

Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 5

Date: Oct 07, 2013 10:00 AM

URL: <http://pirsa.org/13100011>

Abstract:

$$(\square + m^2) E(x) = 0$$

$$E(0, \bar{x}') = 0$$

$$\frac{\partial E}{\partial t}(0, \bar{x}') = \delta^3(\bar{x}')$$

has a unique solution

Pauli-Jordan function $D(x)$

$$(\square + m^2)\phi = 0$$

$$\phi(0, x) = \phi(\bar{x}')$$

$$\dot{\phi}(0, x) = \pi(\bar{x}')$$

$e_0^\infty(\mathbb{R}^3)$



CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
WHEN WORKING ON THE BOARD OF THE BOARD
IF AN EMERGENCY OCCURS
PLEASE CONTACT THE BOARD
PLEASE REPORT TO THE BOARD

$$(\square + m^2) \psi(x) = 0$$

$$E(0, \vec{x}) = 0$$

$$\frac{\partial E}{\partial t}(0, \vec{x}) = \delta^3(\vec{x})$$

has a unique solution

Pauli-Jordan function $D(x)$

$$\psi(x) = \int D(x-y) \underline{f}(y) dt d\vec{x}$$

$$(\square + m^2)\psi(x) = 0$$

$$\phi(0, x) = \phi(\vec{x})$$

$$\dot{\phi}(0, x) = \pi(\vec{x})$$

$e^{\text{bo}}(\mathbb{R}^3)$



CAUTION
DO NOT TOUCH THE BOARD
IF YOU TOUCH THE BOARD
IT WILL BE DAMAGED
PLEASE BE CAREFUL

$$(\square + m^2)\psi(x) = 0$$

$$D(x) = R(x) - A(x)$$

$$\langle f, g \rangle = - \int f(x) D(x-y) g(y) d^4x d^4y$$

$$D(-x) = -D(x)$$

$$\langle f, g \rangle = -\langle g, f \rangle$$

$$= \int_{\mathbb{R}^3} \left[\psi_f(t, x) \frac{\partial \psi_g(t, x)}{\partial t} - \frac{\partial \psi_f(t, x)}{\partial t} \psi_g(t, x) \right] d^3x \quad t = \text{const}$$

$$(\square + m^2)\psi(x) = 0$$

$$D(x) = R(x) - A(x)$$

$$\langle f, g \rangle = (-) \int f(x) D(x-y) g(y) d^4x d^4y$$

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$$\psi_f = \int D(x-y) f(y) d^4y \equiv$$

$$= \int_{\mathbb{R}^3} \left[\psi_f(t, x) \frac{\partial \psi_g(t, x)}{\partial t} - \frac{\partial \psi_f(t, x)}{\partial t} \psi_g(t, x) \right] d^3x = \langle \psi_f, \psi_g \rangle$$

$t = \bar{t}$

$$(\square + m^2)\psi(x) = 0$$

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$t = \bar{t}$

$$\langle f, g \rangle_D = \langle \psi_1, \psi_2 \rangle$$

$$J^m(\psi_1, \psi_2) = \psi_1 \partial^m \psi_2 - (\partial^m \psi_1) \psi_2$$

$$\partial_m J^m = \partial_m \psi_1 \partial^m \psi_2 - \partial^m \psi_1 \partial_m \psi_2 + \psi_1 \square \psi_2 - \square \psi_1 \psi_2 - m^2 \psi_1 \psi_2 + m^2 \psi_1 \psi_2 = 0$$

$$\langle \psi_1, \psi_2 \rangle = \int (\psi_1) \overline{\psi_2} - (\overline{\psi_1}) \psi_2$$

$\psi \rightarrow \hat{\psi} \rightarrow \hat{\psi}$

$$[\psi(x), \psi(y)] = -i D(x-y)$$

$W = e^{i\varphi(P)}$

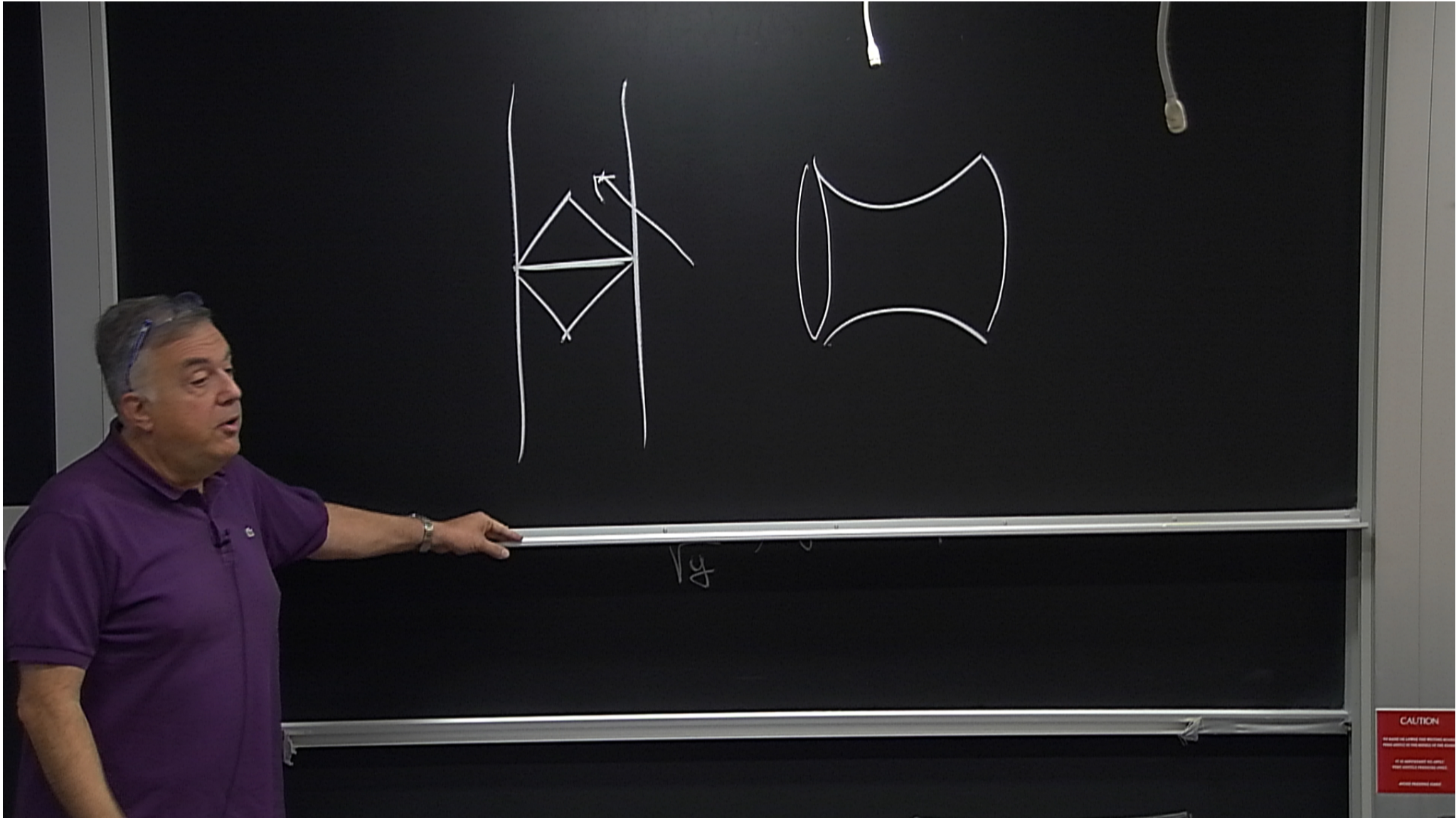
CAUTION
 DO NOT TOUCH THE BOARD
 IT IS IMPROPER TO TOUCH
 THE BOARD SURFACE

$$\langle \psi_1, \psi_2 \rangle = \int (\psi_1) \overline{\psi_2} - (\overline{\psi_1}) \psi_2 \quad \psi \rightarrow \hat{\psi} \rightarrow \hat{\psi}$$

$$[\psi(x), \psi(y)] = -i D(x-y) \quad \leftarrow$$

$W = e^{i\varphi(P)}$

CAUTION
 DO NOT TOUCH THE BOARD
 AT ANY TIME
 IT IS PROHIBITED TO OPEN
 THE BOARD DOOR



$$(\square_g + m^2)E = 0$$

$$E^\pm$$

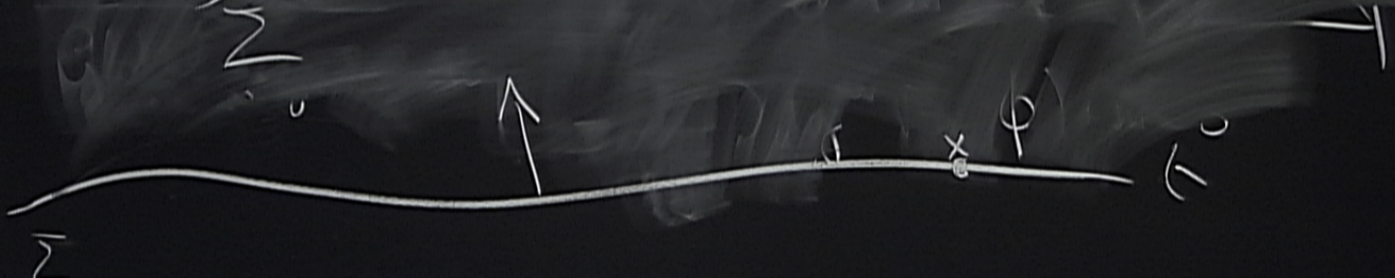
$$\rightarrow E = E^+(x) - E^-(y)$$

Leray - Lichnerowicz

Leray - Lichnerowicz

$$\langle f, g \rangle = - \int f(x) \underline{E(x-y)} g(y)$$

$$\langle \psi_f, \psi_g \rangle = \psi_f \overset{\sum^M}{\Delta} \psi_g$$
$$= - \int \sum_{\mu=0}^M \left(\psi_f \partial_{\mu} \psi_g - (\partial_{\mu} \psi_f) \psi_g \right) \sqrt{h} d^3x$$



Leray - Lichnerowicz

$$\langle f, g \rangle = - \int f(x) \underline{\underline{E(x-y)}} g(y)$$

$$[\phi(x), \phi(y)] = -\underline{\underline{i}} E(x, y)$$

$$\langle \psi_f, \psi_g \rangle = \psi_f \overset{\Delta^m}{\Delta} \psi_g$$

$$= - \int \sum_n \underline{\underline{n^m}} (\psi_f \partial_n \psi_g - (\partial_n \psi_f) \psi_g) \underline{\underline{\sqrt{h}}} d^3x$$



$$\frac{1}{\sqrt{|g|}} \partial_\mu \left(g^{\mu\nu} \sqrt{|g|} \partial_\nu \phi \right) + m^2 \phi = 0$$

$$\frac{1}{\sqrt{|g|}} \partial_\mu \left(g^{\mu\nu} \sqrt{|g|} \partial_\nu \phi \right) + m^2 \phi = 0$$

$$\phi(x) \rightarrow \hat{\phi}(x) \quad \left[\hat{\phi}(x), \hat{\phi}(y) \right] = -i \bar{E}(x-y)$$

CAUTION

$$\langle f, g \rangle = - \int f(x) \underline{\underline{E(x-y)g(y)}} dx$$

$$\rightarrow \langle \phi(x), \phi(y) \rangle = - \underline{\underline{iE(x,y)}} = 0$$

$$\langle \psi_f, \psi_g \rangle = \psi_f \nabla^m \psi_g$$

$$= - \int_{\Sigma} \underline{\underline{n^m}} (\psi_f \partial_m \psi_g - (\partial_m \psi_f) \psi_g) \sqrt{h} d^3x$$

The diagram shows a curved surface labeled Σ . A point x is marked on the surface. A normal vector n^m is shown pointing upwards from the surface. The surface is also labeled with τ_0 at its right end.

$$\langle \psi_1, \psi_2 \rangle = i \int (\psi_1^* \nabla^2 \psi_2 - \nabla^2 \psi_1^* \psi_2) \sqrt{h} d^3x$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE.
IF AT ANY TIME YOU NOTICE
SMELLING OR HEARING NOISES,
STOP IMMEDIATELY.

$$\langle \psi_1, \psi_2 \rangle = i \int (\psi_1^* \nabla^2 \psi_2 - \nabla^2 \psi_1^* \psi_2) \sqrt{h} dx$$

$$C(x, y) = -iE(x, y)$$

CAUTION
 DO NOT TOUCH THE BOARD SURFACE.
 IF AT ANY TIME YOU NOTICE
 ANY DAMAGE TO THE BOARD,
 PLEASE REPORT TO THE
 MAINTENANCE DEPARTMENT.

$$[\phi(x), \phi(y)] = -iE(x, y)$$

$$[\phi(x), \phi(y)]_{\Sigma} = 0$$

$$[n^{\mu} \partial_{\mu} \phi(x), n^{\nu} \partial_{\nu} \phi(y)]_{\Sigma} = 0$$

$$[\phi(x), n^{\mu} \partial_{\mu} \phi(y)]_{\Sigma} = \frac{i}{\sqrt{h}} \delta^3(\vec{x} - \vec{y})$$

$$[n^\mu \partial_\mu \phi(x), n^\mu \partial_\mu \phi(y)]_{\Sigma} = 0$$

$$[\phi(x), n^\mu \partial_\mu \phi(y)]_{\Sigma} = \frac{1}{\sqrt{|h|}} \delta^3(\vec{x} - \vec{y})$$

$$\langle f, \phi \rangle = i \int (f^\dagger(x) n^\mu \partial_\mu \phi - n^\mu \partial_\mu f^\dagger \phi) \sqrt{|h|} d^3x$$

$$[\langle f, \phi \rangle, \langle g, \phi \rangle] = -\langle f, g^* \rangle_{\Sigma}$$

$$\langle f, \phi \rangle = i \int (f^\dagger(x) n^m \partial_m \phi - n^m \partial_m f^\dagger \phi) \sqrt{n} dx^3$$

$$[\langle f, \phi \rangle, \langle g, \phi \rangle] = - \langle f, g^* \rangle \underbrace{\left(\sum \right)}_{\leftarrow}$$

$$\langle u_i, u_j \rangle = \delta_{ij}$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\langle u_i, u_j^* \rangle = 0$$

CAUTION
DO NOT USE LAMP OR WELDING TORCH.
BLIND SPOT AT THE MIDDLE OF THE BOARD.
IT IS DANGEROUS TO APPLY
HIGH VOLTAGE PRESSURE GASES.
AVOID BREASTING AREA

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$$\langle u_i, u_j \rangle = \delta_{ij}$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\langle u_i, u_j^* \rangle = 0$$

$$\phi = \sum (u_i a_i + u_i^* a_i^*)$$

$$[a_i, a_j^*] = \delta_{ij}$$

$$[a_i, a_j] = 0$$

CAUTION
DO NOT TOUCH THE GLASS SURFACE
OR THE SURFACE OF THE WINDOW
IF IT IS NECESSARY TO APPLY
HIGH PRESSURE PRESSURE ONLY
ALLOW PRESSURE ONLY

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$$= \delta_{ij}$$

$$= -\delta_{ij}$$

$$= 0$$

$$\phi = \sum (u_i a_i + u_i^* a_i^*)$$

$$[a_i, a_j^+] = \delta_{ij}$$

$$[a_i, a_j] = 0$$

$$[\phi(x), \phi(y)] = C(x, y)$$

$$= -iE(x, y) = \sum_j (u_j(x) u_j^*(y) - u_j^*(x) u_j(y))$$

$$\phi_{el}(x)$$

$$\{\phi_{el}(x), \phi_{el}(y)\} = -E(x, y)$$

$$[\phi(x), \phi(y)] = C(x, y) = -iE(x, y) \quad \text{CCR}$$

$$\phi(x) \rightarrow \hat{\phi}(x)$$

$$\langle u_i, u_j^* \rangle = 0$$

$$[a_i, a_j] = 0$$
$$[\phi(x), \phi(y)] = C(x, y)$$

$$[\phi(x), \phi(y)] = C(x, y) = -iE(x, y) \quad \text{CCR} \quad \mathbb{R}^3$$

$$\phi(x) \rightarrow \hat{\phi}(x)$$

Commutator
Two point function

$$\langle u_i, u_j^* \rangle = 0$$

$$[\phi(x), \phi(y)] = C(x, y)$$

$$= -iE(x, y) = \sum_j (u_j(x) \dot{u}_j^*(y) - \dot{u}_j(x) u_j^*(y))$$

CAUTION
DO NOT OPEN THE FRONT COVER
UNLESS YOU ARE INSTRUCTED TO DO SO

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$$\phi(x) \rightarrow \hat{\phi}(x)$$

$$|0\rangle \quad \mathcal{P} \hat{\phi}(x) |0\rangle = \mathcal{K}$$

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle = W(x, y)$$

$$\langle 0 | \phi(x_1) \dots \phi(x_n) | 0 \rangle$$

$$\phi(x) \rightarrow \hat{\phi}(x)$$

$$|0\rangle \quad \mathcal{P} \hat{\phi}(x) |0\rangle = \mathcal{H}$$

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle = W(x, y)$$

$$\langle 0 | [\phi(x) \phi(y) - \phi(y) \phi(x)] | 0 \rangle = C(x, y) =$$

$$= W(x, y) - W(y, x)$$

$$W(x, y) - W(y, x) = C(x, y)$$

$$W'(x, y) = W(y, x)$$

$$W(x, y) - W(y, x) = C(x, y)$$

$$W(x, y) = S(x, y) + A(x, y)$$

$$W(x, y) = \frac{W(x, y) + W(y, x)}{2} + \frac{W(x, y) - W(y, x)}{2}$$

$$\frac{G^{(1)}(x, y)}{2} + \frac{C(x, y)}{2}$$

$$\begin{aligned}
 \langle \psi_f, \psi_g \rangle &= \int \psi_f^\dagger \nabla^m \psi_g \\
 &= \int \sum_i n^m (\psi_f^\dagger \partial_m \psi_g - (\partial_m \psi_f) \psi_g) \dots d^3x
 \end{aligned}$$

(D+m)ψ(x,y) = 0
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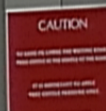
Σ
 Σ

$$W(x, y) - W(y, x) = C(x, y) \quad (\square + m^c) S(x, y) \Rightarrow$$

$$W(x, y) = S(x, y) + A(x, y)$$

$$\underbrace{W(x, y)}_{\text{circled}} = \frac{W(x, y) + W(y, x)}{2} + \frac{W(x, y) - W(y, x)}{2}$$

$$= \frac{G(x, y)}{2} + \frac{C(x, y)}{2}$$



$$\rightarrow \mathcal{D}[\phi(x), \phi(y)] = -\underline{\underline{E(x, y)}} = 0$$

$$W(\bar{f}, g) = \int \underline{\underline{W(x, y)}} \bar{f}(x) g(y) d^4x d^4y$$

$$W(\bar{f}, f) = \|\phi(f) | 0 \rangle\|^2 \geq 0$$

Σ

$$\rightarrow \mathcal{D}[\phi(x), \phi(y)] = -\underline{\underline{E}}(x, y) = 0$$

$$W(\bar{f}, g) = \int \underline{\underline{W}}(x, y) \bar{f}(x) g(y) d^4x d^4y$$

$$W(\bar{f}, f) = \|\phi(f)|0\rangle\|^2 \geq 0$$

Σ

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\langle u_i, u_j^* \rangle = 0$$

$$\int_S \nabla^\mu \phi \nabla_\mu \phi - m^2 \phi^2 \sqrt{g} d^4x = S(x, y)$$

$$[a_i, a_j^\dagger] = \delta_{ij}$$

$$[a_i, a_j] = 0$$

$$\langle \alpha_j | 0 \rangle$$

$$[\phi(x), \phi(y)] = C(x, y)$$

$$= -iE(x, y) = \sum_j (u_j(x) u_j^\dagger(y) - u_j^\dagger(x) u_j(y))$$

CAUTION
 ALL OPERATIONS SHOULD BE PERFORMED WITH CARE AND PRECISION.
 IF A WARNING OR ALERT MESSAGE APPEARS, STOP IMMEDIATELY.
 PLEASE REPORT ANY ISSUES TO THE SUPPORT TEAM.

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$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\langle u_i, u_j^* \rangle = 0$$

$$\int \nabla^2 \phi \nabla \mu \phi - m^2 \phi^2 \sqrt{g} dV$$

$$S(x, y)$$

$$[a_i, a_j^\dagger] = \delta_{ij}$$

$$[a_i, a_j] = 0$$

$$\langle \alpha_j | 0 \rangle$$

$$[\phi(x), \phi(y)] = C(x, y)$$

$$= -iE(x, y) = \sum_j (u_j(x) \dot{u}_j^*(y) - \dot{u}_j^*(x) u_j(y))$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE SURROUNDING AREA
WHILE PRESENTING

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OR THE SURROUNDING AREA
WHILE PRESENTING

$$[\phi(x), \phi(y)] = C(x, y) = -iD(x, y)$$

$$(\square + m')\phi = 0$$

$$\phi \rightarrow \hat{\phi}$$

$$(\square + m') \phi_{\alpha} = 0$$

$$\phi \rightarrow \hat{\phi}$$

$$\left[\begin{array}{l} U(\Lambda, a) \hat{\phi} U(\Lambda, a)^{-1} = \phi(\Lambda x + a) \\ \text{Dirac } P^{\mu} \in V^+ \end{array} \right]$$

CAUTION
DO NOT STAND ON BOARDING BRIDGE
PLEASE STAY OFF OF THE BRIDGE AT ALL TIMES
IF IN EMERGENCY CALL 911
THANK YOU FOR YOUR COOPERATION

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$$[\phi(x), \phi(y)] = C(x, y) = -iD(x, y)$$

$$(\square + m')\phi(x) = 0$$

$$\phi \rightarrow \hat{\phi}$$

$$\int U(\Lambda, a) \hat{\phi} U(\Lambda, a)^{-1} = \phi(\Lambda x + a)$$

$$[D, P^{\mu}] \in V^+$$

CAUTION
 DO NOT TOUCH THE BOARD WHEN
 IT IS HOT OR WHEN THE BOARD IS
 BEING CLEANED. PLEASE CONTACT
 THE SERVICE CENTER FOR ASSISTANCE.
 SERVICE CENTER: 1-800-854-3747

$$\underline{\underline{P}}^n |k\rangle = k^n |k\rangle$$

1-particle
 $\mathcal{H}^{(1)}$

$$U(a)|k\rangle = e^{ika} |\underline{k}\rangle \quad k^0, k^1, k^2, k^3$$

$$\underline{\underline{U(\Lambda)}} |k\rangle = |\underline{\Lambda k}\rangle \quad \Lambda k$$

$$\begin{aligned} U(a)U(\Lambda)|k\rangle &= U(\Lambda)U(\Lambda^{-1}a)|k\rangle \\ &= e^{i(\Lambda k)_0 a} U(\Lambda)|k\rangle = \mathcal{Q} \\ &= |\underline{\Lambda k}\rangle \end{aligned}$$

$$\underline{\underline{P}}^{\mu} |k\rangle = k^{\mu} |k\rangle$$

1-particle
 $\mathcal{H}^{(1)}$

$$U(a) |k\rangle = e^{ika} |k\rangle \quad k^0, k^1, k^2, k^3$$

$$\underline{\underline{U(\Lambda)}} |k\rangle = |\Lambda k\rangle \quad \Lambda k$$

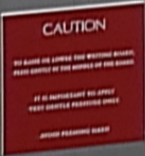
$$\begin{aligned} U(a)U(\Lambda) |k\rangle &= U(\Lambda)U(\Lambda^{-1}a) |k\rangle \\ &= e^{i(\Lambda k) \cdot (\Lambda^{-1}a)} U(\Lambda) |k\rangle = \mathbb{1} \\ &= |\Lambda k\rangle \end{aligned}$$

$$\begin{aligned}
 \mathbb{1} &= \int d\mu(k) |k\rangle\langle k| = \\
 &= U(\lambda)U(\lambda^{-1}) = \mathbb{1} = \int d\mu(k) |k\rangle\langle k|
 \end{aligned}$$

Positivity energy

$$\frac{P^2}{P^0} = P^0 - P^i P^i = m^2$$

$$\int \theta(k^0) \delta(k^2 - m^2) |k\rangle\langle k| = \mathbb{1}$$



$$|k'\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} |k\rangle \langle k|k'\rangle = |k'\rangle$$

$$\langle k|k'\rangle = (2\pi)^3 2k^0 \delta(\vec{k} - \vec{k}')$$

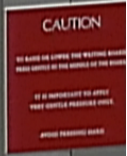
relativistic normalization

$$\mathbb{1} = \int d\mu(k) |k\rangle\langle k| = \int d^3k \frac{\delta(k^0 - \sqrt{k^2 + m^2})}{2k^0} |k\rangle\langle k|$$

Positivity energy

$$P^2 = P^0 P_{\mu} = m^2$$

$$\int \theta(k^0) \delta(k^2 - m^2) |k\rangle\langle k| d^4k$$



$$|k'\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} |k\rangle \langle k|k'\rangle = |k'\rangle$$

$$\langle k|k'\rangle = (2\pi)^3 2k^0 \delta(\vec{k} - \vec{k}')$$

impro

relativistic normalization

$$|f\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} f(\vec{k}) |k\rangle$$

$$\langle f|g\rangle = \int \frac{f^*(k)g(k) d^3k}{(2\pi)^3 2k^0}$$

$$\begin{aligned}
 & \dots = |0\rangle \\
 & |k_1, \dots, k_n\rangle = \frac{1}{n!} \sum_{\pi \in S_n} \langle q_{\pi_1} | k_{\pi_1} \rangle \dots \langle q_{\pi_n} | k_{\pi_n} \rangle
 \end{aligned}$$

$$|k_1, \dots, k_n\rangle = \frac{1}{n!} \sum_{\pi \in S_n} \langle q_1 | k_{\pi(1)} \rangle \dots \langle q_n | k_{\pi(n)} \rangle$$

$1 \dots n$
 $\pi(1) \dots \pi(n)$

$$\rightarrow A^\dagger(k) |k_1, \dots, k_n\rangle = \sqrt{n+1} |k, k_1, \dots, k_n\rangle$$

$$\rightarrow A(k) |k_1, \dots, k_n\rangle = \frac{1}{\sqrt{n}} \sum_{j=1}^n \langle k | k_j \rangle |k_1, \dots, \cancel{k_j}, \dots, k_n\rangle$$

$\mathcal{J}(\mathcal{H}^{(n)})$

$\phi(x)$

$P^2 = P, P_{\mu\nu} = m^2 \delta_{\mu\nu}$ (2/13)

A

$$\begin{aligned}
 (k) \quad |k_1 \dots k_n\rangle &= \frac{1}{\sqrt{n}} \sum_{j=1}^n \langle k | k_j \rangle \\
 & - i \frac{\hbar^2 m}{k^2} t + i k x
 \end{aligned}$$

$$\int e^{-ikx} \left(A(k) + C^*(k, x) A^\dagger \right)$$

$$\hat{\phi}(x) U(x, \Lambda)^{-1} = \phi(\Lambda x + \alpha)$$

$$\rightarrow A^\dagger(k) |k_1, \dots, k_n\rangle = \sqrt{n+1} |k, k_1, \dots, k_n\rangle$$

$$\rightarrow A(k) |k_1, \dots, k_n\rangle = \frac{1}{\sqrt{n}} \sum_{j=1}^n \langle k | k_j \rangle |k_1, \dots, k_j, \dots, k_n\rangle$$

$$\begin{aligned} & \int_{\mathcal{J}(d^3k)} \hat{\phi}(x) \int_{\mathcal{J}(d^3k)} e^{-ikx} (A(k) + A^\dagger(k)) \frac{d^3k}{(2\pi)^3 2\omega} \\ & U(\Lambda, \Lambda) \hat{\phi}(x) U(\Lambda, \Lambda)^{-1} = \hat{\phi}(\Lambda x + a) \end{aligned}$$

$$P^2 = P^\mu P_\mu = m^2 \int_{\mathcal{J}(d^3k)} \theta(k^0) \delta(k^2 - m^2) |k\rangle \langle k| d^4k$$

$$\begin{aligned}
 &= \frac{1}{\sqrt{n}} \sum_{j=1}^n \langle k | k_j \rangle |k_1 \dots k_j \dots k_n\rangle \\
 &Q(k) \quad ikx \quad A(k) \quad \frac{d^3 k}{(2\pi)^3 2\omega} \\
 &Q [A(k), A^+(k')] = \langle k | k' \rangle \delta \\
 &A(k = -k)
 \end{aligned}$$

$$\frac{A(k)}{(2\pi)^3 2\omega} + \frac{A^\dagger(k)}{(2\pi)^3 2\omega}$$

$$\int \frac{d^3k}{(2\pi)^3 2\omega}$$

$$\langle [A(k), A^\dagger(k')] \rangle = \langle k | k' \rangle \delta$$

$$\langle k | = [a(k), a(k')] = \delta(k - k')$$

$$(2\pi)^3 2k^0 \delta(k - k')$$

$$(Nk) |k\rangle \langle k|$$

$$\frac{\delta(k^0 - \sqrt{k^2 + m^2})}{2k^0}$$

$$k^0 \delta(k^2 - m^2) |k\rangle \langle k| \frac{d^4k}{(2\pi)^4}$$

$$\langle \phi(x) \phi(y) \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\omega} e^{-ik(x-y)}$$

$$-iD(x-y) = \frac{1}{(2\pi)^3} \epsilon(k^0) \delta(k^2 - m^2)$$

$$W(x-y) = \frac{1}{(2\pi)^3} \int \Theta(k^0) \delta(k^2 - m^2) d^4k$$



$$U(\omega) U(\Lambda) k = U(\Lambda) U(\Lambda^{-1} \omega) k$$

$$(\Lambda k, a) = (\Lambda^{-1} k, a)$$

$$\langle f | g \rangle = \int_{-\infty}^{\infty} f(k)g(k) dk$$

$$C(x, y) = W(x, y) - W(y, x)$$

$$C(x-y) = W(x-y) - W(y-x)$$

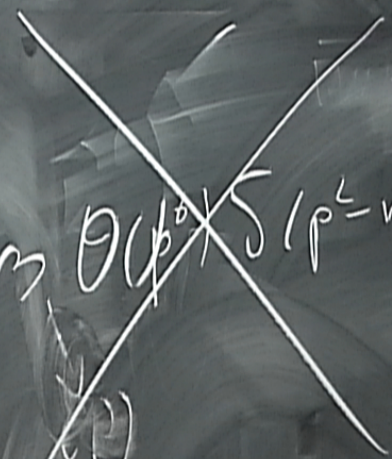
$$C(\xi) = W(\xi) - W(-\xi)$$

$$\xi = x-y$$

$$(1 + m^2) \hat{W}(z)$$

$$\Rightarrow (-p^2 + m^2) \hat{W}(p)$$

$$\langle k | = \alpha \theta(p^0) \delta(p^2 - m^2) + \beta \theta(p^0) \delta(p^2 - m^2)$$



$$(\square + m^2) W(z)$$

$$\Rightarrow (-p^2 + m^2) \widehat{W}(p)$$

$$\langle k | = \theta(p^0) \delta(p^2 - m^2) + p \theta(p^0) \delta(p^2 - m^2)$$

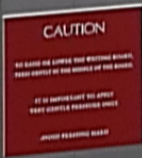
$$W(x, y) = \frac{1}{(2\pi)^3} \int e^{-i(p \cdot x - y)} \theta(p^0) \delta(p^2 - m^2)$$

$$\begin{aligned}
 \langle f | g \rangle &= \int f(x) W(x-y) g(y) d^4x d^4y \\
 f \in \mathcal{L}_0^{\infty}(\mathbb{R}^4, \mathbb{C}) & \\
 &= \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\sqrt{k^2+m^2}} \underbrace{\tilde{f}(-\vec{k}, -\bar{k}) \tilde{g}(-\vec{k}, -\bar{k})}_{\tilde{f}(\vec{k}) \tilde{g}(\vec{k})}
 \end{aligned}$$

Translation - invariant

$$C(x-y) = W(x-y) - W(y-x)$$

$$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p)$$



Translation - invariant

$$C(x-y) = W(x-y) - W(y-x)$$

$$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p)$$

$$\tilde{W}(p) = \Theta(p^0) C(p) =$$

$$\tilde{W}(p) = -\Theta(p^0) C(-p) = \Theta(p^0) C(p)$$

Translation - invariant

$$C(x-y) = W(x-y) - W(y-x)$$

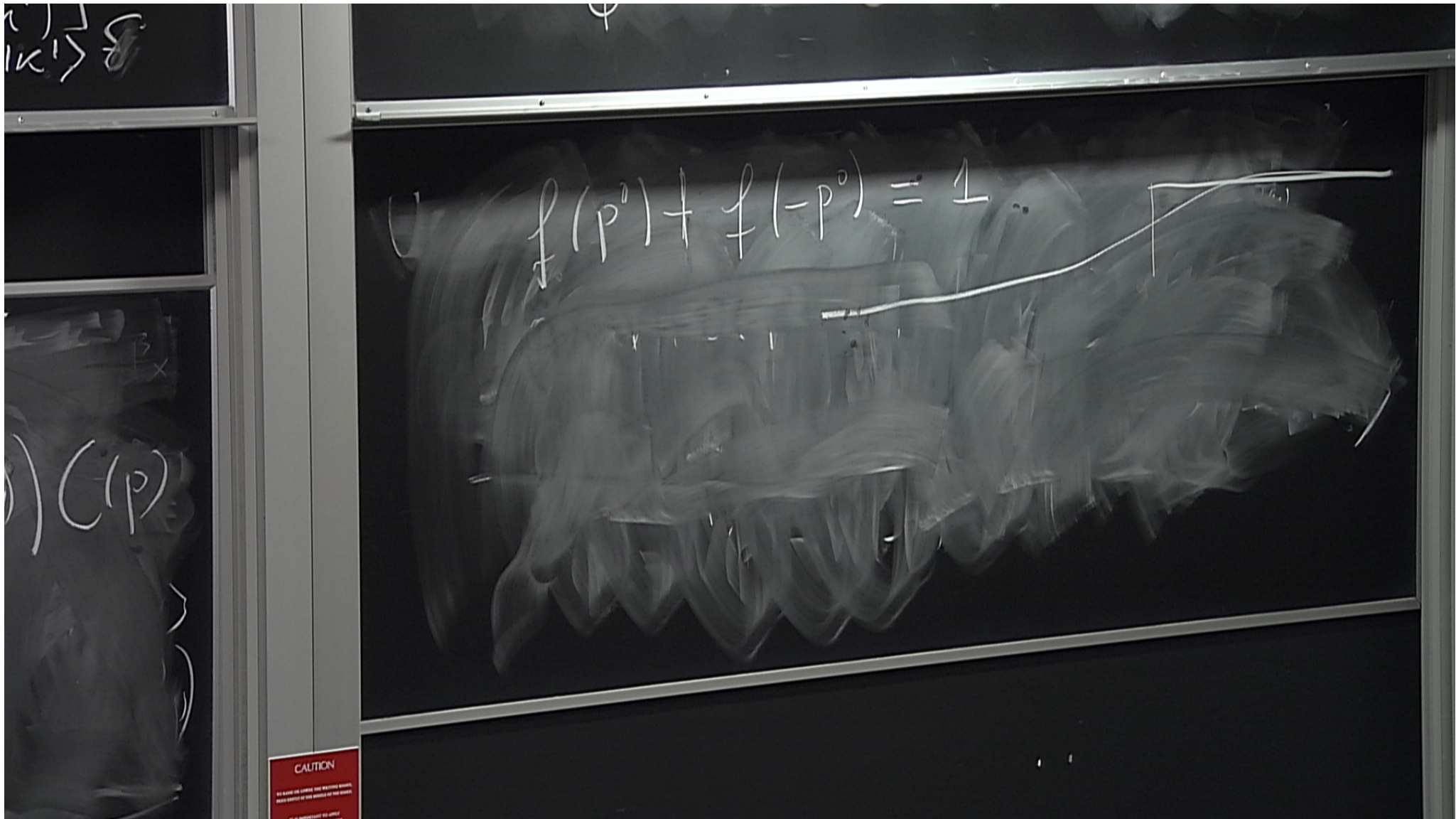
$$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p) = (f(p^0) - f(-p^0)) C(p)$$

$$\tilde{W}(p) = f(p^0) C(p)$$

$$\tilde{W}(-p) = f(-p^0) C(-p) = -f(p^0) C(p)$$

CAUTION

DO NOT USE LENSES FOR VIEWING OBJECTS.
DO NOT STARE AT THE SOURCE OF THE LASER.
IF AN INDIVIDUAL HAS A SEVERE
ALLERGIC REACTION TO LASER
RADIATION, ADVISE THE INSTRUCTOR IMMEDIATELY.



$$f(p^0) + f(-p^0) = 1$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
BECAUSE IT IS HOT TO THE TOUCH

$\beta < 1$
 $\beta < 1$

$C(p)$

$$f(p^0) + f(-p^0) = 1$$
$$f(p^0) = \frac{1}{4 - e^{-\beta p^0}}$$

CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD OR THE BOARD OR THE BOARD

$$f(p^0) + f(-p^0) = 1$$

$$f(p^0) = \frac{1}{1 - e^{-\beta p^0}} + \frac{1}{1 - e^{\beta p^0}} = \frac{e^{-\beta p^0} + e^{\beta p^0}}{2 - e^{-\beta p^0} - e^{\beta p^0}} = 1$$

$$\tilde{W}_\beta(x, y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\epsilon(p^0) \delta(p^2 - m^2)}{1 - e^{-\beta p^0}}$$

$$W_{\beta}(x, y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\epsilon(p^0) \delta(p^2 - m^2)}{1 - e^{-\beta p^0}}$$

$$W_{\beta}(x, y) - W_{\beta}(y, x) = C(x, y)$$

$$\int W_{\beta}(x, y) \bar{f}(x) f(y) d^4x d^4y \geq 0$$

$$W_\beta(x, y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\epsilon(p)}{1 - e^{-\beta \epsilon(p)}} d^3p$$

KMS $T = 1/\beta$

$$W_\beta(x, y) - W_\beta(y, x) = C(x, y)$$

$$\int W_\beta(x, y) \overline{f(x)} f(y) d^4x d^4y \geq 0$$

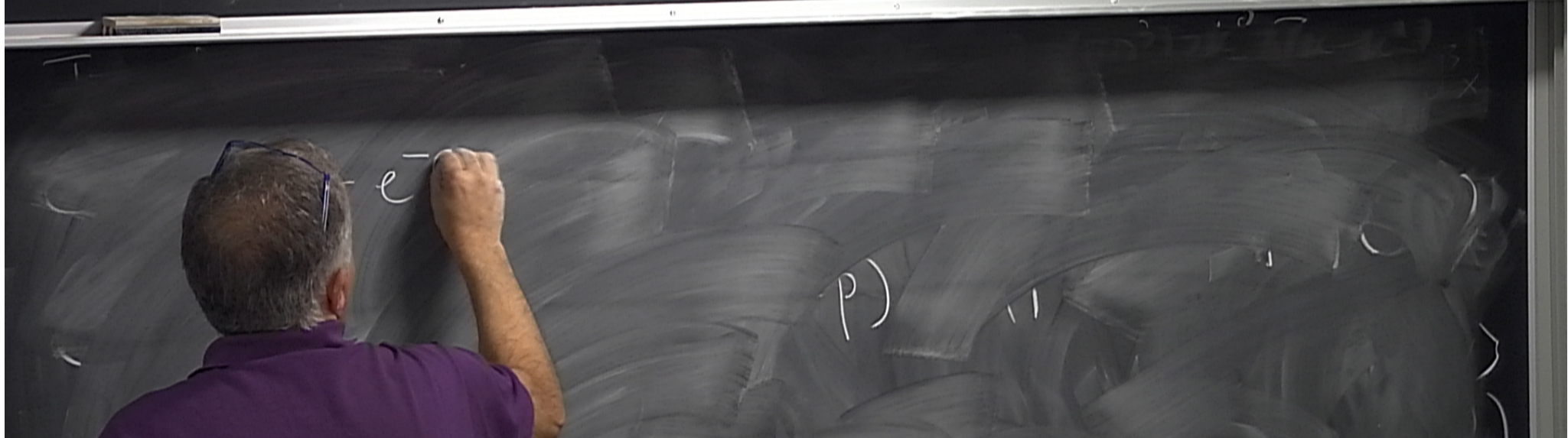
$$G(x, y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\epsilon(p^0) \delta(p^2 - m^2)}{1 - e^{-\beta p^0}}$$

$$G(y, x) - W_\beta(y, x) = C(x, y)$$

$$\int \bar{f}(x) f(y) d^4x d^4y \geq 0$$

CAUTION
 DO NOT TOUCH THE BOARD SURFACE.
 PLEASE NOTICE BY THE NUMBER OF THE BOARD.

$$W_{\theta}(x, y) = \frac{1}{(2\pi)^3} \int e^{-ipx-y} \frac{\theta(p^0)}{1 - e^{-\beta p^0}} + \int \frac{\theta(p^0) e^{\beta p^0}}{1 - e^{-\beta p^0}} e^{-ip(x-y)}$$



$$W_{\theta}(x, y) = \frac{1}{(2\pi)^3} \int e^{-ipx-y} \frac{\theta(p^0)}{1 - e^{-\beta p^0}} + \int \frac{\theta(p^0) e^{\beta p^0} e^{-ip(x-y)}}{1 - e^{-\beta p^0}}$$

$$\frac{1}{1 - e^{-\beta p^0}} = \sum_{n=0}^{\infty} e^{\beta p^0 n}$$

p)

$$\frac{1}{1 - e^{-\beta p^0}} = \sum_{n=0}^{\infty} e^{\beta p^0 n}$$

$$W_{\beta}(x, y) = \sum_{n=0}^{\infty} \frac{1}{e^{i\tau\beta}} \int e^{-i[p(x-y) - i\beta p^0 n]} \{ \theta(p^0) \delta(p^2 - m^2) \}$$

$$= \sum_{n=0}^{\infty} W_{\beta}(t - i\beta n, \vec{x}, s, \vec{y}) + \sum_{n=1}^{\infty} W(s, \vec{y}, t + i n \beta, \vec{x})$$

$$W(x-y) = \frac{1}{(2\pi)^3} \int \Theta(k^0) \delta(k^2 - m^2) d^4k$$

1) $W(t, \vec{x}, s, \vec{y})$

$$\frac{-\beta < \Im t < 0}{\square}$$

$$W(s)$$



$z = w$

$$W(s, \vec{y}, t, \vec{x}) =$$

$$= W(t + i\beta, \vec{x}, s, \vec{y})$$

$$\frac{\int \frac{\Delta}{12} e^{-\beta H} \phi(x) \phi(y)}{\int e^{-\beta H}}$$

ψ

$$= \dots$$