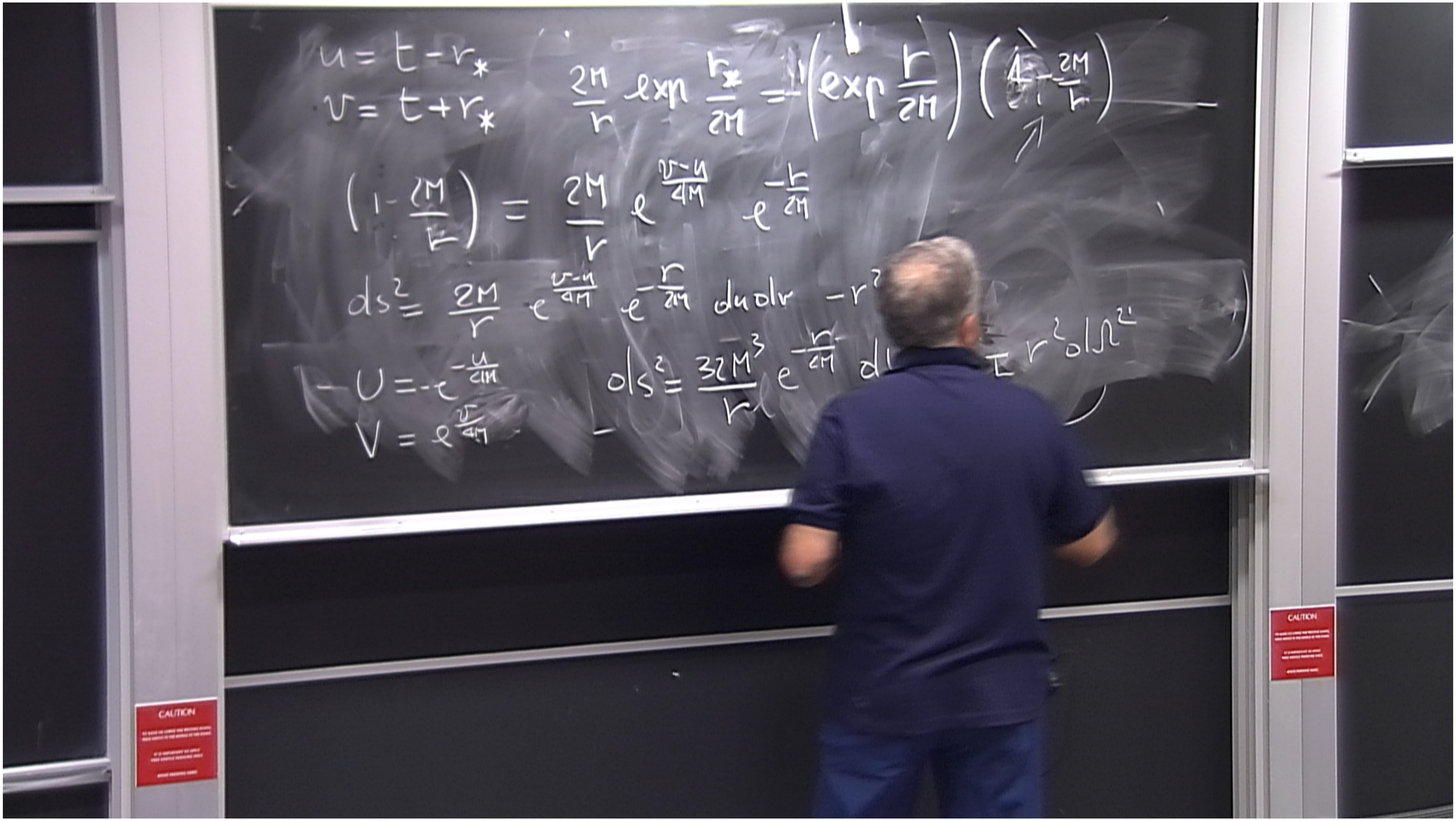

r* = -∞
u = t - r*
v = t + r*


r* = −∞
u = t − r*
v = t + r*
2M/r exp r*/2M

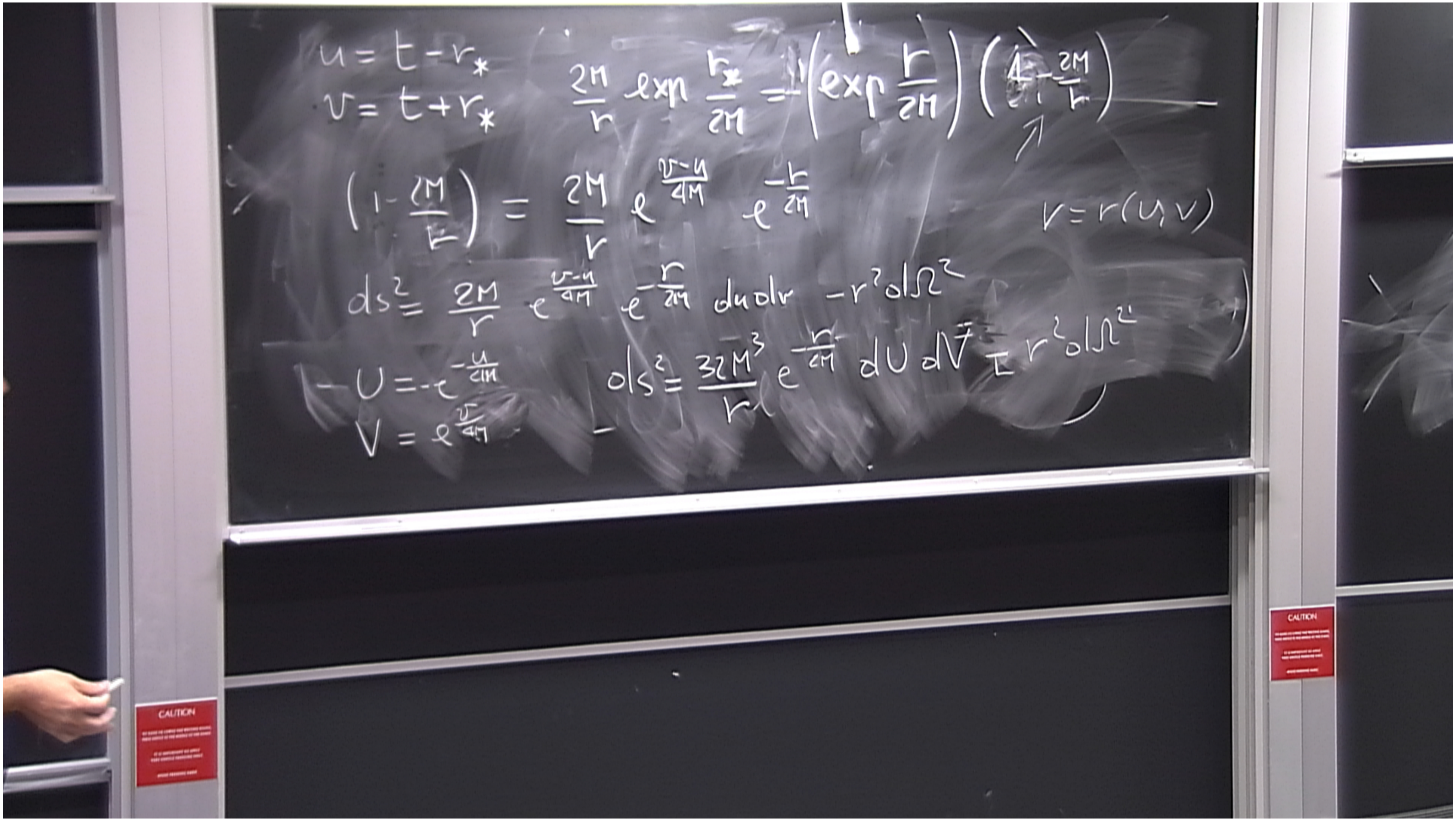

r* = -∞
u = t - r*
v = t + r
exp r*/2M = (exp r/2M)(1 - 2M/r)
CAUTION
CAUTION

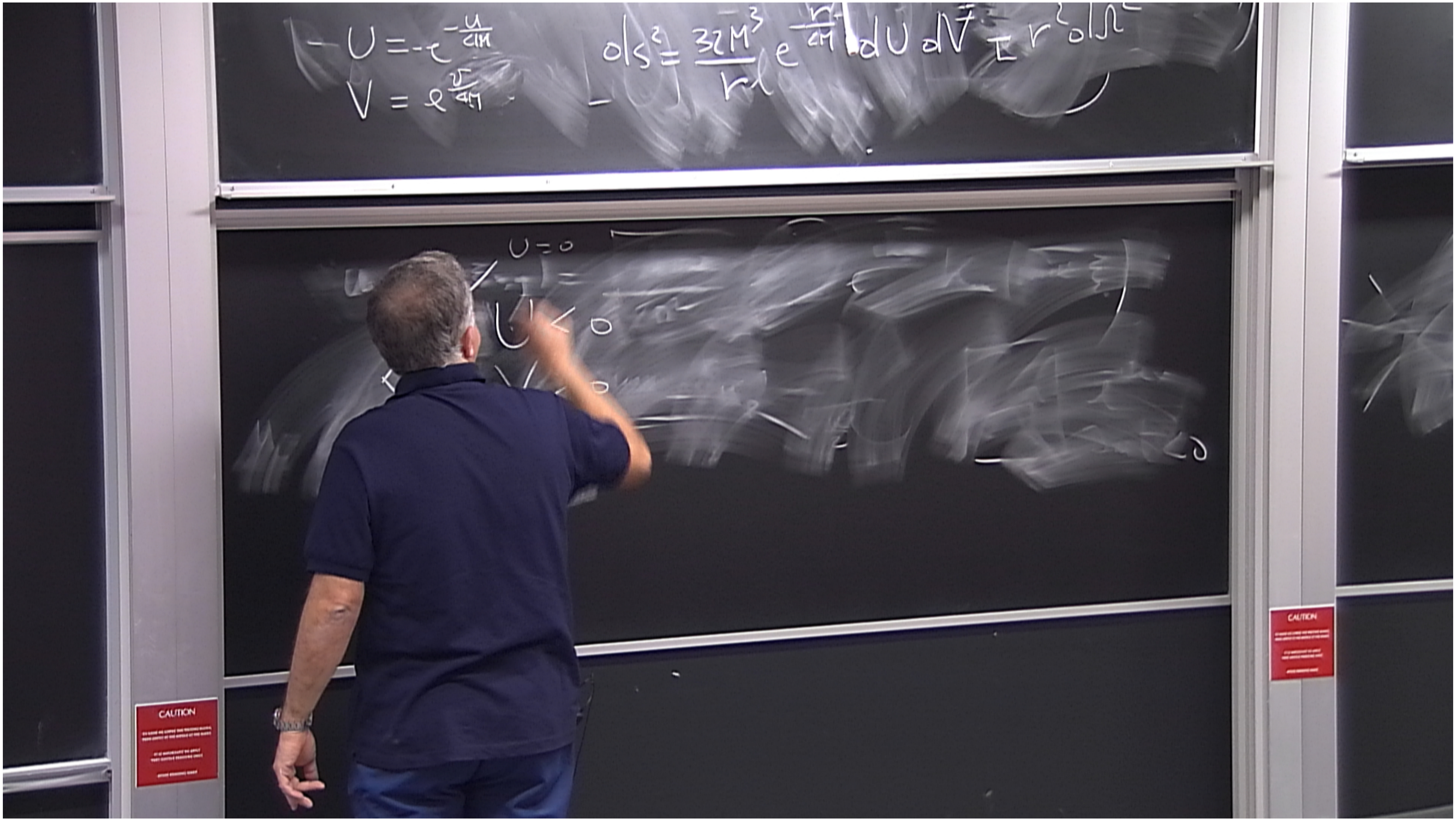

u = t - r*
v = t + r*

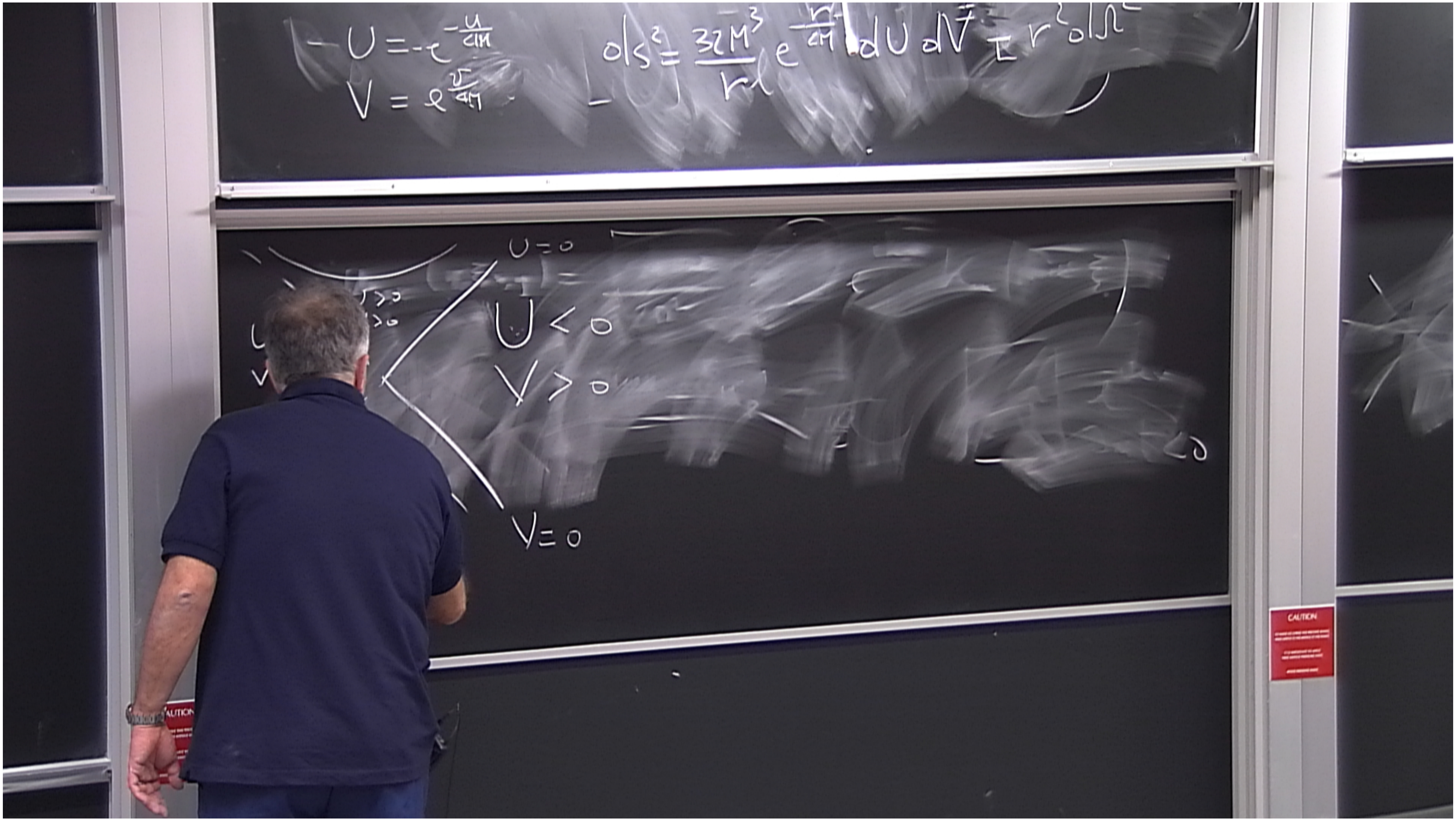

u = t - r*
v = t + r*
ds² = 2M/r
CAUTION

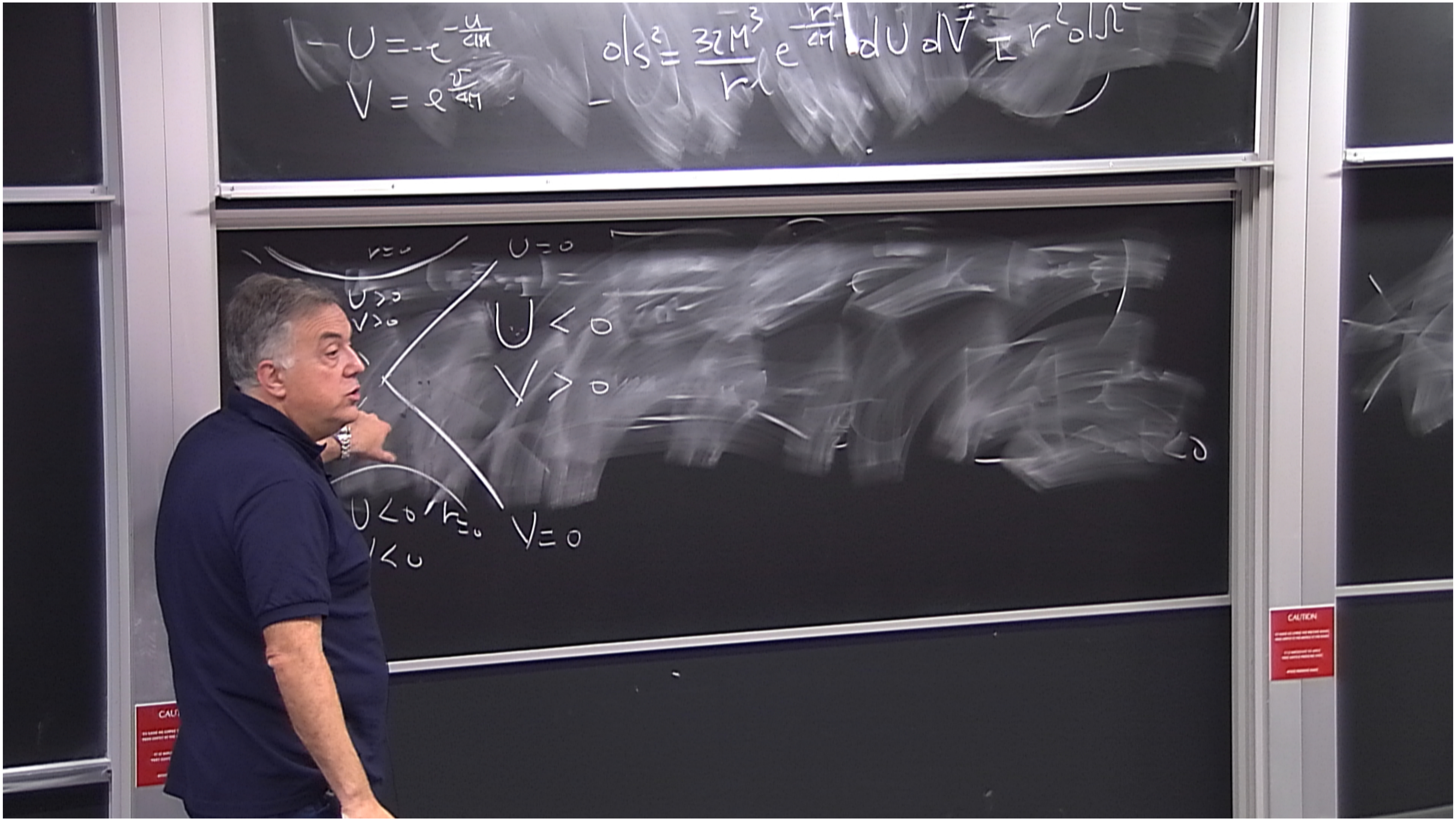


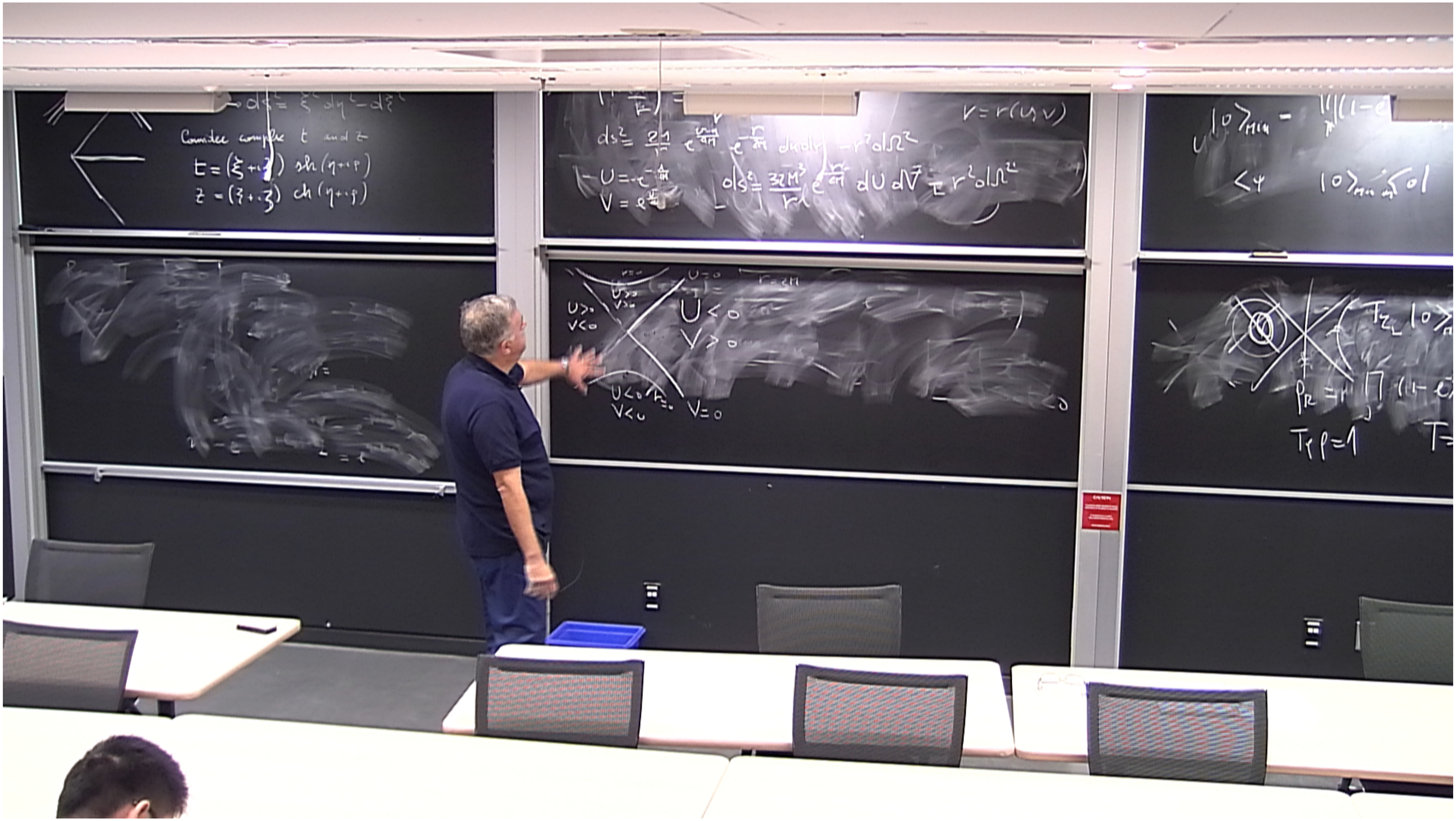
u = t − r*
v = t + r*
CAUTION
CAUTION

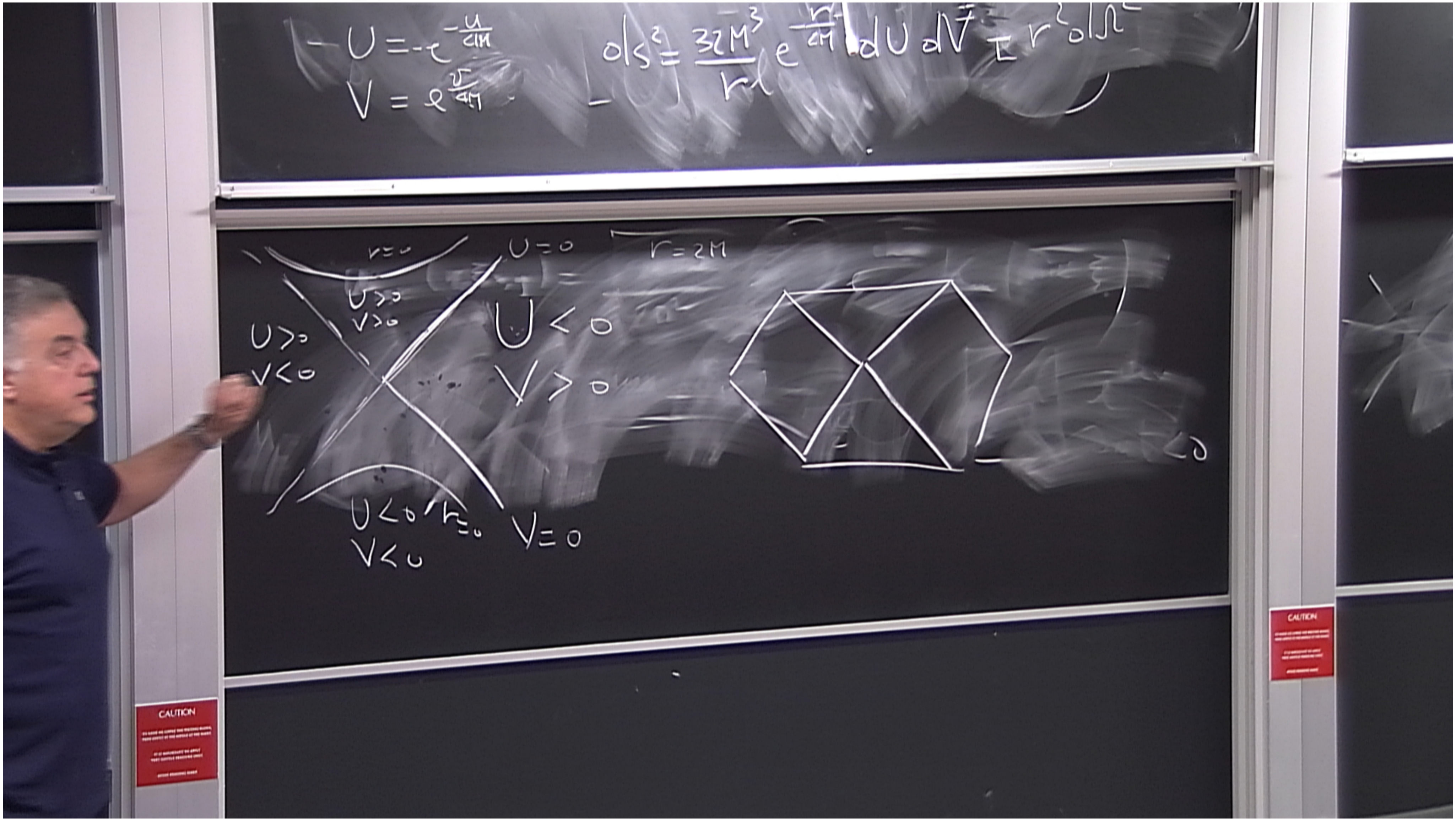

$u = t - r_*$
$v = t + r_*$
$\frac{2M}{r} \exp \frac{r_*}{2M} = \left(\exp \frac{r}{2M}\right)\left(1 - \frac{2M}{r}\right)$
$\left(1 - \frac{2M}{r}\right) = \frac{2M}{r} e^{\frac{v-u}{4M}} e^{-\frac{r}{2M}}$
$r = r(u,v)$
$ds^2 = \frac{2M}{r} e^{\frac{v-u}{4M}} e^{-\frac{r}{2M}} du\, dv - r^2 d\Omega^2$
$U = -e^{-\frac{u}{4M}}$
$V = e^{\frac{v}{4M}}$
$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} dU\, dV - r^2 d\Omega^2$
CAUTION
CAUTION

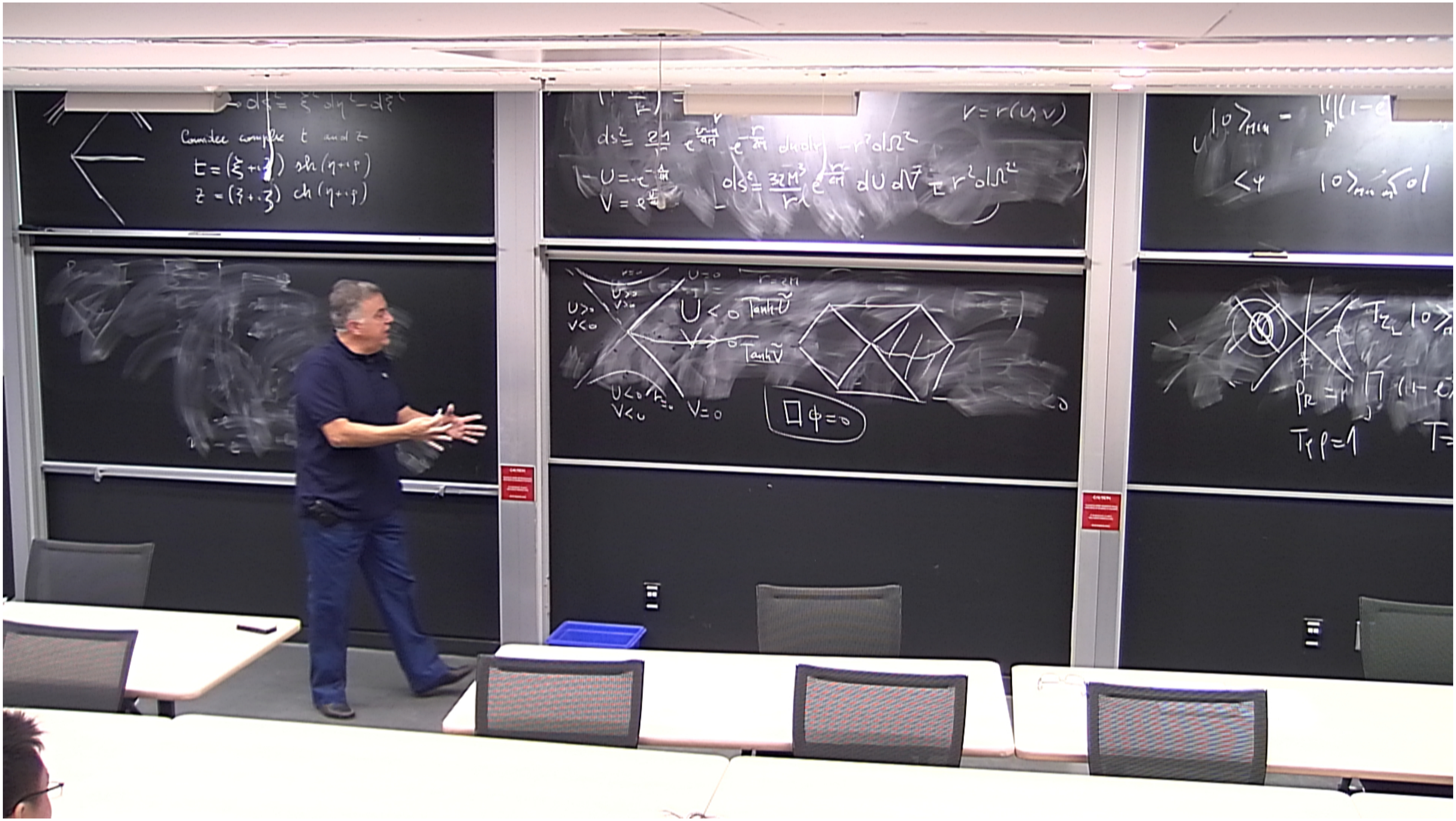

U = 0
U < 0

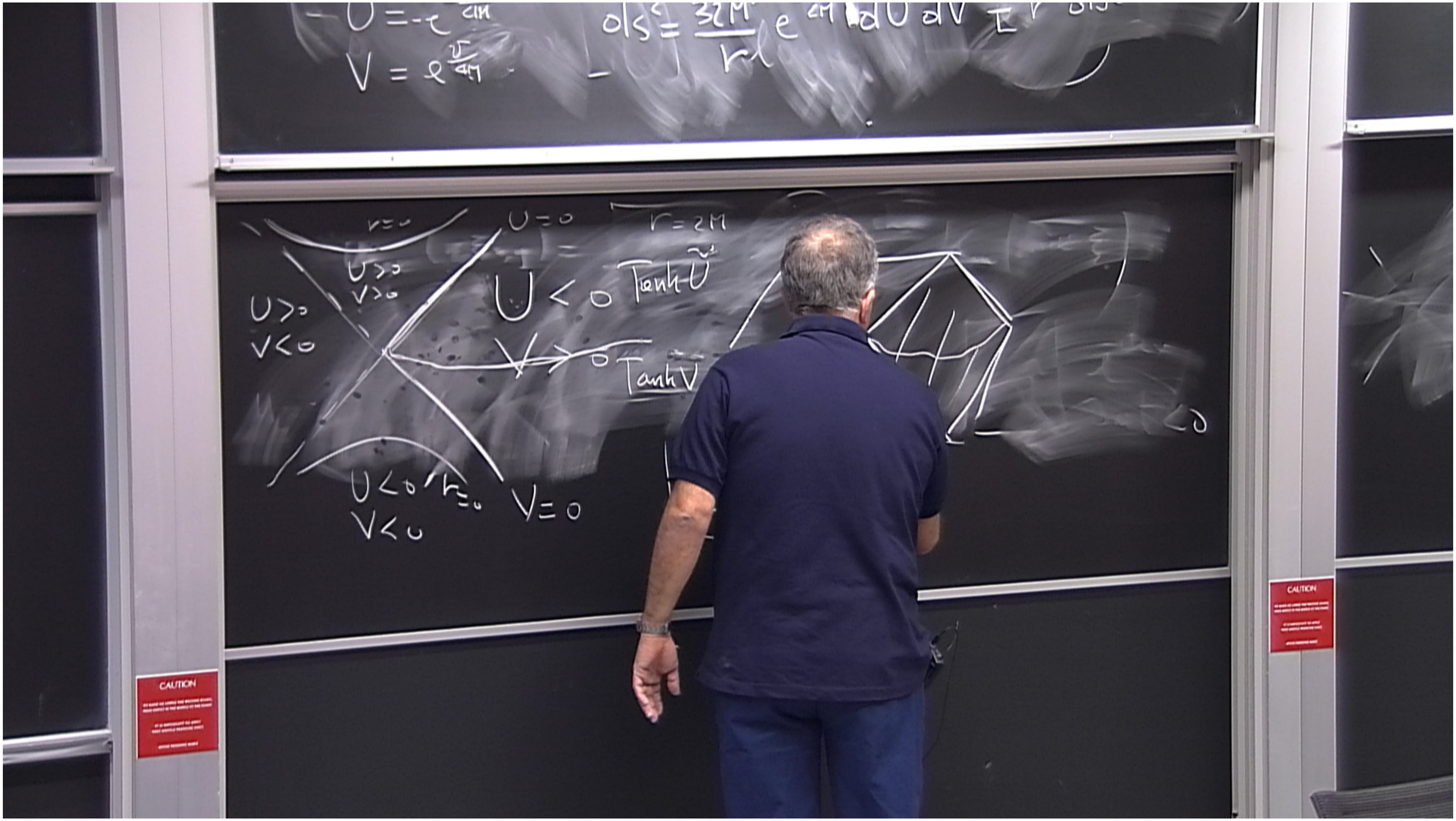

U = 0
U < 0
V > 0
V = 0
CAUTION

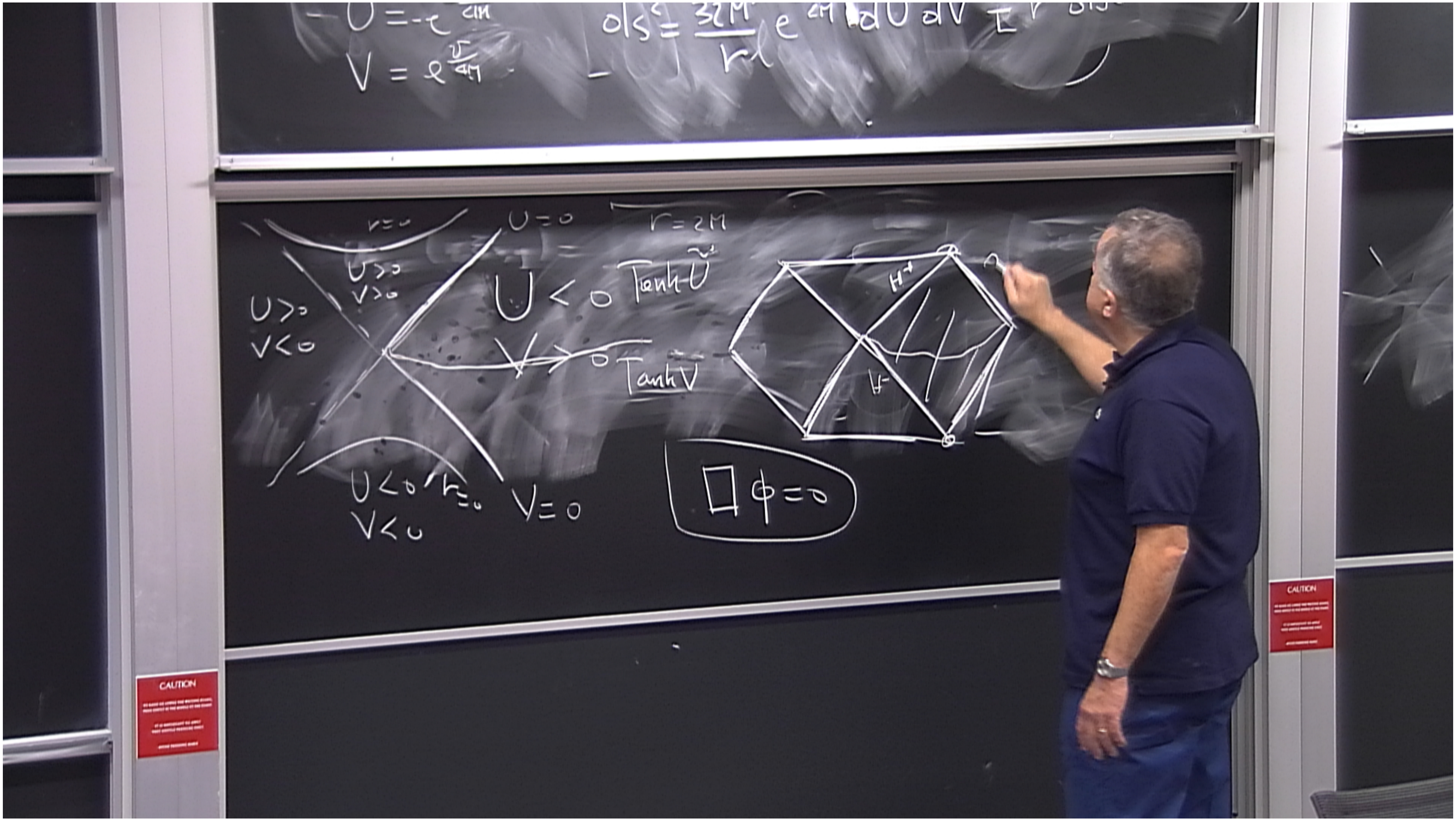

U = −e^{−u/4M}
V = e^{v/4M}
ds² = 32M³/r e^{−r/4M} dU dV + r²dΩ²
r = 0
U = 0
U > 0
V > 0
U < 0
V > 0
U < 0
V < 0
r = 0
V = 0
CAUTION

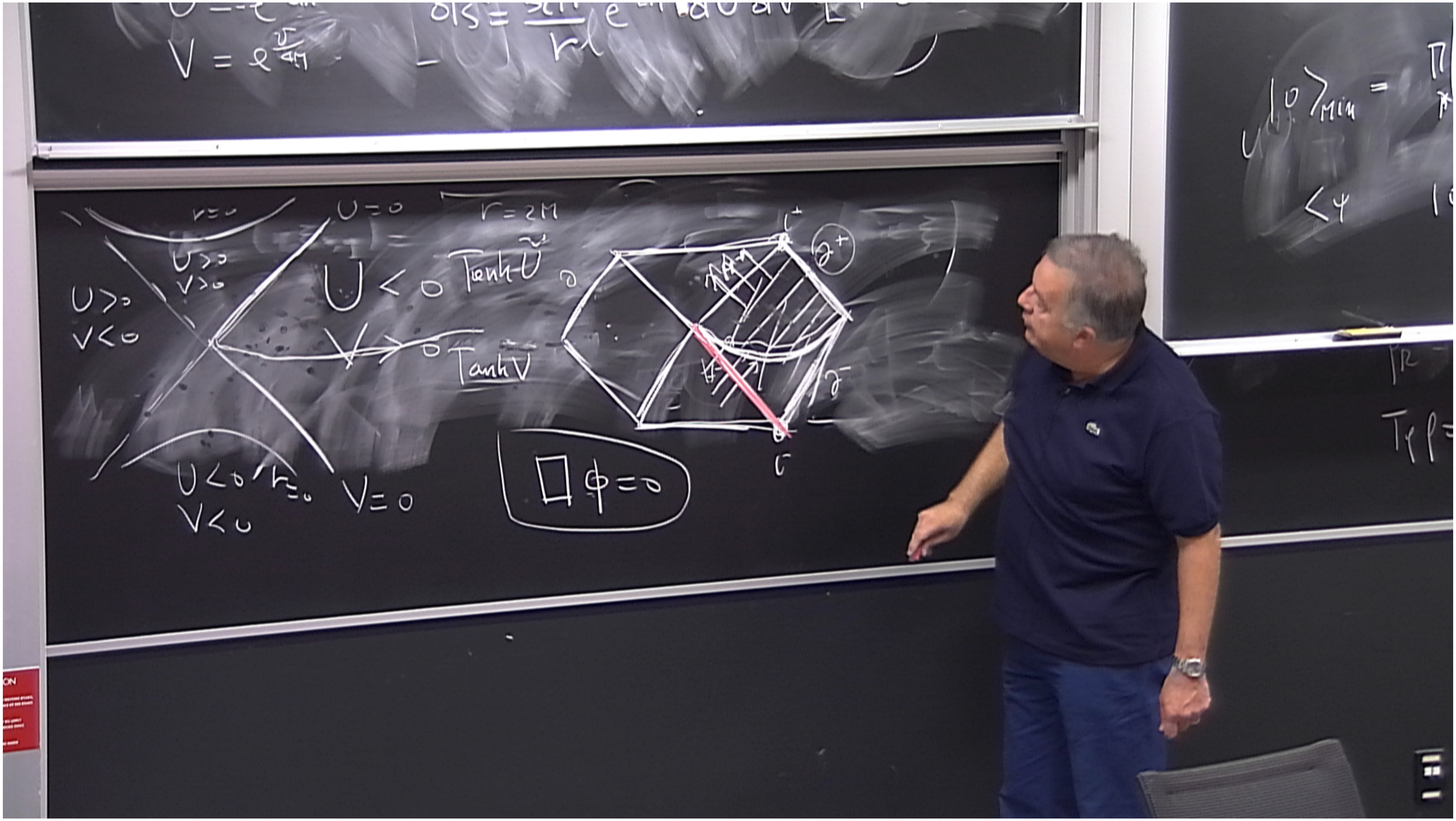


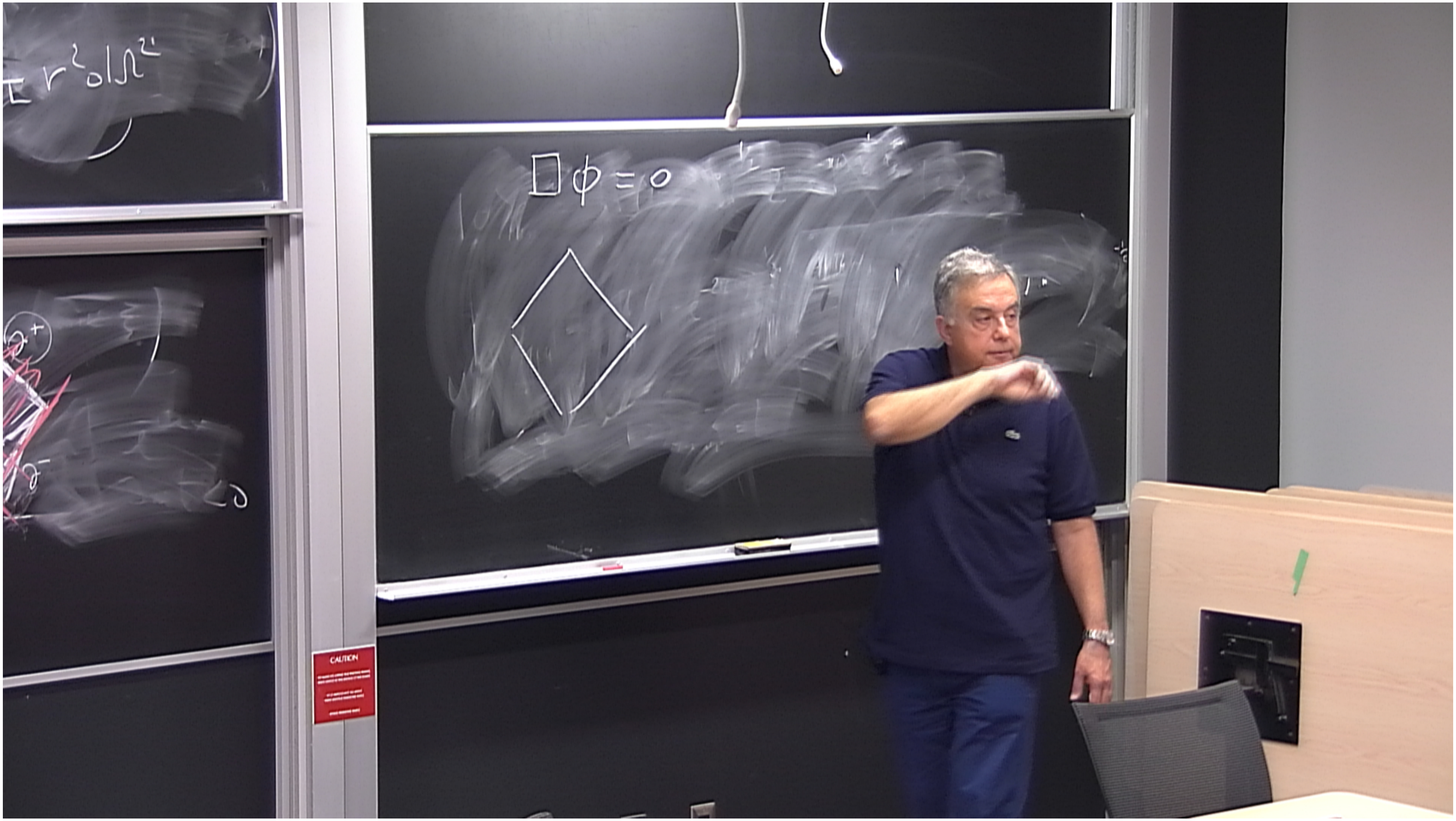

U = 0
r = 2M
U > 0
V > 0
U > 0
V < 0
U < 0
V > 0
U < 0
V < 0
V = 0

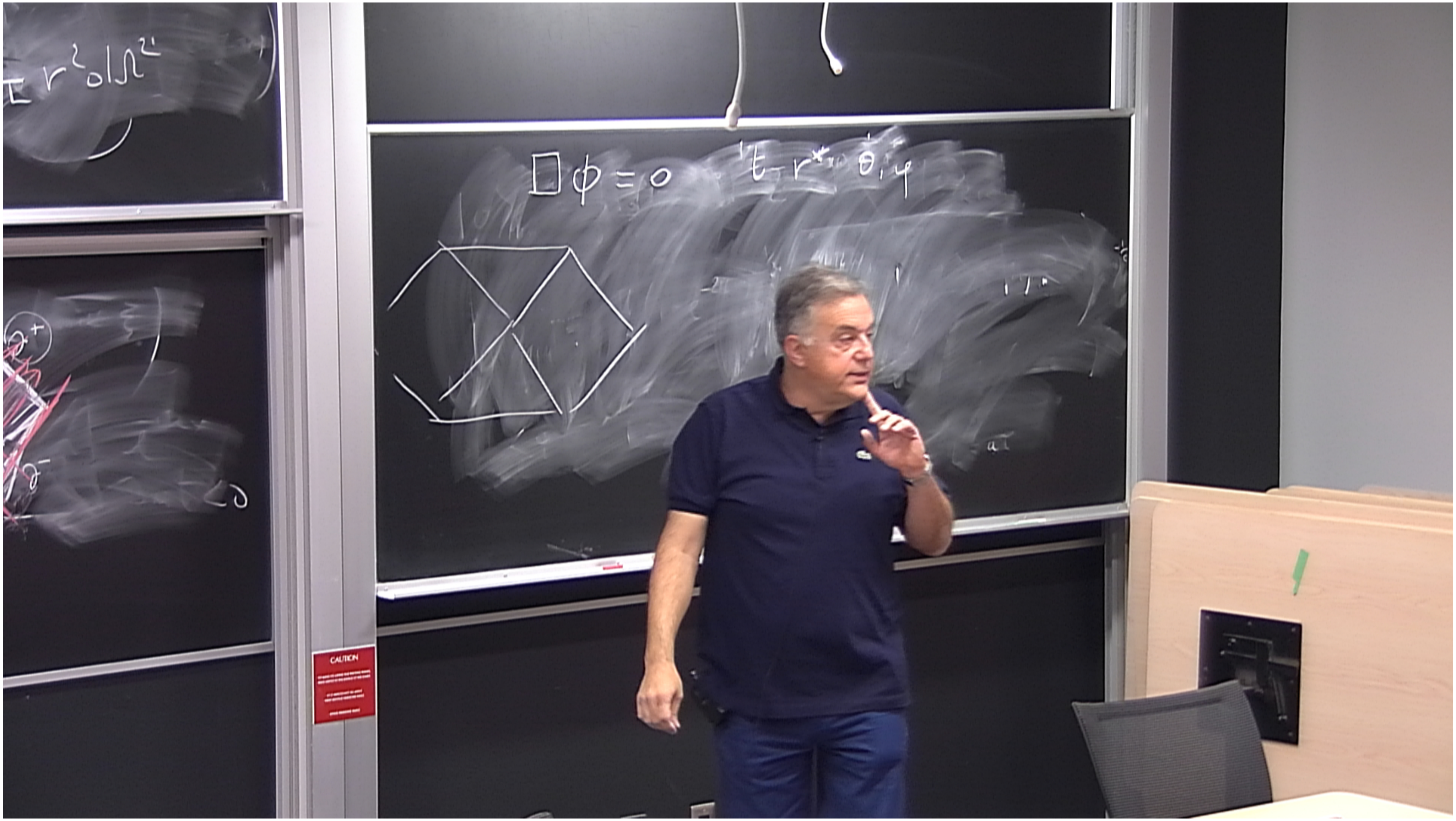

Consider complex t and z
r = r(U,V)
□φ = 0

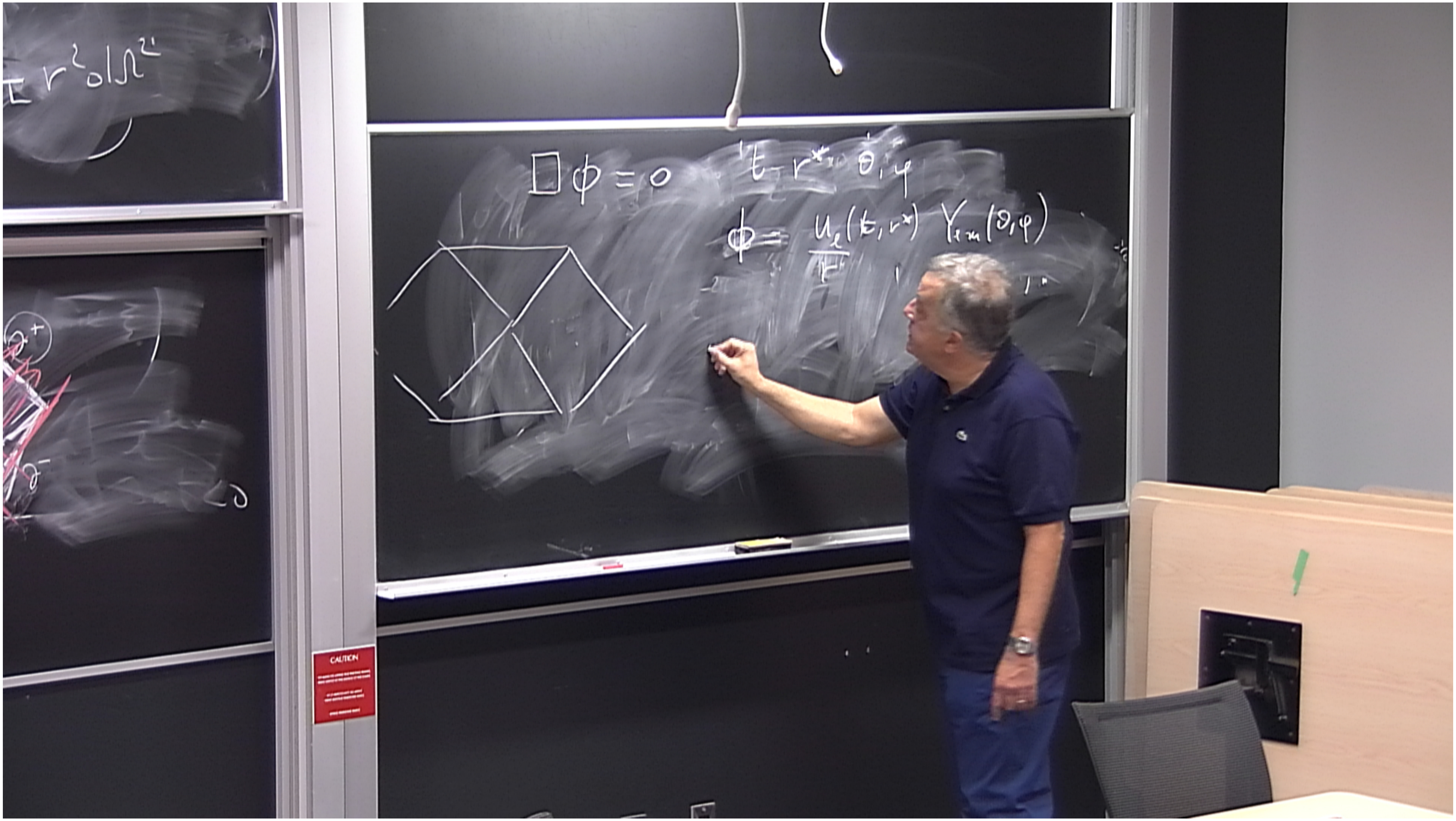

V = e
ds² = 32M / r
r=0
U=0
r=2M
U>0
V>0
U>0
V<0
U<0 Tanh-Ũ
V>0 Tanh V
U<0
V<0
r=0
V=0

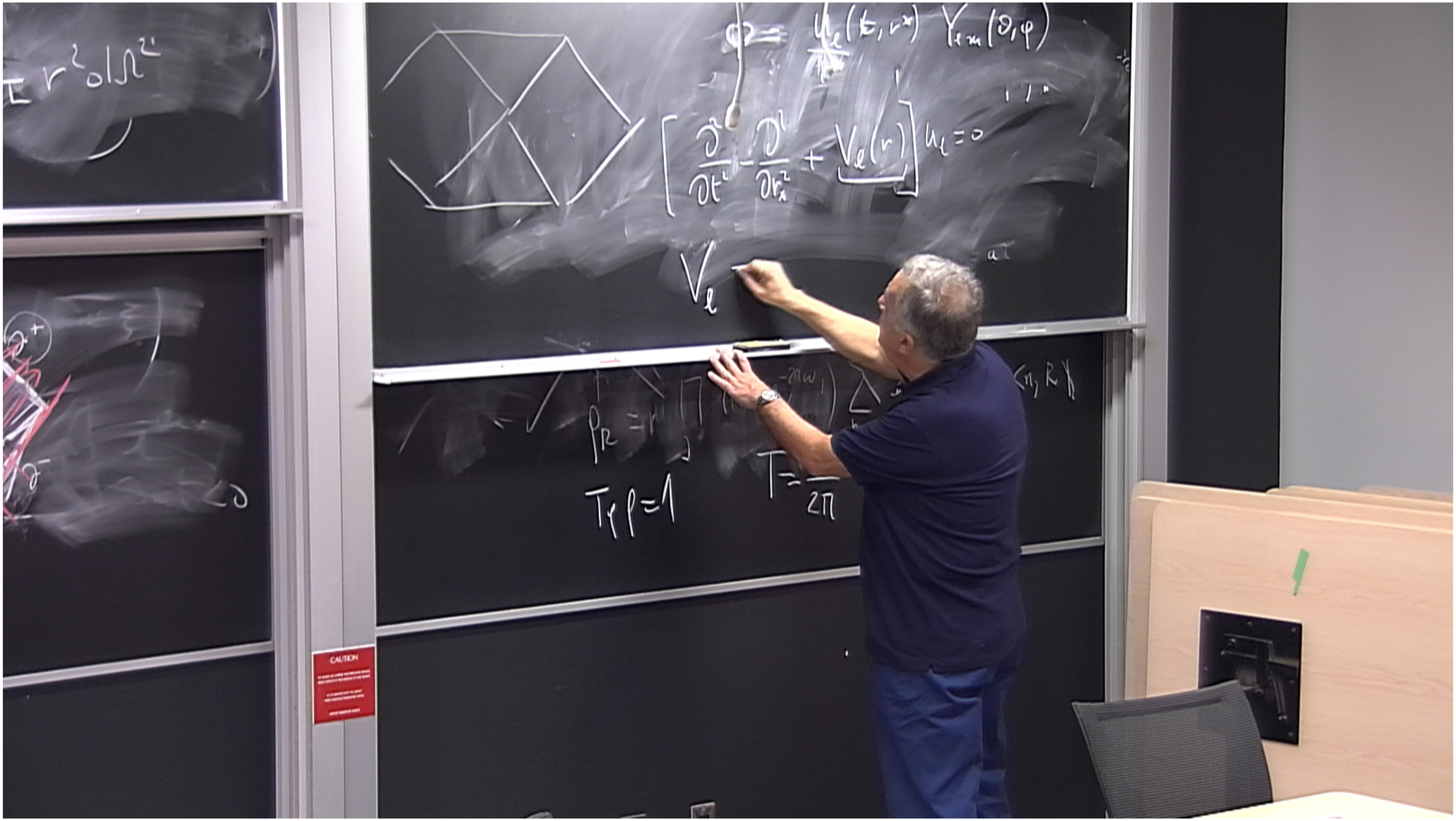

U>0
V<0
U=0
r=2M
V=0
□φ=0
H+
H-

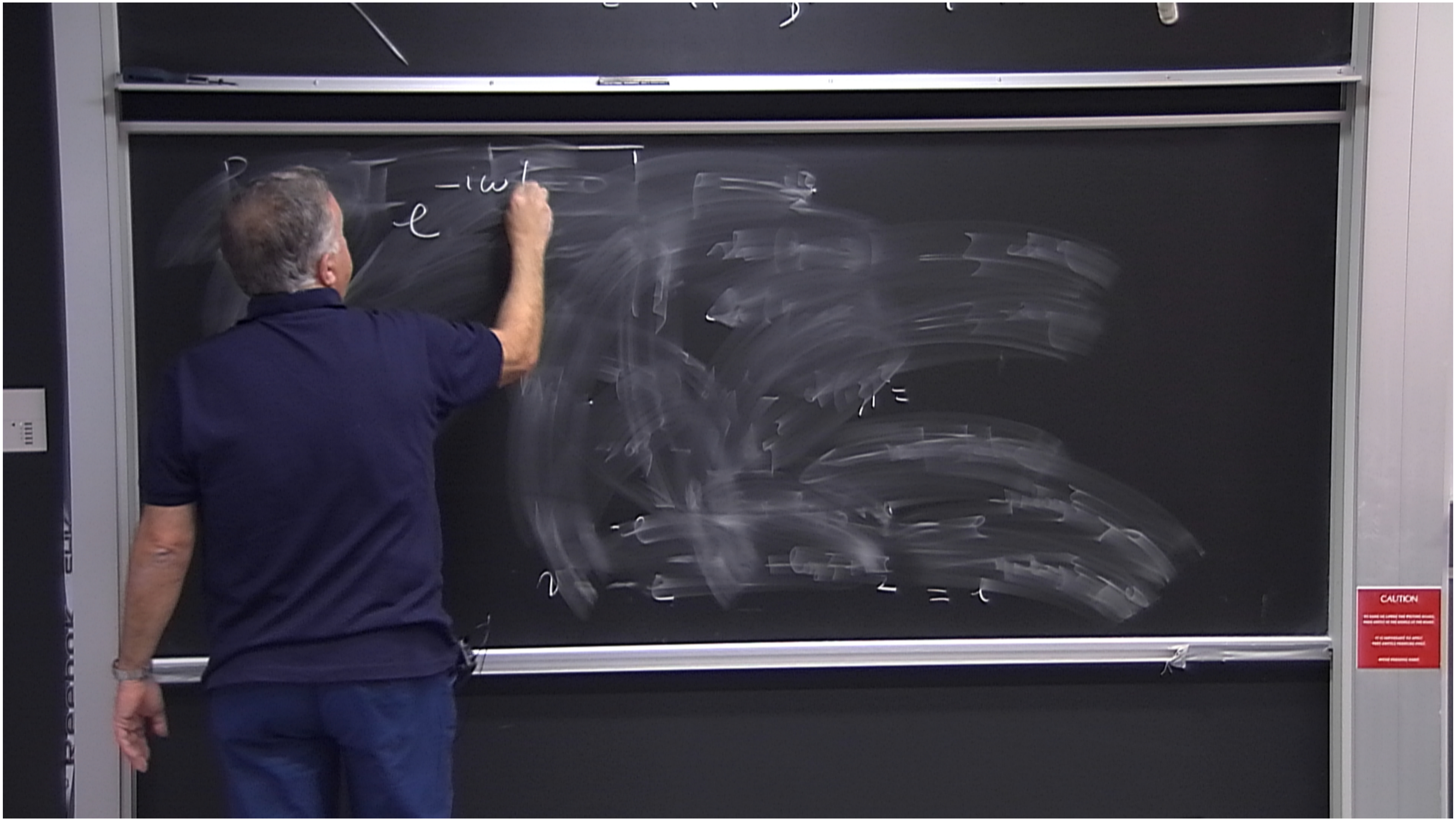


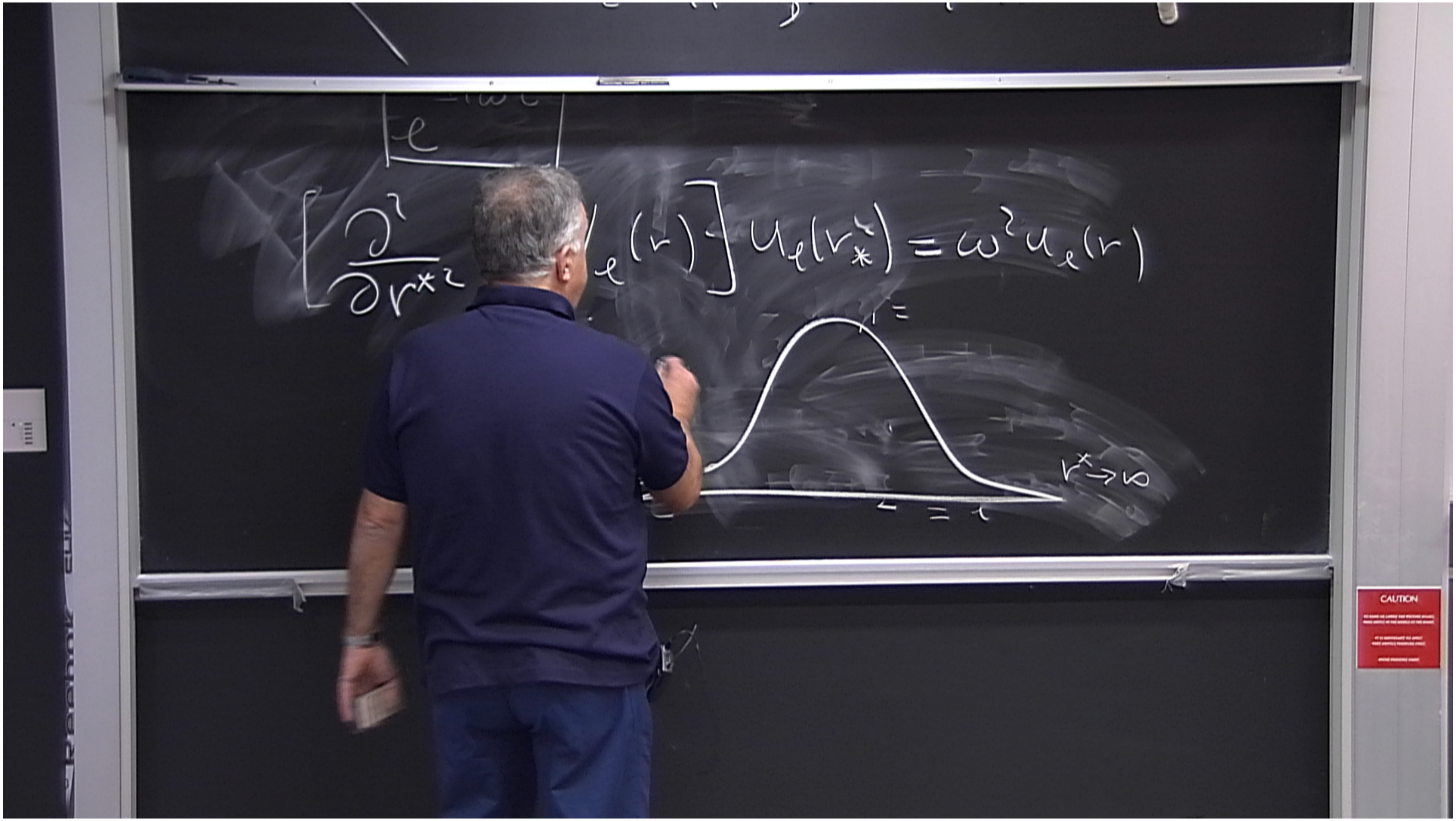

$\Box\phi = 0$

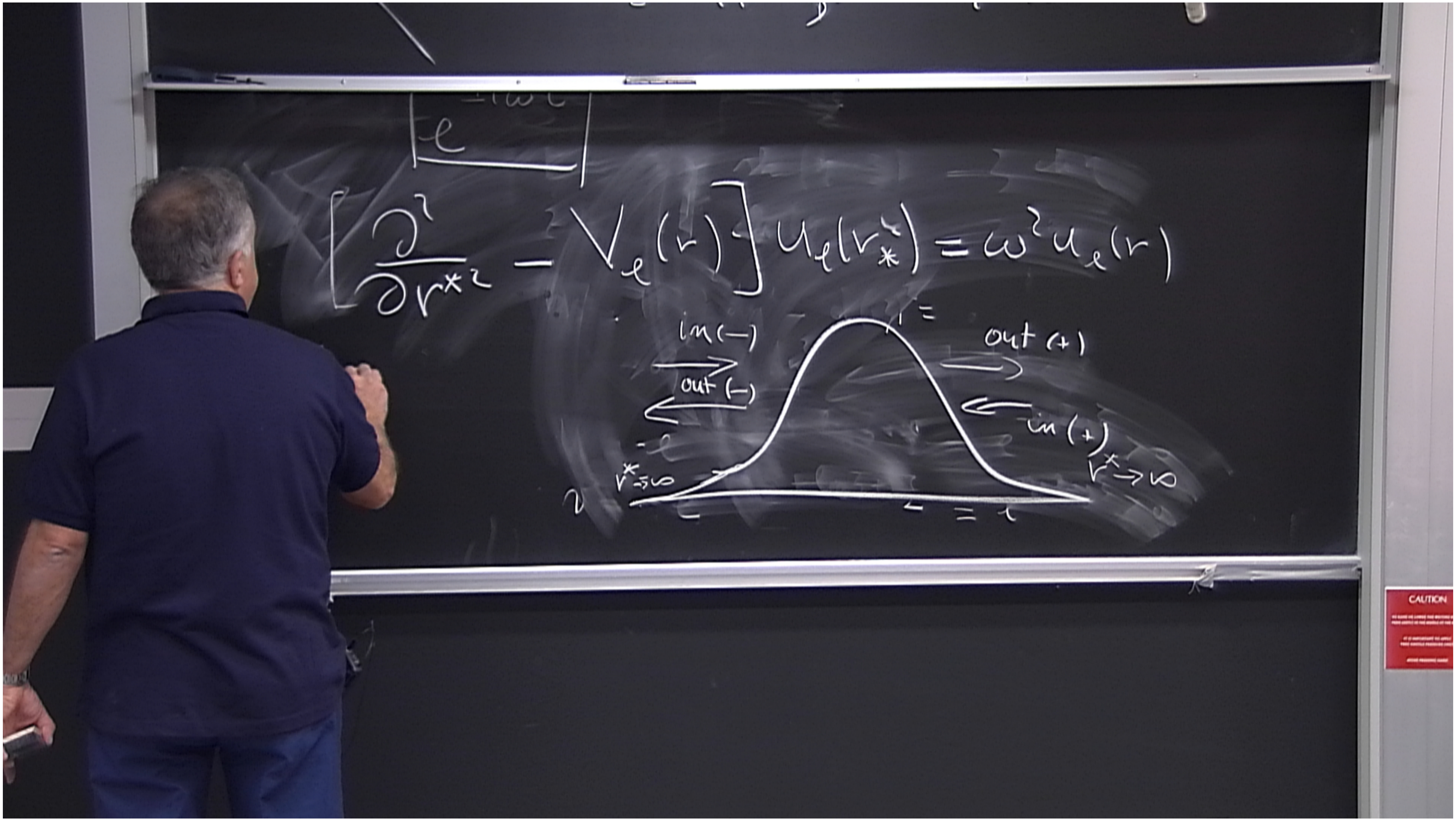

$\Box\phi = 0$

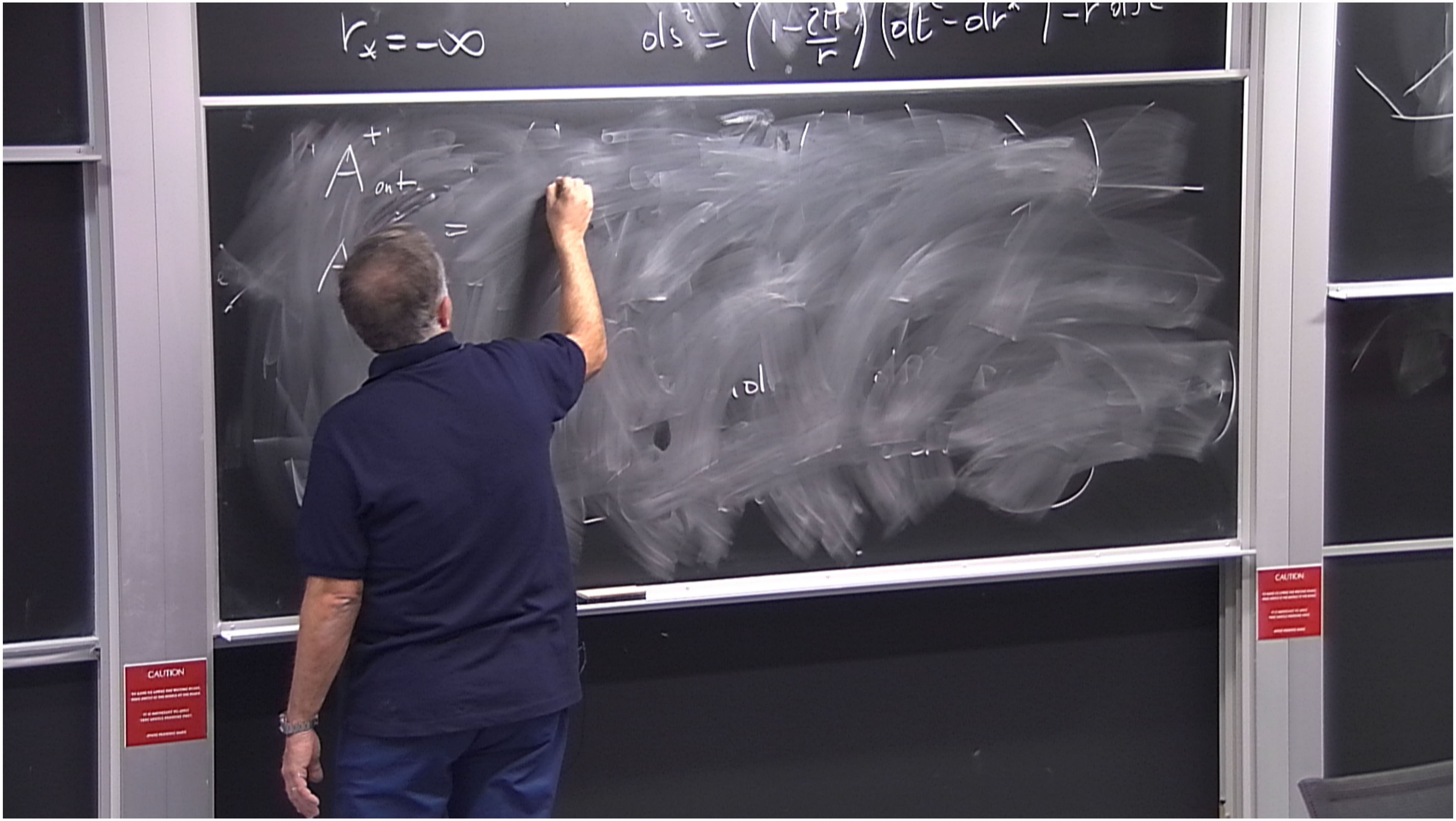

□φ = 0
t, r*, θ, φ
φ = u_l(t, r*)/r Y_lm(θ, φ)


$\Phi = \frac{u_\ell(t, r^*)}{r} Y_{\ell m}(\vartheta, \varphi)$
$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V_\ell(r)\right] u_\ell = 0$
V_ℓ
$\mathrm{Tr}\,\rho = 1$

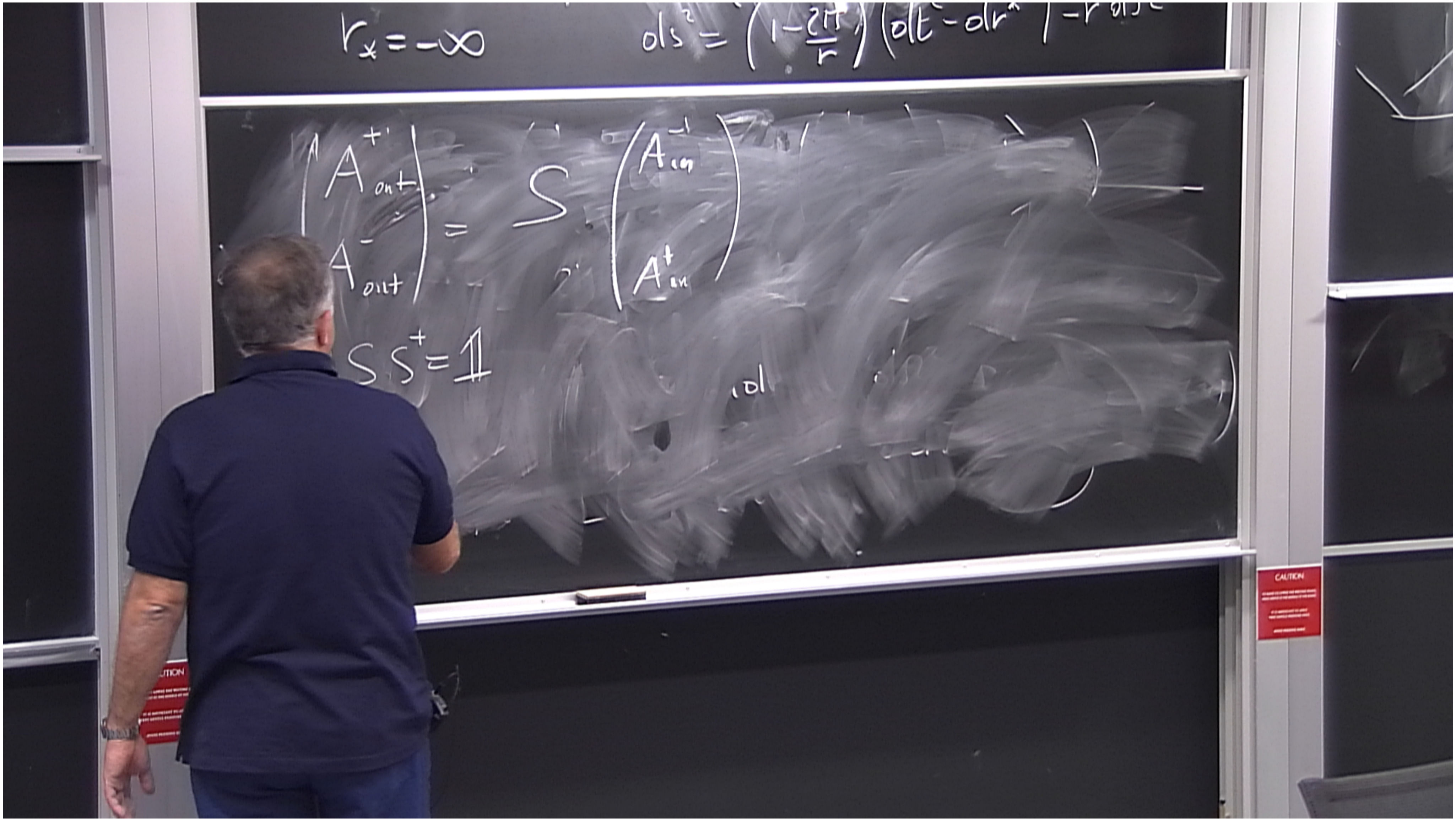




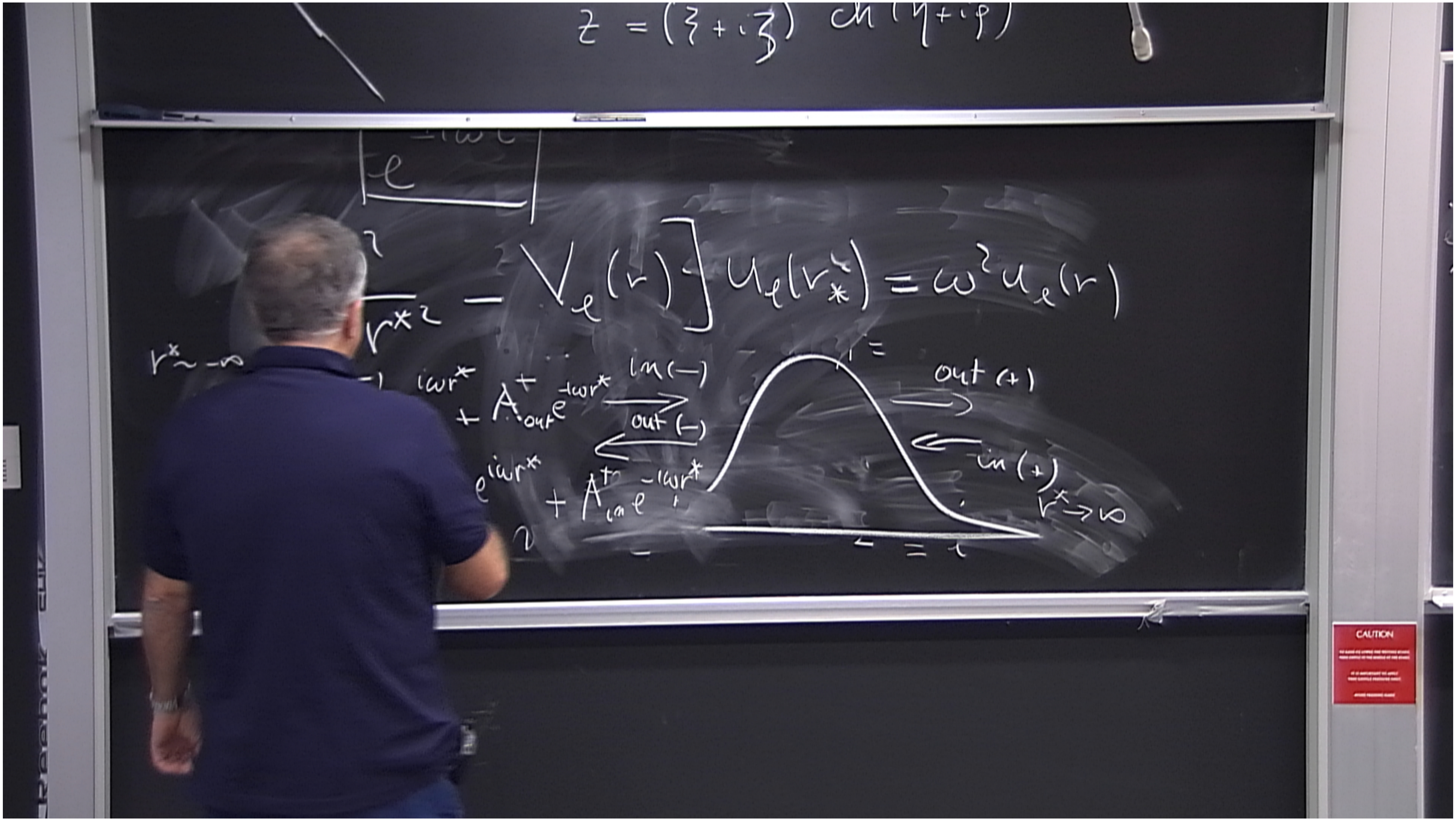

in (−)
out (−)
out (+)
in (+)
CAUTION

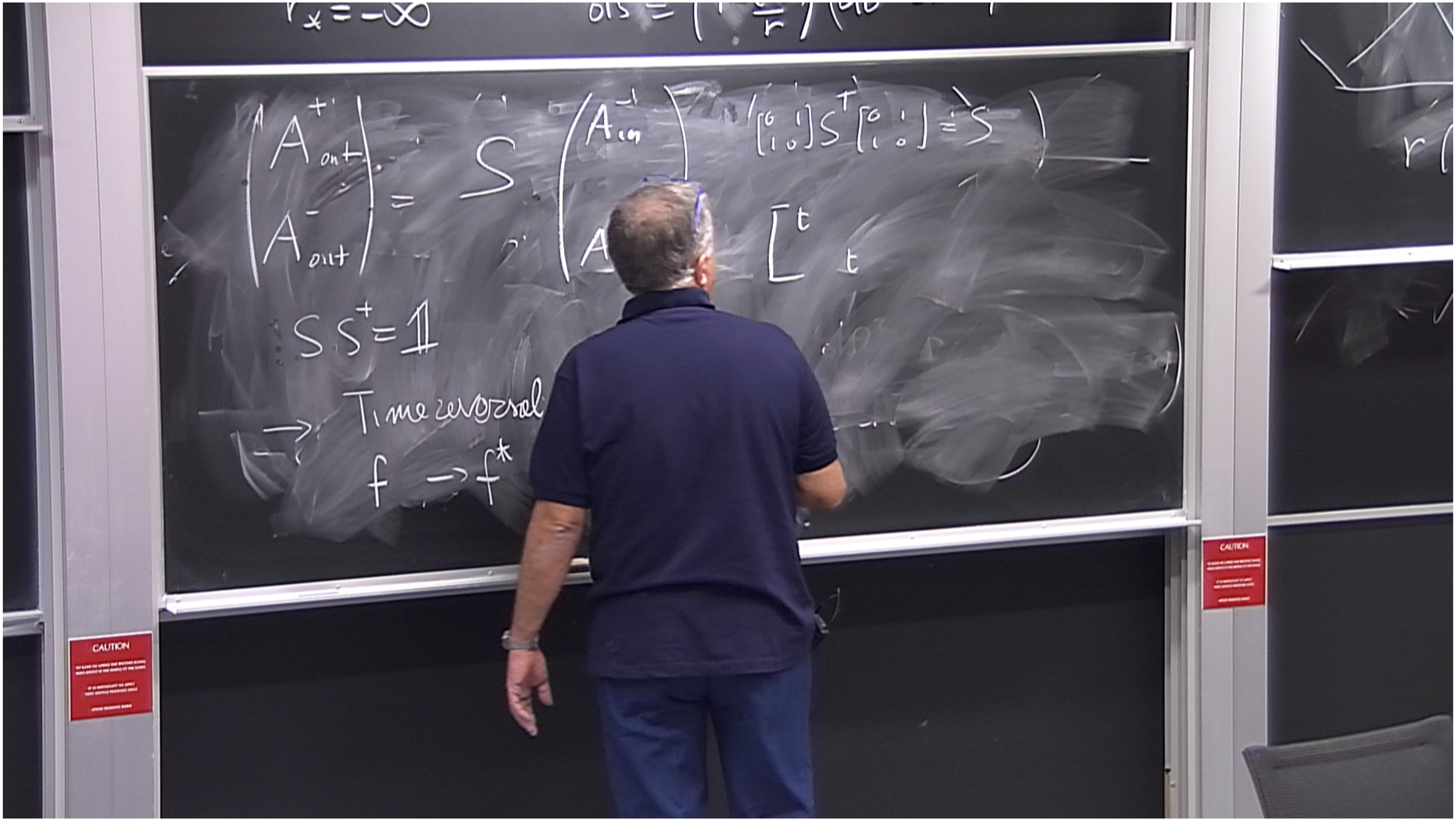


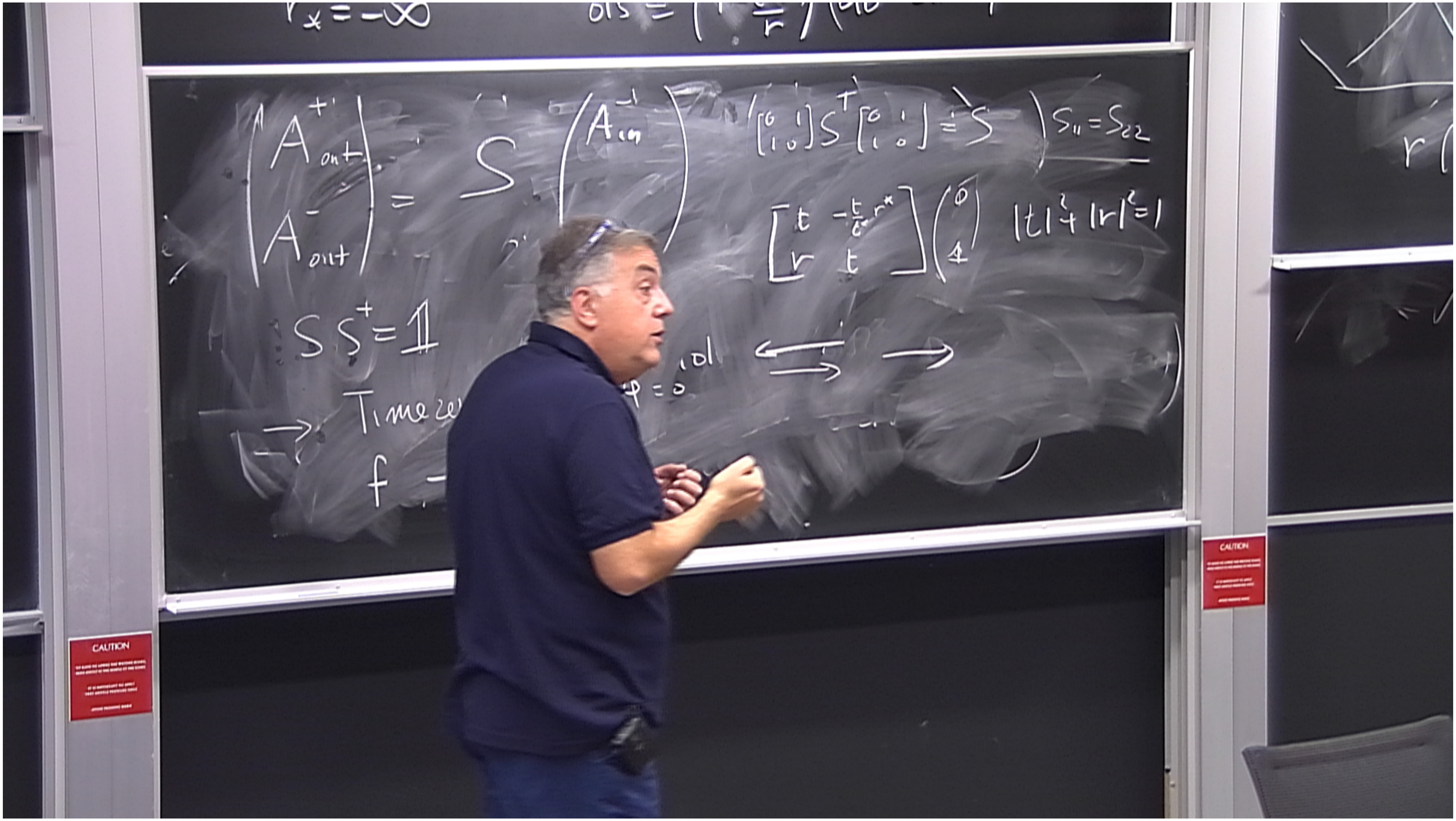
r* = −∞

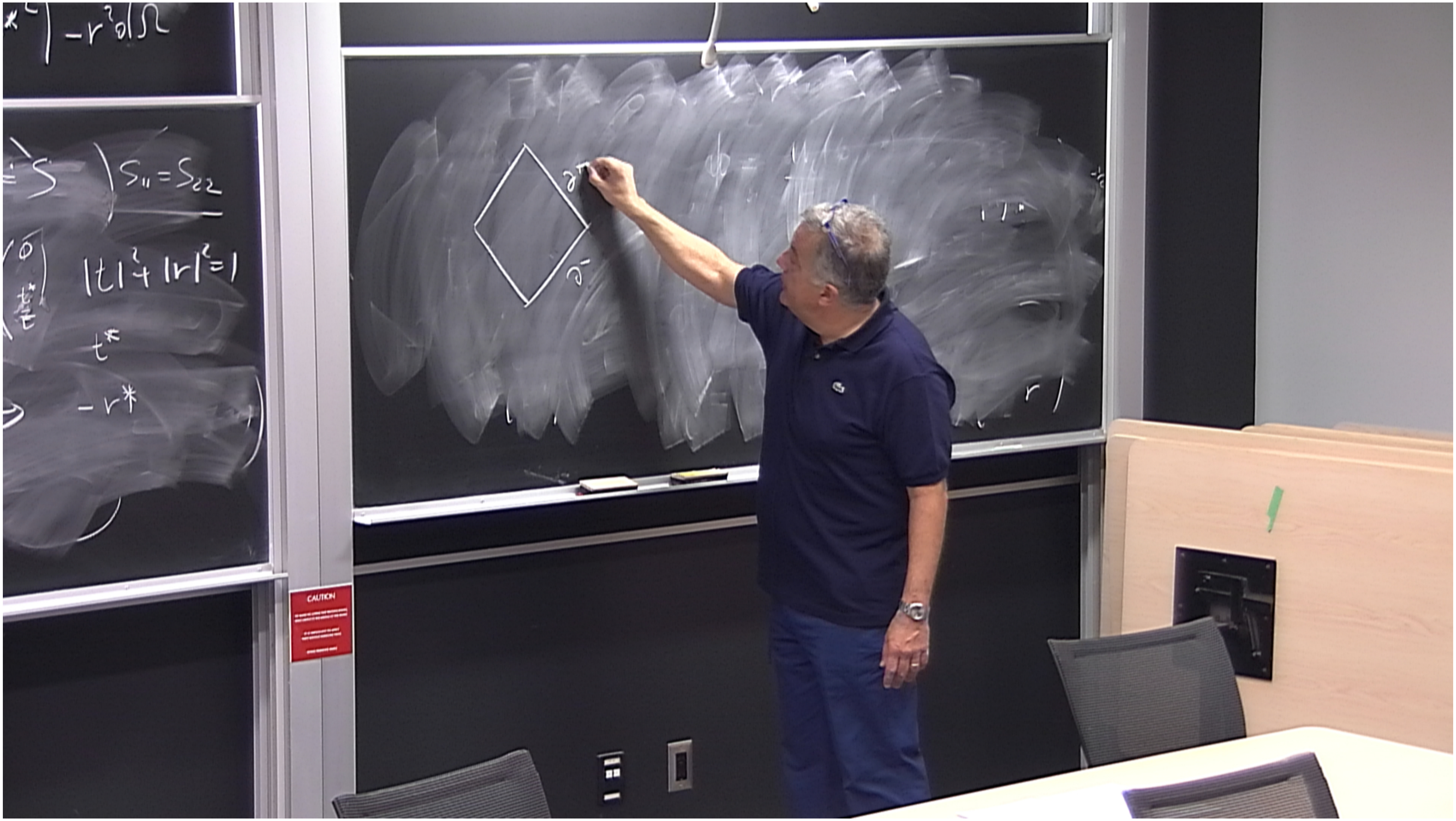


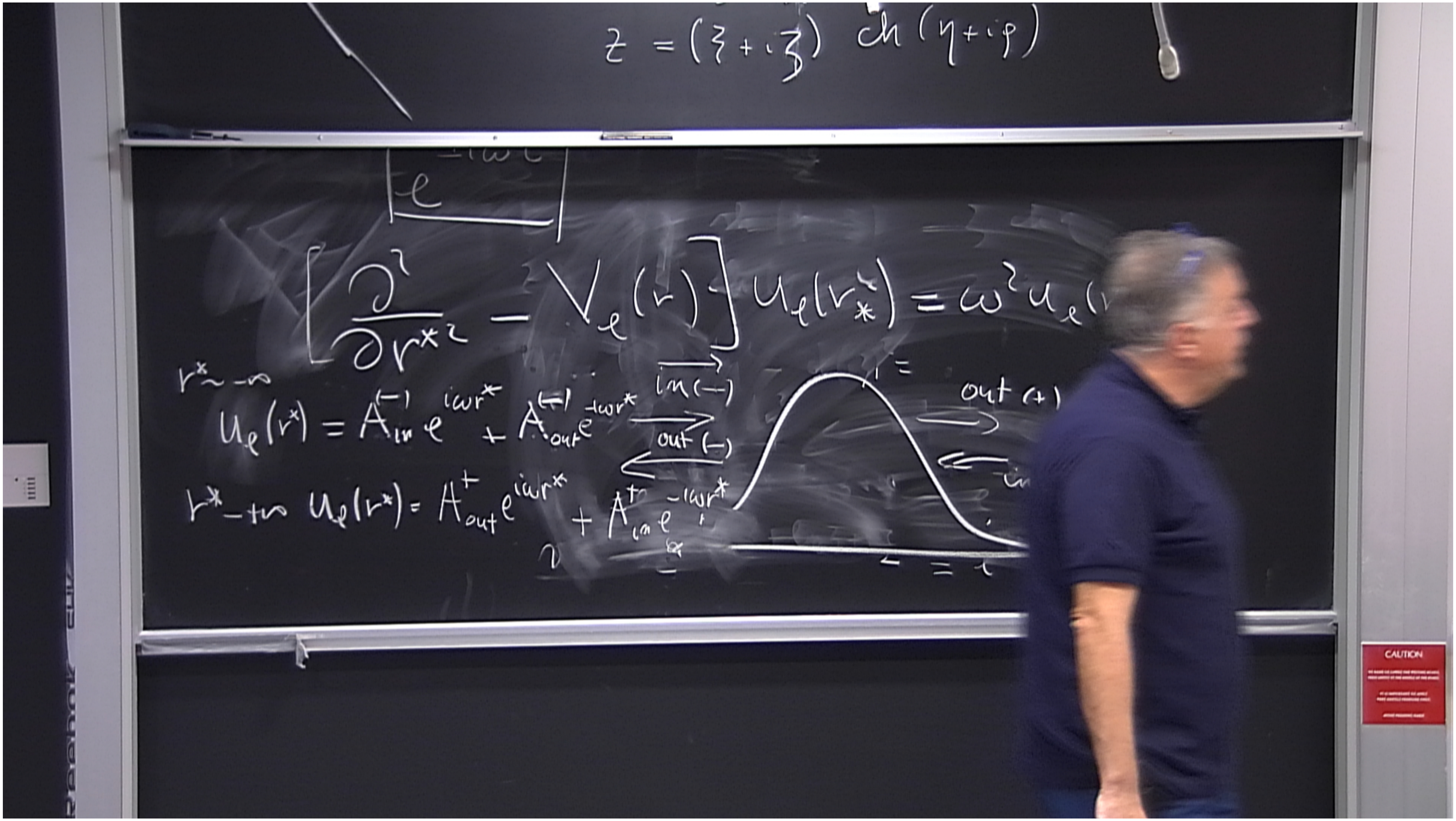
r* = -∞
ds² = (1 - 2M/r)(dt² - dr*²
A+out
A-out
= S
A-in
A+in
SS+ = 1

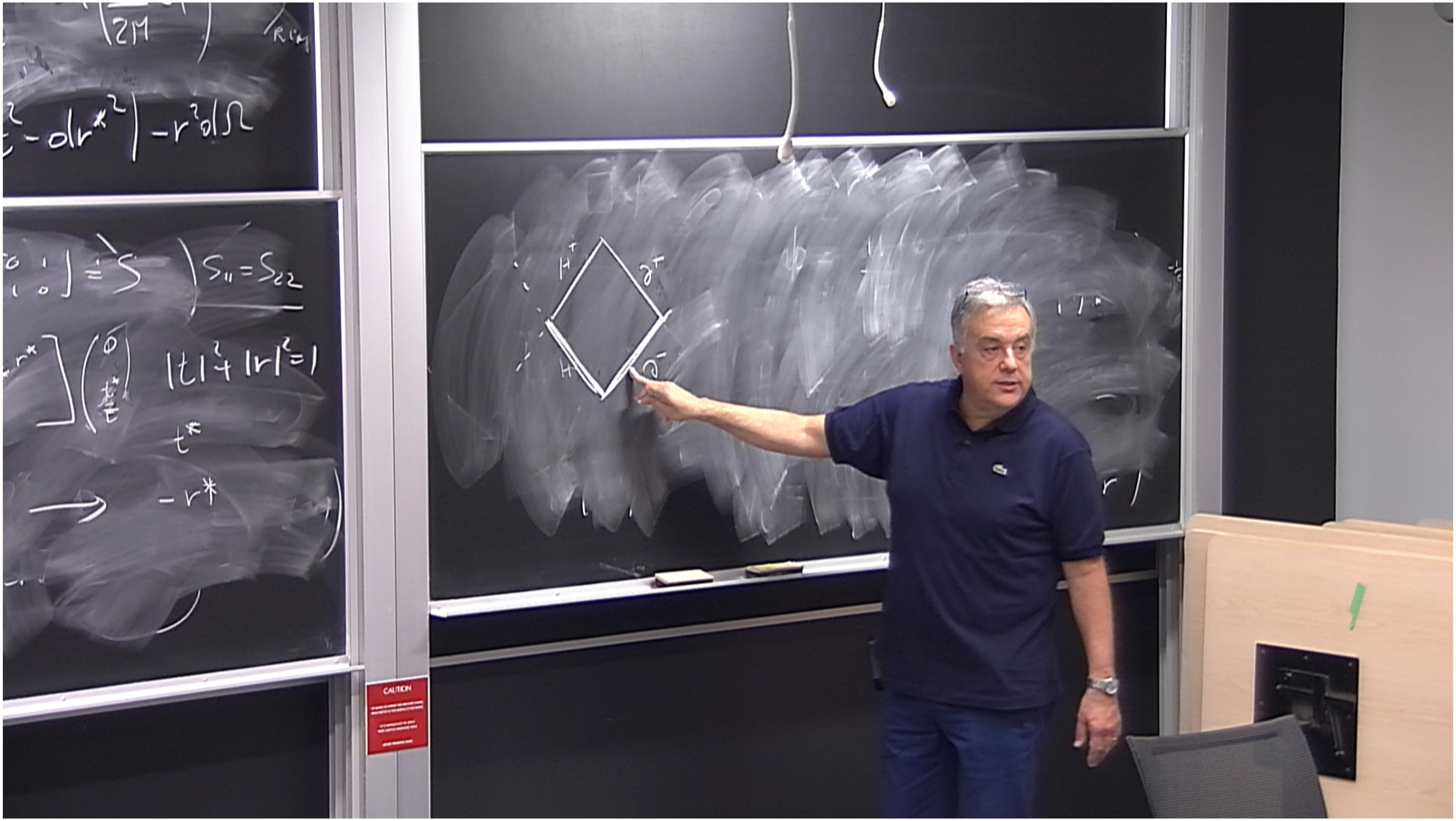

in (−)
out (+)
out (−)
in (+)
CAUTION

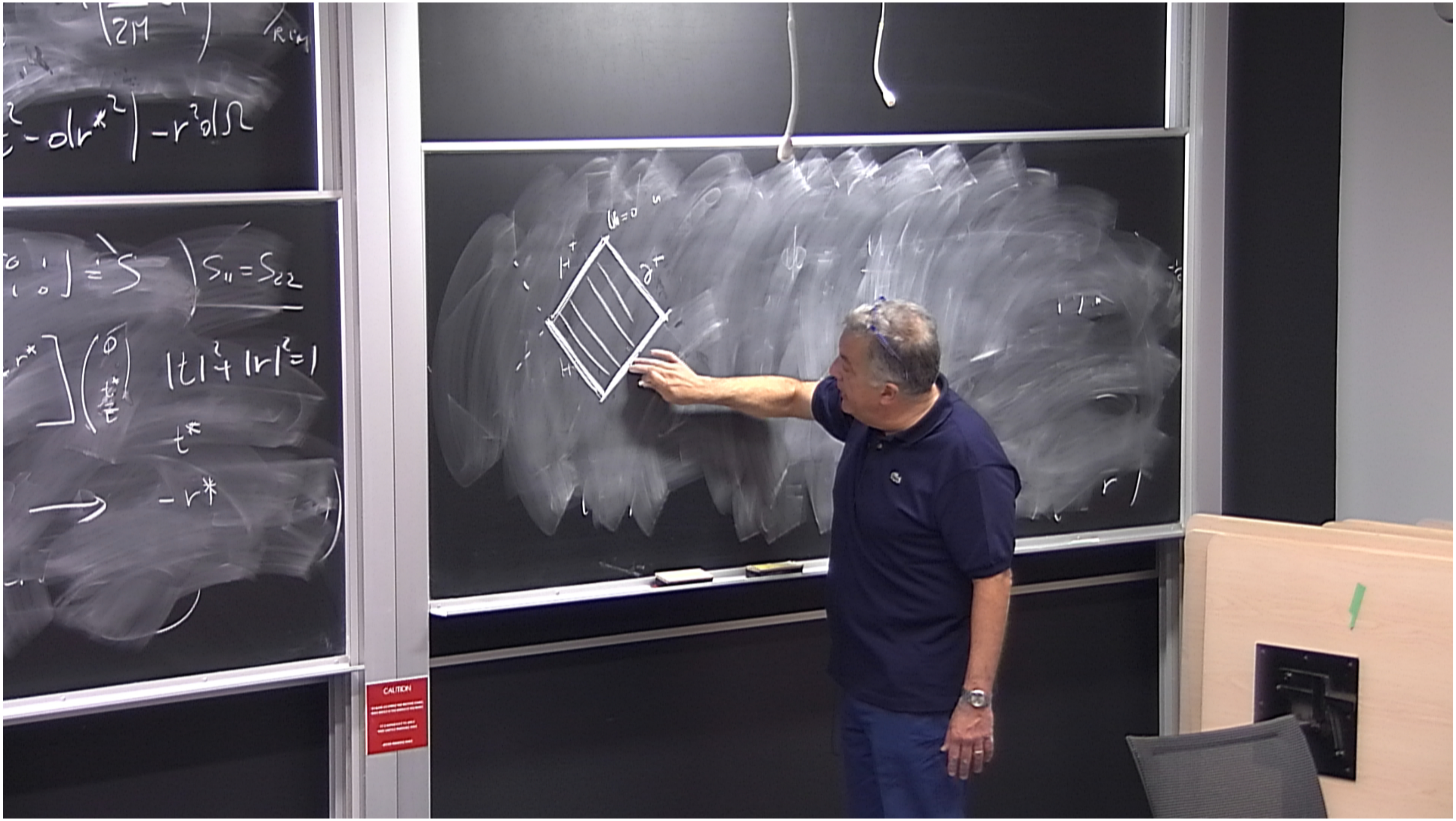

SS+=1
Time reversal
f → f*

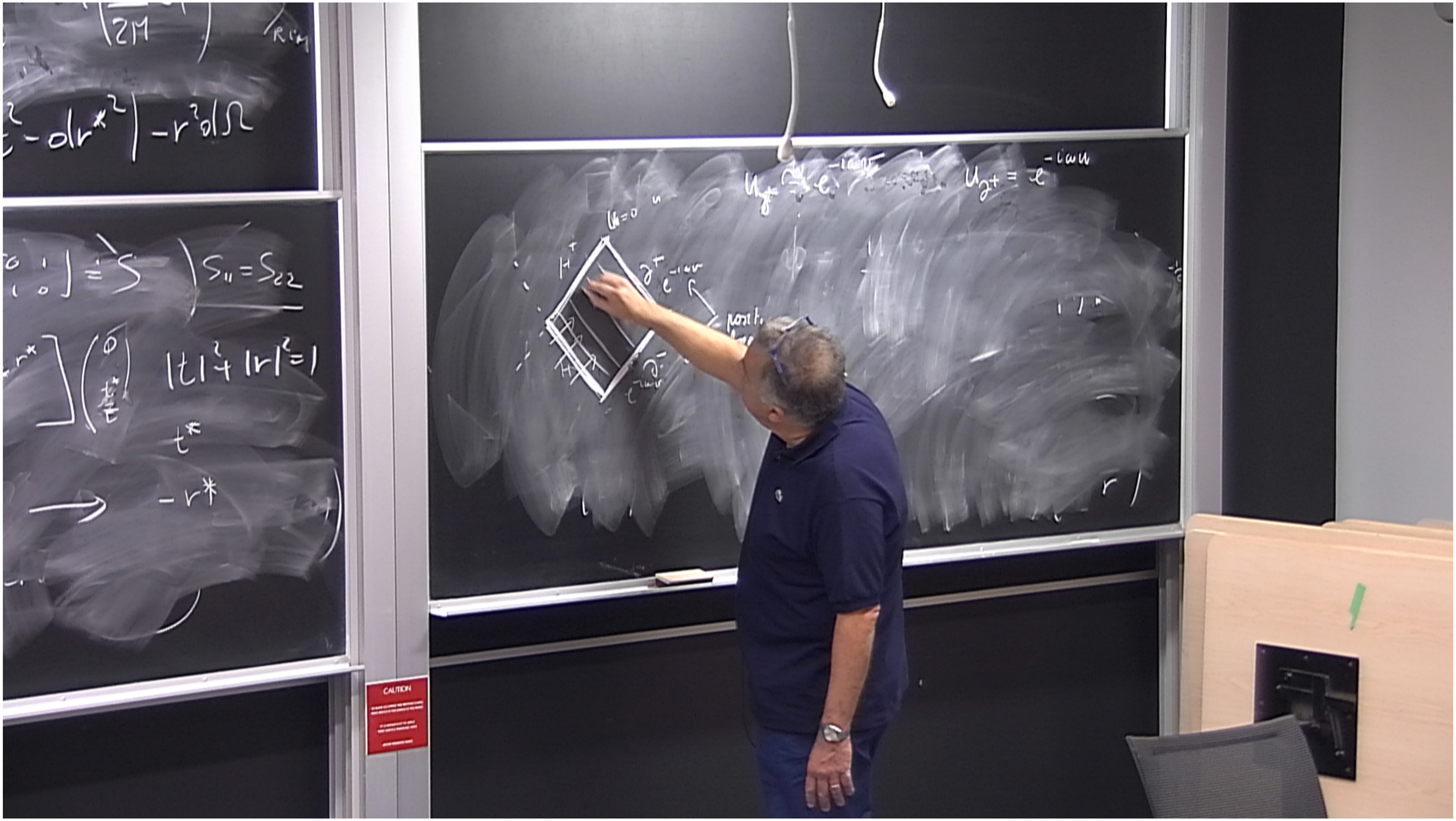

$S S^\dagger = \mathbb{1}$
$|t|^2 + |r|^2 = 1$
$S_{11} = S_{22}$
CAUTION

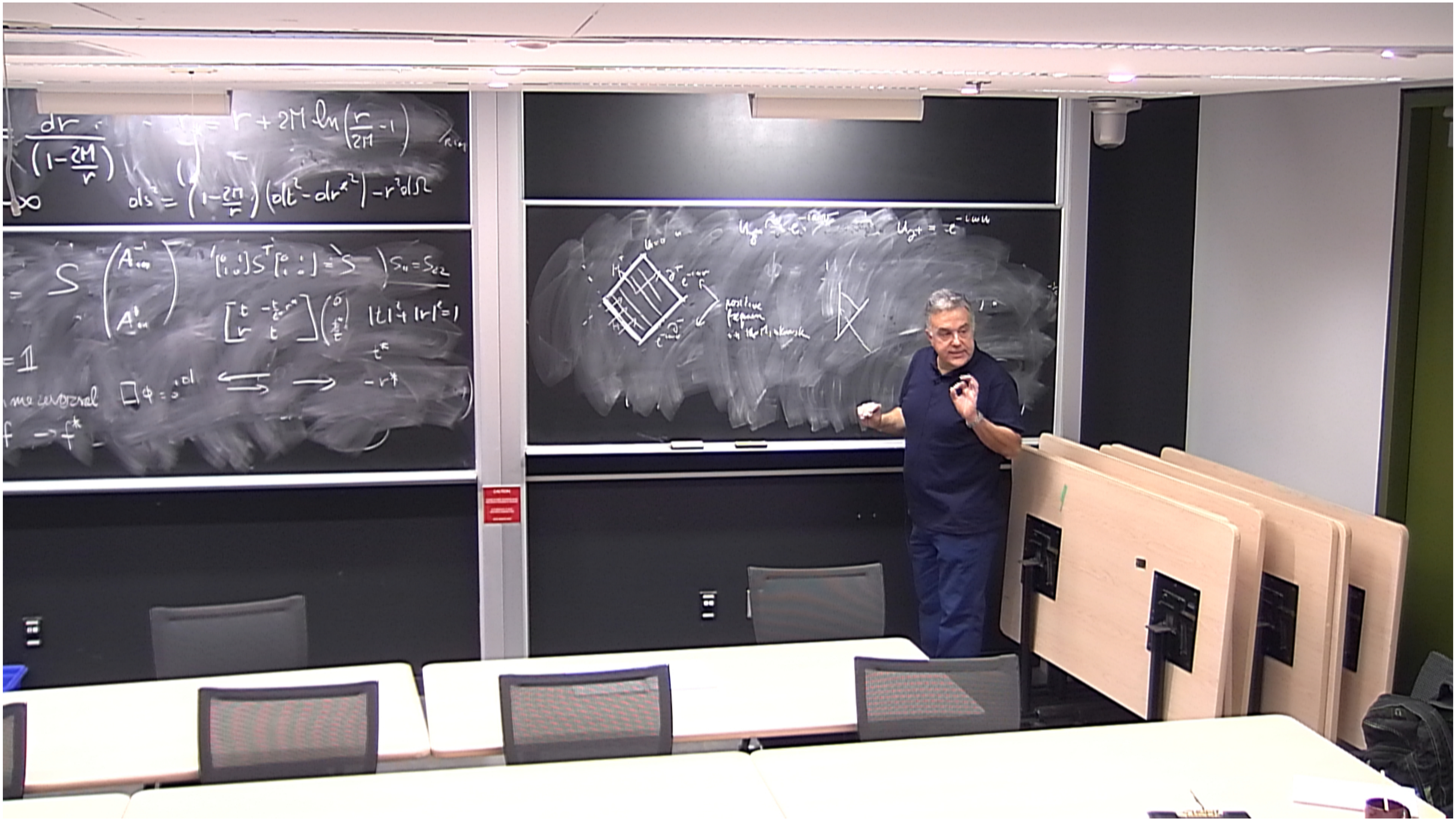

$-r^2 d\Omega$
$S_{11} = S_{22}$
$|t|^2 + |r|^2 = 1$
t^*
$-r^*$
CAUTION

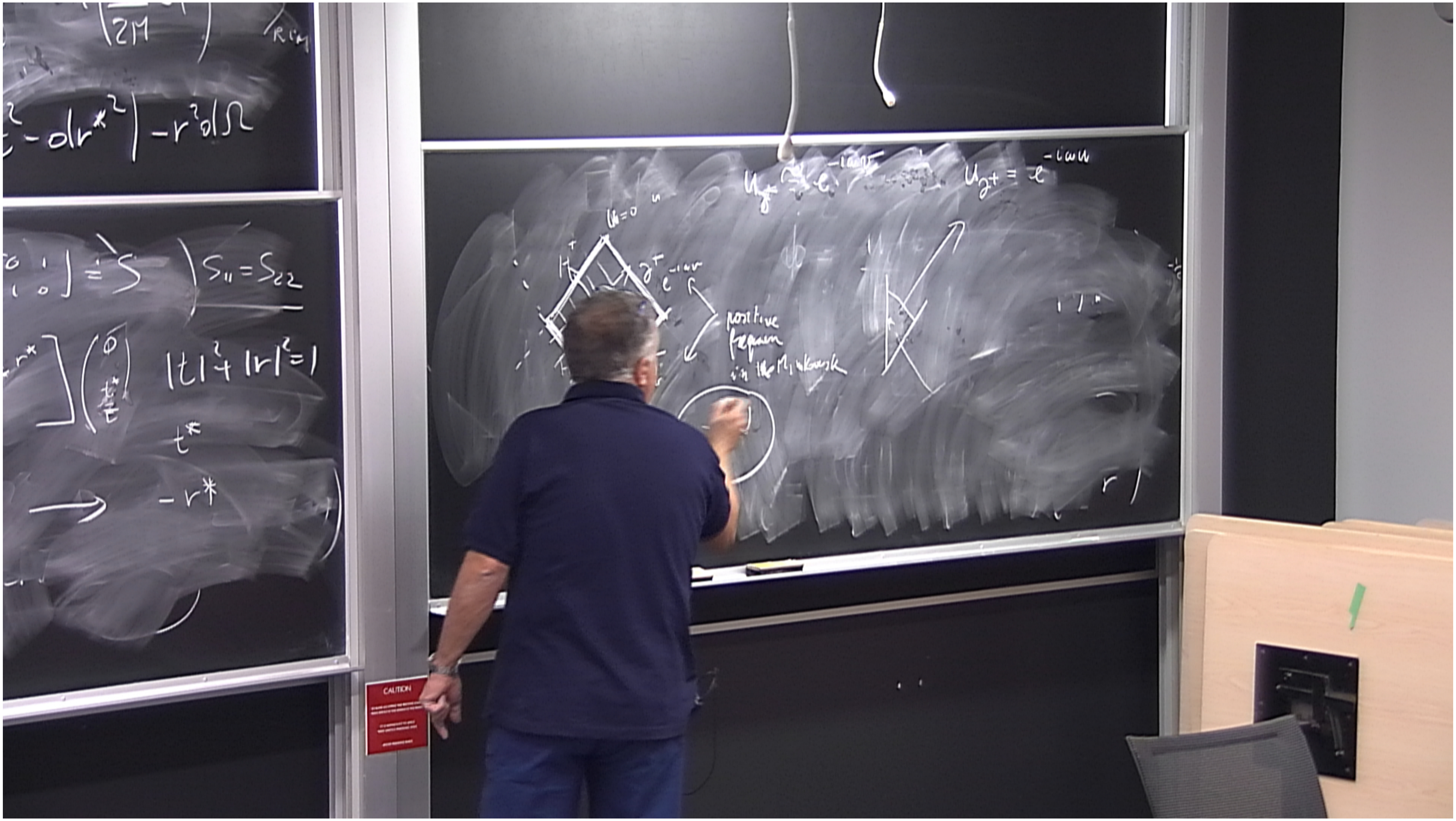

in (−)
out (−)
out (+)
CAUTION

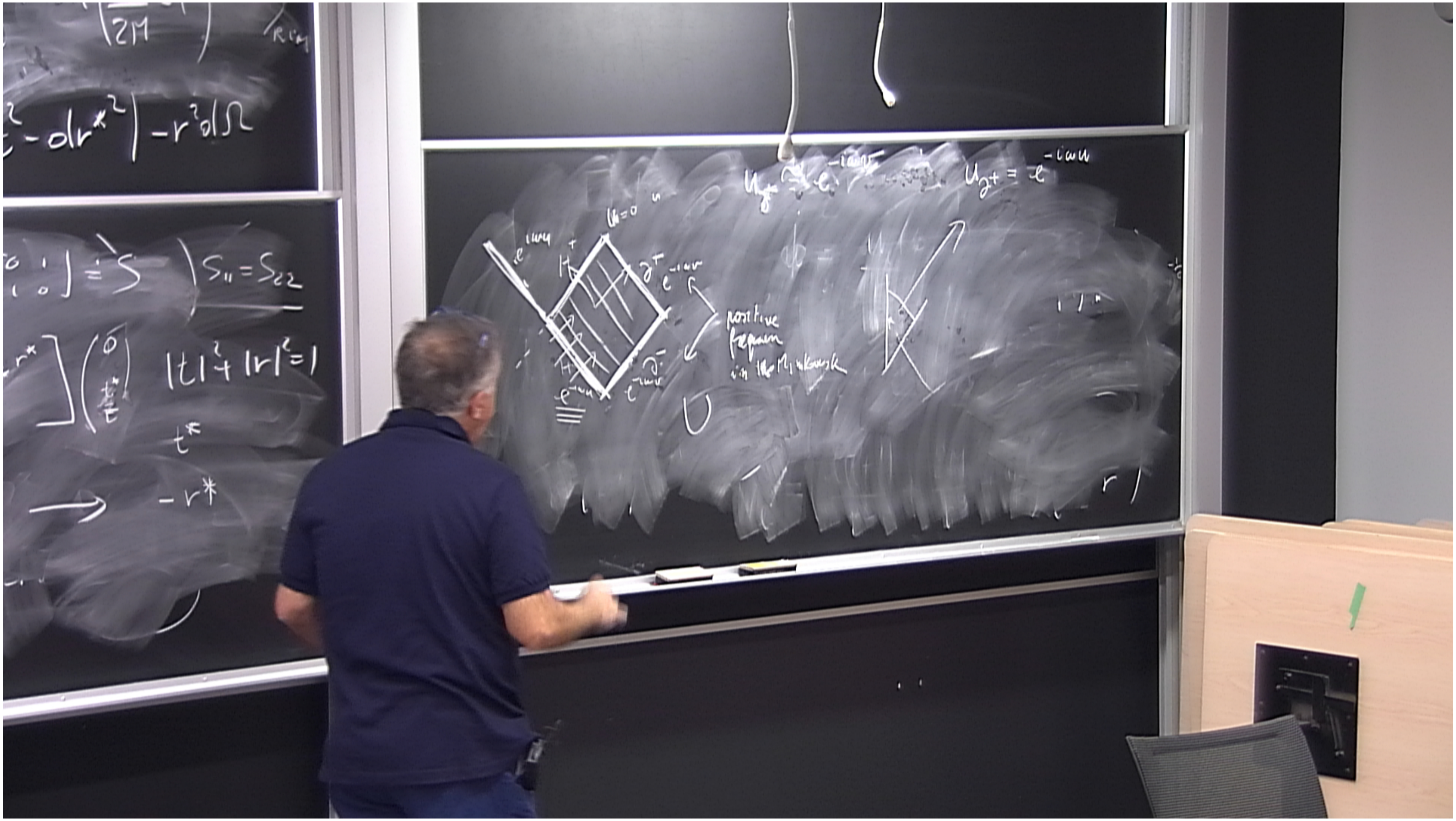


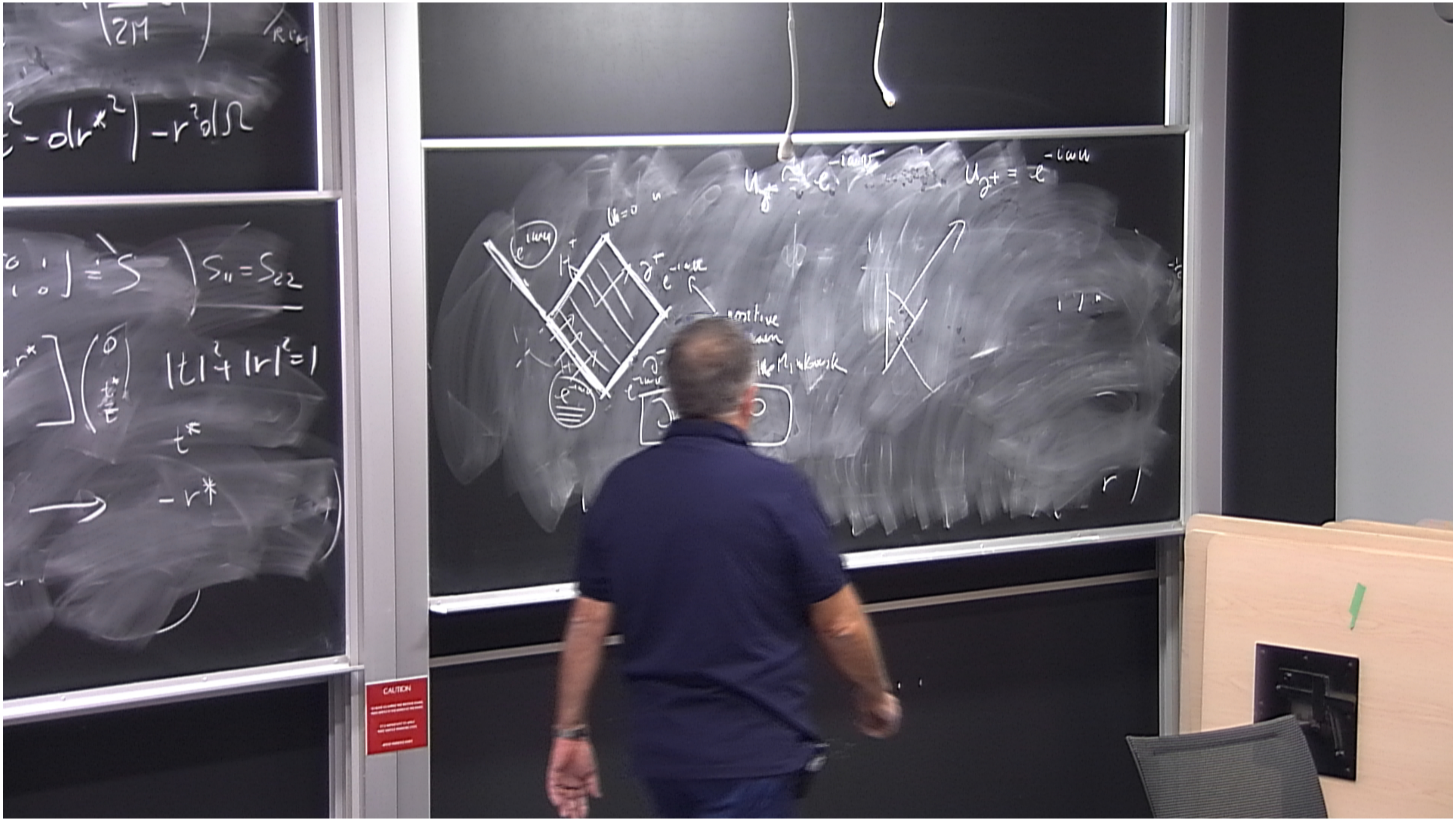


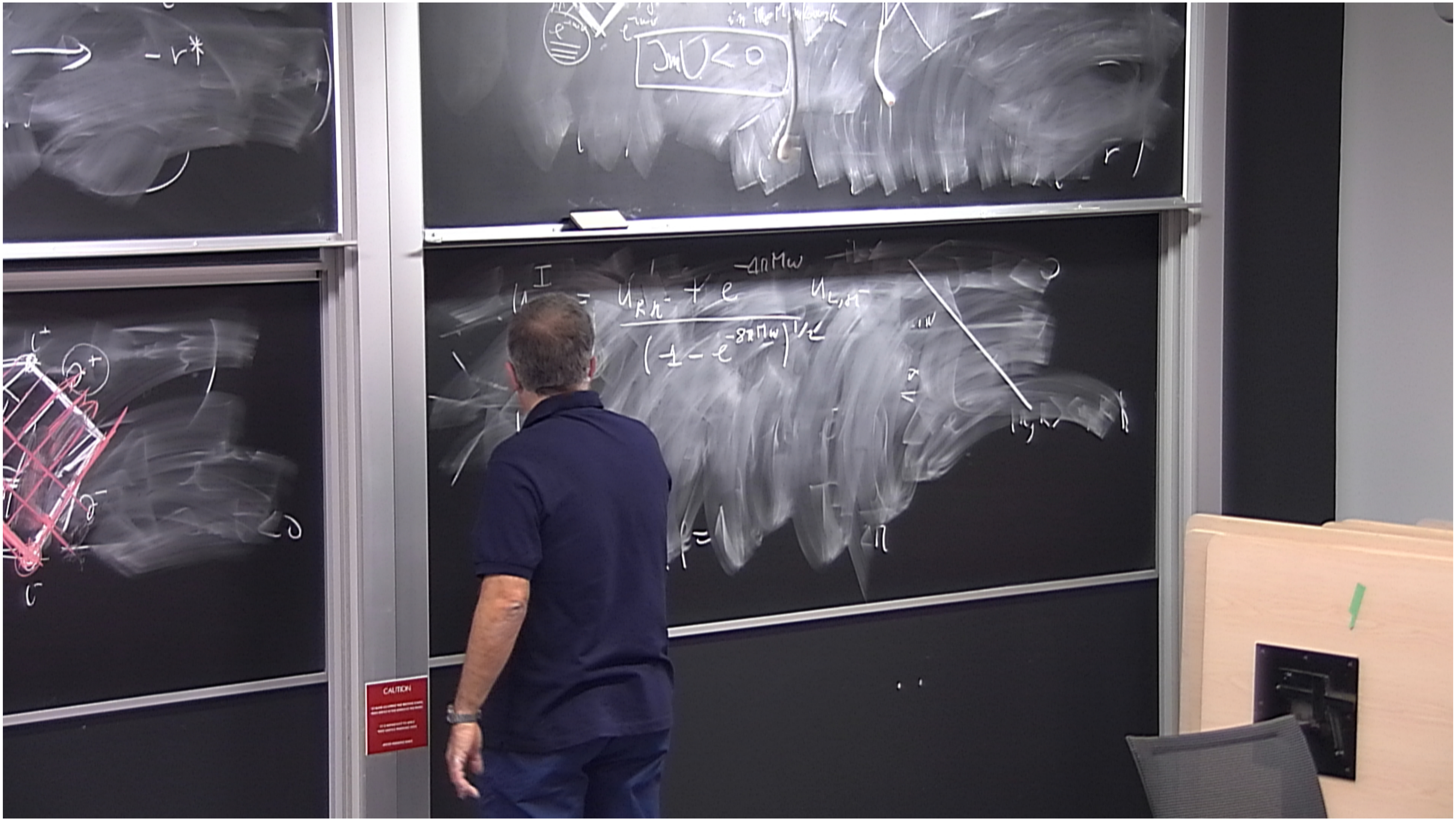

2M
$-dr^{*2}) - r^2 d\Omega$
$S_{11} = S_{22}$
$|t|^2 + |r|^2 = 1$
t^*
$-r^*$
H^+
H^-
$e^{-i\omega v}$
$U_{\mathcal{J}^+} = e^{-i\omega u}$
CAUTION

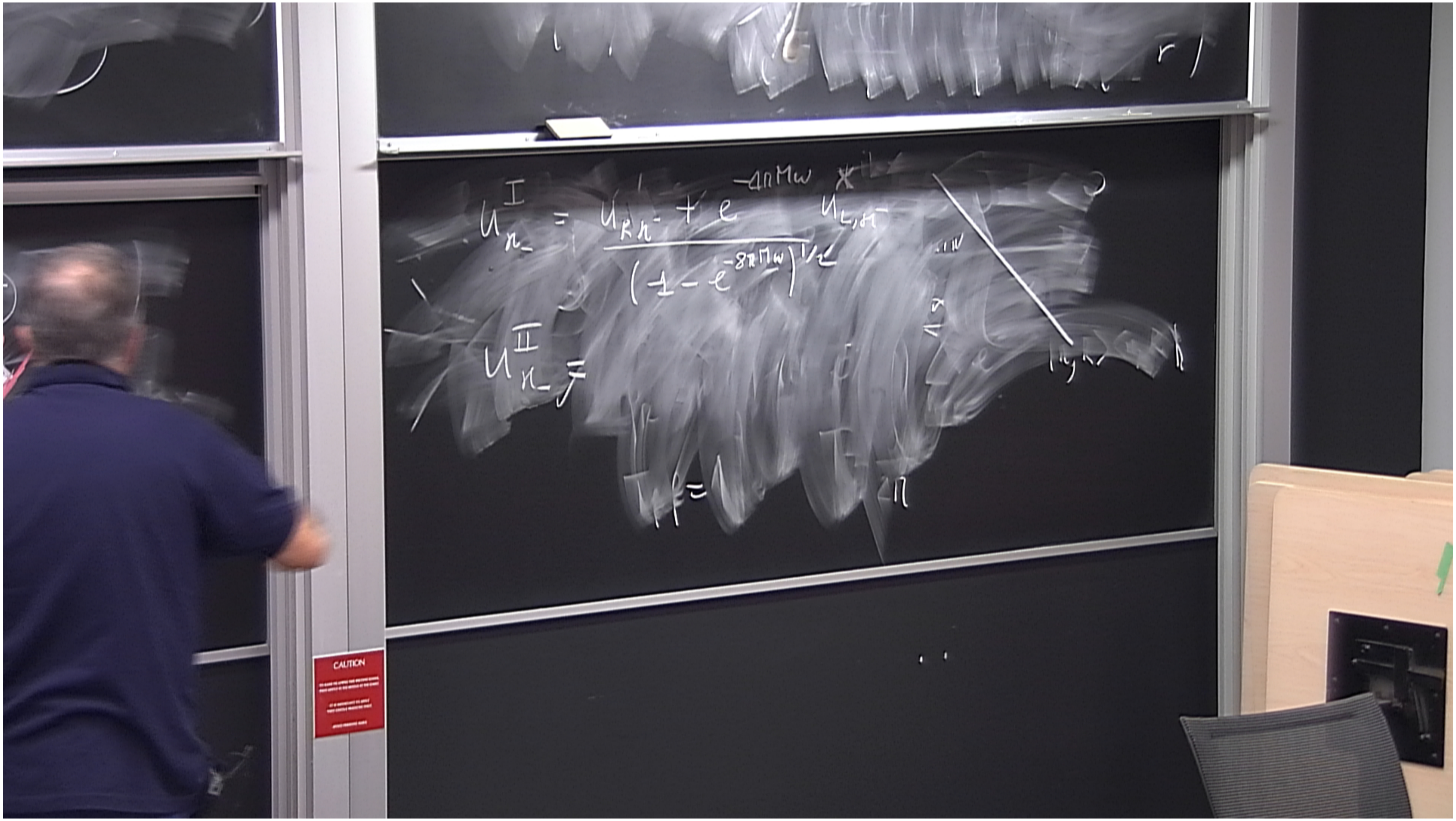




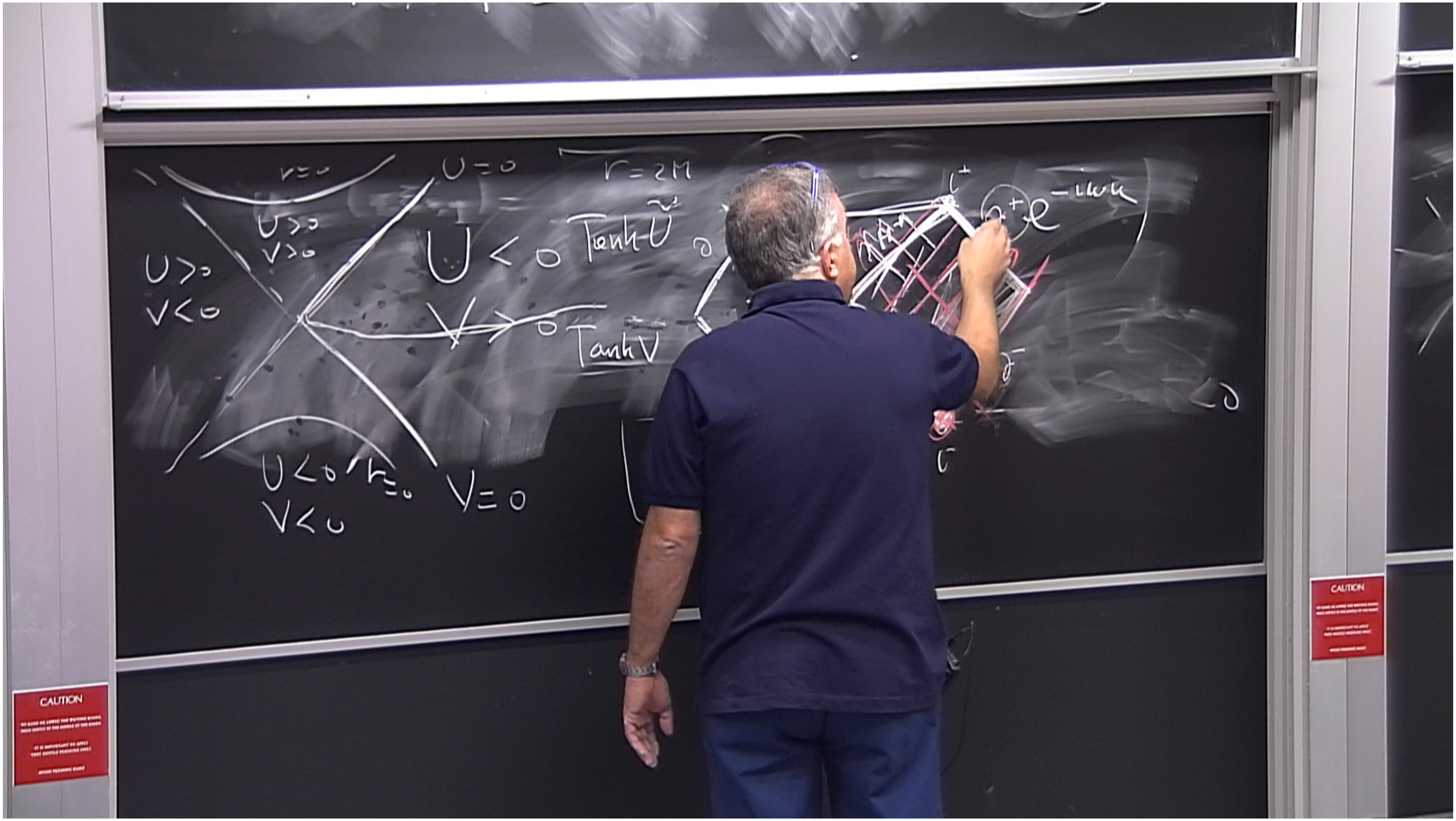

positive
frequen
in the Minkowsk

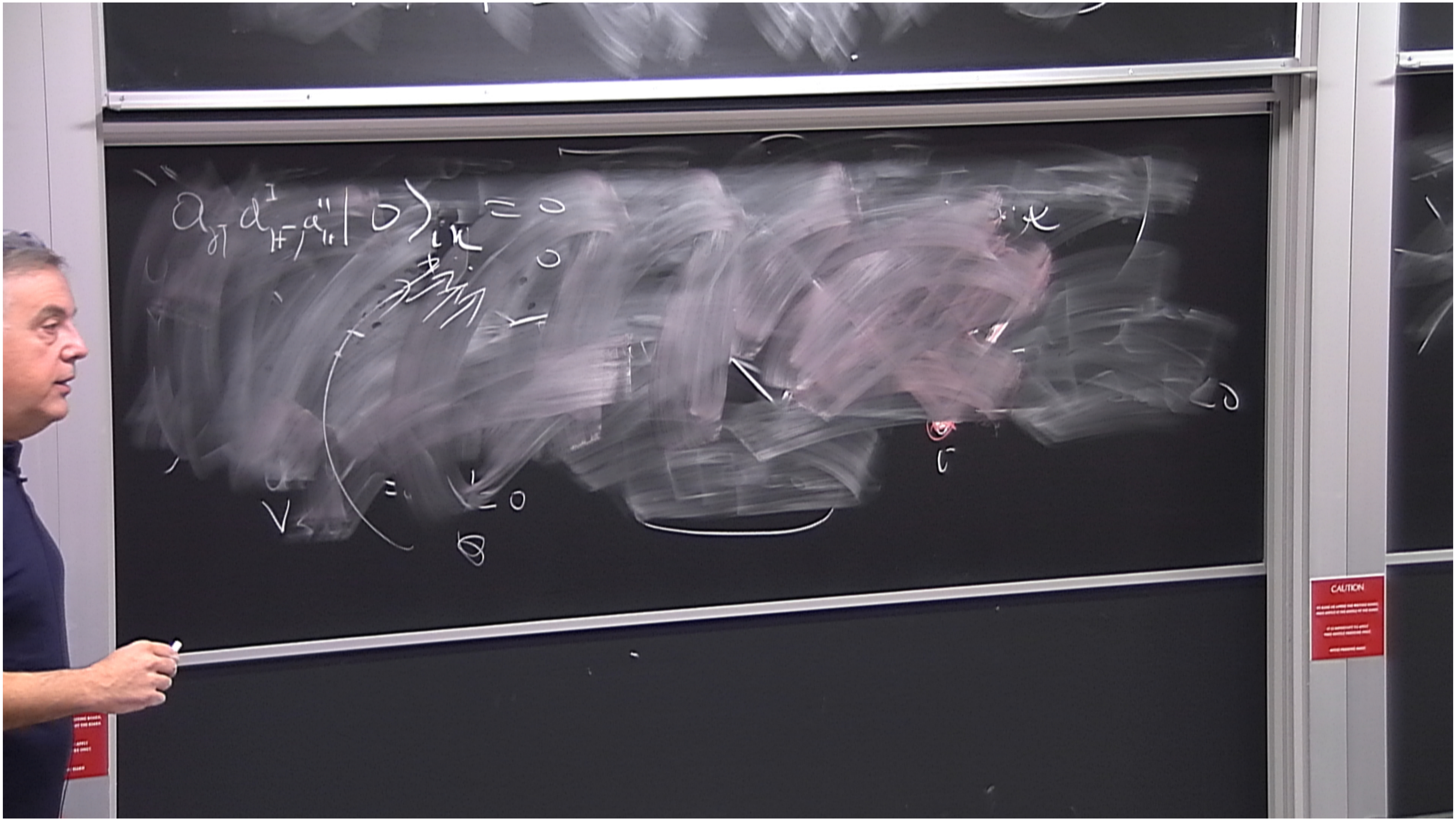


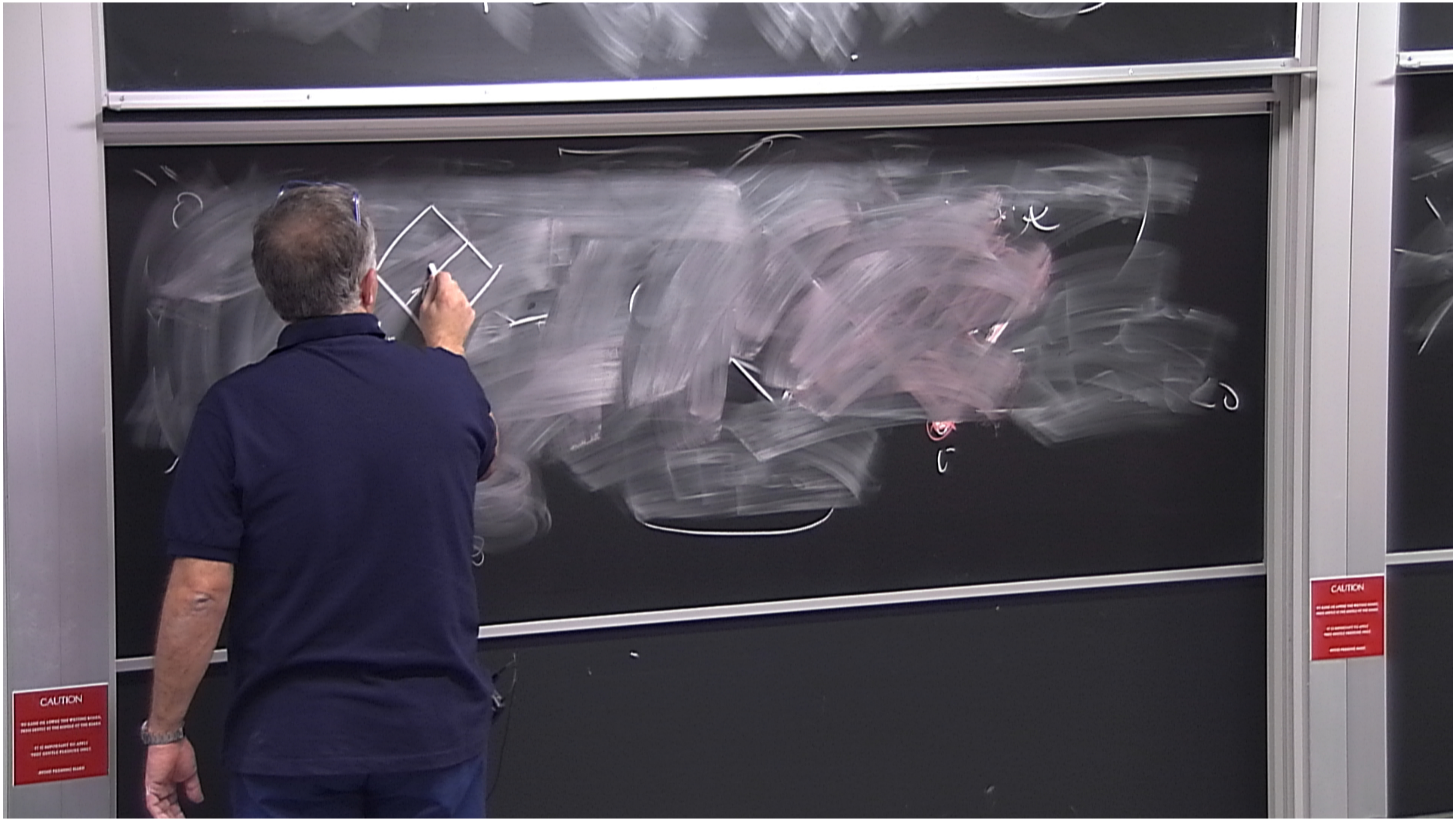

Im U < 0
CAUTION

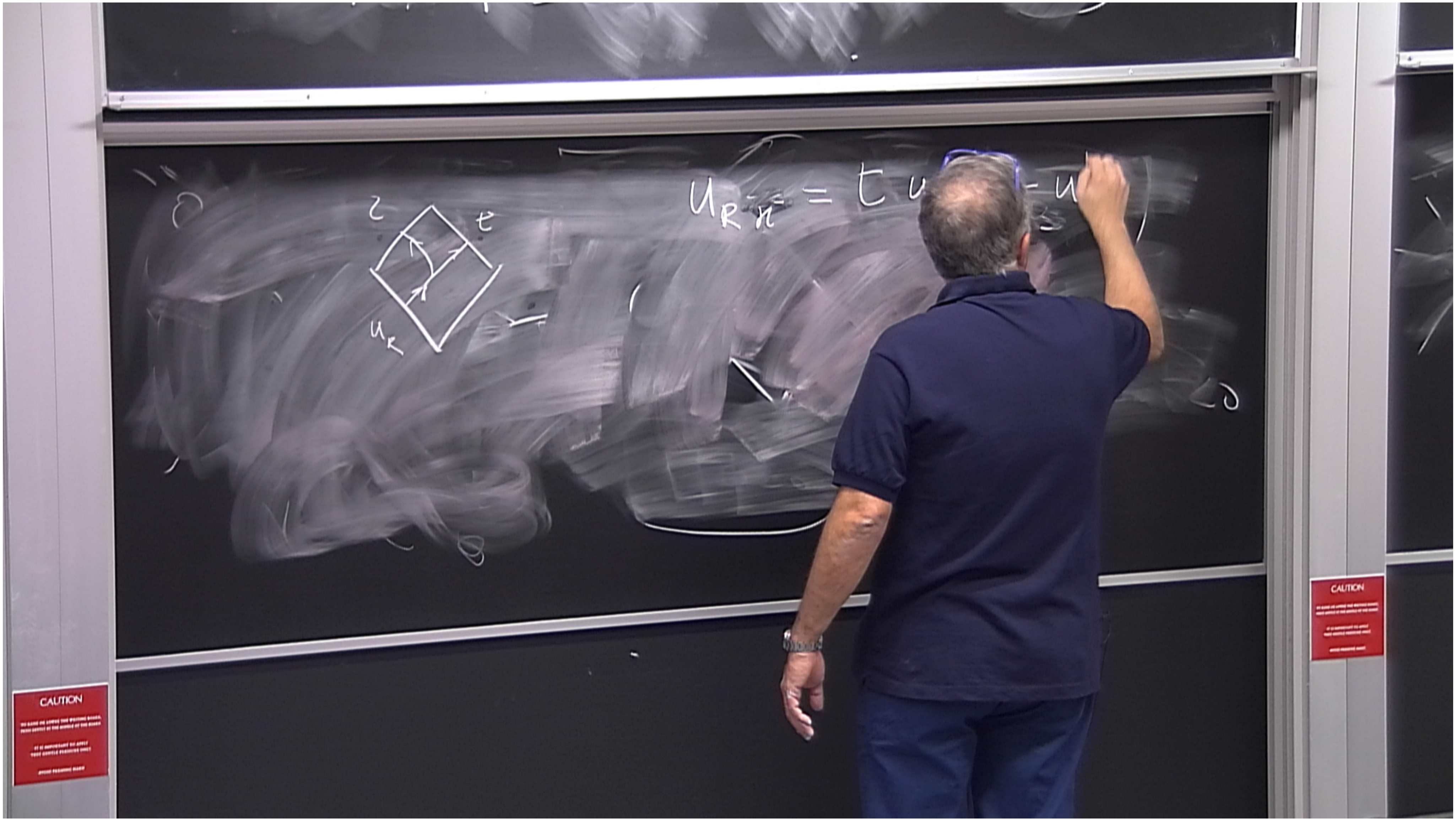


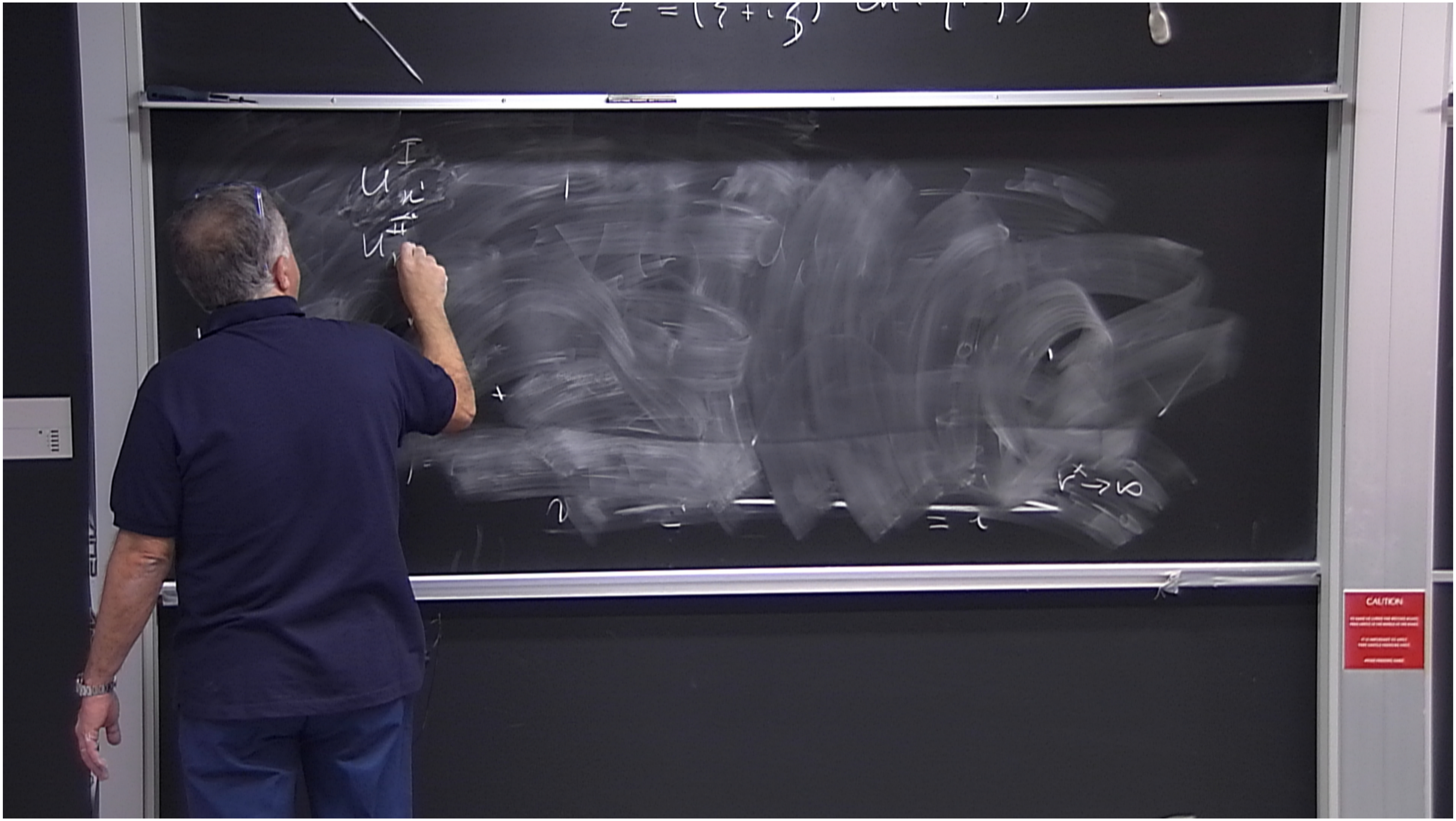
$U^{I}_{R_-} = \frac{U_{Rn^-} + e^{-4\pi M\omega} U^*_{L,H^-}}{(1-e^{-8\pi M\omega})^{1/2}}$
$U^{II}_{R_-} =$
CAUTION

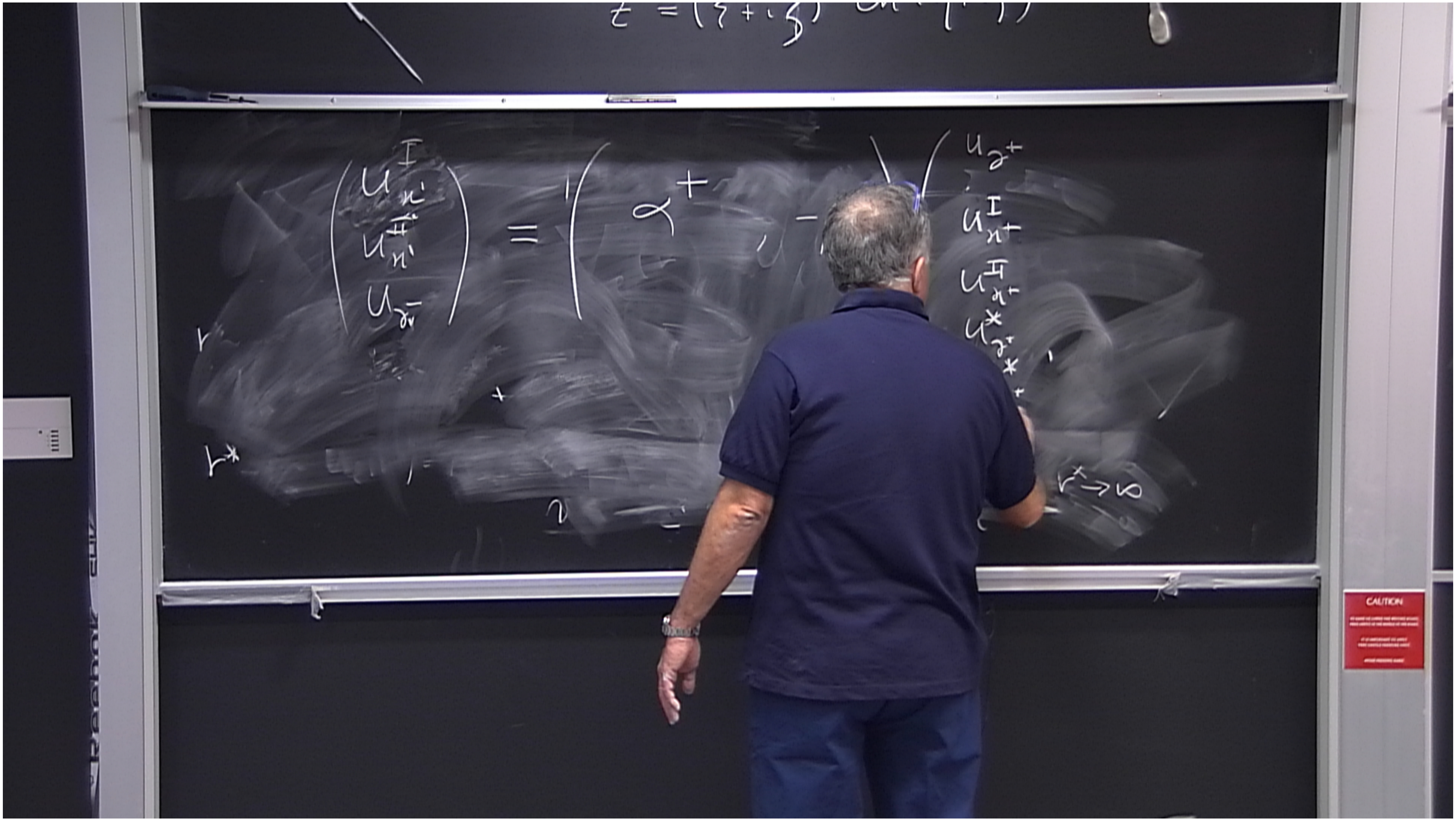

r=0
U=0
r=2M
U>0
V>0
U>0
V<0
U<0
Tanh Ũ
V>0
Tanh V
U<0
V<0
r=0
V=0
CAUTION

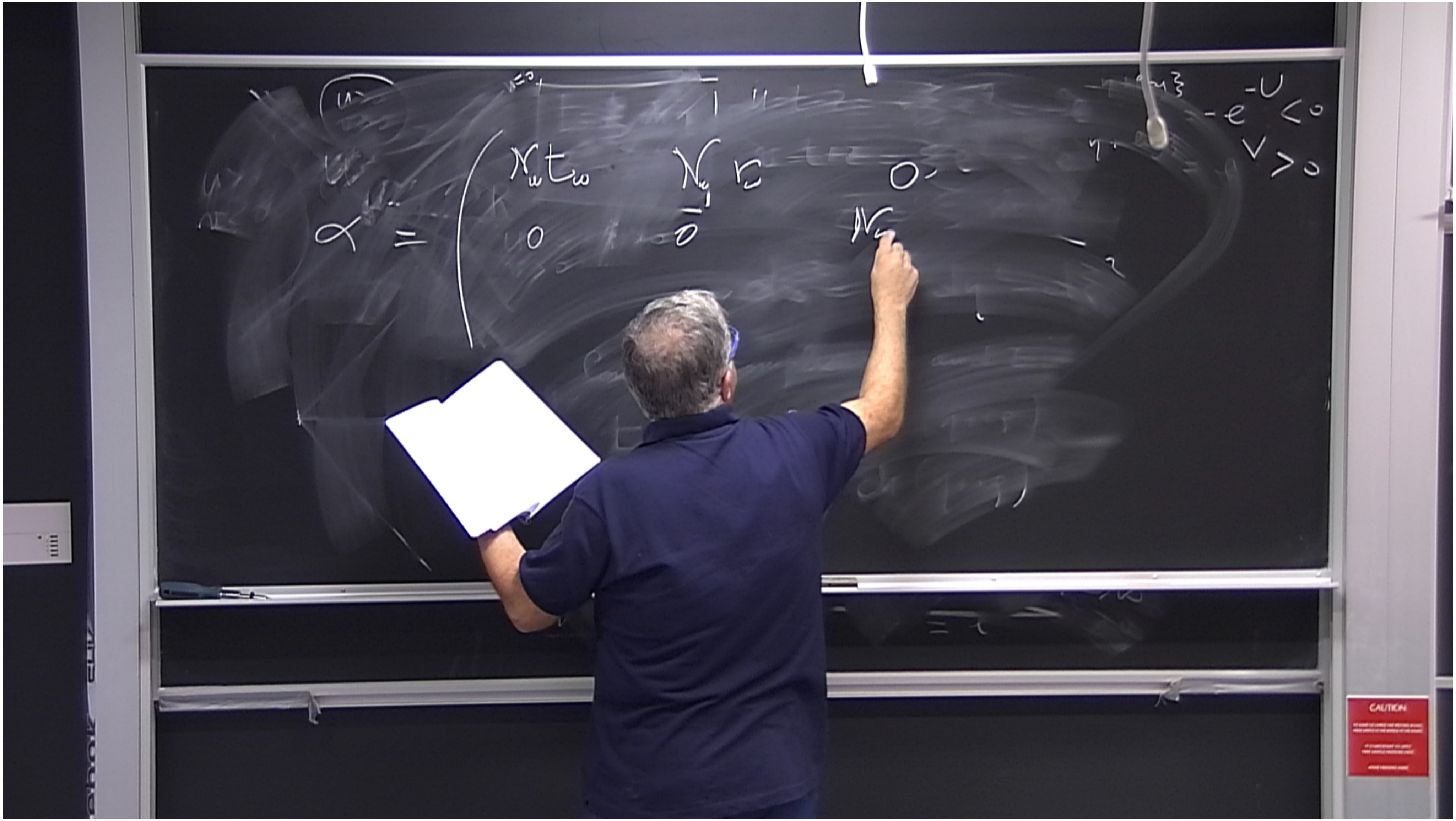

CAUTION

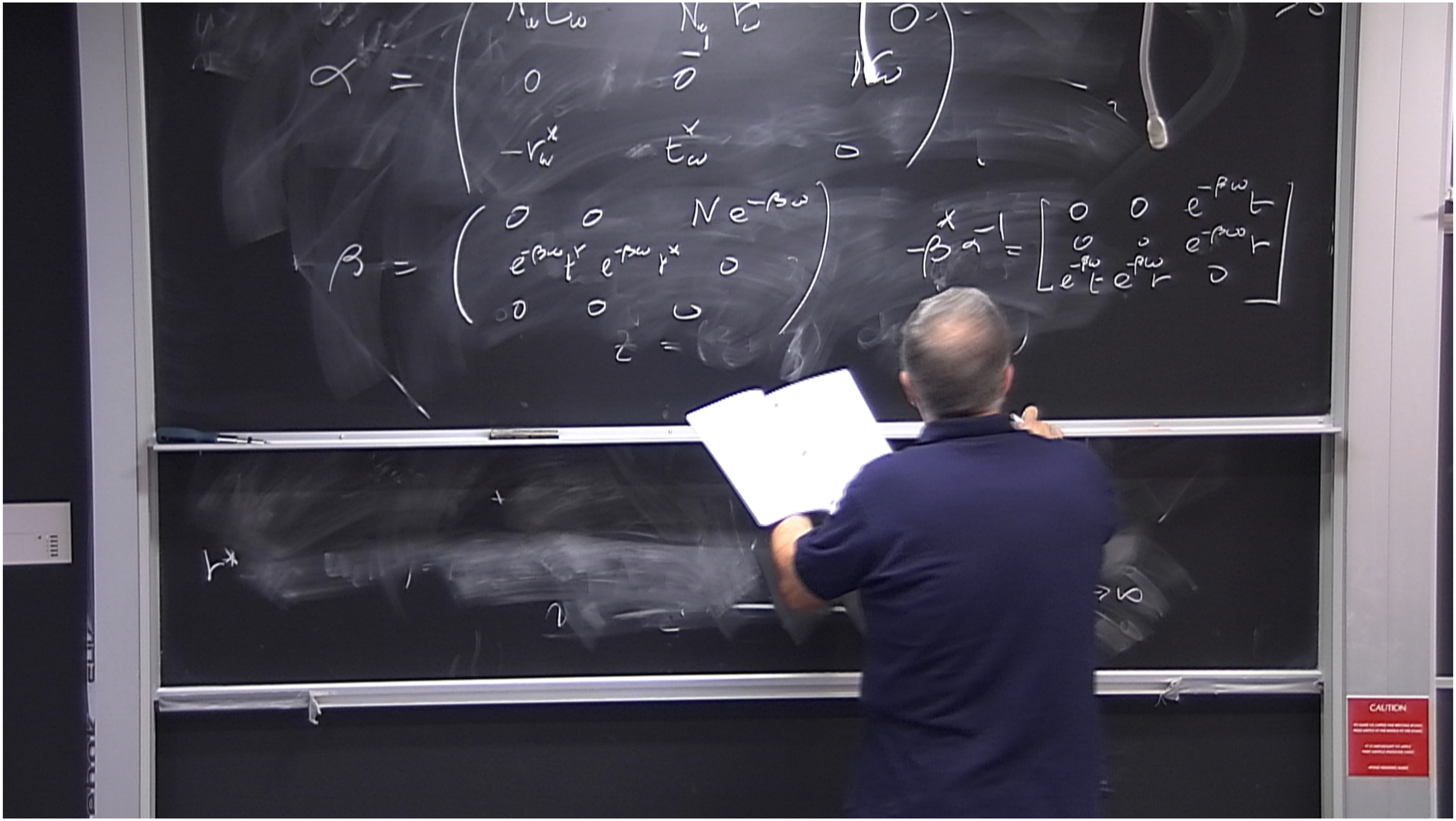


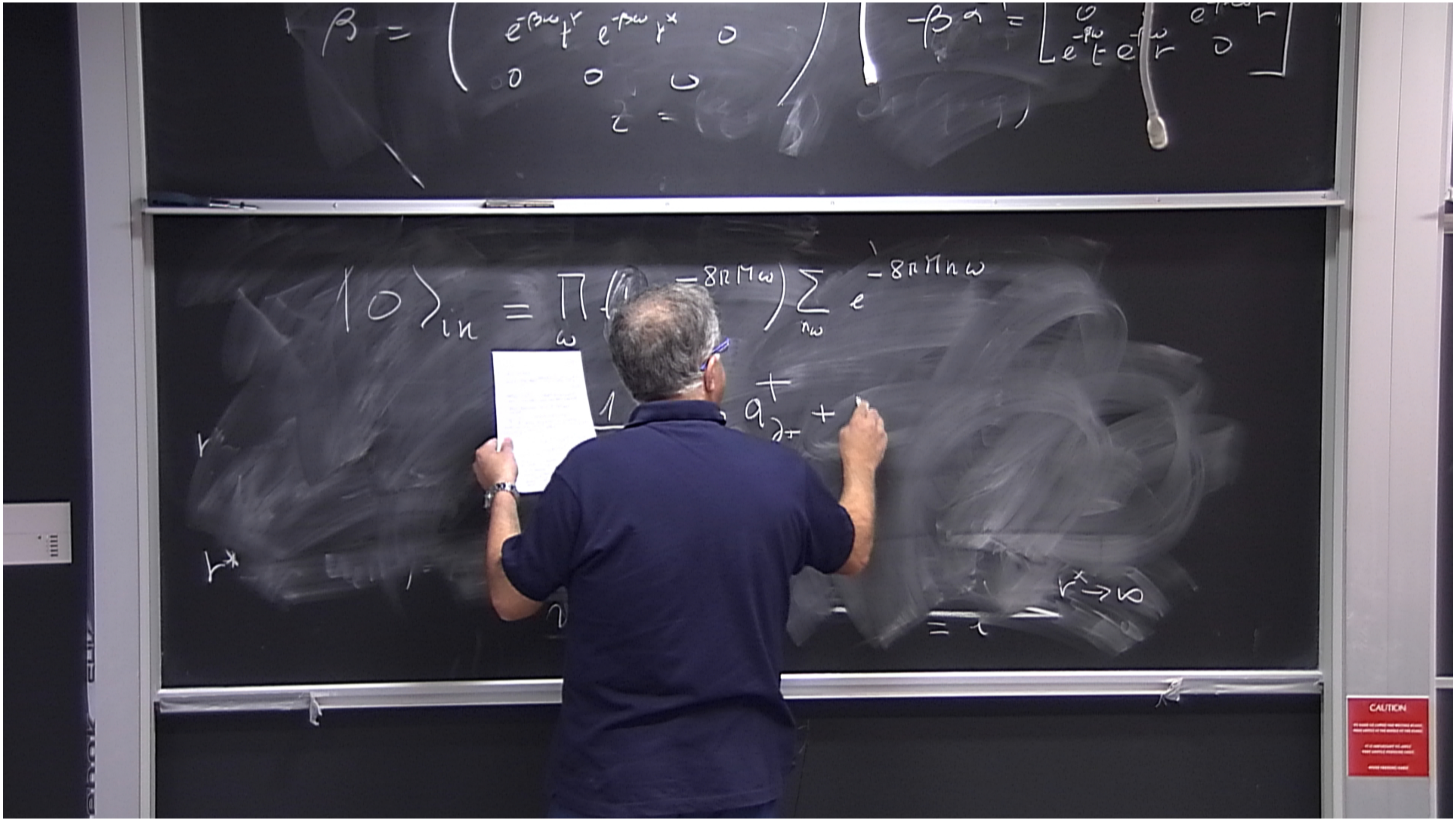


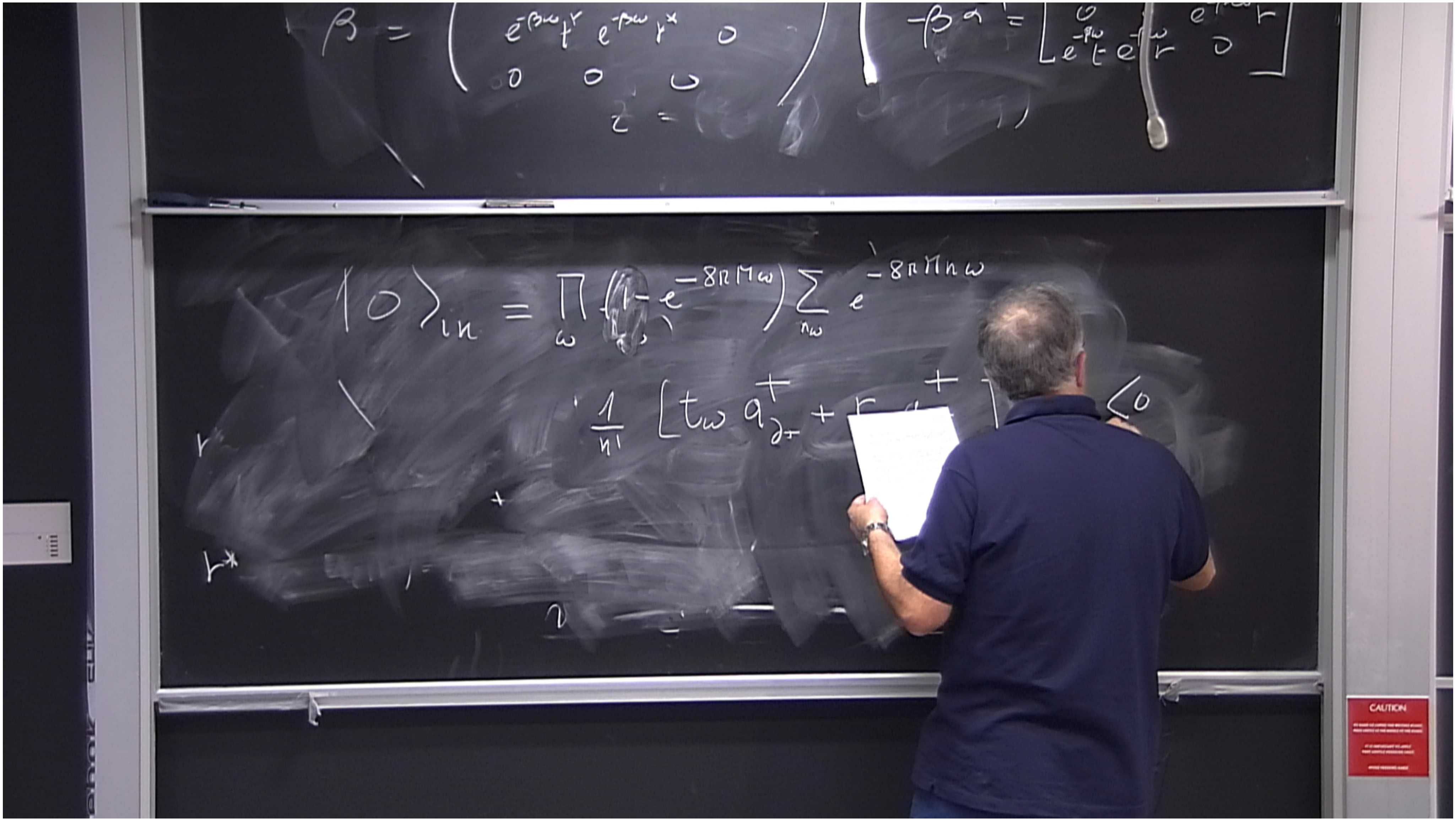


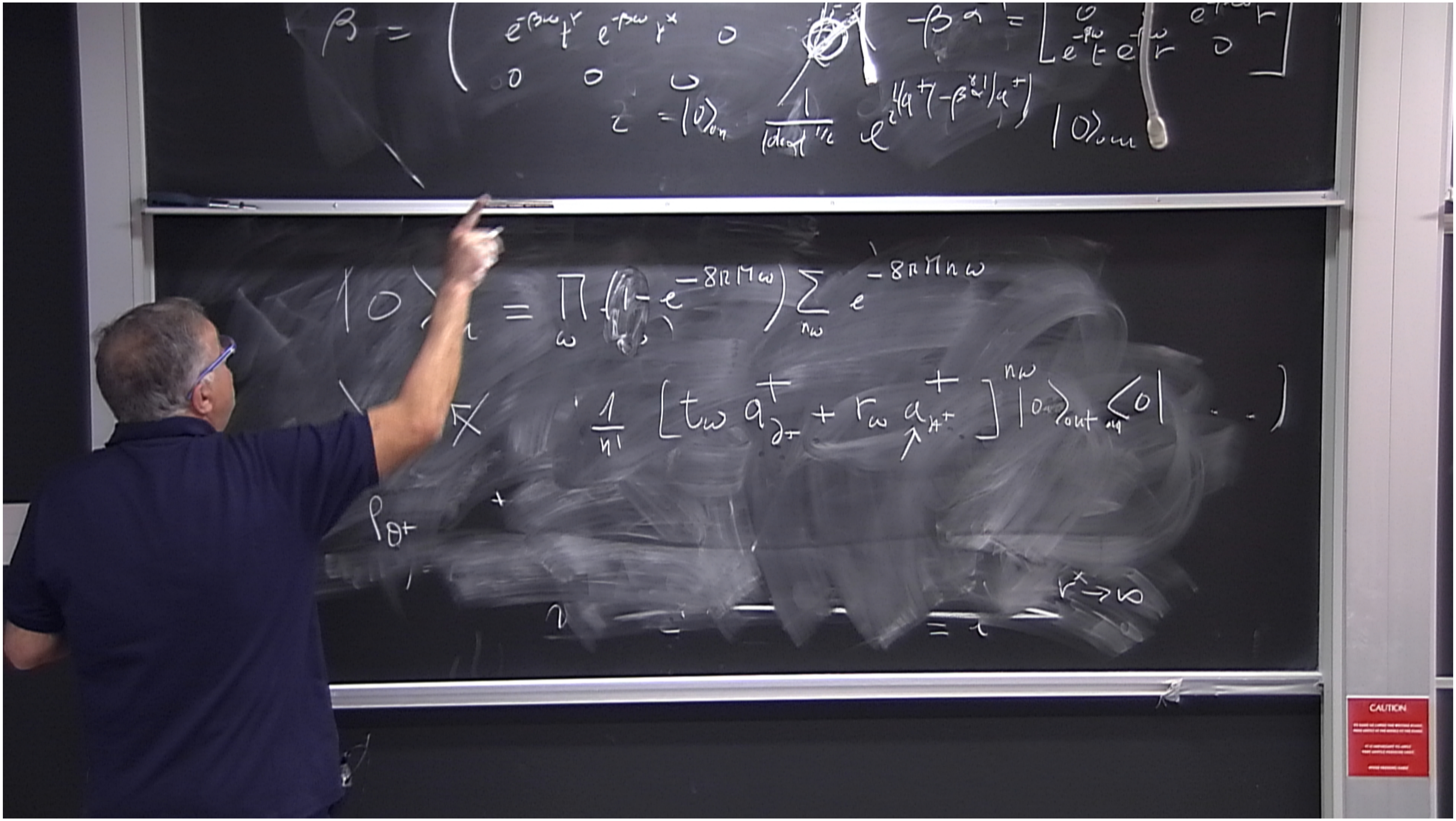


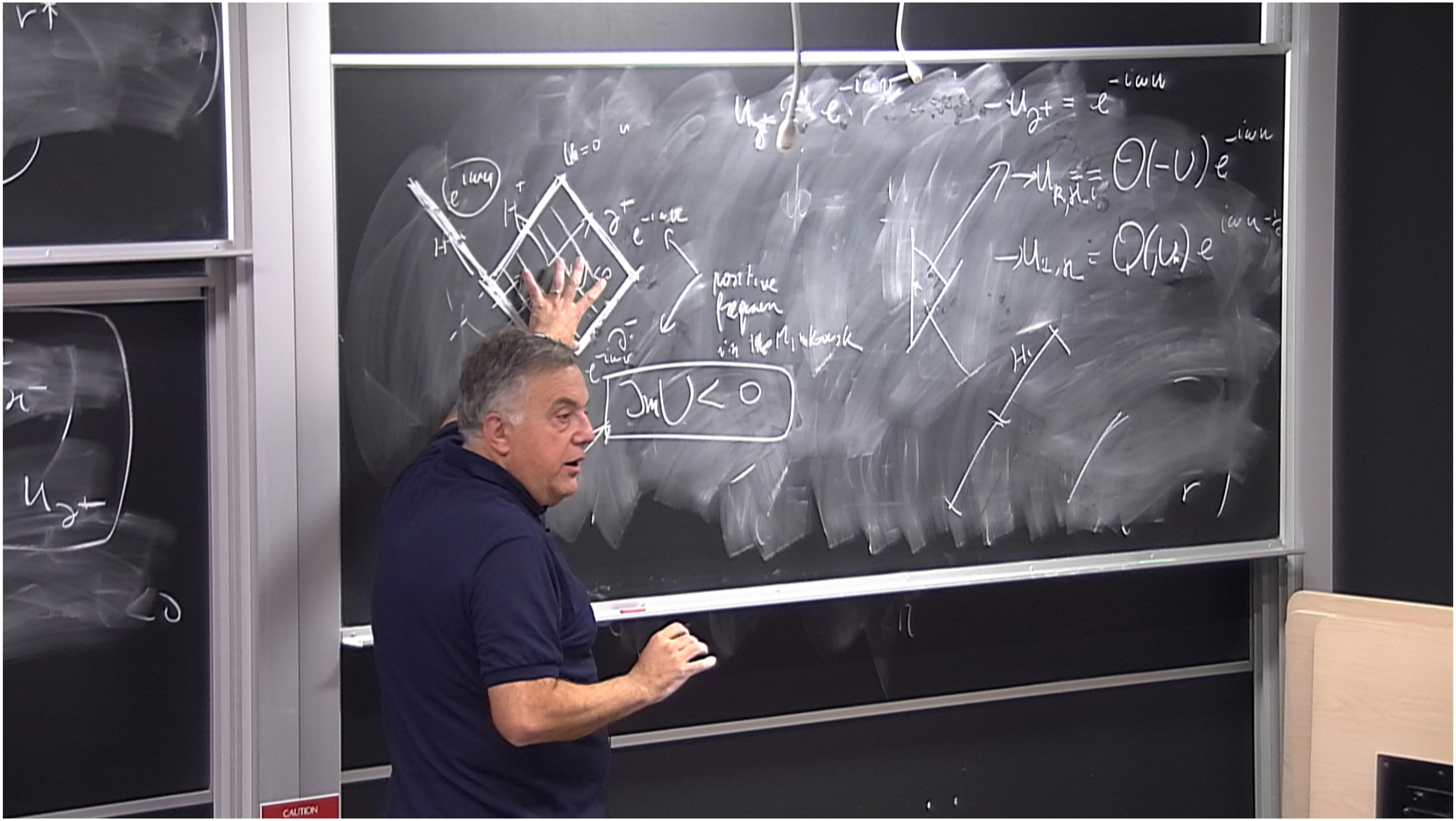

$-e^{-U} < 0$
$V > 0$
CAUTION


α =
β =
$-\beta^* \alpha^{-1} =$




|0⟩_in = Π_ω (1 − e^{−8πMω}) Σ_{n_ω} e^{−8πMnω}
CAUTION


CAUTION


positive
frequen
in the Minkowsk
$\mathrm{Im}\, U < 0$
CAUTION