Title: Peaks, excursion sets and the distribution of dark matter haloes

Date: Sep 03, 2013 11:00 AM

URL: http://pirsa.org/13090062

Abstract: I will review recent developments in our theoretical understanding of the abundance and clustering of dark matter haloes. In the first part of this talk, I will discuss a toy model based on the statistics of peaks of Gaussian random field (Bardeen et al 1986) and show how the clustering properties of such a point set can be easily derived from a generalised local bias expansion. In the second part, I will explain how this peak formalism relates to the excursion set approach and present parameter-free predictions for the mass function and bias of dark matter halos.

OUTLINE

- *Galaxy clustering*
- *The peak formalism: BBKS and beyond*
- *Local bias approach to BBKS peaks*
- *Excursion set peaks and the halo mass function*

Galaxies positions now routinely measured out to high redshift ...

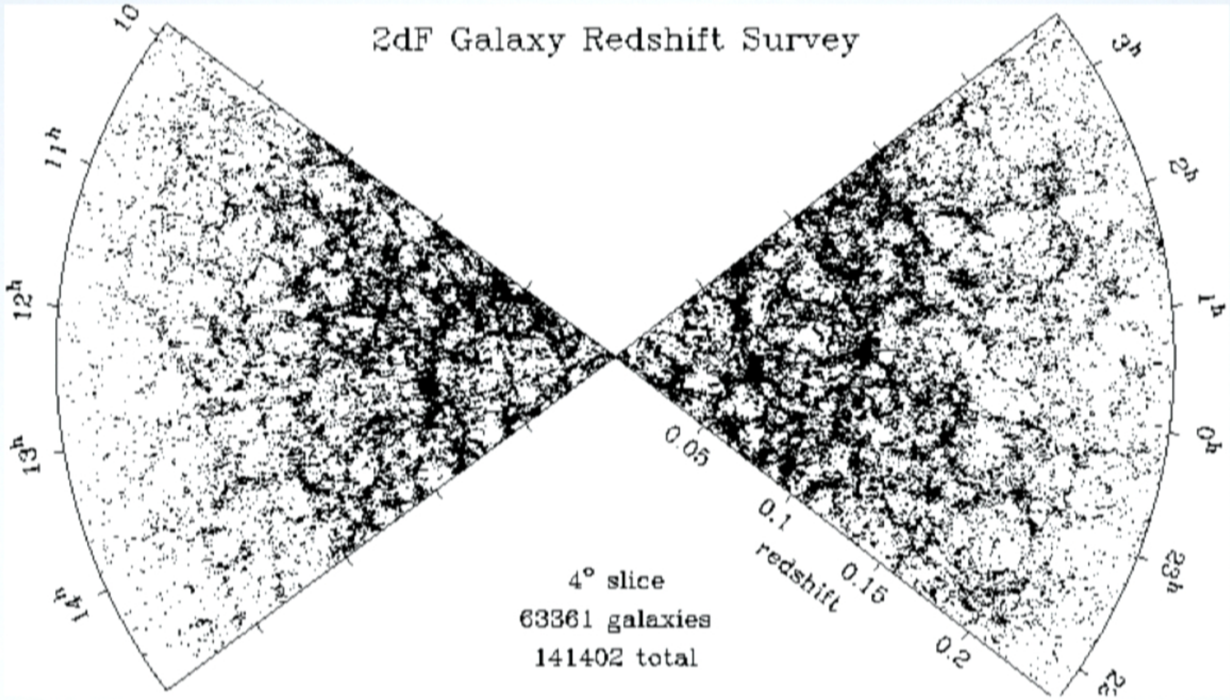

Image credit: 2dF GRS

SCIENCE WITH LARGE GALAXY SURVEYS

A plethora of ongoing and planned galaxy surveys, e.g. BOSS, DES, EUCLID, Pan-STARR, VIPERS, WiggleZ ..., to measure (among others)

- Matter power spectrum: matter content, neutrino masses etc.
- Baryon acoustic oscillation (BAO): dark energy equation of state
- Redshift space distortions: growth rate of structure (probe of gravity)
- Galaxy bispectrum: primordial non-Gaussianity (probe of inflation) etc.

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Require solid understanding of galaxy clustering !

BIAS

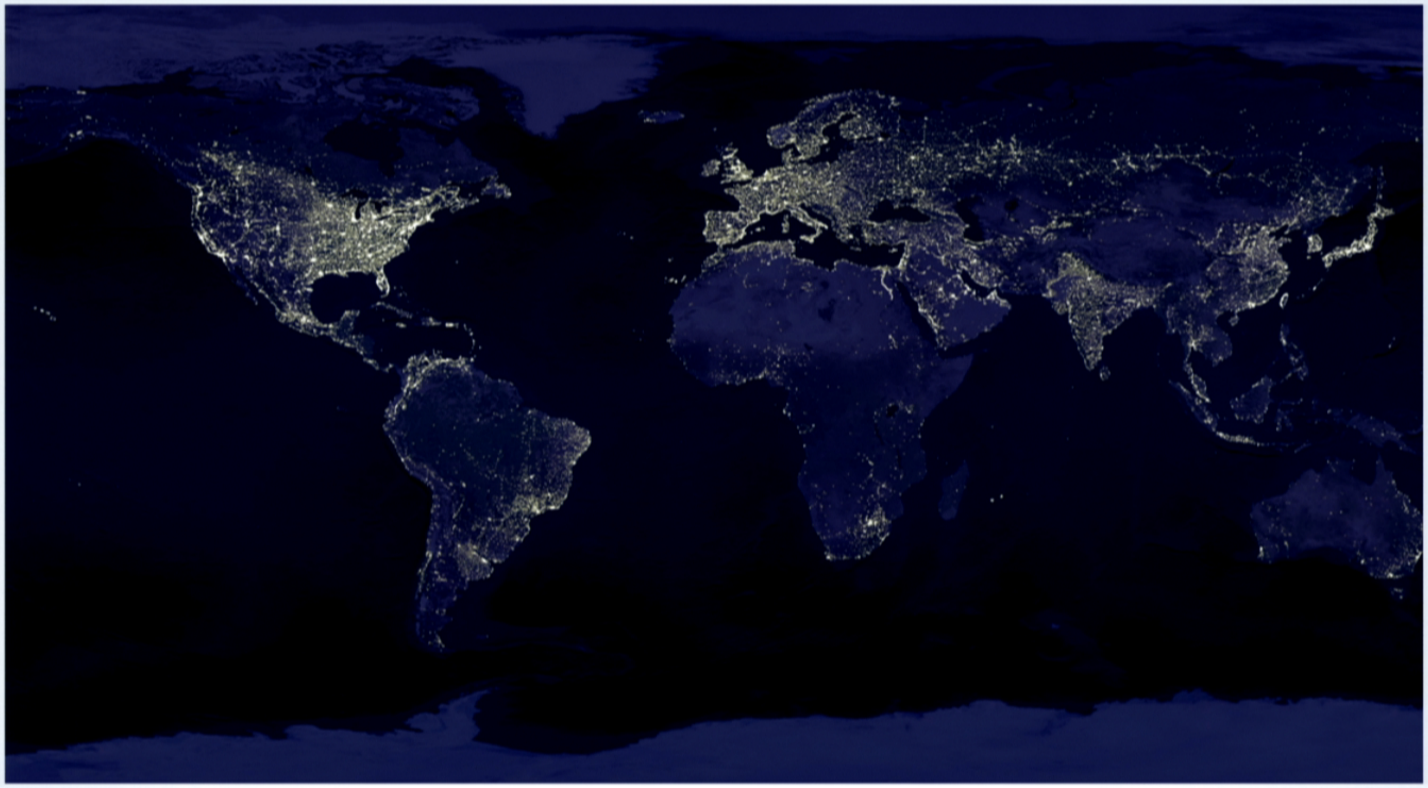

Figure credit: NASA

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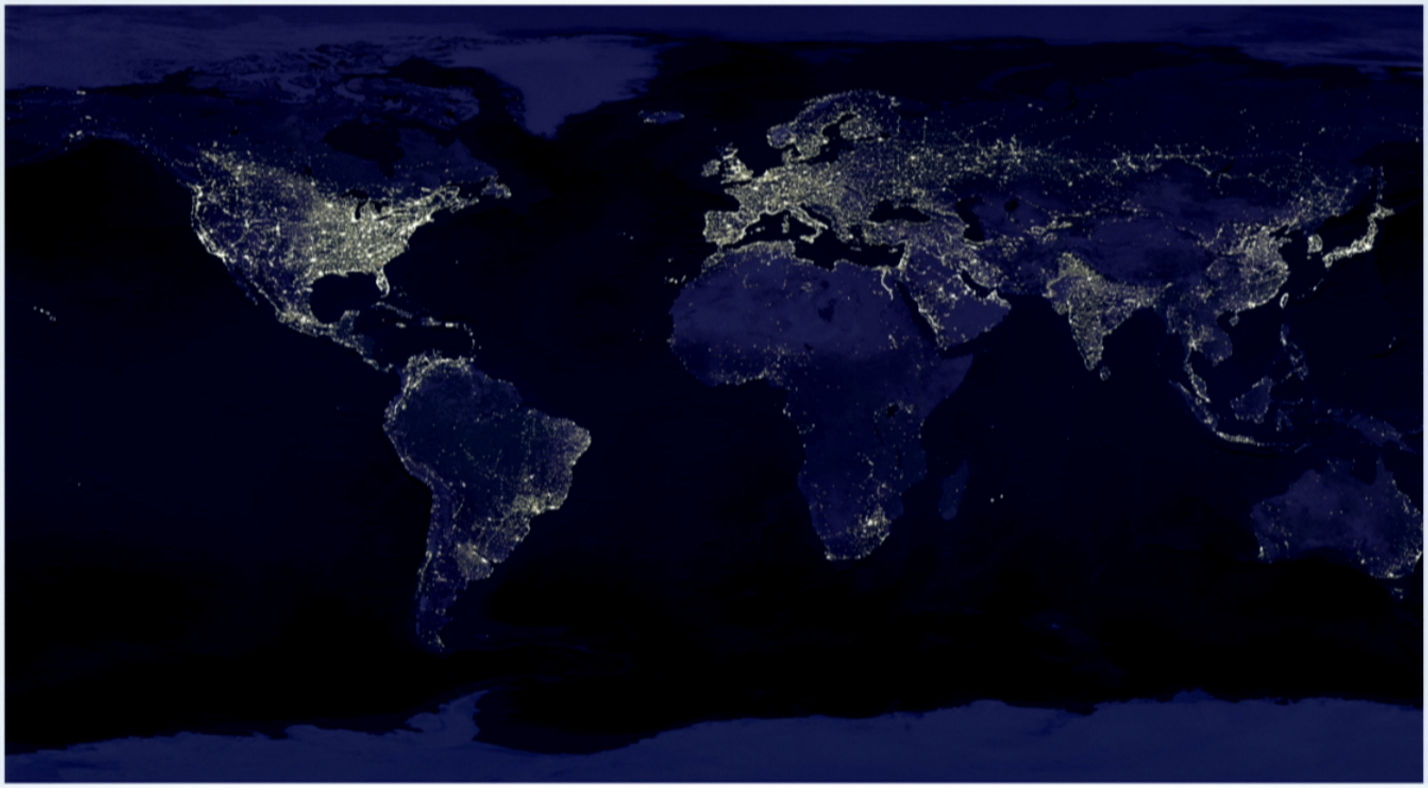


Figure credit: NASA

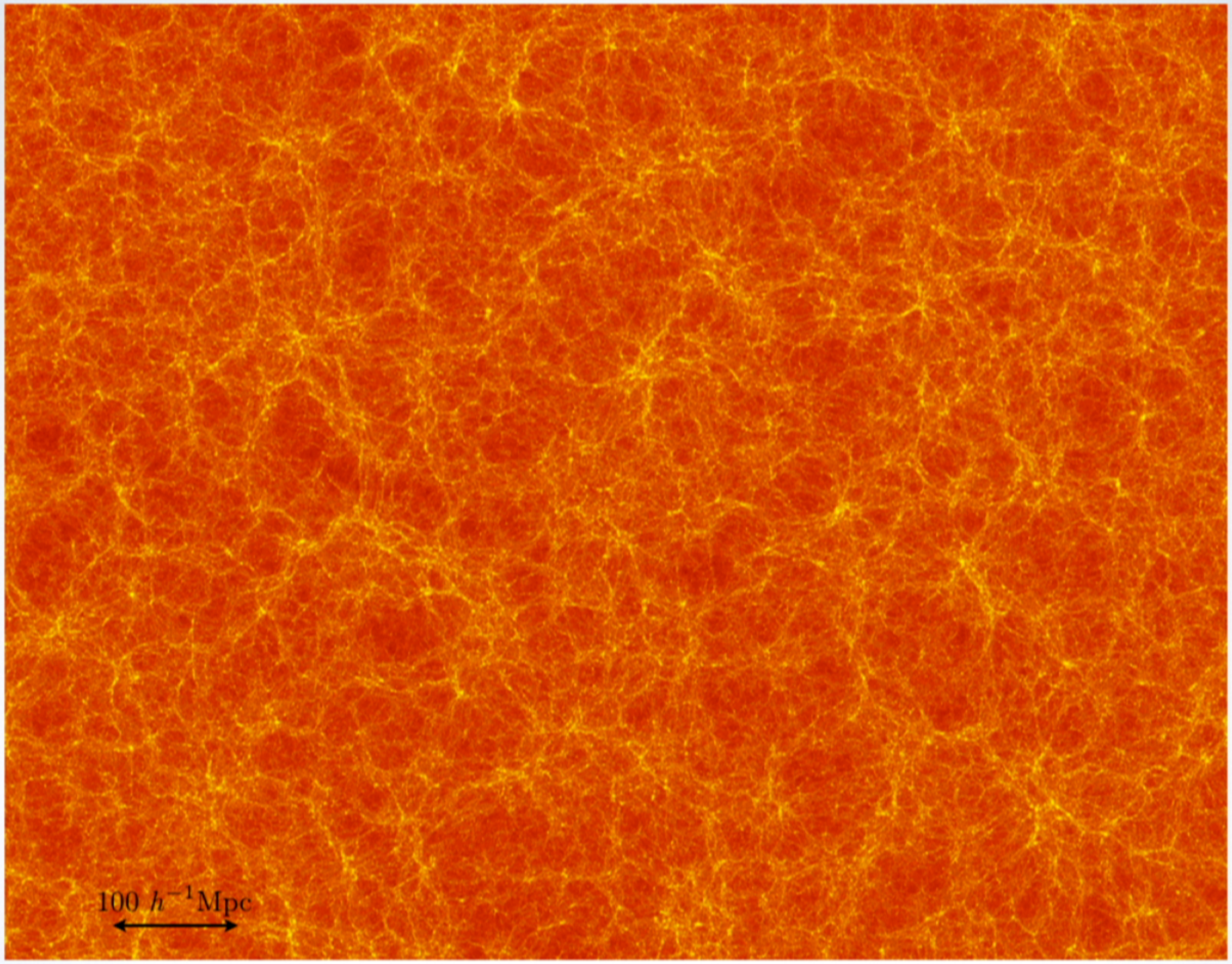
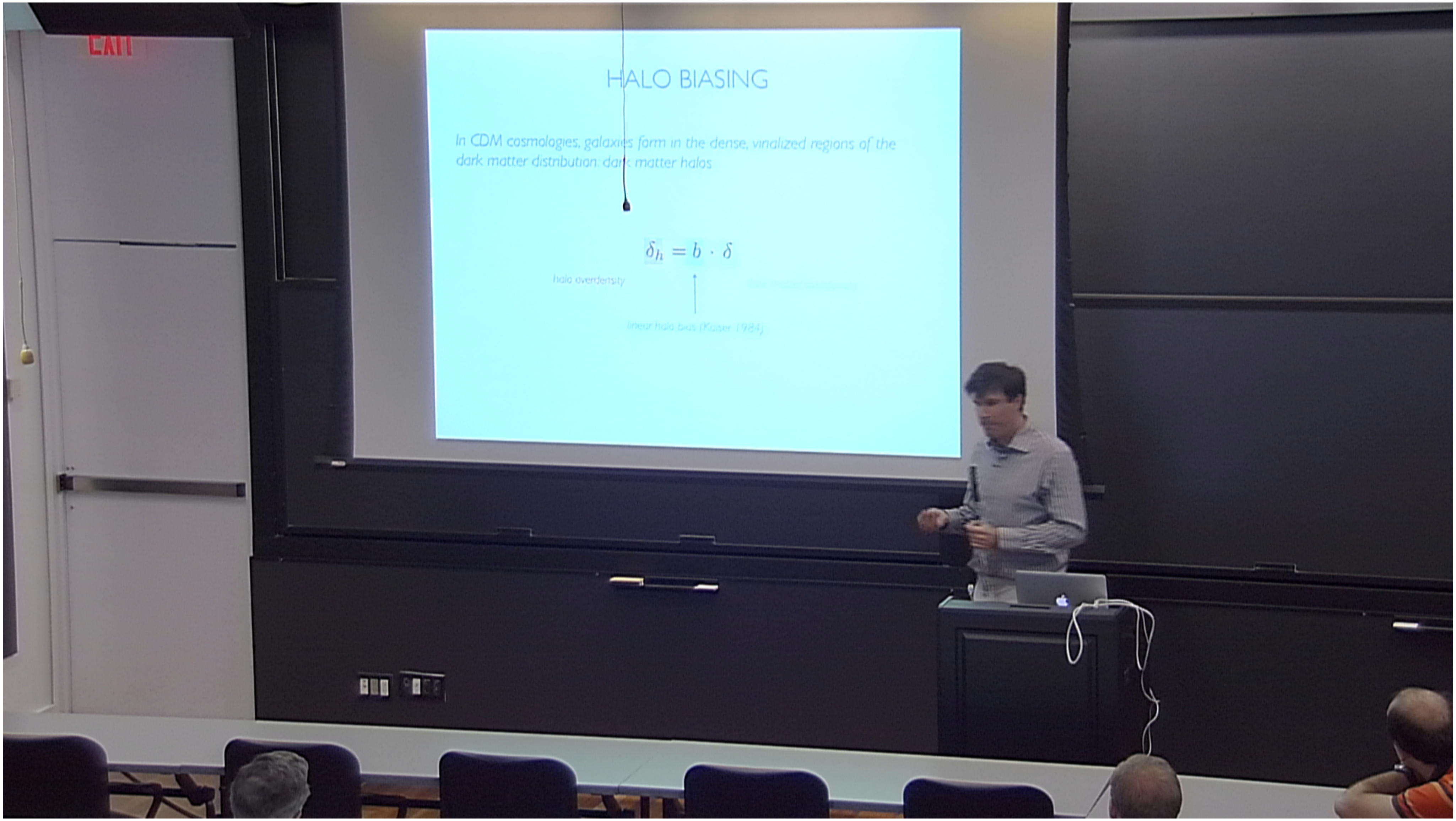


Figure courtesy Ilian T. Iliev


HALO BIASING
In CDM cosmologies, galaxies form in the dense, virialized regions of the dark matter distribution: dark matter halos
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halo overdensity
linear halo bias (Kaiser 1984)

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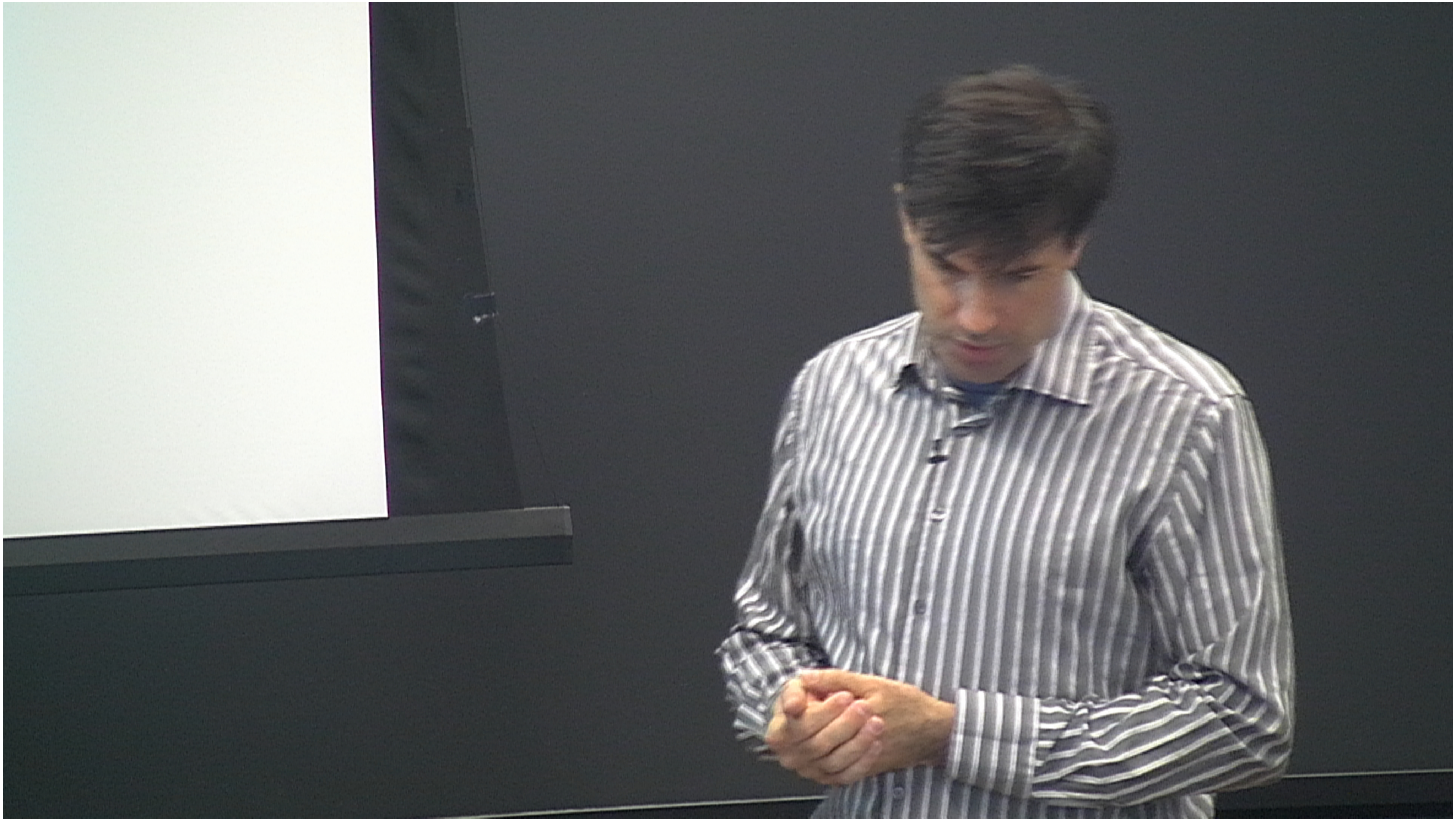

LOCAL LAGRANGIAN BIAS

Simplest bias scheme: dependence on mass density field only (Szalay 1988; Fry & Gaztanaga 1993, ...):

$$\delta_h(\mathbf{x}) = b_1\delta_R(\mathbf{x}) + \frac{1}{2}b_2\delta_R^2(\mathbf{x}) + \frac{1}{6}b_3\delta_R^3(\mathbf{x}) + \dots$$

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For example, 2-point correlation is computed as:

$$\xi_h(|\mathbf{x}_2 - \mathbf{x}_1|) = \langle \delta_h(\mathbf{x}_1) \delta_h(\mathbf{x}_2) \rangle = b_1^2 \, \xi_R(|\mathbf{x}_2 - \mathbf{x}_1|) + \dots$$

TWO REMARKS

- *One could think of a more general expansion:*

$$\delta_h(\mathbf{x}) = b_1\delta_R(\mathbf{x}) + c_1\nabla^2\delta_R(\mathbf{x}) + \frac{1}{2}b_2\delta_R^2(\mathbf{x}) + c_2\left(\nabla\delta_R\right)^2(\mathbf{x}) + c_3\left(\partial_i\partial_j\delta_R - \frac{1}{3}\delta_{ij}\nabla^2\delta_R\right)^2(\mathbf{x}) + \ldots$$

- *Galaxies and dark matter halos are pointlike objects as far as their 3D position is concerned. Does this matter (beyond a Poisson shot noise correction to the power spectrum) ?*

THE PEAK FORMALISM

Consider maxima of the linear density field as a proxy for the formation sites of discrete tracers of the mass distribution such as halos or galaxies (Bardeen et al 1986 = BBKS; ...)


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$$\nu(\mathbf{x}) \equiv \frac{1}{\sigma_0}\delta_s(\mathbf{x}), \qquad \eta_i(\mathbf{x}) = \frac{1}{\sigma_1}\partial_i\delta_s(\mathbf{x}), \qquad \zeta_{ij} \equiv \frac{1}{\sigma_2}\partial_i\partial_j\delta_s(\mathbf{x}), \qquad \delta_s(\mathbf{x}) = \delta_{R=R_s}(\mathbf{x})$$

$$\sigma_n^2 = \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} P_s(k)$$

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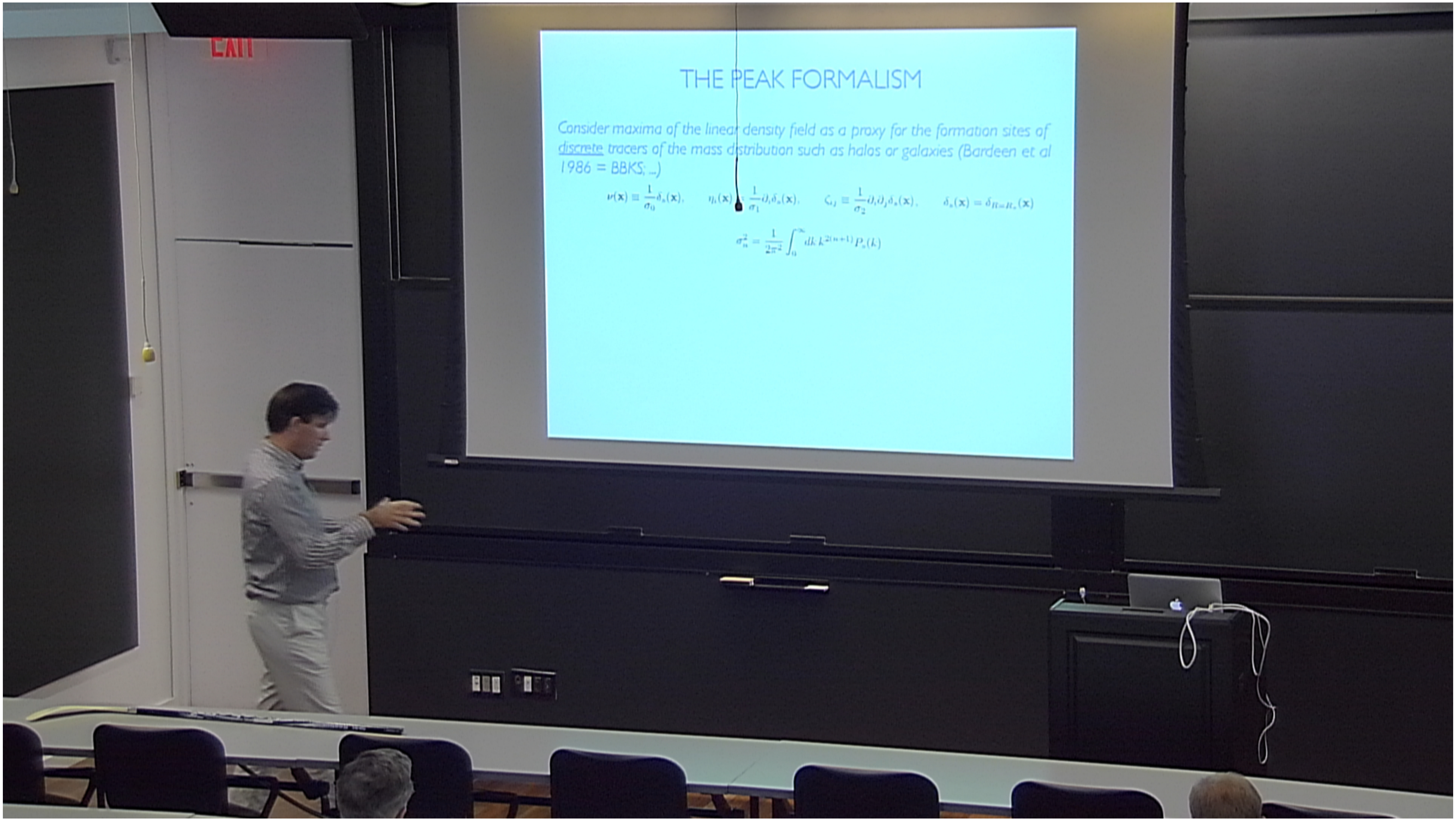
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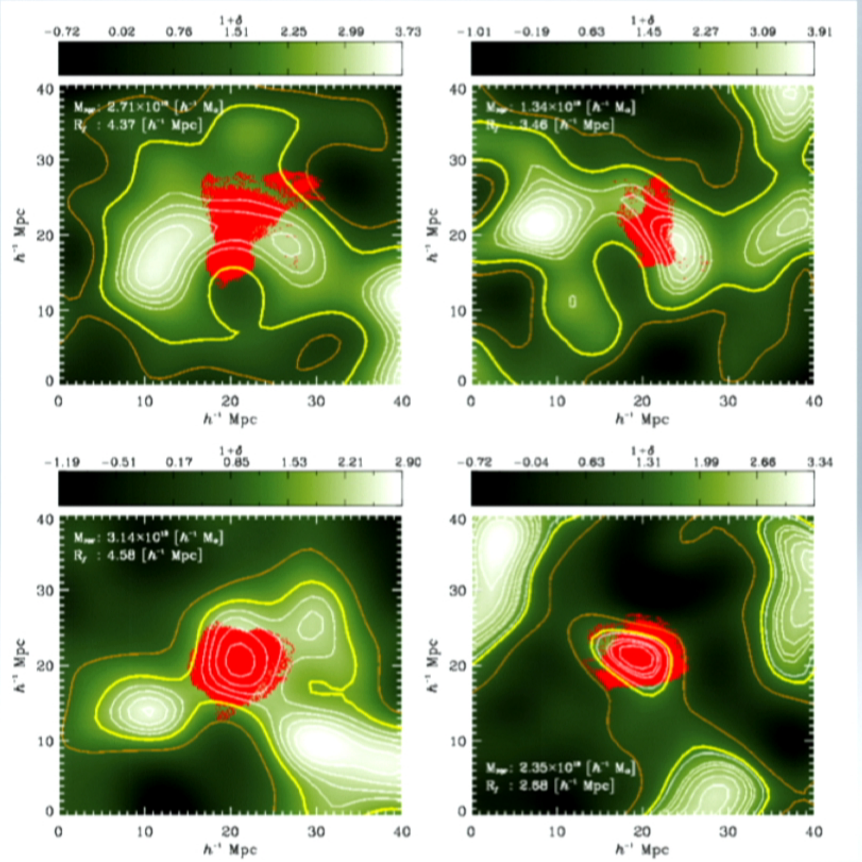
(i) $\nu(\mathbf{x}_p) = \dfrac{\delta_c}{\sigma_0} = \dfrac{1.68}{\sigma_0} \equiv \nu_c$

(ii) $\eta_i(\mathbf{x}_p) = 0$

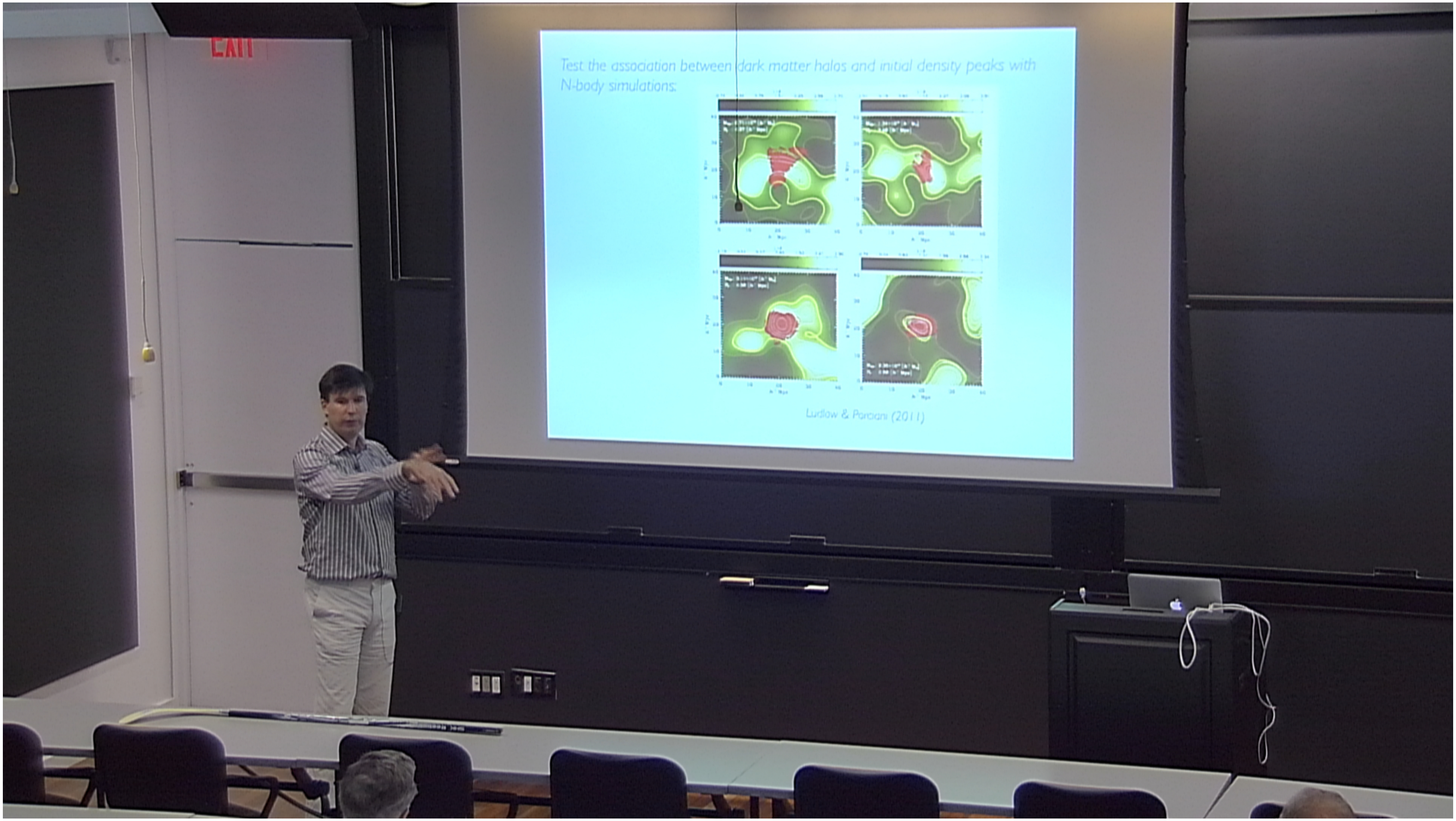
(iii) $\lambda_1(\mathbf{x}_p) \geq \lambda_2(\mathbf{x}_p) \geq \lambda_3(\mathbf{x}_p) > 0$

$\lambda_a(\mathbf{x}) = \text{eigenvalues of } -\zeta_{ij}(\mathbf{x})$

Test the association between dark matter halos and initial density peaks with N-body simulations:



Ludlow & Porciani (2011)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PEAK CORRELATIONS

The "localized" peak number density is (Kac 1943; Rice 1951; BBKS):

$$n_{\rm pk}(\mathbf{x}) = \sum_{\mathbf{x}_p} \delta_D(\mathbf{x}-\mathbf{x}_p) = \frac{3^{3/2}}{R_\star^3}\left|\det\zeta(\mathbf{x})\right|\delta_D\left[\boldsymbol{\eta}(\mathbf{x})\right]\theta_H\left[\lambda_3(\mathbf{x})\right]\delta_D\left(\nu(\mathbf{x})-\nu_c\right)$$


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MEAN NUMBER DENSITY

First computed in BBKS:

$$\langle n_{\rm pk}(\mathbf{x})\rangle \equiv \bar{n}_{\rm pk}(\nu_c) = V_\star^{-1}\, G_0^{(1)}(\gamma_1, \gamma_1\nu_c)\, \frac{e^{-\nu_c^2/2}}{\sqrt{2\pi}}$$

$$V_\star \equiv (2\pi)^{3/2} R_\star^3, \qquad \gamma_1 \equiv \frac{\sigma_1^2}{\sigma_0\sigma_2}$$

$$G_n^{(\alpha)}(\gamma_1, \omega) = \int_0^\infty du\, u^n\, f(u,\alpha)\, \frac{\exp\left[-\frac{(u-\omega)^2}{2(1-\gamma_1^2)}\right]}{\sqrt{2\pi(1-\gamma_1^2)}}$$

$$f(u,\alpha) \equiv \frac{3^2 5^{5/2}}{\sqrt{2\pi}} \left\{\int_0^{u/4} dv \int_{-v}^{+v} dw + \int_{u/4}^{u/2} dv \int_{3v-w}^{v} dw\right\} F(u,v,w)\, e^{-\frac{5\alpha}{2}(3v^2+w^2)}$$

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$u \equiv -{\rm tr}\zeta =$ peak curvature

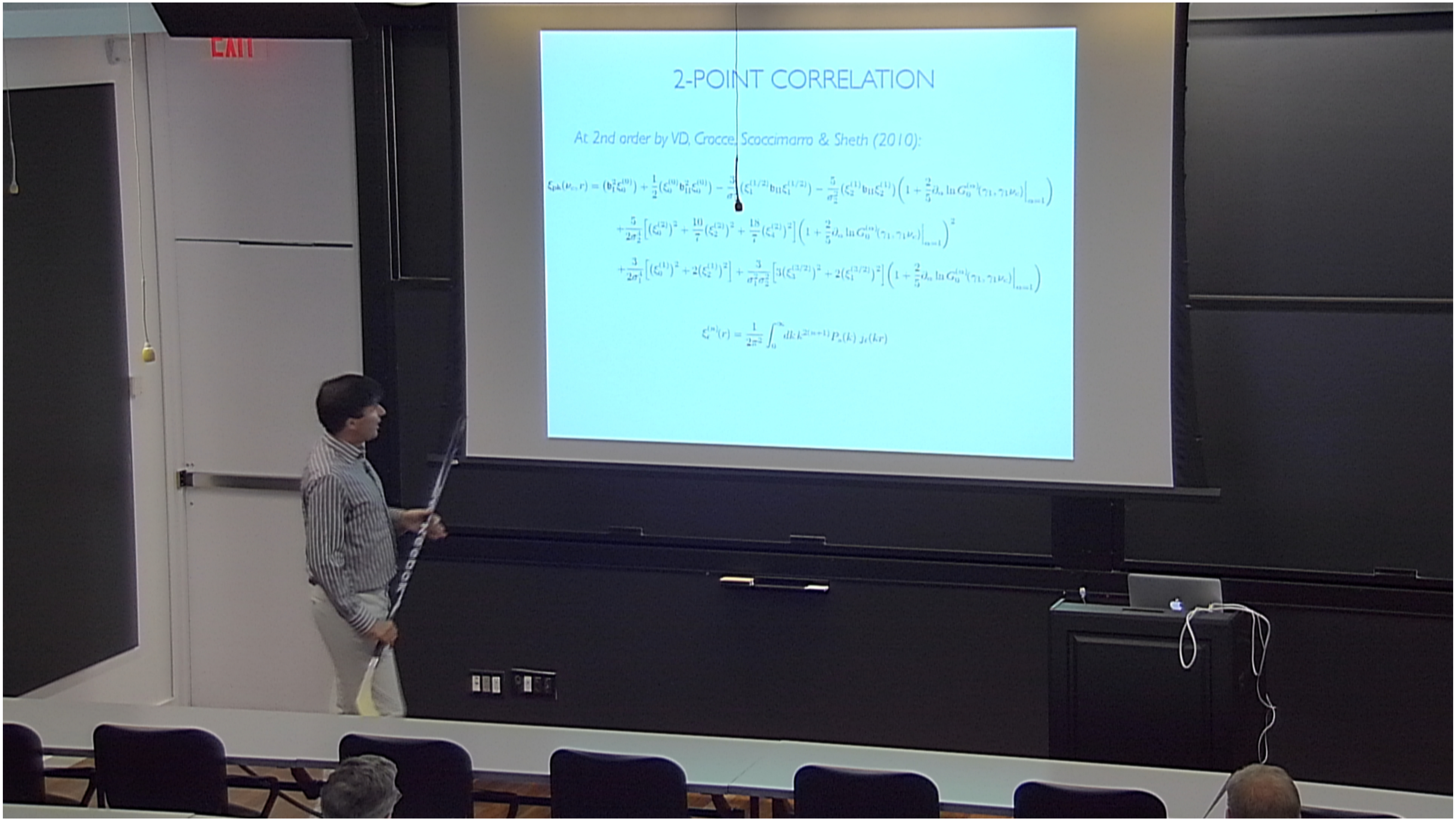
$v, w \sim$ peak ellipticity and prolateness

2-POINT CORRELATION

At 2nd order by VD, Crocce, Scoccimarro & Sheth (2010):

$$\xi_{\rm pk}(\nu_c, r) = \left(\mathfrak{b}_{\rm I}^2 \xi_0^{(0)}\right) + \frac{1}{2}\left(\xi_0^{(0)} \mathfrak{b}_{\rm II}^2 \xi_0^{(0)}\right) - \frac{3}{\sigma_1^2}\left(\xi_1^{(1/2)} \mathfrak{b}_{\rm II} \xi_1^{(1/2)}\right) - \frac{5}{\sigma_2^2}\left(\xi_2^{(1)} \mathfrak{b}_{\rm II} \xi_2^{(1)}\right)\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1\nu_c)\Big|_{\alpha=1}\right)$$
$$+\frac{5}{2\sigma_2^4}\left[\left(\xi_0^{(2)}\right)^2 + \frac{10}{7}\left(\xi_2^{(2)}\right)^2 + \frac{18}{7}\left(\xi_4^{(2)}\right)^2\right]\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1\nu_c)\Big|_{\alpha=1}\right)^2$$
$$+\frac{3}{2\sigma_1^4}\left[\left(\xi_0^{(1)}\right)^2 + 2\left(\xi_2^{(1)}\right)^2\right] + \frac{3}{\sigma_1^2\sigma_2^2}\left[3\left(\xi_3^{(3/2)}\right)^2 + 2\left(\xi_1^{(3/2)}\right)^2\right]\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1\nu_c)\Big|_{\alpha=1}\right)$$

$$\xi_\ell^{(n)}(r) = \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} P_s(k)\, j_\ell(kr)$$


EXIT
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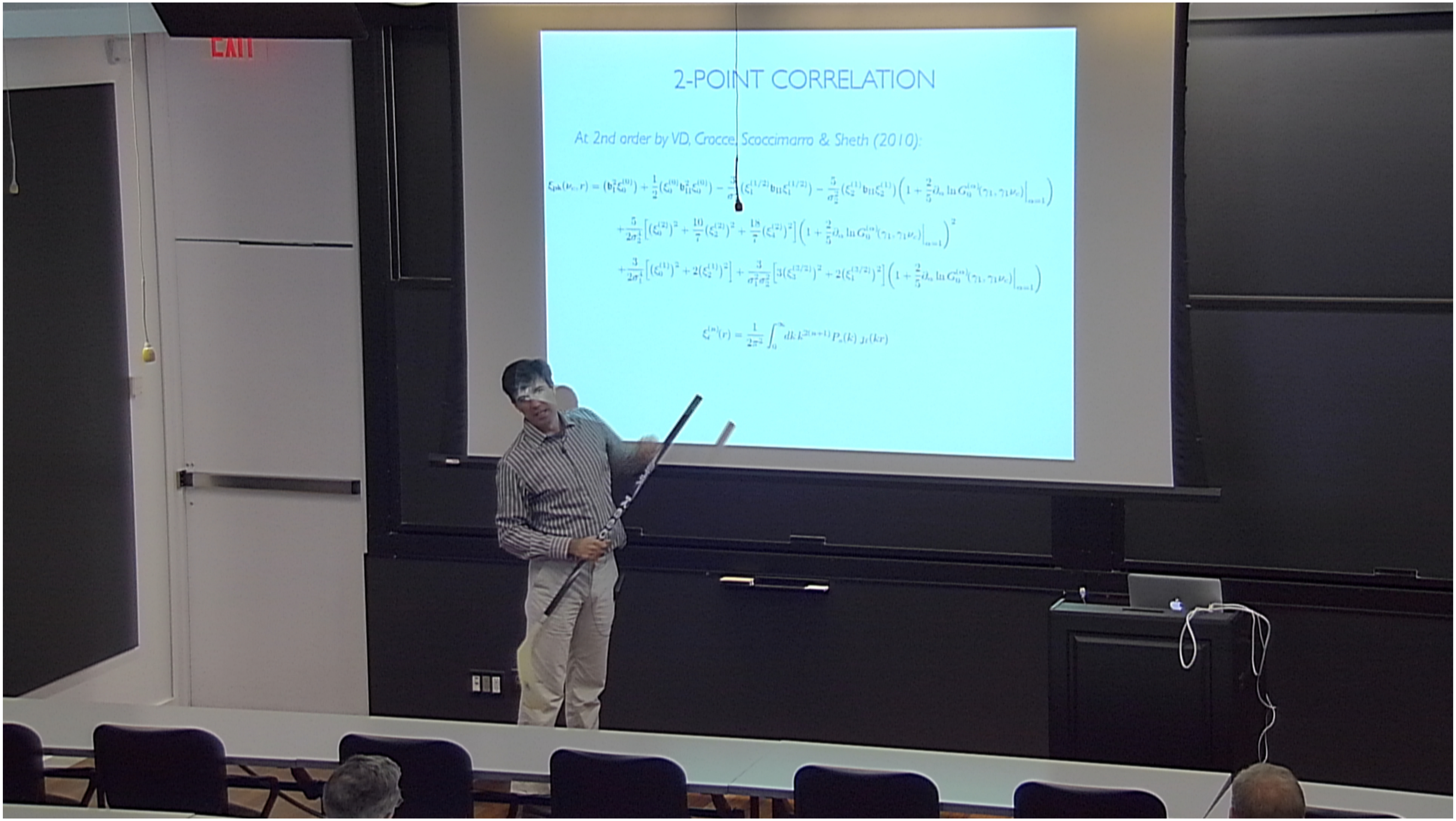
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PEAK BIAS FACTORS

1st order:

$$\mathfrak{b}_{\rm I}(k) = b_{10} + b_{01}k^2, \qquad b_{10} = \frac{1}{\sigma_0}\left(\frac{\nu_c - \gamma_1\bar{u}}{1-\gamma_1^2}\right), \quad b_{01} = \frac{1}{\sigma_2}\left(\frac{\bar{u} - \gamma_1\nu_c}{1-\gamma_1^2}\right)$$

2nd order :

$$\mathfrak{b}_{\rm II}(k_1,k_2) = b_{20} + b_{11}\left(k_1^2 + k_2^2\right) + b_{02}\, k_1^2 k_2^2, \qquad b_{20} = \frac{1}{\sigma_0^2}\left[\frac{\nu_c^2 - 2\gamma_1\nu_c\bar{u} + \gamma_1^2\overline{u^2}}{\left(1-\gamma_1^2\right)^2} - \frac{1}{\left(1-\gamma_1^2\right)}\right]$$

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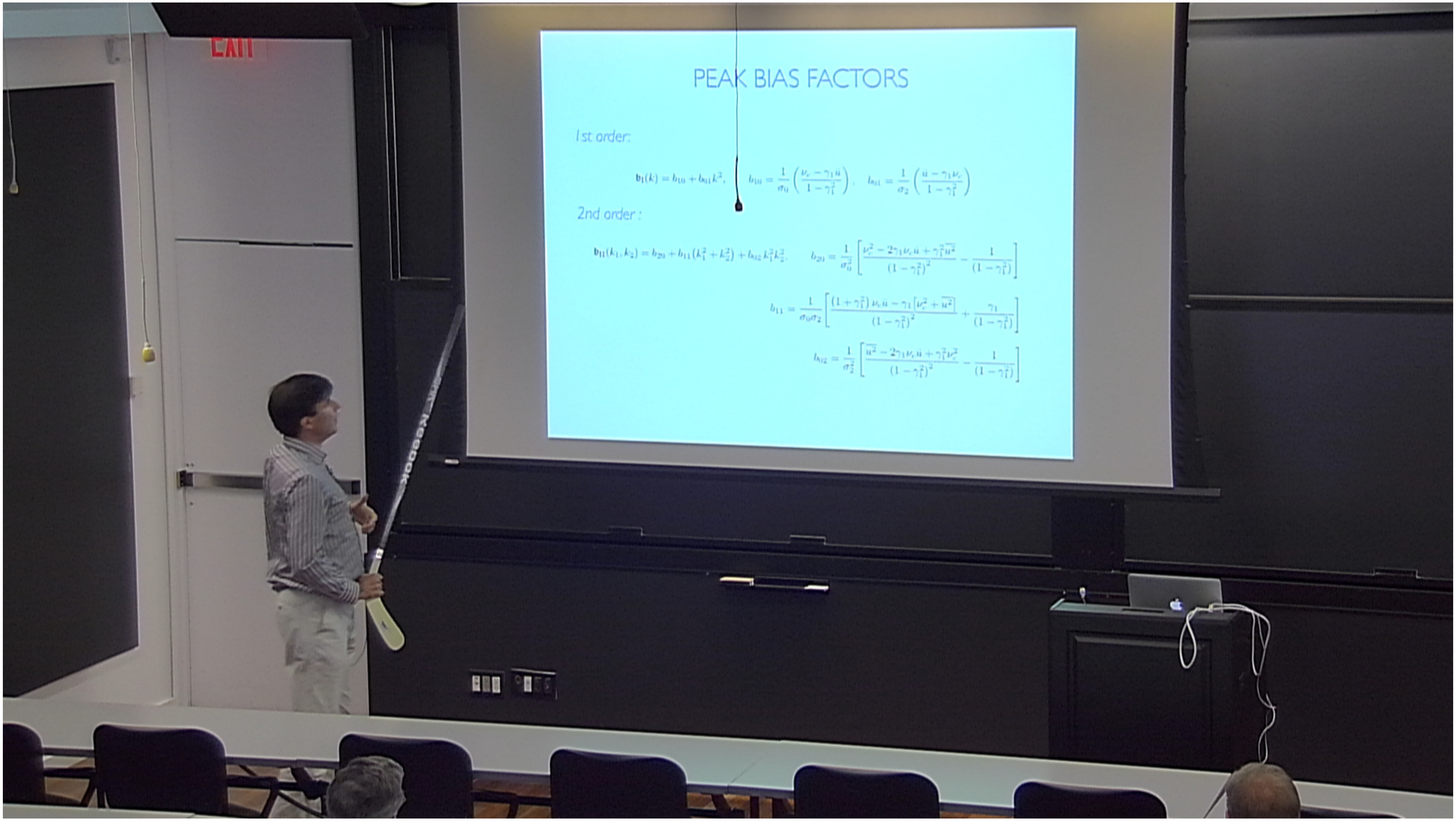

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$\mathbf{b}_{\mathrm{II}}(k_1, k_2) = b_{20} + b_{11}(k_1^2 + k_2^2) + b_{02}k_1^2 k_2^2, \quad b_{20} = \frac{1}{\sigma_0^2}\left[\frac{\nu_c^2 - 2\gamma_1\nu_c\bar{u} + \gamma_1^2\overline{u^2}}{(1-\gamma_1^2)^2} - \frac{1}{(1-\gamma_1^2)}\right]$
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EFFECTIVE LOCAL BIAS EXPANSION (I)

$$\left(\mathfrak{b}_{\rm I}^2\xi_0^{(0)}\right)+\frac{1}{2}\left(\xi_0^{(0)}\mathfrak{b}_{\rm II}^2\xi_0^{(0)}\right):$$

Can be though of as arising from the effective continuous, deterministic local bias expansion:

$$\begin{aligned}\delta_{\rm pk}(\mathbf{x}) &= b_{10}\delta_s(\mathbf{x}) - b_{01}\nabla^2\delta_s(\mathbf{x}) + \frac{1}{2}b_{20}\delta_s^2(\mathbf{x}) - b_{11}\delta_s(\mathbf{x})\nabla^2\delta_s(\mathbf{x}) + \frac{1}{2}b_{02}\left[\nabla^2\delta_s(\mathbf{x})\right]^2 + \dots \\ &= \sigma_0 b_{10}\nu(\mathbf{x}) + \sigma_2 b_{01}u(\mathbf{x}) + \frac{1}{2}\sigma_0^2 b_{20}\nu^2(\mathbf{x}) + \sigma_0\sigma_2 b_{11}\nu(\mathbf{x})u(\mathbf{x}) + \frac{1}{2}\sigma_2^2 b_{02}u^2(\mathbf{x}) + \dots\end{aligned}$$

EFFECTIVE LOCAL BIAS EXPANSION (I)

$$\left(\mathfrak{b}_{\mathrm{I}}^2 \xi_0^{(0)}\right) + \frac{1}{2}\left(\xi_0^{(0)} \mathfrak{b}_{\mathrm{II}}^2 \xi_0^{(0)}\right) :$$

Can be though of as arising from the *effective* *continuous, deterministic local bias expansion:*

$$\begin{aligned}\delta_{\rm pk}(\mathbf{x}) &= b_{10}\delta_s(\mathbf{x}) - b_{01}\nabla^2\delta_s(\mathbf{x}) + \frac{1}{2}b_{20}\delta_s^2(\mathbf{x}) - b_{11}\delta_s(\mathbf{x})\nabla^2\delta_s(\mathbf{x}) + \frac{1}{2}b_{02}\left[\nabla^2\delta_s(\mathbf{x})\right]^2 + \dots \\ &= \sigma_0 b_{10}\nu(\mathbf{x}) + \sigma_2 b_{01} u(\mathbf{x}) + \frac{1}{2}\sigma_0^2 b_{20}\nu^2(\mathbf{x}) + \sigma_0\sigma_2 b_{11}\nu(\mathbf{x})u(\mathbf{x}) + \frac{1}{2}\sigma_2^2 b_{02} u^2(\mathbf{x}) + \dots\end{aligned}$$

SYMMETRIES

- *Peak number density invariant under rotations, so must depend on:*

$$\nu(\mathbf{x})$$

$$u(\mathbf{x}) \equiv -\mathrm{tr}\zeta(\mathbf{x})$$

VD (2013), 1211.4128

2-POINT CORRELATION

At 2nd order by VD, Crocce, Scoccimarro & Sheth (2010):

$$\xi_{\rm pk}(\nu_c,r) = \left(\mathfrak{b}_{\rm I}^2\xi_0^{(0)}\right) + \frac{1}{2}\left(\xi_0^{(0)}\mathfrak{b}_{\rm II}^2\xi_0^{(0)}\right) - \frac{3}{\sigma_1^2}\left(\xi_1^{(1/2)}\mathfrak{b}_{\rm II}\xi_1^{(1/2)}\right) - \frac{5}{\sigma_2^2}\left(\xi_2^{(1)}\mathfrak{b}_{\rm II}\xi_2^{(1)}\right)\left(1+\frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1,\gamma_1\nu_c)\Big|_{\alpha=1}\right)$$
$$+\frac{5}{2\sigma_2^4}\left[\left(\xi_0^{(2)}\right)^2+\frac{10}{7}\left(\xi_2^{(2)}\right)^2+\frac{18}{7}\left(\xi_4^{(2)}\right)^2\right]\left(1+\frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1,\gamma_1\nu_c)\Big|_{\alpha=1}\right)^2$$
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$$\eta^2(\mathbf{x}) \equiv \frac{1}{\sigma_1^2}\left(\nabla\delta_s\right)^2(\mathbf{x})$$

$$\zeta^2(\mathbf{x}) \equiv \frac{3}{2}\mathrm{tr}\left(\tilde{\zeta}^2\right)(\mathbf{x}), \qquad \tilde{\zeta}_{ij} = \zeta_{ij} + \frac{1}{3}u\,\delta_{ij}$$

$$\det\zeta(\mathbf{x})$$

VD (2013), 1211.4128

CORRELATORS

Define $y_\alpha \equiv y(\mathbf{x}_\alpha)$

$$\langle \eta_1^2 \eta_2^2 \rangle = 1 + \frac{2}{3\sigma_1^4}\left[\left(\xi_0^{(1)}\right)^2 + 2\left(\xi_2^{(1)}\right)^2\right]$$

$$\langle \eta_1^2 \nu_2^2 \rangle = 1 + \frac{2}{\sigma_0^2\sigma_1^2}\xi_1^{(1/2)}\xi_1^{(1/2)}$$

$$\langle \zeta_1^2 \eta_2^2 \rangle = 1 + \frac{2}{5\sigma_1^2\sigma_2^2}\left[3\left(\xi_3^{(3/2)}\right)^2 + 2\left(\xi_1^{(3/2)}\right)^2\right]$$

$$\ldots$$

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$$+\frac{5}{2\sigma_2^4}\left[\left(\xi_0^{(2)}\right)^2+\frac{10}{7}\left(\xi_2^{(2)}\right)^2+\frac{18}{7}\left(\xi_4^{(2)}\right)^2\right]\left(1+\frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1,\gamma_1\nu_c)\Big|_{\alpha=1}\right)^2$$

$$+\frac{3}{2\sigma_1^4}\left[\left(\xi_0^{(1)}\right)^2+2\left(\xi_2^{(1)}\right)^2\right]+\frac{3}{\sigma_1^2\sigma_2^2}\left[3\left(\xi_3^{(3/2)}\right)^2+2\left(\xi_1^{(3/2)}\right)^2\right]\left(1+\frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1,\gamma_1\nu_c)\Big|_{\alpha=1}\right)$$

$$\xi_\ell^{(n)}(r) = \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} P_s(k)\, j_\ell(kr)$$

PROBABILITY DENSITIES

Probability distribution $P_1(\mathbf{y})$ *needed to compute* $\bar{n}_{\rm pk}$ *is the multivariate Gaussian*

$$P_1(\mathbf{y})d^{10}\mathbf{y} = \frac{1}{(2\pi)^5|\mathrm{detM}|^{1/2}}e^{-Q_1(\mathbf{y})}d^{10}\mathbf{y}$$

The quadratic form is

$$Q_1(\mathbf{y}) = \frac{\nu^2 + u^2 - 2\gamma_1\nu u}{2(1-\gamma_1^2)} + \frac{3}{2}\eta^2 + \frac{5}{2}\zeta^2$$

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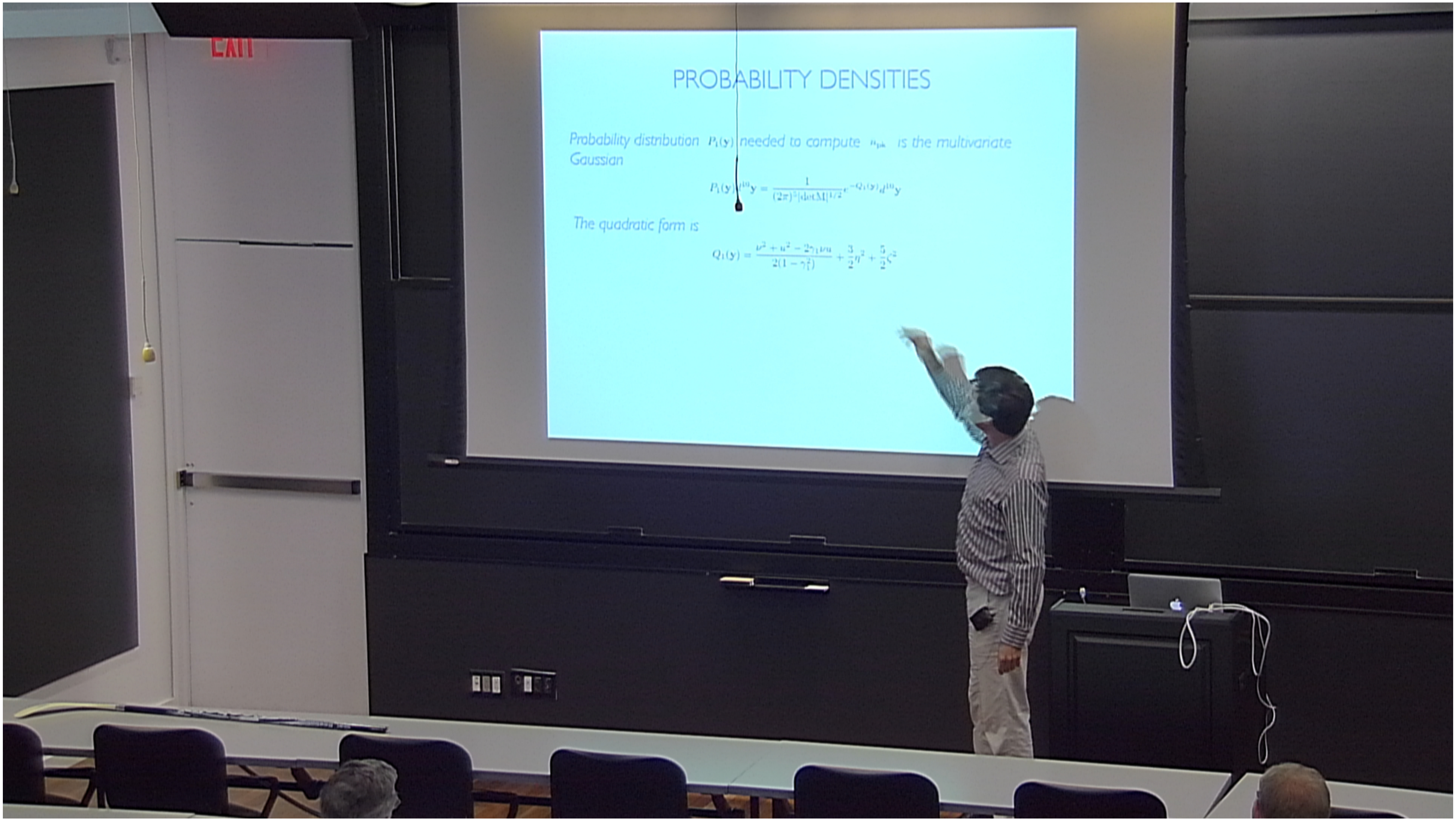
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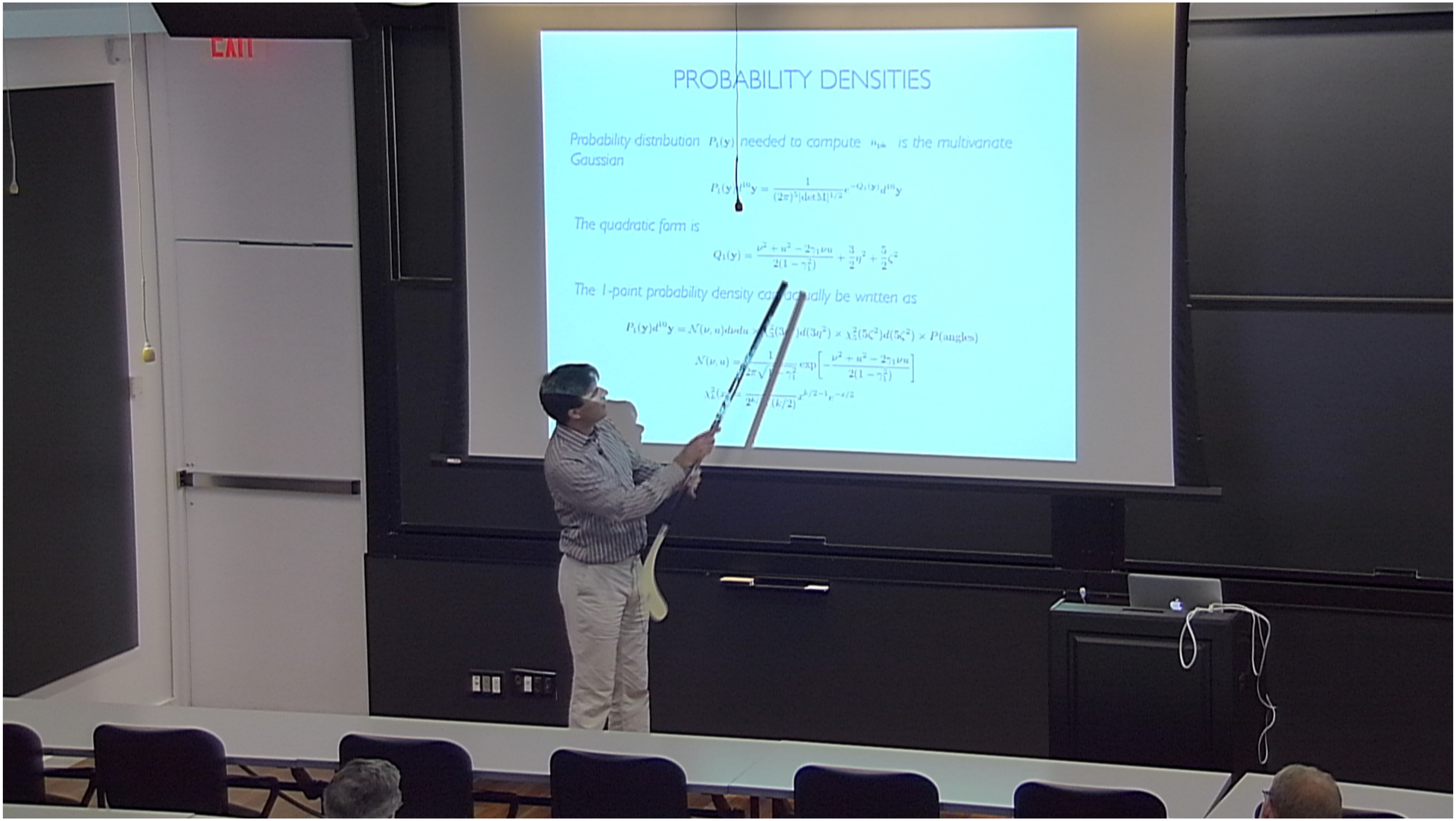

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The 1-point probability density ca ually be written as

GENERALIZING PBS (I)

- *Bias factors* b_{ij} *can be written as derivatives of* $\mathcal{N}(\nu, u)$ *, i.e. they are bivariate Hermite polynomials averaged over all the possible peak configurations:*

$$\sigma_0^i \sigma_2^j b_{ij} \equiv \frac{1}{\bar{n}_{\rm pk}} \int d^{10}\mathbf{y}\, n_{\rm pk}(\mathbf{y}) H_{ij}(\nu, u)\, P_1(\mathbf{y})$$

$$H_{ij}(\nu, u) = \mathcal{N}(\nu, u)^{-1} \left(-\frac{\partial}{\partial \nu}\right)^i \left(-\frac{\partial}{\partial u}\right)^j \mathcal{N}(\nu, u)$$

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- *The same logic must apply to the bias factors associated with* $\eta^2(\mathbf{x})$ *and* $\zeta^2(\mathbf{x})$ *. We thus expect ensemble averages of generalized Laguerre polynomials:*

$$\sigma_1^{2k} \chi_{k0} \equiv \frac{(-1)^k}{\bar{n}_{\rm pk}} \int d^{10}\mathbf{y}\, n_{\rm pk}(\mathbf{y}) L_k^{(1/2)}\!\left(\frac{3\eta^2}{2}\right) P_1(\mathbf{y})$$

$$\sigma_2^{2k} \chi_{0k} \equiv \frac{(-1)^k}{\bar{n}_{\rm pk}} \int d^{10}\mathbf{y}\, n_{\rm pk}(\mathbf{y}) L_k^{(3/2)}\!\left(\frac{5\zeta^2}{2}\right) P_1(\mathbf{y})$$

$$L_n^{(\alpha)}(x) \equiv \frac{x^{-\alpha} e^x}{n!} \frac{d^n}{dx^n}\left(e^{-x} x^{n+\alpha}\right)$$

2-POINT CORRELATION

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$$+\frac{5}{2\sigma_2^4}\left[\left(\xi_0^{(2)}\right)^2 + \frac{10}{7}\left(\xi_2^{(2)}\right)^2 + \frac{18}{7}\left(\xi_4^{(2)}\right)^2\right]\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1\nu_c)\Big|_{\alpha=1}\right)^2$$

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GENERALIZING PBS (II)

- *A straightforward calculation shows that*

$$\chi_{10} = \frac{1}{2\sigma_1^2}\Big(\langle 3\eta^2|\mathrm{pk}\rangle - 3\Big) = -\frac{3}{2\sigma_1^2}$$

$$\chi_{01} = \frac{1}{2\sigma_2^2}\Big(\langle 5\zeta^2|\mathrm{pk}\rangle - 5\Big) = -\frac{5}{2\sigma_2^2}\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1\nu_c)\Big|_{\alpha=1}\right)$$

- *Physical interpretation: "long"-wavelength perturbations locally modulate the mean of the distributions* $\mathcal{N}(\nu, u)$, $\chi_3^2(3\eta^2)$ *and* $\chi_5^2(5\zeta^2)$ *; e.g.*

$$\chi_k'^2(x;\epsilon) = \chi_k^2(x)\sum_{j=0}^{\infty}\frac{\Gamma\left(\frac{1}{2}k\right)}{\Gamma\left(\frac{1}{2}k + j\right)}\left(-\frac{\epsilon}{2}\right)^j L_j^{(\alpha)}\left(\frac{x}{2}\right)$$

χ_{01}:PEAK ASPHERICITY

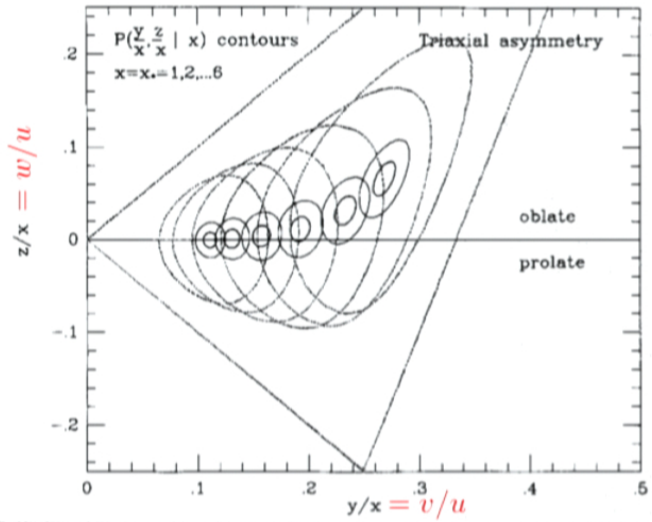

FIG. 7.—The 95%, 90%, and 50% contours of the conditional probability for ellipticity $e = y/x$ and prolateness $p = z/x$ subject to the constraint of given x for peaks (eq. [7.6]). (The x and x_* used here are $1.58 \approx \gamma^{-1}$ times those used in the text, so $\nu = 1, 2, \ldots, 6$ corresponds to the different curves.) This figure demonstrates that, even for high ν, the shapes are triaxial. The values of e and p are constrained to lie in the triangle.

Bardeen et. al. (1986)

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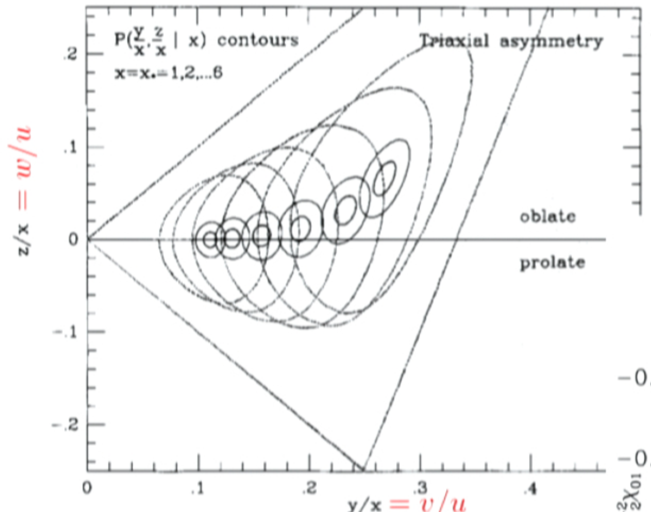

FIG. 7.—The 95%, 90%, and 50% contours of the conditional probability for ellipticity $e = y/x$ and prolateness $p = z/x$ subject to the peaks (eq. [7.6]). (The x and x_* used here are $1.58 \approx \gamma^{-1}$ times those used in the text, so $\nu = 1, 2, \ldots, 6$ corresponds to the different curves.) that, even for high ν, the shapes are triaxial. The values of e and p are constrained to lie in the triangle.

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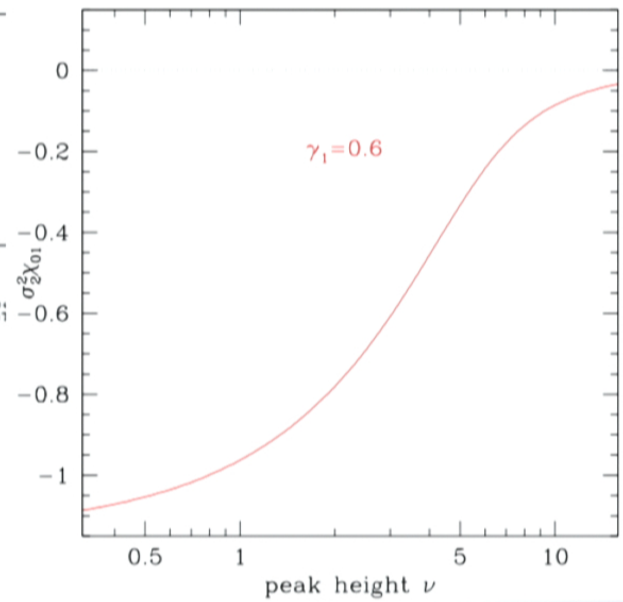
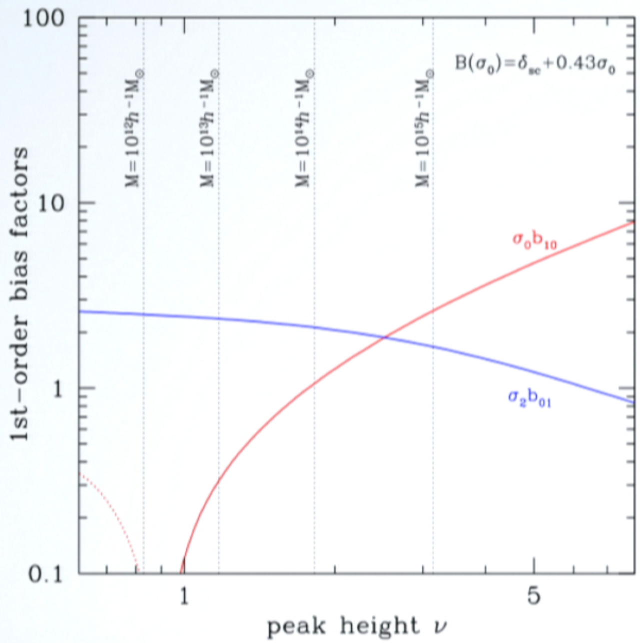
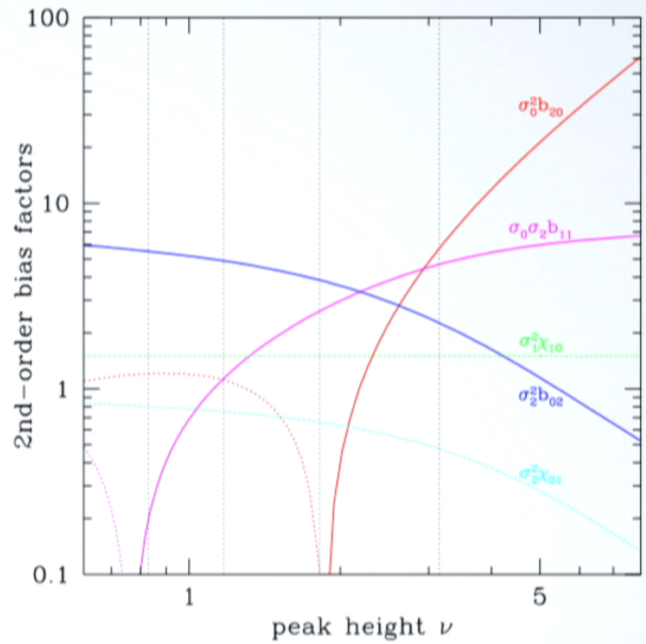


PEAK BIAS FACTORS





EFFECTIVE LOCAL BIAS EXPANSION (II)

$\xi_{\rm pk}(\nu_c, r)$ *can be computed from the local bias expansion*

$$\delta_{\rm pk}(\mathbf{x}) = \sigma_0 b_{10}\nu(\mathbf{x}) + \sigma_2 b_{01} u(\mathbf{x}) + \frac{1}{2}\sigma_0^2 b_{20}\nu^2(\mathbf{x}) + \sigma_0\sigma_2 b_{11}\nu(\mathbf{x})u(\mathbf{x}) + \frac{1}{2}\sigma_2^2 b_{02} u^2(\mathbf{x}) + \sigma_1^2\chi_{10}\eta^2(\mathbf{x}) + \sigma_2^2\chi_{01}\zeta^2(\mathbf{x}) + \dots$$

provided we ignore all correlators involving zero-lag moments

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Speculation: exact to all orders and valid for any point process of a Gaussian random field specified as a set of "localized" constraints

COUNTING DARK MATTER HALOS

Image credit: Acquarius Project (Springel et. al. 2008)

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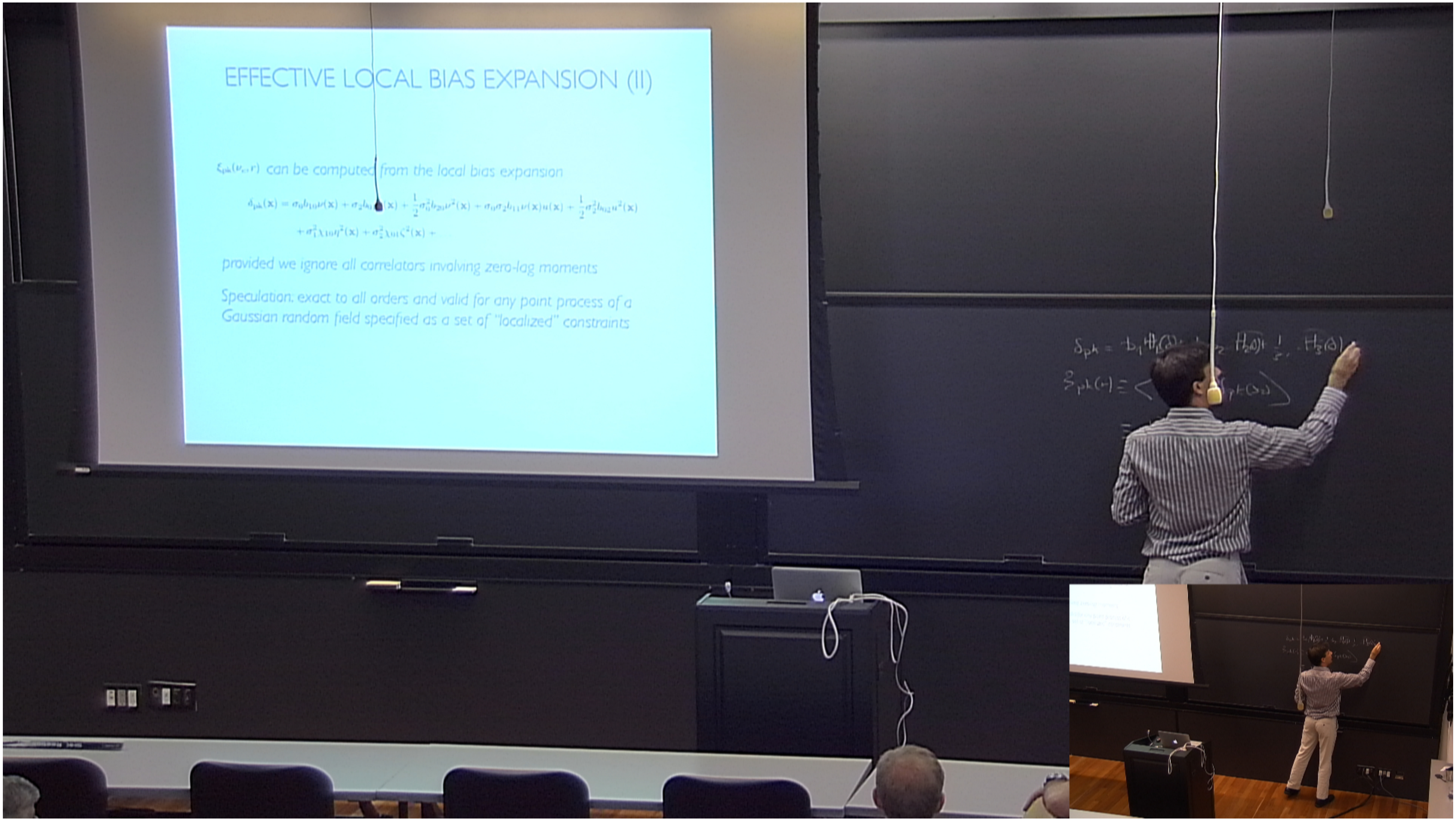
EFFECTIVE LOCAL BIAS EXPANSION (II)
$\xi_{pk}(\nu_c,r)$ can be computed from the local bias expansion
provided we ignore all correlators involving zero-lag moments
Speculation: exact to all orders and valid for any point process of a Gaussian random field specified as a set of "localized" constraints

$$\delta_{pk} = b_1 H_1(\delta) + \frac{1}{2} b_2 H_2(\delta) + \frac{1}{3!} H_3(\delta) + \dots$$

$$\xi_{pk}(r) \equiv \langle \delta_{pk}(\underline{x}_1)\, \delta_{pk}(\underline{x}_2) \rangle$$

$$\equiv b_1^2\, \xi(r)$$

COUNTING DARK MATTER HALOS

Image credit: Acquarius Project (Springel et. al. 2008)

EXCURSION SET PEAKS (I)

How do we associate halos of mass M with peaks of the initial density field ?


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- *Smooth the linear density field with a tophat filter on scale* $R_s \propto (M/\bar{\rho})^{1/3}$
- *Select all the peaks with height* $\nu = \nu_c$ *and (Appel & Jones 1991)*

$$\delta(R_s - \Delta R_s) > \delta_c\,, \quad \delta(R_s + \Delta R_s) < \delta_c$$

Approximation to the first upcrossing condition


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Approximation to the first upcrossing condition
EXIT

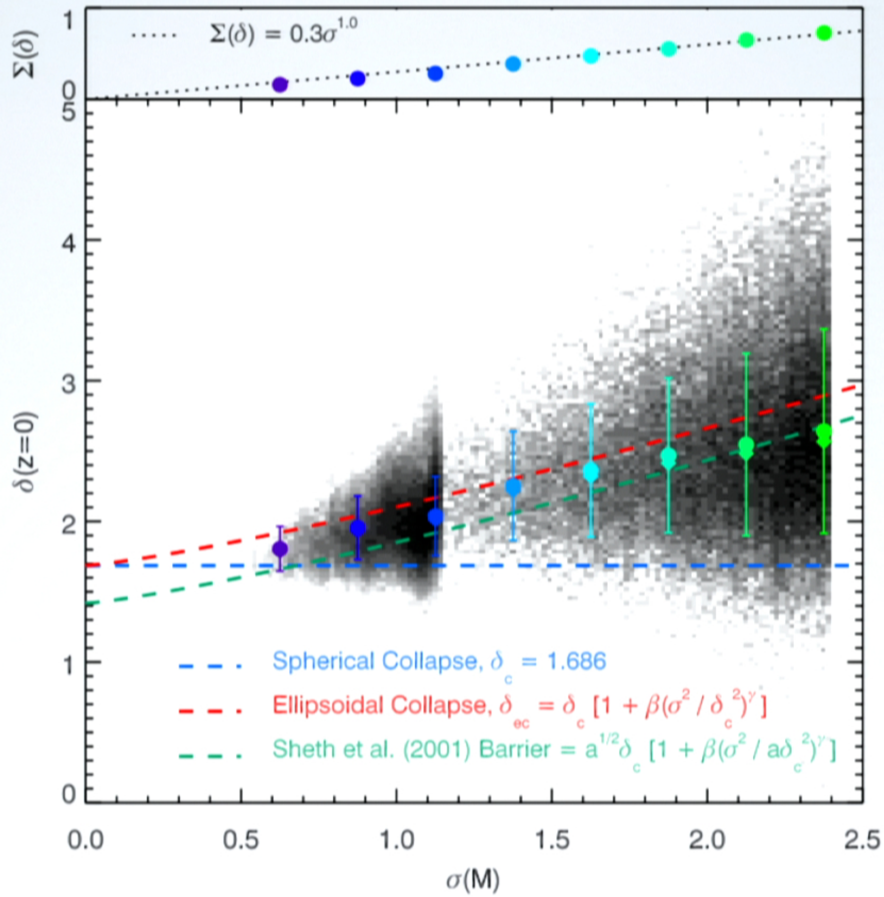
HALO MASS FUNCTION (I)

- *The first (up)crossing distribution is*

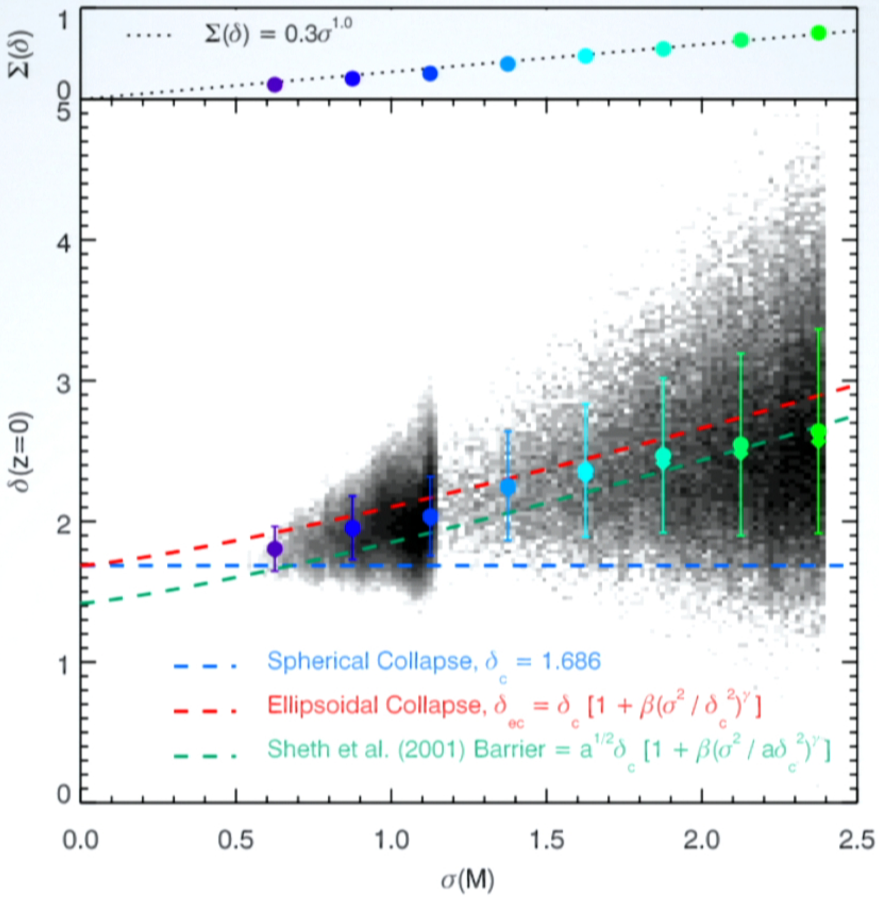
$$f_{\rm ESP}(\nu_c) = \frac{M}{\bar{\rho}} \bar{n}_{\rm UC}(\nu_c, R_s) \frac{dR_s}{d\nu_c}$$

and the halo abundance

$$\bar{n}_{\rm h}(M) = \frac{\bar{\rho}}{M^2} \nu_c f_{\rm ESP}(\nu_c, R_s) \frac{d \ln \nu_c}{d \ln M}$$



Robertson et. al. (2009)



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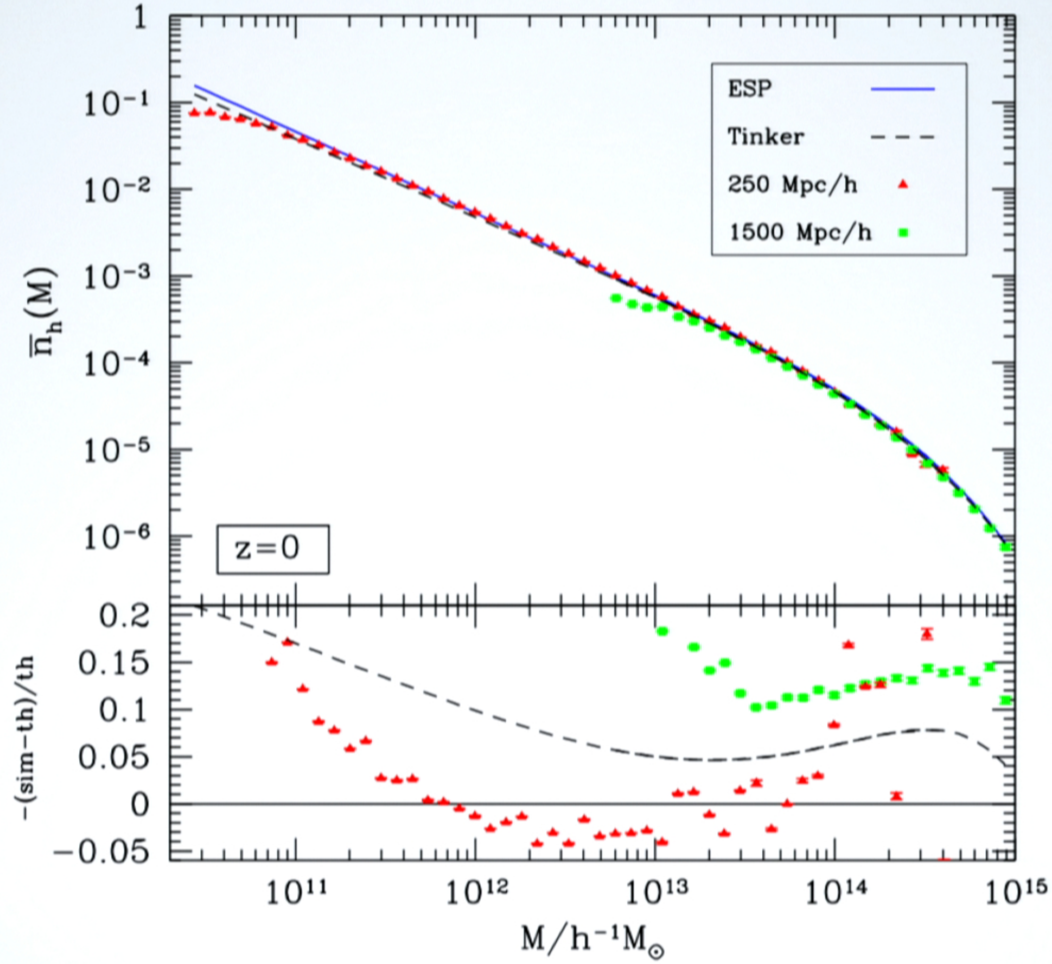

Paranjape, Sheth, VD (2013), Biagetti et. al. (in prep.)