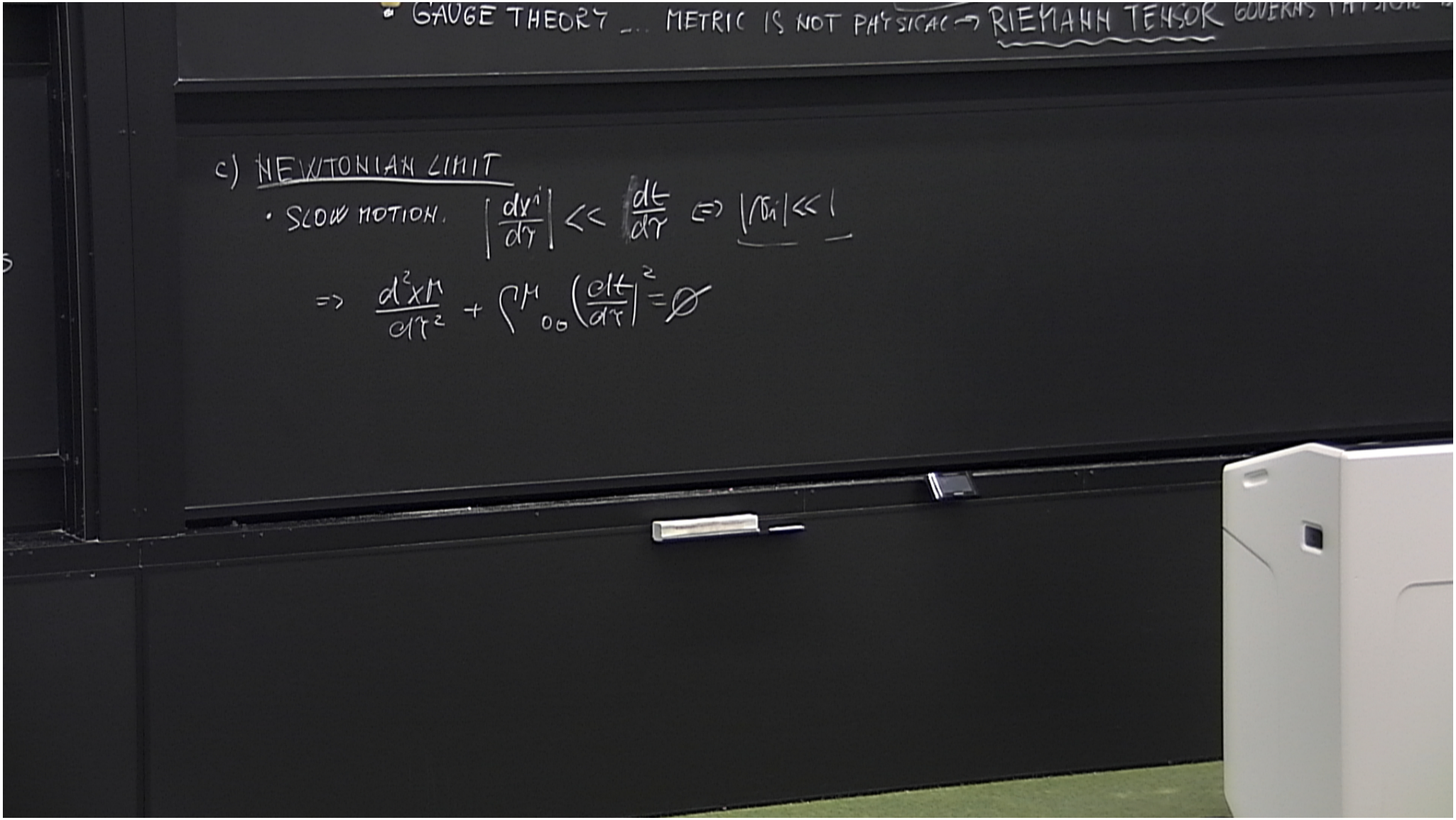
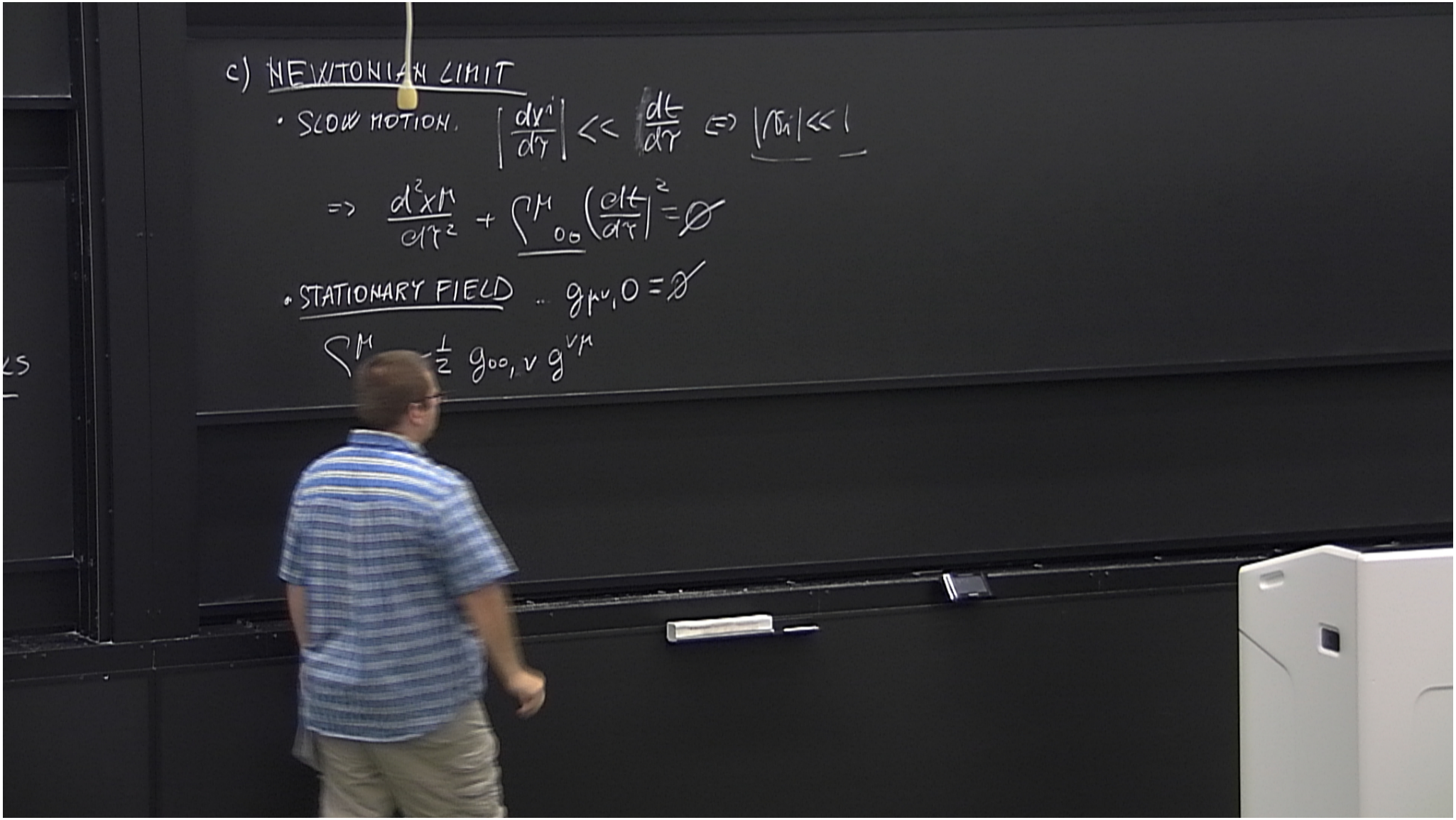
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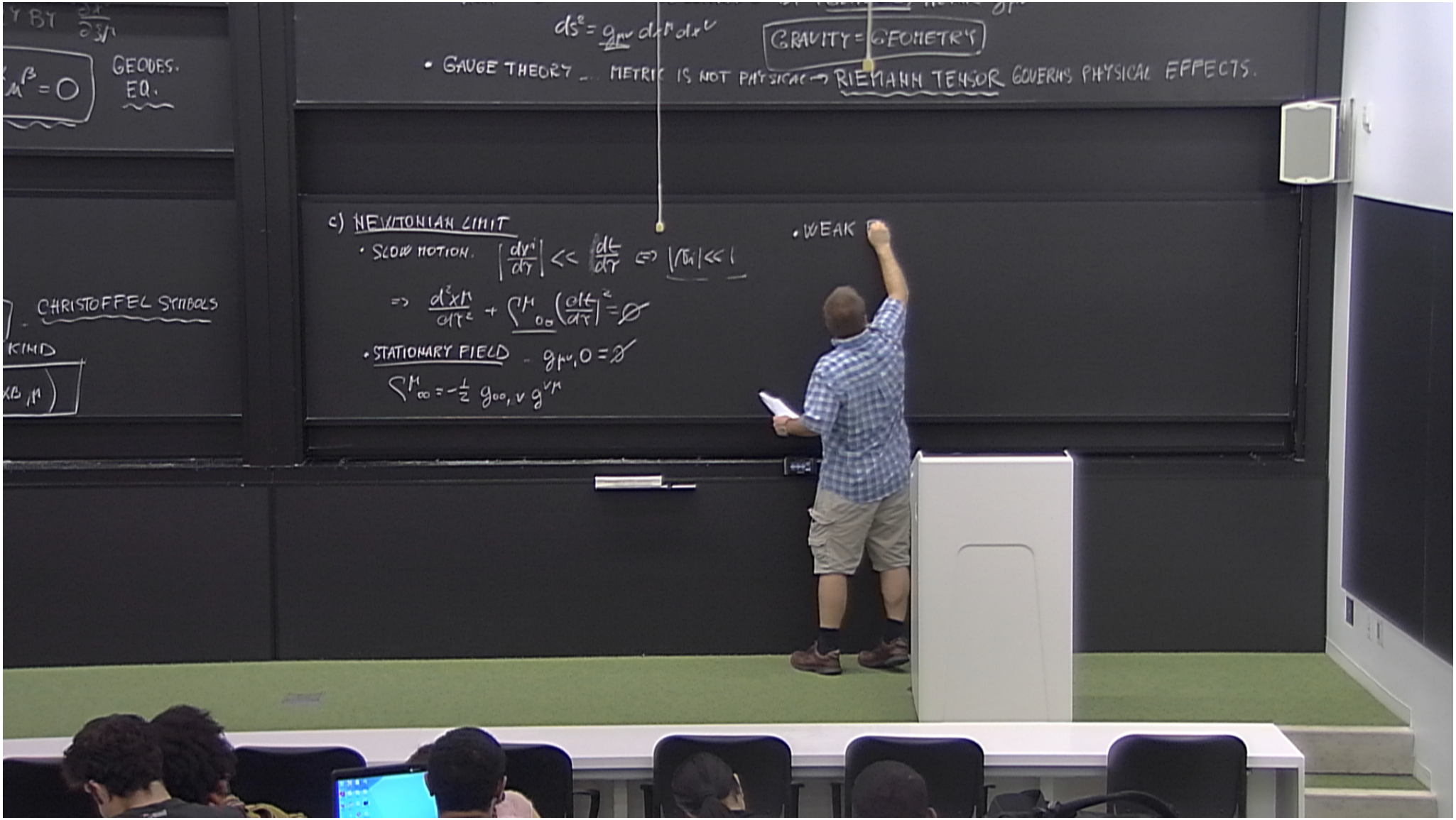
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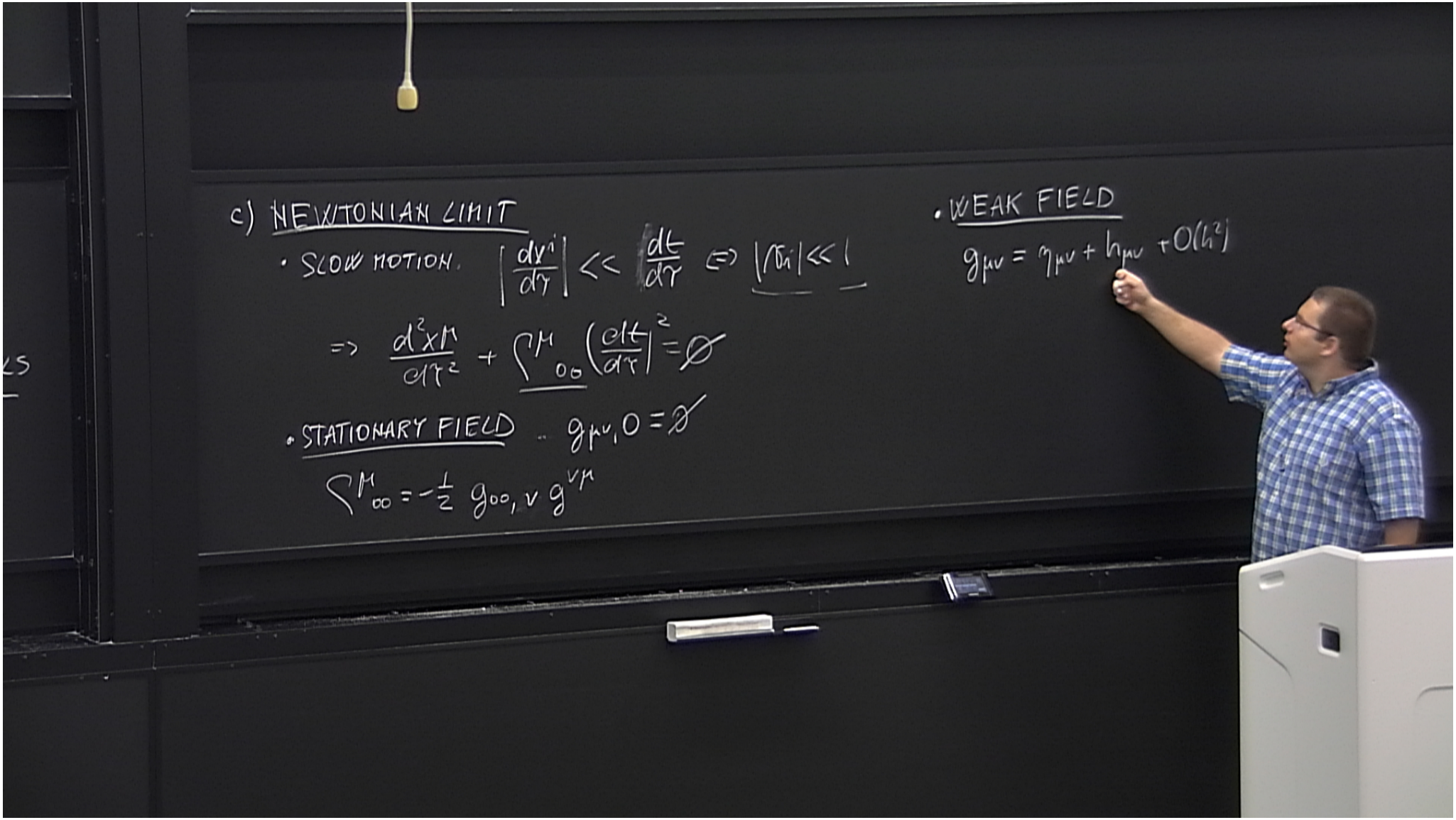
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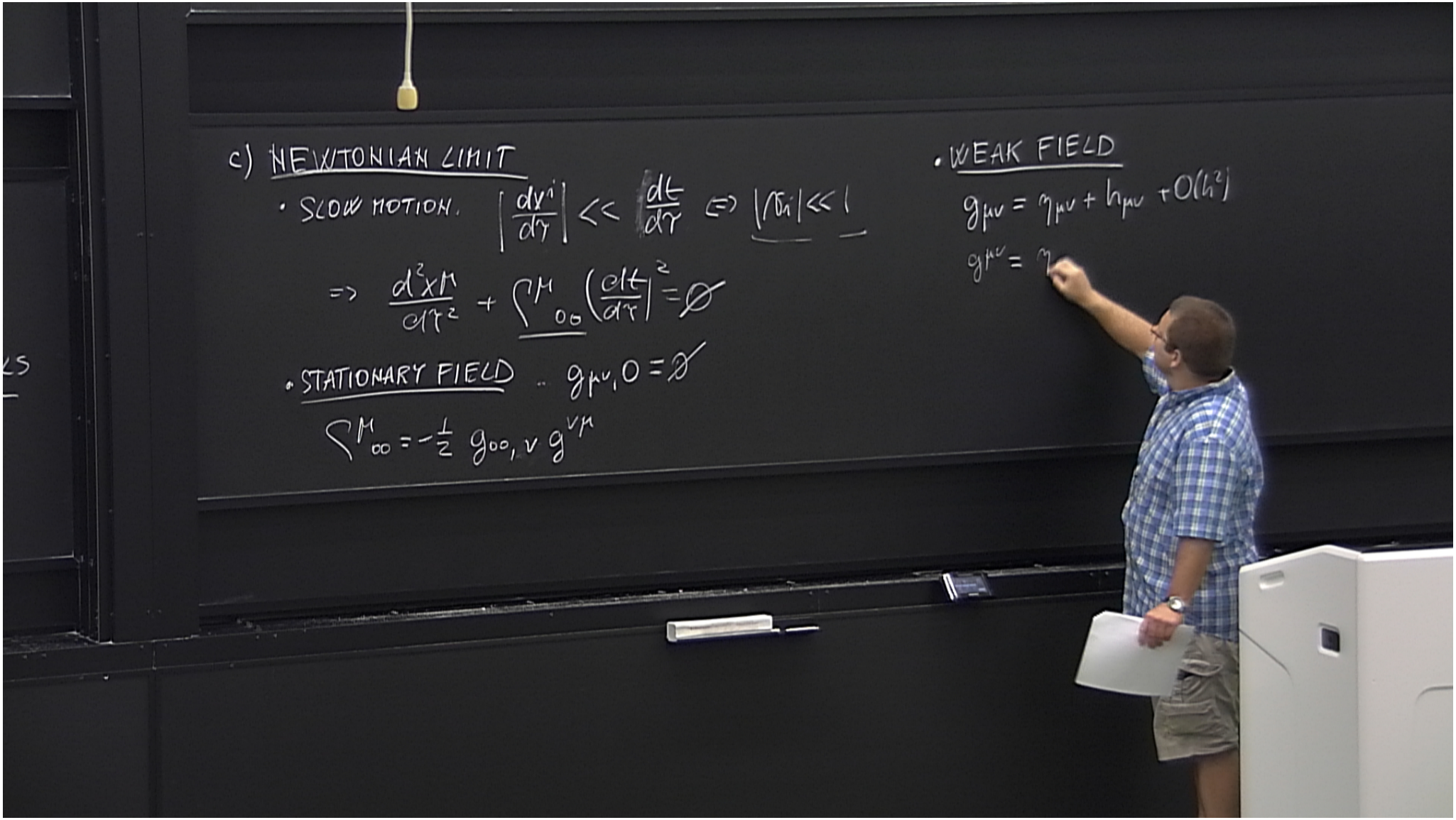
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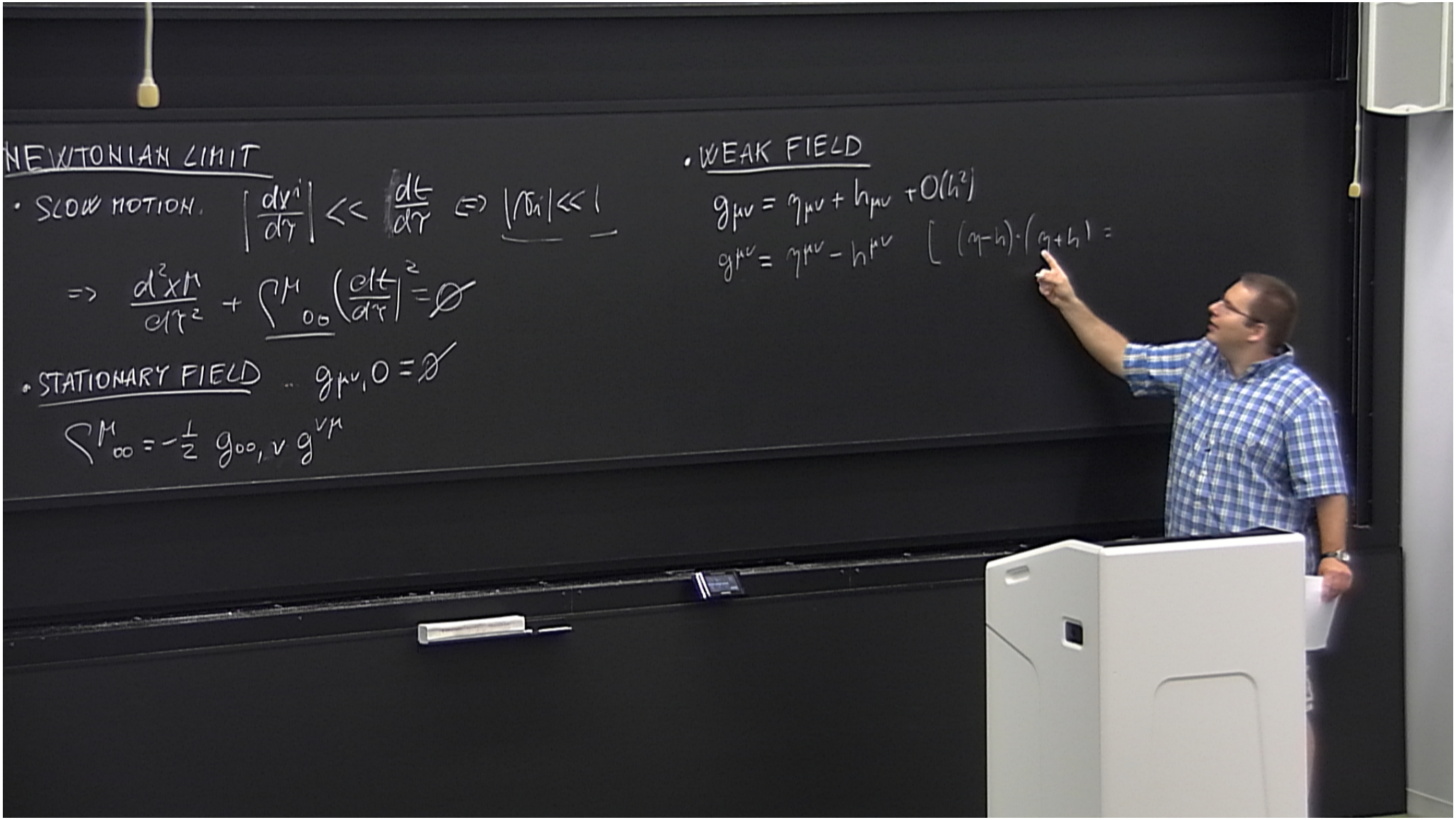

GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR
c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \left|\frac{dt}{d\tau}\right| \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$

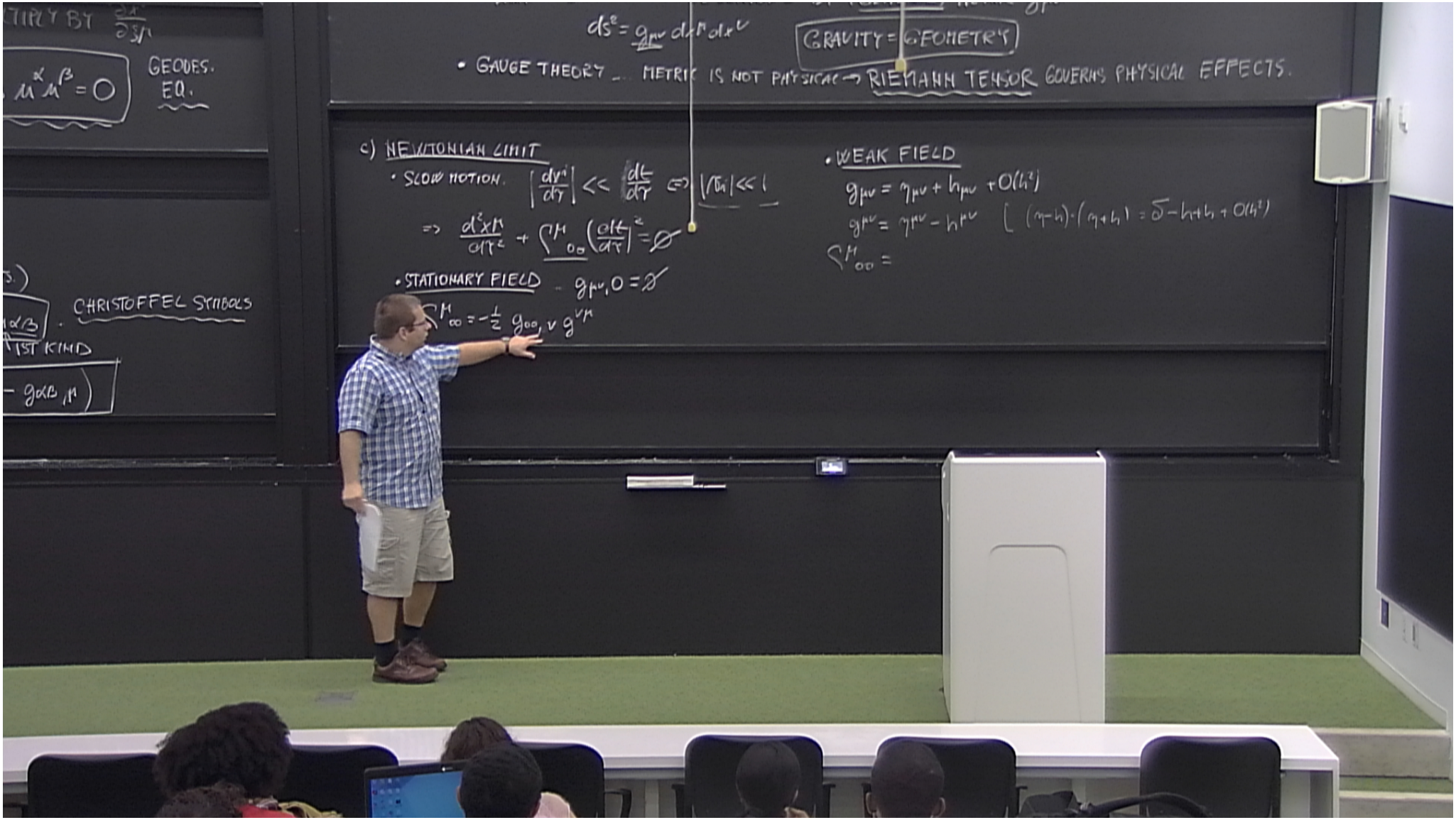

c) NEWTONIAN LIMIT
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$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
Γ^μ $-\frac{1}{2} g_{00,\nu} g^{\nu\mu}$

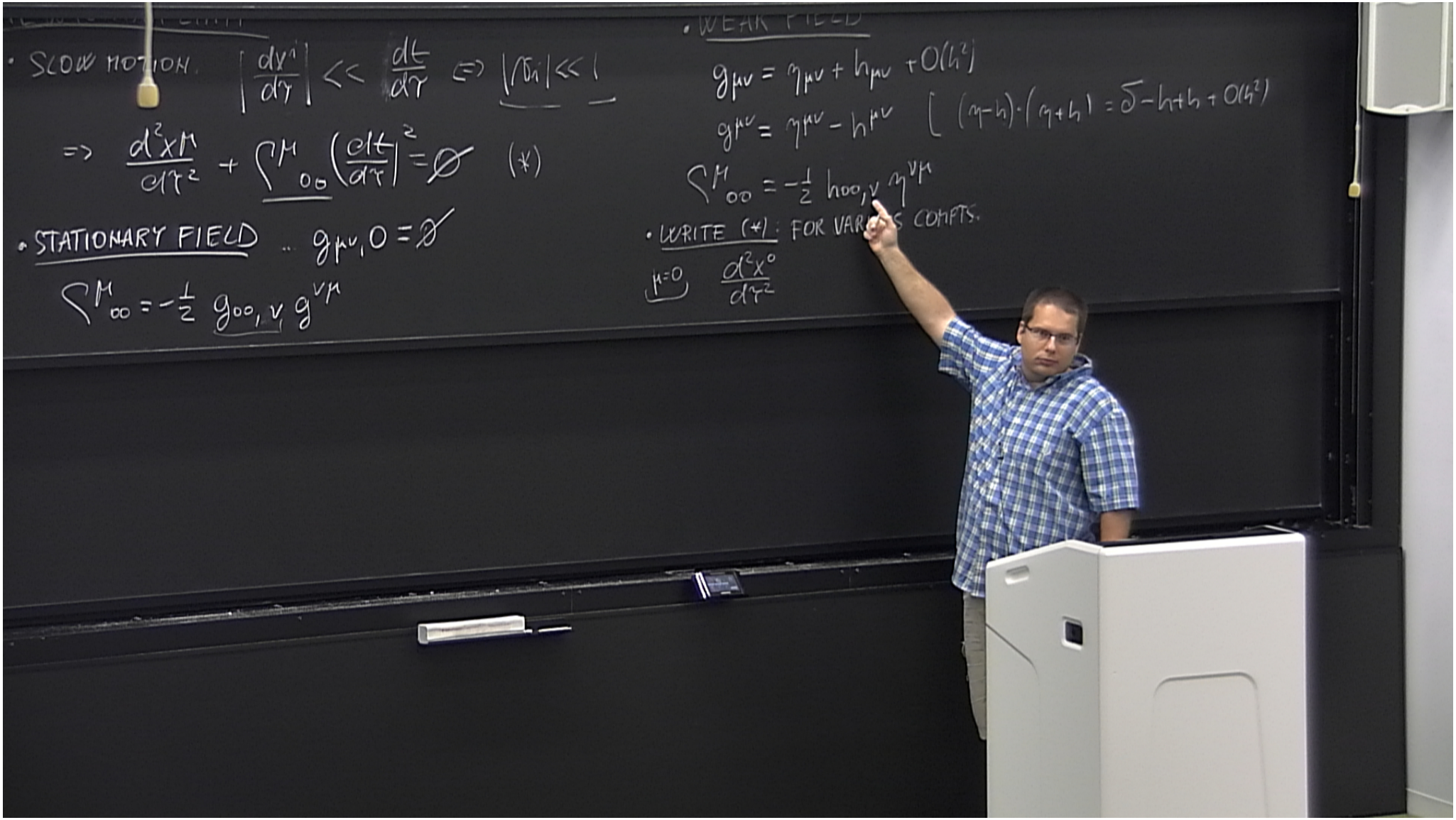

GEODES. EQ.
CHRISTOFFEL SYMBOLS
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
GRAVITY = GEOMETRY
• GAUGE THEORY ... METRIC IS NOT PHYSICAL → RIEMANN TENSOR GOVERNS PHYSICAL EFFECTS.
c) NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_M| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK

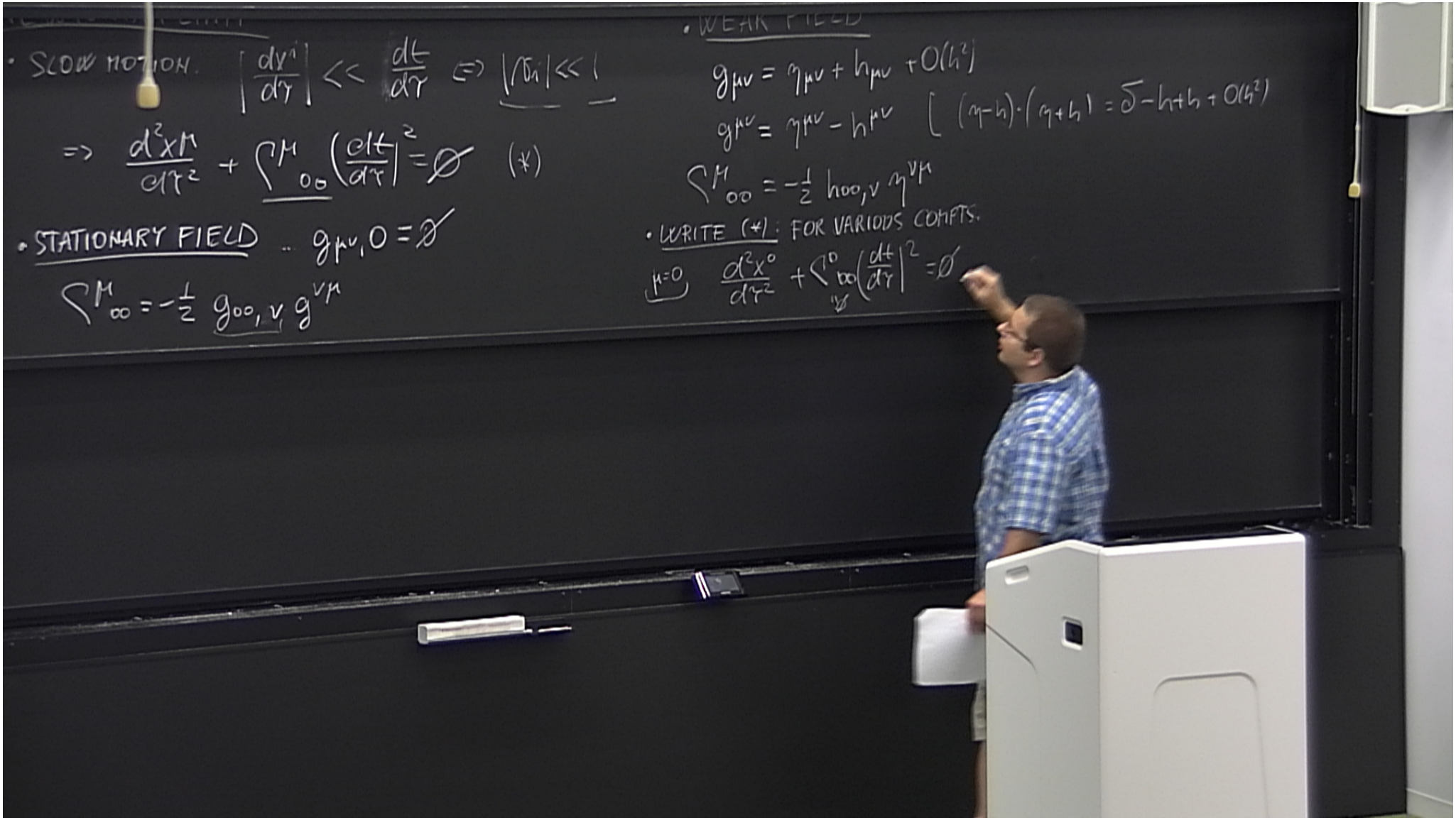

c) NEWTONIAN LIMIT
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$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$

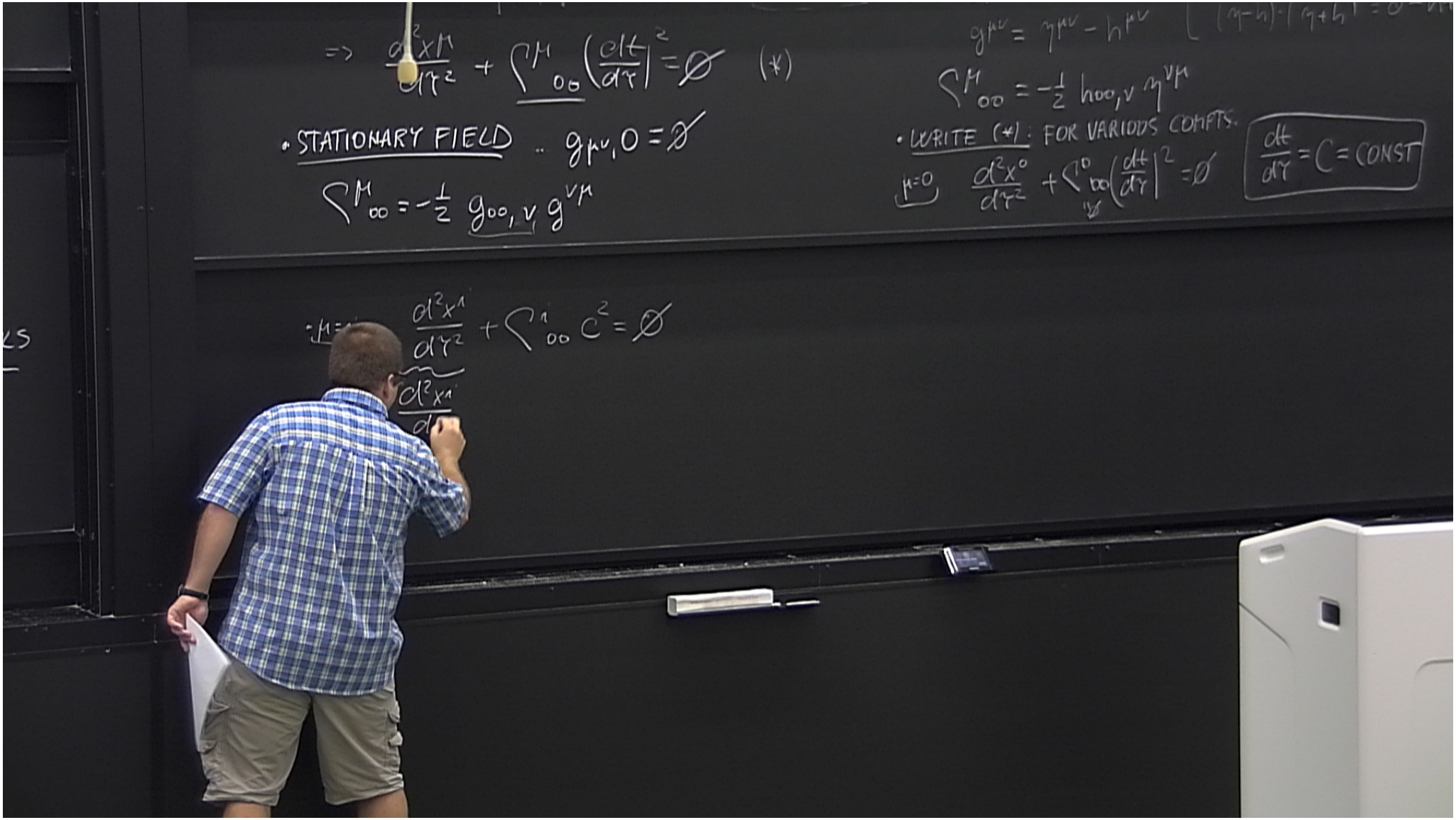

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• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta$

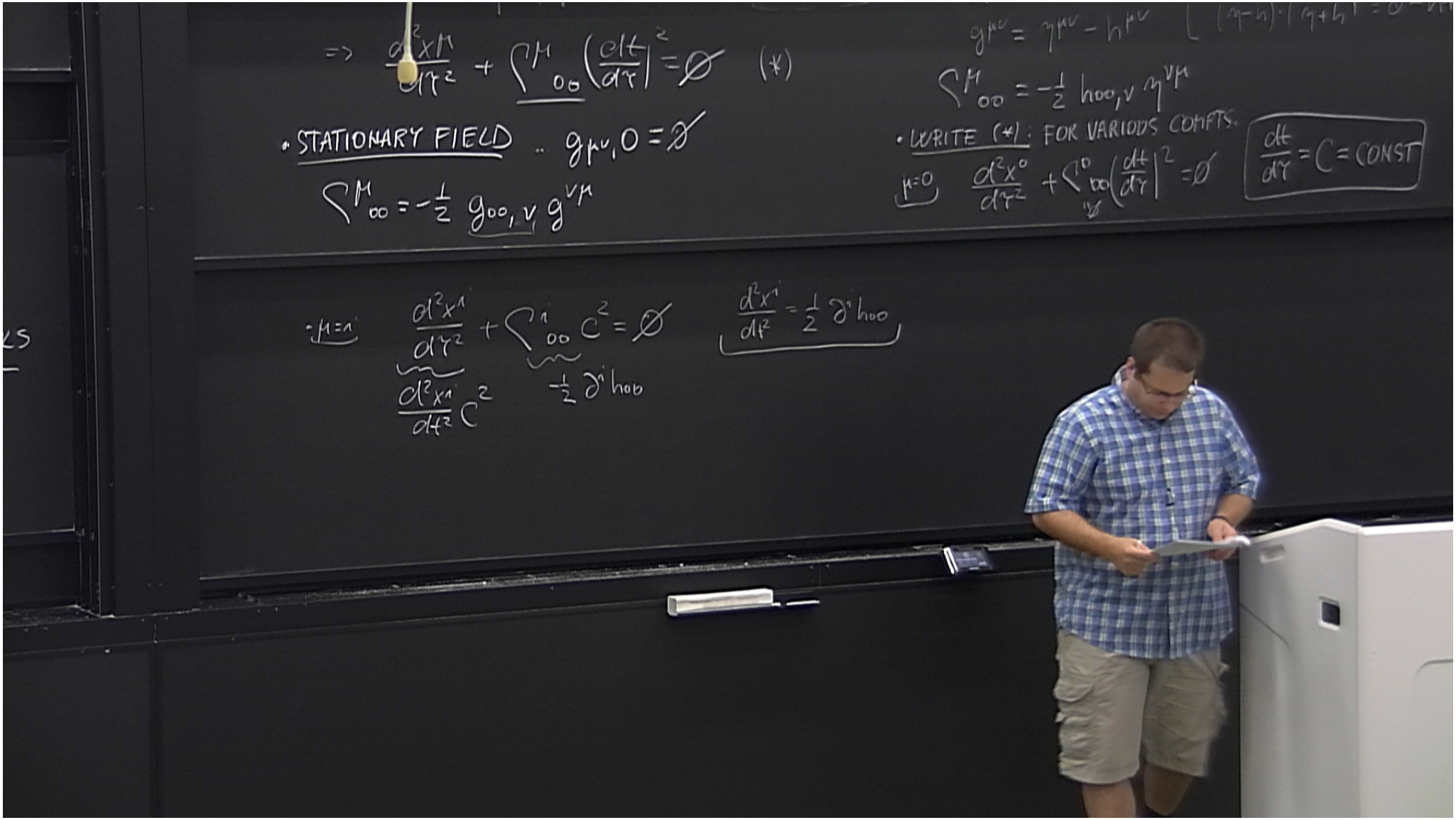

NEWTONIAN LIMIT
• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \left|\frac{dt}{d\tau}\right| \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
• STATIONARY FIELD $g_{\mu\nu,0} = 0$
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$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ [$(\eta - h)\cdot(\eta + h) =$

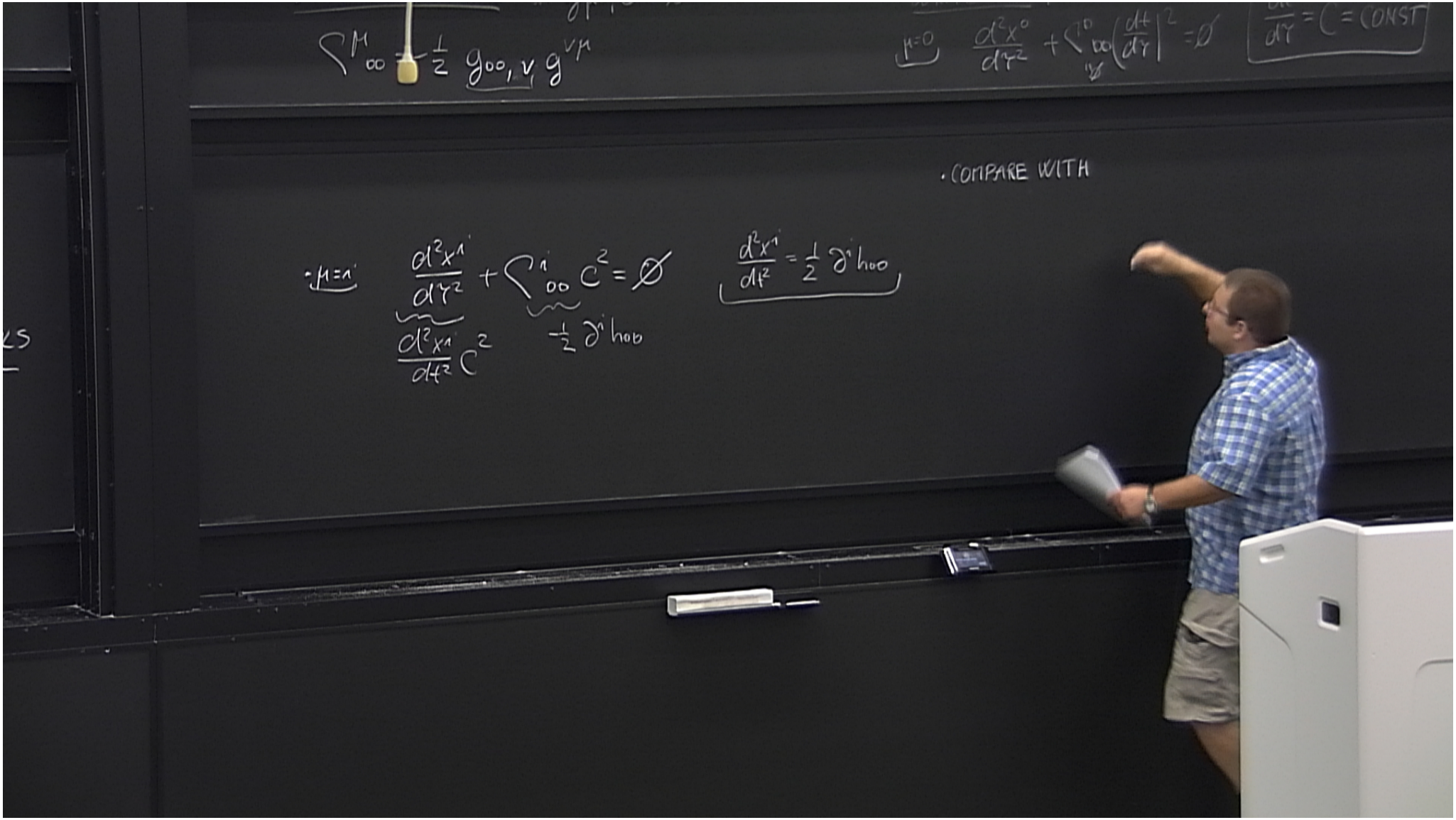

GEODES. EQ.
CHRISTOFFEL SYMBOLS
GAUGE THEORY
METRIC IS NOT PHYSICAL
RIEMANN TENSOR GOVERNS PHYSICAL EFFECTS.
GRAVITY = GEOMETRY
c) NEWTONIAN LIMIT
SLOW MOTION.
STATIONARY FIELD
WEAK FIELD

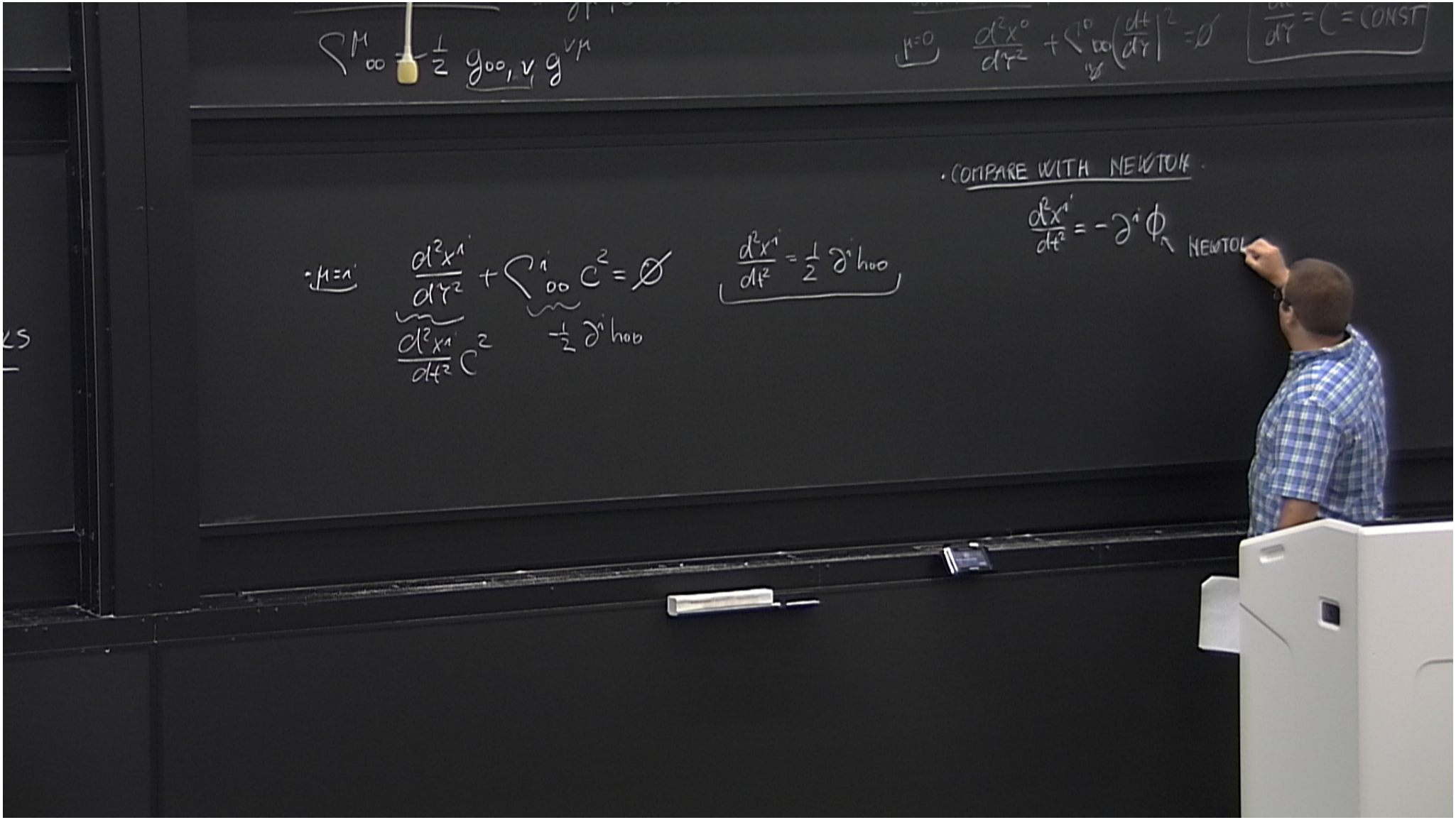

• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
• WEAK FIELD
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ $[(\eta-h)\cdot(\eta+h) = \delta - h + h + O(h^2)]$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*) FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2}$

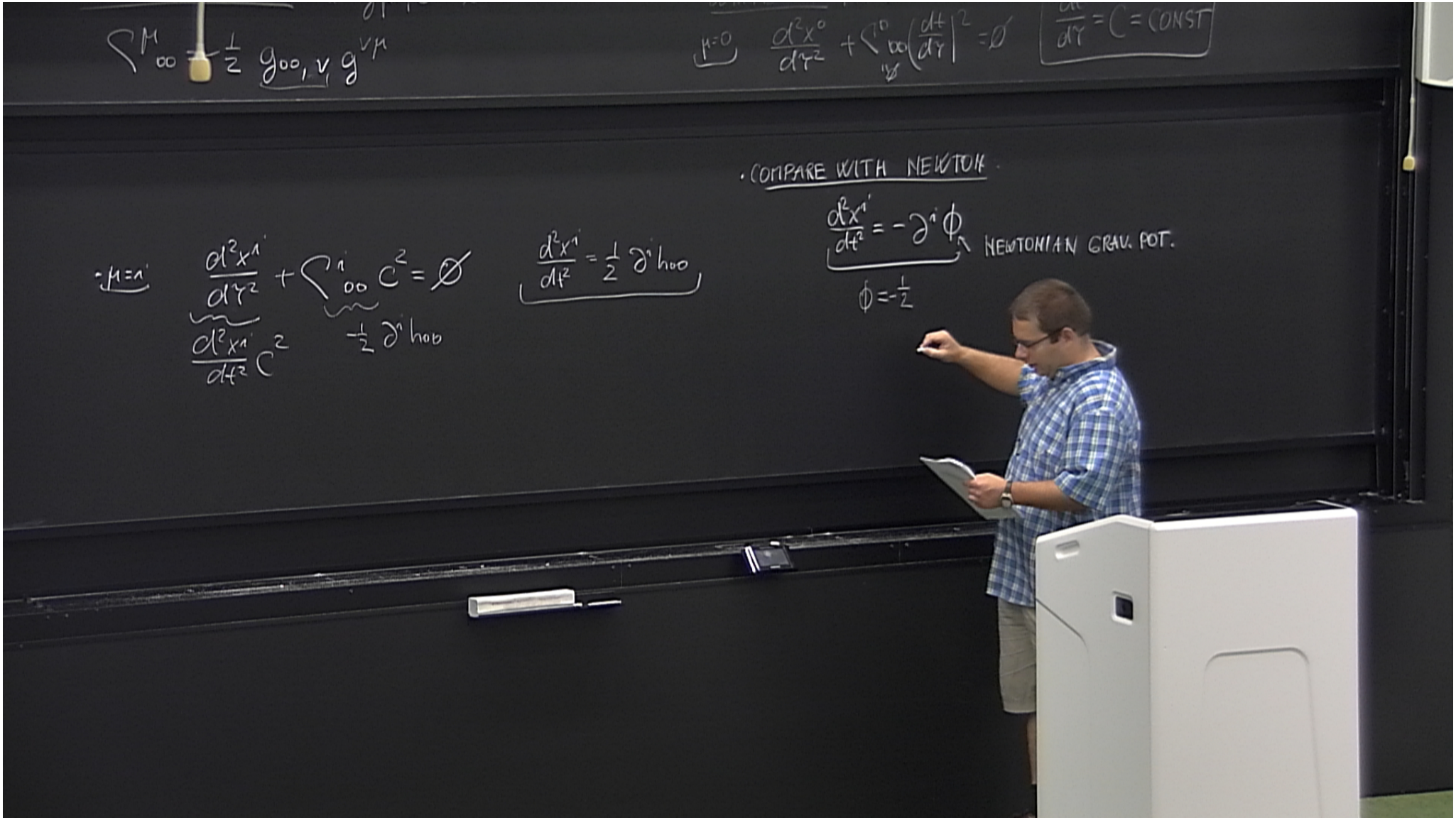

• SLOW MOTION. $\left|\frac{dx^i}{d\tau}\right| \ll \frac{dt}{d\tau} \Leftrightarrow |v_i| \ll 1$
$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
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$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ $[(\eta-h)\cdot(\eta+h) = \delta - h + h + O(h^2)$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*): FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$

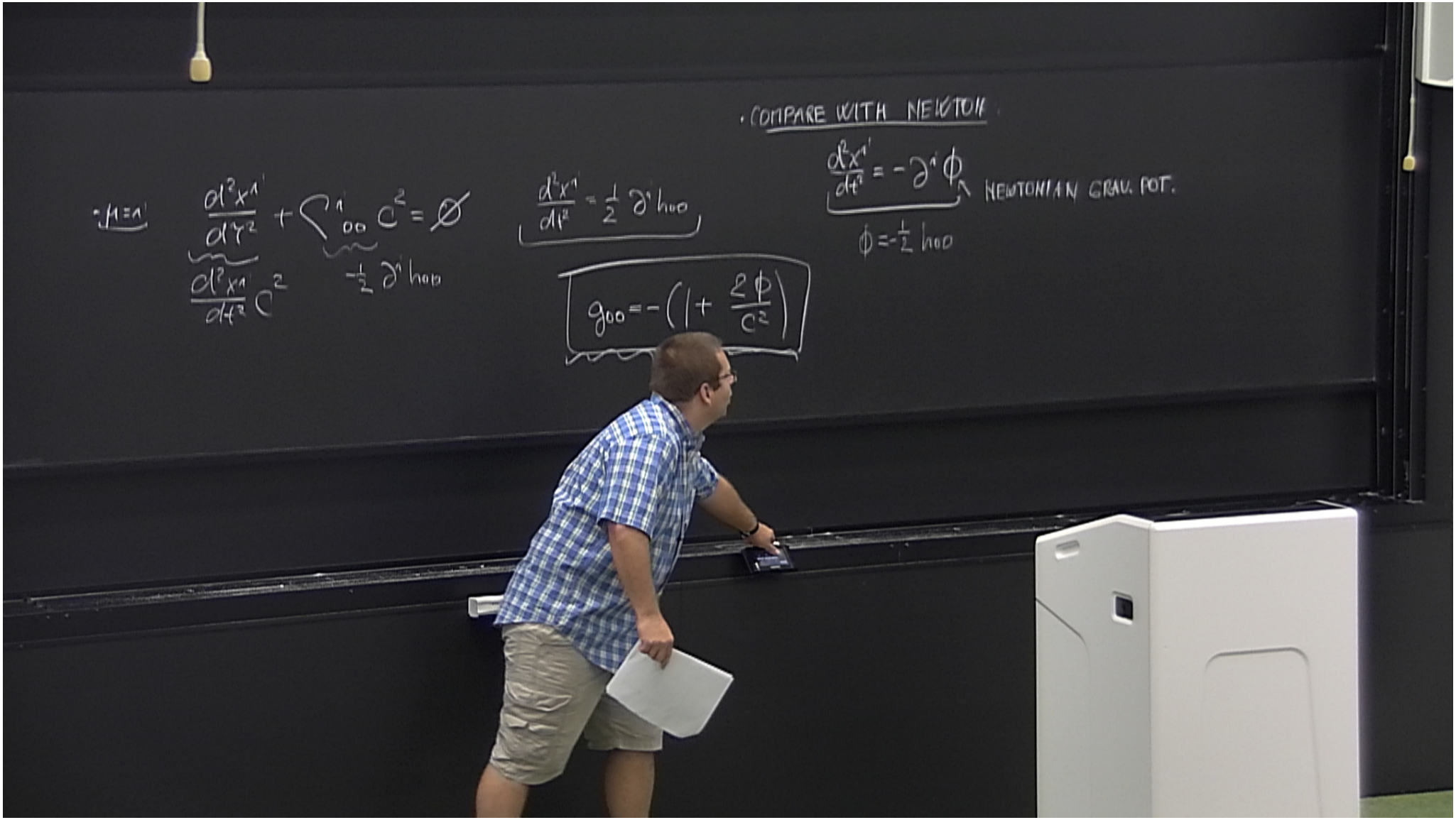

• STATIONARY FIELD
• WRITE (*): FOR VARIOUS COMPTS.
μ=0
dt/dτ = C = CONST

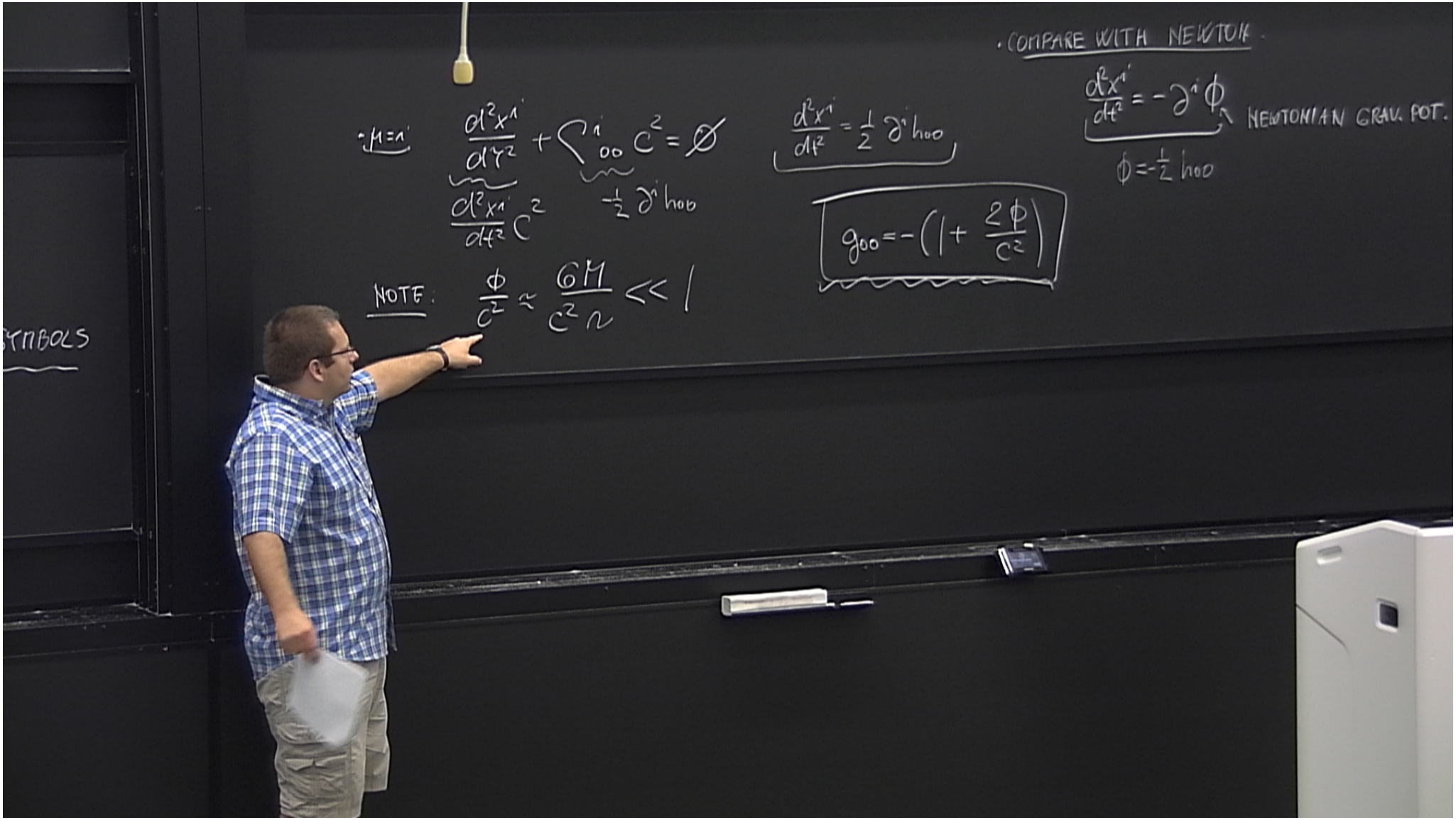

$\Rightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$ (*)
• STATIONARY FIELD .. $g_{\mu\nu,0} = 0$
$\Gamma^\mu_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$
$\Gamma^\mu_{00} = -\frac{1}{2} h_{00,\nu} \eta^{\nu\mu}$
• WRITE (*): FOR VARIOUS COMPTS.
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
$\mu=i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

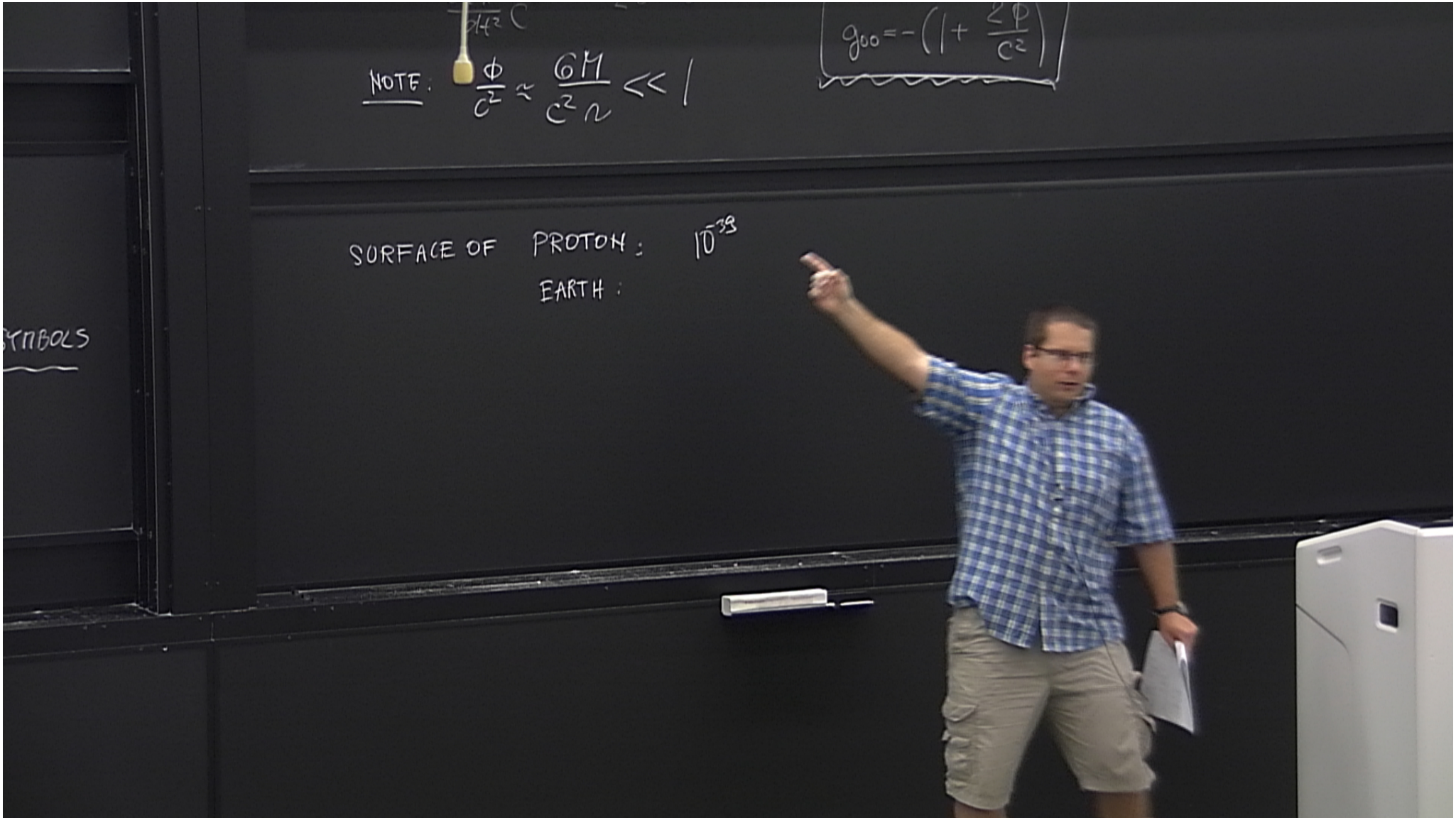

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^{0}_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
• COMPARE WITH
$\mu=i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^{i}_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

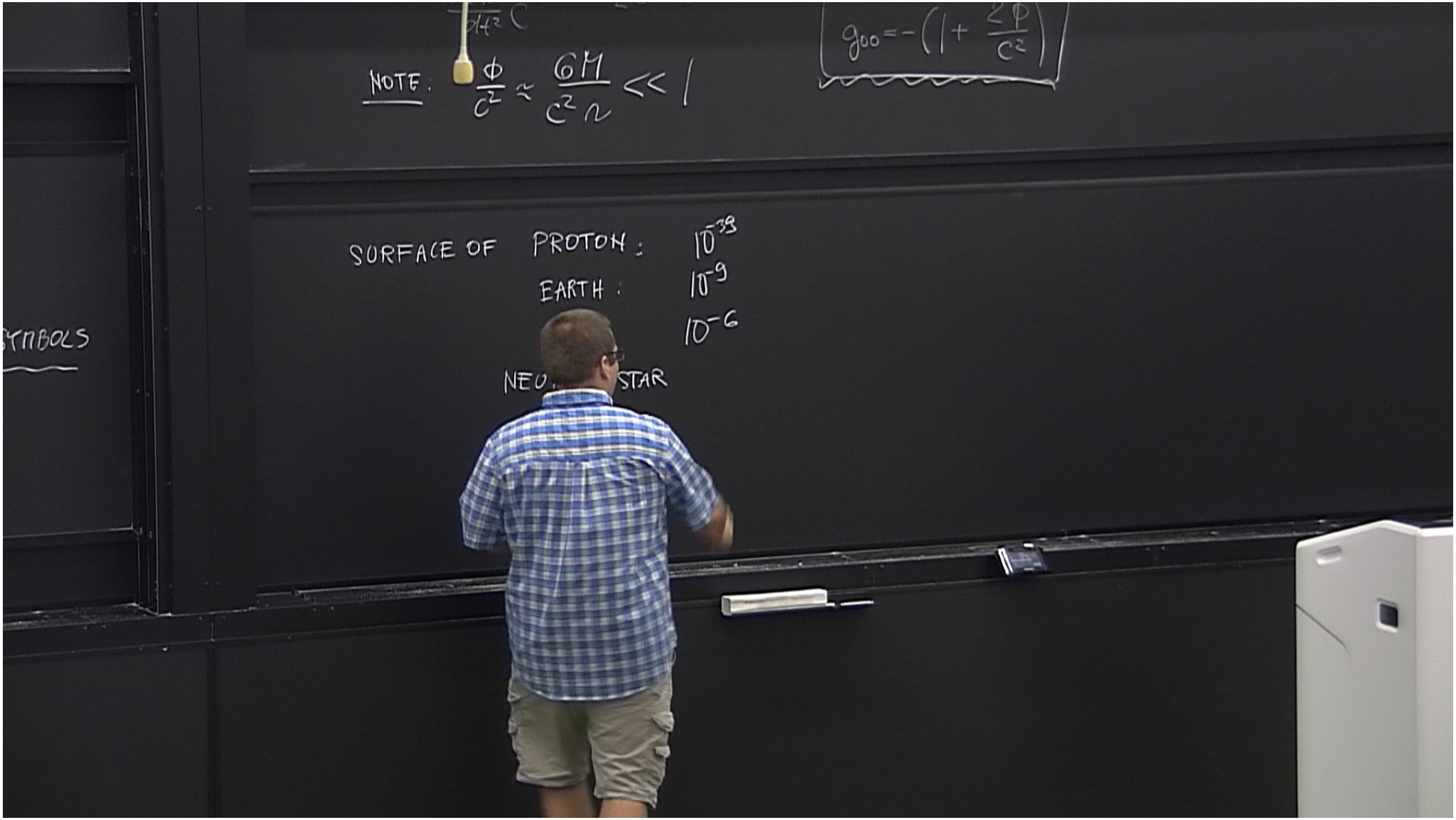

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
$\frac{dt}{d\tau} = C = \text{CONST}$
• COMPARE WITH NEWTON
$\frac{d^2x^i}{dt^2} = -\partial^i \phi$ NEWTO
$\mu = i$ $\frac{d^2x^i}{d\tau^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$

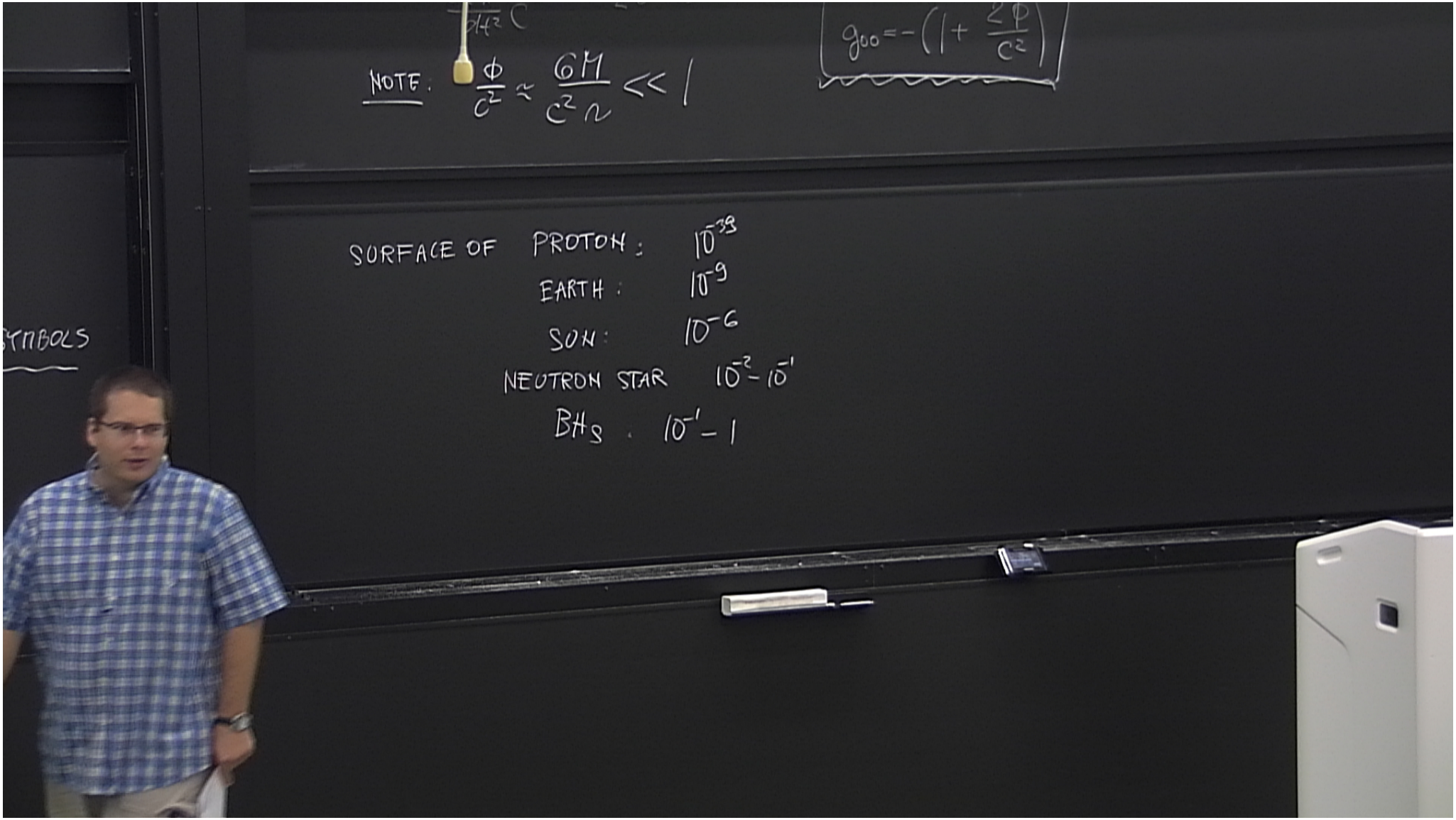

$\Gamma^{\mu}_{00} = -\frac{1}{2} g_{00,\nu} g^{\nu\mu}$
$\mu=0$ $\frac{d^2x^0}{d\tau^2} + \Gamma^0_{00}\left(\frac{dt}{d\tau}\right)^2 = 0$
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$\mu=i$ $\frac{d^2x^i}{dt^2} + \Gamma^i_{00} C^2 = 0$
$\frac{d^2x^i}{dt^2} C^2$ $-\frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2} = -\partial^i \phi$
NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}$

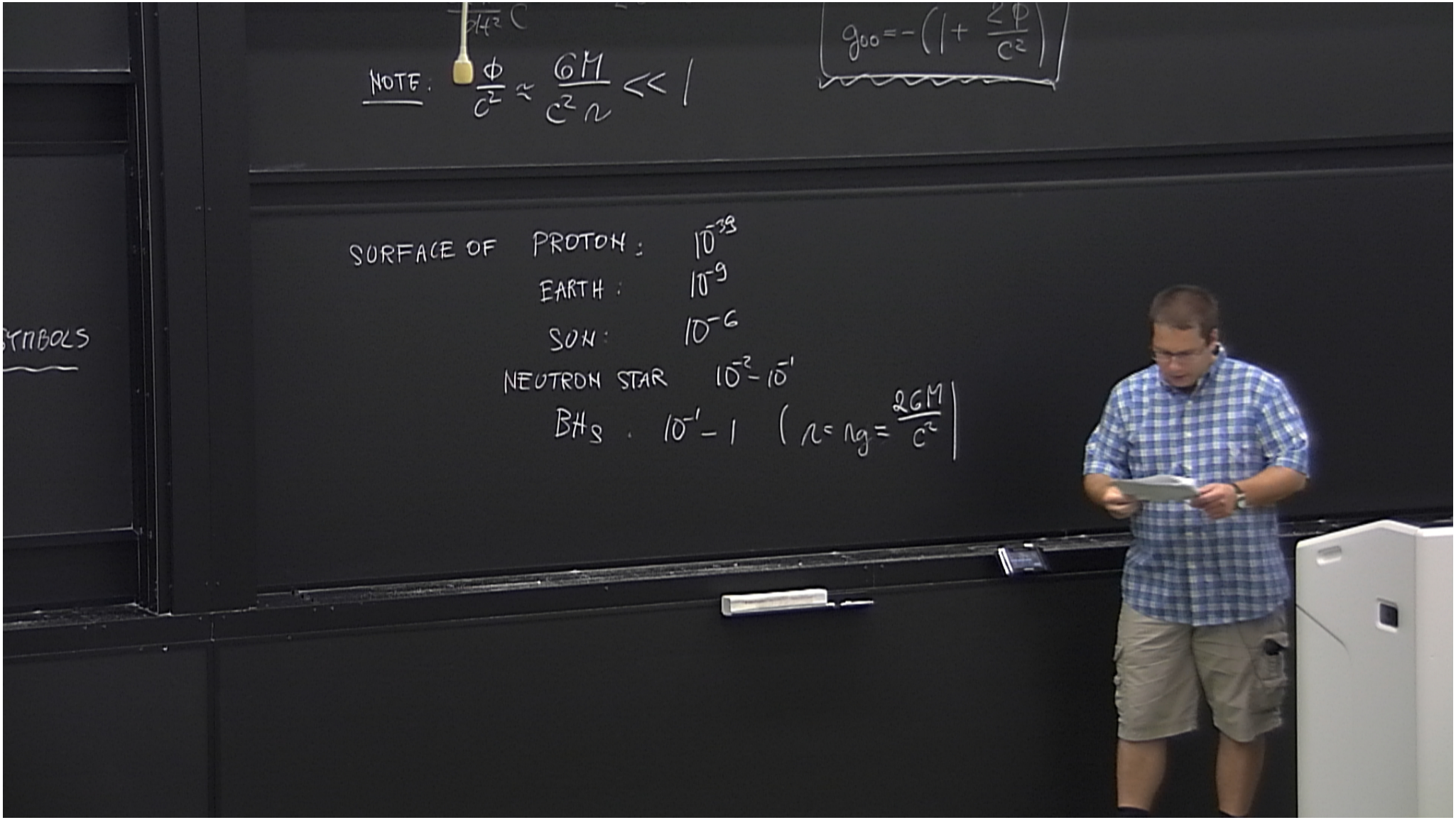

· COMPARE WITH NEWTON
$\frac{d^2x^i}{dt^2} = -\partial^i\phi$
NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}h_{00}$
$\mu = i$
$\frac{d^2x^i}{dt^2} + \Gamma^i_{00}c^2 = 0$
$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$\frac{d^2x^i}{dt^2}c^2$
$-\frac{1}{2}\partial^i h_{00}$
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$

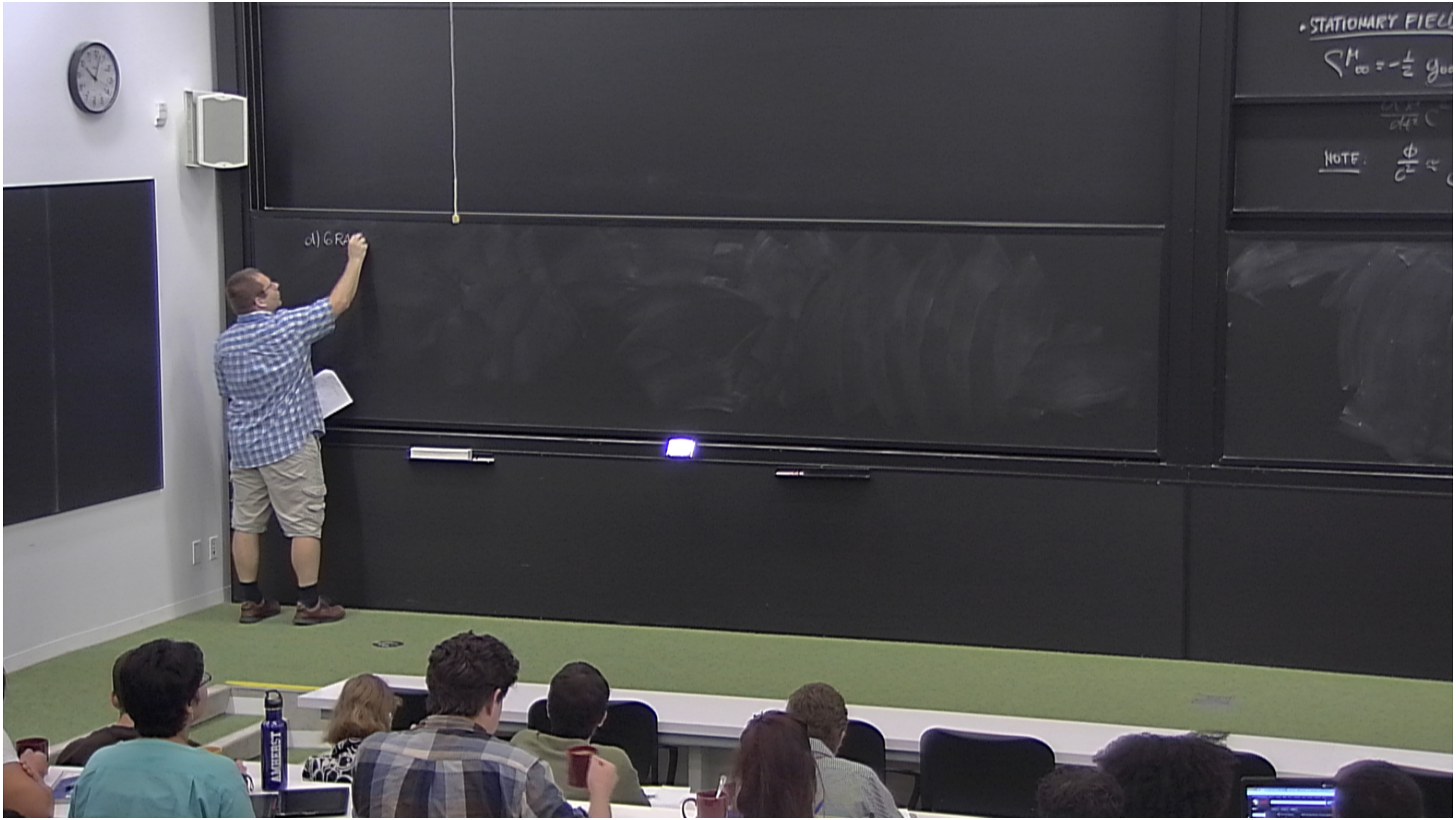

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NEWTONIAN GRAV. POT.
$\phi = -\frac{1}{2}h_{00}$
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$\frac{d^2x^i}{dt^2} + \Gamma^i_{00}c^2 = 0$
$\frac{d^2x^i}{dt^2}c^2$
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$\frac{d^2x^i}{dt^2} = \frac{1}{2}\partial^i h_{00}$
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
NOTE:
$\frac{\phi}{c^2} \approx \frac{GM}{c^2 r} \ll 1$
SYMBOLS

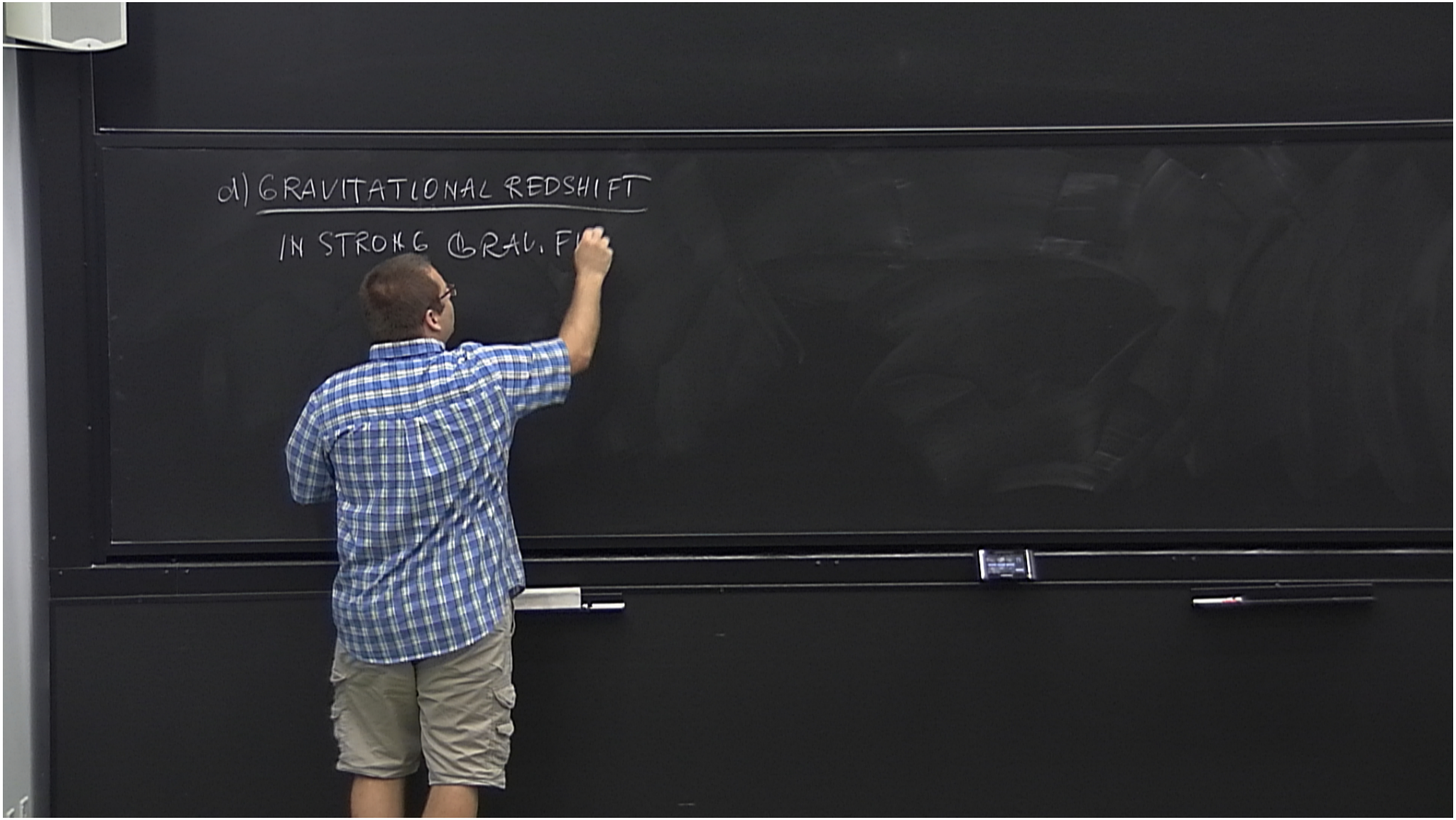

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-39}
EARTH:
SYMBOLS

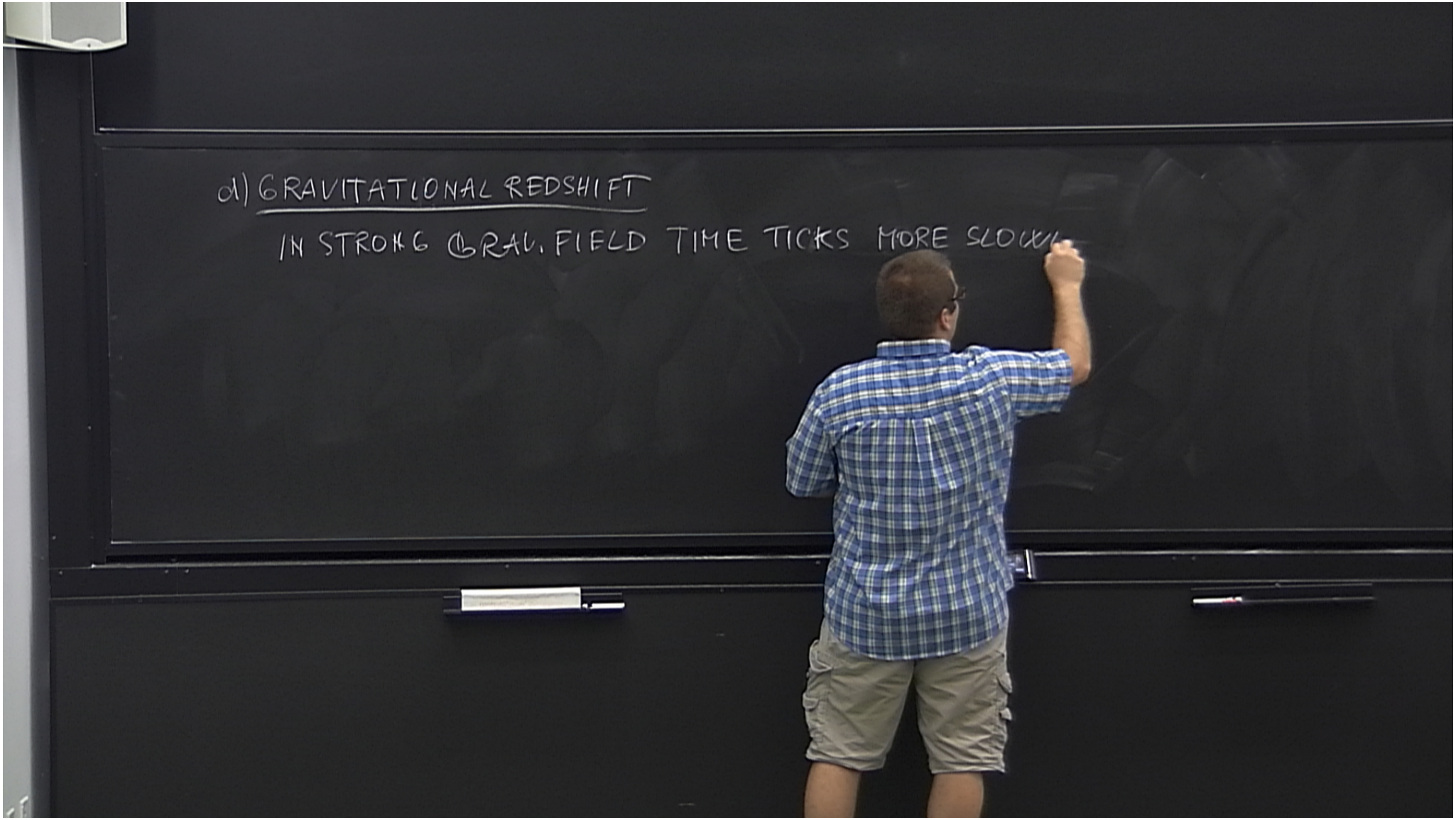

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-38}
EARTH: 10^{-9}
10^{-6}
NEU. STAR
SYMBOLS

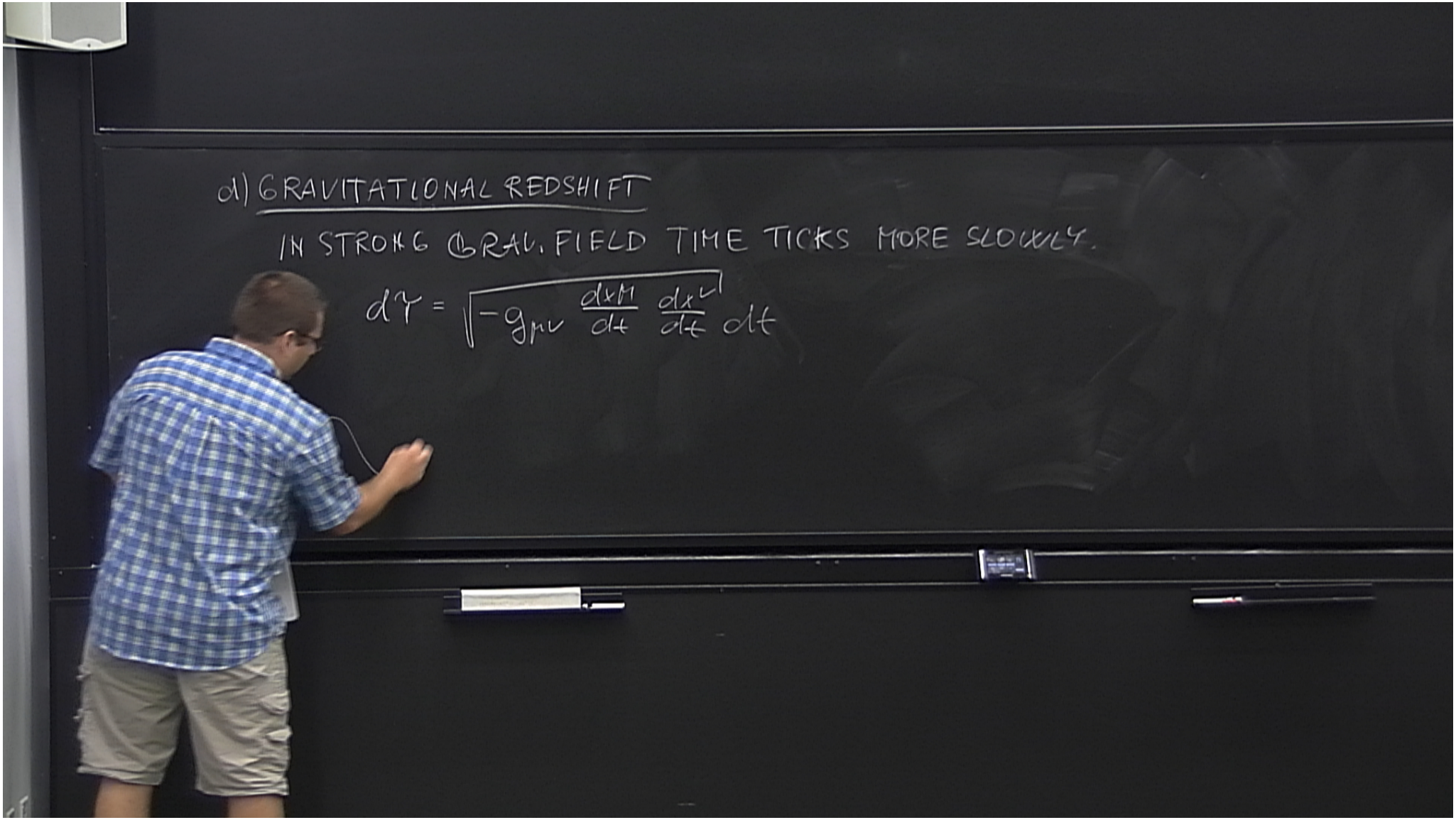

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-38}
EARTH: 10^{-9}
SUN: 10^{-6}
NEUTRON STAR $10^{-2} - 10^{-1}$
BHs $10^{-1} - 1$

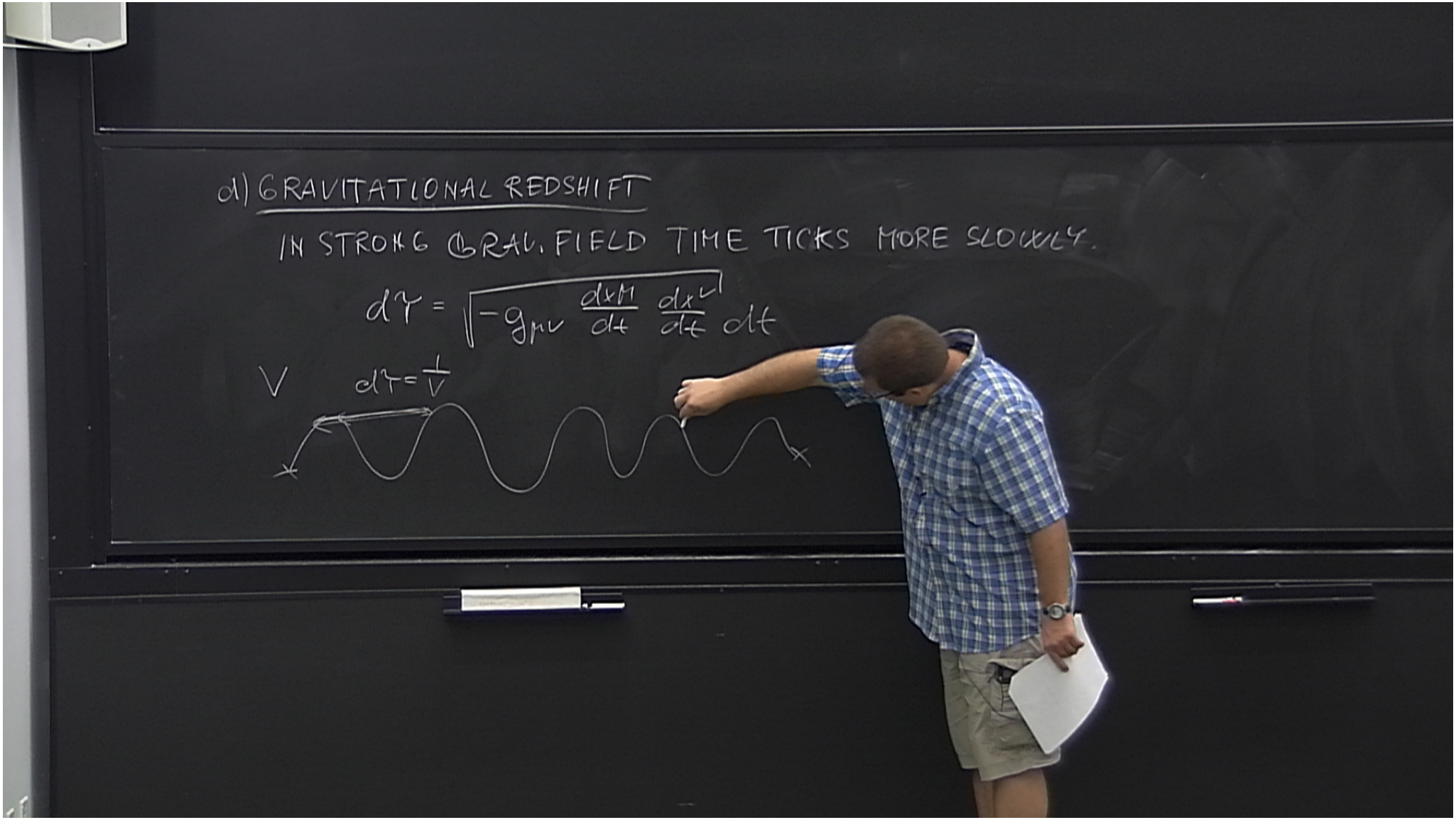

NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{c^2 r} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
SURFACE OF PROTON: 10^{-39}
EARTH: 10^{-9}
SUN: 10^{-6}
NEUTRON STAR $10^{-2} - 10^{-1}$
BHs $10^{-1} - 1$ $\left(r = r_g = \frac{2GM}{c^2}\right)$
SYMBOLS

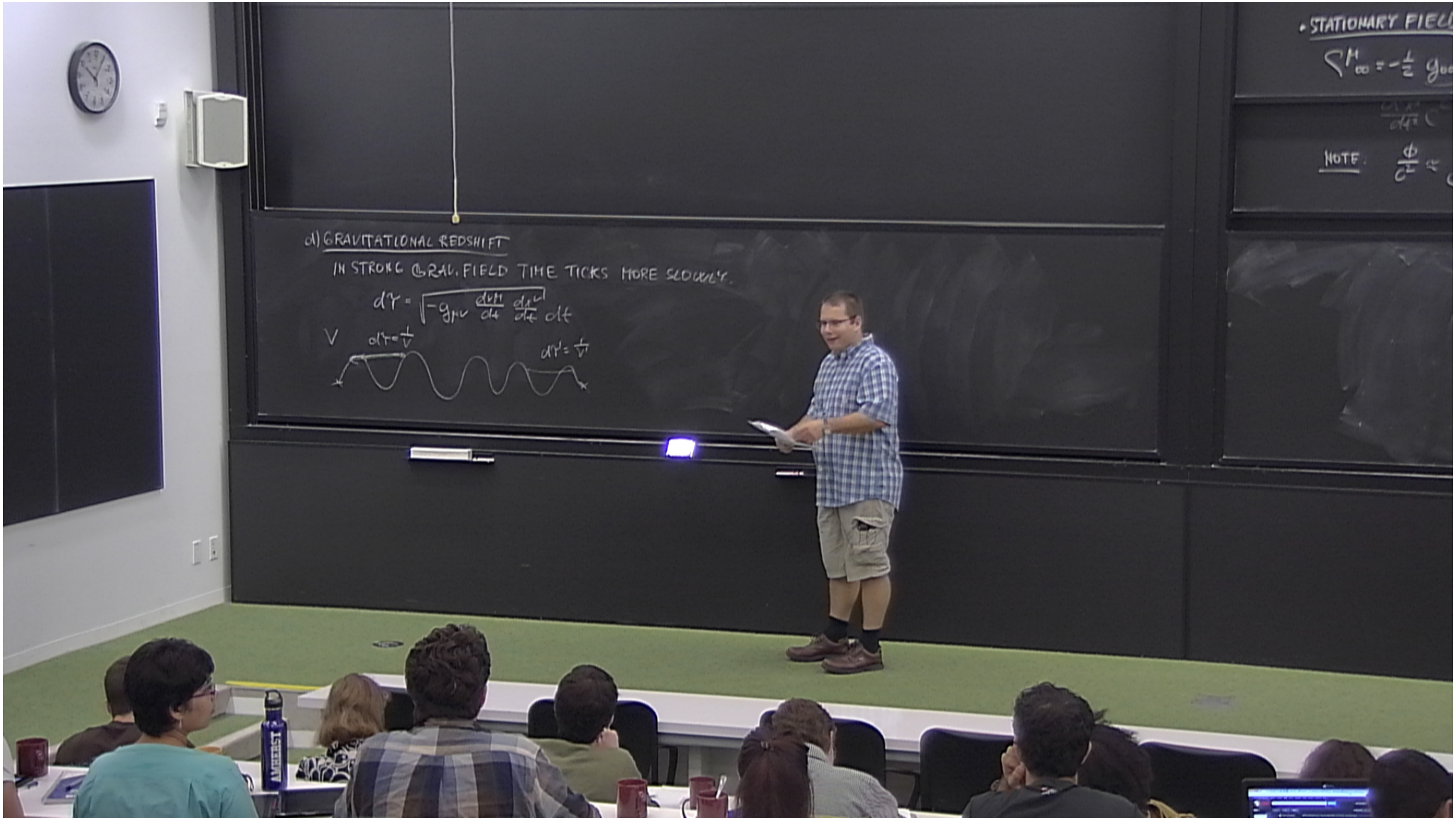

• STATIONARY FIEL
NOTE:

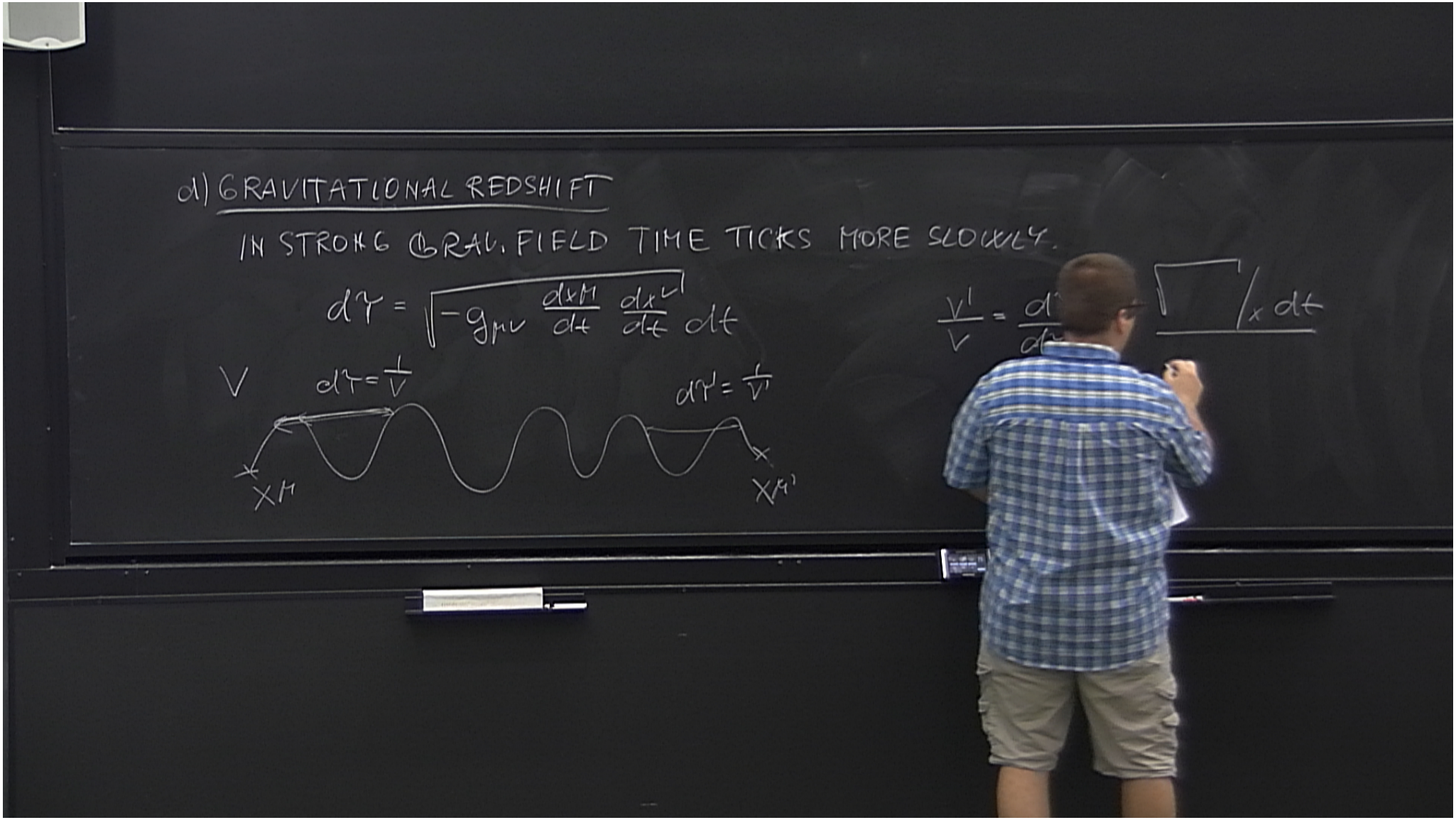

a) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FI

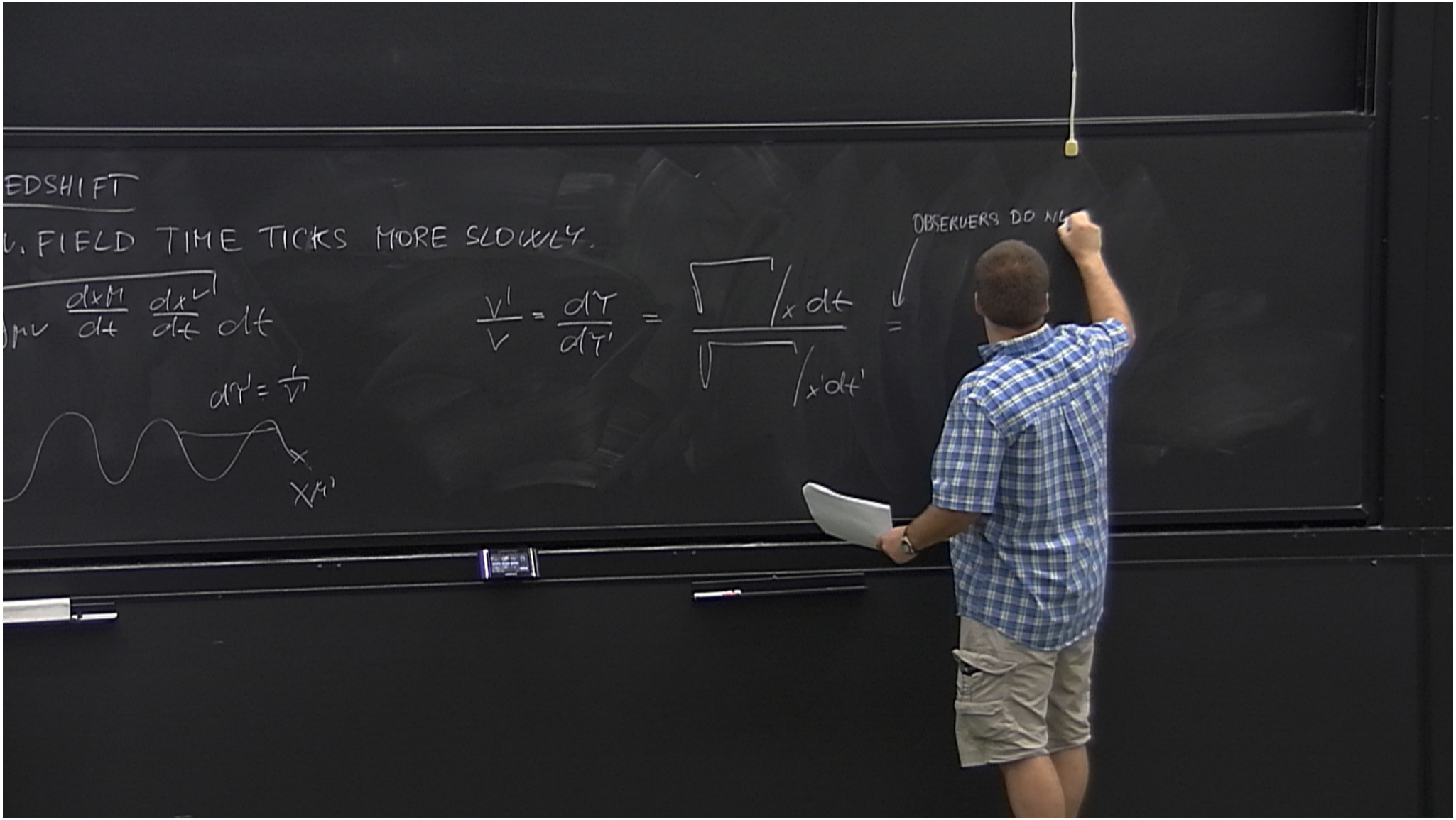

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOW

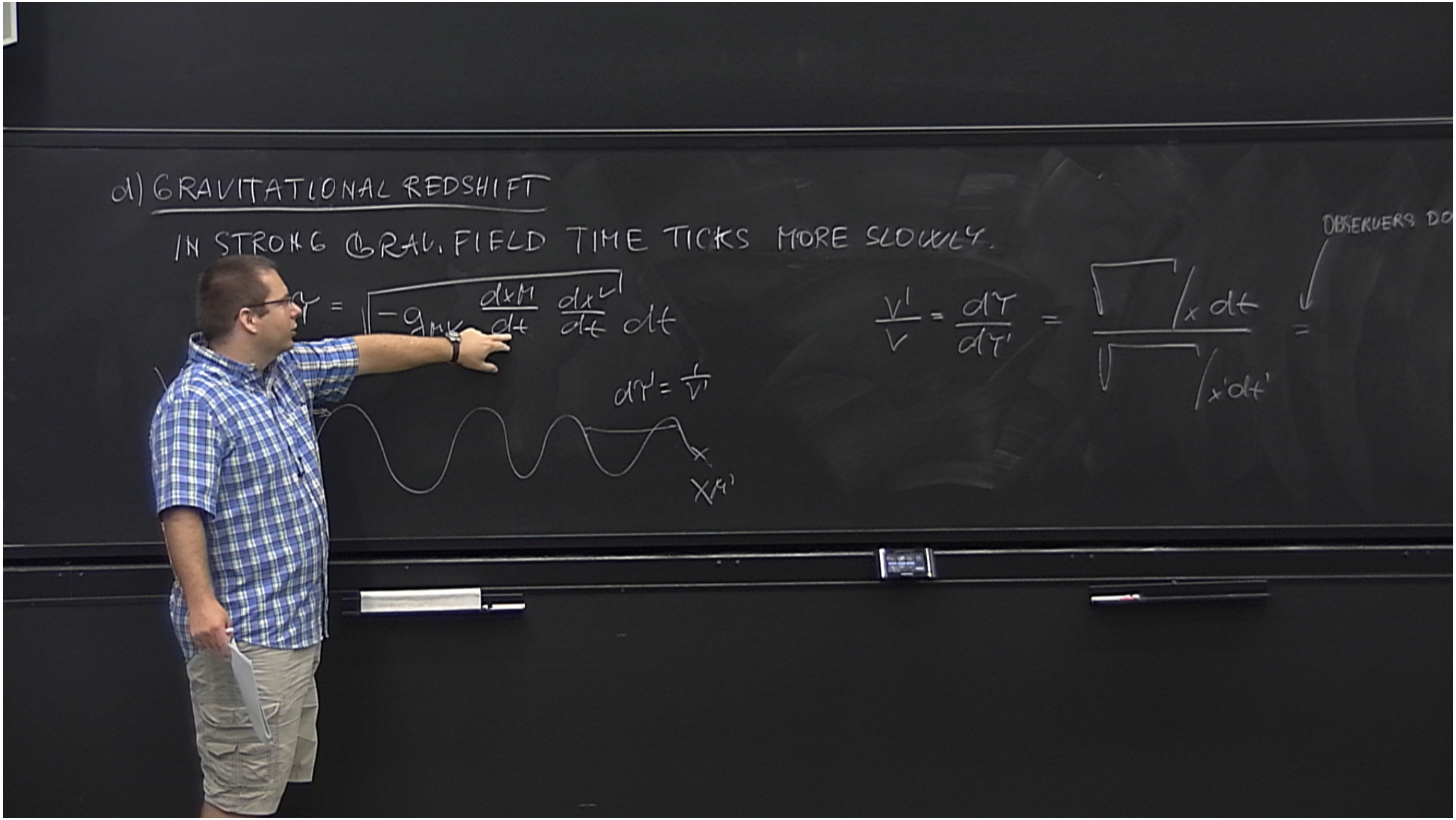

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}\, dt$

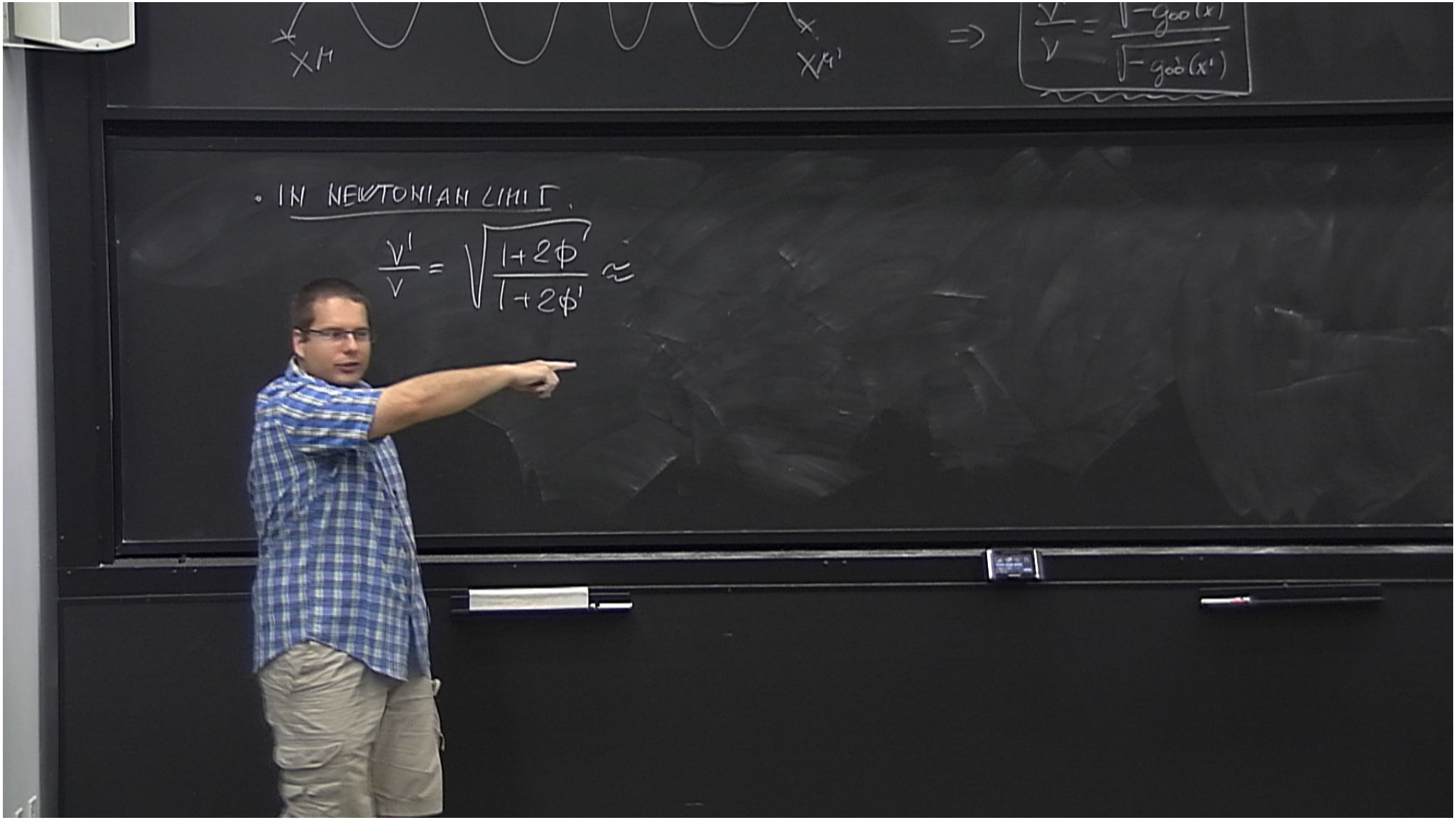

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}\, dt$
V
$d\tau = \frac{1}{\nu}$

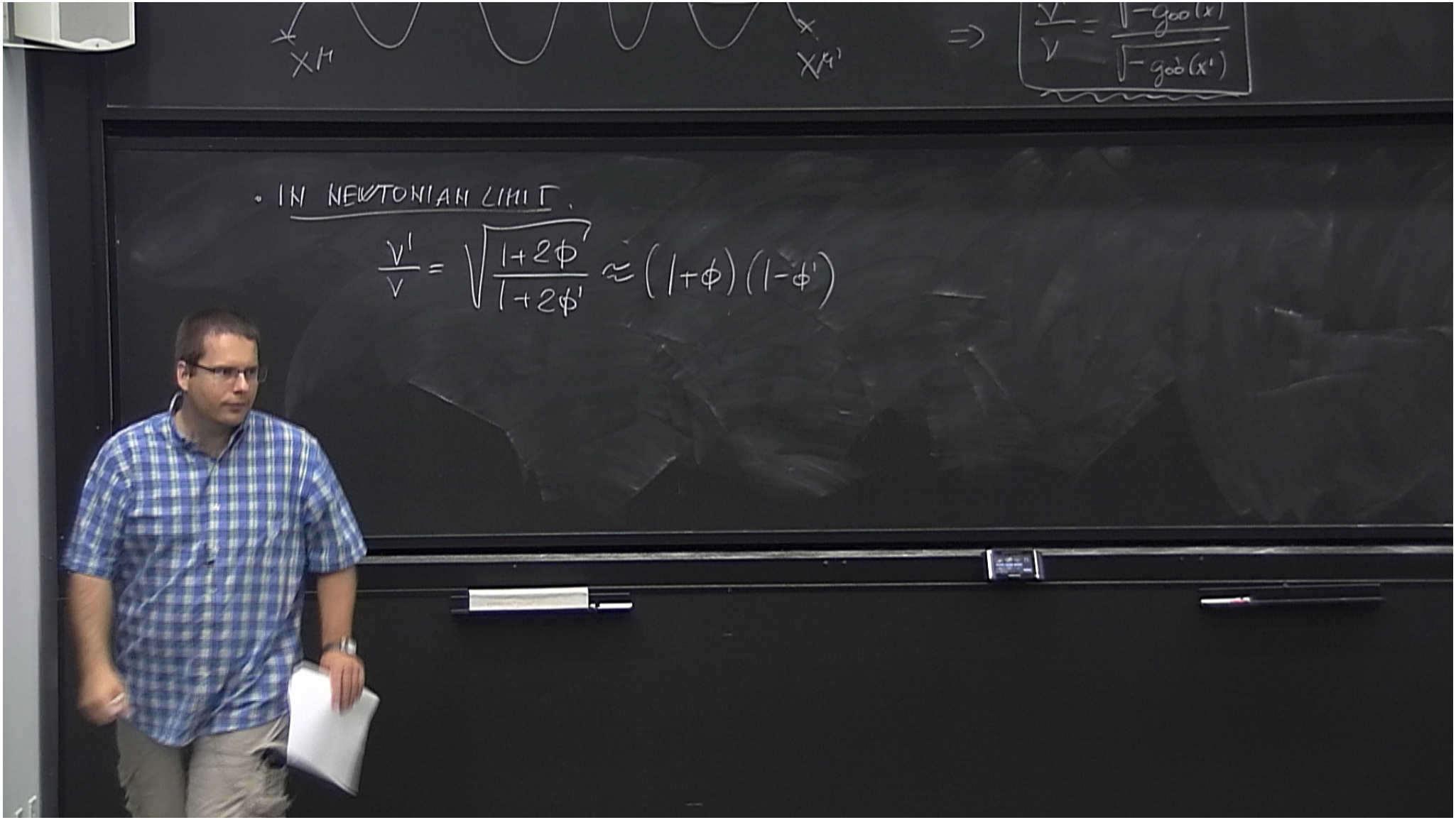

• STATIONARY FIELD
NOTE:
d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$
$d\tau = \frac{1}{\nu}$
$d\tau = \frac{1}{\nu}$

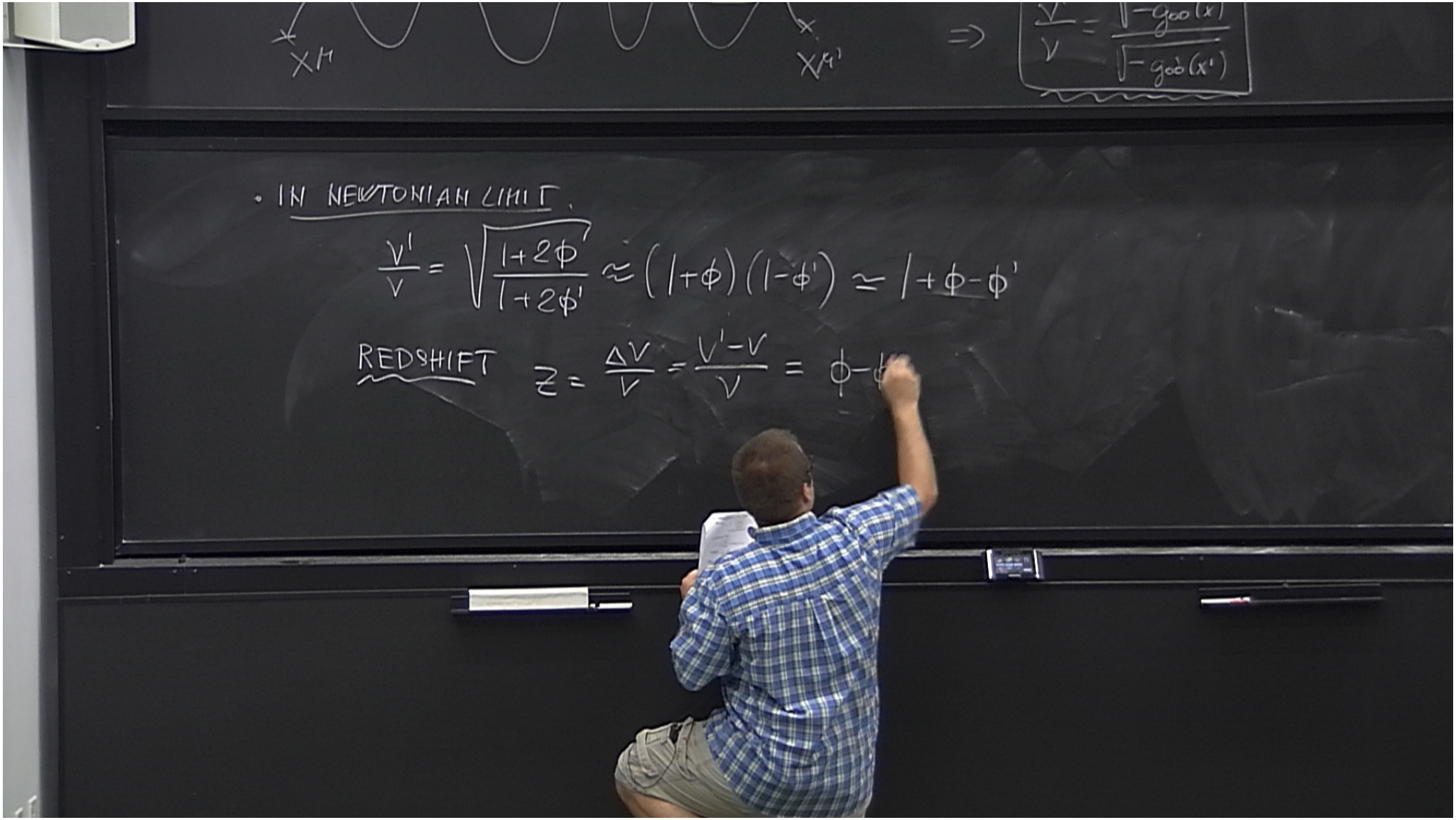

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$
V
$d\tau = \frac{1}{\nu}$
$d\tau' = \frac{1}{\nu'}$
x^μ
$x^{\mu'}$
$\frac{\nu'}{\nu} = \frac{d}{d}$
$/_x\,dt$

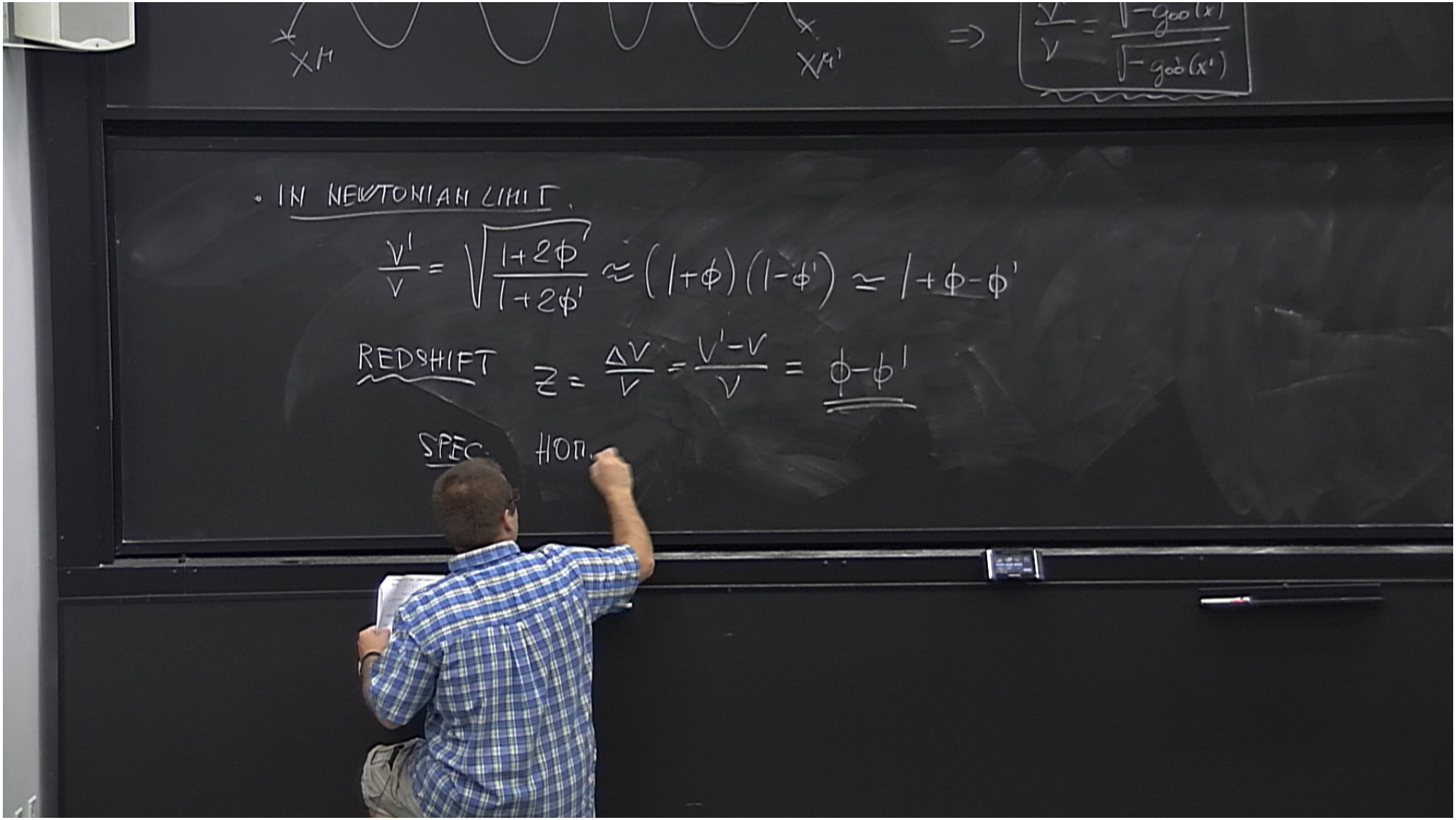

EDSHIFT
. FIELD TIME TICKS MORE SLOWLY.
OBSERVERS DO NO
dτ = 1/ν'
x
x'

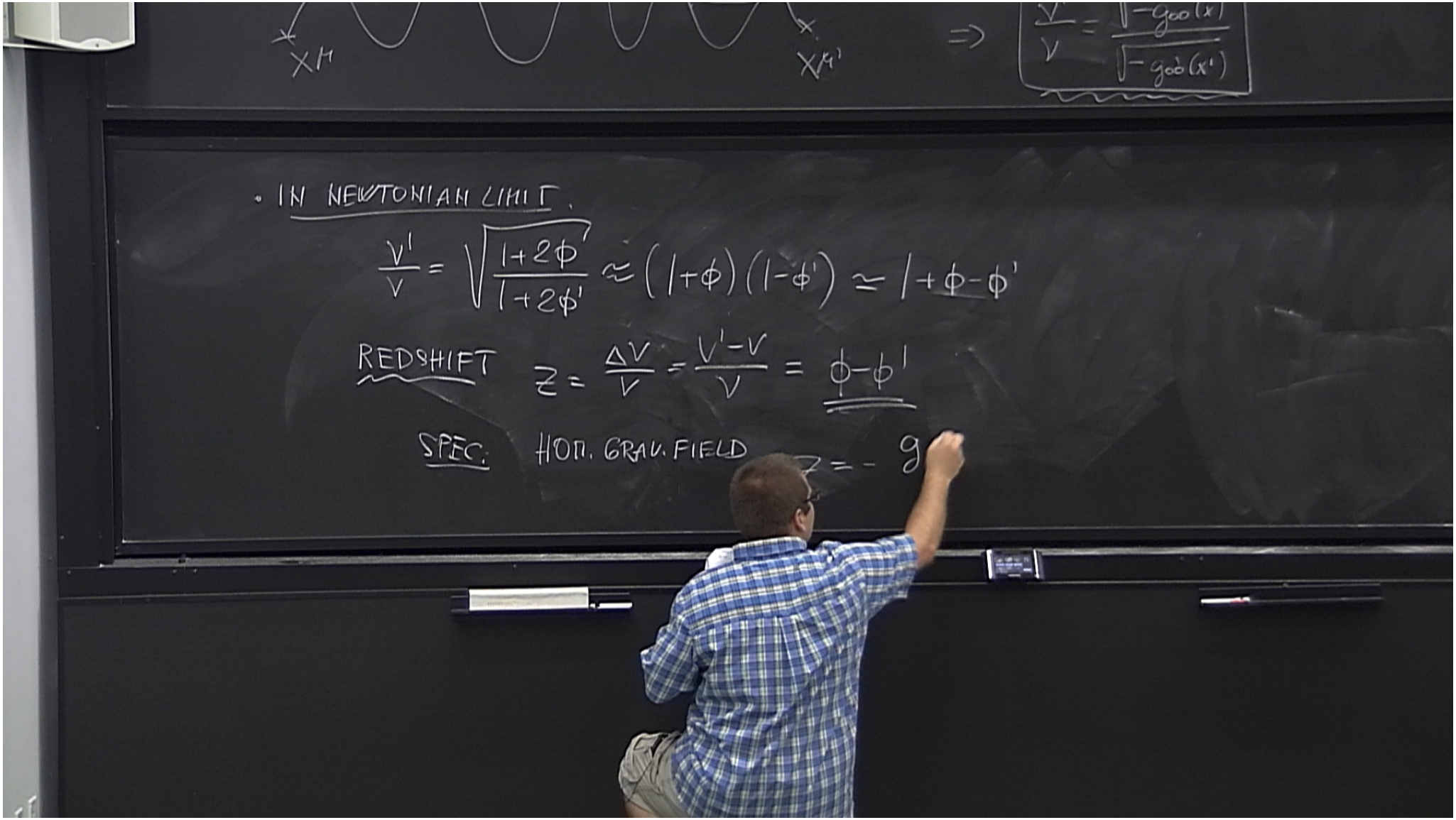

d) GRAVITATIONAL REDSHIFT
IN STRONG GRAV. FIELD TIME TICKS MORE SLOWLY.
OBSERVERS DO
dτ = 1/ν'
X(τ')

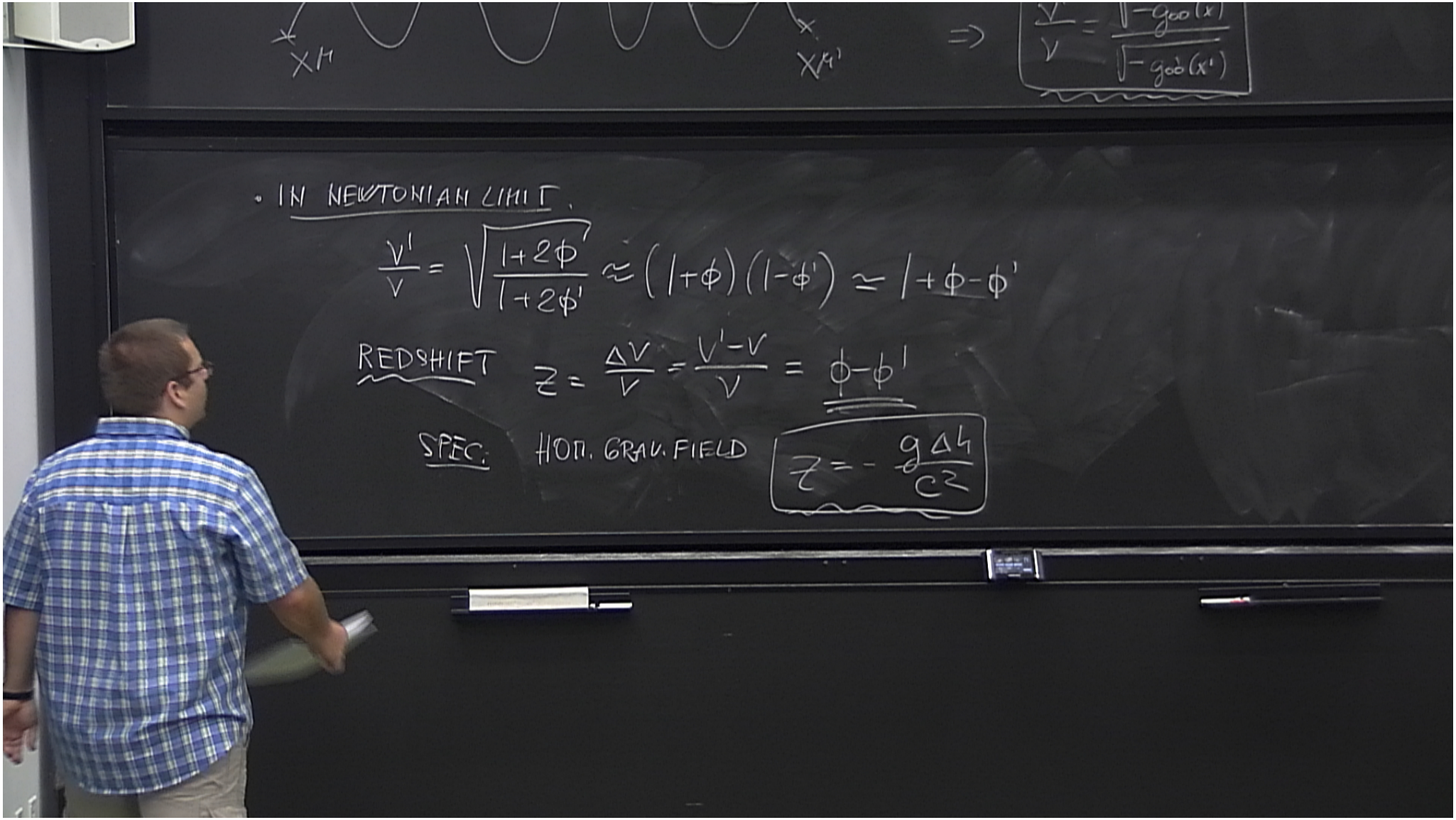

x^{μ}
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq$

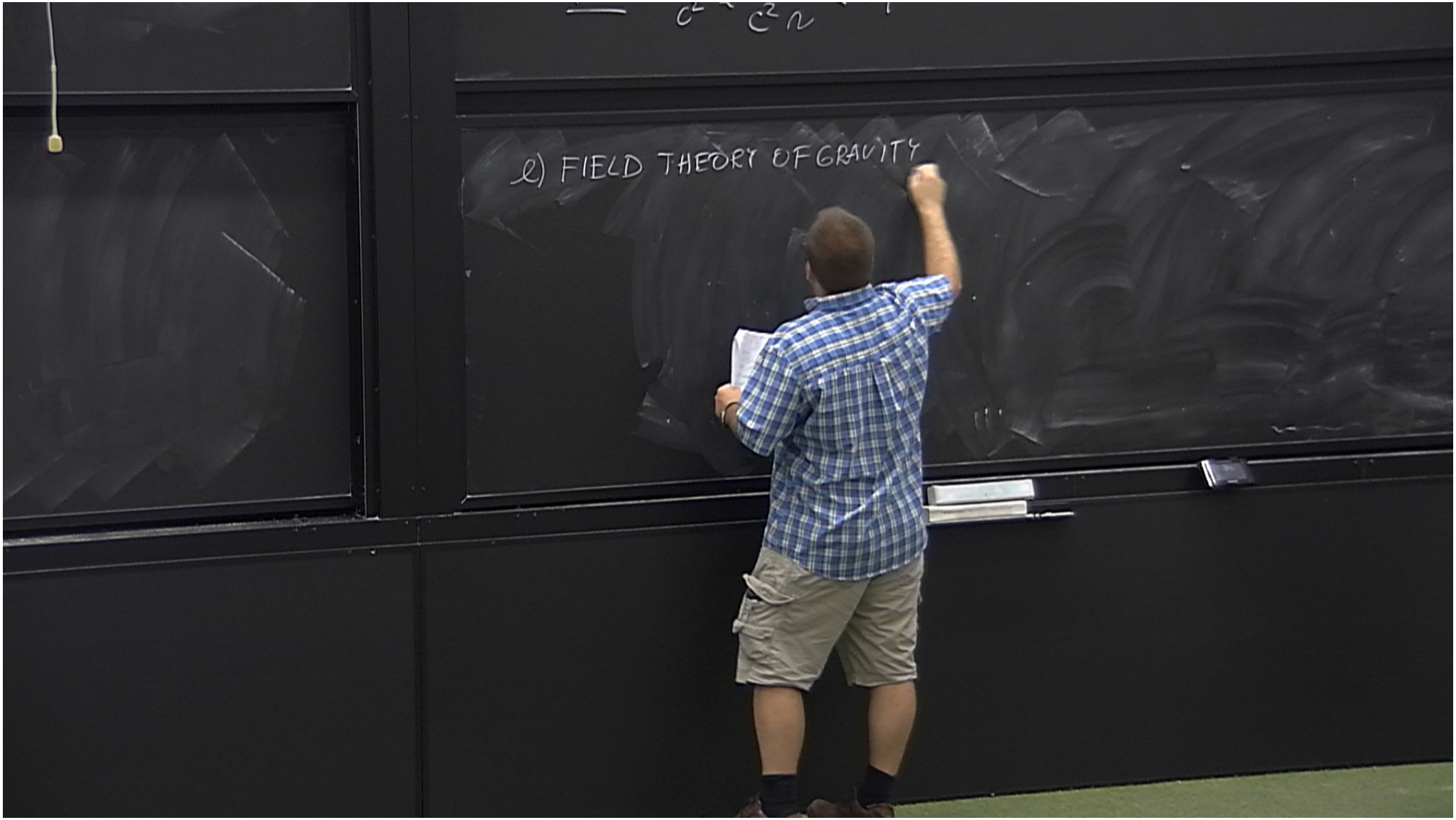

x^μ
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$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi')$


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$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-$

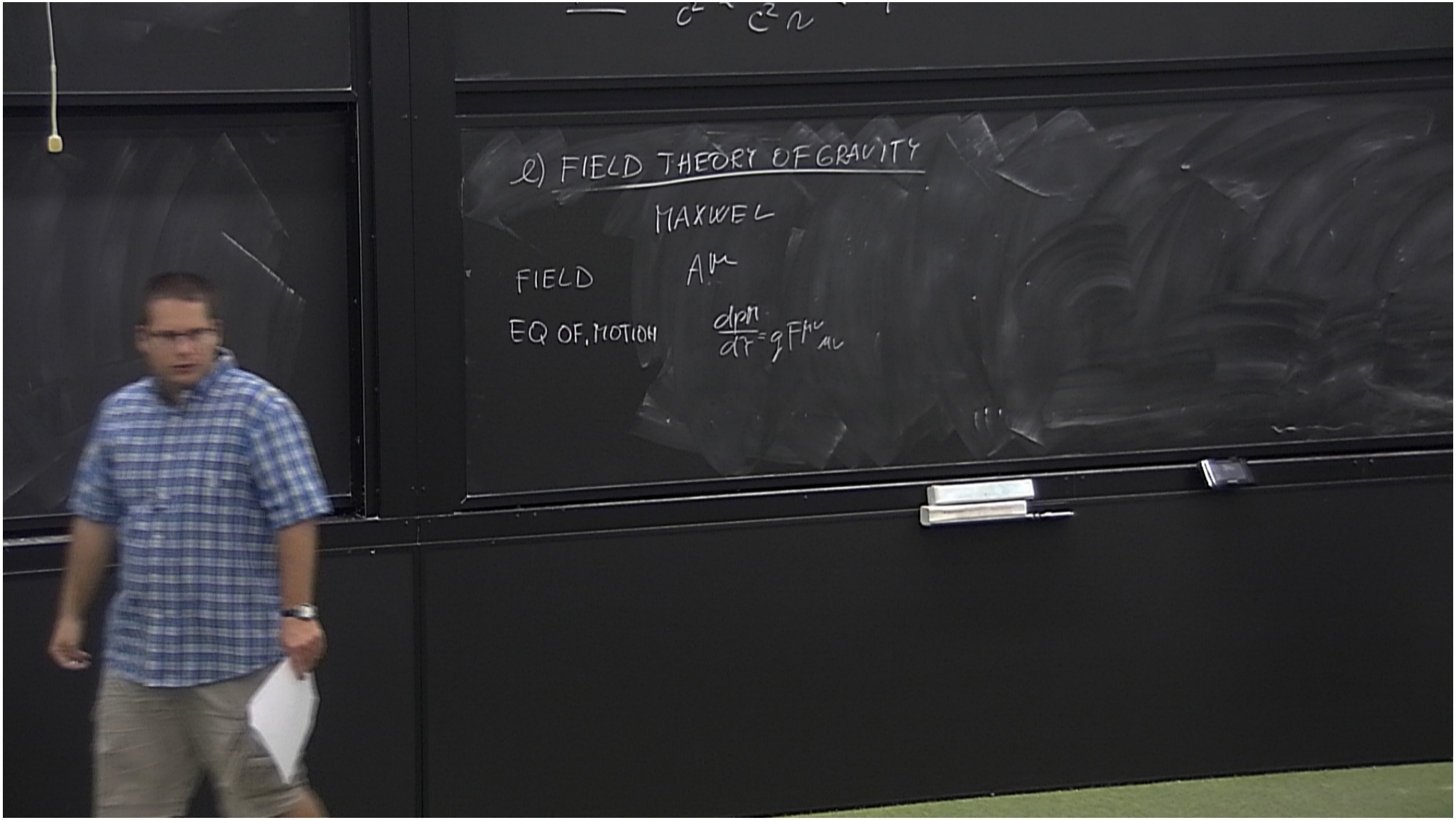

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REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC HOM.

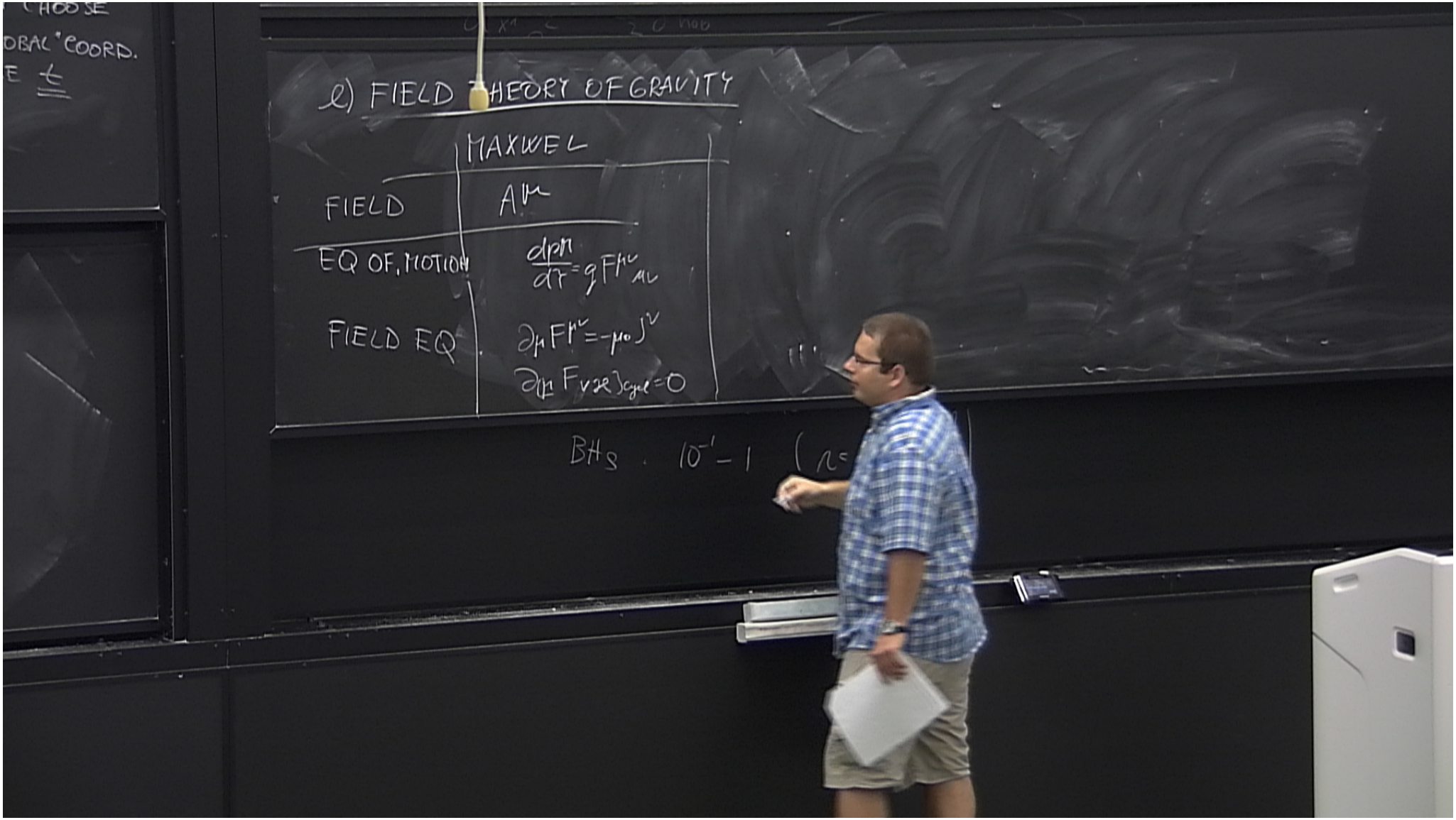

$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC: HOM. GRAV. FIELD $z = -g$

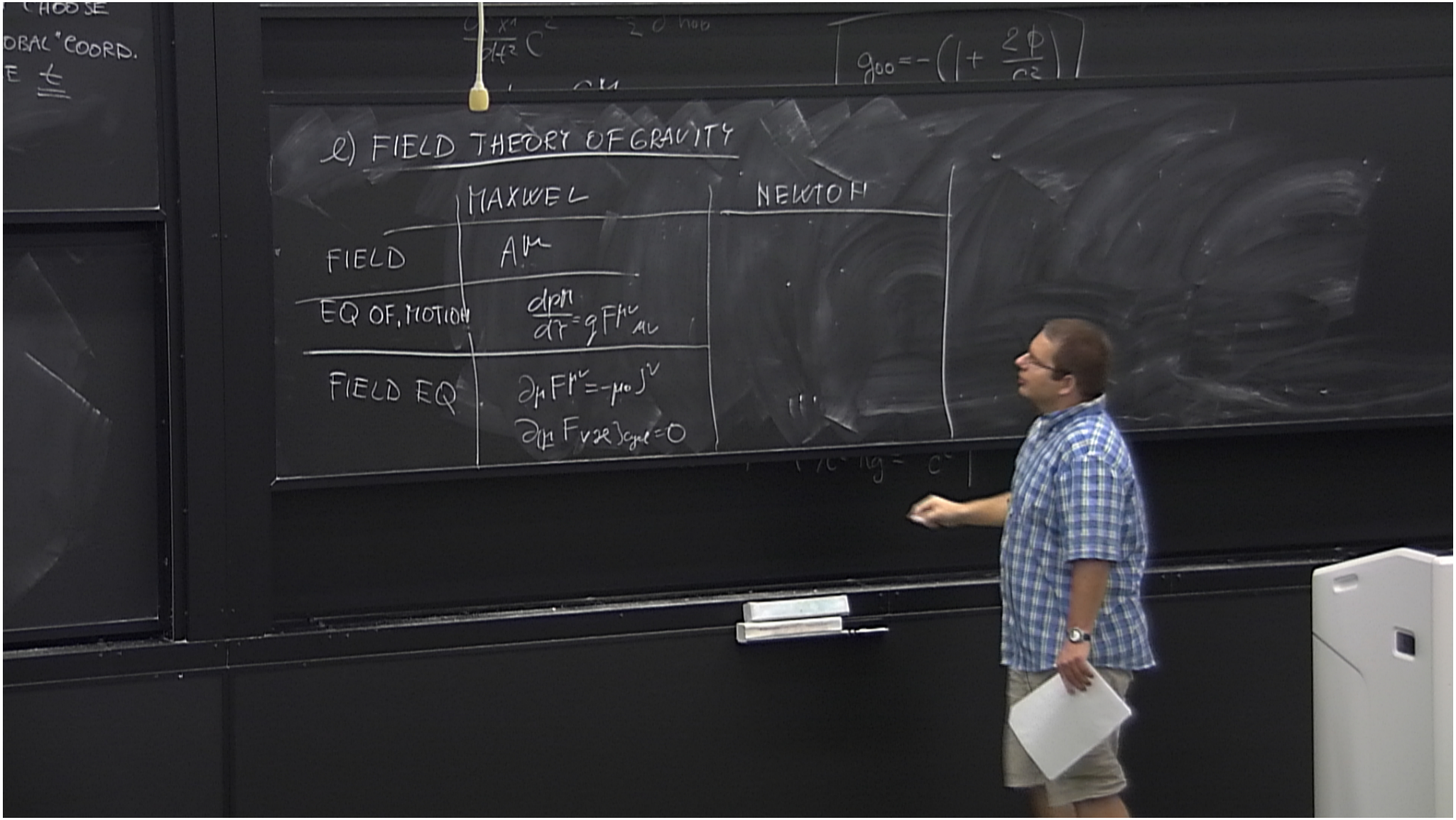

x^μ
$x^{\mu'}$
$\Rightarrow$
$\frac{\nu'}{\nu} = \frac{\sqrt{-g_{00}(x)}}{\sqrt{-g_{00}(x')}}$
• IN NEWTONIAN LIMIT.
$\frac{\nu'}{\nu} = \sqrt{\frac{1+2\phi}{1+2\phi'}} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
REDSHIFT $z = \frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} = \phi-\phi'$
SPEC: HOM. GRAV. FIELD $z = -\frac{g\Delta h}{c^2}$

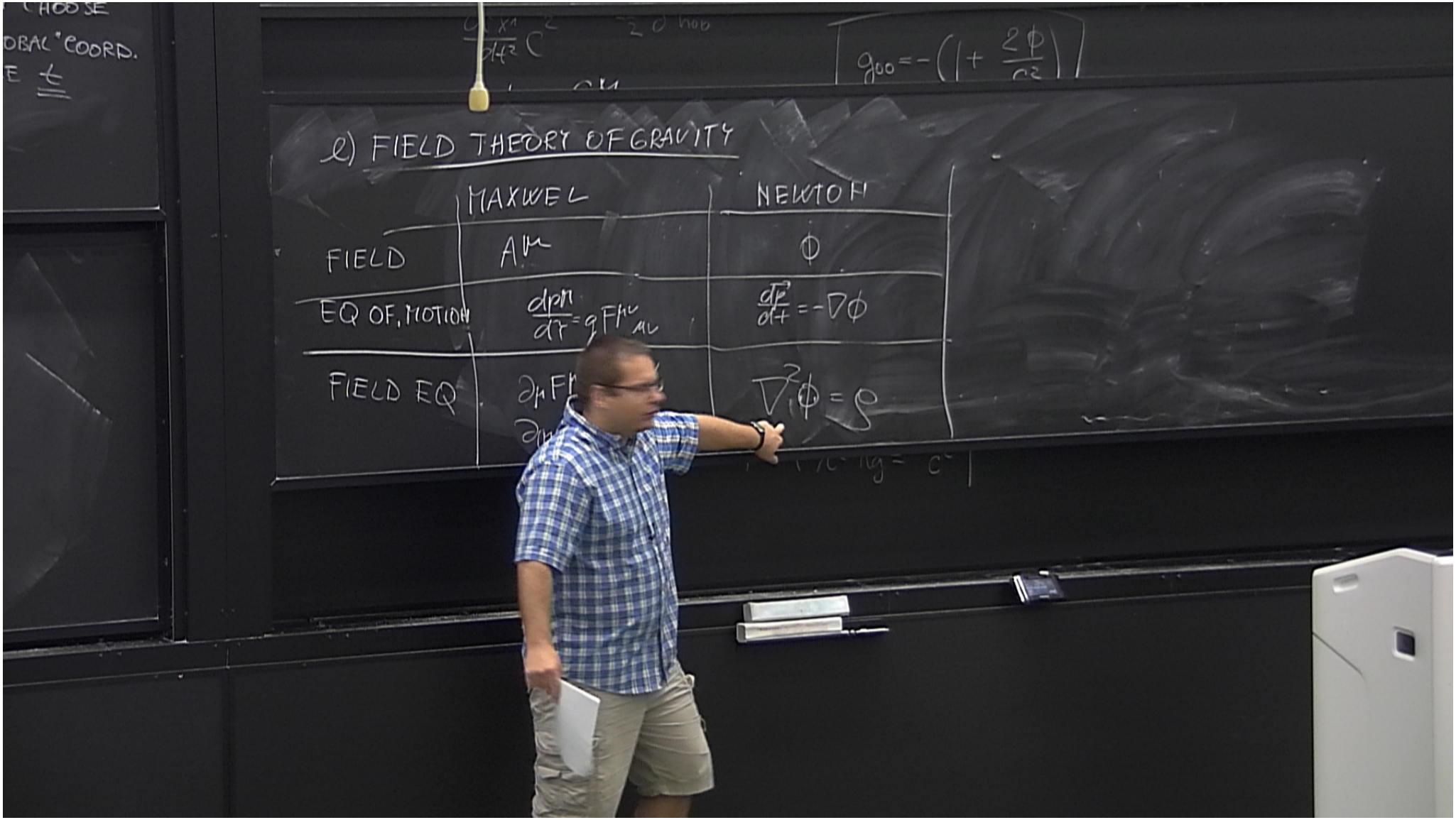

2) FIELD THEORY OF GRAVITY

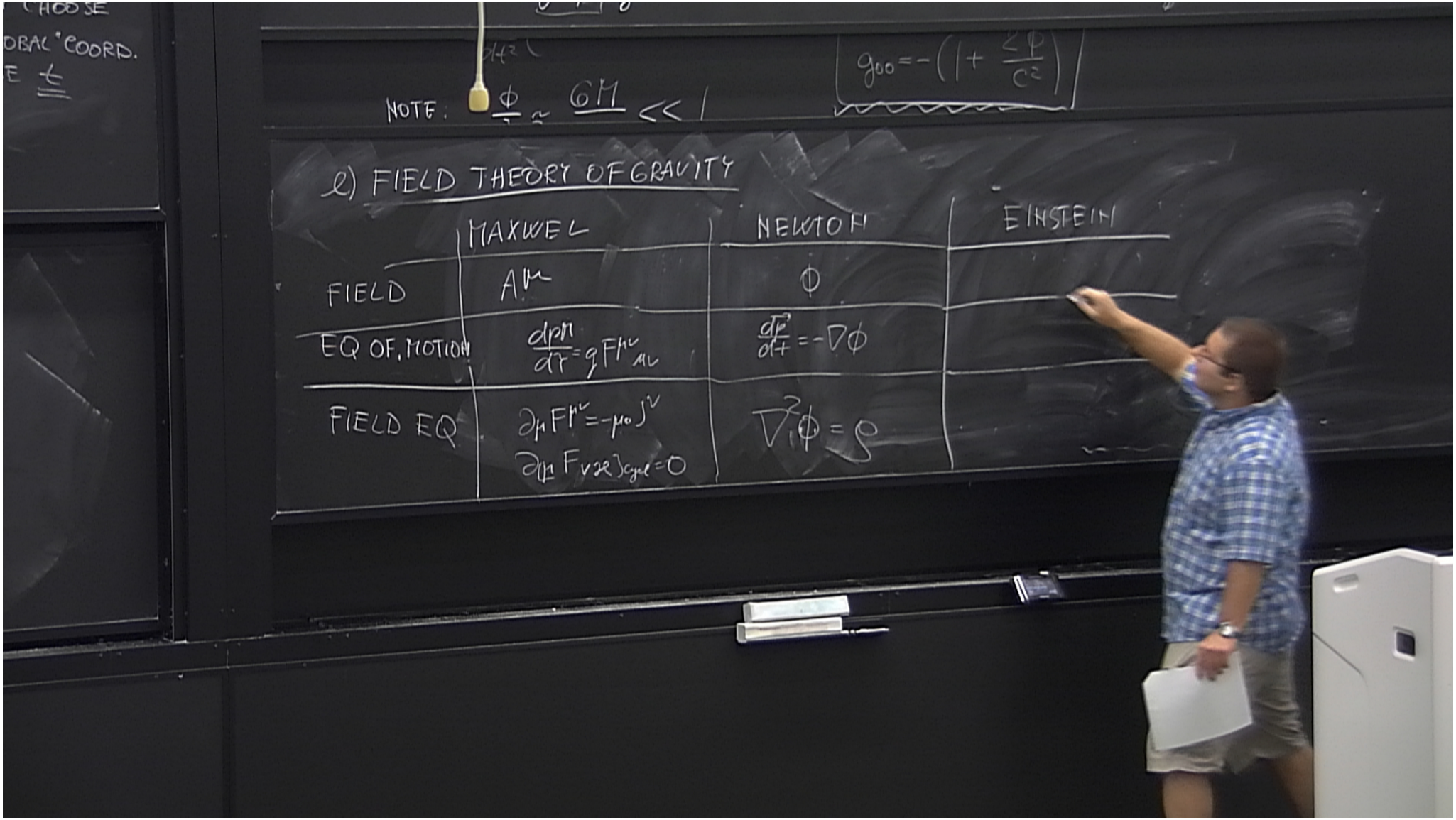

c) FIELD THEORY OF GRAVITY
MAXWEL

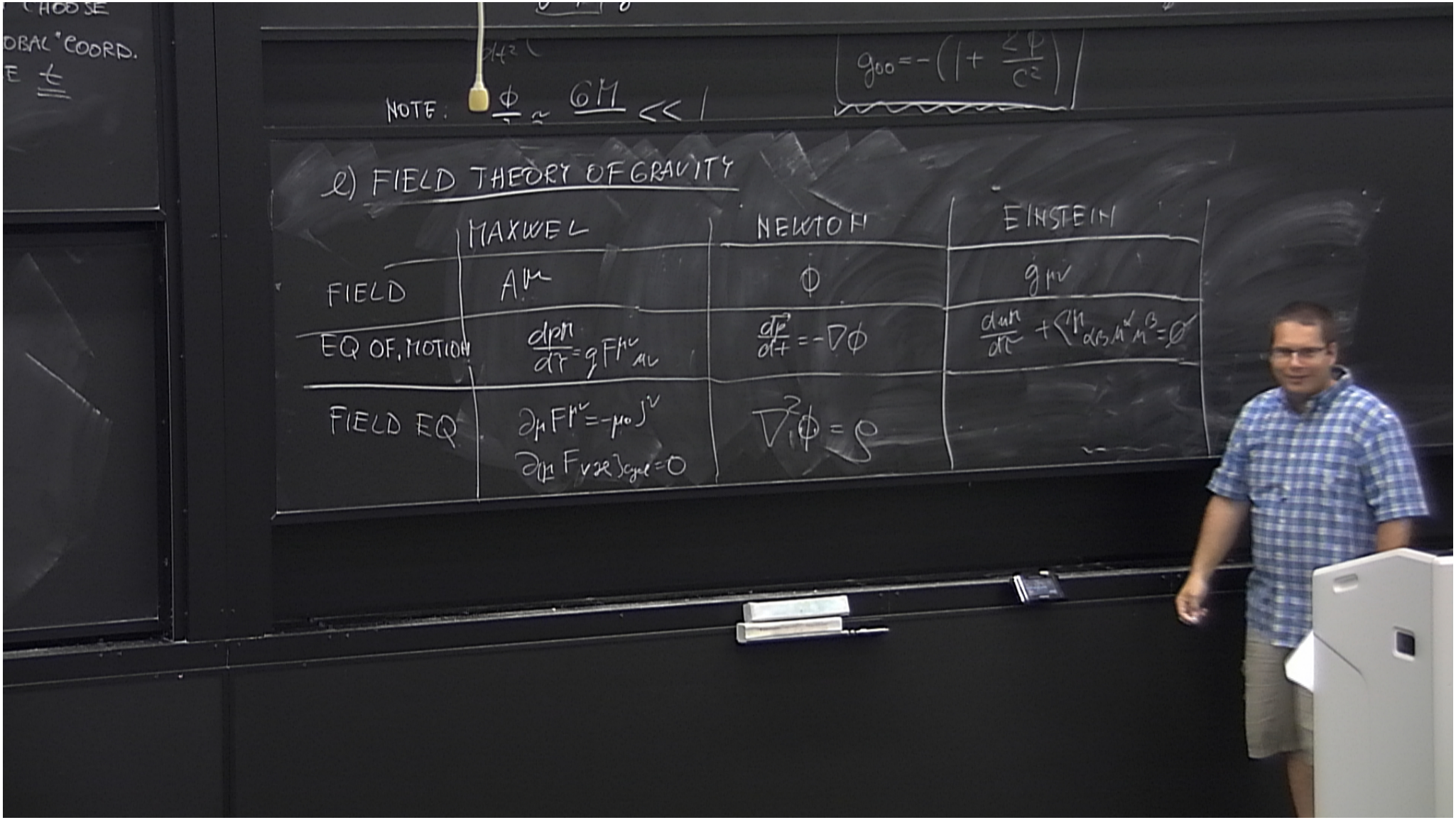

2) FIELD THEORY OF GRAVITY
MAXWEL
FIELD
A
EQ OF. MOTION

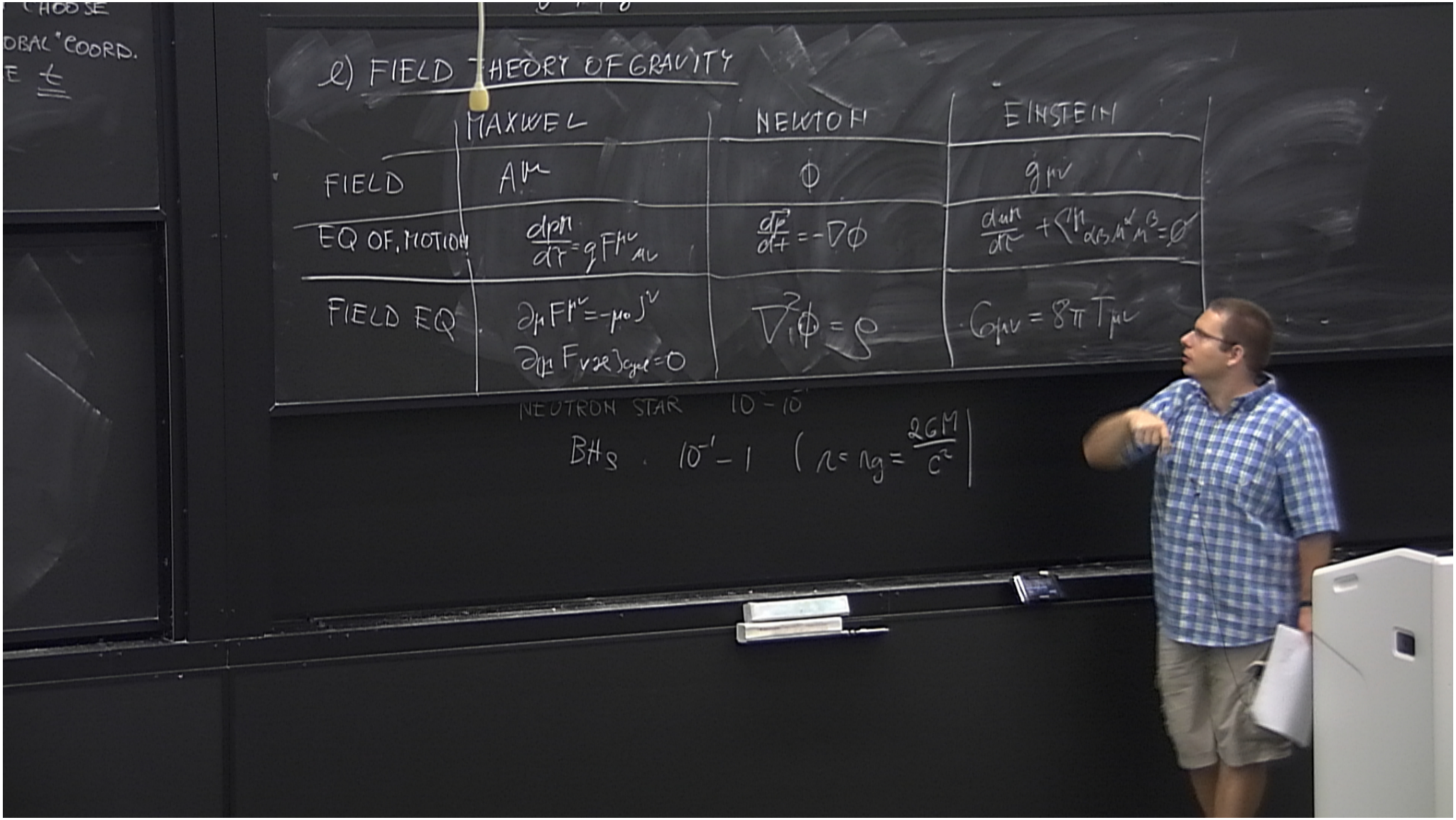

OBAL "COORD.
E ≡
ℓ) FIELD THEORY OF GRAVITY
MAXWEL
FIELD
A^μ
EQ OF. MOTION
$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_\mu F_{\nu\rho]} = 0$
BHs . $10^{-1} - 1$

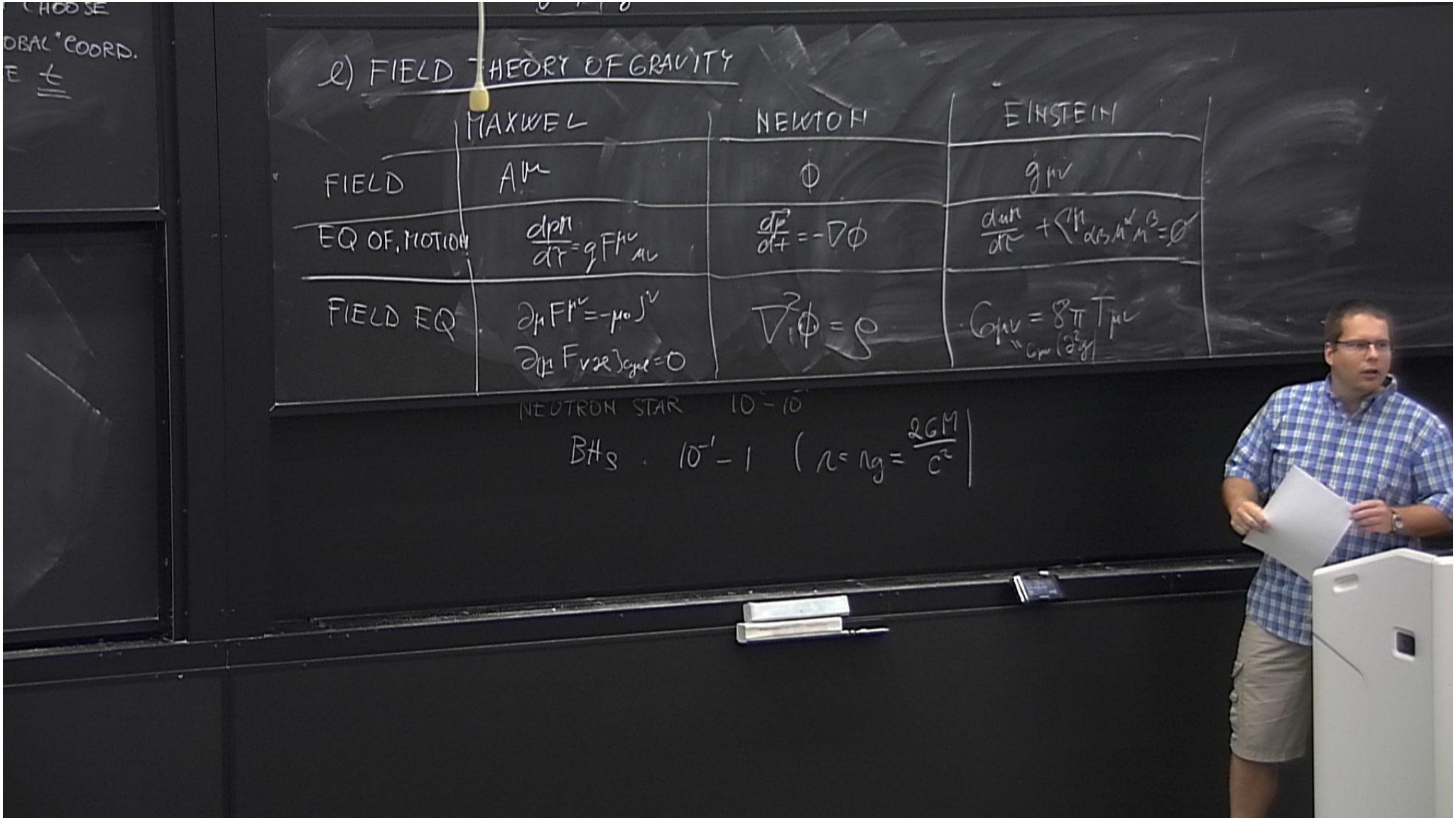

OBAL "COORD.
$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
e) FIELD THEORY OF GRAVITY
MAXWEL
NEWTON
FIELD
A^μ
EQ OF. MOTION
$\frac{dp^\mu}{d\tau} = q F^{\mu\nu} u_\nu$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_\mu F_{\nu\rho}]_{cyc} = 0$

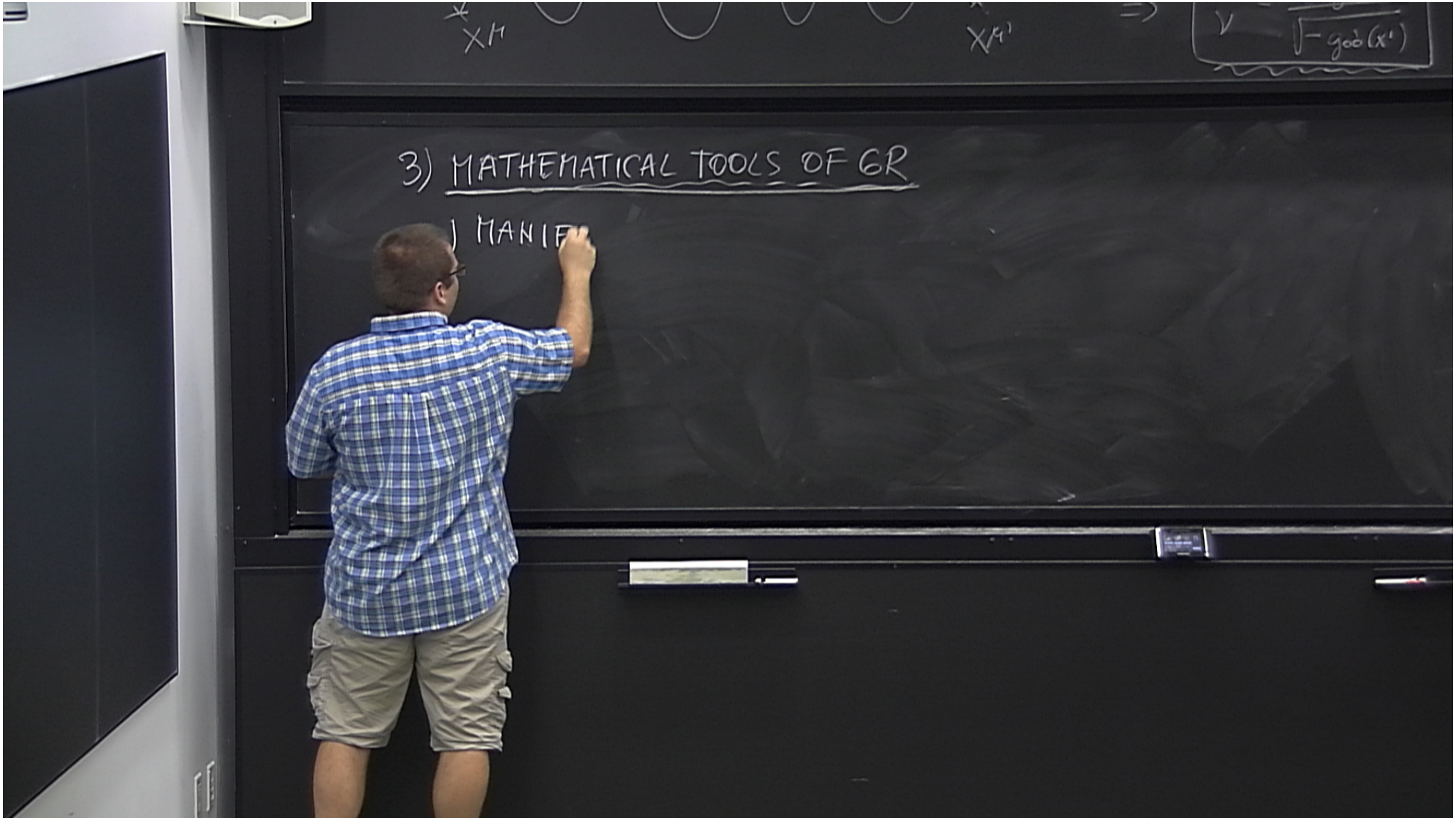

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$g_{00} = -\left(1 + \frac{2\phi}{c^2}\right)$
e) FIELD THEORY OF GRAVITY
MAXWEL
NEWTON
FIELD
A_μ
ϕ
EQ OF. MOTION
$\frac{dp_\mu}{d\tau} = q F^{\mu\nu} u_\nu$
$\frac{d\vec{p}}{dt} = -\nabla\phi$
FIELD EQ
$\partial_\mu F^{\mu\nu}$
$\nabla^2\phi = \rho$

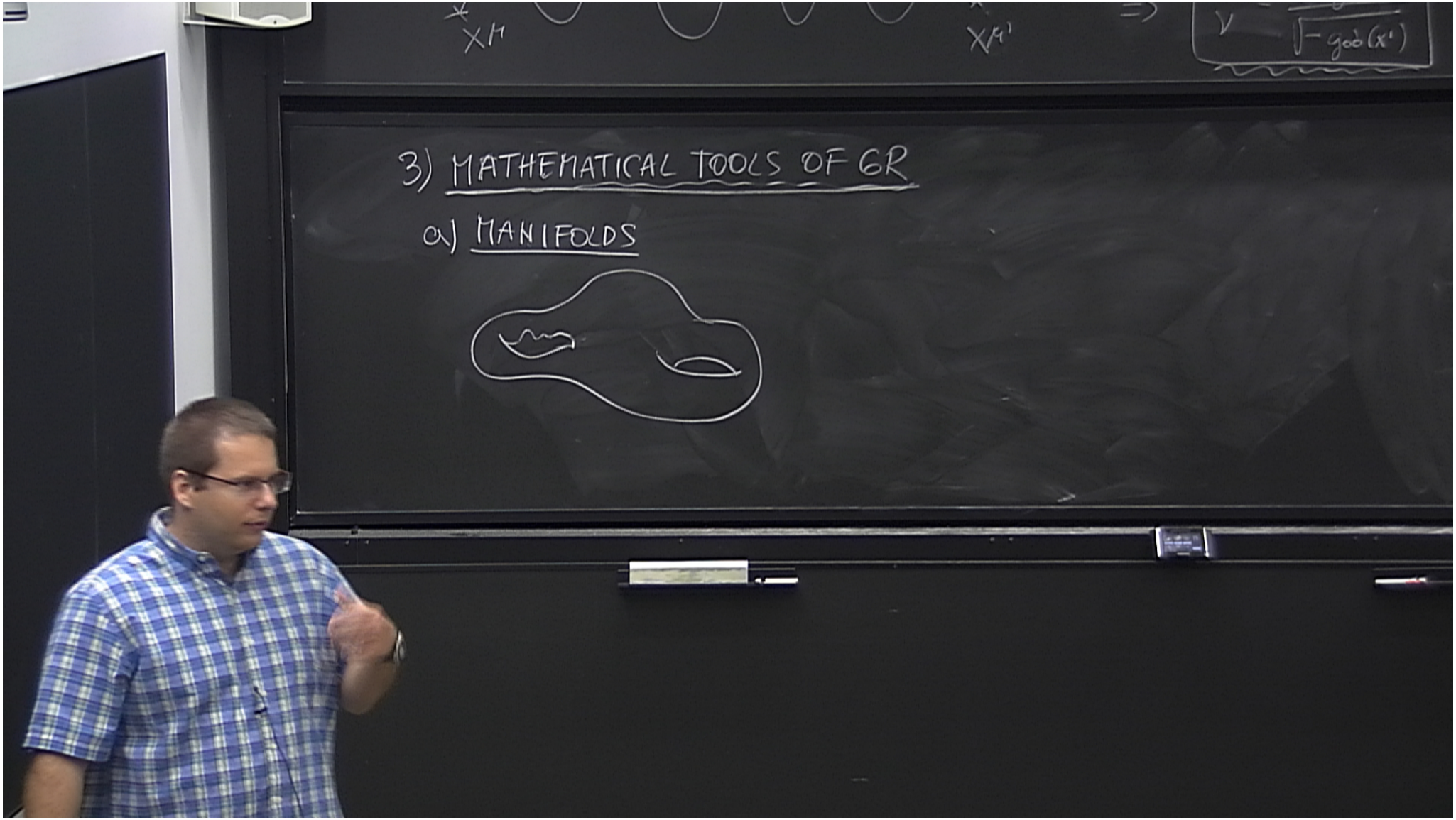

OBAL "COORD.
NOTE: $\frac{\Phi}{c^2} \simeq \frac{GM}{rc^2} \ll 1$
$g_{00} = -\left(1 + \frac{2\Phi}{c^2}\right)$
ℓ) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
A^μ
Φ
EQ OF MOTION
$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}u_\nu$
$\frac{d\vec{p}}{dt} = -\nabla\Phi$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_{[\lambda} F_{\nu\rho]} = 0$
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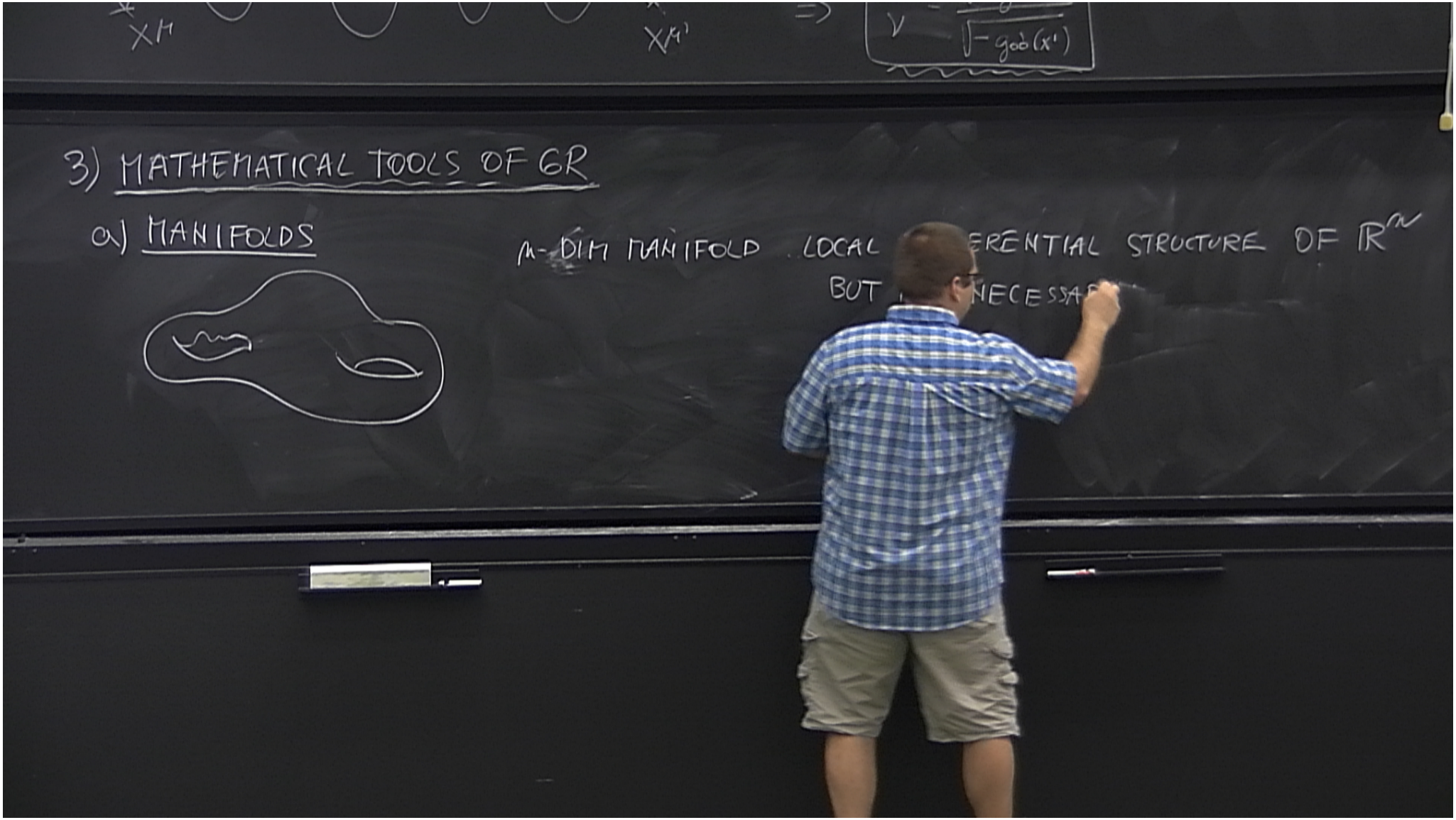

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e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
A^μ
Φ
$g_{\mu\nu}$
EQ OF MOTION
$\frac{dp^\mu}{d\tau} = qF^\mu{}_\nu$
$\frac{d^2\vec{r}}{dt^2} = -\nabla\Phi$
$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0$
FIELD EQ
$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$
$\partial_{[\mu} F_{\nu\rho]} = 0$
$\nabla^2\Phi = \rho$

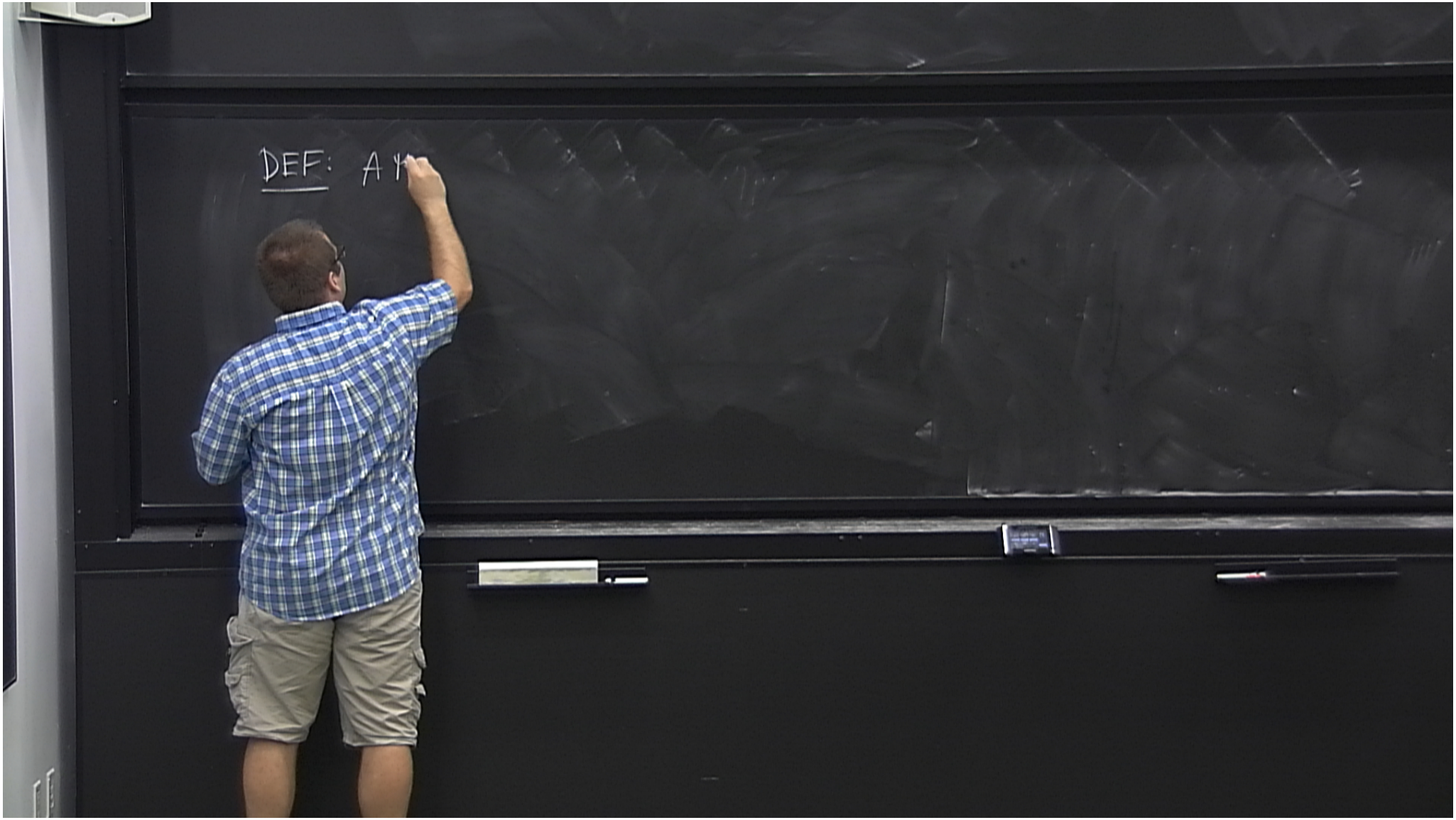

OBAL "COORD.
E ≡
ℓ) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
EQ OF. MOTION
FIELD EQ
NEUTRON STAR 10 - 10
BHs . 10 - 1

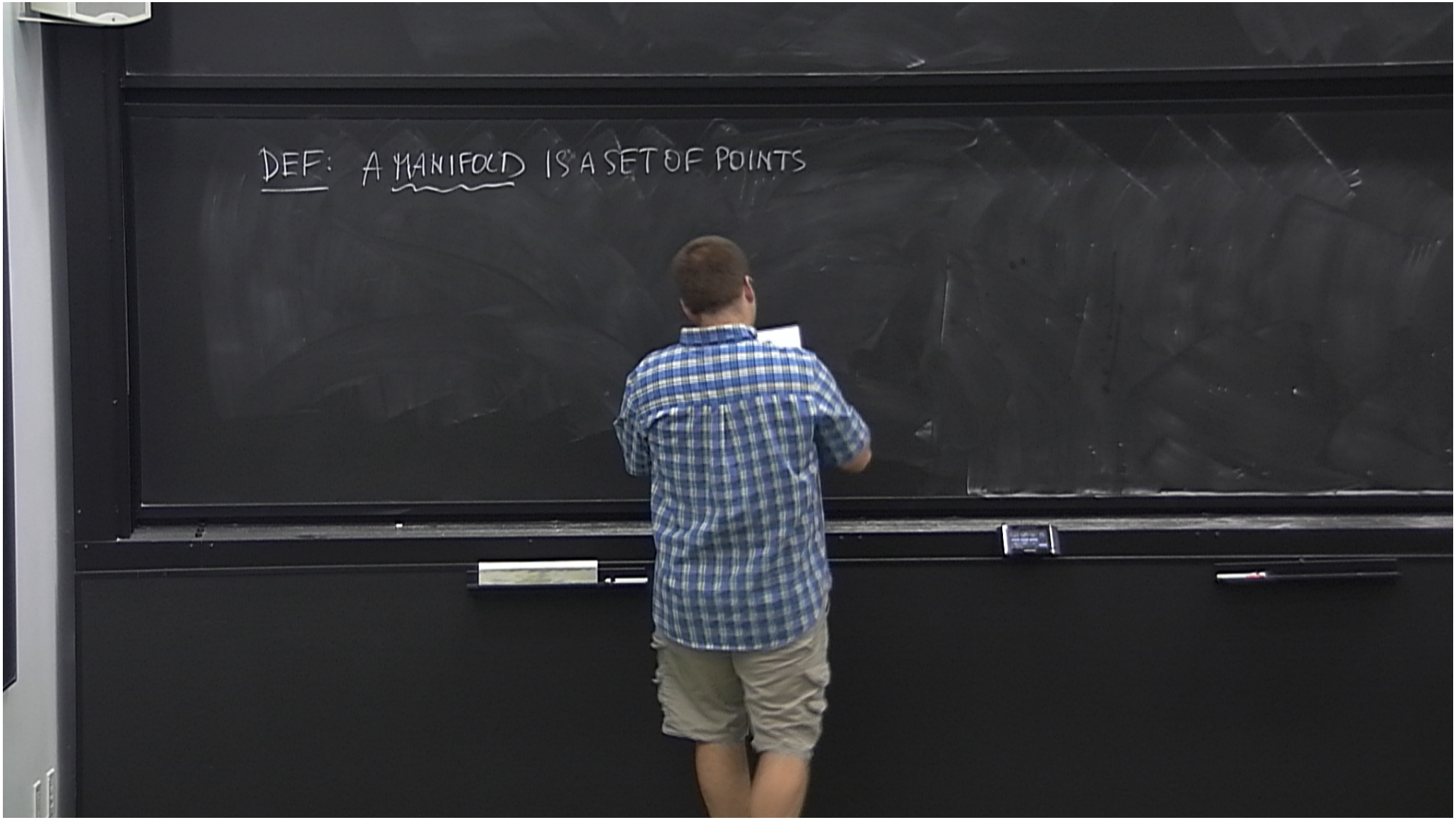

OBAL "COORD.
e) FIELD THEORY OF GRAVITY
MAXWELL
NEWTON
EINSTEIN
FIELD
EQ OF. MOTION
FIELD EQ
NEUTRON STAR
BHs


3) MATHEMATICAL TOOLS OF GR
) MANIF


3) MATHEMATICAL TOOLS OF GR
a) MANIFOLDS


3) MATHEMATICAL TOOLS OF GR
a) MANIFOLDS
M-DIM MANIFOLD
LOCAL
ERENTIAL STRUCTURE OF $\mathbb{R}^n$
BUT
NECESSA


DEF:


DEF: A MANIFOLD IS A SET OF POINTS