Title: Orbifolds and topological defects

Date: Sep 17, 2013 02:00 PM

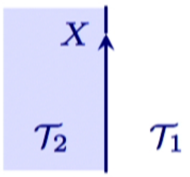
URL: http://pirsa.org/13090061

Abstract: Orbifolding a
2-dimensional quantum field theory by a symmetry group admits an elegant description in terms of defect lines and their junction fields. This perspective offers a natural generalization of the concept of an orbifold, in which the role of the symmetry group is replaced by a defect with the structure of a (symmetric) separable Frobenius algebra. In this talk I will focus on the case of Landau-Ginzburg models, in which defects are described by matrix factorizations. After introducing the generalized twisted sectors and discussing topological bulk and boundary correlators in these sectors, I will present a simple proof of the Cardy condition and discuss some further consistency checks on the generalized orbifold theory. This talk is based on arXiv:1307.3141 with Ilka Brunner and Nils Carqueville.

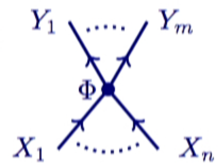
Motivation

- orbifolds with symmetry group G can be described via **defects** A_G
- **generalization**: allow any defect A with appropriate algebraic structure
- standard results on orbifolds recovered in simpler/more conceptual way
  $\rightsquigarrow$ carries over to the generalized setting
- case study: $\mathcal{N} = 2$ **Landau-Ginzburg** models:
  - description of many $\mathcal{N} = 2$ CFTs, stringy regime of **CY compactifications**
  - explicit description of defects via matrix factorizations
  - compute **arbitrary topological correlators** in the generalized orbifold theory: bulk/boundary correlators ($\rightsquigarrow$ **eff. superpotentials, D-brane charges**), defect actions on bulk fields, ...

Defects in 2D field theories



- A **defect** X is a 1D interface between two theories $\mathcal{T}_1$ and $\mathcal{T}_2$ together with a gluing condition on fields
- special case – boundary condition ($\mathcal{T}_1$ or $\mathcal{T}_2$ trivial)
- folding trick – view defect as b.c. in doubled theory $\mathcal{T}_1 \otimes \overline{\mathcal{T}}_2$
  $\overline{\mathcal{T}}_2$ – left-$\leftrightarrow$right-movers interchanged
- defects can form **junctions** $\rightsquigarrow$ junction fields:



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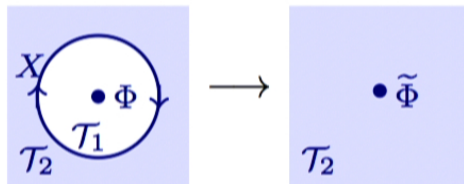
- necessary condition for $\exists$ of top. defects: $c^{(1)} = c^{(2)},\ \bar{c}^{(1)} = \bar{c}^{(2)}$

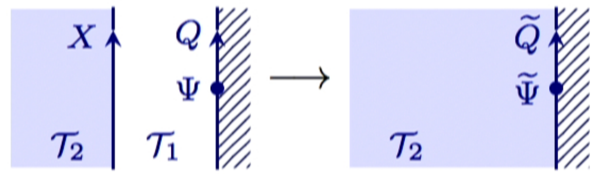
Defects in 2D field theories

- correlators are invariant under deformations of top. defects
- use this to map objects of $\mathcal{T}_1$ to objects of $\mathcal{T}_2$ (and vice versa):

bulk fields:



boundary (fields):



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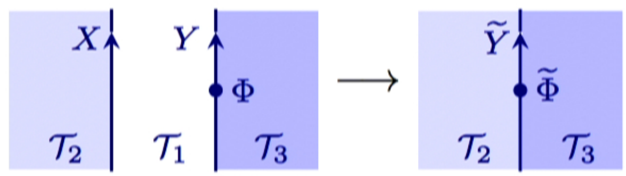

- for non-topological defects these operations are usually singular
- some aplications: RG flows, D-brane monodromies, order-disorder dualities, ...
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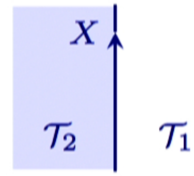

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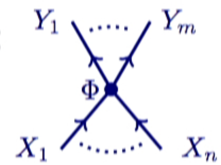

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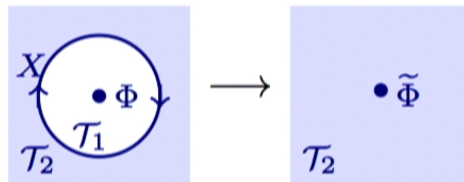

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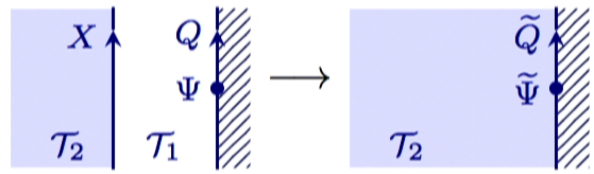

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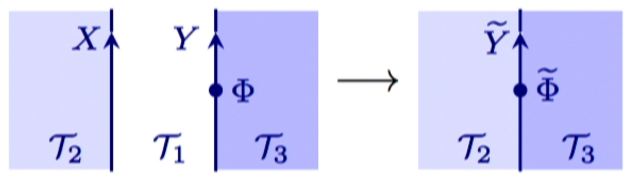

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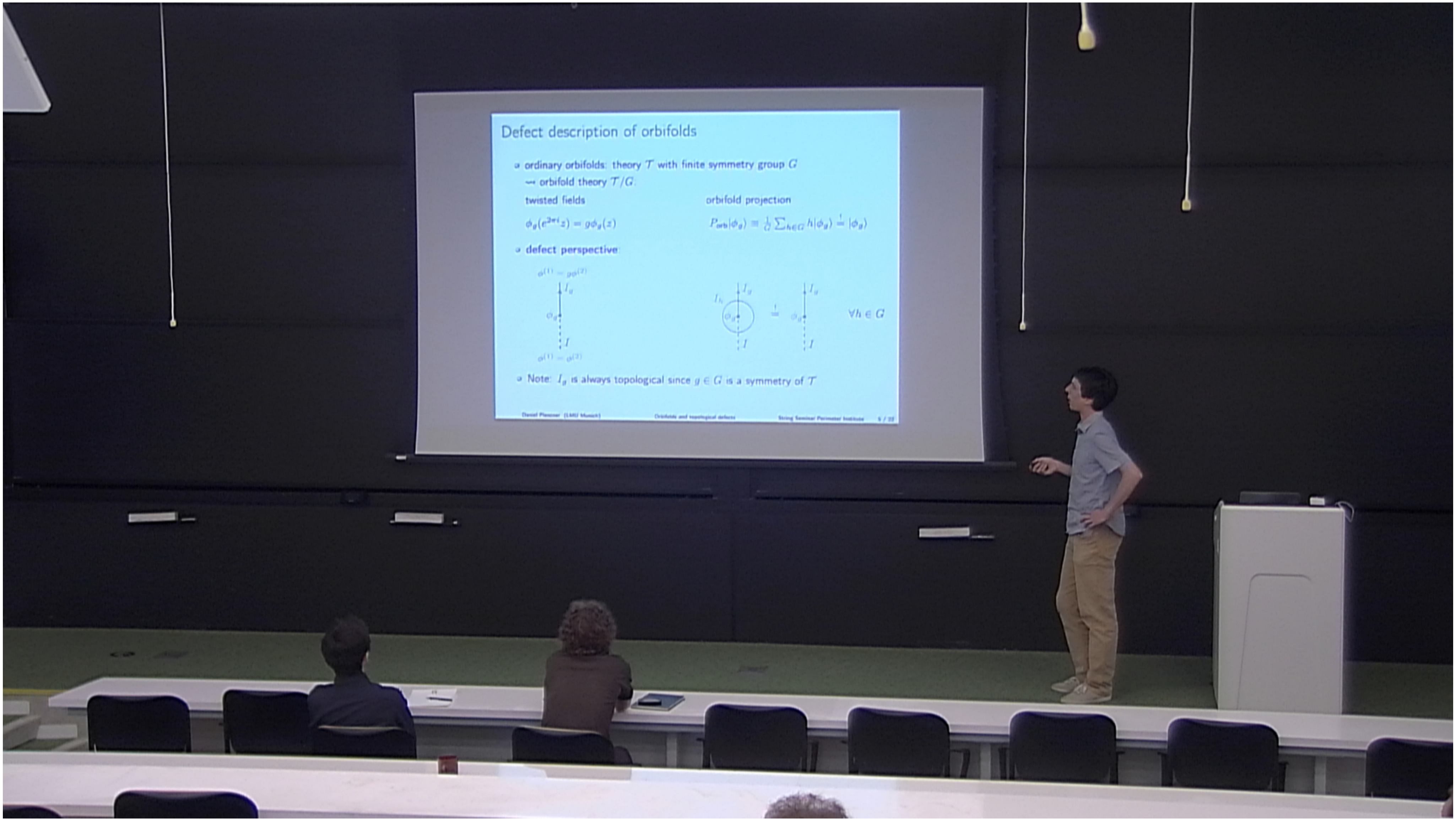

Defect description of orbifolds
ordinary orbifolds: theory $\mathcal{T}$ with finite symmetry group G
$\rightsquigarrow$ orbifold theory $\mathcal{T}/G$:
twisted fields
orbifold projection
$\phi_g(e^{2\pi i}z) = g\phi_g(z)$
$P_{orb}|\phi_g\rangle = \frac{1}{|G|}\sum_{h\in G} h|\phi_g\rangle \stackrel{!}{=} |\phi_g\rangle$
defect perspective:
$\phi^{(1)} = g\phi^{(2)}$
I_g
ϕ_g
I
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I_h
$\forall h \in G$
Note: I_g is always topological since $g \in G$ is a symmetry of $\mathcal{T}$

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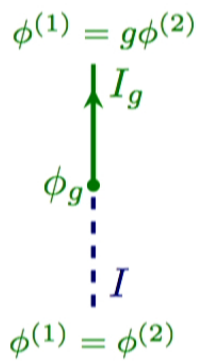

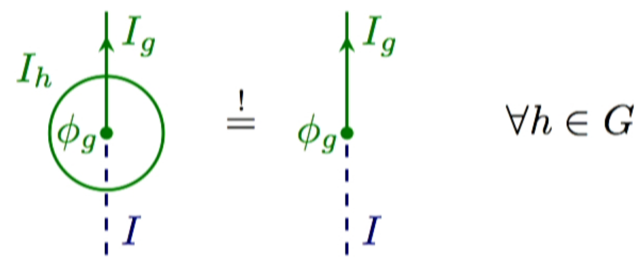

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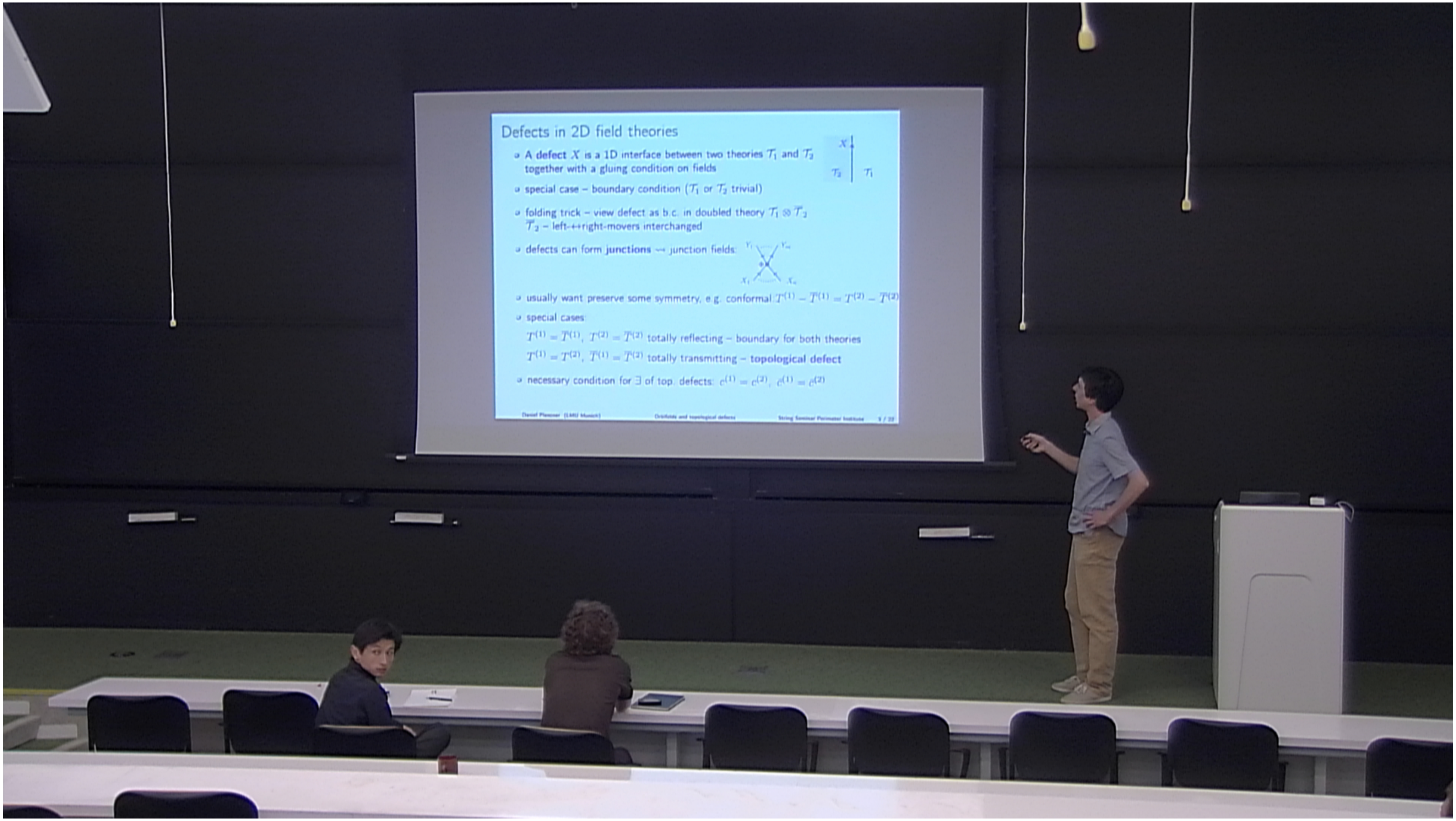
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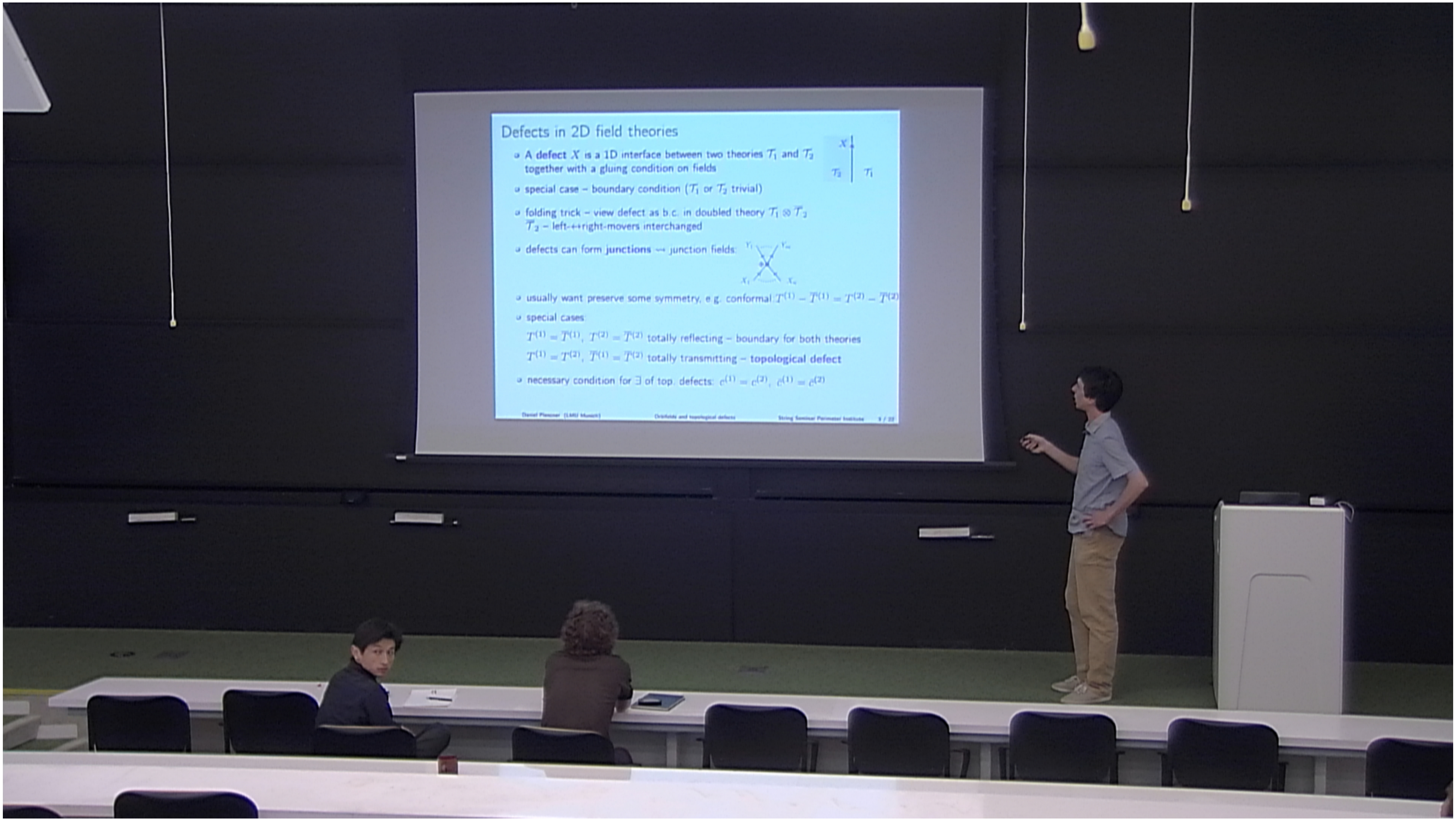
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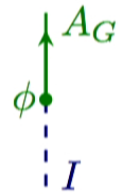
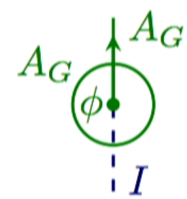
Generalized orbifolds

Assemble I_g to $A_G = \bigoplus_{g \in G} I_g$

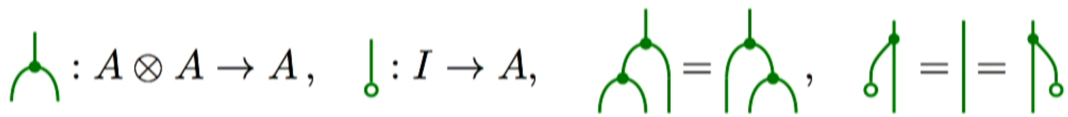
twisted fields – elements in $\mathrm{Hom}(I, A_G)$



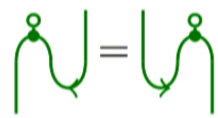
orbifold projector

$P_{\mathrm{orb}}(\phi) =$ 

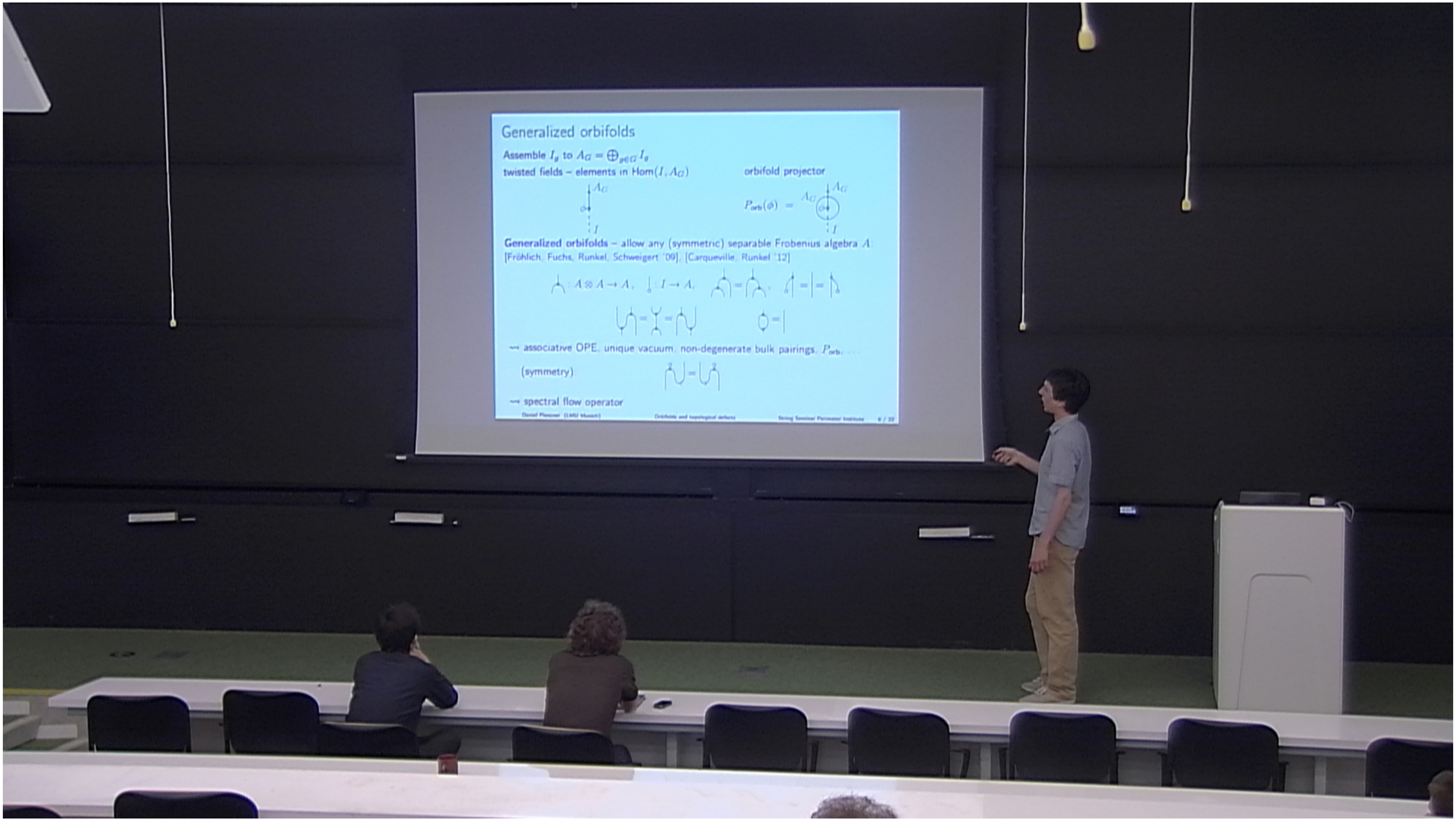
Generalized orbifolds – allow any (symmetric) separable Frobenius algebra A:
[Fröhlich, Fuchs, Runkel, Schweigert '09], [Carqueville, Runkel '12]



$\rightsquigarrow$ associative OPE, unique vacuum, non-degenerate bulk pairings, P_{orb}, ...

(symmetry): 

$\rightsquigarrow$ spectral flow operator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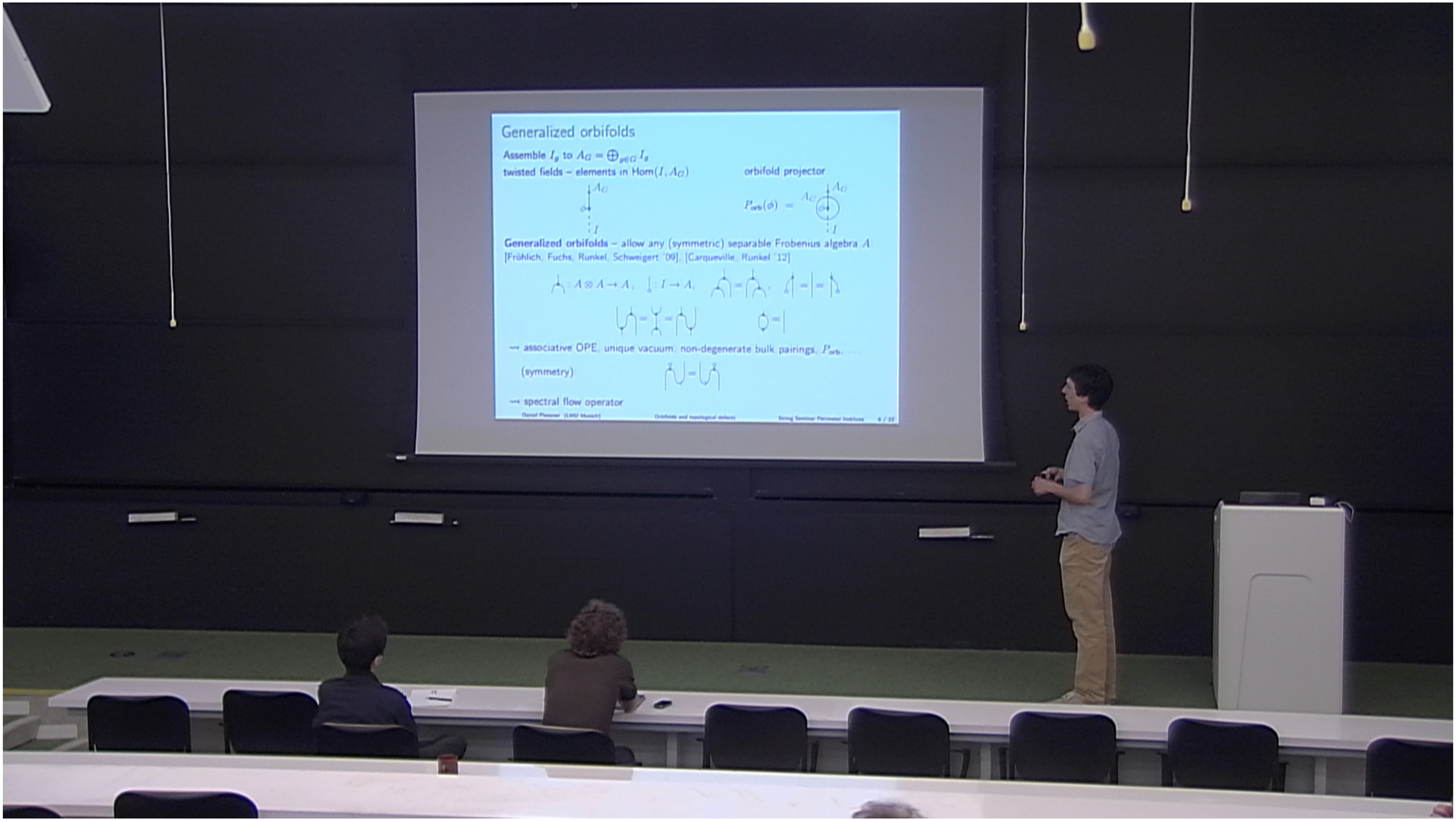
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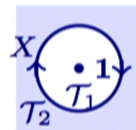
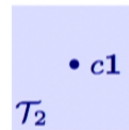
- **important class** $A = X^\dagger \otimes X$, where $X : \mathcal{T}_1 \to \mathcal{T}_2$ is a top. defect with invertible $\dim(X)$, i.e. 

$=$ 

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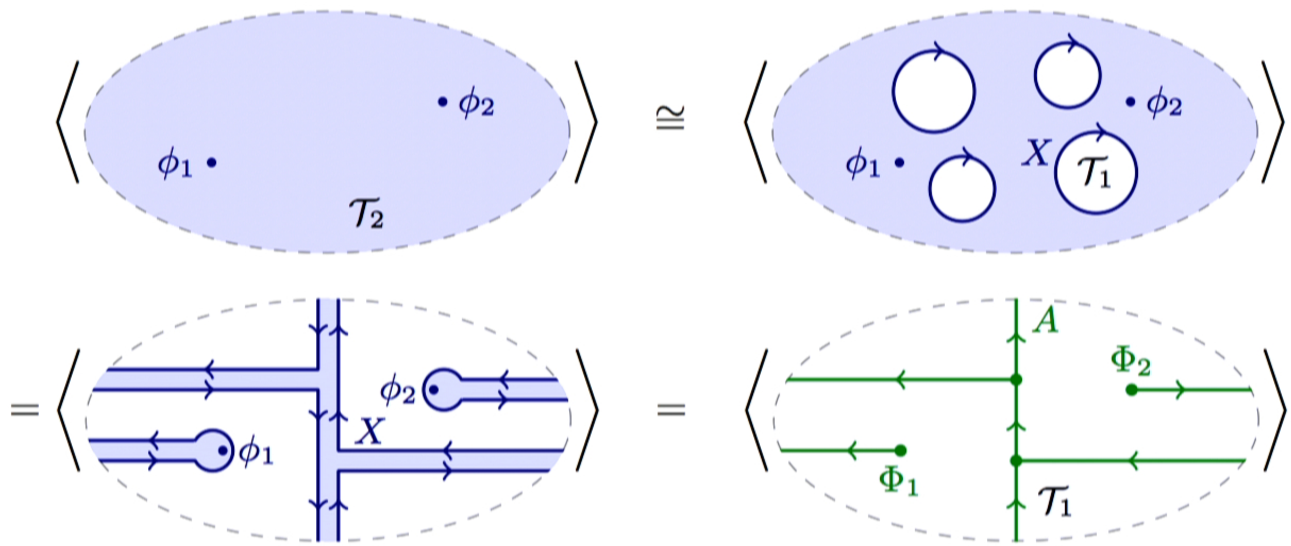

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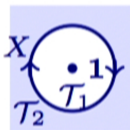
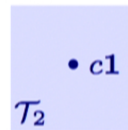
- **important class** $A = X^\dagger \otimes X$, where $X : \mathcal{T}_1 \to \mathcal{T}_2$ is a top. defect with invertible dim(X), i.e. 

$=$ 

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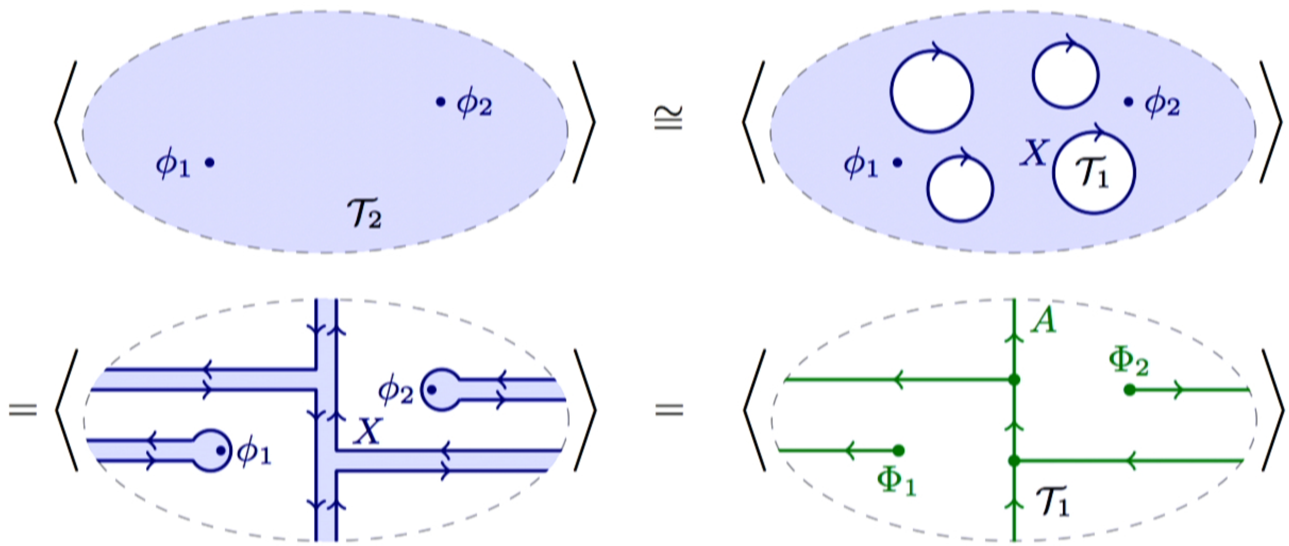

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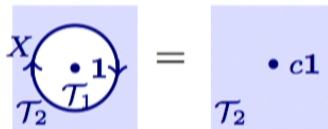
- **important class** $A = X^\dagger \otimes X$, where $X : \mathcal{T}_1 \to \mathcal{T}_2$ is a top. defect with invertible $\dim(X)$, i.e. 

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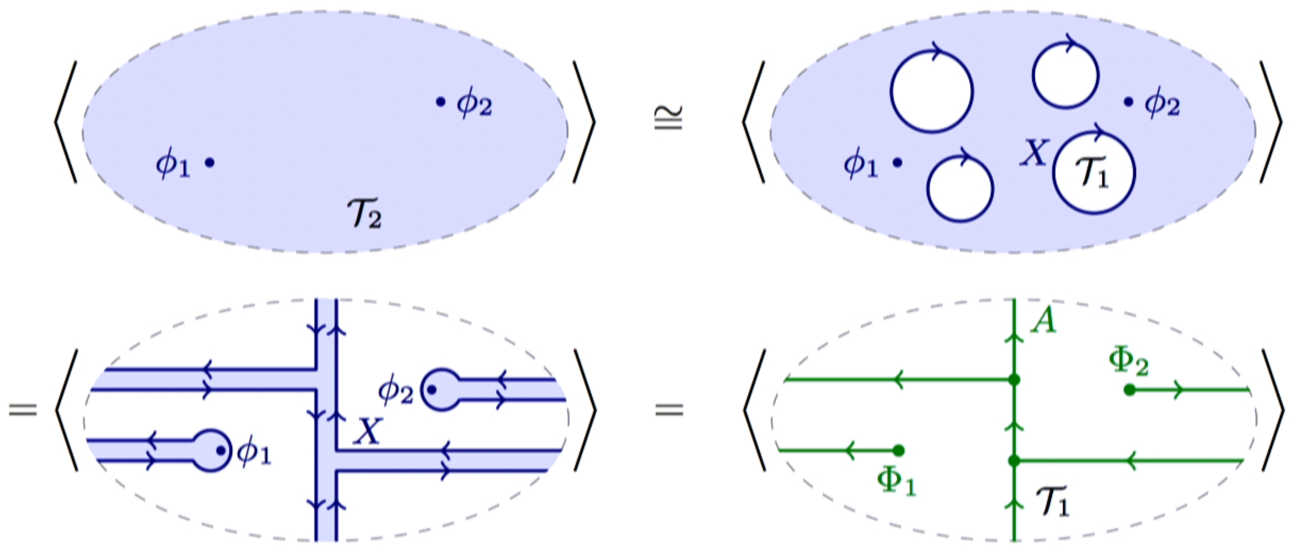

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- similarly with D-branes (defects) – twisted branes in $\mathcal{T}_1 \leftrightarrow$ branes in $\mathcal{T}_2$
  $\rightsquigarrow$ equivalence of D-brane categories

- for **rational CFTs** (e.g. minimal models or WZW models) one has the result: [Fröhlich, Fuchs, Runkel, Schweigert '09]

  Any two consistent rational CFTs with identical central charge and identical left and right symmetry algebras are related by a generalized orbifold construction.

- example: minimal models (ADE classification)

  A and D series related by a $\mathbb{Z}_2$-orbifold

  A and E series related by a generalized orbifold

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Landau-Ginzburg models

- $\mathcal{N} = 2$ LG models are 2D QFTs with action:

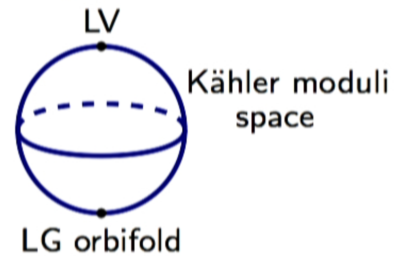
$$S = \int d^2z\, d^4\theta\, K(X_i, \bar{X}_i) + \frac{1}{2}\left(\int d^2z\, d^2\theta\, W(X_i)|_{\bar{\theta}^{\pm}=0} + c.c.\right)$$

  X_i – chiral superfields, $K(X_i, \bar{X}_i)$ – Kähler potential
  $W(X_i)$ – **superpotential**

- LG not conformal, but for W quasi-homogeneous $\rightsquigarrow$ flow to an IR fixed pt.
- **CFT in IR** characterized solely by W. Can extract information about the CFT from properties of W
- e.g. chiral primary fields $\leftrightarrow$ Jacobi ring $\frac{\mathbb{C}[X_i]}{(\partial W)}$

Landau-Ginzburg models

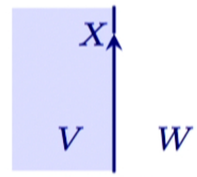
- Many $\mathcal{N} = 2$ CFTs described as IR fixed pts. of LG orbifolds, e.g.:
  - $\mathcal{N} = 2$ minimal models, Kazama-Suzuki models
  - **CY compactifications** in stringy regime of Kähler moduli space, e.g. CY hypersurface $W = 0 \leftrightarrow$ LG orbifold with W and $G = \mathbb{Z}_{\mathrm{deg}(W)}$, generalizations to hypersurfaces in toric varieties, ... [Witten '93]
  - orbifolds/tensor products thereof, ...



- from here on: topologically B-twisted LG models

Defects in LG models

- Defects between LG models with superpotentials $W(x_1,..,x_n)$, $V(z_1,..,z_m)$ described by **matrix factorizations** of $V-W$. [Brunner, Roggenkamp '07] [Khovanov, Rozansky '04] [Kapustin, Li '02]

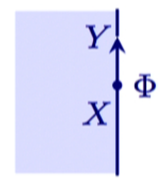


- A matrix factorization of a polynomial $p(u_i)$ is a pair (X, d_X), where X is a $\mathbb{Z}_2$-graded $\mathbb{C}[u_i]$-module and d_X is an odd operator s.t.

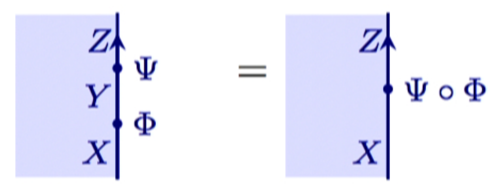
$$d_X^2 = p(u_i) \cdot 1_X$$

- Junction fields Φ from X to Y given by maps in the **cohomology** $\mathrm{Hom}(X,Y)$ of the operator

$$D_{XY}\Phi = d_Y\Phi - (-1)^{|\Phi|}\Phi\, d_X$$



- Defect OPE given by composition of maps



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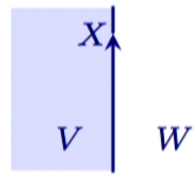

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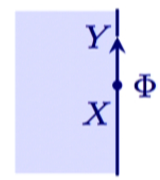

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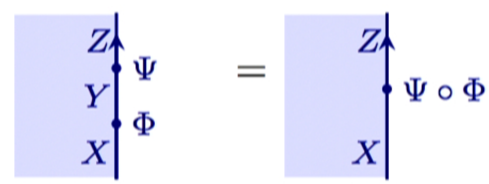
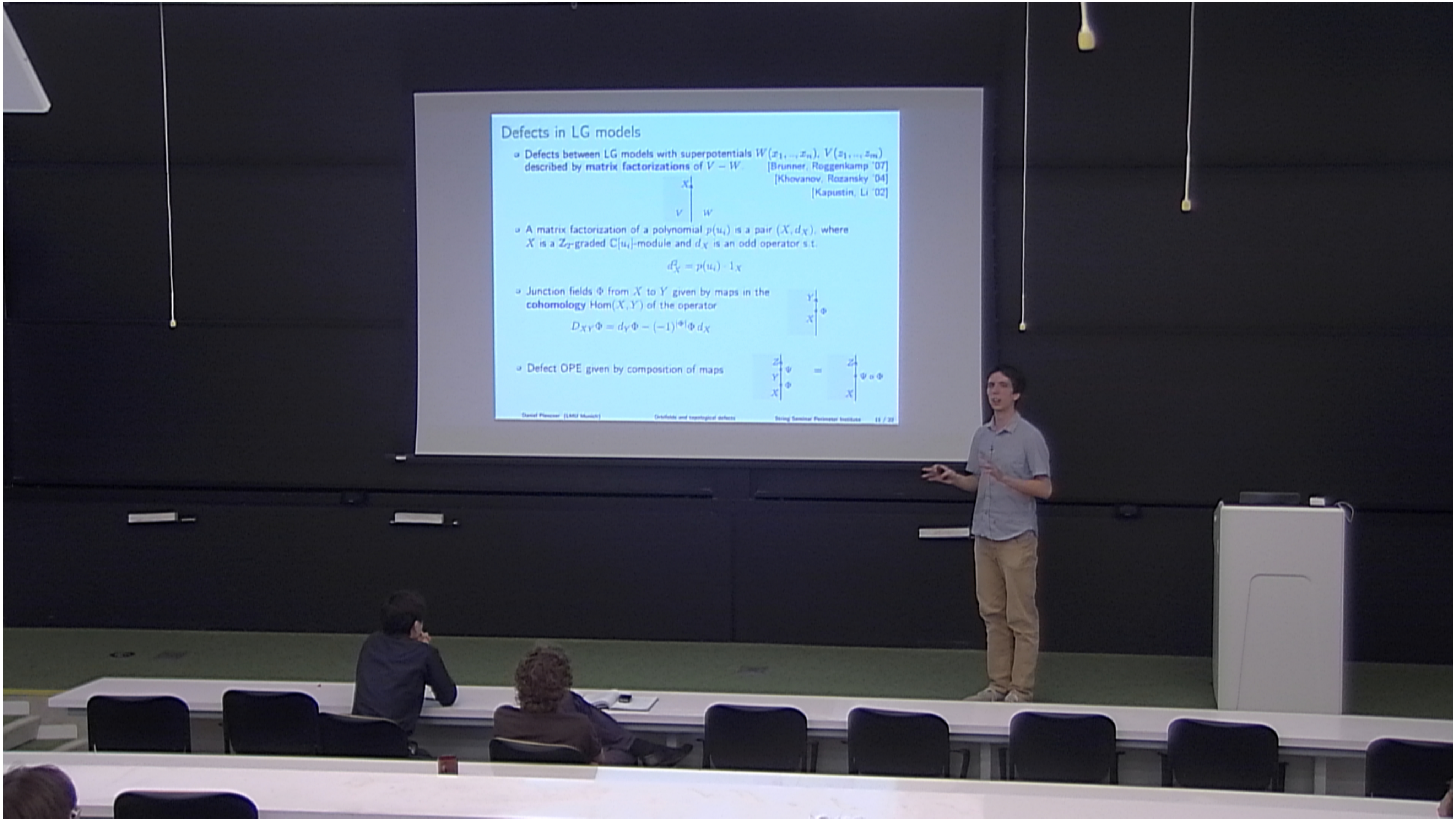
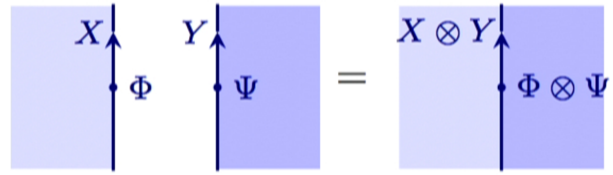



Defects in LG models
Defects between LG models with superpotentials $W(x_1, \ldots, x_n)$, $V(z_1, \ldots, z_m)$ described by **matrix factorizations** of $V - W$.
[Brunner, Roggenkamp '07]
[Khovanov, Rozansky '04]
[Kapustin, Li '02]
X
V W
A matrix factorization of a polynomial $p(u_i)$ is a pair (X, d_X), where X is a $\mathbb{Z}_2$-graded $\mathbb{C}[u_i]$-module and d_X is an odd operator s.t.
$d_X^2 = p(u_i) \cdot 1_X$
Junction fields Φ from X to Y given by maps in the **cohomology** $\mathrm{Hom}(X, Y)$ of the operator
$D_{XY}\Phi = d_Y\Phi - (-1)^{|\Phi|}\Phi\, d_X$
Y Φ X
Defect OPE given by composition of maps
Z Ψ Y Φ X = Z Ψ∘Φ X

Defects in LG models

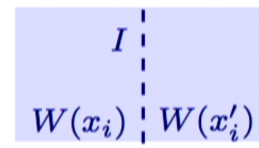
- Defect fusion given by $\otimes$ of m.f.

  $$d_{X\otimes Y} = d_X \otimes 1_Y + 1_X \otimes d_Y$$



- identity defect:

  $$I = \mathbb{C}[x,x']^{\oplus 2}, \qquad d_I = \begin{pmatrix} 0 & x - x' \\ \frac{W(x)-W(x')}{x-x'} & 0 \end{pmatrix}$$



  for one variable. In general:

  $$I = \bigwedge \Big(\bigoplus_i^n \mathbb{C}[x,x'] \cdot \theta_i \Big), \qquad d_I = \sum_{i=1}^n \left((x_i - x'_i) \cdot \theta_i^* - (\partial_{[i]} W) \cdot \theta_i \right)$$

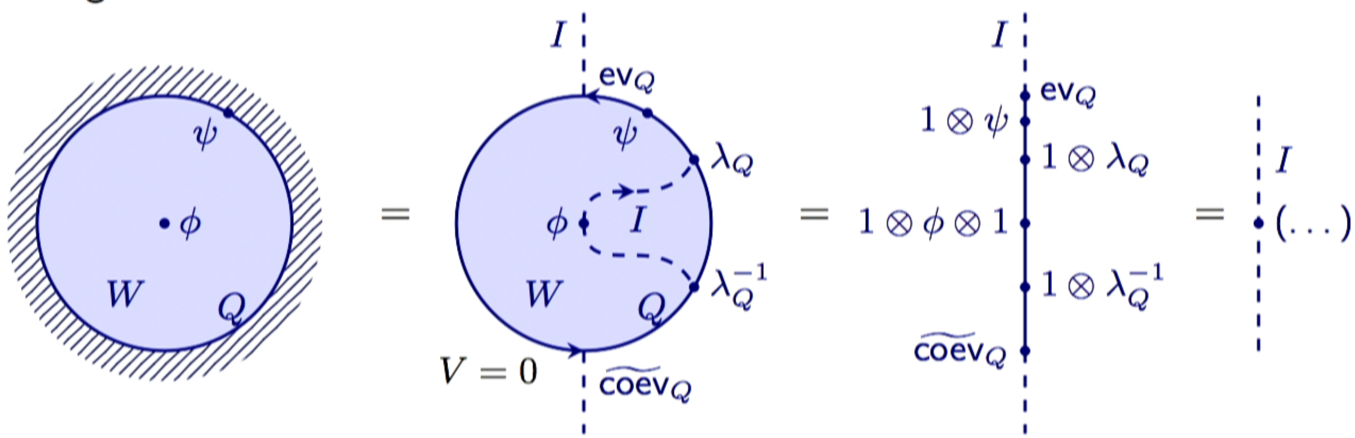
  with $\{\theta_i, \theta_j^*\} = \delta_{ij}$

- symmetry defects – for every $g \in G$ s.t. $W(gx_i) = W(x_i)$ define

  $$_gI = \bigwedge \Big(\bigoplus_i^n \mathbb{C}[x,x'] \cdot \theta_i \Big), \quad d_{_gI} = \sum_{i=1}^n \Big((gx_i - x'_i) \cdot \theta_i^* - (\partial_{[i]} W)\big|_{x \to gx} \cdot \theta_i \Big)$$

Defects in LG models

- one can compute **arbitrary topological correlators** using defects and m.f.
- first express everything in terms of defects, then evaluate using m.f. by composing horizontally ($\otimes$) and vertically ($\circ$)
- e.g. disk correlator:



$$= \mathrm{Res}\left[\frac{\phi\ \mathrm{STr}\left[\partial_{x_1} d_Q \dots \partial_{x_m} d_Q\ \psi\right] \underline{\mathrm{d}x}}{\partial_{x_1} W \dots \partial_{x_m} W}\right]$$

- λ, ev, coev are canonical maps known explicitly for any defect [Carqueville, Murfet '12]
- natural language for top. defects – bicategories with adjoints

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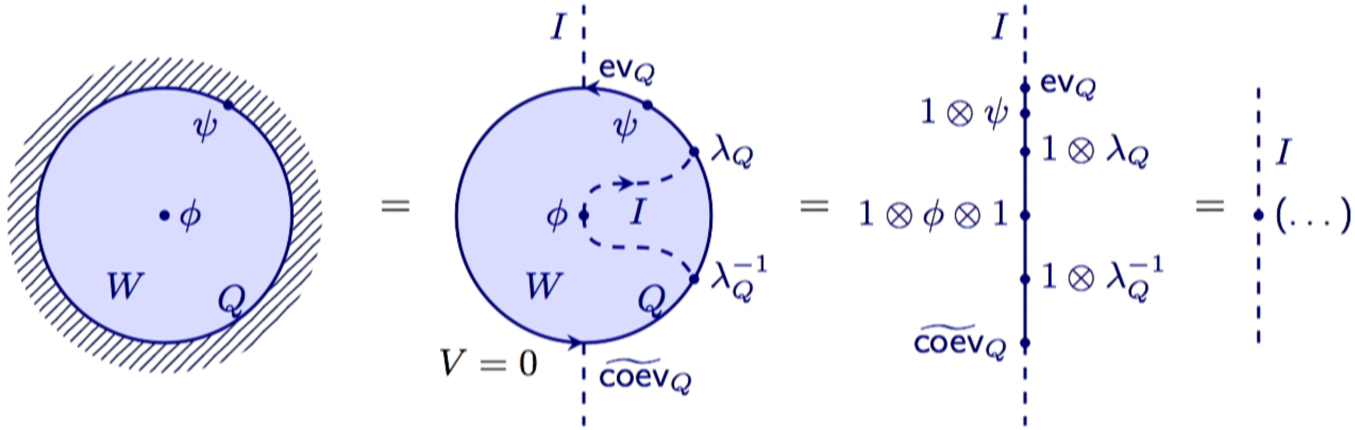

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LG orbifolds – bulk sector

study twisted **RR g.s.** and **(c,c)-fields** [Intriligator, Vafa '90]

unprojected RR g.s. in g-th sector

$|\phi_g\rangle = \prod_{\Theta_i^g \in \mathbb{Z}} (X_i)^{l_i} |0\rangle^g_{RR}$

action of $h \in G$

$hX_i h^{-1} \equiv h_i^j X_i = e^{2\pi i \Theta_i^h} X_i$

$h|0\rangle^g_{RR} = \det(h)\; e^{2\pi i \sum_{\Theta_i^h \in \mathbb{Z}} \Theta_i^g} \; |0\rangle^g_{RR}$

defect/m.f. perspective:

compute cohomology Hom(I, I_g)

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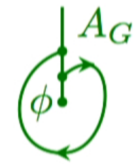
[Brunner, Roggenkamp '07]

evaluate the diagram

$P^{RR}_{\text{orb}}(\phi) =$ 

$\rightsquigarrow$ reproduces the spectrum and phases above [Brunner, Carqueville, DP '13]

(c,c)-fields similarly – unprojected spectrum iso to RR g.s. via spectral flow,

but different representation of G – reproduced by $P^{(c,c)}_{\text{orb}}(\phi) =$ 

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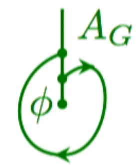
$$P^{RR}_{\text{orb}}(\phi) =$$ 

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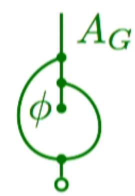
but different representation of G – reproduced by $P^{(c,c)}_{\text{orb}}(\phi) =$ 

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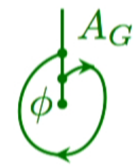
$P_{\text{orb}}^{RR}(\phi) =$ 

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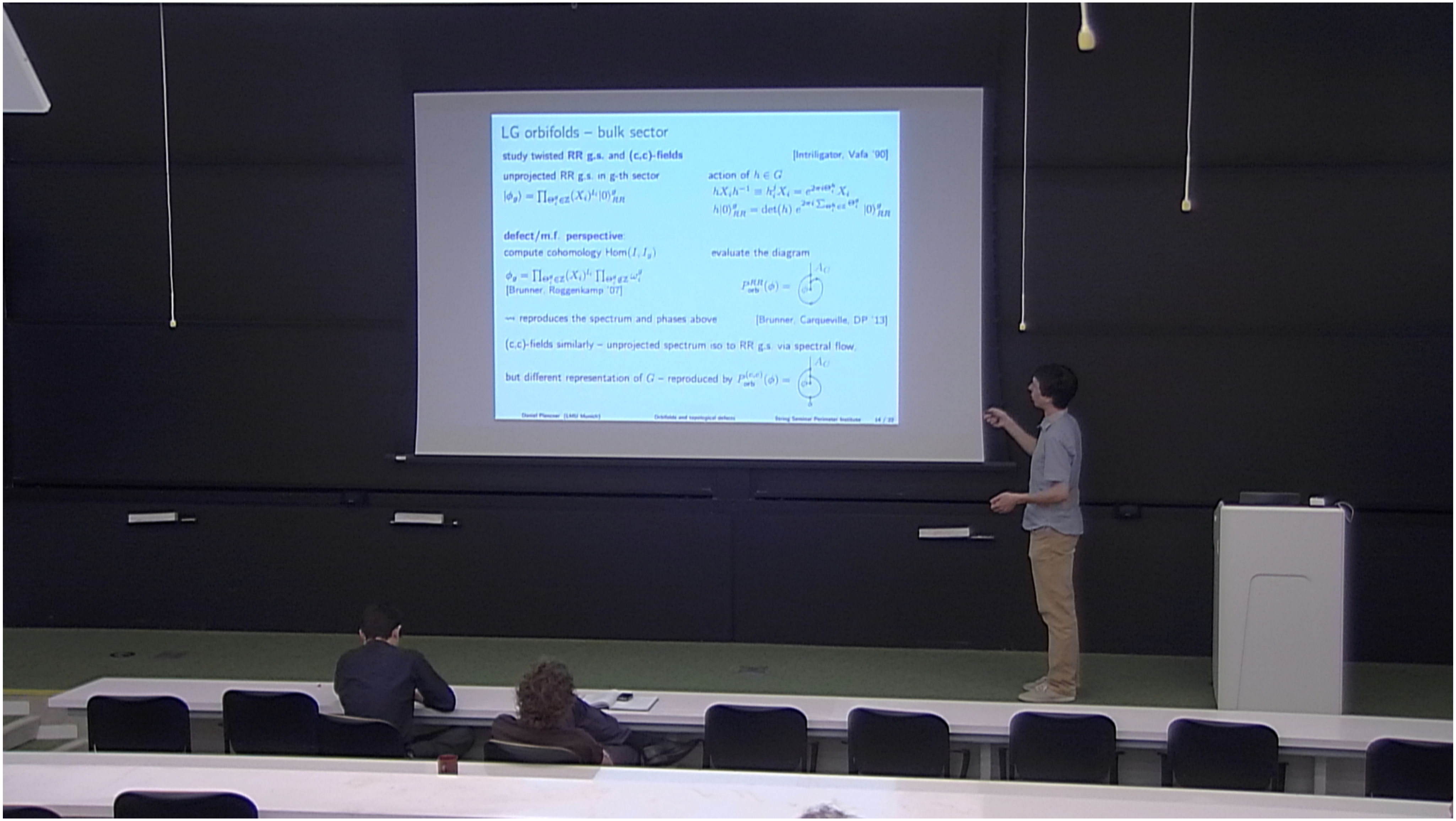
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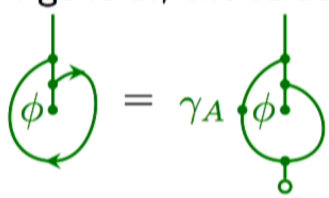
A = X⁺ ⊗ X

Generalized LG orbifolds – spectral flow

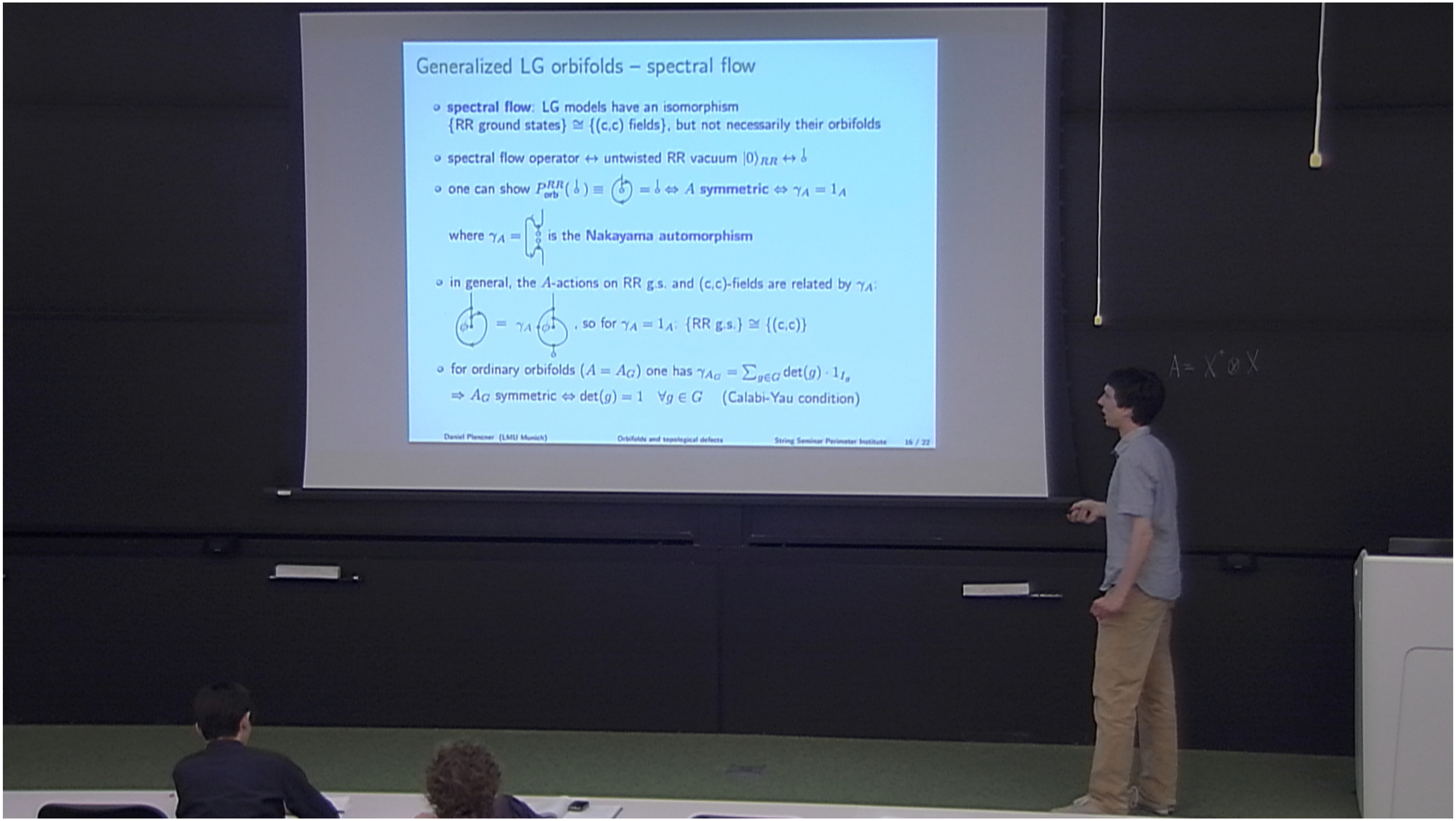
- **spectral flow**: LG models have an isomorphism
  {RR ground states} $\cong$ {(c,c) fields}, but not necessarily their orbifolds
- spectral flow operator $\leftrightarrow$ untwisted RR vacuum $|0\rangle_{RR} \leftrightarrow$
- one can show $P^{RR}_{\text{orb}}(\;) \equiv$  $= \; \Leftrightarrow A$ **symmetric** $\Leftrightarrow \gamma_A = 1_A$

  where $\gamma_A =$  is the **Nakayama automorphism**
- in general, the A-actions on RR g.s. and (c,c)-fields are related by γ_A:

  , so for $\gamma_A = 1_A$: {RR g.s.} $\cong$ {(c,c)}
- for ordinary orbifolds ($A = A_G$) one has $\gamma_{A_G} = \sum_{g \in G} \det(g) \cdot 1_{I_g}$

  $\Rightarrow A_G$ symmetric $\Leftrightarrow \det(g) = 1 \quad \forall g \in G \qquad$ (Calabi-Yau condition)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Daniel Plencner (LMU Munich)
Orbifolds and topological defects
String Seminar Perimeter Institute
16 / 22
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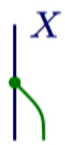
LG orbifolds – boundary sector

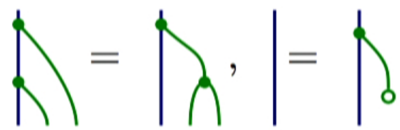
- a boundary Q in LG orbifold is described by a G-**equivariant matrix factorization**, i.e. there is a representation γ of G on Q, s.t.

$$\gamma\, d_Q(gx_i)\,\gamma^{-1} = d_Q(x_i)$$

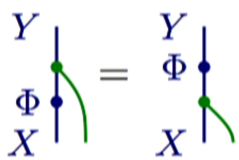
- for generalized orbifolds, boundaries are A-**modules**



$: X \otimes A \longrightarrow X$ s.t.



boundary fields **module maps**



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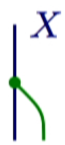

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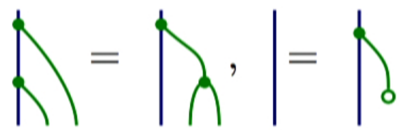

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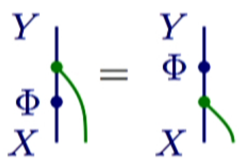

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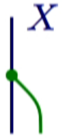

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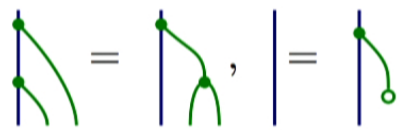

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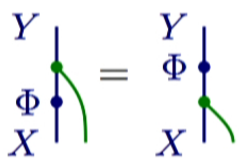


- for $A = A_G$ this reproduces G-equivariant matrix factorizations [Carqueville, Runkel '12]

LG orbifolds – topological disk correlators

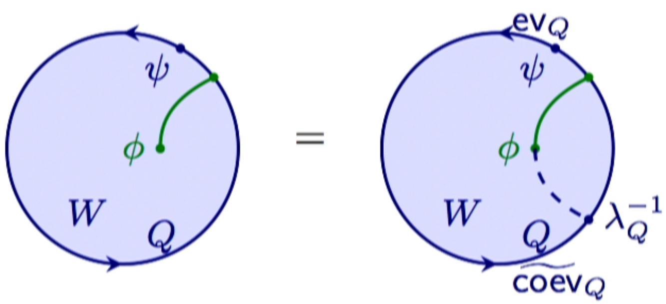
- for ordinary LG orbifolds proposal by [Walcher '04]

$$\langle \phi_g \rangle_Q = \mathrm{Res}\left[\frac{\phi_g^{\text{inv.}}\, \mathrm{STr}[\gamma\, \partial_1 d_{\bar{Q}} \ldots \partial_r d_{\bar{Q}}]}{\partial_1 \overline{W} \ldots \partial_r \overline{W}} \right]$$

with $\phi_g^{\text{inv.}}$ the polynomial part of ϕ_g, and $\overline{W}$, $d_{\bar{Q}}$ are W, d_Q with non-invariant variables set to zero.

checks: Cardy condition, comparison with known D-brane charges

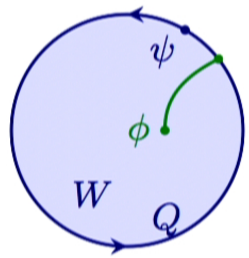
- defect approach:

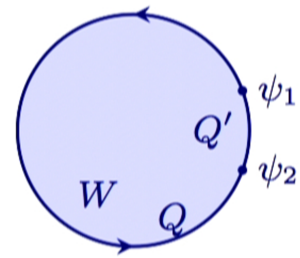


- reproduces above proposal for $A = A_G$

Generalized LG orbifolds – boundary sector

- topological disk correlators:

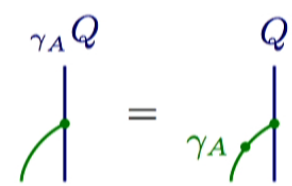
$\langle \phi\psi \rangle_Q =$ 

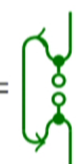
$\langle \psi_1, \psi_2 \rangle_{Q,Q'} =$ 

- consistency checks: boundary paring nondegenerate:

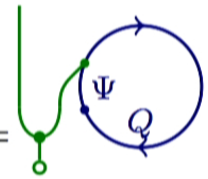
  boundary chiral sector paired with Ramond sector

  $\langle -, - \rangle_{Q,Q'} : \mathrm{Hom}(Q, Q') \times \mathrm{Hom}(Q', {}_{\gamma_A}Q[n]) \longrightarrow \mathbb{C}$

  nontrivial Serre functor $S_A = {}_{\gamma_A}(-)$ twisting the A-action: 

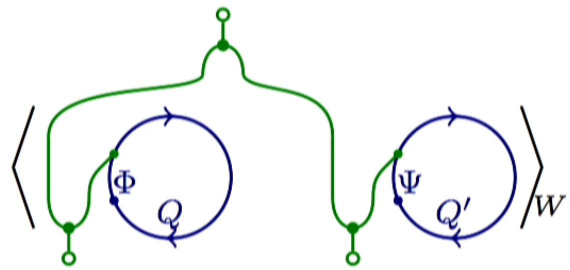
  where again γ_A is the Nakayama automorphism $\gamma_A =$ 

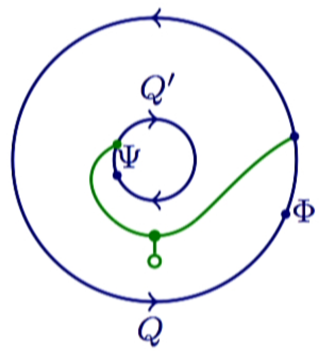
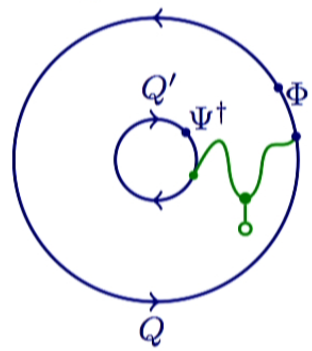
Cardy condition

- **boundary-bulk map** $\beta^{Q}_{\text{orb}}(\Psi) :=$ 

- Theorem: The **Cardy condition** holds for (generalized) LG orbifolds, i.e. $\langle \beta^{Q}_{\text{orb}}(\Phi), \beta^{Q'}_{\text{orb}}(\Psi) \rangle_{(W,A)} = \text{str}({}_{\Psi}m_{\Phi})$

  Proof:

$\langle \beta^{Q}_{\text{orb}}(\Phi), \beta^{Q'}_{\text{orb}}(\Psi) \rangle_{(W,A)} =$  $=$

$=$  $=$  $= \text{str}({}_{\Psi}m_{\Phi})$

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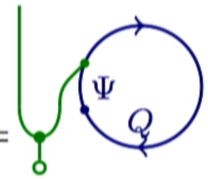
- **boundary-bulk map** $\beta^{Q}_{\text{orb}}(\Psi) :=$ 

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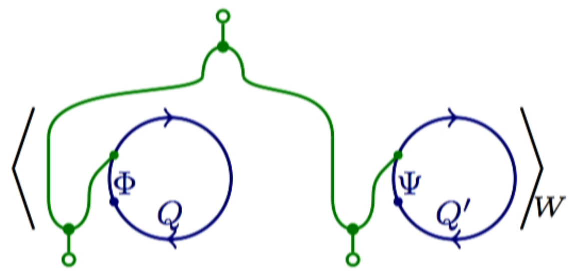
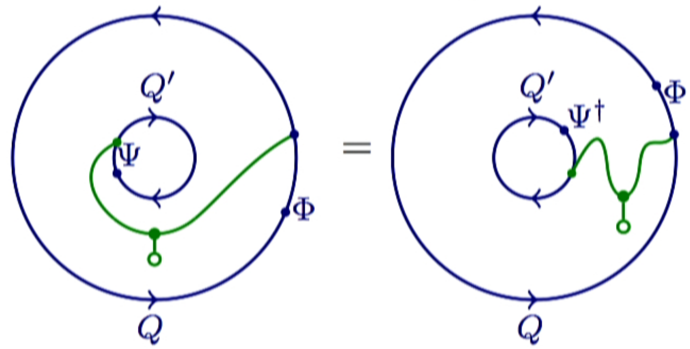
$\langle \beta^{Q}_{\text{orb}}(\Phi), \beta^{Q'}_{\text{orb}}(\Psi) \rangle_{(W,A)} =$  $=$

$=$  $= \text{str}({}_{\Psi}m_{\Phi})$

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Orbifold equivalences between LG models

- Recall: if there is a top. defect $X : V \to W$ with

  s.t. $c \equiv \dim(X) \in \mathbb{C} \setminus \{0\}$, one can describe W as a generalized orbifold of V with $A = X^\dagger \otimes X$

- in LG models amounts to finding matrix factorisation X of $W(x) - V(z)$ s.t.

$$\dim(X) = \mathrm{Res}\left[\frac{\mathrm{STr}\left[\prod_i \partial_{x_i} d_X \ \prod_j \partial_{z_j} d_X\right] \underline{\mathrm{d}z}}{\partial_{z_1} V, \ldots, \partial_{z_m} V}\right] \neq 0$$

- constructed explicitly between A-↔D-type and A-↔E-type singularities [Carqueville, Runkel '12], [Carqueville]

- Task: classify defects with invertible $\dim(X) \rightsquigarrow$ new equivalences between LG models and their D-brane categories ($\mathrm{mod}(X^\dagger \otimes X, V) \cong \mathrm{mf}(W)$) beyond the rational case

Orbifold equivalences between LG models

- Recall: if there is a top. defect $X : V \to W$ with

  s.t. $c \equiv \dim(X) \in \mathbb{C} \setminus \{0\}$, one can describe W as a generalized orbifold of V with $A = X^\dagger \otimes X$

- in LG models amounts to finding matrix factorisation X of $W(x) - V(z)$ s.t.

$$\dim(X) = \operatorname{Res}\left[\frac{\operatorname{STr}\left[\prod_i \partial_{x_i} d_X \; \prod_j \partial_{z_j} d_X\right] \underline{\mathrm{d}z}}{\partial_{z_1} V, \ldots, \partial_{z_m} V}\right] \neq 0$$

- constructed explicitly between A-↔D-type and A-↔E-type singularities [Carqueville, Runkel '12], [Carqueville]

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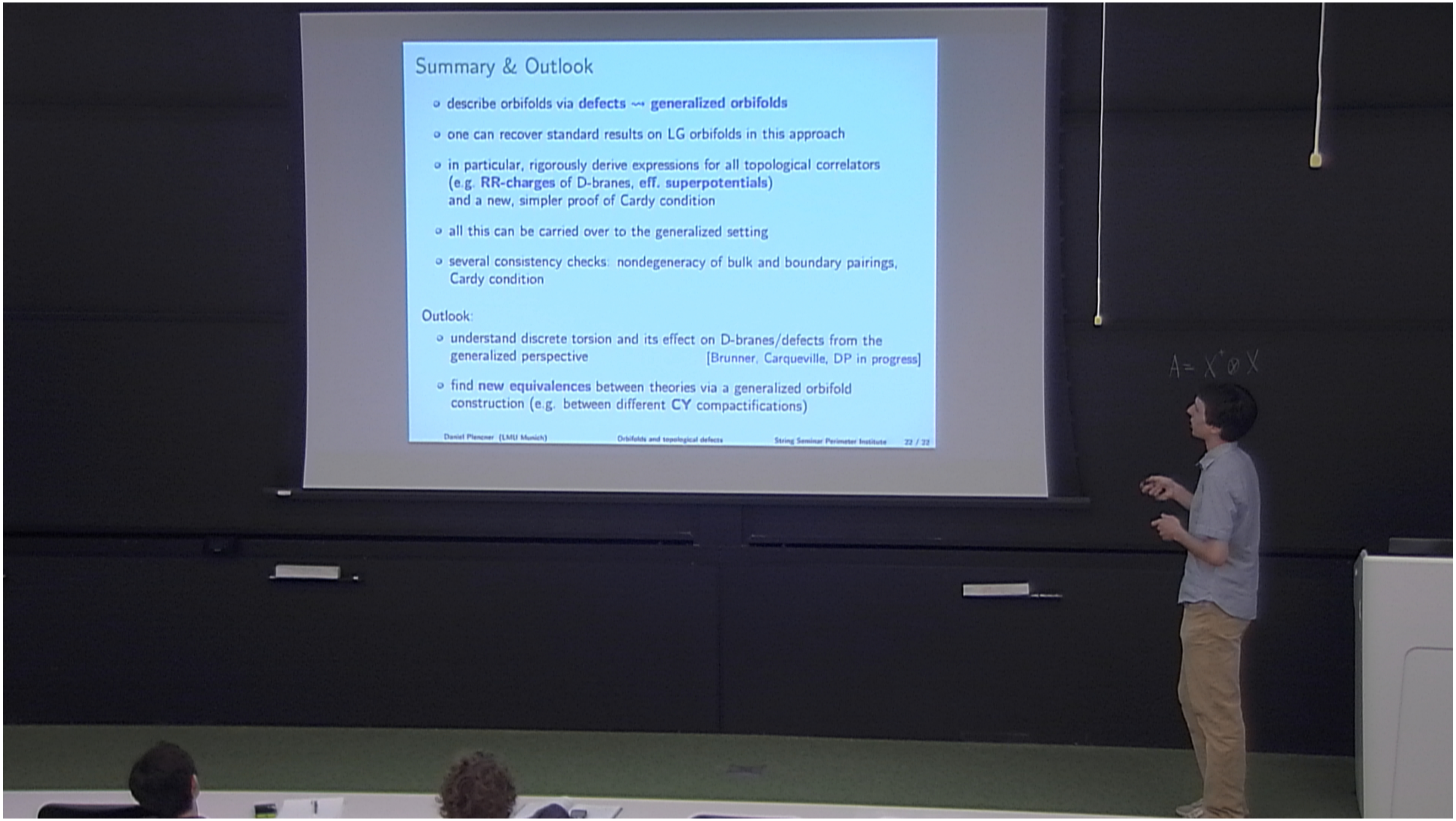
Summary & Outlook

- describe orbifolds via **defects** $\rightsquigarrow$ **generalized orbifolds**
- one can recover standard results on LG orbifolds in this approach
- in particular, rigorously derive expressions for all topological correlators (e.g. **RR-charges** of D-branes, **eff. superpotentials**) and a new, simpler proof of Cardy condition
- all this can be carried over to the generalized setting
- several consistency checks: nondegeneracy of bulk and boundary pairings, Cardy condition

Outlook:

- understand discrete torsion and its effect on D-branes/defects from the generalized perspective [Brunner, Carqueville, DP in progress]
- find **new equivalences** between theories via a generalized orbifold construction (e.g. between different **CY** compactifications)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Daniel Plencner (LMU Munich)
Orbifolds and topological defects
String Seminar Perimeter Institute
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