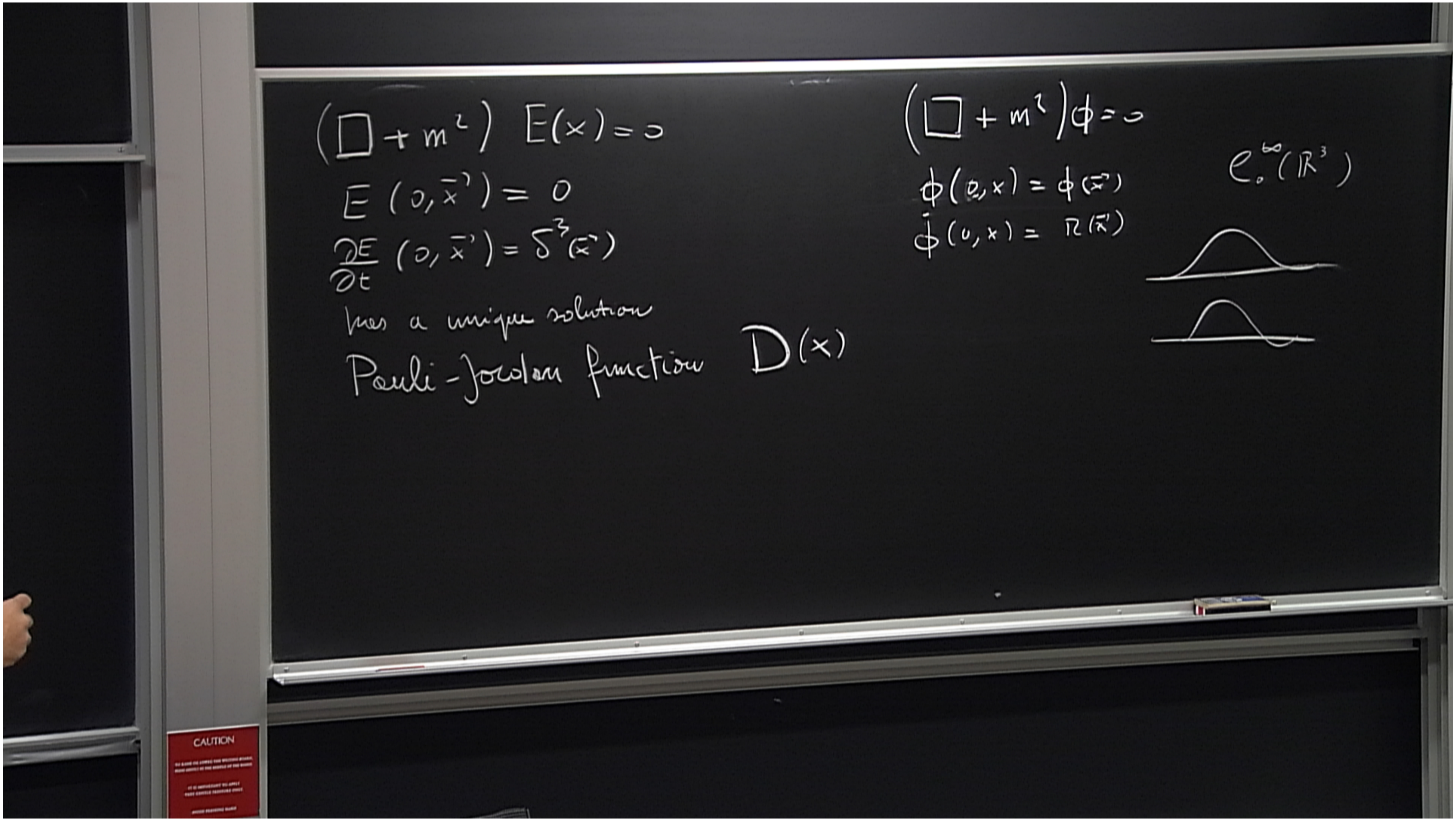
Title: Topics in QFT on Flat and Curved Spacetimes - Lecture 5

Date: Oct 07, 2013 10:00 AM

URL: http://pirsa.org/13100011

Abstract:


$(\Box + m^2)\, E(x) = 0$
$E(0, \vec{x}) = 0$
$\frac{\partial E}{\partial t}(0, \vec{x}) = \delta^3(\vec{x})$
has a unique solution
Pauli-Jordan function $D(x)$
$(\Box + m^2)\phi = 0$
$\phi(0, x) = \phi(\vec{x})$
$\dot{\phi}(0, x) = \pi(\vec{x})$
$C_0^\infty(\mathbb{R}^3)$
CAUTION

$(\Box + m^2)\, E(x) = 0$

$E(0, \vec{x}) = 0$

$\frac{\partial E}{\partial t}(0, \vec{x}) = \delta^3(\vec{x})$

has a unique solution

Pauli-Jordan function $D(x)$

$\phi(x) = \int D(x-y)\, \underline{f(y)}\, dt\, d\vec{x}$

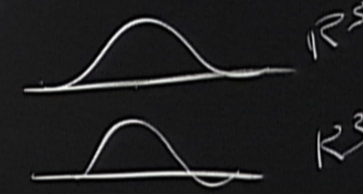
$(\Box + m^2)\phi(x) = 0$

$(\Box + m^2)\phi$

$\phi(0, x) = \phi(\vec{x})$

$\dot{\phi}(0, x) = \pi(\vec{x})$

$C_0^\infty(\mathbb{R}^3)$



$$(\Box + m^2)\psi(x) = 0$$

$$\boxed{D(x) = R(x) - A(x)}$$

$$\langle f, g \rangle = -\int f(x)\, D(x-y)\, g(y)\, d^4x\, d^4y$$

$$D(-x) = -D(x) \qquad \langle f, g \rangle = -\langle g\, f \rangle$$

$$= \int_{\mathbb{R}^3\ t=\text{const}} \left[\psi_f(t,x) \frac{\partial \psi_g(t,x)}{\partial t} - \frac{\partial \psi_f(t,x)}{\partial t} \psi_g(t,x) \right] d^3x$$

$(\Box + m^2)\psi(x) = 0$

$$\boxed{D(x) = R(x) - A(x)}$$

$$\langle f, g \rangle = - \int f(x)\, D(x-y)\, g(y)\, d^4x\, d^4y$$

$D(-x) = -D(x)$ $\quad \langle f, g \rangle = -\langle g\, f \rangle$ $\quad \psi_f = \int D(x-y)\, f(y)$

$$= \int_{\mathbb{R}^3\ t=\text{const}} \left[\psi_f(t,x) \frac{\partial \psi_g(t,x)}{\partial t} - \frac{\partial \psi_f(t,x)}{\partial t} \psi_g(t\,x) \right] d^3x = \langle \psi_f, \psi_g \rangle$$

$t = \bar{t}$

$(\Box + m^2)\varphi(x) = 0$

$$\boxed{D(x) = R(x) - A(x)}$$

$$\langle f, g \rangle = - \int f(x)\, D(x-y)\, g(y)\, d^4x\, d^4y$$

$D(-x) = -D(x)$ $\quad \langle f, g \rangle = -\langle g\, f \rangle$ $\quad \psi_f = \int D(x-y)\, f(y)$

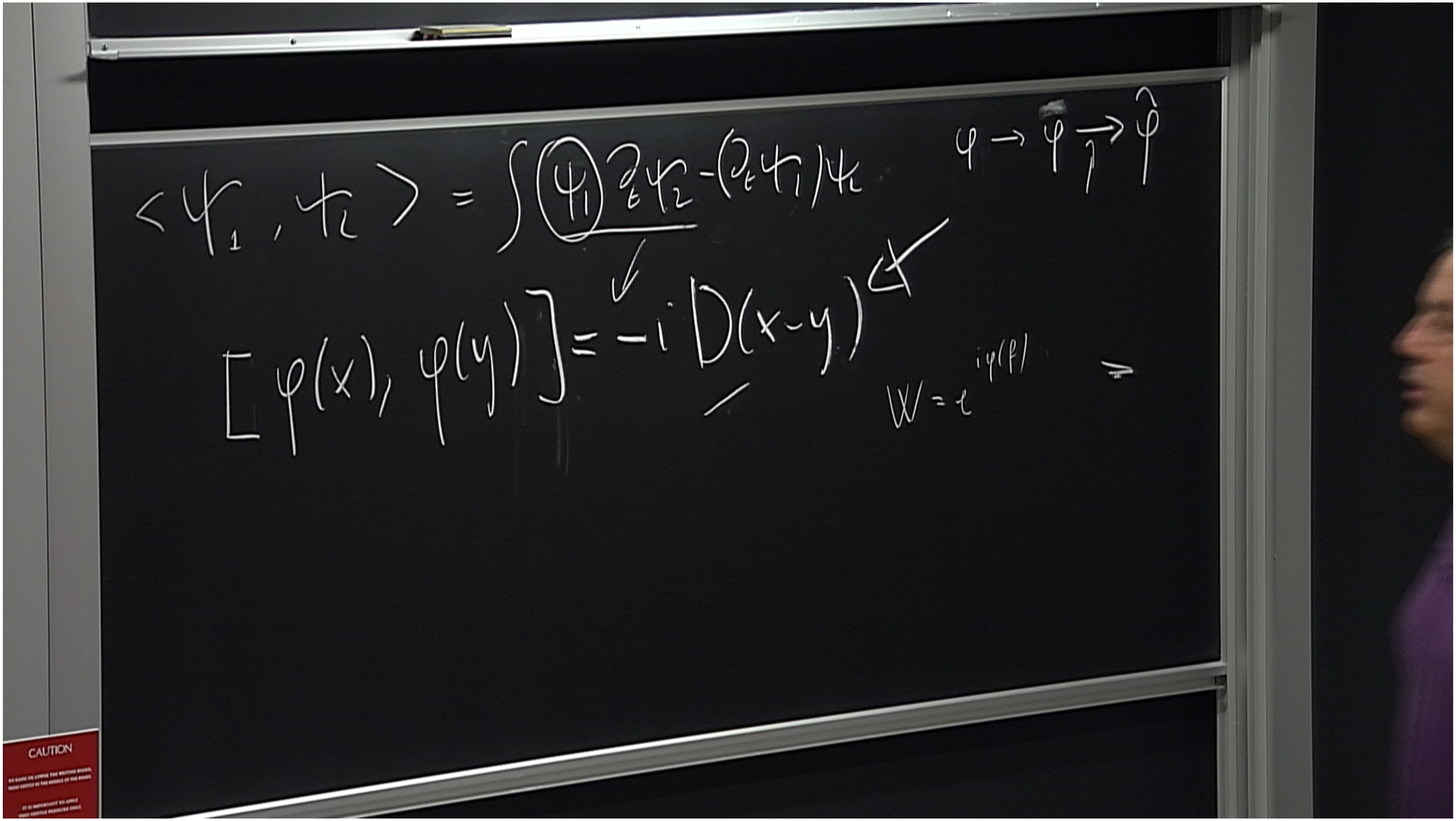
$$= \int_{\mathbb{R}^3\ t=\text{const}} \left[\psi_f(t,x) \frac{\partial \psi_g(t,x)}{\partial t} - \frac{\partial \psi_f(t,x)}{\partial t} \psi_g(t\,x) \right] d^3x = \langle \psi_f, \psi_g \rangle$$

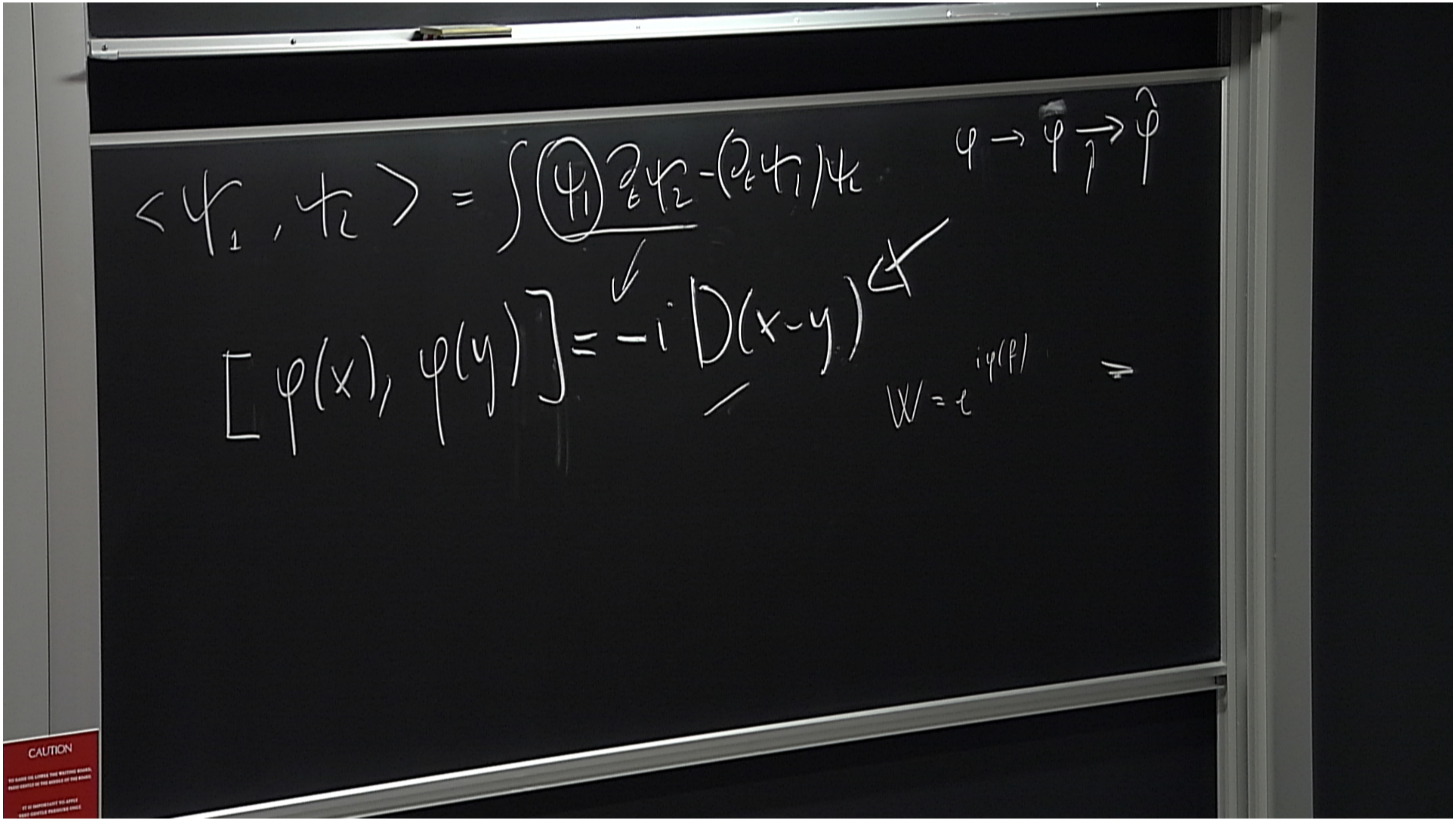
$t = \bar{t}$

$$\langle f, g \rangle_D = \langle \psi_f , \psi_g \rangle$$

$$J^\mu(\psi_1, \psi_2) = \psi_1 \partial^\mu \psi_2 - (\partial^\mu \psi_1) \psi_2$$

$$\partial_\mu J^\mu = \partial_\mu \psi_1 \partial^\mu \psi_2 - \partial^\mu \psi_1 \partial_\mu \psi_2 + \psi_1 \Box \psi_2 - \Box \psi_1 \psi_2 - m^2 \psi_1 \psi_2 + m^2 \psi_1 \psi_2 = 0$$


$\langle \psi_1, \psi_2 \rangle = \int \psi_1 \partial_t \psi_2 - (\partial_t \psi_1) \psi_2$
$\varphi \to \varphi \to \hat{\varphi}$
$[\varphi(x), \varphi(y)] = -i D(x-y)$
$W = e^{i\varphi(f)}$
CAUTION


$\langle \psi_1, \psi_2 \rangle = \int \psi_1 \partial_t \psi_2 - (\partial_t \psi_1)\psi_2$
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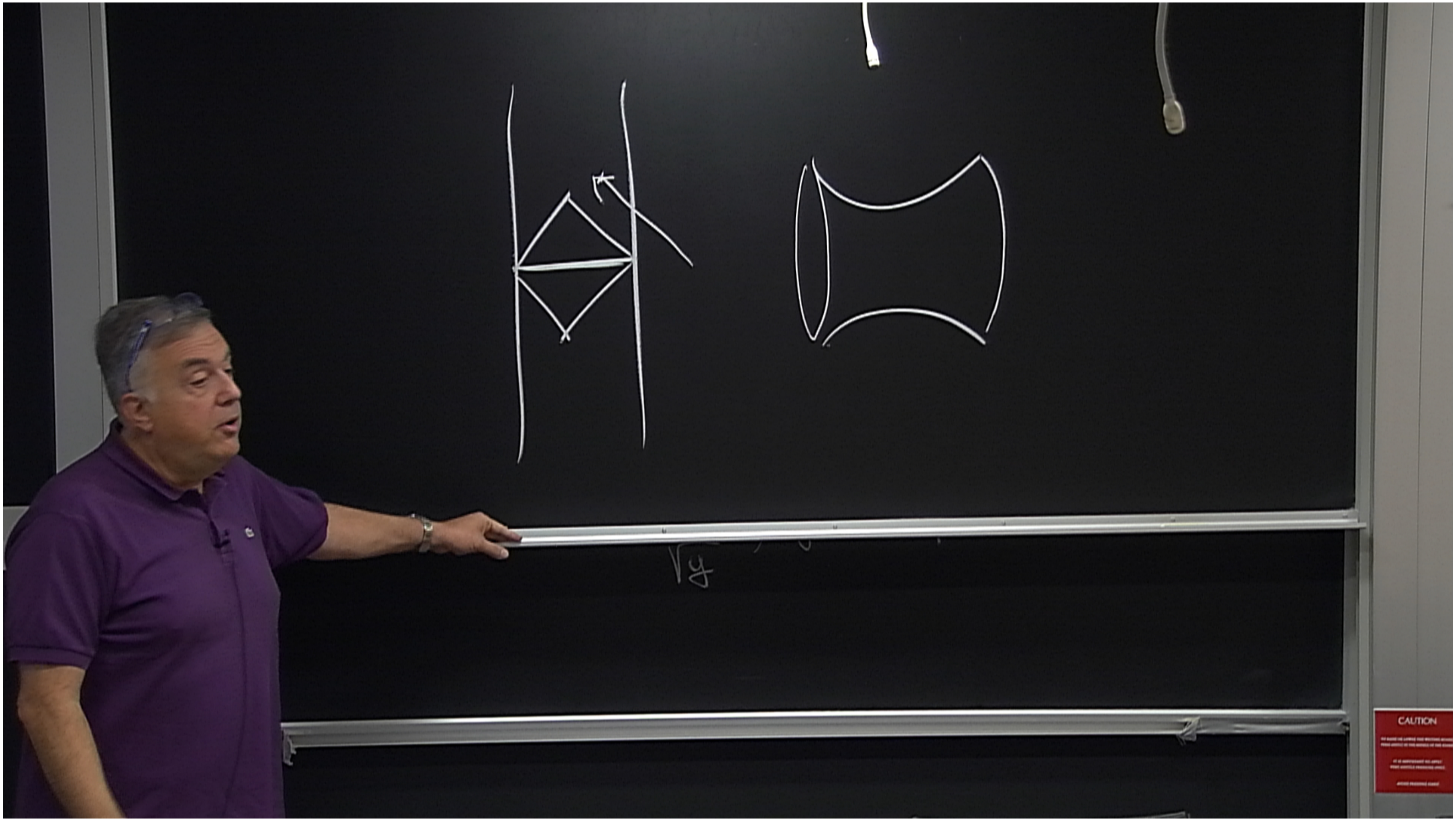
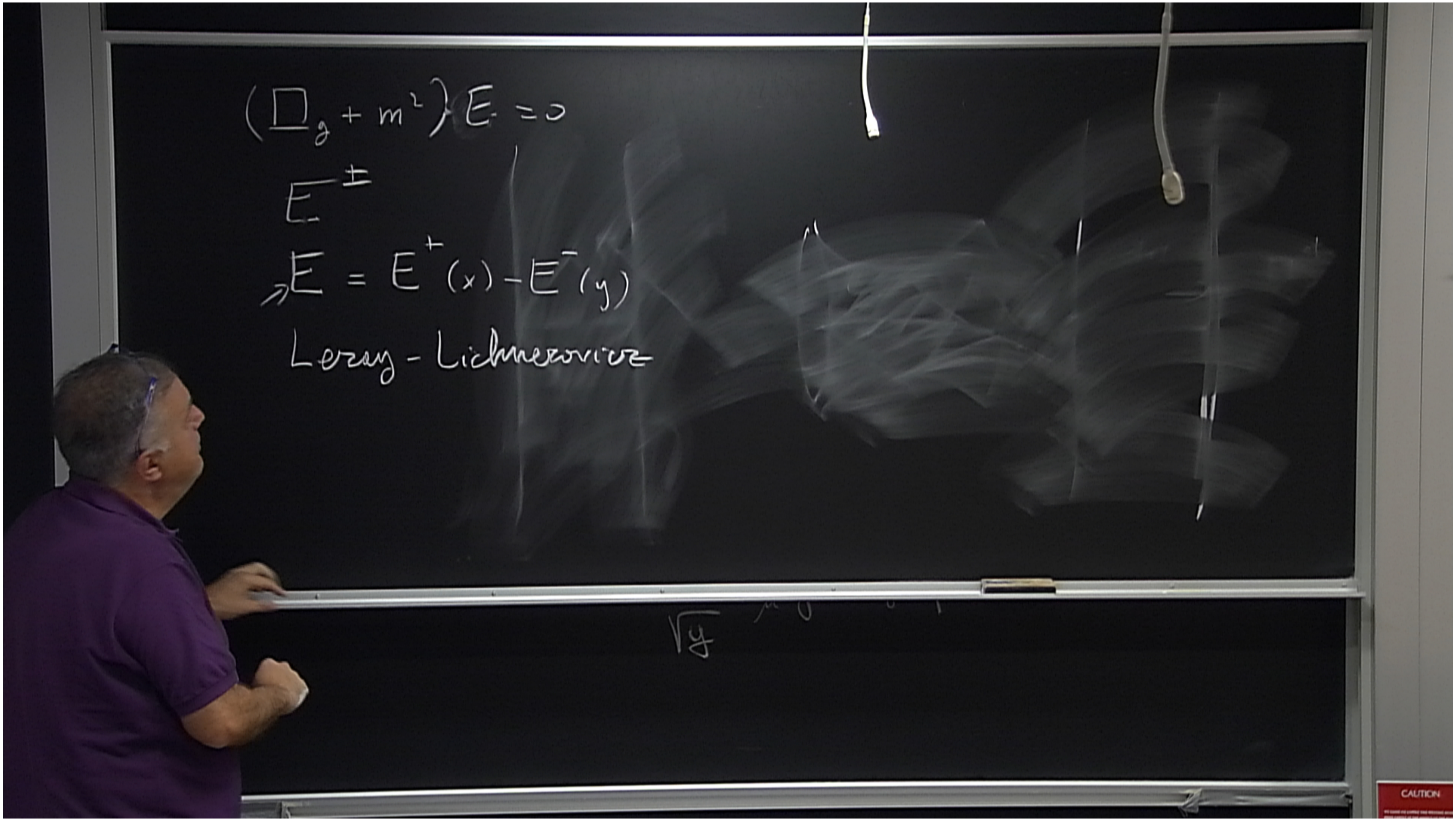
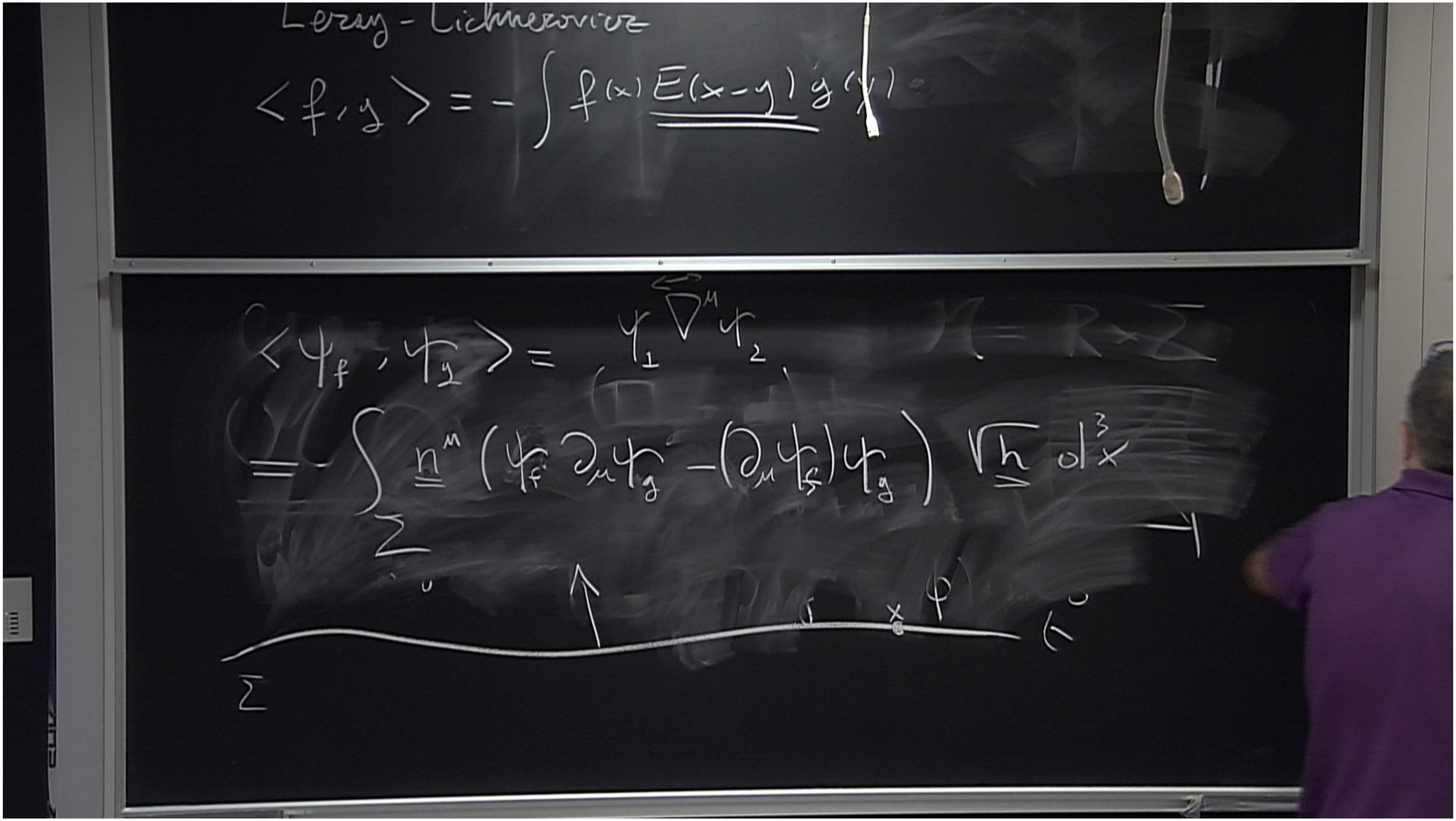
CAUTION




$(\Box_g + m^2) E = 0$
$E^{\pm}$
$E = E^{+}(x) - E^{-}(y)$
Leray-Lichnerowicz


Leray - Lichnerowicz
$\langle f, g \rangle = -\int f(x)\, E(x-y)\, g(y)$
$\langle \psi_f, \psi_g \rangle = \psi_1 \overleftrightarrow{\nabla}^\mu \psi_2$
$= -\int_\Sigma n^\mu \left(\psi_f \partial_\mu \psi_g - (\partial_\mu \psi_f) \psi_g \right) \sqrt{h}\, d^3x$
Σ

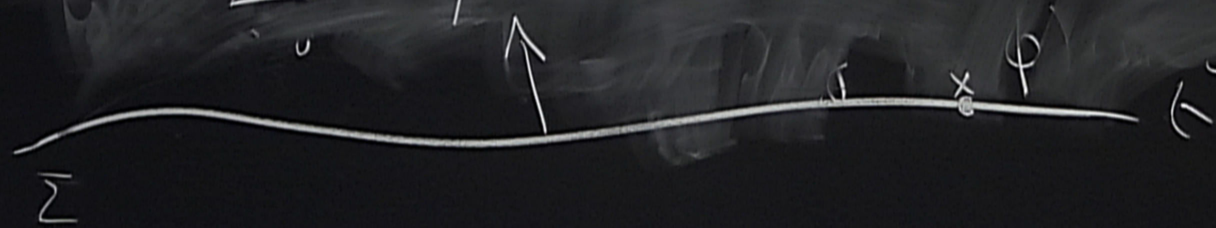
Leray - Lichnerowicz

$$\langle f, g \rangle = -\int f(x) \boxed{E(x-y)} g(y)$$

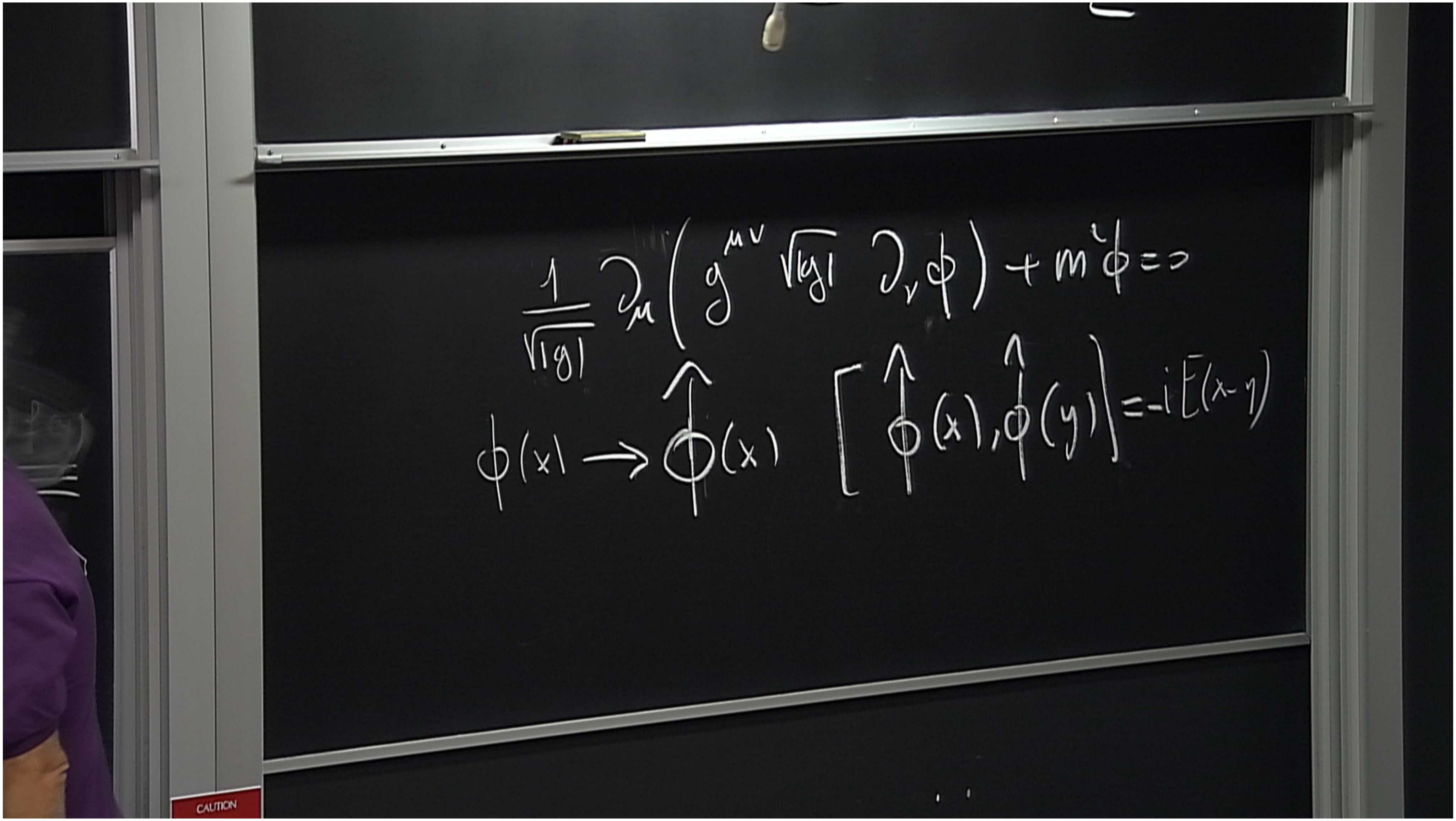
$$[\Phi(x), \Phi(y)] = -i E(x,y)$$

$$\langle \psi_f , \psi_g \rangle = \psi_1 \overleftrightarrow{\nabla}^\mu \psi_2$$

$$= -\int_\Sigma n^\mu \left(\psi_f \partial_\mu \psi_g - (\partial_\mu \psi_f) \psi_g \right) \sqrt{h}\, d^3x$$



$$\frac{1}{\sqrt{|g|}}\partial_\mu\left(g^{\mu\nu}\sqrt{|g|}\,\partial_\nu\phi\right)+m^2\phi=0$$


$\frac{1}{\sqrt{|g|}}\partial_\mu\left(g^{\mu\nu}\sqrt{|g|}\,\partial_\nu\phi\right) + m^2\phi = 0$
$\phi(x) \longrightarrow \hat{\phi}(x)$
$\left[\hat{\phi}(x), \hat{\phi}(y)\right] = -iE(x-y)$
CAUTION

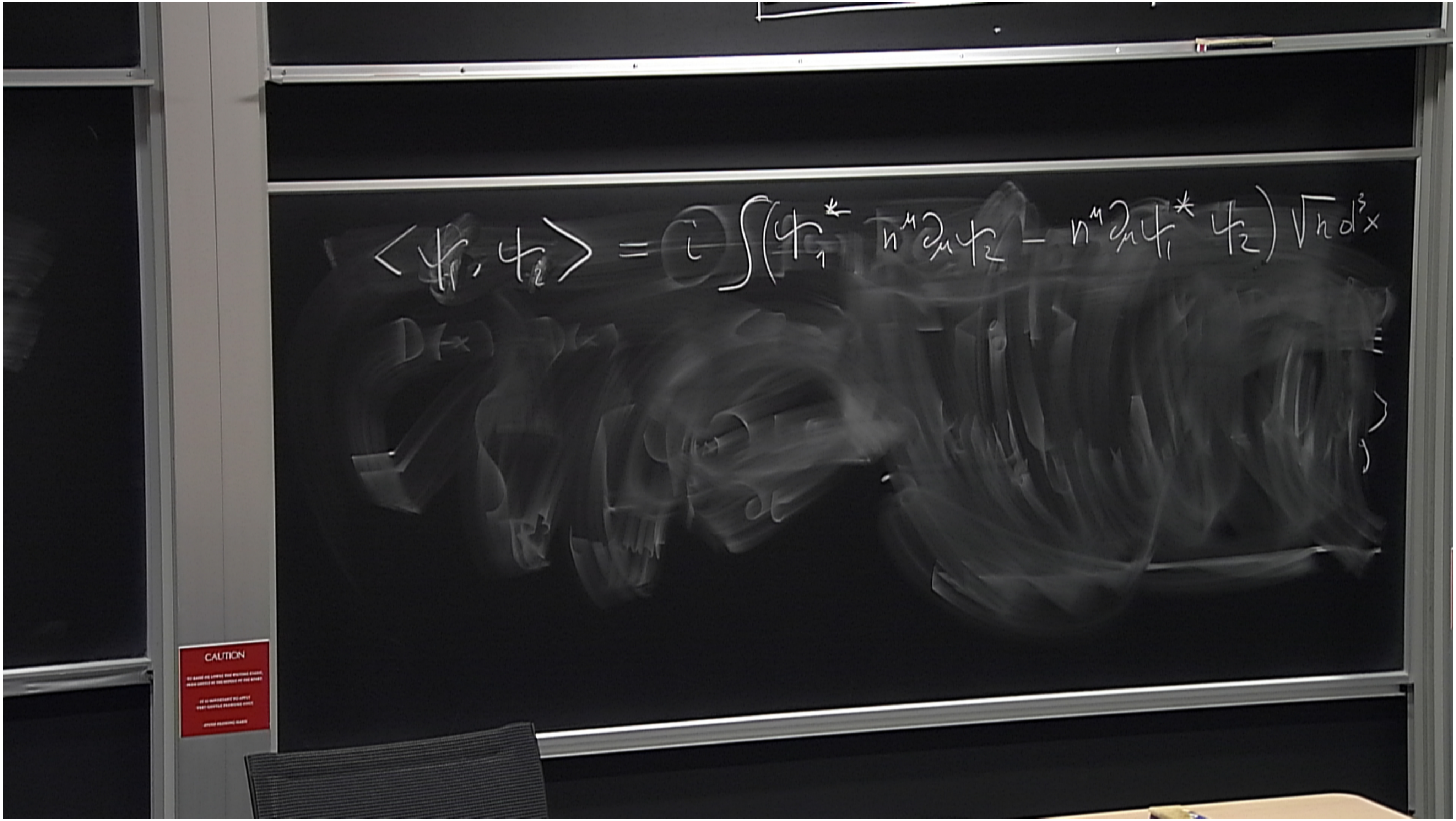
$$\langle f, g \rangle = -\int f(x) \boxed{E(x-y)} g(y)$$

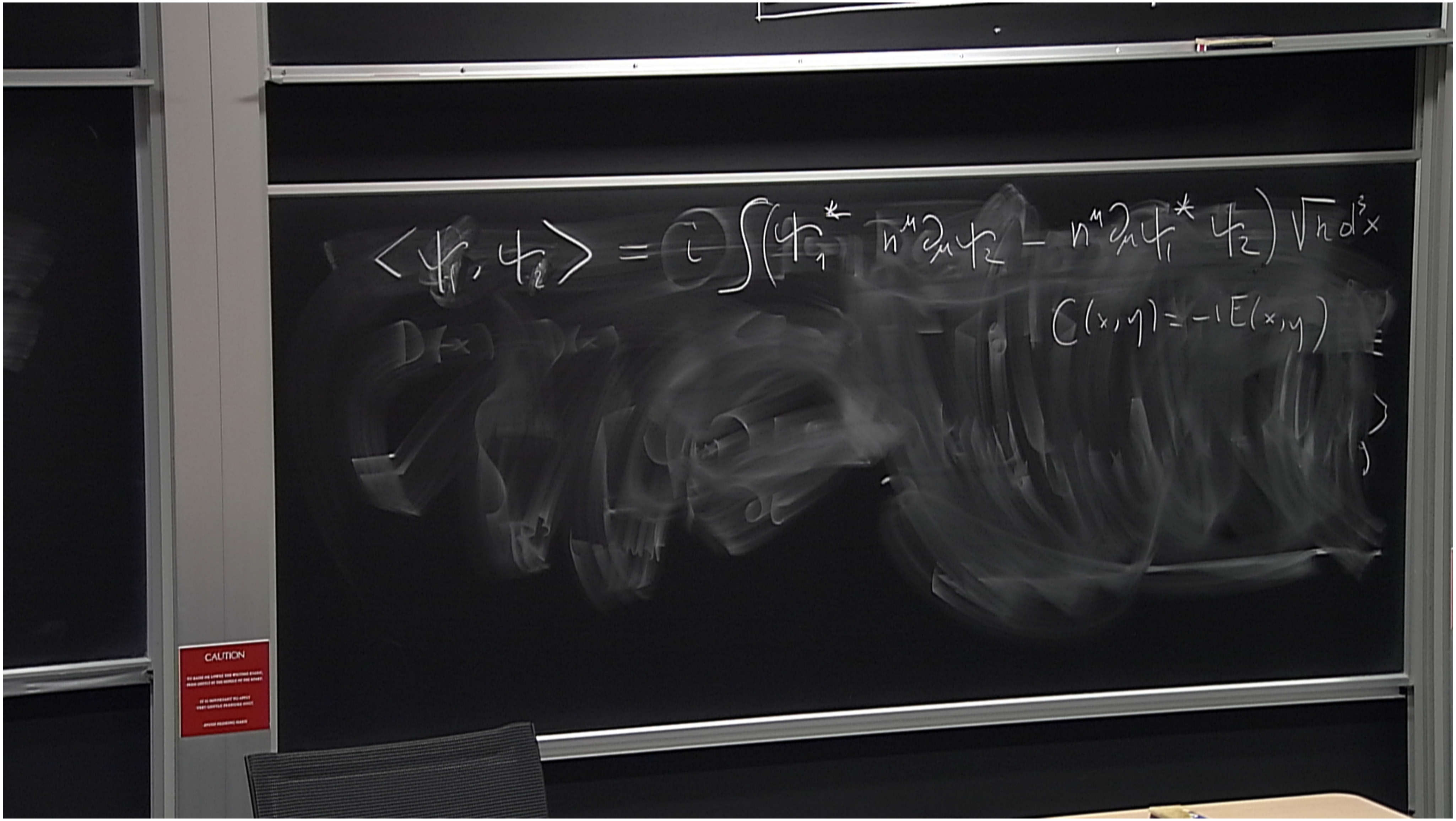
$$\to [\Phi(x), \Phi(y)] = -i E(x,y) = 0$$

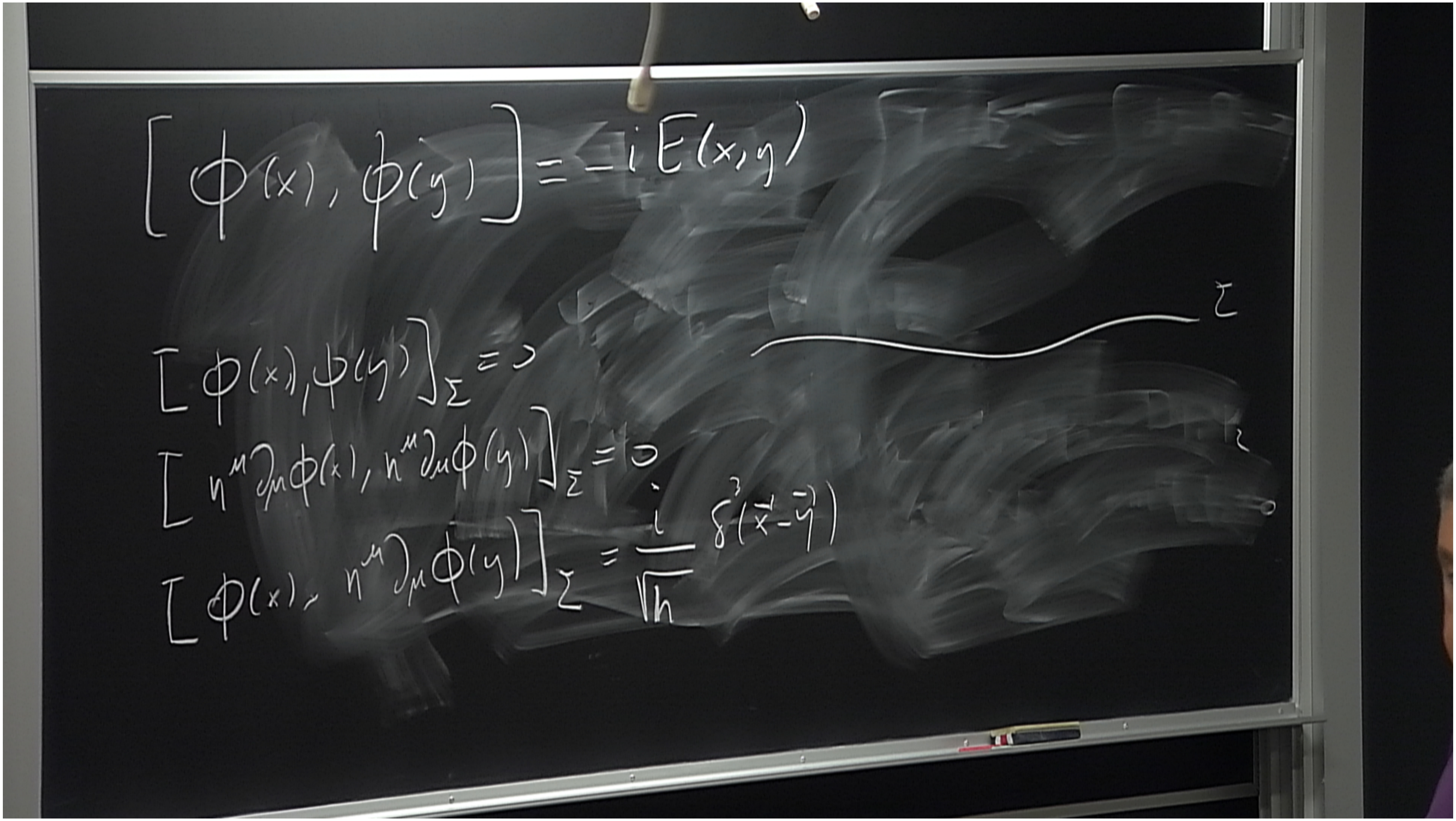
$$\langle \psi_f, \psi_g \rangle = \psi_1 \overleftrightarrow{\nabla}^\mu \psi_2$$

$$= \int_\Sigma n^\mu \left(\psi_f \partial_\mu \psi_g - (\partial_\mu \psi_f)\psi_g\right) \sqrt{h}\, d^3x$$

Σ


$\langle \psi_1, \psi_2 \rangle = i \int (\psi_1^* n^\mu \partial_\mu \psi_2 - n^\mu \partial_\mu \psi_1^* \psi_2) \sqrt{h}\, d^3x$
CAUTION


$\langle \psi_1, \psi_2 \rangle = i \int \left(\psi_1^* \, n^\mu \partial_\mu \psi_2 - n^\mu \partial_\mu \psi_1^* \, \psi_2 \right) \sqrt{h} \, d^3x$
$C(x,y) = -i E(x,y)$
CAUTION


$[\phi(x), \phi(y)] = -iE(x,y)$
Σ
$[\phi(x), \phi(y)]_\Sigma = 0$
$[n^\mu \partial_\mu \phi(x), n^\mu \partial_\mu \phi(y)]_\Sigma = 0$
$[\phi(x), n^\mu \partial_\mu \phi(y)]_\Sigma = \frac{i}{\sqrt{h}} \delta^3(\vec{x} - \vec{y})$

$$[n^\mu\partial_\mu\phi(x), n^\mu\partial_\mu\phi(y)]_\Sigma = 0$$

$$[\phi(x), n^\mu\partial_\mu\phi(y)]_\Sigma = \frac{i}{\sqrt{h}}\delta^3(\vec{x}-\vec{y})$$

$$\langle f, \phi\rangle = i\int \left(f^*(x)\, n^\mu\partial_\mu\phi - n^\mu\partial_\mu f^*\,\phi\right)\sqrt{h}\, d^3x$$

$$[\langle f,\phi\rangle, \langle g,\phi\rangle] = -\underline{\underline{\langle f, g^*\rangle_{\Sigma}}}$$

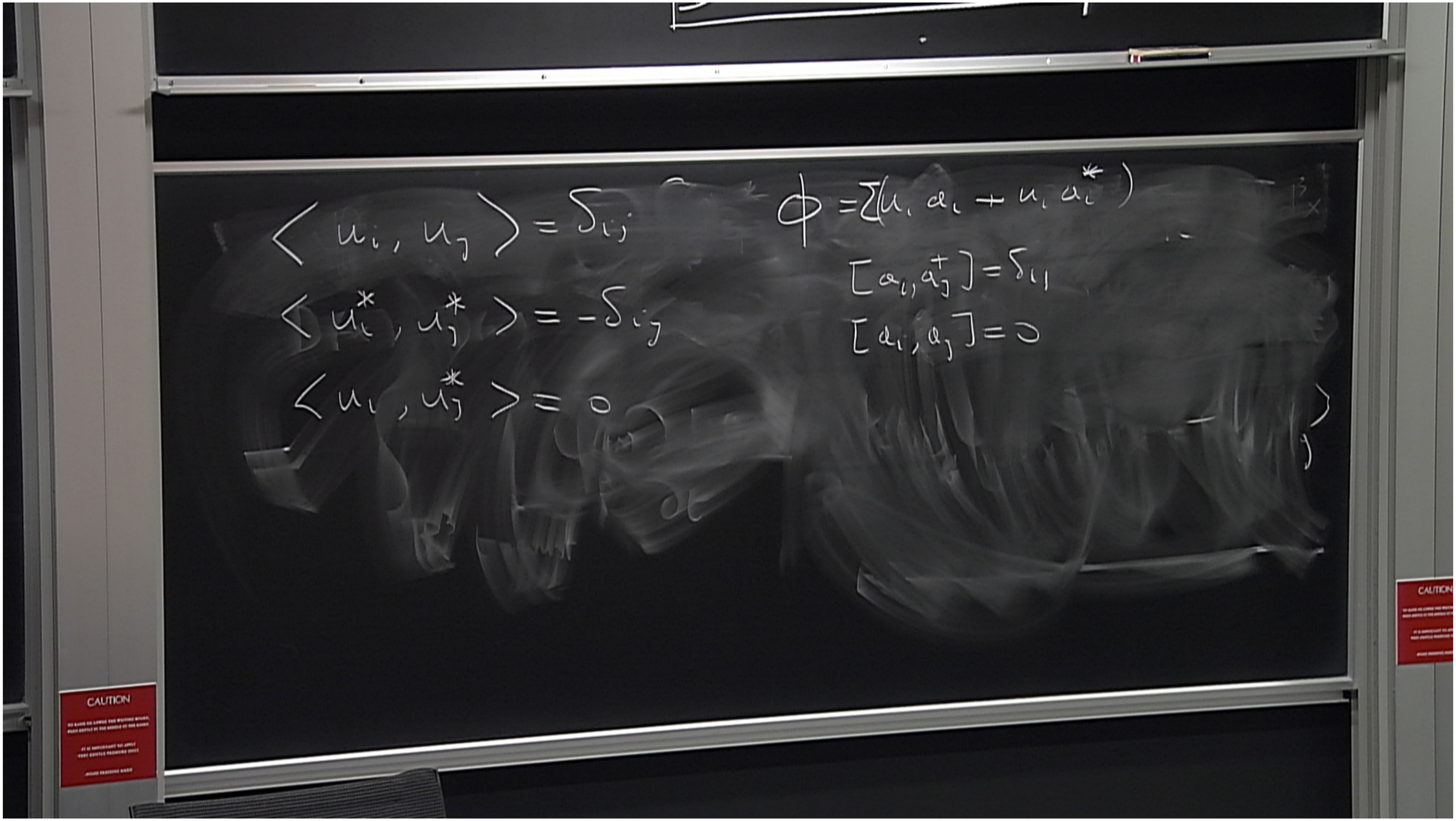
$$\langle f, \phi \rangle = i \int \left(f^*(x)\, n^\mu \partial_\mu \phi - n^\mu \partial_\mu f^* \, \phi \right) \sqrt{h}\, d^3x$$

$$\left[\langle f, \phi \rangle, \langle g, \phi \rangle \right] = - \langle f, g^* \rangle_{\Sigma}$$

$$\langle u_i , u_j \rangle = \delta_{ij}$$

$$\langle u_i^* , u_j^* \rangle = -\delta_{ij}$$

$$\langle u_i , u_j^* \rangle = 0$$



$\langle u_i, u_j \rangle = \delta_{ij}$
$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$
$\langle u_i, u_j^* \rangle = 0$
$\phi = \sum(u_i a_i + u_i a_i^*)$
$[a_i, a_j^+] = \delta_{ij}$
$[a_i, a_j] = 0$

$= \delta_{ij}$

$$\phi = \sum (u_i a_i + u_i a_i^{*})$$

$$[a_i, a_j^{+}] = \delta_{ij}$$

$= -\delta_{ij}$

$$[a_i, a_j] = 0$$

$= 0$

$$[\phi(x), \phi(y)] = C(x,y)$$

$$= -iE(x,y) = \sum_j \left(u_j(x)\, u_j^{*}(y) - u_j^{*}(x)\, u_j(y)\right)$$

$\phi_{cl}(x)$

$\{\phi_{cl}(x), \phi_{cl}(y)\} = -E(x,y)$

$\downarrow$

$[\phi(x), \phi(y)] = C(x,y) = -iE(x,y)$ CCR

$\downarrow$

$\phi(x) \longrightarrow \hat{\phi}(x)$

$[a_i, a_j] = 0$

$\langle u_i, u_j^* \rangle = 0$

$[\phi(x), \phi(y)] = C(x,y)$

$$[\phi(x),\phi(y)] = C(x,y) = -iE(x,y) \quad \text{CCR}$$

$$\phi(x) \to \hat{\phi}(x)$$

Commutator

Two point function

$$[a_i, a_j] = 0$$

$$\langle u_i, u_j^* \rangle = 0$$

$$[\phi(x),\phi(y)] = C(x,y)$$

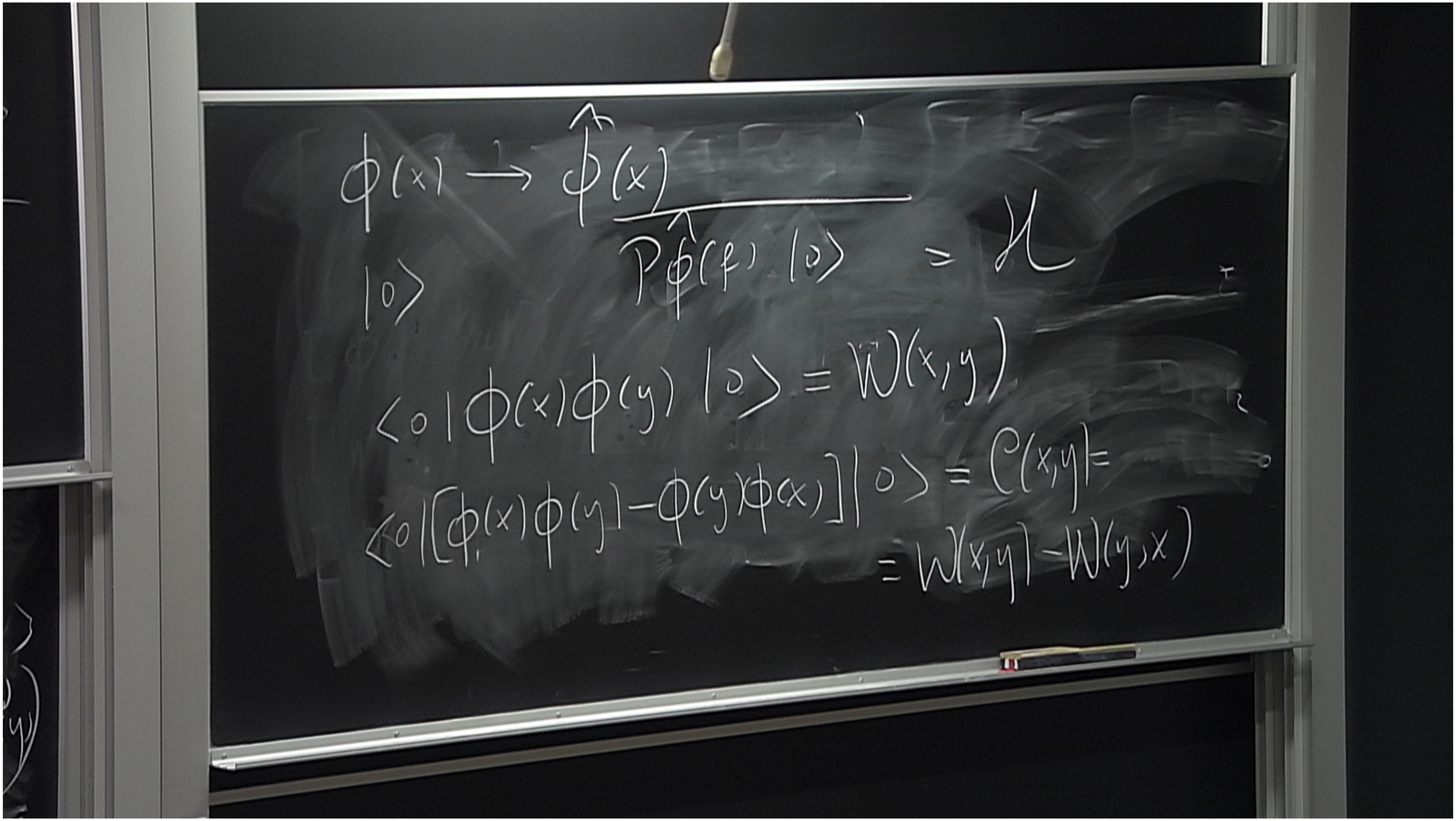
$$= -iE(x,y) = \sum_j \left(u_j(x)\, u_j^*(y) - u_j^*(x)\, u_j(y) \right)$$

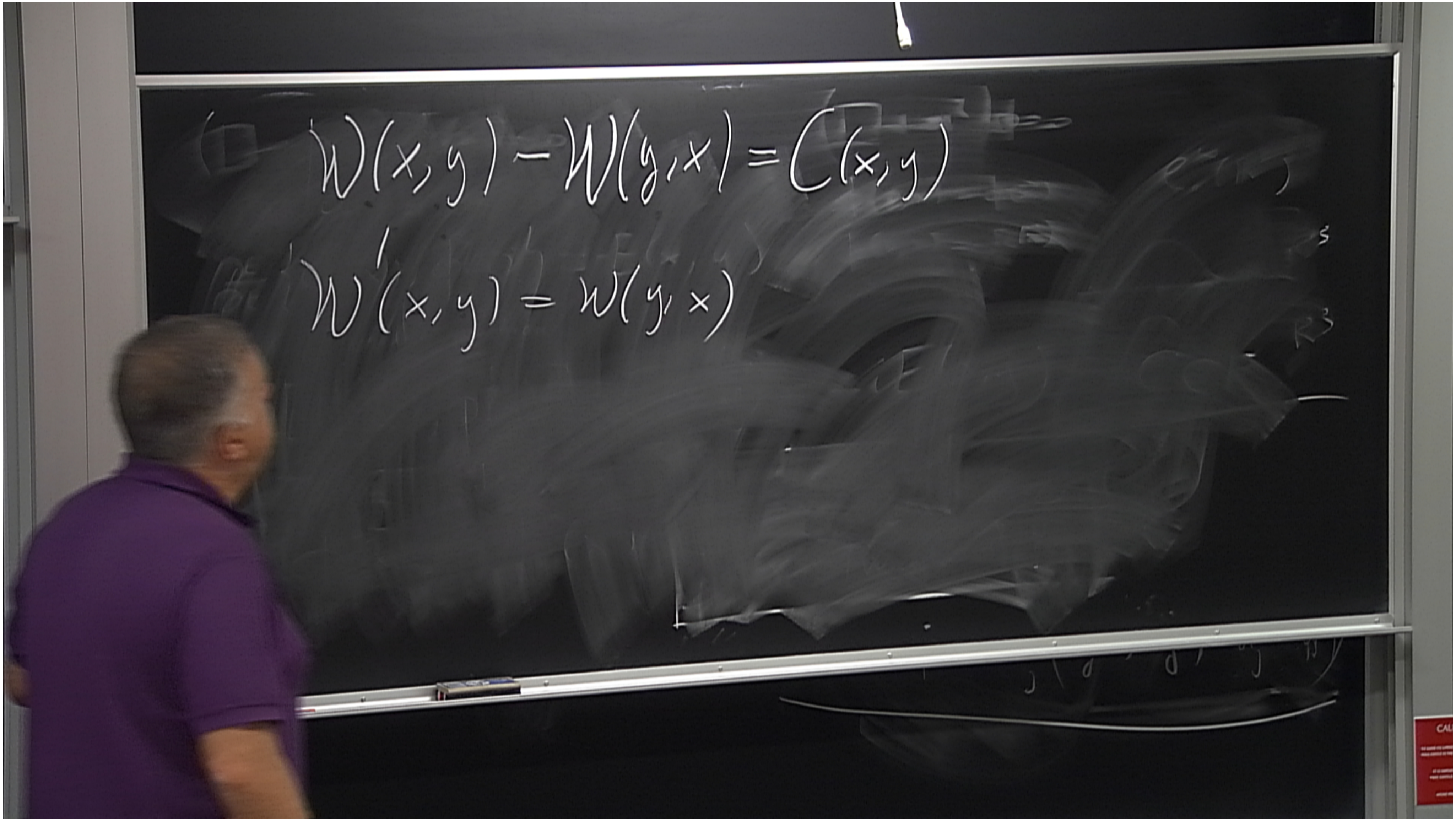
$$\phi(x) \longrightarrow \hat{\phi}(x)$$

$$|0\rangle \qquad \overline{P\hat{\phi}(f)\,|0\rangle} = \mathcal{H}$$

$$\langle 0|\phi(x)\phi(y)\,|0\rangle = W(x,y)$$

$$\langle 0|\phi(x_1)\cdots\phi(x_n)|0\rangle$$


$\phi(x) \longrightarrow \hat{\phi}(x)$
$|0\rangle$
$\overline{P\hat{\phi}(f)|0\rangle} = \mathcal{H}$
$\langle 0|\phi(x)\phi(y)|0\rangle = W(x,y)$
$\langle 0|[\phi(x)\phi(y) - \phi(y)\phi(x)]|0\rangle = C(x,y) =$
$= W(x,y) - W(y,x)$

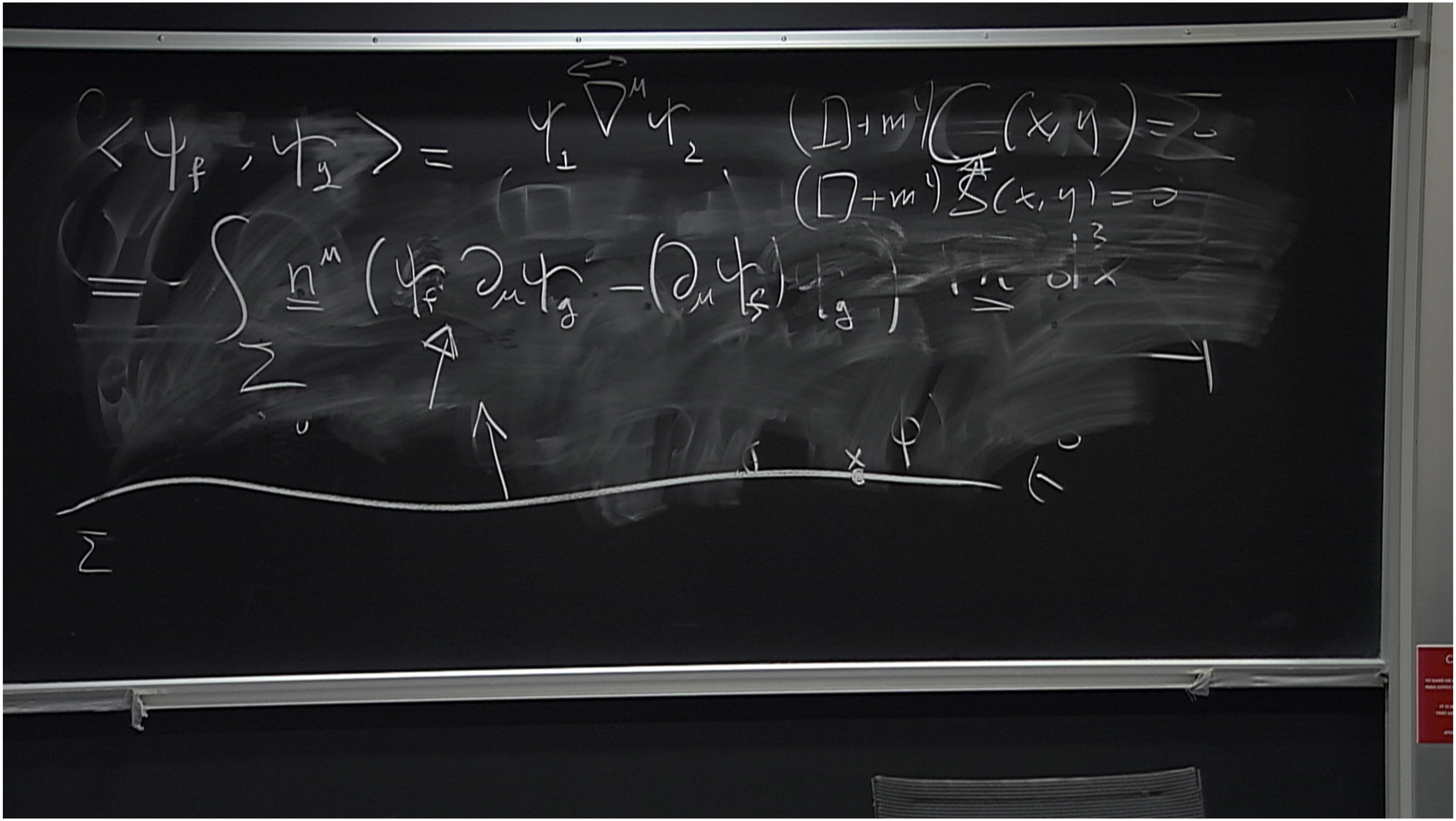

W(x,y) − W(y,x) = C(x,y)
W'(x,y) = W(y,x)

$$W(x,y) - W(y,x) = C(x,y)$$

$$W(x,y) = S(x,y) + A(x,y)$$

$$W(x,y) = \frac{W(x,y) + W(y,x)}{2} + \frac{W(x,y) - W(y,x)}{2}$$

$$\frac{G^{(1)}(x,y)}{2} + \frac{C(x,y)}{2}$$



$\langle \psi_f, \psi_g \rangle = \psi_1 \overleftrightarrow{\nabla}^\mu \psi_2$
$(\Box + m^2) G(x,y) = $
$(\Box + m^2) S(x,y) = 0$
$= \int_\Sigma \underline{n}^\mu (\psi_f \partial_\mu \psi_g - (\partial_\mu \psi_f) \psi_g) \sqrt{h}\, d^3x$
Σ

$$W(x,y) - W(y,x) = C(x,y) \qquad (\Box + m^2)\, S(x,y) = 0$$

$$W(x,y) = S(x,y) + A(x,y)$$

$$W(x,y) = \frac{W(x,y) + W(y,x)}{2} + \frac{W(x,y) - W(y,x)}{2}$$

$$\frac{G^{(1)}(x,y)}{2} + \frac{C(x,y)}{2}$$

$$\to [\phi(x), \phi(y)] = -i E(x,y) = 0$$

$$W(\bar{f}, g) = \int W(x,y)\, \bar{f}(x)\, g(y)\, d^4x\, d^4y$$

$$W(\bar{f}, f) = \|\phi(f)|0\rangle\|^2 \geq 0$$

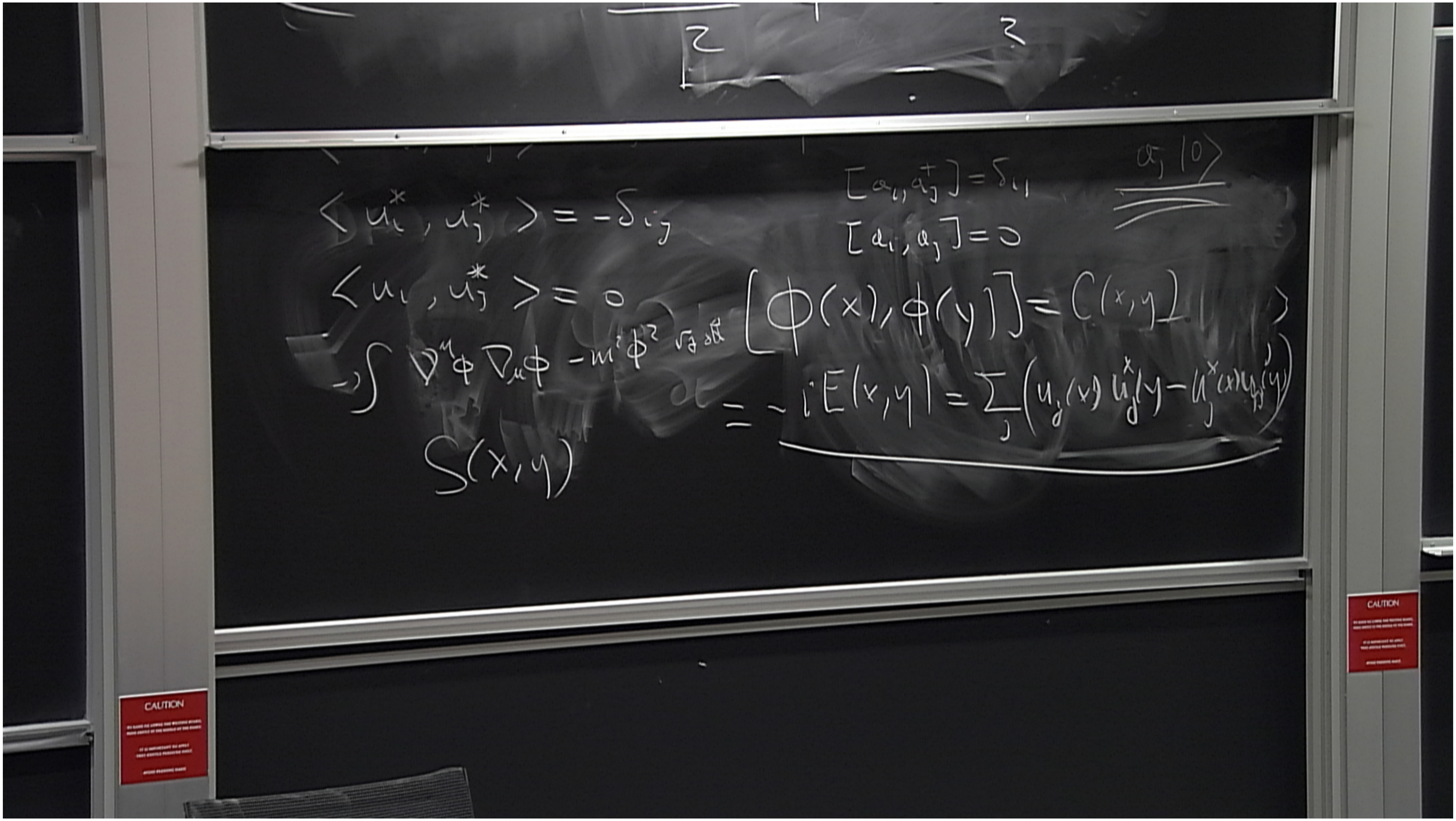
Σ

$$\rightarrow [\Phi(x), \Phi(y)] = -i E(x,y) = 0$$

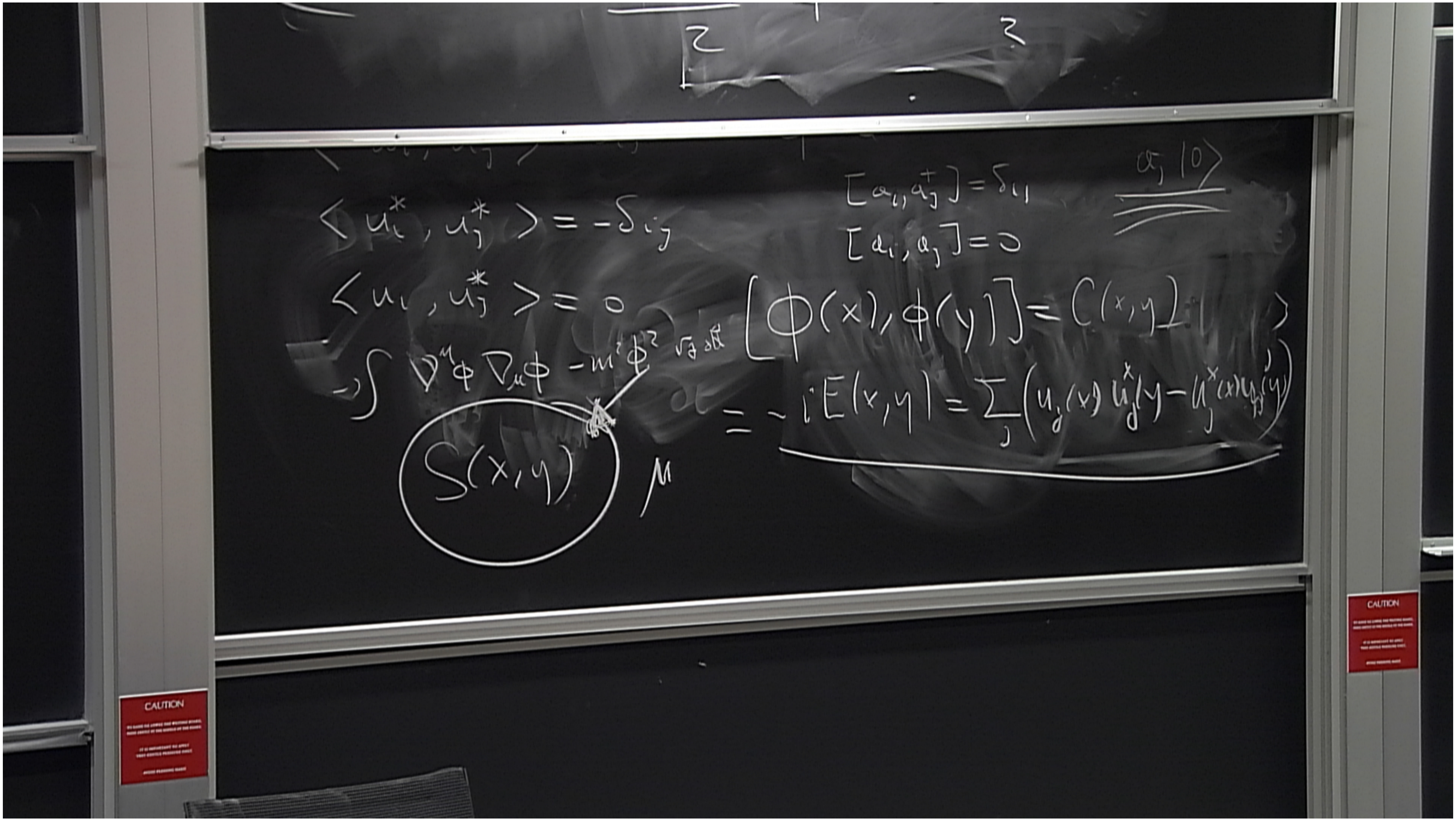
$$W(\bar{f}, g) = \int W(x,y)\, \bar{f}(x)\, g(y)\, d^4x\, d^4y$$

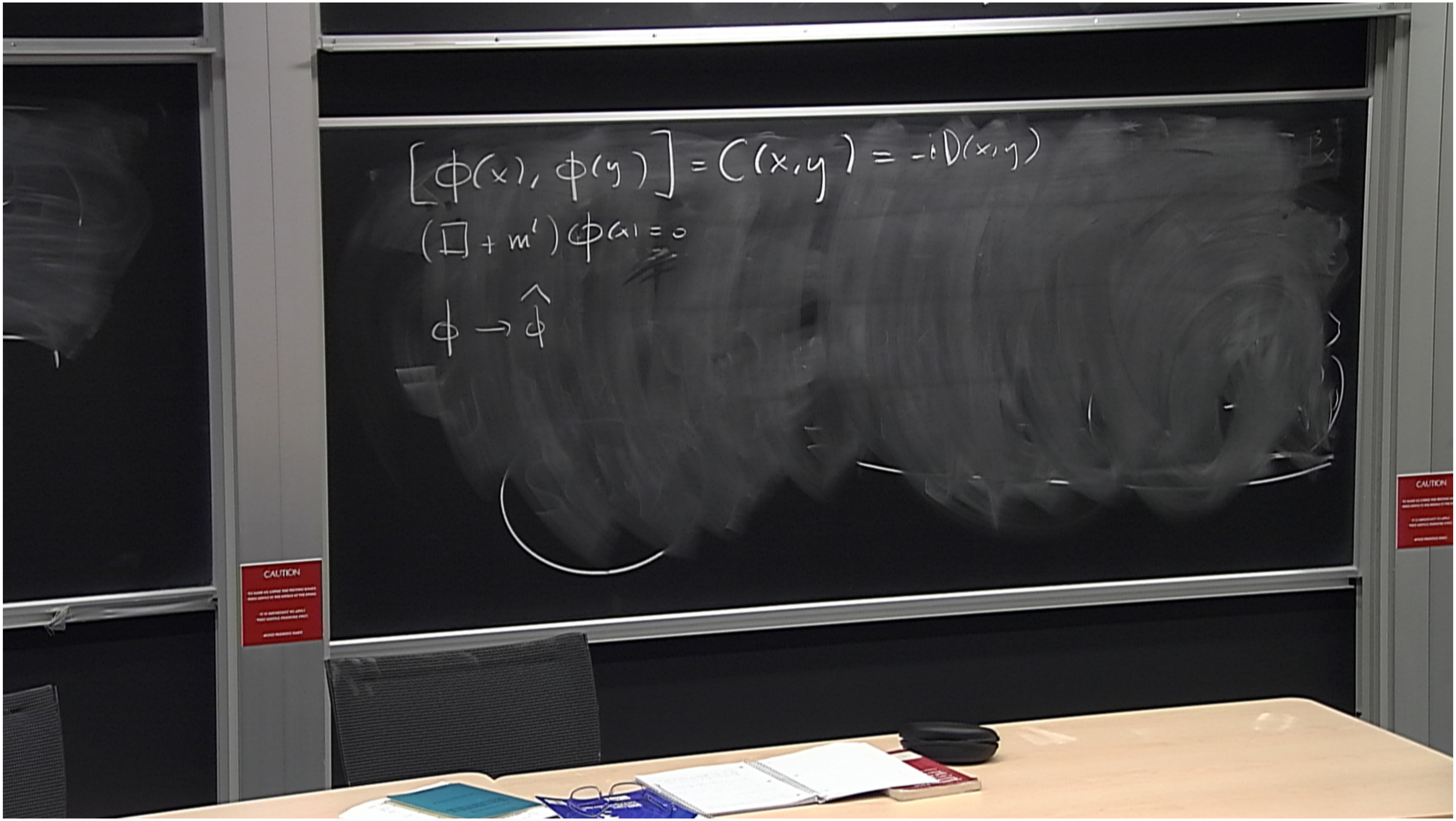
$$W(\bar{f}, f) = \|\Phi(f)|0\rangle\|^2 \geq 0$$

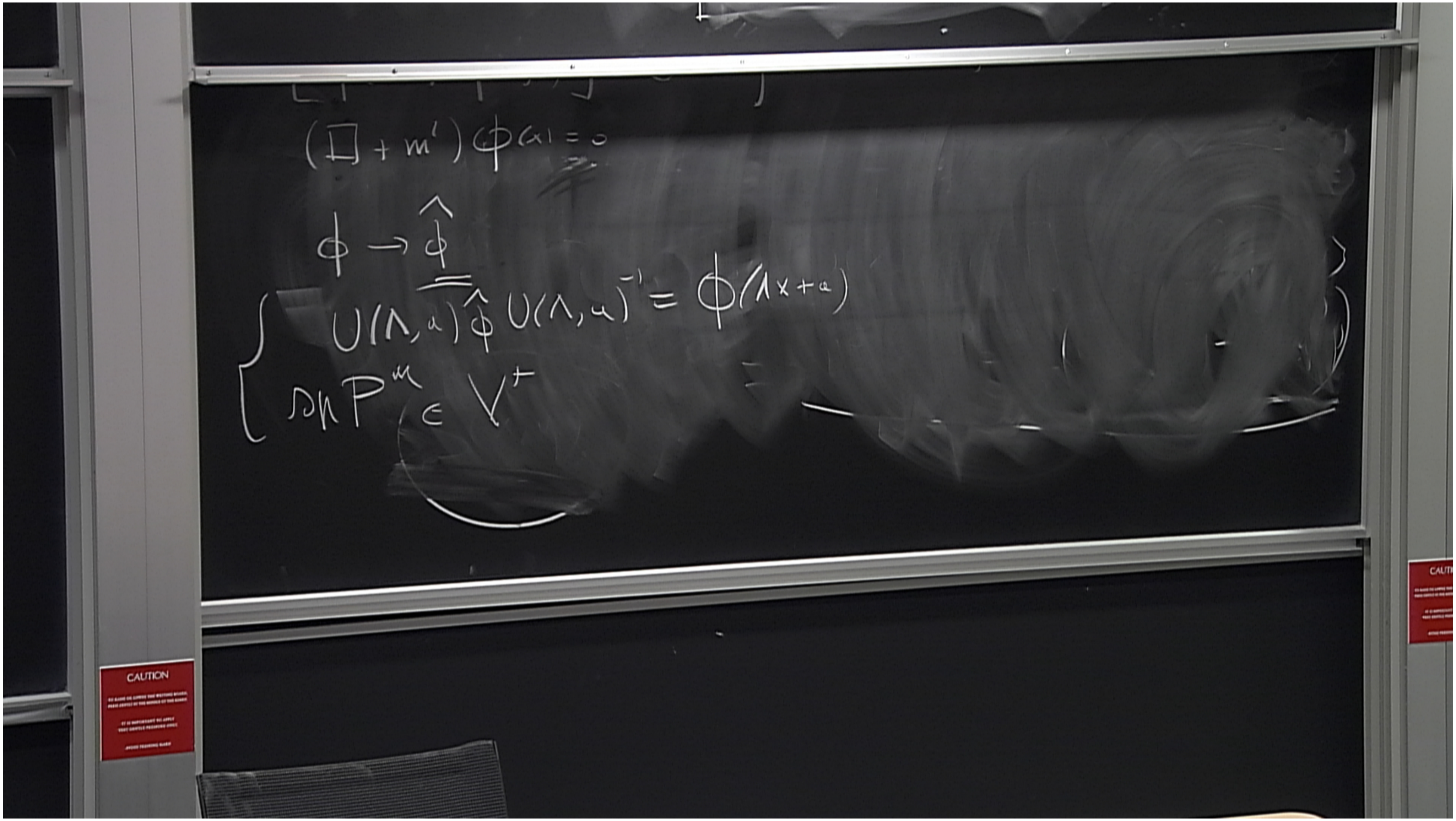
Σ



$[a_i, a_j^+] = \delta_{ij}$
$[a_i, a_j] = 0$
$a_j |0\rangle$
$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$
$\langle u_i, u_j^* \rangle = 0$
$[\phi(x), \phi(y)] = C(x,y)$
$-\int \nabla^\mu \phi \nabla_\mu \phi - m^2 \phi^2 \sqrt{g}\, d^4x$
$= -iE(x,y) = \sum_j (u_j(x) u_j^*(y) - u_j^*(x) u_j(y))$
$S(x,y)$


⟨u_i^*, u_j^*⟩ = -δ_ij
⟨u_i, u_j^*⟩ = 0
[a_i, a_j^†] = δ_ij
[a_i, a_j] = 0
a_j |0⟩
→ ∫ ∇^μφ ∇_μφ - m²φ² √g d⁴x
[φ(x), φ(y)] = C(x,y)
= -iE(x,y) = Σ_j (u_j(x) u_j^*(y) - u_j^*(x) u_j(y))
S(x,y)
CAUTION


$[\phi(x), \phi(y)] = C(x,y) = -iD(x,y)$
$(\Box + m^2)\phi(x) = 0$
$\phi \rightarrow \hat{\phi}$

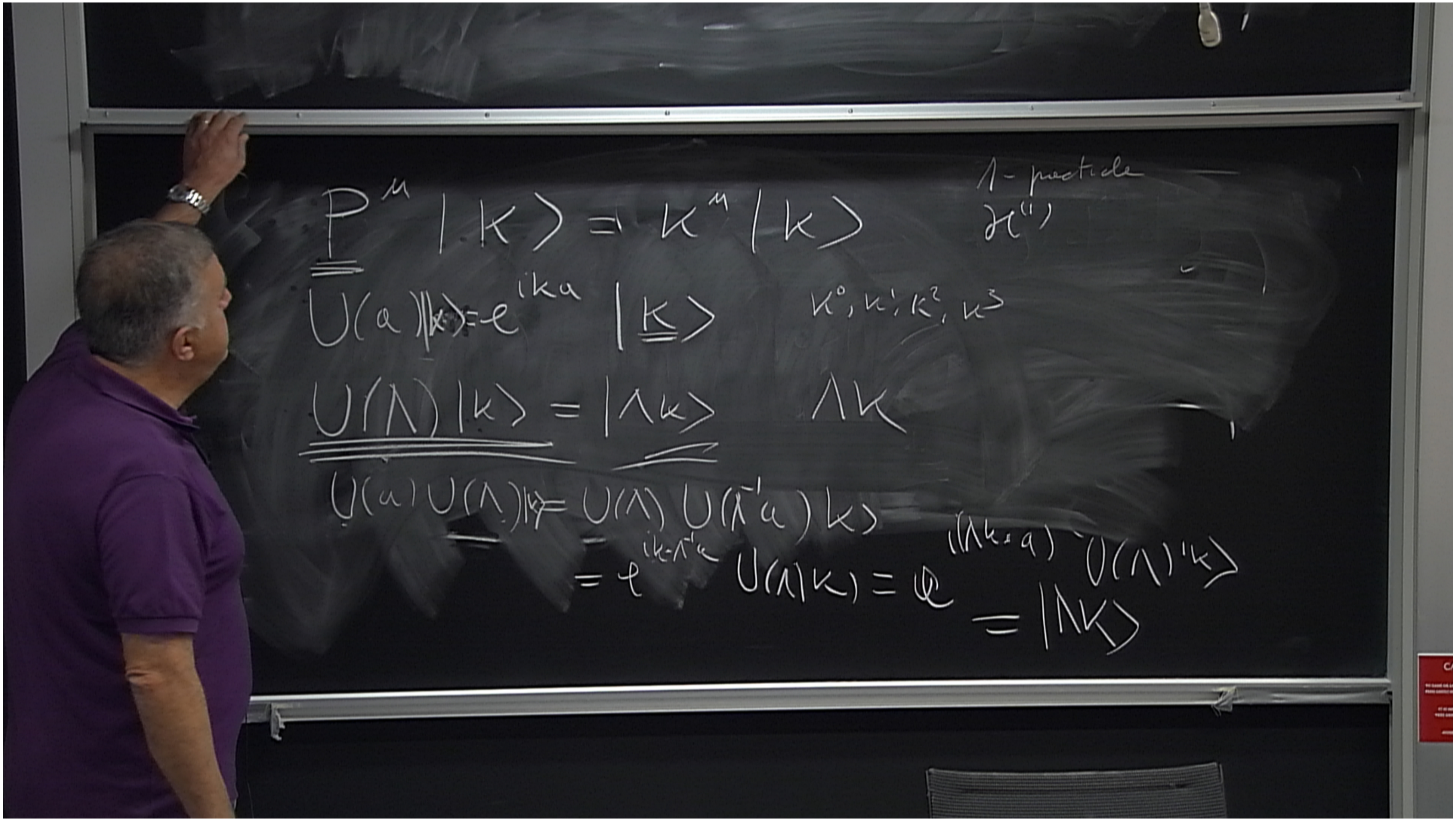

$(\Box + m^2)\phi(x) = 0$
$\phi \to \hat{\phi}$
$U(\Lambda, a)\,\hat{\phi}\,U(\Lambda, a)^{-1} = \phi(\Lambda x + a)$
$\mathrm{sp}\, P^\mu \in V^+$

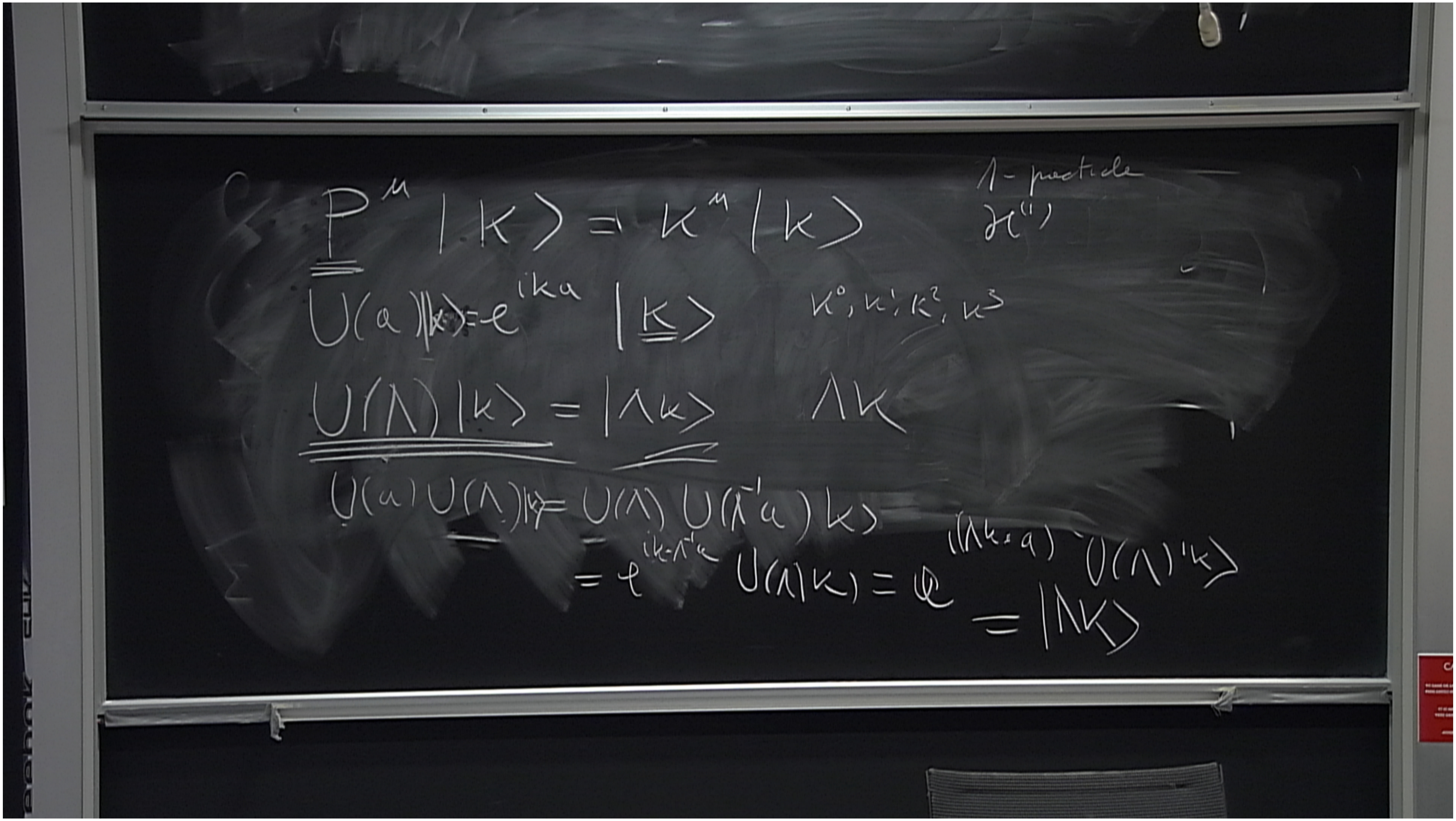
$$[\phi(x), \phi(y)] = C(x,y) = -iD(x,y)$$

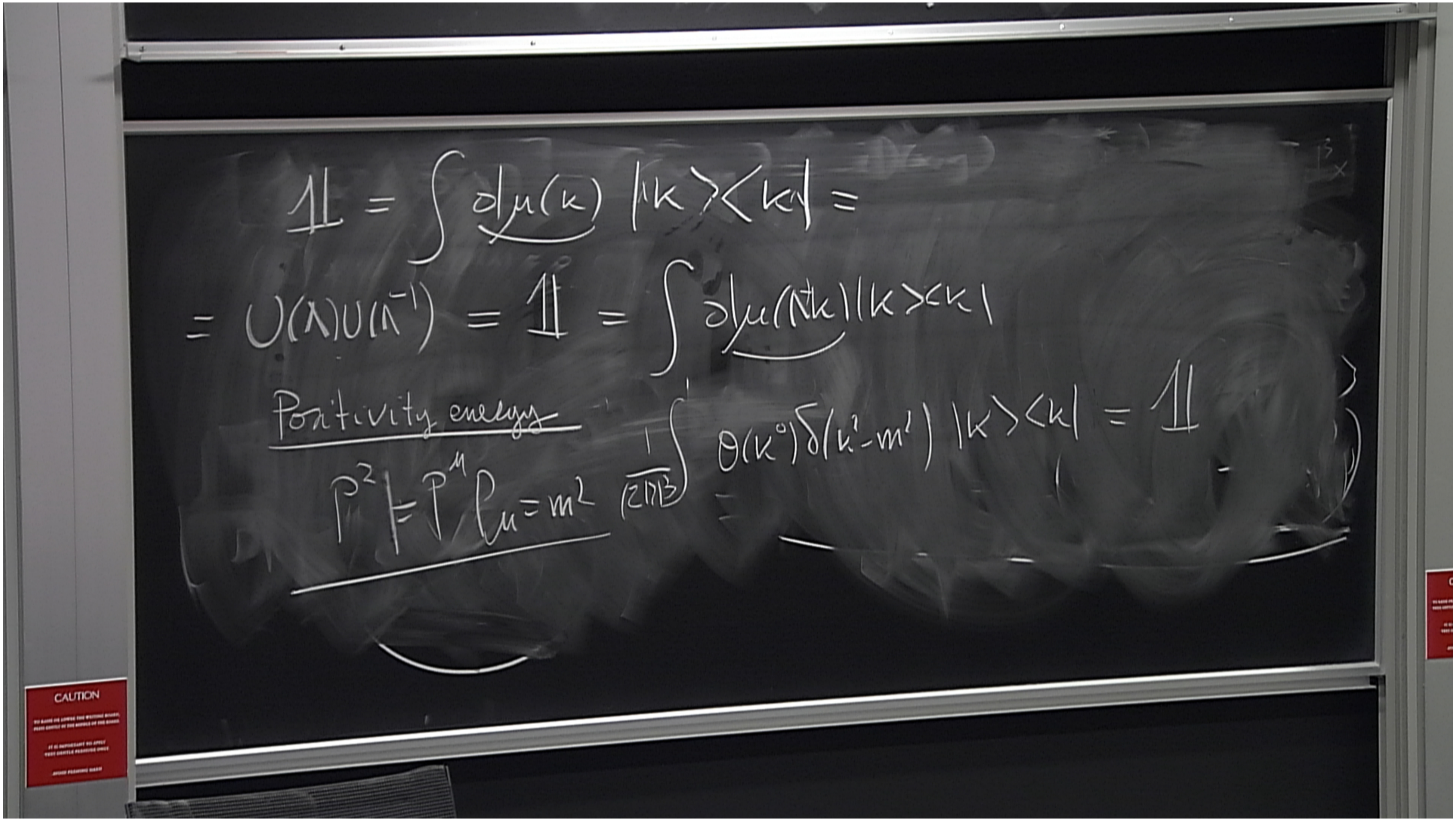
$$(\Box + m^2)\phi(x) = 0$$

$$\phi \to \hat{\phi}$$

$$\begin{cases} U(\Lambda, a)\,\hat{\phi}\,U(\Lambda, a)^{-1} = \phi(\Lambda x + a) \\ \mathrm{Sp}\,P^\mu \in V^+ \end{cases}$$


P^μ |k> = k^μ |k>
1-particle
U(a)|k> = e^{ika} |k>
k^0, k^1, k^2, k^3
U(Λ)|k> = |Λk>
Λk

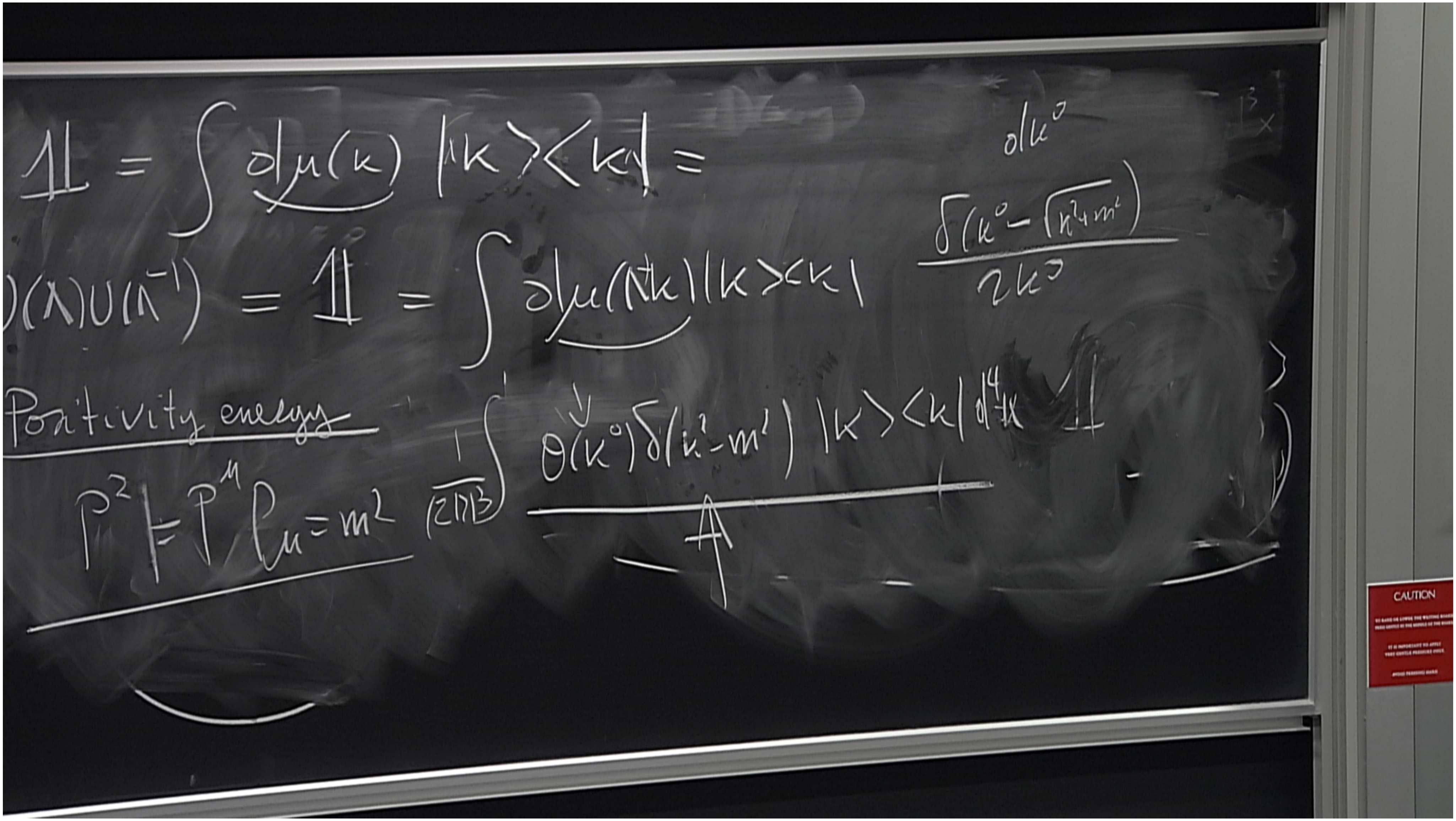

$P^\mu |k\rangle = k^\mu |k\rangle$
1-particle
$\mathcal{H}^{(1)}$
$U(a)|k\rangle = e^{ika} |\underline{k}\rangle$
k^0, k^1, k^2, k^3
$U(\Lambda)|k\rangle = |\Lambda k\rangle$
Λk
$U(a)\,U(\Lambda)|k\rangle = U(\Lambda)\,U(\Lambda^{-1}a)|k\rangle$
$= e^{ik\cdot\Lambda^{-1}a}\,U(\Lambda)|k\rangle = e^{i(\Lambda k\cdot a)}\,U(\Lambda)|k\rangle$
$= |\Lambda k\rangle$


$\mathbb{1} = \int d\mu(k)\, |k\rangle\langle k| =$
$= U(\Lambda)U(\Lambda^{-1}) = \mathbb{1} = \int d\mu(\Lambda k)\,|k\rangle\langle k|$
Positivity energy
$P^2 = P^\mu P_\mu = m^2$
$\frac{1}{(2\pi)^3}\int \theta(k^0)\delta(k^2-m^2)\,|k\rangle\langle k| = \mathbb{1}$

$$|k'\rangle = \int \frac{d^3k}{(2\pi)^3\, 2k^0}\, |k\rangle\langle k|k'\rangle = |k'\rangle$$

$$\langle k|k'\rangle = (2\pi)^3\, 2k^0\, \delta(\vec{k}-\vec{k}')$$

relativistic normalization


$\mathbb{1} = \int d\mu(k)\, |k\rangle\langle k| =$
dk^0
$\frac{\delta(k^0 - \sqrt{k^2+m^2})}{2k^0}$
$U(\Lambda)U(\Lambda^{-1}) = \mathbb{1} = \int d\mu(\Lambda k) |k\rangle\langle k|$
Positivity energy
$P^2 = P^\mu P_\mu = m^2$
$\frac{1}{(2\pi)^3}\int \theta(k^0)\delta(k^2-m^2)\, |k\rangle\langle k|\, d^4k$
CAUTION

$$|k'\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} |k\rangle\langle k|k'\rangle = |k'\rangle$$

impo

$$\langle k|k'\rangle = (2\pi)^3 2k^0 \delta(\vec{k}-\vec{k}')$$

relativistic normalization

$$|f\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} f(\vec{k})|k\rangle$$

$$\langle f|g\rangle = \int f^*(k) g(k) \frac{d^3k}{(2\pi)^3 2k^0}$$

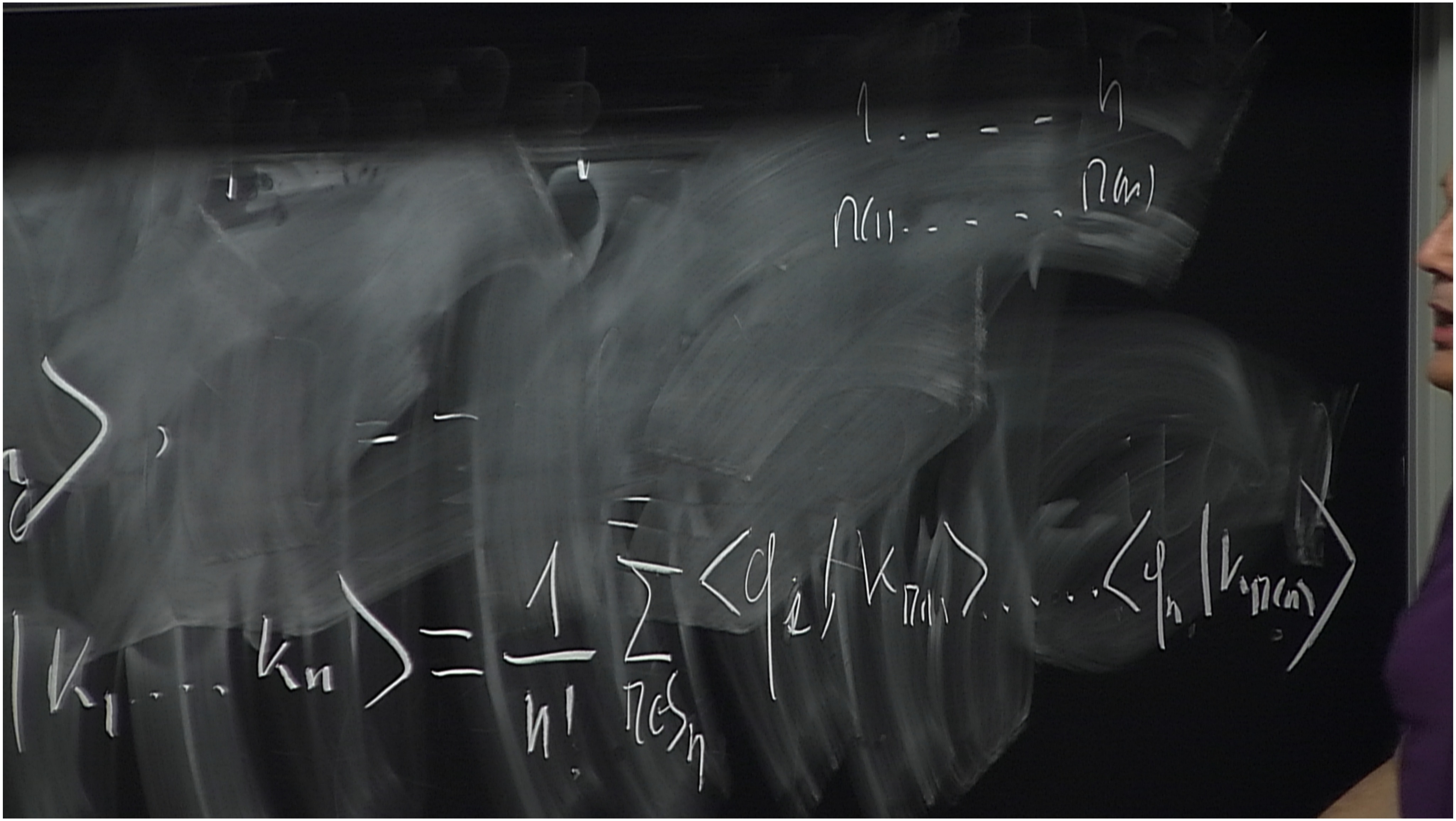
$$1 \dots n$$

$$\pi(1) \dots \pi(n)$$

$$\rangle = |0\rangle$$

$$|k_1 \dots k_n\rangle, =$$

$$\langle q_1 \dots q_n | k_1 \dots k_n \rangle = \frac{1}{n!} \sum_{\pi \in S_n} \langle q_1 | k_{\pi(1)} \rangle \dots \langle q_n | k_{\pi(n)} \rangle$$


$1 \ldots n$
$\pi(1) \ldots \pi(n)$
$|k_1 \ldots k_n\rangle = \frac{1}{n!} \sum_{\pi \in S_n} \langle q_1 | k_{\pi(1)}\rangle \ldots \langle q_n | k_{\pi(n)}\rangle$

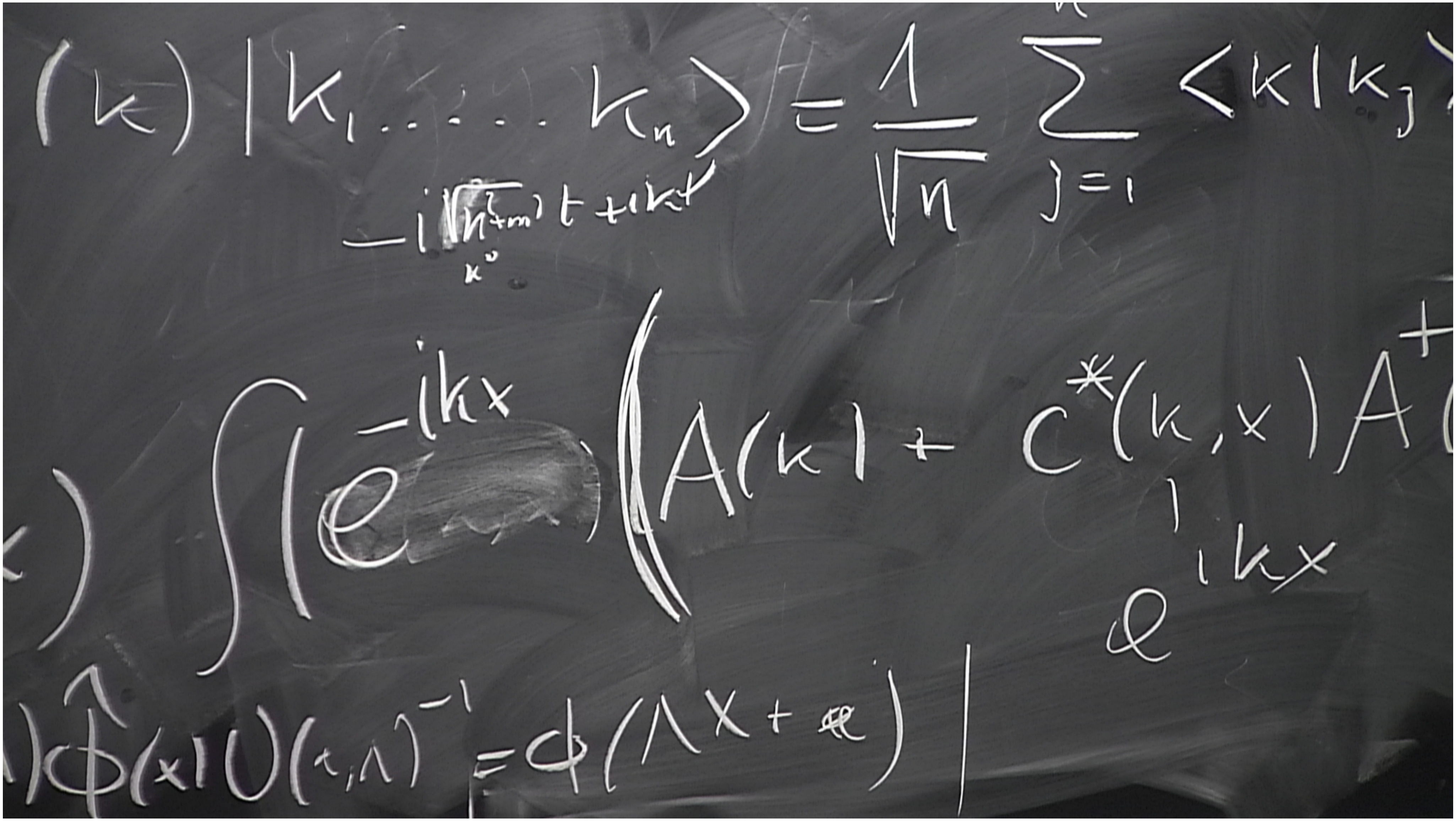
$$\rightarrow A^{+}(k)\,|k_1 \ldots k_n\rangle = \sqrt{n+1}\,|k, k_1 \ldots k_n\rangle$$

$$\rightarrow A(k)\,|k_1 \ldots k_n\rangle = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \langle k | k_j \rangle\, |k_1 \ldots \cancel{k_j} \ldots k_n\rangle$$

$$\mathcal{J}(\mathcal{H}^{(1)})$$

$$\phi(x)$$

$$P^2 = P^{\mu} P_{\mu} = m^2 \quad (2\pi)^3$$



(k) |k_1 . . . . . k_n> = 1/√n Σ_{j=1}^n <k|k_j
−i √(k²+m²) t + i k x
k⁰
) ∫ (e^{−ikx} (A(k) + c*(k,x) A⁺(
e^{ikx}
) φ(x) U(a,Λ)^{−1} = φ(Λx + a)

$$\longrightarrow A^{+}(k)\,|k_1 \ldots k_n\rangle = \sqrt{n+1}\,|k, k_1 \ldots k_n\rangle$$

$$\longrightarrow A(k)\,|k_1 \ldots k_n\rangle = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \langle k|k_j\rangle\,|k_1 \ldots \cancel{k_j} \ldots k_n\rangle$$

$$-i\sqrt{k^2+m^2}\,t + ikx$$

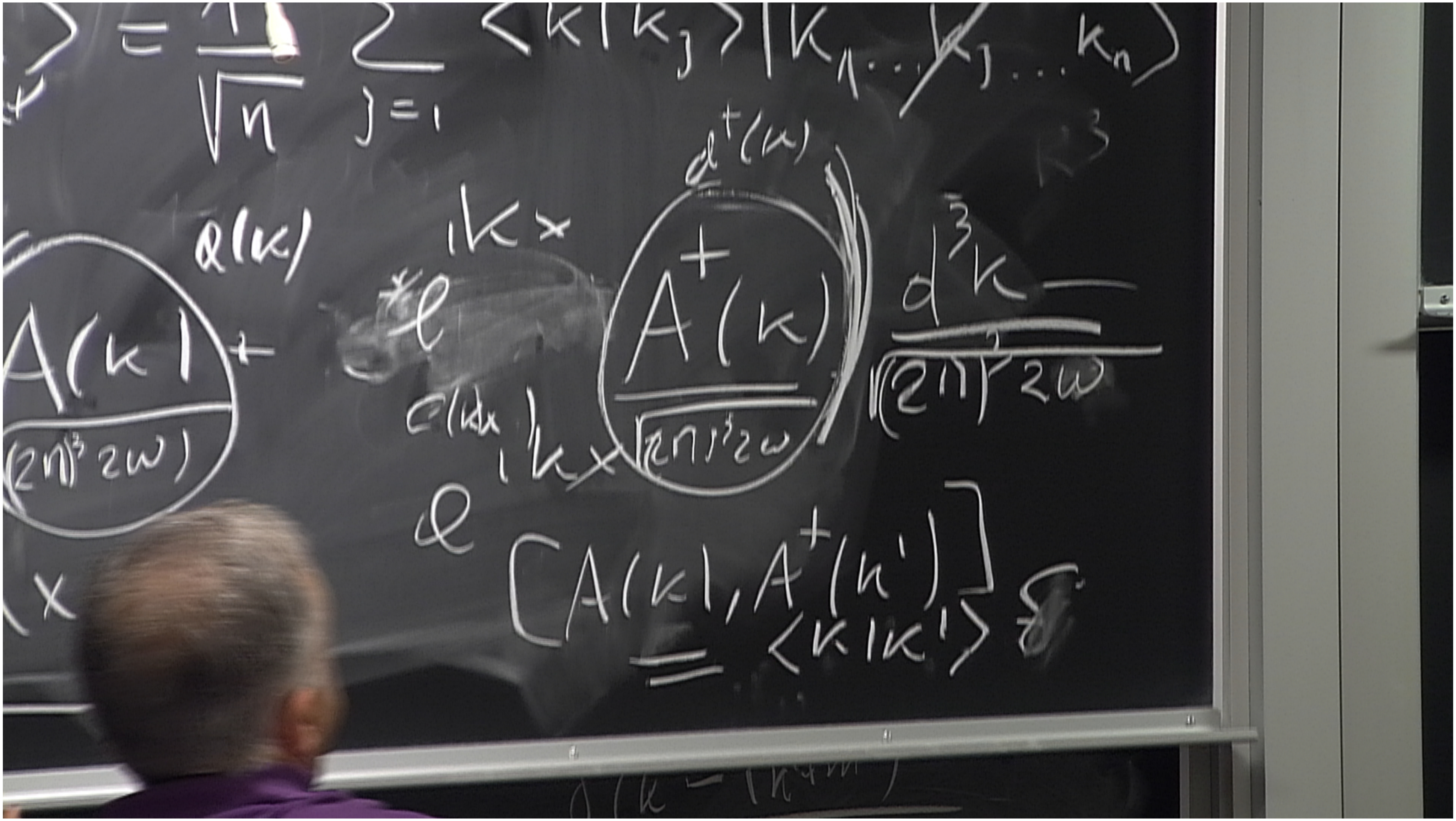
$$\mathcal{J}(\mathcal{H}^{(1)})$$

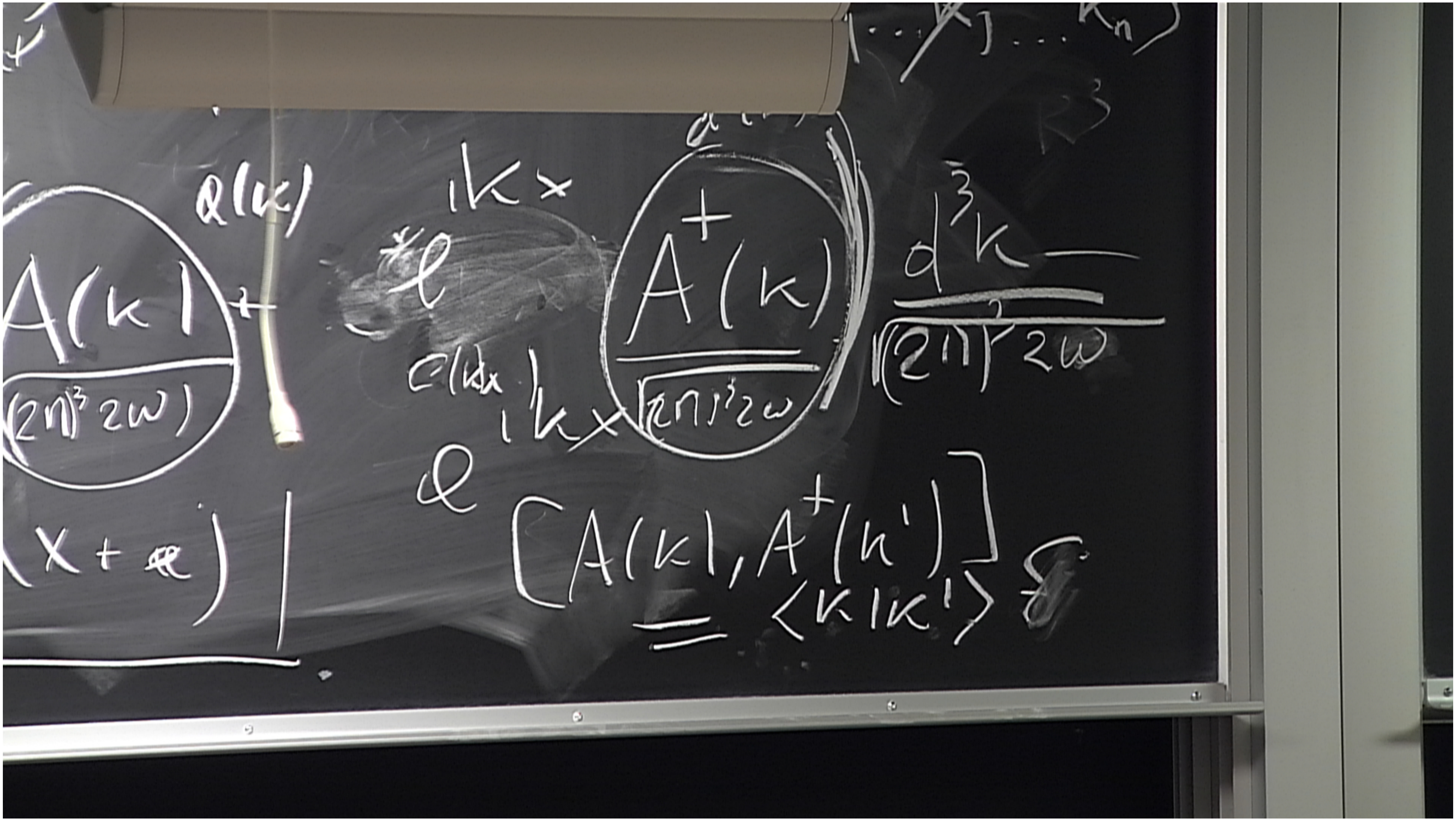
$$\hat{\phi}(x) \int \left(e^{-ikx} A(k) + e^{ikx} A^{+}(k) \right) \frac{d^3k}{(2\pi)^3 2\omega}$$

$$e^{ikx}$$

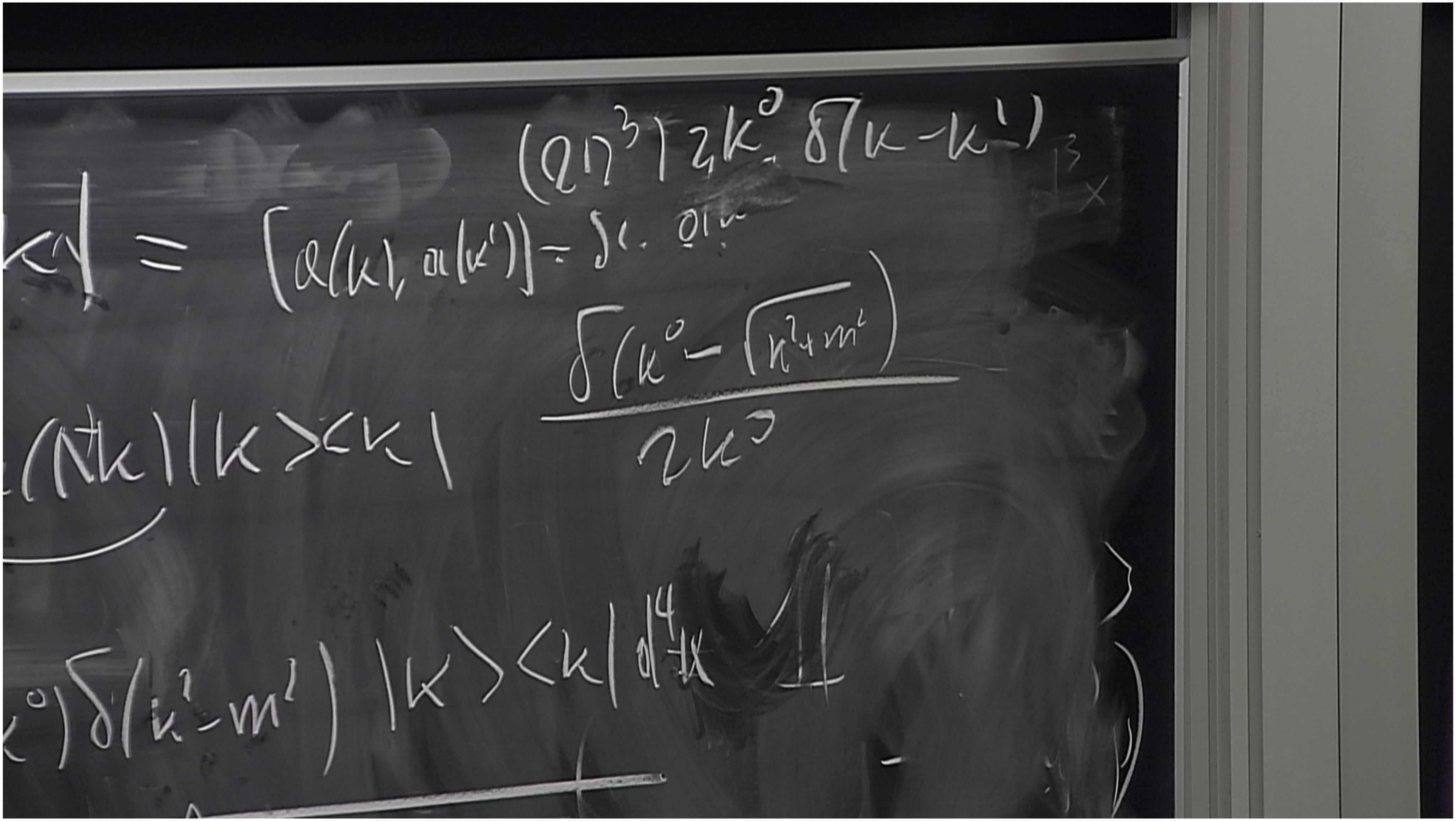
$$U(a,\Lambda)\hat{\phi}(x)U(a,\Lambda)^{-1} = \phi(\Lambda x + a)$$

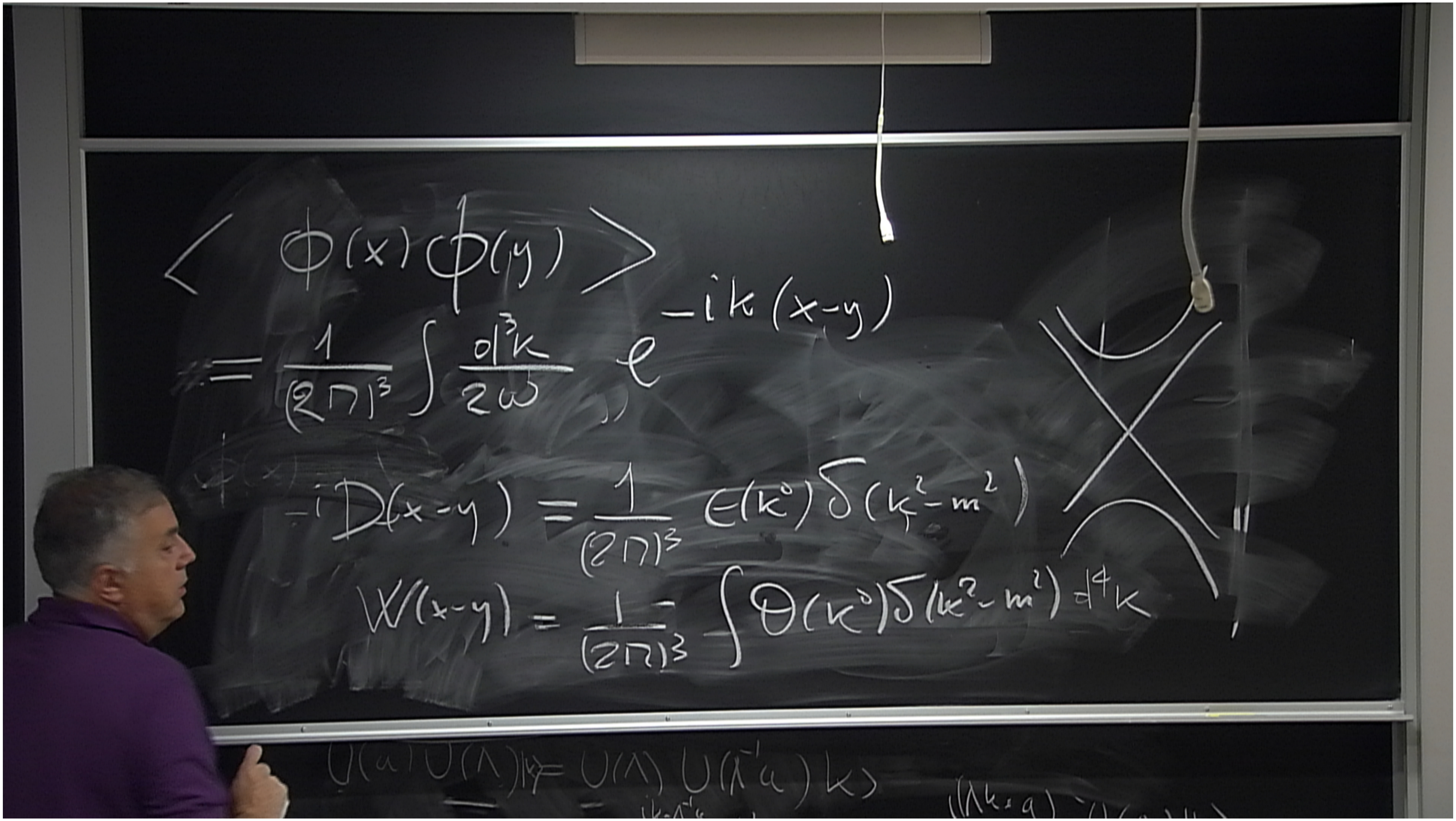
$$P^2 = P^\mu P_\mu = m^2 \quad \frac{1}{(2\pi)^3}\int \theta(k^0)\delta(k^2-m^2)\,|k\rangle\langle k|\,d^4k \quad \mathbb{1}$$





A(k)
(2π)³ 2ω
e ikx
A⁺(k)
d³k
[A(k), A⁺(k')]
= ⟨k|k'⟩


$(2\pi)^3 2k^0 \delta(k-k')$
$[a(k), a(k')] =$
$\frac{\delta(k^0 - \sqrt{k^2+m^2})}{2k^0}$
$|k\rangle\langle k|$
$\delta(k^2-m^2)\ |k\rangle\langle k|\ d^4k$



$\langle \phi(x)\phi(y) \rangle$
$= \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\omega} e^{-ik(x-y)}$
$-iD(x-y) = \frac{1}{(2\pi)^3} \epsilon(k^0)\delta(k^2-m^2)$
$W(x-y) = \frac{1}{(2\pi)^3} \int \theta(k^0)\delta(k^2-m^2)\, d^4k$
$U(a)U(\Lambda)|k\rangle = U(\Lambda)\, U(\Lambda^{-1}a)|k\rangle$

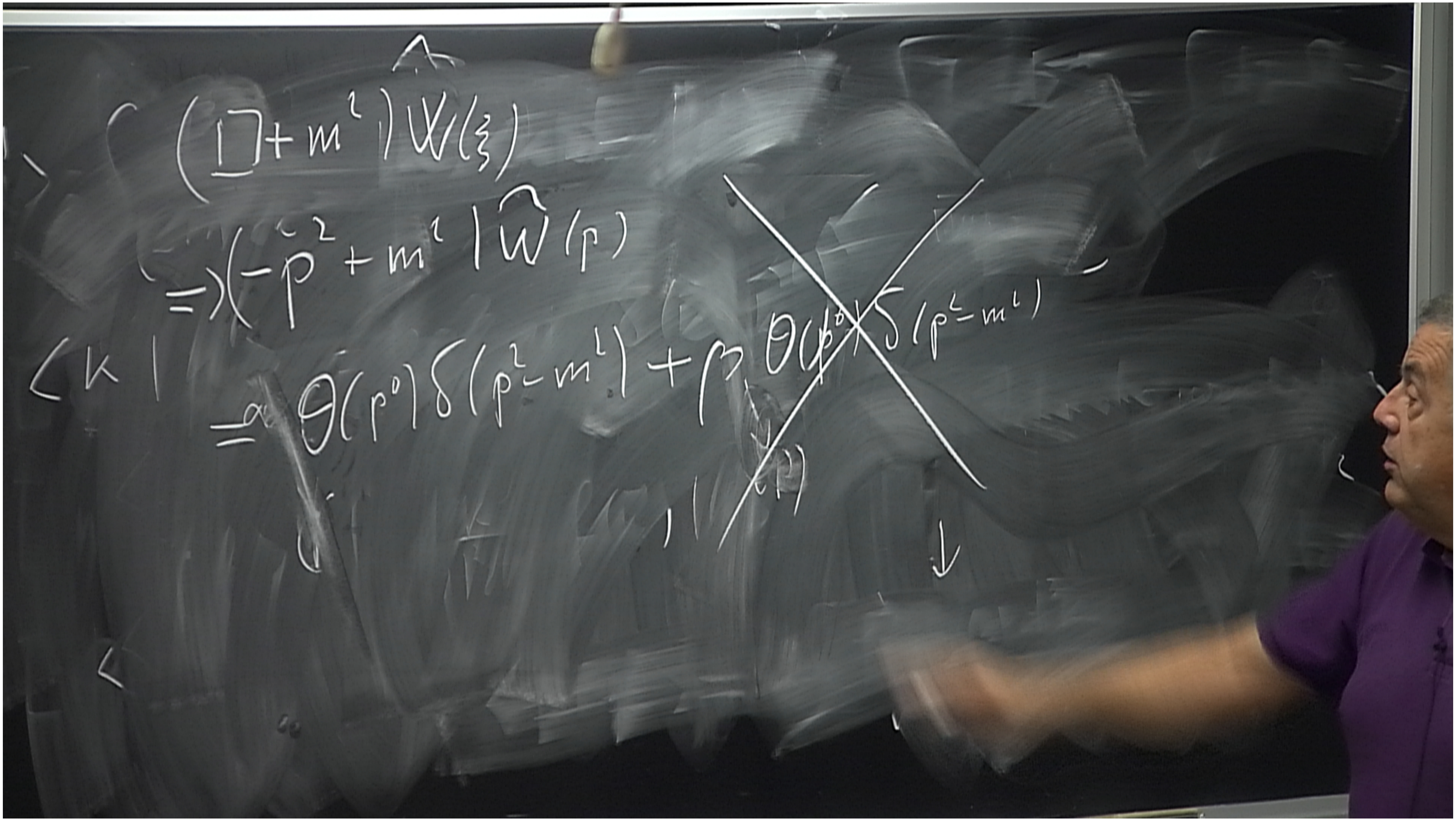
$$\langle f | g \rangle = \int \frac{f^*(k)\, g(k)\, d^3k}{(2\pi)^3\, 2k^0}$$

$$C(x,y) = W(x,y) - W(y,x)$$

$$\xi = x - y$$

$$C(x-y) = W(x-y) - W(y-x)$$

$$C(\xi) = W(\xi) - W(-\xi)$$

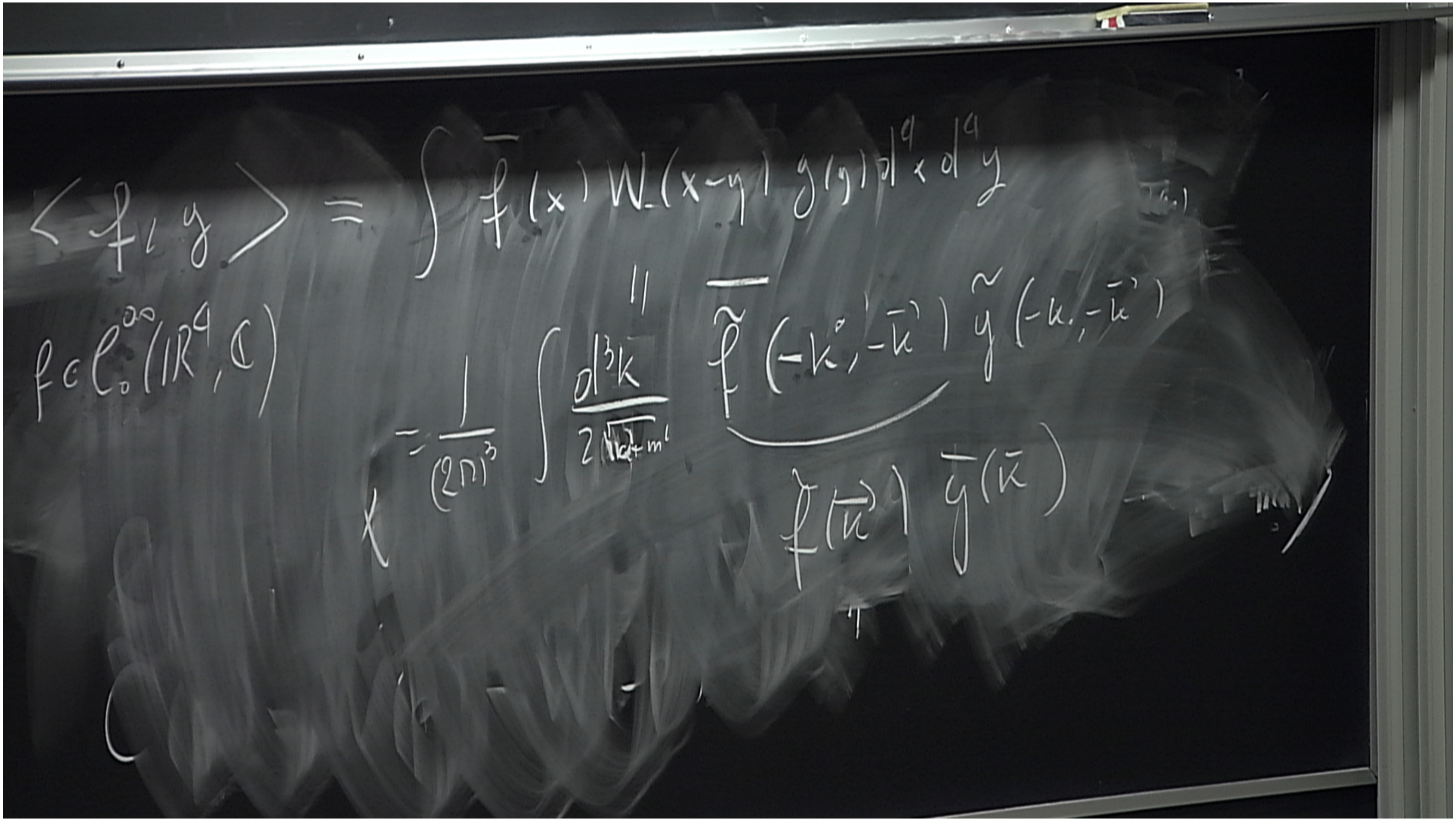

$(\Box + m^2)\, W(\xi)$
$\Rightarrow (-p^2 + m^2)\, \hat{W}(p)$
$= \alpha\, \theta(p^0)\, \delta(p^2 - m^2) + \beta\, \theta(-p^0)\, \delta(p^2 - m^2)$

$$(\Box + m^2)\,W(\xi)$$

$$\Rightarrow (-p^2 + m^2)\,\widehat{W}(p)$$

$$= \alpha\,\theta(p^0)\,\delta(p^2 - m^2) + \cancel{\beta\,\theta(-p^0)\,\delta(p^2 - m^2)}$$

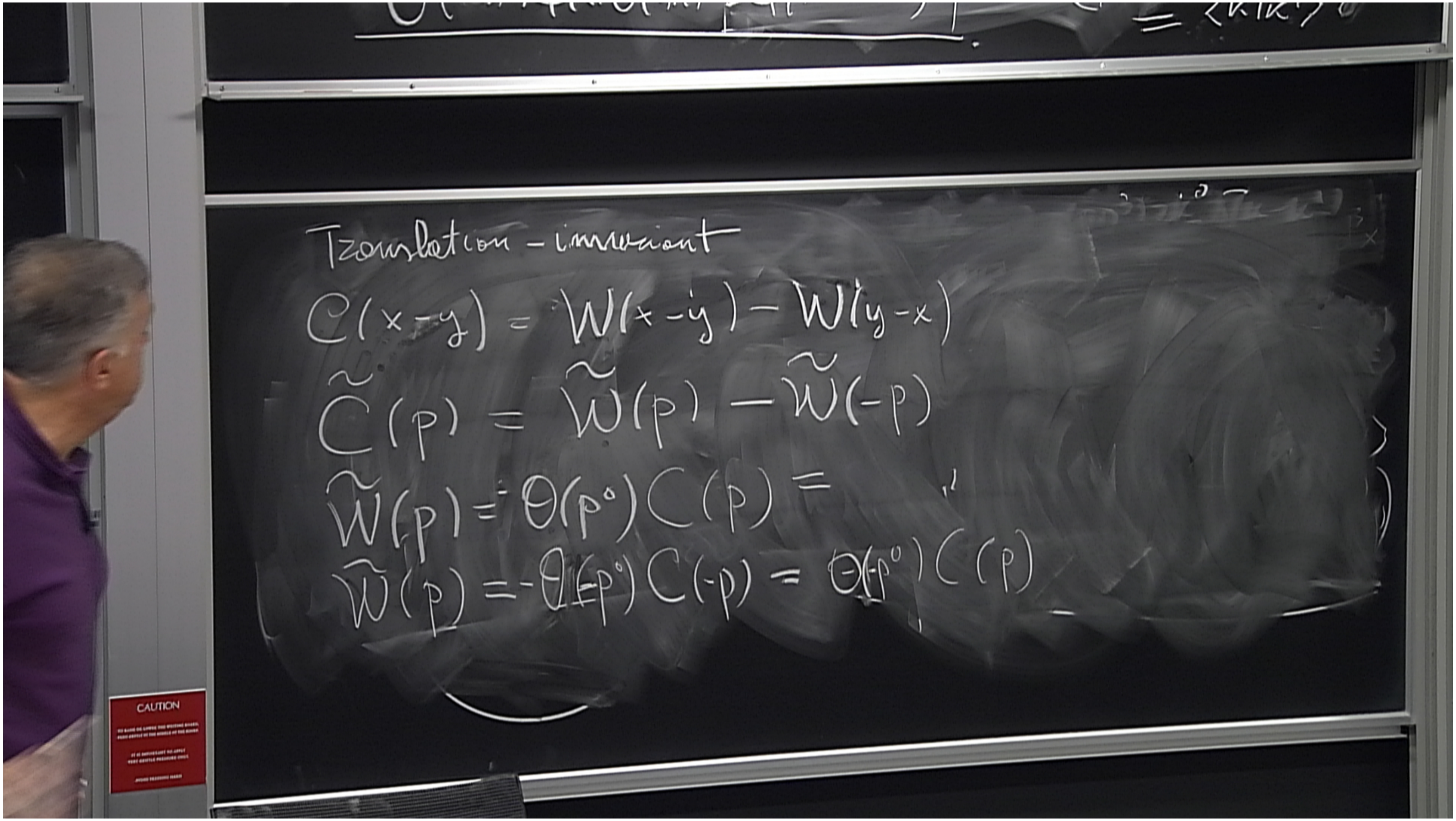
$$W(x,y) = \frac{1}{(2\pi)^3}\int e^{-ip(x-y)}\,\theta(p^0)\,\delta(p^2 - m^2)$$


$\langle f, g \rangle = \int \overline{f(x)}\, W_-(x-y)\, g(y)\, d^4x\, d^4y$
$f \in C_0^\infty(\mathbb{R}^4, \mathbb{C})$
$= \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\sqrt{k^2+m^2}}\, \overline{\tilde{f}(-k_0, -\vec{k})}\, \tilde{g}(-k_0, -\vec{k})$
$\tilde{f}(\vec{k})\, \bar{g}(\vec{k})$

Translation - invariant

$$C(x-y) = W(x-y) - W(y-x)$$

$$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p)$$


Translation - invariant
$C(x-y) = W(x-y) - W(y-x)$
$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p)$
$\tilde{W}(p) = \theta(p^0) C(p) =$
$\tilde{W}(p) = -\theta(-p^0) C(-p) = \theta(p^0) C(p)$
CAUTION

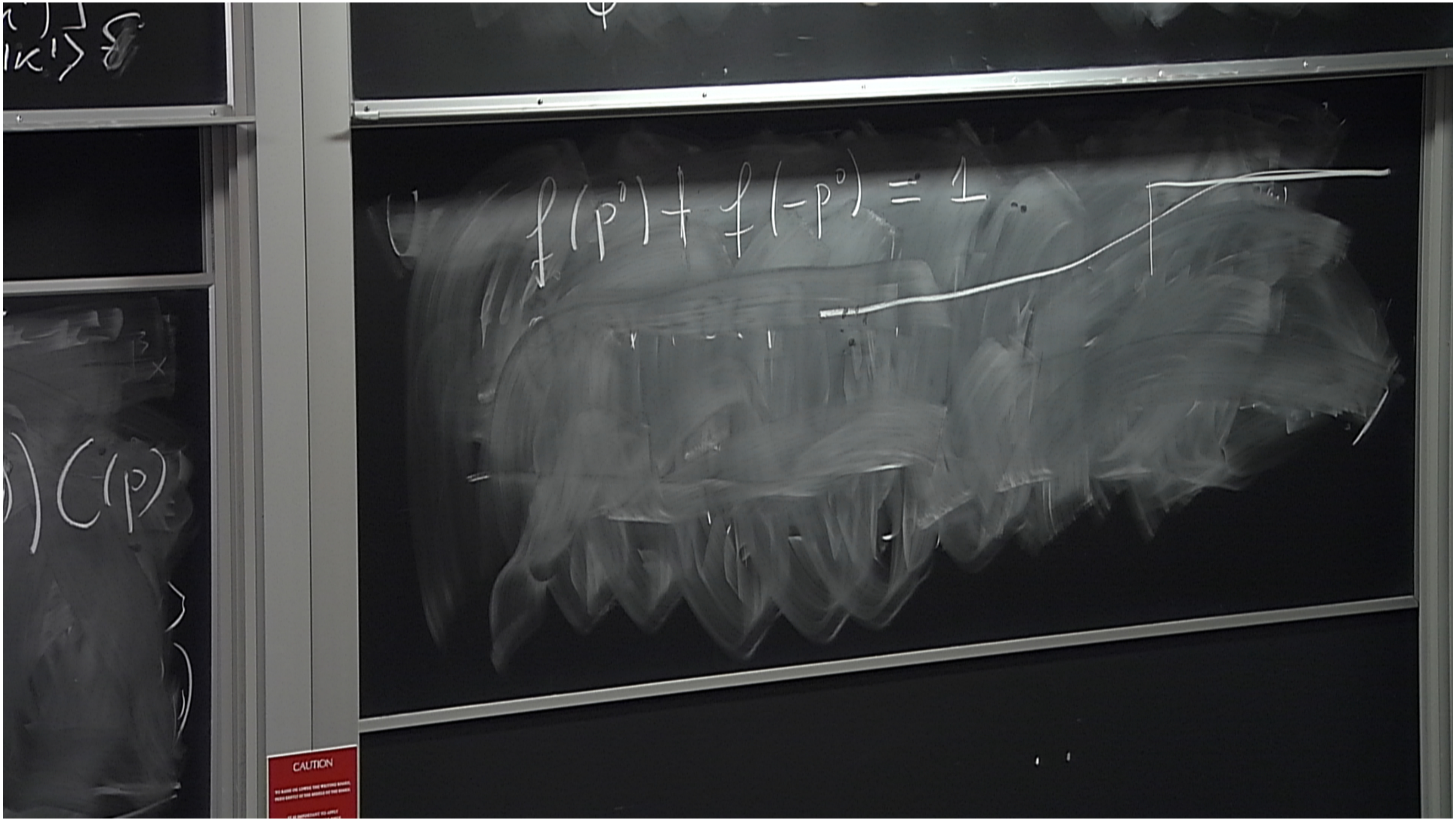
Translation - invariant

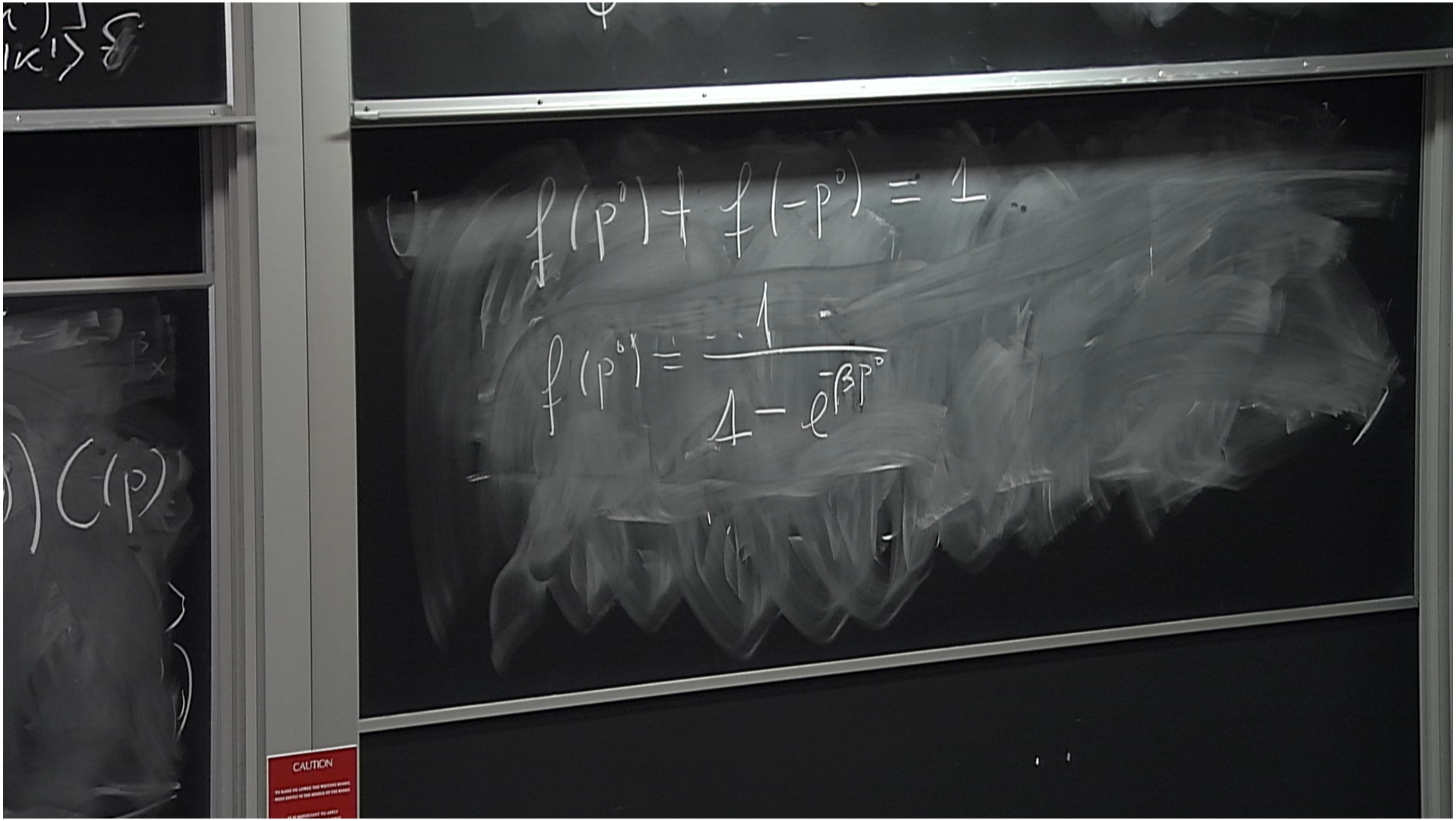
$$C(x-y) = W(x-y) - W(y-x)$$

$$\tilde{C}(p) = \tilde{W}(p) - \tilde{W}(-p) = \left(f(p^0) - f(-p^0)\right) C(p)$$

$$\tilde{W}(p) = f(p^0)\, C(p)$$

$$\tilde{W}(-p) = f(-p^0)\, C(-p) = -f(-p^0)\, C(p)$$


f(p⁰) + f(-p⁰) = 1



$f(p^0) + f(-p^0) = 1$
$f(p^0) = \frac{1}{1 - e^{-\beta p^0}}$
CAUTION

$$f(p^0) + f(-p^0) = 1$$

$$f(p^0) = \frac{1}{1-e^{-\beta p^0}} + \frac{1}{1-e^{\beta p^0}} = \frac{2-e^{-\beta p^0}-e^{\beta p^0}}{2-e^{-\beta p^0}-e^{\beta p^0}} = 1$$

$$W_\beta(x,y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\varepsilon(p^0)\delta(p^2-m^2)}{1-e^{-\beta p^0}}$$

$$W_\beta(x,y) = \frac{1}{(2\pi)^3}\int e^{-ip(x-y)} \frac{\varepsilon(p^0)\,\delta(p^2-m^2)}{1-e^{-\beta p^0}}$$

$$W_\beta(x,y) - W_\beta(y,x) = C(x,y)$$

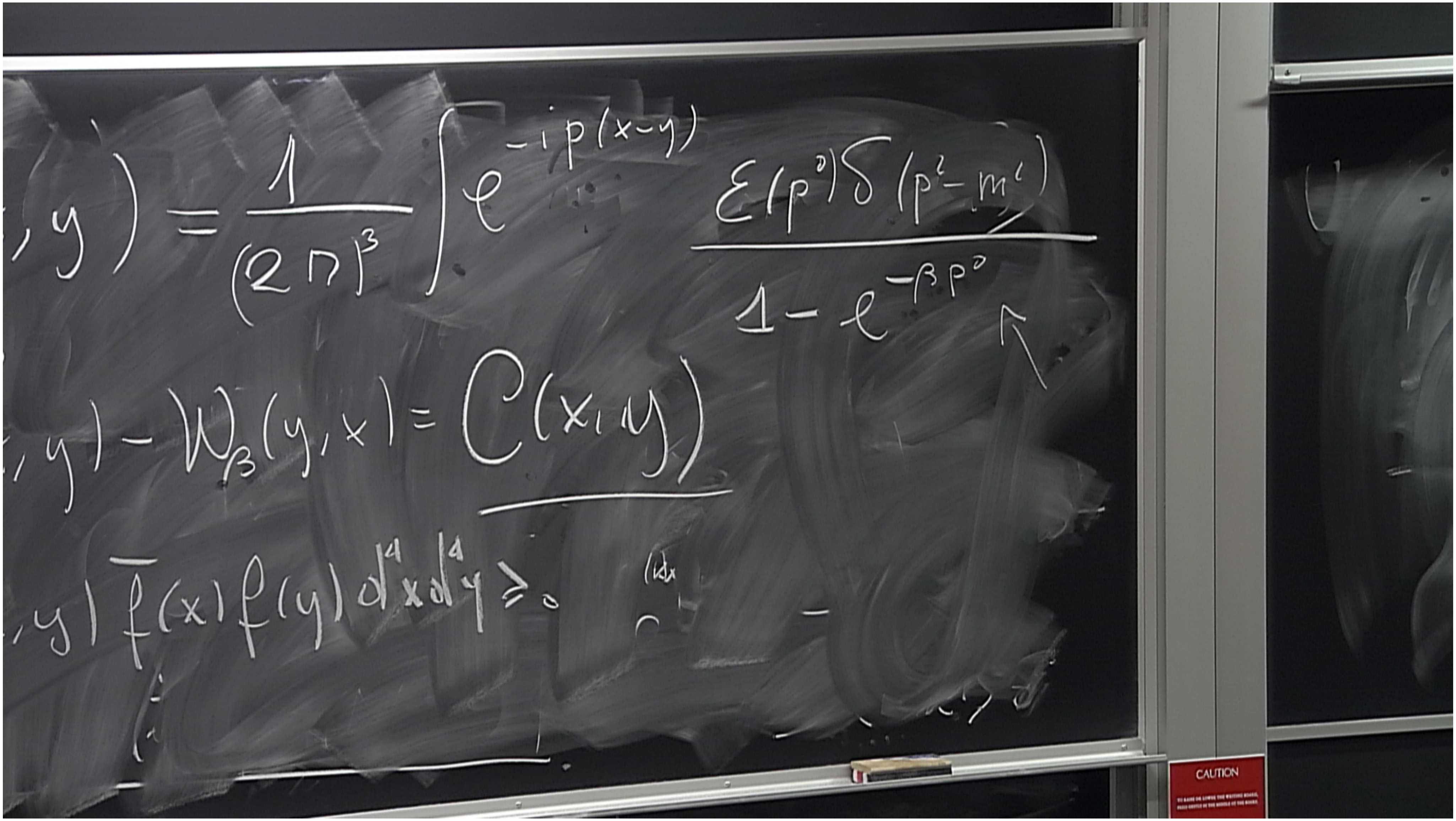
$$\int W_\beta(x,y)\,\bar{f}(x)\,f(y)\,d^4x\,d^4y \geq 0$$

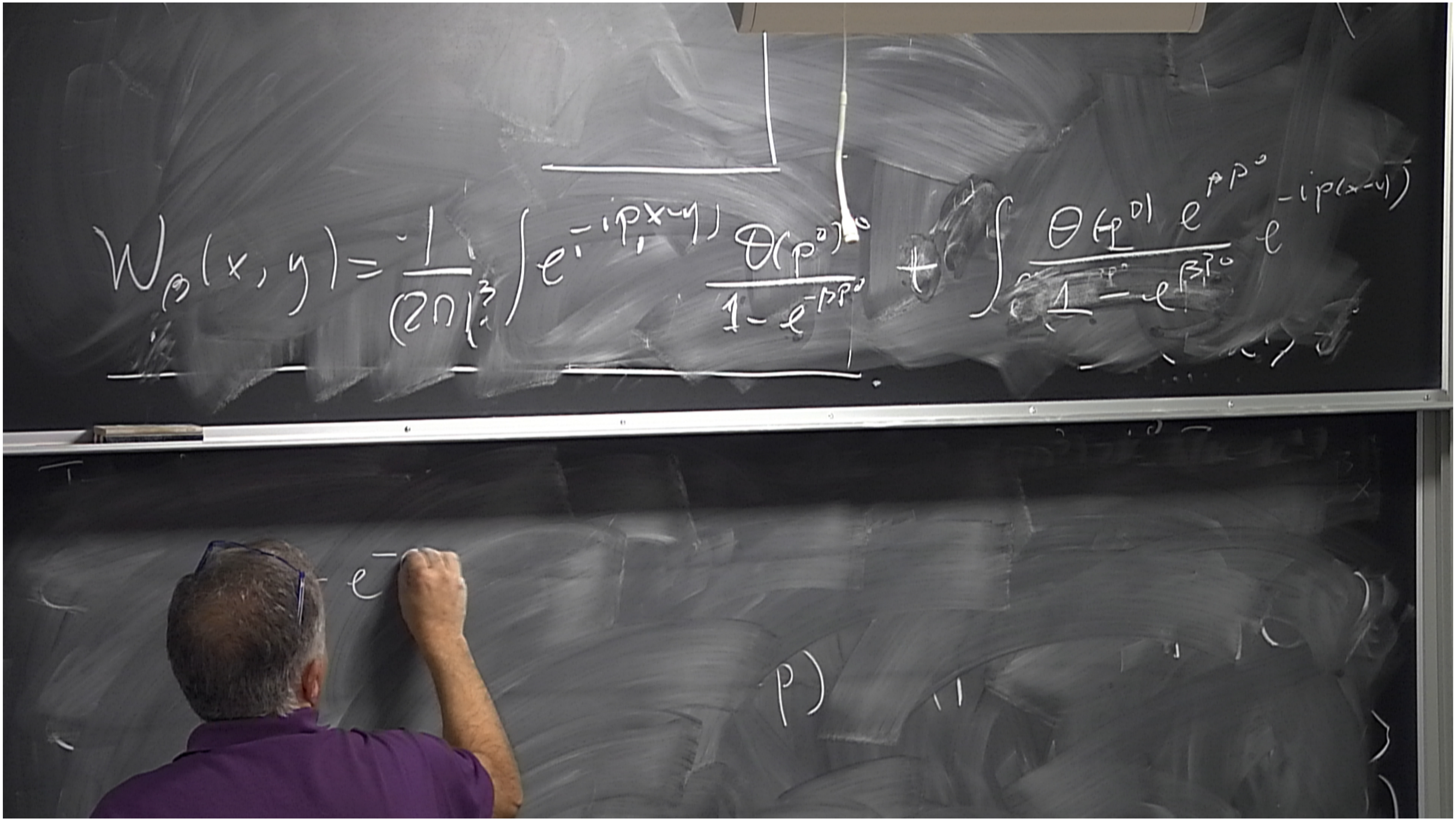
$$W_\beta(x,y) = \frac{1}{(2\pi)^3}\int e^{-ip(x-y)} \frac{\mathcal{E}(p}{1-}$$

KMS $T = 1/\beta$

$$W_\beta(x,y) - W_\beta(y,x) = C(x,y)$$

$$\int W_\beta(x,y)\,\overline{f(x)}\,f(y)\,d^4x\,d^4y \geq 0$$


$, y) = \frac{1}{(2\pi)^3} \int e^{-ip(x-y)} \frac{\varepsilon(p^0)\delta(p^2-m^2)}{1-e^{-\beta p^0}}$
$, y) - W_\beta(y, x) = C(x, y)$
$, y) \bar{f}(x) f(y) d^4x d^4y \geq 0$
CAUTION


$W_\beta(x,y) = \frac{1}{(2\pi)^3}\int e^{-ip(x-y)} \frac{\theta(p^0)}{1-e^{-\beta p^0}} + \int \frac{\theta(-p^0)\, e^{\beta p^0}}{1-e^{\beta p^0}} e^{-ip(x-y)}$

$$W_\beta(x,y) = \frac{1}{(2\pi)^3}\int e^{-ip(x-y)}\,\frac{\theta(p^0)}{1-e^{-\beta p^0}} + \int \frac{\theta(p^0)\, e^{\beta p^0}}{1-e^{\beta p^0}}\, e^{-ip(x-y)}$$

$$\frac{1}{1-e^{-\beta p^0}} = \sum_{n=0}^{\infty} e^{\beta p^0 n}$$

$$\frac{1}{1-e^{-\beta p^0}} = \sum_{n=0}^{\infty} e^{\beta p^0 n}$$

$$W_\beta(x,y) = \sum_{n=0}^{\infty} \frac{1}{(2\pi)^3} \int e^{-i[p(x-y) - i\underline{\beta p^0 n}]}\, \theta(p^0)\,\delta(p^2-m^2)$$

$$= \sum_{n=0}^{\infty} W_\beta(t - i\beta n, \vec{x}, s, \vec{y})$$

$$+ \sum_{n=1}^{\infty} W(s, \vec{y}, t + in\beta, \vec{x})$$

$$W(x-y) = \frac{1}{(2\pi)^3} \int \Theta(k^0)\delta(k^2-m^2)\, d^4k$$

1) $W(t,\vec{x},s,\vec{y})$

$-\beta < \operatorname{Im} t < 0$

$W(s$

$W(s,\vec{y},t,\vec{x}) =$

$= W(t+i\beta,\vec{x},s,y)$

$$\frac{\operatorname{Tr} e^{-\beta H}\phi(x)\phi(y)}{\operatorname{Tr} e^{-\beta H}}$$

$= |k\rangle$