Title: Holography without Strings - Joint Quantum Gravity and String Theory Seminar

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Abstract: A defining feature of holographic dualities is that, along with the bulk equations of motion, boundary correlators at any given time t determine those of observables deep in the bulk. We argue that this property emerges from the bulk gravitational Gauss law together with bulk quantum entanglement as embodied in the Reeh-Schlieder theorem. Stringy bulk degrees of freedom are not required and play little role even when they exist. As an example we study a toy model whose matter sector is a free scalar field. The energy density (\rho) sources what we call a pseudo-Newtonian potential (\Phi) through Poisson's equation on each constant time surface, but there is no back-reaction on the matter. We show the Hamiltonian to be essentially self-adjoint on the domain generated from the vacuum by acting with boundary observables localized in an arbitrarily small neighborhood of the chosen time t. Since the Gauss law represents the Hamiltonian as a boundary term, the model is holographic in the sense stated above.

Holography without strings?

Donald Marolf, UCSB

9/12/2013

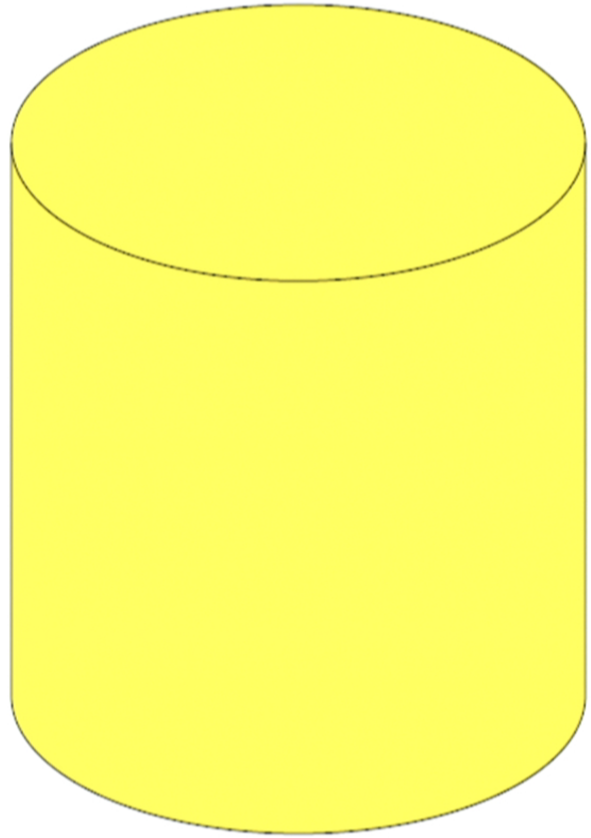
Arxiv: 1308.1977

What makes an AdS bulk theory holographic?

Do strings play a key role? (Other than to make the bulk UV complete.)

Much earlier discussion, and papers by MAGOO, Friedel, D.M., & others.

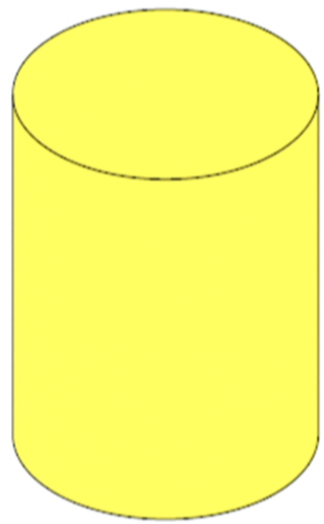
We'll review this, and then add a new technical point.



What do we mean by holographic?

Witten For any local AdS-invariant bulk theory (e.g. scalar fields),

$\beta = \lim_{z \to 0} z^{-\Delta} \phi$ defines a CFT (conformally invariant correlators).



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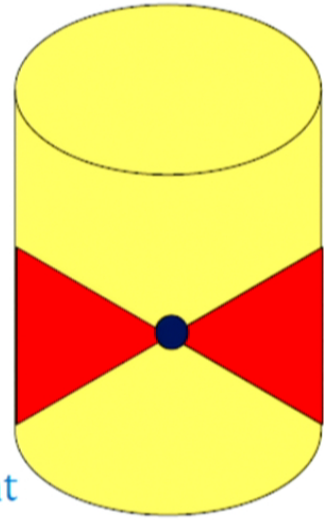
BDHM, BKL(T), Bena, HKLL, KLL, HMPS

At least perturbatively, all info in ϕ can be recovered from β. I.e., this is a duality.



Not holography.
Just solving the equations of motion.

Fourier transform: Bulk EOM means that momenta along boundary determine the radial momentum.

Or, use spacelike Green's function.

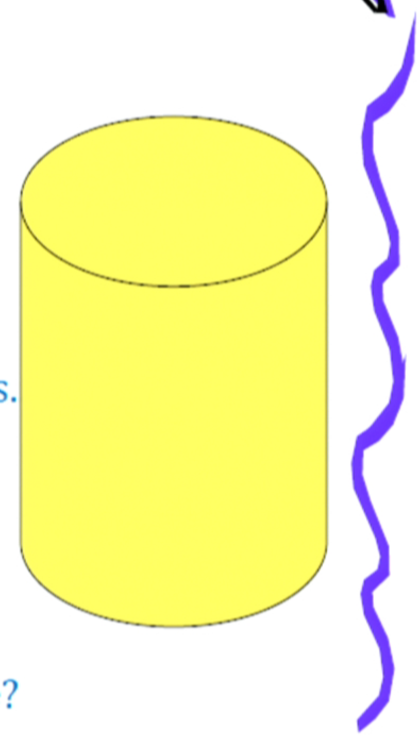
CFT is self-contained ("Unitarity")

The CFT wavefunction contains complete info at each time.

⇕

The CFT observables contain complete info at each time.

Does this mean that boundary limits of bulk observables contain complete info at each t?



Perhaps yes in known examples. CFT's are gauge theories.
Spacelike Wilson loops (and time derivatives)
are a complete set of observables at each time.
Related to strings near the boundary.

Does this signal that strings play a central role?
(I will argue that it does not.)

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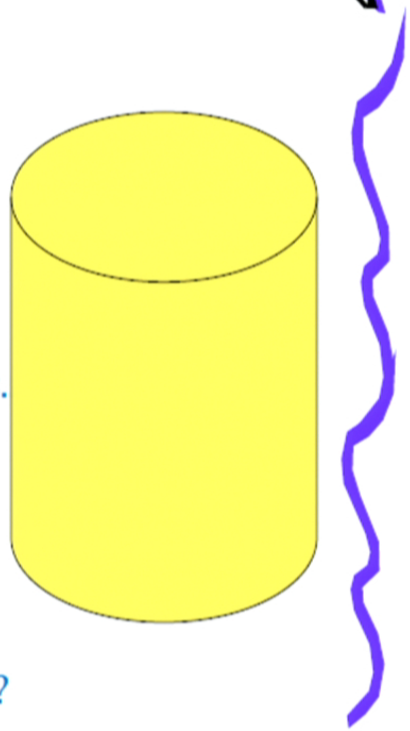
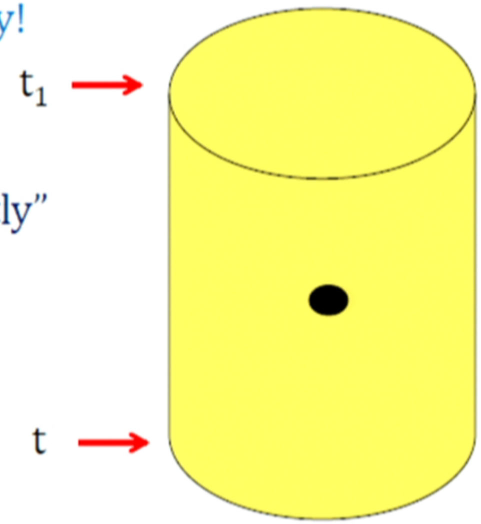


Does this signal that strings play a central role?
(I will argue that it does not.)

Dynamical Completeness requires only f(H).

$$O(t_1) = e^{iH(t_1-t)}\, O(t)\, e^{-iH(t_1-t)}$$

Applies to boundary values of bulk fields if H is determined by such boundary values.
True for any (diffeo-invariant) gravitational theory!
ADM Hamiltonian is a boundary term on-shell.



Gauge theory: Can in principle write RHS “explicitly” as a sum of Wilson loops.

But i) sum is infinite with structure at the cut-off scale. Is this sum more than a formal expression?

And ii) how to they explain excitations localized in the bulk?

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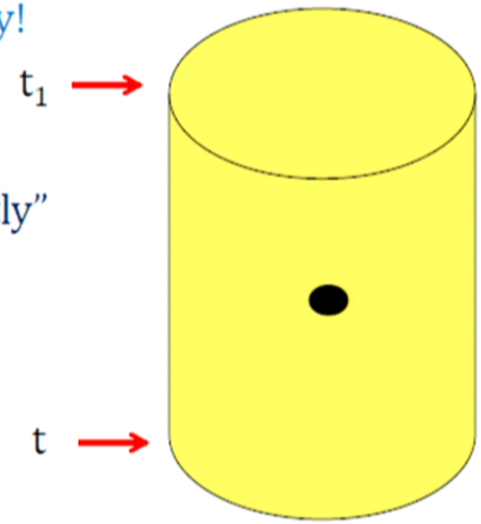

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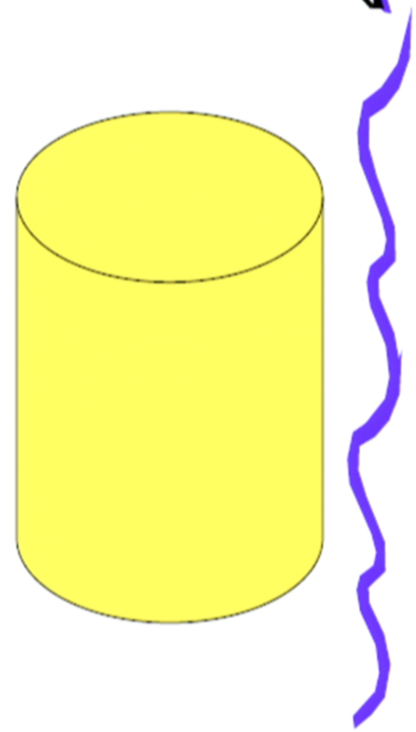
Suggests that any UV complete bulk gravity theory is holographic.

Generally gives T_{ab} as well.

Likely ensures that boundary observables commute at spacelike separation. (Follows from Gao & Wald in the classical limit assuming WEC. Semiclassically follows from GSL. [Wall])

UV completeness may be important due to gravitational collapse and the need for e^{iHt} for all t.



Can we say more?

$$O(t_1) = e^{iH(t_1 - t)}\, O(t)\, e^{-iH(t_1 - t)}$$

This equation is elegant, but formal.
Constructing $e^{iH(t_1 - t)}$ is equivalent to solving the full dynamics!

Is knowledge of H equivalent to knowledge of $e^{iH(t1\text{-}t)}$?
What other CFT properties can we ask the bulk to reproduce?

Familiar Example: QM in a 1-d box

$$H = p^2/2m = -\frac{\partial^2}{\partial x^2}$$

Determines H on smooth functions that vanish near the walls.
These are dense.

But does not determine $e^{iH(t_1-t)}$! Additional boundary conditions are required.

H is symmetric on above domain D, but not self-adjoint.
Self-adjoint *extensions* to $\mathcal{H} \supset$ D exist (in this case!),
but are not unique.

H is *essentially self-adjoint* on D when there is a unique extension to $\mathcal{H} \supset$ D.

Relation to Holography?

Consider CFT.

Let D = { W|0> } for Wilson loops W in some thin time slice $t \in (t_1,t_2)$.

Expect H to be essentially self-adjoint on D for any t_1, t_2.

Sketch of proof that H is essentially self-adjoint on D

D = $\{\beta\, |0\rangle\}$ for β (rescaled) boundary values of bulk fields in some thin time slice $t \in (t_1, t_2)$.

Note that H has a discrete spectrum (after imposing whatever BCs are needed).

Suppose that for each $|E\rangle$ we can find a sequence $|\Psi_N(E)\rangle \in D$ s.t.

i) $|\Psi_N(E)\rangle \to |E\rangle$ in $\mathcal{H}$-norm

ii) $\langle\Psi_N(E)|\,(H-E)^2\,|\Psi_N(E)\rangle \to 0.$

Then H is essentially self-adjoint on D.
I.e., any self-adjoint $\tilde{H}$ that agrees on D with H has $\tilde{H} = H$ on all of $\mathcal{H}$.
(So D already knows about all BCs!)

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Proof: From ii, $<\Psi_N(E)| (\widetilde{H}-E)^2 |\Psi_N(E)> \to 0$. (*)

Consider eigenvectors $|\tilde{E}>$ of $\widetilde{H}$. (Suppose discrete for simplicity.) Write

$$|\Psi_N(E)> = \Sigma_{\tilde{E}} \Psi_N(E,\tilde{E}) \, |\tilde{E}> \quad \text{and} \quad \Sigma_{\tilde{E}} \; (E-\tilde{E})^2 \, |\Psi_{N(E,\tilde{E})}|^2 \to 0.$$

Positive definite, so each term vanishes separately.

But from (i), $\Psi_{N(E,\tilde{E})} \to <\tilde{E}|E>$. So $<\tilde{E}|E> \propto \delta_{\tilde{E},E}$ and $\widetilde{H} = H$.

Sketch of proof that H is essentially self-adjoint on D

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D = {β |0> } for β (rescaled) boundary values of bulk fields in some thin time slice $t \in (t_1,t_2)$.

Idea: Expand in spherical harmonics, work separately with each $\vec{L} = (L, m_1, m_2, \dots)$.

Then study Fourier transform in t. Find functions $g_N(t)$ that vanish outside (t_1,t_2) s.t.

$$|\Psi_N(E)> = \int dt d\Omega \; \overline{g_N}(t) \overline{Y_{\vec{L}}}(\Omega) \, \beta(t, \Omega) \, |0>$$ approximate each |E>.

Looks like we need $\tilde{g}_N$ and g_N both tightly peaked!

Key point: |0> annihilated by negative frequency parts of β.
(Or even by positive frequency parts with $\omega < \omega_{\text{lowest state for } \vec{L}}$.)

Start with single-particle states.
Find lowest |E> for each $\vec{L}$ first.
Higher single-particle states and multi-particle states are similar…

For lowest single-particle |E>, find a sequence $|\Psi_N(E)\rangle \in D$ s.t.

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ii) $\langle\Psi_N(E)|\,(H-E)^2\,|\Psi_N(E)\rangle \to 0.$

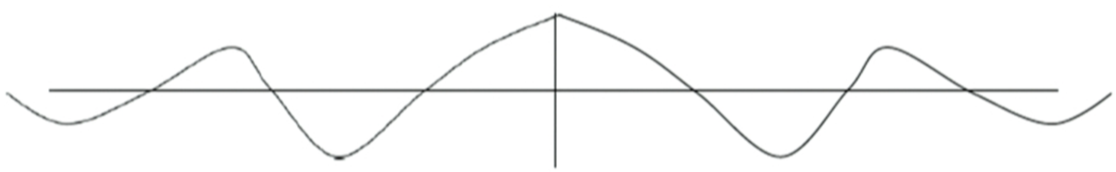
For each E, $\vec{L}$, let $|\Psi_N(E)\rangle = \int dt d\Omega\ \overline{g_N}(t)\overline{Y_{\vec{L}}}(\Omega)\ \beta(t,\Omega)\ |0\rangle$

Warm up: Let $(t_1,t_2) = (-\varepsilon,\varepsilon)$.

Consider

$$f_0 = \theta(t+\varepsilon) - \theta(t-\varepsilon)$$

$$\tilde{f}_0 = \frac{2}{\omega}\sin(\omega\varepsilon)$$



$$f_N(t) \propto e^{-i\frac{\pi N}{\varepsilon}t} \int_{-\infty}^{t} dt'\, e^{+i\frac{\pi N}{\varepsilon}t'}\ f_{N-1}(t')$$

= 0 for t > ε due to zero of f_{N-1} at $\omega = -\frac{\pi N}{\varepsilon}$.

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Similar story for higher states.

For each single-particle E, $\vec{L}$,
let $|\Psi_N(E)\rangle = \int dt d\Omega \; \overline{g_N}(t)\overline{Y_{\vec{L}}}(\Omega)\, \beta(t, \Omega)\, |0\rangle$

These satisfy i) $|\Psi_N(E)\rangle \to |E\rangle$ in $\mathcal{H}$-norm

ii) $\langle\Psi_N(E)|\,(H-E)^2\,|\Psi_N(E)\rangle \to 0.$

Multi-particle states also similar.

Shows that H is essentially self-adjoint on $D = \{\beta\, |0\rangle\}$ for β (rescaled) boundary values of bulk fields in any thin time slice $t \in (t_1, t_2)$.

Mirrors CFT expectations as desired!

Summary

- Reviewed arguments that bulk gravity Gauss law is the critical feature to have a holographic dual.
- Strings may do little beyond providing a UV completion. Expect holography for any non-stringy theories as well.
- Studied a further check (Wald): Expect H_{CFT} to be essentially self-adjoint on D built from |0> by acting with local operators with $t \in (-\varepsilon, \varepsilon)$.
- Confirmed analogous property for bulk free fields (and pseudo-Newtonian model). Should work for perturbative interactions as well.