Title: Quantum States are Consistent Probability Distributions

Date: Sep 12, 2013 11:00 AM

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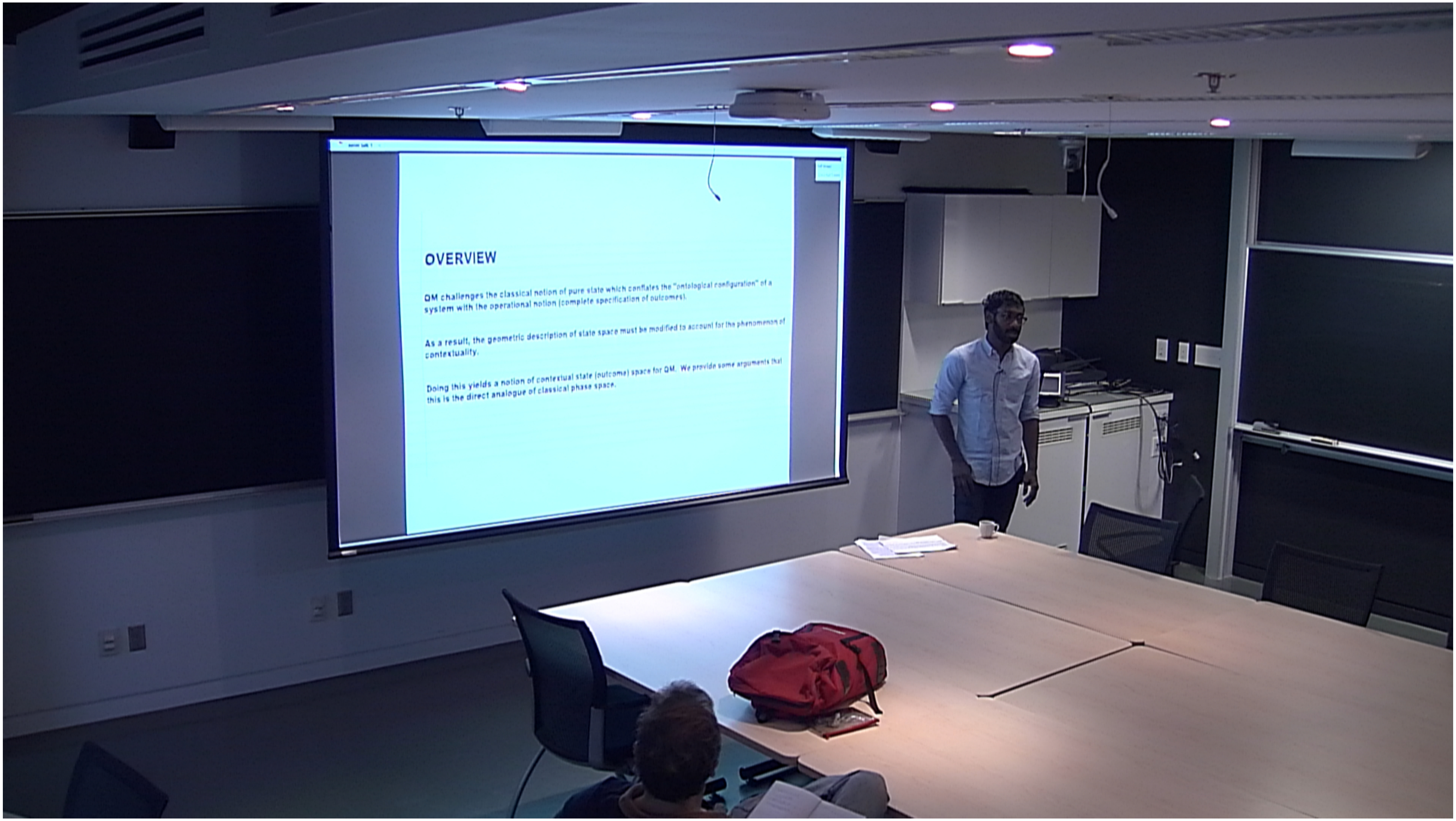
Abstract: We describe a notion of state for a quantum system which is given in terms of a collection of empirically realizable probability distributions and is formally analogous to the familiar concept of state from classical statistical mechanics. We first demonstrate the mathematical equivalence of this new notion to the standard quantum notion of density matrix. We identify the simple logical consistency condition (a generalization of the familiar no-signalling condition) which a collection of distributions must obey in order to reconstruct the unique quantum state from which they arise. In this way, we achieve a formal expression of the common intuition of a quantum state as being classical distributions on compatible observables.


OVERVIEW
QM challenges the classical notion of pure state which conflates the "ontological configuration" of a system with the operational notion (complete specification of outcomes).
As a result, the geometric description of state space must be modified to account for the phenomenon of contextuality.
Doing this yields a notion of contextual state (outcome) space for QM. We provide some arguments that this is the direct analogue of classical phase space.

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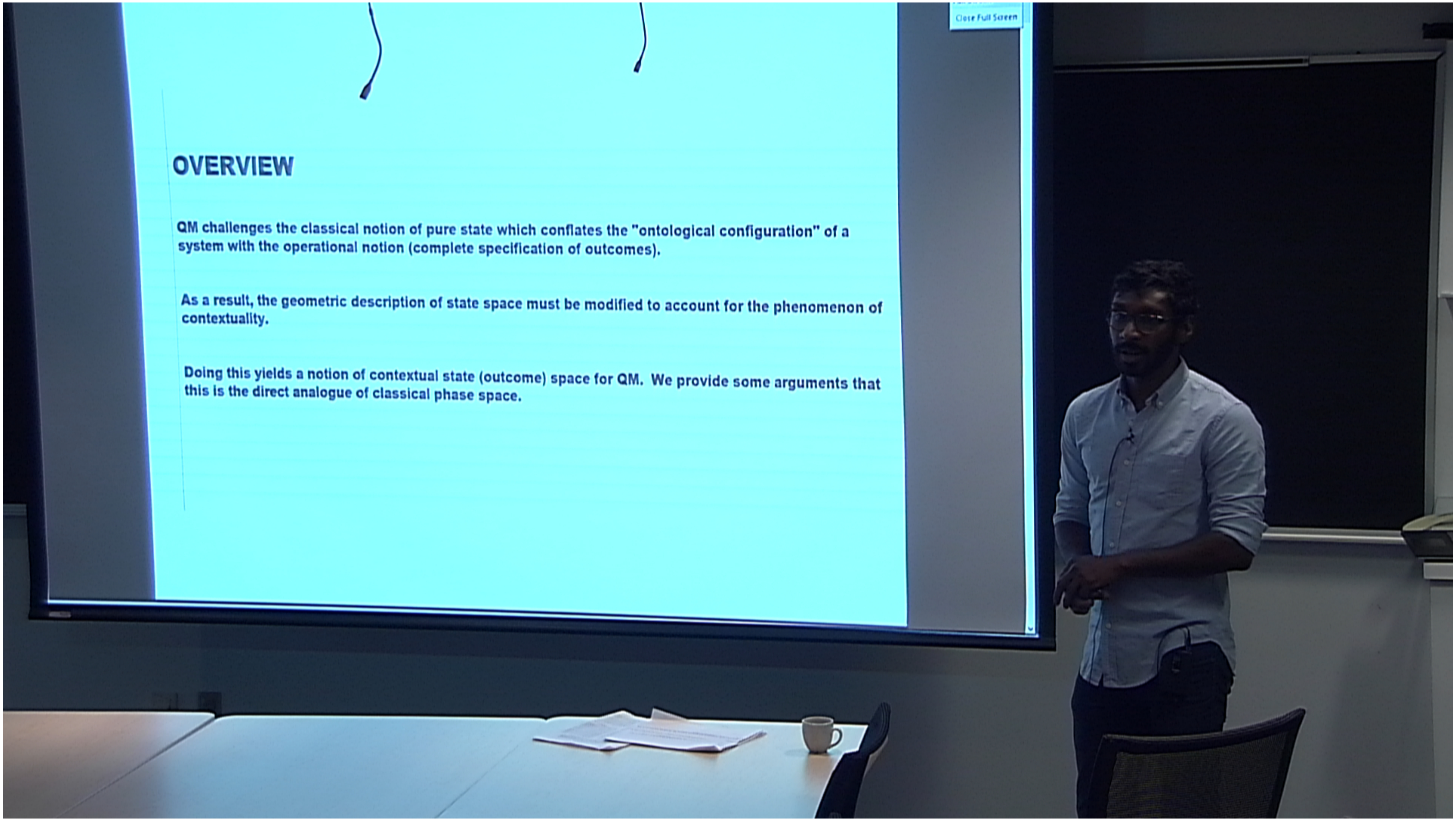
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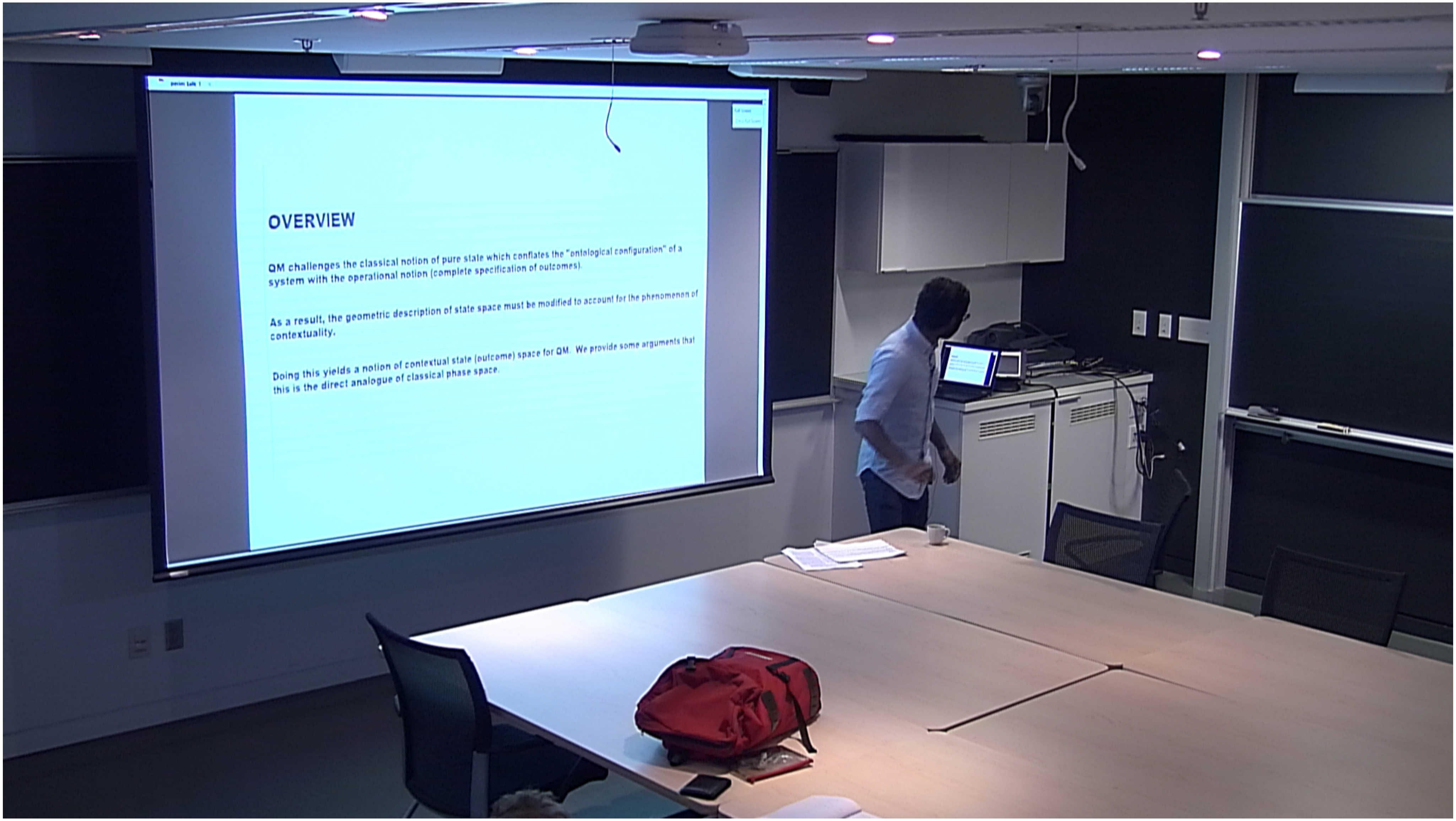
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1) This notion of state space allows for elegant formulations of key theorems of quantum foundations: the Kochen-Specker theorem and Gleason's theorem.

2) The space yields an algorithmic method for extending concepts of classical geometry to their analogues in noncommutative geometry. This confirms the direct analogy between classical state space and the noncommutative state space.

3) When the notion of classical state (a probability distribution) is extended by this algorithm, we get as output, the notion of density matrix.

- This shows us how to recover the usual notion of state as a probability distribution over contextual outcome space, or, equivalently, as a family of probability distributions which is consistent with respect to marginalization.
- Can be generalized to provide a notion of state space for any theory which features contextuality.
- This connects with the fundamental question of which empirical models (i.e. observable correlations) are realizable by quantum mechanics .

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a physical system

↓

A THEORY OF PHYSICS

↓

a mathematical model

- i.e. pendulum, atom, galaxy, human, etc.
- theory specifies regime of applicability

- CM, QM, GR, etc

- mathematical structures (manifolds, vector spaces...) and physical interpretations
- used to generate predictions

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Every mathematical model will, either explicitly or implicitly, contain a description of:

1) all the (distinguishable) possible ways a system can exist, i.e. states

2) what quantities can be measured, i.e observables

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The collection of observables ($C^\infty(M,\mathbb{R})$ $B(H)$)
have algebraic structure... they can be added / multiplied
to get new quantities.

The collection of states (M, H) has geometric
structure, i.e. two states are "close" when they have
similar properties.

States and observables are in duality.

a state provides a valuation for each observable
an observable ... state

the value of observable O for a state s.

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Classical physics can be expressed geometrically.

observables = real-valued functions on the space (continuous, at least)

is a state space "geometry / shape"

i.e set of points + extra structure

points = physical pure states of the system ⟹ specifies experimental outcomes for all observables with certain precision.

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$\longrightarrow \mathbb{R}$

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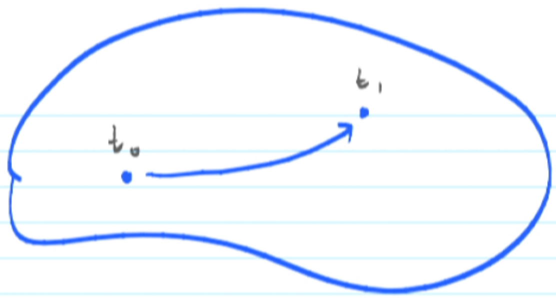

time evolution given by a path in the shape

i.e. in Hamiltonian / symplectic formulations, time evolution is given by flows generated by the energy / Hamiltonian function and the symplectic form

the physics of the system is encoded by the geometry

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Quantum mechanics cannot admit such an elegant description. (caveats : non-contexuality, etc)

This, essentially, is the content of the Kochen-Specker Theorem.

But what if we allow ourselves some liberty in what we consider to be a "geometric space"

VAGUE PROPOSAL: a space is an object which is studied using "geometric tools / intuition"

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GEL'FAND DUALITY

C. H. topological spaces
+ cont. f'ns

commutative, unital C^*-algebras
+ unital *-homomorphisms

an equivalence of categories

$$X \longmapsto C(X)$$

$C(-)$

take the algebra of cont. $\mathbb{C}$-valued functions

$$\Sigma(\mathcal{A}) \longleftarrow\!\!| \ \mathcal{A}$$

Σ

take all unital *-homos $\mathcal{A} \to \mathbb{C}$ with weak topology

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Physically...

$$X \longmapsto C(X) \quad \text{via } C(-)$$

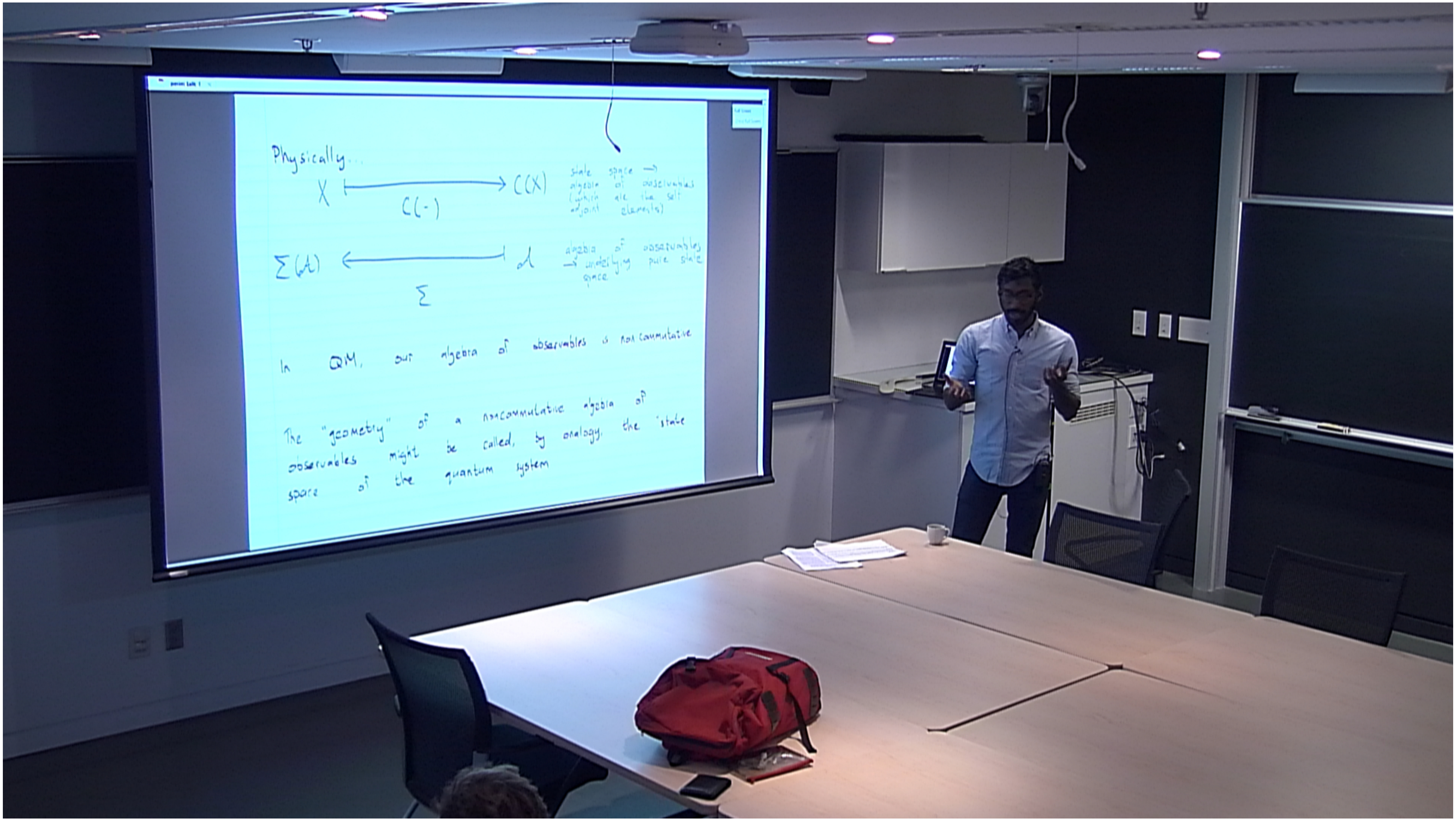
state space $\longrightarrow$ algebra of observables (which are the self-adjoint elements)

$$\Sigma(\mathcal{A}) \longleftarrow\!\!| \ \mathcal{A} \quad \text{via } \Sigma$$

algebra of observables $\longrightarrow$ underlying pure state space

In QM, our algebra of observables is non-commutative

The "geometry" of a noncommutative algebra of observables might be called, by analogy, the "state space of the quantum system



Physically...
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C(-)
state space → algebra of observables (which are the self adjoint elements)
Σ(A) ⟵ A
Σ
algebra of observables → underlying pure state space
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Noncommutative (operator) geometry - NCG - is already an active field of deep mathematical research.

It is actually, in practice, algebra! Operator algebras.

The essential idea: noncommutative C^*-algebras can be thought of as an exotic sort of topological space using Gel'fand duality as the guiding analogy.

i.e. an algebra $\mathcal{A}$ is the algebra of continuous functions from the "noncommutative space of $\mathcal{A}$" to $\mathbb{C}$.

There is no actual topological space X s.t. $\mathcal{A} = C(X)$ (commutative) when $\mathcal{A}$ is noncommutative. We have a "phantom space" and must work with the algebra of f'ns.

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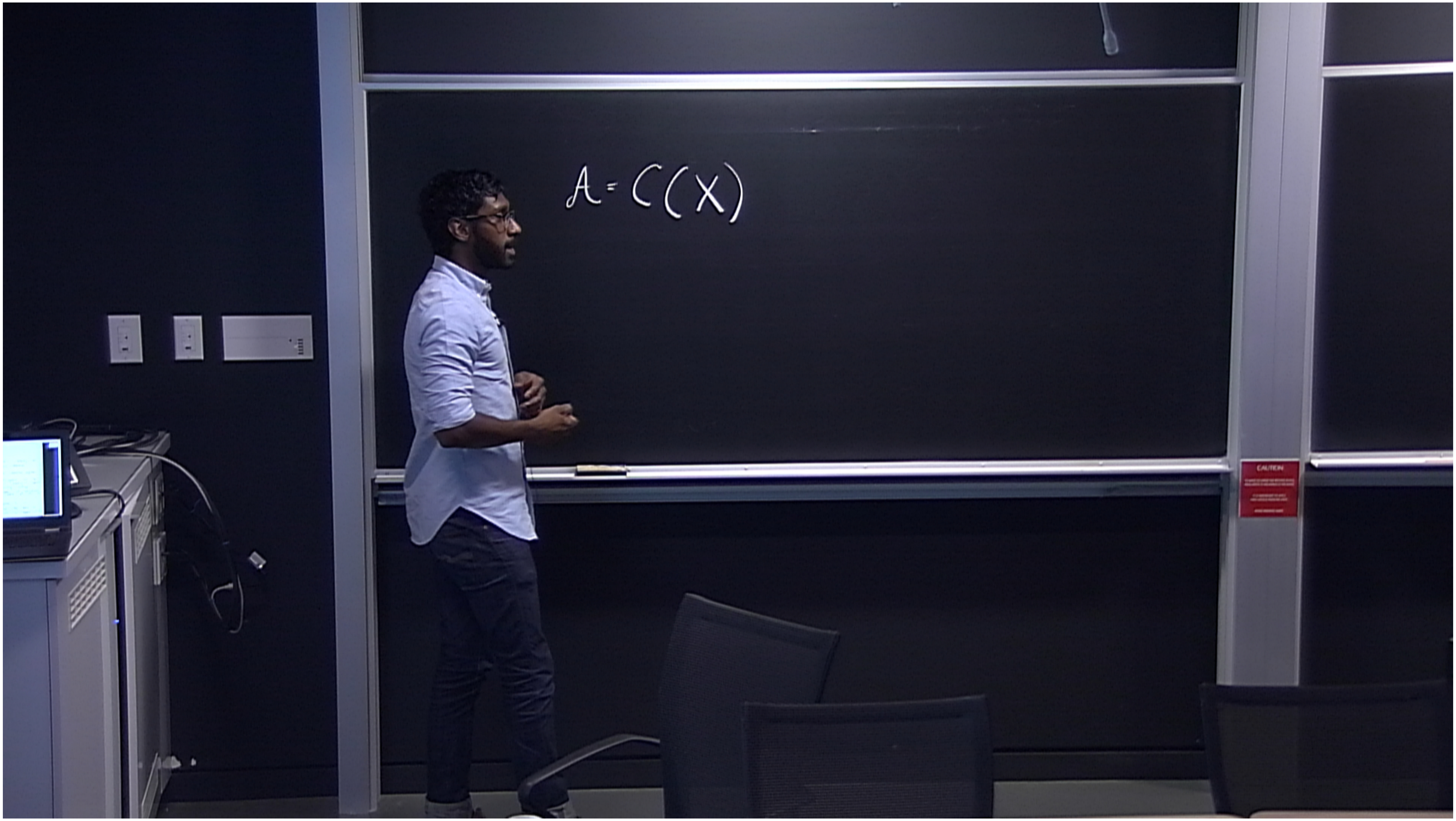
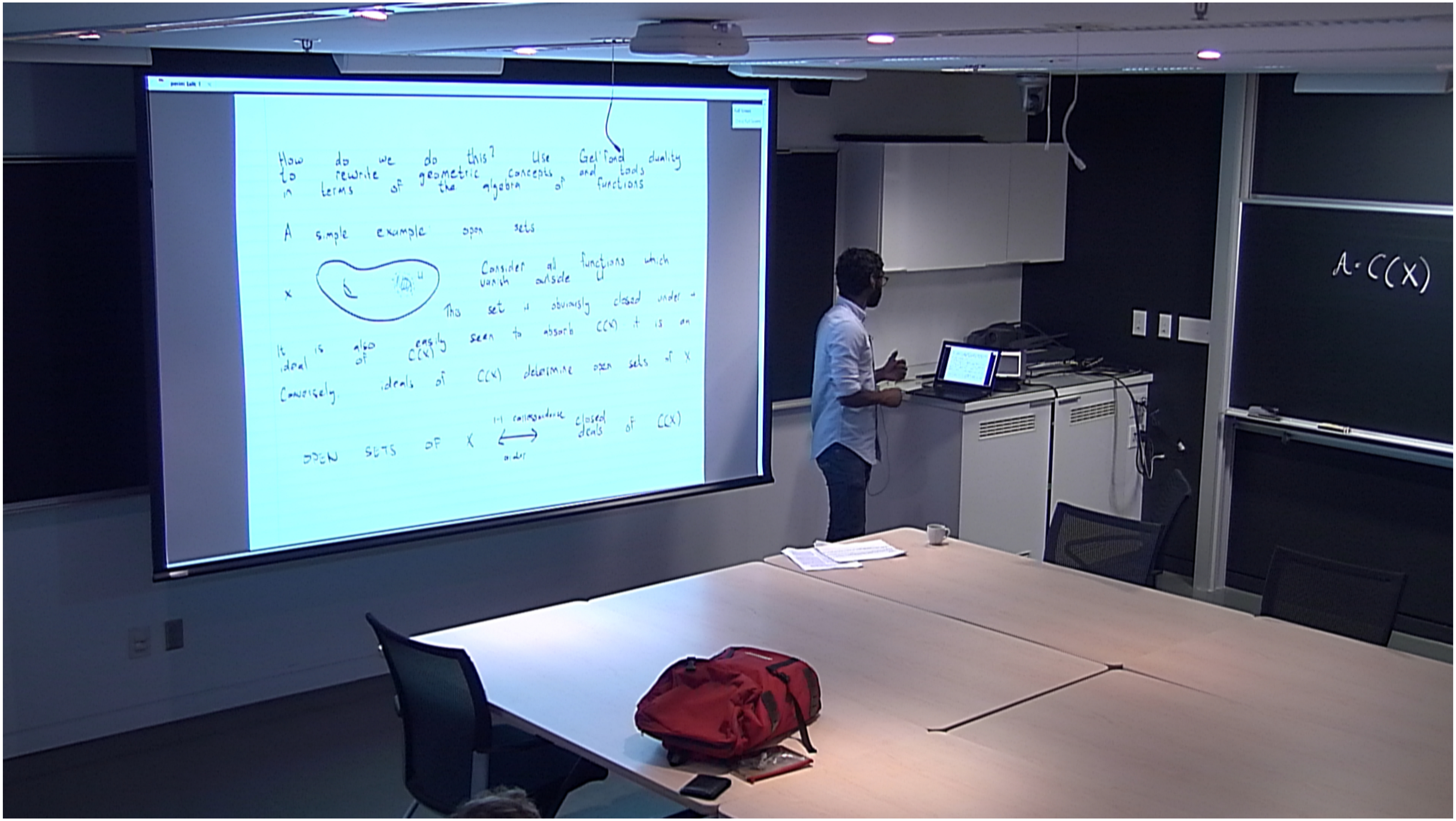

A = C(X)


How do we do this? Use Gel'fand duality
to rewrite geometric concepts and tools
in terms of the algebra of functions
A simple example open sets
Consider all functions which vanish outside U
This set is obviously closed under +
It is also easily seen to absorb C(X) it is an ideal of C(X)
Conversely ideals of C(X) determine open sets of X
OPEN SETS OF X
1-1 correspondence
closed ideals of C(X)
A = C(X)

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A simple example: open sets.

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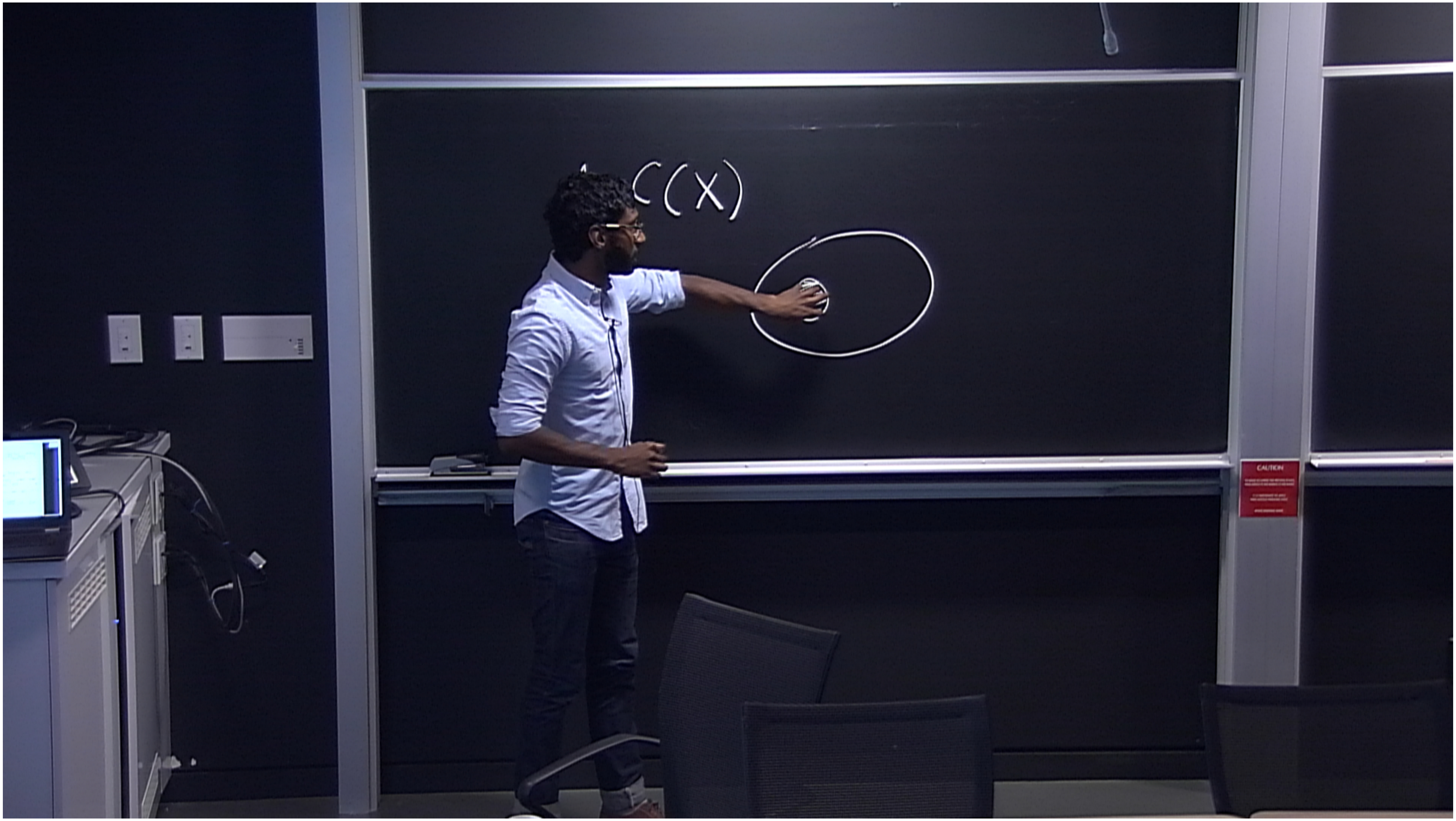
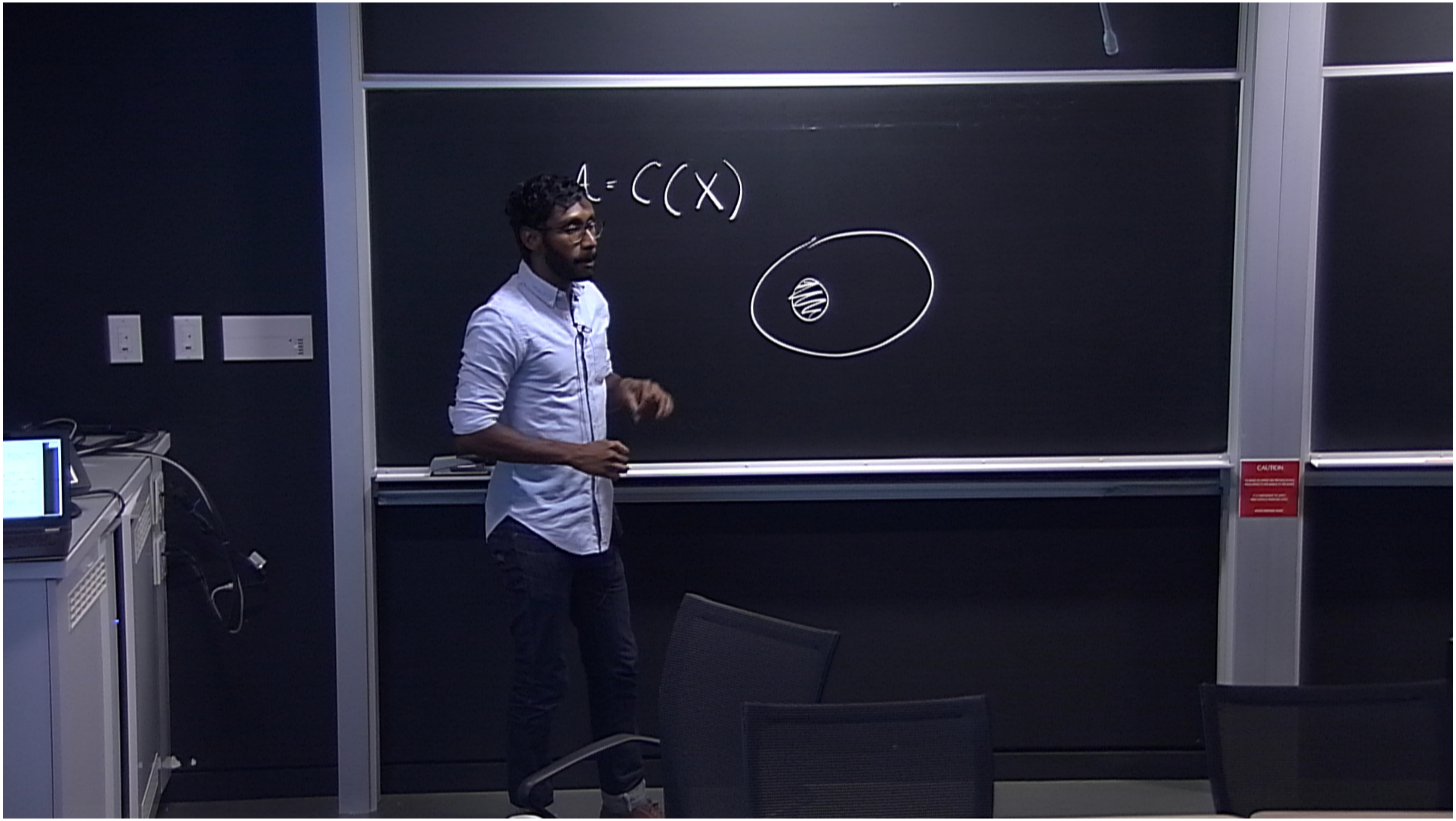
OPEN SETS OF X $\xleftrightarrow[\text{order}]{\text{1-1 correspondance}}$ closed ideals of C(X)


C(X)
CAUTION


A = C(X)

NONCOMMUTATIVE DICTIONARY

Topology	Algebra
continuous $\mathbb{C}$-valued function	element of the algebra
continuous $\mathbb{R}$-valued function	self-adjoint element
open set	closed ideal
vector bundle	finite, projective module
homeomorphism	isomorphism
disjoint union	direct sum
cartesian product	minimal tensor product
Borel measure	positive functional
integral	trace
1-point compactification	unitalization (max)
Stone - Čech compactification	multipliers algebra

and so on...

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A geometrical dual to noncommutative C^*-algebras would provide, for QM, the direct analogue of classical state space.

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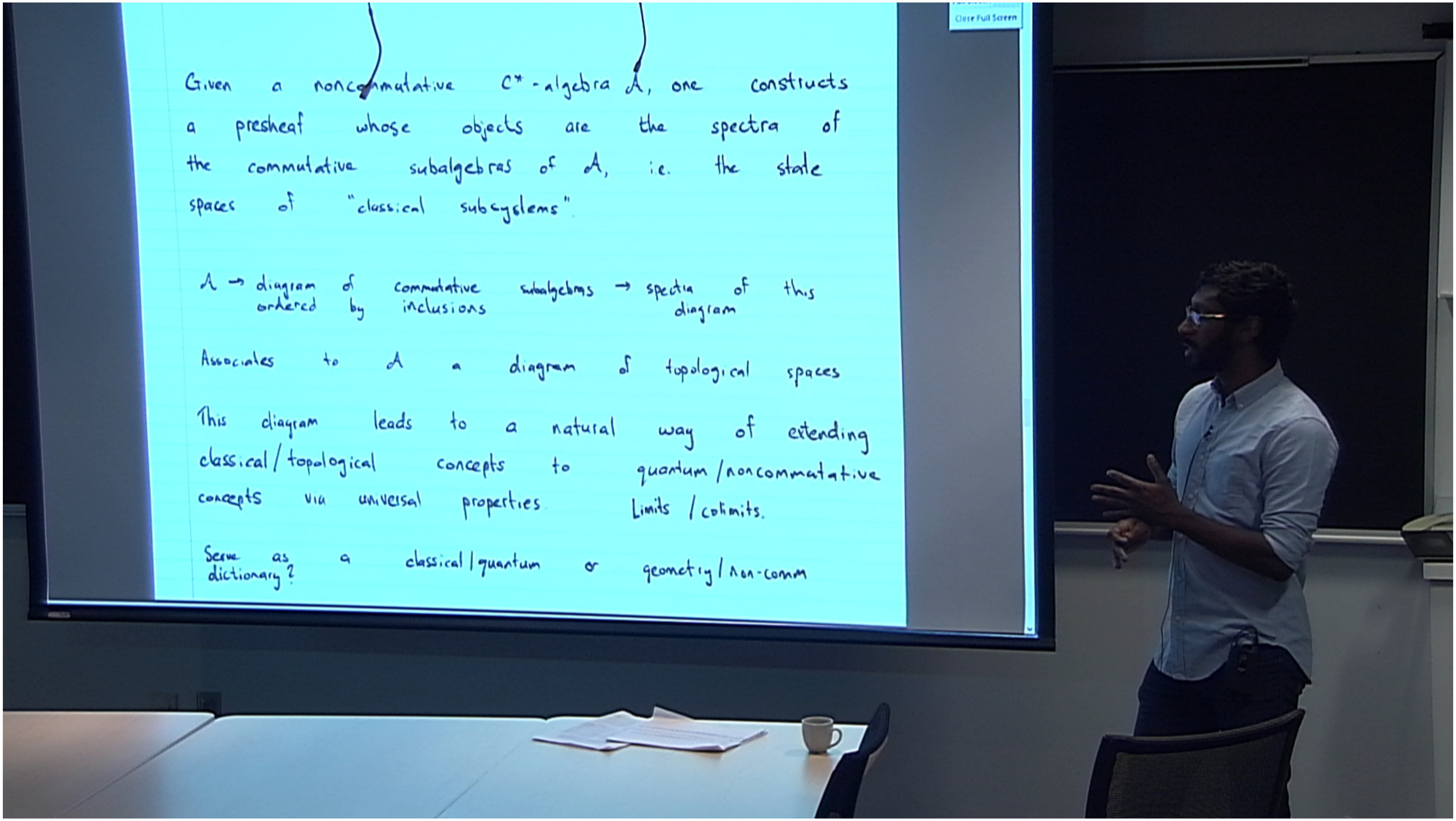
Given a noncommutative C^*-algebra $\mathcal{A}$, one constructs a presheaf whose objects are the spectra of the commutative subalgebras of $\mathcal{A}$, i.e. the state spaces of "classical subsystems".

$\mathcal{A}$ → diagram of commutative subalgebras ordered by inclusions → spectra of this diagram

Associates to $\mathcal{A}$ a diagram of topological spaces

This diagram leads to a natural way of extending classical / topological concepts to quantum / noncommutative concepts via universal properties. Limits / colimits.

Serve as a classical / quantum or geometry / non-comm dictionary?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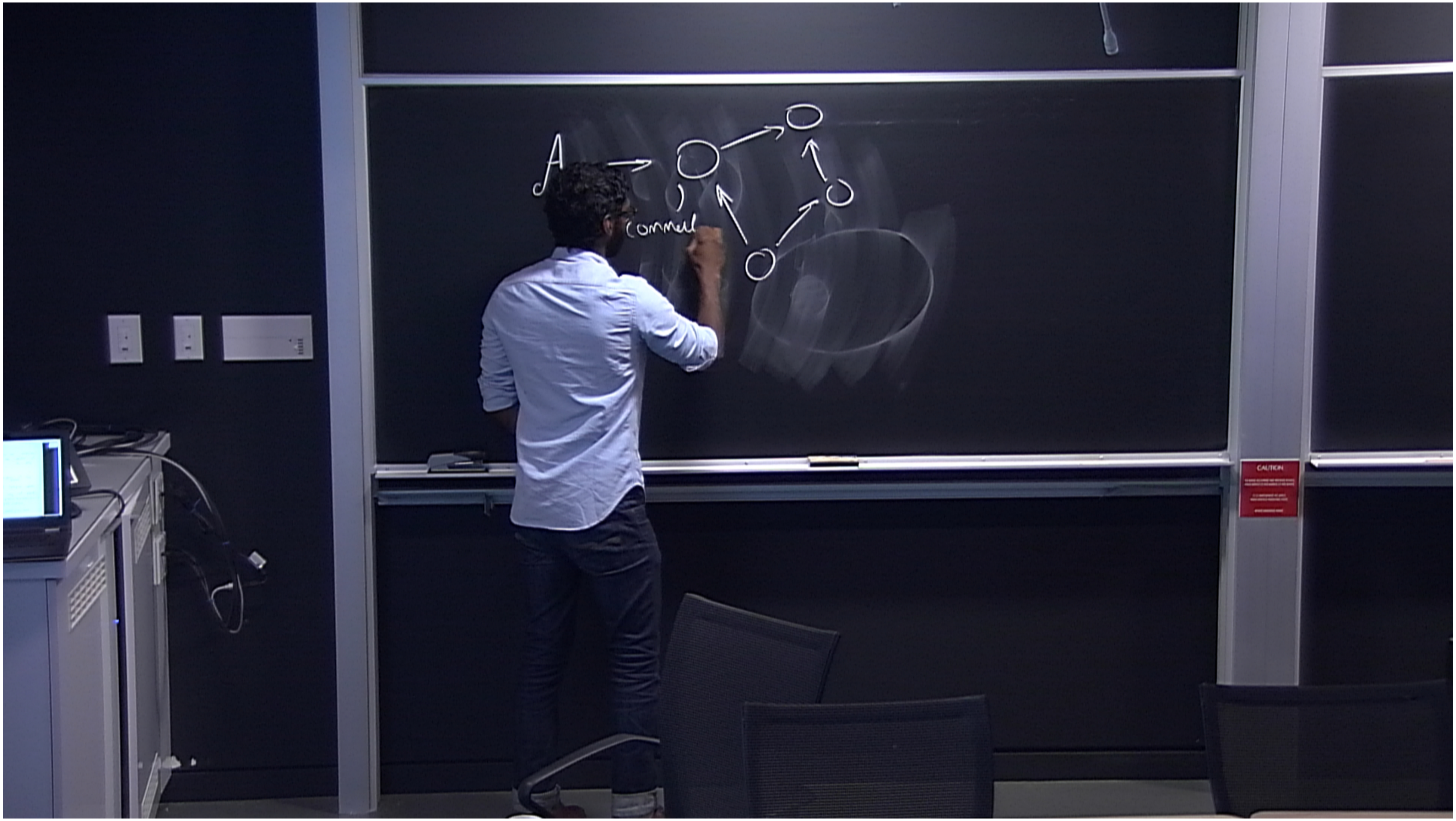
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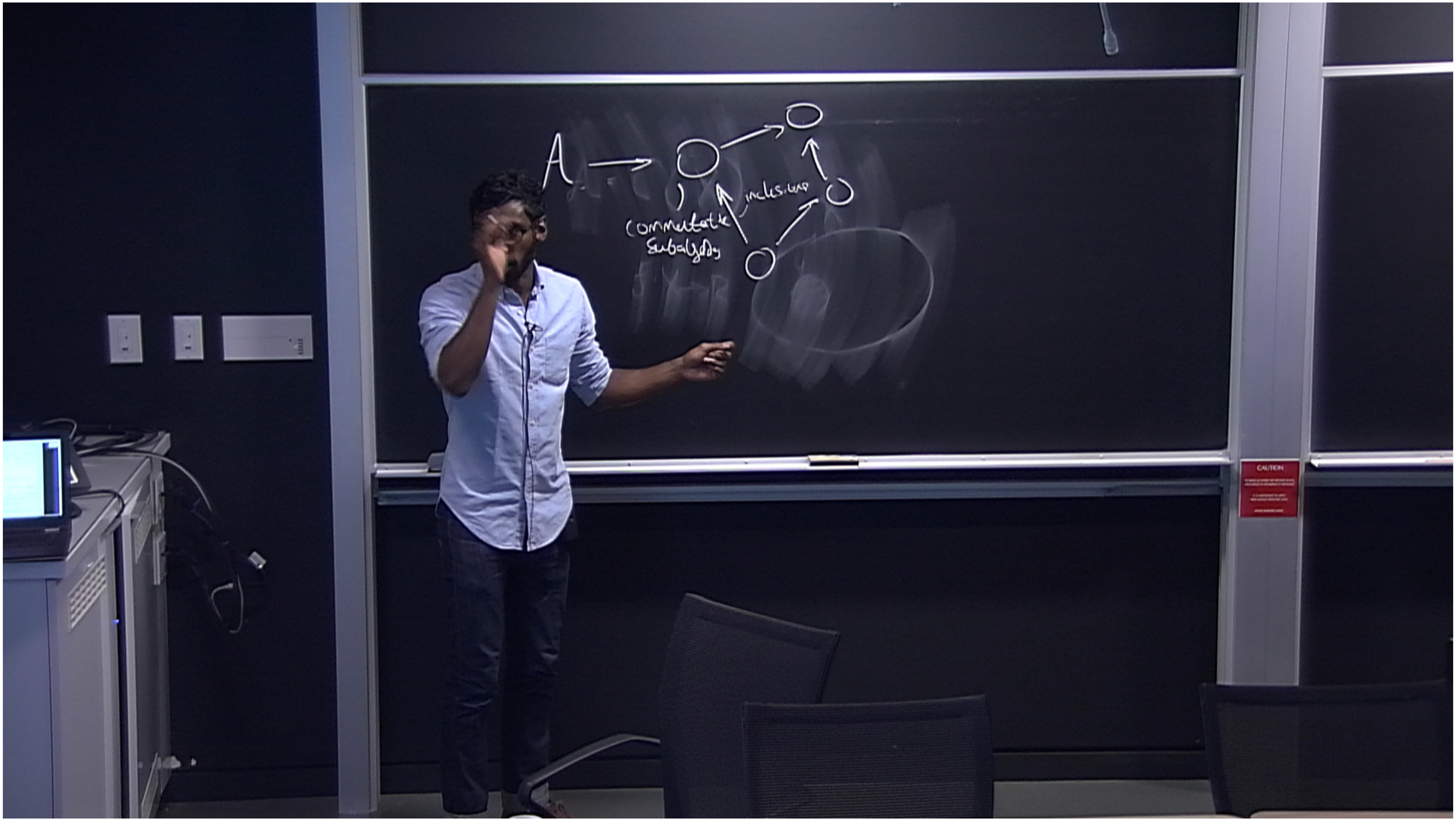
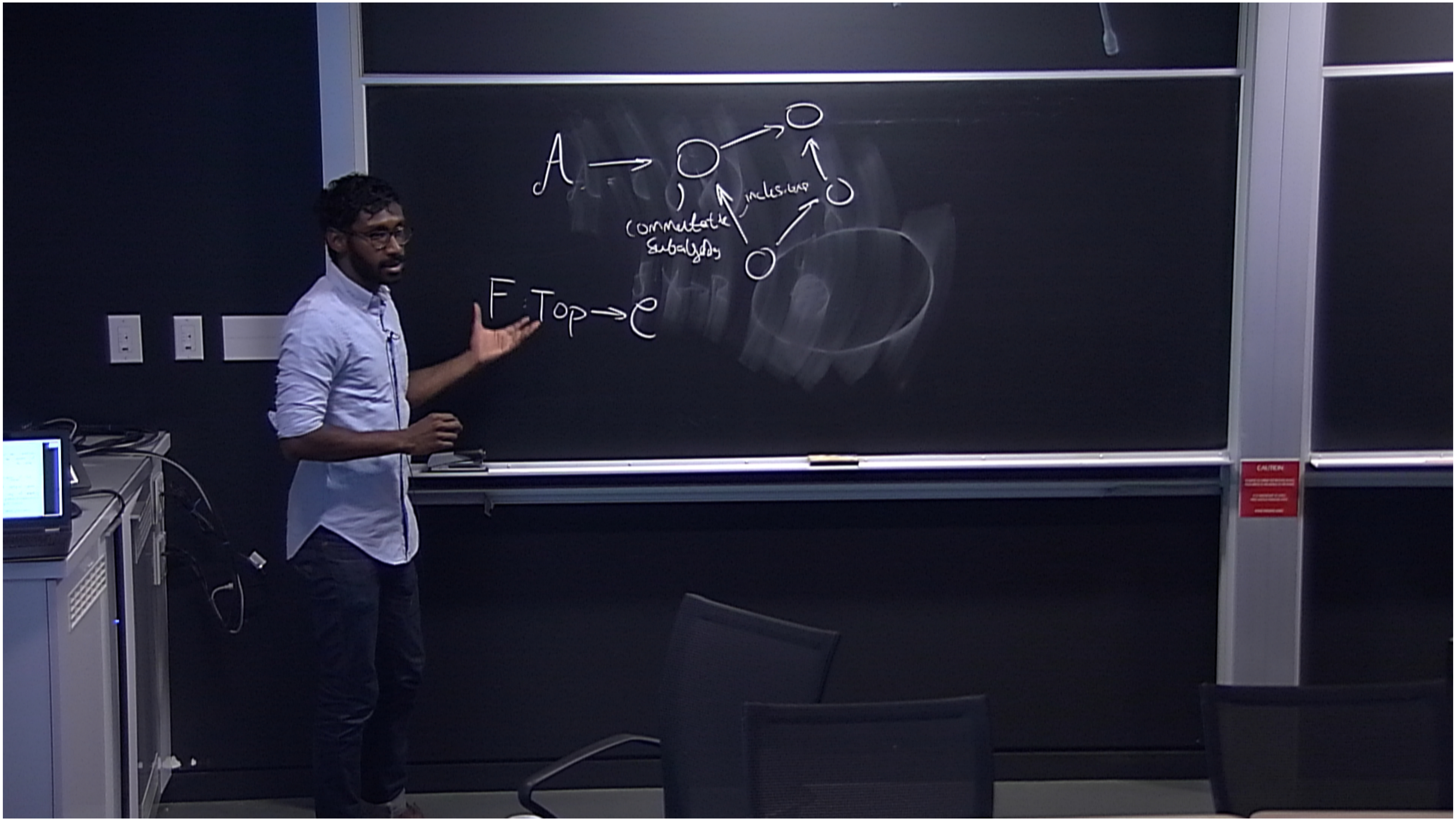
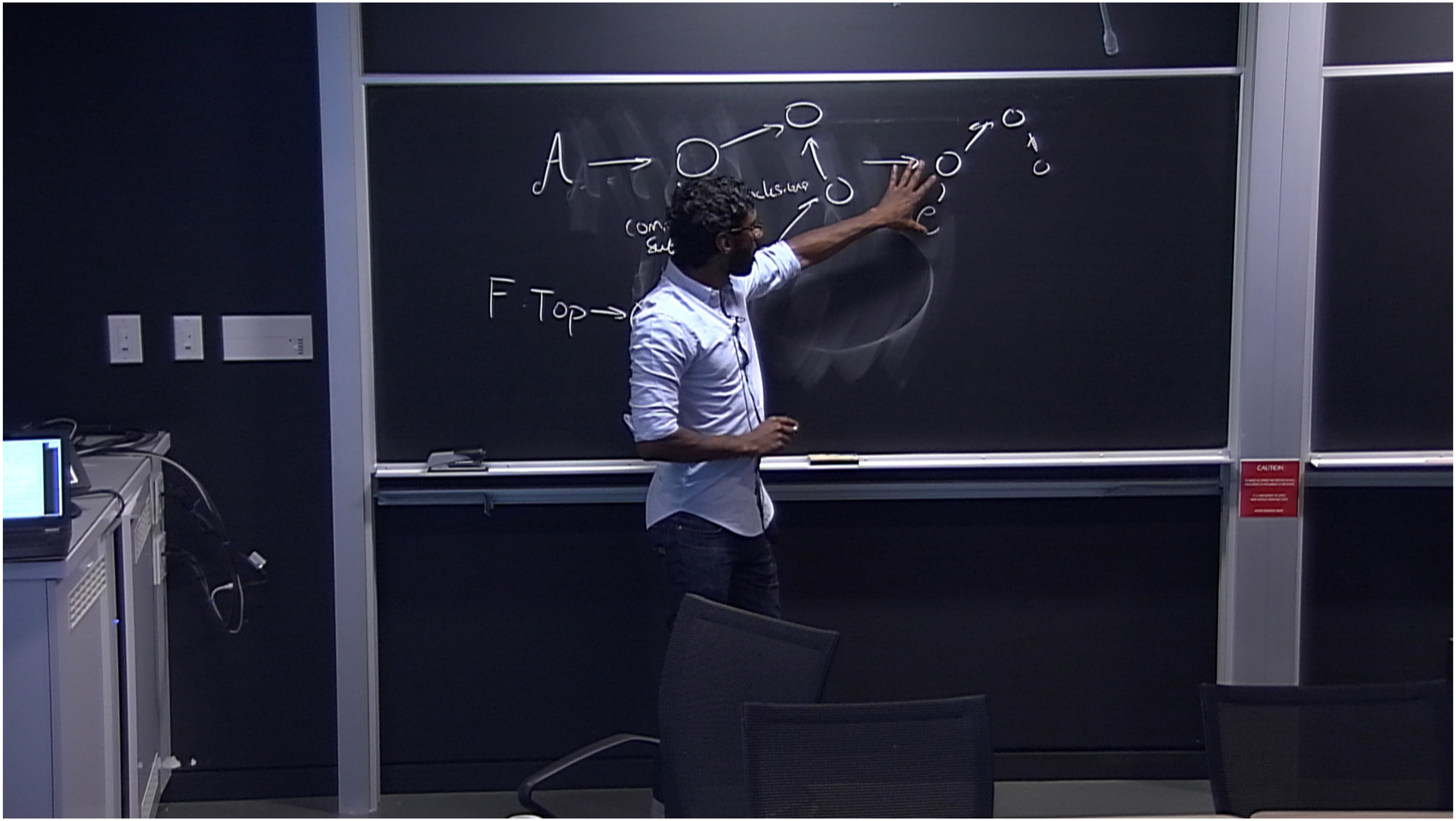
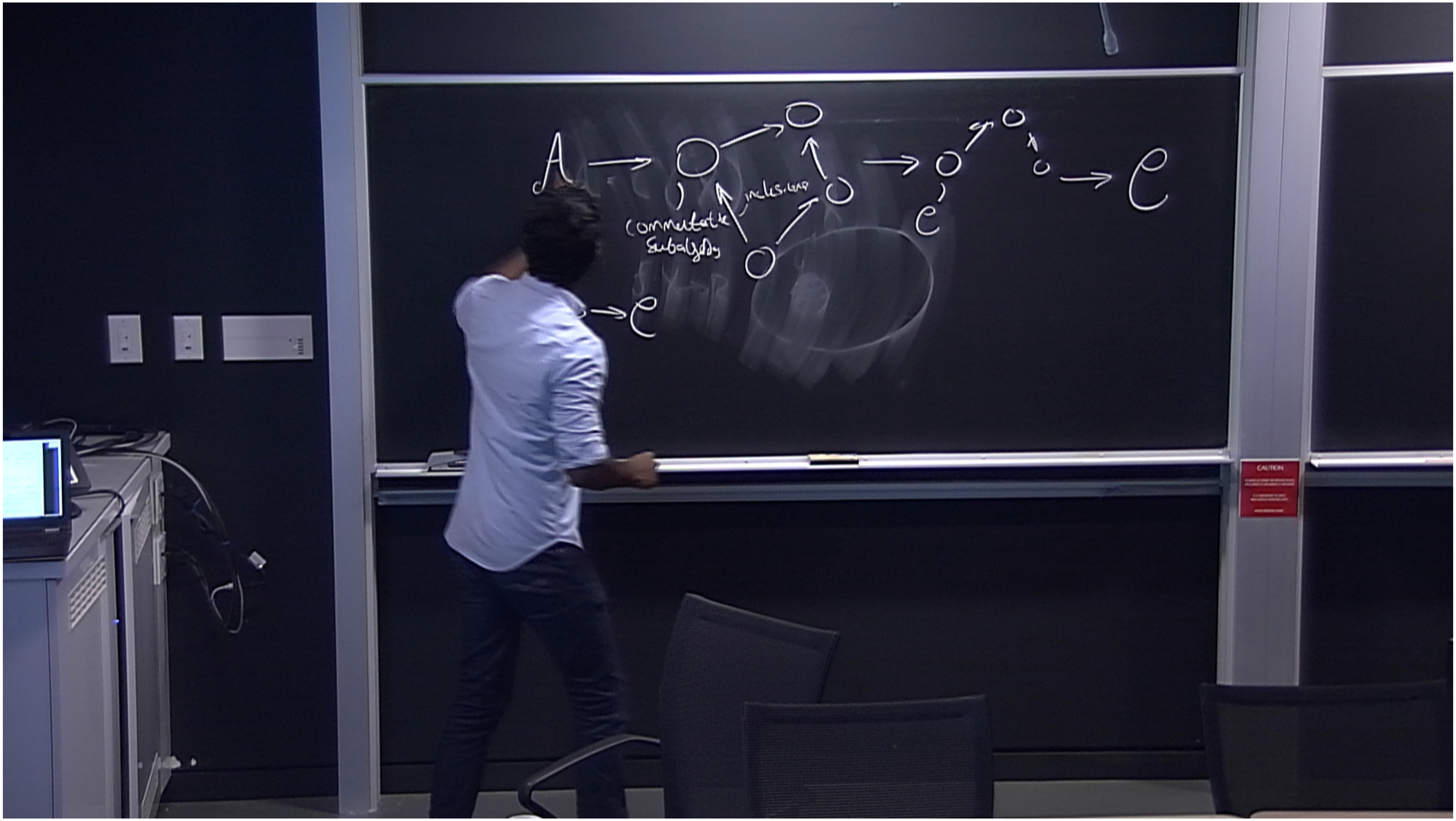

A
inclusion
commutative
Subalgebra



A
commutative
subalgebra
inclusion
F: Top → C

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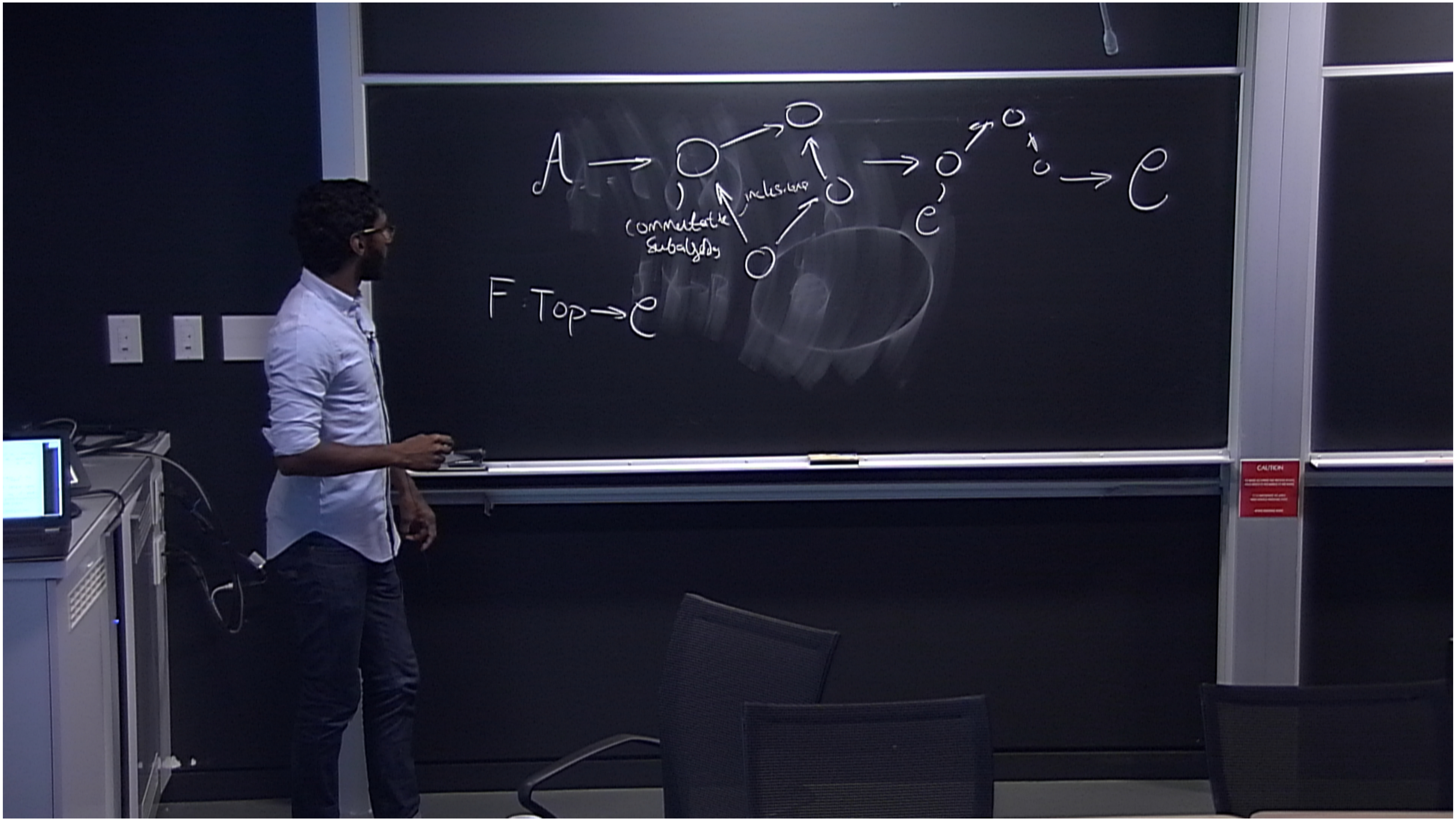
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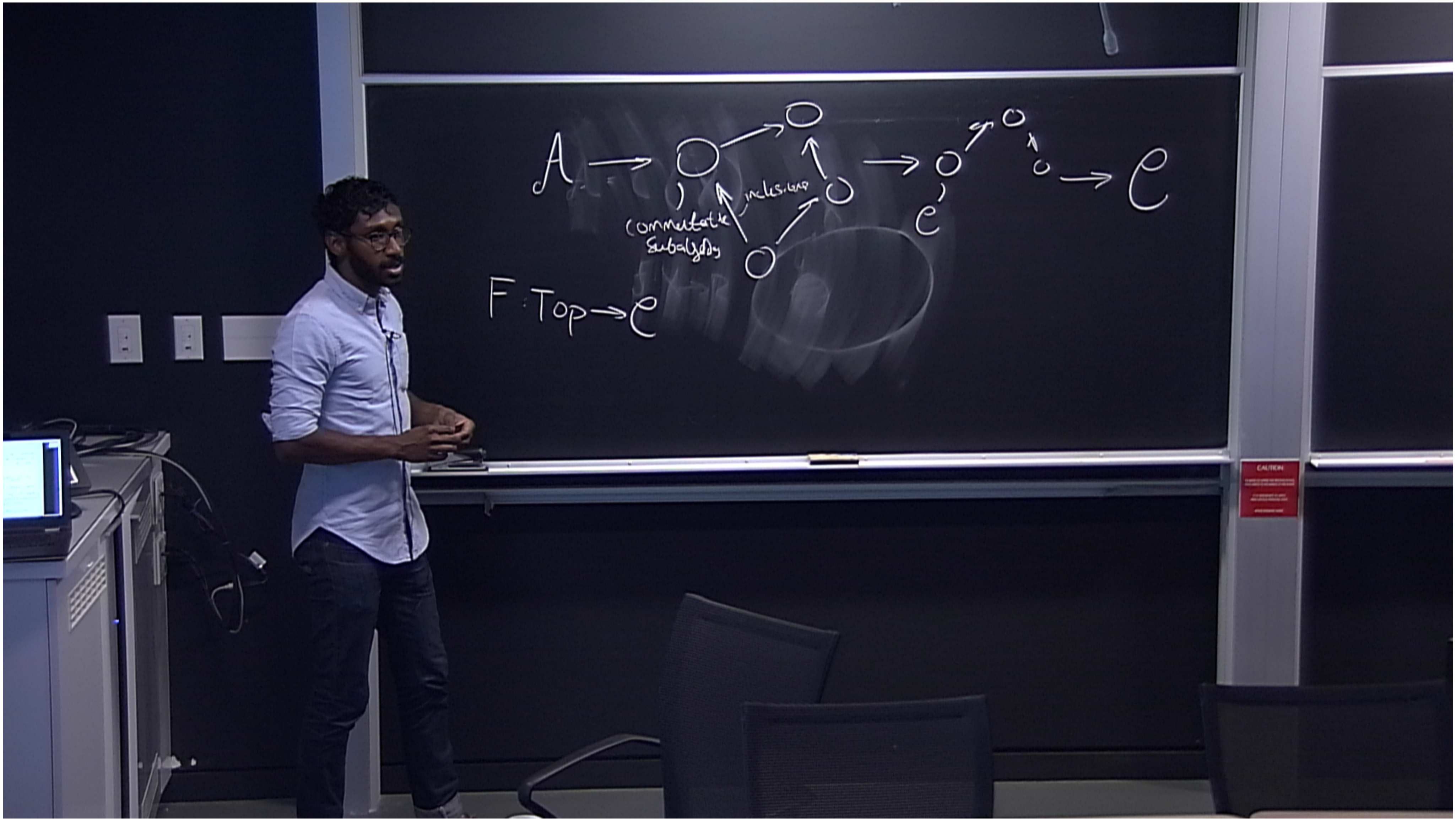

A
commutative subalgebra
inclusions
F : Top → C

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$Id : Top \longrightarrow Top$ i.e. state space

$\widetilde{Id} : vN \longrightarrow Top$

$$= \begin{cases} \Sigma & \text{commutative} \\ \phi & \text{non commutative } (M_2) \end{cases}$$

This is equivalent to the Kochen - Specker Theorem.

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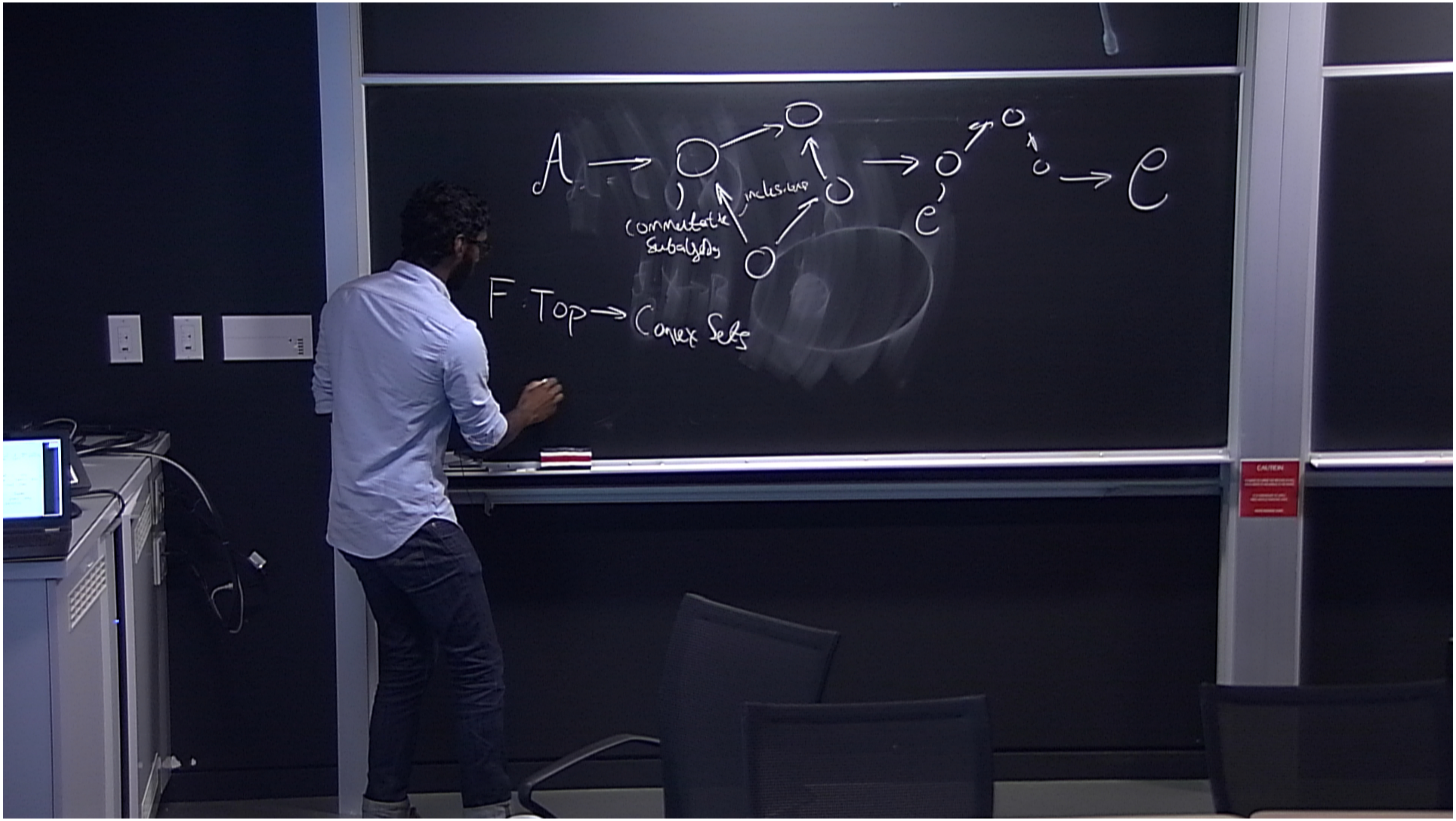
D : Top $\rightarrow$ Convex Sets

assigns to a space: the convex set of all probability distributions (gas)

$\tilde{D}$: vN $\rightarrow$ Convex Sets

assigns to an algebra what is more commonly known as state space (positive, linear functionals of norm 1), i.e. mixed states.

This is equivalent to Gleason's Theorem and gives an interesting picture of quantum states as consistent families of classical distributions.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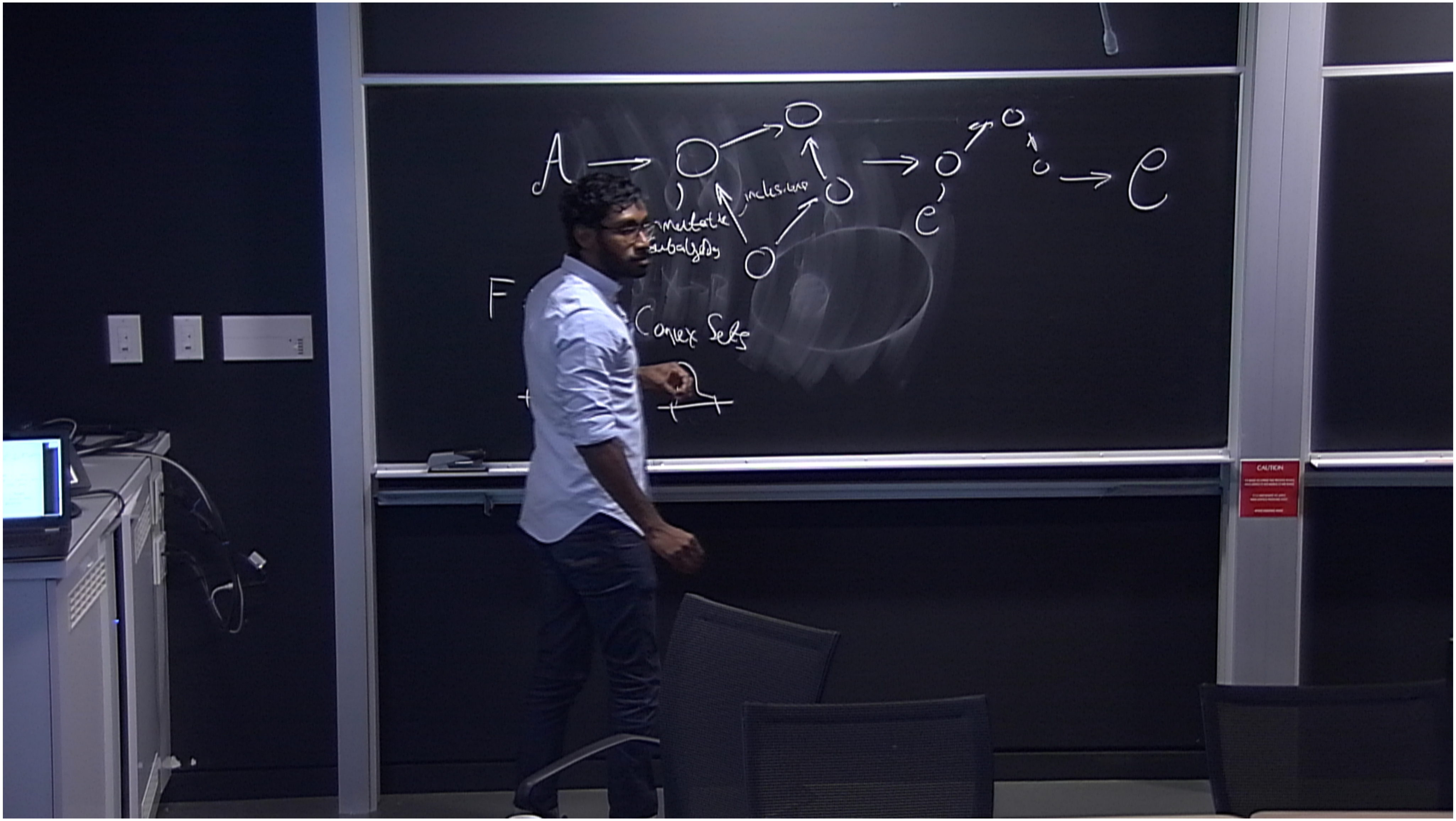
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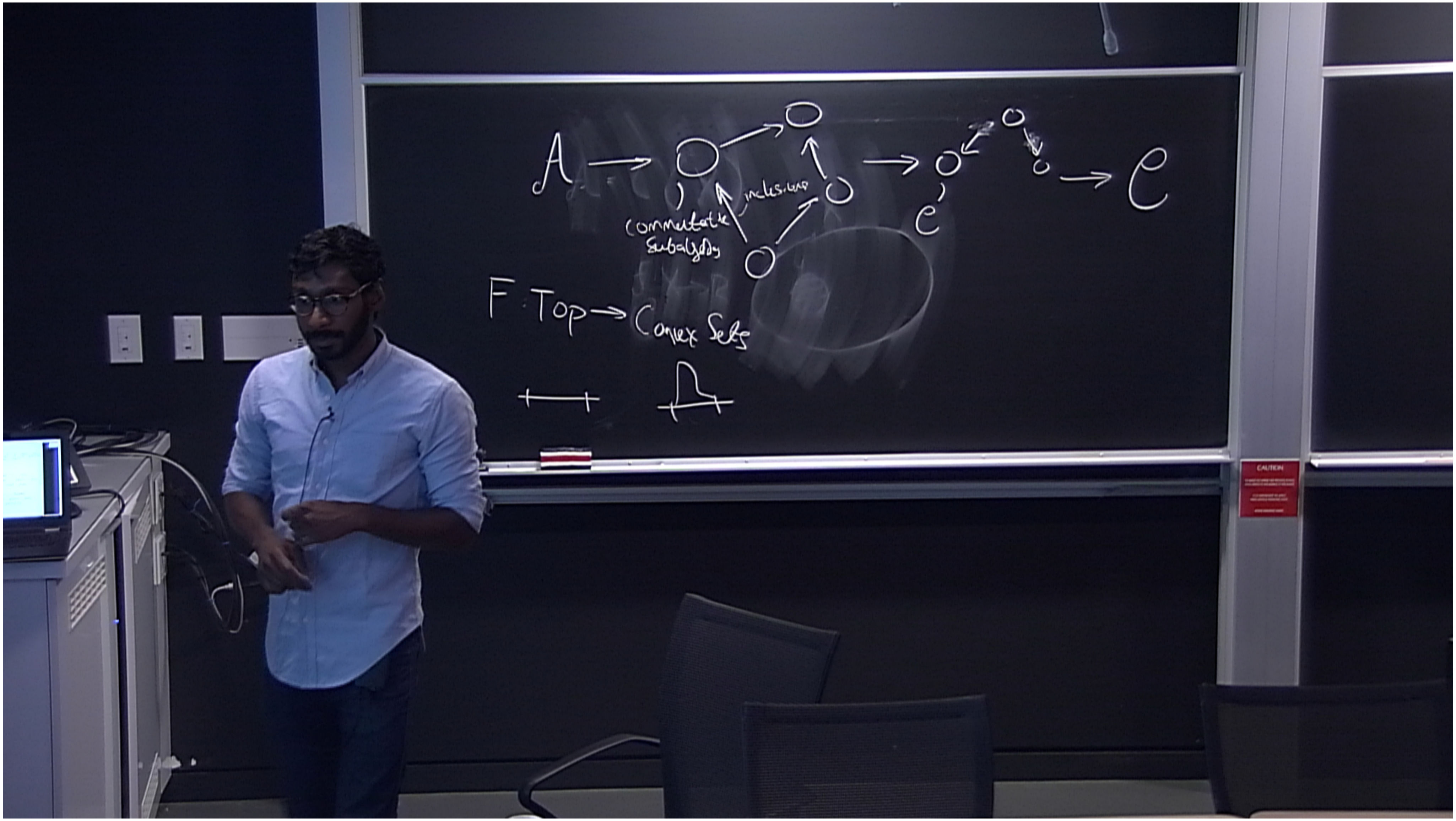
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FOR EMPHASIS

In the case of quantum mechanics, empirical models are states

(any vN algebra)

Borel probability measure at each context which agree on marginalization

↕ mathematical correspondence — Gleason

quantum states (d.m. / plf of norm 1)

This justifies identifying the notion of 'state' with empirical models in more general contextual theories

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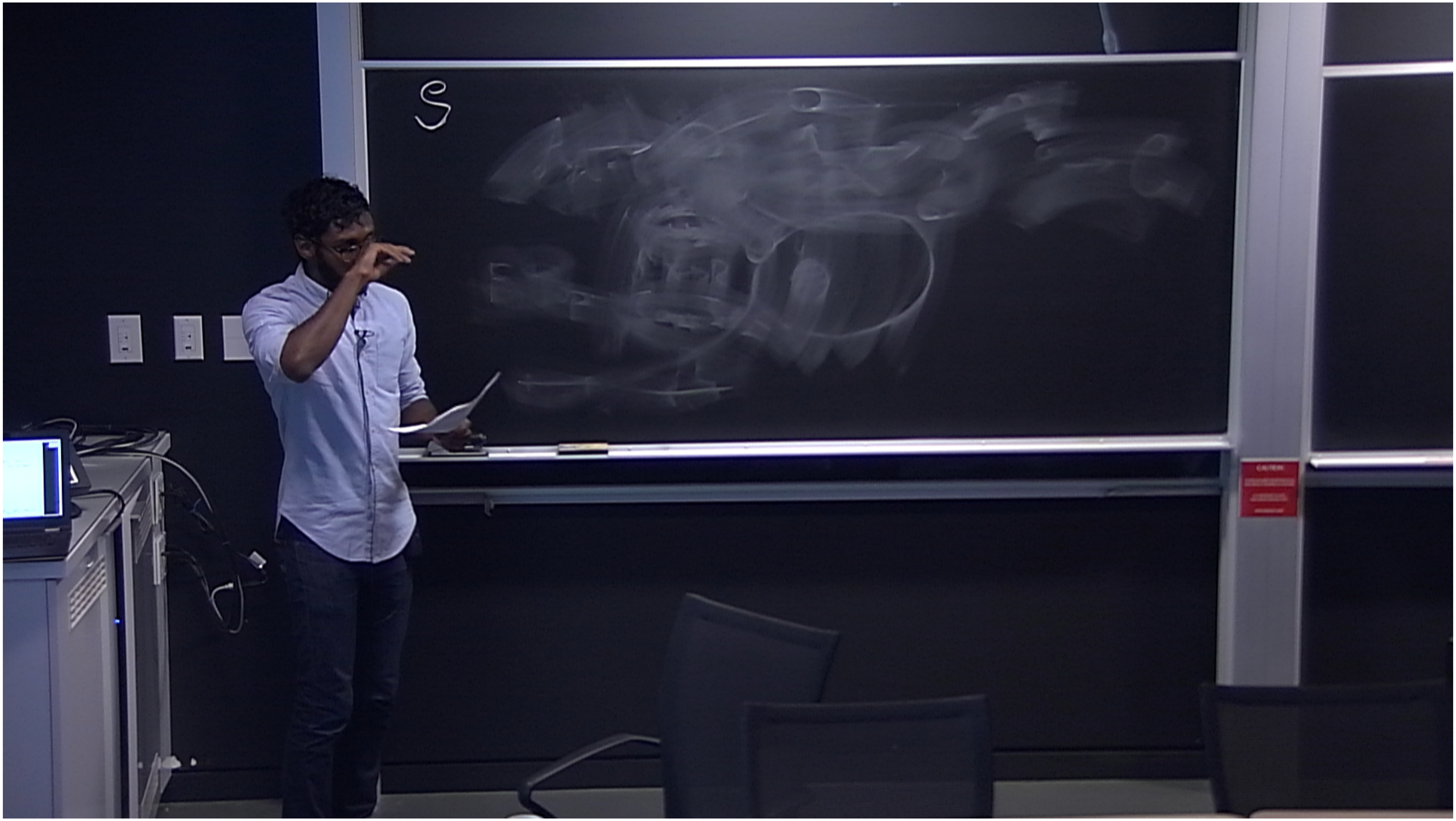
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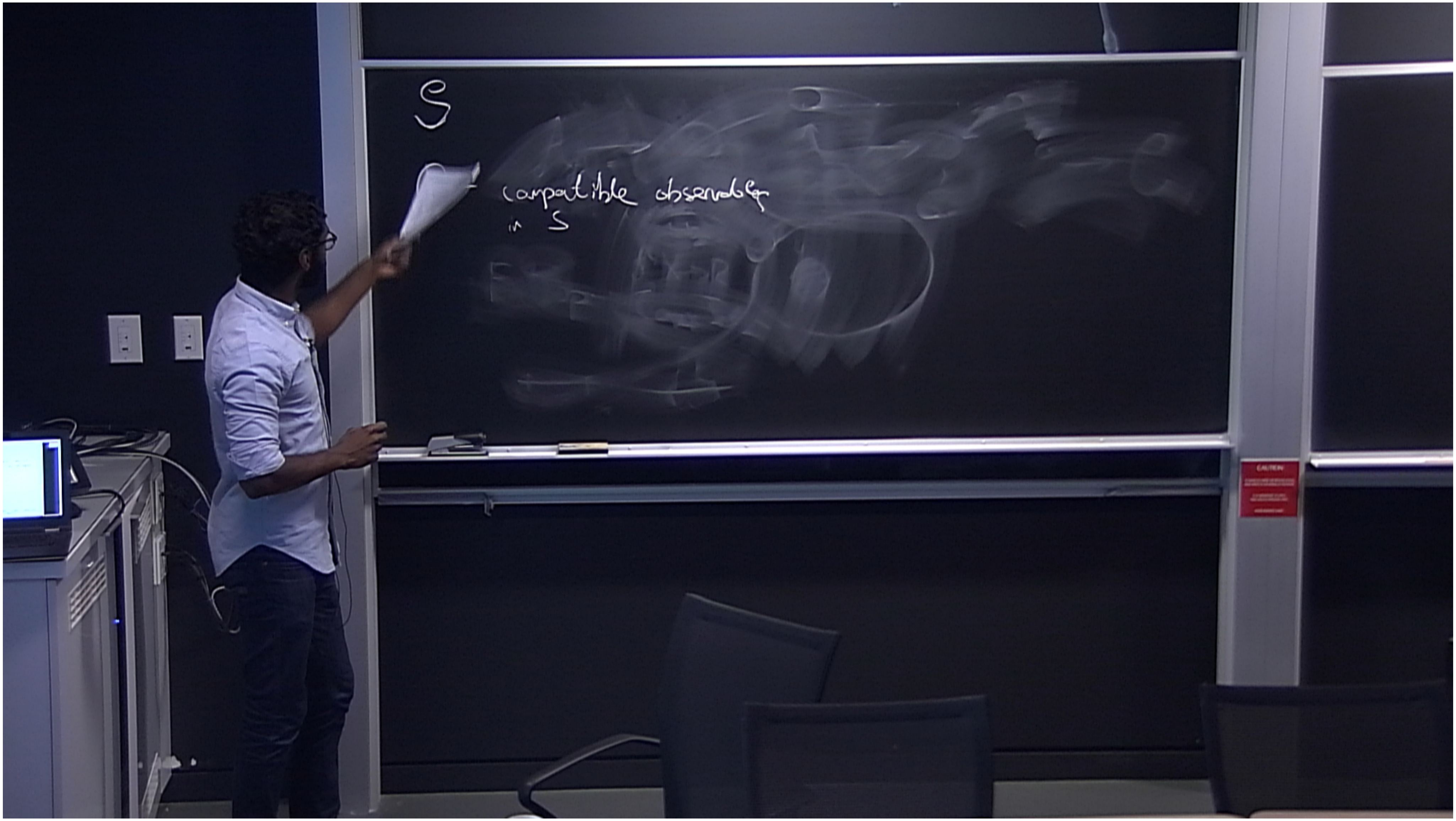
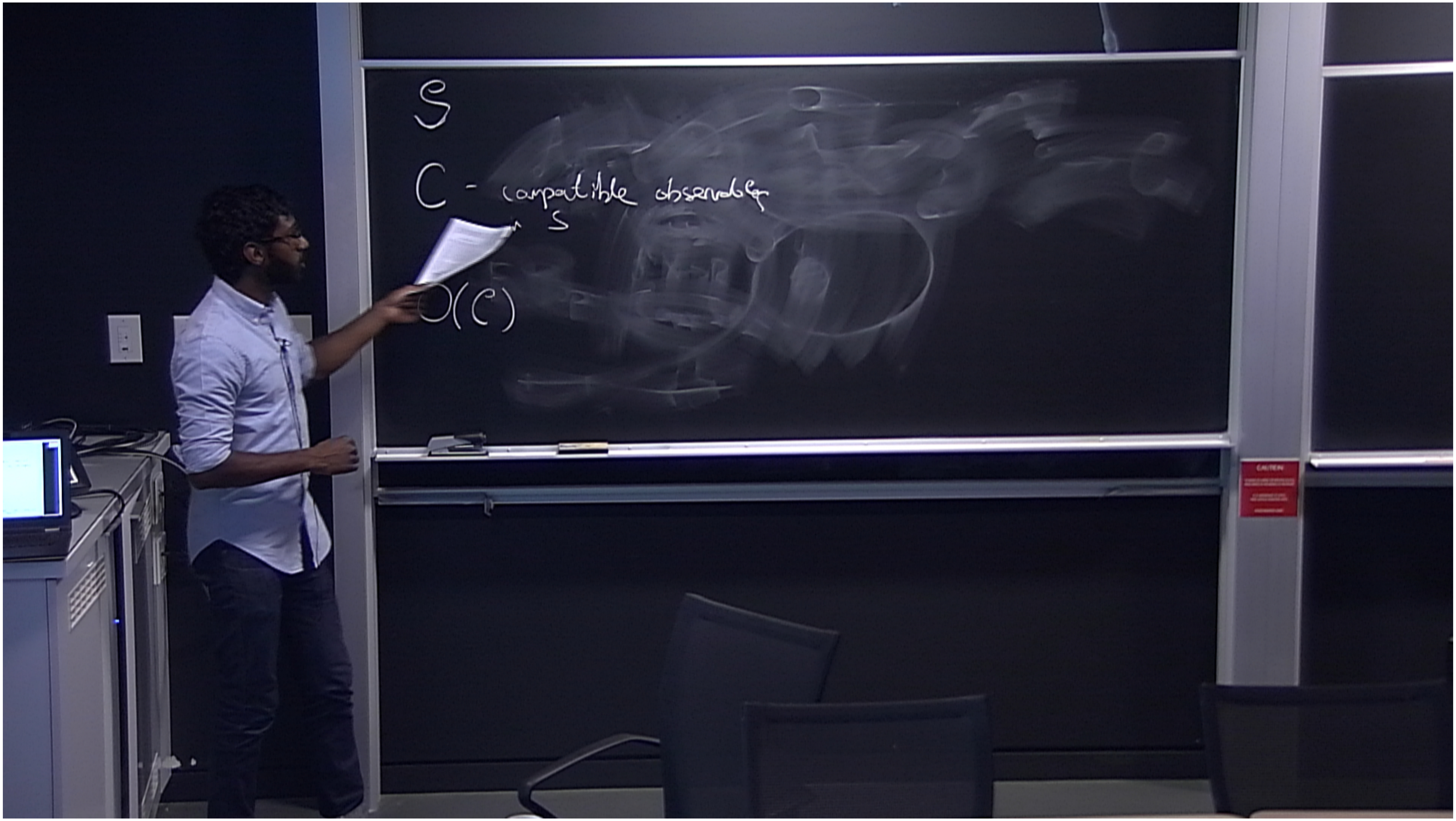
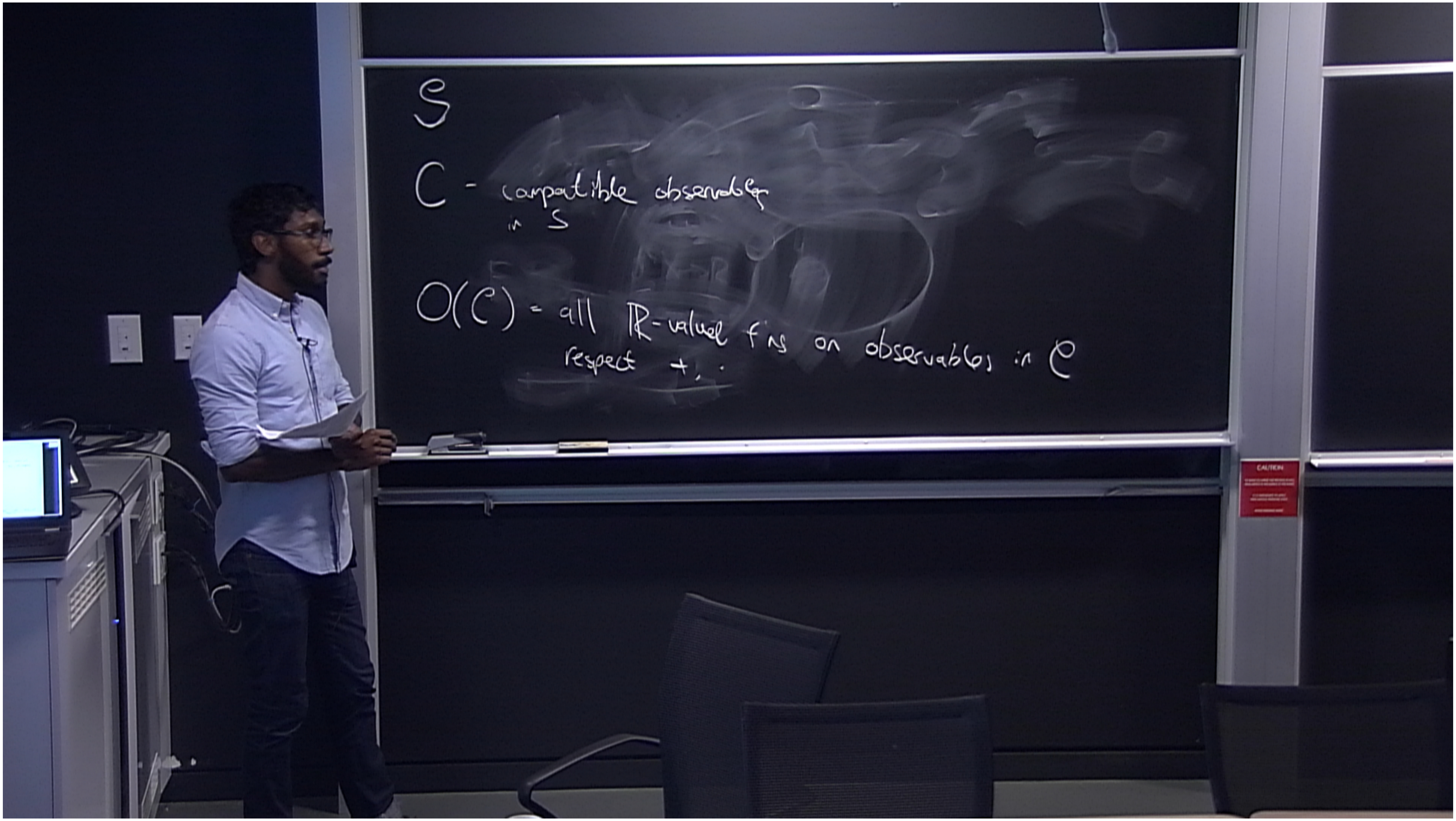
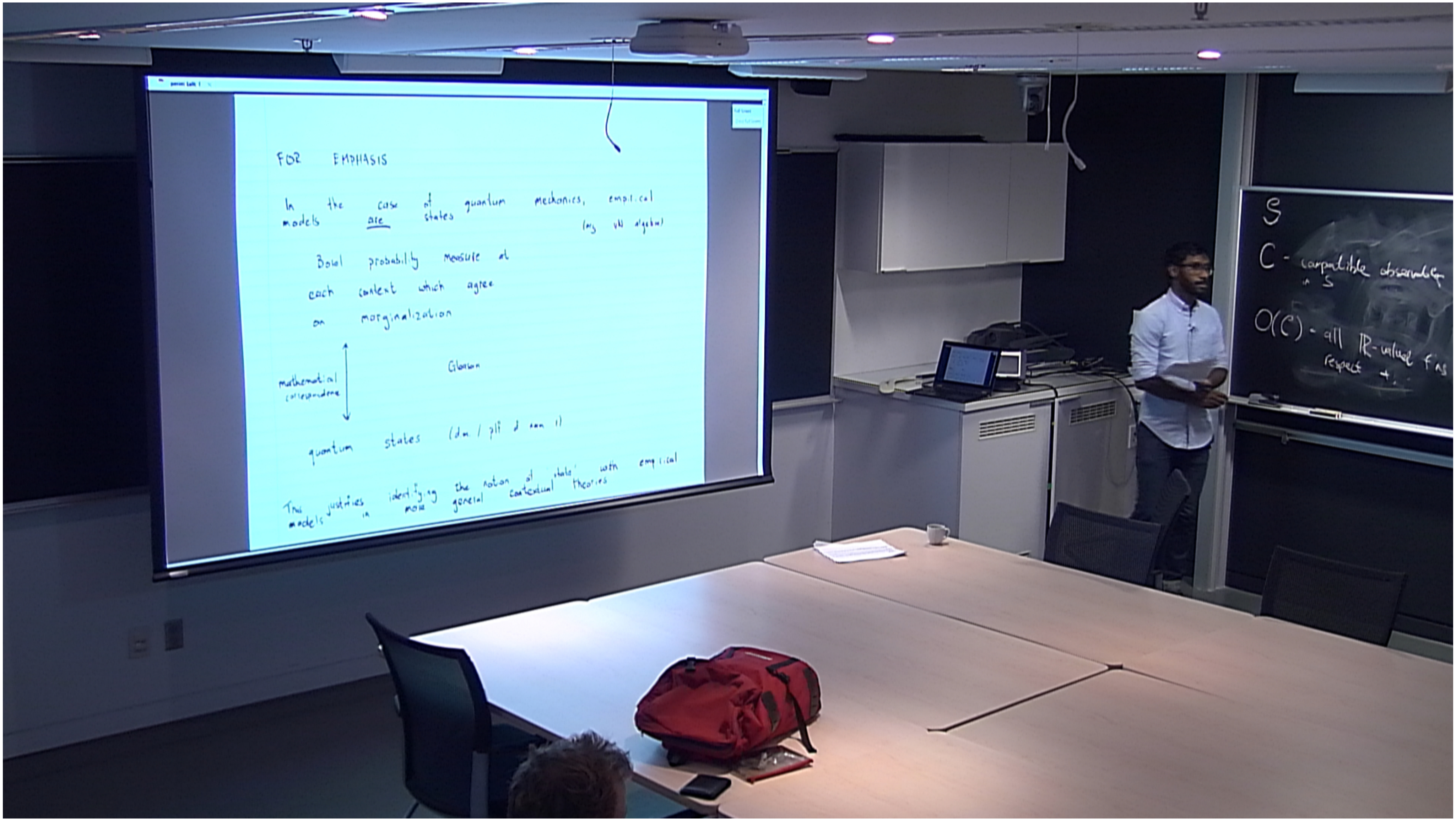

S
compatible observables
in S


S
C - compatible observables
S
O(C)


S
C - compatible observables in S
O(C) = all ℝ-valued f'ns on observables in C
respect +, ·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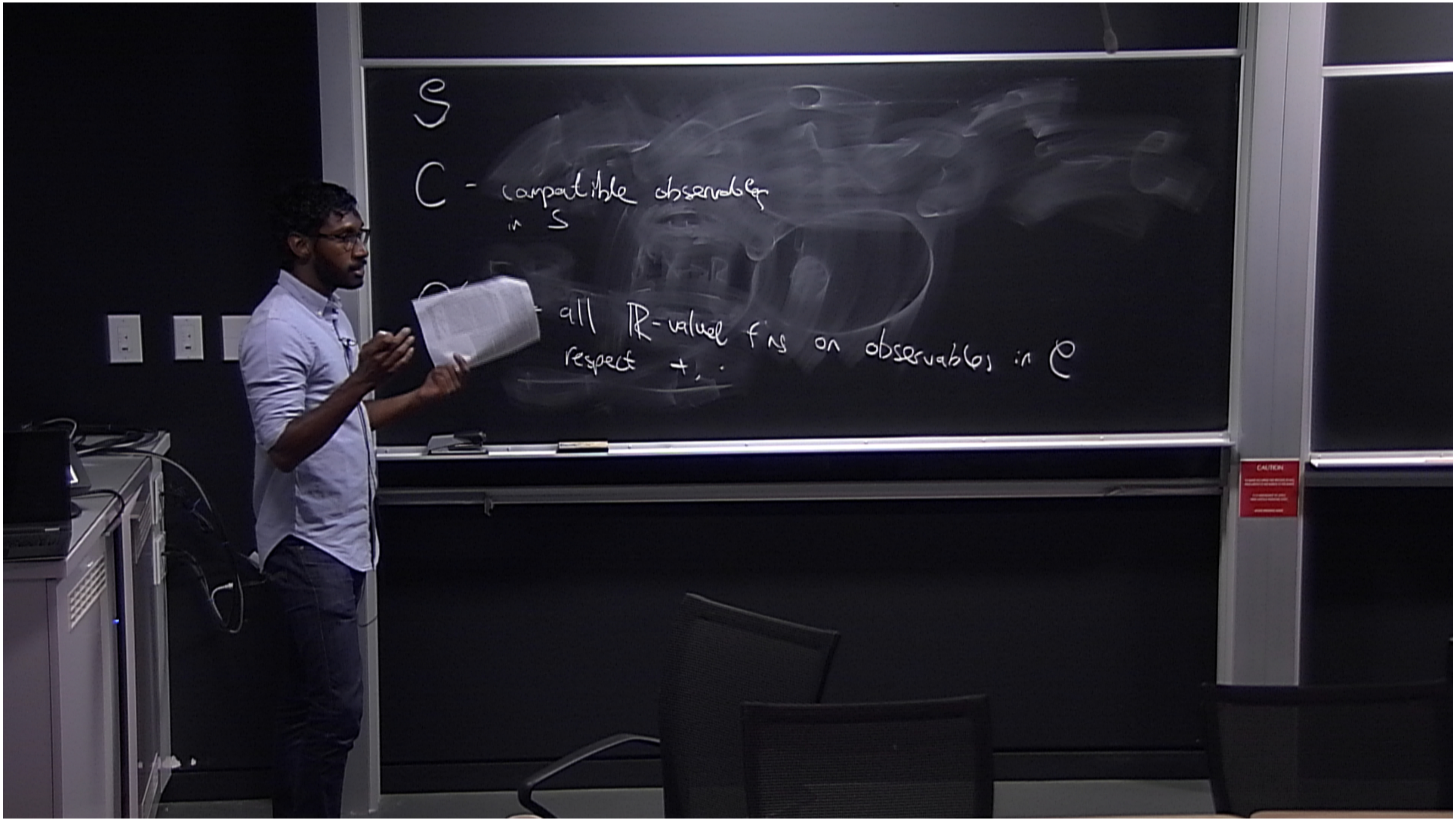
mathematical correspondence
Gleason
quantum states
This justifies identifying the notion of 'state' with empirical
models in more general contextual theories
C - compatible observables
O(C) - all ℝ-valued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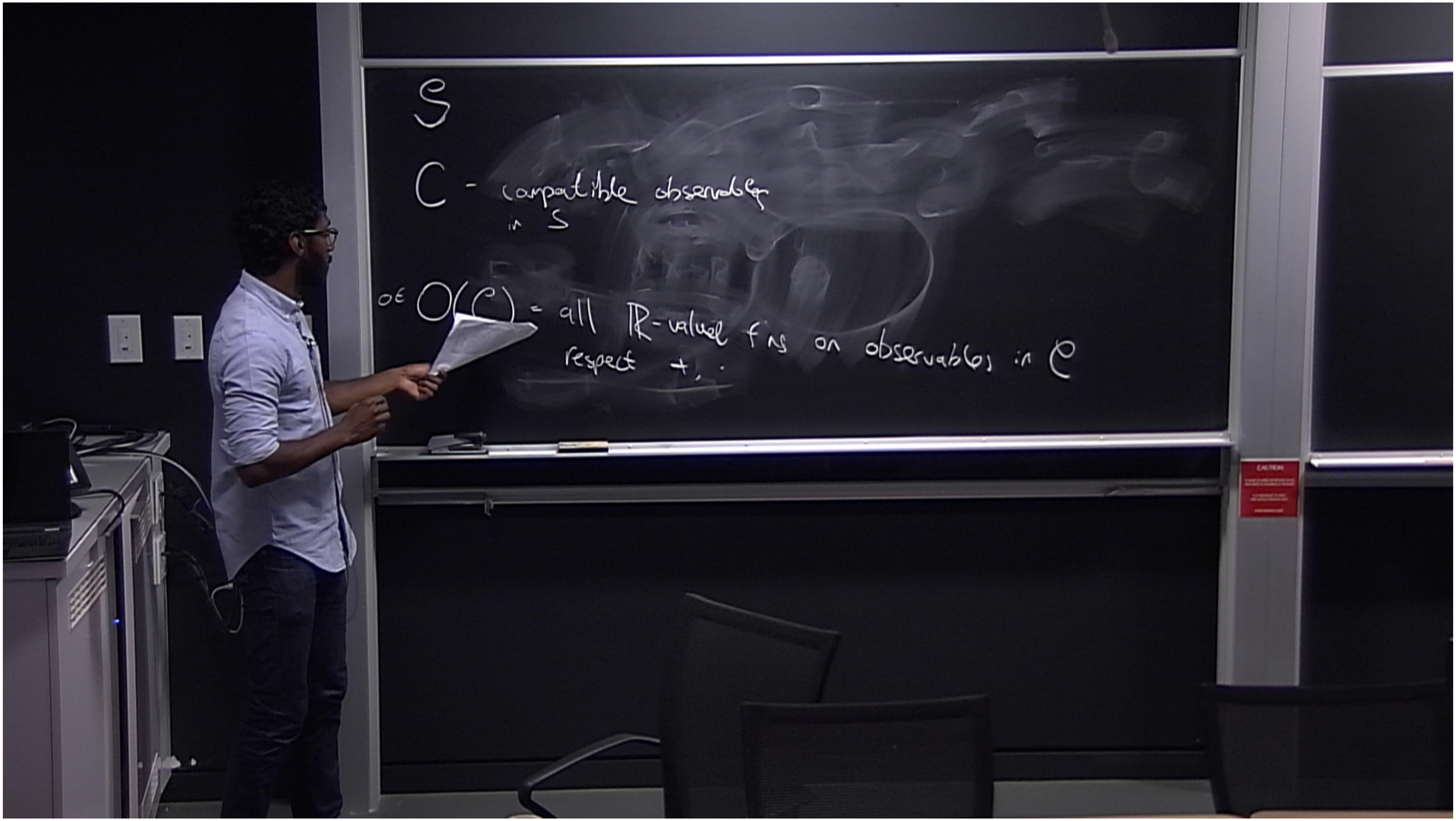
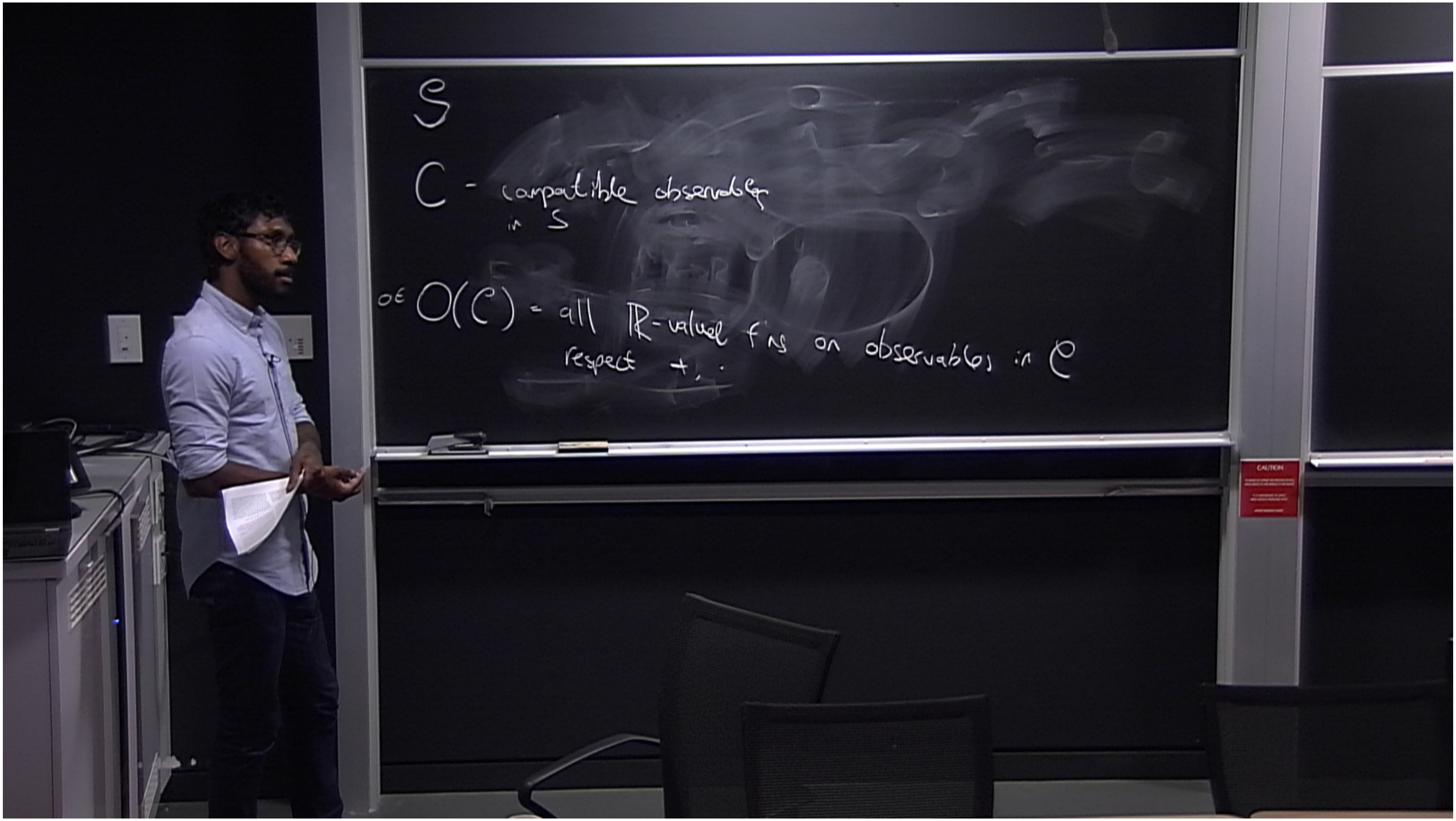
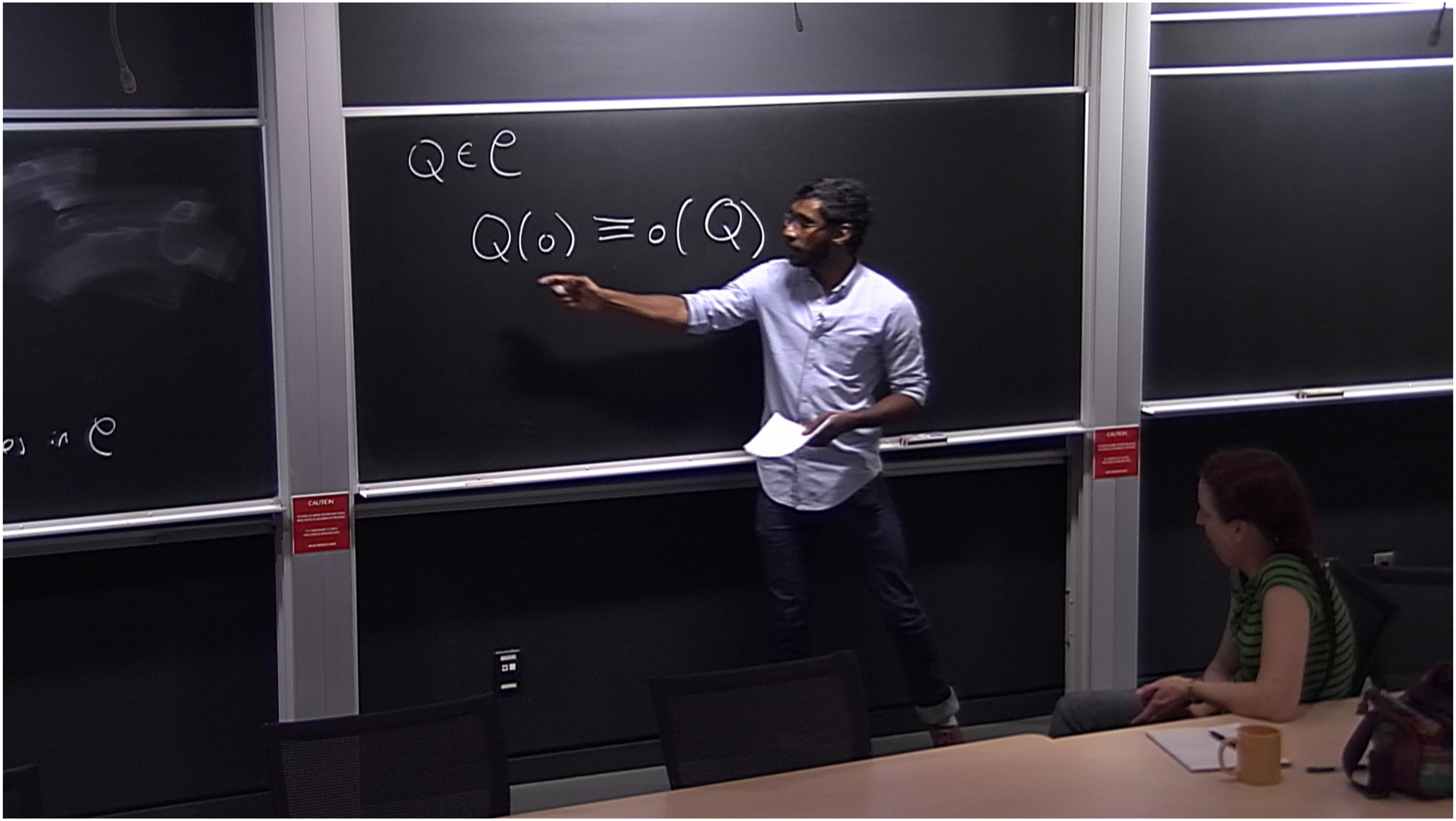
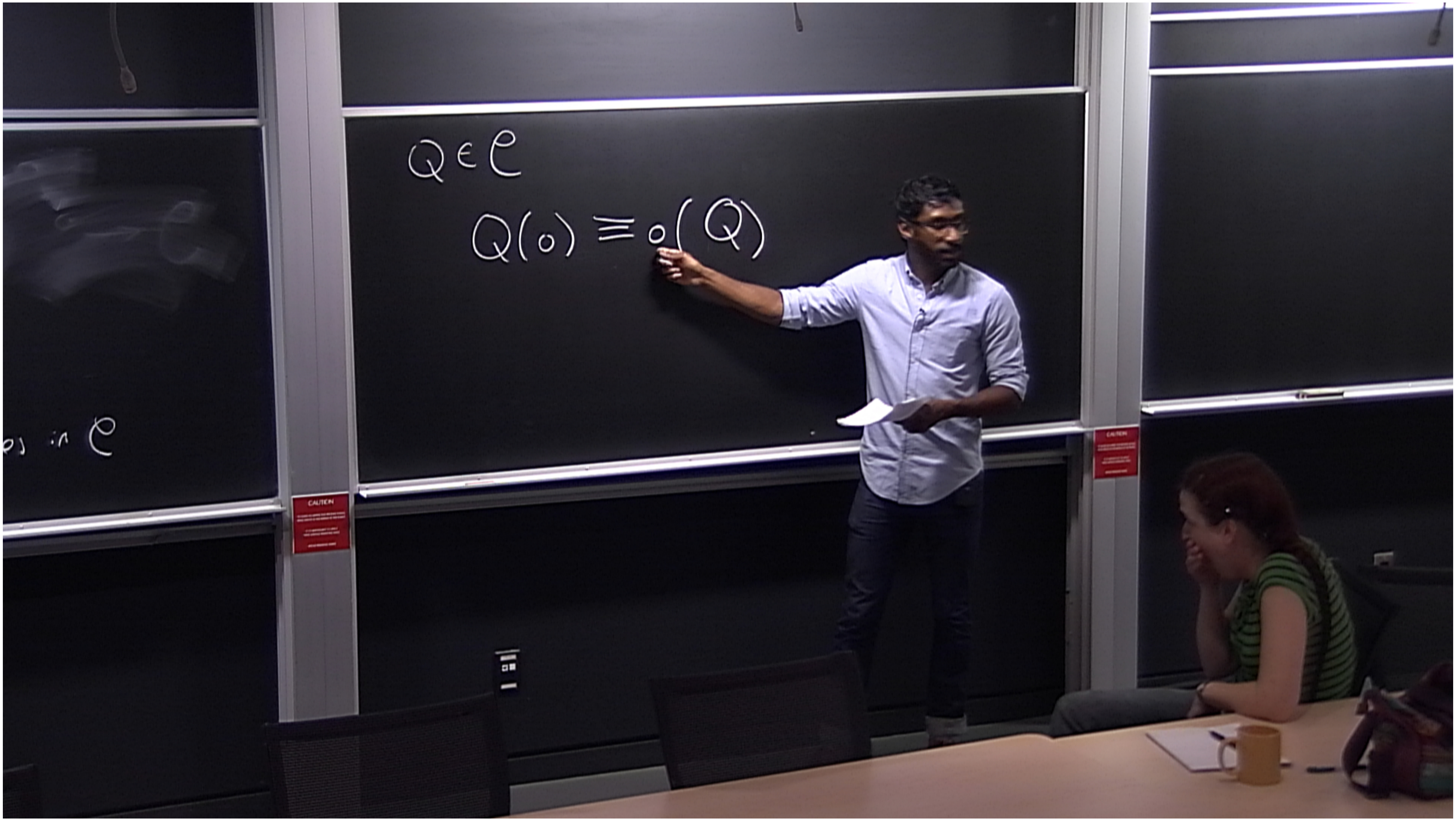
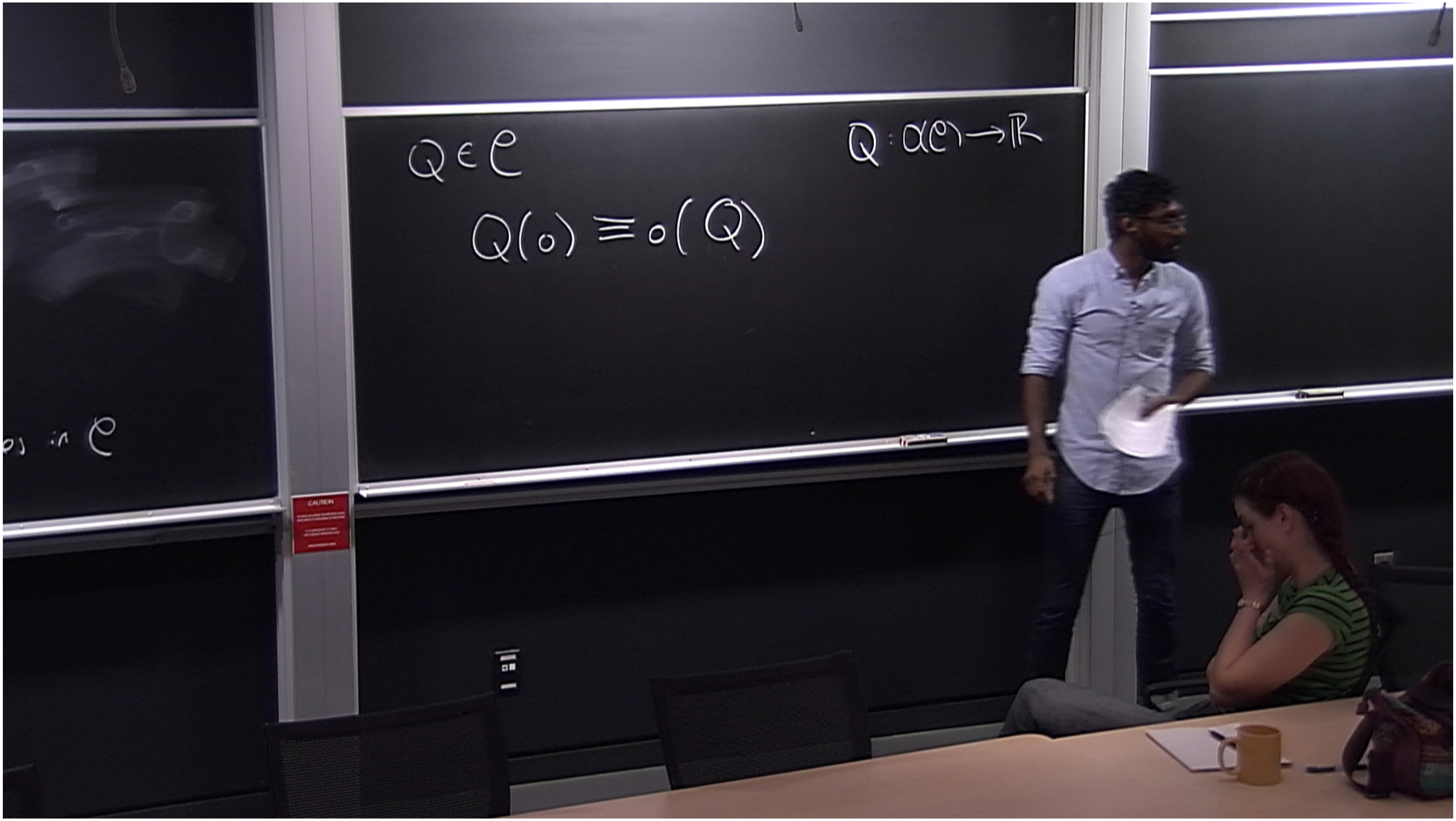
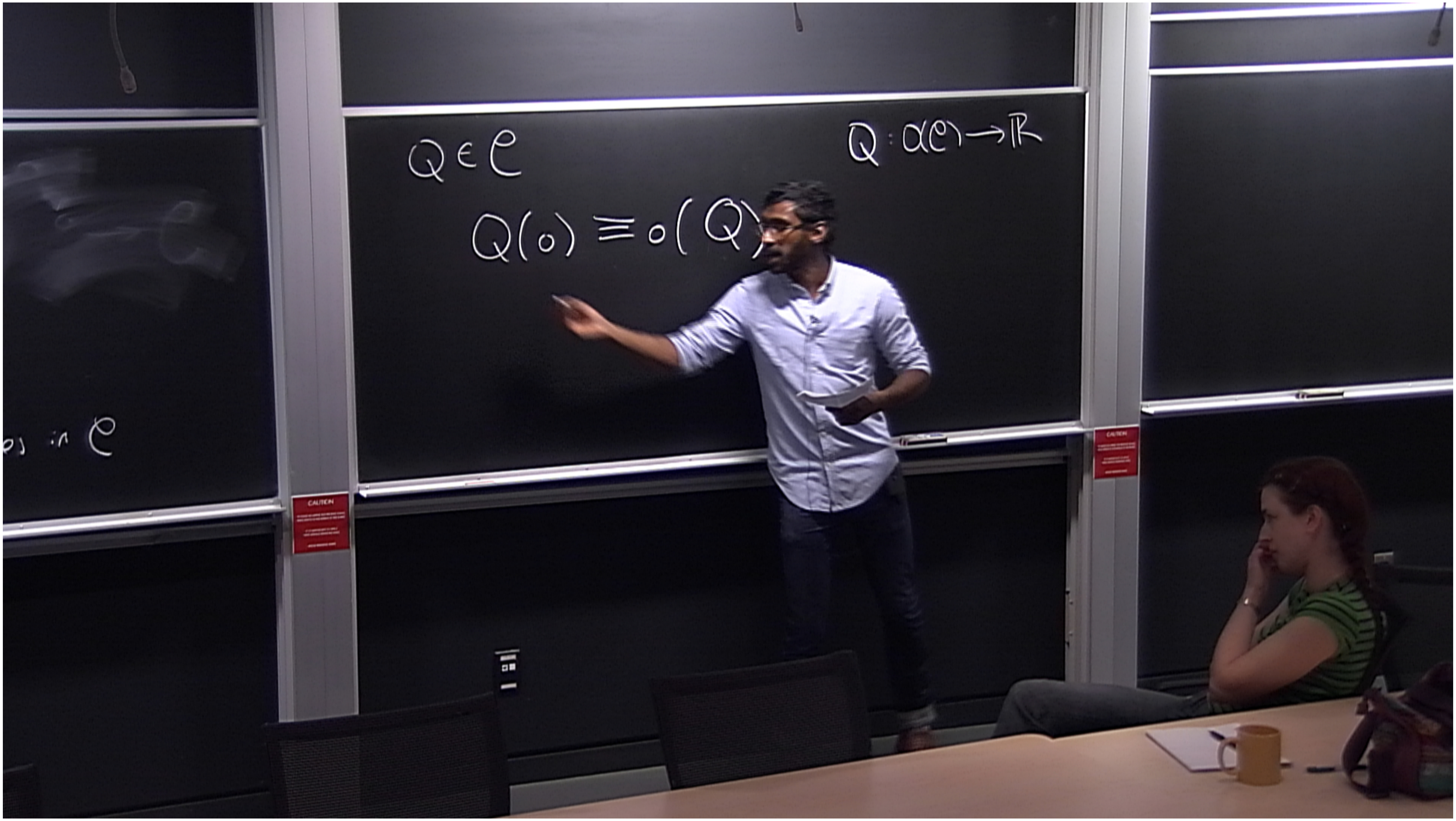

S
C - compatible observables in S
o∈ O(C) = all ℝ-valued fns on observables in C
respect +, ·


S
C - compatible observables
in S
o∈ O(C) = all R-valued fns on observables in C
respect +, ·
CAUTION

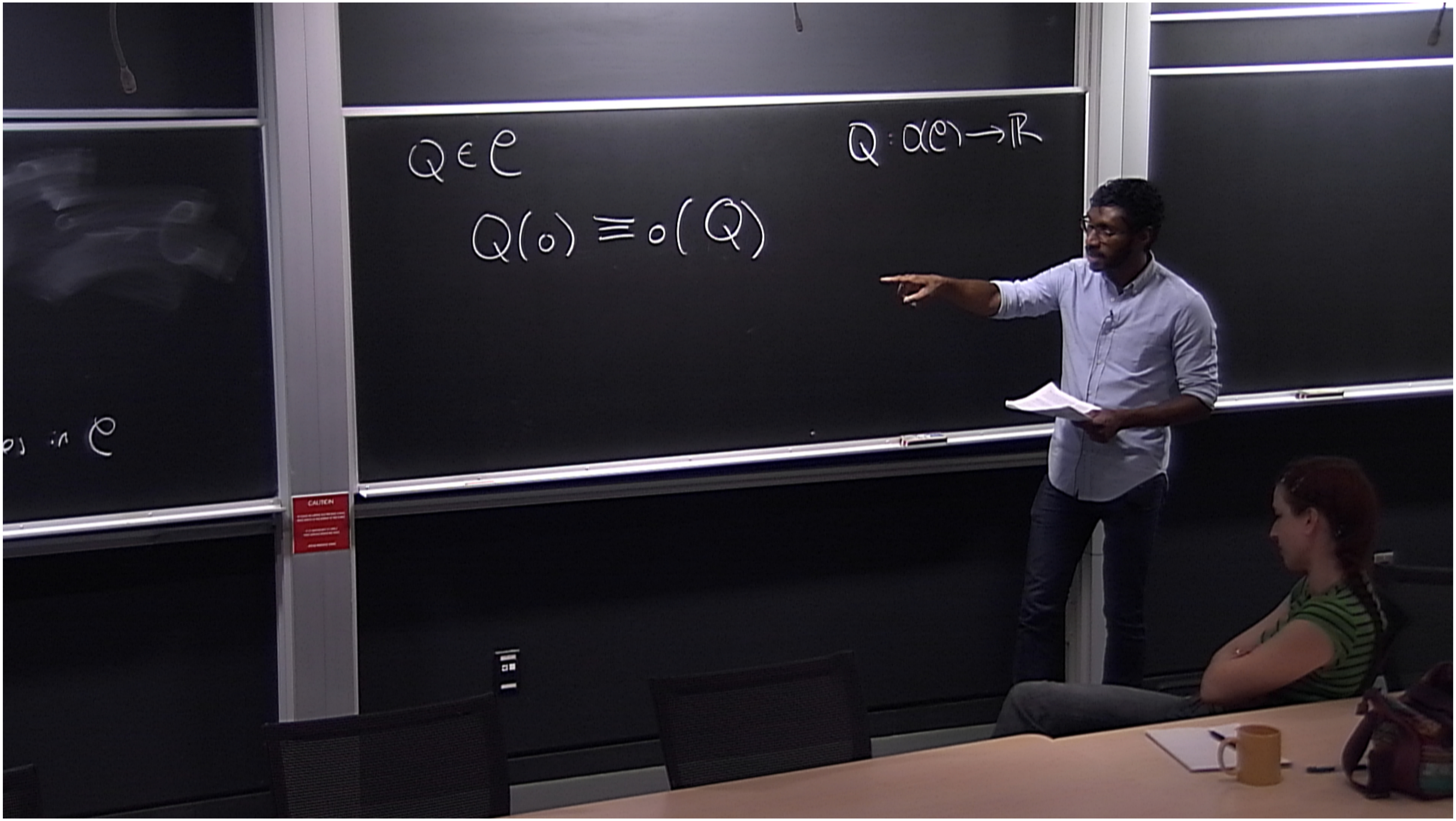

Q ∈ C
Q(o) ≡ o(Q)

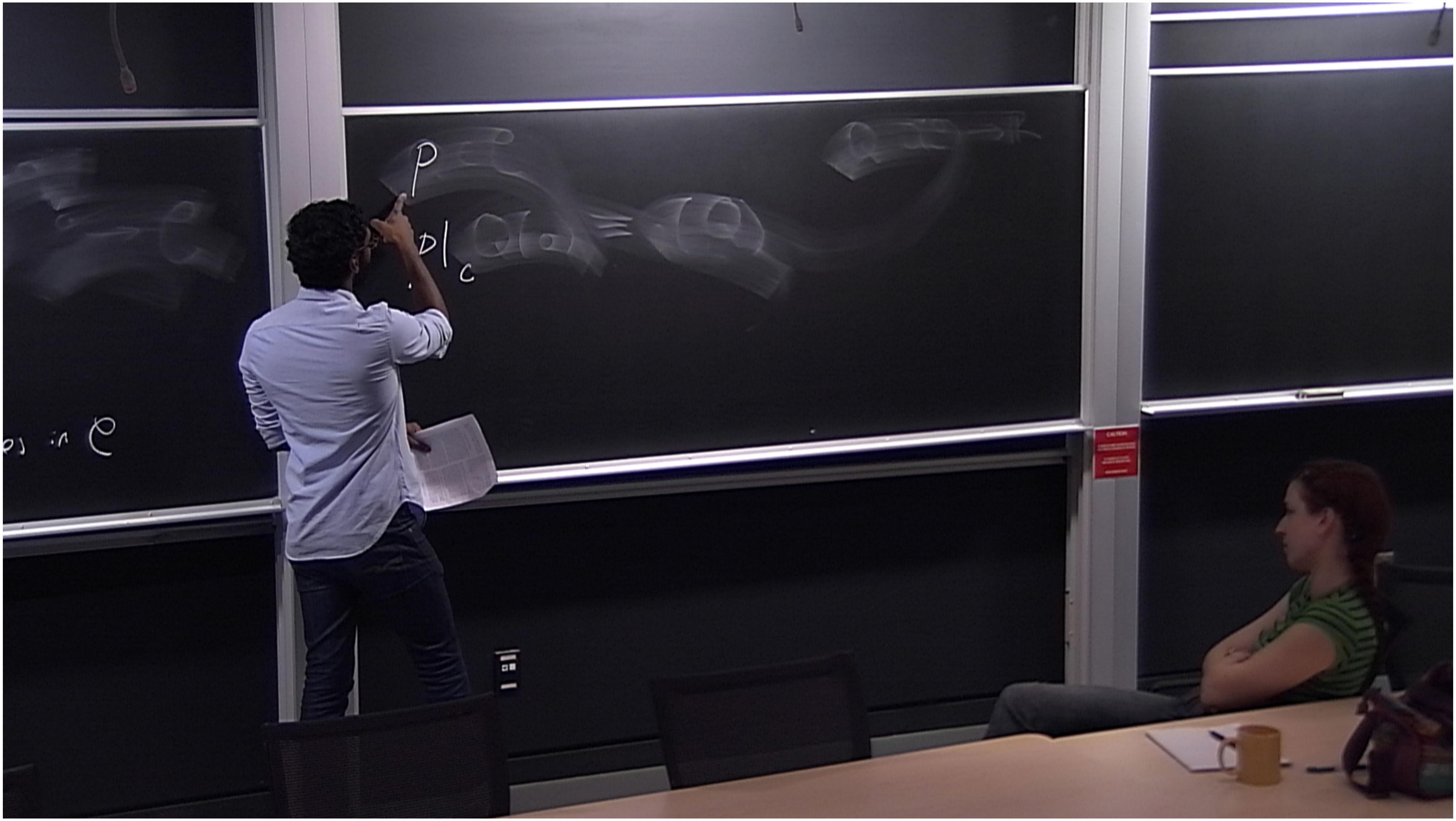

Q ∈ C
Q(0) ≡ 0(Q)

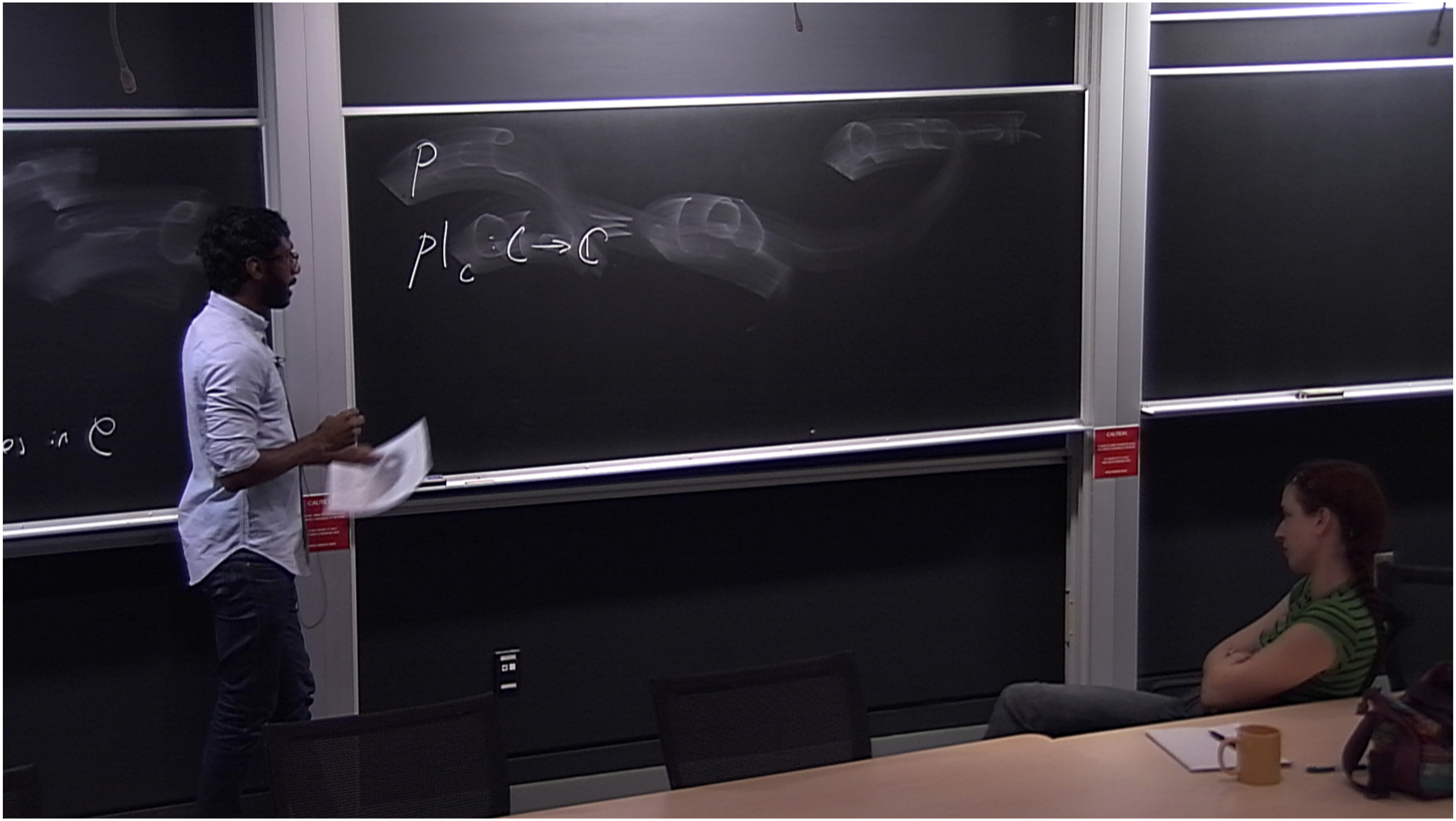

Q ∈ C
Q : α(C) → ℝ
Q(o) ≡ o(Q)
in C
CAUTION

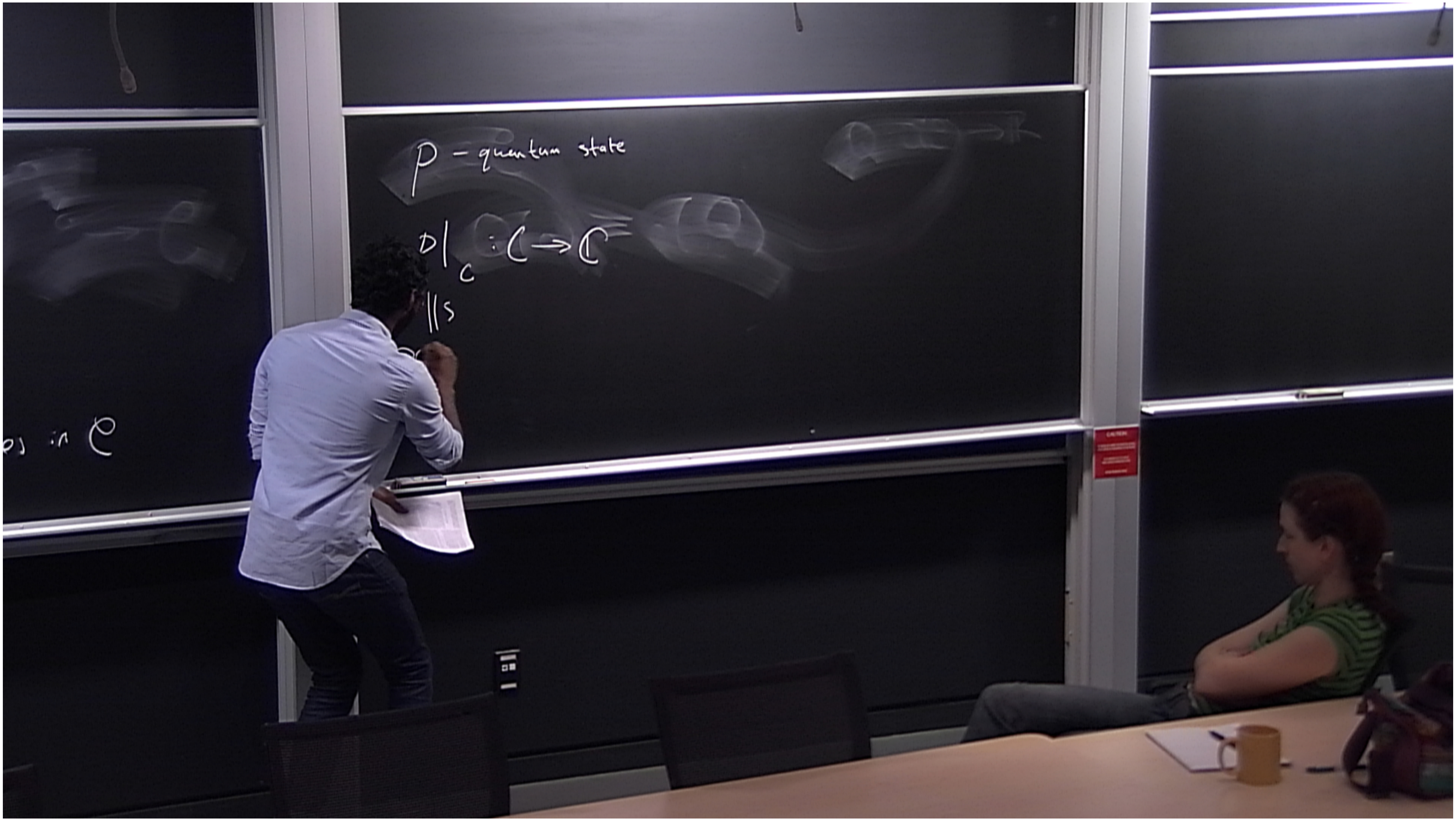


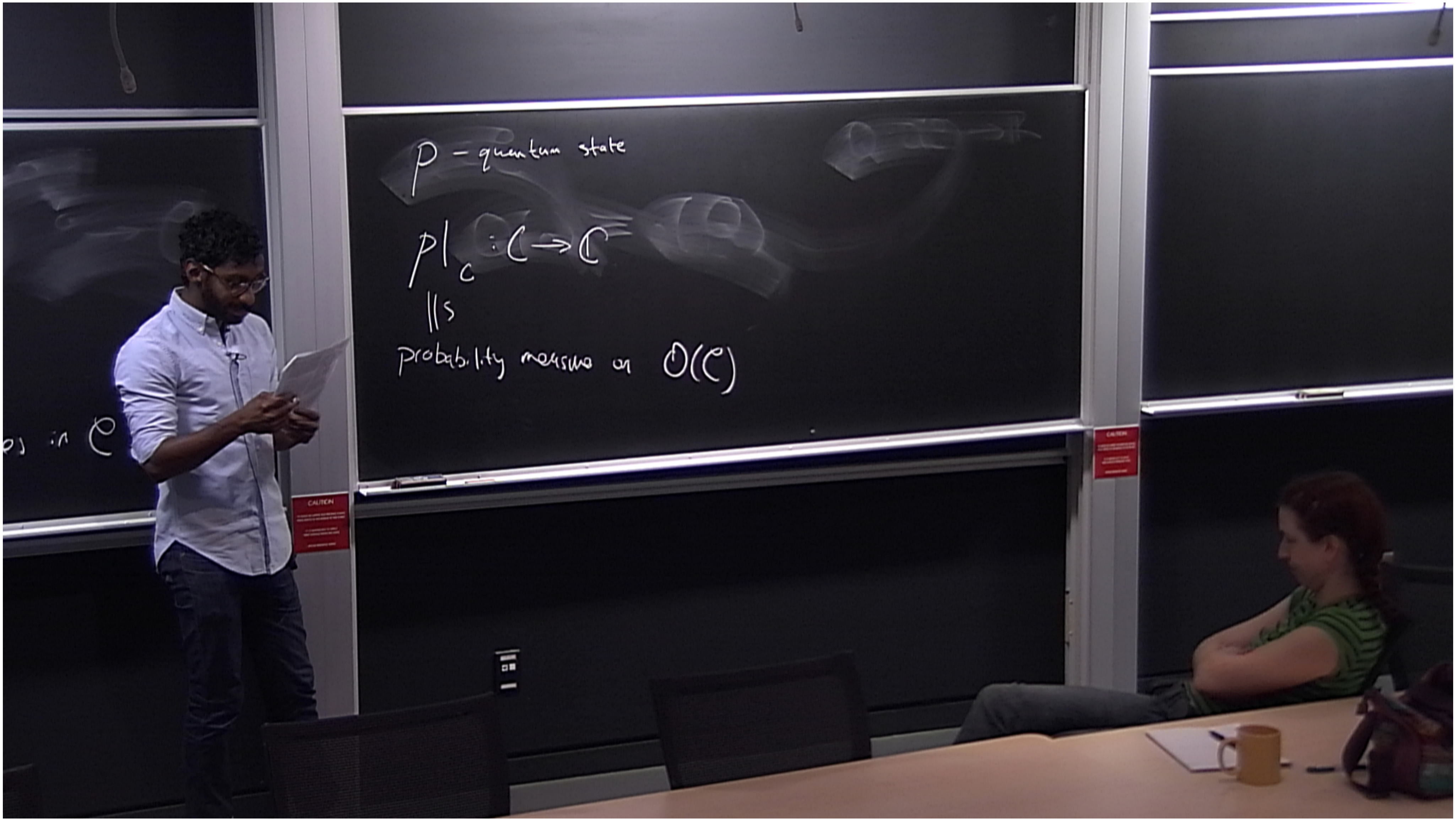
$Q \in \mathcal{C}$
$Q : \alpha(\mathcal{C}) \to \mathbb{R}$
$Q(\circ) \equiv \circ(Q)$
CAUTION

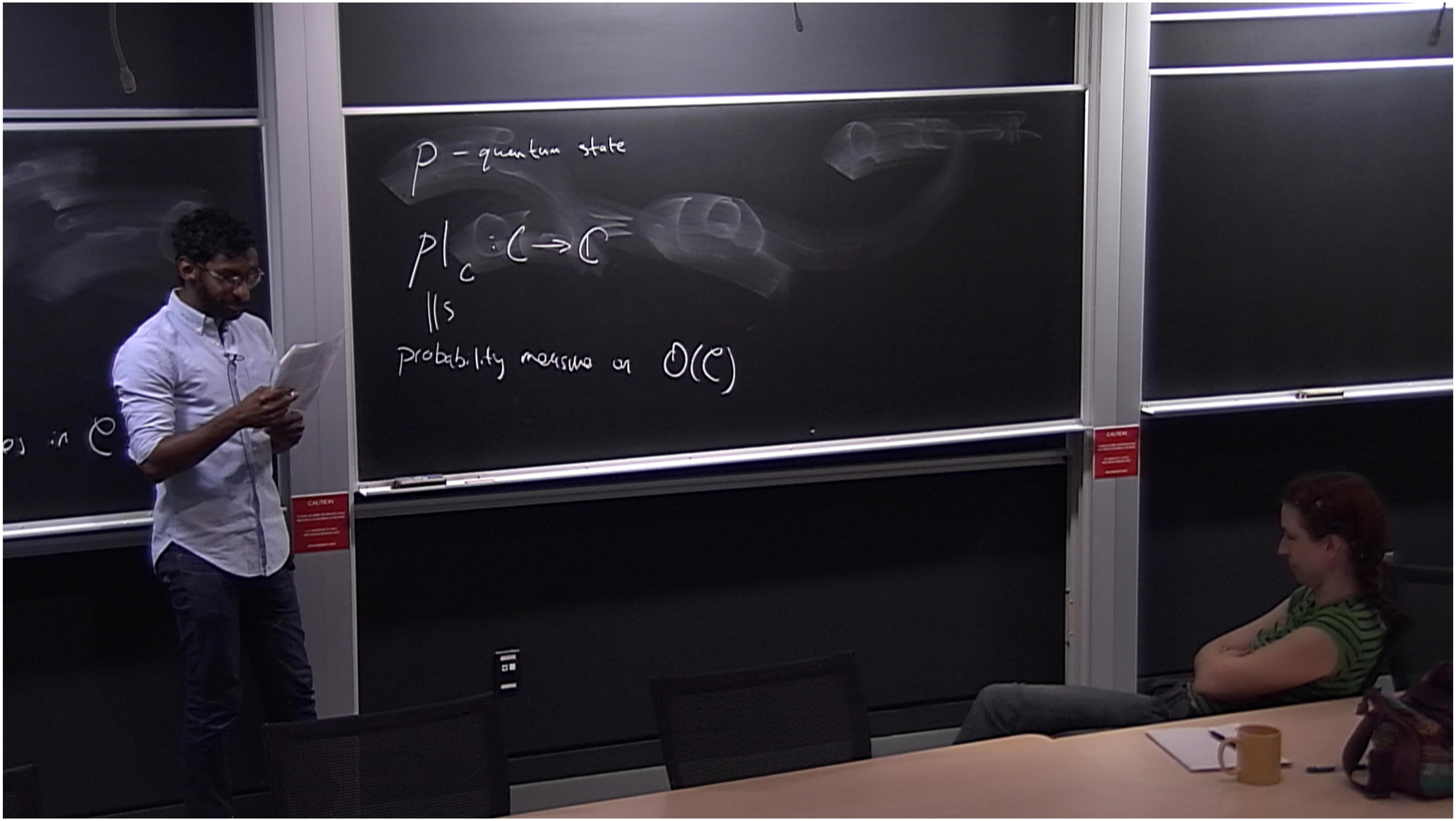

Q ∈ C
Q(o) ≡ o(Q)
Q : A(C) → R
in C




P
$P|_c : C \to \mathbb{C}$
in $\mathcal{C}$


ρ - quantum state
D|c : C → C


ρ - quantum state
ρ|C : C → ℂ
||s
probability measure on O(C)
CAUTION


ρ - quantum state
ρ|_C : C → ℂ
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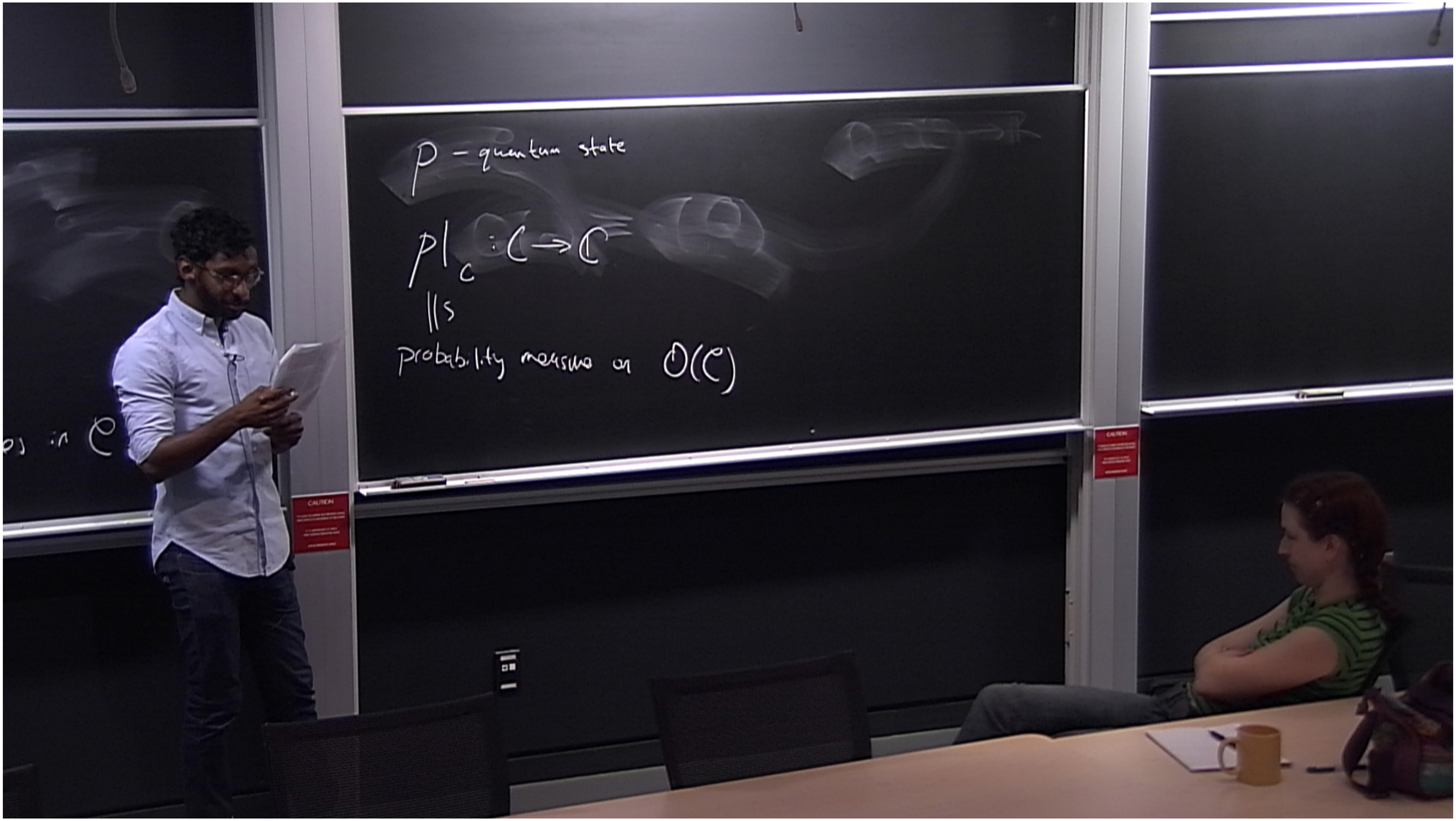
probability measure on O(C)


ρ – quantum state
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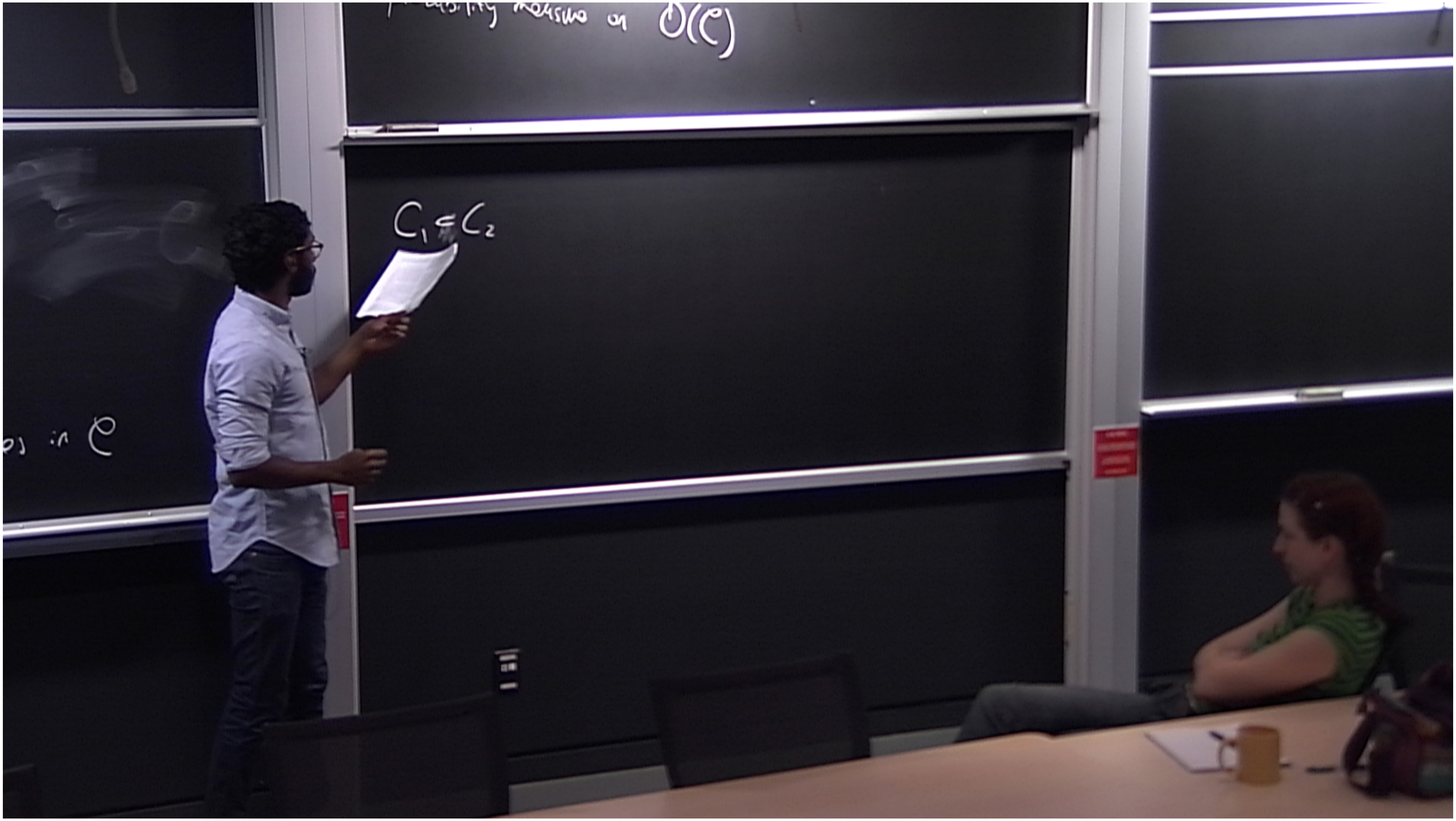
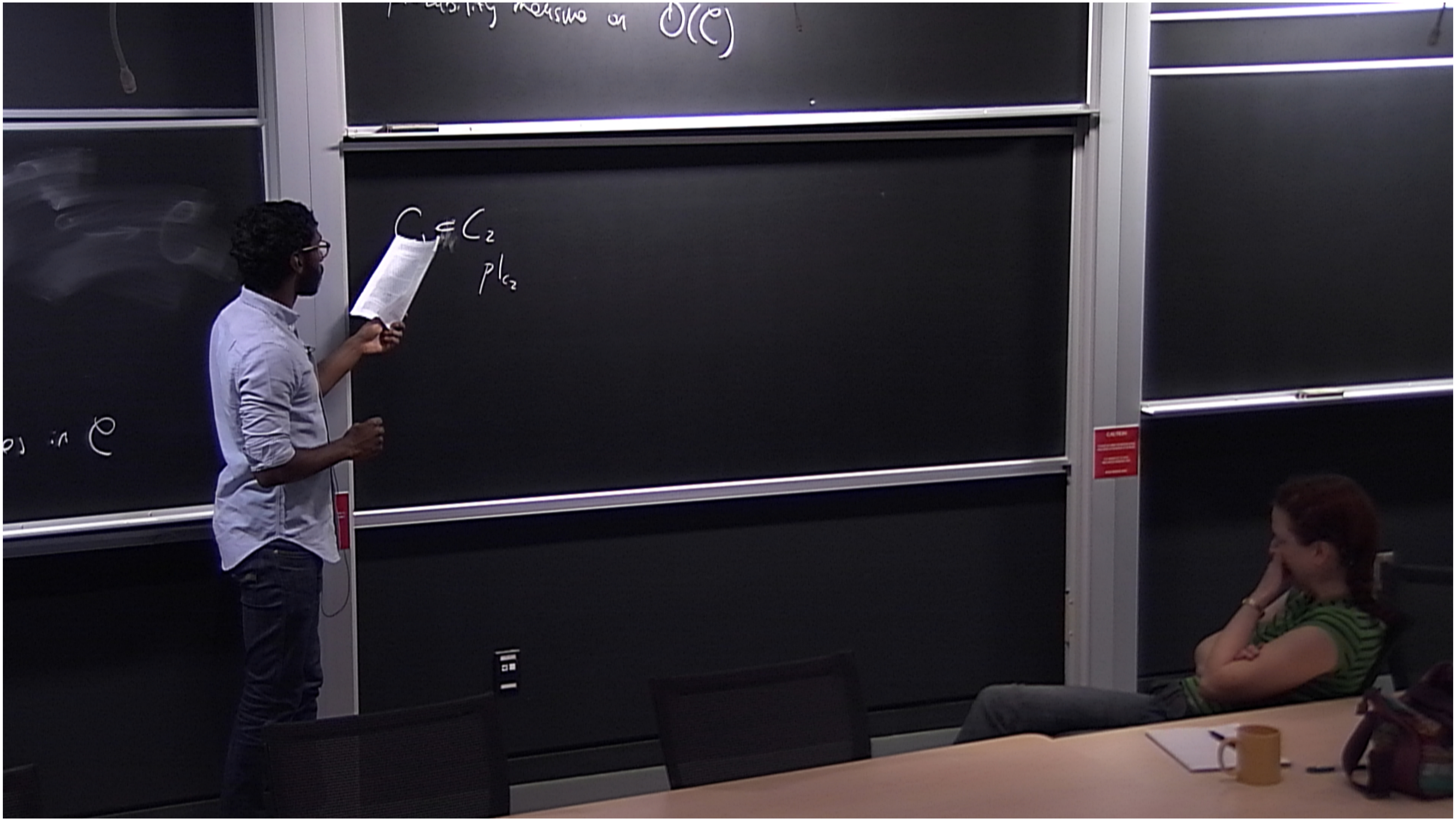
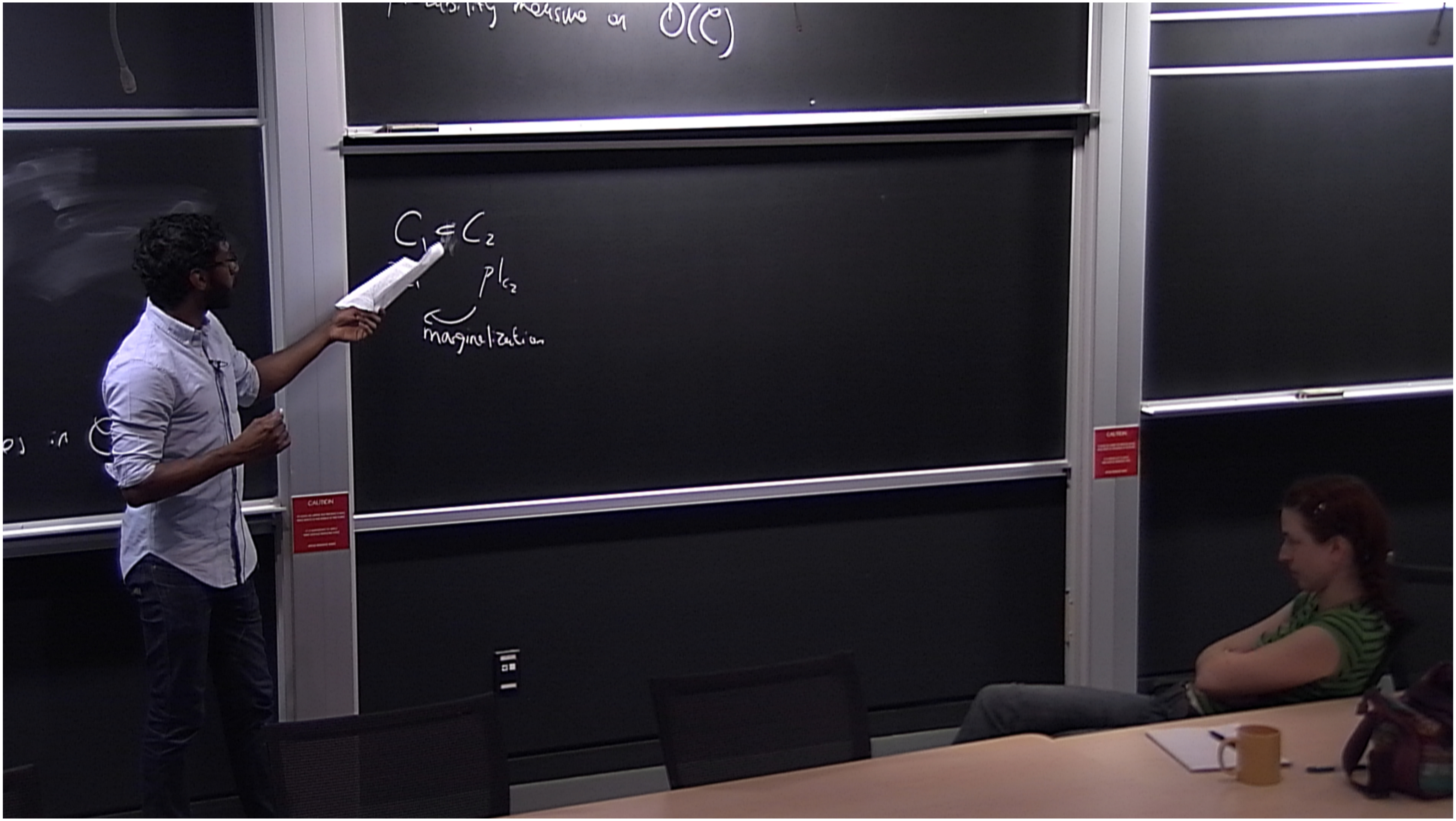
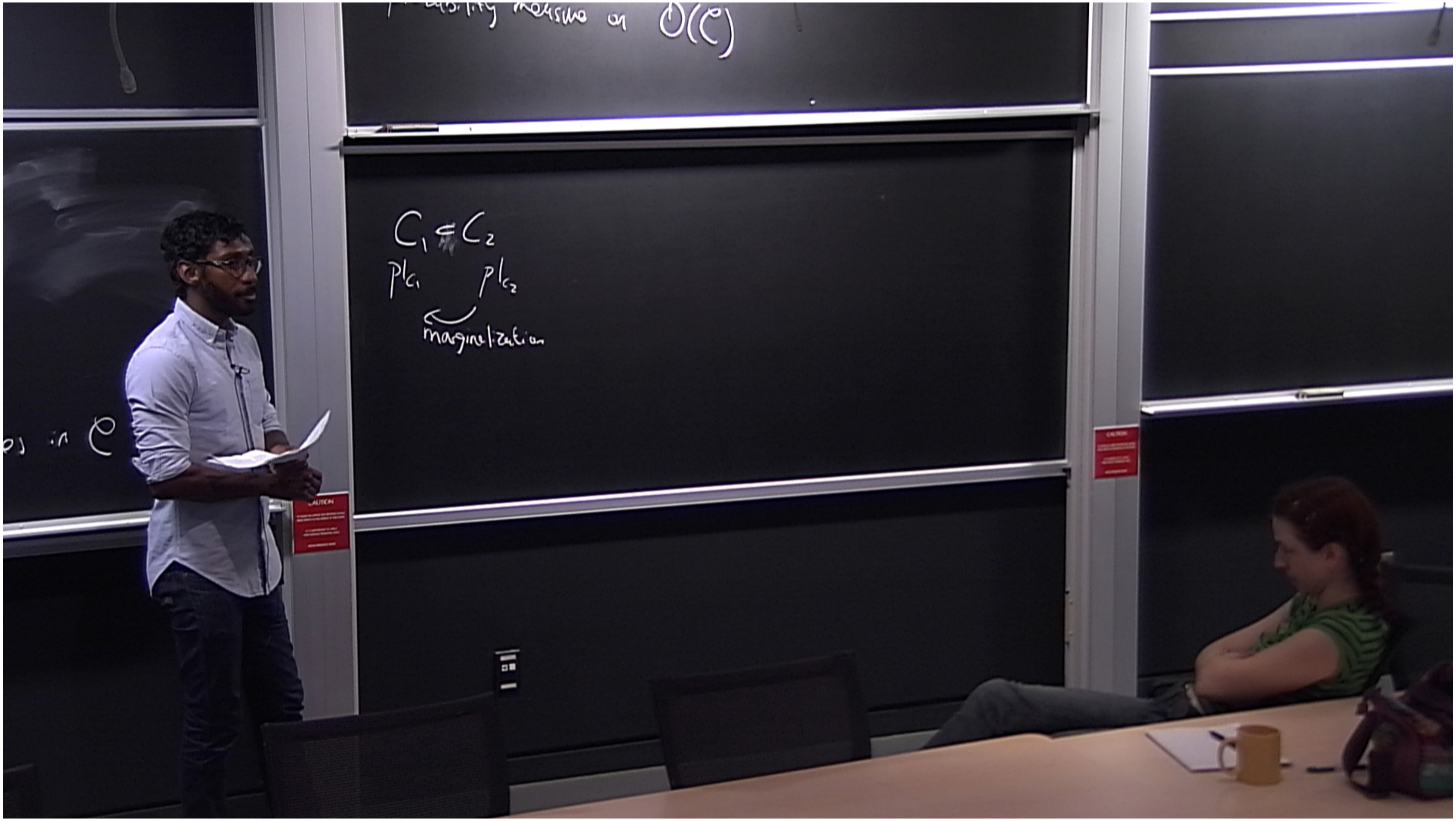
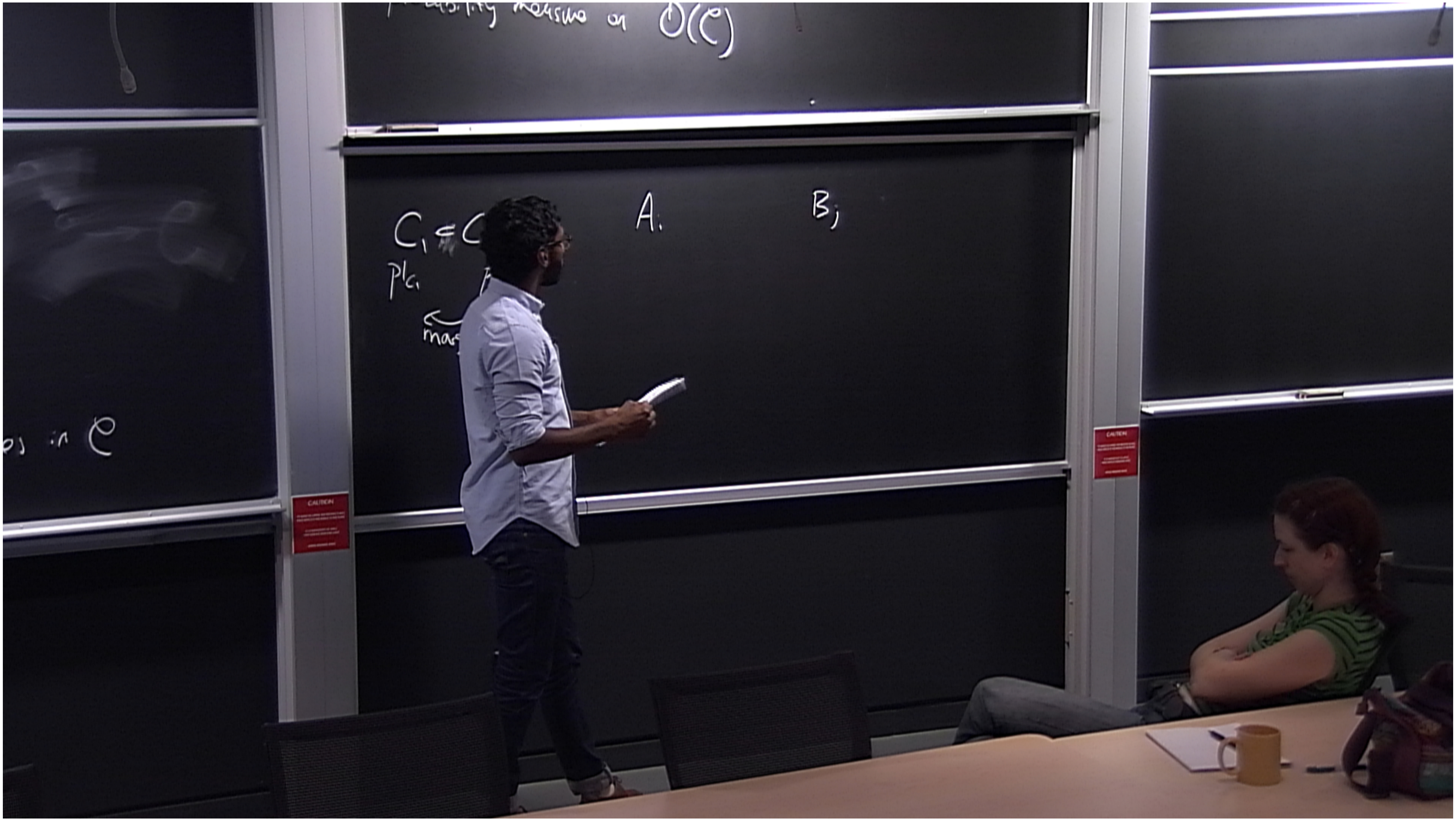
probability measure on O(C)

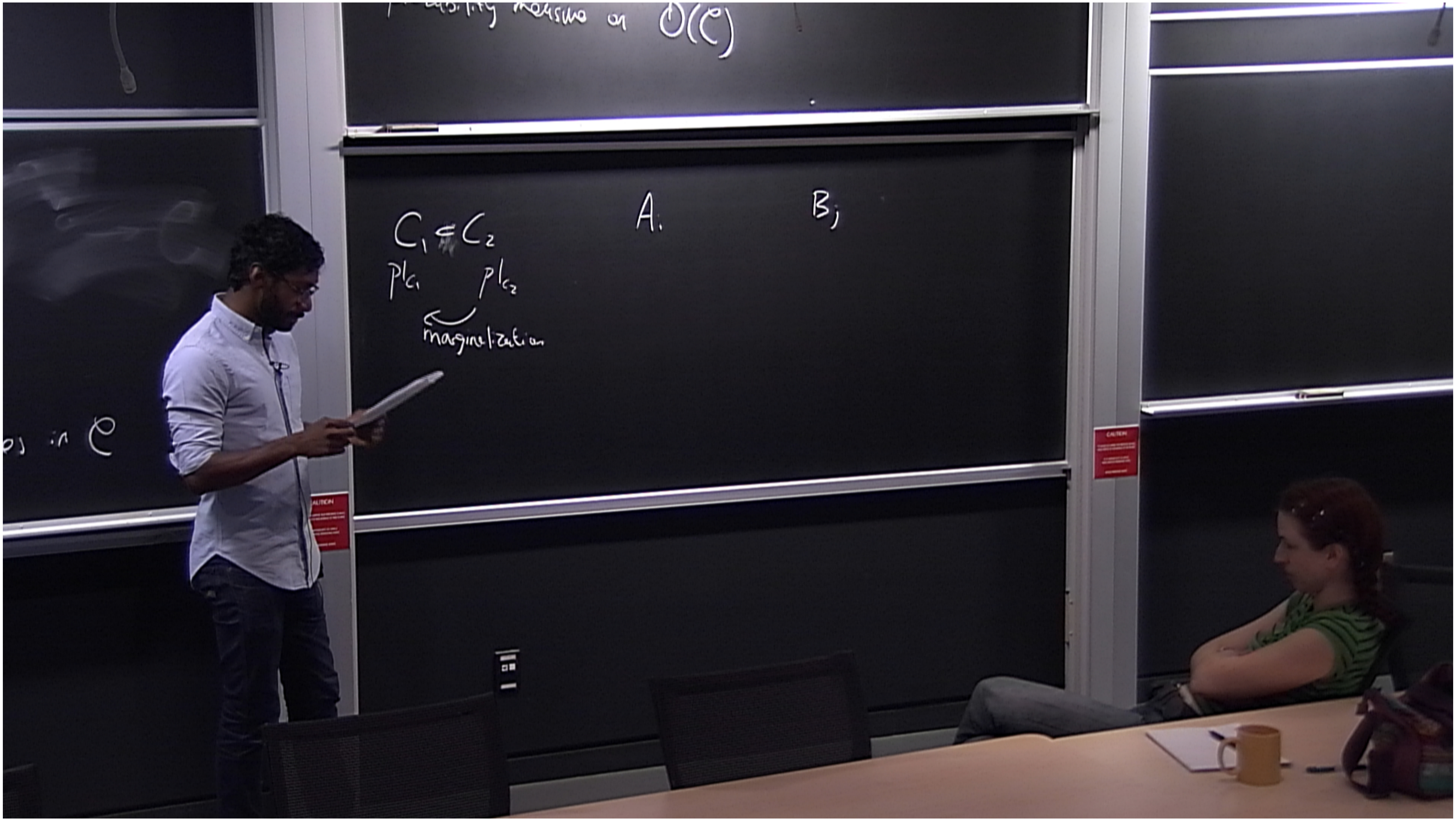

$C_1 \subseteq C_2$


probability measure on $\mathcal{O}(\mathcal{C})$
$C_1 \subseteq C_2$
$p|_{C_2}$
in $\mathcal{C}$

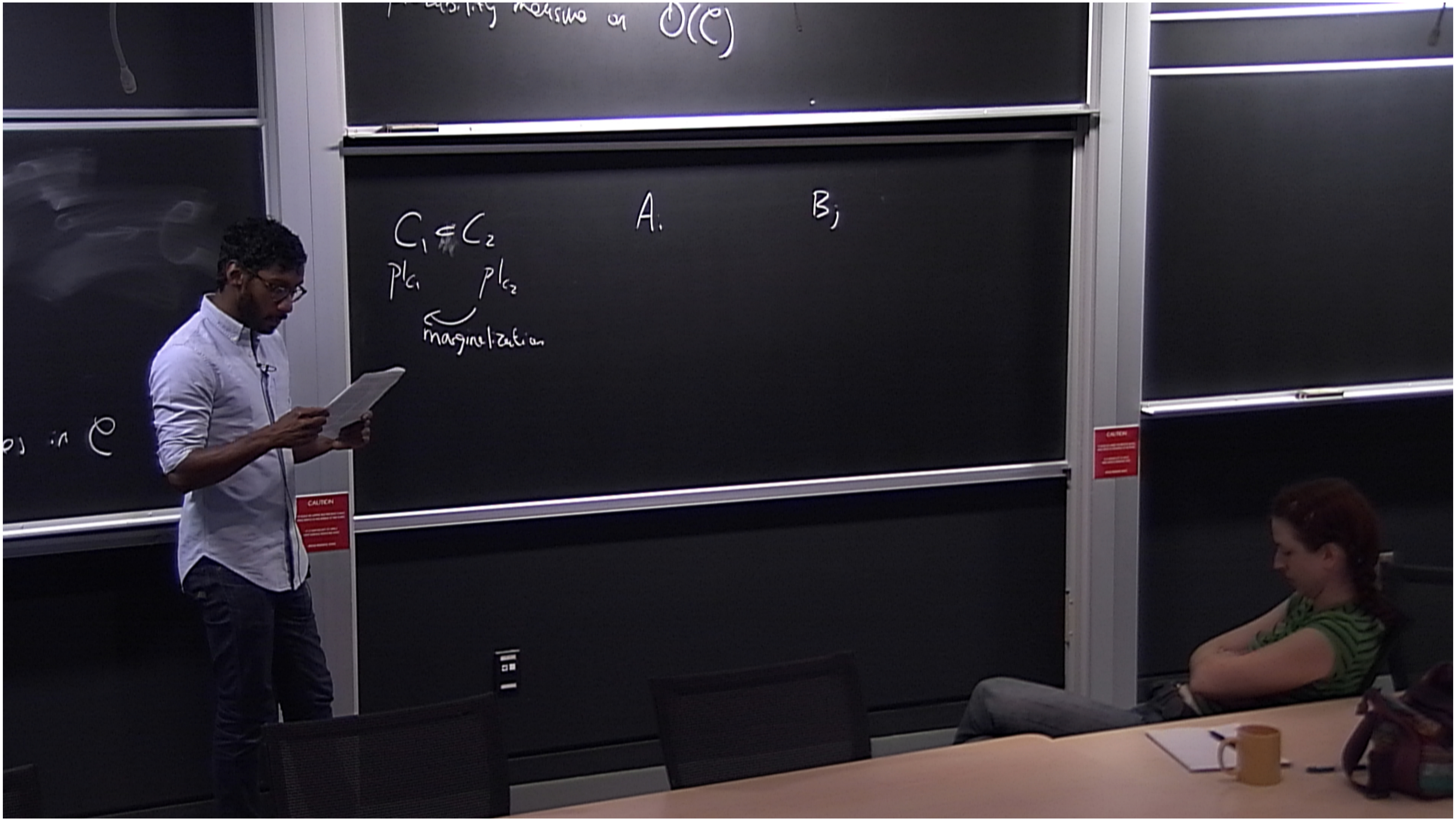

on $\mathcal{O}(C)$
$C_1 \subseteq C_2$
$p|_{C_2}$
marginalization


marginalization


A_i
B_j



$C_1 \subseteq C_2$
$p|_{C_1}$ $p|_{C_2}$
marginalization
A.
B



A.
B
magnalization